

Understanding the challenges of the secondary-tertiary transition in mathematics for economics in higher education: a literature review

IDA LANDGÄRDS-TARVOLL^{†,*}

[†]*Department of Mathematical Sciences, University of Agder, Universitetsveien 25D, 4630, Kristiansand, Norway*

**Corresponding author. Email: Ida.landgards@uia.no*

[Received August 2023; accepted December 2023]

This review paper examines the issues identified by research regarding students transitioning from school mathematics to service mathematics modules within economics education at the tertiary level. Literature was gathered in four steps, mainly through hand-searching strategies from journals, books, conference proceedings and reports. The results show that existing research addresses the challenges of students to cope with changed roles of students and teachers, as well as with new approaches to teaching and assessment, in the transition phase between education levels. Special issues identified for service mathematics in economics education are (i) issues of heterogeneous mathematics background in the student group, (ii) of mismatch in expectations about mathematics level and demand and (iii) the issue of relevance of mathematics. This review helps practitioners gain deeper insights into these multifaceted issues of transition. Furthermore, several potential directions for further research in the field are recommended.

1. Introduction

Mathematics has an important role in and is an integral part of the study of economics in higher education. Weintraub (2002) discusses the history of twentieth-century economics and describes how economics has become mathematized. For many scholars, the mathematization of economics theory has been a way of making the discipline more scientific in its approach. This can be seen as most studies in mainstream peer-reviewed journals include formal mathematical and statistical analysis (Dawson, 2014). Hence, mathematical knowledge and reasoning are presented as having a key role in developing an understanding of key economic principles in documents on academic standards of graduates (bachelor's degree) in economics (in terms of what graduates might know, do and understand); [e.g. Subject Benchmark Statement (UK)];¹ Minimumskrav for Bachelor i økonomi og administrasjon

¹ <https://www.qaa.ac.uk/docs/qaa/subject-benchmark-statements/subject-benchmark-statement-economics.pdf>

(Norway)].² Consequently, undergraduate business and economics studies include a significant mathematics curriculum in the first year of study (Dawson, 2014; Darlington & Bowyer, 2017), covering a wide range of topics from algebra to calculus (Laging & Voßkamp, 2017; Voßkamp, 2023). The intention is that the mathematics modules included in the education of economics scaffold students' understanding and reasoning in the modules on economics (Biza *et al.*, 2022). However, internationally, universities struggle with high dropout rates and failure rates in mathematics among economics students (e.g. Jackson, 2014; Büchele, 2020; Kühling-Thees *et al.*, 2020).

The rationale for the literature review presented in this paper relates to the concern that high dropout and failure rates are devastating challenges for the academic performance of universities, society and the many individual students involved. The reasons for high dropout rates may be discussed from different perspectives. This paper focuses on the problem in relation to the secondary-tertiary transition with the premise that the significant numbers of first year students leaving or not managing to cope with mathematics is an indicator of problematic or unsuccessful secondary-tertiary transition. Secondary-tertiary transition has received increased attention in research literature due to the European policies of widening participation in undergraduate studies, changing from elite to mass higher education systems since the 1960s (Mergner *et al.*, 2019; Anastasakis *et al.*, 2022). However, the secondary-tertiary transition into mathematics within business and economics education is an under-researched area (Hochmuth *et al.*, 2021). This literature review surveys and draws together existing research and knowledge on secondary-tertiary transition in mathematics in relation to university studies in economics and identifies potential lines for further research in the field. The research question is, 'What issues have been focused on in research within the field of secondary-tertiary transition and service mathematics modules that have relevance for economics education at the university level?' With a deeper understanding of the phenomena, practitioners can obtain insights into possible directions for innovative teaching and learning practices that better support students in the transition phase.

2. Central concepts for the review process

The concept of transition is central in this review. In this paper, transition refers to the transition from second stage education to higher education; in the abbreviated form, it is called the secondary-tertiary transition. While all kinds of learning could be seen as a transition from one state to another (Hochmuth *et al.*, 2021), generally, in the literature on mathematics education, the notion of secondary-tertiary transition refers to the passage of students from school to university-level mathematics courses (Gueudet *et al.*, 2016). Transition from secondary to higher education is a process that involves many adjustments by the student. Such transition entails discontinuities, for example, with respect to mathematical content, teaching and learning style and personal and interpersonal adjustments, all affecting teaching and learning phenomena at the tertiary level (Holton & Artigue, 2001; Gueudet & Thomas, 2020). Considering students transitioning into service mathematics (mathematics studied in a field of education other than mathematics at the university level), there is also the discontinuity in different approaches to mathematics linked to different disciplines (González-Martín, 2021). Some personal and interpersonal adjustments, such as new living conditions, gender roles, etc., are crucial aspects of the secondary-tertiary transition that affect all students to some extent, not specifically those on programmes involving the study and use of mathematics (Dyson & Renk, 2006; Gueudet & Thomas, 2020). Thus, this paper restricts

² https://www.uhr.no/_f/p1/i48e11d18-8b76-4089-acb1-511fa13135e4/boa-planvedtatt-av-uhr-oa-november-2018-endelig.pdf

the focus to specific mathematical issues, practices and institutional gaps related to secondary-tertiary transition and to service mathematics for economics education.

Likewise, the concept of service mathematics is crucial in this review study. A service mathematics module is a module which covers mathematics as applied within another disciplinary or professional field. These modules are often compulsory within programmes in fields such as natural sciences, engineering, economics/business, informatics, social sciences, psychology, medicine, etc. (Alpers, 2020; Gueudet & Thomas, 2020; Hochmuth, 2020). Service mathematics is, hence, an offering made explicitly for students in non-mathematical study programmes (Alpers, 2020). Other terms synonymously used to ‘service mathematics’ are ‘mathematics as a service subject’ and ‘mathematics education for non-specialists at the tertiary level’. The field of research into service mathematics has recently featured at international conferences on university mathematics education (Gueudet & Thomas, 2020; Hochmuth *et al.*, 2021). Especially in Europe, issues on service mathematics are much discussed. Alpers (2020) listed the following European competence centres, which were set up to explicitly address research on service mathematics: the Mathematics Education Centre (MEC, UK), the Competence Centre for University Mathematics (KHDM, Germany) and the Centre for Research, Innovation and Coordination of Mathematics Teaching (MatRIC, Norway).

The type of service mathematics module that this review focuses on is ‘mathematics for the education of economics’, including mathematics modules incorporated in both business administration and economics. In some countries (e.g. in Germany), there is a distinct division of the economics field into the two mentioned above; however, in other countries, for example, in the United States and English-speaking countries, the distinction is not clearly made and one simply speaks of economics (Voßkamp, 2023). Typical content of mathematics module(s) in the first year are as follows: basic algebra, linear functions, logarithms and exponentials, financial mathematics, differentiation, partial differentiation, optimization, integrations, matrices and difference equations (Dawson, 2014; Voßkamp, 2023). However, how extensive such modules are and the topics covered vary among universities in one country as well as between countries. In a large comparison study of bachelor programmes in economic in Europe and North America, mathematics was found to be one out of six core modules (Monteiro & Lopes, 2007). Dawson (2014) found that mathematics and statistics modules in the UK constituted 25% to 40% of the first year studies of undergraduate economics. The goal of such mathematics education is to enable students to gain the mathematical competencies needed in the economics programme, which is why it is classified as service mathematics (Alpers, 2020). Synonymously to ‘mathematics for the study of economics’, terms such as ‘mathematics for economists’ and ‘mathematics in economics study programme’ are commonly used.

In this paper, mathematics education within economics education is considered as service mathematics at the institutional level (in the sense of the course structure of universities and related organizational elements). Mathematics for economics has a distinctive character and its existence dates back to the beginning of the nineteenth century (Hodgson, 2013). The approach to mathematics education in economics blends verbal and quantitative reasoning in a special way that can be both practical and abstract (Allgood *et al.*, 2015). This differs from service mathematics in disciplines such as engineering and natural sciences, where a synergistic interplay between mathematics and the respective application field has catalysed progress and fostered a reciprocal enhancement of both domains (Dugger Jr, 1993; Hafni *et al.*, 2020; Pepin *et al.*, 2021). The integration of mathematical instruments, methodologies and ideas from other disciplines into economics education originated from a concerted effort to imbue economic theories with enhanced rigour, accuracy and transparency, ultimately aiming to deepen the understanding and improve predictions of economic models, trends and patterns (Tarasov, 2019). Consequently, mathematical concepts have been reshaped and serve a vital complementary role in

explaining economics theory (Hodgson, 2013). For answering economic questions, economists build models which they base on assumptions (often mathematical simplifications that are reasonable and understandable in economics settings) and prove these using mathematical modelling and mathematical logic (Voßkamp, 2023). Hence, the peculiarity of mathematics for economics is described as ‘The triad of mathematics definition-theorem-proof “mutates” to model-theorem-proof’ (Voßkamp, 2023, p. 54). A feature of mathematics in economics education is its multifaceted representation, encompassing explanations in economic terminology, graphical illustrations and mathematical expressions, necessitating students to proficiently navigate and integrate these varied forms of representations (Ariza *et al.*, 2015; Voßkamp, 2023).

3. Methodology

This research paper is an overview review paper (e.g. Krumsvik, 2016) that retrieves information from scholarly articles, conference proceedings, review studies, book and book chapters and reports related to teaching and learning issues in the secondary-tertiary transition to university mathematics education, with a primary focus on service mathematics module(s) for economists. The aim is to synthesize existing research and knowledge on the transition from secondary to tertiary mathematics in economics education to enhance practitioners’ comprehension of the complex phenomena.

In the literature search, the primary area of search was on transition into mathematics for the education of economics in English and Nordic languages. Due to limited research literature exclusively dedicated to the transition into mathematics within economics education, it was convenient to broaden the search for literature to include transition into service mathematics relevant for economics education. By widening the search criteria, studies where economics programmes were mentioned as a small part of the study, or economics students/educators were part of the sample, as well as studies discussing service mathematics modules such as modules of mathematics for economics studies were included (see Appendix A). The search process was conducted with the following guiding question: ‘What issues have been focused on in research within the field of secondary-tertiary transition and service mathematics modules that have relevance for economics education at the university level?’ Hence, the review is not comprehensive in terms of discussing transition issues of service mathematics courses in general but focuses on the issues concerning the transition into mathematics in economics education.

The literature search process was conducted in four literature gathering steps. As a first step, a review of the state-of-the-art studies on transition issues related to secondary-tertiary transition in mathematics education was conducted. In Google Scholar and the ERIC database, the following keywords were used: mathematics, service mathematics, economics, teaching, learning, secondary-tertiary transition, service mathematics, study entrance phase, business and administration, economics, mathematics for non-mathematicians, applied calculus and mathematics as service subject. The search resulted in five relevant review articles (see Table 1). The articles outline transition issues through several theoretical lenses and related views on the secondary-tertiary transition in mathematics education. The articles consider research on secondary-tertiary transition in all kinds of mathematics courses. However, most of the courses considered are specialist mathematics courses. In addition to the review articles, there is one state-of-the-art article on service mathematics, with a subchapter about transition into service mathematics, published by the SEFI working group. There were no review studies on the topic of mathematics for economics education.

The review studies found in the literature gathering first step were the basis for the start of the literature gathering second step, where the primary research sources concerning economics education were examined. The second step was a manual search or ‘hand-searching’ (e.g. Chapman *et al.*, 2010)

TABLE 1. *Results from the literature gathering process*

Step 1	Step 2	Step 3	Step 4
(De Guzmán, Hodgson <i>et al.</i> , 1998)	(Alpers <i>et al.</i> , 2013)	(Hochmuth <i>et al.</i> , 2021)	(Albeshree <i>et al.</i> , 2022)
(Gueudet, 2008)	(Anastasakis <i>et al.</i> , 2022)	(Hodgen <i>et al.</i> , 2014)	(Bolstad <i>et al.</i> , 2022)
(Gueudet <i>et al.</i> , 2016)	(Ariza <i>et al.</i> , 2015)	(Holton & Artigue, 2001)	(Feudel & Biehler, 2021)
(Bergsten & Jablonka, 2019)	(Arnold & Straten, 2012)	(Jablonka <i>et al.</i> , 2017)	(González-Martín, 2021)
(Di Martino <i>et al.</i> , 2022)	(Asian-Chaves <i>et al.</i> , 2021)	(Johnson & Kuennen, 2006)	(González-Martín <i>et al.</i> , 2023)
(Alpers, 2020)	(Asian-Chaves <i>et al.</i> , 2022)	(Lagerlöf & Seltzer, 2009)	(Göllner & Rück, 2023)
	(Ballard & Johnson, 2004)	(Laging & Voßkamp, 2017)	(Kortemeyer & Biehler, 2023)
	(Briggs <i>et al.</i> , 2012)	(Macbean, 2004)	(Lawson <i>et al.</i> , 2020)
	(Brown-Robertson <i>et al.</i> , 2015)	(Mallik & Lodewijks, 2010)	(Liebendörfer <i>et al.</i> , 2023)
	(Büchele, 2020)	(Mkhatshwa & Doerr, 2018)	(Pepin <i>et al.</i> , 2021)
	(Christensen, 2008)	(Nortvedt & Siqveland, 2019)	(Romo-Vázquez & Artigue, 2023)
	(Cohn <i>et al.</i> , 2001)	(Opstad, 2018)	(Rønning, 2023)
	(Cohn <i>et al.</i> , 2004)	(Opstad, 2021)	
	(Darlington & Bowyer, 2017)	(Pampaka <i>et al.</i> , 2016)	
	(Dawson, 2014)	(Pampaka <i>et al.</i> , 2012)	
	(Dolado & Morales, 2006)	(Peters <i>et al.</i> , 2017)	
	(Flegg <i>et al.</i> , 2012)	(Randahl, 2016)	
	(Gueudet & Thomas, 2020)	(Trenholm <i>et al.</i> , 2019)	
	(Harris <i>et al.</i> , 2015)	(Voßkamp, 2017)	
	(Harvey <i>et al.</i> , 2006)	(Willcox & Bounova, 2004)	
	(Hester <i>et al.</i> , 2014)	(Winsløw <i>et al.</i> , 2018)	
	(Hey, 2005)	(Zetland <i>et al.</i> , 2010)	
	(Hochmuth, 2020)		
			(Bašić & Šipuš, 2023)
			(Feudel, 2017)
			(Feudel, 2018)
			(Feudel, 2023)
			(Gueudet & Quéré, 2018)
			(Mkhatshwa & Doerr, 2016)
			(Mkhatshwa & Doerr, 2015)
			(Nardi, 2016)
			(Nardi & Winsløw, 2018)
			(Voßkamp, 2023)

conducted in several loops. First, the articles cited in and citing the review studies were examined, and, in further loops, relevant articles citing and cited in the articles previously found were examined, and so on. Only articles published after 2000 were included. This process led to an identification of several important studies from a variety of journals from both mathematics and economics education research fields.

The same hand-searching strategy was also applied in the third literature gathering step. First, the main journals on mathematics and economics education were reviewed: *International Journal of Research in Undergraduate Mathematics Education*, *Teaching Mathematics and its Applications*, *Mathematics Education Research Journal*, *International Journal of Mathematical Education in Science and Technology*, *Educational Studies in Mathematics*, *Journal of Economic Education* and *Education Economics*.

Then, the relevant chapters from the *Encyclopaedia of Mathematics Education* (Lerman, 2020), and *Research and Development in University Mathematics Education: Overview* (Durand-Guerrier *et al.*, 2021), produced by the international network for research on didactics of university mathematics, were examined. In addition, the relevant citing and cited articles were reviewed.

Considering the research field on service mathematics is relatively young, it was advantageous to also explore proceedings of mathematics education research conferences. This allowed a more comprehensive understanding of the developing body of knowledge in the area. Hence, in the fourth step of the literature search, research published between 2015 and 2023, relevant for the research question presented above, were extracted from the following conference proceedings: RUME (XIX–XXIV),³ INDRUM (2016, 2018 and 2020),⁴ CERME (8–11),⁵ ICME 13, Calculus in Upper Secondary and Beginning University Mathematics,⁶ SEFI (2015–2022)⁷ and the Learning and Teaching of Calculus across Disciplines.⁸ These sources were chosen from among a range of STEM and economics education journals as the main ones addressing transition issues linked to service mathematics and economics. The list of chosen sources could presumably be complemented with other general economics journals, but the chosen ones should make a sound ground for discussing and observing important issues related to the review question posed. In Table 1, the result from the literature gathering process is outlined. The table presents scholarly articles, conference proceedings, review studies, book and book chapters, as well as reports related to teaching and learning issues in the secondary-tertiary transition to university mathematics education, with a primary focus on service mathematics modules for economists. The research items are organized based on their initial discovery during the literature review process. The entities in Table 1 are arranged alphabetically. Research items that appeared in multiple steps of the literature review process are listed only under the stage in which they were first identified.

The articles were reviewed and the issues identified and discussed in each of them were coded and categorized, drawing on conventional content analysis (e.g. Hsieh & Shannon, 2005), as categories were developed inductively. The codes from the first coding cycle were in the second cycle coding (e.g. Miles *et al.*, 2020) and were re-evaluated and grouped into five categories on two levels of generality, namely, categories of transition issues concerning all types of mathematics courses and categories of transition issues special for the transition into mathematics for the study of economics service mathematics modules. Fig. 1 below illustrates the grouping of codes into categories of issues and the division into the two levels of generality. These results are presented in Table 2 and delineated in the subsequent section.

4. Results

4.1. Secondary-tertiary transition in mathematics education

In the literature on mathematics education, secondary-tertiary transition is discussed in depth, using several different theoretical lenses. In the early review study of De Guzmán *et al.* (1998), the issues of transition were viewed through what they characterize as epistemological, cognitive, socio-cultural

³ <http://sigmaa.maa.org/rume/Site/Proceedings.html>

⁴ <https://hal.archives-ouvertes.fr/INDRUM/page/indrum-proceedings>

⁵ <http://www.mathematik.tu-dortmund.de/~erme/index.php?slab=cerme-proceedings>

⁶ <https://matric-calculus.sciencesconf.org/>

⁷ https://www.sefi.be/?post_type=proceedings

⁸ <https://matriccalconf2.sciencesconf.org/>

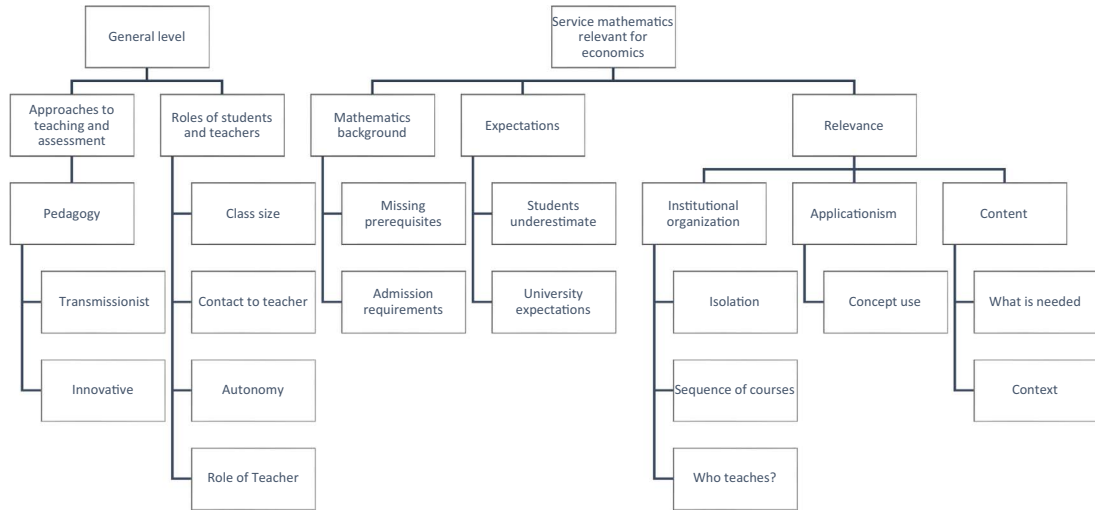


FIG. 1 Illustration of the coding and categorization process.

TABLE 2. Issues identified in the literature on secondary-tertiary transition into service mathematics courses for education of economics divided into two levels of generality

Issues identified in the literature on secondary-tertiary transition into service mathematics courses relevant for the study of economics at the university level.

All types of mathematics course including service mathematics courses

Service mathematics courses specifically

Roles of students and teachers changing
Approaches to teaching and assessment changing

Students' diverse mathematics background
Mismatch in expectations about mathematics level and demand
Relevance of mathematics taught to application area

and didactical perspectives. The work of Gueudet (2008) continued in a similar direction in terms of categorization of transition issues but focused more on the definition of transition based on different theoretical frames. The categories presented by Gueudet (2008) are as follows: thinking modes, mathematical language and communication and didactical transposition and contract. These two review studies were followed up by a later 'state-of-the-art' study by Gueudet *et al.* (2016), concerning transitions at all the stages from preschool to university to workplace. Drawing on the work done by De Guzmán *et al.* (1998), Gueudet (2008), Gueudet *et al.* (2016) and Bergsten & Jablonka (2019) identify another five theoretical frameworks that elucidate the different aspects of transition issues, which are not considered in the other review studies. Additionally, Di Martino *et al.* (2022) present four main themes of mathematics education research concerned with transition: mathematical gap, potential of technology, factors correlating with success (e.g. attitude towards mathematics) and failure in transition (related to students' vision of mathematics).

Together, the review studies give a wide insight into the complex research area of secondary-tertiary transition in mathematics education. However, none of the studies specifically consider the transition into service mathematics courses. Much of the literature analysed in the review studies consider transition

into specialist mathematics courses. Di Martino *et al.* (2022) especially identified significant differences between transition into specialist mathematics courses and into non-specialist mathematics courses and called for the need of further research to investigate this issue. They write, ‘These differences can be related to the different mathematical identities of the first-year students, to the different degrees of discontinuity of mathematical contents (for example, calculus) as they are presented at school and as they are presented in the different degrees, as well as to the different mathematical practices and requests in the different degrees’ (Di Martino *et al.*, 2022, p. 25).

A need for a distinction between the transition into specialist mathematics courses and transition into service mathematics courses is also apparent when considering the different aims of mathematics education. Alpers (2020) makes a rough division of mathematics education within the tertiary level into three categories according to mathematics education goals that are formed by the programme within which education takes place. He distinguishes between mathematics presented within a general basic education (often in upper secondary school), mathematics presented as a scientific subject (as in specialist mathematics and mathematics teacher education courses) and service mathematics. In a subsection (3.3) of his state-of-the-art report about mathematics as a service subject at tertiary level, he reports that research concerning students’ transition into service mathematics has, so far, mostly been concerned with students’ school mathematics background, students’ missing fluency in basic algebra, the effectiveness of different support measures and students’ perceptions of university mathematics.

In fact, besides the subchapter on transition issues into service mathematics including economics as a small part in Alpers (2020), there are no other review studies focusing exclusively on secondary-tertiary transition into service mathematics in general or particularly into mathematics for economics, which is why this review study adds to the literature. Instead, there seems to have been a tradition to quantitatively investigate to what extent mathematics preparation is of importance for studying other economics modules, as there are several studies identifying mathematics knowledge being a significant predictor for student performance and success in different economics modules, for example, Principles of Economics (Lagerlöf & Seltzer, 2009; Brown-Robertson *et al.*, 2015), Statistics (Johnson & Kuennen, 2006; Opstad, 2018), Microeconomics (Anderson *et al.*, 1994; Ballard & Johnson, 2004; Mallik & Lodewijks, 2010; Opstad, 2018), Introductory Economics (Ballard & Johnson, 2004; Dolado & Morales, 2006; Mallik & Lodewijks, 2010), Quantitative Methods (Lagerlöf & Seltzer, 2009), Business Economics and Financial Analysis (Opstad, 2018), etc.

Furthermore, focusing on school mathematics as a predictor for success in economics studies, Asian-Chaves *et al.* (2022) and Opstad (2018) found school mathematics tracks (no mathematics, mathematics for social sciences or technological mathematics track) as significant predictors for success in first-year economics studies. Both these studies raised concerns about the liberal (with respect to mathematics background) admission requirements for tertiary economics studies. Similarly, Arnold & Straten (2012) found school mathematics background accounting for most of the variations in study success across students of economics. Similarly, school mathematics background—in terms of both tracks studied and the grade obtained, together with final school grade—is specified by Laging & Voßkamp (2017) as being significant variables on a mathematics skills test administered to students of economics in the first week of their undergraduate studies.

4.2. *Secondary-tertiary transition with a particular focus on service mathematics for economics education*

There is a growing recognition that a great number of students study mathematics as a part of their university studies. In fact, students studying specialist mathematics are a small minority of students needing

mathematical competences; for example, all STEM-related programmes incorporate varying levels of mathematics (Holton & Artigue, 2001; Nardi, 2016). Internationally, universities face high dropout and failure rates due to students not managing to cope with the mathematical demand of their main application study programme (Hernandez-Martinez, 2016; Nardi & Winsløw, 2018; González-Martín *et al.*, 2023). Despite this, research on service mathematics has been scarce until now, but thanks to several research networks and working groups at international conferences specifically addressing service mathematics issues, the research area is developing (Romo-Vázquez & Artigue, 2023).

In this section, the findings from the literature search on what issues research has identified as important with respect to secondary-tertiary transition into service mathematics modules relevant for economics education are summarized. Table 2 presents the secondary-tertiary issues, which were identified in the literature. They are outlined at two levels of generality, that is, issues relating to transition to mathematics modules of all types, including service mathematics and issues relating specifically to transitions to service mathematics courses relevant for the study of economics at the university level.

4.2.1. Roles of students and teachers changing. On a general level, there is the issue of the respective roles of teachers and students changing in the secondary-tertiary transition phase. Contrary to what students are used to from school, students need to adapt to the larger class sizes, often reorganized classes, the distanced contact with lecturers with little or no opportunity to ask questions and the increased autonomy in terms of time management (up to 60% self-study time in mathematics courses (Göller & Rück, 2023)) and flexible and proactive use of diverse study resources, etc. (De Guzmán *et al.*, 1998; Holton & Artigue, 2001; Gueudet, 2008; Pampaka *et al.*, 2012; Gueudet *et al.*, 2016; Pampaka *et al.*, 2016; Jablonka *et al.*, 2017). Similarly, the change in the role of the teacher in that the teacher is not only a teacher but also a researcher who must divide their time between the two missions, the availability of more than one teacher per course and the teacher as a course organizer and designer has less time to cooperate on pedagogical issues but more time to prepare for teaching lead to multiple challenges for students (De Guzmán *et al.*, 1998; Pampaka *et al.*, 2012; Gueudet *et al.*, 2016; Jablonka *et al.*, 2017). These aspects of transition are common for students entering undergraduate studies in various disciplines (Harvey *et al.*, 2006; Briggs *et al.*, 2012; Schaeper, 2020).

However, for students transitioning into service mathematics modules there is another dimension on the role of the teacher or more precisely, the discipline of the teacher. For example, Voßkamp (2017) found that in Germany approximately half of the courses in mathematics within economics education were provided by educators from the scientific field of mathematics and half by educators from the scientific field of economics. However, Voßkamp (2017) could not draw any conclusion of whether the mathematics modules should be taught internally or externally to the business and economics departments. Similarly, in engineering education, personnel from mathematics departments often have the responsibility for courses in mathematics (Flegg *et al.*, 2012). Willcox & Bounova (2004) noted that in engineering courses, there is a barrier to deep understanding of several phenomena due to the lack of mathematical understanding. This recognition led them to investigate the curricula and communication between the mathematics and engineering faculty. They found that while the members of the engineering faculty were unaware of the details of the mathematics curricula and unfamiliar with the context in which mathematical skills were learnt by the students, the members of the mathematics faculty did not possess a clear understanding of how mathematics is supposed to be taught in engineering courses. Rønning (2023) presented similar findings and proposed ‘division of labour’ as the main message from the teachers in the study. While the mathematician brings the knowledge behind a method, the engineer finds relevant application examples. Flegg *et al.* (2012, p. 718) underscored a pervasive uncertainty in the field of engineering education, noting an absence of a coherent, research-driven consensus on the most effective

methods and timings for teaching mathematics to engineering students, as well as on who is best qualified to do so. This issue of inconsistency in educational practices is also present in the field of economics. Thus, there is a notable heterogeneity among teachers of mathematics modules (Voßkamp, 2023).

4.2.2. Approaches to teaching and assessment changing. The most common mode of teaching and assessment in universities, with the lecturers as ‘transmissionist’, is often mentioned in the literature as posing challenges for first-year students (Gueudet *et al.*, 2016; Peters *et al.*, 2017; Hochmuth, 2020). Jablonka *et al.* (2017) concluded that the teaching format influenced the forms and functions of assessment. They wrote, ‘While in school classrooms students might receive some formative feedback, in undergraduate mathematics courses there is commonly only summative assessment through an obligatory closed book examination’ (Jablonka *et al.*, 2017, p. 72). The transmission kind of pedagogy is still the most common mode of teaching mathematics at the university level (Liebendörfer *et al.*, 2023). However, teaching and curriculum design for undergraduate level mathematics is receiving increased attention in practice and research, which recently has resulted in many innovative teaching and learning approaches (Trenholm *et al.*, 2019; Lawson *et al.*, 2020; Albeshree *et al.*, 2022). Holton & Artigue (2001) point to the ‘either-research-or-teaching’ polarity nowadays being false but acknowledge also that instructors at undergraduate level would benefit from different forms of training to creatively implement and use innovative technological tools, pedagogic strategies and modes of assessment. One example of an innovative approach to first-year mathematics in engineering is the MARTA project at the Norwegian University of Science and Technology, where the interplay between mathematics and a main subject is the focus (Bolstad *et al.*, 2022). In the case of economics, Bašić & Šipuš (2023) discussed innovative teaching and learning opportunities that could be provided by tasks from workplace financial literacy.

To assist students studying mathematics in their transition to university studies, many mathematics departments have established various support measures, such as drop-in centres with tutors (who are more advanced students or teachers) or various types of bridging courses (Gueudet & Thomas, 2020; Pepin *et al.*, 2021). Although such courses and support measures are crucial for many students’ opportunities to succeed in service mathematics modules, this body of literature is left out from this review. The interested reader is directed to the review study by Lawson *et al.* (2020).

4.2.3. Students’ diverse mathematics background. Missing prerequisites and lack of fluency in basic algebra are the most reported transition issues when it comes to service mathematics, and particularly in the case of mathematics for economics (Dawson, 2014; Nardi & Winsløw, 2018; Nortvedt & Siqveland, 2019; Alpers, 2020; Büchele, 2020; Asian-Chaves *et al.*, 2021). Application study programmes usually only set a minimum mathematics admission requirement, if any (Hodgen *et al.*, 2014). In the case of economics studies, it is generally recommended to have studied advanced school mathematics, for example, a social science mathematics track or similar, but only few universities require advanced mathematics in their admission requirements (Dawson, 2014; Darlington & Bowyer, 2017; Asian-Chaves *et al.*, 2021; Opstad, 2021). Accordingly, the first-year student groups of these programmes are heterogeneous in terms of mathematical backgrounds. Generally, many students in service mathematics modules experience discontinuities between school and university mathematics syllabi due to the broad admission requirements (Hodgen *et al.*, 2014). Discontinuities between school and university mathematics syllabi imply that not all students who fulfil the entrance qualifications are prepared for mathematics studies at the university level (Nardi, 2016; Opstad, 2021; Romo-Vázquez & Artigue, 2023).

4.2.4. Mismatch in expectations on mathematics level and demand. The heterogeneity (in terms of mathematics knowledge) issue is closely related to the issue of mismatch in expectations by university

lecturers and students. The problem of students possessing lower mathematical abilities than universities wish them to have has been called ‘the mathematics problem’, a term coined in the middle of the 1990s (Hawkes & Savage, 2000). Not only do universities and lecturers have ambiguous expectations on the students’ mathematical knowledge but often students themselves underestimate the mathematical demands of the degree programmes which causes problems, for example, in terms of lack of confidence and anxiety, which in turn lead to poor performance and high dropout rates (De Guzmán *et al.*, 1998; Dawson, 2014; Hester *et al.*, 2014; Hodggen *et al.*, 2014; Harris *et al.*, 2015; Anastasakis *et al.*, 2022). Voßkamp (2023, p. 52) wrote, ‘Many first-year students are surprised at the beginning of their studies that mathematics modules are compulsory in the context of business administration and economics study programs’. The economics education approach to mathematics demanding students to master both verbal and quantitative reasoning, as described above, surprises students when they begin their university studies. ‘Survey evidence (from the Economics Network Student Survey) suggests that over 80% of students stated there was much more mathematics involved in their economics degree programme than they expected’ (Dawson, 2014, p. 6).

4.2.5. Issues related to relevance of mathematics taught to application area. Another issue with the transition into service mathematics is the issue of relevance of the service mathematics module(s). This issue has been discussed from three angles (see Figure 1) in the literature.

To start with, there is a discussion about whether the service mathematics module(s) should be organized within or outside the department of the main field of education. This issue is closely related the ‘who teaches the module’ question and considerations about mathematics module(s) being isolated from other courses in the degree programme within the first year (Alpers, 2020). There is also an issue of mathematics often being isolated in the sense that there is a lack of connectedness of mathematics curricula and the application studies (Christensen, 2008; Winsløw *et al.*, 2018). Specific mathematics topics are often taught in a separate mathematics module parallel or prior to the major application study modules (Hochmuth, 2020; Hochmuth *et al.*, 2021). This institutionalized separation leads to an epistemological perspective, viewing mathematics as pure and independent of other disciplines and termed ‘applicationism’ by Barquero *et al.* (2013). Applicationism neglects intrinsic dialectics between mathematics and its use in other disciplines (Hochmuth, 2020). The separation causes problems for students, especially in the transition phase but also in later use of mathematics in the application courses. Students find it hard to see the relevance (Christensen, 2008; Winsløw *et al.*, 2018), and, furthermore, ‘Because of the mismatch of practices, it is often not clear which activities and reasoning are allowed, required, or forbidden and, in particular, how students have to interpret symbols in view of a specific task in major-subject-courses’ (Hochmuth, 2020, p. 771). Introductory economics courses in undergraduate programmes demand quantitative knowledge, particularly in terms of analysis/calculus which is why topics such as limits of sequences and functions, continuity, differential calculus and integral calculus are central in the first mathematics module(s) for the study of economics. However, Feudel & Biehler (2021) and Feudel (2017) recognize the applicationism difficulties within economics in the case of the derivative concept. They found that most economics students could not make the connection between the mathematical concept of the derivative and the common economic interpretation of the derivative (the amount of change when increasing production by one unit), although it was covered in their previous calculus module. Derivatives are used for solving optimization problems; they are also used in marginal analysis and for characterizing and describing economics functions (Feudel, 2018). Other studies that found similar results were by Mkhathshwa & Doerr (2015, 2018), who observed that economics students had difficulties in dealing with topics from calculus (instantaneous rate of change) in economic contexts (marginal change). Ariza *et al.*, (2015) further noted the complementarity that exists

between mathematical understanding of mathematical concepts and economic concepts, for example, the relationship between a function and its derivative that is crucial for dealing with marginal analysis.

Although there are many arguments questioning the existence of separate service mathematics modules as they seem to make the transition problematic for many students, there are several arguments as well for them to persist. The financial aspect is strong because usually service mathematics modules bring together large student cohorts (sometimes even from different application study programmes), with diverse mathematics backgrounds, different needs and with different goals, which then legitimize the isolation and applicationist approach to service mathematics (Romo-Vázquez & Artigue, 2023). When service mathematics modules bring together students from different specializations within a single field, there are opportunities to provide relevant examples from that field, in addition to making it easy for a student to change specialization at a later stage as the mathematical competencies for the other specialization have presumably been achieved in the first year (Rønning, 2023). ‘Another argument used in favour of general mathematics courses is that one of the strengths of mathematics is exactly the fact that it is general and that one of the competencies that students should acquire by studying mathematics is to adapt to new and unknown situations’ (Rønning (2023), p. 694).

Additionally, there is the idea of students taking service mathematics module(s) in the first year because it is meant to provide the students with the skills and competences they need for later courses in their application study programmes (Alpers, 2020; Hochmuth *et al.*, 2021). Generally, there is a conviction that it would not be a satisfactory solution only integrating mathematics in the application courses (Kortemeyer & Biehler, 2023). Mathematics is often described as an essential tool when studied as a service subject (Macbean, 2004). Nonetheless, Harris *et al.* (2015, p. 322) argued that ‘mathematics is not generally like a “tool” that can be picked up and applied in various places without adjustment, modelling, and even sometimes rebuilding one’s mathematical knowledge’. Feudel (2023) found that basic economics textbooks frequently feature symbolic methods from calculus (as opposed to engineering textbooks in the case of integrals (González-Martín, 2021)), but the symbolic methods are additionally presented with two further dimensions: their geometric interpretation and their application within economics contexts. Rønning (2023) proposes students’ need to master the interplay between ‘praxeologies’ (as conceived in the Anthropological Theory of Didactics). Particularly, he found students needing not only mathematical techniques or electrical engineering techniques for solving tasks but also an understanding for the justification of the use of the techniques from both disciplines to solve problems in electrical engineering. Several other studies similarly address students’ (in)ability to apply mathematics in the application courses (Christensen, 2008; Alpers *et al.*, 2013; Hester *et al.*, 2014; Harris *et al.*, 2015; Randahl, 2016).

Thus, while there is a shared desire of lecturers and students that examples and models from application courses should be incorporated in the service mathematics module(s) to make the content relevant, interesting and approachable in the transition phase (Dawson, 2014; Gueudet & Thomas, 2020), there is a dilemma of sequencing the modules of study to enable the integration of mathematics into application modules. Service mathematics modules are generally offered in the first year of students’ university studies (Willcox & Bounova, 2004; Alpers *et al.*, 2013; Dawson, 2014). While this leads to the limited availability of modules where the mathematical concepts might be incorporated, it also leads to the limited availability of application programme concepts that students are familiar with and could serve as a basis for discussing the mathematics incorporated (Alpers *et al.*, 2013). Furthermore, Alpers *et al.* (2013) enjoined due caution in the curriculum design process of mathematics module(s) based on application needs. ‘Mathematics has a coherent structure on its own which also must be taken into account to avoid the impression that mathematics is just a set of unrelated “chunks” which might be useful in certain models’ (Alpers *et al.*, 2013, p. 62). Communication between mathematics lecturers and educators in the

application disciplines is crucial, and further research is needed on the use of mathematics in different disciplines (Hochmuth, 2020; Romo-Vázquez & Artigue, 2023).

Several studies also examined what the content of service mathematics module(s) should be. The literature is concerned with mathematics being taught in the service mathematics module(s) without references to why students should learn it; nor is there a discussion of how mathematics should be thought to be appropriate later in the application courses (Gueudet & Quéré, 2018; Feudel & Biehler, 2021; González-Martín, 2021; Hochmuth *et al.*, 2021; Pepin *et al.*, 2021). For example, Mkhathshwa & Doerr (2016) analysed six business calculus textbooks and found that students had limited opportunities to reason with quantities (e.g. sales discount) and to work with realistic, context-based economic analyses involving several quantities. Again, the lack of communication between service mathematics lecturers and application programme lecturers exacerbates the issue, as lecturers in the application courses do not seem well informed about what mathematics has been taught to the students and in what way (Willcox & Bounova, 2004). Mills (2015) investigated the curricular integration of mathematics in the economics courses. By using an online survey instrument, she found that business faculty members perceived concepts of differentiation, optimization and rate of change as most necessary for students to succeed in subsequent business courses. Other studies in economics education have critically discussed the need for the use of graphs, algebra or both in teaching the demand curve in, for example, the course on principles of economics. While Hey (2005) suggested only using graphs, other studies mentioned the possibility that graphs may reduce learning, as learning to understand graphs takes time, but they also found that most students felt that graphs were helpful in understanding economics (Cohn *et al.*, 2001, 2004). Zetland *et al.* (2010) have argued that what is most important for students not to get confused by the combination of algebra and graphs is the quality of teaching economic concepts, whatever the method used. However, there exists very little research on curricular integration of mathematics; hence, more research is needed for deeper insights into how mathematical concepts are used and understood in the field of economics and also how students can be supported in the transition from school mathematics to disciplinary studies and ‘use’ of mathematics within further studies in economics.

5. Discussion

The main aim of the study was to review what issues that have been focused on in research within the field of secondary-tertiary transition and service mathematics modules that have relevance for economics education at the university level. The review study identified several important aspects of the secondary-tertiary transition phase for practitioners to consider when designing, implementing and facilitating teaching and learning of mathematics within economics education at the university level.

The most reported transition issue in mathematics for the study of economics pertains to the absence of prerequisites and inadequate fluency in basic algebra. Many quantitative studies have demonstrated the importance of school mathematics grade, school mathematics track or both for better performance in economics studies. Still, although heavily debated, few economics programmes set admission restrictions to mathematics tracks studied in school. Instead, there is a vast increase in various support measures and interventions to address the heterogeneous mathematics background among first-year students (Lawson *et al.*, 2020). While bridging, preparation courses or both are usually made compulsory in the US (e.g. Bahr, 2008), attendance in such modules are usually voluntary in Europe and not formally assessed. Furthermore, students are usually allowed to continue their studies regardless of how they performed pre-course offers (Büchele, 2020). The discussion and presentation of various support measures in the transition phase is crucial but too extensive to include in this review study. However, the literature provided here emphasizes the importance of acknowledging that a significant gap exists for many

students in terms of mathematical content covered in school and in the service mathematics module(s) in economics education. Hence, when it comes to transition into mathematics module(s) in economics programmes, the discontinuity is not the shift in the kind of mathematics to be mastered, described by Tall (1991) as a move from elementary to advanced mathematical thinking (definition-theorem-proof), which is the case of students transitioning to specialist mathematics courses wherein much focus is put on formal mathematics and proof and proving. Instead, many first-year economics students just did not have the opportunity to study previously the mathematics needed for successful transition into the study of mathematics for economics, where the focus is on modelling and mathematical logic (model-theorem-proof) and various representations of mathematics and their interplay. However, while there is the shared knowledge that many students lack prerequisites for a successful transition, there is a lack of research outlining the mathematical needs—both in terms of content and level—of first-year economics students.

Accordingly, the mismatch in institutional requirements and local expectations of prior student experience, in addition to many students underestimating the mathematics demand of the economics study programme, are aspects of the secondary-tertiary transition that become apparent right from the time students enter their university studies. Together with missing opportunities of previously having studied the prerequisite mathematics expected by the course structure, students must also cope with the more general transition issues (i.e. yielding all types of mathematics courses) of changes in the roles of students and teachers' and 'the changes in the approaches to teaching and assessment'. The literature suggests factors such as larger class sizes; class reorganizations; distanced and limited communication with lecturers; assessment and students' increased autonomy in terms of time, choices of resources, etc. as being demanding for the novice student to be successful in mathematics at the tertiary level. In the case of mathematics within economics education, there is the additional concern that the lecturer for service mathematics module(s) might not be from the application study programme but belongs to the mathematics department, with little or no contact with lecturers from the main field of economics. This problem applies equally to engineering programmes also (Willcox & Bounova, 2004; Flegg *et al.*, 2012; Rønning, 2023). While there is no consensus on who would be best suited to teach mathematics module(s) to economics students (Voßkamp, 2017), it is a shared understanding that course organization, content structure, the discipline of the lecturer and, above all, communication between the parties involved are all crucial in supporting students in their transition from school-level mathematics to service mathematics module(s) tailored for economics. For practitioners and stakeholders to make informed curriculum changes, introduce critical thinking and use technological resources in future teaching, there is a need for further research in these directions.

Closely related to the concern regarding whether the mathematics lecturer should belong to the mathematics or the economics department is the transition issue about relevance. This issue broadens the transition phase to viewing the transition phase as lasting from school mathematics to the use of mathematics in the economics programme. Practical aspects of this discussion are whether there should be an institutionalized separation of distinct mathematics module(s) and also how to operationalize the sequencing of modules and the integration of economics in the mathematics module(s) and vice versa. However, the core of the discussion is the relevance and understanding of mathematics in the course. Researchers highlight the challenges students face in comprehending mathematical concepts in the context of economics. Difficulties arise in establishing the connections between mathematical concepts and their corresponding interpretation in economics (Ariza *et al.*, 2015; Mkhatshwa & Doerr, 2018; Feudel & Biehler, 2021). Furthermore, not only are concepts often treated and understood differently in the disciplines but research also points to a missing focus on important mathematics topics such as quantities (Mkhatshwa & Doerr, 2016; Voßkamp, 2023). In the case of economics education, a discernible research gap exists, necessitating this line of research.

Although it was not part of this literature review study, one could ‘zoom out’ once more and take a broader perspective in which the transition phase could be seen as lasting from school mathematics right through to the use of mathematics in the workplace, acknowledging that workplace demands are relevant to university economics education. Here, research and practitioners discuss the ‘gaps in skills’ that exist with respect to mathematics learnt at the level of university studies contrary to what is needed in the workplace (Carreira *et al.*, 2002; Kent *et al.*, 2007). When considering the role of mathematics in the modern society, it is apparent that mathematics is present (in a variety of forms as part of technology, embedded in computer systems, graphs, tables, symbol manipulation, data handling, modelling, etc.), yet it is often invisible (Noss *et al.*, 2000; Kent *et al.*, 2007; FitzSimons, 2013). Gravemeijer *et al.* (2017, p. 107) emphasize, ‘today, basically all mathematical operations that are taught in primary, secondary, and tertiary education can be performed by computers and are performed by computers in the world outside school’. Kent *et al.* (2007) argue that in today’s workplaces employees need techno-mathematical literacies: ‘They (employees) need to be able to appreciate the computer outputs from a mathematical perspective, while interpreting them in their context, and recognizing the parts that the IT system has kept hidden’ (Kent *et al.*, 2007, p. 79). Through the theoretical lens of ATD, Romo-Vázquez & Artigue (2023) indicate that characterizing workplace praxeologies and conditions need to be done first to facilitate relevant service mathematics education. Certainly, further exploration is necessary to delve into the diverse array of work environments and roles in which economists operate. However, it is left as a future research agenda because, examining the mathematics employed, whether explicitly or implicitly, necessitates a profound understanding of the specific field, as, typically, workers do not perceive their tasks as mathematical in nature (FitzSimons, 2013).

In conclusion, the issues identified in the literature on service mathematics for economics programmes make it possible and perhaps necessary to view the transition phase as involving several steps, all interacting and affecting each other. It goes from school mathematics to service mathematics courses within economics education, through to the use of mathematics in the application courses and further to the use of mathematics at the workplace. This result aligns with the thoughts of Gueudet (2008), who stated that determining the precise occurrence of the secondary-tertiary transition phase is challenging, and perhaps impossible, considering it is important to acknowledge that the transition phase continues beyond the period spanning from the end of secondary school to the commencement of university studies.

By synthesizing the literature on transition issues into service mathematics modules for economics education, a secondary aim was to enhance practitioners’ understanding of the complex phenomena. The issues highlighted above are crucial for practitioners to consider as they affect students’ study progression, motivation and understanding of economics, as well as the financial situation of universities. However, as mentioned previously, much research is still needed in this field. Especially, this review has highlighted the need for research focusing on mathematical requirements in terms of content and level for first-year students of economics, research on concept discrepancies between the mathematics and the economics, innovative teaching pedagogies and the application of technology in economics education. Additionally, to ensure the relevance of the mathematics modules for future societal needs, it is crucial to explore how mathematics is utilized in the workplace, particularly in contexts that are pertinent to university-level economics education.

6. Conclusion

This review summarizes the issues that research has identified regarding mathematics in the secondary-tertiary transition phase for economics students. The process of transition of students from school mathematics to service mathematics within economics education is both multifaceted and demanding.

The issues identified suggest broadening the perspective of transition as constituting certain steps from school mathematics to service mathematics courses within economics education, through to the utilization of mathematics in economics and in the workplace. All steps are interconnected, influencing each other in various ways. These issues are discussed in this article. Additionally, the article underscores the necessity for more in-depth studies on specific mathematical content and proficiency needed by first-year economics students, discrepancies in concept usage between mathematics and economics, innovative instructional methods and the integration of technology in economics education. To guarantee that mathematics modules remain pertinent to future societal demands, it is also imperative to investigate the application of mathematics in workplace settings.

Acknowledgement(s)

The author wishes to express her thankfulness to the anonymous reviewers for their insightful and constructive feedback, which has significantly enhanced the quality of this article.

REFERENCES

- ALBESHREE, F., AL-MANASIA, M., LEMCKERT, C., LIU, S. & TRAN, D. (2022) Mathematics teaching pedagogies to tertiary engineering and information technology students: a literature review. *Int. J. Math. Educ. Sci. Technol.*, 53, 1609–1628.
- ALLGOOD, S., WALSTAD, W. B. & SIEGFRIED, J. J. (2015) Research on teaching economics to undergraduates. *J. Econ. Lit.*, 53, 285–325.
- ALPERS, B. (2020) *Mathematics as a service subject at the tertiary level. A state-of-the-art report for the Mathematics Interest Group*. Brussels: European Society for Engineering Education.
- ALPERS, B., DEMLOVA, M., FANT, C.-H., GUSTAFSSON, T., LAWSON, D., MUSTOE, L., OLSEN-LEHTONEN, B., ROBINSON, C. & VELICHOVA, D. (2013) *A framework for mathematics curricula in engineering education: a report of the mathematics working group*. Brussels: European Society for Engineering Education.
- ANASTASAKIS, M., ZAKYNTHINAKI, M., TRUJILLO-GONZÁLEZ, R., GARCÍA-ALONSO, I. & PETRIDIS, K. (2022) An activity theory approach in explaining engineering students' difficulties with university mathematics. *IJMEST*, 53, 1571–1587.
- ANDERSON, G., BENJAMIN, D. & FUSS, M. A. (1994) The determinants of success in university introductory economics courses. *J. Econ. Educ.*, 2, 99–119.
- ARIZA, A., LLINARES, S. & VALLS, J. (2015) Students' understanding of the function-derivative relationship when learning economic concepts. *MERJ*, 27, 615–635.
- ARNOLD, I. J. & STRATEN, J. T. (2012) Motivation and math skills as determinants of first-year performance in economics. *J. Econ. Educ.*, 43, 33–47.
- ASIAN-CHAVES, R., BUITRAGO, E. M., MASERO, I. & YÑIGUEZ, R. (2021) Advanced mathematics: an advantage for business and management administration students. *Int. J. Manag. Educ.*, 19, 1472–8117.
- ASIAN-CHAVES, R., BUITRAGO, E. M., MASERO, I. & YÑIGUEZ, R. (2022) Mathematical background as a success factor in economics and business degrees. *CSR*, 24, 758–772.
- BAHR, P. R. (2008) Does mathematics remediation work?: a comparative analysis of academic attainment among community college students. *Res. High. Educ.*, 49, 420–450.
- BALLARD, C. L. & JOHNSON, M. F. (2004) Basic math skills and performance in an introductory economics class. *J. Econ. Educ.*, 35, 3–23.
- BARQUERO, B., BOSCH, M. & GASCÓN, J. (2013) The ecological dimension in the teaching of mathematical modelling at university. *RDM*, 33, 307–338.
- BAŠIĆ, M. & ŠIPUŠ, Z. M. (2023) Connecting mathematics and economics: the case of the integral. *Proceedings of The Learning and Teaching of Calculus Across Disciplines* (T. DREYFUS, A. S. GONZÁLEZ-MARTÍN, E. NARDI, J. MONAGHAN & P. W. THOMPSON eds). Bergen: MatRIC, pp. 89–92.

- BERGSTEN, C. & JABLONKA, E. (2019) Understanding the secondary-tertiary transition in mathematics education: contribution of theories to interpreting empirical data. *Proceedings of CERME11* (U. T. JANKVIST, M. VAN DEN HEUVEL-PANHUIZEN & M. VELDHUIS eds). Utrecht: Freudenthal Group & Freudenthal Institute, Utrecht University and ERME.
- BIZA, I., GONZÁLEZ-MARTÍN, A. S. & PINTO, A. (2022) ‘Scaffolding’ or ‘filtering’: a review of studies on the diverse roles of calculus courses for students, professionals and teachers. *Int. J. Res. Undergrad. Math. Educ.*, 8, 389–418.
- BOLSTAD, T., HØYVIK, I.-M., LUNDHEIM, L., NOME, M. & RØNNING, F. (2022) Study programme driven engineering education: interplay between mathematics and engineering subjects. *Teach. Math. Appl.*, 41, 329–344.
- BRIGGS, A. R. J., CLARK, J. & HALL, I. (2012) Building bridges: understanding student transition to university. *Qual. High. Educ.*, 18, 3–21.
- BROWN-ROBERTSON, L., NTEMBE, A. & TAWAH, R. (2015) Evaluating the “underserved student” success in economics principles courses. *J. Econ. Econ. Educ. Res.*, 16, 13–24.
- BÜCHELE, S. (2020) Bridging the gap – how effective are remedial math courses in Germany? *Stud. Educ. Eval.*, 64, 100832.
- CARREIRA, S., EVANS, J., LERMAN, S. & MORGAN, C. (2002) Mathematical thinking: studying the notion of ‘transfer’. *Proceedings of PME26* (A. D. COCKBRUN & E. NARDI eds). Norwich: International group for the psychology of mathematics education.
- CHAPMAN, A. L., MORGAN, L. C. & GARTLEHNER, G. (2010) Semi-automating the manual literature search for systematic reviews increases efficiency. *Health Inf. Libr. J.*, 27, 22–27.
- CHRISTENSEN, O. R. (2008) Closing the gap between formalism and application—PBL and mathematical skills in engineering. *Teach. Math. Appl.*, 27, 131–139.
- COHN, E., COHN, S., BALCH, D. C. & BRADLEY JR., J. (2001) Do graphs promote learning in principles of economics? *J. Econ. Educ.*, 32, 299–310.
- COHN, E., COHN, S., BALCH, D. C. & BRADLEY JR., J. (2004) The relation between student attitudes toward graphs and performance in economics. *AEX*, 48, 41–52.
- DARLINGTON, E. & BOWYER, J. (2017) Students’ views of A-level mathematics as preparation for degree-level economics. *Citizsh. Soc. Econ. Educ.*, 16, 100–116.
- DAWSON, P. (2014) *Skills in Mathematics and Statistics in Economics and tackling transition*. The Higher Education Academy.
- DE GUZMÁN, M., HODGSON, B. R., ROBERT, A. & VILLANI, V. (1998) Difficulties in the passage from secondary to tertiary education. *Proceedings of ICM* (A. K. LOUIS, U. REHMAN & P. SCHNEIDER eds). Berlin: Journal der Deutschen Mathematiker-Vereinigung.
- DI MARTINO, P., GREGORIO, F. & IANNONE, P. (2022) The transition from school to university in mathematics education research: new trends and ideas from a systematic literature review. *Educ. Stud. Math.*, 1–28.
- DOLADO, J. J. & MORALES, E. (2006) Which factors determine the grades of undergraduate students in economics? Some evidence from Spain. *IZA Discussion Papers, No. 2491*. Bonn: Institute for the Study of Labor (IZA).
- DUGGER JR., W. E. (1993) The relationship between technology, science, engineering, and mathematics. *Proceedings of the Annual Conference of the American Vocational Association*.
- DURAND-GUERRIER, V., HOCHMUTH, R., NARDI, E. & WINSLØW, C. (2021) *Research and Development in University Mathematics Education: Overview Produced by the International Network for Research on Didactics of University Mathematics*. New York: Routledge.
- DYSON, R. & RENK, K. (2006) Freshmen adaptation to university life: depressive symptoms, stress, and coping. *J. Clin. Psychol.*, 62, 1231–1244.
- FEUDEL, F. (2017) Students’ interpretation of the derivative in an economic context. *Proceedings of CERME10* (T. DOOLEY & G. GUEUDET eds). Dublin: DCU Institute of Education and ERME.
- FEUDEL, F. (2018) $C'(x) = C(x+1) - C(x)$?—Students’ connections between the derivative and its economic interpretation in the context of marginal cost. *Proceedings of INDRUM2018* (V. DURAND-GUERRIER, R. HOCHMUTH, S. GOODCHILD & N. M. HOGSTAD eds). Kristiansand: University of Agder and INDRUM.

- FEUDEL, F. (2023) What knowledge related to the derivative is commonly used in basic economics textbooks? – Selected results from a praxeological analysis. *Proceedings of the Learning and Teaching of Calculus Across Disciplines* (T. DREYFUS, A. S. GONZÁLEZ-MARTÍN, E. NARDI, J. MONAGHAN & P. W. THOMPSON eds). MatRIC: Bergen.
- FEUDEL, F. & BIEHLER, R. (2021) Students' understanding of the economic interpretation of the derivative in the context of marginal cost. *Int. J. Res. Undergrad. Math. Educ.*, 437–468.
- FITZSIMONS, G. E. (2013) Doing mathematics in the workplace: a brief review of selected literature. *Adults Learn. Math.*, 8, 7–19.
- FLEGG, J., MALLET, D. & LUPTON, M. (2012) Students' perceptions of the relevance of mathematics in engineering. *Int. J. Math. Educ. Sci. Technol.*, 43, 717–732.
- GÖLLER, R. & RÜCK, H. G. (2023) Emotions in self-regulated learning of first-year mathematics students. *Practice-Oriented Research in Tertiary Mathematics Education* (R. BIEHLER, M. LIEBENDÖRFER, G. GUEUDET, C. RASMUSSEN & C. WINSLØW eds). Cham: Springer, Cham, pp. 23–44.
- GONZÁLEZ-MARTÍN, A. S. (2021) $VB - VA = \int AB \, dx$ $V_B - V_A = \int A^b f(x) \, dx$. The use of integrals in engineering programmes: a praxeological analysis of textbooks and teaching practices in strength of materials and electricity and magnetism courses. *Int. J. Res. Undergrad. Math. Educ.*, 7, 211–234.
- GONZÁLEZ-MARTÍN, A. S., BARQUERO, B. & GUEUDET, G. (2023) Mathematics in the Training of Engineers: Contributions of the Anthropological Theory of the Didactic. *Practice-Oriented Research in Tertiary Mathematics Education* (R. BIEHLER, M. LIEBENDÖRFER, G. GUEUDET, C. RASMUSSEN & C. WINSLØW eds). Cham: Springer, Cham, pp. 559–579.
- GRAVEMEIJER, K., STEPHAN, M., JULIE, C., LIN, F.-L. & OHTANI, M. (2017) What mathematics education may prepare students for the society of the future? *Int. J. Sci. Math. Educ.*, 15, 105–123.
- GUEUDET, G. (2008) Investigating the secondary–tertiary transition. *Educ. Stud. Math.*, 67, 237–254.
- GUEUDET, G. & QUÉRÉ, P.-V. (2018) “Making connections” in the mathematics courses for engineers: the example of online resources for trigonometry. *Proceedings of INDRUM2018* (V. DURAND-GUERRIER, R. HOCHMUTH, S. GOODCHILD & N. M. HOGSTAD eds). Kristiansand: University of Agder and INDRUM.
- GUEUDET, G. & THOMAS, M. O. (2020) Secondary-tertiary transition in mathematics education. *Encyclopedia of Mathematics Education* (S. LERMAN ed). Cham: Springer International Publishing, pp. 762–766.
- GUEUDET, G., BOSCH, M., DI SESSA, A. A., KWON, O. N. & VERSCHAFFEL, L. (2016) *Transitions in Mathematics Education*. Hamburg: Springer Nature.
- HAFNI, R., HERMAN, T., NURLAELAH, E. & MUSTIKASARI, L. (2020) The importance of science, technology, engineering, and mathematics (STEM) education to enhance students' critical thinking skill in facing the industry 4.0. *Journal of Physics: Conference Series*, 1521, 042040.
- HARRIS, D., BLACK, L., HERNANDEZ-MARTINEZ, P., PEPIN, B., WILLIAMS, J. & WITH THE TRANSMATHS TEAM (2015) Mathematics and its value for engineering students: what are the implications for teaching? *Int. J. Math. Educ. Sci. Technol.*, 46, 321–336.
- HARVEY, L., DREW, S. & SMITH, M. (2006) The first-year experience: a review of literature for the Higher Education Academy. *Report for Higher Education Academy*. Sheffield: Centre for Research and Evaluation.
- HAWKES, T. & SAVAGE, M. (2000) Measuring the mathematics problem. *Report from Seminar at the Møller Centre Cambridge*. London: Engineering Council.
- HERNANDEZ-MARTINEZ, P. (2016) “Lost in transition”: alienation and drop out during the transition to mathematically-demanding subjects at university. *Int. J. Educ. Res.*, 79, 231–239.
- HESTER, S., BUXNER, S., ELFRING, L. & NAGY, L. (2014) Integrating quantitative thinking into an introductory biology course improves students' mathematical reasoning in biological contexts. *CBE-Life Sci. Educ.*, 13, 54–64.
- HEY, J. D. (2005) I teach economics, not algebra and calculus. *J. Econ. Educ.*, 36, 292–304.
- HOCHMUTH, R. (2020) Service-courses in university mathematics education. *Encyclopedia of Mathematics Education*, 770–774.

- HOCHMUTH, R., BROLEY, L. & NARDI, E. (2021) Transitions to, across and beyond university. *Research and Development in University Mathematics Education* (V. DURAND-GUERRIER, R. HOCHMUTH, E. NARDI & C. WINSLØW eds). Norwich: Routledge, pp. 191–215.
- HODGEN, J., MCALINDEN, M. & TOMEI, A. (2014) *Mathematical Transitions*. York: Higher Education Academy.
- HODGSON, G. M. (2013) On the complexity of economic reality and the history of the use of mathematics in economics. *Filosofía de la Economía*, 1, 25–45.
- HOLTON, D. & ARTIGUE, M. (2001) *The Teaching and Learning of Mathematics at University Level: An ICMI Study*, vol. 7. Berlin: Springer Science & Business Media.
- HSIEH, H.-F. & SHANNON, S. E. (2005) Three approaches to qualitative content analysis. *Qual. Health Res.*, 15, 1277–1288.
- JABLONKA, E., ASHJARI, H. & BERGSTEN, C. (2017) “Much palaver about greater than zero and such stuff” – first year engineering students’ recognition of university mathematics. *Int. J. Res. Undergrad. Math. Ed.*, 3, 69–107.
- JACKSON, K. (2014) Dealing with students’ diverse skills in maths and stats. *Handbook*. The Economics network.
- JOHNSON, M. & KUENNEN, E. (2006) Basic math skills and performance in an introductory statistics course. *J. Stat. Educ.*, 14, 1–15.
- KENT, P., NOSS, R., GUILLE, D., HOYLES, C. & BAKKER, A. (2007) Characterizing the use of mathematical knowledge in boundary-crossing situations at work. *Mind Cult. Act.*, 14, 64–82.
- KORTEMAYER, J. & BIEHLER, R. (2023) Analyzing the interface between mathematics and engineering in basic engineering courses. *Practice-Oriented Research in Tertiary Mathematics Education* (R. BIEHLER, M. LIEBENDÖRFER, G. GUEUDET, C. RASMUSSEN & C. WINSLØW eds). Cham: Springer, Cham, pp. 669–692.
- KRUMSVIK, R. (2016) *En doktorgradsutdanning i endring: et fokus på den artikkelbaserte ph.d.-avhandlingen*. Bergen: Fagbokforl.
- KÜHLING-THEES, C., HAPP, R., ZLATKIN-TROITSCHANSKAIA, O. & PANT, H. A. (2020) The impact of entry preconditions on student dropout and subject change in business and economics. *Student Learning in German Higher Education* (O. ZLATKIN-TROITSCHANSKAIA, H. A. PANT, M. TOEPFER & C. LAUTENBACH eds). Wiesbaden: Springer, pp. 351–370.
- LAGERLÖF, J. N. M. & SELTZER, A. J. (2009) The effects of remedial mathematics on the learning of economics: evidence from a natural experiment. *J. Econ. Educ.*, 40, 115–137.
- LAGING, A. & VOßKAMP, R. (2017) Determinants of maths performance of first-year business administration and economics students. *Int. J. Res. Undergrad. Math. Ed.*, 3, 108–142.
- LAWSON, D., GROVE, M. & CROFT, T. (2020) The evolution of mathematics support: a literature review. *Int. J. Math. Educ. Sci. Technol.*, 51, 1224–1254.
- LERMAN, S. (2020) Socioeconomic class and socioeconomic status in mathematics education. *Encyclopedia of Mathematics Education* (S. LERMAN ed). Cham: Springer International Publishing AG, pp. 785–790.
- LIEBENDÖRFER, M., BÜDENBENDER-KUKLINSKI, C., LANKEIT, E., SCHÜRMANN, M., BIEHLER, R. & SCHAPER, N. (2023) Framing goals of mathematics support measures. *Practice-Oriented Research in Tertiary Mathematics Education* (R. BIEHLER, M. LIEBENDÖRFER, G. GUEUDET, C. RASMUSSEN & C. WINSLØW eds). Cham: Springer, Cham, pp. 91–117.
- MACBEAN, J. (2004) Students’ conceptions of, and approaches to, studying mathematics as a service subject at undergraduate level. *Int. J. Math. Educ. Sci. Technol.*, 35, 553–564.
- MALLIK, G. & LODEWIJKS, J. (2010) Student performance in a large first year economics subject: which variables are significant? *Econ. Pap.: J. Appl. Econ. Policy*, 29, 80–86.
- MERGNER, J., LEIŠYTĖ, L. & BOSSE, E. (2019) The widening participation agenda in German higher education: discourses and legitimizing strategies. *Social Inclusion*, 7, 61–70.
- MILES, M. B., HUBERMAN, A. M. & SALDAÑA, J. (2020) *Qualitative Data Analysis: A Methods Sourcebook*, 4th edn. Los Angeles, California: SAGE Publications.
- MILLS, M. (2015) Business faculty perceptions of the calculus content needed for business courses. *Proceedings of RUME18* (T. FUKAWA-CONNELLY, N. E. INFANTE, K. KEENE & M. ZANDIEH eds). Pittsburg: the Special Interest Group of the Mathematical Association of America on Research in Undergraduate Mathematics Education, pp. 231–237.

- MKHATSHWA, T. P. & DOERR, H. M. (2015) Students' understanding of marginal change in the context of cost, revenue, and profit. *Proceedings of CERME 9-Ninth Congress of the European Society for Research in Mathematics Education* (K. KRAINER & N. VONDROVA eds). Prague: Charles University in Prague, Faculty of Education and ERME, pp. 2201–2206.
- MKHATSHWA, T. P. & DOERR, H. M. (2016) Opportunity to learn solving context-based tasks provided by business calculus textbooks: An exploratory study. *Proceedings of RUME19* (T. FUKAWA-CONNELLY, N. E INFANTE, K. KEENE & M. ZANDIEH eds). Pittsburg: the Special Interest Group of the Mathematical Association of America on Research in Undergraduate Mathematics Education, pp. 1124–1132.
- MKHATSHWA, T. P. & DOERR, H. M. (2018) Undergraduate students' quantitative reasoning in economic contexts. *Math. Think. Learn.*, 20, 142–161.
- MONTEIRO, H. & LOPES, A. F. (2007) A benchmarking of the undergraduate economics major in Europe and in the United States. *Int. Rev. Econ. Educ.*, 6, 9–26.
- NARDI, E. (2016) Teaching mathematics to non-mathematicians: what can we learn from research on teaching mathematicians. *Proceedings of ICME13* (G. KAISER ed). Cham: Springer.
- NARDI, E. & WINSLØW, C. (2018) INDRUM2016 special issue editorial. *Int. J. Res. Undergrad. Math. Ed.*, 4, 1–7.
- NORTVEDT, G. A. & SIQVELAND, A. (2019) Are beginning calculus and engineering students adequately prepared for higher education? An assessment of students' basic mathematical knowledge. *Int. J. Math. Educ. Sci. Technol.*, 50, 325–343.
- NOSS, R., HOYLES, C. & POZZI, S. (2000) Working knowledge: mathematics in use. *Education for Mathematics in the Workplace* (A. BESSOT & J. RIDGWAY eds). Dordrecht: Springer, Dordrecht, pp. 17–35.
- OPSTAD, L. (2018) Success in business studies and mathematical background: the case of Norway. *J. Appl. Res. High. Educ.*, 10, 399–408.
- OPSTAD, L. (2021) Factors explaining business students' performance in an introductory mathematics course. What are the impacts of gender, academic ability, personality traits, and attitudes towards mathematics? *Adv. Educ. Sci.*, 3, 23–43.
- PAMPAKA, M., WILLIAMS, J. & HUTCHESON, G. (2012) Measuring students' transition into university and its association with learning outcomes. *Br. Educ. Res. J.*, 38, 1041–1071.
- PAMPAKA, M., WILLIAMS, J. & HOMER, M. (2016) Is the educational 'what works' agenda working? Critical methodological developments. *Int. J. Res. Method Educ.*, 39, 231–236.
- PEPIN, B., BIEHLER, R. & GUEUDET, G. (2021) Mathematics in engineering education: a review of the recent literature with a view towards innovative practices. *Int. J. Res. Undergrad. Math. Ed.*, 7, 163–188.
- PETERS, J., HOCHMUTH, R. & SCHREIBER, S. (2017) Applying an extended praxeological ATD-Model for analyzing different mathematical discourses in higher engineering courses. *Proceedings of the Didactics of Mathematics in Higher Education as a Scientific Discipline Conference* (R. BIEHLER, R. HOCHMUTH & H.-G. RÜCK eds). Kassel: Kompetenzzentrum hochschuldidaktik matematik.
- RANDAHL, M. (2016) *Engineering students approaching the mathematics textbook as a potential learning tool – opportunities and constraints* Ph.D. Thesis., Norway: University of Agder.
- ROMO-VÁZQUEZ, A. & ARTIGUE, M. (2023) Challenges for research on tertiary mathematics education for non-specialists: where are we and where are we to go? *Practice-Oriented Research in Tertiary Mathematics Education* (R. BIEHLER, M. LIEBENDÖRFER, G. GUEUDET, C. RASMUSSEN & C. WINSLØW eds). Cham: Springer, Cham, pp. 535–557.
- RØNNING, F. (2023) Learning mathematics in a context of electrical engineering. *Practice-Oriented Research in Tertiary Mathematics Education* (R. BIEHLER, M. LIEBENDÖRFER, G. GUEUDET, C. RASMUSSEN & C. WINSLØW eds). Cham: Springer, Cham, pp. 603–620.
- SCHAEFER, H. (2020) The first year in higher education: the role of individual factors and the learning environment for academic integration. *Higher Educ.*, 79, 95–110.
- TALL, D. (1991) *Advanced Mathematical Thinking* (D. TALL ed), vol. 11. Berlin: Springer Science & Business Media.

- TRENHOLM, S., PESCHKE, J. & CHINNAPPAN, M. (2019) A review of fully online undergraduate mathematics instruction through the lens of large-scale research (2000–2015). *Primus*, 29, 1080–1100.
- VOBKAMP, R. (2017) Mathematics in economics study programmes in Germany: structures and challenges. *Proceedings of Didactics of Mathematics in Higher Education as a Scientific Discipline* (R. BIEHLER, R HOCHMUTH & H.-G. RÜCK eds). Kassel: Kompetenzzentrum hochschuldidaktik matematik.
- VOBKAMP, R. (2023) Calculus in mathematics for economists. *Proceedings of the Learning and Teaching of Calculus Across Disciplines* (T. DREYFUS, A. S. GONZÁLEZ-MARTÍN, E. NARDI, J. MONAGHAN & P. W. THOMPSON eds). Bergen: MatRIC.
- WEINTRAUB, E. R. (2002) *How Economics Became a Mathematical Science*. Durham, North Carolina: Duke University Press.
- WILCOX, K. & BOUNOVA, G. (2004) Mathematics in engineering: identifying, enhancing, and linking the implicit mathematics curriculum. *Proceedings of the 2004 American Society for Engineering Education Annual Conference & Exposition*. Salt Lake City: American Society for Engineering Education.
- WINSLØW, C., GUEUDET, G., HOCHMUTH, R. & NARDI, E. (2018) Research on university mathematics education. *Developing Research in Mathematics Education* (T. DREYFUS, M. ARTIGUE, D. POTARI, S. PREDIGER & K. RUTHVEN eds). Norwich: Routledge, pp. 60–74.
- ZETLAND, D., RUSSO, C. & YAVAPOLKUL, N. (2010) Teaching economic principles: algebra, graph or both? *Am. Econ.*, 55, 123–131.

Ida Landgärds-Tarvoll is an Assistant Professor of mathematics education at University of Agder (UiA) in Kristiansand, Norway. She has been teaching the service mathematics course for economics students since 2017 and is doing research within that field as well. She is an active member of the Norwegian Centre for Excellence in Education – Centre for Research, Innovation and Coordination of Mathematics Teaching (MatRIC), based at UiA.