UiA University of Agder

Optimization of Rotor Tilts in a Floating Offshore Wind Farm

Optimizing tilt angles to maximize power output for individual FOWT in a wind farm using PyWake

MAGNUS VIKSE

SUPERVISOR
Professor João Leal

University of Agder, 2024
Faculty of Engineering and Science
Department of Engineering Sciences

Master

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Abstract

This thesis aims to optimize the total power output of a wind farm with 11 turbines by adjusting individual tilt angles for each turbine, and looking at what scenarios provide optimal tilts. This is done using PyWake to simulate the wind farm by implementing site and turbine conditions, as well as both Jensen (NOJ) and Bastankhah Gaussian (BG) wake deficit models. Three optimization algorithms, simplicial homology global optimization (shgo), Dual Annealing (DA), and Differential Evolution (DE) were compared during the initial simulations, using -30°and 30°as tilt bounds. The initial simulations included two wind directions of 180°and 190°, multiple wind speeds, and distances between turbines. Furthermore, the comparative simulations were done using only BG and DE, as well as a reduced amount of distances and wind speeds. These simulations consisted of changing turbines, increasing turbine power, multiple wind directions, and adding more rows. From the results, it can be seen that there seems to be a power gain for optimized tilts, and it becomes more prominent at either a low wind speed, below 8 $\frac{m}{s}$, or low distances between turbines, below 6D distance. It can also be concluded that the greatest power gain compared to 0 tilt came from BG, at a 3D distance and a wind speed of 9 $\frac{m}{s}$, with a gain of 3.71 MW, corresponding to 28.6%. Furthermore, it can be concluded that the furthest up and downstream turbines typically only resulted in optimized tilt angles of 0, while the remaining turbines had various tilts, and regularly reached maximum or minimum tilt.

Sammendrag

Denne oppgaven har som mål å optimere total effekt av en vind farm bestående av 11 turbiner ved å endre på individuelle tiltvinkler for hver turbin, og se på hvilke scenariorer som gir optimale tilt vinkler. Dette er gjort ved å bruke PyWake til å simulere vindfarmen ved å implementere lokasjon og turbin egenskaper, samt Jensen (NOJ) og Bastankhah Gaussian (BG) vakemodeller. Tre optimeringsalgoritmer, simplicial homology global optimization (shgo), Dual Annealing (DA) og Differential Evolution (DE) ble sammenlignet under de initielle simuleringene, med -30°og 30°som grenseverdier for tilt. De initielle simuleringene bestod av to vindretninger, 180°og 190°, samt flere vindhastigheter og distanser mellom turbinene. Videre ble sammenlignende simuleringer gjennomført ved å kun bruke BG og DE, samt et redusert antall distanser og vindhastigheter. Disse simuleringene bestod av å endre turbiner, øke kraften på turbiner, flere vindretninger og å legge til flere rader. Resultatene indikerer at det er en kraftøkning ved å optimere tilt på turbinene, og dette kan spesielt sees på lave vindhastigheter, lavere enn 8 $\frac{m}{s}$, og kortere distanser mellom turbinene, under 6D. Det kan også konkluderes med at den største kraftøkningen, sammenlignet med 0 tilt, kom fra BG ved en distance på 3D og en vindhastighet på 9 $\frac{m}{s}$ med en økning på 3.71 MW, som tilsvarer en økning på 28.6%. Videre kan det konkluderes med at turbinene lengst nedstrøms og oppstrøms hadde optimale tilt vinkler på 0, mens de resterende turbinene hadde variende tilt vinkler, og nådde ofte grenseverdiene for maksimum og minimum tilt.

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Abbreviation

AEP - Annual Energy Production

AIS - Automatic identification system

API - Application Programming Interface

BEM - Blade Element Momentum

BG - Bastankhah Gaussian

CSA - Classical Simulated Annealing

DA - Duan Annealing

DE - Differential Evolution

DTU - Technical University of Denmark

xD - x Diameter distance

EI - Energy Institute

FOWT - Floating Offshore Wind Turbine

FSA - Fast Simulated Annealing

GWEC - Global Wind Energy Council

IEA - International Energy Agency

IRENA - International Renewable Energy Agency

LCOE - Levelized Cost Of Energy

NOJ - Niels Otto Jensen

PDF - Probability Density Function

RWT - Reference Wind Turbine

SA - Simulated Annealing

shgo - simplicial homology global optimization

UN - United Nations

Chapter 1

Introduction

1.1 Background

As global energy consumption is continuously increasing, with the Energy Institute (EI) finding an average increase of 1.4% [1] in primary energy consumption between 2012-2022, a need to further develop sustainable energy sources is vital. United Nations (UN) states in development goal 7 that access to affordable, reliable, sustainable, and modern energy for all within 2030 [2]. This development goal along with the Paris climate agreement to limit the global increase in temperature due to global warming by 1.5 °C, or well below 2 °C, above pre-industrial levels [3], sets the stage for fast development of renewable energy. To achieve these goals, renewable energy sources must be implemented in new projects and take over fossil energy sources already implemented in the energy mix. This is further enhanced by the predictions of the International Energy Agency (IEA), which estimated a peak in fossil fuel consumption, including coal, oil, and natural gas by 2030[4]. Due to the natural intermittency of most renewable energy sources, typically weather-dependent, it is clear that as of today there is not a single renewable energy source that solely can be substituted for fossil energy sources sustainably. On the other hand, renewable energy sources, such as wind, hydro, and solar power, are well-functioning as complementary energy sources to provide stable power to the grid. This was acknowledged by the then Director-General of the International Renewable Energy Agency (IRENA) 'Reliability is not a function of individual generation technologies, but a function of the electricity system as a whole.' [5].

Wind power has been used for centuries, both as early day means of transportation with sailing, and later for mechanical work as windmills. In later years, with the implementation of electricity and increasing demand for electric power, these windmills were adapted to wind turbines. WindEurope found an increase of 17 GW of installed capacity in Europe in 2023, resulting in a total of 272 GW capacity installed in Europe at the end of 2023. This capacity led to 19% of the produced electricity within the EU coming from wind power. Furthermore, they estimate Europe to provide over 500 GW wind energy capacity installed by 2030[6].

Recent years have shifted the placement of wind turbines from onshore to offshore, and one benefit to moving them offshore is due to the area needed for a wind farm. With the wind turbine diameter reaching over 250-meter[7], and an efficient wind farm needing around 7 diameters distance between them, it is clear that a wind farm needs a lot of area with the increase in wind power. Globally in 2022, 64.3 GW of offshore wind capacity was installed, which stood for 7.1% of the total wind capacity installation. Global Wind Energy Council (GWEC) also predicts an increase in installed capacity of 380 GW from 2022-2032[8]. This shift can also be seen in the installed capacity in Europe, as well as the energy outlook from WindEurope. From the 17 GW installed in 2023, 3 GW installed offshore, resulting in a total of 13% (34 GW) of the total installed wind capacity in Europe is offshore[6]. Floating Offshore wind is considered by many to be the next chapter in wind energy[9], as potential new project sites are no longer limited to shallow water locations. Floating wind farms are still in the early phase when compared with bottom fixed wind offshore and particularly onshore wind, and there is still continuous learning for offshore wind. This is emphasized by the Levelized Cost Of Energy (LCOE) for offshore wind which in 2023 was at $\frac{80\$}{MWh}$ for fixed offshore wind, and $\frac{270\$}{MWh}$ for floating. However, this is estimated to reach levels of $\frac{51\$}{MWh}$ and $\frac{67\$}{MWh}$ for fixed and floating in 2050[10].

Hywind Tampen opened full production in 2023, marking the start of the biggest floating offshore wind farm. Hywind Tampen consists of 11 8 MW turbines, resulting in a total of 88 MW [11]. A topic that has gotten some attention the recent years is the effect of tilt on individual turbines in a wind farm. As a floating wind turbine is exposed to wind, it will tilt with the incoming wind speed. As a result, the wind turbine area facing the wind will be reduced due to this tilt angle and decrease the power output. Recent studies have researched how tilt can impact energy output on a wind farm, where a tilted upstream turbine may have a positive impact on downstream turbines, and possibly the whole wind farm. This also raises interest in what the optimal tilt angles for individual turbines are in a wind farm to maximize production.

1.2 Floating Offshore Wind Farm

Floating Offshore Wind Turbine (FOWT) has the advantage compared to onshore wind and bottom fixed wind in that it is further out of general sight, generally has fewer natural obstructions provides a more stable wind flow, and is not limited to shallow water increasing global implementation possibilities. FOWT is still in its early stages, especially cost being a current drawback. Since there are few large-scale wind farms operational today, there are certain aspects that will need further investigation in the times moving forward. One of these is rotor tilt.

1.2.1 Rotor tilt

Rotor tilt is the angle between the rotor and the horizontal axis. Some turbines have a standard rotor tilt of around 5°to counter a tower strike, which is when a turbine blade hits the tower. This can occur if the rotor is tilted too far forward. The rotor tilt along with the pitching of a platform will influence the effective rotor tilt [12]. A FOWT property is that the tower is not fixed at to ground or the seabed, and due to it being floating, the incoming wind pitches the platform and tilts the turbine in the wind speed direction resulting in a smaller cross-section area thus reducing power output. An illustration of tilt can be seen in 1.1, where a positive rotor tilt is shown when the rotor is tilting back, with the front being the direction of the nose cone. This illustration shows the effective rotor tilt, as the whole tower is tilting, which can be the case for FOWT.

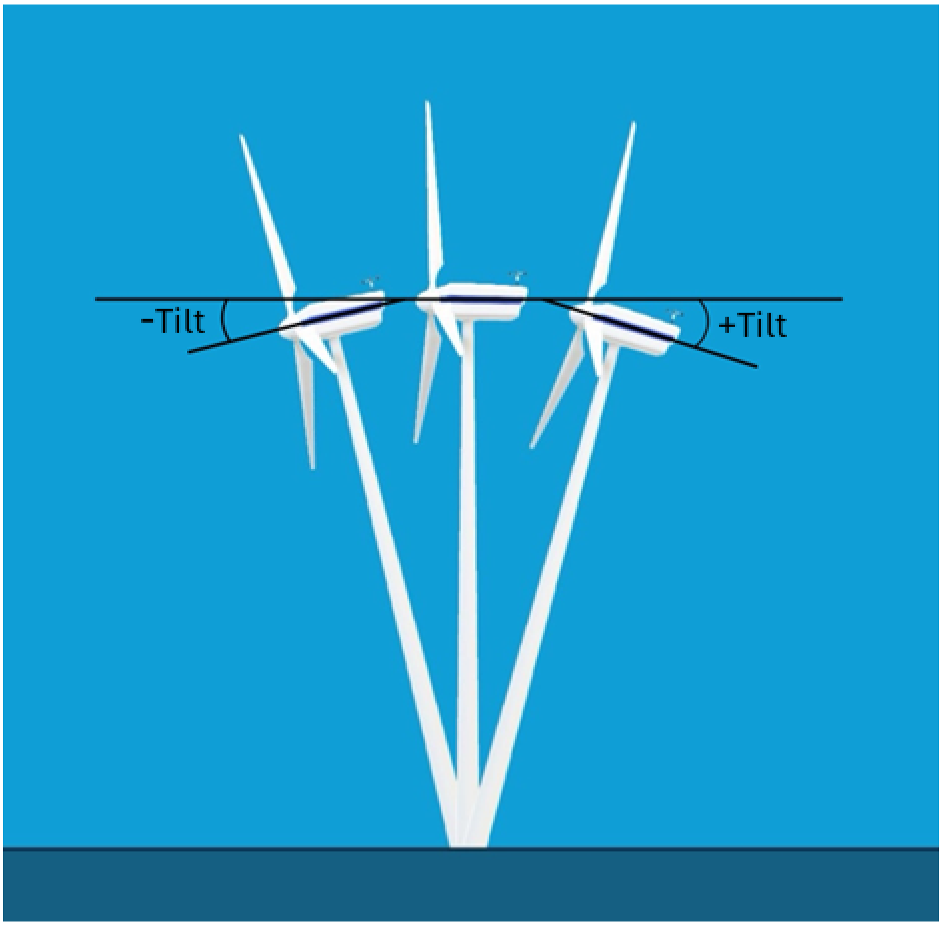

Figure 1.1: Illustration of effective rotor tilt, showing a positive tilt angle when the turbine is tilted back

1.3 Related research

There is an increasing interest in tilt within both academia and industry and there seems to be no decrease in interest with the mentioned increase of FOWT. Various areas including different simulation models and layouts have been researched, as well as wake steering using tilt.

Nanos et al.[12] tested ballast control for vertical wake deflection on two DTU 10 MW turbines with different platforms. They found that power gains reached up to 2.6% for two aligned turbines with distance between 10-12 diameter distance, and a 8% increase for the two turbines at 6-8 diameter distance. Due to a standard rotor uptilt of 5°, this corresponds to a -15°pitch on the platform.

Leikvoll[13] researched both power and AEP with various layouts, number of turbines, distances, and wake deficit models. She only looked at positive tilt angles using a 15 MW wind turbine. She found, using a Bastankhah Gaussian wake deficit model, a tilt angle of 10°and PyWake, both gains and losses for different power cases. She found a gain of 2.84% for a three-turbine array, with a loss of 2% for one front and two back, and two front and one back turbine layout.

Fleming et al.[14] researched various methods of wake mitigation control using SOWFA for two NREL-5 MW turbines, where one of the methods was the tilt. They used a 7-diameter distance between the turbines, and among other techniques calculated power at various tilts. At 30°tilt, they found an increase in power of 7%.

Annoni et al.[15] researched a broad spectrum of tilt angles, distances, and offsets of turbines to see how impactful tilt control could be for a wind farm using Sowfa. They simulated both 2 and 3 NREL-5 MW turbines and only changed the tilt of the upstream turbines.

Furthermore, they spaced the turbines with 7-diameter distances and found a power gain on three turbine arrays of 13% when both upstream turbines tilted 25°, and a gain of -3.6% when both were tilted -25°. A positive tilt is here considered a turbine tilting forward.
Cutler et al.[16] researched both a 2-turbine array, similar to Annoni et al., and an optimization of the Princess Amalia wind farm, a 60-turbine wind farm, with both active and fixed tilt, to maximize the Annual Energy Production (AEP). This optimization was done by optimizing individual turbines and fixing this tilt to each turbine, as well as an active tilt control. For both studies, the 2-turbine array and Princess Amalia wind farm, they used NREL-5 MW turbines. They showed promising results with increases of 2.77% and 13.64% increase in AEP for fixed and active tilt respectively. The tilt angles were optimized using SNOPT, a non-linear algorithm that uses sequential quadratic programming, with tilt boundaries at -30 and 30°. They also ran a two-turbine array simulation using a modified Bastankhah Gaussian wake deficit model, with the upstream turbine at an angle of 25°and -25°, which resulted in a power gain of around 6% and -3% at 0 spanwise offset.These percentages are compared to an upstream turbine of -5°.

1.4 Motivation

Since there are scenarios where a tilt angle on turbines can increase total power output for multiple wind turbines, such as a wind farm, it raises a few questions regarding which scenarios an increase in power can be expected. These scenarios include differences in wind speeds, wind direction, turbine types, and various rows of turbines in a wind farm with optimized tilts to maximize total power output. These scenarios will be investigated in this thesis, as well as two different wake deficit models, using PyWake.

1.5 Research Questions

From the previous sections, 4 questions can be raised, which this thesis aims to answer.

Research Question 1

Are there optimal tilt angles in a wind farm that can increase the power output, or will the optimal always be 0?

Research Question 2

At what wind speeds and distances between turbines are there optimal tilt angles?

Research Question 3

Will different turbine types, turbine sizes, wind directions, or additional rows provide the same optimal tilt angles?

Research Question 4

To what extent are the results dependent on different wake deficit models used?

Chapter 2

Theory

2.1 Wind Energy

Wind energy is the movement of air in the form of kinetic energy. Due to air being a fluid, it is possible to apply laws related to fluid mechanics when converting the kinetic energy in the wind to other forms of energy, for instance, mechanical energy in a wind turbine. When looking at wind energy, especially for turbines, it can quickly become complex to accurately calculate turbine-wind interactions. To make computations more manageable, a Blade Element Momentum method can be implemented for simplifications, which combines 1D momentum theory and blade element theory to locally calculate forces. 1D momentum theory is a simplified model that considers a rotor to be an ideal permeable disc, which means it is frictionless, and that is has no rotational velocity component in the wake. Since 1D momentum is considered an ideal disc, it makes it possible to derive relationships between the incoming velocity, axial velocities, wake velocity, thrust, and power from the shaft. Blade element theory consists of dividing a turbine blade into sections, calculating local forces on each section, and then they are integrated for the whole blade or rotor. For BEM to more accurately calculate these forces, two corrections, Prandtl and Glauert, need to be added [17].

2.1.1 Wind Power

Wind power is a general description of how much power is in the wind. This is a theoretical power output and does not account for any losses when converting energy, or take into account realistic maximum power output, which will later be described as Betz law. It does however give a base starting point for further power calculations and can be described as in Equation 2.1

$$P = \frac{1}{2} * \rho * A * v^3 \tag{2.1}$$

where ρ is the air density, A is the area used for power calculation, and v is the velocity. This shows very clearly how impactful the wind speed is when calculating wind power as it is cubed. Thus a slight increase in wind speed will result in a significant increase in power level [18].

2.1.2 Turbine Power

When going from potential wind power to actual turbine power, a fundamental law, the Betz law, must be considered. Betz law was established by Albert Betz in 1919 and defines the maximum power a turbine can output from the wind in an open-flow environment. Betz law gives a limit of $\frac{16}{27} \approx 59.3\%$, and it is limited by mass and momentum conservation [19]. If a turbine were to have 100% output efficiency, all kinetic energy would be transferred from the wind resulting in stationary air behind the turbine due to lack of velocity. This would further result in zero power output in the turbine, due to zero velocity in the wind. When calculating turbine power, a power coefficient is included in the wind power formula, resulting in the following Equation 2.2

$$P = \frac{1}{2} * \rho * C_P * \pi * r^2 * v^3 \tag{2.2}$$

where ρ is the density of air, C_P is the power coefficient, which includes the Betz limit and other turbine-specific variables [18], and r is the radius of the turbine, or turbine blade length. This is a slightly simplified equation and does not include generator or gearbox efficiency.

2.1.3 Thrust

Thrust force in a wind turbine is an axial force and is a product of a pressure drop over the rotor, due to wind and turbine interactions. From the thrust force, it is possible to calculate a thrust coefficient, which is a dimensionless value of how efficiently a wind turbine interacts with the incoming wind. Equation 2.3 shows how the thrust coefficient is calculated

$$C_t = \frac{T}{\frac{1}{2} * \rho * v^2 * A} \tag{2.3}$$

where T is the thrust force, ρ is the air density, v is the wind velocity, and A is the swept area of the wind turbine [17] [20].

2.2 Site Conditions

Due to the wind speeds major impact on power output for a wind turbine, great care must be taken when selecting the site where wind turbines should be placed. There are global, local, natural, and human-made components that influence the wind conditions at a site, where some include mountain ranges, valleys, and buildings. This careful consideration makes it easier to implement wind turbines in sites that suit the needs they are aimed at, either being more constant winds at a medium velocity, or more fluctuating wind velocity considering both wind speed and frequency.

2.2.1 Wind Rose

When considering a site a wind rose is a useful tool. It efficiently visualizes the wind at a site and can provide information regarding wind speed, frequency, and wind direction. As with most data analysis tools, a wind rose only gives results based on the quality of input data. Some ways to increase the quality of the result could be to use data from a reliable source and include a lot of data, ideally stretching over a few years. A wind rose can be quite useful when designing the layout of wind turbines at a site, and optimally placing the individual turbines according to the most prominent wind direction, referring to frequency and velocity.

2.2.2 Weibull Distribution

A typical way of mathematically distributing wind speed is by using a Weibull distribution, by applying the Weibull Probability Density Function (PDF) in Equation 2.4

$$f(u) = \frac{k}{A}\left(\frac{u}{A}\right)^{k-1} \exp\left[-\left(\frac{u}{A}\right)^k\right] \quad (k > 0, u > 0, A > 1). \tag{2.4}$$

Which consists of two site-specific parameters, k and A, which represent the shape and scale parameters. These can be estimated by using curve fitting methods, but since the Weibull PDF is non-linear, it requires non-linear algorithms to estimate the Weibull k and A values. From these algorithms, there have been some simplifications, which make the calculation less complex, and make it possible to calculate both Weibull k and A values from the two Equations 2.5 and 2.6

$$k = \left(\frac{\sigma}{\bar{u}}\right)^{-1.086} \tag{2.5}$$

$$A = \frac{\bar{u}}{\Gamma\left(1 + \frac{1}{k}\right)} \tag{2.6}$$

where σ is standard deviation, $\bar{u}$ is mean wind speed, and Γ is a mathematical function that generalizes a factorial function to complex numbers, which can be expressed shown in Equation 2.7 [21]

$$\Gamma(y) = \int_0^{\infty} e^{-x} x^{y-1}\, dx \tag{2.7}$$

2.3 Wake

Wake is the resulting turbulent air after it interacts with a structure. This could for instance be air behind a wind turbine, where a deficit occurs in the resulting downstream wind speed because of the blade turbine interacting with the air. Research and experience have provided a general rule spacing turbine with a distance of around 6-8 diameters in a wind farm for the wind to reduce the loss of power due to wake [22][23]. Wake is still a complex phenomenon, and there are several methods to calculate wake, which differ in complexity and usability. Today, various models calculate wake deficits, two of which are the Jensen and Bastankhah Gaussian models.

2.3.1 Niels Otto Jensen

Niels Otto Jensen (NOJ) model, or simply Jensen model, is a top-hat wake deficit model, which is indicated by a top-hat shape on the wake in a downstream deficit profile, which can be seen in Figure 2.1. It gets a top-hat shape due to the constant velocity profile across the wake and has a clear boundary between the wake and free flow. A wake model of Jensen can be seen in Figure 2.2 which shows the constant deficit values along the crosswind distance at a given downwind distance. The wake velocity is calculated using Equation 2.8

$$V = U * (1 - 2a(\frac{r_o}{(r_o + \alpha x)})^2) \tag{2.8}$$

where V is wake velocity, U is the incoming velocity, a is the axial induction factor, r_o is the radius of the wake, x is the downstream distance, and α is an entrainment constant, also known as wake decay constraint, with typically a value of 0.1 [24].

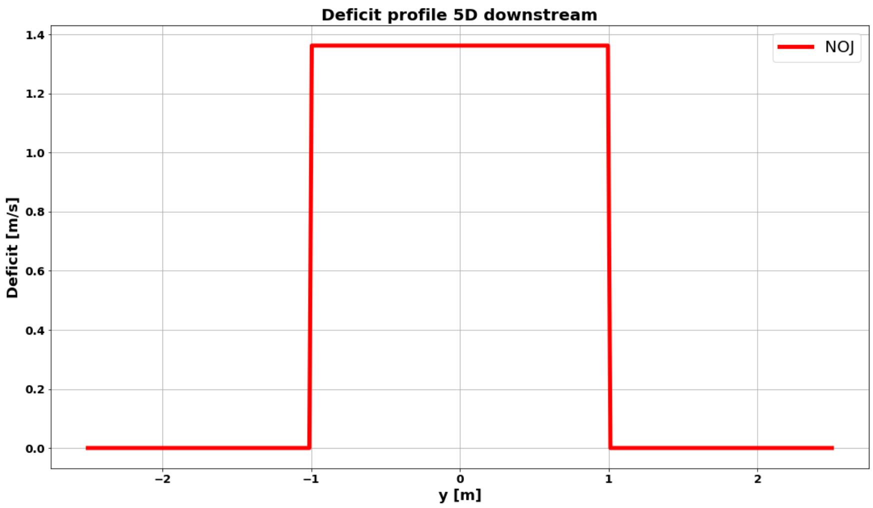

Figure 2.1: Wake deficit profile of NOJ 5D downstream, with the top-hat shape

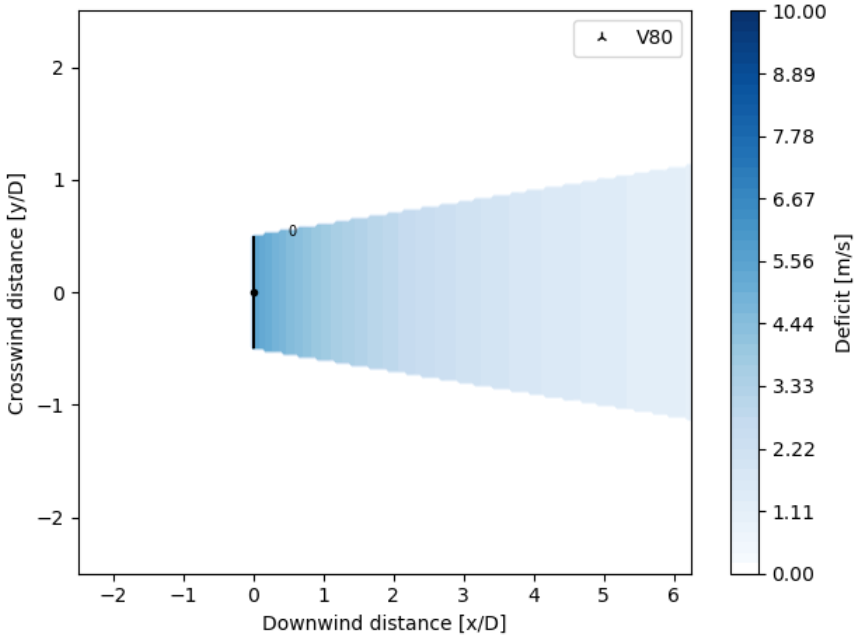

Figure 2.2: Example of a NOJ wake deficit shape from DTU[25]

2.3.2 Bastankhah Gaussian

Bastankhah Gaussian (BG) is an analytical model which uses a Gaussian distribution for the wake deficit and derives from the conservation of mass and momentum and is generally expressed as found Equation 2.9

$$\frac{\Delta U}{U_\infty} = C(x) e^{-\frac{x^2}{2\sigma^2}} \tag{2.9}$$

Where $C(x)$ is the max velocity deficit at the center of wake, and is found using Equation 2.10

$$C(x) = 1 - \sqrt{1 - \frac{C_T}{8 * (\frac{\sigma}{d_0}^2}} \tag{2.10}$$

Where the width of the wake, σ, is found using Equation 2.11

$$\frac{\sigma}{d_0} = k * \frac{x}{d_0} + \epsilon \tag{2.11}$$

where k is a wake expansion parameter, ϵ is a constant equal to 0.2β, where β is a function of the C_T of a given turbine. As a result, the velocity deficit can be found using Equation 2.12

$$\frac{\Delta U}{U_\infty} = \left(1 - \sqrt{1 - \frac{C_T}{8 * (\frac{k*x}{d_0} + \epsilon)^2}}\right) * exp\left(-\frac{1}{2(\frac{k*x}{d_0} + \epsilon^2}\left\{\left(\frac{z - z_h}{d_0}\right)^2 + \left(\frac{y}{d_0}\right)^2\right\}\right) \tag{2.12}$$

where C_T is the thrust coefficient of a wind turbine, x is the distance to the calculation point downstream, z_H is the hub height of a turbine, y is the calculation point on the crosswind axis, and z is the height of the calculation point [26].
Figure 2.3 shows the characteristic Gaussian distribution 5D downstream, and Figure 2.4 shows the wake deficit model. and

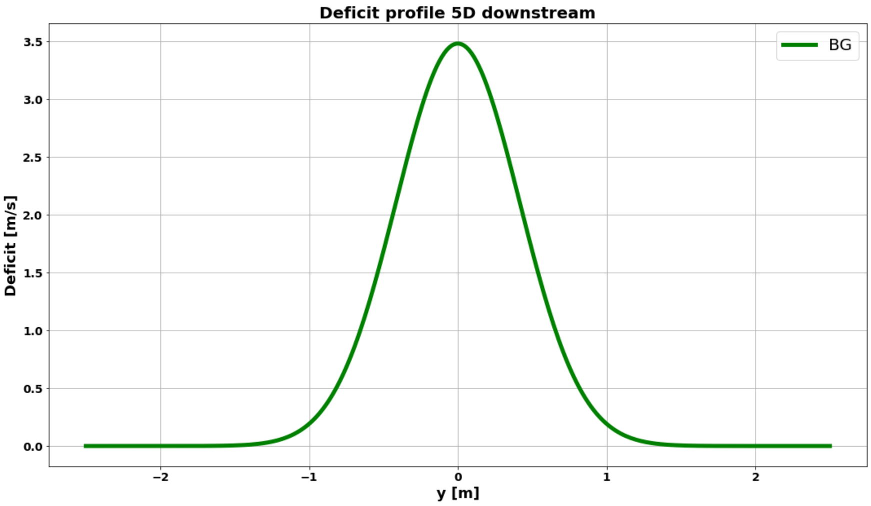

Figure 2.3: Wake deficit profile of BG 5D downstream, with the Gaussian shape

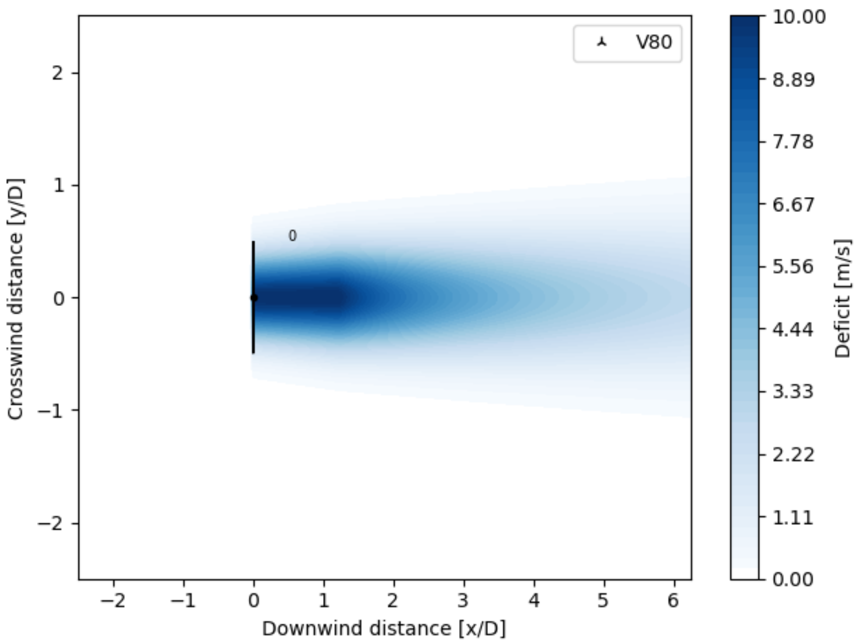

Figure 2.4: Example of a BG wake deficit shape from DTU [25]

2.4 Optimization

Optimization is a method used to find the global or local optimum for an objective function, typically by using an algorithm to find an optimal solution by changing some variables. This can be both maximum or minimum values, depending on the objective function, such as maximum values for power output and minimum values for cost. There are multiple algorithms capable of solving an optimization problem, among other simplicial homology global optimization (shgo), Dual annealing (DA), and Differential Evolution (DE).

2.4.1 shgo

Shgo is a relatively new algorithm, first published in 2018, that is based on topographical global optimization (TGO). TGO is an iterative algorithm that uses clustering to describe the hypersurface of an objective function and sets it in a topography matrix. Shgo creates a simplicial complex that covers the search area before applying homology to assess the topology of the simplicial complex. Furthermore, it uses the information from the homology as a foundation to globally narrow the search areas before using local optimization of the narrowed areas, before updating the simplicial complex. This process is repeated until converges to a global maximum. Shgo can also have different sampling methods such as simple and Sobol. Simple sampling is based on simple random sampling, resulting in each point has an equal probability of being selected, while Sobol sampling generates the points using Sobol sequence, which is a low discrepancy sequence that more efficiently covers the full search area [27] [28].

2.4.2 DA

Dual annealing (DA) is a stochastic approach derived from Xian et al.[29] where they extended Simulated Annealing (SA), on the Thomson model. It puts together a generalization of Classical Simulated Annealing (CSA) and Fast Simulated Annealing (FSA) to combine both local and global optimization techniques. Inspired by the annealing process from metallurgy, where a material is heated and slowly cooled under controlled conditions to change the physical properties of the material. DA uses a modified version of the Cauchy-Lorentz distribution for the shape controller parameter, which is found using Equation 2.13

$$g_{q_v}(\Delta x(t)) \propto \frac{T_{q_v}(t)^{\frac{D}{3-q_v}}}{\left[1+(q_v-1)\frac{(\Delta x(t))^2}{T_{q_v}(t)^{\frac{2}{3-q_v}}}\right]^{\frac{1}{q_v-1}+\frac{D-1}{2}}} \tag{2.13}$$

where g_{q_v} is a probability density function, q_v is a shape-defining parameter, t is the artificial time, $\Delta x(t)$ us a trial jump distance of a variable $x(t)$, $T_{q_v}(t)$ an artificial temperature, and D is the dimension of variable space. This is used for the initial condition, and after applying the distribution function, the acceptance probability is found using Equation 2.14

$$p_{q_a} = \min\left\{1, [1-(1-q_a)\,\beta\Delta E]^{\frac{1}{1-q_a}}\right\} \tag{2.14}$$

where p_{q_a} is an acceptance probability parameter, β is a control parameter that controls the possibility of accepting a worse solution throughout the optimization process, and ΔE is the difference between the current solution and the new solution. Furthermore, the artificial temperature is reduced by using Equation 2.15

$$T_{q_v}(t) = T_{q_v}(1)\frac{2^{q_v-1}-1}{(1+t)^{q_v-1}-1} \tag{2.15}$$

By putting these global and local optimization methods together, it can efficiently optimize and escape local minima [30].

2.4.3 Differential Evolution

Differential Evolution (DE) was developed by Storn and Price in 1995 and is based on using evolutionary strategies to find improved results through iterations. DE is widely used today, mostly due to it being user-friendly with a self-organizing nature. It is also stochastic, can search a wide range of candidate areas, and does not use gradient methods. Initially, DE takes a random population, set within the boundaries, before it runs it through three operations, Mutation, Crossover, and selection. Storn and Price define DE using 2.16

$$x_{i,G}, i = 1, 2, ..., NP \tag{2.16}$$

where $x_{i,G}$ is the ith individual in a population, where G in number of generations, i is the individual index within the population, and NP is the size of the population [31] [32].

Mutation

To provide some variation in the iterations, and avoid local max or min values, a mutation provides a broader search area by continuously mutating a given size of the population. Storn and Price defined the mutation by generating a mutant vector for each target vector by using Equation 2.17

$$v_{i,G+1} = x_{r_1,G} + F * (x_{r_2,G} - x_{r_3,G}) \tag{2.17}$$

where $v_{r_1,G+1}$ is the mutated vector, r_1, r_2, r_3 are integers, mutually different, and random indexes ranging from 1,2,...,NP, F is a factor with a positive value, and can be seen as the mutation rate, as it controls the impact of the difference between the two integers[31].

Crossover

To further diversify the results, a crossover between parent vectors is implemented, and a trial vector is formed by Equation 2.18

$$u_{i,G+1} = (u_{1i,G+1}, u_{2i,G+1}, ..., u_{Di,G+1} \tag{2.18}$$

where the jth trial vector is defined as shown in Equation 2.19

$$u_{j,i,G+1} = \begin{cases} v_{j,i,G+1} & \text{if } (\text{randb}(j) \leq CR) \text{ or } j = \text{rnbr}(i), \\ x_{j,i,G} & \text{if } (\text{randb}(j) > CR) \text{ and } j \neq \text{rnbr}(i), \end{cases} \tag{2.19}$$

$$j = 1, 2, \ldots, D.$$

where $randb(j)$ is the jth evaluation of a random number generator with outcome $\epsilon[0, 1]$, CR is a crossover constant within $\epsilon[0, 1]$, $rnbr(i)$ is an index which is randomly selected within a range of 1, 2, ... , D, where D is the number of parameters. This ensures that the trial vector gets at least one parameter from the mutation parameter [31].

Selection

Lastly, a selection process decides how the trial vector compares to the target vector. If the trial vector provides a more optimal solution, the target vector, $x_{i,G}$ is set to the trial vector $u_{i,G+1}$. Else, $x_{i,G}$ is kept as the target vector [31].

Chapter 3

Method

3.1 Overview of simulations

Table 3.1 provides an overview of the different simulations showing what the different simulation variables were set to during each simulation. Figure 3.1 shows a flowchart of the initial simulation, and how the steps are connected. It shows how the different wake models are optimized, and compared to 0 tilt values, different optimization methods, and finally the two different wake models. Furthermore, all simulation codes can be found in Appendix A.

Table 3.1: Overview of the different simulation cases

Simulation	Wd [°]	Wake Model	Optimizer	Turbine model	P_{Turb} [MW]
Initial 180	180	NOJ and BG	DE, DA and shgo	DTU	10
Initial 190	190	NOJ and BG	DE, DA and shgo	DTU	10
Different Turbine	180	BG	DE	IEA	10
Different Size Turbine	180	BG	DE	IEA	15
Varying Wd	200, 210, 220, 270	BG	DE	DTU	10
3 Rows	180	BG	DE	DTU	10
4 Rows	180	BG	DE	DTU	10

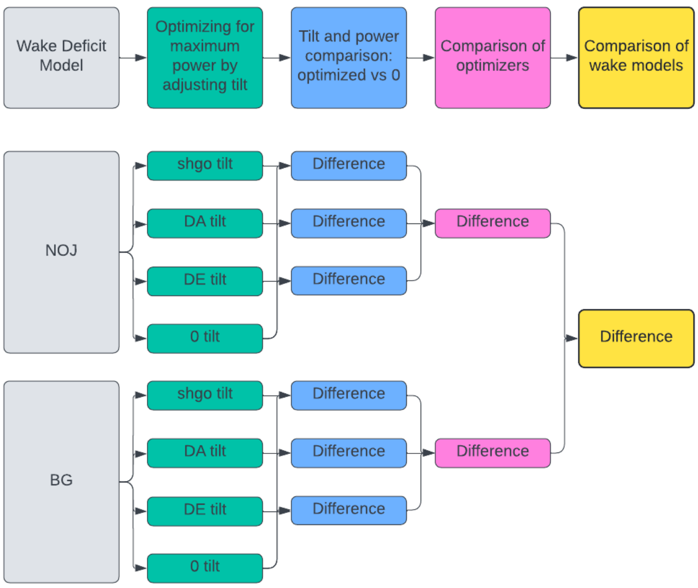

Figure 3.1: Flowchart of the process comparing optimizers and wake deficit models

3.2 Data Gathering and processing

Firstly, site data were gathered to implement a site component in PyWake. As Hywind Tampen is located between Gullfaks and Snorre, it was decided to use wind data from Gullfaks using historical weather data from Open-Meteo, which is an open-source weather Application Programming Interface (API). Four years of wind speed and wind direction at a height of 100m from Gullfaks from 2020 to 2023 were exported. The wind direction was divided into 12 sectors, and the frequency of each sector was calculated. Furthermore, both standard deviation and mean values of the wind speed were calculated, which were applied to the calculation of the Weibull A and k values for the Weibull distribution. These wind data were also used to generate the wind rose in Figure 3.2.

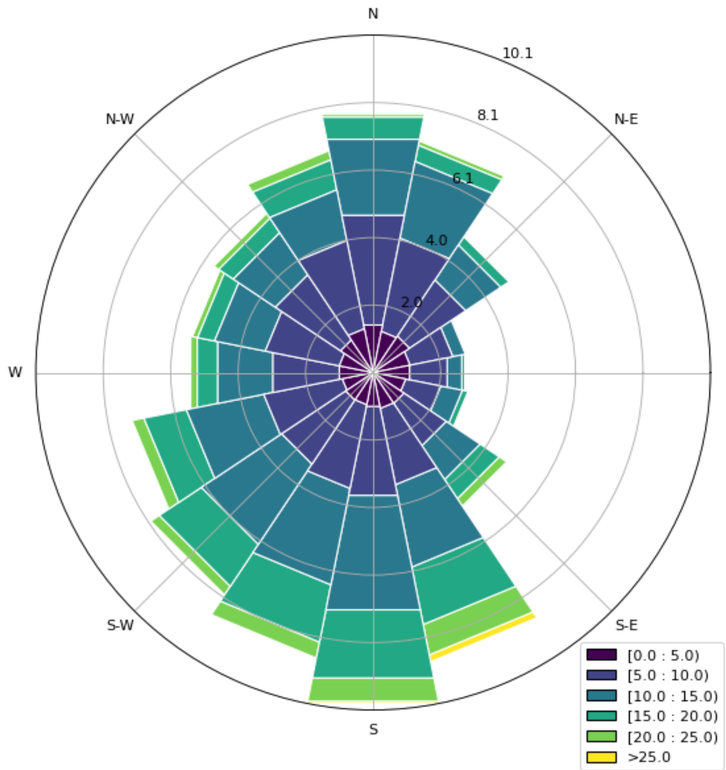

Figure 3.2: Wind Rose from Gullfaks, 2020-2023

3.2.1 Wind Farm Layout

This thesis uses a wind farm layout inspired by Hywind Tampen, and Figure 3.3 shows the actual turbine positions in Hywind Tampen using Automatic Identification System (AIS) data from MarineTraffic. The wind farm consists of 11 turbines in two rows. The layout is defined using xD, where x is a given distance and D is the turbine diameter, for defining the space between each turbine in a row and between the neighboring turbine row. The second row also has turbines separated by the same xD values, but these are offset by a half distance in the y-direction compared to the first row, resulting in a zig-zag pattern between the rows. Even though the relation between the x and y directional space between the turbines is constant, they are both adjusted according to a set distance and a turbine rotor diameter. Table 3.2 shows how the layout is implemented in the code, and how the layout is dynamic with the changing distance and diameter. The positions are negative to match turbine 1 in the simulations with turbine 1 from Hywind Tampen. Accordingly, row numbering starts from the rights, so row 1 includes turbines 1-6. Figure 3.4 shows a layout with 7D distance and the NOJ wake deficit model.

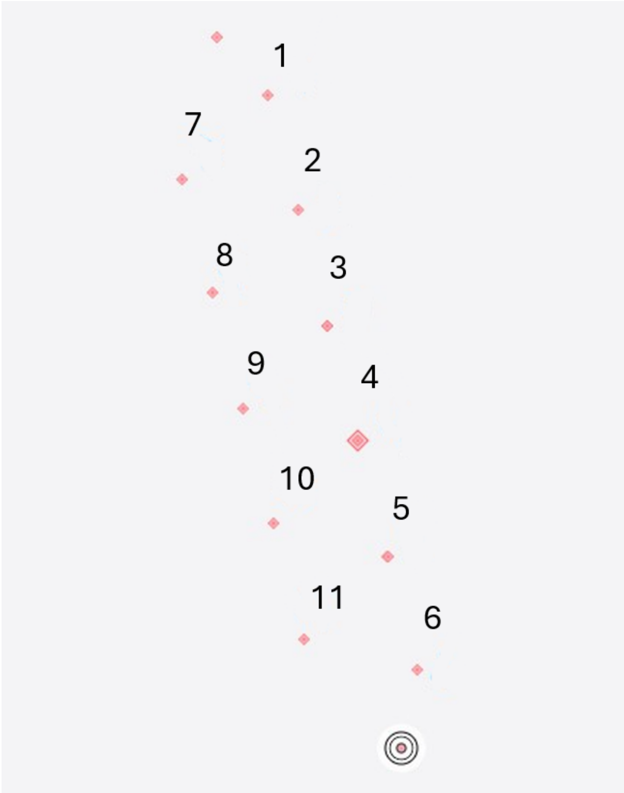

Figure 3.3: Layout of Hywind Tampen from MarineTraffic [33]

Table 3.2: Turbine Layout showing how the turbine positions are defined by a set distance and turbine diameter

Turbine	Position [x,y]
1	[0,0]
2	[0,-distance*diameter]
3	[0,-2*distance*diameter]
4	[0,-3*distance*diameter]
5	[0,-4*distance*diameter]
6	[0,-5*distance*diameter]
7	[-distance*diameter,-0.5*distance*diameter]
8	[-distance*diameter,-1.5*distance*diameter]
9	[-distance*diameter,-2.5*distance*diameter]
10	[-distance*diameter,-3.5*distance*diameter]
11	[-distance*diameter,-4.5*distance*diameter]

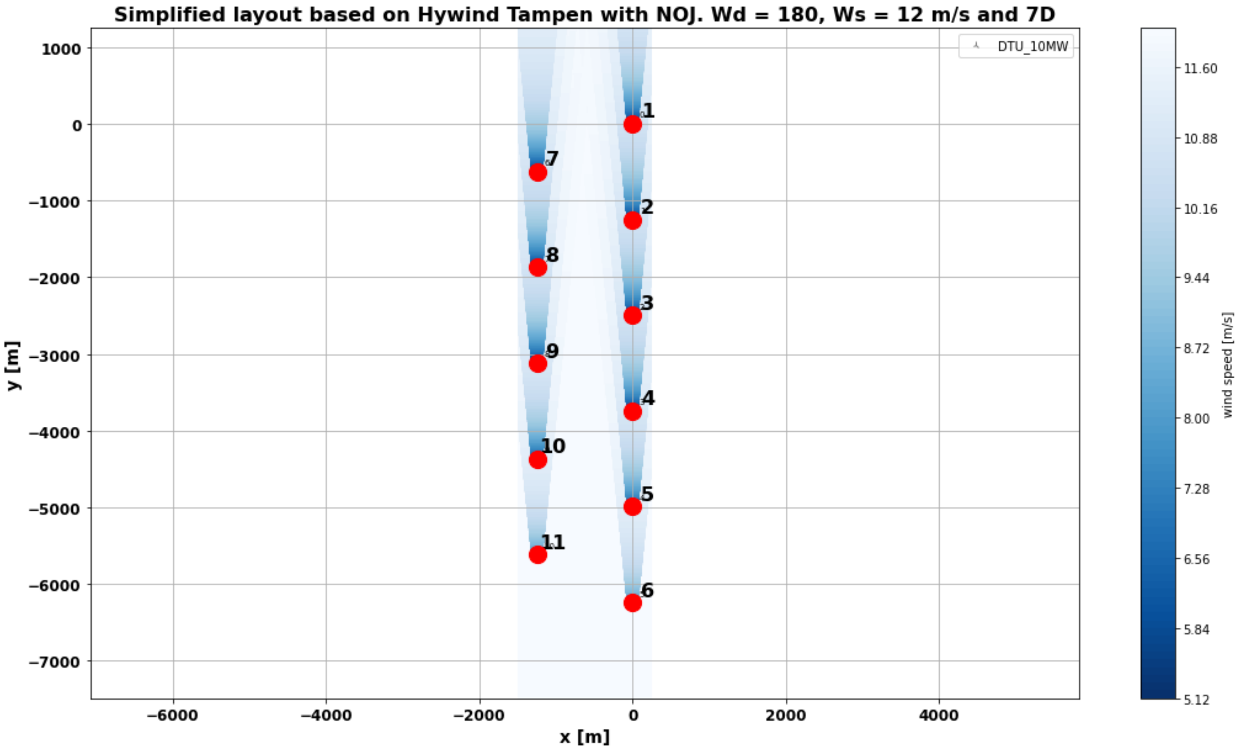

Figure 3.4: Layout of wind farm model used for simulations, with NOJ wake model, 7D distance between turbines, 180°wind direction, and 12 m/s wind speed using PyWake.

The wind rose from Gullfaks in Figure 3.2 indicates the most prominent wind direction is from the south. To give a clear representation of how the individual tilts are affecting the power output, a 'worst-case' layout was used to maximize the wake deficit. This was done by adjusting the original layout of Hywind Tampen slightly, creating two straight vertical rows of turbines aligned with the most prominent wind direction of 180°.

3.3 Simulation tool - PyWake

PyWake is a Python-based, open-source wind farm simulation package that considers wake effects when calculating power output, annual Energy Production (AEP), or visualizing wake flow for single turbines or wind farms. PyWake was developed at the Technical University of Denmark (DTU) and is a versatile tool due to the numerous components in its library in combination with the ability to create specific components easily. Some components include; site conditions, wind turbine, wake deficit models, and turbulence intensity. Information and guide for PyWake and parameters were provided by DTU [25].

Figure 3.5 shows the structure of how PyWake uses the main components to calculate the wind farm model. It shows how the initial values are implemented in the wind farm model before going through the Site conditions and returning new local values to the wind farm model.

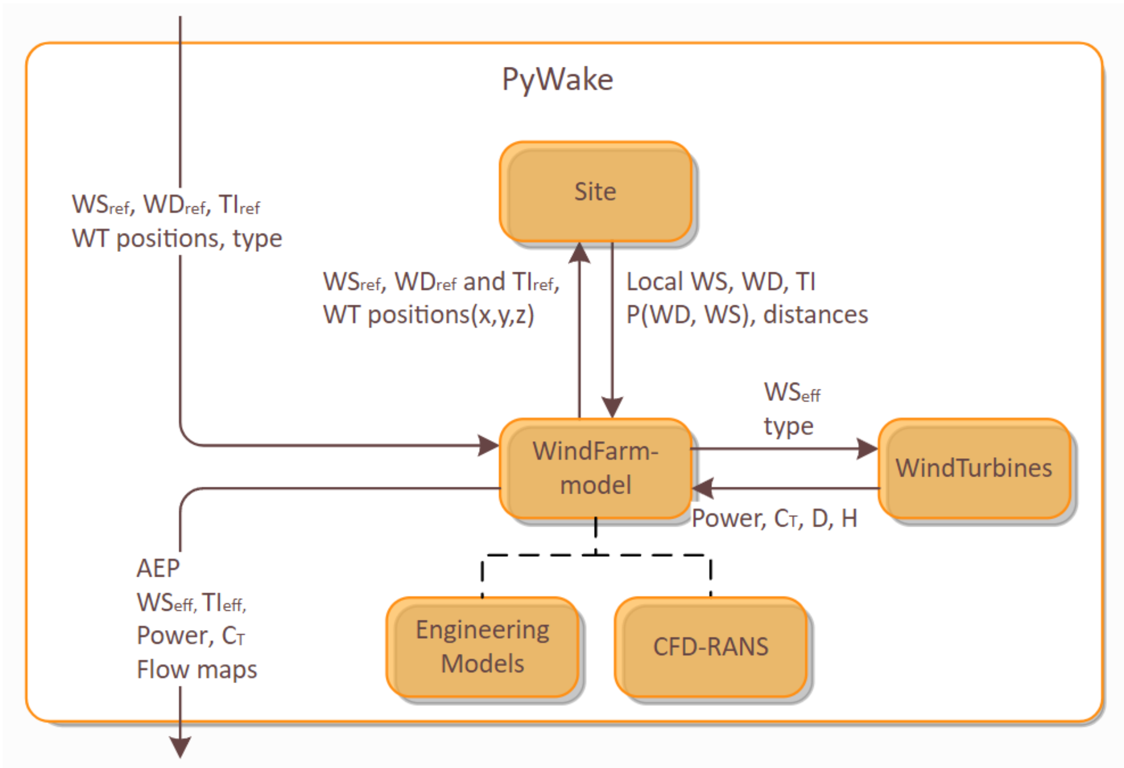

Figure 3.5: Architecture of PyWake from DTU PyWake page[25]

Site Object

A XRsite is used for these simulations as it is the most general and flexible. XRsite requires Weibull A and k values as well as sector frequency as inputs. Additionally, optional parameters can be adjusted such as reference wind speed, reference wind direction, and turbulence intensity. After these inputs have been implemented into the site, it provides the wind farm model with local wind conditions as well as calculated downwind, crosswind, and vertical distance between turbines.

Wind Turbine Object

After the site object supplies the wind farm model with the new data, values from the wind farm model are then inputted into the Wind Turbines object, which provides the wind farm model with turbine characteristics such as the C_t curve, power curve, hub height, and rotor diameter. For this thesis a custom turbine will be defined, using turbine characteristics which will be defined later. From the Wind Turbines object, these turbine characteristics are outputted back into the wind farm model.

Wake Deficit Models

To further use the variables from the wind farm model, the wind farm data can be processed through engineering models, which is where the wake deficit models are implemented. Both NOJ and BG were imported and implemented for various simulations, and both these required Site and Wind Turbine components, which have been defined earlier as *site* and *custom_turbine*. These wake deficit models provide the wind farm model with wake deficits, effectively providing a new effective wind speed for each turbine.

After all objects and models had been implemented, it was possible to define a wind farm simulation, shown in Figure 3.6. As can be seen in this definition, the wind farm simulation requires both an input tilt and wind speed. Moreover, *sim_res* provides a calculation based on the variables, resulting in individual power for all the turbines in MW.

```
def wind_farm_simulation(tilts, wind_speed):
    sim_res = wake(location_x, location_y, tilt=np.array(tilts).reshape((num_turbines, 1)), wd=wind_direction, ws=wind_speed)
    individual_power_output = sim_res.Power.values
    flattened_output = individual_power_output.flatten()
    return flattened_output / (10**6)
```

Figure 3.6: Definition of wind farm simulation in Python

3.4 Optimization

Initially, an in-house optimization code was provided, alternated, and tested to fit this thesis. Using a Genetic Algorithm (GA) within a loop to find the optimal objective function, which is maximum power for the wind farm, by changing the tilts of each turbine within a boundary of minimum and maximum tilt. After multiple attempts, this GA did not converge to a stable result, indicating either a multi-solution problem, local maximums close to the global maximum, or not enough computation power to effectively provide a sufficient result. As each of the iterations in the optimizer had to run through the PyWake simulation before reevaluating the result, making changes to the best results by mutating, evolving, and introducing random immigrants, it was eventually deemed not sufficient enough to use for this optimization.

Instead of continuing the use of the in-house optimization *Scipy.optimize* was applied. This was chosen as *Scipy.optimize* provides already established optimizers, which can call separate optimizers more efficiently. This drastically reduced the computation time from multiple hours for each run with the previous code, to well under an hour for each simulation. This allowed running multiple optimizers through more numerous cases which increased the number of results.

3.4.1 Objective Function

A key function for an optimization algorithm is the objective function, which defines the purpose of the optimization. For this thesis, the objective function is set to maximize total power output by changing the tilt angles for each turbine individually. For practical reasons, this has been separated into two functions, $obj\_function(tilt\_angles)$ and $wrapped\_obj\_function$ which can be seen in Figures 3.7 and 3.8. The $obj\_function$ calculates the total power output and keeps it negative for optimizing, due to these algorithms optimizing for minimum value, whereas the $wrapped\_obj\_function$ tracks the positive values to a list. They are separated to indicate functions called later.

```
def obj_function(tilt_angles):
    total_power_output = wind_farm_simulation(tilt_angles, wind_speed)
    return -np.sum(total_power_output)
```

Figure 3.7: Objective Function to optimize tilts for maximum power output as defined in the code

```
def wrapped_obj_function(x):
    result = obj_function(x)
    obj_function_values.append(result*(-1))
    return result
```

Figure 3.8: Wrappped Objective Function as defined in the code

3.4.2 Optimization Algorithm

Before implementing the algorithms, some boundaries have to be set to limit the optimizers. These are set as *bounds* and are defined by a minimum and maximum tilt. Furthermore, the optimizers are in two loops, the outer loop consisting of wind speeds among a wind speed list, and the inner loop is for the selection of optimizer. The optimizers have the same inputs, and the function for DE can be seen in Figure 3.9, where both the $obj\_function$ and *bounds* are predefined. The optimization is set within the $optimize.differential\_evolution$ function, but keeps recalling the $obj\_function$, and also then the $wind\_farm\_simulation$ to provide the maximum power for each generation of tilts. In other words, the wind farm model is run for each iteration to provide new power outputs based on the tilts from the optimization. When the optimization has been completed, the maximum power value for the wind farm, along with the individual turbine power and corresponding tilts are stored. The values are stored in a data frame, and exported to a .csv file for further processing.

```
if optimizer == 'DE':
    result = optimize.differential_evolution(obj_function, bounds)
```

Figure 3.9: DE function within the algorithm

3.5 PyWake verification

To verify that PyWake could provide tilt angles, a visual verification was done, and shown in Figure 3.10, which shows each turbine having an individual tilt, with the incoming wind speed coming from negative y-axis. This figure shows a side-view of the turbine layout, which is why the axes are y and z. The wake deficit was also tested to see if PyWake implements an effective wind speed for each turbine, which would affect the power output for the individual turbines. This can be seen in Figure 3.11 which shows the power distribution for the individual wind turbines for the wind farm at a 5D distance between turbines, wind speed of 7 $\frac{m}{s}$ and 0 tilt. This figure shows that PyWake successfully implements wake effects, as the power output of the upstream turbines, turbines 6 and 11, is significantly higher than the remaining downstream turbines.

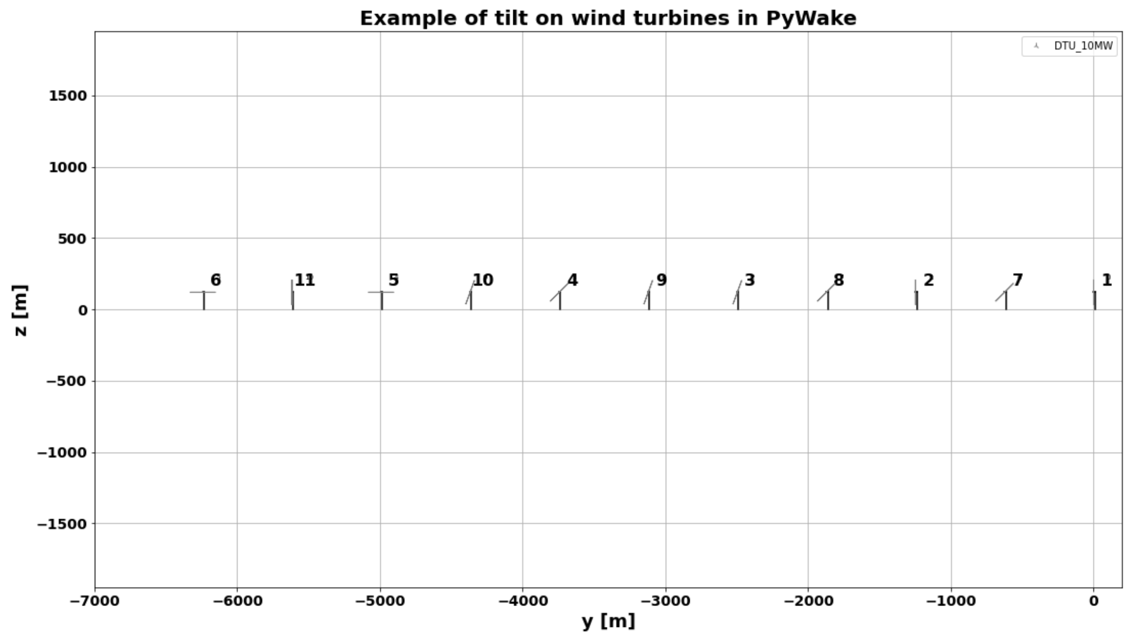

Figure 3.10: Example of tilts in PyWake

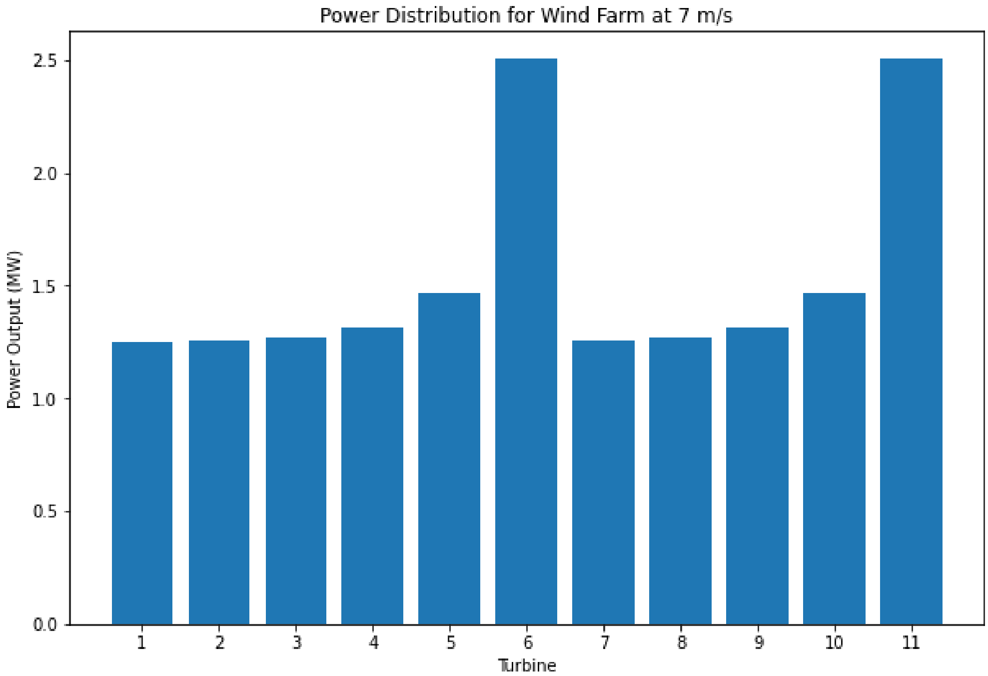

Figure 3.11: Power distribution for the wind farm with ws 7 $\frac{m}{s}$, 5D distance and 0 tilt.

3.6 Initial simulations

The initial simulation sets the base for the remaining thesis and will be used for comparison to other results within the thesis, and also with other researchers' relevant work. Parameters that are defined in the initial simulation will be similar for all simulations unless stated otherwise in their respective section. All simulations ran with a corresponding 0 tilt simulation which serves as benchmark values, and codes for initial simulation, for both wind directions and 0 tilts can be found in Appendix A.2 - A.4.

3.6.1 Site object - initial

To quantify the results, distances ranging from 1D - 10D, 15D, and 20D were implemented to vary the layout of the wind farm. Furthermore, wind speeds of 1, 3, 4 - 13, 20, and 25 $\frac{m}{s}$ were used for each distance This covers a broad area of wind speeds within the turbine wind speed range, but also below cut in and at cut out velocity to understand if Pywake manages these wind speeds. The simulations where performed for both Wind directions of both 180°and 190°. Moreover, a standard turbulence intensity of 0.1 was implemented.

3.6.2 Wind Turbine Object - initial

The DTU 10 MW 178m Reference Wind Turbine (RWT) turbines were used for the initial simulation, and data containing key turbine information were implemented in Table 3.3 [34]. Furthermore, the C_T and power curve for all turbines used can be seen in Appendix B. These turbines are slightly larger than the Siemens Gamesa SG 8.0-167 DD used in Hywind Tampen. These turbines from DTU were selected mainly for two reasons. First, due to the lack of turbine data for the Siemens Gamesa turbines used in Hywind Tampen, and second to take into consideration the increase in turbine size which is expected to continue to increase.

Table 3.3: Turbine properties DTU 10 MW 178 RWT v1 from [34]

Property	Value
Rated power [MW]	10
Rotor diameter [m]	178.3
Hub Height [m]	119
Cut in, rated, cut out Ws [$\frac{m}{s}$]	4, 11.4, 25

3.6.3 Wake Deficit Models - initial

For comparison reasons, both NOJ and BG were implemented, and the full range of distances, wind speeds, and wind directions were simulated using both wake deficit models. As these wake deficit models are calculated differently, they are both used to compare the differences between them.

3.6.4 Optimization - initial

For the initial simulation, all three optimizers were included and the final tilt, individual power, and total power results for all three optimizers were stored for comparison. The tilt boundaries, the minimum and maximum, were set to -30°and 30°for the regular simulation, and -0.01°and 0.01°for the benchmark simulations. The benchmark simulation bounds could not be set to 0°and 0°, as the optimization would not run with both being 0.

3.6.5 Verification

A verification simulation was performed to verify the results from the optimizers. This was done to ensure that both the optimizers and PyWake ran consistently and gave similar results. This verification was only done using BG at a reduced number of distances and wind speeds. This was considered sufficient as it is only a verification.

3.7 Comperative simulations

To further investigate and verify the results from the initial simulation, various alternative simulations were performed. These include the same size turbine from a different manufacturer, increased size of the turbine, various wind directions, and increasing number of turbines by adding more rows. When continuing these alternative simulations, a few simplifications from the initial simulations were made. As with the verification simulations, only the BG wake model was considered, along with only optimizing using DE. This could be done as the results provided by all optimizers were close to identical, and it was done to reduce simulation time. These alternative simulations also used fewer distances and wind speeds, with distances ranging from 4D to 7D and wind speeds from 4-13 $\frac{m}{s}$.

3.7.1 Simulation with different turbines

As PyWake considers the C_T and power curve of a turbine, a different type of turbine with the same rated power was to be simulated to see if there is any difference in the tilt and power. It was decided to compare results with the IEA 10MW 198 RWT from the National Renewable Energy Laboratory (NREL), as this turbine represents an updated version of the DTU 10 MW used in the initial simulations. After the simulation, the values were extracted to an Excel spreadsheet, and the difference in tilt between the initial simulation results and the new turbine results was calculated. Furthermore, updated turbine characteristics, including updated C_T, power curve values, and diameter were implemented in the code to get the correct input. These turbine characteristics can be seen in Table 3.4 which are given from NREL [35]. The code can be seen in Appendix A.5.

Table 3.4: Turbine properties IEA 10 MW 198 RWT from [35]

Property	**Value**
Rated power [MW]	10
Rotor diameter [m]	198
Hub Height [m]	119
Cut in, rated, cut out Ws [$\frac{m}{s}$]	4, 11, 25

3.7.2 Simulation of turbine with increase power rating

In addition to different turbine models, a turbine with increased power was tested in the form of the IEA 15MW 240 RWT from NREL. Simulating using a turbine with increased power was done to see how the individual tilts of a wind farm consisting of larger wind turbines behave compared to the turbines with smaller rated power. As the distance between turbines within each simulation is related to the diameter of the turbines, the relative distances between individual turbines are the same for both simulation cases. Updated turbine characteristics, taken from NREL [36], for the IEA 15 MW turbine was implemented in the code and can be seen in Table 3.5. The code can be seen in Appendix A.6.

Table 3.5: Turbine properties IEA 15 MW 240 RWT from [36]

Property	**Value**
Rated power [MW]	15
Rotor diameter [m]	240
Hub Height [m]	150
Cut in, rated, cut out Ws [$\frac{m}{s}$]	3, 10.6, 25

3.7.3 Simulation with various wind directions

Several wind directions are considered to simulate the effect at varying wind directions. The initial simulation consisted of wind incoming from 180°, which from the wind farm layout can be considered the least favorable wind direction. Additionally, a wind direction from 190°is considered. Furthermore, for a more realistic overview of how the tilt can affect the overall power output, various wind directions were tested. For this purpose, wind directions of 200°, 210°, 220°, and 270°were also simulated. The code can be seen in Appendix A.7.

3.7.4 Simulations with larger wind farm

Lastly, two simulations were done with an increased number of rows. This was done to see if an increase in rows influence the wake interactions between the rows to affect power output and tilts.

Simulation with one extra row of turbines

A new row identical to the first row was added, consisting of six turbines, next to the second row, negative on the x-axis. This resulted in a total turbine count of 17 turbines. Figure 3.12 shows the layout for 3 rows of turbines in PyWake with the BG wake deficit model. This wind farm uses the same indexing as earlier simulations, with the new row adding in turbines 12-17, where 12 is the furthest downstream turbine. The turbine layout can be seen in the code in Appendix A.8.

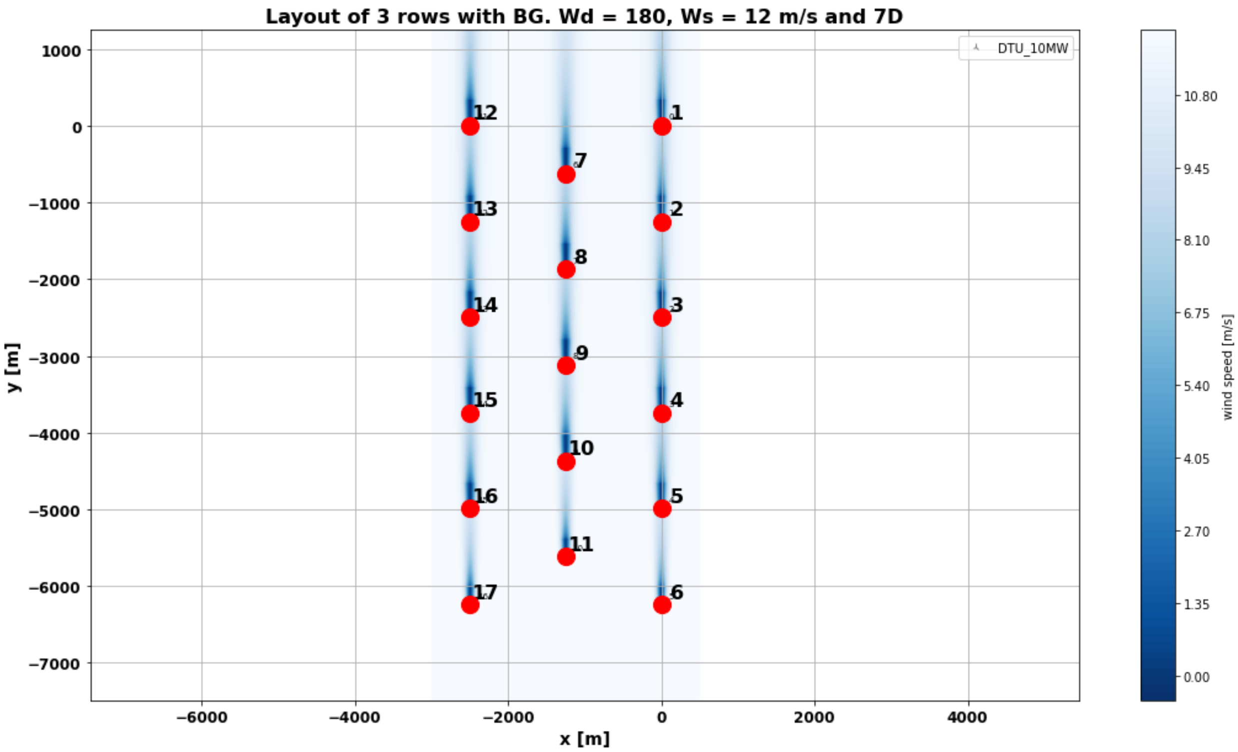

Figure 3.12: Layout of wind farm with 3 rows

Simulation with two extra rows of turbines

A final simulation was done with 4 rows. For this layout, a row identical to the second row (consisting of 5 turbines) was added next to the new third row from '3 Rows' scenario (negative on the X-axis). This resulted in a total turbine count of 22 turbines. Figure 3.13 shows the layout for the wind farm with 4 rows in PyWake using the BG wake deficit model. This wind farm uses the same indexing as earlier simulations, with the new row adding in turbines 18-22, where 22 is the furthest downstream turbine. The turbine layout can be seen in the code in Appendix A.9.

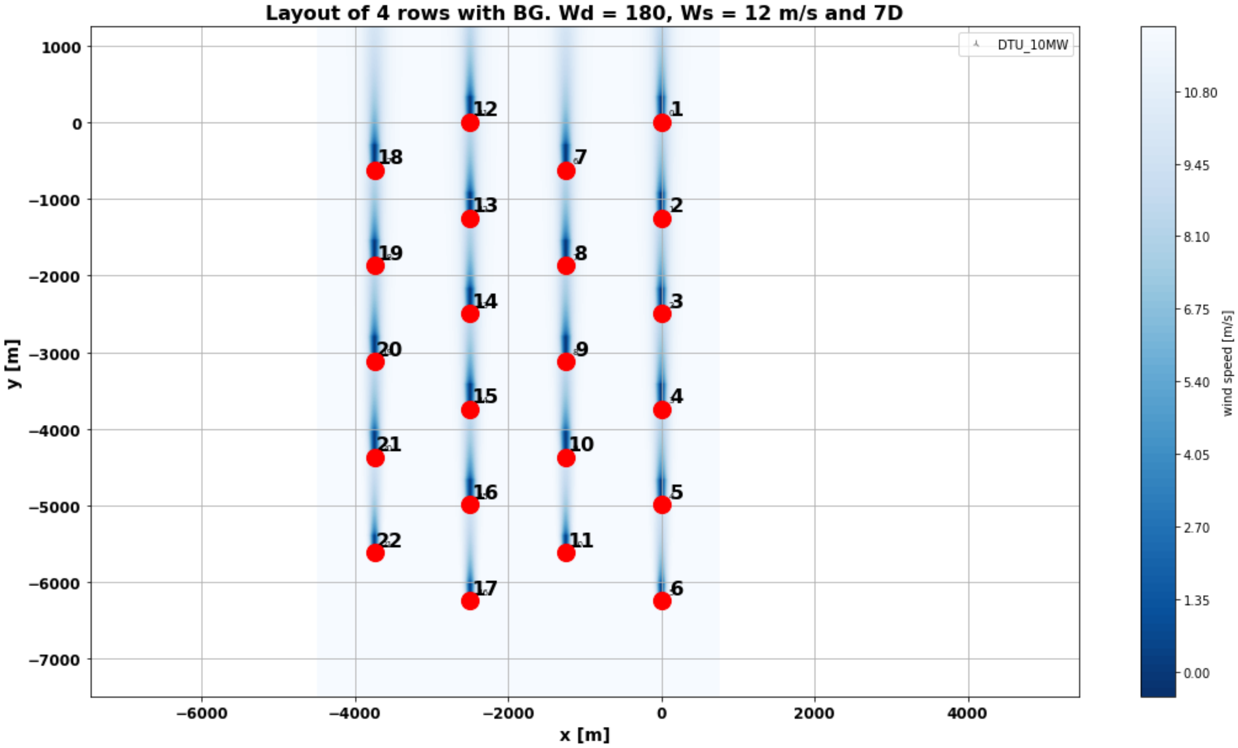

Figure 3.13: Layout of wind farm with 4 rows

Chapter 4

Results and Discussion

A key aspect when looking at all the simulation results is to see if tilting the turbines has had an increase in total power output. This can be verified by comparing benchmark tilt, which corresponds to all turbines having 0 tilt, to the new total power with optimized tilts. If the difference is zero, there has been no power gained by the optimized tilts.

4.1 Initial simulations

This section will go through a comparison of different optimizers, wake deficit models, The initial simulation includes simulating multiple wind speeds and distances between turbines for both NOJ and BG wake deficit models, using all three optimization methods, and with two wind directions.

4.1.1 Comparing Optimizers

NOJ

Tables 4.1 and 4.2 consist of results using the NOJ wake deficit model and shows two distances, 4D and 6D, between turbines, and three wind speeds, 5, 7, and 8 $\frac{m}{s}$. Furthermore, the results for each optimization method for each turbine are separated by columns named by the optimization methods; shgo, DA, and DE. $Ptot$ shows the total power output for the wind farm, using optimal tilts, while $Pbench$ shows the benchmark power output with each turbine having 0 tilt. $Difference$ shows the difference between $Ptot$ and $Pbench$, where a positive value is a result of $Ptot$ being higher than $Pbench$. For 6D distance at 7 and 8 $\frac{m}{s}$, all tilts are 0, resulting in similar values for both total and benchmark power. This results in the remaining distance and wind speed combinations having some differences in the tilt and power for one or more turbines.

Starting with 4D, there are some differences between the optimizers across all three wind speeds. At a wind speed of 5 $\frac{m}{s}$, turbines 9 and 10, shgo provide a different power output compared to DA and DE. Turbine 9 provides around 0.25 MW lower power output compared to the two other optimizers, while turbine 10 provides an increase of 0.1 MW. For these turbines, there is also a significant difference in turbine tilt, with turbine 9 having a tilt of -18.66°, compared to 0 tilt for the two remaining optimizers. At the same wind speed and distance, turbines 2 and 5 for all three optimizers and 8 and 10 for DA and DE have opposite tilt angles, but equal power output.

When looking at the distance of 6D and a wind speed of 8 $\frac{m}{s}$, all three optimization methods provide varying tilts for turbines 3, 4, and 5, different power output for turbines 1-5, and opposite tilts for turbines 9 and 10. This difference in tilt and power for turbines 1-4 might be connected to the tilt angle affecting the downstream turbines, as these five turbines are all located in the first row. As for the actual value of the differences, the optimizers vary in providing the highest power and tilt. Overall, *Ptot* is relatively similar for all optimizers across both distances and all wind conditions, with the highest difference being 0.7 MW for 4D distance at 5 $\frac{m}{s}$, where shgo gives a *Ptot* of 4.23 MW, and DA and DE gives 4.3 MW.

There are some differences in individual turbine power across the three optimization methods, with shgo occasionally having power differences compared to DA and DE. This is especially visible in 6D distance at 5 $\frac{m}{s}$ wind speed, where there is a power difference in 6 of the turbines, turbines; 1-4, 7, and 8, and slightly visible in 4D distance at the same wind speed, with turbines 9 and 10.

There are multiple instances where one of the optimizers provides a polar opposite tilt angle than the remaining two, but all these are found in the 4D distance between turbines, with this occurring for turbines 2, 5, 8, and 10 at both 5 and 7 $\frac{m}{s}$. All these four turbines are the second turbines and the second to last turbines in both rows.

Table 4.1: Tilt and Power results from the different optimization methods for 4D and various wind speeds using NOJ

	shgo		DA		DE	
Turbine	**Tilt [°]**	**Pturb [MW]**	**Tilt [°]**	**Pturb [MW]**	**Tilt [°]**	**Pturb [MW]**
			Ws5			
1	0.00	0.50	0.00	0.50	0.00	0.50
2	-30.00	0.16	-30.00	0.16	30.00	0.16
3	-12.32	0.21	-12.32	0.21	-12.32	0.21
4	0.00	0.47	0.00	0.47	0.01	0.47
5	-25.63	0.14	25.63	0.14	25.63	0.14
6	0.02	0.80	0.00	0.80	0.03	0.80
7	0.01	0.50	0.00	0.45	0.00	0.45
8	-30.00	0.17	-23.13	0.15	23.13	0.15
9	-18.66	0.23	0.00	0.48	0.00	0.48
10	-8.37	0.24	25.85	0.14	-25.85	0.14
11	-0.01	0.80	0.01	0.80	0.01	0.80
Ptot [MW]	4.23		4.30		4.30	
Pbench [MW]	3.89		3.89		3.89	
Difference [MW]	0.34		0.41		0.41	
			Ws7			
1	0.00	1.01	0.00	1.01	0.00	1.01
2	-4.53	1.10	4.55	1.10	4.56	1.10
3	-13.28	1.04	-13.27	1.04	-13.27	1.04
4	-14.77	1.05	-14.77	1.05	-14.78	1.05
5	-15.17	1.13	15.17	1.13	15.18	1.13
6	0.00	2.51	0.00	2.51	0.00	2.51
7	0.00	1.02	0.00	1.02	0.00	1.02
8	-4.51	1.11	4.49	1.11	4.52	1.11
9	-13.25	1.07	-13.25	1.07	-13.25	1.07
10	-14.54	1.15	14.54	1.15	14.55	1.15
11	0.00	2.51	0.00	2.51	0.00	2.51
Ptot [MW]	14.70		14.70		14.70	
Pbench [MW]	14.59		14.59		14.59	
Difference [MW]	0.11		0.11		0.11	
			Ws8			
1	0.00	1.61	0.00	1.68	0.00	1.63
2	0.01	1.70	-0.01	1.91	0.00	1.73
3	-8.56	1.71	-22.12	1.30	0.05	2.02
4	-12.05	1.71	-0.02	1.81	24.79	1.30
5	9.61	2.02	-5.83	2.08	0.00	2.11
6	0.00	3.73	0.00	3.73	0.00	3.73
7	0.00	1.62	0.00	1.62	0.00	1.62
8	0.01	1.72	-0.03	1.72	0.00	1.72
9	-7.86	1.78	7.86	1.78	7.85	1.78
10	8.17	2.05	-8.16	2.05	8.17	2.05
11	0.00	3.73	-0.01	3.73	0.00	3.73
Ptot [MW]	23.39		23.42		23.42	
Pbench [MW]	23.36		23.36		23.36	
Difference [MW]	0.03		0.06		0.06	

Table 4.2: Tilt and Power results from the different optimization methods for 6D and various wind speeds using NOJ

	shgo		**DA**		**DE**	
Turbine	**Tilt [°]**	**Pturb [MW]**	**Tilt [°]**	**Pturb [MW]**	**Tilt [°]**	**Pturb [MW]**
			Ws5			
1	0.00	0.53	0.00	0.37	0.00	0.37
2	-26.08	0.21	0.00	0.39	-0.01	0.39
3	0.00	0.37	0.00	0.56	-0.04	0.56
4	0.00	0.38	28.10	0.20	28.10	0.20
5	0.00	0.42	0.00	0.42	0.00	0.42
6	-0.01	0.80	0.00	0.80	-0.01	0.80
7	0.00	0.54	0.00	0.39	-0.01	0.37
8	-26.31	0.21	0.00	0.55	0.01	0.37
9	0.00	0.38	-27.85	0.20	0.04	0.38
10	0.00	0.42	0.00	0.42	0.02	0.42
11	0.01	0.80	-0.01	0.80	0.01	0.80
Ptot [MW]	5.05		5.10		5.07	
Pbench [MW]	5.03		5.03		5.03	
Difference [MW]	0.02		0.07		0.04	
			Ws7			
1	0.00	1.44	-0.01	1.44	0.00	1.44
2	0.00	1.45	-0.02	1.45	0.00	1.45
3	0.00	1.46	-0.01	1.46	0.00	1.46
4	0.00	1.49	0.00	1.49	0.00	1.49
5	0.00	1.63	0.01	1.63	0.00	1.63
6	0.00	2.51	0.00	2.51	0.00	2.51
7	0.00	1.45	0.00	1.45	0.00	1.45
8	0.00	1.46	-0.01	1.46	0.00	1.46
9	0.00	1.49	-0.03	1.49	0.00	1.49
10	0.00	1.63	0.00	1.63	0.00	1.63
11	0.00	2.51	0.00	2.51	0.00	2.51
Ptot [MW]	18.50		18.50		18.50	
Pbench [MW]	18.50		18.50		18.50	
Difference [MW]	0.00		0.00		0.00	
			Ws8			
1	0.00	2.34	0.00	2.34	0.00	2.34
2	0.00	2.35	0.00	2.35	0.00	2.35
3	0.00	2.36	0.01	2.36	0.00	2.36
4	0.00	2.41	0.00	2.41	0.00	2.41
5	0.00	2.58	0.00	2.58	0.00	2.58
6	0.00	3.73	0.00	3.73	0.00	3.73
7	0.00	2.35	0.00	2.35	0.00	2.35
8	0.00	2.36	0.00	2.36	0.00	2.36
9	0.00	2.41	0.00	2.41	0.00	2.41
10	0.00	2.58	0.00	2.58	0.00	2.58
11	0.00	3.73	0.00	3.73	0.00	3.73
Ptot [MW]	29.20		29.20		29.20	
Pbench [MW]	29.20		29.20		29.20	
Difference [MW]	0.00		0.00		0.00	

BG

Tables 4.3 and 4.4 shows the tilt and power results using BG wake deficit model and are set up identically to Tables 4.1 and 4.2, with comparing results for the three optimizers. The results for BG show similar indications as NOJ, but these results provide more variation in tilt between the optimizers, and particularly shgo provides some differences in tilt and power output. Again, the majority of differences are seen within 4D distance, and most prominently is the total power output at a wind speed of 8 $\frac{m}{s}$, where shgo has a total power output of 11.98 MW, whereas DA and DE has a power output of 12.30 MW, a difference of 0.32 MW. This is a result of all individual turbines, except turbines 6 and 11, having different turbine outputs for shgo compared to DA and DE. This is further emphasized by the tilt difference for most of the turbines providing different power, turbines 2-5, 9, and 10. It is also worth noting that shgo provides a power output close to the benchmark value, despite having a significant tilt on six turbines. When comparing this to the increased power from DA and DE, which only have tilt on four turbines yet still provide a power gain, it shows that there is an optimum tilt angle for each turbine. There are also some sporadic differences between shgo and the remaining two optimizers across the remaining wind speeds and distances, especially in the 4D distance at a wind speed of 7 $\frac{m}{s}$, where turbines 1-5 have different power output compared to DA and DE. Overall, their differences are quite small, besides the quite significant difference in total power at 4D and 8 $\frac{m}{s}$, and particularly DA and DE provide similar results.

Table 4.3: Tilt and Power results from the different optimization methods for 4D and various wind speeds using BG

	shgo		**DA**		**DE**	
Turbine	**Tilt [°]**	**Pturb [MW]**	**Tilt [°]**	**Pturb [MW]**	**Tilt [°]**	**Pturb [MW]**
Ws5						
1	0.00	0.24	0.00	0.38	0.00	0.38
2	-29.96	0.00	-30.00	0.04	-30.00	0.04
3	0.00	0.24	16.48	0.12	16.65	0.12
4	-30.00	0.13	26.02	0.16	26.08	0.16
5	-30.00	0.00	0.00	0.02	0.00	0.02
6	0.00	0.80	0.00	0.80	0.00	0.80
7	0.00	0.38	0.00	0.38	0.00	0.24
8	-30.00	0.04	-30.00	0.04	12.84	0.00
9	-26.99	0.15	26.56	0.15	0.00	0.27
10	-30.00	0.00	0.00	0.02	-0.17	0.02
11	0.00	0.80	0.00	0.80	0.00	0.80
Ptot [MW]	2.79		2.92		2.85	
Pbench [MW]	2.66		2.66		2.66	
Difference [MW]	0.13		0.27		0.20	
Ws7						
1	0.00	0.31	0.00	0.82	0.00	0.82
2	0.00	0.88	-30.00	0.10	-30.00	0.10
3	-30.00	0.09	-11.80	0.88	-11.80	0.88
4	-14.24	0.69	-30.00	0.05	-30.00	0.05
5	-30.00	0.14	-15.25	0.25	15.25	0.25
6	0.00	2.51	0.00	2.51	0.00	2.51
7	0.00	0.87	0.00	0.87	0.00	0.87
8	-30.00	0.09	30.00	0.09	30.00	0.09
9	-13.90	0.70	13.90	0.70	13.90	0.70
10	-30.00	0.14	30.00	0.14	30.00	0.14
11	0.00	2.51	0.00	2.51	0.00	2.51
Ptot [MW]	8.91		8.90		8.90	
Pbench [MW]	7.18		7.18		7.18	
Difference [MW]	1.73		1.72		1.72	
Ws8						
1	0.00	0.88	0.00	0.54	0.00	0.54
2	-30.00	0.17	0.00	1.02	0.00	1.02
3	-17.57	0.69	-30.00	0.14	-30.00	0.14
4	-30.00	0.18	-17.87	0.43	-17.87	0.43
5	-13.84	0.49	0.00	0.56	0.00	0.56
6	0.00	3.73	0.00	3.73	0.00	3.73
7	0.00	0.55	0.01	1.01	0.00	1.01
8	0.00	0.93	30.00	0.14	-30.00	0.14
9	-30.00	0.17	17.61	0.43	-17.61	0.43
10	-18.24	0.44	0.00	0.56	0.00	0.56
11	0.00	3.73	0.00	3.73	0.00	3.73
Ptot [MW]	11.98		12.30		12.30	
Pbench [MW]	11.96		11.96		11.96	
Difference [MW]	0.02		0.34		0.34	

Table 4.4: Tilt and Power results from the different optimization methods for 6D and various wind speeds using BG

	shgo		DA		DE	
Turbine	**Tilt [°]**	**Ptot [MW]**	**Tilt [°]**	**Ptot [MW]**	**Tilt [°]**	**Pturb [MW]**
			Ws5			
1	0.00	0.42	0.00	0.42	0.00	0.43
2	-28.20	0.05	28.20	0.05	30.00	0.06
3	0.00	0.44	0.00	0.44	4.48	0.16
4	-30.00	0.06	30.00	0.06	0.00	0.44
5	-8.16	0.17	-8.15	0.17	29.80	0.05
6	0.00	0.80	0.00	0.80	0.00	0.80
7	0.00	0.42	0.01	0.42	0.01	0.42
8	-28.21	0.05	-28.21	0.05	-28.21	0.05
9	0.00	0.44	0.00	0.44	0.00	0.44
10	-29.91	0.04	-29.91	0.04	29.91	0.04
11	0.00	0.80	0.00	0.80	0.00	0.80
Ptot [MW]	3.69		3.69		3.70	
Pbench [MW]	3.23		3.23		3.23	
Difference [MW]	0.46		0.46		0.47	
			Ws7			
1	0.00	0.69	0.00	0.69	0.00	0.69
2	0.00	0.70	0.00	0.70	0.00	0.70
3	0.00	0.71	0.00	0.71	0.00	0.71
4	0.00	0.74	0.00	0.74	0.00	0.74
5	0.00	0.78	0.00	0.78	0.00	0.78
6	0.00	2.51	0.00	2.51	0.00	2.51
7	0.00	0.70	0.00	0.70	0.00	0.70
8	0.00	0.71	0.00	0.71	0.00	0.71
9	0.00	0.74	0.00	0.74	0.00	0.74
10	0.00	0.78	0.00	0.78	0.00	0.78
11	0.00	2.51	0.00	2.51	0.00	2.51
Ptot [MW]	11.58		11.58		11.58	
Pbench [MW]	11.58		11.58		11.58	
Difference [MW]	0.00		0.00		0.00	
			Ws8			
1	0.00	1.13	0.00	1.13	0.00	1.13
2	0.00	1.14	0.00	1.14	0.00	1.14
3	0.00	1.16	0.00	1.16	0.00	1.16
4	0.00	1.21	0.00	1.21	0.00	1.21
5	0.00	1.29	0.00	1.29	0.00	1.29
6	0.00	3.73	0.00	3.73	0.00	3.73
7	0.00	1.14	0.00	1.14	0.00	1.14
8	0.00	1.16	0.00	1.16	0.00	1.16
9	0.00	1.21	0.00	1.21	0.00	1.21
10	0.00	1.29	0.00	1.29	0.00	1.29
11	0.00	3.73	0.00	3.73	0.00	3.73
Ptot [MW]	18.18		18.18		18.18	
Pbench [MW]	18.18		18.18		18.18	
Difference [MW]	0.00		0.00		0.00	

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4.1.2 Optimization verification

To verify the results of the different optimizers, an identical simulation was done a second time using a reduced number of distances and wind speeds. These results provided nearly identical values when compared to the initial results, with only sporadic differences in some individual tilts. The overall power output was identical for all variables. This indicates three well-functioning optimization algorithms that operate consistently with the PyWake models. These results can be found in Appendix C

4.1.3 Comparing Wake Deficit Models

To compare the results between the two wake deficit models, a compressed table is seen in Table 4.5, which includes distances between 3-8D and wind speeds from 4-9 $\frac{m}{s}$. These values were selected to give insight into how the increase in distance and wind speeds affect the power output. Only the power is compared at this stage, as the differences in power between optimized and benchmark values provide insight into the power gain by both NOJ and BG. In the first column, the wind speeds are given followed by $Ptot$, which is the optimized power output of the wind farm. Furthermore, $Pbench$ refers to the wind farm power output when all tilts are 0. $Difference$ Is the difference in power between $Ptot$ and $Pbench$, where a positive value is a result of $Ptot$ being higher than $Pbench$. Lastly, $Difference$[%] represents the power difference in percentage gain or loss compared to $Pbench$.

Overall, it is clear that the NOJ wake deficit model provides a higher power output, both for optimized tilts and for 0 tilt. The highest power gain for NOJ is 0.99 MW at wind speed 6 $\frac{m}{s}$ and a distance of 3D. For BG the highest power gain is also the overall highest gain with an increase in power of 3.71 MW at a wind speed of 9 $\frac{m}{s}$ at a distance of 3D. For percentage gain the highest gain for NOJ is 16.12% at the same wind speed and distance as the power gain of 6 $\frac{m}{s}$ and 3D, while BG again has the overall highest percentage gain of 28.60% at a wind speed of 9 $\frac{m}{s}$ and a distance of 3D.

Increasing distance and wind speed overall reduces the difference between optimized tilts and 0 tilt, both in percentage and power. This is visualized when looking at the table, and seeing a decrease in difference with increasing distance and wind speed. This is as expected since an increase in distance will reduce the wake effects, thus reducing the potential power gain from tilting.

Table 4.5: Power comparison NOJ and BG over selected wind speeds and distances between turbines

Wind speeds	**3D**		**4D**		**5D**		**6D**		**7D**		**8D**	
4.00 $\frac{m}{s}$	**NOJ**	**BG**	**NOJ**	**BG**	**NOJ**	**BG**	**NOJ**	**BG**	**NOJ**	**BG**	**NOJ**	**BG**
Ptot [MW]	1.60	0.92	1.83	1.11	1.98	1.23	2.11	1.33	2.22	1.46	2.32	1.59
Pbench [MW]	1.48	0.87	1.71	1.06	1.93	1.18	2.08	1.29	2.21	1.44	2.31	1.59
Difference [MW]	0.12	0.06	0.12	0.05	0.05	0.05	0.03	0.04	0.01	0.02	0.01	0.01
Difference [%]	8.15	6.57	7.02	4.56	2.59	4.28	1.44	3.24	0.45	1.63	0.43	0.46
5.00 $\frac{m}{s}$	**NOJ**	**BG**	**NOJ**	**BG**	**NOJ**	**BG**	**NOJ**	**BG**	**NOJ**	**BG**	**NOJ**	**BG**
Ptot [MW]	3.80	2.62	4.30	2.85	4.71	3.32	5.07	3.70	5.66	3.97	6.14	4.19
Pbench [MW]	3.50	2.32	3.89	2.66	4.23	3.05	5.03	3.23	5.66	3.50	6.14	3.80
Difference [MW]	0.30	0.30	0.41	0.20	0.48	0.27	0.04	0.47	0.00	0.47	0.00	0.39
Difference [%]	8.50	12.93	10.54	7.41	11.35	8.96	0.80	14.71	0.00	13.51	0.00	10.36
6.00 $\frac{m}{s}$	**NOJ**	**BG**	**NOJ**	**BG**	**NOJ**	**BG**	**NOJ**	**BG**	**NOJ**	**BG**	**NOJ**	**BG**
Ptot [MW]	7.10	4.74	8.20	5.87	9.50	6.26	10.59	6.68	11.65	7.71	12.46	8.67
Pbench [MW]	6.11	4.74	8.10	4.58	9.49	5.20	10.59	6.49	11.65	7.71	12.46	8.67
Difference [MW]	0.99	0.00	0.10	1.29	0.01	1.06	0.00	0.19	0.00	0.00	0.00	0.00
Difference [%]	16.12	0.00	1.23	28.29	0.11	20.44	0.00	2.87	0.00	0.00	0.00	0.00
7.00 $\frac{m}{s}$	**NOJ**	**BG**	**NOJ**	**BG**	**NOJ**	**BG**	**NOJ**	**BG**	**NOJ**	**BG**	**NOJ**	**BG**
Ptot [MW]	12.10	8.21	14.70	8.90	16.85	9.66	18.50	11.58	19.96	13.36	21.19	14.95
Pbench [MW]	11.67	7.49	14.59	7.18	16.84	9.66	18.50	11.58	19.96	13.36	21.19	14.95
Difference [MW]	0.43	0.72	0.11	1.72	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Difference [%]	3.69	9.62	0.75	23.98	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00
8.00 $\frac{m}{s}$	**NOJ**	**BG**	**NOJ**	**BG**	**NOJ**	**BG**	**NOJ**	**BG**	**NOJ**	**BG**	**NOJ**	**BG**
Ptot [MW]	19.50	12.53	23.42	12.30	26.81	15.14	29.20	18.18	31.10	20.56	32.70	22.75
Pbench [MW]	19.12	9.76	23.36	11.96	26.81	15.14	29.20	18.18	31.10	20.56	32.70	22.75
Difference [MW]	0.38	2.78	0.06	0.34	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Difference [%]	1.99	28.37	0.26	2.81	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
9.00 $\frac{m}{s}$	**NOJ**	**BG**	**NOJ**	**BG**	**NOJ**	**BG**	**NOJ**	**BG**	**NOJ**	**BG**	**NOJ**	**BG**
Ptot [MW]	29.10	16.68	34.81	17.67	39.37	22.20	42.49	26.02	44.85	29.57	46.92	32.52
Pbench [MW]	28.78	12.97	34.83	17.67	39.37	22.20	42.49	26.02	44.85	29.57	46.92	32.52
Difference [MW]	0.32	3.71	-0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Difference [%]	1.10	28.60	-0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Figures 4.1 and 4.2 show the difference between total power for optimized tilt and power for zero tilt, and how they vary for different distances and wind speeds. Figure 4.1 shows how the power difference is influenced with a NOJ wake deficit model. As seen in the figure, it becomes clear that there is a bigger power difference at lower distances between turbines and wind speeds. Furthermore, Figure 4.2 shows the power differences using the BG wake deficit model, and the lower distances between turbines provide the highest power difference. The maximum power difference moves towards lower wind speeds with an increase in distance. This is prominent for distances 3D, 4D, and 5D, where the maximum power is reached at respectfully 9, 7, and 5 $\frac{m}{s}$. Comparing both wake deficit models, it becomes clear that BG provides the highest power gain, with the maximum power gain being over 3.5 times greater for BG than NOJ. Overall they both provide similar indications, with a decrease in power gain with increasing distance and wind speeds.

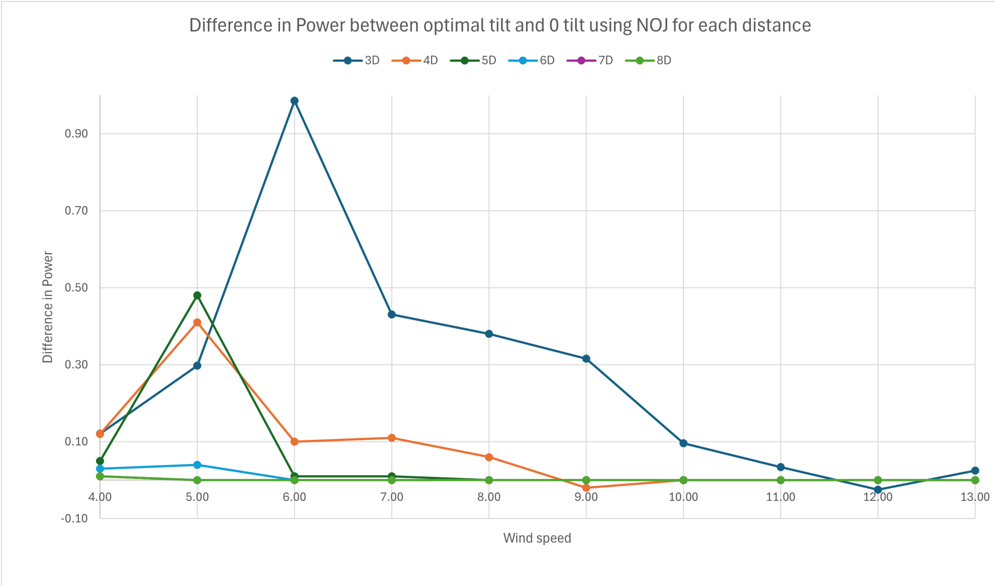

Figure 4.1: Difference in Power between P_{opt} and P_{0tilt} for NOJ

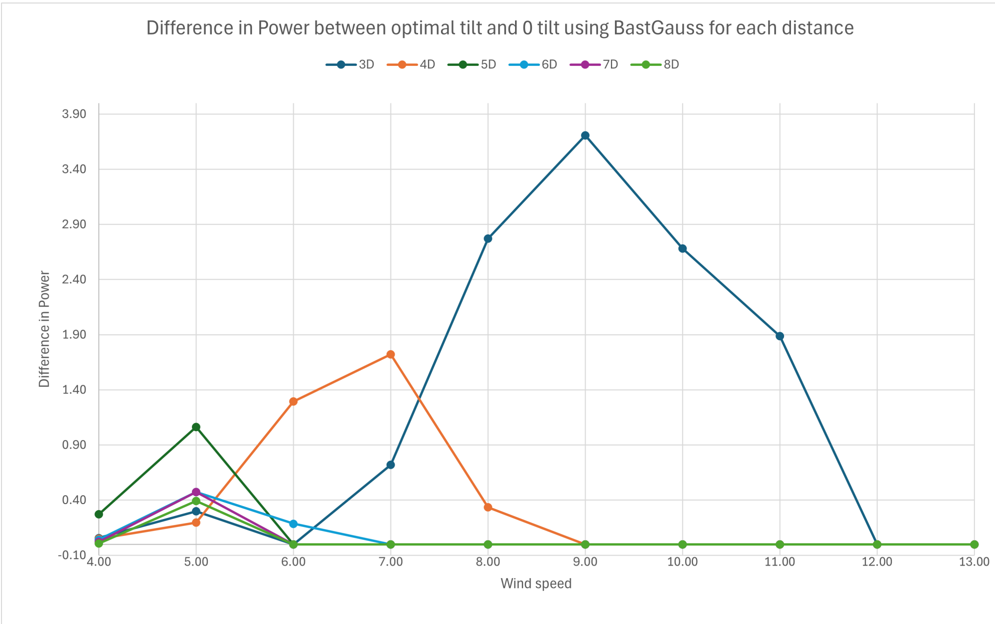

Figure 4.2: Difference in Power between P_{opt} and P_{0tilt} for BG

4.1.4 Difference in tilt

Compressing the results presented in Table 4.5 even further provided a more easy presentation of the tilts for each turbine. The resulting figures 4.3 - 4.6 showcase the tilt angles using BG and the DE optimization from Tables 4.3 and 4.4. These figures are separated by the distance between turbines from 4D to 7D distance and with a wind speed range between 5-8 $\frac{m}{s}$. As shown in the figures, the wind speeds are separated by color, with 5 $\frac{m}{s}$ in blue, 6 $\frac{m}{s}$ in orange, 7 $\frac{m}{s}$ in green and 8 $\frac{m}{s}$ in light-blue. The boundaries for the plot are set to similar values as the tilt thresholds, -30°and 30°.

Figure 4.3 shows some differences between the tilts, resulting in an increased power output. Nearly all turbines have a significant tilt angle at multiple wind speeds, except turbines 1, 6, 7, and 11. These turbines are the first and last turbines of each row, thus being the only turbines that do not have wake interactions both up and downstream. Further, it can be seen that both maximum and minimum tilt thresholds, or close to them, occur for several turbines over multiple wind speeds. This includes turbines 2, 3, 4, 5, 8, and 10, which leaves only turbine 9 as the only turbine not having close to 0 or threshold tilts, throughout all the wind speeds.

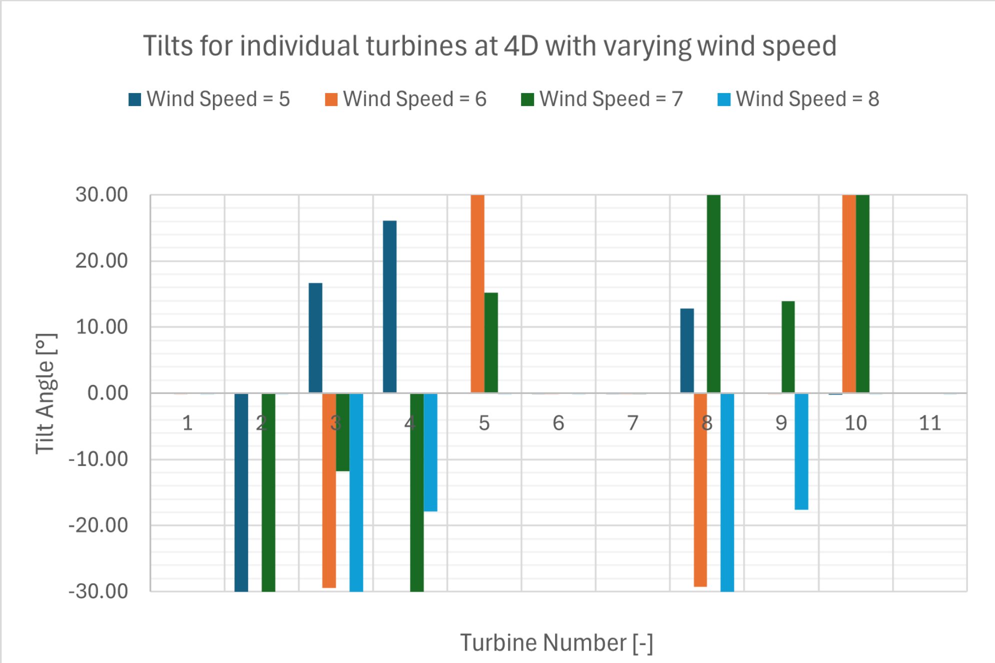

Figure 4.3: Tilts for individual turbines at 4D distance with wind speed in range 5-8 $\frac{m}{s}$

Figure 4.4 shows the turbine tilts at 5D distance, and it clearly shows that wind speeds of 7 and 8 $\frac{m}{s}$ result in 0°tilt for all turbines. The remaining wind speeds of 5 and 6 $\frac{m}{s}$ show turbines 2, 4, 8, and 10 having a tilt for both wind speeds, while only 6 $\frac{m}{s}$ has a slight tilt on turbine 5. Most of the tilts are reaching maximum or minimum tilt, particularly the tilts for 6 $\frac{m}{s}$ where all turbines with tilt, except turbine 5, provide a maximum or minimum tilt.

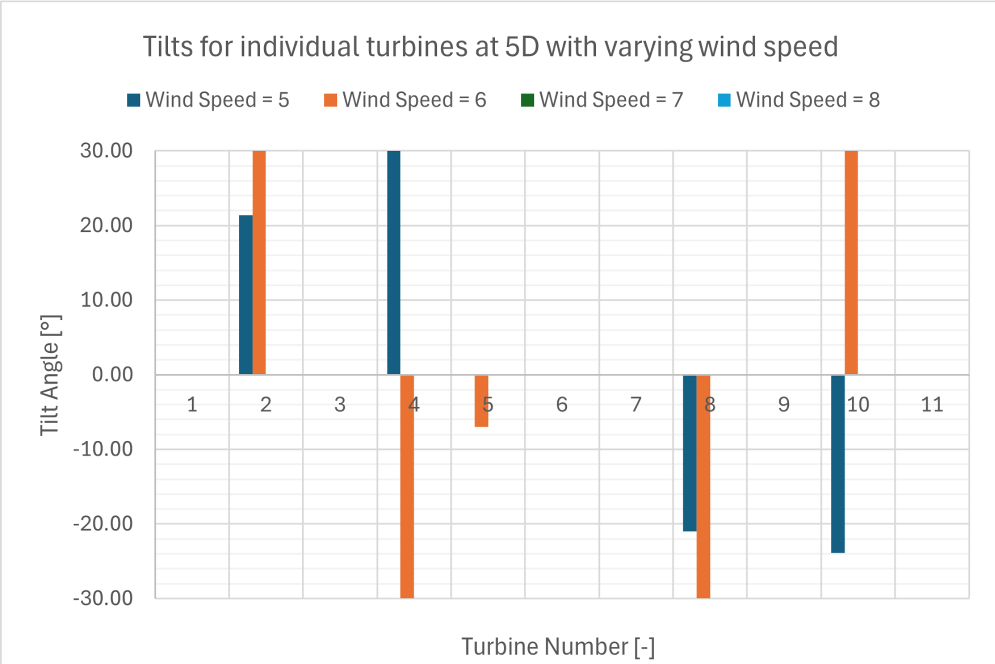

Figure 4.4: Tilts for individual turbines at 5D distance with wind speed in range 5-8 $\frac{m}{s}$

Figure 4.5 shows the turbine tilts at a 6D distance, and also for this distance only the low wind speeds of 5 and 6 $\frac{m}{s}$ result in a tilt angle other than 0. It differentiates from the results in 5D distance by having a higher total number of turbines with a maximum or minimum tilt, with the addition of turbines 3, 5, and 9, and the reduction of turbine 4. It also shows a decrease in turbines with a tilt value for wind speed of 6 $\frac{m}{s}$, where only turbines 3 and 9 have any tilt, and both these are at maximum tilt.

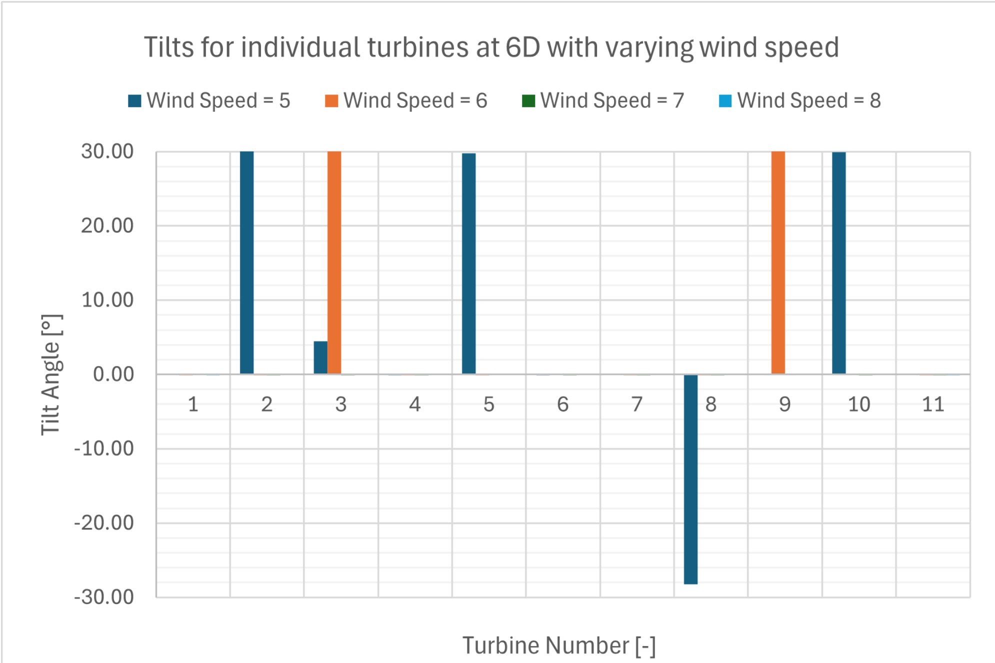

Figure 4.5: Tilts for individual turbines at 6D distance with wind speed in range 5-8 $\frac{m}{s}$

Figure 4.6 shows the turbine tilts at a 7D distance, with a wind speed of 5 $\frac{m}{s}$ solely having turbines with a tilt other than 0. It provides quite similar results as for 6D except for turbines 2 and 10 switching from maximum tilt to minimum tilt. Four out of the five remaining turbines, 2, 5, 8, and 9, provide maximum or minimum tilt, while the remaining turbine 3 provides around 7.5°.

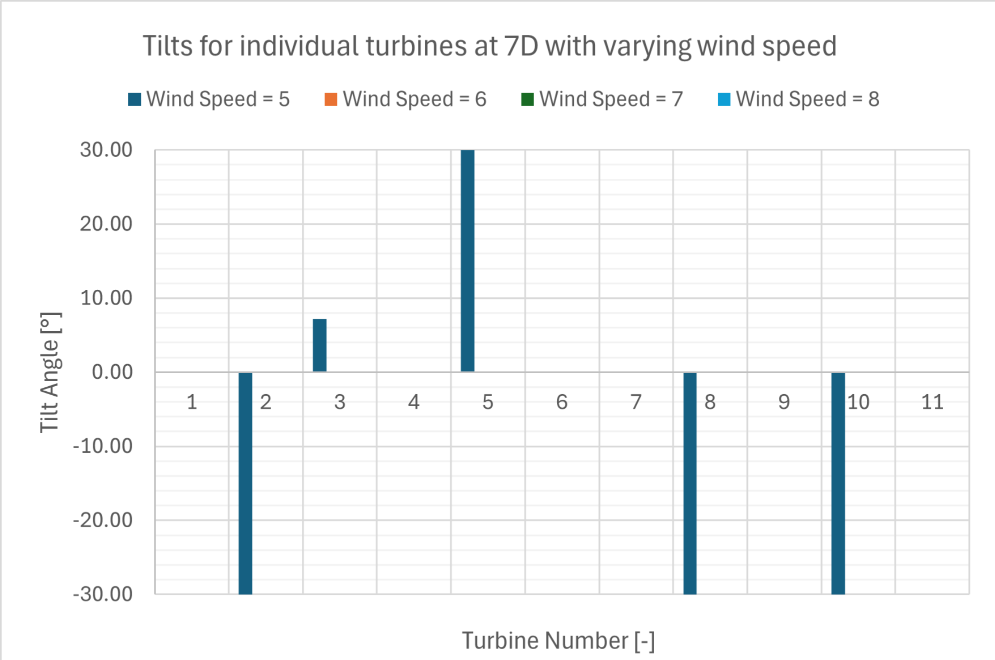

Figure 4.6: Tilts for individual turbines at 7D distance with wind speed in range 5-8 $\frac{m}{s}$

In general, it can be seen that there were no tilts for the furthest up and downstream turbines, specifically turbines 1, 6, 7, and 11. Furthermore, in most of the cases where a turbine was tilting, the tilt was at its maximum or minimum value as set by the boundaries.

4.1.5 190°wind direction

In addition to wind coming from a 180-degree direction, a 190-degree was simulated to show how the tilt is affected by alternative wind directions, across the different optimization methods. Table 4.6 shows the result of two distances, 4D and 6D, across 3 wind speeds, 5, 7, and 8 $\frac{m}{s}$. This table shows the tilt and power output for each turbine across the different parameters, as well as summing up the total power in $Ptot$ and comparing it to the 0 tilt values of $Pbench$. For these distances and wind speeds, the optimal tilt is zero for all the values, resulting in the same power output for both zero and optimal tilt. All three optimization methods provide similar results, both for tilt and power output for all individual turbines. These results indicate how turbine tilts are affected by the wake, and the sensitivity of potentially increasing power output from tilted turbines compared to zero-tilt turbines.

Table 4.7 shows the result of the power difference between wind directions of 180°and 190°. It shows that there is a difference in power between them and that the overall power output is higher for a wind direction of 190°, even though it does not provide any increased power from the tilt. This is particularly seen for BG, where a wind direction of 190°consistently provides more than double the power output compared to a wind direction of 180°. The greatest difference can be seen in 5D distance at 5 $\frac{m}{s}$, where the difference is 22.1 MW, which is a significant difference. For NOJ, the gap between the two wind directions is smaller, but a wind direction of 190°still results in an overall higher power output. The greatest difference in power between the wind directions can be found at 6D at 8 $\frac{m}{s}$ where the power difference is 6.2 MW.

Table 4.6: Tilt and power of the different optimization models using wake deficit model BG for wind direction 190°

Distance = 4D - Wd190						
Ws5	**shgo**		**DA**		**DE**	
Turbine	**Tilt [°]**	**Pturb [MW]**	**Tilt [°]**	**Pturb [MW]**	**Tilt [°]**	**Pturb [MW]**
1	0.00	0.50	0.00	0.50	0.01	0.50
2	0.00	0.52	0.00	0.52	0.00	0.52
3	0.00	0.53	0.00	0.53	0.00	0.53
4	0.00	0.53	0.00	0.53	0.00	0.53
5	0.00	0.53	0.00	0.53	0.00	0.53
6	0.00	0.80	0.00	0.80	0.00	0.80
7	0.00	0.53	0.00	0.53	0.00	0.53
8	0.00	0.53	0.00	0.53	0.00	0.53
9	0.00	0.53	0.00	0.53	0.00	0.53
10	0.00	0.53	0.00	0.53	0.00	0.53
11	0.00	0.80	0.00	0.80	0.00	0.80
Ptot [MW]	6.31		6.31		6.31	
Pbench [MW]	6.31		6.31		6.31	
Difference [MW]	0.00		0.00		0.00	
Ws8	**shgo**		**DA**		**DE**	
Turbine	**Tilt [°]**	**Pturb [MW]**	**Tilt [°]**	**Pturb [MW]**	**Tilt [°]**	**Pturb [MW]**
1	0.00	2.71	0.00	2.71	0.00	2.71
2	0.00	2.82	0.00	2.82	0.00	2.82
3	0.00	2.83	0.00	2.83	0.00	2.83
4	0.00	2.83	0.00	2.83	0.00	2.83
5	0.00	2.88	0.00	2.88	0.00	2.88
6	0.00	3.73	0.00	3.73	0.00	3.73
7	0.00	2.83	0.00	2.83	0.00	2.83
8	0.00	2.83	0.00	2.83	0.00	2.83
9	0.00	2.83	0.00	2.83	0.00	2.83
10	0.00	2.88	0.00	2.88	0.00	2.88
11	0.00	3.73	0.00	3.73	0.00	3.73
Ptot [MW]	32.90		32.90		32.90	
Pbench [MW]	32.90		32.90		32.90	
Difference [MW]	0.00		0.00		0.00	

Table 4.7: Power comparison for wind directions 180°and 190°

Wind speeds	**4D**				**5D**				**6D**			
4.00 m/s	**NOJ**	**BG**	**NOJ190**	**BG190**	**NOJ**	**BG**	**NOJ190**	**BG190**	**NOJ**	**BG**	**NOJ190**	**BG190**
Max opt [MW]	1.83	1.11	2.12	2.34	1.98	1.23	2.33	2.66	2.11	1.33	2.51	2.83
0 tilt [MW]	1.71	1.06	2.10	2.34	1.93	1.18	2.33	2.66	2.08	1.29	2.51	2.83
Difference [MW]	0.12	0.05	0.02	0.00	0.05	0.05	0.00	0.00	0.03	0.04	0.00	0.00
Difference [%]	7.02	4.56	0.97	0.00	2.59	4.28	0.08	0.00	1.44	3.24	0.00	0.00
5.00 m/s	**NOJ**	**BG**	**NOJ190**	**BG190**	**NOJ**	**BG**	**NOJ190**	**BG190**	**NOJ**	**BG**	**NOJ190**	**BG190**
Max opt [MW]	4.30	2.92	5.20	6.31	4.71	3.32	6.18	7.48	5.10	3.70	6.96	8.08
0 tilt [MW]	3.89	2.66	5.04	6.31	4.23	3.05	6.18	7.48	5.03	3.23	6.96	8.08
Difference [MW]	0.41	0.27	0.17	0.00	0.48	0.27	0.00	0.00	0.07	0.47	0.00	0.00
Difference [%]	10.54	10.04	3.30	0.00	11.35	8.96	0.00	0.00	1.39	14.71	0.00	0.00
6.00 m/s	**NOJ**	**BG**	**NOJ190**	**BG190**	**NOJ**	**BG**	**NOJ190**	**BG190**	**NOJ**	**BG**	**NOJ190**	**BG190**
Max opt [MW]	8.20	5.91	10.60	12.71	9.50	6.27	12.53	14.72	10.59	6.68	13.84	15.72
0 tilt [MW]	8.10	4.58	10.60	12.71	9.49	5.20	12.53	14.72	10.59	6.49	13.84	15.72
Difference [MW]	0.10	1.33	0.00	0.00	0.01	1.07	0.00	0.00	0.00	0.19	0.00	0.00
Difference [%]	1.23	29.13	0.00	0.00	0.11	20.55	0.00	0.00	0.00	2.87	0.00	0.00
7.00 m/s	**NOJ**	**BG**	**NOJ190**	**BG190**	**NOJ**	**BG**	**NOJ190**	**BG190**	**NOJ**	**BG**	**NOJ190**	**BG190**
Max opt [MW]	14.70	8.91	18.48	21.43	16.85	9.66	21.28	24.59	18.50	11.58	23.24	26.05
0 tilt [MW]	14.59	7.18	18.48	21.43	16.84	9.66	21.28	24.59	18.50	11.58	23.24	26.05
Difference [MW]	0.11	1.73	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Difference [%]	0.75	24.12	0.00	0.00	0.06	0.00	0.00	0.00	0.00	0.00	0.00	0.00
8.00 m/s	**NOJ**	**BG**	**NOJ190**	**BG190**	**NOJ**	**BG**	**NOJ190**	**BG190**	**NOJ**	**BG**	**NOJ190**	**BG190**
Max opt [MW]	23.42	12.30	29.18	32.90	26.81	15.14	32.82	37.22	29.20	18.18	35.37	39.13
0 tilt [MW]	23.36	11.96	29.18	32.90	26.81	15.14	32.82	37.22	29.20	18.18	35.37	39.13
Difference [MW]	0.06	0.34	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Difference [%]	0.26	2.81	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

4.2 Comperative simulations

As all three optimization algorithms provided relatively similar results for tilts and power outputs, it was decided to only use DE for the remaining simulations. For a wake deficit model, it was also decided to only use BG. This was due to them both providing similar overall trends, but BG had higher power gains. As most of the data over a certain distance and wind speed provided little power gain, it was also decided to use distances between 4D - 7D and wind speeds between 4 - 13 $\frac{m}{s}$.

4.2.1 Comparison with different Turbine

The intention of comparing with a different turbine model is to check for variations in tilt and power behavior. This was done by comparing DTU and IEA 10 MW turbines using BG wake deficit model and DE optimizing model. Tables 4.8, 4.9, and 4.10 shows the results for distances of 4D and 5D and wind speeds of 5, 7, and 8 $\frac{m}{s}$. This table shows the tilt and power result of the simulation given as $Tilt$ and $Pturb$, with the corresponding total power, benchmark power, and difference. In addition, $\Delta Tilt$ shows the difference in tilt between the two turbines, and it is calculated by taking the results from Tables 4.3 and 4.4 and subtracting the results from Tables 4.8, 4.9, and 4.10, resulting in a positive value indicating higher tilt value for the DTU 10 MW turbine.

The IEA 10 MW has approximately zero tilt for all turbines through both distances and all three wind speeds, except at 4D distance with a wind speed of 5 $\frac{m}{s}$ where 7 out of 11 turbines have optimum power output at a tilt. Turbine 1 has a tilt, which is interesting considering it is the furthest downstream turbine in the first row, due to its wake not impacting the wind farm output since it does not interact with any other turbine.

Furthermore, there are some differences between the two turbines, both in power and tilt. Particularly when the wind speed is 5 $\frac{m}{s}$, there are positive and negative direction variations. As seen earlier, the values seem to stabilize with increasing wind speed and distance. The maximum difference in tilt can be found at turbine 2 at 4D and 5 $\frac{m}{s}$, with a difference of 54.30°. Why there is such a significant difference in tilts between the two turbines could be related to the slightly higher thrust coefficient and power output of the IEA 10 MW, which can seen in Appendix B. Another factor could be the difference in turbine diameter between the two turbines, 178 m for DTU and 198 m for IEA, which could impact the wake deficit of the downstream turbines.

Table 4.8: Tilt and power between DTU and IEA 10 MW turbines using BG wake deficit model and DE optimizer

Distance = 4D				**Distance = 5D**			
Ws5				**Ws5**			
Turbine	**Tilt [°]**	**Pturb [MW]**	**ΔTilt [°]**	**Turbine**	**Tilt [°]**	**Pturb [MW]**	**ΔTilt [°]**
1	-3.74	0.04	3.75	1	0.00	0.08	0.00
2	24.30	0.04	-54.30	2	0.00	0.08	21.40
3	-29.23	0.04	45.88	3	0.00	0.10	0.00
4	-7.31	0.04	33.39	4	0.00	0.13	30.00
5	0.00	0.07	0.00	5	0.00	0.20	0.00
6	0.00	1.07	0.00	6	0.00	1.07	0.00
7	9.54	0.04	-9.55	7	0.00	0.08	0.00
8	11.13	0.04	1.70	8	0.00	0.10	-21.03
9	24.88	0.04	-24.88	9	0.00	0.13	0.00
10	0.00	0.07	-0.17	10	0.00	0.20	-23.89
11	0.00	1.07	0.00	11	0.00	1.07	0.00
Ptot [MW]		2.56		**Ptot [MW]**		3.24	
Pbench [MW]		2.56		**Pbench [MW]**		3.24	
Difference[MW]		0.00		**Difference[MW]**		0.00	

Table 4.9: Tilt and power between DTU and IEA 10MW turbines using BG wake deficit model and DE optimizer

Distance = 4D				**Distance = 5D**			
Ws7				**Ws7**			
Turbine	**Tilt [°]**	**Pturb [MW]**	**ΔTilt [°]**	**Turbine**	**Tilt [°]**	**Pturb [MW]**	**ΔTilt [°]**
1	-0.01	0.35	0.01	1	0.00	0.68	0.00
2	0.00	0.36	-30.00	2	0.00	0.69	0.00
3	0.00	0.39	-11.80	3	0.00	0.71	0.00
4	0.00	0.43	-30.00	4	0.00	0.76	0.00
5	0.00	0.51	15.25	5	0.00	0.84	0.00
6	0.00	3.15	0.00	6	0.00	3.15	0.00
7	0.01	0.36	-0.01	7	0.00	0.69	0.00
8	0.00	0.39	30.00	8	0.00	0.71	0.00
9	0.00	0.43	13.90	9	0.00	0.76	0.00
10	0.00	0.51	30.00	10	0.00	0.84	0.00
11	0.00	3.15	0.00	11	0.00	3.15	0.00
Ptot [MW]		10.05		**Ptot [MW]**		13.00	
Pbench [MW]		10.05		**Pbench [MW]**		13.00	
Difference[MW]		0.00		**Difference[MW]**		0.00	

Table 4.10: Tilt and power between DTU and IEA 10MW turbines using BG wake deficit model and DE optimizer

Distance = 4D				**Distance = 5D**			
Ws8				**Ws8**			
Turbine	**Tilt [°]**	**Pturb [MW]**	**ΔTilt [°]**	**Turbine**	**Tilt [°]**	**Pturb [MW]**	**ΔTilt [°]**
1	0.00	0.64	0.00	1	0.00	1.06	0.00
2	0.00	0.66	0.00	2	0.00	1.07	0.00
3	0.00	0.70	-30.00	3	0.00	1.10	0.00
4	0.00	0.77	-17.87	4	0.00	1.16	0.00
5	0.00	0.85	0.00	5	0.00	1.30	0.00
6	0.00	4.72	0.00	6	0.00	4.72	0.00
7	0.00	0.66	0.00	7	0.00	1.07	0.00
8	0.00	0.70	-30.00	8	0.00	1.10	0.00
9	0.00	0.77	-17.61	9	0.00	1.16	0.00
10	0.00	0.85	0.00	10	0.00	1.30	0.00
11	0.00	4.72	0.00	11	0.00	4.72	0.00
Ptot [MW]		16.04		**Ptot [MW]**		19.80	
Pbench [MW]		16.04		**Pbench [MW]**		19.80	
Difference[MW]		0.00		**Difference[MW]**		0.00	

Simulation result of higher rated turbine

Tables 4.11 and 4.12 show the simulation result using the IEA 15 MW turbine for a selection of distances between turbines and wind speeds, and the tilt comparison of the DTU 10 MW. This table is structured similarly to Table 4.8, with the $Tilt$ and $Pturb$ showcasing the individual tilt and power output for the turbine and $\Delta Tilt$ showing the calculated tilt difference between values from the initial simulation in Tables 4.3 and 4.4. Furthermore, $Ptot$ and $Pbench$ give the total power output for optimized tilts and zero-tilt and the resulting difference between them. Again, generally, it can be seen that tilts appear at lower wind speeds and distances. A difference in this result is that there is a slightly increased power output for the optimal tilt results compared to the 0 tilt, which can be seen in three of the simulation results, with the most significant being at 5D distance at 5 $\frac{m}{s}$, with a power increase of 0.28 MW which is an increase in power of around 7.2%. It is also noteworthy that at a 4D distance and a wind speed of $5\frac{m}{s}$, turbines 1 and 7 have tilts of -15.56°and -2.02°, which again is interesting due to it being the furthest downstream turbine. Furthermore, when looking at both the furthest upstream turbines at the same distance and wind speed, both turbines 6 and 11 have 0 tilt. When looking at $\Delta Tilt$, there are quite some differences in tilt between the two turbines. There are significant differences in tilt for all the variables, except for a distance of 5D with wind speeds of 7 and 8 $\frac{m}{s}$, where the $\Delta Tilt$ is 0 for all the turbines.

Table 4.11: Tilt and power between DTU 10 MW and IEA 15 MW turbines. Using BG wake model and DE optimizer

Distance = 4D				**Distance = 5D**			
Ws5				**Ws5**			
Turbine	**Tilt [°]**	**Pturb [MW]**	**ΔTilt [°]**	**Turbine**	**Tilt [°]**	**Pturb [MW]**	**ΔTilt [°]**
1	-15.56	0.07	15.56	1	0.00	0.17	0.00
2	-14.74	0.07	-15.26	2	30.00	0.07	-8.60
3	-26.67	0.07	43.32	3	0.00	0.18	0.00
4	-7.91	0.07	33.99	4	-30.00	0.07	60.00
5	-19.69	0.07	19.69	5	0.00	0.19	0.00
6	0.00	1.43	0.00	6	0.00	1.43	0.00
7	-2.02	0.07	2.02	7	0.00	0.09	0.00
8	8.36	0.07	4.48	8	0.00	0.18	-21.03
9	-2.09	0.07	2.09	9	-30.00	0.07	30.00
10	0.13	0.07	-0.30	10	0.00	0.19	-23.89
11	0.00	1.43	0.00	11	0.00	1.43	0.00
Ptot [MW]		3.49		**Ptot [MW]**		4.07	
Pbench [MW]		3.49		**Pbench [MW]**		3.80	
Difference[MW]		0.00		**Difference[MW]**		0.28	
Ws7				**Ws7**			
Turbine	**Tilt [°]**	**Pturb [MW]**	**ΔTilt [°]**	**Turbine**	**Tilt [°]**	**Pturb [MW]**	**ΔTilt [°]**
1	0.01	0.27	-0.01	1	0.00	0.71	0.00
2	-0.01	0.28	-29.99	2	0.00	0.73	0.00
3	0.00	0.31	-11.79	3	0.00	0.75	0.00
4	0.00	0.38	-30.00	4	0.00	0.82	0.00
5	0.00	0.58	15.25	5	0.00	1.01	0.00
6	0.00	4.34	0.00	6	0.00	4.34	0.00
7	0.00	0.30	0.00	7	0.00	0.73	0.00
8	0.00	0.37	30.00	8	0.00	0.75	0.00
9	-0.01	0.65	13.91	9	0.00	0.82	0.00
10	30.00	0.27	0.00	10	0.00	1.01	0.00
11	0.00	4.34	0.00	11	0.00	4.34	0.00
Ptot [MW]		12.09		**Ptot [MW]**		16.01	
Pbench [MW]		12.04		**Pbench [MW]**		16.01	
Difference[MW]		0.05		**Difference[MW]**		0.00	

Table 4.12: Tilt and power between DTU 10MW and IEA 15MW turbines. Using Bastankhah Gaussian wake model and DE optimizer

Distance = 4D				**Distance = 5D**			
Ws8				**Ws8**			
Turbine	**Tilt [°]**	**Pturb [MW]**	**ΔTilt [°]**	**Turbine**	**Tilt [°]**	**Pturb [MW]**	**ΔTilt [°]**
1	0.00	0.56	0.00	1	0.00	1.19	0.00
2	0.00	0.59	0.00	2	0.00	1.21	0.00
3	0.00	0.69	-30.00	3	0.00	1.25	0.00
4	0.00	1.10	-17.87	4	0.00	1.35	0.00
5	30.00	0.56	-30.00	5	0.00	1.65	0.00
6	0.00	6.48	0.00	6	0.00	6.48	0.00
7	0.00	0.65	0.00	7	0.00	1.21	0.00
8	0.00	0.97	-30.00	8	0.00	1.25	0.00
9	-30.00	0.36	12.39	9	0.00	1.35	0.00
10	0.00	1.00	0.00	10	0.00	1.65	0.00
11	0.00	6.48	0.00	11	0.00	6.48	0.00
Ptot [MW]		19.42		**Ptot [MW]**		25.05	
Pbench [MW]		19.25		**Pbench [MW]**		25.05	
Difference[MW]		0.17		**Difference[MW]**		0.00	

4.2.2 Different Wind Directions

Wind directions 200°, 210°, 220°, and 270°provided 0 tilt for all turbines and selections of data. This resulted in the total power output being equal to the benchmark power, and also the difference in tilt compared to the initial result being the tilts of the initial results. These Tables can be found in Appendix D

4.2.3 3 Rows

Table 4.13 shows the result for a wind farm layout with 3 turbine rows, presenting 4D and 5D distances with 5 and 7 $\frac{m}{s}$ wind speeds. The $Tilt$ and $Pturb$ still show the resulting tilt and power for each turbine, but the $\Delta Tilt$ for turbines 12-17 are calculated using tilt values from turbines 1-6 from the initial simulation. These can be compared as the turbines are placed similarly, with only the placement on the x-axis being different. When comparing the tilt values for the corresponding turbines, 1 and 12, 2 and 13, and so on, the tilt results are either almost identical or with similar values but opposite signs. This can be seen between turbines 2 and 13 at a 4D distance and wind speed of 5$\frac{m}{s}$, with their tilt being -30 and 30°. For $\Delta Tilt$ these opposite angles make for some extreme values, such as -60°for turbine 13 at a distance of 4D and wind speed of 5 $\frac{m}{s}$. Furthermore, there are differences in $\Delta Tilt$, and it can be seen that the tilt difference is spread throughout the distances, wind speeds, and turbines. For a distance of 5D and wind speed of 7 $\frac{m}{s}$ there is consistency with both all tilt and $\Delta Tilt$ angles being 0. Furthermore, the difference between $Ptot$ and $Pbench$ indicates a power increase for all scenarios, with the exception being 5D distance at a wind speed of 7$\frac{m}{s}$, where all tilts are 0, thus the difference being zero. The highest increase in total power can be seen in the 4D distance at 7$\frac{m}{s}$ with a 2.68 MW increase, which is almost a 25% increase in power compared to 0 tilt. Moreover, all the first and last turbines in each row provide 0 tilt, with the turbines being 1, 6, 7, 11, 12, and 17.

Table 4.13: Tilt and power between turbines in 3 rows

Distance = 4D				**Distance = 5D**			
Ws5				**Ws5**			
Turbine	**Tilt [°]**	**Pturb [MW]**	**ΔTilt [°]**	**Turbine**	**Tilt [°]**	**Pturb [MW]**	**ΔTilt [°]**
1	0.00	0.38	0.00	1	0.01	0.34	-0.01
2	-30.00	0.04	0.00	2	21.40	0.03	0.00
3	16.59	0.12	0.06	3	0.00	0.39	0.00
4	-26.06	0.16	52.14	4	30.00	0.08	0.00
5	-0.05	0.02	0.05	5	0.02	0.11	-0.02
6	0.00	0.80	0.00	6	0.00	0.80	0.00
7	0.00	0.24	0.00	7	-0.01	0.34	0.01
8	-0.86	0.00	13.70	8	21.03	0.03	-42.06
9	0.00	0.27	0.01	9	0.00	0.37	0.00
10	0.07	0.02	-0.24	10	23.89	0.03	-47.77
11	0.00	0.80	0.00	11	0.00	0.80	0.00
12	0.01	0.38	-0.01	12	0.01	0.34	-0.01
13	30.00	0.04	-60.00	13	-21.41	0.03	42.81
14	16.75	0.12	-0.11	14	0.00	0.39	0.00
15	26.11	0.16	-0.04	15	30.00	0.08	0.00
16	-0.02	0.02	0.03	16	-0.01	0.11	0.01
17	0.00	0.80	0.00	17	0.00	0.80	0.00
Ptot [MW]		4.38		**Ptot [MW]**		5.07	
Pbench [MW]		3.98		**Pbench [MW]**		4.61	
Difference [MW]		0.39		**Difference [MW]**		0.45	
Ws7				**Ws7**			
Turbine	**Tilt [°]**	**Pturb [MW]**	**ΔTilt [°]**	**Turbine**	**Tilt [°]**	**Pturb [MW]**	**ΔTilt [°]**
1	0.00	0.82	0.00	1	0.00	0.47	0.00
2	-30.00	0.10	0.00	2	0.00	0.48	0.00
3	11.79	0.88	-23.59	3	0.00	0.50	0.00
4	-30.00	0.05	0.00	4	0.00	0.54	0.00
5	-15.25	0.25	30.51	5	0.00	0.58	0.00
6	0.00	2.51	0.00	6	0.00	2.51	0.00
7	0.00	0.87	0.00	7	0.00	0.48	0.00
8	30.00	0.09	0.00	8	0.00	0.50	0.00
9	-13.90	0.70	27.80	9	0.00	0.54	0.00
10	30.00	0.14	0.00	10	0.00	0.58	0.00
11	0.00	2.51	0.00	11	0.00	2.51	0.00
12	0.00	0.93	0.00	12	0.00	0.47	0.00
13	-30.00	0.05	0.00	13	0.00	0.48	0.00
14	18.04	0.25	-29.83	14	0.00	0.50	0.00
15	0.00	0.77	-30.00	15	0.00	0.54	0.00
16	-30.00	0.14	45.25	16	0.00	0.58	0.00
17	0.00	2.51	0.00	17	0.00	2.51	0.00
Ptot [MW]		13.54		**Ptot [MW]**		14.73	
Pbench [MW]		10.86		**Pbench [MW]**		14.73	
Difference [MW]		2.68		**Difference [MW]**		0.00	

4.2.4 4 Rows

Lastly, Tables 4.14 and 4.15 contains the results from a wind farm layout containing 4 rows. It follows the same format as for 3 rows, with turbines 12-22 being compared to turbines 1-11 from the initial simulation in $\Delta Tilt$. These results follow the same pattern as for 3 rows, with the highest difference, thus power increase, being in the 4D distance at a wind speed of $7\frac{m}{s}$, with a total increase of 3.48 MW, which is just above a 24% increase. It also shows that the tilt and power increase for a distance of 5D at wind speed $7\frac{m}{s}$ is 0, while the remaining scenarios have one or more turbines tilting, which provides a power increase. There are some bigger variations in $\Delta Tilt$ in these results, which could be a result of the optimization having an increased number of turbines to find an optimal tilt, potentially increasing the number of solutions giving the same power output. It can still be seen that the difference in turbine tilt varies with the different turbines across the distances and wind speeds, but turbines 8 and 15 provide some tilt difference across three variations of distances and wind speeds. It can be seen that the turbines furthest up and downstream, turbines 1, 6, 7, 11, 12, 17, 18, and 22 provide 0 tilt for all scenarios.

Table 4.14: Tilt and power between turbines in 4 rows

Distance = 4D				**Distance = 5D**			
Ws5				**Ws5**			
Turbine	**Tilt [°]**	**Pturb [MW]**	**ΔTilt [°]**	**Turbine**	**Tilt [°]**	**Pturb [MW]**	**ΔTilt [°]**
1	0.00	0.38	0.00	1	0.00	0.36	0.00
2	-30.00	0.04	0.00	2	-26.74	0.00	48.14
3	-16.66	0.12	33.30	3	0.00	0.39	0.00
4	26.08	0.16	0.00	4	30.00	0.08	0.00
5	-0.01	0.02	0.01	5	0.01	0.11	-0.01
6	0.00	0.80	0.00	6	0.00	0.80	0.00
7	0.00	0.38	0.00	7	0.00	0.34	0.00
8	-30.00	0.04	42.84	8	21.03	0.03	-42.06
9	26.99	0.15	-26.99	9	0.00	0.37	0.00
10	25.19	0.00	-25.36	10	-23.89	0.03	0.00
11	0.00	0.80	0.00	11	0.00	0.80	0.00
12	0.00	0.38	0.00	12	0.00	0.41	0.00
13	30.00	0.04	-60.00	13	-30.00	0.05	51.40
14	16.54	0.12	0.11	14	0.37	0.14	-0.37
15	-26.49	0.16	52.57	15	-15.00	0.18	45.00
16	-17.48	0.00	17.48	16	-0.02	0.11	0.02
17	-0.01	0.80	0.01	17	0.00	0.80	0.00
18	0.01	0.24	-0.01	18	0.00	0.34	0.00
19	-9.18	0.00	22.02	19	-21.03	0.03	0.00
20	0.00	0.27	0.00	20	0.00	0.37	0.00
21	0.02	0.02	-0.19	21	-23.89	0.03	0.00
22	0.00	0.80	0.00	22	0.00	0.80	0.00
Ptot [MW]		5.73		**Ptot [MW]**		6.57	
Pbench [MW]		5.31		**Pbench [MW]**		6.09	
Difference[MW]		0.42		**Difference[MW]**		0.48	

Table 4.15: Tilt and power between turbines in 4 rows

Distance = 4D				**Distance = 5D**			
Ws7				**Ws7**			
Turbine	**Tilt [°]**	**Pturb [MW]**	**ΔTilt [°]**	**Turbine**	**Tilt [°]**	**Pturb [MW]**	**ΔTilt [°]**
1	0.00	0.93	0.00	1	0.00	0.47	0.00
2	-30.00	0.05	0.00	2	0.00	0.48	0.00
3	18.04	0.25	-29.83	3	0.00	0.50	0.00
4	0.00	0.77	-30.00	4	0.00	0.54	0.00
5	-30.00	0.14	45.25	5	0.00	0.58	0.00
6	0.00	2.51	0.00	6	0.00	2.51	0.00
7	0.00	0.87	0.00	7	0.00	0.48	0.00
8	-30.00	0.09	60.00	8	0.00	0.50	0.00
9	-13.90	0.70	27.80	9	0.00	0.54	0.00
10	-30.00	0.14	60.00	10	0.00	0.58	0.00
11	0.00	2.51	0.00	11	0.00	2.51	0.00
12	0.00	0.77	0.00	12	0.00	0.47	0.00
13	-30.00	0.12	0.00	13	0.00	0.48	0.00
14	8.67	0.91	-20.46	14	0.00	0.50	0.00
15	30.00	0.05	-60.00	15	0.00	0.54	0.00
16	-15.20	0.25	30.45	16	0.00	0.58	0.00
17	0.00	2.51	0.00	17	0.00	2.51	0.00
18	0.00	0.87	0.00	18	0.00	0.48	0.00
19	30.00	0.09	0.00	19	0.00	0.50	0.00
20	-13.90	0.70	27.80	20	0.00	0.54	0.00
21	30.00	0.14	0.00	21	0.00	0.58	0.00
22	0.00	2.51	0.00	22	0.00	2.51	0.00
Ptot [MW]		17.84		**Ptot [MW]**		19.32	
Pbench [MW]		14.36		**Pbench [MW]**		19.32	
Difference[MW]		3.48		**Difference[MW]**		0.00	

4.3 Comparison and Discussion

4.3.1 Initial simulation

Even though there are power differences between the two wake deficit models, they showcase the same trend regarding power and tilt. The resulting power outputs indicate a higher influence of tilt at lower wind speeds and distances between turbines. A difference between the two wake deficit models is that NOJ appears to be more influenced by the wind speed, while BG is more influenced by the distance between the turbines when regarding the difference in tilted power compared to 0 tilt power.

The initial results indicate that there might be a slight power gain for floating wind turbines using optimal tilt angles for all turbines, compared to a zero-tilt wind farm. When looking at the percentage gain, it indicates a relatively big increase in power. This needs to be considered with the actual power gain, and at what wind speeds and the distances between turbines to give a realistic overview of the gain. For instance, at wind speed 4 $\frac{m}{s}$ the highest percentage gain is 8.15% for NOJ at 3D, and when looking at actual power gain, the value is 0.12 MW. Figure 4.7 visualizes the relation between wind speed, distances, and percentage gain, using values from Table 4.5. It emphasizes the findings of where the potential gain can be found, and that a combination of high wind speeds and distances results in 0% gain.

When comparing power output for 180°and 190°wind direction, since 190°always provided higher power output when compared to optimized tilted 180°, it shows that for this layout a difference in turbine layout could be more beneficial when strictly looking at power output.

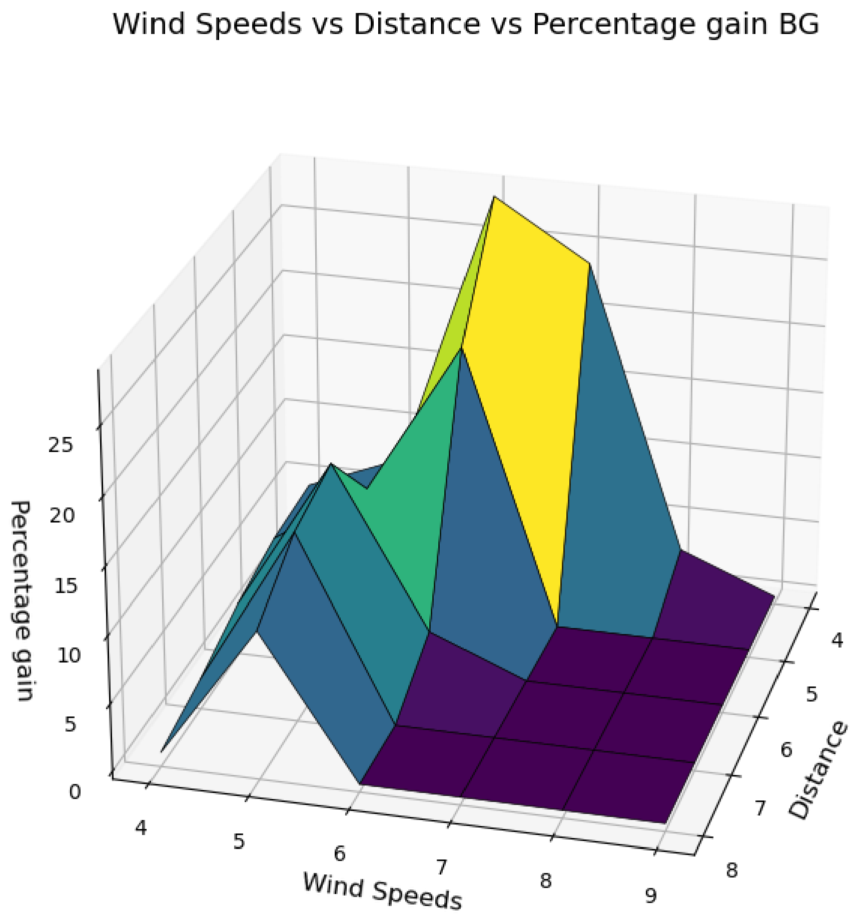

Figure 4.7: 3D plot showing wind speeds, distances and percentage power gain

4.3.2 Comparing with different turbines

As mentioned in the results for both IEA 10 and 15 MW turbines, they provided tilt angles for both turbines 1 and 7, which are the turbines located furthest downstream, at 4D distance and 5 $\frac{m}{s}$ wind speed. This is an interesting result, especially considering the high tilt angles provided, with the maximum being -15.56°. This can be a result of the low wind speed and distance, resulting in small variations in power output. As the Ptot and Pbench are equal, it shows that there is no overall power loss for the wind farm, and implies that the power loss is small enough to be insignificant. These findings also further enhance the possibility of having multiple tilts providing maximum power output. It is also interesting that corresponding turbines provide such different tilt angles since the overall conditions are similar. There is a difference in diameter size between the turbines, CT-curve, and height for the IEA 15 MW compared to the two 10 MW turbines which could be the explanation. Even so, the wind farm layout is dynamic throughout the different distances between turbines and is defined by the turbine diameters. This gives further insight into the difficulty of accurately simulating and predicting wake effects for different turbines.

4.3.3 Comparing different wind directions

As all power outputs for all the different wind directions resulted in zero power gain from tilt, and there were no tilts, there is a clear indication of how much impact the wind direction has on tilts for a wind farm. Furthermore, this indicates that optimal tilts might only provide higher power output when exposed to unfavorable wake conditions.

4.3.4 Comparing with different amount of rows

As the increased amount of rows is spaced out similarly to the initial results, the overall trends should be the same, which they are. There are some differences in the tilts between the increased rows and the initial simulation, except for two turbines in the layout with 4 rows, there was an inconsistency in which turbines had a difference in tilt. Two reasons for these differences could be wake deficit across the rows, especially at lower distances between turbines, and the increased number of tilts for DE to take into consideration while optimizing for maximum power output.

4.3.5 Comparing with other studies

Table 4.16 shows the tilt and power values for all BG at 6D distance and 5 $\frac{m}{s}$, and represents the data which will be used for comparison. Table 4.17 includes some studies done on tilt and the resulting power gain or loss from them. These studies include various wake or simulation models, but cases including BG and Simulator for Wind Farm Applications (SOWFA) were included in the comparison. The tilts also differ from the sources, and Annoni et al. [15] use polar opposite tilts when referring to tilt angles in their study. For this study, the tilts used for comparison can be seen in 4.16. All sources use distances in a range between 6-8D, while Cutler et al. [16], while this thesis has an increased number of turbines compared to the others, with 11 compared to 2-3. Lastly, the gain in percentage varies for all studies. This thesis provides the highest percentage gain with 14.7%, while closely followed by Annoni et al.[15] at 13%. Annoni et al. also present the worst percentage gain, with a value of -3.6% resulting in a loss, with the turbines tilting in the opposite direction. Compared to the other studies, particularly studies using BG, it is expected that this thesis has a higher gain, as this thesis aimed to optimize tilts individually within a significant range of -30 to 30°. Particularly compared to Leikvolls[13], who only included positive tilt as a result of incoming wind. It is also worth noting that Annoni et al.[15] compared their gain values to a 5°tilt, or -5°tilt for their study, and not to a 0°as done in this thesis.

Table 4.16: Tilts and power using BG at 6D and 5 $\frac{m}{s}$ used for comparison

Turbine no.	**Tilt [°]**	Pturb [MW]
1	0.00	0.43
2	30.00	0.06
3	4.48	0.16
4	0.00	0.44
5	29.80	0.05
6	0.00	0.80
7	0.01	0.42
8	28.21	0.05
9	0.00	0.44
10	29.91	0.04
11	0.00	0.80
Ptot [MW]		3.70
Pbench [MW]		3.23
Difference [MW]		0.47
Difference [%]		14.55

Table 4.17: Results from various studies

Source	**This thesis**	**Cutler et al.[16]**	**Nanos et al.[12]**	**Leikvoll[13]**	**Annoni et al.[15]**	**Fleming et al.[14]**
Turbine size [MW]	10	5	10	15	5	5
Wake model/Simulation	BG	BG	CFD/Wind tunnel	BG	SOWFA	SOWFA
Tilt [°]	Varying	20-25	-15	10	-25, 25 *	-30
Distance	6D	7	6-8D	7D	7D	7D
No. Turbines	11	2	2	3	3	2
Gain [%]	14.6	6	7-8	2.84	13, -3.6	7

4.4 Limitations

When simulating and using optimization algorithms there will always be a discussion between accuracy and efficiency computational-wise. Increasing the accuracy comes at a cost of time and or resources which is not always available. Engineering models and built-in optimizers bypass these by being more time-efficient, making them more manageable. The engineering models used for wake deficit have their limitations by using simplifications and assumptions. It might not always fully capture the complexity of a wake, especially near-wake. This might influence particularly the results with low distance between turbines, both within each row, but also across each of the rows. This can further be affected by a simplified wake distribution, where the NOJ wake deficit is constant for a given downwind distance, and the BG has a Gaussian distribution. Furthermore, there has not been implemented any deflection, turbulence, or ground models, which makes for a further simplification of the wake interactions.

Chapter 5

Conclusions

5.1 Research Questions

RQ1

Are there optimal tilt angles in a wind farm, or will the optimal always be 0?

- There seem to be optimal tilt angles for the turbines, but only when the distances between turbines are smaller than 6D and wind speeds are lower than 8 $\frac{m}{s}$. There also seem to be some turbines that always have an optimal tilt of 0, with both the turbines furthest upstream and downstream typically having an optimal tilt of 0. However, there does not seem to be a single tilt angle for each turbine which provides the optimal result. The optimal tilts vary with the changing parameters, indicating no global solutions. Furthermore, it can be seen that there are some occasions where a wind farm may benefit from an optimized rotor tilt of the turbines.

RQ2

At what wind speeds and distances between turbines are there optimal tilt angles?

- Turbines subject to greater wake deficits seem to be able to provide a power increase by having optimal tilt angles. In other words, a higher percentage of power gain can be seen at smaller distances between turbines, less than 6D downstream, and lower wind speeds, less than 8 $\frac{m}{s}$.

RQ3

Will different turbine types, turbine sizes, wind directions, or additional rows provide the same optimal tilt angles?

-There did not seem to be any conclusive results from this thesis for all individual turbines. However, minimum and maximum tilts of -30 and 30°were often represented for one or more of the turbines when the results provided some optimal tilts. Moreover, the furthest upstream and downstream turbines for each row rarely were provided with a tilt.

RQ4

To what extent are the results dependent on different wake deficit models used?
NOJ provides a higher total power output compared to BG, but when looking at percentage

gain for optimized tilts, BG has a generally higher percentage gain. The overall trends between the two wake deficit models are relatively similar, with the majority of power increases being found at lower wind speeds and distances between turbines. So from a power output perspective, the results are heavily dependent on the wake deficit model, with the greatest difference in power increase between the wake deficit models being 3.4 MW at 3D distance with a wind speed of $9\frac{m}{s}$ and a direction of 180°.

Chapter 6

Recommendations

6.1 Compare realistic tilt with real data

Get real data from an actual wind farm and compare the simulated data with the actual data. This is mainly interesting to verify how well the simulation models work and makes it possible to tune the simulations for particularly that specific site. Preferably having power data for each turbine, along with actual tilt values from the turbine structure. This would also further increase the realism of the data, as it could implement more degrees of freedom on the floating turbine, as well seeing the impact of a dynamic turbine at actual floating conditions.

6.2 CFD

As many have stated before, and some already have tested, a CFD analysis would be highly beneficial to implement a more accurate model and simulate how the dynamic response of the turbine would be at different conditions. This could also provide information on what different engineering models provide the most accurate data. It would also be interesting to see if an active tilt control can be implemented, either through rotor tilt, or pitch of the platform, and how quickly it adjusts to changes in wind speed and direction , and how this impacts the overall output.

6.3 Economical point

A new question related to comparing power gain from tilts to power loss from wake deficit could also be seen from a more economical perspective. This perspective could include multiple advantages, such as a wind farm covering a smaller area, shared mooring or anchor systems, and easier maintenance, and disadvantages such as higher loads on the turbines and structure from both the tilted angles and the wake deficit.

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Appendix A

Python scripts:

Only 1D distance between turbines are provided for the codes. To change the distance, simply change *distance* = to any distance. Furthermore, since most of the codes as identical, only the full code will be shown for the initial NOJ and BG, before only providing the differences done for each code.

A.1 Initial code for all simulations

```
import pandas as pd
from scipy import optimize
import numpy as np
from py_wake import NOJ
from py_wake.deficit_models.gaussian import BastankhahGaussian
from py_wake.wind_turbines import WindTurbine
from py_wake.wind_turbines.power_ct_functions import PowerCtTabular
from py_wake.site import XRSite
import xarray as xr
from scipy.special import gamma

#------------------Inputs for PyWake---------------

# Load site data from Gullfaks, taken from OpenMeteo
Site = pd.read_csv('open-meteo-61.20N2.17E0m.csv')

# Assign sector based on wind direction
def assign_sector(direction):
    return np.floor(direction / 30) % 12

Site['Sector'] = Site['wind_direction_100m'].apply(assign_sector)

# Calculate wind direction frequency
frequency_per_sector = Site['Sector'].value_counts().sort_index() / ...
   Site.shape[0]
percentage_df = ...
   frequency_per_sector.reset_index().rename(columns={'index': 'Sector', ...
   'Sector': 'Percentage'})

# Prepare lists to store values
sector_data = {'Sector': [], 'Std': [], 'Mean': [], 'k': [], 'A': []}

# Iterate through unique sectors and filter for current sector
for sector in np.unique(Site['Sector']):
    sector_data_filtered = Site[Site['Sector'] == sector]

    # Calculate standard deviation and mean
```

```
    std_value = sector_data_filtered['windspeed_100m'].std()
    mean_value = sector_data_filtered['windspeed_100m'].mean()

    # Calculate k and c using the wind speed data
    if std_value > 0 and mean_value > 0:
        k_value = (std_value / mean_value) ** (-1.09)
        A_value = mean_value / gamma(1 + 1/k_value)
    else:
        k_value = None
        A_value = None

    # Append the results
    sector_data['Sector'].append(sector + 1)
    sector_data['Std'].append(std_value)
    sector_data['Mean'].append(mean_value)
    sector_data['k'].append(k_value)
    sector_data['A'].append(A_value)

# Convert to DataFrame
sector_results_df = pd.DataFrame(sector_data)

#Weibull values
f = percentage_df['Percentage']
A = sector_results_df['A']
k = sector_results_df['k']
wd = np.linspace(0, 360, len(f), endpoint=False)

#Turbulence Intensity
ti = .1

# Site with wind direction dependent weibull distributed wind speed
my_site = XRSite(
    ds=xr.Dataset(data_vars={'Sector_frequency': ('wd', f), 'Weibull_A': ...
       ('wd', A), 'Weibull_k': ('wd', k), 'TI': ti},
                  coords={'wd': wd}))

#------------------Defining Functions---------------
# Simulation function (to be used as GA fitness function)
def wind_farm_simulation(tilts, wind_speed):
    sim_res = wake(location_x, location_y, ...
       tilt=np.array(tilts).reshape((num_turbines, 1)), ...
       wd=wind_direction, ws=wind_speed)
    individual_power_output = sim_res.Power.values
    flattened_output = individual_power_output.flatten()
    return flattened_output / (10**6) #MW


# Objective Function: Maximize the total power output
def obj_function(tilt_angles):
    total_power_output = wind_farm_simulation(tilt_angles, wind_speed)
    return -np.sum(total_power_output)

def wrapped_obj_function(x):
    result = obj_function(x)
    obj_function_values.append(result*(-1))
    return result
```

A.2 Initial simulation - NOJ

```
#Turbine values
num_turbines = 11
diameter = 178.3
hub_height = 119

# Load and filter data for wind speed between 3-25 m/s
DTU_10MW = pd.read_csv("DTU_10MW_178_RWT_v1.csv")
filtered_DTU_10MW = DTU_10MW[(DTU_10MW['Wind Speed'] >= 3) & ...
    (DTU_10MW['Wind Speed'] <= 25)]

# Sort out data needed
u = filtered_DTU_10MW['Wind Speed']
power = filtered_DTU_10MW['Power']
ct = filtered_DTU_10MW['Ct']

# Initialize wind turbine and site
custom_turbine = WindTurbine('DTU_10MW', diameter, hub_height, ...
    PowerCtTabular(u, power, 'kW', ct))
site = my_site
wfm = NOJ(site, custom_turbine)
wake = wfm

#------------------Simulation characteristics---------------


#Tilt Thresholds
tilt_min = -30
tilt_max = 30

#Site data
wind_speeds = [1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 20, 25]
wind_direction = 180
distance = 1
optimizers = ['shgo', 'DA', 'DE']

# Define turbine coordinates
location_x = [0, 0, 0, 0, 0, 0, -distance*diameter, -distance*diameter, ...
    -distance*diameter, -distance*diameter, -distance*diameter,]
location_y = [0, -distance*diameter, -2*distance*diameter, ...
    -3*distance*diameter, -4*distance*diameter, -5*distance*diameter, ...
    -0.5*distance*diameter, -1.5*distance*diameter, ...
    -2.5*distance*diameter, -3.5*distance*diameter, -4.5*distance*diameter]



#-----------------------------The ...
    algortihm-------------------------------------------

# Bounds for each tilt angle, assuming tilt_min and tilt_max are defined
bounds = [(tilt_min, tilt_max)] * num_turbines

results = []

result = dict()

obj_function_values = []
for wind_speed in wind_speeds:
    for optimizer in optimizers:

        # Select and run optimizer
        if optimizer == 'DE':
```

```
            result = optimize.differential_evolution(obj_function, bounds)
        elif optimizer == 'DA':
            result = optimize.dual_annealing(obj_function, bounds)
        elif optimizer == 'shgo':
            result = optimize.shgo(obj_function, bounds, n=1, iters=1)
        else:
            raise ValueError(f"Unknown optimizer: {optimizer}")

        # Process and store results
        optimized_tilts = result.x
        individual_powers = wind_farm_simulation(optimized_tilts, wind_speed)

        results.append({
            'wind_speed': wind_speed,
            'optimizer': optimizer,
            'optimized_tilts': optimized_tilts,
            'optimized_power': -result.fun,
            'individual_powers': individual_powers,
        })


results_df = pd.DataFrame(results)


tilt_columns = [f'tilt_{i+1}' for i in range(num_turbines)]
individual_power_columns = [f'power_turbine_{i+1}' for i in ...
    range(num_turbines)]

results_df[tilt_columns] = ...
    pd.DataFrame(results_df['optimized_tilts'].tolist(), ...
    index=results_df.index)
results_df[individual_power_columns] = ...
    pd.DataFrame(results_df['individual_powers'].tolist(), ...
    index=results_df.index)

results_df.drop('optimized_tilts', axis=1, inplace=True)
results_df.drop('individual_powers', axis=1, inplace=True)

column_order = ['wind_speed', 'optimizer', 'optimized_power'] + ...
    tilt_columns + individual_power_columns
results_df = results_df[column_order]

filename = f'NOJ_{distance}D.csv'
results_df.to_csv(filename, index=False)
```

0 tilt

```
#Turbine values
num_turbines = 11
diameter = 178.3
hub_height = 119

# Load and filter data for wind speed between 3-25 m/s
DTU_10MW = pd.read_csv("DTU_10MW_178_RWT_v1.csv")
filtered_DTU_10MW = DTU_10MW[(DTU_10MW['Wind Speed'] >= 3) & ...
    (DTU_10MW['Wind Speed'] <= 25)]

# Sort out data needed
u = filtered_DTU_10MW['Wind Speed']
power = filtered_DTU_10MW['Power']
ct = filtered_DTU_10MW['Ct']
```

```
tilt_ws = filtered_DTU_10MW['Tilt']

# Initialize wind turbine and site
custom_turbine = WindTurbine('DTU_10MW', diameter, hub_height, ...
    PowerCtTabular(u, power, 'kW', ct))
site = my_site
wfm = NOJ(site, custom_turbine)
wake = wfm

#------------------Simulation characteristics---------------


#Tilt Thresholds
tilt_min = -0.01
tilt_max = 0.01

#Site data
wind_speeds = [1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 20, 25]
wind_direction = 180
distance = 1
optimizers = ['shgo', 'DA', 'DE']

# Define turbine coordinates
location_x = [0, 0, 0, 0, 0, 0, -distance*diameter, -distance*diameter, ...
    -distance*diameter, -distance*diameter, -distance*diameter,]
location_y = [0, -distance*diameter, -2*distance*diameter, ...
    -3*distance*diameter, -4*distance*diameter, -5*distance*diameter, ...
    -0.5*distance*diameter, -1.5*distance*diameter, ...
    -2.5*distance*diameter, -3.5*distance*diameter, -4.5*distance*diameter]



#-----------------------------The ...
    algortihm-------------------------------------------

# Bounds for each tilt angle, assuming tilt_min and tilt_max are defined
bounds = [(tilt_min, tilt_max)] * num_turbines

results = []

result = dict()

obj_function_values = []
for wind_speed in wind_speeds:
    for optimizer in optimizers:

        # Select and run optimizer
        if optimizer == 'DE':
            result = optimize.differential_evolution(obj_function, bounds)
        elif optimizer == 'DA':
            result = optimize.dual_annealing(obj_function, bounds)
        elif optimizer == 'shgo':
            result = optimize.shgo(obj_function, bounds, n=1, iters=1)
        else:
            raise ValueError(f"Unknown optimizer: {optimizer}")

        # Process and store results
        optimized_tilts = result.x
        individual_powers = wind_farm_simulation(optimized_tilts, wind_speed)

        results.append({
```

```
            'wind_speed': wind_speed,
            'optimizer': optimizer,
            'optimized_tilts': optimized_tilts,
            'optimized_power': -result.fun,
            'individual_powers': individual_powers,
        })


results_df = pd.DataFrame(results)


tilt_columns = [f'tilt_{i+1}' for i in range(num_turbines)]
individual_power_columns = [f'power_turbine_{i+1}' for i in ...
    range(num_turbines)]

results_df[tilt_columns] = ...
    pd.DataFrame(results_df['optimized_tilts'].tolist(), ...
    index=results_df.index)
results_df[individual_power_columns] = ...
    pd.DataFrame(results_df['individual_powers'].tolist(), ...
    index=results_df.index)

results_df.drop('optimized_tilts', axis=1, inplace=True)
results_df.drop('individual_powers', axis=1, inplace=True)

column_order = ['wind_speed', 'optimizer', 'optimized_power'] + ...
    tilt_columns + individual_power_columns
results_df = results_df[column_order]

filename = f'NOJ_0_tilt_{distance}D.csv'
results_df.to_csv(filename, index=False)
```

A.3 Initial simulation - BG

```
#Turbine values
num_turbines = 11
diameter = 178.3
hub_height = 119

# Load and filter data for wind speed between 3-25 m/s
DTU_10MW = pd.read_csv("DTU_10MW_178_RWT_v1.csv")
filtered_DTU_10MW = DTU_10MW[(DTU_10MW['Wind Speed'] >= 3) & ...
    (DTU_10MW['Wind Speed'] <= 25)]

# Sort out data needed
u = filtered_DTU_10MW['Wind Speed']
power = filtered_DTU_10MW['Power']
ct = filtered_DTU_10MW['Ct']

# Initialize wind turbine and site
custom_turbine = WindTurbine('DTU_10MW', diameter, hub_height, ...
    PowerCtTabular(u, power, 'kW', ct))
site = my_site
wfm = BastankhahGaussian(site, custom_turbine)
wake = wfm

#------------------Simulation characteristics---------------
```

```
#Tilt Thresholds
tilt_min = -30
tilt_max = 30

#Site data
wind_speeds = [1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 20, 25]
wind_direction = 180
distance = 1
optimizers = ['shgo', 'DA', 'DE']

# Define turbine coordinates
location_x = [0, 0, 0, 0, 0, 0, -distance*diameter, -distance*diameter, ...
    -distance*diameter, -distance*diameter, -distance*diameter,]
location_y = [0, -distance*diameter, -2*distance*diameter, ...
    -3*distance*diameter, -4*distance*diameter, -5*distance*diameter, ...
    -0.5*distance*diameter, -1.5*distance*diameter, ...
    -2.5*distance*diameter, -3.5*distance*diameter, -4.5*distance*diameter]



#-----------------------------The ...
    algortihm-------------------------------------------

# Bounds for each tilt angle, assuming tilt_min and tilt_max are defined
bounds = [(tilt_min, tilt_max)] * num_turbines

results = []

result = dict()

obj_function_values = []
for wind_speed in wind_speeds:
    for optimizer in optimizers:

        # Select and run optimizer
        if optimizer == 'DE':
            result = optimize.differential_evolution(obj_function, bounds)
        elif optimizer == 'DA':
            result = optimize.dual_annealing(obj_function, bounds)
        elif optimizer == 'shgo':
            result = optimize.shgo(obj_function, bounds, n=1, iters=1)
        else:
            raise ValueError(f"Unknown optimizer: {optimizer}")

        # Process and store results
        optimized_tilts = result.x
        individual_powers = wind_farm_simulation(optimized_tilts, wind_speed)

        results.append({
            'wind_speed': wind_speed,
            'optimizer': optimizer,
            'optimized_tilts': optimized_tilts,
            'optimized_power': -result.fun,
            'individual_powers': individual_powers,
        })


results_df = pd.DataFrame(results)
```

```
tilt_columns = [f'tilt_{i+1}' for i in range(num_turbines)]
individual_power_columns = [f'power_turbine_{i+1}' for i in ...
    range(num_turbines)]

results_df[tilt_columns] = ...
    pd.DataFrame(results_df['optimized_tilts'].tolist(), ...
    index=results_df.index)
results_df[individual_power_columns] = ...
    pd.DataFrame(results_df['individual_powers'].tolist(), ...
    index=results_df.index)

results_df.drop('optimized_tilts', axis=1, inplace=True)
results_df.drop('individual_powers', axis=1, inplace=True)

column_order = ['wind_speed', 'optimizer', 'optimized_power'] + ...
    tilt_columns + individual_power_columns
results_df = results_df[column_order]

filename = f'BastGauss_{distance}D.csv'
results_df.to_csv(filename, index=False)
```

0 tilt

```
#Turbine values
num_turbines = 11
diameter = 178.3
hub_height = 119

# Load and filter data for wind speed between 3-25 m/s
DTU_10MW = pd.read_csv("DTU_10MW_178_RWT_v1.csv")
filtered_DTU_10MW = DTU_10MW[(DTU_10MW['Wind Speed'] >= 3) & ...
    (DTU_10MW['Wind Speed'] <= 25)]

# Sort out data needed
u = filtered_DTU_10MW['Wind Speed']
power = filtered_DTU_10MW['Power']
ct = filtered_DTU_10MW['Ct']
tilt_ws = filtered_DTU_10MW['Tilt']

# Initialize wind turbine and site
custom_turbine = WindTurbine('DTU_10MW', diameter, hub_height, ...
    PowerCtTabular(u, power, 'kW', ct))
site = my_site
wfm = BastankhahGaussian(site, custom_turbine)
wake = wfm

#------------------Simulation characteristics---------------


#Tilt Thresholds
tilt_min = -0.01
tilt_max = 0.01

#Site data
wind_speeds = [1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 20, 25]
wind_direction = 180
distance = 1
optimizers = ['shgo', 'DA', 'DE']

# Define turbine coordinates
location_x = [0, 0, 0, 0, 0, 0, -distance*diameter, -distance*diameter, ...
```

```
    -distance*diameter, -distance*diameter, -distance*diameter,]
location_y = [0, -distance*diameter, -2*distance*diameter, ...
    -3*distance*diameter, -4*distance*diameter, -5*distance*diameter, ...
    -0.5*distance*diameter, -1.5*distance*diameter, ...
    -2.5*distance*diameter, -3.5*distance*diameter, -4.5*distance*diameter]



#-----------------------------The ...
    algortihm-------------------------------------------

# Bounds for each tilt angle, assuming tilt_min and tilt_max are defined
bounds = [(tilt_min, tilt_max)] * num_turbines

results = []

result = dict()

obj_function_values = []
for wind_speed in wind_speeds:
    for optimizer in optimizers:

        # Select and run optimizer
        if optimizer == 'DE':
            result = optimize.differential_evolution(obj_function, bounds)
        elif optimizer == 'DA':
            result = optimize.dual_annealing(obj_function, bounds)
        elif optimizer == 'shgo':
            result = optimize.shgo(obj_function, bounds, n=1, iters=1)
        else:
            raise ValueError(f"Unknown optimizer: {optimizer}")

        # Process and store results
        optimized_tilts = result.x
        individual_powers = wind_farm_simulation(optimized_tilts, wind_speed)

        results.append({
            'wind_speed': wind_speed,
            'optimizer': optimizer,
            'optimized_tilts': optimized_tilts,
            'optimized_power': -result.fun,
            'individual_powers': individual_powers,
        })


results_df = pd.DataFrame(results)


tilt_columns = [f'tilt_{i+1}' for i in range(num_turbines)]
individual_power_columns = [f'power_turbine_{i+1}' for i in ...
    range(num_turbines)]

results_df[tilt_columns] = ...
    pd.DataFrame(results_df['optimized_tilts'].tolist(), ...
    index=results_df.index)
results_df[individual_power_columns] = ...
    pd.DataFrame(results_df['individual_powers'].tolist(), ...
    index=results_df.index)

results_df.drop('optimized_tilts', axis=1, inplace=True)
results_df.drop('individual_powers', axis=1, inplace=True)
```

```
column_order = ['wind_speed', 'optimizer', 'optimized_power'] + ...
    tilt_columns + individual_power_columns
results_df = results_df[column_order]

filename = f'BastGauss_0_tilt_{distance}D.csv'
results_df.to_csv(filename, index=False)
```

From this point, only differences in the code will be shown, as the remaining code is similar.

A.4 Initial simulation - NOJ and BG 190

```
#Site data
wind_speeds = [1, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 20, 25]
wind_direction = 190
distance = 1
optimizers = ['shgo', 'DA', 'DE']
```

A.5 Initial simulation - IEA 10 MW

```

#Turbine values
num_turbines = 11
diameter = 198
hub_height = 119

# Load and filter data for wind speed between 3-25 m/s
IEA_10MW = pd.read_csv("IEA_10MW_198_RWT.csv")
filtered_IEA_10MW = IEA_10MW[(IEA_10MW['Wind Speed'] >= 2) & ...
    (IEA_10MW['Wind Speed'] <= 25)]

# Sort out data needed
u = filtered_IEA_10MW['Wind Speed']
power = filtered_IEA_10MW['Power']
ct = filtered_IEA_10MW['Ct']


# Initialize wind turbine and site
custom_turbine = WindTurbine('IEA_10MW', diameter, hub_height, ...
    PowerCtTabular(u, power, 'kW', ct))
site = my_site
wfm = BastankhahGaussian(site, custom_turbine)
wake = wfm

...

wind_speeds = [ 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]
wind_direction = 180
distance = 1
optimizers = ['DE']
```

A.6 Initial simulation - IEA 15 MW

```
#Turbine values
num_turbines = 11
diameter = 240
hub_height = 150

# Load and filter data for wind speed between 3-25 m/s
IEA_15MW = pd.read_csv("IEA_15MW_240_RWT.csv")
filtered_IEA_15MW = IEA_15MW[(IEA_15MW['Wind Speed'] >= 2) & ...
    (IEA_15MW['Wind Speed'] <= 25)]

# Sort out data needed
u = filtered_IEA_15MW['Wind Speed']
power = filtered_IEA_15MW['Power']
ct = filtered_IEA_15MW['Ct']


# Initialize wind turbine and site
custom_turbine = WindTurbine('IEA_15MW', diameter, hub_height, ...
    PowerCtTabular(u, power, 'kW', ct))
site = my_site
wfm = BastankhahGaussian(site, custom_turbine)
wake = wfm
```

A.7 Initial simulation - Wind directions

```
#Turbine values
num_turbines = 11
diameter = 178.3
hub_height = 119

# Load and filter data for wind speed between 3-25 m/s
DTU_10MW = pd.read_csv("DTU_10MW_178_RWT_v1.csv")
filtered_DTU_10MW = DTU_10MW[(DTU_10MW['Wind Speed'] >= 3) & ...
    (DTU_10MW['Wind Speed'] <= 25)]

# Sort out data needed
u = filtered_DTU_10MW['Wind Speed']
power = filtered_DTU_10MW['Power']
ct = filtered_DTU_10MW['Ct']
tilt_ws = filtered_DTU_10MW['Tilt']

# Initialize wind turbine and site
custom_turbine = WindTurbine('DTU_10MW', diameter, hub_height, ...
    PowerCtTabular(u, power, 'kW', ct))
site = my_site
wfm = BastankhahGaussian(site, custom_turbine)
wake = wfm

#Site data
wind_speeds = [ 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]
wind_direction = 200
distance = 4
```

```
optimizers = ['DE']

...

#Site data
wind_speeds = [ 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]
wind_direction = 210
distance = 4
optimizers = ['DE']

...

#Site data
wind_speeds = [ 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]
wind_direction = 220
distance = 4
optimizers = ['DE']

...

#Site data
wind_speeds = [ 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]
wind_direction = 270
distance = 4
optimizers = ['DE']
```

A.8 Initial simulation - 3 Rows

```
#Turbine values
num_turbines = 17
diameter = 178.3
hub_height = 119

...

# Define turbine coordinates
location_x = [0, 0, 0, 0, 0, 0, -distance*diameter, -distance*diameter, ...
    -distance*diameter, -distance*diameter, -distance*diameter, ...
    -2*distance*diameter, -2*distance*diameter, -2*distance*diameter, ...
    -2*distance*diameter, -2*distance*diameter, -2*distance*diameter ]
location_y = [0, -distance*diameter, -2*distance*diameter, ...
    -3*distance*diameter, -4*distance*diameter, -5*distance*diameter, ...
    -0.5*distance*diameter, -1.5*distance*diameter, ...
    -2.5*distance*diameter, -3.5*distance*diameter, ...
    -4.5*distance*diameter, 0, -distance*diameter, -2*distance*diameter, ...
    -3*distance*diameter, -4*distance*diameter, -5*distance*diameter]
```

A.9 Initial simulation - 4 Rows

```
#Turbine values
num_turbines = 22
diameter = 178.3
```

```
hub_height = 119

...

# Define turbine coordinates
location_x = [0, 0, 0, 0, 0, 0, -distance*diameter, -distance*diameter, ...
    -distance*diameter, -distance*diameter, -distance*diameter, ...
    -2*distance*diameter, -2*distance*diameter, -2*distance*diameter, ...
    -2*distance*diameter, -2*distance*diameter, -2*distance*diameter, ...
    -3*distance*diameter, -3*distance*diameter, -3*distance*diameter, ...
    -3*distance*diameter, -3*distance*diameter ]
location_y = [0, -distance*diameter, -2*distance*diameter, ...
    -3*distance*diameter, -4*distance*diameter, -5*distance*diameter, ...
    -0.5*distance*diameter, -1.5*distance*diameter, ...
    -2.5*distance*diameter, -3.5*distance*diameter, ...
    -4.5*distance*diameter, 0, -distance*diameter, -2*distance*diameter, ...
    -3*distance*diameter, -4*distance*diameter, -5*distance*diameter, ...
    -0.5*distance*diameter, -1.5*distance*diameter, ...
    -2.5*distance*diameter, -3.5*distance*diameter, -4.5*distance*diameter]
```

Appendix B

C_T and Power Curves

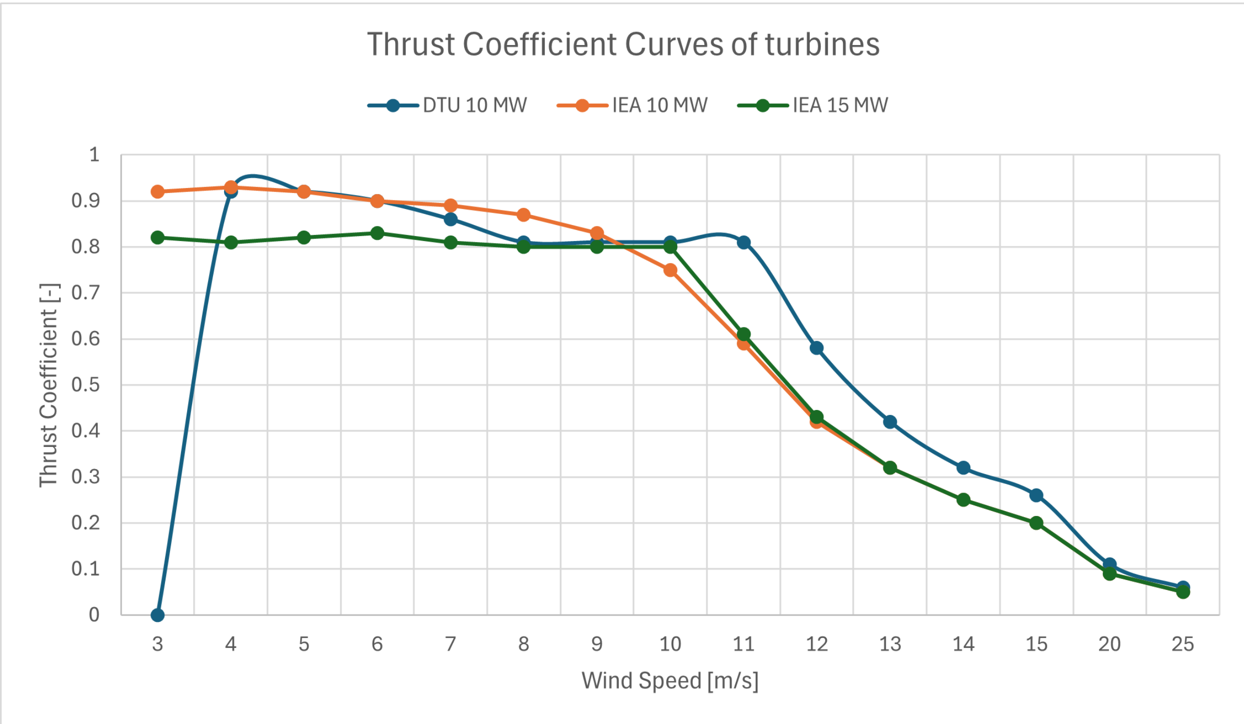

Figure B.1: C_T curves for DTU 10 MW, IEA 10 MW, and IEA 10 MW

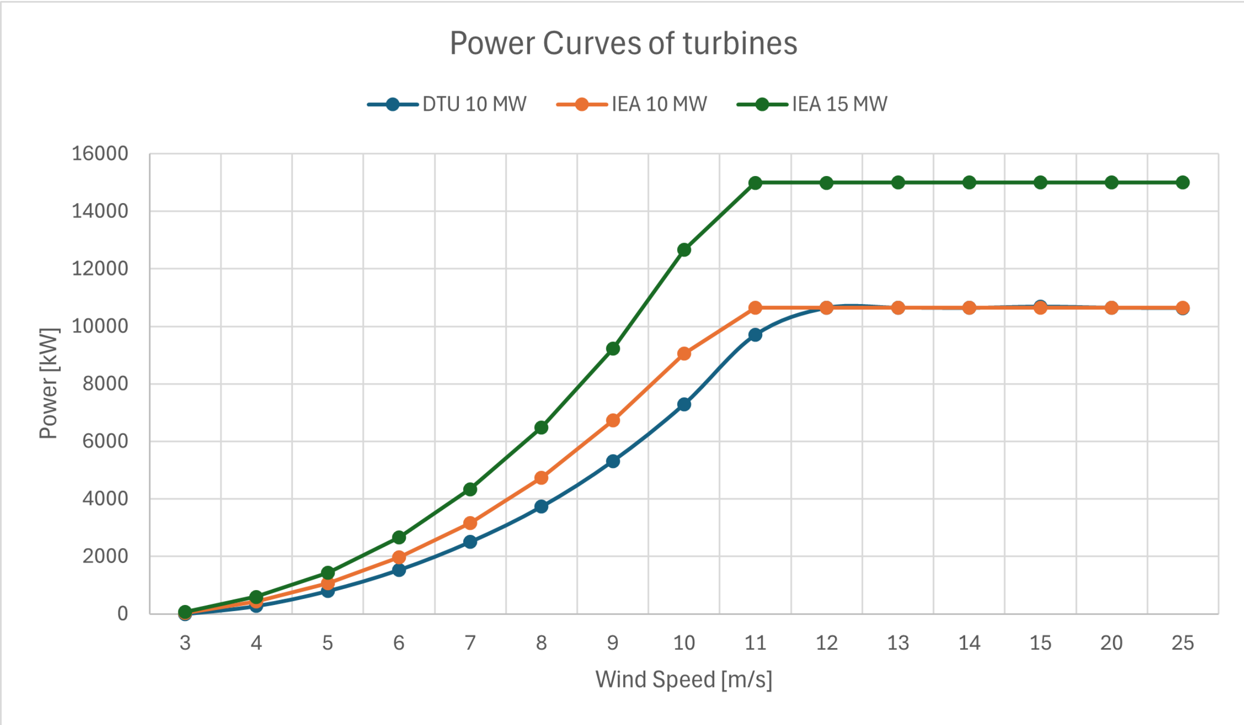

Figure B.2: Power curves for DTU 10 MW, IEA 10 MW, and IEA 10 MW

Appendix C

Optimization Verification

Table C.1: Difference in tilt and power for verification of optimization models. Using Bastankhah Gaussian wake model and selected distances and ws

4D distance							6D distance						
Ws5	**shgo**		**DA**		**DE**		**Ws5**	**shgo**		**DA**		**DE**	
Turbine	**Tilt [°]**	**Pturb [MW]**	**Tilt [°]**	**Pturb []**	**Tilt [°]**	**Pturb [MW]**	**Turbine**	**Tilt [°]**	**Pturb [MW]**	**Tilt [°]**	**Pturb [MW]**	**Tilt [°]**	**Pturb [MW]**
1	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	-0.01	-0.02	0.00	0.02
2	0.00	0.00	-60.00	0.00	-60.00	0.00	2.00	0.00	0.00	58.20	-0.01	58.20	0.01
3	0.00	0.00	-0.16	0.00	33.28	0.00	3.00	0.00	0.00	4.49	0.28	4.48	-0.28
4	0.00	0.00	-0.05	0.00	0.01	0.00	4.00	0.00	0.00	30.01	-0.38	30.00	0.38
5	0.00	0.00	0.00	0.00	0.00	0.00	5.00	0.00	0.00	-37.95	0.12	21.64	-0.12
6	0.00	0.00	0.00	0.00	0.00	0.00	6.00	0.00	0.00	0.00	0.00	0.00	0.00
7	0.00	0.00	0.00	0.00	0.00	0.00	7.00	0.00	0.00	0.00	0.00	0.01	0.00
8	0.00	0.00	0.00	0.00	23.98	0.00	8.00	0.00	0.00	-56.43	0.00	0.00	0.00
9	0.00	0.00	53.12	0.00	0.00	0.00	9.00	0.00	0.00	0.00	0.00	0.00	0.00
10	0.00	0.00	0.00	0.00	-0.19	0.00	10.00	0.00	0.00	-59.81	0.00	0.00	0.00
11	0.00	0.00	0.00	0.00	0.00	0.00	11.00	0.00	0.00	0.00	0.00	0.00	0.00
Ptot [MW]	0.00		0.00		0.00		**Ptot [MW]**	0.00		-0.01		0.01	
Ws7	**shgo**		**DA**		**DE**		**Ws7**	**shgo**		**DA**		**DE**	
Turbine	**Tilt [°]**	**Pturb [MW]**	**Tilt [°]**	**Pturb [MW]**	**Tilt [°]**	**Pturb [MW]**	**Turbine**	**Tilt [°]**	**Pturb [MW]**	**Tilt [°]**	**Pturb [MW]**	**Tilt [°]**	**Pturb [MW]**
1	0.00	0.00	0.00	-0.03	0.00	-0.10	1.00	0.00	0.00	0.00	0.00	0.00	0.00
2	0.00	0.00	0.00	0.03	-60.00	0.05	2.00	0.00	0.00	0.00	0.00	0.00	0.00
3	0.00	0.00	-21.34	0.59	6.24	0.63	3.00	0.00	0.00	0.00	0.00	0.00	0.00
4	0.00	0.00	-30.00	-0.72	-30.00	-0.72	4.00	0.00	0.00	0.00	0.00	0.00	0.00
5	0.00	0.00	14.75	0.11	-14.75	0.11	5.00	0.00	0.00	0.00	0.00	0.00	0.00
6	0.00	0.00	0.00	0.00	0.00	0.00	6.00	0.00	0.00	0.00	0.00	0.00	0.00
7	0.00	0.00	0.00	0.00	0.00	0.00	7.00	0.00	0.00	0.00	0.00	0.00	0.00
8	0.00	0.00	60.00	0.00	0.00	0.00	8.00	0.00	0.00	0.00	0.00	0.00	0.00
9	0.00	0.00	27.80	0.00	27.80	0.00	9.00	0.00	0.00	0.00	0.00	0.00	0.00
10	0.00	0.00	0.00	0.00	60.00	0.00	10.00	0.00	0.00	0.00	0.00	0.00	0.00
11	0.00	0.00	0.00	0.00	0.00	0.00	11.00	0.00	0.00	-0.01	0.00	0.00	0.00
Ptot [MW]	0.00		-0.02		-0.03		**Ptot [MW]**	0.00		0.00		0.00	
Ws8	**shgo**		**DA**		**DE**		**Ws8**	**shgo**		**DA**		**DE**	
Turbine	**Tilt [°]**	**Pturb [MW]**	**Tilt [°]**	**Pturb [MW]**	**Tilt [°]**	**Pturb [MW]**	**Turbine**	**Tilt [°]**	**Pturb [MW]**	**Tilt [°]**	**Pturb [MW]**	**Tilt [°]**	**Pturb [MW]**
1	0.00	0.00	0.01	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
2	0.00	0.00	0.00	0.00	0.00	0.00	2.00	0.00	0.00	0.00	0.00	0.00	0.00
3	0.00	0.00	0.00	0.00	-60.00	0.00	3.00	0.00	0.00	0.00	0.00	0.00	0.00
4	0.00	0.00	-35.74	0.00	0.00	0.00	4.00	0.00	0.00	0.00	0.00	0.00	0.00
5	0.00	0.00	0.00	0.00	0.00	0.00	5.00	0.00	0.00	0.00	0.00	0.00	0.00
6	0.00	0.00	0.00	0.00	0.00	0.00	6.00	0.00	0.00	0.00	0.00	0.00	0.00
7	0.00	0.00	0.00	0.56	0.01	0.56	7.00	0.00	0.00	0.00	0.00	0.00	0.00
8	0.00	0.00	30.00	-0.34	-30.00	-0.34	8.00	0.00	0.00	0.00	0.00	0.00	0.00
9	0.00	0.00	17.61	-0.11	-17.61	-0.11	9.00	0.00	0.00	0.00	0.00	0.00	0.00
10	0.00	0.00	0.00	0.00	0.00	0.00	10.00	0.00	0.00	0.00	0.00	0.00	0.00
11	0.00	0.00	0.00	0.00	0.00	0.00	11.00	0.00	0.00	0.00	0.00	0.00	0.00
Ptot [MW]	0.00		0.11		0.11		**Ptot [MW]**	0.00		0.00		0.00	

Appendix D

Results Wind directions

Table D.1: Difference in tilt and power for wind direction 200°and 210°at various wind speeds

Distance = 4D							
Ws5 - Wd200				**Ws5 - Wd210**			
Turbine	**Tilt [°]**	**Pturb [MW]**	**ΔTilt [°]**	**Turbine**	**Tilt [°]**	**Pturb [MW]**	**ΔTilt [°]**
1	0.01	0.45	-0.01	1	0.00	0.39	0.00
2	0.00	0.45	-30.00	2	0.01	0.39	-30.01
3	0.00	0.47	16.65	3	0.00	0.39	16.64
4	0.00	0.79	26.08	4	-0.01	0.39	26.09
5	0.00	0.79	0.00	5	0.00	0.80	0.00
6	0.00	0.80	0.00	6	0.00	0.80	0.00
7	0.00	0.79	0.00	7	0.00	0.80	0.00
8	0.00	0.79	12.84	8	0.00	0.80	12.84
9	0.00	0.79	0.00	9	0.00	0.80	0.00
10	0.00	0.79	-0.16	10	0.00	0.80	-0.17
11	0.00	0.80	0.00	11	0.00	0.80	0.00
Ptot [MW]		7.72		**Ptot [MW]**		7.17	
Pbench [MW]		7.72		**Pbench [MW]**		7.17	
Difference [MW]		0.00		**Difference [MW]**		0.00	
Ws7 - Wd200				**Ws7 - Wd210**			
Turbine	**Tilt [°]**	**Pturb [MW]**	**ΔTilt [°]**	**Turbine**	**Tilt [°]**	**Pturb [MW]**	**ΔTilt [°]**
1	0.00	1.58	0.00	1	0.00	1.46	0.00
2	0.00	1.58	-30.00	2	0.00	1.46	-30.00
3	0.00	1.62	-11.80	3	0.00	1.46	-11.80
4	0.00	2.50	-30.00	4	0.00	1.46	-30.00
5	0.00	2.50	15.25	5	0.00	2.51	15.25
6	0.00	2.51	0.00	6	0.00	2.51	0.00
7	0.00	2.50	0.00	7	0.00	2.51	0.00
8	0.00	2.50	30.00	8	0.00	2.51	30.00
9	0.00	2.50	13.90	9	0.00	2.51	13.90
10	0.00	2.50	30.00	10	0.00	2.51	30.00
11	0.00	2.51	0.00	11	0.00	2.51	0.00
Ptot [MW]		24.78		**Ptot [MW]**		23.38	
Pbench [MW]		24.78		**Pbench [MW]**		23.38	
Difference [MW]		0.00		**Difference [MW]**		0.00	
Ws8 - Wd200				**Ws8 - Wd210**			
Turbine	**Tilt [°]**	**Pturb [MW]**	**ΔTilt [°]**	**Turbine**	**Tilt [°]**	**Pturb [MW]**	**ΔTilt [°]**
1	0.00	2.44	0.00	1	0.00	2.28	0.00
2	0.00	2.44	0.00	2	0.00	2.28	0.00
3	0.00	2.48	-30.00	3	0.00	2.28	-30.00
4	0.00	3.72	-17.87	4	0.00	2.29	-17.87
5	0.00	3.72	0.00	5	0.00	3.73	0.00
6	0.00	3.73	0.00	6	0.00	3.73	0.00
7	0.00	3.72	0.00	7	0.00	3.73	0.00
8	0.00	3.72	-30.00	8	0.00	3.73	-30.00
9	0.00	3.72	-17.61	9	0.00	3.73	-17.61
10	0.00	3.72	0.00	10	0.00	3.73	0.00
11	0.00	3.73	0.00	11	0.00	3.73	0.00
Ptot [MW]		37.18		**Ptot [MW]**		35.25	
Pbench [MW]		37.18		**Pbench [MW]**		35.25	
Difference [MW]		0.00		**Difference [MW]**		0.00	

Table D.2: Difference in tilt and power for wind direction 220°and 270°at various wind speeds

Distance = 4D							
Ws5 - Wd 220				**Ws5 - Wd 270**			
Turbine	**Tilt [°]**	**Pturb [MW]**	**ΔTilt [°]**	**Turbine**	**Tilt [°]**	**Pturb [MW]**	**ΔTilt [°]**
1	0.00	0.60	0.00	1	0.00	0.80	0.00
2	0.00	0.60	-30.00	2	0.00	0.80	-30.00
3	0.00	0.60	16.65	3	0.00	0.80	16.65
4	0.00	0.60	26.08	4	0.00	0.80	26.08
5	0.00	0.80	0.00	5	0.00	0.80	0.00
6	0.00	0.80	0.00	6	0.00	0.80	0.00
7	0.00	0.80	0.00	7	0.00	0.80	0.00
8	0.00	0.80	12.84	8	0.00	0.80	12.84
9	0.00	0.80	0.00	9	0.00	0.80	0.00
10	0.00	0.80	-0.17	10	0.00	0.80	-0.17
11	0.00	0.80	0.00	11	0.00	0.80	0.00
Ptot [MW]		8.01		**Ptot [MW]**		8.79	
Pbench [MW]		8.01		**Pbench [MW]**		8.79	
Difference [MW]		0.00		**Difference [MW]**		0.00	
Ws7 - Wd 220				**Ws7 - Wd 270**			
Turbine	**Tilt [°]**	**Pturb [MW]**	**ΔTilt [°]**	**Turbine**	**Tilt [°]**	**Pturb [MW]**	**ΔTilt [°]**
1	0.00	2.03	0.00	1	0.00	2.51	0.00
2	0.00	2.03	-30.00	2	0.00	2.51	-30.00
3	0.00	2.03	-11.80	3	0.00	2.51	-11.80
4	0.00	2.03	-30.00	4	0.00	2.51	-30.00
5	0.00	2.51	15.25	5	0.00	2.51	15.25
6	0.00	2.51	0.00	6	0.00	2.51	0.00
7	0.00	2.51	0.00	7	0.00	2.51	0.00
8	0.00	2.51	30.00	8	0.00	2.51	30.00
9	0.00	2.51	13.90	9	0.00	2.51	13.90
10	0.00	2.51	30.00	10	0.00	2.51	30.00
11	0.00	2.51	0.00	11	0.00	2.51	0.00
Ptot [MW]		25.67		**Ptot [MW]**		27.57	
Pbench [MW]		25.67		**Pbench [MW]**		27.57	
Difference [MW]		0.00		**Difference [MW]**		0.00	
Ws8 - Wd 220				**Ws8 - Wd 270**			
Turbine	**Tilt [°]**	**Pturb [MW]**	**ΔTilt [°]**	**Turbine**	**Tilt [°]**	**Pturb [MW]**	**ΔTilt [°]**
1	0.00	3.09	0.00	1	0.00	3.73	0.00
2	0.00	3.09	0.00	2	0.00	3.73	0.00
3	0.00	3.09	-30.00	3	0.00	3.73	-30.00
4	0.00	3.09	-17.87	4	0.00	3.73	-17.87
5	0.00	3.73	0.00	5	0.00	3.73	0.00
6	0.00	3.73	0.00	6	0.00	3.73	0.00
7	0.00	3.73	0.00	7	0.00	3.73	0.00
8	0.00	3.73	-30.00	8	0.00	3.73	-30.00
9	0.00	3.73	-17.61	9	0.00	3.73	-17.61
10	0.00	3.73	0.00	10	0.00	3.73	0.00
11	0.00	3.73	0.00	11	0.00	3.73	0.00
Ptot [MW]		38.48		**Ptot [MW]**		41.04	
Pbench [MW]		38.48		**Pbench [MW]**		41.04	
Difference [MW]		0.00		**Difference [MW]**		0.00	