Students' mathematical modelling with the aid of digital technologies

An Activity and Affordance Theory perspective

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Preface

The research presented in this thesis was conducted in the Department of Mathematical Sciences at the University of Agder (UiA) under the supervision of Professor Said Hadjerrouit and Professor John Monaghan. Undertaking this doctoral study has been an invaluable research apprenticeship and wide-ranging learning experience. My engagement with the field of mathematical modelling with digital technologies in this thesis stems from my past experiences, particularly during my master's thesis project. Researching students' mathematical modelling with digital technologies, I considered it an excellent opportunity to explore different interactions within students' mathematical modelling with the aid of digital technologies in group settings. This undertaking has allowed me to study several theoretical perspectives about mathematical modelling, technology-mediated activities in students' mathematical modelling, and group work in mathematical modelling, among others. This has fed my curiosity in the mathematical modelling field and made it an excellent experience for my learning and development as a researcher.

A lot of work and time has been invested in coming up with this thesis, and with the support of others, it was much easier to overcome the difficulties and stress in this process. As such, I am grateful to many people for their support and guidance through this undertaking. My primary debt of gratitude goes to God as my source of life and strength. I further wish to acknowledge the academic guidance and moral support of my supervisors, Professor Said Hadjerrouit and Professor John Monaghan. My supervisors have been a fundamental force for the realization of this thesis, and I am highly thankful to them for their encouragement, generous guidance, and constructive and valuable feedback, among others.

The students also deserve special thanks for their willingness to participate in this project. I want to thank all the principals of the different schools for allowing me to situate this study in these schools and the teachers who supported the project in many different ways. I am thankful to Professor Martin Carlsen, Professor Thomas Gjesteland and Stig Eriksen for linking me to some students in their class when I needed participants for the pilot studies. I thank Gjermund Torkildsen for connecting me to one of the schools in this project. I am also grateful to my colleague, Nils-Jacob Herleiksplass, for reviewing my codes during the inter-coder reliability process. I thank Camilla Rødstøl for helping translate Norwegian to English during the transcriptions.

My research taught me that no task can be completed in professional isolation without interaction with others. The discussions that arose from differences of opinion or approach, particularly criticism, were sometimes time-consuming and even painful to deal with at some point. However, it has not only kept me from making mistakes but also added clarity. I am grateful for the opportunity to participate in seminars, workshops, and conferences organized by MERGA, MatRIC, and other international research communities. At some point, I used labels from modelling competences in describing students' modelling processes from an Activity Theory perspective, which received a lot of criticism with the argument that competences frameworks on modelling cycles describe cognitive activities. Thus, the cognitive approach finds cognitive barriers and never any socio-cultural barriers in the modelling process. I thank Associate Professor Jorunn Reinhardtsen (and others whose names I cannot mention for some reason) for the discussions concerning situating modelling competences in an Activity Theoretical framework. After a series of arguments and discussions, I devised modelling actions to describe students' mathematical modelling processes from an Activity Theory perspective. Professor John Monaghan also put me in contact with Professor Tom Lowrie, with whom I had a great conversation about interaction sequences in group activities. I sincerely thank my excellent colleagues at the Department of Mathematical Sciences for numerous helpful discussions and engagements. I am also thankful to the non-teaching staff of the Department of Mathematical Sciences, Faculty of Engineering and Science administrators, and UiA library for creating a supportive and conducive research environment.

Finally, I want to express my gratitude to my parents and siblings. You have supported me greatly during my stay in Norway, even though you have been far away! I am also thankful to my friends and all other acquaintances – close or more peripheral – for being such great social and academic motivators. *Tusen takk*!!

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Abstract

Digital technologies in mathematical modelling activities are receiving increased attention in curriculum and policy documents in Norway and internationally. An important point arising from diverse interests in technology and students' mathematical modelling activities is the various forms of interactions that emerge within students' activities generated by digital technologies. However, the majority of research on technology-mediated activities in mathematical modelling is still approached from a cognitive perspective.

This study takes another perspective, a socio-cultural perspective combining Cultural-Historical Activity Theory (CHAT) with Affordance Theory. This dissertation explored how secondary school students solve mathematical modelling tasks with the aid of digital technologies. This research study focuses on different forms of interactions within students' mathematical modelling activities with the aid of digital technologies in group settings. Three subgoals were formulated from the aim of the study: Examining the various forms of interactions within the students' activities; investigating the students' working processes; and examining students' interactions with digital technologies.

The study is framed within a qualitative research paradigm and adopts an ethnographical case study research design. It involves four secondary schools in southern Norway. Empirical data were collected through video recordings, screen capture, fieldnotes and students' worksheets. A combination of inductive and deductive approaches was used in the data analysis.

The study highlights three major findings. Firstly, it shows that the elements of CHAT are seen as a collective system interacting with each other, in contrast to cognitive approaches focusing on heuristics and modelling processes. Secondly, the findings indicate that students' activities in mathematical modelling can be categorized as modelling actions and operations towards the object of solving a mathematical modelling task, from a CHAT perspective. Digital technologies also played an essential role in the modelling actions that emerged. Thirdly, the findings highlight the emergence of technological, mathematical and socio-cultural affordances and constraints of the digital technologies in the students' activities.

The study contributes to research in mathematics education and draws various implications out of the major findings regarding students' use of digital technologies in mathematical modelling activities. These implications include the potential influence of digital technologies in group interactions, students' tendency

in selecting or using particular digital technologies, the impact of different types of mathematical modelling tasks on interaction dynamics, and students' roles in mathematical modelling activities. Future research is also suggested to explore the expansion of students' mathematical modelling activities with the aid of digital technologies, teachers' roles in the students' activities, and considerations of other theoretical perspectives.

Sammendrag

Digitale teknologier i matematisk modelleringsvirksomhet får økt oppmerksomhet i læreplaner og politiske dokumenter i Norge og internasjonalt. Et viktig poeng som oppstår fra ulike interesser for teknologi og elevenes matematiske modelleringsaktiviteter er de ulike formene for interaksjoner som dukker opp innenfor elevenes aktiviteter generert av digitale teknologier. Imidlertid er størstedelen av forskningen relatert til teknologimedierte aktiviteter innen matematisk modellering fortsatt tilnærmet fra et kognitivt perspektiv.

Denne studien tar et annet perspektiv, et sosiokulturelt perspektiv som kombinerer Cultural-Historical Activity Theory (CHAT) med Affordance Theory. Denne avhandlingen utforsket hvordan ungdomsskoleelever løser matematiske modelleringsoppgaver ved hjelp av digitale teknologier. Denne forskningsstudien fokuserer på ulike former for interaksjoner innenfor elevenes matematiske modelleringsaktiviteter ved hjelp av digitale teknologier i gruppemiljøer. Tre delmål ble formulert fra målet med studien: Undersøke de ulike formene for interaksjoner innenfor studentenes aktiviteter; undersøke studentenes arbeidsprosesser; og undersøke studentenes interaksjoner med digitale teknologier.

Studien er innrammet innenfor et kvalitativt forskningsparadigme og tar i bruk et etnografisk casestudie-forskningsdesign. Det involverer fire ungdomsskoler i Sør-Norge. Empiriske data ble samlet inn ved hjelp av videoopptak, skjermfangst, feltnotater og elevenes arbeidsark. En kombinasjon av induktive og deduktive tilnærminger ble brukt i dataanalysen.

Studien fremhever tre hovedfunn. For det første viser funnene at elementene i CHAT blir sett på som et kollektivt system som samhandler med hverandre, i motsetning til kognitive tilnærminger med fokus på heuristikk og modelleringsprosesser. For det andre indikerer funnene at elevenes aktiviteter i matematisk modellering kan kategoriseres som modelleringshandlinger og operasjoner mot målet om å løse en matematisk modelleringsoppgave, fra et CHAT-perspektiv. Digitale teknologier spilte også en viktig rolle i modelleringshandlingene som dukket opp. For det tredje fremhever funnene fremveksten av teknologiske, matematiske og sosiokulturelle fordeler ("affordances") og begrensninger ("constraints") for de digitale teknologiene i studentenes aktiviteter.

Studien bidrar til forskning innen matematikkundervisning og trekker ulike implikasjoner ut av de hovedfunnene om elevenes bruk av digitale teknologier i matematiske modelleringsaktiviteter. Disse implikasjonene inkluderer den potensielle påvirkningen av digitale teknologier i gruppeinteraksjoner, elevenes tendens til å velge eller bruke bestemte digitale teknologier, innvirkningen av ulike typer matematiske modelleringsoppgaver på interaksjonsdynamikk, og elevenes roller i matematiske modelleringsaktiviteter. Fremtidig forskning foreslås også for å utforske utvidelsen av elevenes matematiske modelleringsaktiviteter ved hjelp av digitale teknologier, lærernes roller i elevenes aktiviteter og betraktninger av andre teoretiske perspektiver.

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1 Introduction

This doctoral dissertation explores how secondary school students solve mathematical modelling tasks with the aid of digital technologies. The dissertation mainly focuses on the different forms of interactions within the students' activities. Four Norwegian schools participated in this research project, and the results are based on an analysis of classroom observations of the students' activities.

This introductory chapter provides the rationale and overview of this research study. Section 1.1 presents the background of the study, followed by the motivation of the study in Section 1.2. In Section 1.3, I present the research goals by highlighting the aims and motives of this research. Section 1.4 presents the research questions, and Section 1.5 presents the structure of the dissertation.

1.1 Background for the study

Applying digital technologies in the teaching and learning of mathematics, resulting from technological advancements, has increasingly gained importance in education systems (Greefrath et al., 2018). I will define and describe digital technologies in Section 2.2. However, for now, I share the views of Greefrath et al. (2018) that these technologies are digital media in the likes of computers, tablets, and handheld devices, among others, that can be used to support the teaching and learning of mathematics to some extent. Olofsson et al. (2020) argue that in the last decades, there has been an increase in the use of digital technologies in upper secondary schools. Furthermore, this development is crucial as it enables students to participate in and contribute to a highly digitalized society. The National Council of Teachers of Mathematics (2000) also argues that technology is essential in teaching and learning mathematics and that it might influence the mathematics taught, enhancing students' learning. Drijvers (2003) highlights that the introduction of digital technologies has provided opportunities for exploring mathematical situations and opening new previously inaccessible horizons. That is, facilitating students' investigations and discoveries. For instance, Ellington (2006) points out how digital technology (like graphical calculators) improved students' operational and problem-solving skills when this technology formed an integral part of testing and instruction. Genlott and Grönlund (2016) are also of the view that students perform better when digital technologies are well integrated with the curriculum. However, the benefit of using digital technologies in mathematics education itself is a subject of debate (Drijvers, 2018). For instance, the report by the Organization for Economic Co-operation and Development (OECD) on students' achievement and the use of digital technologies shows that "despite considerable investments in computers, internet connections and software for educational use, there is little solid evidence that greater computer use among students leads to better scores in mathematics and reading" (OECD, 2015, p. 145).

The report from OECD highlights that several things need to be considered when integrating digital technologies into mathematics education. For instance, understanding the relation between the user and the technology, in this case, how an individual uses the technology for a specific task (Artigue, 2002; Trouche, 2005). Drijvers (2015) highlights some factors to consider when integrating digital technologies into the education system: the design of digital technology and the corresponding tasks and activities, the role of the teacher, and the educational context. Concerning the first factor, we can ask, 'What forms of activities support the use of digital technologies?'. There are several activities, but one such activity is mathematical modelling, a process that maps real-world situations in mathematical terms to find a real-world solution (discussed in Section 2.1). Stillman (2007) reports that digital technology allows more authentic modelling situations. Furthermore, the literature supports the assertion that digital technologies impact students' modelling processes (Molina-Toro et al., 2019). Despite the impact of digital technologies in mathematical modelling, Strässer (2007) warns that these technologies should not only be considered as a means to enhance students' modelling abilities and to enrich the students' experience of applications and modelling since using such technologies might profoundly change the scope and way mathematics is used within the society. Monaghan (2016b) also critiques ideas of integrating digital technologies in mathematical modelling, pointing out the complex nature of technologies in this area (in reality). Mathematical modelling with digital technologies plays an essential role in our societies; for instance, many decisions taken during the COVID-19 (novel coronavirus disease) pandemic were based on mathematical models generated by digital technologies (Romano et al., 2020; Alguliyev et al., 2021).

The teaching and learning of mathematical modelling have played an essential role in mathematics education worldwide over the years (Schukajlow et al., 2018). One of several reasons for this importance could be that today's schools face the challenge of preparing students to live, work and prosper in this rapidly changing world. It is through mathematical modelling that a lot of mathematics might be used in careers beyond school (Blum et al., 2007). Many countries responding to this global challenge have adopted a national curriculum that focuses on developing 21st-century skills, and Norway is not an exception (Bakken & Andersson-Bakken, 2021). In the new Norwegian mathematics curriculum (student-centered), implemented in the autumn of 2020, “modelling and application” is incorporated as one of the core elements (Ministry of Education and Research, 2019). This implementation raises many questions and issues to explore in the Norwegian context. For instance, Bakken and Andersson-Bakken (2021) investigate if and how the tasks in science and language arts textbooks in Norwegian upper secondary schools have changed after the curriculum reform. Their results show that although the curriculum has changed, school tasks have mostly stayed the same. These tasks do not give students sufficient opportunities to practice the competences highlighted in the new curriculum. Berget (2022) also examines mathematical modelling in textbook tasks and national exams in light of the new curriculum. The findings indicate different perspectives on mathematical modelling in the curriculum, the textbook tasks, and the national exam, where only parts of the modelling process are included. None of these studies above touches on mathematical modelling with the aid of digital technologies, although there are some studies in this field which are sparse in the Norwegian context.

Of course, research in mathematical modelling and the use of digital technology is a topic that has been introduced previously. However, there are many unanswered questions about the involvement of digital technologies in mathematical modelling. For instance, Greefrath et al. (2018) raise the question, ‘How does the effective acquisition of modelling competence (defined in Sub-Section 2.1.2) differ as a function of student educational levels when using digital technologies?’. Blum (2002, p. 167) also raises a general question: “How should technology be used at different educational levels to effectively develop students’ modelling abilities and to enrich the students’ experience of open ended mathematical situations in applications and modelling?” English et al. (2016) assumed in the literature that there is still a research gap since questions of these likes have not yet been answered satisfactorily. Indeed, answering such questions requires a broad variety of research and theoretical frameworks. However, research in this area is often done from a cognitive perspective focusing on heuristics and modelling processes (often schematized in a cyclic diagram—modelling cycle, defined in Sub-Section 2.1.1) (Cevikbas et al., 2021). Vos and Frejd (2022) emphasize that many researchers use the modelling cycle as an analytical tool to analyze empirical data in light of the different phases that the cycle distinguishes.

For instance, Wess et al. (2021) analyzed students' modelling competencies and listed common difficulties or errors encountered by students in certain phases of the cycle. Vos and Frejd (2022) argue that a research result will be primarily cognitive if an analytic framework has a cognitive focus. In summing up the contributions of the systematic literature survey, Cevikbas et al. (2021) acknowledge that many papers in the literature point to the vital need for further theoretical work on conceptualizing modelling competences. Vos and Frejd (2022) argue that with only an emphasis on cognitive aspects, a research might not capture other important aspects that play a role in mathematical modelling. As such, they suggested some aspects like a dimension for metacognitive strategies, a dimension for digital technology use, and a dimension for social norms. Regarding a dimension for social norms, the modelling discussions should also consider interactions within modelling activities (e.g., student—student and teacher—student interactions). For instance, the quality of peer interactions might determine the outcome of the activity (Hernandez-Martinez & Harth, 2015).

This research study contributes to the ongoing discussions by exploring how secondary school students solve mathematical modelling tasks with the aid of digital technologies from a socio-cultural perspective. This is done by looking at the students' activity (as a whole) while paying attention to dimensions such as digital technology usage and social norms. The main focus of this study is on different forms of interactions taking place in classrooms in which active approaches to the use of digital technologies are supported. The interactions identified in this study are student—student (influenced to some extent by digital technologies and tasks), student—digital technology, student—tasks (influenced to some extent by digital technologies), digital technology—tasks, and others. The dimensions and the forms of interactions do not exist in isolation, as they interact within the students' activities. As such, I subscribe to Cultural-Historical Activity Theory (CHAT—presented in Section 4.1) to study the different forms of interactions. Another aspect is that one needs more than CHAT to explore the relationship between student—digital technology interactions, and I subscribed to an Affordance Theory (defined and explained in Section 4.3). Of course, other theories could be used to study these interactions, but I chose Affordance Theory while considering its compatibility with CHAT (this justification is presented in Sub-Section 4.4.1). English et al. (2016, p. 406) point out that 'there is still limited emphasis on the affordances of technology' in the literature on mathematical modelling with digital technologies. Affordance Theory is one of the means of analyzing affordances of digital technologies in mathematics education. Bower and Sturman (2015) argue that educators need to analyze the affordances and constraints of digital technologies to make them useful in the educational context. Thus, affordances and constraints perspective helps understand the opportunities and challenges of integrating digital technology into the education system.

I will now present my motivation for this research study.

1.2 Motivation for the study

The research goals (as stated in the next section) mainly stem from my past experiences. The first time I heard about mathematical modelling was a course (TMA4195—Mathematical Modelling) I took during my master's degree in mathematical sciences at the Norwegian University of Science and Technology (NTNU). This course was an introduction to generic principles and methods to formulate mathematical models of systems and processes in science and engineering. The second time I heard about mathematical modelling was a course (MA-424—Working Methods in Mathematics) I took during my master's degree in mathematics education at the University of Agder (UiA). One aspect of interest in this course was an overview of the role of problem-solving and mathematical modelling in mathematics research and curricula. The mathematical modelling course at UiA was under mathematics didactics compared to the one I took at NTNU (under mathematical analysis). Reflecting on the modelling course at UiA and comparing it with my experiences as a student from the primary to the secondary level, I found that most of the mathematical tasks I worked on in the past had no connection with the real world. The ones that came close were some word problems that I worked on but were more mathematical and did not relate much to the real world.

With this background, I decided to conduct a research study in mathematical modelling for my master's thesis at UiA. I chose to study "how upper secondary school students solve algebraic word problems in the area of mathematical modelling" (Afram, 2019). I analyzed students' solutions to algebraic word problems and a mathematical modelling task, in this project. In the analysis, one of the students expressed his opinion that there is no need to learn about things (for instance, function analysis and trigonometry) that cannot be used in the future. This student also pointed out the need to learn more equations (algebra) as they could solve a realistic problem with that. From this point of view, engaging students in modelling activities might motivate them to learn the mathematics subjects. Another student's conception during the interview was that the modelling task could have been easier when using the computer rather than drawing the graph with paper-and-pencil. I was motivated by this and other issues to explore the students' working activities further as they solve mathematical modelling tasks using digital technologies. Again, in the master thesis, I only analyzed students' working sheets and interviews (conducted a week after the classroom activities), which was a limitation since I could not give a detailed account of the students' interactions. As such, I was motivated in the current research study to do a classroom observation through video recordings (for details about the students' working processes). Also, my engagement with upper secondary school students in my previous research influenced choosing participants for the current study.

I was also motivated to study the relationship between the students and digital technology in the current research study based on the course (MA-421—Digital Tool in Mathematics Teaching) I took at UiA. One component of interest in this course is the theoretical background and insights from research on using digital tools in learning and teaching mathematics. From this theoretical background, I got to understand how affordances and constraints emerge from the interaction between students and digital technologies. Another important reason for choosing this current research study is the new Norwegian mathematics curriculum, which has 'applications and modelling' as one of its core elements. Implementing this new curriculum raises many questions and issues to explore in the area of modelling with digital technologies. Again, this research study supports using digital technologies and active learning approaches promoted by Center for Research, Innovation and Coordination of Mathematics Teaching (MatRIC) at UiA.

With this motivation for the study and the background for the study, I will now present my research goals in the forthcoming section.

1.3 Research goals: Aims and motives

Following the discussion in Section 1.1, I raised two crucial concerns. The first concern was about the new Norwegian curriculum and the opportunity to explore the area of mathematical modelling with digital technologies. Generally, research in this area in the Norwegian context after implementing the new curriculum is sparse. This research could contribute to the literature in advancing knowledge in this area. The second concern I raised was that much of the research in this area follows a cognitive perspective focusing on its heuristics and modelling processes. As such, this research contributes to the ongoing discussion by exploring mathematical modelling with the aid of digital technologies from a socio-cultural perspective. Combining the two issues, I study mathematical modelling with the aid of digital technology in the Norwegian context by subscribing to the socio-cultural perspective. I do this by exploring the several factors (such as group work, digital technology, nature of tasks, and roles adopted by students) affecting students' mathematical modelling and not just looking at cognitive barriers.

Selecting a researchable issue means the researcher must determine the study's goals based on empirical settings and previous research in the field. These goals are gradually articulated and refined into relevant research questions according to the research's direction. Bryman (2016) highlights that many researchers begin their research with a general idea (goals) in which they are interested, and research questions guide the researcher to consider more specific issues they want to find out about much more precisely and rigorously. Research questions should be coherent with the choice of theoretical framework and methodology (Radford, 2008a). In this case, I initially formulated my primary research goal, and I also acknowledged that this would have to evolve into more specific research questions during the entire duration of the research process:
Explore how secondary school students solve mathematical modelling tasks with the aid of digital technologies.

Furthermore, sub-goals are formulated from this primary goal. Before presenting the sub-goals, I define students' activity as "a group of secondary school students solving mathematical modelling tasks with the aid of digital technologies". The sub-goals form the themes in which I derive the research questions. The three sub-goals are listed as follows:

- *Examine the various forms of interactions taking place within the students' activity*.
- *Investigate the students' working processes in the students' activity*.
- *Examine students' interactions with digital technologies in the students' activity*.

In light of these aims and motives, I will present the research questions of this study in the forthcoming section.

1.4 Research questions

In this section, I present the research questions organized into three themes. These themes are derived from the sub-goals of this research study. The first theme relates to the different interactions occurring within the students' activities. Students' activities in this study are referred to as "a group of secondary school students solving two mathematical modelling tasks with the aid of digital technologies". There are several components within the students' activities (e.g., digital technologies used, characteristics of the students, types of tasks, etc.), and these components interact with each other. This aspect covers the entire students' activities (a dimension for digital technology use and social norms, among others). The second theme concerns the students' solution processes and the role of digital technologies in these processes. Thus, it zooms into the students' activities and pays particular attention to students' actions emerging as they work on the mathematical modelling tasks using digital technologies. The third theme involves the relationship between the students and digital technologies. Through this relation, affordances and constraints of digital technologies emerge. Thus, the third theme zooms in further into the students' activities and pays particular attention to students' actions resulting from their engagement with digital technologies.

Figure 1.1 summarizes the link between the research questions, showing that the second research question (RQ2a and RQ2b) is embedded in the first research question (RQ1), and the third research question (RQ3) is further embedded in the second research question. The terms used in the research questions are defined in greater detail in the following sections and subsections: mathematical modelling tasks in 5.5, digital technologies in 2.2, contingencies and students' interactions in 2.4.1, mediation in 4.1, emergence in 4.3.1 and 4.3.2, modelling actions in 2.1.3 and 4.4.2, mathematical modelling activity in 2.1, affordances and constraints in 4.3.4 and 4.4.1, technology-based model/solution in 2.2.1 and 5.5.

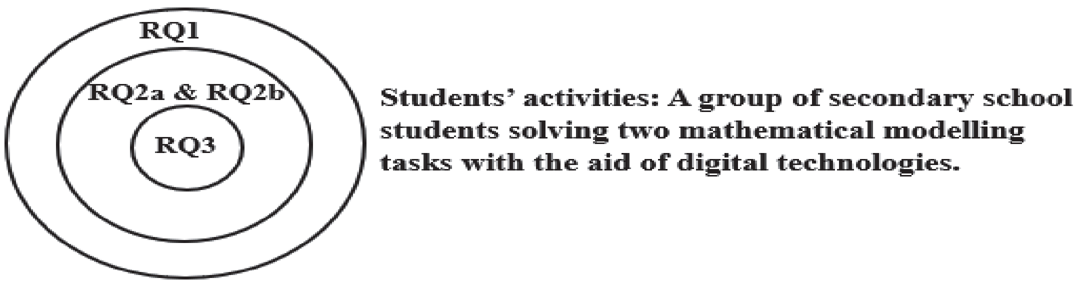


Figure 1.1: Connection between RQ1, RQ2a, RQ2b, and RQ3.

I analyze the following research questions from an Activity Theory perspective:

Theme 1: **Students' mathematical modelling activities**

RQ1: *How do students solve mathematical modelling tasks with the aid of digital technologies?*

This overreaching question is split into four more specific sub-research questions:

RQ1a: *What digital technologies did the students use in solving the two mathematical modelling tasks?*

RQ1b: *What contingencies were shown in the student interactions when solving the two mathematical modelling tasks with the aid of digital technologies?*

RQ1c: *What are the rules that mediate students' mathematical modelling activities when solving the two mathematical modelling tasks with the aid of digital technologies?*

RQ1d: *What roles did the students adopt in the student interactions when solving the two mathematical modelling tasks with the aid of digital technologies?*

Theme 2: **Emergence of modelling actions and the role of digital technologies**

RQ2a: *What modelling actions emerge during the mathematical modelling activities of the students?*

RQ2b: *What part do the uses of digital technologies play within the modelling actions that emerge?*

Theme 3: **Emergence of affordances and constraints of digital technologies in mathematical modelling activities**

RQ3: *What affordances and constraints of the digital technologies emerge as the students develop a technology-based model/solution?*

I analyze RQ1 using Engeström's expanded mediational triangle (see Section 4.2). In this case, I consider three forms of interactions:

1. Mediating artefacts/tools for the subject-object interactions, where I look at the digital technologies the students used and the interaction contingencies that were shown. This addresses RQ1a and RQ1b.
2. Rules for the subject-community interaction. The rules could be explicit or implicit. This addresses RQ1c.
3. Division of labour for the community-object interaction, where I look at the roles students adopt in the students' interactions. This addresses RQ1d.

I analyze RQ2a and RQ2b using Leont'ev's three hierarchical layers of an activity (see Sub-Section 4.4.2). I analyze RQ3 using the combination of Leont'ev's three hierarchical layers of an activity and Affordance Theory (see Sub-Section 4.4.1).

I will now present the structure of this dissertation.

1.5 The structure of the dissertation

There are eight chapters in this dissertation. Chapter 1 briefly presents the background for the research study, my motivation behind the studies, the research goals, the research questions, and an outline of the thesis's structure.

Chapter 2 provides a review of relevant literature. I start this chapter by discussing mathematical modelling and highlighting different perspectives on mathematical modelling, modelling competence, and ontology and epistemology of modelling competence. This is followed by a review of digital technologies and their use in mathematical modelling activities. The chapter concludes by discussing mathematical task design and group work/activity.

Chapter 3 presents the Norwegian education system as the context of the research study. I discuss the Norwegian educational system, mathematics education in Norway, and mathematical modelling and applications in the Norwegian mathematics curriculum. This is followed by discussing the schools and cooperating teachers in this research study.

Chapter 4 deals with the theoretical framework applied in this research study. I started by presenting an introduction to CHAT and how I adapted it to this study. This is followed by a discussion of Affordance Theory, highlighting the emergence, perception and actualization of affordances and constraints in mathematics education. The compatibility of the adopted theories is also presented.

Chapter 5 concerns methodological issues such as the research paradigm, the research design and strategy, the context of the study, the digital technologies and mathematical modelling tasks used in this study, and data collection methods. I also discuss aspects like data analysis strategy and management, presentation of analysis, validity and trustworthiness, and ethical considerations.

Chapter 6 presents the research study's results. It provides an overview of data analysis in tabular form. Then, it describes the research findings in embedded case studies of four groups drawn from four schools. The four cases are discussed around a structure that helps to address the research questions. The chapter concludes by discussing a cross-case analysis of the four cases.

Chapter 7 presents a discussion by revisiting the research questions in light of the emerging issues in the cases presented in the results section and as they stand to the theoretical and research literature. The chapter also presents significant issues arising from the research and my reflection on using theoretical perspectives and their link with the research findings.

Chapter 8 concludes the thesis. It first presents a summary and reflection on the thesis's quality. Then, the study's limitations, implications, and proposal for further research are presented.

2 Literature Review

This chapter presents the review of relevant literature in this research study. The chapter has five parts, and the first part presents a review of mathematical modelling, highlighting the different perspectives on mathematical modelling and modelling competence with its ontology and epistemology in Section 2.1. The second part, Section 2.2, explores the literature on digital technologies in mathematics education. The third part, Section 2.3, examines the literature on mathematical task design, highlighting the types of tasks, design elements of mathematical tasks, students' perspectives on designed tasks, and the role of digital technologies in designing mathematical tasks. The fourth part, Section 2.4, presents the literature on group work in mathematical modelling activities. The final part, Section 2.5, presents the summary of the chapter.

2.1 Mathematical modelling

The notion of mathematical modelling depends, amongst other things, on the theoretical perspective adopted. Several different approaches towards applications and modelling are not newly discovered. Thirty-eight years ago, Kaiser-Meßmer (1986) pointed out that different perspectives could be distinguished within applications and modelling (Kaiser & Sriraman, 2006). Kaiser and Sriraman (2006, p. 302) argued that there is no "homogeneous understanding of modelling and its epistemological backgrounds within the international discussion on modelling". The literature highlights different perspectives and approaches to mathematical modelling. The inclusion of mathematical modelling in curricula in schools and universities varies widely. Some of the goals of its inclusion might be: using mathematical modelling as a vehicle to teach mathematical concepts and procedures; promoting mathematics as a human activity answering problems of a different nature that might give rise to the emergence of mathematical concepts, notions and procedures; providing experiences that contribute to education for life after school; questioning the role of mathematical models in society and the environment; motivation to learn mathematics; amongst others (Stillman, 2019, p. 4). On the issue of how mathematical modelling should be handled in teaching situations, Julie and Mudaly (2007) argued that central to the debate on this issue is whether mathematical modelling should be used as a vehicle for the development of mathematics (modelling-as-vehicle) or treated as content in and of itself (modelling-as-content) (ibid.).

In the following subsection, I will discuss the different theoretical approaches to mathematical modelling in mathematics education.

2.1.1 Different perspectives on mathematical modelling

Stillman (2019) presented four examples of current theoretical lines of inquiry in mathematical modelling: local theories (*prescriptive modelling*, *modelling frameworks/cycles* and *modelling competencies*) and a general line of inquiry (anticipatory metacognition). Geiger and Frejd (2015) argued that within the research conducted in mathematical modelling, the local theories (mainly modelling cycle and modelling competencies) are the most frequently used theoretical approaches compared to other approaches. That is, a strong emphasis is placed on developing these local theories rather than general theories from outside the field. I will briefly discuss an overview of each of the perspectives listed above, drawing on different literature in the line of argument, and discuss other perspectives afterwards.

Prescriptive modelling. Meyer (1984) used *descriptive* and *prescriptive* modelling to describe models used for different modelling purposes. A *descriptive* model describes or predicts how something does or will work, and a *prescriptive* model is meant to help us choose the best way for something to work (ibid., p. 61). Alternatively, another name for *prescriptive* models is *normative* models. According to Niss (2015), the processes of *descriptive* modelling are typically represented by one of several or similar versions of the modelling cycle. In this case, most of the modelling processes follow descriptions of the modelling cycle from a cognitive perspective. Thus, much attention is given to the transitions between the modelling phases or nodes of the modelling cycle. Stillman (2019) pointed out that modelling cycles used in theoretical and empirical research are limited concerning adequately capturing all processes involved in *prescriptive* modelling (ibid., p. 7). Although a *normative* description of the modelling process is seen as an ideal way of modelling, Boromeo Ferri (2006) argued that an empirical description (what happens within the students' activity) differs from the *normative* description. Niss (2015) recommended focusing on theoretical and empirical research on *prescriptive* modelling while considering tasks of higher complexity.

Modelling frameworks/cycles. Much academic work has been done on modelling cycles (a description of the modelling process as a cyclic activity). See Figure 2.1 for an often-used example of a modelling cycle from Blum and Leiß (2007). There are other modelling cycles (Blomhøj & Jensen, 2003; Perrenet & Zwaneveld, 2012), and these cycles may have fewer or more phases (and other wordings) in the modelling process compared to the seven phases shown in Figure 2.1.

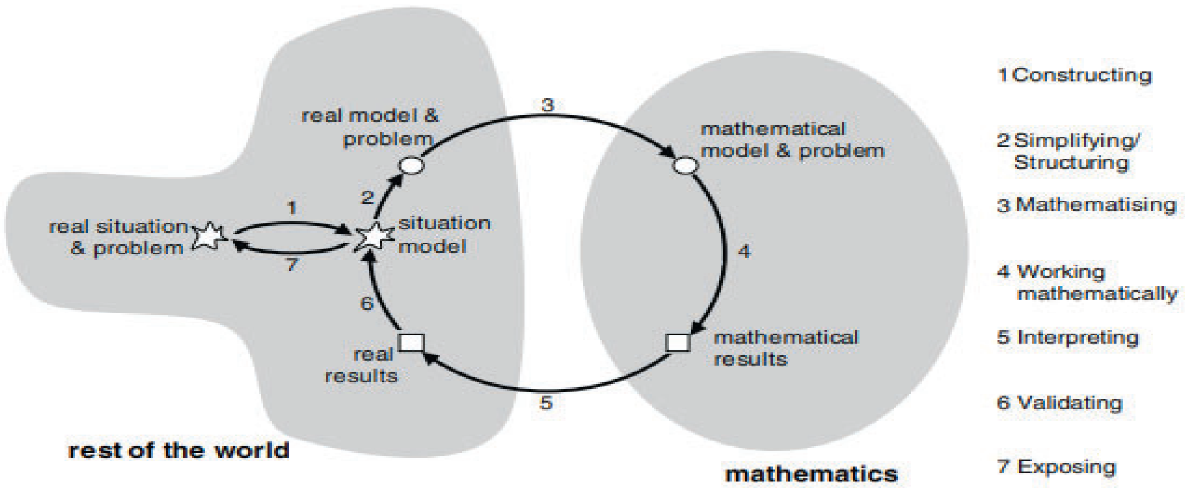


Figure 2.1: The modelling cycle by Blum and Leiß (2007).

The modelling cycle in Figure 2.1 illustrates (in an ideal way) steps in a modelling process from a cognitive point of view. Niss and Blum (2020) emphasized that it cannot be stressed enough that the depiction of the cognitive processes in Figure 2.1, involved in performing modelling, is an analytic reconstruction of what must happen in principle (ibid.). It must be noted that students follow routes other than what is described in modelling cycles when they are given a modelling task. Niss and Blum argue that the modelling cycle in Figure 2.1 (or other cycles) is (are) not a description of the path a concrete modeller will necessarily take in actual practice. For instance, the student's specific modelling routes in a given context in the study by Blum and Boromeo Ferri (2009) were different from the general modelling cycle depicted in Figure 2.1 (that is, the empirical description of modelling shows that the modelling cycle in Figure 2.1 is not linear); nonetheless, most phases are observed in students' activities. Stillman (2019) argued that a modelling cycle is a theoretical description of what real-world modelling involves and raised the question, 'Do we really need separate cycles for modelling with technology?'. Other studies describe modelling cycles while including a stage for technology within the cycle (see Sub-Section 2.2.1).

Other studies have opposing voices on the generality of modelling cycles (Albarracin et al., 2019). For instance, Cai et al. (2014) pointed out that a modelling cycle might show only some of the actual work done by students in a mathematical activity. Some studies (Ärlebäck, 2009; Czocher, 2016; Albarracin et al., 2019) highlight the difficulties in the qualitative identification of the stages of the modelling process corresponding to each episode of students' work. For this reason, Albarracin et al. (2019) analyzed the video recordings of four groups of students working on mathematical modelling tasks using Modelling Activity Diagrams (MAD). MAD (proposed by Ärlebäck (2009)) is an analytical tool for characterizing students' choices and actions in a mathematical modelling activity. The components of MAD comprise reading, making models/modelling, estimating, calculating, validating, and writing. These activities are used to characterize the modelling processes (Ärlebäck, 2009; Albarracin et al., 2019). Although these activities in MAD might help qualitatively identify the stages of the modelling processes, they do not include the role played by digital technologies in these activities. As such, this research study (in a different way) presents students' modelling processes as emerging actions and provides technology's role in these emerging actions (see Sub-Sections 2.1.3 and 4.4.2).

Modelling competencies. There are several definitions relating to modelling competence, and most often, these descriptions are based on phases in the modelling cycle. Niss et al. (2007) described modelling competence as follows:

> Mathematical modelling competency means the ability to identify relevant questions, variables, relations, or assumptions in a given real-world situation, to translate these into mathematics and to interpret and validate the solution of the resulting mathematical problem about the given situation, as well as the ability to

analyze or compare given models by investigating the assumptions being made, checking properties and scope of a given model etc. (ibid., p. 12)

That is, Niss and colleagues see modelling competence as the ability to perform specific appropriate actions in modelling situations to construct and investigate mathematical models. Kaiser (2007) sees this description as 'modelling ability' and argues that modelling competence must include the willingness to work out problems through mathematical modelling. Kaiser (2007) further distinguishes modelling competencies based on phases in the modelling cycle (see 'modelling competence' in Table 2.1 on page 26). In addition to this, Maaß (2006) highlights metacognition as an essential issue for developing modelling competencies (*metacognitive modelling competencies*). Vos and Frejd (2022) argue that metacognitive strategies are needed to regulate and coordinate the many processes (both individual and group processes) in modelling. Within modelling activity, aims and outcomes of the modelling processes need to be coordinated and regulated considering goals in the task and resources present, amongst others. Vos and Frejd further point out that different metacognitive strategies can be linked to each phase in Figure 2.1. For instance, strategies to understand and reformulate the problem and to use additional information can be linked to 'constructing'. From a research point of view, analyzing metacognitive strategies requires a theoretical framework different from that used for cognitive activities. However, metacognitive strategies and cognitive activities are intertwined (ibid.). I will further discuss modelling competencies in Sub-Section 2.1.2.

Anticipatory metacognition. Metacognition has been considered necessary in mathematical modelling (Maaß, 2006; Vorhölter, 2018), especially in reflecting on actions when addressing a real-world problem. The reflection on actions focuses on the mathematics employed and the modelling undertaken (Stillman, 2019). Stillman (2019) emphasized that new development in this area is *anticipatory metacognition* and describes it as "a reflection that points forward to actions yet to be undertaken, that is, noticing possibilities of potentialities" (ibid., p. 9). Noticing possibilities of potentialities can be an exciting idea to discuss from affordances and constraints perspective (that is, perceived and actualized actions, see Sub-Section 4.3.3). Back to the discussion, Stillman (2019, p. 9) highlighted three distinct dimensions that are encompassed within *anticipatory metacognition*: *meta-metacognition, implemented anticipation*, and *modelling oriented noticing*. *Meta-metacognition* results from teachers' thinking about (or reflecting on) the appropriateness/effectiveness of their students' metacognitive activity during mathematical modelling activity and subsequently acting, bearing this in mind. *Implemented anticipation* results from the successful use of foreshadowing and feedback loops to govern actions in decision-making during mathematization (see phase 3 in Figure 2.1). Niss (2010a) presented *implemented anticipation* as a pre-mathematization process for which the modeller needs to project himself/herself into a situation that does not quite exist yet. Thus, relevant future steps projected back onto the current actions and can also be put as 'what a modeller does now determines what they will do next'. *Modelling oriented noticing involves* 'noticing' how mathematicians and educators act when operating within the field of modelling from mathematical and pedagogical points of view. Geiger et al. (2018) empirically test some aspects of the theoretical dimensions of anticipatory metacognition and recommend that further research will be worth pursuing.

There have been other perspectives in mathematical modelling than those discussed above. For instance, Blum (2015) presented some perspectives (categorized by Kaiser and colleagues) of mathematical modelling according to their aims. Blum first distinguishes four groups of justifications for including applications and modelling in curricula and everyday teaching and then links these justifications to different perspectives in mathematical modelling. These justifications are listed as follows:

- *Pragmatic* justification: In order to understand and master real-world situations, suitable applications and modelling examples have to be explicitly treated (requires concrete authentic examples).
- *Formative* justification: Competencies can also be advanced by engaging in modelling activities; in particular, modelling competency can only be advanced in this way, and argumentation competency can be advanced by

"reality-related proofs (requires cognitively rich examples, accompanied by meta-cognitive activities).

- *Cultural* justification: Relations to the extra-mathematical world are indispensable for an adequate picture of mathematics as a science comprehensively (requires authentic examples that show students how strongly mathematics shapes the world or epistemologically rich examples that shed some light on mathematics as a science).
- *Psychological* justification: Real-world examples may contribute to raising students' interest in mathematics, to motivate or structure mathematical content, to better understand it and to retain it longer (requires either interesting examples for motivation or illustration purposes, to make mathematics attractive for students or mathematically rich examples that serve the purpose to make specific mathematical topics better comprehensible) (ibid., p. 81)

Blum conceptualized 'perspective' as a pair (aim | suitable examples). Furthermore, with this idea, Blum distinguishes between six perspectives, namely:

- (pragmatic | authentic) → *applied* modelling
- (formative | cognitively rich) → *educational* modelling
- (cultural with an emancipatory intention | authentic) → *socio-critical* modelling
- (cultural concerning mathematics | epistemologically rich) → *epistemological* modelling
- (psychological with marketing intention | motivation) → *pedagogical* modelling
- (psychological | mathematically rich) → *conceptual* modelling

Kaiser and Sriraman (2006), Blomhøj (2009), and Abassian et al. (2020) discuss these perspectives differently. Kaiser and Sriraman (2006) distinguished between different perspectives on modelling in mathematics education based on analyses of literature mainly generated by the International Commission on Mathematical Instruction (ICMI) and the International Community of Teachers of Mathematical Modelling and Applications (ICTMA) activities, amongst others. These perspectives are *realistic*/*applied*, *contextual*, *educational*, *socio-critical*, *epistemological*/*theoretical*, and *cognitive* modelling. This list (specifically, the wording used) slightly differs from Blum's list above. For instance, Kaiser and colleagues' use of *educational* modelling might capture both *pedagogical* and *conceptual* modelling described by Blum. Nonetheless, these perspectives are classified according to their central aims concerning modelling and other factors. I will present an overview of each of the perspectives drawing on other literature:

Epistemological/theoretical modelling. Mathematical modelling under this perspective is perceived as a lens to establish general theories for teaching and learning mathematics (Blomhøj, 2009), and the only goal of this perspective is the development of mathematical understanding (Abassian et al., 2020). Blomhøj (2009) listed two very different examples of such theories: Realistic Mathematics Education (RME; often used to conduct research within this perspective) and the Anthropological Theory of Didactics (ATD). García et al. (2006), from the theoretical perspective of ATD, argued that modelling is not considered an aspect or dimension of mathematics; instead, mathematical activity is essentially a modelling activity. They point out that their view of modelling is meaningful if a precise meaning is given to the modelling activity and if the idea of modelling includes both extra-mathematical modelling (real-world problems) and intra-mathematical modelling (pure mathematics-related problems such as the geometrical representation of algebraic and arithmetical expressions). Following this argument, Kaiser and Sriraman (2006) emphasized that modelling is not limited to the mathematizing of non-mathematical issues. Freudenthal (1973) distinguished between two forms of mathematization, local and global, and further explains that in global mathematization, the process of mathematizing is viewed as part of the development of mathematical theory. Similarly, Treffers (1987) also distinguished between the two forms of mathematizing: vertical (working inside mathematics) and horizontal (from reality to mathematics) mathematizing.

Realistic/applied modelling. Mathematical modelling under this perspective is viewed as applied problem solving for which a strong emphasis is put on real-life situations (Blomhøj, 2009). Students usually work with realistic and authentic (defined in Sub-Section 2.3.1) real-life modelling from this perspective. Kaiser and Sriraman (2006) emphasized that modelling can be understood as an activity to solve authentic problems and not as the development of mathematical theory (in contrast to the epistemological/theoretical modelling perspective). Furthermore, they viewed modelling as a way to help students understand the relevance of mathematics in everyday life and acquire competencies that enable them to solve real mathematics problems. These competencies are classified as modelling competence. Blomhøj (2009) argued that the students' modelling work should be supported using relevant digital technologies under this perspective. In academic literature, a modelling competence approach is often used more frequently than the other theoretical modelling approaches (Geiger & Frejd, 2015). From a realistic modelling perspective, Kaiser and Schwarz (2006) studied mathematical modelling as a bridge between school and university. Their results show that complex modelling examples are not only reserved for high-performing students but, on the contrary, average students in ordinary schools could carry it out.

Educational modelling. This perspective puts the structuring of learning processes (didactical) and fostering the understanding of concepts (conceptual) into the foreground of interest (Kaiser & Sriraman, 2006). According to Blomhøj (2009), this perspective does not only have the goal of developing mathematical modelling competencies, such as *realistic* modelling but also of learning mathematics. From this perspective, Blomhøj and Kjeldsen (2006) viewed modelling as a suitable didactical choice considers some of the central challenges in curricular reforms (for instance, introducing some concepts and their development). Under this perspective, Zbiek and Conner (2006, p. 89) develop a "diagrammatic model of mathematical modelling as a process that allows for mathematical understandings to be identified as learners are engaged in modelling tasks". Again, the empirical findings of Blomhøj and Kjeldsen (2013) showed that modelling activities open a window to the students' images of the mathematical concepts involved.

Socio-critical modelling. Kaiser and Sriraman (2006) referred to this perspective as the socio-cultural dimensions of mathematics, which are closely associated with *ethnomathematics* (mathematics, which is practised among identifiable cultural groups). This perspective highlights the role of mathematics and the function of mathematical modelling in society (ibid., 306). Barbosa (2006) argued from this perspective that the modelling processes developed in different approaches (for instance, *realistic* and *educational* modelling perspectives) are inadequate when describing students' modelling activities and that there must be a focus on students' discourse in modelling activities. Reflexive discussions amongst students form part of the modelling process, which might promote critical thinking. According to Kaiser and Sriraman (2006), critical thinking should be promoted as a central goal of teaching in mathematics pedagogy.

Contextual modelling. According to Kaiser and Sriraman (2006), *contextual* modelling is a long-standing perspective of solving word problems, especially in the American realm. In this perspective, Doerr (2006) argued that the tasks used in modelling activities reveal students' thought processes through descriptions, explanations, justifications, and representations students develop as they engage with the tasks. The above activity is the Model Eliciting Activity (MEA) (Lesh & Doerr, 2003; Doerr, 2006; Doerr & Lesh, 2011). MEA is defined as "a problem-solving activity constructed using specific principles of instructional design in which students make sense of meaningful situations, and invent, extend, and refine their own mathematical constructs" (Kaiser & Sriraman, 2006, p. 306). Doerr and Lesh (2011) acknowledge that there is more to modelling than just MEAs. In conclusion, Abassian et al. (2020) pointed out an overlap between the *contextual* modelling perspective and the goal of modelling from the *educational* perspective, and the fundamental difference is the emphasis on MEA.

Cognitive modelling. This perspective looks at the analysis of modelling processes with a cognitive focus (Kaiser & Sriraman, 2006). Blomhøj (2009, p. 10) argued that the main interest in this perspective "is to understand which cognitive functions are activated in the individual student's mathematical modelling activities". That is, identifying the cognitive barriers in the modelling processes. Blomhøj (2009) argued that the *cognitive* perspective is closely related to the *educational* perspective and the goal of developing mathematical modelling competence. Kaiser and Sriraman (2006) highlighted that this perspective is not a normative approach connected to the goals of teaching modelling in school, but rather, it starts from a descriptive position (see “prescriptive modelling” on page 12). Looking at the different types of modelling situations, Boromeo Ferri (2006) points out that the structure of tasks used in modelling activities influences, to some extent, the distinction between phases of the modelling processes. For example, from a cognitive perspective, Palharini and de Almeida (2015) studied how the interaction between modelling tasks and students’ mathematical thinking occurs. They found that students’ engagement in modelling tasks might promote the development of advanced mathematical thinking.

Another perspective that has yet to be explicitly captured in the list so far is *ethnomodelling* (which originates in ethnomathematics). This term expresses the relationship between culture and mathematics (D'Ambrosio, 2001). Rosa and Orey (2013) described *ethnomodelling* as the study of ideas and procedures elaborated by members of distinct cultural groups, and this involves the mathematical practices developed, used and presented in diverse situations in the daily life of the members of these groups. This process allows the group members to study mathematics as a system taken from their reality. Furthermore, they pointed out that *ethnomodelling* can be considered part of critical mathematics education (Skovsmose & Nielsen, 1996) because it might provide a learning process in which teachers encourage the use of multiple sources of knowledge from different cultural contexts. Furthermore, the acquired knowledge here is centred, oriented and grounded on the students’ cultural backgrounds (ibid.).

I will end this section by addressing the question, ‘*Which of the perspectives discussed above do I adopt in this research study?*’. Firstly, I have to consider the current Norwegian curriculum and its’ description of modelling and applications since this research study is in the Norwegian context. According to the Ministry of Education and Research (2019), modelling and applications are described as (a Google translate of the Norwegian text):

> A model in mathematics is a description of reality in mathematical language. The students must have an insight into how models in mathematics are used to describe

everyday life, working life and society in general. Modelling in mathematics is about creating such models. It is also about critically assessing whether the models are valid and their limitations, assessing the models in light of the original situations and assessing whether they can be used in other situations. Applications in mathematics are about the pupils' gaining insight into how they should use mathematics in various situations, both inside and outside the subject.

The description above follows Blum's pragmatic justification for including applications and modelling in curricula. Thus, students gain insight into how models in mathematics are used to describe everyday life when suitable applications and modelling examples are explicitly treated. This explanation falls mainly under the *realistic/applied* modelling perspective. The Norwegian curriculum also highlights model creation, critically assessing the validity of the model, its limitations and generalization of the model, which corresponds to some of the elements in *modelling frameworks/cycles* and *modelling competence* description (local theories: see pages 13 and 14). However, this curriculum does not explicitly mention these local theories, although we can link the descriptions in the curriculum to the elements of these local theories. I will return to this issue in Section 3.3, but before that I will discuss modelling competence in the forthcoming subsection.

2.1.2 Modelling competence

The notion of mathematical competence is broad and encompasses more specifically defined competencies (such as modelling competence); as such I will first discuss mathematical competence before presenting modelling competence.

Mathematical competence. Kilpatrick and Lerman (2014) argued that there are many different theoretical approaches to mathematical competence, and no single conceptual framework exists. Niss (2003, p. 6) defined *mathematical competence* as "the ability to understand, judge, do, and use mathematics in a variety of intra- and extra-mathematical contexts and situations in which mathematics plays or could play a role". Weinert (2001) identified seven ways competence is defined or theoretically interpreted: general cognitive competencies, specialized cognitive competencies, the competence-performance model, modifications of the competence-performance model, cognitive competencies and motivational action tendencies, objective and subjective competence concepts, and action competence (Kilpatrick & Lerman, 2014). Kilpatrick and Lerman (2014) emphasized that competence frameworks in mathematics education fall primarily into Weinert's specialized cognitive competencies (the cognitive prerequisites that must be available for an individual to perform well in a particular content area). Blomhøj and Jensen (2007) argued that the discussions above are good reasons for applying competence as an analytical concept in mathematics education; however, to transform it into a developmental tool, we need to be more specific (I will further expound on this statement below). Boesen et al. (2018) proposed six competencies with particular content areas in assessing mathematical competence: problem-solving, reasoning, procedural, representation, connection and communication competency. The categories mentioned above describe cognitive activities. In addressing the question, 'What does it mean to master mathematics?' The Danish KOM (Competencies and the Learning of Mathematics) project identifies eight competencies: mathematical thinking, representation, symbols and formalism, communication, aids and tools, reasoning, modelling and problem handling (Niss, 2003; Niss et al., 2016; Niss & Højgaard, 2019) (see Figure 2.2 below).

Each competency highlighted in Figure 2.2 overlaps each of the other seven competencies but is also distinct. For instance, modelling with digital technologies touches both modelling and aids and tools competency (discussed further in Sub-Section 2.2.1). Niss and Højgaard (2019) argued that competencies can be applied normatively for designing curricula in any mathematics education context and level. Such normative use of the competency framework can be seen in the new Norwegian mathematics curriculum (K20), which has the following core elements: exploring and problem solving; modelling and applications; reasoning and argumentation; representation and communication; abstraction and generalization; mathematical knowledge areas; and programming and algorithmic thinking (Ministry of Education and Research, 2019).

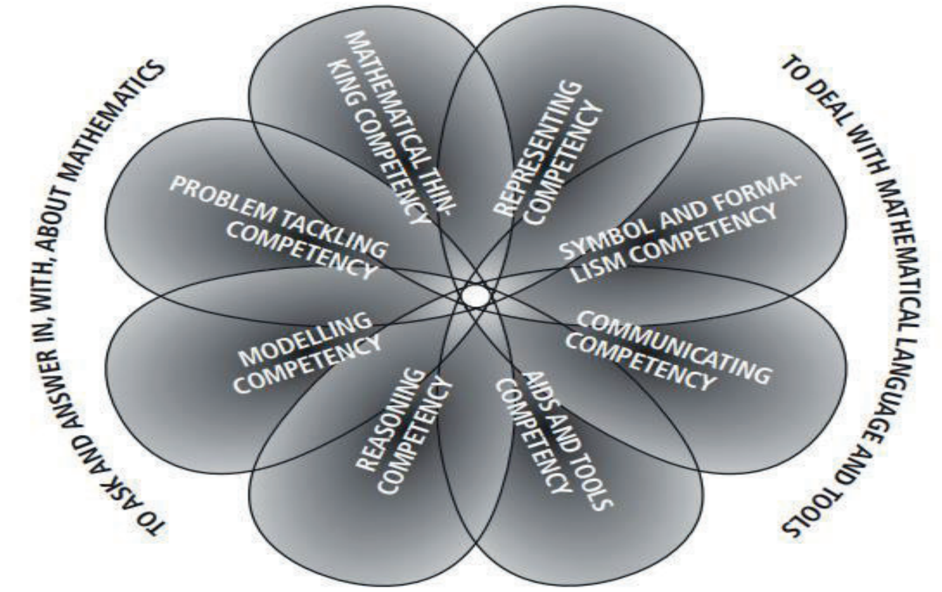


Figure 2.2: A visual representation of the competence framework (Niss & Højgaard, 2019, p. 19).

Niss and Højgaard (2019) suggested that the competency framework can also be used in describing the ongoing teaching and learning of mathematics. Classroom observation is a means that can be used directly to research these competencies in teaching and learning mathematics, but there are some challenges to be noted and addressed (Schlesinger & Jentsch, 2016; Schoenfeld et al., 2018; Ing & Webb, 2012). One such challenge concerns the observer ratings of the observations conducted by internal (teachers or students) or external (researchers or educators) observers. I have addressed this challenge in this study in Sub-Section 5.9.1. One competency content of interest that I will discuss now is modelling competence.

Modelling competence. I briefly discussed modelling competence in Sub-Section 2.1.1 (see page 14), but I will discuss it in more detail here. Blomhøj and Jensen (2003) explained modelling competence using cyclic activities, and by modelling competence, they meant "being able to autonomously and insightfully carry through all aspects of a mathematical modelling process in a certain context" (ibid., p. 126). According to Niss and Højgaard (2019), modelling competence focuses on mathematical models and modelling, which is when mathematics is used in dealing with extra-mathematical questions, context, and situations. Mathematical models "refers to purposeful mathematical descriptions of situations, embedded within particular systems of practice that feature an epistemology of model fit and revision" (Lesh & Lehrer, 2003, p. 109). Thus, models are mathematical representations of reality. Geiger et al. (2022) argued that modelling competency can be understood as the capacity to undertake all aspects of mathematical modelling holistically. All the definitions above seem to capture the same thing: the aspects needed to master a modelling process. Again, these definitions of modelling competence are based on competence frameworks on modelling cycles describing congnitive activities, which can be observed in (or deduced from) students' speech, gestures, writings, reactions and other explicit or implicit expressions. This competence framework identifies congnitive barriers in the modelling process but never any socio-cultural barriers. That is, the framework detaches individual modellers from cultural-historical contexts.

Wess et al. (2021) argued that modelling competence is not a one-dimensional construct but can be interpreted as a combination of different sub-competencies. The German group (e.g., Blum, Kaiser, Maaß) has done much work on developing a comprehensive concept of modelling competencies based on sub-competencies and their evaluation (Kaiser & Brand, 2015). Table 2.1 below describes the sub-competencies of modelling. The sub-competencies described in Table 2.1 highlight the other seven aspect of Niss's (2003) competency framework (see Figure 2.2 on page 24). Theoretical reflections from different literature point to many possible sub-competencies of modelling competencies; however, Maaß (2006) argued that it can be challenging to gain empirical evidence about which sub-competencies are needed to carry out a modelling process.

Concerning the description of modelling and applications in the new Norwegian mathematics curriculum (see the last paragraph on page 21), the phases in Figure 2.1 (on page 13) and the modelling competencies and sub-competencies in Table 2.1 are recognized in the description of the curriculum. The summary of the description is creating models (a description of reality in mathematical language), critically assessing the models' validity and limitations, assessing the models in light of original situations, and assessing whether they can be used in other situations. Creating models is associated with setting up a mathematical model, as it requires the translation of the real situation into a mathematical model. To critically assess the model's validity, the mathematical question(s) within the mathematical model need to be solved while recognizing the model's limitations. Assessing the model in light of the original situation is associated with interpreting the mathematical results in a real situation and generalizing the solution to suit a different context. So, the modelling processes in the new Norwegian curriculum are prominent in explaining modelling, like in the definitions of modelling competence.

Modelling Competence	Sub-Competencies
Understand the real problem	Making assumptions about the problem and simplifying the situation. Identifying relevant questions in the given real-world situation. Looking for available information and differentiating between relevant and irrelevant information. Recognizing quantities that influence the situation (by naming them and identifying key variables). Constructing relations between the variables.
Set up a mathematical model	Translating the real problem into the mathematical world and eventually GeoGebra (the computer world). Mathematizing relevant quantities and their relations. Simplifying relevant quantities and their relations (that is, reducing their number and complexity). Choosing appropriate mathematical notations and representing the situations graphically.
Solve the mathematical questions within the model	Using mathematical knowledge to solve the problem. Observing the effect of parameters on the graph (For instance, using sliders in GeoGebra to vary parameters to see the effects on the function/s on the graph). Manipulating mathematical figures and shapes dynamically to see what happens.
Interpret the mathematical results in a real situation	Using appropriate mathematical language to communicate the solutions. Generalizing the solution to suit a different context.
Validate the solution	Critically check and reflect on found solutions. Reflecting on other ways of solving the problem. Going through the modelling process if the solution does not fit the situation.

Table 2.1: Structure of Modelling Competence (based on Maaß (2006) and Blum & Kaiser (1997)).

There are different perspectives on modelling competence, aside from the one reported by the German group (see the second paragraph on page 25). Putting all the perspectives together, Kaiser and Brand (2015) summarized them as:

1. The introduction of modelling competencies in an overall comprehensive concept of competencies within the Danish KOM project.
2. The assessment of modelling skills and the development of assessment instruments (as within British-Australian researchers).
3. Development a comprehensive concept of modelling competencies based on sub-competencies and their evaluation (predominantly among German researchers).
4. Integrating metacognition into modelling competencies (predominantly among Australian researchers) (ibid., p. 135).

Within these perspectives shaping the discourse on modelling competencies, Cevikbas et al. (2021) distinguished two primary approaches: ‘a holistic understanding’ and ‘analytic description’ of modelling competencies. Niss and Blum (2020, pp. 80-81) respectively label these approaches as *top-down* and *bottom-up*. According to Niss and Blum (2020), the first perspective corresponds to *top-down*, whereas the second and third perspectives correspond to the *bottom-up*. The fourth perspective does not represent a specific approach but is placed somewhere between *top-down* and *bottom-up*.

Top-down (modelling competenc*y*): This approach deals with a comprehensive, overarching entity called *the* modelling competenc*y* in the singular. In this perspective, there exists such a distinct, recognizable, and more or less well-defined entity. Moreover, there is the possibility (resulting from closer analysis) of identifying major components of and other elements in this entity. In this case, *the* modelling competenc*y* is the primary object, whilst the major components (sub-competencies) are derived (secondary objects).

Bottom-up (modelling competenc*ies*): This approach deals with a set of distinct and separate modelling competenc*ies* in the plural without, in the first place, seeing them as instances, aspects or components of a comprehensive, overarching modelling competenc*y*. And these competencies are tightly linked to the modelling cycle.

Although Niss and Blum (2020) placed the fourth perspective between *top-down* and *bottom-up*, Kaiser and Brand (2015) assigned the fourth perspective to the analytic approach (*bottom-up*). I will delve into this issue by discussing *holistic* and *atomistic* approaches to the development of modelling competency. According to Blomhøj and Jensen (2003), a *holistic* approach needs a full-scale modelling process, where students work on the modelling process in its entity. In contrast, in an *atomistic* approach students concentrate on selected phases of the modelling process. Thus, students work to develop one (or few) sub-competencies at a time (especially mathematizing and analyzing models). Blomhøj and Jensen (2003) emphasized a balance between the two approaches, as none of them is seen as adequate. Kaiser and Brand (2015, p. 146) argued that the complex construct [modelling competency] consists of a global, overarching modelling competency and several sub-competencies. Niss and Blum (2020) see this description as combining the *top-down* and the *bottom-up* definitions.

For instance, Maaß (2006) argued that different empirical studies hint at single factors which seem to influence modelling competencies. For instance, mathematical skills, knowledge about the modelling process, a sense of direction, and working in groups, amongst others, are sometimes seen as distinct factors that seem to have a positive impact on the development of modelling competencies and aspects which have not yet been considered might have a significant influence (ibid., p. 119). Blomhøj and Jensen (2007) argued that progress in mathematical modelling competency needs to be described in more than one dimension. Blomhøj and Jensen analytically distinguish between at least three different dimensions in mathematical modelling competency, namely:

- A dimension describing the *degree of coverage*, meaning which parts of the modelling process the students are working with and at what level of reflection.

- A dimension that has to do with the *technical level* of the students' activities involved in the modelling process, meaning what kind of mathematics they use and how flexibly they do it.
- A dimension that has to do with variation in the situations and contexts in which the students can activate their mathematical modelling competency (*radius of action*) (ibid., p. 51).

Analyzing progress in mathematical modelling competency might require an interplay between the dimensions, for instance, the parts of the modelling process the students are working with and the mathematics they use in that part. So far, the discussion on modelling competence still emphasizes cognitive aspects, which may not capture other aspects that play a role in mathematical modelling. Vos and Frejd (2022) pointed out that other dimensions in the form of metacognitive skills, digital technology use and social norms play a role in mathematical modelling. For instance, Cevikbas et al. (2021) argued that metacognitive skills are needed to monitor modelling activities, specifically the many processes in modelling—both individual and group processes (Maaß, 2006; Stillman, 2011; Vorhölter, 2018). Within the Australian modelling discussion groups (Galbraith, Stillman), much work has been done on integrating metacognition into modelling competencies (Kaiser & Brand, 2015). I will continue with the discussion of the metacognitive dimension (while discussing more of the digital technology dimension in Section 2.2 and the social norms dimension in Section 2.4).

The term metacognition does not have a standardized definition (Desoete & De Craene, 2019). However, Maaß (2006) described it as knowledge about an individual's thinking up to self-regulation in problem-solving. Maaß (2006) identified, in a qualitative study, misconceptions: (a) setting up the real model, (b) setting up the mathematical model, (c) the mathematical solution, (d) the interpretation and validation, (e) and general misconceptions, although most of the students developed appropriate metacognitive modelling competencies (ibid., p. 134). In Maaß's studies, students' meta-knowledge was measured by analyzing interviews and concept maps, which the students had to create (and the quality of meta-knowledge in most cases was related to performance in modelling). Vorhölter (2018) emphasized that Maaß's work focused on metacognitive knowledge and not strategies. Vorhölter argued that knowledge and skills are essential but insufficient for solving a problem successfully and that other aspects influence metacognitive strategies during group work. For example, aspects like students' motivation, task difficulty, group members, digital technology used, and others (ibid.). Vos and Frejd (2022) linked different metacognitive strategies to each of the phases of the modelling cycle in Figure 2.1 (on page 13). Table 2.2 below illustrates phases in the modelling process with indicative dimensions for cognitive activities, metacognitive strategies, and digital technologies used.

Students would have to read the intentions beneath the task description (sometimes problematic) at the start of their work and anticipate what they can do to reach a satisfying answer. From Table 2.2, in each cognitive activity, the students can expect unexpected situations for which they might reflectively change the initial plans; to do this, they need to anticipate, reflect, plan, and monitor, amongst others. Vos and Frejd (2022) emphasized that from a research point of view, a theoretical framework should be different from that of cognitive activities to analyze metacognitive strategies. However, metacognitive and cognitive activities are intertwined (ibid.). Another way of studying metacognitive strategies in the students' activity while considering other aspects (such as digital technology use and social norms, amongst others) is through the lens of Activity Theory. From an Activity Theory perspective, these aspects can be viewed as components interacting with each other as a whole and not distinct.

I will further address this issue in Chapter 4. However, in the following subsection, I will discuss the ontology and epistemology of modelling competence (based on an activity theorist's epistemology and ontology).

	Cognitive Activities	Metacognitive Strategies	Digital technologies used
1	Constructing	strategies to understand and reformulate the problem, to use additional information	Interpret task sheet, investigate resources (e.g., Wikipedia, Google search, Google maps, etc.).
2	Simplification /structuring	strategies to select and organize information, develop	Experiment with sketch & drawing tools (e.g., GeoGebra), spreadsheets, etc.

		plans, anticipate later actions, to monitor progress	
3	Mathematizing	strategies to organize information, develop & implement plans, to monitor progress	Visualize and organizing with spreadsheets, plotter, GeoGebra, etc.
4	Working mathematically	strategies to implement plans, to monitor progress	Calculate & simulate with GeoGebra, calculators, spreadsheets, etc.
5	Interpreting	strategies to interpret results, to face unplanned outcomes	Visualize with presentation tools, etc.
6	Validating	strategies to verify results, to invite critique, to evaluate the process and products	Control using information resources, etc.
7	Exposing	strategies to present results, to communicate and convince	Present a report using digital tools (e.g., Microsoft Word, etc.).

Table 2.2: Phases in the modelling process with indicative dimensions for cognitive activities, metacognitive strategies and digital technologies used (based on Vos & Frejd (2022)).

2.1.3 Ontology and epistemology of modelling competence

I will discuss the ontology of modelling competence in light of *top-down* and *bottom-up* approaches (see page 27). Niss and Blum (2020) argued that these approaches are ontologically distinct. Thus, a *top-down* approach corresponds to an empirically well-delineated entity found in the real world. Moreover, we can say that someone possesses this competency. In the other view, separate modelling competencies constitute the primary notions within the *bottom-up* approach, each of which exists (in conceptual and empirical terms) independently of the other competencies; furthermore, an individual may possess some of these competencies and not others.

I have carefully and deliberately used Niss and Blum's (2020) own words rather than paraphrasing them. I do this for the reason of dwelling on the ontological stances reflected in the words or phrases: "set of distinct and separate modelling competencies", "someone possesses this competency", and "each of which exists". These highlighted phrases sound a bit like someone out there possesses this competency. However, I do not believe or claim that the literature on modelling competencies sees competencies as entities floating about in an ethereal space that students somehow grasp hold of. Again, Niss and Blum (2020) provided a rich ontology in both approaches and see competency or competencies as existing entities. However, I argue that these competencies are transient entities that emerge and then disappear. I will further explore this statement by turning to activity theorist epistemology and ontology, but before that, I will present mathematical modelling as a mathematical activity.

Mathematical modelling is a subset of mathematical activity. Dundar et al. (2012) described mathematical modelling as a conversion activity of a real-world problem in a mathematical form. There is no clear definition for mathematical activity. However, we can describe some components and types of mathematical activities. According to Cuoco et al. (1996), mathematical activity could include pattern-seeking, experimenting, describing, tinkering, inventing, visualizing, conjecturing, and guessing. On the other hand, Plaxco and Wawro (2015) defined five mathematical activity categories: defining, proving, relating, example-generating, and problem-solving. All these components in the two descriptions above are highlighted in the modelling competences described in Table 2.1 (on page 26). The term activity here is used in the sense of Activity Theory (Leont'ev, 1981a). Activity in Cultural-Historical Activity Theory (CHAT) is the analytic unit for understanding human performances, such as their practices, the sense they make, or their actions (Roth, 2012). Kaptelinin et al. (1995) pointed out that "activity cannot exist as an isolated entity" and "the very concept of activity implies that there is an" individual or collective 'subject' who acts, for which the "activity is directed at something" (ibid., p. 191). As describe by Leont'ev (1974), the concept of activity refers to the subject-object interaction mediated through tools and societal relations and tools (defined and explained in Section 2.2 and Sub-Section 4.1.1). In the sense of CHAT, mathematical activity can be described as a cultural or historically developing sociocultural activity comprising several components and aspects or object-oriented, collective, and culturally mediated human activity. Mathematical modelling can also be viewed as an activity in the sense of CHAT, and it is through this activity that modelling competencies emerge.

Now, back to the discussion on the ontology of modelling competence, I will expound on a CHAT ontological interpretation of modelling competence for this research study. I then regard modelling competenc(y)(ies) as "cultural forms of reflection". Furthermore, I do so by turning to Radford's (2008b, p. 215) ontological basis for *knowledge objectification*, the "social process through which students become progressively conversant with cultural forms of reflection". Radford (2008b, p. 221) again "suggests that mathematical objects are historically generated during the course of the mathematical activity of individuals". To be precise, Radford further highlights that "mathematical objects are fixed patterns of reflexive human activity incrusted in the ever-changing world of social practice mediated by artefacts" (ibid., p. 222). In his argument, Radford used a conceptual object such as a circle, but I would say that his argument applies to modelling competenc(y)(ies). I argue that modelling competence is a phenomenon that emerges within an activity since competencies are not seen as entities floating about in an ethereal space that students somehow grasp hold of. Thus, this phenomenon is not seen as a general manifestation but emerges within an activity. For instance, modelling competencies always emerge in students' interactions with digital technologies during modelling activities. As such, one cannot know (in advance) which modelling competence will emerge in a particular situation/context, although we can draw on experience to anticipate the emergence of potential modelling competencies (that is not always the case). An example in Blum and Boromeo Ferri (2009) showed that the students' specific modelling routes, in a given context, differed from the defined general modelling cycle. Niss and Blum (2020) point out that the ideal modelling process described in a modelling cycle is not an analytic reconstruction of what must happen in principle.

If we then interpret competencies as emerging actions performed by students within an activity, then one might raise the issue that "when students get a mathematical modelling task and say, they could take approach A or B (approach A is somewhat faster, but the students carry out approach B because the teacher wants them to use GeoGebra—a dynamic mathematics software, discussed in Section 5.4); *can we then conclude that the students only have the competency to* *do approach B, because that was the emerging action, although the students can carry out approach A*? More generally, how do we then analyze professional modellers' competencies – they can potentially solve many problems in many different ways. However, they will not always show all these options in an activity. *Further to this, when looking at the emerging actions, does that mean that he/she only has limited competencies when only one approach shows up at the moment we are analyzing*?" I will address these issues in the discussion below.

Taking a closer look at modelling competence, when students engage in a mathematical modelling task (successfully) during a modelling activity, they employ historically accumulated knowledge that they have appropriated. One can view this through the lens of the modelling process. However, the activity, actions and operations (this is defined in Sub-Section 4.1.2) are specific to the context of the task – the knowledge is *situated* (Lave & Wenger, 1991). For instance, Boromeo Ferri (2006) argued that an individual might stay mainly between the real model and mathematical model concerning Problem 1 compared to Problem 2.

- Problem 1—contains more information in connection with given numbers in the task, and numbers which have to be adding through extra-mathematical knowledge.
- Problem 2—have fewer numbers given, but inner-mathematical knowledge is available on an implicit level and must be recognized and used for solving (ibid., p. 93).

Back to the argument, the formulation and mathematization of the students depend on their extant knowledge when meeting the task. Furthermore, there is a dialectical relation between the artefacts (mathematical concepts, digital technologies, among others) they employ and their mathematical analysis. For example, in the same task, one student might draw a function to represent the data with GeoGebra, and another might use Excel/spreadsheet to generate data to solve the task. Again, the interpretation and evaluation of the students are related to their perception of the task's demands. Generally, students' actions emerging within an activity might depend on the rules of the activity (e.g., whether a teacher tells the students to use approach A or B), the technology used, the characteristics of the students, and groupwork, among others. All these components acts as a whole in influencing students' actions.

This series of arguments could be conceived of in terms of *top-down* and *bottom-up* perspectives of modelling competence. However, as mentioned above, I argue that modelling competence is an emergent phenomenon. Thus, from the student's perspective, the competence disappears once the task is solved. However, remnants of the competence may be retained, and throughout solving multiple tasks, some actions may be routinized. I conclude that if students face a new task, which might require historically accumulated knowledge that the students have not appropriated, then what is called modelling competence will not be available to the students. For instance, students solving Task A (see Appendix B on page 355) might measure the air distance between the different locations. These same students might again measure the air distance between cities while working on Task 2 (see Sub-Section 5.5.2). However, the new task (Task 2) does not require only the air distance but also the consideration of actual road distance and other factors. Suppose these students work on multiple tasks similar to Task A for some time. In that case, some of their actions may be routinized (they might even develop different approaches to solving this type of task over time), and upon meeting Task 2, their appropriated historically accumulated knowledge comes into play (although the emerging actions are specific to the context of the current task).

Considering Niss's (2010a) notion of implemented anticipation about relevant future steps that are projected back onto current actions. I would say that these future steps result from students appropriated historically accumulated knowledge and components such as digital technology used, students' characteristics, group work, and others governing the emerging actions in decision-making within modelling activities. Furthermore, the student's actions emerging here, I will call *modelling actions* in an Activity Theory sense. I will return to the notion of *modelling actions* in Sub-Section 4.4.2. In conclusion, I view the modelling process (for instance, modelling cycle and competence) as a tool for analyzing students' modelling work (Vos & Frejd, 2022) or a lens through which we can view the historically accumulated knowledge that the students have appropriated.

The idea of *modelling actions* might, in a broader sense, describe the emerging actions, the goals these actions are directed towards, and the operations done within these actions of a modelling activity.

Another dimension of concern within students' modelling activity is the digital technology used. I will discuss this dimension in the following section.

2.2 Digital technologies

Digital technologies are electronic tools, systems, resources and devices that help generate, store or process data (Ibem & Laryea, 2014). Digital technologies are *tools* (digital tools), and from the perspective of CHAT, they are used in goal-directed actions. In this case, digital tools mediate the activities of individuals (see Section 4.1). In mathematics education, Greefrath et al. (2018) referred to digital tools as digital media such as computers, tablets, or hand-held devices that can be used to support the learning and teaching of mathematics in some specific way (ibid., p. 234). From the descriptions above, I argue that digital tools are computer hardware, software tools, networks, calculators, and mobile technologies, among others, and the primary concern of using such tools in mathematics education is not simply becoming fluent with these tools but ensuring the learning and teaching of mathematics. Some perspectives or frameworks describe students' use of digital tools in mathematics education: instrumental approach, co-action, co-construction of tools, affordances and constraints (discussed in Section 4.3), and others. These frameworks uniquely acknowledge an interaction between the user(s) and the tool where one is not wholly formed without the other.

Instrumental approach. This approach was developed to view students' activity in technology-enhanced environments from a French didactics perspective. Verillon and Rabardel (1995) proposed an approach that distinguishes an *artefact* from an *instrument*. According to Trouche (2005), an *artefact* is a material or abstract object aiming to sustain human activity in performing a task (for instance, a calculator and an algorithm for solving quadratic equations are *artefacts*). In contrast, an *instrument* is what the subject *builds* from the *artefact*. Thus, the *artefact* becomes a mediating tool, and an *instrument* emerges once a meaningful relationship develops between the *artefact* and the student(s) as they work on a specific task. The *building* process in Trouche's definition of an *instrument* is called *instrumental genesis*. *Instrumental genesis* refers to a complex process linked to the characteristics of the *artefact* (its potentialities and constraints) and the subject's activity, his/her knowledge and former work methods (ibid., p. 144). This *instrumental genesis* has two components (Artigue, 2002): *instrumentalization* and *instrumentation*. *Instrumentalization* is the transformation of the *artefact* into an *instrument* such that the potentialities of the *artefact* for performing specific tasks are recognized. *Instrumentation* is the process (taking place) within the user in order to use the *instrument* for a specific task. In summary, Trouche (2005) points out that a subject builds an *instrument* to perform a type of task, and this *instrument* is thus composed of both an *artefact* and the subject's schemes, allowing him/her to perform tasks and control his/her activity. According to Drijvers and Gravemeijer (2005), the process of *instrumental genesis* is two-dimensional in that, on the one hand, the possibilities and constraints of the *artefact* shape the conceptual development of the user and, on the other hand, change how he/she uses the *artefact*, in some cases may even lead to changing the *artefact* or customizing it (ibid., p. 168). We can label a teacher's activity towards promoting students' *instrumental genesis* as *instrumental orchestration* (Trouche, 2003, 2005). Social aspects of learning are recognized within this process, and *instrumental genesis* takes place in a social context. Although *instrumental genesis* is often a social process, the utilization schemes are individual, according to Drijvers and Gravemeijer (2005).

Co-action with digital technologies. Moreno-Armella et al. (2008) introduced the concept of *co-action* (in the study of shifting from static to dynamic media in mathematics classrooms), which meant that "a user can guide and simultaneously be guided by a dynamic software environment". This concept describes a fluid activity in that students within a dynamic environment perform specific actions upon the environment while their subsequent actions are guided by the environment (ibid). For instance, if a student drags a vertex of a rectangle in GeoGebra, the medium re-acts to the student's action, producing a new object, and this might stimulate a new action from the student (if, for instance, the student wants to have a sizeable rectangle). According to Moreno-Armella and Hegedus (2009), "the student and the medium re-act to each other and the iteration of this process" can be described as *co-action* between the student and the medium (ibid., p. 510). Carreira et al. (2013) applied this concept to study students' modelling of linear functions, mainly on how GeoGebra stimulates a geometrical approach.

Doerr and Zangor's co-construction of tools. Doerr and Zangor's (2000) idea of *co-construction* stems from a perspective in which psychological aspects of learning are coordinated with the social aspects through students' interactions with mathematical tasks, each other and their teacher within the social context of the classroom. Critical aspects of this social context include the tools (for instance, graphical calculator) and the norms for tool usage (the calculator as computational, transformational, data collection and analysis, visualizing, and checking tool), which emerge as students and teachers interact with the tool and each other. Doerr and Zangor argued that "it is through these interactions that the meaning of the graphing calculator as a tool for mathematical learning within the classroom is constructed by both teacher and students". They further highlight a limitation: the tendency for students to use their calculator (tool) as private devices regularly leads to a breakdown of group interactions (defined and explained in Section 2.4) (ibid.).

Each framework described above acknowledges that the purpose and use of a tool are inseparable from its user and the context (the activity in which the tool is used). However, these frameworks do not outline the role of digital technologies as mediating tools for social interaction among peers (in transforming an artefact into a tool), although these frameworks describe processes for forming a tool and students' use of tools in mathematics classroom environments. *What, then, is the role of digital technologies in mediating socially oriented mathematical practice?* In order to consider the roles digital technologies play in mediating collaborative (defined and explained in Section 2.4) practice, it is helpful to start with Taylor's (1980) description of the three ways in which technology is used in education:

- As a tutor—for teaching or providing feedback.
- As a tool—for doing mathematics or presenting ideas.
- As a tutee—programming or teaching the computer (ibid.).

Building on these descriptions, other literature considers another role of digital technology as an *actor* (e.g., artificial intelligence/AI systems and other non-human actors). Digital technology as an *actor* means the capacity of the technology (actor) to act in a given environment (Van Vaerenbergh & Pérez-Suay, 2022). Thus, there is no difference between human beings and computers, although they have different capacities (Borba & Villarreal, 2006). In this regard, we can view humans and computers as actors/agents from an Actor-Network Theory (ANT) perspective. Actor-Network Theory (ANT) provides a framework that explains that everything exists in a network of interactive relationships, including people, technology and non-living or inanimate objects (Callon, 1984; Law, 1992; Latour, 2005). ANT proposes that human and nonhuman agents are equally important, and both agents can influence the development of social-ecological systems (Dwiartama & Rosin, 2014).

Digital technologies used in students' mathematical activities might influence students' collaborative discussions. For instance, Geiger et al. (2010) emphasized that digital technologies (such as GeoGebra, Excel, etc.) can be used to mediate productive collaborative interaction (in mathematical modelling activities) even though these technologies have not been specifically designed to support collaboration. Goos et al. (2003) extensively researched how digital technologies can facilitate collaborative inquiry in small-group interactions or whole-class discussions. Furthermore, they categorized and illustrated four roles of technology in teaching and learning interactions: *technology as master*—students might be subservient to the technology if their knowledge and usage are limited to a narrow range of operations over which they have technical competence; *technology as servant*—technology is used as a fast, reliable replacement for mental or pencil-and-paper calculations, but the tasks of the classroom remain unchanged; *technology as partner*—technology is used creatively to increase the power students exercise over their learning, for example, by providing access to new kinds of tasks or new ways of approaching existing tasks; and *technology as extension of self*—involves users incorporating technological expertise as a natural part of their mathematical and/or pedagogical repertoire (ibid.). Discussing some issues in designing environments that support computer-supported collaborative learning, Stahl (2006) points out that cognitive tools (defined on page 41) for collaborative settings are essentially different from cognitive tools for individuals. Stahl outlines some considerations concerning the differences:

- The use of cognitive tools by a collaborative community takes place through many-to-many interactions among people, not by individuals acting on their own,
- The cognition the tools foster is inseparable from the collaboration they support and others (ibid., p. 104).

The considerations above support a socio-cultural perspective on learning. Geiger et al. (2010) argued that using digital technologies to mediate collaborative learning is consistent with a socio-cultural perspective on intellectual development, where learning takes place via social interaction and is supported by cultural artefacts or tools. In this case, there is a more robust and complex interaction between technologies and humans, which can also change the interaction between humans. Borba and Villarreal (2006) presented that human thinking is reorganized by different media, such as computers and their evolving interfaces. By reorganization, Borba and Villarreal meant that computers do not substitute humans, nor are they juxtaposed (placed side by side) with them. They further argued that these computers interact and are actors in knowing. Again, these computers form part of a collective that thinks and are not simply neutral tools or have some peripheral role in producing knowledge (ibid., p. 2). Borba and Villarreal further suggest that "humans are constituted by technologies that transform and modify their reasoning and, simultaneously, these humans are constantly transforming these technologies". In this case, there is an inter-shaping relationship between students and technologies. Moreover, per the argument above, Borba and Villareal dismiss the dichotomization of humans and technology and assert that knowledge is produced by a collective composed of human-with-media or human-with-technologies and not by individual humans alone or collectives composed only by humans (ibid.). Another perspective of discussing this relationship between students and digital technologies is through the affordances and constraints of digital technologies (see Section 4.3). In the next paragraph, I will discuss some categorizations of digital technologies.

Categories of digital technologies. In the last decades, there has been an increase in the use of digital technologies in secondary schools (Olofsson et al., 2020). Olofsson et al. (2020) argued that this development is essential as it enables students to participate in and contribute to a highly digitalized society. There are hundreds of digital technologies/tools in mathematics education. These digital tools have different functions, for instance, tools that include software for algebra and calculus (e.g., computer algebra systems—CAS), for 2D and 3D geometry (dynamic geometry systems—DSG), for statistics (e.g., R programming), among others (Drijvers, 2020). I will first categorize digital tools under *digital resources*. *Digital resources* can be described as any resource/material in digitalized form (either internet or offline resources). For example, calculators, computers and mathematics software are digital resources (Artigue, 2007). These resources can be further categorized as *hardware technologies* and *educational technologies*. Hardware technologies include calculators, laptops, smartphones, and others. Davies and West (2014) considered educational technology as any tool that might help students accomplish specified learning goals (not always the case in reality).

This technology includes both instructional technologies (focusing on technologies teachers employ to provide instruction) and learning technologies (focusing on technologies learners use to accomplish specific learning objectives) (ibid.). We can further distinguish educational technologies as *standard educational software* (e.g., Wolfram Alpha, GeoGebra, Excel, Stack, Numbas, Aplusix, and others) and *online educational technologies* (e.g., Khan Academy, massive open online courses—MOOCs, Google Search and Maps, among others).

Pepin et al. (2017) distinguished between *digital curriculum resources* and *educational technologies* (in terms of *curriculum material*). *Curriculum material* is broad as it encompasses several things ranging from little things (like worksheets) to a full-blown curriculum scheme. Nonetheless, Pepin and colleagues based their distinction on curriculum materials that are specifically organized systems of digital resources in electronic formats that articulate a scope and sequence of curricular content. In this case, "it is the attention to sequencing—of grade-, or age-level learning topics, or of content associated with a particular course of study (e.g., algebra)—to cover (all or part of) a curriculum specification, which differentiates digital curriculum resources from other types of digital instructional tools or educational software programme". There is an overlap in this distinction as digital curriculum resources use these other types of tools and software (ibid., p. 647).

Digital technologies in mathematics education can also be described as *cognitive tools*. Jonassen (1992) described *cognitive tools* as mental and computational devices that support, guide, and extend the thinking processes of their users. According to Mayes (1992), a *cognitive tool* is a device or technique for focusing the student's analytical processes. Jonassen (1992) argued that many *cognitive tools* (such as cognitive and metacognitive learning strategies) are internal to the learner, while computer-based devices and environments that extend learners' thinking processes are external. These computer-based devices and environments can engage learners in meaningful cognitive processing of information. Some digital technologies (e.g., GeoGebra, Excel, Wolfram Alpha, etc.) can also be viewed as learning tools which have been adapted and developed to support learning as compared to other regular or task-specific tools (e.g., basic calculator, etc.) (ibid.). In summary, *cognitive tools* are tools that can facilitate cognitive processing or tools that support a learning process.

There are other categorizations of digital technologies based on their *designed features* or *usability features* (usability is defined and explained in Sub-Section

4.3.4) (Sedig & Sumner, 2006; Goodwin & Highfield, 2013; Sinclair & Baccaglini-Frank, 2015). These characterizations might help in comparing the different digital technologies. The affordances and constraints (defined in Section 4.3) concept might help evaluate digital technologies to see whether some characterizations are present in the tool in a way relevant to the mathematical concept under study. For instance, GeoGebra and graphical calculators have different design/usability features. An individual cannot manipulate the graph or function on the graphical calculator unless he/she changes the equation keyed in. In contrast, such changes can be made in GeoGebra, whether in its 'Algebra view' or 'Graphical view'. However, these affordances make sense only if an individual can use (or reach) what the digital tool affords. In the following subsection, I will discuss mathematical modelling with the aid of digital technologies.

2.2.1 Mathematical modelling with the aid of digital technologies

In this subsection, I will discuss empirical studies relevant to this study on mathematical modelling with the aid of digital technologies. Greefrath et al. (2018) argued that there is a general assumption in the literature that mathematical models used during modelling activities are influenced to some extent by digital technologies available to students, not just students' mathematical knowledge and abilities. However, developing a model does not depend only on the skills embedded in certain digital technologies but also demands mathematical knowledge (Berry, 2002). Over two decades ago, Zbiek (1998) identified that the digital technologies used did not aid in generating, choosing, or validating function models and that the student's knowledge and understanding of the mathematical concepts were vital. From these arguments, we can say that the skills of using particular digital technology and students' mathematical knowledge are both necessary in developing a model.

What is the role of digital technologies in the students' modelling processes? Siller and Greefrath (2010) described using digital technologies in mathematical modelling activities (see Figure 2.3 below). Figure 2.3 shows a unilateral view of digital technology usage exclusively between the mathematical model and the mathematical solution. However, in reality, students use digital technologies at every stage of the modelling process, depending on the nature of the task. For instance, students working on Task 2 in this study (see Sub-Section 5.5.2) might start searching for the cities' positions even before setting up a mathematical model. Based on the results of a qualitative study on digital technology usage in the modelling cycle, Greefrath et al. (2018) showed an example of how GeoGebra was used at the different stages of the modelling process (see Figure 2.4). Figure 2.4 shows how digital technology was used at different points in the modelling cycle, contrary to the description in Figure 2.3. The writings in red, in Figure 2.4, describe the usability features of digital technology. Following the cognitive activities in a modelling cycle (see Figure 2.1 on page 13), Vos and Frejd (2022) suggested analytic dimensions for metacognition, digital tools used, and social norms that interact differently and modify the cognitive modelling activities. For instance, Table 2.2 (on page 31) shows the phases in the modelling process with indicative dimensions for cognitive activities, metacognition, and the use of digital technologies.

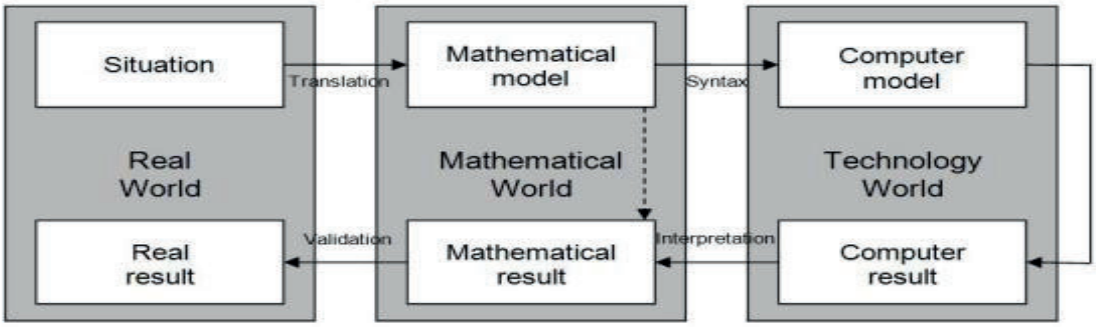


Figure 2.3: Extended modelling cycle – regarding technology when modelling (Siller & Greefrath, 2010).

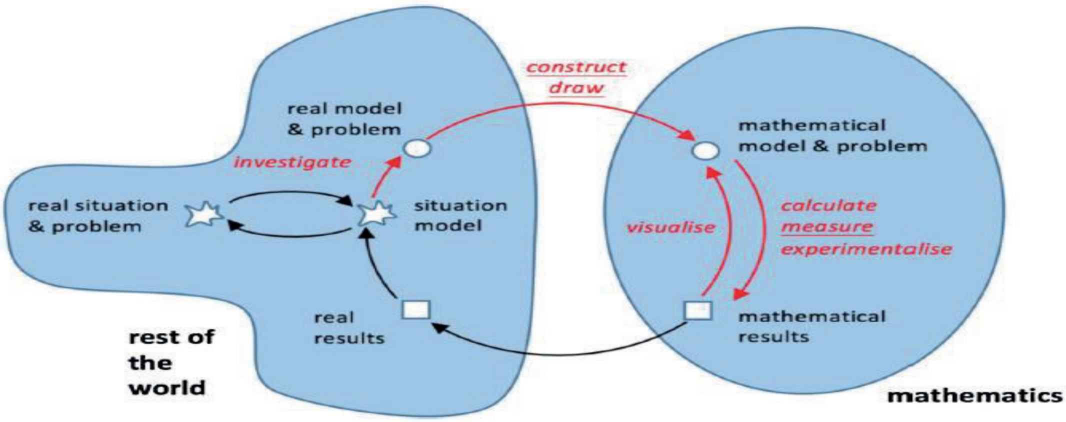


Figure 2.4: Digital technology usage in the modelling cycle (Greefrath et al., 2018).

I will now discuss some empirical results of mathematical modelling with the aid of digital technologies. Mousoulides et al. (2007) examined students' mathematization processes and their interaction with digital technologies during modelling activities. This study is a case study of one group of three 12-year-old students who worked for two 40-minute sessions on a modelling task. Results from the study show that the software assisted the students in developing the necessary mathematical constructs and processes (a similar study was conducted by Mousoulides (2011) with one class of 14-year-old students). Greefrath and Siller (2017) undertook a theoretical (the concept of modelling competence and simulations) and empirical examination of modelling with digital technologies in mathematics instruction. This is a case study of four pairs of students in grade 10 at a secondary school in Germany, observed as they worked on a reality-based assignment using GeoGebra. Results from the study show that the students used GeoGebra at different points in the modelling process.

Carreira et al. (2013) examined students' (8th graders) approaches to contextualized problems in a modelling activity and found that these students skipped much of the algebra work and chose a geometrical approach while working on the problem. Thus, the students obtained an algebraic equation to the problem in the 'Algebra View' in GeoGebra through a geometrical representation they set up in the modelling process. The graphical representation features of some digital technologies might support the various processes in students' modelling activities (Pead et al., 2007). Arzarello et al. (2012) argued that these digital technologies provide students with diverse representation options (such as graphic, symbolic and numeric) while working on mathematical modelling tasks. These digital technologies do not only offer diverse representation options but also might have controlling options. Moreno-Armella et al. (2008) pointed out that these controlling options might be the selection, dragging, pointing, clicking, and grabbing (by mouse movements) options these digital technologies provide as students engage with mathematical modelling tasks. For instance, students in a modelling activity can drag the vertices of well-constructed objects/figures to observe if parameters are preserved upon manipulation.

To summarize these arguments above, Gallegos and Rivera (2015) pointed out that digital technologies might promote the transition between different stages of the modelling cycle. However, the studies mentioned above do not account for other social factors besides digital technologies mediating the students' activity. Furthermore, some studies address this issue from the perspectives of some adopted theories. Gallegos and Rivera (2015) suggested that in future investigations, some attention should be given to the technological and collaborative work of the students and not only the mathematical competencies. So, using CHAT as a theoretical lens in this research study, I address the issue of mathematical modelling with the aid of digital technologies, where I look at the forms of interactions within the students' modelling activity.

I have so far presented discussions on digital technologies and their use in mathematical modelling activities, and it is equally important to understand students' tendency to select specific digital technologies while working on mathematical tasks. As such, in the following subsection, I will discuss students' tendency to use specific digital technologies while working on mathematical tasks.

2.2.2 Choice of digital technologies in students' activities

My use of the word 'choice' here (and elsewhere) is without meaning (in regard to the adopted theories used in this study) but should be understood as 'free choice'. I consider choice of digital technologies in students' activities as the students' tendency of selecting or using a particular technology. Generally, there are several reasons behind students' tendency to select or use a particular digital technology regularly (Margaryan et al., 2011) in a mathematical activity. For instance, a students' interest in using a certain digital technology might stemmed out from their experience in the classroom (Jacinto & Carreira, 2017). That is, students following how their teacher uses digital technology quite often in a way of illustrating some examples during mathematics lessons. Geiger et al. (2002) also points out that students might prefer to use a particular digital technology over another simply because these students use this particular digital technology often and are more familiar with its operation. However, there can be situations where students must decide on the digital technology to use (from a variety of different digital technologies) if, for instance, the teacher uses several digital technologies in classroom activities. In such a situation, students might decide on using a particular digital technology for some reason. For instance, Flehantov and Ovsiienko (2019) and Flehantov et al. (2022) argued that students use Excel/spreadsheet for numerical calculations and representation of numerical results in the form of tables, and GeoGebra for visual representation and drawing of functions. An approach (known or unknown) a student uses in problem solving is determined by the type of digital technology used. Jacinto and Carreira (2017) argued that unknown ways of tackling a problem might be revealed if students are allowed to choose digital technologies of their choice. In this case, the nature of digital technologies might give students different options while working on a task.

For instance, Hoyles (2018, p. 211) pointed out that digital technologies such as dynamic and graphical tools allow “mathematics to be explored in diverse ways, from different perspectives”. For example, students might easily be aware of what varies and what does not, through the reflection on and the manipulation of a sketch on a graph. In such a situation, students are likely to become aware of what to focus on in the process of dragging their sketch and knowing if the constraints of the problem are indeed satisfied while also becoming aware of invariants and possible relationships between the elements under dragging.

Combining two or more digital technologies in a single problem solving activity might be beneficial to students’ learning. Flehantov and Ovsiienko (2019) emphasized that working with several digital technologies might give an advantage over using only one. For instance, the simultaneous use of Excel and GeoGebra might improve the academic achievement of students in a mathematical modelling activity (ibid.). Abramovich (2022) asserted that problem solving that is technology-enabled might include the use of multiple digital technologies in support of a single mathematical task. Monaghan (2016a) demonstrated how several tools could be used in working on a single mathematics task. Given a single task about the bisecting of an angle, Monaghan (2016a) showed how tools such as a straight edge and a compass, a protractor, a dynamic geometry system, and a book (which sounds a bit strange) could be used in bisecting an angle. Although several tools provide the platform of solving a single task, students must still choose between the different tools while working on a task (and they do so for several reasons). Several influences might affect students’ decision if their teacher allows these students to choose a digital technology being used for an assignment. Some digital technologies (e.g., GeoGebra) offer students different platforms for working on a task. GeoGebra, for instance, has the graphic view, algebra view, spreadsheet (or numeric) view, among others for which students can choose any of the views depending on the nature of the task, the students’ preference, and others. Hegedus et al. (2017) pointed out that the features of a digital technology might support individual preferences and approaches while working on a task. For instance, GeoGebra offers a platform for both numerical and graphical views, and a student might subscribe to any of them depending on the strategy they adopt. That is, if a student prefers to key in some numbers and generate the data they might go for the numerical view of the digital technology or the graphical view if the students prefer sketching and manipulating a graph.

Anastasakis et al. (2017) argued that the decision made by students in choosing certain tools is goal oriented; and so, the students might choose a certain tool based on the demands of the tasks. A problem solving strategy adopted by students might also influence students' choice of digital technology. Yerushalmy (2000) reported on a problem solving strategy (function representations) where students might start with a graphical representation of variations, used later on to analyze patterns of numbers by watching the behaviour of the increment, moves on to analysis and construction of relations between quantities that are defined, to accurate graphs and then to explicit expressions (ibid.). And in these stages, a student might use a particular tool or tools in each of the stages above. Choosing a certain tool might also be influenced by prior experience (Gueudet & Pepin, 2018) and the simplicity and efficiency of the tool (Hillesund, 2020). That is, students might choose certain digital technologies based upon ease of using them. Students' choices in using certain digital technologies might also be based on their experiences of using them; and if these technologies resulted in a positive experience (i.e., having more confidence of using them), then it is likely for the students to use the same technology again (Owens-Hartman, 2015).

The studies above (and others elsewhere) mostly touch on digital technologies in mathematical problem-solving activities and not necessarily in modelling activities. Furthermore, in the activities of the studies above, the students were either allowed or instructed to use certain digital technologies. In general, there is a limited literature base related to the issue of students' selection and/or switching between digital technologies when learning or doing mathematics (Geiger et al., 2002). A part of this thesis seeks to extend knowledge in this domain through the analysis of students' activities, in their selection and/or switching between digital technologies in a mathematical modelling activity, where the students are allowed to use any tool (no restriction of digital technologies use). Without any restriction on digital technologies use, there might be some dynamics in the students' solution process (particularly, on how and why students choose certain tools over others).

The task given to the students in this research study was prepared by the researcher (although adapted from other literature), and based on this, it is reasonable to discuss mathematical task design. In the forthcoming section, I will present mathematical task design, where I discuss the types of mathematical tasks, design elements of mathematical tasks, students' perspectives on task design, the role of digital technologies in designing mathematical tasks, and some considerations in designing a tool-based task.

2.3 Mathematical task design

The tasks that are used in the classroom form the basis for students' learning (Doyle, 1988) and might give some opportunities for students' thinking. Stein and Smith (1998) argued that the tasks that ask students to perform a memorized procedure routinely led to one type of opportunity for student thinking whilst the other led to a different set of opportunities for student thinking (that is, the tasks that require students to think conceptually and that stimulates students to make connections) (ibid., p. 269). Doyle (1983) pointed out that tasks influence students' by directing their attention to particular aspects of content (or concepts they have been taught) and by specifying ways of processing information (ibid., p. 161). In this case, the tasks given to students might help them better understand the concepts being taught, so the task given might define what the students learn.

Shimizu et al. (2010) pointed out that mathematics tasks are essential vehicles for such a purpose in classroom instruction that aims to enhance students' learning. The role of mathematical tasks in stimulating students' cognitive processes is crucial in achieving quality mathematics instruction (ibid., p. 1). On the other hand, Christiansen and Walther (1986, p. 262) highlighted that "even when students work on assigned tasks supported by carefully established educational contexts and by corresponding teacher actions, learning as intended does not follow automatically from their activity on the tasks". Over the years, several research activities have been on task design in mathematics education. There have been different perspectives into which researchers look at task design in these activities. Watson and Ohtani (2015) emphasized that attention to task design is significant from several perspectives regarding mathematics education and practice. For instance, from a cognitive perspective, the detail and content of tasks have a significant effect on learning; from a cultural perspective, tasks shape the students' or learners' experience of the subject and their understanding of the nature of mathematical activity; and from a practical perspective, tasks are the bedrock of classroom life, that is, the "things to do" (ibid., p. 3).

2.3.1 Types of mathematical tasks

Tasks are critical in teaching and learning (National Council of Teachers of Mathematics, 2000). The National Research Council (2001) claimed that the quality of teaching depends on the kind of cognitively demanding tasks the teacher selects. Zaslavsky and Sullivan (2011) argued that the tasks used by teachers are mediating tools and that it is through and around tasks that teachers and students communicate and learn mathematical ideas. In mathematics education, issues concerning the selection of mathematical tasks for instruction in the classroom are of great interest to researchers (Berisha & Bytyqi, 2020; Lithner, 2017; Watson & Ohtani, 2015; Sullivan et al., 2009). Moreover, a proper selection, among other things, of tasks might lead to one type or a different set of learning opportunities for students thinking (Stein & Smith, 1998). For instance, Berisha and Bytyqi (2020) argued that solving familiar tasks does not often require (any kind of) conceptual understanding and that a proper selection of tasks, among other things, might cover a wide range of learning outcomes.

To discuss the types of mathematical tasks, I will first define and explain mathematical tasks. Doyle (1988) defines academic tasks (not specific to mathematics) in terms of four components: a product, operations to produce the product, resources, and the significance or weight of a task in the accountability systems of a class. These components sum up to four aspects of work in a class: a goal state or end product to be achieved; a problem space or set of conditions and resources available to accomplish the task; the operations involved in assembling and using resources to reach the goal state or generate the product; and the importance of the task in the overall work system of the class (ibid., p. 169). From this definition, we can refer to a mathematical task as the product students are expected to produce and the operations and resources that students need to use or are expected to use when generating the product for assessment. Mathematical tasks are given to assess the students or further help the students understand mathematical ideas or content taught. From this perspective, Stein et al. (1996) defined a mathematical task as a classroom activity for the purpose of directing students' attention to a particular mathematical idea or content (ibid., p. 460). The National Council of Teachers of Mathematics (2000) considered mathematical tasks to be things constructed by the teacher (or from the textbook) and presented to the students as a way of learning and doing mathematics. Christiansen and Walther (1986) perceived tasks as proposals and challenges set by the teacher (or taken from the textbook) for the students to achieve a specific goal. That is, what a teacher wants to achieve in a classroom determines the kind of tasks which are given to the students. Furthermore, one should remember that not all mathematical tasks offer the same learning opportunities (Stein et al., 2009).

Berisha and Bytyqi (2020) argued that different types of mathematical tasks have different weights, roles and potential in mathematics teaching and learning (ibid., p. 752). Mathematical tasks can be categorized depending on how we look at or analyze the tasks. Furthermore, the categories of mathematical tasks depend on the contextual features, forms of representation, answer forms required, the mathematical activity involved, the level of cognitive demand and others (Zhu & Fan, 2006; Glasnovic Gracin, 2018; Bayazit, 2013). There are several frameworks and methods for categorizing mathematical tasks based on the perspectives above (Berisha & Bytyqi, 2020); however, the perspective used in this study is adopted according to the views of Zhu and Fan 2006; Glasnovic Gracin, 2018 Bayazit, 2013; Sahlberg and Berry, 2003.

I will now present the types of mathematical tasks according to their contextual features, forms of presentation, answer forms, and group tasks in school mathematics. Moreover, these categories are linked together (to some extent).

Contextual features. Berisha and Bytyqi (2020) argued that categorizing mathematical tasks based on the type, amount, and nature of the context embedded in the task is a frequent dimension of task categorization. Glasnovic Gracin (2018) referred to contextual features as the extent and ways real-world experiences are incorporated into the tasks. Real-world contexts play the role of linking school mathematics tasks with students' actual experiences. Zhu and Fan (2006) put mathematical tasks in different classifications, and the classification of interest here is the distinction between *application* and *non-application* tasks. A non-application task is unrelated to the real world (or any practical background in everyday life) (for example, see the example task under "symbolic presentation" in Table 2.3). In contrast, an application task is rooted in the context of a real-life situation. Zhu and Fan further distinguished application tasks into two different categories: *fictitious* and *authentic*. Fictitious/realistic application tasks contain conditions and data made by the one formulating the tasks (see Task 1 in Sub-Section 5.5.1), whilst authentic application tasks contain conditions and data from real-life situations or collected by the students themselves from their daily lives (see Task 2 in Sub-Section 5.5.2) (ibid., pp. 613-614).

Forms of presentation. Mathematical tasks can be differentiated according to the form of presentation. The presentation of tasks could be verbal, symbolic (purely mathematical), visual, or combined (Zhu & Fan, 2006; Berisha & Bytyqi, 2020). Zhu and Fan (2006) argued that the categorization above is based on the representation forms of a problem describing the situation's setting and the data presentation for the task/question (ibid., p. 615). These representation forms of mathematical tasks are linked to the forms of communication in the mathematics classroom, for which Berisha and Bytyqi (2020) described the link as interrelated and that they depend on each other. Berisha and Bytyqi further pointed out that different and multiple presentation and communication forms might facilitate the development of representation, modelling and communication skills. For instance, Friedlander and Tabach (2001) highlighted that the forms of presentation can potentially enhance the learning process. However, each form of presentation has disadvantages, and a combined use might cancel out these disadvantages. Table 2.3 presents a description of each of the forms of presentation with an example.

<table>
<tr><th>Forms of Presentation</th><th>Description</th><th>Example</th></tr>
<tr><td>Symbolic (Pure mathematical)</td><td>Tasks presented in symbolic form, or if the stem of a problem/task includes only mathematical expressions.</td><td>Solve the following system of simultaneous equations:
$4y + 3x = 100$
$4y - 19x = 12$</td></tr>
<tr><td>Verbal</td><td>Tasks presented in textual form, or if the stem is entirely verbal, namely, in written words only.</td><td>Task 1 and 2
(See Section 5.5)</td></tr>
<tr><td>Visual</td><td>If the stem simply consists of figures, pictures, graphs, charts, tables, diagrams, maps, etc.</td><td><table><tr><td>y</td><td>7</td><td>17</td><td>31</td><td>51</td><td>83</td></tr><tr><td>x</td><td>1</td><td>3</td><td>5</td><td>7</td><td>9</td></tr></table>Draw a graph for the data given in the table.</td></tr>
<tr><td>Combined</td><td>Tasks are presented in combined modes of two or three presentation forms.</td><td>Example: Visual and Verbal</td></tr>
</table>

		 A farmer has stacked up straw bales like in the photo above. You can assume that all straw bales are 1.5m in diameter and that they always sink 20cm into the layer of straw bales below them. *Make a labeled drawing and set up a formula that you can use to calculate the height of the stack. You do not need to calculate the height* (Hankeln et al., 2019).

Table 2.3: Examples of tasks with different forms of presentation (adapted from Zhu and Fan (2006)).

Answer forms. Mathematical tasks can be categorized based on the answer type required. Glasnovic Gracin (2018) classified mathematical tasks as open-ended (tasks with several/many correct answers, see Task 2 in Sub-Section 5.5.2), close-ended (tasks that have only one answer, see Task 1 in Sub-Section 5.5.1), and multiple-choice tasks (tasks that offer a limited number of defined response options) (Zhu & Fan, 2006; Yeo, 2007; Glasnovic Gracin, 2018). The open-ended and close-ended task can be further distinguished. For instance, a task can be open at the start but can have either a close-end or an open-end (likewise, a task with a close start). Monaghan et al. (2009) argued that the open-start tasks are different from many tasks classified as open-ended tasks. The openness or closeness of a task can be linked to the teaching approach in the classroom. Berisha and Bytyqi (2020) pointed out that the answer type required by the tasks used for class instruction might indicate whether the teaching approach is open or closed. The openness of a teaching approach creates a rich environment for creativity, autonomy and deeper levels of conceptual understanding, whilst the other relies mainly on fully defined tasks that usually have only one way of solving (only one solution) (ibid., p. 752).

Group tasks in school mathematics. Sahlberg and Berry (2003) classified different types of group tasks. The task under this classification falls (somehow) under the categories described above. Table 2.4 describes a typology of group tasks in school mathematics with examples. The left column of the table presents the scope of mathematical tasks and categories, and the right column presents an example corresponding to the categories. The middle column describes the nature of each task type (see Sub-Section 2.4.2 for further discussion of Table 2.4).

Category	Nature of Task	Example
Drilling basic skills	Close in terms of method and outcomes.	Solve $3x + 2 = 7$
Applying a formula or algorithm	Typically closed in terms of outcomes and also the methodology.	Ella wanted to buy new shoes. The initial price was 480.99 Nok and there was a 30% sale in that store. They gave an additional 20% discount on shoes. How much money did Ella need for her shoes? What percentage was the actual reduction?
Measuring and collecting data	There is some openness regarding methodology but rather closed in terms of outcomes.	The Burglar (Sahlberg & Berry, 2003, p. 145)
Real problem solving	Real problems are those encountered in everyday life. They may or may not involve mathematical models. The openness of these tasks may vary from closed to open.	A simple model of a drink can is a cylinder of radius *r* with circular ends. If its volume is 330 ml. find an expression for *h*, the height of the can, and hence an expression for *A*, the total surface area of the can. Find the dimensions of the can that has the smallest surface area for this volume. Comment on your answer.
Mathematical modelling	Modelling tasks are typically real problems that require mathematical principles and formulas to solve the task. Tasks are open in terms of procedures and outcomes.	You have a roll of kitchen paper and one sheet of identical paper. Estimate the number of sheets that your kitchen roll has in total.
Mathematical Investigations	Basic investigations are often closed regarding outcomes but open to various methods. Extended investigations are typically open tasks.	Choose a two-digit number. Reverse the number to give a new number. Find the difference between the two numbers. Repeat for other numbers. Is the difference always the same? What do you notice about your answers? Can you find a rule? ….

Designing projects and studies in mathematics	Projects and studies are the most open mathematical tasks. The openness includes the setting of questions and the selection of methods.	It's your birthday. Your parents agree that you can have a party to celebrate. You can have the event at home, or you can book a hall so that more of your friends can come. Plan your event. Make sure that you arrange for enough food and drink. Be careful to include everything in your estimate of the cost. Decide how you are going to report your work. You may do a poster, a written report, a diary or something else.

Table 2.4: A classification of group tasks in school mathematics (adapted from Sahlberg and Berry (2003)).

In conclusion, this subsection presents a framework for categorizing and classifying mathematical tasks. In Section 5.5, I will discuss the mathematical tasks used in this study with the categories presented above. In the forthcoming subsection, I will discuss a framework that describes the design elements of mathematical tasks.

2.3.2 Design elements of mathematical tasks

The tasks designed for students focus on the nature of the mathematics that is taught. Sullivan et al. (2015) pointed out that the potential and appropriateness of a task might depend on the student's prior experiences, the pedagogic purpose, and the teacher's and student's expectations. The age range of the students can also be factored in when designing a task (ibid., p. 90). For instance, Jaworski et al. (2011) described a task based on a specific content of mathematical knowledge but modified to suit students from grade 1 to 13, respectively. Several frameworks exist for task design dilemmas and suitability criteria in designing and analyzing mathematical tasks. However, I will discuss a few of the task design dilemmas and suitability criteria framework relevant to this study. The design dilemmas and the suitability criteria are adopted from Sullivan et al. (2015). Sullivan et al. (2015) presented five task design dilemmas: *context*, *language*, *structure*, *distribution*, and *level of interactions*.

Context as a dilemma. In this dilemma, Barbosa and de Oliveira (2013) highlighted three possibilities for mathematical tasks: pure mathematics, semi-reality and reality. Zhu and Fan (2006) categorized these features of mathematical tasks as application and non-application tasks (see page 50). Tasks 1 and 2 (see Section 5.5) used in this study have semi-reality and reality contextual features, respectively. Barbosa and de Oliveira argued that conflict arises when students discuss the adequacy of the context for a task to achieve its learning goal. Sullivan et al. (2015) emphasized that a task with realistic context (to some extent) maximizes students' engagement. Furthermore, the context's level of reality might diminish the task's potential to achieve the intended learning. For instance, Brady and Jung (2022) argued that the choice of tasks might stimulate students' interest in solving the problem. However, these students might end up discussing more about the reality of the task and less about the mathematical ideas or concepts for the intended learning. The reality level of the context of Tasks 1 and 2 does not detract from the potential of the task from the mathematics to be learned. Nonetheless, Task 2 requires additional elements. Thus, it requires the students to make decisions in the face of uncertainty. Furthermore, students are forced to reconcile their answers with reality, but ultimately, the intended learning of the mathematical concepts is achieved.

Language as a dilemma. Barbosa and de Oliveira (2013) referred to language under the design dilemma as the level of rigour tied to the task. Under this dilemma, Sullivan et al. (2015) argued that mathematical precision is part of the desired learning, and the students need clarity to support the learning. For instance, Task 2 (see Sub-Section 5.5.2) in this study was redesigned to suit the Norwegian context. Again, Tasks 1 and 2 were translated into Norwegian to deal with any form of language barrier. Language is essential in task design as it leads to a form of understanding. Polya (2004) argued that the first step to problem-solving is understanding the problem. Understanding the problem text is a crucial step that might lead to a correct mathematical computation (Boonen et al., 2016; Lewis & Mayer, 1987). However, understanding the problem text can be subjective as it might depend on the nature of the task and the characteristics of the students, among others. Reusser (1988) pointed out that students sometimes solve problems correctly, even without understanding them. At times, students tend to focus more on what to do with the numbers in the problem situation if, for instance, the problem text has some numeric elements. In a classic example, many students from different parts of the world tend to solve the unsolvable problem "There are 26 sheep and 10 goats on a ship. How old is the captain?" by combing the numbers given in the problem (Verschaffel et al., 2000, p. 3).

Structure as a dilemma. According to Barbosa and de Oliveira (2013), structure refers to the degree of task openness. A task is closer if more supplementary questions are posed to guide students' actions. Sullivan et al. (2015) argued that questions can be posed in such a way that they scaffold student engagement with a task in a more prescribed way on the one hand and, on the other hand, allow students greater opportunity to make strategic decisions on pathways and destinations for themselves (ibid., p. 93). For instance, Task 1 is closer, whilst Task 2 is more open. Again, what constitutes openness of a task is a subject of discussion (Monaghan et al., 2009). Glasnovic Gracin (2018) puts the structure of tasks into three categories based on their answer types (see page 52).

Distribution as a dilemma. Barbosa and de Oliveira (2013) referred to distribution to the selection content to be focused on tasks, that is, what is expected to be taught in a task. This dilemma is linked to the level of cognitive demand as described by Stein and Smith (1998). Stein and Smith (1998) developed a framework that describes the hierarchy of tasks that develop from memorization to procedures without connections to procedures with connections to doing mathematics tasks. For instance, regarding Tasks 1 and 2, the students will be doing mathematics when creating their solutions and considering the solutions of others (group members). In converting the word problem into the mathematical form and solving it, the students would perform procedures with connections. When they identify the commonalities and differences between the questions, they would be doing mathematics at this point (Sullivan et al., 2015). One must note that the control of what is to be explored by the students while approaching an open-ended task (such as Task 2) could be unpredictable (Barbosa & de Oliveira, 2013).

Levels of interaction as a dilemma. According to Barbosa and de Oliveira (2013), the interaction level refers to how the task positions students and teachers. Sullivan et al. (2015) pointed out that a task does not exist by itself and that its implementation is influenced by the nature of the intended interactions between the participants (in this case, the teacher and the students). Clark et al. (2014) argued that the problem type might lead to effective group interaction and activity. Considering these considerations, the tasks used in this study (Task 1 and 2) were intended to engage the students in discussions with peers and then the teacher (especially in the introductory activity, see Section 5.3) at some stages of their solution process. Barbosa and de Oliveira (2013) highlighted that teachers are expected to keep themselves far from students' doings on a structured task (close-ended task), whilst teachers interact more with students if the task does not have many scaffolds (ibid., p. 546). These arguments could be an expectation, but they are not always the case. The teacher's role must be carefully considered when a teacher interacts more with students in an activity in which the task given does not have many scaffolds. Calor et al. (2022) suggested a small group scaffolding tool that might assist teachers regarding how and when to support students in group interactions/activities. These tools are contingency to the group, phasing out content support when the group can continue independently, and transferring responsibility for learning to the group.

Designers and teachers sometimes confront these task design dilemmas and make some choices while considering mathematical tasks for students. In some instances, the teacher might make decisions that the designer did not intend or anticipate (Sullivan et al., 2015). For instance, Son and Kim (2015) investigated teachers' selection and enactment of problems and tasks from textbooks and highlighted some critical issues in teacher decision-making. That is a situation where a teacher might not understand the underlying philosophy of the textbook used. In this case, there is a difference between the curriculum goals and the enacted tasks goals. In the Norwegian context, Berget (2022) examined mathematical modelling in textbook tasks and national exams in light of the new curriculum. The findings indicate different perspectives on mathematical modelling in the curriculum, the textbook tasks and the national exam, where only parts of the modelling process are included. From these arguments, Giménez et al. (2013) suggested a suitable framework for the analysis of tasks: e*pistemic*, *cognitive*, *interactional*, *mediational*, *affective*, and *ecological* suitability.

Epistemic suitability. According to Giménez et al. (2013, p. 581), "epistemic suitability refers to the extent to which the mathematics taught is good mathematics". In this case, the specific content of the curriculum (and the institutional mathematics) is used as a reference in designing a task (Sullivan et al., 2015). One question we can ask is, 'Does the designed task meet the curriculum goals?'. For instance, the nature of the task used in this study allows the students to go through all the modelling processes as intended by the curriculum. Task 1 addresses critical mathematical concepts as they connect with the concept of functions to some extent. Sullivan et al. (2015) highlighted that the student's prior experience should be factored in and not only considering the content of the curriculum when designing a task. In the case of Task 2, students at the lower secondary level might only consider the mathematical aspect of the task without reconciling their mathematical results with reality, as compared to students at the upper secondary level. Garfunkel and Montgomery (2016) argued that students in the higher grades might take more sophisticated information into account while solving mathematical modelling tasks compared with students in the lower grades.

Cognitive suitability. Giménez et al. (2013) described cognitive suitability as the reflection of "the degree to which the teaching objectives and what is actually taught are consistent with the student's developmental potential, as well as the match between what is eventually learnt and the original targets" (ibid., p. 581). Sullivan et al. (2015) emphasized that the objectives of the mathematics lessons and what the students have been taught previously play a significant role in task design. For instance, regarding Tasks 1 and 2, the students have already been taught functions, geometry, and others (a prerequisite to solving both tasks).

Interactional suitability. This category "relates to the extent to which the forms of interaction enable students to identify and resolve conflicts of meaning and promote independent learning" (Giménez et al., 2013, p. 581). Barbosa and de Oliveira (2013) described a similar concept (referred to as "levels of interaction", see page 56), and Sullivan et al. (2015) pointed out that this interaction could be between teacher and students, student and the peers, and the student and the task.

Mediational suitability. Giménez et al. (2013, p. 581) described mediational suitability as "the availability and adequacy of the material and temporal resources required by the teaching/learning process". The availability of some digital technologies (e.g., GeoGebra, Excel/spreadsheet, Google Maps, and others) might prompt students to produce a technology-based model/solution. Again, Sullivan et al. (2015) argued that the cognitive demands of a task might be evident through the level and type of engagement in group activity. For instance, the context of both tasks used in this study might require role-playing the situation, so the nature of the task becomes apparent. Some students might be taking the position of a member of one of the three cities in Task 2 during group interaction.

Affective suitability. According to Giménez et al. (2013, p. 581), "affective suitability reflects the students' degree of involvement (interest, motivation, etc.)

in the study process”. Middleton (1995) argued that one of the characteristics that influence effective responses between teachers and students is the arousal and control levels in determining the intrinsic motivation for mathematics activities. Furthermore, Sullivan et al. (2015) described this as an opportunity for students’ decision-making (that is, the choices that the students can make). Interest and arousal could be important determinants of student motivation. As Brady and Jung (2022) argued, some realistic tasks could stimulate students’ interest in solving a problem. For instance, in Task 2, the students not only do the math but also must make decisions in the face of uncertainty. Thus, students might decide what is ‘fair’ and imagine themselves in a related situation.

Ecological suitability. Giménez et al. (2013, p. 581) referred to ecological suitability as “the degree of compatibility between the study process and the school’s educational policies, the curricular guidelines, and the characteristics of the social context, etc.” Considering Tasks 1 and 2 (and the other tasks in Appendix B), the problems are redesigned to suit the Norwegian context, and these tasks are compatible with the core elements of the Norwegian mathematics curriculum. For instance, Tasks 1 and 2 allow the students to go through all the mathematical modelling processes (as intended by the curriculum) and not only focus on an aspect of the modelling process (Berget, 2022). Sullivan et al. (2015) argued that the aims of mathematics curricula are challenging to discern. Sometimes, the link between the task and the mathematics curriculum is more tenuous and requires the teacher’s intervention in making things explicit.

The two frameworks discussed in this section describe the elements and design considerations for mathematical tasks. Of course, other frameworks may be similar or different from the two frameworks discussed in this study. I will further discuss these frameworks in Section 5.5 in light of the tasks used in this study. In the design frameworks presented above, little is discussed about students’ perspectives on task design considerations. As such, I will present the literature on students’ perspectives on task design in the forthcoming section.

2.3.3 Students’ perspective on task design

Ainley and Margolinas (2015) argued that focusing on the impact of students’ perceptions of tasks on their mathematical learning raises some relevant questions if we do not only look at the intentions of task designers and teachers who select these tasks. Furthermore, researching these questions might help to minimize the gap between teacher intentions and student mathematical activity. Using contextualized word problems in mathematical activities might make learning more accessible by giving meaning to mathematical ideas or concepts (showing their usefulness). However, using context can distract students' attention from the mathematical ideas (ibid., p. 116). Gerofsky (1996) argued that in some instances, students tend to ignore the story elements in the task and use the mathematics they have just learned to transform the word problem into an arithmetic or algebraic form and solve the problem to find an answer (ibid., p. 39). Cooper and Dunne (2000) gave a task (the shopping item) that students needed to recognize the hidden structure of the problem (a pair of simultaneous equations). However, one student used the first equation and the knowledge of the price of cola she had gained from everyday shopping to generate the solution (ibid., p. 41). In this case, this student used the price she recently paid rather than using the information given in the task.

Jaworski et al. (2012) analyzed the teaching goals and students' epistemological positions and highlighted fundamental differences between students' perceptions of the teaching they experience and the goals of the teaching team. Ainley and Margolinas (2015) gave an example where students had their conception (different from the teacher's) when asked to solve the question, "Is -1 the square of a number?". The teacher expected the answer 'it is not possible'; however, one student thought it was possible since $-(1)^2 = -1$ (resulting from students' previous learning) (ibid., p. 120). Again, if we consider the solution of a linear equation in mathematics instruction, there might be the possibility of different perspectives regarding the teacher and the students. For instance, the teacher might put it in the broader perspective of the usefulness beyond the classroom walls or achieving deep levels of understanding. In contrast, the students might simply be motivated by getting an answer. Although this example applies to mathematical instruction, a similar concept applies to mathematical task design. It is necessary to consider the goal of mathematical tasks as intended by the teacher and student's perspective. Ainley and Margolinas (2015) pointed out that students' perspective changes the very definition of a task.

How can we close the gap (which might arise) between teachers' (and task designers') intentions and the students' perceptions concerning task design? From the example (is -1 the square of a number?) above, students previous learning led them to overgeneralize the situation in a way the teacher had not anticipated. Ainley and Margolinas (2015, p. 127) argued that "deciding when it is, or is not, appropriate to generalize a mathematical idea can be challenging for students".

Students who are only exposed to problems that require a single answer might face challenges while solving problems with several solutions. Schukajlow et al. (2015) argued that the performance of students might improve as they engage in an activity of constructing several solutions to a problem. Savard et al. (2013) commented that students' progress in solving problems that require holistic analysis, but teachers tend to revert to more traditional formats with a single answer. One example of this traditional setting is a teacher explicitly giving all requirements (or kinds of behaviours) expected of the students, which might be counterproductive if the teacher's aim is for the students to develop creative, independent mathematical thinking and others (Ainley & Margolinas, 2015). Coles and Brown (2013) addressed this tension by pointing out that "the more the desired behaviours in students are specified, the less these behaviours are likely to emanate from the students' own awareness" (ibid., p. 184). Ainley and Margolinas (2015) summarized the above discussion on research addressing student perspective in task design into two different perspectives: *reflective* and *emergent* task design.

Reflective task design. This perspective considers students' perspectives in task design that takes place in a reflective space away from the classroom. In this case, the designer anticipates students' learning or what they might learn as they engage in the tasks. Palhares et al. (2013) emphasized the need for cognitive analysis of tasks to adapt their connections to the student's abilities as best as possible. A means of accessing students' abilities in task design is through a prior analysis of the mathematical knowledge involved. Calleja (2013) reported factoring students' likely responses while designing tasks. Furthermore, these likely responses might lead to a change in a mathematical problem. Ainley and Margolinas (2015) emphasized that a simple change in a mathematical problem can make a big difference in the cognitive demands of students. For instance, Problem 1 in the study by Mousoulides (2011) has a realistic context involving three unknown cities; this problem was transformed into another problem (see Task 2 in Sub-Section 5.5.2) with authentic context. That is, giving actual names to the three cities students can identify (using Google Maps). In Problem 1, the students might construct a rectangle with its vertices representing the unknown cities and find the midpoint of these vertices. At the same time, the transformed task (Task 2) is less about finding the midpoint of the vertices (representing the unknown cities) and more about estimation in terms of the time of travel, the population, and others.

Emergent task design. This perspective considers students' perspectives on task design within the classroom. Ainley and Margolinas (2015) argued that this perspective "concerns the ways in which teachers develop tasks during the flow of classroom activity, in response to the actions of students" (ibid., p. 133). According to Bikner-Ahsbahs and Janßen (2013), emergent tasks refer to the situation in which "the teacher conceives the mathematical potential of a learning opportunity and translates it into a task" (ibid., p. 154). The teacher does that to maintain students' interest. This aspect of task design is seen in this study's *introductory activity* (see Section 5.3). In the *introductory activity*, the students solved Tasks A and B (see Appendix B on page 355) by themselves, and the teacher (and/or researcher) provided some support in moments of difficulty. These supports were not straightforward but can be classified as emergent tasks. Thus, the teacher poses the difficulty within the initial task as a new task for the students/group or asks other group members if they can address this difficulty. In such a situation, Calor et al. (2022) suggested that small group scaffolding tools might assist teachers in how and when to offer students support in group interactions/activities (see Sub-Section 2.4.2 under "levels of interaction" as a dilemma). Emergent task design might be helpful in the learning process. However, Ainley and Margolinas (2015) highlighted that it can be challenging for teachers in specific moments/situations.

So far, I have discussed the literature on task design, highlighting types of tasks, design elements, and students' perspectives on task design. A few of the categories relevant to this study have been discussed, and there are several categories to consider in mathematical task design. At this point, I have not yet discussed the role of digital technologies in designing mathematical tasks. The literature on the role of digital technologies in task design is relevant to this study, as the participants solve mathematical modelling tasks using digital technologies. As such, I will present the literature on the role of digital technologies in task design in the forthcoming section.

2.3.4 The role of digital technologies in designing mathematical tasks

Digital technologies are mediating artefacts that mediate the activities of individuals (see Sub-Section 4.1.3). These artefacts are directed towards an activity with a motive. According to Cole (1996), when an artefact is used in a goal-directed action, it becomes a tool (see Sub-Section 4.1.1). Tools in the context of mathematical task design can broadly be viewed as physical or virtual artefacts that have the potential to enhance mathematical understanding (Leung & Bolite-Frant, 2015). Leung and Bolite-Frant (2015) described a tool-based task:

> A tool-based task is seen as a teacher/researcher design aiming to be a thing to do or act on in order for students to activate an interactive tool-based environment where teachers, students, and resources mutually enrich each other in producing mathematical experiences (ibid., p. 192).

From this description, teachers/researchers design these tasks based on their intentions of what they want the students to learn. For instance, Monaghan (2016a) presented how four different tools can be used on a single task, bisecting angles. These tools have different affordances to students' mathematical learning. Hence, a teacher might decide which tools to use based on his/her intentions of what he/she would like the students to learn. From these arguments, it can be said that different epistemological approaches to mathematical learning might have different implications for designing tool-based tasks. Leung and Bolite-Frant (2015) put these approaches into two perspectives using Sfard's (2008) two metaphors for learning, *participationist* and *acquisitionist*:

> A participationist orientation would favor design with potential for students to participate in the construction of mathematical knowledge/ experiences, whereby a more acquisitionist orientation would favor design that explores and discovers established mathematical knowledge (Leung & Bolite-Frant, 2015, p. 192).

Revisiting the example by Monaghan (2016a, p. 13), considering the use of 'straight edge and a compass' and 'dynamic geometry software' for bisecting an angle. In this situation, students might follow a teacher's given construction procedure or check the validity of a given theorem while using a straight edge and a compass. On the other hand, students might construct their geometrical models to explain a specific mathematical phenomenon using dynamic geometry software. This argument depends on the affordances that each tool affords. Monaghan and Trouche (2016) highlighted the existence of a mismatch between tasks and tools. For instance, the actions of sketching a function using graphing software require techniques (typing digital-mathematics, keying, dragging and zooming in or out) quite different from the techniques of sketching the same function with a paper-and-pencil approach (construct a table of x and y values, select suitable scales, draw axes on graph paper, plot the points, and sketch the graph by interpolating values between points.). However, in another given task, a lot more mathematical actions might be done with the graphing software than paper-and-pencil. Again, one tool can afford different perspectives of teachers' intentions towards students' learning. For instance, Leung and Bolite-Frant (2015) argued that the same tool could be used in two task designs at opposite epistemological poles.

There are several theoretical frameworks for designing tool-based tasks, and a few are mentioned in Section 2.2. However, for this study, I chose an Activity and Affordance Theory framework for designing tool-based tasks (see Chapter 4). One can study the relationship between the tool and the subjects/students from an Affordance Theory perspective. Another way to look at the relationship between tool use and the pedagogic environment is by looking at the interrelationships in the structural representation of Activity Theory (e.g., Engeström's mediational triangle, see Figure 4.3 on page 96). For example, consider students' mathematical activity in constructing a geometric figure/shape. The components in Figure 4.3 will represent: the subject is a class of students, the object of the activity is to construct a geometric figure/shape, the tool (mediating artefact) is GeoGebra, the rules are how to use GeoGebra's functionalities, the community is the teacher and students, and the division of labour is organized by pedagogical approaches and arrangements. In such a scenario, Leung and Bolite-Frant (2015) argue that task design might focus on how GeoGebra can assist students in creating a learning process comprising a possible connection of activity routes that might lead to possible outcomes.

I will now discuss some considerations in designing tasks that use tools. Many tools are used in mathematics education, but I will only focus on selected digital technologies while discussing these considerations for this study. There are several considerations in designing tool-based tasks. However, I will discuss four considerations Leung and Bolite-Frant (2015) reported: *epistemological and mathematical*, *tool representational*, *pedagogical*, and *discursive* considerations.

Epistemological and mathematical consideration. Under this consideration, Leung and Bolite-Frant (2015) argued that different epistemological approaches to mathematical knowledge have different implications on task design. That is, the tasks designed for students are based on the kind of mathematical knowledge the teacher wants the students to achieve, and this also affects the type of tool used. Drawing on Sfard's participationist and acquisitionist epistemological orientation, Leung and Bolite-Frant (2015) emphasized that the first might favour a tool-based design that can lead students to participate in the construction of shared mathematical experiences. In contrast, the latter might lead students to explore and consequently construct personal mathematical knowledge. From this argument, we can say that different tools used in task design might have different epistemological stances (depending on their affordances). Similarly, the same tool can be used in task designs with different epistemological stances. Reflecting on these instances, it can be a challenge in tool-based task design when considering possible epistemological orientations and the type of mathematical knowledge the tool can afford. For instance, a tool like GeoGebra used in task design might cover a sizeable epistemic spectrum from drawing precise, robust geometrical figures (shared mathematical experiences) to exploring new geometric theorems and developing argumentation discourse. In summary, the kind of knowledge to be acquired by the students might lead to the type of epistemological approach adopted and the tool that will be used.

Tool representational considerations. The nature of school mathematics is symbolic. As such, Leung and Bolite-Frant (2015) argued that the way a chosen tool represents mathematical knowledge is at the heart of tool-based task design. They further suggested some questions of consideration that might be of interest for a mathematical topic: "How far away from the expected symbolic representation is in the tool's potential to represent the mathematical concept?" and "Is the tool capable enough of representing the targeted mathematical knowledge parallel to the corresponding symbolic representation?" (ibid., p. 195). These questions highlight the gap between the symbolic representation within the mathematics curriculum and the tool. Morgan et al. (2009) described this gap as *epistemological distance*. Epistemological distance constitutes the difference between the affordances for meaning when considering the distance between representations (ibid.). For instance, Wassie and Zergaw (2019) pointed out that some of the commands used in the input bar of GeoGebra are not user-friendly (especially for individuals with no prior programming experience). In another example, there is a significant distance between the mathematics of the standard curriculum and the representation in GeoGebra version 4.4. One example is using x^2 rather than the superscription for powers x^2, which can form a barrier for some students (the same applies to Excel/spreadsheet, especially using it as a tool for learning algebra). To address this issue, the students in this study used GeoGebra 6, which has an interface that corresponds much with the standard curriculum.

Pedagogical considerations. Under this consideration, Leung and Bolite-Frant (2015) argued that a suitable pedagogical environment must support tool-based task design. Different types of tools afford different mathematical task activities. For instance, a tool like GeoGebra with the 'graphics view', 'algebra view', and 'spreadsheet view' combines these views into a single dynamic multi-representational tool, and this may open up a rich pedagogical space for task design to explore. When considering a tool-based task, there is a need to consider the distance, classified by Morgan et al. (2009) as *pedagogic distance*, between the form of pedagogy represented in the tool's environment and that of the classroom into which it is being introduced. As such, Leung and Bolite-Frant (2015) argued that familiarity with a tool and how to use it effectively to teach and learn are also critical pedagogical considerations for tool-based task design. For instance, students at secondary schools in Norway are familiar with the GeoGebra software, so a tool-based task design with such a tool may support group activities and is also cost-efficient. That is, the pedagogy represented in GeoGebra's environment is not far from the pedagogy represented in the classroom.

Discursive considerations. Leung and Bolite-Frant (2015) emphasized that practising using a tool to accomplish a task involves the formation of appropriate tool-based vocabularies to develop utilization routines. Thus, the designed tool-based tasks should bring about discourses for mathematical knowledge mediated by tools in the mathematics classroom. For instance, when students work in groups to complete mathematical modelling tasks with digital technologies, the discourses that emerge during the student's activities mediated by these digital technologies must relate to the mathematics knowledge/content. These discourses emerging within the students' activities must relate to the mathematical content as a *shared goal* (Roschelle & Teasley, 1995; Granberg & Olsson, 2015). Granberg and Olsson (2015) argued that to create a *shared goal*, the students have the tool to look at the same thing as they negotiate and agree on the appearance of the mathematical representation generated by the tool. The students might also use the tool as a *reference tool* (during a mathematics discourse) to visually demonstrate their ideas to one another. For instance, a student might suggest a function/equation to their peers and use GeoGebra to represent this function graphically. Similarly, in the discursive consideration, a tool can be used to *observe and repair divergencies* during a mathematics discourse (Roschelle & Teasley, 1995; Granberg & Olsson, 2015). Granberg and Olsson (2015) point out that to *observe and repair* *divergences*, the tool is used to maintain shared mathematical knowledge and ideas through the verification of ideas or to settle disagreements by performing tests and referencing, among others, during a mathematics discourse.

In summary, much has been discussed about the dimension for digital technology use (so far), not the social norms dimension, which is also relevant in mathematical modelling processes. As such, in the forthcoming section, I will present a discussion on group work in relation to mathematical modelling activities.

2.4 Group work/activity

There are several definitions for group work/activity (depending on the perspective one adopts). In this study, I will take the approach where a teacher forms a group (same or mixed ability/attainment regarding exam scores) to work on a mathematical task. In this perspective, Cohen and Lotan (2014, p. 1) defined group work/activity as "students working together in a group small enough so that everyone can participate in a clearly assigned learning task". Furthermore, the type of task might determine the level of participation among group members (Cohen, 1994; Sahlberg & Berry, 2003). There are different types of group work/activities, and some examples are cooperative, collaborative, problem-based and team-based learning, amongst others. Cooperative and collaborative are the best-known and most researched among these types.

Cooperative learning is seen as a type of group work where participants split a task into subtasks among themselves and independently work on these subtasks. That is a division of work between students in a joint activity situation (Hadjerrouit, 2012). Judd et al. (2010) argued that *collaborative learning* involves the mutual engagement of participants in a coordinated effort to solve a task (collectively). According to Witney and Smallbone (2011), *collaborative learning* can be described as a learning process generated by small, interdependent groups of students who work together as a team with shared problem-solving (ibid., p. 103). The group work/activity used in this research study is a form of *collaborative learning*. Thus, I design an environment intended for achieving mutuality (sharing of an action or equal contribution). However, students in the group contribute differently (see Section 5.3). Hadjerrouit (2012) emphasized that through collaboration, students contribute to each other's learning, which might create a social learning environment that is more fruitful than the addition of individual work. Hadjerrouit further pointed out that *collaborative learning* is grounded in

Vygotsky's socio-constructivist learning theory, which assumes learning occurs through collaboration and information sharing in authentic contexts. Furthermore, how students learn is shaped by their relationships with others (ibid.).

Mathematical modelling and problem-solving are often accompanied by group work (collaborative learning). Interactions generated by group activities are often suitable for mathematical modelling (Ikeda & Stephens, 2001). The literature on collaborative group work (hereafter referred to as "group work/activity") in problem-solving and mathematical modelling shows how students' interactions contribute positively to their learning. For instance, Laal and Ghodsi (2012) documented the potential benefits of group work to students learning. Sahlberg and Berry (2002) pointed out that group work might produce equal academic outcomes among all group members compared to more traditional methods of instruction. On the other hand, Thom (2020) argued that not all contexts in which students interact with each other benefit students' levels of learning.

In mathematical modelling with the aid of digital technologies, interactions generated by group activities affect and are affected by digital technologies. Digital technologies as mediating artefacts might impact sense-making in group interactions as students work on mathematical modelling tasks. For instance, the results by Granberg and Olsson (2015) showed that GeoGebra provided students with a shared working space and feedback that became the subject of these students' creative reasoning. The results from Zengin (2021) also showed how GeoGebra impacted sense-making in group interactions. However, other studies highlight the complex nature of group interactions in this area. For instance, Lowrie (2011) reported on the tensions between using genuine artefacts and group interactions when some students were challenged to solve a realistic mathematics problem. Clark et al. (2014) suggested that the problem type might lead to effective group interaction and activity. For instance, some problem types might motivate the students to engage positively with their peers in a group activity. Sahlberg and Berry (2003) presented a list of problem types and their associated interactions (I will further develop this argument in Sub-Section 2.4.2). Brady and Jung (2022) argued that the choice of tasks, having a client outside the classroom for whom the solution to the problem is meaningful and relevant, might stimulate students' interest in solving the problem. Students adopt different roles in group activities while working on a mathematical modelling task. In some group activities, temporary roles are assigned by the teacher to embody particular divisions of labour in group work (Radinsky, 2008). Thus, a teacher might assign differentiated roles to each group member (Smith, 1996; Rosser, 1998; Crawford, 2001) within a group activity (e.g., the role of task recorder, process recorder, coordinator, checker and timekeeper). The roles identified in this study are characterizations of the observed patterns of students' participation in a group activity.

What are (some of) the roles students adopt in group activity in mathematical modelling with the aid of digital technologies? There are several roles students adopt in group interaction during mathematical modelling activities. For instance, one student might be taking charge of the computer activities (mainly when a group works with only one computer), and another student might be taking records with paper-and-pencil. Other roles can also be observed within group interactions (through classroom observations), such as *questioning and challenging*, *leading* roles, and others. *Questioning and challenging* ideas are seen as one of the factors in the success of group interaction (Goos et al., 2002; Paterson & Watt, 2014). Sahlberg and Berry (2003) pointed out the need for students to challenge each other's thinking to develop new concepts. In a group interaction, a student might question another student's idea(s) (in an attempt) to understand his/her thinking, and the response to the question might also be seen as an attempt to clarify, elaborate, evaluate, or justify one's thinking. Hernandez-Martinez and Harth (2015) argued that sometimes, the student whose idea(s) is challenged might not be able to respond to the challenge. Lowrie (2011) gave a potential reason that sometimes a student might not have the confidence to justify his convictions; hence, his/her idea(s) may lose impact within group interactions. The *leading* role describes the student dominating the communications within group interactions. The student dominating sometimes leads the other group members through his/her ideas. Esmonde (2009) highlighted that high-achieving students often dominate in group interactions. Costley (2021) argued that students might take on different roles and contribute to different levels and that students with high levels of motivation are more likely to contribute to group work through planning and sharing of information, amongst others (ibid., p. 4).

Other roles that might emerge within group interactions are *suggesting*, *supporting*, *opposing*, and *non-contributing* (described in Table 9.5 in Appendix E). Through classroom observations, I categorized these roles in the analysis of the pilot studies (see Sub-Section 5.6.4) and the empirical data of the main studies. In an *opposing* role, students do not agree to or accept their peers' comments and (or) introduce their ideas/solutions while solving a common task. With the *non-contributing* role, the student does not contribute entirely to the group work or contributes at some point but remains silent most of the time. Costley (2021) emphasized that students primarily gain an advantage from just being a member of a group and not necessarily by making significant contributions to that group (ibid.), although Tsay and Brady (2012) pointed out that students who actively participate in group work learn more and that the more students interact (with each other), the greater their learning. In a *suggesting role*, the students usually recommend an idea to assist the idea of the other student. In contrast, in a *supporting role*, the student only agrees to comments without adding anything new or critically assessing it.

In the forthcoming section, I will present the trends/patterns within group interactions. I will label these trends/patterns as interaction sequences. These interaction sequences depend much on the characteristics of the students.

2.4.1 Group interaction sequences

What are the patterns in group interactions as students work on mathematical tasks? Several patterns occur within group interactions. Jones and Gerard (1967) distinguished four categories of patterns (interaction sequences) that might occur within social interaction. They named these patterns social contingencies, a conceptual framework that deals with contingencies among replies. These interaction sequences are *pseudocontingency*, *asymmetrical*, *reactive* and *mutual* contingency. *Pseudocontingency* describes a limited cases of social interaction where members within the group follow their pre-established plans. Thus, each individual's responses are primarily determined by his/her pre-established plan. In an *asymmetrical* contingency, Student A's responses are primarily determined by self-produced stimuli (plans, strategies, or ideas). In contrast, Student B's responses are determined predominantly by social stimuli produced by Student A. In Peter-Koop's (2002) study, high-performing students frequently dominated this category. In *reactive* contingency, neither Student A nor Student B follows self-produced stimuli; instead, the interaction occurs in a sequence in which each student's response is almost entirely dependent on the preceding response of the other. Concerning *mutual* contingency, this is a situation where sense-making and conversations are mutually driven. Thus, each response is partially determined by the proceeding response of the other and partly by the individual's internal and self-produced stimulation. In Peter-Koop's (2002) study, this category is infrequent and tends to occur when high-performing students work together. In an example of mathematics education, Peter-Koop (2002) utilized these four contingencies in monitoring students' lines of thought as they engage in group activity. Lowrie (2011) also utilized these contingencies in analyzing group interactions in collaborative learning in mathematics tasks.

On the other hand, Mercer (1994) identified three distinct talk types or interaction categories as he explored the quality of talk in computer-assisted collaborative activity. These talk types are *disputational*, *cumulative* and *exploratory*. *Disputational* talk is characterized by disagreement and individualized decision-making. In the *cumulative* talk, the speaker builds positively but uncritically on what the other person said, and repetitions, confirmations, and elaborations characterize this discourse. In *exploratory* talk, members engage critically but constructively with each other's ideas (Wegerif & Mercer, 1997). Falloon and Khoo (2014), in exploring young students' (1st grade) talk in iPad-supported collaborative learning environments, described different elements under the talk types identified by Mercer (see Table 2.5 below). Table 2.5 is an extract from the literature by Falloon and Khoo (2014). Table 2.5 presents Mercer's talk types with some elements (active subcode) described by Falloon and Khoo (2014) for analyzing students' talk. There are more elements described in the original table. However, the elements in Table 2.5 are for the purpose of analyzing empirical data in this study. For instance, in the original table, an element 'competitive/defense' describes students competing for time on the device ('my turn, your turn'). However, in this study, the students are secondary school students, and this element is not generally observed in the students' interactions.

Talk Type	Active subcode	Talk description
Cumulative	Affirmative/ agreement	Talk that is supportive and affirming. Non-critical. Agreement with what was suggested without cause to review or challenge. Passive and compliant.
	Consensus/ clarification	Talk that builds understanding of suggestions or ideas but in a non-critical, non-challenging and non-expansive way.
	Elaboration	Questions are asked to seek further detail about how to do things or clarify why a partner suggests a particular course of action.
Disputational	Individualized	Talk that indicates possessiveness of own contribution. Unwilling to consider other's suggestions for improvement or change.
Exploratory	Critically constructive	Talk that indicates respectful cognitive engagement with and consideration and critical review of others' ideas in a way, leading to improved decision-making or content.

		Constructive critique focuses on the ideas or suggestions, not the person.
	Negotiated/ debated	Talk that demonstrates tentative ideas being offered and debated. Student(s) receptive to change if an excellent supporting reason(s)/case can be made by other(s). Different perspectives are acknowledged and synthesized into a collective response.
	Justification	Talk that seeks justification of perspectives or ideas being offered, focusing on how they will improve decision-making or output quality. Reasons for suggestions are pursued through probing questioning or offering alternatives.

Table 2.5: Talk type classifications, subcodes and description (adopted from Falloon and Khoo (2014)).

The group interaction framework by Jones and Gerard (1967) described a more general situation considering the contingencies among replies, while the framework by Mercer (1994) described in a more detailed way the kind of talk emerging from students' group interactions (in a computer environment). To study the patterns in group interactions as students work on a mathematical modelling task using digital technologies, I combined the two frameworks (Jones & Gerard, 1967; Mercer, 1994) to analyze the empirical data in this research study. Mercer takes a socio-cultural perspective based on Vygotsky and other socio-culturalists, while the perspective taken by Jones and Gerard is not clear. However, Jones and Gerard's interpretations of the different interaction sequences somewhat relate to Mercer's description. For instance, Falloon and Khoo's (2014) descriptions of the elements in Table 2.5 help combine the two frameworks. That is, *disputational* talk (individualized) is more associated with *pseudocontingency*, where the students' responses are individualized, and group members might be unwilling to consider other suggestions for improvement or change. Again, *pseudocontingency* can co-occur with another contingency involving the use of digital technology in group interaction. For instance, in a group of three students, Student A might interact with the computer (using a different strategy) while Student B and C communicate together (using another strategy) as they work on the same task. Thus, the interaction between Student A—computer and Student B—Student C is a pseudocontingency. However, the interaction between Student B and C could also be another form of contingency or interaction sequence.

Cumulative talk (affirmative/agreement, consensus/clarification, elaboration) is associated with *asymmetrical* contingency, where the response of Student A to

Student B's comments/ideas within a group interaction is affirmative and non-critical. This second response could also be a repetition of the first response for clarity, and this response is non-critical and non-challenging. Again, this second response could also be a question to seek further details about a member's earlier suggestion or ideas. *Exploratory* talk has both elements of *reactive* (critically constructive, justification) and *mutual* (negotiation) contingency. In *reactive* contingency, a group member considers and critically reviews another member's ideas, leading to improved decision-making or content. The reasons for suggestions (justifications) are pursued through probing questions or offering alternatives, and it must be noted that each individual's response is almost entirely dependent on the preceding response of the other. The description for *reactive* contingency here might be slightly different from Peter-Koop's (2002) description. In Peter-Koop's description, students react (often very spontaneously) to comments from their peers without developing and contributing their strategies. However, in this study, the emphasis is on the reaction, which is critically reviewing the ideas of others, and from this perspective, I relate *reactive* contingency to an *exploratory* talk. Concerning *mutual* contingency, different perspectives are acknowledged and synthesized into a collective response.

The issue of high-achieving students dominating discussions within group interactions discussed above can take a different turn. For instance, scaffolding has been contrived as the interaction between a more knowledgeable leader and the learner. However, similar but different sorts of scaffolding (but still effective) can occur between peers of complementary skills. In this case, students can come together with different skills and scaffold each other (Panselinas & Komis, 2009). Thus, in a mixed attainment group, there can be a possibility that a low-achieving student might complement a high-achieving student based on his/her unique skills (and not necessarily a high-achieving student only dominating in group interactions). Sahlberg and Berry (2003) argued that a well-designed task might lead to an interaction where students stimulate each other with ideas at the edge of their knowledge, understanding and skills, other than a high-achieving student only helping a low-achieving student to perform basic tasks. Another perspective of addressing this issue is Wenger's conception of *community of practice*, which offers a possible model for a classroom that could facilitate learning through social interaction (Olitsky, 2007). Olitsky (2007) argued that this perspective describes learning as taking place within a collective activity in which individuals provide scaffolding for each other to acquire the skills and knowledge for participation.

Summarizing the discussion on group interaction sequences, I would say the frequency of the interaction sequences within group activities might depend on the nature of the task (which I will further discuss in the forthcoming section), digital technologies used (see pages 38—40), and others. Different interaction sequences might emerge in different stages of the students' modelling activities. Drawing on the discussions on mathematical task design in Section 2.3, I will now discuss group work/activity concerning task design.

2.4.2 Group work/activity and task design

In this subsection, I will discuss the types of mathematical tasks (see Sub-Section 2.3.1) concerning group work/activity (there are other perspectives one can consider). Sahlberg and Berry (2003) addressed the question, 'What kinds of mathematical tasks are suitable for small group learning?'. The type of tasks selected for group activity depends on the teacher's/researcher's intentions towards students' learning. Furthermore, the way students work together in groups and the interaction quality also depends on task design. Sahlberg and Berry argued that if the objective of the task is to develop relatively routine mathematical skills (recalling facts, learning basic algorithms and rules, among others), then the designed task will take a particular form different from a designed task (with the objective) to develop higher order skills (mathematical thinking, creative problem solving or conceptual understanding). Moreover, the type of interaction among students working in small groups in both designs will also be different. For instance, when students work together in a group to solve a routine task, in many instances, the high-performing student often tends to help the low-performing student to perform the basic tasks better. In this case, the one who constructs the explanation and the other who receives the explanation both benefit if the quality of the explanation is conceptually rich. On the other hand, when students work together in a group to solve a conceptual understanding task, there might be a kind of interaction in which students stimulate each other with ideas at the edge of their knowledge, understanding and skills (ibid., p. 73).

Cohen (1994) distinguished between two types of tasks using the notion of exchange models: *limited exchange model* and *equal exchange model*. These models describe the quality and type of exchange/interaction when students work in small groups (Sahlberg & Berry, 2003). Cohen (1994) described a limited exchange model as a pattern in group interaction where the main reason for interaction lies in supplying information on how to proceed and content information, and the information is likely to flow from academically more robust students to weaker students (ibid., p. 64). This interaction usually is suitable for routine tasks. For instance, consider an example given by Sahlberg and Berry (2003), considering a group of three students who solve routine tasks such as solving quadratic equations. Each student might likely take turns to solve the problem while the other student might watch to check that the method and solution are correct (and that they understand what is happening). These students might also solve the same problem and then discuss the solution (ibid., p. 73).

Cohen (1994) argued that if we consider a less routine task with more conceptual objectives, then the pattern model of working together (in group work) might be an equal exchange model. To create equal exchange in group work, one needs to design a task where a single student within the group cannot easily do the task alone (ibid., p. 64). In this case, members within the group might find it necessary to exchange ideas freely for the common goal of getting the right solution. Sahlberg and Berry (2003) pointed out that a task that demands multiple abilities from group members might produce an equal exchange form of interaction. If we consider Task 2 (see Sub-Section 5.5.2) in this study, it is unlikely that a single student within the group can sit down and do the problem straight away by working alone. There are many issues to think about, such as, what is the definition of 'fairness' in the task, whether it is about the population, the time/distance of travel, and so on. As such, Task 2 might provide a very different interaction type that is much richer than a routine task. Task 1 (see Sub-Section 5.5.1) in this study might not yield similar results as there is a possibility that a single member within the group can solve it alone. From the definition of the types of exchange models above, we can associate the equal exchange model with reactive and mutual contingencies and the limited exchange model with asymmetrical and pseudo- contingencies.

Sahlberg and Berry (2003) presented some types of mathematical tasks and their corresponding type of exchange model. Table 2.6 below is an extract from this book. Table 2.6 presents a classification of the suitability of group mathematical tasks for collaborative student group work. The nature of each task category with a corresponding example is already presented in Table 2.4 (on page 54). Equal exchange in Table 2.6 describes the extent to which the designed task affords equal contributions from all members within the group as against limited exchange, which describes situations where one or more students know(s) the answer and tell(s) the other group members.

Category of Mathematical Task	Type of Exchange Model
Drilling basic skills	Very limited
Applying a formula or algorithm	Limited in how to proceed and checking results
Measuring and collecting data	Provides some opportunities for equal exchange of ideas and opinions
Real problem solving	Equal exchange
Mathematical modelling	Several opportunities for rich equal exchange
Mathematical Investigations	Several opportunities for rich equal exchange
Designing projects and studies in mathematics.	Rich equal exchange

Table 2.6: A classification of group mathematical tasks using the notion of exchange models (Sahlberg & Berry, 2003, p. 74).

So far, I have discussed interaction sequences and group work relating to task design without mentioning digital technologies. This does not mean the technology is separated from the students' activities; it is an integral part (see Section 2.2).

2.5 Summary of the chapter

This chapter has provided a general overview of current research in mathematical modelling with the aid of digital technologies. The chapter highlights different perspectives on mathematical modelling, modelling competencies, digital technologies, task design, and group interactions. While the field of mathematical modelling with the aid of digital technologies is attracting interest from researchers, and a lot of work has been done from a cognitive perspective, a mature theory of studying this field from a socio-cultural perspective is still emerging. Several studies suggest other dimensions such as metacognitive strategies, digital technology used, social norms, and the designed task, not only focusing on cognitive activities in mathematical modelling. As such, more empirical studies are needed to address these dimensions from a socio-cultural perspective. The suggested dimensions are not viewed as separate components but are seen as interacting with each other as a whole. That is, the inseparability of cognitive activity from the metacognitive strategies, the designed tasks, the social norms, and the digital technologies that help mediate the students' activity is consistent with a Vygotskian view of the social nature of learning. From this point of view, I will discuss in Chapter 4 how I put all these dimensions together (as a whole) using both Activity and Affordance Theory. Before this discussion, in the next chapter (Chapter 3), I will present the Norwegian educational context, which is the context in which the study was conducted.

3 The Norwegian Educational Context

One needs to know the context of a situation in order to understand it fully. Hence, I will elaborate on the context of the participants in this research study, which is the Norwegian educational context. Section 3.1 presents an overview of the Norwegian school system. Section 3.2 elaborates on mathematics education in Norway, highlighting mathematics education at the secondary school level. Section 3.3 presents mathematical modelling and applications in the Norwegian mathematics curriculum, particularly at the secondary school level. Section 3.4 describes the schools where the research took place with the cooperating teachers and research cohort. The chapter ends with a summary in Section 3.5.

3.1 The Norwegian school system

The information about the Norwegian school system given here is strictly taken from the Ministry of Education and Research (2019), Onstad and Kaarstein (2015) and Feagles and Dickey (1994). The Norwegian education system is governed by national legislation, and the Ministry of Education and Research (*Kunnskapsdepartementet*) (involving expert groups) is responsible for carrying out national educational policy at all levels of education, including preschool (for children up to age five). The curriculum used in Norway is centralized and comprises all subjects for Grades 1 to 13. Within the framework set by the curriculum, local schools and teachers have considerable freedom to make their own decisions regarding organization and instructional methods. Kindergarten or preprimary school is neither compulsory nor (totally) free in Norway (although every child has a right to attend). Following preprimary school, every child has the legal right to 13 years of education, of which the first 10 years (Grades 1 to 10) are compulsory and free. The next three years of education (Grade 11 to 13) are not compulsory but are still free. Most students are generally enrolled in public schools, as private schools play a minor role in Norwegian education. Mathematics is a compulsory subject between Grade 1 to 10, and one of the common subjects at the upper secondary school. Currently, there is a new Norwegian curriculum (*Kunnskapsløftet* 2020) for all school years.

I will now present mathematics education in the Norwegian curriculum in the forthcoming section.

3.2 Mathematics education in Norway

Mathematics is one of the prominent subject in the Norwegian school curriculum, and it is one of the core subjects covered on national examinations in the 10th grade. Over the years, longitudinal studies such as the Trends in International Mathematics and Science Study (TIMSS) and the Programme for International Students Assessment (PISA) have shown a setback in mathematical performance among students in both lower and upper secondary school, particularly in the topics of numbers and algebra (Lie et al., 1997; Grønmo et al., 2004; Kjærnsli et al., 2004, 2007; Grønmo & Onstad, 2009; Grønmo et al., 2010, 2011). Espeland (2017) pointed out that the Norwegian results from these comparative studies are generally considered unsatisfactory and have stimulated much debate in Norway. As a result of such debates, Norway has adopted a new curriculum for all school years. The new Norwegian mathematics curricula are organized under competence aims and assessment for each school year (Grades 1 to 10) and the upper secondary education. The Norwegian mathematics curriculum has six core elements: exploration and problem-solving; modelling and applications; reasoning and argumentation; representation and communication; abstraction and generalization; and mathematical fields of knowledge. The new curriculum emphasizes that (a Google translate of the Norwegian text):

> The students must work more with methods and ways of thinking to understand the subject better. Numbers and numerical understanding are the foundation of what students must master during primary school. Personal finance, measurement and statistics are essential areas where numbers are used in realistic contexts. Programming and algorithmic thinking also become part of the subject.

The text above highlights understanding the subject, applying it in a realistic context, and using digital technologies in such inquiries. One may ask, “Are there any major changes between the new and the previous curricula, in terms of materials used?” which is a relevant question. Nonetheless, Bakken and Andersson-Bakken (2021) conducted a study in this area a year after implementing the curriculum. Bakken and Andersson-Bakken investigated if and how the tasks in science and language arts textbooks in Norwegian upper secondary schools have changed after the curriculum reform. A content analysis was conducted of around 5,067 tasks in science and language arts textbooks compared to tasks (in textbooks) published before the reform. The results from the study show that there was only a marginal change in tasks in each subject. That is, although the curriculum has changed, the tasks used in schools have not changed that much, and these tasks do not present the students with sufficient opportunities to practice the competencies highlighted in the new curriculum. Moreover, what was the focus of the previous Norwegian mathematics curriculum? Pedersen (2015) presented some studies from TIMSS in the Norwegian context and pointed out that:

> Previous analyses have revealed that the Norwegian curriculum for upper secondary school mathematics places the most significant emphasis on applying procedures and methods and that the curricular objectives, to a far lesser degree, describe activities such as analyzing, investigating, assessing, discussing, proving, modelling and generalizing" (ibid., p. 74).

From the text above, the focus of the previous curriculum differs from the core elements of the new curriculum. Suppose the task used within the previous curriculum is used again within the new curriculum without any modification (to suit the current curriculum). In that case, the competencies highlighted by the new curriculum might not be realized. The results from Pedersen's studies show that (concerning the previous curriculum), Norwegian upper secondary school students tend to perform weakly on items that place high demands on symbol manipulation. However, these students' strengths are in tasks placed in an extra-mathematical (applied) context. In this case, the students can generate the mathematical expressions needed to find a solution but have a low ability to manipulate symbolic expressions. Early school leaving (or non-completion) in the upper secondary school is another concern. Markussen et al. (2011) argued that Norwegian upper secondary schools do not (to some extent) recognize the significant variation in knowledge and skills among students transitioning from lower to upper secondary schools. Thus, pedagogical differentiation is not used to the necessary extent, and as such, most of these students are treated as if they could cope with the demands of upper-secondary education. As a result of this issue (and other factors), students might leave/drop out of upper secondary school (ibid., p. 240).

In recent times, Haugan et al. (2019) conducted a longitudinal study of factors predicting students' intentions to leave upper secondary school. Their results show that the student's grades from elementary school, parental and teacher support, and school engagement in upper secondary school are important factors (to some extent) that lead to early school leaving (dropout). The issues highlighted here seem to give an impression of Norwegian education (regarding the transition between lower and upper secondary school) in general and not specific to mathematics education. However, the discussed issue might also apply in a mathematics subject context. For instance, teachers might have different beliefs and practices in mathematics teaching (Nilsen, 2010) at both levels, leading to differences in school engagement. Feldlaufer et al. (1988) pointed out that the differences in a classroom environment might contribute to the changes and behaviours sometimes associated with a transition from one level to another (e.g., from the elementary school level to the lower secondary school level). These differences could be the class task organization, competition and social comparison among students and others. In the next paragraph, I will present some competence aims in the Norwegian mathematics curriculum relevant to this study.

The participants in this current research study are students in lower and upper secondary school levels, specifically Vg1 (first year in upper secondary school), Vg2 (second year in upper secondary school), and Grade 9. The students in both Vg1 (Grade 11) and Vg2 (Grade 12) are under the program for general studies taking 1T (theoretical mathematics—year 1) and R1 (mathematics for science—year 2) mathematics, respectively. Competence aims are defined in the mathematics curriculum for Grade 9, Vg1 (1T mathematics) and Vg2 (R1 mathematics). A summary of a section of the competence aims highlight students' application of mathematics to realistic/authentic problems using digital technologies. For instance (at the Vg1 level), students are to identify variable quantities in different contexts while creating models with digital tools.

In the forthcoming section, I will present mathematical modelling and applications in the Norwegian curriculum.

3.3 Mathematical modelling and applications in the Norwegian mathematics curriculum

Mathematical modelling has been part of the Norwegian curriculum for decades (Berget & Bolstad, 2019) and might have a different approach. Twenty-nine years ago (and years before), modelling, experimentation, and exploration were among the nine aims in upper secondary school mathematics (Berget, 2022). Frejd and Bergsten (2018) pointed out that mathematical modelling has been incorporated in different ways in national curricula. For instance, the approaches towards mathematical modelling in Norway implicitly describe mathematical modelling where the terms are used but not explained or used at all (ibid., p. 119). Thus, the

Norwegian curriculum does not use the term modelling explicitly but says students should work with problems in a realistic context. However, the curriculum is explicitly connected to information technology. For instance, an appropriate use of graphic calculators and computers in the modelling process at the upper secondary level (Antonius, 2004).

The discussion above is based on the previous curriculum, and I will now discuss the new Norwegian curricula in the context of mathematical modelling and applications. I have already described the curriculum's core element of "modelling and application" (see the last paragraph on page 21). The steps in a modelling cycle (see Figure 2.1 on page 13) can be recognized in the description of "modelling and applications" in the curriculum: to convert a real situation into a mathematical model, to solve this model and assess the validity of the model, to interpret and generalized the model, among others. The description of mathematical modelling highlighted in the curriculum is prominent in explaining modelling, as in the definitions of modelling cycle and competencies. In summary, the Norwegian curriculum does not use terms like modelling competencies and cycle(s); however, the descriptions can relate to these terms.

One may ask, "Which of the steps of a modelling cycle are needed to solve textbook modelling tasks or tasks from Norwegian national examinations?". To address this question, I will turn to Bakken and Andersson-Bakken's (2021) findings in the previous section. Thus, the tasks used have not changed much regarding the previous and current curricula. In the context of mathematical modelling and applications, Berget (2022) examined mathematical modelling in textbook tasks and national exams in light of the new curriculum. The findings indicate different perspectives on mathematical modelling in the curriculum, the textbook tasks and the national exam. Thus, in the textbook and national exams, only parts of the modelling process are included as compared to what is intended by the curriculum. For instance, Berget (2022) presented an example of a task analyzed through the steps in a modelling cycle (see Figure 2.1). Figure 3.1 below shows an analysis of a textbook task through the lens of a modelling cycle. The steps such as *constructing*, *simplifying*, *mathematizing*, *validating* and *exposing* (that is, steps 1, 2, 3, 6 and 7, respectively, in the far right of Figure 3.1) are missing out, whilst the only steps present are *working mathematically* and *interpreting* (that is, steps 4 and 5 respectively in the far right of Figure 3.1).

From Figure 3.1, the dataset is already presented, and the students are asked to make a linear model. To make this model, the students must only work mathematically on the given numbers using digital technology (e.g., GeoGebra). The task in Figure 3.1 does not ask for validation and argument on why the model should be accepted. The answers to the preceding sub-questions in Figure 3.1 must be translated from the mathematical model. As such, there is no need to expose the model, as no assumptions are made by the students in this situation. Berget (2022) emphasized that several examples of tasks, like Figure 3.1, are seen in textbooks and exam tasks. Concerning Tasks 1 and 2 (see Section 5.5) in this study, the task is formulated in everyday language, and there are no given steps on how the students should solve it. Thus, the students decide and identify critical components themselves and mathematize the situation. Berget argued that only a few examples of textbook tasks relate to Tasks 1 and 2 in this study.

One of several reasons why schools continue to use tasks such as the one presented in Figure 3.1 might be the strong bias against mathematical modelling. At the same time, greater attention is given to high-level mathematics such as theorems, proofs, formulas, and others at the upper secondary level (particularly in some European countries) (Stillman, 2007). There could also be the possibility that teachers have to prepare their students for national exams and that greater attention is given to tasks similar to what the students will meet in the exam. Artaud (2007) argued that the teaching process must be accorded extra time if mathematical modelling (concerning Task 1 and 2) is added to the ordinary didactical system. Antonius (2004) also points out that it is still hard to find time for modelling activities, which are very time-consuming in the Nordic context.

The table shows driving length and tire tread depth for 7 summer tires.

Driving length in 1000 km	14	17	24	35	37	38	39
Tire tread depth in mm	5.7	6.5	4.0	1.9	2.7	1.9	2.3

a) Make a linear model for the tread depth $f(x)$ in mm as a function of driving length x in 1000 km
b) Use the model to calculate what the tread depth can be for new tires
c) The minimum legal tread depth for summer tires is 1,6 mm. How long can you legally drive before changing tires, according to this model?
d) What is the domain of the model, for legal car drivers?
e) Recommended tread depth is 3.0 mm. How many per cent shorter than legal driving length is the driving length if you follow this advice?

1) NO
2) NO
3) NO
4) YES = 1
5) YES = 1
6) NO
7) NO

Figure 3.1: Analysis of textbook task through the lens of a modelling cycle (Berget, 2022, p. 60).

In the next section, I will present the background of the schools and students that participated in this research study.

3.4 Schools and cooperating teachers in this study

Four schools participated in this research study, and students from these schools are of different levels (but all are at the secondary school level). These schools are located in the southern part of Norway (*Agder*). Students in these schools can use different digital technologies in problem-solving (e.g., GeoGebra, Excel, and others). The students also have their personal computers that they use for mathematics lessons. All the schools follow the same national curriculum (although task selection is specific to each teacher), as presented in the previous sections. Before this research study, the students had not solved tasks such as Tasks 1 and 2 (see Section 5.5). However, they have experience tackling tasks like those in Figure 3.1 (especially the students at the upper secondary level).

I will present a specific description of each school in the following subsections. Pseudonyms are used for schools, teachers and students. The reports under each school are strictly from the teachers' reflections during my visits to the schools between Autumn 2020 and Autumn 2021 (see Table 5.3 on page 137).

3.4.1 School A

School A is a public upper-secondary school specializing in general studies and vocational education programs. School A is a medium-sized school by Norwegian standards with a ceiling of a little over 600 students.

Ten (7 males and 3 females) second-year students (Vg2) from School A participated in the research study. These students had been together for almost a year and a half. The age of these students at the time of the research was between 16 and 17 years, of which the majority were 17. The students were taking the R1 mathematics subject in Autumn 2021. Furthermore, they covered topics like logarithms, functions, equations (linear, quadratic, and exponential functions), and trigonometry. The students had geometry at the lower secondary level.

The students have five 45-minute mathematics lessons weekly. They usually solve a mathematics task in between 10 and 30 minutes (depending on the task, either flexible, allowing students to explore and be creative or restricted). The students usually work in groups, although some prefer to work individually. Working in groups, the students have the perception that all group members should contribute. Regarding assessments, students must learn and use the methods taught or use their methods if they can argue for them.

The teacher expects that students should learn and understand mathematics, but these students might have different goals (to pass the test/exams and not necessarily learn mathematics). Concerning technology, the school allows students to use it. However, the teacher believes that technology takes away the mathematics, and sometimes the teacher struggles as she wants the students to learn mathematics. The teacher believes that much time is spent on technology since the students have less experience using digital technologies (e.g., programming) at the lower secondary (an issue in the change of the curricula).

Of the ten students who participated in the studies in School A, three volunteered to form the focus group (*Group A*) during data collection. I will briefly summarize this focus group based on their teacher's account.

Group A. Thea, Rolf and Kåre were members of Group A, and within this group, Thea and Rolf were assigned grade 4, whilst Kåre was assigned grade 3 after the exams/test. In the Norwegian performance scale, a *high performance* is between grades 5 and 6, an *average performance* is between grades 3 and 4, and a *low performance* is between grades 1 and 2. To further differentiate Thea and Rolf, the teacher noted that Thea performs higher than Rolf, but Rolf has the highest mathematical understanding but has a little lower motivation within the group.

3.4.2 School B

School B is a public upper-secondary school specializing in general studies and vocational education programs. School B is a large school with a population of approximately 1500 students.

Twenty-eight (14 males and 14 females) first-year students (Vg1) from School B participated in the research study. These students have known each other since the start of the Autumn 2021 semester (but some knew each other before, as they were in the same class at the lower secondary school). The age of these students at the time of the research was between 16 and 17 years, of which the majority were 16. The students were taking the 1T mathematics subject in Autumn 2021. Furthermore, they were expected to cover topics like basic calculation (fractions, multiplication, division, integers, and others), equations (linear and quadratic), factorization, linear polynomial functions, exponential functions, regression, percentage calculations, rate and programming. The students had geometry at the lower secondary level (not in the syllables at the upper secondary level).

The students have five 45-minute mathematics lessons weekly. They usually solve a mathematics task in between 10 and 25 minutes (depending on the task—flexible or restricted). The students usually work in pairs and sometimes in groups of four. The students expect every member of the group to participate. Regarding assessments, the students usually send a screenshot of their work to the teacher. The teacher also makes a checklist of what she expects of the students. With the new syllables, the students are taught the tools and can choose which to use depending on the situation. The teacher does not tell the students to solve a problem in a specific way but expects them to show what they did and why.

The teacher believes the students have different backgrounds/levels and does not expect everyone to pass. The teacher expects the students to learn and understand mathematics rather than just pass the exam. However, they must pass (minimum, grade 4) to enrol in the International Baccalaureate (IB) program the following year. Moreover, some students like mathematics and want to learn further (sometimes have exciting thoughts). Regarding using digital technologies, students often use computers at school. However, the teacher wants them to understand how to solve problems algebraically (paper-and-pencil), and not just press the button with the computer without understanding what is happening mathematically. The teacher also feels that much time is spent on technology since the students have less experience.

Out of the 28 students who participated in the studies in School B, 4 volunteered to form the focus group (*Group B*) during data collection. I will briefly summarize this focus group based on their teacher's account.

Group B. Emil, Thor, Ella, and Tore were Group B members, and the teacher assigned Emil grade 5, Thor grade 4, Ella grade 3, and Tore grade 1 after the exams/test. The teacher sees Emil as a high-performing student who works in a different way than 'school math' and thinks logically and outside the box. Thor is a medium-plus performing student and good at logical thinking, but he is not that good at using symbols and takes time to understand them. Ella is a medium/ average performing student who performs better than Tore (struggling in math).

3.4.3 School C

School C is a public upper-secondary school specializing in general studies and vocational education programs. School C is a large school by Norwegian standards, with a population of approximately 900 students.

Seventeen (6 males and 11 females) first-year students (Vg1) from School C participated in the research study. These students have known each other since the start of the Autumn 2021 semester. The age of these students at the time of the research was between 16 and 17 years, of which the majority were 16. The students were taking the 1T mathematics subject in Autumn 2021. They were expected to cover topics like functions, algebra, equations, 2nd-degree expression, percentage calculations, fractions, square roots, and programming. The students had geometry at the lower secondary level.

The students have five 45-minute mathematics lessons weekly. They usually solve a mathematics task in between 2 and 10 minutes (depending on the task—flexible or restricted). The students usually work individually, and the teacher sometimes instructs or encourages them to work together. The students believe that they know a little, and so do their peers, so they build on each other's knowledge. Very often, the one who knows most about the technology grabs the computer, so they get even better than the others. Regarding assessments, the teacher marks or assesses and gives feedback on most of the tasks the students hand in. The teacher also assesses the class participation of the students.

The teacher expects the students to do well as they should (within their limits). The teacher encourages them to work with the things until they understand them. However, the student's primary goal is to get good grades. Aside from this goal, some students think mathematics is fun. Regarding using digital technologies, the school follows the curriculum. With the new curriculum, the school can only include some things now, and it will take some years to get all the mathematics teachers in the school to that level. Again, the teacher emphasized that the current group of students are struggling more than the previous group because they have lost a lot during the COVID-19 pandemic. The teacher believes there is not enough time to teach technology because the basics of mathematics are not very good. Thus, the students must first know the logic or structure of algebra, and GeoGebra will be an excellent tool for visualizing and finding solutions and explorations.

Out of the 17 students who participated in the studies in School C, 3 volunteered to form the focus group (*Group C*) during data collection. I will briefly summarize this focus group based on their teacher's account.

Group C. Nils, Anna, and Jørn were Group C members, and the teacher assigned them grade 5 after the exams/test. The teacher did not give further information about this group or the hierarchies within the performance bands.

3.4.4 School D

School D is a private primary (Grade 5 to 7) and lower secondary (Grade 8 to 10) school. School D is a small school by Norwegian standards with a ceiling of approximately 105 students.

Eighteen (5 males and 13 females) Grade 9 students from School D participated in the research study. The students comprise two schools that came together in the 5th grade (but some students have known each other since the 1st grade). The age of these students at the time of the research was between 14 and 15 years, of which the majority were 14. These students took the 9th-grade mathematics subject in Autumn 2021, which comprises functions, logical reasoning, math for the national tasks, and others. The students were familiar with algebra and equations from the 7th and 8th grades (and will have it in both the 9th and 10th grades). The students also worked on geometry each year.

The students have five 45-minute mathematics lessons weekly. They usually solve a mathematics task in between 10 and 15 minutes (depending on the task). The students usually work with open tasks, which allow them to explore (being creative). The students usually work in groups of three. Some students prefer working individually, while others prefer working in groups. The school has a big focus on group work. The students expect their peers to work and expect everybody to take responsibility within the group. Regarding assessments, oral, written and student-to-student (students giving feedback to each other) forms of assessment are usually used.

The teacher believes these students' expectations are low since many students struggle with mathematics. Furthermore, the entire school has a big expectation of students' achievements (students have achieved higher results over the years). The teachers see the current class as lower than the other classes. The teacher emphasized that some students in this class might want to pass the exam, and some have been working hard to get a higher grade (and find mathematics exciting). Regarding using digital technologies, the school has a big focus on the use of digital technologies, and there is a one-to-one Chrome book (computer) for every student. The students have been working with different kinds of software and smart screens (big screens). The students have worked with GeoGebra and many Google applications (like Google Sheets, documents, and presentations, among others) for many years. Digital technology is incorporated in all subjects at School D. Students often use digital tools but can work without them.

Of the 18 students who participated in the studies in School D, 3 volunteered to form the focus group (*Group D*) during data collection. I will briefly summarize this focus group based on their teacher's account.

Group D. Olga, Hege and Lena were members of Group D, and the teacher assigned Olga grade 6, Hege grade 4 and Lena grade 3 after the exams/tests. The teacher described Olga as far from the others (high-performing student), Hege as an average plus student, and Lena as average.

3.5 Summary of the chapter

This chapter has provided an overview of the Norwegian educational context and the schools participating in this research study. The chapter highlights the structure of the Norwegian school system, mathematics education in the Norwegian context, mathematical modelling and applications in the Norwegian curriculum, and a description of the schools taking part in the research project. One of the significant issues raised in this chapter is the use of tasks (from the previous curricula) in the current implementation of the new curricula, which does not present the students with sufficient opportunities to practice the competencies highlighted in the new curricula. Another significant issue raised is about the mathematical modelling tasks from textbooks, which have a different perspective on mathematical modelling compared to what the new curriculum intends. In this case, only parts of the modelling process (and not the whole) are included in the textbook tasks.

There are some similarities and differences in the descriptions of the schools. Students in Schools A, B, and C were not very experienced in using digital technologies from previous years due to the COVID-19 pandemic and the implementation of the new curriculum. These students lacked some aspects of the mathematics subject as well. Students in School D were experienced in using digital technologies, which was a primary focus of the school. Groups A, B, and D (respectively from School A, B, and D) were made up of mixed attainment students (based on their scores in the exam), whilst Group C (from School C) was made up of high-performing students.

Having presented the context of the students in the schools in which the study was conducted, I will, in the next chapter (Chapter 4), present the theoretical framework providing knowledge of what is already known through previous research, thus guiding the analysis and interpretations of the data generated.

4 Theoretical Framework

This chapter addresses the theoretical basis of the research study. I repeat to the reader the aim of this research, which is to explore how secondary school students solve mathematical modelling tasks with the aid of digital technologies. Different theories are used in studying mathematical modelling (Buchholtz, 2013). For instance, the commognition theory (Ärlebäck & Frejd, 2013), Anthropological Theory of Didactics (Garcia et al., 2006), and others. These theories help understand modelling issues (with digital technologies). Silver and Herbst (2007) argue that (the concept of) theory has been defined in several ways and can be seen and used differently in diverse contexts and areas.

Theories in research can provide models and frameworks as tools for describing and understanding a particular phenomenon (Nesher, 2015). In research, theories can be seen in quite different ways: theory as a lens (Simon, 2009; Nesher, 2015; Niss, 2007); theory as a tool (Assude et al., 2008; Simon, 2009); or theory as an object (Assude et al., 2008). Furthermore, theories are categorized into three common levels: grand theories, middle-range theories and local theories. Again, theories help researchers formulate the research problem and questions, the study's design, the method adopted, the analysis of data, and the interpretation of results (Silver & Herbst, 2007; Bikner-Ahsbahs & Prediger, 2010). For this reason, I present in this chapter Cultural-Historical Activity Theory (CHAT) as a lens to study students' use of digital technologies in mathematical modelling activities. This thesis focuses on interactions within the students' activities, and as such, a socio-cultural approach (in this case, CHAT) with a focus on actors and action is relevant. I will also show how supplementary theories, such as Affordance Theory and concepts like modelling actions (coined from the ontology of modelling competence in the sense of CHAT) complement CHAT in this study.

The chapter opens with an introduction to CHAT in Section 4.1. Section 4.2 presents an argument for adopting CHAT perspective stance in this research study. Section 4.3 elaborates on Affordance Theory, highlighting the emergence, perception and actualization of affordances and constraints, and affordances and constraints in mathematics education. Section 4.4 elaborates on the compatibility between the adopted theories, where I shed light on the compatibility between CHAT and Affordance Theory and the interpretation of modelling actions from CHAT perspective. The chapter ends with a summary in Section 4.5.

4.1 Introduction to Cultural-Historical Activity Theory (CHAT)

To explore how secondary school students solve mathematical modelling tasks with the aid of digital technologies, I tend to consider the situation as students' activity. And I address issues such as 'Can students' activity be broken down into smaller units?', 'what role does digital technologies play in students' activity?', among others. To address these questions, the students' activity needs a more elaborate concept of activity, which can be offered by Activity Theory.

Activity Theory refers to various theories rooted in Soviet approaches to psychology 100 years ago. 'Activity' is an everyday word for 'doing something'; however, the *activity* in Activity Theory retains this meaning but conceives it as the interaction of a person(s) (*subject*) with things around them (*mediating artefacts*) to achieve a particular outcome (the *object* of the activity). Activity Theory is often referred to as CHAT. *Activity* in CHAT is the analytic unit for understanding human performances, such as their practices, the sense they make, or their actions (Roth, 2012). The concept of *activity* is fundamental in that its representations can be used for many different functions (Moran, 2003), and one of such functions is human-computer interactions. For instance, Nardi (1996) discusses using Activity Theory as a framework to study human-computer interactions. There are many versions of Activity Theory. However, three that are highly referenced are associated with Vygotsky, Leont'ev and Engeström (chronologically). These versions of the development of CHAT can be organized into three generations.

I will present the different CHAT versions stated above in the forthcoming subsections.

4.1.1 First generation CHAT

Lev Vygotsky's sociocultural approach is considered the first generation of CHAT. In the 1920s, Vygotsky focused on *activity* in his considerations of consciousness as a problem for psychology. In CHAT, social and cultural reality is constructed, and according to Vygotsky (1978), acceptable knowledge is the interpretation of the participant's actions and thinking in a given context. CHAT draws from the idea that all social actions are mediated by language, discourse, and other cultural means (Jones et al., 2016). Cole (1996, p. 108) states that the central thesis of CHAT is that "the structure and development of human psychological processes emerge through culturally mediated, historically developing, practical activity", and these three components are interrelated. The idea of mediation is usually illustrated by the mediational triangle (see Figure 4.1).

In Figure 4.1, *tools* are intermediate between the *subject* and the *object*. *Tools* are sociohistorically formed means and modes through which a person is connected to other people and assimilates the experiences of humanity. "*Tools* shape how human beings interact with reality" (Kaptelinin et al., 1995, p. 192). Monaghan et al. (2016) define an *artefact* as a material object humans make for specific purposes. According to Cole (1996), when an *artefact* is used in a goal-directed action, it becomes a *tool*. According to Vygotsky, there are two distinct *tools*: *technical* and *psychological*. Kaptelinin et al. (1995) highlight that "*technical tools* are intended to manipulate physical objects (e.g., a hummer)" whilst "*psychological tools* are used by human beings to influence other people or themselves (e.g., the multiplication table, a calendar, among others)" (ibid., p. 192). One should note that *tools* and *concepts* are two different things in Activity Theory, and that only *tools* can mediate. For instance, 'logarithm' is a mathematical concept, but a logarithm table is a material tool. In other words, a logarithm table is a material manifestation of the logarithm concept that can mediate between subject and object (e.g., solve problems) or between people (e.g., teach solution procedures). Vygotsky claimed that *activity* always involves *mediating artefacts* (e.g., tools, language, and other people). Thus, the *subject*(*s*) interact with an *object* with a motive (*object-oriented activity*-the unit of analysis).

Another essential construct in Lev Vygotsky's theory of learning and development is the *Zone of Proximal Development* (ZPD). The concept of ZPD relates to the difference between what a student can achieve independently and what this student can achieve with guidance and encouragement from a skilled partner (Vygotsky, 1978). Thus, a student can reach a learning goal by solving tasks (not every task, but ones beyond the student's capabilities) with more competent peers (or the teacher). Furthermore, through collaboration within a student group, the gap between what a student can learn on his/her own and what this student can learn with help from others can be bridged. In an example, Hernandez-Martinez and Harth (2015) studied "an Activity Theory analysis of group work in mathematical modelling" and pointed out that peers' ideas proposed at each moment in time (in group interactions) become the tool that the group use in their process of sense-making as they work on a mathematical modelling task.

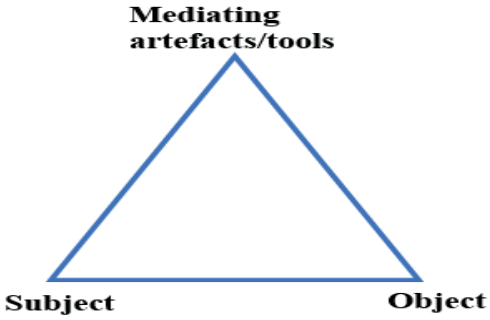


Figure 4.1: The mediational triangle (Vygotsky, 1978).

The first generation of CHAT focuses on individuals performing actions in a sociocultural setting (Engeström, 2001). That is, the unit of analysis remained individually focused on Vygotsky's work. This was a limitation in Vygotsky's work. However, Leont'ev explains in detail the key difference between an individual action and a collective activity. In the forthcoming subsection, I will present Leont'ev's version of CHAT (second generation).

4.1.2 Second generation CHAT

The second generation of CHAT is based on the work of Leont'ev. As Leont'ev (1974) described, the concept of *activity* refers to the subject-object interaction mediated through tools and societal relations. The second generation of CHAT shifts the focus of analysis from individual tool-mediated action to the level of collective human *activity*. Leont'ev (1981b) argues that individual actions are senseless and unjustified if we do not consider collective activity. Kaptelinin et al. (1995) point out "that *activity* cannot exist as an isolated entity", and the very concept of *activity* implies that there is an individual or collective 'subject' who acts, for which the *activity* is directed at something (ibid., p. 191). The principle of object-orientedness states that all human *activities* are directed towards their *objects* (with a specific motive), and the activities are differentiated by their respective *objects* (Kaptelinin, 2017). Thus, *activities* are comprised of actions for which these actions are directed towards a motive or goal (Leont'ev, 1978). For instance, when students solve mathematical modelling tasks by modelling with digital technologies, their main goal might be to develop a technology-based model/solution. An *activity* cannot be without a motive; however, an unmotivated *activity* has the motive concealed subjectively or objectively. The motive and *object* of the *activity* ensure a meaningful *activity* (Williams & Goos, 2012).

Leont'ev (1977) presented a theoretical model that analyzes human *activity* in a three-level hierarchy. This theoretical model explains the structure of human activities regarding human functioning. Figure 4.2 below shows Leont'ev's three hierarchical layers of an *activity*, where the top level concerns the whole *activity*, which is driven by an object-related *motive*. The *object* gives direction to an *activity* and distinguishes one *activity* from another. For instance, the *motive* behind the classroom *activity* involving solving a linear equation may differ whether one considers it from the teacher's or the student's perspective. The teacher might put it in the broader perspective of the usefulness beyond the classroom walls, whereas the students might be motivated by passing the exam.

The second level in Figure 4.2 concerns the individual *actions* that translate an *activity* into reality. Developing an *activity* into separate actions often results from the *division of labour* among the participants. In Leont'ev's perspective, separate partial results are achieved by separate participants in the collective labour *activity*, where these separate partial results are *goals* to which the *actions* are directed. The *actions* of the participants are also interwoven in the *motive* of the activity. Kaptelinin et al. (1995) explain that *actions* are conscious processes oriented towards *goals*, and these *goals* are the *objects* of actions. Thus, *goals* are functionally subordinated to other *goals*, which may be subordinated to still other *goals* (and so on). Nonetheless, we finally reach a top-level *goal* (the first level in Figure 4.2), which is not subordinate to any other *goal* as we move up the hierarchy of *goals*. This top-level *goal* (called the *motive*) is the object of the whole activity. In summary, *goals* are the *objects* human *activities* are directed at, while *motives* are the *objects* which motivate human *activities*. Moving down the hierarchy of *actions* (as shown in Figure 4.2), we cross the border between conscious and automatic processes, and these automatic processes for which individuals are not aware of are responsive to actual conditions (ibid., p. 193).

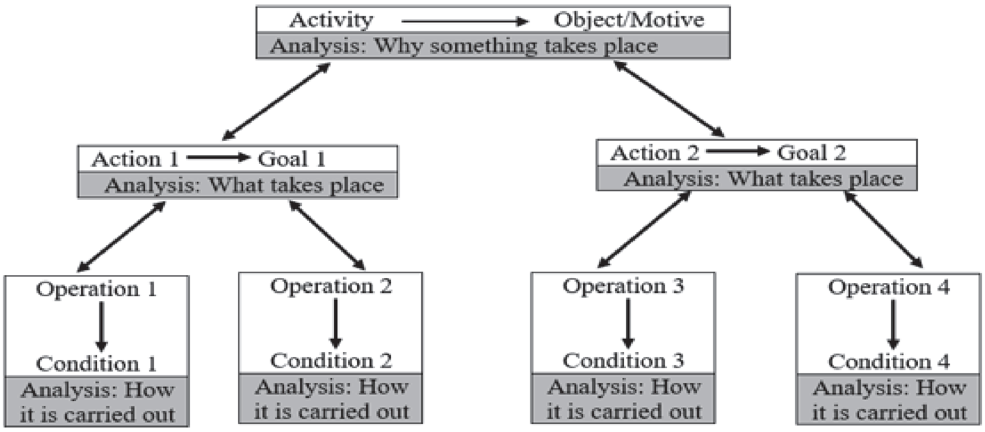


Figure 4.2: Hierarchical levels of an activity (Leont'ev, 1977).

The third level in Figure 4.2 deals with the operational aspects, which are the objective circumstances under which the *actions* are carried out. The *operations* at a more subconscious level describe the methods by which goal-directed *actions* are carried out. *Operations* are routine processes (they do not have their own goals) providing an adjustment of actions to ongoing situations. They are oriented toward the conditions under which the subject (both individual and collective) is trying to attain a *goal*. In summary, *activities* (driven by *motives*) are performed through specific *actions* directed towards *goals*, which, in succession, are implemented through certain *operations* (Kaptelinin et al., 1995). For instance, Tyskerud et al. (2017) used this principle to investigate "teachers' lesson study" as they study the development of mathematics teachers' professional practice.

In summary, Leont'ev explains in detail the key difference between an individual action and a collective activity. However, he never graphically expanded Vygotsky's original model into a model of a collective system (Engeström, 2001). Hence, I will present Engeström's version of CHAT (third generation), highlighting his graphical expansion of Vygotsky's original model.

4.1.3 Third generation CHAT

Engeström (1987) extends Vygotsky's focus on mediation through signs and *tools* to multiple forms of mediation and extends Leont'ev's frame to *activity systems*, which include the community and social rules underlying *activity* (see Figure 4.3 below). Leont'ev explicitly highlights that *activities* can be carried out by individual human beings and social entities (collective subjects). However, he does not systematically explore the structure and development of collective *activities* (Kaptelinin, 2017). Engeström presents a conceptual model of collective *activity* (*activity system*, see Figure 4.3). Figure 4.3 presents Engeström's extended triangular mode of an *activity system*. An activity system incorporates the tool mediation and the societal mediation in an *activity*, as proposed by Leont'ev, with the addition of societal dimensions in the triangular model of this tool mediation. Engeström (third generation) focuses on the collective system and considers students solving problems with *tools* as an *activity system* by interactions between *subject*, *object*, *community*, *rules*, *division of labour* and *mediating artefacts/tools*. Engeström presents the *activity system* as an indivisible 'whole', and the unit of analysis is the same as the *activity system*.

The *subjects* engage in this object-oriented *activity* (with the reason) to obtain an outcome. The state of the *object* in Figure 4.3 is subject to change, and when the change happens, then the result is an outcome leading to the realization of the reasons for the *activity*. The *rules*, the *community*, and the *division of labour* represent the social or collective elements of the *activity*, which interact between them and, along with the *mediating artefacts/tools*, mediate the *activity*. The activities of the students are driven by motives which are then directed towards an *object*. Figure 4.3 shows multiple forms of mediation. For instance, the top triangle (subject-tool-object) is a mediational triangle; the lower right triangle (division of labour-community-object), the *division of labour* mediates the object-oriented actions of the *community*; the lower left triangle (subject-rules-community), the social *rules* (norms and conventions) are mediational means. The rules regulate an activity by setting standards for human actions. In mathematics education, some researchers distinguish between *implicit* and *explicit rules* (Núñez, 2009). *Implicit rules* are associated with Yackel and Cobb's (1996) concept of 'sociomathematical norms'. These rules can be the gesture of raising hands in the classroom (Núñez, 2009) and students setting boundaries in group work (Hernandez-Martinez &

Harth, 2015). *Explicit rules* are associated with conditions set by authorities (e.g., school/institution). These rules could be assessments, formant of examination, and time constraints. The *community* comprises the subjects and other individuals (who may interact directly or indirectly with the subjects) who are brought together by a shared *object*. *Division of labour* shows how participants in an *activity* divide the task to reach the *object* of the *activity*.

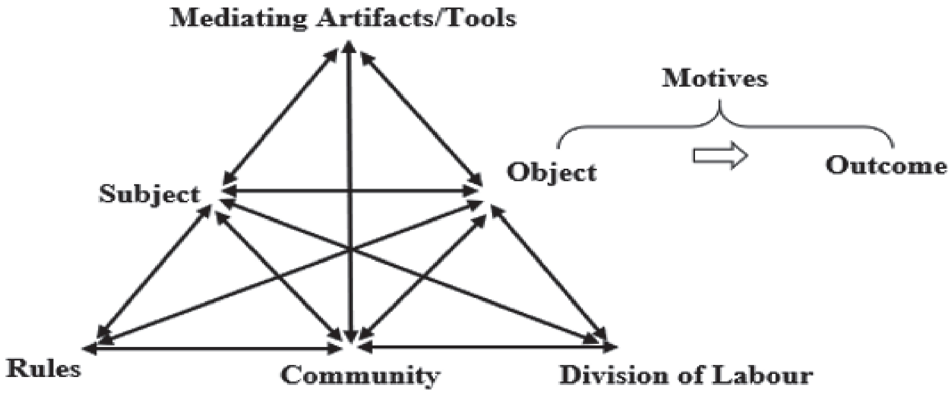


Figure 4.3: Engeström's expanded mediational triangle (Engeström, 2001).

I will demonstrate how I used CHAT theoretical perspective in this research study.

4.2 Adopting CHAT perspective stance

This thesis focuses on interactions; as such, adopting a socio-cultural approach (in this case, CHAT) that centres on actors and actions is relevant. The advantage of adopting CHAT perspective is how it can provide a particular conceptual overview of an activity. Activity (hereafter referred to as students' activity) in this research study is a group of secondary school students solving two mathematical modelling tasks with the aid of digital technologies. CHAT helps in making sense of human practice in real settings (Roth et al., 2009) and acknowledges the cultural origins of human learning and cognition (Roth, 2020). In clarifying the cultural-historical stance on human activity and cognition, Roth (2012, p. 102) points out that CHAT "does not require us to make hypotheses about the contents of peoples' minds but asks us to study societal relations that are the origin of anything that might be attributable to the individuals and their minds". Given this, I do not only look at how the students solve mathematical modelling tasks on a cognitive level but rather consider the whole context surrounding the students' activities. Thus, considering the contextual factors helps to understand the interactions and actions that emerge in the students' activities.

To address the first research question and its sub questions (RQ1, RQ1a, RQ1b, RQ1c and RQ1d, see Section 1.4 on page 7), Engeström's version of CHAT captures the whole students' activity in the form of multiple mediations. Table 4.1 below presents the possible description of the elements in Figure 4.3 in this research study. The descriptions are adapted from Kain and Wardle (2014) and Hashim and Jones (2007). The elements in Table 4.1 are not in isolation but interact with each other as a whole. Rules in Table 4.1 help address RQ1c, while the division of labour helps address RQ1d. The mediating artefact/tool in Table 4.1 helps address RQ1a and RQ1b. One missing point in Table 4.1 is classifying group work/interaction as a mediating artefact/tool. Sahlberg and Berry (2003) argue that some tasks demand multiple abilities from group members. In this case, a single student within the group cannot sit down and solve the problem by working alone. Thus, group work/interaction can be a tool in solving the task (see Vygotsky's idea of ZPD, the last paragraph on page 91).

To summarize this section, I would say that although Engeström's version of CHAT captures the whole activity in the form of multiple mediations, introducing the concept of modelling actions might help better understand the subject-tool-object interaction in Figure 4.3. The operationalization of CHAT and modelling actions helps address the second research question (RQ2a & RQ2b, see Section 1.4). I will address this in Sub-Section 4.4.2. Again, zooming in on the student—tool interaction, an Affordance Theory might help to explore this interaction. This will help address the third research question (RQ3). Furthermore, I will address this in Sub-Section 4.4.1.

Elements in the Activity System	Students' Activity System
Subject	Secondary school students (both lower and upper secondary). These students share a common problem space or object. The students are the focus of the study.
Mediating artefacts/tools	GeoGebra, Excel/spreadsheet, calculator, Google Maps, Google Search, group work/interactions, and others. These are physical objects and systems that the subjects use to accomplish the activity.
Object	Solving mathematical modelling tasks. The immediate goal of the activity or intended activity.
Outcome	Technology-based model/solution. Long-term goal of the activity.
Motive	To develop a technology-based solution/model.

	Purposes or reasons for the activity.
Rules	Norwegian mathematics curriculum, school regulations, norms/ expectations, expectations of peers, time constraints, and others. These are sets of conditions that help determine how and why individuals may act and result from social conditioning.
Community	Students (teachers, researchers, and school leaders may interact directly or indirectly with the students). The people and groups whose knowledge, interest, stakes, and goals shape the activity.
Division of labour	Students' roles in group work and their relation to other participants in the community (see Section 2.4). This provides for the distribution of actions and operations among the community.

Table 4.1: Elements of the Activity System in this research study.

Before addressing the compatibility issues of the adopted theories and concepts, I will discuss Affordance Theory in the forthcoming section.

4.3 Affordance Theory (AT)

The concept of *affordances*, originated by James Gibson (1977), denotes action possibilities provided to the actor by the environment. That is, Gibson (1977) defines *affordance* as what the environment offers the organism what it provides/furnishes, either for good or ill. *Affordance*, in this case, refers to the relationship between an object's physical properties and a user's characteristics, enabling specific interactions between the user and the object (Hadjerrouit, 2017). Object here differs from those described in Sections 4.1 and 4.2. Gibson (2014) argues that affordances emerge in perception from the relation between the organism(s) and the environment. From the definitions above, we can say that an *affordance* is not a property of an object but rather defined in the relation between the user and the object. For instance, a vehicle door allows (affords) opening if an individual can reach the handle. However, for a toddler, the vehicle door does not allow opening if he/she cannot reach the handle. An example with digital technology, Geogebra affords the conversion of numeric values to a function/graph if an individual can select the tabular information in the 'Spreadsheet View' and select 'list of points', which converts the tabular information into a graphical representation. The concept of *affordance* was introduced to the human-computer interaction community by Norman (1988) to describe the perceived and actual properties of the user interface of a tool to determine just how it could be used.

It is essential to highlight that affordance does not happen in isolation; it goes along with constraints. That is, both affordances and constraints have a dialectic relation between them. Hadjerrouit (2020) argues that affordances are not without constraints, and when one thing is afforded, something else is simultaneously constrained. Thus, affordances and constraints are not separable because constraints are complementary and not the opposite of affordances (ibid., p. 367). Brown et al. (2004) point out that affordances describe the potential for action, whilst constraints describe the structure for action. For instance, the graphical calculator affords mathematical representation, and one is constrained in terms of representational control. Thus, one can directly alter only a function's algebraic or numerical representations, not the graphical representation (ibid.). In an educational setting, Kennewell (2001) emphasizes that teachers can orchestrate the affordances and constraints of the settings of digital technologies to (deliberately) constrain novice learners. This act allows intended learning to occur, resulting from the gap between the intended ability (needed to achieve the task outcome) and the existing ability of the learner. For instance, a teacher can take the bisector angle command out of GeoGebra so that the students have to construct the bisector themselves, which might ultimately foster mathematical thinking.

If there are such things as affordances and constraints, then the question is 'How do they exist?'. I will address this issue in the forthcoming subsections, but I will discuss the notion of emergence first.

4.3.1 The notion of emergence

Van Lier (2004) argues that *emergence* could happen when simple organisms or elements reorganize themselves into more complex systems, and these systems could adapt to changing conditions, whereas the simpler forms do not have such adaptive abilities. That is, *emergence* occur when the parts of the organism(s) interact. In this case, the organism(s) attains new properties its parts do not have on their own (ibid). In the mathematics education context, for instance, students do not only understand the concept of functions when the teacher only talks about it, but they do so by participating in certain practices like proving, generating conjectures, working with examples, group discussions, and others. In this way, their interaction with the social environment brings about the gradual emergence of understanding the concept of functions. From the ontological basis, Font et al. (2013) argue that "mathematical objects emerge from the practices performed by people within particular contexts, communities, cultures or institutions". Thus, it through mathematical practice (activity) "in which individuals gain their experience and in which mathematical objects emerge" (ibid., p. 104). Based on the descriptions above, I define *emergence* in this study as an occurrence resulting from students' interactions with their social environment.

I will present the emergence of affordances and constraints in the forthcoming subsection.

4.3.2 Emergence of affordances and constraints

From the argument in Sub-Section 4.3.1, I would say that the emergence of affordances is the action possibilities that the organism/perceiver can do with the object/tool within an activity. In the context of mathematics education, affordances emerge during individual(s) interactions with a mathematical object during an activity. That is, the relation between the characteristics of an individual and specific properties of the mathematical object fosters the emergence of affordances. Radford (2008b, p. 221) suggests that "mathematical objects are historically generated during the course of the mathematical activity of individuals". From these arguments, I argue that affordance is a phenomenon that emerges within an activity and is not seen as a general manifestation. In the case of mathematical modelling activities with digital technologies, affordances emerge in the students' interactions with digital technologies. Moreover, one cannot know which affordances will emerge in a particular situation/context in advance. However, we can draw on past experiences to anticipate the emergence of potential affordances (which is not always the case). For instance, although GeoGebra affords data generation, students might end up using an Excel/spreadsheet to generate their data based on the nature of the task and the student's characteristics, among others. It could also be the results of the students' modelling process (e.g., differences in the ideal modelling route and students' actual modelling route, Blum & Boromeo Ferri, 2009). In other instances, a student might know that the tool can afford him/her while working on a task, but they cannot achieve that (since the student cannot use/reach what he/she perceived).

The last example above leads to a discussion of the perception and actualization of affordances and constraints in the forthcoming subsection.

4.3.3 Perception and actualization of affordances and constraints

Osiurak et al. (2017) argue that affordances can be objective and subjective. That is, affordances are objective since they exist independently of the act of perception but also subjective since the frame of reference is the individual's action capabilities (ibid). Affordances are not the properties of the object; however, the properties of the object are necessary conditions for affordances. Markus and Silver (2008) argue that affordances should be perceived by the individual(s) before they can be acted on or actualized. For instance, the affordances that emerge when a group of students solve mathematical modelling tasks with GeoGebra can be put into two categories: the students being aware of the existence of the action potential of GeoGebra (perceived affordances), and when the students can turn the potential of GeoGebra into action (actualized affordances). In this case, affordances could be potentials for actions that might or might not occur, depending on the goal of the individuals (goal-oriented actors, Strong et al., 2014).

Affordance actualization is the action potential of the digital technologies turned into actual actions (Anderson & Robey, 2017), bringing an effect, an outcome attributed to the actualization of an affordance (Bernhard et al., 2013). Strong et al. (2014, p. 70) define affordance actualization as "the actions taken by actors as they take advantage of one or more affordances through their use of the technology to achieve immediate concrete outcomes in support of organizational goals". Individuals interacting with digital technologies are goal-oriented actors, and affordance actualization might vary from one individual to another. Volkoff and Strong (2017) argue that "actualization relates to a particular individual actor and details regarding the specific actions that actor will take or has taken" (ibid., p. 137). For instance, different affordance actualizations emerge when two different groups of students solve mathematical modelling tasks using digital technologies, and this depends on the students' characteristics and goals.

There are several arguments that affordances are perceived before being actualized (Anderson & Robey, 2017; Bernhard et al., 2013) and actualized without perceiving it (Strong et al., 2014; Volkoff & Strong, 2013; Wang et al., 2018). This is somewhat complicated as Anderson and Robey (2017, p. 102) argue that "affordance actualization observed in practice is not fully explainable through perception and goals alone because there are times when a user is both aware of an affordance and has a current goal that the affordance could support, and yet the affordance is not actualized". This could result from the characteristics of the students, the features of the digital technologies, and the characteristics of the classroom environment (Strong et al., 2014). For example, Hadjerrouit and Nnagbo (2021) subscribed to the view that perceived and actualized affordances are two distinct processes as they explore the formative feedback of digital technology for teaching and learning mathematics. In this study, I acknowledge that both perceived and actualized affordances are two distinct processes, and nonetheless, in most instances, affordances are perceived before being actualized.

From the arguments above, I would say that the codes (see Table 9.7 in Appendix E.3) generated for the analysis regarding the third research question (RQ3, see Section 1.4) in this research study are perceived affordances resulting from pilot studies and existing literature. It is without doubt that not all the perceived affordances are actualized. I will now present affordances and constraints in mathematics education.

4.3.4 Affordances and constraints in mathematics education

In mathematics education, several affordances and constraints emerge. Turner and Turner (2002) described a three-layer definition of affordances that emerge. These layers are *perceived*, *ergonomic* and *cultural* affordance. Chiappini (2013) applied these notions to Alnuset (a digital technology for high school algebra). According to Chiappini (2013), *perceived* affordance concerns the evaluation of the basic usability aspects of controls that mediate interaction with the tool, while *ergonomic* affordance concerns affordance for the embodied actions involved in solutions of tasks to the context where the tool is used. *Cultural* affordance concerns the cultural teaching and learning objectives underlying the tool use (ibid., p. 95). Kirschner et al. (2004) describe a three-layer definition of affordances similar to the framework by Turner and Turner (2002). These layers are *technological* affordances covering usability issues, *educational* affordances facilitating teaching and learning, and *social* affordances fostering social interactions. From these arguments, Hadjerrouit (2019) categorize affordances according to *technological*, *pedagogical* and *socio-cultural* levels in mathematics education. And these concepts are applied in this research study while acknowledging that the affordances that might emerge will depend on the student's characteristics and knowledge level, the type of tasks, the classroom environment, the teacher's characteristics, fellow students, and other factors. To be precise, *pedagogical* affordances are broad and consist of other affordances at different levels, such as the student, classroom, mathematics tasks, and mathematics subject levels (Pierce & Stacey, 2010). Thus, due to the scope of this study, I consider *mathematical* affordances (which are at the mathematics tasks level).

Individuals perceiving affordances and constraints in their interaction with digital technologies within mathematical modelling activity provides an opportunity for a broader perspective in understanding mathematical modelling with digital technologies. I will discuss *technological*, *mathematical* and *socio-cultural* affordances and constraints from this perspective. *Technological* affordances concern the *usability* features of digital technologies. *Usability* is concerned with whether a system allows for the accomplishment of a set of tasks efficiently and effectively that satisfies the user (Kirschner et al., 2004). Some digital technologies at the functional level might help to construct diagrams, perform calculations, and draw graphs and functions, among others (Takači et al., 2015). Hadjerrouit (2019) argues that *technological* affordances are a pre-requisite of a tool leading to *mathematical* affordances, in this case providing support. Pierce and Stacey (2010) highlight that several *mathematical* affordances emerge at the mathematics subject level. For instance, linking representations (moving fluidly between geometric, numeric and graphic representations), simulating real situations (using dynamic diagrams, dragging, and collecting data for analysis), exploring regularity and variation (strategically varying computations, searching for patterns, observing effect of parameters), among others (ibid., p. 7). For example, Mousoulides (2011) argues that digital technology, like GeoGebra, assists students in broadening their explorations and visualization skills concerning some mathematical concepts. Finally, curricular and other social issues are considered on the *socio-cultural* level. Hadjerrouit (2019) emphasizes that digital technology should provide opportunities to concretize the mathematics subject curriculum. Furthermore, the tool should be tied to teaching mathematics in schools, supporting mathematics learning at all levels. *Socio-cultural* affordances also foster social interactions during group activities in mathematical modelling. Kirschner et al. (2004) used the term *social affordances* to describe socio-cultural affordances, as the properties of digital technologies acting as social-contextual facilitators relevant to the students' social interactions.

The students in this research study solve two mathematical modelling tasks (see Section 5.5) in groups, and there is a need to describe the type of affordances that emerge within the students' activity. Leonardi (2013) differentiates between different affordances that could emerge: *individual*, *collective* and *shared*. *Individual* affordances are actualized by one actor acting independently of others, while *collective* affordances involve many people doing different things to accomplish a joint goal. *Shared* affordance is the same affordance being actualized by many people in similar ways: group members interact and depend on each other to accomplish a joint goal (Leonardi, 2013; Volkoff & Strong, 2017). Considering the context (see Section 5.3) of this study, I subscribe to *shared* affordances because the students share one common working space (using only one computer). In contrast, *collective* affordances might come into play when the students use their individual computers while solving the task.

In the forthcoming section, I will present the compatibility between CHAT and Affordance Theory and CHAT's interpretation of modelling actions.

4.4 Compatibility between the adopted theories

Compatibility of theories raises the question of networking theories (a complex issue that cannot be addressed sufficiently in this study). Bikner-Ahsbahs (2016) argues that networking of theories means building relations among theories and that networking allows for explicitly working with different theories to benefit from their theoretical strengths (ibid.). Radford (2008a) argues that a connection between theories can happen at the level of principles or as a combination/ coordination of these theories depending on their compatibility (e.g., CHAT and Commognition) or incompatibility (e.g., CHAT and Constructivism or Theory of Didactic Situations—TDS) of their theoretical premises. Networking CHAT and other theories can also happen at the methodologies and research questions level.

I will now present the compatibility between CHAT and Affordance Theory.

4.4.1 Compatibility between CHAT and Affordance Theory

In this subsection, I will discuss affordances from a CHAT point of view. To understand how CHAT and Affordance Theory can work together to explain the complexity of students' mathematical modelling with the aid of digital technologies, an overview of the main ideas of each theory is needed (which I have discussed in Section 4.1 and 4.3 respectively). Using CHAT and Affordance Theory in conjunction is rare (not enough work is done in this situation). Few studies are reporting on instances where an explicit connection between these two theories can be made (Albrechtsen et al., 2001; Bærentsen & Trettvik, 2002; Martinovic et al., 2013; Fredriksen, 2021). It is essential to find a connection between these theories in mathematical modelling using digital technologies, as this contributes to the theories themselves. Figure 4.4 below summarizes the connection between CHAT and Affordance Theory based on the discussions of these theories in earlier sections. Figure 4.4 presents a general overview of 'viewing the affordances of digital technologies and the activities mediated with digital technologies in mathematical modelling through the lenses of Activity and Affordance Theory'. I will further explain Figure 4.4 in the paragraphs below.

Pedersen and Bang (2016) point out that Affordance Theory needs CHAT to understand the societal nature of the individual-environment relationship. Considering the ontological basis for CHAT and affordances, Albrechtsen et al. (2001) argue that Activity Theory and Gibsonian thinking share the basic idea that perception is not conducted inwards (a general manifestation) and that it is connected with action and only through acting do people perceive their environment (ibid., p. 15). Connecting CHAT and Affordance Theory based on their ontology is quite broad, and a deep philosophical discussion on this issue cannot be addressed in sufficient detail in this study. Thus, Affordance Theory has no exact ontology (Blewett & Hugo, 2016). There are two distinct ontologies concerning CHAT: one is realism (we assume reality exists but make some interpretations of that reality), which is linked with activity systems—Engeström, and another is anti-realism associated with Radford (2008b, 2013). However, arguing from the 'emergence' (see Sub-Sections 4.3.1 and 4.3.2) perspective, we can say that both affordances and activity emerge. Affordances emerge in the relation between the individuals and the tool in an activity (through the perception of the individuals). Albrechtsen et al. (2001) argue that CHAT insists that human actions and perceptions are mediated by a variety of tools. Thus, CHAT gives a valuable means for understanding the tools and how they are shaped in a dialectical relationship with the changing practice of use (ibid., p. 15).

Kuswara et al. (2008) argue that Activity Theory and the concept of affordances concern how people interact with the world. For instance, considering a group of students working on mathematical modelling tasks with the aid of digital technologies, one can view the socially mediated aspects of group work (e.g., group activities mediated with digital technologies) through Activity Theory and how student(s) within the group utilizes the environment (affordances of digital technologies) to perform their contribution (see Figure 4.4). In this case, a change in the form of activity is reflected by a change in which affordances and constraints are utilized (ibid.). CHAT is concern with the socio-historical dimension of an individual's interaction with the environment.

Albrechtsen et al. (2001) argue that Gibsonian thinking relates to the level of operations in Leont'ev's (1977) three-level hierarchy description of an activity. The level of operation describes the things to be performed or modes of using tools. Bærentsen and Trettvik (2002) emphasize that at least some of the misunderstanding of the concept of affordances is caused by the fact that Gibson focused mainly on the perceptual side of the concept, thus leaving the organism's activity as a largely implicit precondition. That is, Gibson did not account for the internal dynamic structure of activity as suggested by Leont'ev. Furthermore, Gibson's "focus was on the perceptual requirements of the operational realization of activity, the information available in the environment that lets the organism control locomotion and simple forms of object related activities" (ibid.). Pedersen and Bang (2016) point out that there should be a theoretical need to overcome an undifferentiated notion of activity in order to bring Affordance Theory forward concerning not only the operational level of activities but human activity as such. A distinction between using digital technologies in an activity might help categorize affordances from a CHAT perspective. For instance, Bødker's (1991) distinction between the complementary aspects of the use of digital technologies.

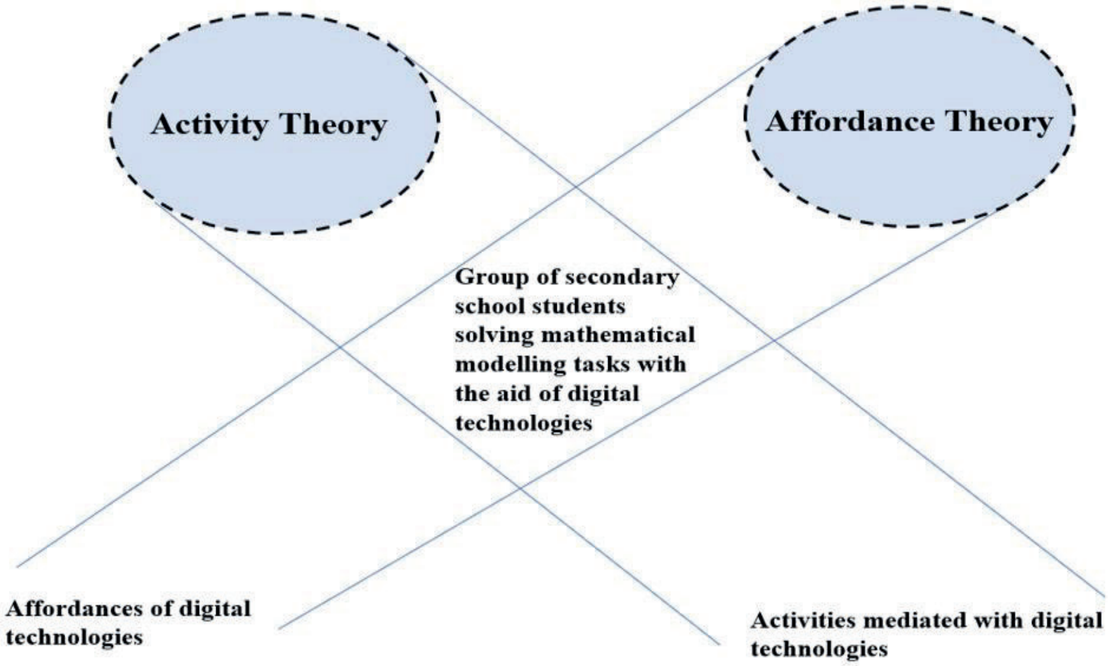


Figure 4.4: Viewing the affordances of digital technologies and the activities with digital technologies in mathematical modelling through the lenses of Activity and Affordance Theory (adapted from Martinovic et al., 2013).

I will present an argument on how I view affordances and constraints emerging in the students' activity concerning operations, actions and activity. I consider students' interaction with digital technologies conditioned by the usability features of the digital technologies to form the operational level in Leont'ev model. The students perform several (individual) actions within the activity as they develop a technology-based model/solution. For instance, students' use of actual data to create a model is seen as an action where the goal is to develop a real model (from the researcher's perspective). This action and other actions forms the second level of Leont'ev's model. Group work and interaction with peers form part of the collective level that facilitates the actions for the common motive of developing a technology-based model/solution. I consider the joint mathematical discourse or interaction taking place through collaboration within the group to be the activity level in Leont'ev's model.

How do affordances and constraints relate to Leont'ev's model? Relating affordances and constraints to Leont'ev's model, at the *operational* level, *technological* affordances and constraints relate to the usability features of digital technologies. Examples of these usability features could be the ability to draw graphs and functions, construct diagrams, solve equations, perform calculations, and more. One can perceive these usability features when engaging with digital technologies, and eventually actualize them if one can use/reach what the digital technologies afford. At the *action* level, students might perceive *mathematical* affordances and constraints as they solve mathematical modelling tasks. Thus, they perceive mathematical affordances and constraints of connecting mathematical representations, exploring regularity and variations of the graph, simulating and visualizing mathematical concepts, and others. In the *activity* level, the final level, *socio-cultural* affordances and constraints relate to the norms controlling the students' activity as they interact with themselves (student-student) and digital technologies (e.g., student-GeoGebra) in solving the mathematical modelling tasks. This interaction might induce affordances and constraints at a collective level. For instance, digital technologies might stimulate student cooperation as they develop a technology-based model/solution.

The socio-cultural affordances and constraints emerging from the students' activities at the activity level could be a common focus (shared knowledge and creating a shared goal), observing and repairing divergencies (Roschelle & Teasley, 1995; Granberg & Olsson, 2015), authority of the digital technology (personalizing of problems in group situations) (Lowrie, 2011), among others (these categories are coded under "socio-cultural affordances and constraints" in Table 9.7, in Appendix E.3). To explain further, in *creating a shared goal*, the students have the facility (digital technology) to look at the same thing as they negotiate and agree on the appearance of the mathematical representation generated by the digital technology. They also might use digital technology as a *reference tool* to visually demonstrate their individual ideas to one another. For instance, a student might suggest a function/equation to their peers and use GeoGebra to represent this function graphically. To *observe and repair divergences*, digital technology is used to maintain shared knowledge and ideas through the verification of ideas or settling disagreements by performing tests, and referencing, among others. The *authority of digital technology* describes situations where students only accept an answer from the digital tool as the correct answer. *Personalizing problems* is based on an individual's interest, which could be the problem-solving strategies (Yerushalmy, 2000) adopted or the choice of mathematical representation and representational types offered by digital technology. In an example in mathematics education, Fredriksen (2021) combines Leont'ev's model and Affordance Theory in investigating the affordances of a flipped mathematics classroom, and the results highlight affordances for mathematics learning at the three levels of Leont'ev's version of CHAT.

In summary, from Affordance Theory, affordances and constraints exist (emerge in an activity) through the perception of the individual/s. However, the theory does not account for the environment (social aspects) in which affordances and constraints emerge. CHAT provides the social aspect of the activity in which affordances and constraints emerge (if we connect the two theories, as shown in Figure 4.4). In the forthcoming subsection, I will present the interpretation of modelling actions from a CHAT perspective.

4.4.2 Modelling actions in CHAT perspective

Following the discussion in Sub-Section 2.1.3, where I presented the ontology of modelling competence, I will discuss the interpretation of modelling actions from a CHAT perspective in this section.

The focus of this research study expands from considering the cognitive levels in the students' modelling processes (modelling cycles and competencies) to embracing the whole context in which students' actions in the form of modelling actions emerge. Thus, individual actions are senseless and unjustified if we do not consider collective activity (Leont'ev, 1981b). Again, the student's activities are oriented towards an object with a specific motive (Leont'ev, 1978). Williams and Goos (2012) point out that the motive/object of the activity ensures that the activity is meaningful and integrates both emotional and cognitive aspects. Given this, putting the processes from the substantive mathematics on one side and the contexts of practical activity in which they make 'sense' on the other might leave the metacognitive aspect high-and-dry. Thus, it elevates metacognition but detaches it from the context and the affective, which is the motive and emotions (ibid.). Considering the collective activity, in this study, the actions identified in the students' activity of solving mathematical modelling tasks include:

- breaking the tasks into manageable parts,
- searching for a model,
- finding a solution for the model,
- explaining the results in real terms and
- checking the results for adequacy.

The respective identified goals for which these actions are directed to are to understand the problem text or real situation, to set up a mathematical model, to solve the mathematical questions within the model, to interpret the mathematical results in real situation, to validate the solution, among others (presented as modelling competences, see Table 2.1). These actions and goals characterizes the students' modelling processes in a mathematical modelling activity. I consider the categories of actions and goals (presented above) as just names used for the purpose of reporting or describing the students' modelling processes, other than anything intrinsic in the ontology of modelling competence (see Sub-Section 2.1.3). The goals are viewed through the observations of the students' actions emerging. The students' actions in this study are not a cyclic activity (as described in Figure 2.1 on page 13) but rather depends on the characteristics of the students, the nature of the task and the digital technologies used. In this case, the result of an action might lead to the next action. For instance, if the students perform the action of breaking the task into manageable parts and identify the model in this process, they might perform another action of finding a solution for that model (skipping the action, searching for a model).

I will now present how modelling actions are analyzed in this research study. Modelling actions are manifested in the students' activities of solving mathematical modelling tasks with the aid of digital technologies, and the environment in which the students solve these tasks plays a role in the modelling actions that emerge. One of the sub-goals in this study is to investigate the students' working processes in the students' activity (see Section 1.3), which corresponds to the second research questions (RQ2a and RQ2b, see Section 1.4). That is, the study tends to investigate the modelling actions emerging in light of the circumstances that form the setting of the students' activity. To achieve this purpose, modelling action must be considered as a process instead of a product. That is, modelling actions are not a general manifestation (definite) as different groups of students have different modelling actions emerging. As an analytical tool, CHAT helps understand how modelling actions are conceptualized as a process. When students engage in a mathematical modelling task during a modelling activity, they employ historically accumulated knowledge that they have appropriated, which we can view through the lens of modelling process (that is, the cognitive activities in modelling cycles and competencies—Table 2.1 on page 26). Niss and Blum (2020) explain that a modelling cycle should be understood as an analytic reconstruction of the steps of modelling necessarily present, explicitly or implicitly, as an instrument for capturing and understanding the principal processes of mathematical modelling (ibid., p. 14). In this case, the modelling cycle is a tool for analyzing (among others) some essential aspects of modelling. However, zooming out and observing modelling actions emerging in a modelling activity might yield analytical results that could be social (in nature). Thus, the modelling actions emerging through the lens of modelling process are viewed, and attention is paid to the context in which these actions emerge.

Solving a mathematical modelling task using digital technologies can be seen as an activity (*modelling activity*) in the perspective of CHAT, and to solve the task comprises a combination of action(s) (*modelling actions*) and operations (*modelling operations*). In summary, *modelling activities* driven by motives are performed through specific *modelling actions* directed towards goals, which are implemented through certain *modelling operations* in succession (see Figure 4.5). Figure 4.5 presents a model for mathematical modelling activity categorized in the three-level hierarchy of activity. I will further explain this figure in Table 4.2. Students go through different processes in developing a technology-based model/solution for a modelling task. For instance, a group of students solving a modelling task perform an action of 'breaking the task into manageable parts' with the goal 'to understand the problem text or the real situation'. In this process, individual actions are directed towards a common goal, and in the end, all these actions form a collective action. The students' actions are observed through their utterances, interactions with the computer, and writings with paper-and-pencil.

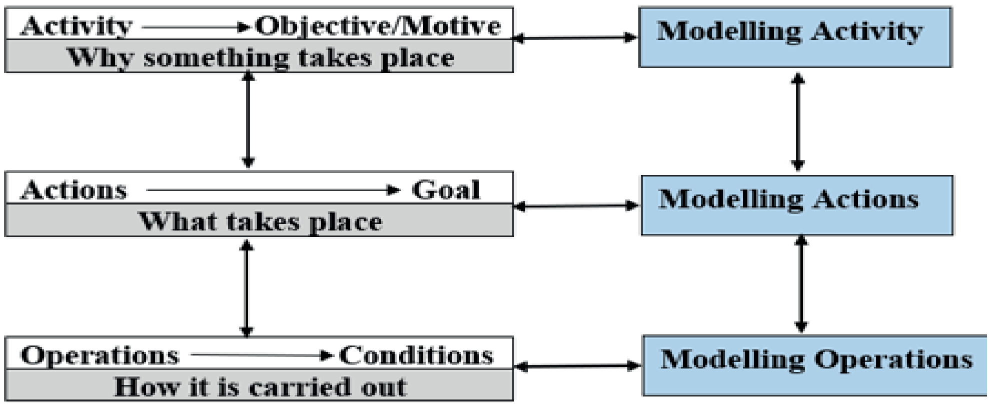


Figure 4.5: A model for modelling activity, actions and operations in CHAT perspective

I will relate the goals directing the students' actions of solving the mathematical modelling task to the categories of modelling competencies described in Table 2.1 (to report or describe the students' modelling processes). Analyzing the students' actions in this case enables us to make sense of the emerging modelling actions. This analysis also facilitates making sense of the factors of the environment contributing to the students' actions (if any). Carrying out the students' actions (*modelling actions*) of solving the mathematical modelling tasks involves some execution of operations (*modelling operations*). This execution of operations is considered parallel to Leont'ev's operation-condition layer in Figure 4.2. For instance, to breaking the task into manageable parts, the students perform several operations, for example, searching for available information, recognizing relations between variables, and constructing relations between the variables, among others. This operational level enables us to analyze the role tools (mainly, digital technologies in our case) play in the execution of these operations. I relate the kind of operations the students execute in achieving the goals (categories of modelling competence) to the categories of sub-competencies of modelling in Table 2.1 (thus, viewing the emerging operations through the lens of sub-competencies). For the reader, I repeat that the categories (sub-competencies) are just names used to report or describe the students' modelling processes, other than anything intrinsic to the modelling competence's ontology. Table 4.2 below further explains the model in Figure 4.5. Thus, it presents a model for modelling activity, actions and operations from a CHAT perspective in mathematics education.

The actions and operations in CHAT imply that the students' actions of solving the task and the operations executed are connected. For instance (see Table 4.2), if we consider the students' action of 'breaking the tasks into manageable parts', the students perform some operations like 'looking for available information and differentiating between relevant and irrelevant information'. In looking for available information, the students might use Google Search or Google Maps (an example of digital technology), and the information they get influences how they differentiate between relevant and irrelevant information. The information the students gather affects the forthcoming action, for example, 'searching for a model'. Furthermore, new operations are formed as the students search for a model. These processes are not straightforward, and they could be back and forth; for instance, the operations the students perform in 'searching for a model' might not lead to the next action of 'finding a solution for the model' but instead going back again to 'breaking the tasks into manageable parts' to understand the mathematical modelling task further. This could be a result of a failed operation. In summary, the modelling actions and operations listed in Table 4.2 are not always the case, as the emerging actions are specific to the nature of the task, the characteristics of the students, and the digital technology used, among others.

Modelling Activity	Group of secondary school students solving mathematical modelling tasks by modelling with digital tools.				
Object/ Motive	Solve a mathematical modelling task/Develop a technology-based model or solution.				
Modelling Actions	Breaking the task into manageable parts.	Searching for a model.	Finding a solution for the model.	Explaining the results in real terms.	Checking the results for adequacy.
Goals	Understand the problem text or real situation.	Set up a mathematical model.	Solve the mathematica l questions within the model.	Interpret the mathematic al results in real situation.	Validate the solution.
Modelling Operations	Making assumptions for the problem and simplifying the situation. Identifying relevant questions in the given real-world situation. Looking for available information and differentiating between relevant and irrelevant information. Recognizing quantities that influence the situation. Constructing relations between the variables.	Translating the real problem into a mathematical problem (such as, equation, function, diagram, figure, table, term, etc.) and using technological tools to model the problem. Mathematizing relevant quantities and their relations. Simplifying relevant quantities and their relations. Choosing appropriate mathematical notations and representing the situations	Using mathematical knowledge to solve the problem. Observing the effect of parameters on the graph. Manipulating mathematical figures and shapes dynamically to see what happens.	Using appropriate mathematical language to communicate about the solutions. Identifying the correct meaning of aspects of the mathematical results/model. Generalizing the solution to suit a different context.	Critically checking and reflecting on found solutions. Reflecting on other ways of solving the problem. Going through the working process if the solution does not fit the situation.

Table 4.2: A model for modelling activity, actions, and operations in CHAT perspective.

4.5 Summary of the chapter

This chapter has provided a general overview of Cultural-Historical Activity Theory (CHAT) and Affordance Theory from the perspective of students' mathematical modelling using digital technologies. Activity in CHAT is an analytic unit for understanding human performances such as their practices, the sense they make, or their actions (Roth, 2012). Students' mathematical modelling activities have been defined in the sense of CHAT. That is, a cultural activity or historically developing sociocultural activity comprising several components and aspects, or in other words, as an object-oriented, collective, and culturally mediated human activity. From Engeström's version (third generation) of CHAT, I consider a group of secondary school students as an activity system with several interactions between the components subject, objects, community, rules, division of labour, and mediating artefacts/tools. These components forming an activity system is an indivisible whole.

A CHAT interpretation of modelling actions has been presented in this chapter. Leont'ev (second generation of CHAT) presented a theoretical model that analyzes human activity in three-level hierarchy (activity, actions, and operations) and that all human activities are oriented towards an object with a specific motive. Through these hierarchy, I viewed students' mathematical modelling activities as modelling activities, modelling actions and modelling operations. Modelling actions are seen as emerging in the students' activities and are considered a process instead of a product. Again, modelling actions and operations emerging are specific to the nature of the task, characteristics of the students, digital technologies used, and others.

Affordances and constraints regarding technological, mathematical and socio-cultural (in mathematics education) have been presented in this chapter. Affordances and constraints emerge within an activity, and both perceived and actualized affordances are two distinct processes, but in most instances, affordances are perceived before being actualized. Another aspect of the students' mathematical modelling activities is their relationship with the digital technology they use, which we can view through Affordance Theory. With a combination of CHAT (Leont'ev's version) and Affordance Theory as a lens, we can view the affordances of digital technologies and the activities mediated with digital technologies in mathematical modelling. Thus, CHAT and Gibson's thoughts about affordances share the basic idea (from their ontological basis) that perception is not conducted inwards and is connected with action (Albrechtsen et al., 2001). From an emergence perspective we can also say that affordance and constraints and the activity itself emerge.

The next chapter (Chapter 5) presents this study's epistemological, ontological and methodological issues. Thus, the methods used to gather and analyze data and the theoretical underpinnings for these methods. In mathematics education research, the theoretical framework, methodology and research questions are closely related. Given this, Radford (2008a) explains that a well-connected theoretical framework and its methodology help distinguish between relevant and irrelevant data. Furthermore, to tackle a research question, the question needs to be framed in the form that the theory can deal with (ibid.).

5 Methodology

This chapter presents an outline of the fundamental epistemological, ontological and methodological issues relating to the research design of this study. This research aims to study how secondary school students solve mathematical modelling tasks with the aid of digital technologies, and three research questions (see Section 1.4) guide the research. This research is conducted within a qualitative paradigm based on an activity theorist epistemology and ontology.

The chapter opens with a presentation of the research paradigm in Section 5.1, followed by an elaboration of the adopted research design and strategy in Section 5.2. Section 5.3 presents the context of the study and sheds light on my role as a researcher during the data collection process. Section 5.4 presents the digital technologies the students used in solving the mathematical modelling tasks. Section 5.5 presents the mathematical modelling tasks the participants worked on. Section 5.6 further elaborates on methods for data collection, while Section 5.7 addresses the strategies concerning data analysis and management. Section 5.8 presents the data analysis before I reflect on the validity and trustworthiness of the research design in Section 5.9 and ethical considerations in Section 5.10. The chapter ends with a summary in Section 5.11.

5.1 Research paradigm

There are several definitions and different usages of 'paradigm' as there seems to be no agreement among philosophers, educators, and scientists, among others, on a precise definition of paradigm. However, in this research, I subscribe to the definition by Thomas S. Kuhn, which has been used across different fields of study. Kuhn (2012) in the 1960s described a paradigm as "the entire constellation of beliefs, values, techniques, and so on shared by the members of a given community". In this case, the research paradigm constitutes established models in a research community in which researchers subscribe to their beliefs in making sense of the phenomena under study. According to Guba and Lincoln (1994), research paradigms can be characterized by the way scientists respond to three basic questions:

1. Ontological question: What is the form and nature of reality, and, therefore, what is there that can be known about it?
2. Epistemological question: What is the nature of the relationship between the knower or would-be knower, and what can be known?

3. Methodological question: How can the inquirer (would-be knower) go about finding out whatever he/she believes can be known? (ibid., p. 108).

In the forthcoming sub-subsections, I will address the questions above in light of this research study. CHAT is the overarching framework for this study, and I subscribe to an activity theorist epistemology and ontology. According to Vygotsky (1978), knowledge is socially constructed in historical and cultural contexts; thus, knowledge primarily originates from the social sphere. Furthermore, the choice of research paradigm must be consistent with this presumption. This study explores how secondary school students solve mathematical modelling tasks using digital technologies. As such, I consider reality to be somehow embedded in the cultural expression that these students are part of, and it is not appropriate to choose a research paradigm that considers humans as mere predictable units (Cohen et al., 2017). For instance, we cannot view affordances and constraints of digital technologies in students' activities as cause and effect but rather see it as emerging within the activity depending on the characteristics of the students and the nature of the task, among others. Again, I acknowledge that the teacher and students form their own culture with their unique expressions and attributes, although there is an external shaping of this culture. Thus, if we consider that the participants/humans interact with a particular social world (which has its own norms and cultural expressions), then the notion of truth (which is problematic in itself) and validity needs to be considered in a much broader sense. In this view, Enerstvedt (1989) considers truth as a negotiation between culturally informed humans or constructed in and by human activity.

This study explores specific human behaviour, such as the forms of interactions (e.g., student—student and student—tools, among others) within group activities of secondary school students. The study focuses on interactions in students' activities, which requires interpreting events, and an endeavour that is more closely aligned with a *qualitative* methodology is essential. As the research paradigm needs to reflect the nature of what is being studied, I find it appropriate to choose a *qualitative* research paradigm, where video recordings of group activities, screen capture software, and field notes are central data sources. These data are elaborated through systematic coding and descriptive analyses (Bryman, 2016). Qualitative research involves an interpretative and naturalistic approach to the world, that is, an inquiry process conducted in a natural setting to understand a social problem by reporting detailed views of informants (Denzin & Lincoln, 2005;

Creswell, 2007). Qualitative research calls for a subjective dimension in processes involving the interpretation of empirical data (Cohen et al., 2017). Subjectivity here is linked to both the research object and the researcher. Patton (2002) argues that traditions of the qualitative paradigm are broad and encompass other more specifically defined paradigms. One such paradigm is the *naturalistic* paradigm, and from the theoretical stance perspective, Moschkovich and Brenner (2000) highlight that this paradigm can be defined with the assumption that meaning is socially constructed and negotiated in practice. In this view, research endeavors in naturalistic inquiry focus on how individuals behave in natural settings while engaging in life experiences (and reality is subjective). Now, viewing meaning as socially constructed, Given (2008, p. 548) explains this concept by emphasizing "human interaction, and the context in which those interactions occur, as the basis for how one comes to know or understand phenomena". Thus, to understand or explore how secondary school students solve mathematical modelling tasks using digital technologies, we cannot undermine the characteristics of the students, the nature of the task, the kind of digital technologies used, and group work, among others. This study is conducted in the participants' natural environment (the students' classroom during lesson hours). Although it is a natural environment, the presence of the researcher and the task given to the students are outside this natural environment. Selecting students for this study should be purposive in ensuring that these students have direct experience with the issues or the topic under study.

Another paradigm to consider is the *interpretive* paradigm. Kivunja and Kuyini (2017, p. 33) explain that the interpretive approach tries "to understand and interpret what the subject is thinking or the meaning s/he is making of the context". Given this, I try to understand the viewpoints of the participants being observed rather than the observer's viewpoint. As such, I interpret the empirical data from the participants' perspectives and my impressions, interpretations, and meanings that are inferred from the empirical data (due to the subjective nature of qualitative research). Again, *ethnographic* paradigm is another broad term within the qualitative paradigm. Cohen et al. (2017, p. 292) argue that "an ethnography is a descriptive, analytical, and explanatory study of the culture (and its components), values, beliefs, and practices of one or more groups". Bryman (2016) argues that ethnographic researchers immerse themselves in the group/society they study. In this study, I participated in the culture of the students and the teacher as a researcher. I played the role of *observer-as-participant* (defined in Sub-Section 5.3.1) in an ethnographic sense. Thus, I observed the teacher and students in their natural settings and gathered data through videotape, screen-capture software and field notes. Immersing myself in their culture, which I hold, gives me ample grounds to provide an authentic interpretive perspective towards the study. Cohen et al. (2017) emphasize that understanding participants' interactions with the world around them shouldn't come from the outside but rather from the inside. In this case, the individuals' behaviour can only be understood by the researcher sharing their frame of reference. Subscribing to the ethnographic perspective has specific methodological implications (Patton, 2002). Thus, finding methods to explore the norms, understanding, and assumptions of participants in the culture being studied.

To be more specific on the methodological position, I consider ontological, epistemological, and unit of analysis issues in light of the research paradigm.

5.1.1 Ontology

Ontology is the study of being and its assumptions concerned with what constitutes reality (Scotland, 2012). Mertens (2020, p. 10) phrases the ontological question as "what is the nature of reality?" The deep philosophical roots of this question cannot be addressed in sufficient detail in this thesis. Given CHAT, the nature of reality is socially constructed. Engeström (2001) explains in the second principle (multi-voicedness of activity system) of CHAT that "an activity system is always a community of multiple points of view, traditions and interests" (ibid., p. 136). From the naturalistic perspective, Moschkovich and Brenner (2000) view reality as multiple constructed realities, and to understand learners on their terms; it is essential to consider multiple points of view of their activities. This study involves humans acting within a social environment and investigates how secondary school students solve mathematical modelling tasks using digital technologies. Given CHAT, I consider reality as social constructions comprising actions, operations, and contextual conditions, among others, of the students' activities. Hence, the phenomena under study can only develop within an activity, not a general manifestation (definite). For instance, modelling actions and affordances emerge within an activity depending on the characteristics of the students, the nature of the task, and the digital technology used, among others. Furthermore, my role as a researcher is to interpret the variety of impressions from the students' activities into a specific body of knowledge, as argued by Cohen et al. (2017).

It might be possible to interpret these ontological assumptions' consequences differently. However, I am convinced that the argument above shows that the CHAT's ontological assumptions apply to this study without causing any ontological contradictions.

5.1.2 Epistemology

Schwandt (1997) defines *epistemology* as the study of the nature of knowledge and justification. Mertens (2020, p. 10) phrases the epistemological question as "What is the nature of knowledge and the relationship between the knower and the would-be known?" From the naturalistic perspective, Lincoln and Guba (1985) argue that the knower and known are interactive and, to some extent, inseparable. In this case, we cannot describe the students solving the tasks, the context in which they solve them, and the researcher separately. Instead, we consider them as mutually interacting. Lincoln and Guba (1985) explain this mutual dependency in that human beings are always in relationships—with one another and the investigator. One cannot study people without considering these relationships.

In the historical and cultural context, acceptable knowledge is what we interpret about the real world in a social context (Vygotsky, 1978). Considering the question phrased by Mertens, there is a link between ontological and epistemological issues pointing to the fact that the nature of knowledge depends on what one knows. Corbin and Strauss (2015) argue that the nature of knowledge is not definite outside the participants in the first assumption of their ontological position. Thus, the external and the internal worlds are created and recreated through interaction (ibid.). Following the arguments above, I will emphasize that the nature of knowledge under study in this research only develops within the students' activities. The students' activities in this study entail communications and mediation through the use of digital technologies in solving mathematical modelling tasks, and the students' experiences evolve through interactions, negotiations and shared perspectives. The evolving experiences are seen as knowledge, which is constructed socially, resulting from the personal experiences of the researcher's engagement with the students in their natural settings.

5.1.3 Unit of analysis

The methodological question has been phrased by Mertens (2020, p. 10) as "How can the knower go about obtaining the desired knowledge and understanding?" An appropriate unit of analysis (UoA) should be defined to address this question since this influences the sample size and sampling strategies (Patton, 2002). The UoA determines the kind of data to be collected. As Patton (2002, p. 228) points out, "the primary focus of data collection will be on what is happening to individuals in a setting and how individuals are affected by the setting". In this study, these individuals are secondary school students in a specific environment or setting. Säljö (2009, p. 206) emphasizes that the UoA is "the choice of a conceptualization of a phenomenon that corresponds to a theoretical perspective or framework". Wertsch (1998) argues that in the socio-cultural perspective, mediated action or individual-acting-with-mediational-means is the UoA. Now, the environment or setting that the students belong to in this study involves solving mathematical modelling tasks in a group using digital technologies. Within this environment, several mediated actions occur, for instance, communications between the students, students' accomplishments of the modelling tasks, digital technologies usage, and others. In CHAT, the UoA is the activity system (Engeström, 2001), and putting the individuals together with their environment or setting forms an activity system. Given this, the UoA in this study is called the *students' activity system* (which is described below):

A Group of secondary school students solving two mathematical modelling tasks with the aid of digital technologies.

The description above is the UoA for the entire research study. However, there are some shifts in the analysis as I zoom in on more specific constituents due to the complex nature of the study. That is, looking at the students' activity system as analytical components (Engeström version of CHAT) and the three-level hierarchy of activity (Leont'ev version of CHAT). Firstly, the students' activity system is broken into analytical components of subject, mediating artefacts/tools, object, rules, community and division of labour. This is done to study how secondary school students solve mathematical modelling tasks using digital technologies. Furthermore, the components (mentioned earlier) are seen as an indivisible whole; that is, they interact with each other within the system. Secondly, I will analyze the actions and operations within the students' activities. This is done to study the emergence of modelling actions, affordances and constraints by observing the actions and operations of the students within the activities. The shifts described above are necessary as Säljö (2009) argues that shifts in UoA should be considered meaningful to study the many facets of complex issues of learning activities (in our case, students' mathematical modelling activities).

Now, with a clear definition of the UoA in this research, the research questions (see Section 1.4) will be addressed while considering the UoA in light of the theoretical framework. The forthcoming sections address the methodological issues involving the adopted research paradigm in this research study.

5.2 Research design and research strategy

Bryman (2016) defines *research design* as a framework for collecting and analyzing data. That is, the research design gives the framework created to find answers to the research questions. According to Yin (2014) when selecting a research design, the researcher should consider these three conditions:

1. the type of research question posed,
2. the extent of control a researcher has over actual behavioral events,
3. and the degree of focus on contemporary as opposed to entirely historical events (p. 9).

In this research, the questions asked are of the form 'how' and 'what' in an exploratory sense. As a researcher, I have little control over the activities of the students. That is, I prepare mathematical modelling tasks for the students, and these students solve the tasks using digital technologies without any support from me (the researcher) or the teacher. Yin (2014, p. 14) asserts that a case study design is suitable when "a how or why question is being asked about a contemporary set of events, over which a researcher has little or no control". A 'what' type of question could also be under a case study design if they are more exploratory (and/or explanatory). Now, the forms of questions described by Yin under the case study design are more explanatory as they deal with operational links traced over time instead of measuring mere frequencies.

Case study design

According to Miles and Huberman (1994, p. 25), a *case* is "a phenomenon of some sort occurring in a bounded context". Stake (1995) refers to a *case* as a bounded system. Thus, a case study is the study of the particularity and complexity of a single case, to understand its activity within critical circumstances (ibid.). Given the definitions above, a case study has a close relationship with the UoA; for that matter, the UoA is the case itself (case is the same as the UoA in activity theorists' perspective). Yin (2014, p. 16) defines *case study design* as "an empirical inquiry that investigates a contemporary phenomenon (the case) in depth and within its real-world context, especially when the boundary between phenomenon and context may not be clearly evident". For instance, in this study, defining the boundary between the phenomenon (modelling actions, affordances and constraints) and the context (how they emerge in the students' activities) is not clearly evident or predictable. Given this, case study design is helpful in studies of this nature. In addressing the research questions in this study, the case study design helps me make sense of modelling actions, affordances and constraints that emerge through observing students' interactions with digital technologies while working on mathematical modelling tasks. In this case, I analyze the actions and operations of the students and the components in the students' activity system holistically.

Case study research characteristically emphasizes natural settings rather than artificial situations. In this study, the students are observed in their natural setting: an existing mathematics class in a secondary school where the researcher designs the mathematical modelling tasks and spontaneous groups are formed (and dissolved after the activity). Case studies may be single or multiple. Yin (2014) argues that multiple case studies generate more compelling results than one. The selection of cases in multiple case studies follows a replication, not a sampling logic. This means that "each case must be carefully selected so that it either predicts similar results (a literal replication) or predicts contrasting results but for anticipatable reasons (a theoretical replication)" (ibid., p. 54). It is argued from the premise that modelling actions, affordances and constraints (see Section 4.4) emerge in a particular situation/context. However, one can anticipate the potential emergence of these phenomena by drawing on experiences. The cases in this study have some differences on which predictions might be based. The cases involve four different secondary schools with different classroom environments, students' characteristics, knowledge levels, and others, and the selection of these cases might predict contrasting results in a theoretical replication sense.

How is sampling done in the adopted case study research design? Sampling in this research study is done at two levels: the *contex*t and the *participants* (Yin, 2014). The first sampling level relates to the choice of schools (see Section 3.4), and this sampling is considered purposive (Cohen et al., 2017) in the sense that the selected context is relevant to answering the research questions. These schools use the same curriculum (see Sections 3.2 and 3.3) but also have some differences. The choice of these different schools gives some diversity for *triangulation* (defined in Section 5.9) purposes in the empirical data. Students in these schools are also more familiar with the GeoGebra software (the primary digital technology used in this study; see Section 5.4), and this influenced the choice of schools since it takes time to learn how to use a technological/mathematical tool considering the time limit of the research study. Another form of sampling adopted at this level is convenience sampling, where these schools were selected because they were geographically accessible for the research study within the resources available to the project. Again, these schools were self-selected regarding the curriculum, digital tool criterion, and willingness to participate in the research. As such, invitations were sent to the majority of the public secondary schools in the south of Norway, and only four schools responded positively to the invitation.

The second level of sampling relates to the choice of participants/students. Students from different schools volunteered to participate in this study, which is considered volunteer sampling (Murairwa, 2015). The project resources, the COVID-19 pandemic, and the results from the pilot studies (see Sub-Section 5.6.4) did not permit video recordings of all the groups within a single classroom across the schools. For this reason, only one group of students was targeted (*focus group*) for the video recordings in each of the four schools. These groups are randomly selected from amongst the students who volunteered. Thus, the students who were willing to participate in the video recordings (the rest of the students in the same classroom participated without being recorded on tape). The sampling forms discussed above serve the purpose of the study since the study is a case study emphasizing the depth of study rather than the breadth of study and also focusing on the particular and not the general. To conclude my choice of adopting a case study research, Cohen et al. (2017) remark that investigators must be vigilant as case study research is prone to selective reporting. That is, picking some pieces of evidence which only support a particular conclusion, which might misrepresent the credibility of a case. This form of bias is addressed in the research study through my critical reflection and awareness of how my interests and experiences could affect the interpretation. As such, I use the triangulation method (see Section 5.9), which I believe helps minimize the extent of observer bias.

The methods of collecting data in this study have some elements of ethnography, so I classify it as an *ethnographical case study* (Moschkovich & Brenner, 2000). More about ethnographical case study will be discussed in the subsequent sections, but before that, I will first discuss some criticisms of case study research. Case study research has been criticized for its non-representativeness and lack of statistical generalizability despite the many advantages and the description of case study research as discussed above. Yin (1984) points out that case studies are often tagged as difficult to conduct and produce a massive amount of documentation (in particular, case studies of an ethnographic or longitudinal nature). Zainal (2007) adds that case studies provide minimal basis for scientific generalization since they use few subjects. In response to these criticisms, Flyvbjerg (2006) points out that case studies provide a generalization through "the force of example". For instance, case study research might offer an example of students' activities and the conditions of these activities. In this situation, Bassey (1999) considers the generalization here as "fuzzy generalization". That is, "fuzzy generalization arises from studies of singularities and typically claims that it is possible or likely or unlikely that what was found in the singularity will be found in similar situations elsewhere" (ibid., p. 12). Having discussed issues concerning case study research, I will discuss the context of the research study in the next section.

5.3 Context

This study involves four secondary schools located in the southern part of Norway. These schools use the same curriculum, implemented in Autumn 2020, and have mathematical modelling and applications as one of its core elements. The participants are in lower and upper secondary schools (see Table 5.1). The students in the lower secondary school take the mathematics subject at the ninth grade, while the students in the first-year upper secondary school take the 1T (theoretical mathematics) course. The students in the second-year upper secondary school take the R1 (mathematics for science) course (see Section 3.2). The participants are between the ages of 14 and 17 years. Approximately 73 students (32 males and 41 females) voluntarily participated in this study. Furthermore, out of the 70 students, 10 students were randomly selected as the *focus group* (See Section 3.4).

What is the role of the teacher in this research study? One of the factors considered in integrating digital technology into the education system is the role of the teacher (Drijvers, 2015), and their knowledge of the tool and understanding of the principles behind its use is necessary (Watson et al., 1993). As such, at the beginning of the project, I met the teachers at each school and discussed and solved the mathematical modelling tasks. The role of the teacher here is to act as a facilitator (providing students with help/hints when necessary). I labelled this stage of the students' activity the *introductory activity*. In the *introductory activity*, the students were allowed to engage with some examples of mathematical modelling tasks (Tasks A, B, and C in Appendix B). The idea of engaging the students in the introductory activity was that the students were not familiar with such tasks. At the beginning of the introductory activity, the teacher briefly introduced the activities, and then the example tasks were presented to the students on paper. The students were permitted to ask questions, but the teacher and/or researcher were not supposed to give a direct answer. However, the students mostly solved the tasks in groups during the introductory activity. Sometimes, they called for help from the teacher or researcher. In situations like this, the teacher or researcher only gives hints to the students so they can figure out the answer themselves. The students in the introductory activity mostly used GeoGebra while working on the example tasks, and the average time spent on each task was approximately 20 minutes.

The next activity was the *main activity*. In the *main activity*, the students solve a set of mathematical modelling tasks (Tasks 1 and 2, see Section 5.5) without the help of either the teacher or the researcher. In the main activity, the teacher distributed the task sheet to the groups and gave some instructions. Thus, each group solves the task and digitally sends their reports to the teacher (see Appendix D for the solution reports of the focus groups and the other groups in the same classroom in each school). The expected time for each task was 20 minutes. At the end of the main activity, each group presented their results to the class. The researcher did not engage with the students during the main activity but only took notes. The students were allowed to solve the tasks alone (in a group) so that there would be no interruption in the flow of their discussion. For this reason, some groups used more than 20 minutes on a single task but were not interrupted (since the researcher was interested in the outcome). One primary reason for allowing the students to work alone was that these students were the focus of the study and not the teacher (or the researcher). The students were allowed to use a digital technology of their choice while working on Tasks 1 and 2. Thus, digital technologies were not imposed on the students; instead, they were allowed to choose freely which technology suits them best. Empirical data for this research study was collected during the main activity, for which the focus was on one particular group (focus group) among the other groups within the class. All the data were collected during the Autumn 2021 semester. In the *introductory* and *main* activity, the students solve the mathematical modelling tasks in groups of 3-4. The groups in the introductory activity were maintained for the main activity. The main activity was conducted a few days (in some cases, a week) after the introductory activity. Lou et al. (2001) point out that small groups are more likely to collaborate than large groups. Therefore, the students in this study worked in groups of 3-4 (depending on the class size). The students' joint work (group work) was an object of study, and as such, the students were not given any instructions on how to organize their work (or were assigned roles while working on the tasks).

Table 5.1 below presents a description of the students in the different schools. *Group A* are students in the second year at the upper secondary school (12th grade), while students in *Group B* and *C* are students in the first year at the upper secondary school (11th grade). The students in *Group D* are in the lower secondary school (9th grade). The time spent by each group on both tasks in the *main* activity is recorded in Table 5.1. The table again presents the hierarchies within the bands of performance (high, average and low). The teachers of the groups gave information on the hierarchies within performance based on the grades assigned to the students. In the Norwegian performance scale (or grading scale), a high performance is between grades 5 and 6, an average performance is between grades 3 and 4, and a low performance is between grades 1 and 2. To further differentiate between the performance scale, the teacher for *Group A* highlights that Thea performs higher than Rolf, although they were both assigned grade 4. However, Rolf has the highest mathematical understanding but has a little lower motivation within the group. The teacher in Group C did not give any further information besides the assigned grades. Groups A, B, and D consist of students with different attainments (mixed-achievement), while Group C consists of students with the same attainment (same-achievement) or a group of high-performing students. Mixed/same-achievement is a product of the Norwegian school system and is not a rigorous measure. The empty spaces or spaces marked by a dash (-) represent no performance score in Table 5.1.

Group	Level	Students	Performance						Time	
			High		Average	Low			Task 1	Task 2
			6	5	4	3	2	1		
A	2nd year Upper secondary school (12th grade)	Thea	-	-	X	-	-	-	25	35
		Rolf	-	-	X	-	-	-		
		Kåre	-	-	-	X	-	-		
B	1st year Upper secondary school (11th grade)	Emil	-	X	-	-	-	-	20	40
		Thor	-	-	X	-	-	-		
		Ella	-	-	-	X	-	-		
		Tore	-	-	-	-	-	X		
C	1st year Upper secondary school (11th grade)	Nils	-	X	-	-	-	-	8	31
		Anna	-	X	-	-	-	-		
		Jørn	-	X	-	-	-	-		
D	Lower secondary school (9th grade)	Olga	X	-	-	-	-	-	17	18
		Hege	-	-	X	-	-	-		
		Lena	-	-	-	X	-	-		

Table 5.1: A summary of the level, performance of each member of the different groups, and the time (in minutes) used on each task.

Table 5.2 below presents the tools each group member engaged with while working on Tasks 1 and 2. The tools are a calculator device, computer (GeoGebra, Excel/spreadsheet, Google Maps, Google Search, and calculator software on computer), and paper-and-pencil. In each group, the students took turns using these tools. I further differentiate between high, medium, and low tool usage in the table. High tool usage is when the students use the tool on most occasions, whilst medium usage is when the students use the tool on some occasions. Low tool usage is when the students use the tool in just one instance.

Group	Students	Tool usage
A	Thea	Computer (high) and paper-and-pencil (medium)
	Rolf	Computer (medium)
	Kåre	Calculator device (high) and paper-and-pencil (medium)

B	Emil	Computer (high)
	Thor	Computer (high), calculator device (high) and paper-and-pencil (medium)
	Ella	Computer (low) and calculator device (low)
	Tore	Paper-and-pencil (medium)
C	Nils	Computer (high)
	Anna	Computer (high)
	Jørn	-
D	Olga	Computer (high)
	Hege	Computer (medium)
	Lena	-

Table 5.2: A summary of each group member's tool usage in the students' activities.

I will now present the role of the researcher during both the *introductory* and the *main* activity in the forthcoming subsection.

5.3.1 The researcher's role

Classroom observation is the means I used in researching the activities of the students. As a researcher, I play different roles in the students' activities (introductory and main activities, described in the section above). In the introductory activity, I played the role of *observer-as-participant*. In this role, Gold (1958), in his typology of roles in participant observation, explains that the researcher contributes to some extent to discussions and activities. However, the most important part is taking notes. This role allows the researcher to participate in the students' activities as desired. Thus, I play this role by giving help/hints to students (by going around the class) when they call for it (other than that, I only observed and took field notes). The teacher was the primary facilitator, going around the class and providing help/hints when the students called for it. The students mostly asked the teacher instead of the researcher for help in the face of challenges while working on the tasks. When the students asked the researcher for help a few times, the teacher was already engaged with other groups (and these students could not wait). This is to say that the students were more comfortable asking the teacher than the researcher (as the researcher is not part of the students' natural settings in the classroom).

In the main activity, I play the role of a *complete observer*. In this role, Gold (1958) explains that the researcher only observes and takes field notes. In this case, the researcher is completely hidden as he/she observes the participants' activities

(Kawulich, 2005). I play this role by taking field notes (whilst the students work alone without support). Taking field notes is challenging as one might not be able to grasp all of the exchanges among the students, so I used video recordings during the observation. The teacher and the students were aware of my intentions in both roles that I played. Thus, before both activities, I met the teacher and students, presented the details of the whole activity, and afterwards, they signed a consent form (see Appendix A). Some challenges come with the two roles that I played during the students' activities. An example of such challenges was my presence in the classroom and my use of the video camera. That is, the students act differently than they usually do in class, and the presence of video cameras might cause stress among the students. I will address this issue and how I minimize this challenge in Section 5.9. Another challenge concerns *observer ratings*. Observer ratings in this study are discussed in two dimensions: the information an observer has about the participants and the coding of students' participation in an activity. Schlesinger and Jentsch (2016) highlight that external observers (researchers) usually have only a little information about the class and the students compared with the internal observer (teacher). Concerning this research study, the only information I knew about the students is the general information online concerning the curriculum and the regulations outlined by the Norwegian Ministry of Education and Research. As such, I had several meetings with the respective teachers seeking extra information about the participants, which is relevant to the study (see Section 3.4). Coding the observed activities of students (from the researcher's perspective) could be biased. However, this bias could be minimized if another rater codes the observed data with the same guidelines (Ing & Webb, 2012). I had another researcher who coded a section of the data from the students' activities (see Sub-Section 5.9.1).

I used fieldnotes, video recordings, and screen capture software in this classroom observation (see Section 5.6). In the following section, I will now present the digital technologies the students used while working on Task 1 and 2.

5.4 GeoGebra, Excel/spreadsheet, calculator, Google Search and Google Maps

In this subsection, I will discuss the literature on the digital technologies the students in this study used in their mathematical modelling activities while highlighting some particular usability features of these digital technologies.

GeoGebra. Wassie and Zergaw (2019) point out that GeoGebra is an interactive mathematics software that can be used for teaching and learning mathematics (mainly algebra and geometry). GeoGebra has several usability features, and some examples are its potential to enable multiple representations of concepts, allowing the insertion of images (for example, taking a screenshot from Google Maps and inserting it into GeoGebra for analysis), among others. Pereira et al. (2017) emphasize that the effectiveness of GeoGebra might improve students' understanding of some mathematical concepts (or geometric figures). For instance, Anabousy et al. (2014) show how GeoGebra is essential in visualizing and understanding the effects of varying parameters through function transformations in different representations. GeoGebra allows students with different learning styles to flourish (Wassie & Zergaw, 2019) due to its multiple representation features. However, some challenges come with GeoGebra's use in students' activities. For instance, some of the commands used in the input bar of GeoGebra might not be user-friendly (see "tool representational consideration" on page 65).

Excel/spreadsheet. Microsoft Excel is a spreadsheet developed by Microsoft which features calculation or computation capabilities (arithmetic operations) and graphing tools (for displaying data), among others. Evans (2000) reports that Excel offers some pedagogical advantages for learning simulation. For instance, students could modify a data set's price, cost, or profit and observe the corresponding dynamic graphical output. Flehantov and Ovsiienko (2019) analyze the simultaneous use of Excel and GeoGebra in mathematical modelling and how it might improve students' learning outcomes. The students who use Excel mainly represent their data in table form and analyze the obtained numerical results and graphs by changing the input parameters of their model. On the other hand, the students who only use GeoGebra created visual representations while interactively using sliders to change the model's input parameters. The students that use Excel and GeoGebra mainly use Excel for numerical calculations and representation of numerical results in tables while using GeoGebra for visual representation and analysis of dynamic motion characteristics (ibid.).

Calculator. A calculator is an electronic device (or computer software) used to perform calculations (such as addition, multiplication, subtraction, division, and others). Several types of calculators (e.g., scientific calculators, graphing calculators, and others) could be used to perform calculations ranging from basic arithmetic to complex mathematics. Stacey and Groves (1994) argue that the presence of calculators might allow students to work with more significant numbers and solve more realistic problems. Thus, the calculator might provide the opportunity to compute large numbers quickly and perform error-prone and time-consuming calculations when done by hand (paper-and-pencil) (Pierce & Stacey, 2010). Retnawati et al. (2019) point out that integrating calculator utilization into students' activities might enhance students' participation in mathematics learning. However, Strässer (2007) warns that digital technologies (such as calculators) might put the mathematics in a 'black box'. For instance, students might not know the algorithm behind the computation made by the calculator.

Google Maps and Google Search. Google Search is an internet search engine, while Google Maps is a web mapping platform which offers locations (of countries, cities, towns, and others), satellite imagery, street maps, route planning for travelling (by foot, car, bike, air and public transportation), among others. Both Google Maps and Google Search are applications offered by Google, and these applications can be used in mathematics education. For instance, Fesakis et al. (2018) in their study used Google Maps to design a mathematical trail for students in primary education. According to Shoaf et al. (2004, p. 6), "mathematics trail is a walk to discover mathematics". That is a part of outdoor education, where students discover and solve mathematical problems on real objects while walking.

In the next section, I will present a discussion on the two mathematical modelling tasks used in the main activity.

5.5 The mathematical modelling task

This section presents two mathematical modelling tasks (Task 1 and 2) used in the main activity. These two tasks are adapted (adopted in the case of Task 1) from the study by Mousoulides (2011) and are different from what the students usually work on at school (e.g., see Figure 3.1). The two tasks were chosen to study secondary school students' mathematical modelling activities using digital technologies. The subsections below present an a priori analysis of Tasks 1 and 2, respectively.

5.5.1 Task 1

Solar power car: A car making company is launching a new solar powered car. Recent market research showed that one hundred people would buy the car for a selling price of €5000. Further, the market research showed that for every €100 price increase, people's interest in buying the car would decrease by one person. Find the best-selling price for the car, so as to maximize the company's sales revenue. Send a letter explaining how you solved the problem to the company's sales manager.

Task 1 (presented in the box above) has a realistic context (see 'contextual features' on page 50), linking school mathematics tasks with real-life experiences. The task is formulated in everyday language, and students are not told how to solve the task. Thus, a step-by-step account of how the students should solve the task is not given, nor are sub-questions directing students on specific things that need to be done. In such a situation, the task might not provoke the steps in the modelling cycle (see, for instance, Figure 3.1 on page 82). Task 1 contains more information (than Task 2) in connection with given numbers in the task, which have to be added through the application of mathematical formulas (described as inner-mathematical knowledge—Boromeo Ferri, 2006, 2007), such as developing a function from a given information. Task 1 is closed-ended (having only one answer) but can also be viewed as an open-ended task (depending on the argument the students might give). The task requires students to develop quadratic function models for finding the best-selling price for a solar-powered car. However, students might use different approaches while working on the task. Students must decide and identify critical components from the task themselves and mathematize. Thus, identifying the number of people buying the car (*persons*) and the price at which they buy the car (*price*). Students might quite easily use the formula *persons**price* for calculating the total amount of money. The students would have to further calculate, interpret and validate their answers (as the task requires a report on how the students solve it). Students will produce calculations and reasoning using their previous knowledge, and they can be of a different nature or approach. I will present some typical approaches that the students might take in solving the task:

1. The students might directly apply algebraic expressions. Thus, an algebraic formula for all the identified variables. Students might further look for a graphical representation of these formulas.
2. The students might solve the task numerically. Thus, the initial data set is entered, and the entire data is generated on a spreadsheet.
3. Students might first enter and generate their data on a spreadsheet and then represent their generated data on the graph. This approach links the different representations: numeric/table, graph, and equation.

The students might have other approaches different from the ones above. The approach students choose might be influenced by the kind of digital tool used, the working style of the students or characteristics of the students, amongst others.

5.5.2 Task 2

Building a shopping center: The authorities of three towns (Kristiansand, Lillesand and Vennesla) are planning to build a mega shopping center that will serve the needs of their citizens. Identify the optimal place for the shopping center location so that the needs of the three towns are served in a fair way. Send a letter to the ministry in charge explaining and documenting your solution.

Task 2 (presented in the box above) has an authentic context (see 'contextual features' on page 50), linking school mathematics tasks with real-life experiences. The task is formulated in everyday language, and students are not told how to solve the task. Task 2 does not have numerical values in its presentation as compared to Task 1 but might require extra-mathematical knowledge (result of experiences) (Boromeo Ferri, 2006, 2007). The original task (Problem 1 by Mousoulides (2011)) contains four unknown towns, where the students have to find an optimal location for building an airport that reasonably serves the needs of these unknown towns. The new task (Task 2) is presented so the context is relevant (something the students can relate to). Task 2 is open-ended and purposefully not well-defined, and students might have to make some necessary hypotheses to clarify the problem. That is, the task for students is not only to determine one possible answer but to find a way to resolve a societal issue. Again, the task raises issues about what constitutes "fairness" and how social considerations such as the population of each city, the travel distance, and the nature of the area, among others, are integral aspects of the solution. Task 2 might also require students' ability to locate the known cities on Google Maps, find an optimal position (applying some geometry), and argue for this position while considering some factors (e.g., population ratio). Although Task 2 might contain little mathematics initially, the range of arguable solutions might be thought-provoking. I will present some typical approaches that the students might take in solving Task 2:

1. The students might solve the task geometrically. Thus, locating the known cities on Google Maps and importing a screenshot of the identified positions into GeoGebra for further analysis. The students might connect the three unknown cities, forming a triangle, and find the middle point of this triangle (either by circumcircle/circumcenter of a triangle approach or the median of a triangle or centroid approach).
2. The students might locate the three known cities on Google Maps and then conduct some analysis. Thus, looking for the population, actual travel time, distance between these cities, and the optimal location. The students might use the ratio of the values (regarding the population, time and distance of travel) to estimate the optimal location. Some students might further consider the nature of the possible location, whether in the middle of the woods, river or mountain. The students might also generalize their model by applying the first approach if they do not consider other factors.

I will now present the methods for data collection in this research study during the observation of the students as they work on both Task 1 and 2.

5.6 Methods for data collection

The empirical data considered as appropriate for the purpose of investigation is influenced by the choice of research paradigm. This study has elements of interpretive, naturalistic and ethnographic paradigms, and it is relevant to seek evidence considering human activity in its natural setting (Patton, 2002). Thus, to address the research questions, I will use qualitative methods. Fusch et al. (2017) point out that ethnographic case studies can employ a wide range of methods in collecting empirical data for the purpose of triangulation. The primary method for data collection was classroom observation. Thus, I observed groups of students solving mathematical modelling tasks using digital technologies in their natural settings. Table 5.3 presents a timeline of the research process in this study. The timeline includes the time frame in which the teacher and students were informed until empirical data was collected and analyzed. The table starts with the beginning of the research project, where consent was sought, and through with the classroom observations. The empirical data was collected during the main activity.

Time-Frame	Activity	Procedure	Students Involve
Letters were sent to secondary schools in the Agder region (Southern Norway) – in August 2020.			
Had a first meeting with the schools that responded positively to the letter (4 schools) – between August 2020 and October 2020			
Meetings with the school heads and the teachers at the different schools – between January 2021 and June 2021.			
*NSD approval for data collection (1st pilot study) – April 2021			
Conducted the first pilot study (4 university students) – May 2021			
Scheduled the date and time for the introductory and main activity with the teachers – June 2021			
*NSD approval for data collection (main study) – August 2021			
*NSD approval for data collection (2nd pilot study) – August 2021			
Teachers and students signed the consent forms – September 2021			
Conducted the second pilot study (4 university students) – September 2021			
October 2021 (week 42)	Introductory	The students in School A worked on some tasks (see Appendix B).	10
October 2021 (week 43)	main	The students in School A worked on Tasks 1 and 2 (see Section 5.5).	10
November 2021 (week 46)	Introductory	The students in School B worked on some example tasks.	28
November 2021 (week 46)	main	The students in School B worked on Tasks 1 and 2.	27
November 2021 (week 47)	Introductory	The students in School C worked on some example tasks.	15
November 2021 (week 47)	main	The students in School C worked on Tasks 1 and 2.	16
November 2021 (week 47)	Introductory	The students in School D worked on some example tasks.	14
November 2021 (week 48)	main	The students in School D worked on Tasks 1 and 2.	15
In-depth analysis of recorded videos, screen-capture software and fieldnotes.			

*Table 5.3: Time-frame of the research process (*NSD – Norwegian Centre for Research Data).*

I will present and justify why I used the methods, fieldnotes, video recordings, and screen capture software in the classroom observations.

5.6.1 Fieldnotes

Fieldnotes were used as one of the data collection methods throughout the research process (in and out of the classroom observation). Notes were first taken during the meetings with the teachers at the respective schools. In these meetings, each teacher described the students regarding the courses they have taken and their performances, among others (see Section 3.4). Again, most notes were taken during the introductory activities, as the video camera was only used during the main activity. The notes taken at this stage help to account for the students' activities. In addressing the first research question (RQ1, see Section 1.4), the fieldnotes help give a detailed account of the component 'subject of the activity', as these descriptions are not captured in the video recordings. To make sense of the interaction sequences (see Sub-Section 2.4.1) and the component 'division of labour' (roles adopted by the students) in the students' activity (as captured in the video recordings), the fieldnotes about the students' performances helps in understanding these phenomena to some extent.

5.6.2 Video and audio recording

I used video and audio recordings to collect data from classroom observations to address the three research questions. The approximate time of recordings of the students' activities was 18 and 31 minutes for Task 1 and 2, respectively (see Table 5.1). The video recordings provided evidence of the students' interactions during the main activities. This evidence helped reveal the students' modelling processes as they worked on Tasks 1 and 2. The recordings also shed light on the types of digital technologies the students used, the kind of interaction sequences that took place, the roles the students adopted, and the rules that guided the students within the students' activities. The video recordings gave a detailed account of the communication between the students (in the form of suggestions, questions, answers, arguments, etc.), some gestures like pointing to the computer screen, computer activities, their use of other devices such as calculator devices (which cannot be captured by screen capture software), among others. Furthermore, through this, I identified the modelling actions that emerged and the parts that the uses of digital technology played within these actions that emerged. Again, the video recording gives a detailed account of the students' interactions with digital technologies, and it is from these interactions that I identified the affordances and constraints of the digital technologies that emerged.

5.6.3 Screen capture software

The screen capture software (TechSmith Camtasia screen recorder) was used as one of the data collection methods in the classroom observations. This data collection form complements the video recordings as it provides information about how the students solve Tasks 1 and 2 on the computer. Although the position of the video camera captured the students working on the computer, their actual work was unclear due to the students' movement (blocking the camera view). As such, I used the screen capture software, which gave a detailed account of the students' work on the computer. In addressing the issues concerning the students' interactions with digital technologies in the three research questions, the screen capture software gives the needed evidence. The screen capture software only gave evidence of interaction with digital technologies such as GeoGebra, Excel/spreadsheet, computer calculator, Google Search and Google Maps. Digital technologies, such as calculator devices, were captured by the video recordings.

The instruments used for collecting data were first tested. I tested and modified these instruments by conducting two pilot studies. The forthcoming subsection presents the pilot studies conducted in this research.

5.6.4 Pilot studies

I conducted two pilot studies before the introductory and main activity (see Table 5.3). The pilot studies mainly focused on testing my data collection instruments. That is, it checks the efficiency of the instruments and what needs to be measured to address the research questions. At the end of the pilot studies, I got critical feedback from the participants and my supervisors. This feedback was an invaluable means of seeking further guidance on dealing with emerging practical and methodological issues (and the modifications required for the main study). I will now present the two pilot studies below:

Pilot study 1. The first pilot study was conducted with two pairs of university students. These students volunteered to partake in this pilot study (after visiting different classes and talking to students about the research project). Three participants were students in the mathematics education program, and the last was in the engineering program. The participants formed two random groups, with two students in each group. The activities were conducted in two hours (one hour for the introductory and one hour for the main activity). Both groups were in the same classroom (small size room) simultaneously. In the introductory activity, the groups solve three example tasks (see Tasks A, B, and C in Appendix B). The groups solved the task alone but were allowed to ask for support from the researcher in the face of uncertainties. The groups almost solved the task without asking for any support. The students continued with the main activity after completing the introductory activity. The students solve Tasks 1 and 2 (see Section 5.5) in the main activity. At this stage, I video-recorded the activities of the two groups. I used two video cameras, each directed towards one of the respective groups. Again, the students installed screen capture software (CamStudio, free screen recording software) on their computers before the main activity. In the main activity, the groups were supposed to solve the task without support from the researcher. There were some challenges concerning data collection after the main activity. The first challenge was the video recordings. The audio was unclear, although one can see the students' actions in the recordings. There were times that I often heard the discussions of the first group in the recordings of the second group (voice-over). Furthermore, it was not easy to transcribe the recordings. Another challenge had to do with the screen recordings. The audio recordings from the screen capture software had issues similar to those of the video camera recordings described above. Again, the interface of the screen recordings was white (showing no screen activities). Considering these challenges, I conducted the second pilot study using different instruments and setups.

Pilot study 2. The second pilot study was conducted with two pairs of university students. The participants were all students in the mathematics education program. These students volunteered to participate in the second pilot study (after visiting some classes and talking to students about the research project). The participants formed two random groups, with two students in each group. The exact time (two hours) used in the first pilot study was used in the second pilot study. The students also solved the same tasks as the other students did in the first pilot study in the introductory and main activities. To solve the issue of voice-over in the second pilot study, each group worked alone in separate classrooms (one group per day). For that reason, I could clearly hear each group's discussions while having a clear view of their actions in the recordings. I used a new screen capture software (TechSmith Camtasia screen recorder) recommended by the IT help at UiA. Each group installed the screen capture software a day before the activities. During the introductory activity, each group recorded the screen activities. The researcher played the screen recordings (checking the interface) before the main activity (which was then re-recorded). At the end of the main activity, the screen recordings clearly showed the students' activities on the computer. It was easier to transcribe the recordings of the students' activities.

I used the same setup and instruments used in the second pilot study in the main study to overcome the challenges in the first pilot study. Although knowing this setup would work for the main study, some challenges still needed to be addressed. That is, I had no intention of having each group representing the respective schools (see Section 3.4) to solve the task in a separate classroom. That is, taking these groups of students from their natural settings and placing them in a created new setting. So, the question was, 'How do I videotape the groups without having similar challenges as in the first pilot study?'. To address this issue, I planned to spread the groups in the classroom and use two video cameras. I also used an additional microphone and sound filter. One camera was directed towards the groups' computer activities, and the other camera pointed at the table so I could see what they wrote with paper-and-pencil. Furthermore, a microphone and sound filter on their working table to get a clear audio recording from each group. The screen-captured software also had an audio recording feature, so in the end, I had three different audio recordings (with the same group discussions and interactions). It was easier to transcribe the recordings of the groups' activities, although these groups were in the same classroom as other groups.

In the forthcoming section, I will present the processes governing how I managed and analyzed the empirical data of this research study.

5.7 Data analysis strategy and data management

5.7.1 Transcriptions

The first question I addressed was, 'How do I store the data?'. Van den Eynden et al. (2011) argued that a data storage strategy is essential because digital storage media are inherently unreliable. In order to prevent unauthorized persons from accessing the raw data, the video recordings were stored on the University of Agder server (for which the researcher was the only person who had access). At the end of the students' activities (main activity), I had approximately 3 hours and 14 minutes of video recordings (and screen recordings). In addition, data from fieldnotes and copies of the students' working sheets (with paper-and-pencil) were included. The students mostly communicated in English and sometimes

Norwegian (especially Group D). However, all the excerpts in the thesis are presented in English translations. I started the data management by listening and watching the recordings several times and following the students' working processes. The recordings were transcribed verbatim. Pseudonyms were used during the transcriptions for anonymity. I did the transcriptions relating to Groups A, B and C myself. During the transcriptions, there were times when the students used some Norwegian words, which were easy to translate. Group D mostly communicated in Norwegian; as such, the transcriptions were done by a native Norwegian speaker. To avoid violating the agreement with the participants (see Appendix A), this native speaker only had access to the audio version of the recordings, not the video (students could not be identified by the voice recordings). The video recordings were deleted entirely six months after the research.

5.7.2 Coding

To start coding the empirical data, I first partition the students' activities (the transcribed data) into *episodes*. The *episodes* do not follow any category but the researcher's judgment from the classroom observations. An *episode* in this research study describes an event in a particular time frame in the students' activity. For instance, in the first 3 minutes (episode 1) of the students' activity, the students identified and classified the variables in the mathematical modelling task. In the next 4 minutes (episode 2), the students presented the mathematical problem on the computer, and so forth. Partitioning the recorded students' activities into episodes helped me analyze the empirical data (see Appendix C for the partitioned episodes of the activities of all the groups), as I did not look at all the empirical data at once. The analysis was done for each episode, and in the end, all the episodes were put together to have a holistic analysis of the entire empirical data. There are similar ways of partitioning students' activities into episodes for coding and analysis. For instance, to analyze young students' talk in the iPad-supported collaborative learning environment, Falloon and Khoo (2014) partitioned the data sets into episodes, and these episodes were reviewed several times to fully understand the nature of the talk occurring. In another literature, Monaghan and Ozmantar (2006) divided protocol excerpts into episodes describing a student's activity in the study of abstraction and consolidation. Now, to describe phenomena such as modelling actions and affordances and constraints emerging, it was helpful to categorize the materials. That is, describing the phenomena concerning the theoretical framework and identifying specific nuances and patterns in the empirical data. As such, I describe the coding in this research study as *theory-informed inductive coding*. This coding strategy is an interplay between the theoretical framework and the empirical data. In working inductively, Patton (2002, p. 468) explains that "the analyst moves back and forth between the logical construction and the actual data in a search for meaningful patterns".

I developed a coding template based on the research question and the theoretical framework (deductive coding). This template serves as a data management tool for organizing segments of similar or related text (to assist in the interpretation of data). I tested the initial codes in a pilot study. The predefined codes were then modified and used in the main study. Predefined codes do not often fully cover the empirical data, so I allowed themes to emerge directly from the data (inductive coding) (Fereday & Muir-Cochrane, 2006). For instance, considering RQ3 (see Section 1.4), I had 'observing and repairing divergences' defined under socio-cultural affordances. However, this definition could not help explain an episode in the students' activities. Hence, I introduced a new code, 'observing and improving strategies', which best described the particular episode. The theory-driven codes were modified over time to define the students' activities clearly. I also invited another person to code a section of the empirical data, after which we compared the results and modified the codes (see Section 5.9.1). Ultimately, the codes were organized into themes that seemed to say something about the research questions (Fereday & Muir-Cochrane, 2006; Braun & Clarke, 2006). I have presented the codes for the three research questions in Table 9.5, 9.6 and 9.7, respectively, in Appendix E. Concerning RQ1, there were 17 theory-driven codes and 5 inductive codes emerging from the empirical data (see Table 9.5 on page 388). Concerning RQ2, there were 23 theory-driven codes and 1 inductive code emerging from the data (see Table 9.6 on page 394). Concerning RQ3, there were 8 theory-driven codes and 3 inductive codes emerging from the data (see Table 9.7 on page 398).

In the following subsection, I will present the analysis of the empirical data.

5.7.3 Data analysis

Following the coding strategy described in the prior subsection, I used a combined technique of inductive and deductive thematic analysis to analyze the empirical data. According to Rice and Ezzy (1999), thematic analysis involves the identification of themes through careful reading and re-reading of the data, and the emerging themes become the categories for the analysis. Themes are allowed to emerge directly from the empirical data as well as following predefined themes from the prior theory. The categories for the analysis in this study were mainly developed to support answering the research questions (see Section 1.4). Tables 9.5, 9.6 and 9.7 in Appendix E present the categories and their definition/description for analyzing the empirical data. The codes in these tables represent the categories, with sub-codes that describe them. These sub-codes represent the themes emerging from the empirical data. The themes/sub-codes for the 'role of digital technologies' in Table 9.6 are the same for 'technological affordances and constraints' in Table 9.7. To analyze the data, I used Table 9.5 (corresponding to the first research question) to look at each episode of the students' activity. In this case, I started with the activities of Group A for both Task 1 and 2, then moved on to Group B, C and D. While using the table of codes, I allowed some themes to emerge directly from the empirical data (as explained in the subsection above). After analyzing the data corresponding to the first research question, I repeated the same procedure using Table 9.6 (corresponding to the second research question) to look at each episode of the students' activity for both Task 1 and 2 (from Group A to D). Again, I repeated the above procedure for the third research question using Table 9.7.

I will now discuss how I present the analysis in the forthcoming section.

5.8 Presentation of the analysis

I will present the analysis of empirical data in Chapter 6. In Section 6.1, I will present an analysis overview in table form (in a structure that addresses RQ1). In Sections 6.3, 6.4, 6.5 and 6.6, I will present the analysis of Groups A, B, C and D in the form of a report with evidence from the data extracts. The structure of each report is arranged in an order that addresses the research questions. Table 5.4 presents an overview of the data collection, the analysis, and the interpretation of data and how these are linked to the three research questions.

RQ1: How do students solve mathematical modelling tasks with the aid of digital technologies?
RQ1a: What digital technologies did the students use in solving the two mathematical modelling tasks?
RQ1b: What contingencies were shown in the student interactions when solving the two mathematical modelling tasks with the aid of digital technologies?
RQ1c: What are the rules that mediate students' mathematical modelling activities when solving the two mathematical modelling tasks with the aid of digital technologies?

RQ1d: What roles did the students adopt in the student interactions when solving the two mathematical modelling tasks with the aid of digital technologies?	
Data collection	*Video recordings (recorded conversations and actions) *Screen capture software (computer activities) *Fieldnotes *Students' worksheets
Data Analysis	Both inductive and deductive analysis: Categories developed in the data based on the research question and theoretical framework. Engeström's activity system.
Data Interpretation	Activity system: subject (characteristics of the students), object (solving mathematical modelling task), *mediating artefact/tools* (*digital technologies and group work/interactions sequences*), community (group of students), *rules* (*implicit and explicit*), and *division of labour* (*roles adopted by the students*).
RQ2a: What modelling actions emerge during the mathematical modelling activities of the students? **RQ2b: What part do the uses of digital technologies play within the modelling actions that emerge?**	
Data collection	*Video recordings *Screen capture software *Fieldnotes *Students' worksheets
Data Analysis	Both inductive and deductive analysis: Categories developed in the data based on the research question and theoretical framework. Leont'ev's three-level hierarchy: activity, actions, and operations Modelling actions description: Breaking the task into manageable parts, searching for a model, … Technical features of the digital technology assisting the model development
Data Interpretation	Breaking the task into manageable parts (seeking information, construct relations, …), searching for a model (simplified model, represent the mathematical problem in the computer world, …), … Draw geometric objects and functions, visualize, measure, ….
RQ3: What affordances and constraints of the digital technologies emerge as the students develop a technology-based model/solution?	
Data collection	*Video recordings *Screen capture software *Fieldnotes
Data Analysis	Both inductive and deductive analysis: Categories developed in the data based on the research question and theoretical framework. Leont'ev's three-level hierarchy: activity, actions, and operations. Affordance model (technological, mathematical, and socio-cultural affordances and constraints).
Data Interpretation	Technological affordances: drawing graphs and functions, constructing diagrams, performing calculations, … Mathematical affordances: exploring regularities, simulation and visualization, mathematical representations, ….

	Socio-cultural affordances: common focus, observing and repairing divergencies, ….

Table 5.4: Data collection, analysis and interpretation linking the research questions.

The research quality criteria should be considered when going through the processes described in the table above. As such, in the forthcoming section, I will address the issue of validity and trustworthiness in my research study.

5.9 Validity and trustworthiness

Over the years, qualitative research methodology in education has been criticized for lacking rigour, credibility, and trustworthiness (Cohen et al., 2017). Questions concerning what constitutes quality and rigour in qualitative remain contentious. What are the quality criteria that could ensure the integrity of my research findings? To address this question, I will discuss four criteria of trustworthiness that qualitative researchers must establish (Bryman, 2016). These criteria are credibility, transferability, dependability and confirmability:

Credibility. The main question to address under credibility is, 'How credible are the findings?'. Lincoln and Guba (1985) recommend triangulation as an essential technique in assessing the credibility of the research findings. A Triangulation method increases the trustworthiness of a research study (Patton, 2002). Table 5.5 presents the categories of triangulation, their description/definition, and how it is realized/applied in this research study.

Triangulation	Description	Applied in this research study
Data/source triangulation	Examining the consistency of data sources under the same method. Exploring divergences according to types of data sources.	Under the same methods of collecting data, I gathered data from four secondary schools. This helped in identifying the consistencies in the empirical data.
Investigator triangulation	Using multiple observers or coders. That is, using another investigator to review findings.	I invited another researcher to code a section of the empirical data (see Sub-Section 5.9.1). Again, my supervisors reviewed all the processes in this research study.
Theoretical triangulation	Using alternative theoretical	I used Engeström's version of CHAT while looking at the students' activity system. I

	perspective in interpreting data.	used Leont'ev's version of CHAT and the construct modelling actions while looking at the students' modelling processes. I also used a combination of Leont'ev's version of CHAT and Affordance Theory while looking at the students' engagement with digital technologies.
Methodological triangulation	Checking the consistency of findings using different methods. Exploring divergences between findings from different methods.	Video recordings, screen-captured software data, and fieldnotes were the source of data. That is, the video recordings gave evidence of the students' interactions, whilst the screen recordings provided information on the students' computer activities, which corresponded to the students' interactions.

Table 5.5: Triangulation as applied in this research study.

Transferability. The main question under transferability is, 'Can the findings be transferred or applied to other contexts?'. This question is challenging to address as Lincoln and Guba (1985, p. 316) argue that whether the findings "hold in some other context, or even in the same context at some other time, is an empirical issue". Bryman (2016, p. 384) argues that qualitative research findings "tend to be oriented to the contextual uniqueness and significance of the aspect of the social world being studied". As such, qualitative investigators need to produce a thick description (a rich account of the details of a culture) to permit careful comparison. In this study, I consider analytical generalization and not statistical generalization. Thus, the study takes place in a specific learning environment within a particular context, and my goal is to produce a rich description and depth of study rather than breadth of study. Another issue is the question of what constitutes a proper rich/thick description. However, I will bypass this philosophical issue and immerse myself in the ideas of Lincoln and Guba (1985, p. 316) that an investigator "can provide only the thick description necessary to enable someone interested in making a transfer to reach a conclusion about whether transfer can be contemplated as a possibility". For this reason, I provide a database in the results section in Chapter 6 and codes in Appendix E, which I believe will make transferability judgments possible on the part of potential appliers.

Dependability. The main question under dependability is, 'Are the findings likely to apply at other times?' Thus, will this research study's results be similar if repeated in the same context with similar methods and participants? In addressing this issue, I argue that operationalizations of this research study have been made understandable for replication, that is, the application of Activity Theory, Affordance Theory, and modelling actions construct. A clear description of what it entails for secondary school students to solve mathematical modelling tasks with digital technology has been given in Section 4.2. Section 4.3 and Sub-Section 4.4.1 clearly describe what it entails for the affordances and constraints of digital technologies to emerge in the students' activities. Lastly, a clear description of what it entails for modelling actions to emerge in the students' activities has been given in Sub-Sections 2.1.3 and 4.4.2. Again, the procedures (selection of participants, video recording transcripts, data analysis decision, among others) used in this research study are well documented.

Confirmability. The main question under confirmability is, 'Have the findings largely or to a high degree been influenced by the researcher's point of view and values?'. That is, while recognizing that complete objectivity is impossible in qualitative research, Bryman (2016, p. 386) points out that it should be apparent that the investigator "has not overtly allowed personal values or theoretical inclinations to sway the conduct of the research and the findings deriving from it". The findings of this research study are the result of the ideas and experiences of the participants rather than the characteristics and preferences of the researcher; however, I do present in the results chapter my impressions, interpretations, and meanings that are inferred from the empirical data. Thus, I do so by reporting a detailed description of the research study, including an in-depth methodological description where I acknowledge the methods' shortcomings and potential effects on the study. Again, interpreting the findings in light of the theoretical framework in this research study has been a very long process, evolving through conversations with my supervisors and colleagues and participation in seminars and conferences, among others.

5.9.1 Interrater Reliability

Intercoder reliability refers (IR) to the extent to which two or more independent coders agree on coding the content of interest with an application of the same coding scheme (Cho, 2008; Lombard et al., 2002). IR is a standard measure of research quality where a low level of IR suggests weakness in coding methods. This might result from poor operational definitions, unclear coding categories, and poor coder training. The narrative below shows the IR process and the percentage of the coding agreement of this study.

Nils-Jakob Herleiksplass (a PhD fellow at the University of Stavanger) and I had meetings that lasted approximately six hours and thirty minutes in the space of three days (16th – 18th August 2022) to check the reliability of the coding schema that I used for the analysis of empirical data. The meetings took place after I had finished analyzing the first school (Group A) concerning the three research questions (see Section 1.4). Nils-Jakob was the best choice for this work as we've known each other through three PhD courses (theory, methodology, and measuring teachers' and students' mathematical competence course) we took together. Nils-Jakob has some knowledge about my studies, as we were mostly paired together to read and criticize each other's work within our courses. A week before our meeting, I sent the table of codes (coding schema, see Appendix E) and their definitions, as well as an example of the analysis, to Nils-Jacob to have an overview of what our meeting entails. In the meeting, I first explained the progress of my studies and the empirical data I collected to Nils-Jakob. I also explained to him the codes developed with their definitions and how I analyzed the empirical data with those codes, and he asked questions of clarity afterwards. Nils-Jakob agreed to the codes after explaining further and answering the questions, and then he started coding the empirical data alone. The coding was done for only the data set of Group A, which is a transcription of approximately an hour of video and screen capture recordings of students solving Tasks 1 and 2 (see Section 5.5). The coding was done in three parts concerning the research questions.

On the coding relating to the first research question, Nils-Jakob pointed out that some codes need to be well defined or expanded (for instance, the sub-code 'suggesting role = RSS', see Table 9.5 in Appendix E.1 on page 383) to capture some parts of the empirical data. We both agreed to use a new sub-code, 'supporting role = RSX', to explain some of the students' roles. Table 5.6 below compares the frequency of code entries by the researcher and Nils-Jakob concerning the first research question (RQ1). The red codes represent the updated or modified codes, whilst the new codes are marked in blue. The codes in Table 5.6 are defined and described in Table 9.5 (see Appendix E.1). At the end of the process, I used 150 codes, whilst Nils-Jakob used 134 concerning RQ1. The percentage of our coding agreement for both Task 1 and 2 was 89.33% $\left(\approx \frac{81+53}{95+55} \times 100\%\right)$.

Categories	Codes	Task 1		Task 2	
		Researcher	Nils-Jakob	Researcher	Nils-Jakob
Ratify the objective	RO	2	2	2	2
Digital technology	DTG	**13**	**9**	-	-
	DTE	-	-	-	-
	DTC	**10**	**8**	-	-
	DTM	-	-	3	3
	DTS	-	-	3	3
Pseudocontingency	PCI	**3**	**4**	-	-
Asymmetrical contingency	ACA	**10**	**9**	2	2
	ACC	**9**	**6**	**2**	**1**
	ACE	3	3	**1**	**-**
Reactive contingency	RCC	1	1	4	4
	RCJ	-	-	**1**	**-**
Mutual contingency	MCN	1	1	3	3
Roles of students	RSL	12	12	5	5
	RSO	2	2	**5**	**6**
	RSQ	**3**	**4**	**4**	**3**
	RSS	**8**	**6**	**6**	**5**
	RSN	**6**	**7**	2	2
	RSX	**12**	**7**	**12**	**14**
Total		**95**	**81**	**55**	**53**

Table 5.6: A comparative table of the researcher's and Nils-Jakob's code entries in relation to RQ1.

On the coding relating to the second research questions (RQ2a and RQ2b), Nils-Jakob disagreed with the codes marked in red in Table 5.7. We discussed it, and I explained to him why I used these codes. Nils-Jakob agreed and pointed out that he was confused with the definitions and that I should clarify them in the final version. Upon agreeing to the updated version, Nils-Jakob coded the selected empirical data alone and afterwards, we compared our coding. Table 5.7 below compares the frequency of code entries by the researcher and Nils-Jakob about the second research question. The codes in Table 5.7 are defined and described in

Table 9.6 (see Appendix E.2 on page 388). The percentage of our coding agreement for both Task 1 and 2 was 91.36% $\left(\approx \frac{48+26}{53+28} \times 100\%\right)$.

Categories	Codes	Task 1		Task 2	
		Researcher	Nils-Jakob	Researcher	Nils-Jakob
Breaking the task into manageable parts	BAS	2	2	2	2
	BCR	1	1	1	1
	BSI	1	1	4	4
	BRQ	1	1	**1**	**-**
Searching for a model	SMT	1	1	-	-
	SMR	1	1	**1**	**-**
	SMS	1	1	-	-
	SMA	1	1	-	-
Finding a solution for the model	FK	1	-	-	-
	FA	-	-	7	7
	FE	8	8	-	-
	FM	**10**	**7**	-	-
Explaining the results in real terms	EA	-	-	-	-
	EG	-	-	-	-
	EM	1	1	1	1
Checking the results for adequacy	CR	**7**	**6**	4	4
Role of digital technology	RTC	6	6	-	-
	RTR	1	1	3	3
	RTM	-	-	2	2
	RTE	**9**	**8**	-	-
	RTG	1	1	-	-
	RTV	-	-	2	2
	RTA	-	-	-	-
Total		**53**	**48**	**28**	**26**

Table 5.7: A comparative table of the researcher's and Nils-Jakob's code entries in relation to RQ2a and RQ2b.

On the coding relating to the third research question (RQ3), Nils-Jakob disagreed with the codes marked in red in Table 5.8. I explained to him why I used these codes. Nils-Jakob agreed and emphasized that the codes need to be updated for clarity. In our discussion, we both agreed that the code SOR does not match the description in the empirical data. Hence, a new code (SOI) was introduced. Upon agreeing to the updated version, Nils-Jakob coded the selected empirical data alone and afterwards, we compared our coding. Table 5.8 below compares the frequency of code entries by the researcher and Nils-Jakob about the third research question. The codes in Table 5.8 are defined and described in Table 9.7 (see Appendix E.3 on page 394). The percentage of our coding agreement for both Task 1 and 2 was 98.44% $\left(\approx \frac{45+18}{46+18} \times 100\%\right)$.

Categories	Codes	Task 1		Task 2	
		Researcher	Nils-Jakob	Researcher	Nils-Jakob
Technological affordances	TC	6	6	-	-
	TR	1	1	5	5
	TM	-	-	2	2
	TG	3	3	-	-
	TE	7	7	-	-
	TV	-	-	1	1
	TAG	-	-	-	-
Mathematical affordances	MU	-	-	-	-
	MC	1	1	-	-
	MA	-	-	5	5
	MS	-	-	-	-
	ML	3	3	-	-
	MR	8	8	-	-
	MAR	6	6	-	-
Socio-cultural affordances	SC	7	7	5	5
	SOR	-	-	-	-
	SOI	3	3	-	-
	SA	**1**	**-**	-	-
Total		**46**	**45**	18	18

Table 5.8: A comparative table of the researcher's and Nils-Jakob's code entries in relation to RQ3.

In the forthcoming section, I will address key ethical considerations associated with this research study.

5.10 Ethical considerations

Cohen et al. (2017) argue that questions concerning validity and trustworthiness are also ethical issues; however, in this section, I will focus on the rights and needs of the participants in this study. Bryman (2016) highlights that researchers should pay attention to some ethical principles, such as harm to participants, lack of informed consent, invasion of privacy and involvement of deception in a research project. I will now address each of the principles in this research study (and it also applies to the pilot studies).

Harm to participants. Bryman (2016, p. 126) points out that the harm in a research study could include: "physical harm; harm to participants' development; loss of self-esteem; stress", among others. In this research study, the possible harm to participants might be my presence in the classroom, causing disturbances in the students' working process during the classroom observations. Again, using a video camera might cause stress among the students. As such, I fixed a video camera (without recording) in the introductory activity for the students to get used to it before the main activity, where I recorded the students' activities. Regarding my presence in the classroom, I first met the students and discussed my intent to conduct this research. During the students' activities, I tried not to stand right behind the students but only observed them from afar.

Informed Consent. Bryman (2016, p. 129) highlights that "research participants should be given as much information as might be needed to make an informed decision about whether or not they wish to participate in a study". This research project is registered and approved by the Norwegian Centre for Research Data (NSD). The NSD has a mandate to ensure the personal protection of participants in a research project by demanding a certain level of ethical considerations from the researcher. At the beginning of the study, invitations for participation were sent to many schools, and only four schools volunteered to participate (see Section 3.4). The schools, teachers, and students were well-informed about the research project. Thus, the informed consent of the participants was obtained in a process where the participants signed an informed consent form (see Appendix A). Participation in the activities was voluntary, and written parental consent was required for participants under age 16.

Invasion of privacy. Bryman (2016, p. 132) points out that the "issue of privacy is invariably linked to issues of anonymity and confidentiality in the research process". The participants' privacy and anonymity were ensured by introducing pseudonyms, while the personal information concerning participants was kept confidential. I ensured that all necessary precautions were taken, ensuring that no unauthorized person(s) got access to the raw data. Thus, the personal data was stored on an external hard drive and the University of Agder server. The students and teachers were informed to request the deletion of their data at any time.

Deception. Deception occurs when researchers represent their work as something other than what it is (Bryman, 2016). As such, participants were informed about the aim of this research study. That is, to examine the relation between the students and the digital technologies and the modelling actions that will emerge during the modelling activities. The research questions I will address at the end of the study were clearly stated in the consent form (see Appendix A). The participants could ask further questions about the research project before signing the consent forms.

5.11 Summary of the chapter

This chapter addressed the study's ontological, epistemological and related methodological issues. The study is framed within a qualitative research paradigm based on activity theorist epistemology and ontology. The research design and strategy that have been used in addressing the research questions are presented in this chapter. That is, an ethnographical case study research design was adopted in this study. The context of the study, the digital tools and a prior analysis of the mathematical modelling task used are clearly presented in this chapter. A presentation of an overview of the data collection and analysis procedures followed. Thus, empirical data were collected in Autumn 2021 through video recordings (recorded conversations and actions of students), screen capture software (computer activities), students' worksheets and fieldnotes, and an inductive-deductive approach was used in the data analysis. After that, I discussed validity and trustworthiness issues related to the adopted research design in detail. Finally, I presented some ethical considerations regarding the principle of informed consent, harm to participants, invasion of privacy and deception.

The next chapter (Chapter 6) presents the research study's results. In Chapter 6, I will present an overview of data analysis in table form and a structure for each group's case study reports (followed by each group's report).

6 Results

This chapter presents the analysis of data corresponding to the research questions. The research aims to study how secondary students solve mathematical modelling tasks with the aid of digital technologies. In particular, to study the different interactions within the students' mathematical modelling activities. The analysis involves students' language (suggestions, questions, answers, arguments, etc.) and actions (gestures and interaction with digital technologies). The data sources for the analysis consist of recorded conversations (video recordings), computer activities (screen capture software), students' worksheets and fieldnotes.

This chapter consists of three parts, and the first part (Section 6.1) presents the data analysis overview (in table form) corresponding to the first research question. The second part (Sections 6.2, 6.3, 6.4, 6.5 and 6.6) presents the structure and report of the groups. Thus, Section 6.2 presents a structure for reporting the different case studies, whilst the other sections (Sections 6.3, 6.4, 6.5 and 6.6) present the case study reports for Groups A, B, C and D, respectively. The report for each group consists of a detailed analysis. The third part (Section 6.7) presents the cross-case analysis of the groups. The chapter ends with a summary in Section 6.8. I repeat that pseudonyms were given to the students, and the results from this study cannot be generalized or conceived of as an attempt to paint a general picture of the situation; instead, it should be understood as a "detailed and in-depth description so that others can decide the extent to which findings…are generalizable to another situation" (Cohen et al., 2007, p. 137).

6.1 Overview of data analysis

The summary of results corresponding to the first research question is presented in a tabular form in this section. These results consist of the types of digital technologies used in solving Task 1 and 2 (see Section 5.5), the interaction sequences within the students' activities (see Sub-Section 2.4.1), rules mediating the students' activities (see Sub-Section 4.1.3) and the roles taken by the students in the activities (see Section 2.4). I will present the results corresponding to the first research questions (RQ1, RQ1a, RQ1b, RQ1c and RQ1d—see Section 1.4) in the following four sub-sections. Considering the activity system (see Table 4.1 on page 98), the components subject (students), mediating artefacts/tools (digital technologies and group work/interactions), object (solving mathematical modelling tasks), rules (time constraints, among others), community (group of students), and division of labour (students' roles), interact with each other in the activity system. Sub-Section 6.1.1 and 6.1.2 report on physical (digital technologies—RQ1a) and non-physical (group work/interaction—RQ1b) tools that mediate the subject and object interactions. Sub-Section 6.1.3 reports on the rules mediating subject and community interactions (RQ1c). Sub-Section 6.1.4 reports on the division of labour that mediates the community and object interactions (students' roles—RQ1d).

6.1.1 Digital technologies

Table 6.1 below presents an overview of the types of digital technologies that the students used while working on Tasks 1 and 2. Recall that the students worked in groups and were allowed to use any digital technology of their choice, although they mostly used GeoGebra during the introductory activity (see Section 5.3). The digital technologies that mediated the students' activities were GeoGebra, Excel/spreadsheet, calculator, Google Maps and Google Search. These sets of digital technologies are coded in Table 9.5 (in Appendix E.1) for the analysis. From Table 6.1, the students used one or several digital technologies to solve Tasks 1 and 2. The spaces marked by (X) in Table 6.1 show that the group used the corresponding digital technology as they worked on the respective task, whilst the empty spaces marked by a dash (-) represent no use of the corresponding digital technology. I will further explain this table in Sub-Sections 6.3.1, 6.4.1, 6.5.1 and 6.6.1 with some excerpts from the video recording transcriptions and screen recordings (or computer activities) as evidence.

Group	Task	Digital Technology				
		GeoGebra	Excel/Spreadsheet	Calculator	Google Maps	Google Search
A	1	X	-	X	-	X
	2	-	-	-	X	X
B	1	X	-	X	-	-

	2	X	-	X	X	X
C	**1**	-	X	-	-	-
	2	X	-	-	X	X
D	**1**	X	X	X	-	-
	2	X	-	-	X	-

Table 6.1: The digital technologies each group used while working on Tasks 1 and 2.

The forthcoming subsection summarizes the results on the dynamics of the interaction sequences recorded within the students' group work activities.

6.1.2 Interaction sequences

Table 6.2 below summarizes results relating to the interaction sequences (group interactions) in the students' activities (see Sub-Section 2.4.1). Recall that the activities of the groups in both tasks (Task 1 and 2) were divided into episodes (see Sub-Section 5.7.2 and Tables 9.1, 9.2, 9.3 and 9.4 in Appendix C) for the analysis of the empirical data. Each group had different episodes for Tasks 1 and 2, depending on their solution process and the type of interaction that took place. The categories of interaction sequences are coded in Table 9.5 for the analysis (see Appendix E.1). Table 6.2 gives an overview of the frequency of the interaction sequences relating to Tasks 1 and 2 that occurred during the students' activities. I do not present the interaction sequence as it occurred in each episode but only the total count in the table form. The spaces marked by a dash (-) show that the corresponding interaction sequence did not occur within the students' activities. I will further explain this table in Sub-Sections 6.3.1, 6.4.1, 6.5.1 and 6.6.1 with some excerpts from the video recording transcriptions and screen recordings as evidence.

Group Interactions/Interaction Sequences	Group							
	A		B		C		D	
	Task		Task		Task		Task	
	1	2	1	2	1	2	1	2
Pseudocontingency	-	-	-	-	-	-	-	-
Asymmetrical contingency	7	2	6	1	-	1	1	1
Reactive contingency	1	4	-	5	-	2	3	-
Mutual contingency	1	3	1	2	2	2	2	3

Asymmetrical contingency + Pseudocontingency	3	-	-	-	-	-	-	-
Total (episodes)	12	9	7	8	2	5	6	4

Table 6.2: The frequency of the group interactions/interaction sequences that emerged within the students' activities (episodes) relating to Tasks 1 and 2.

The forthcoming subsection summarizes the results of the rules that mediated the students' activities.

6.1.3 Rules

Table 6.3 below presents an overview of the rules (explicit and implicit) that were observed in the student's activities. The table presents two explicit rules (time constraints and no restriction of digital technologies) and one implicit rule (dismissing comments/suggestions). These rules are coded in Table 9.5 (in Appendix E.1) for the analysis. There were no restrictions on digital technologies in the student's activities; as such, the students used several digital technologies while working on Tasks 1 and 2 (see Table 6.1). Thus, all the groups used at least two digital technologies while working on both tasks. The spaces marked by (X) in Table 6.3 show that a specific rule was observed, whilst the empty spaces marked by a dash (-) represent no observed rule. I will further explain this table in Sub-Sections 6.3.1, 6.4.1, 6.5.1 and 6.6.1 with some excerpts from the video recording transcriptions and screen recordings (or computer activities) as evidence.

Group	Task	Rules		
		Explicit		Implicit
		Time constraint	*No restrictions on digital technologies*	*Dismissing comments/ suggestions*
A	1	X	X	X
	2	-	X	-
B	1	-	X	X
	2	X	X	X
C	1	-	X	-
	2	-	X	-
D	1	-	X	-
	2	-	X	-

Table 6.3: The explicit and implicit rules observed in the students' activities relating to Tasks 1 and 2.

The forthcoming subsection summarizes the results of the roles adopted by the students within the group work activities. The results on the roles adopted by the students help to address RQ1d.

6.1.4 Roles of students

Table 6.4 summarizes results relating to the roles adopted by the students in Groups A, B, C and D, respectively. Different roles are adopted by the students (see Section 2.4) within each episode of the students' activities; for instance, in an episode, a student might be opposing and/or suggesting ideas as they work on the task. Table 6.4 gives an overview of the frequency of roles adopted by the students (in the respective groups) as they work on Tasks 1 and 2. The categories of roles adopted are coded in Table 9.5 (Appendix E.1) for the analysis. In Table 6.4, Task 1 is marked in red whilst Task 2 is marked in green; the first block in grey describes the adopted roles of Group A members; the second block in white describes the adopted roles of Group B members; the third block in grey describes the adopted roles of Group C members; and the fourth/final block in white describes the adopted roles of Group D members. The spaces marked by a dash (-) show that the student did not adopt the corresponding role. I will further explain this table in Sub-Sections 6.3.1, 6.4.1, 6.5.1 and 6.6.1 with some excerpts from the video recording transcriptions and screen recordings as evidence.

Group/ Student		Task	Roles of Students					
			Leading	Opposing	Suggesting	Questioning & challenging	Supporting	Non-contributing
A	Thea	1	11	1	1	-	-	-
		2	5	3	1	-	1	-
	Kåre	1	-	1	3	3	9	-
		2	-	1	3	2	6	-
	Rolf	1	1	-	4	-	3	5
		2	-	1	1	1	5	2
	Thor	1	5	-	-	-	2	-
		2	1	2	4	1	7	-
	Emil	1	1	-	4	2	5	-

B		2	-	2	2	1	7	-
	Tore	1	-	-	-	-	7	-
		2	-	-	1	-	7	-
	Ella	1	-	1	2	-	5	1
		2	-	2	3	1	5	1
C	Nils	1	-	-	2	-	2	-
		2	1	-	4	-	4	-
	Anna	1	-	-	2	-	2	-
		2	-	-	3	1	5	-
	Jørn	1	-	-	1	-	-	2
		2	-	-	2	3	2	1
D	Hege	1	-	-	5	2	5	-
		2	-	1	3	-	4	-
	Olga	1	1	1	4	-	5	-
		2	1	-	3	-	3	-
	Lena	1	-	2	2	-	6	-
		2	-	1	1	-	4	-

Table 6.4: The frequency of roles taken by each group member within the groups while working on Tasks 1 and 2.

So far, I have not presented an analysis of data in a tabular form corresponding to the second and third research questions (RQ2a, RQ2b and RQ3). I will present such tables before the reports in Sub-Sections 6.3.2, 6.3.3, 6.4.2, 6.4.3, 6.5.2, 6.5.3, 6.62 and 6.6.3. Before doing this, I will present the structure for the case study reports in the forthcoming section. The reports are structured in a form corresponding to the three research questions.

6.2 A structure for the case study reports

This section presents the structure for the case study reports in Sections 6.3, 6.4, 6.5 and 6.6, which provide data and information that will allow me to address the research questions in Chapter 7. The case study descriptions, analysis, and interpretations of how secondary students solve mathematical modelling tasks using digital technologies are presented from the students' perspectives. On the other hand, interpretations that are inferred from the empirical data are also presented. I subscribe to Flyvbjerg's (2006, p. 238) idea that 'the case story is itself the result'. A well-defined case constitutes a purposeful, holistic and context-sensitive unit of analysis. This principle has guided my delineation of the unit of analysis throughout this study. For the reader, I repeat the unit of analysis (see Sub-Section 5.1.3), which is 'a group of secondary school students solving mathematical modelling tasks with the aid of digital technologies'. Within this unit of analysis, I look at the actions and interactions within the students' activities. The research questions in this study are formulated in such a way as to address the issue of actions and interactions taking place within the students' activities.

Recall that the research questions are presented (Section 1.4) in three foci: (i) students' mathematical modelling activities (see Sub-Section 4.1.3 and Section 4.2) – RQ1; (ii) emergence of modelling actions and the role of digital technologies (see Sub-Sections 2.1.3 and 4.4.2) – RQ2a and RQ2b; (iii) emergence of affordances and constraints of digital technologies in mathematical modelling activities (see Sub-Sections 4.3.4 and 4.4.1) – RQ3. These three foci became the structuring themes. Recall also that I analyzed the research questions from an Activity Theory perspective. The categories under the three foci are coded in Tables 9.5, 9.6 and 9.7 (see Appendix E), corresponding to RQ1 (RQ1a, RQ1b, RQ1c and RQ1d), RQ2a and RQ2b, and RQ3, respectively. Below is the list of the themes and sub-themes supplementing it:

1. Students' mathematical modelling activities
    a. Subject of the activity
    b. Community
    c. Object of the activity
    d. Mediating artefacts/tools for the subject-object interaction
        i. Digital technologies
        ii. Group interactions
    e. Rules for the subject-community interaction
        i. Explicit rule
        ii. Implicit rule
    f. Division of labour for the community-object interaction
        i. Roles adopted by students.
2. Emergence of modelling actions and the role of digital technologies
    a. Breaking the task into manageable parts
        i. Role of digital technology
    b. Searching for a model
        i. Role of digital technology
    c. Finding a solution for the model
        i. Role of digital technology
    d. Explaining the results in real terms
        i. Role of digital technology
    e. Checking the results for adequacy
        i. Role of digital technology

3. Emergence of technological, mathematical, and socio-cultural affordances and constraints
    a. Technological affordances and constraints
        i. Researching
        ii. Measuring
        iii. Visualizing
        iv. Geometric construction
        v. Experimenting/Changing
        vi. Data entry and generation
        vii. Calculating
    b. Mathematical affordances and constraints
        i. Clarification
        ii. Analyzing
        iii. Simulating and visualizing
        iv. Linking representations
        v. Regularity and variations
        vi. Arithmetic and statistics
    c. Socio-cultural affordances and constraints
        i. Common focus
        ii. Observing and improving strategies
        iii. Authority of the digital technology

The case study reports of the groups are addressed along the three themes listed above. Under each theme and sub-theme, I will present only aspects of the data or sections that represent the entire data as evidence (recorded dialogue). For instance, if there are seven instances where the students 'ratify their objective' of solving Tasks 1 or 2 throughout the data set, I will report on all but only present one out of the seven instances as evidence. Thus, the one example or evidence that captures all the other six instances recorded in the data set. Again, I will give a detailed account of the activities in Group A, but for other groups (Groups B, C and D), I will only report on issues different from Group A. That is, not repeating instances similar to that of Group A. I will now present the report for Group A.

6.3 Case study report: Group A (Thea, Kåre and Rolf)

This section offers a detailed description of the case study report of the first school (Group A; see Sub-Section 3.4.1). The narrative is presented in an order as highlighted in Section 6.2. Group A are second-year upper secondary school students (Grade 12). Group A comprises three students aged between 16 and 17 years. The reports in Sub-Sections 6.3.1, 6.3.2 and 6.3.3 correspond to RQ1

(RQ1a, RQ1b, RQ1c and RQ1d), RQ2a & RQ2b and RQ3 respectively. I will first start the report on the students' (Group A's) mathematical modelling activities.

6.3.1 Students' mathematical modelling activities

Analyzing the research questions from an Activity Theory perspective, I consider the students' mathematical modelling activities an activity system. The activity system (hereafter referred to as 'activity') analyzed in this research report is a group of secondary school students solving mathematical modelling tasks with the aid of digital technologies. Details about the students are presented in Sub-Section 3.4.1. I will present each of the components of the activity system (see Sub-Section 4.1.3 and Section 4.2). These components are interrelated (an indivisible whole), but I linearly present them.

Subject of the activity

Three students (Thea, Kåre, and Rolf) volunteered as the focus group (Group A). The teacher describes this group as a mixed-achievement group (see Table 5.1 on page 129 for the grades assigned to each student). Table 5.1 also shows the approximate time the students used on both tasks. Table 5.2 (on page 130) shows the students' different roles concerning tool usage while working on both tasks.

Community

The community of the activity was made up of students (the teacher and researcher were only visible in the introductory activity; see Section 5.3). Three groups of students worked together in the classroom, of which particular attention was paid to the focus group (Group A). The community was formed spontaneously to solve Tasks 1 and 2 (see Section 5.5) and then dissolved.

Object of the activity

The researcher assumes that the object of the activity is to solve the mathematical modelling tasks with digital technologies and write a report. The students confirm this objective at some points in their engagement with both tasks. For instance, Table 6.5 shows a part of the transcription aligned with codes showing a certain point where participants ratify their objective of solving Task 1. In Table 6.5, from the recorded dialogue relating to Task 1, Thea ratifies the main object of the activity as she points out that they need to find the best-selling price of the car to maximize the company's sales revenue. Kåre supported Thea's idea and suggested that the maximum revenue is above fifty people. Thea then tests their function with fifty people buying the car and the corresponding price at which they buy it.

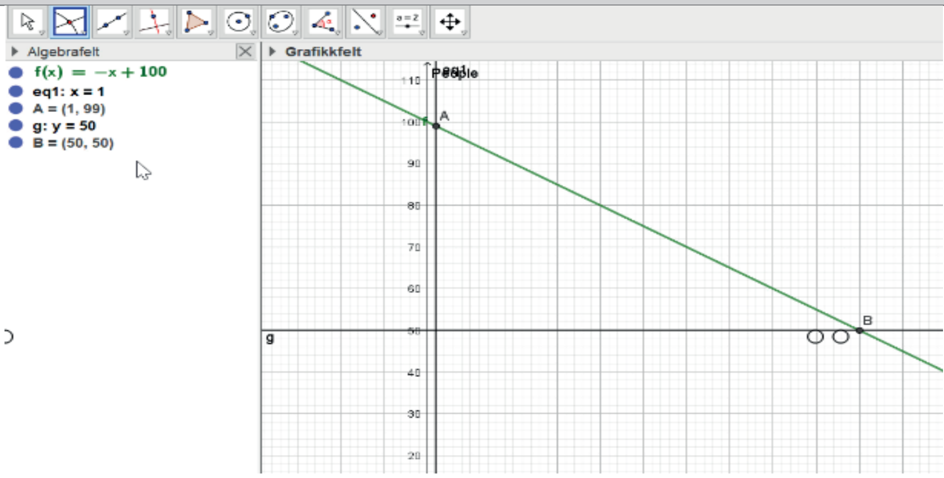
Code	Ratify the objective (Ratifying the objective of solving the mathematical modelling task)
Task	Task 1
Context	The students test their graph or function with 50 people to find the maximum company sales. They looked for the price of the car when 50 people bought it.
Recorded dialogue	*Thea*: Yeah. Good, now we are going to find out when the price is erm… when the maximum company sell is, so…. *Kåre*: I think it should be over 50 because on the 50 we have too many people. *Thea*: Should we try? If we try 50 [writes *y* = 50 on the graph, and found the point of intersection with the line *f*(*x*) = - *x* + 100], then we can find here [points at point B on the graph], fifty fifty
Image	

Table 6.5: Sample Data Aligned with Codes (Objective of the Activity): Group A.

Mediating artefacts/tools for the subject-object interaction

The mediating artefacts/tools mediating the students' activities are physical (digital technologies) and non-physical (group work) tools. I will first present a report on the digital technologies that Group A used while working on both tasks.

Digital technologies

Group A used GeoGebra, a calculator on a mobile phone and Google Search while working on Task 1 and used Google Maps and Google Search while working on Task 2 (see Table 6.1 on page 157). Tables 6.6 and 6.7 below show a part of the transcription aligned with codes, showing examples of different times when GeoGebra and the calculator mediated the interactions between the students and solving Task 1. From Table 6.6, the recorded dialogue shows that the students used GeoGebra to solve Task 1 after analyzing it and knowing what they needed to do.

In the example, the students made a linear graph with persons on the y-axis and the car price on the x-axis. They started with 0, equivalent to 5000 euros on the x-axis, and 1 cm on the x-axis is equivalent to 100 euros. They also made a function $f(x) = -x + 100$ to represent the number of people who buy the car at a given price. For instance, y = 70 (people) intersects the function f at x = 30. Thus, 70 people will buy the car at 8000 euros (30 times 100 plus 5000) and later use the calculator to find the total revenue (e.g., 70 multiplied by 8000, see Table 6.7).

Code	Digital technology (GeoGebra: graphing and visualization)
Task	Task 1
Context	The students made a function representing the number of people buying the car at a given price.
Recorded dialogue	*Thea*: Yeah, it begins with zero. *Kåre*: At 5000. *Thea*: Yeah, it is. *Kåre*: And then it decreases by one person every 100 euros. *Thea*: Ok ... Then we need to find a graph. *Kåre*: Yeah. .... *Kåre*: Like this [Points to the x and y axis in GeoGebra, draws a graph with paper-and-pencil and writes f(x) = 100x representing the graph] *Thea*: Erm no, then you say that erm… its going down with a 1000… If you understand. *Kåre*: So, it will be naturally in there, right? 'Konstantledd' [constant term] or something? .... *Thea*: Yeah, it's going to be on the x-axis, it's not a constant. Do you have any ideas? [Thea asks Rolf if he has any idea] … If we try… I just try something [Draw the graph of the function f(x) = - x + 100 in GeoGebra]. Erm, it goes down by one person, if we just try, I don't think this is the right... *Rolf*: It could be true. *Thea*: Yeah, if we think that 5000 is zero then when [writes x = 1 on the graph] *Rolf*: Should be 99 or something. *Kåre*: So, what you are showing here is erm… you lose one person per 100.

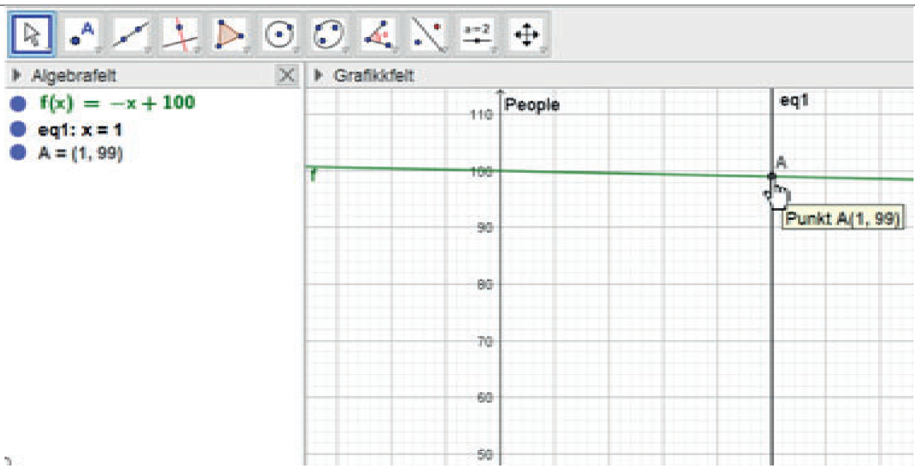
Image	

Table 6.6: Sample Data Aligned with Codes (Digital Technology—GeoGebra): Group A.

The students used the calculator on their phones to calculate some values while working on Task 1, as these values were significant and time-consuming to calculate by hand. In the example (from the recorded dialogue relating to the ‘calculator’ in Table 6.7), Kåre uses the calculator on his mobile phone to calculate the total revenue of seventy people buying the car for 8000 euros. Again, the students used Google Search to seek information about some Norwegian words during the discussion (as English is their second language). For instance, from the recorded dialogue relating to Task 1 in Table 6.14 on page 177, Thea uses Google Translate to find the meaning of ‘konstantledd’ in English during their discussion.

Code	Digital technology (Calculator: solving or computing mathematical calculations)
Task	Task 1
Context	Thea gives the figures needed for the calculations, and Kåre uses the calculator on his mobile phone for the computations.
Recorded dialogue	*Thea*: So, 8000 multiplied with 70? *Kåre*: [Uses the calculator on his mobile phone] 560000.
Image	

Table 6.7: Sample Data Aligned with Codes (Digital Technology—Calculator): Group A.

Concerning Task 2, and from the recorded dialogue in Table 6.8, the students used Google Maps to locate the optimal place to build the shopping centre. In the example, Thea sought the approval of her group to use Google Maps to solve the task. The group members agreed and mutually located the three cities on the map. This led to choosing an ideal location based on the three cities on the map, but more was needed as the task required that the optimal location should serve the three cities fairly.

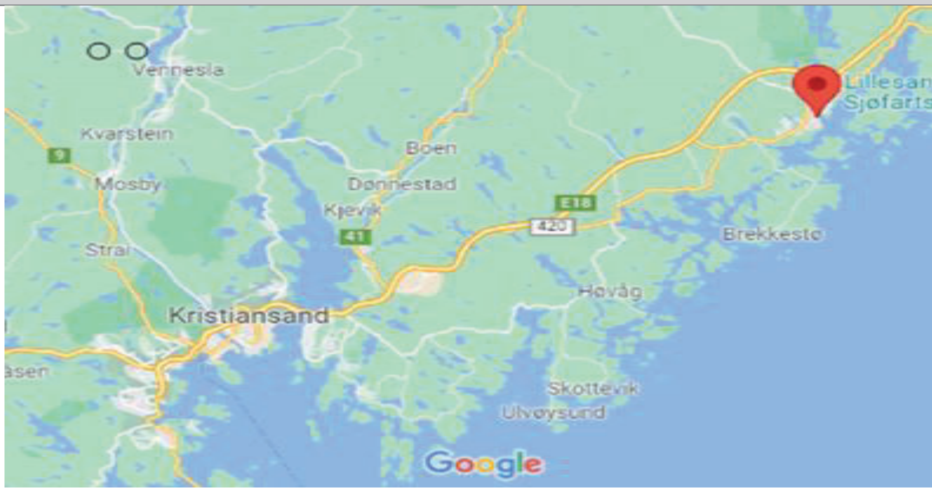
Code	Digital technology (Google Maps: Locating the positions of the three cities).
Task	Task 2
Context	Thea opens Google Maps after getting approval from the group members. The group then located the three cities on the map.
Recorded dialogue	*Thea*: Yes, a moment. Should we find the map, do you agree? *Rolf*: Yes. *Thea*: [Opens Google Maps, and searches for Lillesand]. Lillesand, my cousin has a cabin there. *Kåre*: So, this is Kristiansand and there is Vennesla.
Image	

Table 6.8: Sample Data Aligned with Codes (Digital Technology—Google Maps): Group A.

To solve the issue of fairness, the students looked at the population of each city. That is, the students used Google Search to find the population of each city after locating the cities on the map. In the example (see Table 6.9), Thea and the other group members searched for the population of Lillesand, Vennesla and Kristiansand. Google Maps and Google Search as mediating artefacts encouraged the students to make connections to real-life experiences. For instance, Thea, while searching for Lillesand with Google Maps, remembers her cousin's cabin in Lillesand, and this connection might have established some motivation towards solving this open-ended task.

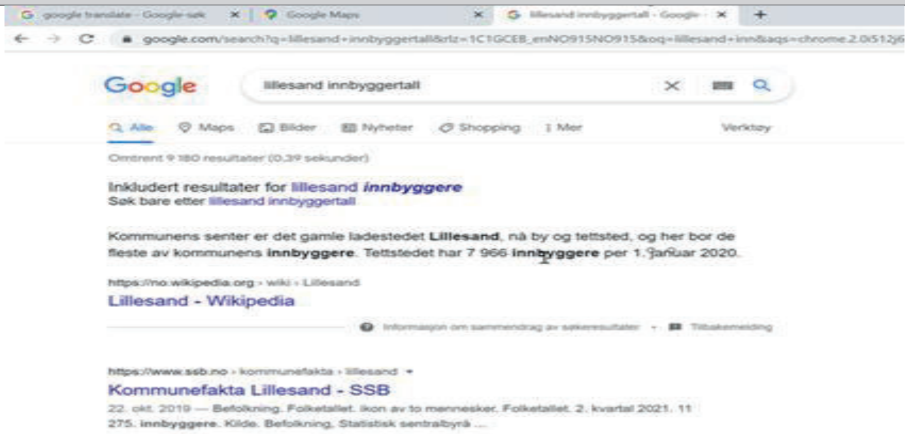
Code	Digital technology (Google Search: searching for the population of each city)
Task	Task 2
Context	Thea searches for the population of the cities on Google Search after locating each city on Google Maps.
Recorded dialogue	*Thea*: Yes, but if we are going to look in the population erm [Google Search; the population of Lillesand] *Kåre*: 7966. *Thea*: It's not so much. *Rolf*: Its 14000 in Vennesla, isn't it? *Thea*: Yes… they have many I think people living there in the summer also. *Rolf*: It's almost 15000. *Kåre*: 14935. *Rolf*: Oh my God… 111000.
Image	

Table 6.9: Sample Data Aligned with Codes (Digital Technology—Google Search): Group A.

I will now present a report on the dynamics of group interaction that occurred in Group A's activities.

Group Work

Group work is a mediational means in the student's activities. Different interactive processes occurred within the students' group work. The behaviour of one student is affected by that of another student in the social interactions that occur. Table 6.2 (in Sub-Section 6.1.2) presents the number of times the different interaction categories or sequences occurred in the episodes of the students' activities. From Table 6.2, the number of asymmetrical contingencies was seen more in the episodes of the Group A activities relating to Task 1 than in Task 2. In the asymmetrical contingencies that were identified in the student's activities, Thea was mostly leading the group through her ideas. At the same time, Rolf (not often) and Kåre were usually affirmative (only agreeing) and not critical (not reviewing and challenging). That is, Kåre and Rolf provided strong support within the group while much of the discussion was orchestrated by Thea. From Table 6.2, one episode describes reactive, and another describes mutual contingencies in the activities relating to Task 1. Thus, there were instances where the comments or ideas of each member were strongly influenced by the proceeding social stimuli and another instance where sense-making and conversation were mutually driven. In the activities relating to Task 2, four episodes describe reactive contingencies, and three describe mutual contingencies. A contingency of interest was where both pseudocontingency and asymmetrical contingency occurred within the same episode. Table 6.10 below shows a part of the transcription aligned with codes showing an example of both pseudocontingency and asymmetrical contingency taking place within the same episode of the students' activities. In Table 6.10, and from the recorded dialogue, we see Thea engaging with Kåre in an asymmetrical contingency, where Kåre is supportive and affirms the ideas of Thea. On the other hand, Rolf was interacting with the computer and not contributing to the discussion between Thea and Kåre. Three interactions occurred here: the interaction between Thea and Kåre (asymmetrical contingency), Rolf and the computer, and Rolf – computer and Thea – Kåre (pseudo contingency). The interaction between Rolf – computer and Thea – Kåre is individualized in the sense that each pair is unwilling to consider the other's suggestions for improvement or change.

Code	Asymmetrical contingency + Pseudocontingency
Task	Task 1
Context	Thea and Kåre were computing the product of 70 people and a price of 8000 with the calculator, while Rolf tried to find a faster way of finding the product with GeoGebra.
Recorded dialogue	*Thea*: Yes, if we try something higher, like this, 8000 per car. *Kåre*: Yeah, 8000 times the people. *Rolf*: [Turns the computer towards himself and starts to write f(g) = x*y at the algebra section in GeoGebra and gets the feedback "Ulovlig funksjon" (illegal function)]. *Thea*: Multiplied with 70 because erm we find out that erm y x is 70… oh no… *Rolf*: [Tried again for f(e) = x*y and gets the same feedback] I'm so confused. *Thea*: Are you kidding. *Kåre*: Is times 30. Try and put it on top, maybe. *Thea*: Yeah, so… but if we try the 70… then it's going to be 70… no! no.

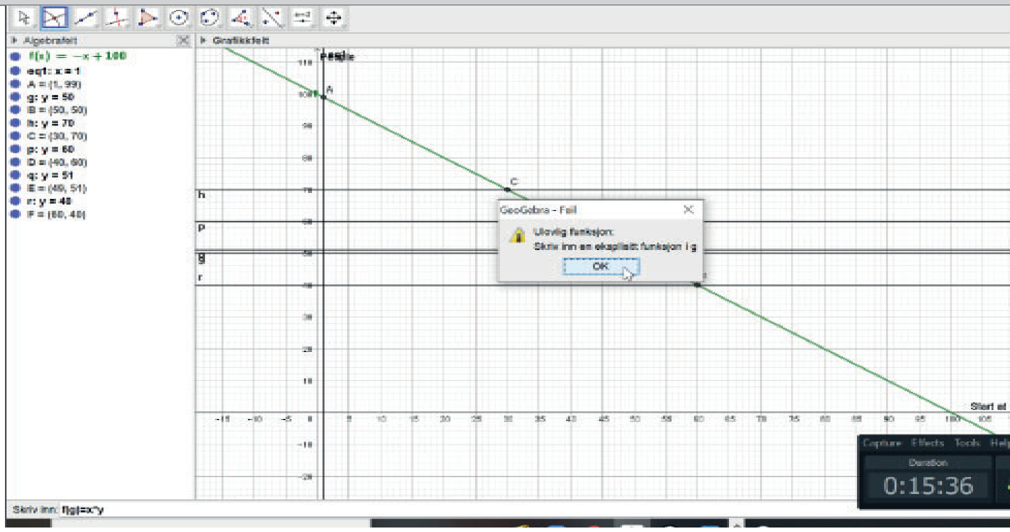
	Kåre: 240000 [Uses the calculator on his mobile phone to multiply 30 by 8000], which is way less than if you will do it with 50.
Image	

Table 6.10: Sample Data Aligned with Codes (Asymmetrical contingency + Pseudocontingency): Group A.

Rules for the subject-community interaction

The rules of the activity are sets of conditions that influence how/why the participants act within the activities. The rules of the activity could be explicit but often also implicit. Table 6.3 in Sub-Section 6.1.3 presents the explicit and implicit rules observed in the students' activities.

Explicit rule

One explicit rule in the students' activities was time constraints. The average time to solve a task during the practice phase (introductory activity) was 20 minutes. So, Group A was expected to use a similar time frame to solve Tasks 1 and 2. At a particular point in the students' activities, it was observed how time constraints impeded the solution process. That is, from Except 6.3.1 (on page 173), Thea dismisses Rolf's comments about using the spreadsheet to generate the data while working on Task 1, and the reason was that they are already close to finding the answer. This incident might result from time constraints (or students' preference for the solution strategy). Another explicit rule was that the students were allowed to use any digital technology available to solve both tasks (the technology was not imposed on them). As such, it was observed that the students used different digital technologies while working on both tasks (see Table 6.1 for the kinds of digital technologies used by Group A while working on Task 1 and 2, respectively). Again, the students were expected to work in a group on a single computer and produce a single group report, which they did (see Appendix D.1 for the solution report of Group A and the other groups in School A).

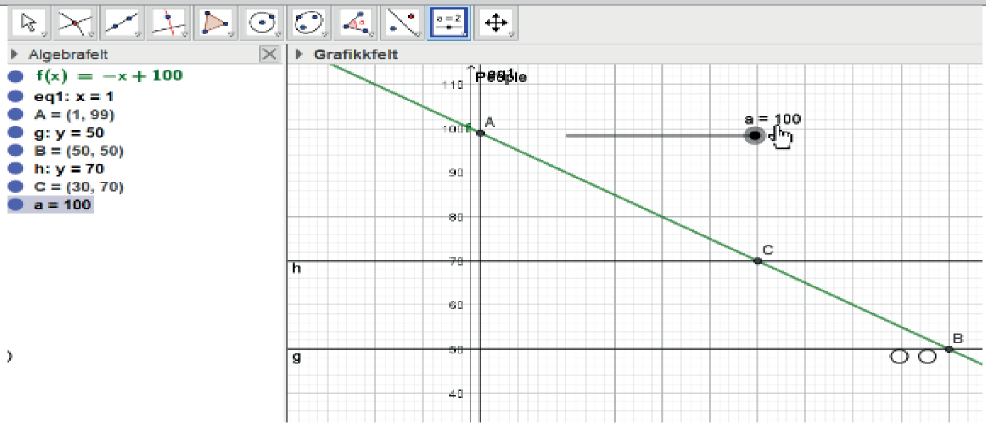
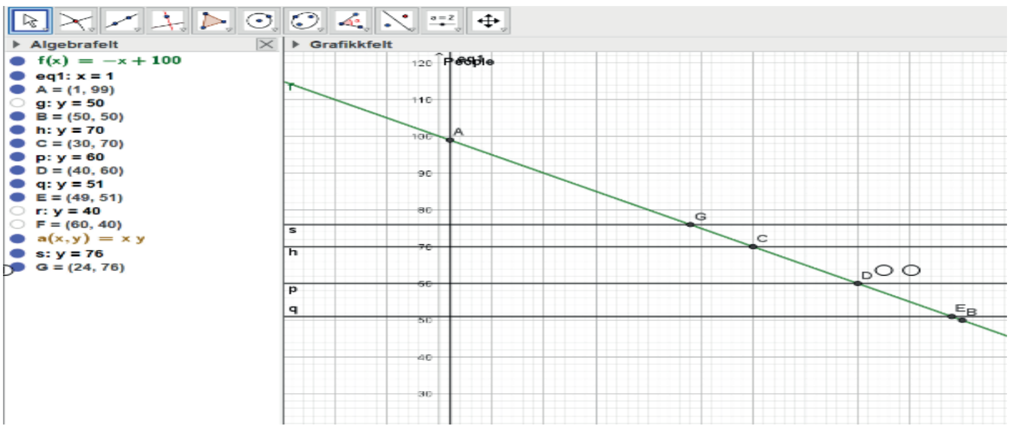
Code	Implicit rule
Task	Task 1
Context	A student thought of a faster way to find values but was unsure what to do. This thought triggered another student to suggest a procedure, but that did not materialize. The students then reverted to the previous way of finding the maximum price.
Recorded dialogue	*Rolf*: But isn't it like a faster way to find that out. I feel like there is, but I don't have any idea how to do it. *Thea*: We can make sliders, I think… We can try. *Kåre*: I don't know. *Thea*: Erm [Makes a slider a = 100, but the slider has no effect on the graph] … We just try something else [writes y = 60 on the graph and found the point of intersection with the line f(x) = - x + 100]. Here 40 multiplied with 100, 4000 so it's not more. So, I think we should try … *Rolf*: Try 100.
Image	 

Table 6.11: Sample Data Aligned with Codes (Implicit rule): Group A.

Implicit rule

Several implicit rules could be identified in the student's activities. However, an example of an implicit rule that was observed is the dismissing of comments or suggestions when they do not fit into the current strategy. For instance, Table 6.11

shows a part of the transcription aligned with codes showing a point where new ideas not fitting with previous ideas were dismissed as the students solved Task 1. In Table 6.11, from the recorded dialogue relating to Task 1, Rolf thought of the fastest way to find out the maximum revenue (but was not confident enough to put forward his thoughts), and this triggered Thea to come up with the idea of making sliders (see the first image in Table 6.11; $a = 100$). Thea made a slider, but it did not affect the graph as the slider was not well-defined. That is, the slider $(a = 100)$ has no link with the function $(f(x) = -x + 100)$. The students dismissed this new idea and reverted to the previous strategy of solving the task by testing their function with sixty people buying the car and the price at which they bought it (see the second image in Table 6.11).

I will now present a report on the different roles adopted by the students in Group A's activities.

Division of labour for the community-object interaction

Forms of distribution of actions and operations among the students were identified. The students had roles concerning tool usage that were constant throughout the activities (see Table 5.2). Other roles in the form of leading, opposing, suggesting, supporting, non-contributing, and questioning and challenging changed at different times during the student's activities. Table 6.4 (on page 160) gives the frequency of roles taken by each member of Group A while working on both tasks.

Roles adopted by students

Working on Task 1, Thea mostly took the leading role (dominating in the communications), and the group discourse was centred around her ideas, whilst Kåre mostly took the supporting role. Kåre mostly agreed with Thea on her ideas and sometimes questioned them if they were not unclear. Rolf took the non-contributing role in most of the episodes of the group's activities. Although Rolf was not contributing, he observed the group and sometimes came in to give a suggestion or support an idea. There were times when Rolf's suggestions were not accepted because he did not know how to put those ideas into effect, as the teacher describes him (under 'subject of the activity') as one with the highest mathematical understanding, and nonetheless has a little lower motivation. On the other hand, when Rolf knew precisely what to do or how to put his ideas into effect, the student dominating the group did not accept those ideas. In the build-up to this situation, Thea started with a problem-solving strategy that she was comfortable with, thus starting with graphical representation and later analyzing patterns of numbers and observing the increment in revenue. Rolf came in with an idea to efficiently generate the data, but Thea had personalized the problem-solving strategy; see the example below:

Rolf:	Oh! we could have done all of it with the 'regneark'(spreadsheet).
Thea:	Yeah, that's right.
Rolf:	And then just try with the
Thea:	We didn't think about it.
Rolf:	Or we just… I mean we can do it now; it might take a shorter time.
Thea:	Do you think?
Rolf:	I think so.
Thea:	But we are already done, though.

Excerpt 6.3.1

From Excerpt 6.3.1, Thea dismissed Rolf's comments. She returned to the existing idea, thinking they were already close to the answer (which might also be a time factor, described under 'rules for the subject-community interaction' on page 170). Subscribing to Rolf's suggestions might have helped the group generate their data with the spreadsheet and find a function representing it. The features of GeoGebra allow multiple problem-solving strategies. However, the approach used by the group depends on the representational choice of the student taking the leading role, especially when they think they are close to finding the answer. Working on Task 2, Thea mostly took the leading role and sometimes opposed the input of other group members. Kåre and Rolf mostly supported the ideas of Thea and sometimes made some suggestions during the group interactions.

Before presenting Group A's report on 'student's mathematical modelling activities', I presented a tabular data analysis overview (see Section 6.1) corresponding to the first research questions. I will similarly present a data analysis overview corresponding to the second and third research questions. After this overview, I will present Group A's report. I will first start with the report on the emerging modelling actions and the role digital technologies play in these actions.

6.3.2 Emergence of modelling actions and the role of digital technologies

The report here corresponds to RQ2a and RQ2b (see Section 1.4). Drawing on CHAT (Leont'ev's three hierarchical layers of an activity, Sub-Section 4.1.2), I analyze the modelling actions emerging within the episodes of the students' activities. From a CHAT perspective, solving mathematical modelling tasks using digital technologies is seen as a combination of actions and operations. Actions and operations are identified from the empirical data in the form of students' utterances, writings with paper-and-pencil, and engagement with digital technologies. The actions of the students are divided into categories. These categories are coded in Table 9.6 in Appendix E.2.

Tables 6.12 and 6.13 present the summary of results relating to the modelling actions (RQ2a) and the role of digital technologies (RQ2b) that emerged in the episodes of Group A, as they work on Task 1 and 2, respectively. The tables are presented in the form that characterizes students' mathematical modelling processes, just as Ärlebäck (2009) and Albarracin et al. (2019) used Modelling Activity Diagrams (MAD) to characterize students' choices and actions in a mathematical modelling activity. The presentation of the tables in this section follows the same structure. Thus, the first part of the table shows the modelling actions emerging within the episodes, and the second part shows the role of digital technologies within the modelling actions that emerged. The digital technologies identified under this subsection are the same ones listed in Sub-Section 6.1.1, but the focus here is on their role in the emerging modelling actions. In each table, I use the same colour (**X** or sometimes both **X** and **X** when there are different actions and corresponding roles of tools) to represent the modelling action and the corresponding role of digital technology. I also use **X** to represent the modelling action without any corresponding role of digital technology. The spaces marked by a dash (-) in the tables show that neither modelling actions nor the role of digital technology were identified in the particular episode. I will further explain Tables 6.12 and 6.13 with some excerpts from the video recording transcriptions and screen recordings as evidence.

Modelling actions	Episodes of the students' activity											
	1	2	3	4	5	6	7	8	9	10	11	12
Breaking the task into manageable parts	X	X	-	-	-	-	-	-	-	-	-	-
Searching for a model	X	X	-	-	-	-	-	-	-	-	-	-

Finding a solution for the model	-	-	X	X	X	X	X	X	X	X	X	X
Explaining the results in real terms	-	-	-	-	-	-	-	-	-	-	-	X
Checking the results for adequacy	-	-	-	-	X	X	X	X	-	X	-	X
Role of digital technologies												
Calculating (Mobile phone calculator)	-	-	-	-	-	-	X	X	X	X	X	X
Researching (Google Search)	-	X		-	-	-	-	-	-	-	-	-
Experimenting/Changing (GeoGebra)	-	-	X	X	X	X	X	X	X	-	X	X
Geometric construction (Geogebra)	-	X	-	-	-	-	X	X	-	-	-	-

Table 6.12: The modelling actions that emerged in Group A's activities (regarding Task 1) and the role of the digital technologies.

Modelling actions	**Episodes of the students' activity**								
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Breaking the task into manageable parts	X	X	-	X	-	X	X	-	-
Searching for a model	X	-	-	-	-	-	-	-	-
Finding a solution for the model	-	X	X	-	X	X	X	X	X
Explaining the results in real terms	-	-	-	-	-	-	-	-	X
Checking the results for adequacy	-	X	X	-	-	X	-	-	X
Role of technological tools									
Researching (Google Search & Maps)	X	-	-	X	-	-	X/X	-	-
Measuring (Google Maps)	-	-	-	-	-	X	X	-	-
Visualizing (Google Maps)	-	-		-	-	X	X	-	-

Table 6.13: The modelling actions that emerged in Group A's activities (regarding Task 2) and the role of the digital technologies.

I will present the analysis of the emerging modelling actions of Group A according to the categories in Tables 6.12 and 6.13. I repeat that these categories are just names used to report or describe the students' modelling processes, other than anything intrinsic in the ontology of modelling competence (see Sub-Section 2.1.3). I have explained my use of these categories in Sub-Section 4.4.2. I will present each category from both tables and the role digital technology played. These categories are interrelated, but I linearly present them.

Breaking the task into manageable parts

In both Task 1 and 2, Group A broke the task into manageable parts with the goal of understanding the problem text (real situation) or finding meaning in what had been read from the problem text. Under the category "breaking the task into manageable parts" in Table 6.12 (relating to Task 1) and 6.13 (relating to Task 2), there were different times in the episodes where the students performed an action of breaking the task into manageable parts. In Table 6.12, it was counted in episodes 1 and 2 (out of 12 episodes), while in Table 6.13, it was counted in episodes 1, 2, 4, 6 and 7 (out of 9 episodes). This shows that Group A went back and forth on the problem while working on Task 2 compared to Task 1. Tables 6.14 and 6.15 below show a part of the transcription aligned with codes showing specific points where participants perform the actions of breaking the tasks into manageable parts towards the goal of understanding or having a clearer view of what the task demands.

As stated earlier (from Table 6.12), the first two episodes describe a situation where the students first read the problem text and simplified it as they constructed relations between the variables identified while working on Task 1. The students simplified the problem by recognizing the quantities that influence the problem and then constructed a relation between the identified quantities in the form of a graph. The students build on this in the subsequent episodes of the students' solution process. In the students' discussions in the other episodes, they had to use Google Search to find a word's meaning at a certain point, as these students have English as their second language. Looking at the example in Table 6.14, Thea searched for the meaning of konstantledd (a word in Norwegian which means constantly articulated) as she explained the linear graph to Kåre. The digital technology here, which happened to be Google Search, was used to seek information about the word 'konstantledd'.

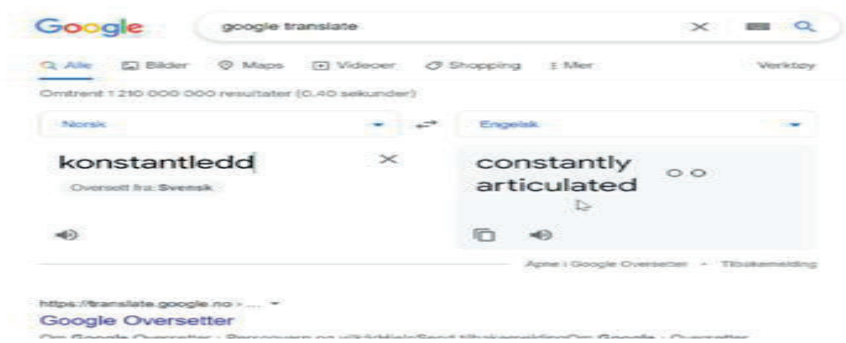
Code	Breaking the task into manageable parts
Task	Task 1
Context	The students made a function representing the number of people that will buy the car at a given price, and they searched for the meaning of 'konstantledd', the constant variable in the linear equation.
Recorded dialogue	*Thea*: [Opens GeoGebra] We going to GeoGebra… Should we say erm… on the y-axis, it's going to be 100 persons. *Kåre*: Like this [Points to the x and y axis in GeoGebra, draws a graph on paper and writes f(x) = 100x] *Thea*: Erm no, then you say that erm… its going down with a 1000… If you understand. *Kåre*: So, it will be naturally in there, right? 'Konstantledd' or something? *Thea*: Shouldn't 100 be the 'konstantledd'. *Kåre*: That's the thing you begin with. *Thea*: No, erm [Google Search the meaning of konstantledd: which means constantly articulated]. The constantly articulated is the last thing [meaning the constant variable in the linear equation]. *Kåre*: You right, that's what I meant, which is the last thing. So, we just need to find what we gonna write in there.
Image	

Table 6.14: Sample Data Aligned with Codes (Breaking the task into manageable parts): Group A—Task 1.

In the example in Table 6.15 relating to Task 2 below, the students were searching for the positions of the three cities on Google Maps in the act of recognizing quantities and simplifying the problem text. The students constructed a relation as Rolf pointed out that the cities' positions form a triangle. The students build on this idea as they analyze the positions of the three cities on Google Maps in the other episodes. In the other episodes of Group A's activities, the students perform new operations of seeking information and recognizing new quantities to better understand the demands of the problem situation. For instance, searching for the population of the three cities and the time of travel between the cities.

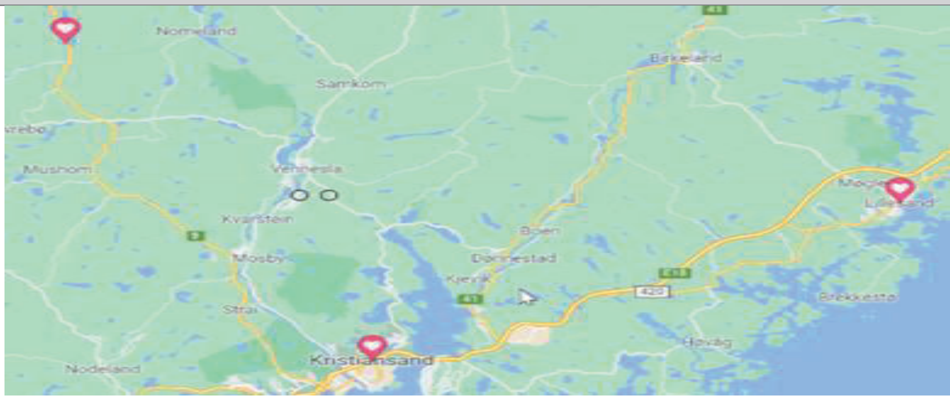
Code	Breaking the task into manageable parts
Task	Task 2
Context	The students suggested the optimal location after reading the mathematical modelling task. The students also located the three cities on Google Maps.
Recorded dialogue	*Kåre*: [Reads Task 2 aloud]. I don’t think it is supposed to be Lillesand. *Thea*: Yeah, it’s far away [laughs]. *Kåre*: Yeah, so it should be like above Vennesla somewhere. Can you show me where Lillesand is on the map? *Thea*: Yes, a moment. Should we find the map, do you agree? *Rolf*: Yes. *Thea*: [Opens Google Maps, and searches for Lillesand]. Lillesand, my cousin has a cabin there. *Kåre*: So, this is Kristiansand and there is Vennesla [Pointing to the map]. *Thea*: We save it [Saves Lillesand on Google Maps]. *Rolf*: It’s like a triangle then. Thea: Yes, it is [Searched and saved the locations Vennesla and Kristiansand on Google Maps].
Image	

Table 6.15: Sample Data Aligned with Codes (Breaking the task into manageable parts): Group A—Task 2.

Role of digital technology

Regarding Task 1, Group A used Google Search to seek information about a word (konstantledd) in their discussion. Regarding Task 2, Group A used Google Maps and Google Search to seek information about the identified variables.

I will now present the next category, ‘searching for a model’, that emerged in the activities of Group A.

Searching for a model

The students set up a mathematical model by translating a suitably simplified real situation into a mathematical form, such as equations, functions, diagrams, and others. Group A performed an action of searching for a model with the goal of setting up a mathematical model. The students performed this action in one or two of the episodes in the activities. In Table 6.12 (relating to Task 1), it was counted in both episodes 1 and 2 (out of 12 episodes), while in Table 6.13 (relating to Task 2), it was only counted in the first episode (out of 9 episodes). This shows that the students did not come back to search for a new model or update their model but went ahead with the initial model they searched for. Working on Task 1, Group A translated the problem text or situation into a mathematical problem and then represented the mathematical problem in the technological world. Thus, the students represented the mathematical problem in GeoGebra by drawing a function for the number of people buying the car at a given price. Drawing the function can be likened to searching for a model, which is directed towards the goal of setting up a mathematical model. See, for instance, the dialogue in Table 6.6 (on page 166). Thea suggested a function representing the number of people that buy the car, and the peers agreed with her suggestion. The suggested function was used in the model creation, and the students worked with this function in the subsequent episodes. On the other hand, the model that the students set up for Task 2 is described in Table 6.15 above. From the dialogue in Table 6.15, the students used Google Maps to set up their model by researching the positions of the three cities (which form a triangle). In the subsequent episodes, the students later analyzed the optimal place to build the shopping mall based on the positions of these cities.

Role of digital technology

Relating to Task 1, Group A used GeoGebra for geometric construction (a graphical representation of a function or equation) as they performed the action of searching for a model. Regarding Task 2, Group A used Google Maps to seek information about the positions of the three cities.

The following presentation is the report on the category ‘finding a solution for the model’ that emerged in Group A’s activities.

Finding a solution for the model

The students work mathematically as they apply heuristics strategies in the form of operations as they find a solution for the model they set up. Finding a solution for the model is an action that is directed towards the goal of solving the mathematical questions within the model or solving the model mathematically. The students performed this action in several of the episodes in the activities. In

Table 6.12 (relating to Task 1), it was counted in 10 episodes (out of 12 episodes), while in Table 6.13 (relating to task 2) it was counted in 7 episodes (out of 9 episodes). This shows that many students' actions were about finding a solution for their model. Tables 6.16 and 6.17 below show a part of the transcription aligned with codes showing examples where participants find solutions for the model towards the goal of solving the mathematical questions within the model.

To find the highest revenue of the car-selling company, Group A used the trial-and-error method by analyzing patterns of numbers after finding a function that represent the number of people buying the car. Looking at the example in Table 6.16, Thea suggested they try 80 people buying the car to see if that gives them the maximum revenue. They found the corresponding price of the car to be 7000 euros and later multiplied that by the number of people using the calculator.

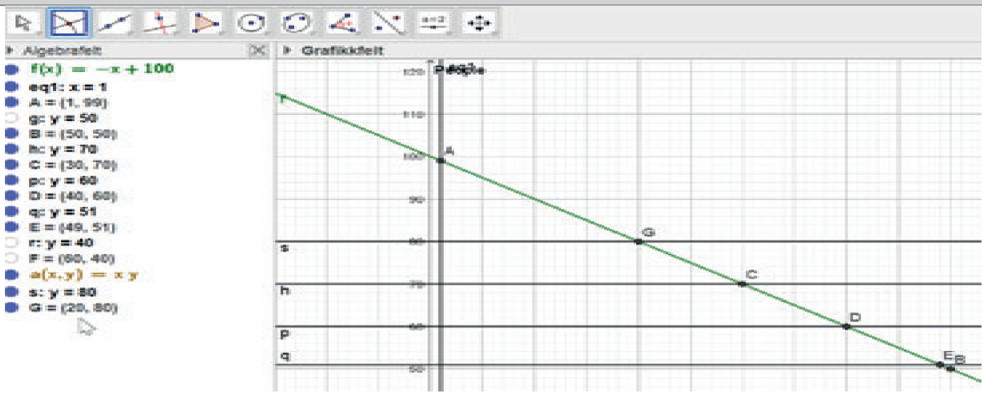
Code	Finding a solution for the model
Task	Task 1
Context	The students test their function with 80 people buying the car and later compute the total revenue (the product of the number of people and the price of the car).
Recorded dialogue	*Thea*: Its lower, so we should try 80 [writes y=80 in GeoGebra] for example. *Kåre*: Yeah. *Thea*: Do you agree with the way we are doing it? *Rolf*: Yeah. *Thea*: Its 20, so erm 7000 *Kåre*: Times? *Thea*: Erm times 80. *Kåre*: [Uses a calculator on mobile phone to compute 80*7000] it's the same. *Thea*: It's the same! Is it? *Kåre*: I think it is. *Thea*: So, if we try 85
Image	

Table 6.16: Sample Data Aligned with Codes (Finding a solution for the model): Group A—Task 1.

In working on Task 2, the students solve the mathematical questions within the model through the analysis of the model by reconciling their model with reality to arrive at a better model. The students did not solve the mathematical questions within the model in Task 2 by observing the effects of parameters on the graph or computing mathematical operations as they did in Task 1. They did so by analyzing the model while considering and comparing distances, travel time and the environment around the three cities. Looking at the example in Table 6.17 (relating to Task 2), the students considered the environment around the chosen optimal place, not just the geographical middle of the three cities. The students then settled on two optimal locations.

Code	Finding a solution for the model
Task	Task 2
Context	The students analyzed the position of the optimal place for the shopping mall on Google Maps. They also switch to the satellite on Google Maps to visualize their choice of the optimal place.
Recorded dialogue	*Thea*: But if we are trying to look at this task, I think we should not have… I don't think we should place it in the middle geographic, because here [Point at the area close to Kjevik] its very many woods and things like that and Tveit is here [pointing to the place between Ryen and Dønnestad]. *Rolf*: You can turn on the satellite. *Thea*: Yeah [Switched to the satellite mode] … So, almost no one is living here [Points at the area around Kjevik]. *Kåre*: Yeah.
Image	

Table 6.17: Sample Data Aligned with Codes (Finding a solution for the model): Group A—Task 2.

Role of digital technology

Relating to Task 1, Group A used GeoGebra to experiment/change the numbers as they looked for the maximum revenue whilst using the calculator to compute the product of the number of people and the corresponding car price. Regarding Task 2, Group A used Google Maps to research the three cities, visualize the environment, and measure the distances between the cities. The students also used Google Search to research the population of all the cities.

Explaining the results in real terms

Interpreting the mathematical results in real situations involves translating the mathematical solution into the real solution. In this case, students might take the mathematical aspects or results and re-evaluate them regarding the real-world problem. Explaining the results in real terms is an action directed towards the goal of interpreting the mathematical results in a real situation, and the students performed this action once in the activities. In Tables 6.12 and 6.13 (regarding Tasks 1 and 2, respectively), the category 'explaining the results in real terms' was counted in the last episode (out of 12 and 9 episodes, respectively). This shows that the students performed this when they were sure of their mathematical results. Tables 2.18 and 6.19 below show a part of the transcription aligned with codes showing examples where participants explained their results in real terms.

In finalizing their results, Group A wrote a report where they interpreted the mathematical results in real terms. From the recorded dialogue in Table 6.18, Thea wrote a report with input from her peers. Group A explained that the maximum revenue would be 562500 euros if the car-selling company sold the car at 7500 euros for 75 people. Group A further explained how they arrived at the result by showing the linear graph they used. On the other hand, from Table 6.19 (regarding Task 2), the students reconcile their results with reality, giving meaning to their results. They explained their analysis of how they chose the optimal location based on the distance, travel time, and not just the geometrical centre of the three cities. Ultimately, the students settled on two optimal locations, giving some reasons.

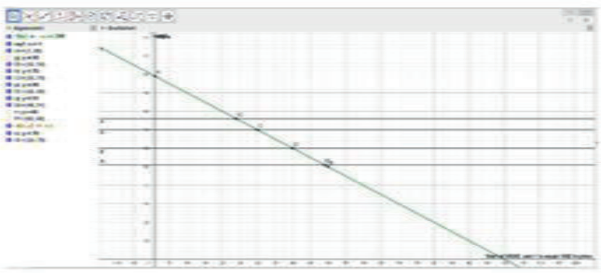
Code	Explaining the results in real terms
Task	Task 1
Context	The students verified their answers after finding that the company gets the maximum revenue if 75 people buy the car at 7500. They also wrote a final report after their findings.
Recorded dialogue	Thea: Yeah, we can try one higher and one lower to be absolutely sure [changes y = 74 at the algebra section], erm 26, erm 7600 multiplied with 74. Kåre: [Uses the calculator on his mobile phone] its just 100 less. Thea: Yes, and one higher [changes y = 76 at the algebra section]. Kåre: How much? Thea: Its… 24 erm… Rolf: 7400 Thea: Yeah, thanks. Kåre: Times? Thea: Erm 76 Kåre: [Uses the calculator on his mobile phone]. Thea: Its lower, so that's the best, the price is 7500 and its 75 people buying it, then the company is getting most money out of this price…… Should we go to the next one? Kåre: Yeah Thea: [Writes the report for Task 1]
Image	Task 1  The maximal revenue would be if the company sold each car for 7500£ for 75 people and the end income would be 562 500£. We found out of this when we made a linear graph, f(x)=-x+100. On the x-axes was the 0, 5000£ and then when x is 1 it meant that we multiplied 1 by 100£ and then pluses 5000£ with 100£. And the y-axes were people.

Table 6.18: Sample Data Aligned with Codes (Explaining the results in real terms): Group A—Task 1.

Below is the written text by Group A in Table 6.18 (the 'image' section):

<u>The text under the graph</u>

The maximal revenue would be if the company sold each car for 7500£ for 75 people and the end income would be 562 500£.

We found out of this when we made a linear graph, $f(x) = -x + 100$. On the x-axes was the 0, 5000£ and then when x is 1 it meant that we multiplied 1 by 100£ and then pluses 5000£ with 100£. And the y-axes were people.

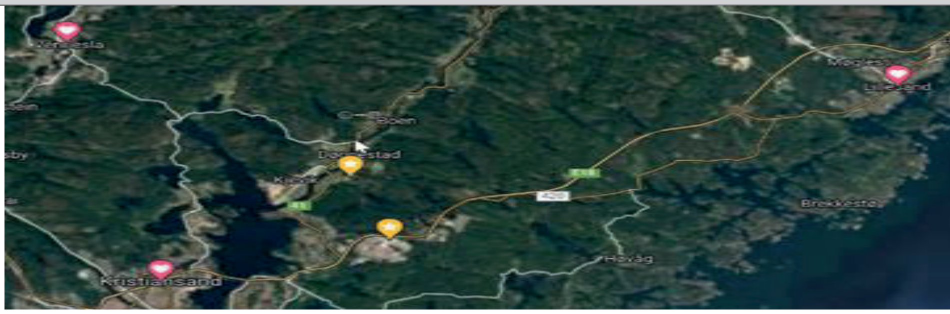
Code	Explaining the results in real terms
Task	Task 2
Context	The students were discussing their final ideas about the position of the optimal place. They also wrote a report after agreeing on where to place the shopping centre.
Recorded dialogue	*Thea*: Is very center. But should we put it closer to Lillesand? *Rolf*: No, Lillesand can just chill up there. *Kåre*: Why shouldn't we put it there [Pointing at the area around Kjevik on the map]. *Thea*: Because it's very much sound from the airport. *Rolf*: We should place the airport somewhere else. *Kåre*: Right [laughs]. *Thea*: [Saves two possible locations in yellow on the map]. *Rolf*: [Writes the final report with input from peers].
Image	 Our first location is on the mega center that already exist which is Sørlandssenteret. The center is located good in terms of population. It is easy to drive to the Sørlandssenteret from all the cities. It seems far away from Lillesand, but the highway (I mean E18) makes it much easier for drivers to come to our new mega center. If we don't have the opportunity to place it on a location where a mega center already exist, then we would have placed it near where Kjevik is. We located our mega center at the other side of Kjevik so that people will not get disturbed by the noises of the airplanes from the airport. There is also a camping place and a beach nearby. People from these camping places can visit the mega center easily which will help the tourism to grow faster in Agder. It is also easy to find where Kjevik is because of the airport which is popular.

Table 6.19: Sample Data Aligned with Codes (Explaining the results in real terms): Group A—Task 2.

Below is the written text by Group A in Table 6.26 (the 'image' section):

<u>The text under the map</u>

Our first location is on the mega center that already exist which is Sørlandssenteret. The center is located good in terms of population. It is easy to drive to the Sørlandssenteret from all the cities. It seems far away from Lillesand, but the

> highway (I mean E18) makes it much easier for drivers to come to our new mega center. If we don't have the opportunity to place it on a location where a mega center already exist, then we would have placed it near where Kjevik is. We located our mega center at the other side of Kjevik so that people will not get disturbed by the noises of the airplanes from the airport. There is also a camping place and a beach nearby. People from these camping places can visit the mega center easily which will help the tourism to grow faster in Agder. It is also easy to find where Kjevik is because of the airport which is popular.

Role of digital technology

Regarding Tasks 1 and 2, Group A wrote their final report on a Word Document (not included in the list of digital technologies). The other groups that worked alongside Group A also wrote their final report in a Word Document (see Appendix D.1 for the solution reports of all the groups in School A).

I will now present the final category, 'checking the results for adequacy', that emerged in the activities of Group A.

Checking the results for adequacy

The solution validation involves checking the results in the situation model for adequacy, which is done within the solution process and/or at the end of the solution process. To check the results for adequacy, the students might reflect on other ways of solving the problem and critically check their final results. Checking the results for adequacy is an action directed towards validating the solution. The students performed this action in several of the episodes in the activities. In Table 6.12 (relating to Task 1), it was counted in 6 episodes (out of 12 episodes), while in Table 6.13 (relating to Task 2) it was counted in 4 episodes (out of 9 episodes). This indicates that at different times (relating to both Task 1 and 2) in the activities, the students checked their results (either initial or final results). For instance, from the recorded dialogue in Table 6.18 (regarding Task 1) above, for Group A to be sure of their answer, Thea suggested they try a number higher and another lower than the number that yielded the maximum revenue. There were also instances where Rolf, a member of Group A, suggested other ways or strategies of solving the task after reflecting on the group's initial strategy (for example, see Excerpt 6.3.1 on page 173 or the dialogue in Table 6.10). Concerning Task 2, there were instances in Group A's activities where the students critically reflected on the optimal locations suggested by their peers. For instance, looking at the dialogue in Table 6.17, the students had to switch to the satellite on Google Maps to view the surroundings of the proposed optimal location before drawing any conclusion.

Role of digital technology

Regarding Task 1, Group A used GeoGebra to experiment/change the numbers whilst using the calculator to compute the product of the number of people and the corresponding car price as they validated their results. Group A also used GeoGebra to construct a new function geometrically after critically reflecting on the solution strategy.

I will now present a report on the affordances and constraints of the digital technologies that emerged in the activities of Group A. As I did with the emergence of modelling actions, I will similarly present a data analysis overview corresponding to the third research question before presenting the report.

6.3.3 Emergence of technological, mathematical and socio-cultural affordances and constraints

The report here corresponds to RQ3 (see Section 1.4). The frequency of actualized technological, mathematical and socio-cultural affordances and constraints that emerged in Group A's activities regarding Tasks 1 and 2 are presented in Table 6.20 and 6.21, respectively. I analyze the affordances and constraints emerging within the students' activities by drawing on CHAT (Leont'ev's three hierarchical layers of an activity, Sub-Section 4.1.2). Solving mathematical modelling tasks with digital technologies is seen as a combination of actions and operations from CHAT perspective. Actions and operations are identified from the empirical data in the form of students' utterances and engagement with digital technologies.

Tables 6.20 and 6.21 present the summary of results relating to the frequency (or number of counts) of actualized technological, mathematical and socio-cultural affordances and constraints that emerged in the episodes of Group A, as they work on Task 1 and 2, respectively. The presentation of tables in this section follows the same structure. The first part of the tables presents the technological affordances and constraints, the second part presents the mathematical affordances and constraints, and the third and final part presents the socio-cultural affordances and constraints. The technological affordances identified under this subsection are listed in Tables 6.12 and 6.13 (roles of digital technologies). However, the narrative of the results is done in the Affordance Theory perspective. The categories of affordances and constraints emerging are coded in Table 9.7 (in

Appendix E.3) for the analysis. The spaces marked by a dash (-) in the tables show the technological, mathematical and socio-cultural affordances and constraints that are not actualized in Group A's activities. I will further explain Tables 6.20 and 6.21 with some excerpts from the video recording transcriptions and screen recordings as evidence.

Affordances & Constraints	Digital Technologies				
	GeoGebra	Excel	Calculator	Google search	Google maps
Technological Affordances					
Calculating	-	-	10	-	-
Researching	-	-	-	1	-
Geometric construction	4	-	-	-	-
Experimenting/Changing	11				
Mathematical Affordances					
Clarification	-		-	1	-
Linking representations	4	-	-	-	-
Regularity and variations	11	-	-	-	-
Arithmetic and statistics	-	-	10	-	-
Socio-cultural Affordances					
Common focus	7				
Observing and improving strategies	3	-	-	-	-
Authority of the digital technology	1	-	-	-	-

Table 6.20: The frequency of actualized technological, mathematical, and socio-cultural affordances that emerged in Group A's activities regarding Task 1.

Affordances & Constraints	Digital Technologies				
	GeoGebra	Excel	Calculator	Google search	Google maps
Technological Affordances					
Researching	-	-	-	2	7
Measuring	-	-	-	-	3
Visualizing	-	-	-	-	2
Mathematical Affordances					

Analyzing	-	-	-	2	9
Socio-cultural Affordances					
Common focus	-	-	-	1	4

Table 6.21: The frequency of actualized technological, mathematical, and socio-cultural affordances that emerged in Group A's activities regarding Task 2.

I will present the analysis of Group A's emerging affordances and constraints according to the categories listed in Table 6.20 and 6.21. These categories are just names that report or describe emerging affordances and constraints. I have explained these categories in Sub-Sections 4.3.1 and 4.3.2. Again, the categories are interrelated, but I linearly present them.

Technological affordances and constraints

The operations performed by the students are characterized by the usability features of the digital technologies, and these operations are done at the subconscious level (see Sub-Sections 4.3.3 and 4.1.2). Digital technologies afford the students different ways in their solution processes as they interact with the digital technologies while working on Tasks 1 and 2. The leading digital technologies Group A used while working on Task 1 were Google Search, GeoGebra and a calculator on the mobile phone. Group A also used Google Maps and Google Search while working on Task 2 (see Table 6.1). From Tables 6.20 and 6.21, the technological affordances and constraints recorded are researching, measuring, visualizing, geometric construction, experimenting/changing, data entry and generation (ø – representing unidentified categories in the students' activity), and calculating. Affordances do not happen in isolation; they go along with constraints (see Section 4.3). The categories listed above appear to be affordances, but I consider them both affordances and constraints. For instance, GeoGebra affords a construction of a function and also has the constraint that an undefined function cannot be constructed. That is, GeoGebra can construct a function if one can define this function. I will now present each of the categories of technological affordances and constraints (the argument here also applies to mathematical and socio-cultural affordances and constraints in this section and elsewhere):

Researching. Google Search used in Task 1 allows retrieving information on the internet for one of the variables in the proposed equation. From Table 6.14 (on page 177), Thea searched the meaning of 'konstantledd' as Kåre thought the equation should be $f(x) = 100x$. Thea then used the translation retrieved from Google Search to explain that the constant value should be 100 and, therefore, the last variable in the equation (that is, $f(x) = -x + 100$). Regarding Task 2, Google Search and Maps allow the retrieval of information about the population and position of the cities, respectively (see Tables 6.9 and 6.15, on pages 168 and 178, respectively). Thus, Group A used Google Maps to search for the position of the three cities and Google Search to retrieve information about the cities' population.

Measuring. Google Maps afforded the measuring of time of travel between the cities, and the dialogue in Table 6.22 below shows how Thea measures the time of travel between Lillesand and Kristiansand. These measurements influenced Group A's choice of the optimal position for the shopping centre.

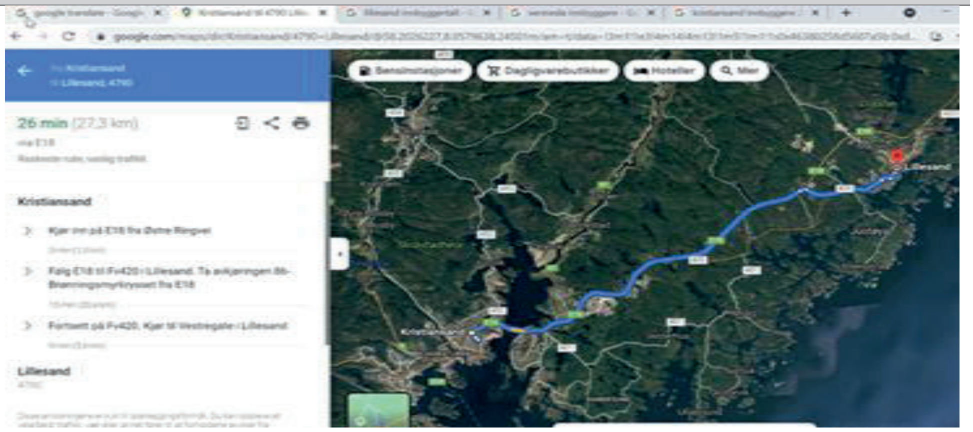
Code	Technological affordances
Task	Task 2
Context	The students measured the travel time between the cities as they analyzed the best position for the shopping centre.
Recorded dialogue	*Thea*: Its faster… I think it is not so long to go from Lillesand to Kristiansand with the new highway [Searched for the minutes and distance of travel between Lillesand and Kristiansand and records it]. *Kåre*: 26 minutes. *Rolf*: Its nothing, you can just drive there. *Kåre*: It is just above what it will take from Vennesla to Kristiansand.
Image	

Table 6.22: Sample Data Aligned with Codes (Technological affordances): Group A—Task 2.

Visualizing. Google Maps afforded Group A the visualization (see the dialogue in Table 6.17 on page 181) of the location of the three cities as they turn on the satellite. With this functionality of Google Maps, the students could access the environment surrounding their choice of the optimal place for the shopping centre.

Geometric construction. GeoGebra afforded the drawing of a function from the identified variables while Group A worked on Task 1. Thea drew the function $f(x) = -x + 100$ with GeoGebra, representing the number of people buying the car. Based on this function, they found the maximum revenue for the car-selling company (see the dialogue in Table 6.6 on page 166). In other instances, GeoGebra affords the drawing of functions. However, these functions did not materialize as they did not yield the expected results, although the students perceived there could be a function that could give them a much faster result. For instance, in the recorded dialogue in Table 6.10 (on page 170), Rolf tried to key in the function $f(g) = x * y$ but got an 'illegal function' feedback, meaning he had to key in the function g before there could be a result. It is impossible to draw an undefined function in GeoGebra; that is a constraint. This constraint helps the students find ways to define a function properly to attain a desired result. The students perceived that they could find the product of the number of people buying the car and the selling price of the car. However, that did not materialize as they could not define the function. GeoGebra could afford the drawing of the function if the students could key in the correct variables. At another point, they keyed in the correct variables for the function that defines the car's selling price. However, they still did not attain the desired results. From the recorded dialogue in Table 6.23 below, Rolf keyed in function $y = 100x + 5000$ as he taught about the car's selling price. However, he did not attain the desired results as no function was defined to combine the two functions $f(x) - x + 100$ and $y = 100x + 5000$.

Code	Technological affordances
Task	Task 1
Context	A student suggested a function for the price of the car but did not know how to combine the function with the function that describes the number of people who buy the car.

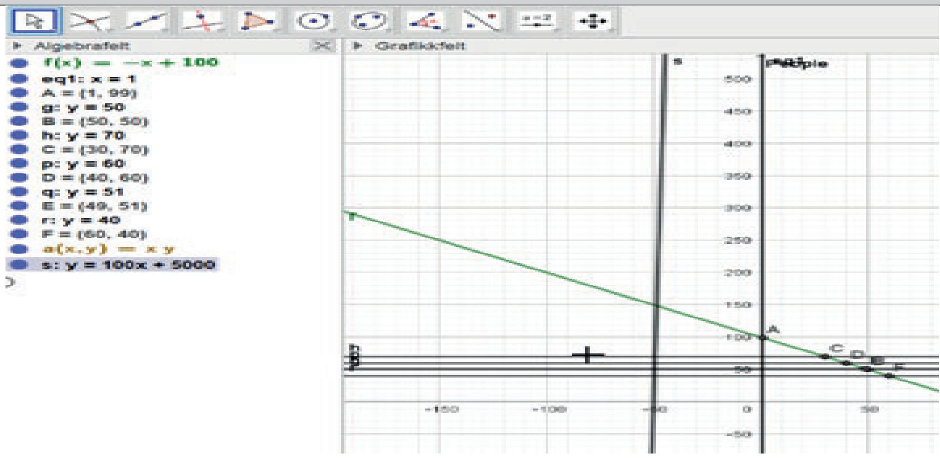
Recorded dialogue	*Rolf*: [Writes the function y = 100x + 5000 in the algebra section and reduces the size of the graph]. *Thea*: So, we should go over 50 and… or between 100 people and 50 people applies …What have you done? *Rolf*: I just wanted to draw a new graph so that we can maybe take erm… I don't think it's right, cos… it can be over the border, I mean go over 100. There might be more money… I just forgot it actually. *Thea*: I don't understand the graph.
Image	

Table 6.23: Sample Data Aligned with Codes (Technological affordances): Group A—Task 1.

Experimenting/Changing. GeoGebra afforded the students the changing of parameters of the function ($f(x) = -x + 100$) as they observed its effects while working on Task 1. The students tried out different numbers while finding the maximum company revenue. The students often used this 'experimenting' procedure with different numbers to find the maximum company revenue. In one instance, from the recorded dialogue in Table 6.11 (on page 171), the students could not find a slider as an efficient way of finding the correct value instead of keying the numbers one after the other, even though they perceived that the features of GeoGebra could afford them that. This was because there was no clear definition for the slider they made; hence, it did not affect the function as supposed.

Data entry and generation. This category was not identified in the student's activities in both tasks.

Calculating. The calculator on the students' mobile phones afforded the calculation of the product of the number of people and the corresponding price of the car (concerning Task 1). From the recorded dialogue in Table 6.16 (on page 181), Kåre calculates the product of 80 people buying the car at a selling price of 7000 euros using the calculator on his mobile phone. Group A used this process repeatedly until they arrived at the desired value. That is, changing the values in GeoGebra and computing the product of the values with a calculator.

I will now present the report on the mathematical affordances and constraints that emerged in the activities of Group A.

Mathematical affordances and constraints

The technological affordances and constraints of the digital technologies at the operational level support the mathematical affordances emerging at the action level, which are done at a conscious level. Several mathematical affordances emerged as the students interacted with digital technologies while working on Tasks 1 and 2. Technological affordances and constraints provide support for the mathematical affordances and constraints; that is, students' actions in the solution process are conditioned by the features of the digital technologies at the operational level. From Tables 6.20 and 6.21 (on pages 187 and 188), the mathematical affordances and constraints are clarification, analyzing, simulating and visualizing (ø), linking representations, regularity and variations, and arithmetic and statistics.

Clarification. From the recorded dialogue relating to Task 1 in Table 6.14 (on page 177), Thea uses the translation of 'konstantledd' retrieved from Google Search to explain the terms in the equation that was put forth. The retrieving of information or the meaning of a mathematical term during a group interaction might help bring out the understanding of a mathematical concept in a mathematics discourse.

Analyzing. The primary mathematical task level affordance that emerged during the solution process of Task 2 was analyzing. To create a model, the students used Google Maps to locate all three cities. The position of the three cities on the map forms the basis of the analysis (see Table 6.8 on page 167). The students also considered the distances and travel time between the cities during the analysis. Thus, the time of travel played a significant role as they chose the optimal place to build the shopping centre. Another factor considered in the analysis in deciding on the optimal place was the number of people living within the cities. Google Search allows the students to search for factual information about the number of people living within the cities (see Table 6.9 on page 168). With this information, the students decided on the city to which the optimal place should be close.

Simulating and visualizing. This category was not identified in the student's activities in both tasks.

Linking representations. Working on Task 1, the students moved between a function/equation and a graphical representation (see Table 6.6 on page 166). In the graphical representation view in GeoGebra, the students saw the increasing and decreasing features of the function as they searched for the maximum revenue of the car-selling company. In another case (see Table 6.23 on page 191), they moved between an algebraic equation and a graphic representation of this equation (as they constructed a new function). The constraint here was that the students could not connect the previous function ($f(x) = -x + 100$) and the new function ($y = 100x + 5000$) or manipulate them, although GeoGebra can afford them that, but they did not perceive it. Again, GeoGebra gives feedback, which, if followed carefully, could help derive or form the desired function. Looking at Table 6.10 (on page 170), the students got the feedback 'illegal function' when they tried to find the product of the number of people and the selling price of the car. Following the feedback, another instruction was that one of the variables should be defined. Drawing students' attention to such instructions might help manipulate functions/equations and foster mathematical thinking in mathematical discourse.

Regularity and variations. The students explored the regularity and variations in the solution model by experimenting with their functions with different numbers. That is, the students observed the effect of the changed parameters on the graph as they searched for the maximum revenue of the car-selling company (see Table 6.16 on page 181). This step was repeated until they found the desired result. In another instance, during the solution process, the students use sliders in GeoGebra to vary the parameters to see the effects on the graph (see Table 6.11 on page 171). This could have been an effective way of observing the changes in value compared to the keying of numbers, one after the other. However, the slider did not have a clear definition and did not affect the graph.

Arithmetic and statistics. The calculator on the students' mobile phones at the mathematical task level afforded numerical computations such as multiplication.

The students could compute numerical computations manually when the numbers are small but use the calculator for more significant numbers (see Table 6.16 on page 181).

I will now present the report on the socio-cultural affordances and constraints that emerged in the activities of Group A.

Socio-cultural affordances and constraints

At the activity level, socio-cultural affordances emerged in the joint mathematical discourse or interactions that occur through group collaboration. The interaction between the students and the digital technologies induced affordances at a collective level. The digital technologies stimulated cooperation between the students as they worked on the mathematical modelling tasks. The socio-cultural affordances that emerged or actualized in the students' activity were common focus (in both Task 1 and 2), observing and improving strategies (only in Task 1), and authority of the digital technology (only in Task 1) (see Tables 6.20 and 6.21 on pages 187 and 188, respectively).

Common focus. Working on Task 1, the students shared the same computer and had the facility to look at the same thing and point at what was presented on the computer. The students negotiated and agreed on the function ($f(x) = -x + 100$) and its graphical representation. They agreed on a shared goal through a flow of turn-taking, dialogue and action. To visually demonstrate their reasoning to one another, they used GeoGebra as a reference tool by looking at the coordinate axis and sketching with paper-and-pencil ($f(x) = 100x$) in relation to the coordinate axis (see Table 6.6 on page 166). To respond to one another, they needed to interpret and evaluate the visualized ideas, which Thea came up with the idea that 100 should be the constant value. Thea, responding to the initial function proposed by Kåre, used GeoGebra for reference in anchoring her proposition by suggesting the function $f(x) = -x + 100$. Rolf and Kåre accepted by discussing and evaluating the proposed function (see Table 6.6). Similar dialogue was observed in some of the episodes of the students' collaborative work, and the students used GeoGebra for reference to visualize their reasoning during the mathematical discourse. Working on Task 2, the students used both Google Search and Google Maps as they analyzed the population, distance and time of travel between the cities. These digital technologies helped the students in creating a shared goal. For instance, in Table 6.9 (on page 168), we see Thea searching for the population of

Lillesand and Kåre communicating the search results. That is, the technology created a platform for the students to look at the same thing and communicate about it. Again, Google Maps served as a reference tool as the students analyzed the positions of the three cities. From Table 6.15 (on page 178), we see Thea searching for the position of the cities on the map, Kåre pointing at the cities on the screen, and Rolf visualizing his reasoning by joining the points of the cities by hand and concluding that it will form a triangle.

Observing and improving strategies. Digital technologies could be used to maintain and improve shared knowledge and ideas at a collective level. There were instances where the students found themselves in a situation where they wanted to find an efficient way of finding the maximum revenue of the company instead of going through the numbers or values one after the other. In one example (see the recorded dialogue in Table 6.11 on page 171), Rolf thought of the fastest way to find out the maximum revenue, which made Thea come up with the idea of making sliders. The students made a slider, but this did not materialize as GeoGebra has the constraint that the slider must be well-defined to affect the function. Their solution strategy could have changed if the students could define this slider. The students had similar dialogue in other instances where they tried to make a new function (but it was not well-defined). At a time when the function was well defined, they were not able to manipulate or put together the two functions (see Table 6.23 on page 191). There was no divergence in the students' solution process. However, their solution needed improvement to be more efficient. Thus, the students did not get into a situation marked by uncertainty or diverging from their strategy in the solution process, where they might need the digital technology to verify knowledge or settle disagreements by performing some tests with the technology.

Authority of the digital technology. From Excerpt 6.3.1 (page 173), Rolf proposes using Excel/spreadsheet to generate the data efficiently. At the same time, Thea continues with the previous or existing strategy as she has personalized the problem-solving strategy. Thea's strategy starts with a graphical representation and later analyses patterns of numbers and observes the car-selling company's revenue increment. Thea dismisses Rolf's comments and goes back to the strategy described above, thinking they are already close to the answer (which might also be a time factor, as discussed under 'Rules for the subject-community interaction' on page 170). Subscribing to Rolf's suggestions might have helped the group generate their data with the spreadsheet and possibly find a function representing this data. The features of GeoGebra allow multiple problem-solving strategies. However, the approach used by the group depends on the representational choice of the students taking the leading role (discussed under 'Division of labour for community-object interaction' on page 172), especially when they think they are close to finding the answer.

In summary, I have reported on the activities of Group A along three themes. That is, the students' (or Group A's) mathematical modelling activities, the modelling actions emerging in Group A's activity and the role digital technology plays, and the affordances and constraints of the digital technologies emerging in Group A's activities. In the following sections, I will present the case study reports of the other groups (Group B, C and D). For the reader, I repeat that in the following sections, I will only present issues different from Group A (since the report on Group A is a detailed report). I will not repeat some expressions used in Group A's report in the other reports. For instance, the expression "activity system, hereafter referred to as activity" (see the first paragraph in Sub-Section 6.3.1 on page 163). In chronological order, I will first begin with Group B's report.

6.4 Case study report: Group B (Emil, Thor, Ella and Tore)

This section describes the second school's case study report (Group B; see Sub-Section 3.4.2). The narrative is presented in an order as highlighted in Section 6.2. Group B consists of first-year upper secondary school students (Grade 11). Group B comprises four students aged between 16 and 17 years. The reports in Sub-Sections 6.4.1, 6.4.2 and 6.4.3 correspond to RQ1, RQ2a & RQ2b and RQ3 respectively. I will first start the report on the students' (Group B's) mathematical modelling activities.

6.4.1 Students' mathematical modelling activities

Details about Group B are presented in Sub-Section 3.4.2. I will present each of the components of the activity system (see Sub-Section 4.1.3 and Section 4.2).

Subject of the activity

Four students (Emil, Thor, Ella and Tore) volunteered as the focus group (Group B). The teacher describes this group as mixed-achievement (see Table 5.1 for the grades assigned to each student). Table 5.1 also shows the approximate time the students used in solving Tasks 1 and 2. The four students adopted different roles concerning tool usage while working on both tasks (see Table 5.2 on page 130).

Community

The community of the activity was made up of students. Seven groups worked together in the classroom (see the group reports of all the groups in Appendix D.2), of which particular attention was paid to the focus group (Group B). The community was formed spontaneously to solve Tasks 1 and 2 and then dissolved.

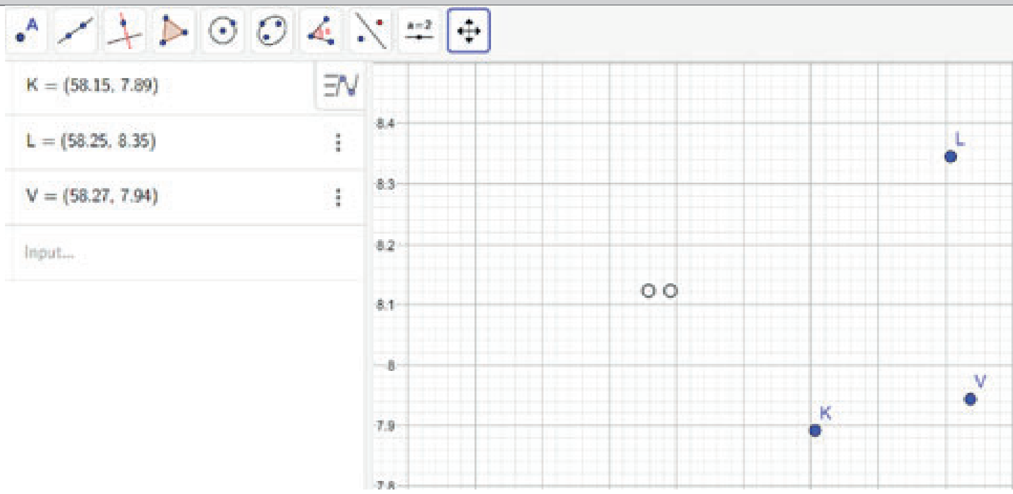
Code	Ratify the objective (Ratifying the objective of solving the mathematical modelling task)
Task	Task 2
Context	The students in this episode made lines to connect the three points representing the three cities in GeoGebra as they discussed the optimal location.
Recorded dialogue	*Tore*: Now put like a line around, like a [pointing to the figure below] … like last time, so we know that which one it closest. *Emil*: Uuh ok. Do you wanna make it like a triangle [connecting the points in the figure below by hand], or? *Thor*: Erm triangle, maybe not. *Emil*: No, that's not what we need. *Tore*: Like we did something last time, like put a… *Thor*: No but those are with four points. *Emil*: I don't fully understand the question, what do we need to find. *Thor*: We need a mega store. *Emil*: Aha. *Thor*: We have a mega store; we need to place it in the optimal place. *Emil*: Aha, the optimal place, ok, sounds good.
Image	

Table 6.24: Sample Data Aligned with Codes (Objective of the Activity): Group B.

Object of the activity

The researcher assumes that the object of the activity is to solve the mathematical modelling tasks with digital technologies and write a report. The students confirmed this objective at some points in their engagement with both tasks. For instance, Table 6.24 above shows a part of the transcription aligned with codes showing a certain point where participants ratify their objective of solving Task 2. In Table 6.24 (from the recorded dialogue), Thor ratifies the main object of the activity as he points out that they need to find an optimal position to place the mega shopping centre, for which the other group members agreed.

Mediating artefacts/tools for the subject-object interaction

The mediating artefacts or tools mediating the students' activities are physical (digital technologies) and non-physical (group work) tools. I will first present a report on the digital technologies Group B used while working on both tasks

Digital technologies

Group B used GeoGebra and a calculator device while working on Task 1 and used Google Maps, Google Search, GeoGebra and a calculator device while working on Task 2 (see Table 6.1 on page 157). Table 6.25 below shows a part of the transcription aligned with codes showing examples of different times GeoGebra and a calculator device mediated the interactions between the students and solving Task 1. The recorded dialogue in Table 6.25 is partitioned into three sections/parts (separated by ......). The first part shows the students using the spreadsheet view of GeoGebra to present their data after analyzing the task and knowing what they need to do (see Image 1 in Table 6.25). The second part of the dialogue shows the students constructing a function from their data set (see Image 2 in Table 6.25). Thus, the students made a linear graph, $g(x) = -0.01 + 150$, with persons on the y-axis and the price of the car on the x-axis. In this case, if the car is sold at $x = 5000$ then the number of people that buy the car will be $g(5000) = 100$. The third part of the dialogue shows the students trying out some numbers with the function they made (see Image 3 in Table 6.25). Thus, the students inserted $y = 50$ in the graph and found the corresponding value (the price of the car), $x = 10000$. The students then used a calculator to compute the product of these values (i.e., $50 \times 10000 = 500000$).

Concerning Task 2, Tables 6.26 and 6.27 below show a part of the transcription aligned with codes showing examples of different times Google

Maps, Google Search, GeoGebra, and a calculator device mediated the interactions between the students and solving Task 2. Table 6.26 shows the students searching for the three cities on Google Maps and transferring the coordinates of each city into GeoGebra for further analysis. The students also searched for the population of each city using Google Search (on a mobile phone).

Code	Digital technology (**GeoGebra** & **Calculator**)
Task	Task 1
Context	The students first put the information gathered into the spreadsheet view in GeoGebra. The students then made a linear graph representing their data. The students inserted some values in their function as they searched for the maximum company sales revenue.
Recorded dialogue	*Emil*: [Completes the values in the spreadsheet]. Now we highlight this, go here… "two variable regression analysis" [highlights the values and selects regression in GeoGebra, see Image 1]. And now these are the points. *Thor*: Which one will fit best? I think… try linear first. *Emil*: I think linear might look like… let's find out [Selects the linear of best fit for the points]. *Tore*: Yeah. *Thor*: Erm export that to GeoGebra or the graphic sheet. .... *Emil*: [Draws a new function, see Image 2] It's still decreasing over here. *Thor*: Yeah, but it seems more of a erm manageable rate. *Emil*: Yeah. Thor: So, we need to find the optimal price. *Emil*: Ok, at a 100 people this top point is a 5000 [points to the top point in Image 2], and then every... so the x-axis is the… .... *Thor*: Profit is more important than the people. So, let's look at 50 people, y equals to 50. *Emil*: 50 people, ok. I will just change this one to 50 [Changes y = 1 into y = 50, see Image 3]. *Tore*: I think that will be 10000. Thor: So, that's 10000 times [writes the calculations on paper]. *Tore*: 50. *Emil*: 50 people will buy the car for 10000. *Thor*: 1000 times 50 [Use a calculator for the calculation], that would be… *Ella*: 500000.

Image

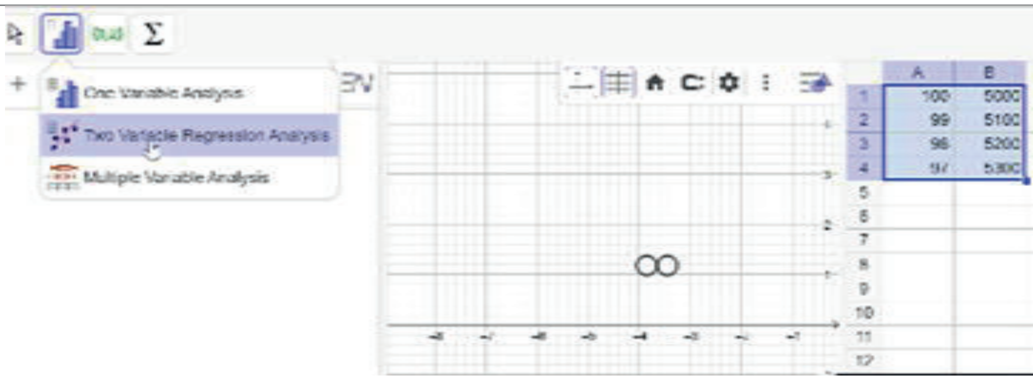


Image 1

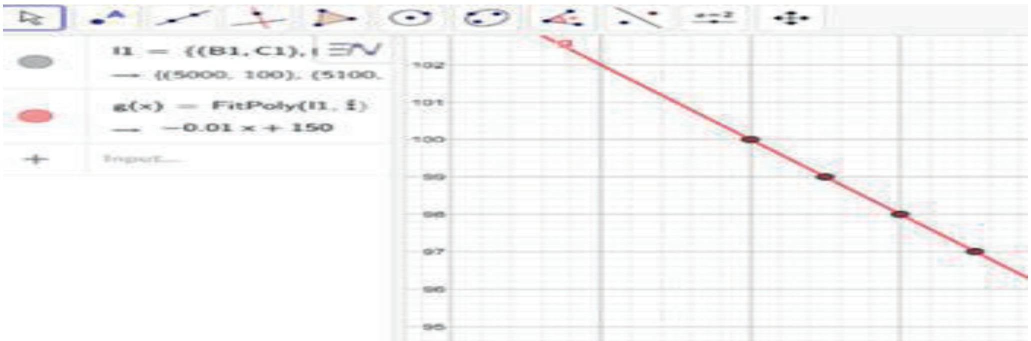


Image 2

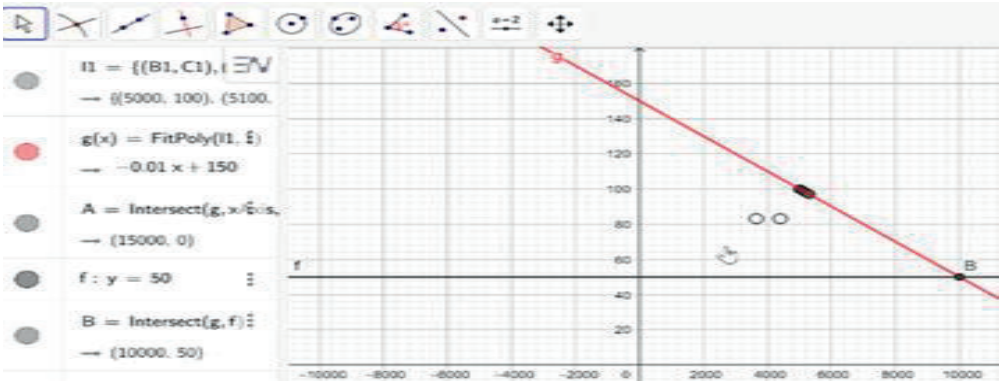


Image 3

Table 6.25: Sample Data Aligned with Codes (Digital Technology—GeoGebra & Calculator): Group B—Task 1.

Code	Digital technology (**Google Maps** & **Google Search**)
Task	Task 2
Context	The students searched for the population and position of the three cities with Google Search and Google Maps, respectively. The students also copied the coordinates from Google Maps into GeoGebra.
Recorded dialogue	Emil: Ok, Google Maps [Search Kristiansand in Google Maps see the image below] and [copies the coordinates from Google Maps into GeoGebra, see the image in Table 6.25]. Ella: Now go to Lillesand and find the coordinates. Emil: [Repeat the same process for Lillesand and Vennesla].

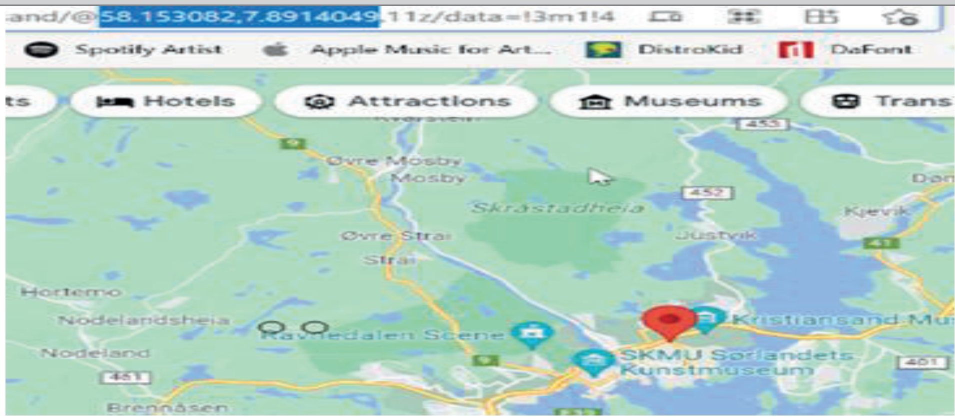
	Thor: Do you think we can use our phones for like erm… while we are doing that as well, just to multitask. Emil: What do we use it for? Thor: For calculation. Emil: Oh yeah, that should be fine. Thor: Ok [search for the population of Kristiansand with Google Search on phone].
Image	

Table 6.26: Sample Data Aligned with Codes (Digital Technology—Google Maps & Search): Group B—Task 2.

The recorded dialogue in Table 6.7 is partitioned into two sections/parts (separated by **……**). The first part of the dialogue shows the students searching for Point D (the middle point of the three cities) and discussing the fairness of the placement of this point (see Image 1 in Table 6.27). The second part of the dialogue shows students searching for another point (Point M) that connects the three cities (see Image 2 in Table 6.27) and discussing the best position of this point. They did so by calculating the median of distances between the three cities and Point M. At the end of their analysis, the students transferred the coordinates of Point M into Google Maps and located the position of Point M on the map.

Code	Digital technology (**GeoGebra** & **Calculator**)
Task	Task 2
Context	The students copied the coordinates of the position of the three cities from Google Maps and inserted them into GeoGebra. They then searched for the middle point as the optimal point for the shopping centre.
Recorded dialogue	Thor: That's not the point we gonna put it but it is close. Ella: Point D has the coordinates 58.23 and 8.06 [see Image 1]. Thor: But Tore: I thought D right now is the point that is closest to all of them, like fair. Thor: It is, but Emil: No, I would say this is false because this… like from D to V is definitely shorter than D to L.

Tore: Yeah, but it needed to be like that.
Thor: Yeah, but L is smaller than V.
Emil: Exactly.

……….

Ella: Now it's in the middle [see point M in Image 2]
Thor: What is the middle erm…
Tore: I think to the left, maybe.
Thor: We need to find the median.
Emil: Median of?
Thor: 0.183
Emil: Ooh, yeah.
Thor: Plus [uses the calculator to add 0.183, 0.124, and 0.287, and divide by 3].
Emil: But we are keeping… It sounds like we wanted it closer to Kristiansand.
Thor: Yeah.

Image

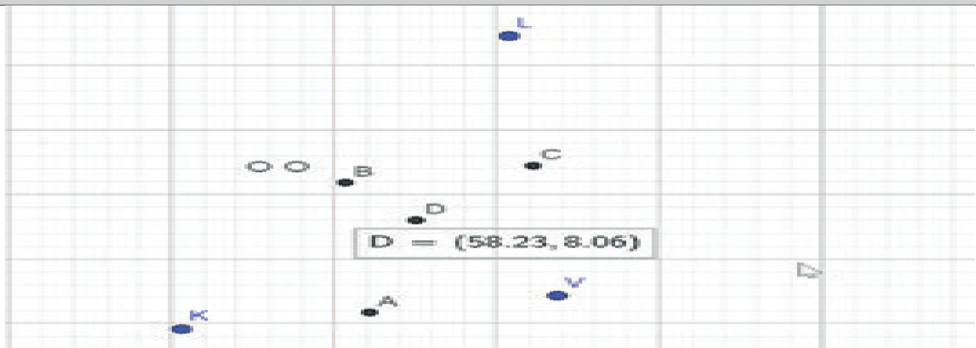


Image 1

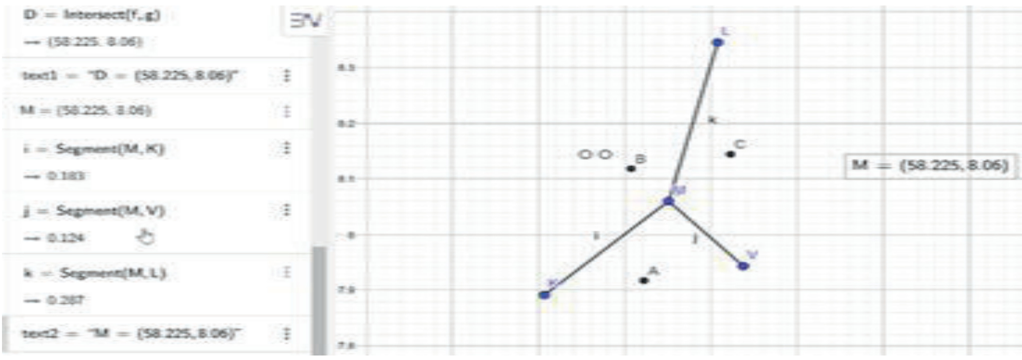


Image 2

Table 6.27: Sample Data Aligned with Codes (Digital Technology—GeoGebra & Calculator): Group B—Task 2.

Group work

Table 6.2 (on page 158) presents the number of times the different interaction categories or sequences occurred in the episodes of the students' activities. From Table 6.2, 6 out of 7 episodes describe asymmetrical contingency in the activities relating to Task 1, whilst 1 out of 8 episodes describe this contingency in the activities relating to Task 2. That is, the interaction that took place in the activities of Task 1 was more supportive, affirming, non-challenging, and non-critical than the activities relating to Task 2. Again, 0 out of 7 and 1 out of 7 episodes describe respectively reactive and mutual contingencies in the activities relating to Task 1, and 5 out of 8 and 2 out of 8 episodes describe respectively reactive and mutual contingencies in the activities relating to Task 2. That is, the interaction that took place in the activities of Task 2 was more critical and demonstrated tentative ideas being offered and debated compared to the activities relating to Task 1.

Rules for the subject-community interaction

Table 6.3 in Sub-Section 6.1.3 presents the explicit and implicit rules observed in the students' activities.

Explicit rule

One explicit rule in the students' activities was that they worked in a group (or as a group and produced a single group report), and they were allowed to use any digital technology available to solve both tasks (the technology was not imposed on them). Table 6.1 shows the different digital technologies that Group B (and the other groups) used while working on Tasks 1 and 2. Another explicit rule was time constraints. The average time used in solving a single task during the practice phase (introductory activity) was 20 minutes. So, Group B was expected to use a similar time frame to solve Tasks 1 and 2. At a particular point in the students' activities (relating to Task 2), it was observed how time constraints impeded the solution process. For example, see Excerpt 6.4.1 below:

Ella:	Yeah, and we are thinking the wrong way because the question is fair and that's certainly not fair.
Thor:	We try to do it as fairly as possible.
Tore:	We failed, so
Ella:	As fairly as possible?
Thor:	We couldn't, this is mathematically the fairest
Ella:	If don't think about the (fucking) road…
Thor:	We mess up, we didn't do it correctly, but we could do this erm more thoroughly and more correct if we had more time, which we do not have right now.
Emil:	[Writes the report with the support of peers].

Excerpt 6.4.1

From Excerpt 6.4.1, Thor points out that the group accepts the optimal location they found (mathematically, the fairest). In contrast, another group member (Ella) argues that they must consider the roads before making the final decision. However, the group went on to write the final report due to time constraints (although they used 40 minutes to solve the task).

Implicit rule

Several implicit rules could be identified in the student's activities (relating to both Task 1 and 2). However, an example of an implicit rule observed is dismissing comments or suggestions (which do not fit in the current strategy) without further analysis. For instance, Excerpt 6.4.2 below shows a part of the transcription showing a certain point where a new idea was dismissed without analyzing it as the students solve Task 1:

Thor:	Because you can think of this like say *f* is 5000 first… 5000 times 100 [writes the calculations 5000*100 = 500000 on paper].
Emil:	I was thinking like we could do a ratio too, if we say like, ok let's have car prices on the left and the people on the right. We know the ratio is 5000 to 100 [writes 5000 : 100 on paper].
Thor:	I don't know if that works, but maybe.
Ella:	Try it your way [referring to Thor].
Thor:	Ok, we will try it my way. So, this is like the lowest price we can go, that's erm 500000, that will be the revenue. So, let's go to the lowest turning point, highest price lowest selling point. Where y = 1, so one person will buy the car.
Emil:	Ok.

Excerpt 6.4.2

From Excerpt 6.4.2, Thor comes up with multiplying the number of people buying the car with the price at which they buy the car. Emil also suggested that the group look at the ratio between the number of people buying the car and the price of the car. However, Ella rejects Emil's suggestion without trying it out, so the group turns to Thor's initial idea (or the strategy they began with).

Division of labour for the community-object interaction

The students had roles that were constant throughout the activities (see Table 5.2 on page 130). Other roles in the form of leading, opposing, suggesting, supporting, non-contributing, and questioning and challenging changed at different times during the student's activities. Table 6.4 in Sub-Section 6.1.4 gives the frequency of roles taken by each member of Group B as they worked on Tasks 1 and 2.

Roles adopted by students

From Table 6.4, relating to Task 1, Thor mostly took the leading role (about five times in the episodes and Emil only once in the episodes), whilst the others mostly took the supporting role. Relating to Task 2, Thor took the leading role only once in the episodes, and this change might be due to the nature of both tasks. Again, members of Group B were recorded opposing each other during the activities relating to Task 2, except for Tore (the student assigned the lowest grade within the group). This also applies to 'questioning & challenging', where each of the members at least took this role either in Task 1 or 2 (or both), except Tore (same with the 'suggesting role', where Tore only took this role once in the episodes relating to Task 2 compared to the other group members). Once, in the episodes relating to Tasks 1 and 2, Ella (the only female in the group) took the non-contributing role.

I will now present a report on the modelling actions of Group B that emerged, and the role digital technologies played in these actions. This report corresponds to RQ2a and RQ2b.

6.4.2 Emergence of modelling actions and the role of digital technologies

The students' actions are divided into categories, coded in Table 9.6 in Appendix E.2. I will present the analysis of the emerging modelling actions of Group B according to the categories in Tables 6.28 and 6.29 below. The description of Tables 6.28 and 6.29 is similar to Tables 6.12 and 6.13 (see the second paragraph on page 174). I will further explain Tables 6.28 and 6.29 with some excerpts from the video recording transcriptions and screen recordings as evidence.

Modelling actions	Episodes of the students' activity						
	1	2	3	4	5	6	7
Breaking the task into manageable parts	X	X	-	-	-	-	-
Searching for a model	-	X	X	-	-	-	-
Finding a solution for the model	-	-	X	X	X	X	X
Explaining the results in real terms	-	-	-	-	-	-	X
Checking the results for adequacy	-	-	-	-	-	-	X
Role of technological tools							
Calculating (Calculator device)	-	-	-	-	X	X	X
Experimenting/Changing (GeoGebra)	-	-	X	X	X	X	X
Geometric construction (Geogebra)	-	X	X	-	-	-	-

Table 6.28: The modelling actions that emerged in Group B's activities (regarding Task 1) and the role of the digital technologies.

Modelling actions	Episodes of the students' activity							
	1	2	3	4	5	6	7	8
Breaking the task into manageable parts	X	-	-	-	-	-	X	-
Searching for a model	X	X	-	-	-	-	-	-
Finding a solution for the model	-	X	X	X	X	X	X	X
Explaining the results in real terms	-	-	-	-	-	-	-	X
Checking the results for adequacy	-	-	-	-	-	X	X	X
Role of technological tools								
Calculating (Calculator device)	-	-	-	-	-	X	-	-
Researching (Google maps and search)	X	-	-	-	-	-	X	-
Experimenting/Changing (GeoGebra)	-	X	X	-	X	-	-	-
Geometric construction (Geogebra)	X	-	-	-	-	-	-	-
Visualizing (GeoGebra)	-	-	-	X	-	-	-	-

Table 6.29: The modelling actions that emerged in Group B's activities (regarding Task 2) and the role of the digital technologies.

I will present each category in Tables 6.28 and 6.29 and the role of digital technology.

Breaking the task into manageable parts

Under the category "breaking the task into manageable parts" in Table 6.28 (relating to Task 1) and 6.29 (relating to Task 2), there were different times in the episodes where the students performed an action of breaking the task into manageable parts. In Table 6.28, it was counted in episodes 1 and 2 (out of 7 episodes), while in Table 6.29, it was counted in episodes 1 and 7 (out of 8 episodes). This shows that Group B started their activities relating to Task 1 by breaking the task into manageable parts, but relating to Task 2, the students then went back to this category after working on the task for some time. Thus, Group B searched for extra information to understand the task demands better. For example (see Excerpt 6.4.3 on page 209), Ella searched for the travel distance as extra information while the group analyzed their proposed optimal position.

Role of digital technology

Regarding Task 1, Group B did not use any digital technology under this category; the students only discussed the task and wrote down the variables needed to make a model (see the first episode in Table 6.28). Regarding Task 2, Group B used

Google Maps and Google Search to seek information about the identified variables (see Table 6.26 on page 201).

I will now present the next category, 'searching for a model', that emerged in the activities of Group B.

Searching for a model

The students performed the action of searching for a model in one or two of the episodes in the activities. In Table 6.28 (relating to Task 1), it was counted in both episodes 2 and 3 (out of 7 episodes), while in Table 6.29 (relating to Task 2), it was only counted in both episodes 1 and 2 (out of 8 episodes). From these results, the students searched for a model after breaking the task into manageable parts. The students also did not come back to search for a new model or update their model but went ahead with the initial model they searched for. Working on Task 1 (see Table 6.25 on page 200), Group B translated the problem text or situation into a mathematical problem and then represented the mathematical problem in the technological world. Thus, the students represented the mathematical problem in GeoGebra by inserting some data points (in the spreadsheet view) and drawing a function (in the graphic view) that represents these data points. Working on Task 2 (see Table 6.26 and 6.27 on pages 201 and 202), the students looked for the positions of the three cities on Google Maps and transferred the coordinates of these positions into GeoGebra. Later, in the subsequent episodes, the students searched for the optimal position by manipulating the points in GeoGebra.

Role of digital technology

Relating to Task 1, Group B used GeoGebra for geometric construction (a graphical representation of a function or equation) as they performed the action of searching for a model. Regarding Task 2, Group B used GeoGebra for a geometric construction (a graphical representation of the position of the three cities). That is, using Google Maps to search for the position of the three cities and transferring the coordinate points into GeoGebra.

The following presentation is on the report on the category 'finding a solution for the model' that emerged in Group B's activities.

Finding a solution for the model

In the activities, the students performed this action in several of the episodes (compared to other categories of modelling actions). In Table 6.28 (relating to Task 1), it was counted in 5 episodes (out of 7 episodes), while in Table 6.29 (relating to task 2) it was counted in 7 episodes (out of 8 episodes). This shows that many students' actions were about finding a solution for their model. Considering the activities relating to Task 1, Group B used the trial-and-error method by analyzing patterns of numbers after finding a function that represents the number of people buying the car. Thus, the students inserted a number in the function and found the corresponding value, for which they computed the product of these two values using the calculator (see, for example, the dialogue in Table 6.25). In working on Task 2, the students inserted the coordinate points of the three cities into GeoGebra (forming a triangle) and searched for the middle point (Point M; see the dialogues in Table 6.26 and 6.27). Group B used the 'median of a triangle (centroid)' approach to construct the centre of the triangle. Thus, they first searched for the midpoint of each side of the triangle and then connected these midpoints to the point (one of the three cities) opposite the corresponding midpoint (for instance, A – L, B –V, and C – K in Image 1 of Table 6.27). The segments (A, L), (B, V) and (C, K) meet at point D, and the students considered this point as the midpoint (optimal point) of the triangle. The students later made a duplicate point (Point M) for point D and then transferred this point from GeoGebra back to Google Maps and searched for the position of this point on the map. Although the students considered point M/D the optimal point, if you construct a circle with point M/D as the centre, the circle will not pass through all the corner points of the triangle.

Role of digital technology

Relating to Task 1, Group B used GeoGebra to experiment/change the number as they looked for the maximum revenue whilst using the calculator to compute the product of the number of people and the corresponding car price. Regarding Task 2, Group B used GeoGebra to search for the middle point by changing (or experimenting) points and the calculator to compute the median of the distances between the middle point and the points representing the three cities. Again, the students used GeoGebra to visualize the coordinates of the middle point.

The following presentation is on the report on the category 'explaining the results in real terms' that emerged in Group B's activities.

Explaining the results in real terms

In Table 6.28 (relating to Task 1) and 6.29 (relating to Task 2), the category 'explaining the results in real terms' was counted in the last episode (out of 7 and 8 episodes, respectively). This shows that the students performed this action when they were sure about their mathematical results. In finalizing their results, Group B wrote a report where they interpreted the mathematical results in real terms (see Group B's report in Appendix D.2). From this report (relating to Task 1), Group B explained that the optimal selling price per car is 7500 euros, increasing the company’s sales revenue by 62500 euros if they sell 75 cars (instead of 100 cars). On the other hand (relating to Task 2), Group B chose Dønnestad as the optimal place (the mathematical central area) to build the shopping mall. The students also pointed out that this place might not be the most optimal location, and with more resources and time, they might find a fairer place for all three cities.

Role of digital technology

Regarding Tasks 1 and 2, Group B wrote their final report in a Word Document. The other groups that worked alongside Group B also wrote their final report in a Word Document (see Appendix D.2 for the solution reports of all the groups in School B).

I will now present the final category, ‘checking the results for adequacy’, that emerged in the activities of Group B.

Checking the results for adequacy

The students checked the results for adequacy in some of the episodes. In Table 6.5 (relating to Task 1), it was counted in the last episode (out of 7 episodes), while in Table 6.9 (relating to Task 2), it was counted in episodes 6, 7 and 8 (out of 9 episodes). This shows that the students performed this action at the end of the activities relating to Task 1 and then closer to the end (both initial and final results) of the activities relating to Task 2. For instance, Excerpt 6.4.3 shows a part of the transcription showing a certain point where the students checked their results for adequacy as they worked on Task 2:

Ella: Yeah, and I am checking how long to drive [Searched for the time of travel between the optimal place and Vennesla on Google maps]. Its 8 minutes from Vennesla to drive, and…

Emil: To where?

Ella: To that location [Referring to the marked location on the map].

Ella: And for Kristiansand it’s 17.

Thor: Ok, that’s pretty far.

Ella: And for Lillesand it’s 31.

Thor: [laughs] maybe they got some good stores.

Excerpt 6.4.3

From Excerpt 6.4.3, Ella searched for the travel distance between the optimal place for the shopping centre and the three cities. The students then discussed the optimal position (on the issue of fairness) by acknowledging the travel time from the cities to this position/place. However, due to time constraints, the students settled on this position at the end of their discussion (see Excerpt 6.4.1 on page 203).

Role of digital technology

Relating to Task 1, Group A used GeoGebra to experiment/change the numbers whilst using the calculator to compute the product of the number of people and the corresponding car price, validating their results. Regarding Task 2, Group B used Google Search to seek information on the travel time between the optimal position and the three cities.

I will now present a report on the affordances and constraints of the digital technologies that emerged in the activities of Group B, and this corresponds to RQ3.

6.4.3 Emergence of technological, mathematical and socio-cultural affordances and constraints

The frequency of actualized technological, mathematical and socio-cultural affordances and constraints that emerged in Group B's activities regarding Tasks 1 and 2 are presented in Table 6.30 and 6.31, respectively. I will present the analysis of Group B's emerging affordances and constraints according to the categories listed in Tables 6.30 and 6.31. The description of Tables 6.30 and 6.31 is similar to Tables 6.20 and 6.21 (see the last paragraph on page 186). I will further explain Tables 6.30 and 6.31 with some excerpts from the video recording transcriptions and screen recordings as evidence.

Affordances & Constraints	Digital Technologies				
	GeoGebra	Excel	Calculator	Google search	Google maps
Technological Affordances					
Calculating	-	-	4	-	-
Geometric construction	2	-	-	-	-
Experimenting/Changing	6	-	-	-	-
Mathematical Affordances					
Linking representations	2	-	-	-	-
Regularity and variations	6	-	-	-	-
Arithmetic and statistics	-	-	4	-	-
Socio-cultural Affordances					

Common focus	**6**	-	-	-	-
Authority of the digital technology	**2**	-	-	-	-

Table 6.30: The frequency of actualized technological, mathematical, and socio-cultural affordances that emerged in Group B's activities regarding Task 1.

Affordances & Constraints	**Digital Technologies**				
	GeoGebra	**Excel**	**Calculator**	**Google search**	**Google maps**
Technological Affordances					
Calculating	-	-	**1**	-	-
Researching	-	-	-	**3**	**3**
Measuring	-	-	-	-	**1**
Geometric construction	**1**	-	-	-	-
Experimenting/Changing	**5**	-	-	-	-
Visualizing	**1**	-	-	-	
Mathematical Affordances					
Analyzing	-	-	-	**3**	**4**
Simulating and visualizing	**1**	-	-	-	-
Linking representations	**1**	-	-	-	-
Regularity and variations	**5**	-	-	-	-
Arithmetic and statistics	-	-	**1**	-	-
Socio-cultural Affordances					
Common focus	**5**	-	-	-	**1**

Table 6.31: The frequency of actualized technological, mathematical, and socio-cultural affordances that emerged in Group B's activities regarding Task 2.

I will now start with a presentation on technological affordances and constraints emerging in the activities of Group B.

Technological affordances and constraints

The digital technologies Group B used while working on Task 1 were GeoGebra and a calculator device. Group B also used Google Maps, Google Search, GeoGebra and a calculator device while working on Task 2 (see Table 6.1). From Tables 6.30 and 6.31, the technological affordances and constraints recorded are researching, measuring, visualizing, geometric construction, experimenting/changing, data entry and generation (ø) and calculating.

Researching. Regarding Task 2, Google Maps and Google Search afford the retrieval of information about the cities' position/location and population, respectively, as the students interact with these technologies (see Table 6.26 on page 201).

Measuring. Relating to Task 2, Google Maps affords the measure of time of travel between the cities and the optimal place for the shopping centre. For instance, the dialogue in Excerpt 6.4.3 (on page 209) shows how Ella measures the travel time between Dønnestad (the optimal location) and Vennesla.

Visualizing. Regarding Task 2, GeoGebra allowed Group B to visualize a point in the graphic view. Thus, the students made a duplicate (Point M in Image 2 in Table 6.27 on page 202) of the middle point (Point D in Image 1 in Table 6.27). The students then showed (making it visible) the coordinate points of the duplicate point in the graphic view of GeoGebra.

Geometric construction. GeoGebra afforded the drawing of a function from the identified variables while Group B worked on Task 1. From Table 6.25 (on page 200), the students drew the function $g(x) = -0.01x + 150$ with GeoGebra, representing the number of people buying the car. To draw this function, the students inserted some data points and then generated an equation representing these data points. The students then found the maximum revenue for the car-selling company based on this function. Again, from Table 6.27, the students constructed a figure (see Image 3) representing the optimal location connected to the three cities.

Experimenting/Changing. GeoGebra afforded Group B the changing of parameters of function ($g(x) = -0.01x + 150$) as they observed its effects while working on task 1. In this case, the students tried out different numbers while finding the maximum company revenue. For instance, the third part of the recorded dialogue in Table 6.25 shows the students trying out their function with $y = 50$ (that is, 50 people buying the car). Relating to Task 2, the students first inserted the coordinates points of the cities into GeoGebra, and they searched for the middle point that connects the three cities. The students did so by experimenting or changing the coordinate points several times until they were satisfied with the optimal point (see Table 6.27).

Data entry and generation. This category was not identified in the student's activities in both tasks.

Calculating. Relating to Task 1, the calculator device afforded the calculation of the product of the number of people and the corresponding price of the car (see the third part of the recorded dialogue in Table 6.25). Relating to Task 2, the calculator device afforded the calculation of the median of the distances between the optimal point and the cities (see the second part of the recorded dialogue in Table 6.27).

Mathematical affordances and constraints

From Tables 6.30 and 6.31, the mathematical affordances and constraints recorded are clarification (ø), analyzing, simulating and visualizing, linking representations, regularity and variations, and arithmetic and statistics.

Clarification. This category was not identified in the student's activities in both tasks.

Analyzing. Group B reconciled their model with reality (relating to Task 2). They did so by inserting the coordinates of the optimal point in GeoGebra into Google Maps. The students also compared the time of travel between the optimal location and the three cities, as well as the population of these cities (see Tables 6.26 and 2.27).

Simulating and visualizing. Working on Task 2, the students made duplicates of points. They moved (or manipulated) these points dynamically to see the changes in the distances between the optimal location and the three cities (see Table 2.27).

Linking representations. Working on Task 1, the students moved between numeric, equations and graphical representation. That is, the students inserted some data points in the spreadsheet view and constructed a graph representing these data points in the graphical view in GeoGebra, and the algebraic view also presented a function that represents both the data points and the graph (see Table 2.25). Relating to Task 2, the students could represent the coordinates (from

Google Maps) of the three cities in a graphical view in GeoGebra (see Table 6.26 and 6.27).

Regularity and variations. Relating to Task 1, The students explored the regularity and variations in the solution model by experimenting with their function with different numbers. That is, the students observed the effect of the changed parameters on the graph as they searched for the maximum revenue of the car-selling company. This step was repeated until they found the desired result (Table 6.25). Similarly, relating to Task 2, the students manipulated or changed the coordinate points in the graphical view in GeoGebra several times until they were satisfied with the optimal point (see Table 6.27).

Arithmetic and statistics. At the mathematical task level, the calculator device allows numerical computations such as multiplication (in Task 1) and average/median of a set of numbers (Task 2).

Socio-cultural affordances and constraints

The socio-cultural affordances and constraints that emerged or actualized in the students' activity were common focus (in both Task 1 and 2), observing and improving strategies (ø) and authority of the digital technology (only in Task 1) (see Table 6.30 and 6.31).

Common focus. The students shared the same computer and had the facility to look at the same thing and point at what was presented on the computer. For instance, to visually demonstrate their reasoning to one another, the students used GeoGebra as a reference tool by looking at the three points in the graphical view and sketching a triangle by hand gesture (see Table 6.24 on page 197).

Observing and improving strategies. This category was not identified in the student's activities in both tasks.

Authority of the digital technology. From Excerpt 6.4.2 (on page 204), Emil came up with the idea of finding the ratio between the number of people buying the car and the price of the car. At the same time, Ella suggested that the group continue with the existing strategy (proposed by Thor). In this case, the other group members (except Emil) had personalized the problem-solving strategy and had not considered other strategies. A similar situation occurred towards the end of the activities relating to Task 1; see Excerpt 6.4.4 below:

Thor: That's more profit than that.

Ella: Try 70 then.

Emil: Oh, you know what we can do.

Thor: Yeah.

Emil: We can make a function to find out the revenue. Can we do like erm…

Thor: We could.

Ella: Try 70 first.

Excerpt 6.4.4

From Excerpt 6.4.4, Emil came up with the idea of making a function representing the revenue. However, Ella insisted they continue with the ongoing strategy (although Thor, taking the leading role, was interested in this new idea). There might be a possibility of Ella insisting the group continue with the ongoing strategy if she thinks they are close to finding the answer. From this narrative, it appears that the features of GeoGebra allow multiple problem-solving strategies. However, the approach used by the group depends on the strategy with which some students are comfortable (or fits with the existing strategy).

In summary, I have reported on the activities of Group B along three themes. That is, Group B's mathematical modelling activities, the modelling actions emerging in Group B's activity and the role digital technology plays, and the affordances and constraints of the digital technologies emerging in Group B's activities. In the following sections, I will present the case study reports of Groups C and D. As stated earlier (before Section 6.4), I noted that I will only present issues different from Group A while presenting the report of Group B (and the other groups). In the same way, I will only present issues different from Groups A and B while writing the report for Groups C and D.

6.5 Case study report: Group C (Nils, Anna and Jørn)

This section offers a description of the case study report of the third school (Group C; see Sub-Section 3.4.3). The narrative is presented in an order as highlighted in Section 6.2. Group C is made up of first-year upper secondary school students (Grade 11). Group C comprises three students aged between 16 – 17 years. The reports in Sub-Sections 6.5.1, 6.5.2 and 6.5.3 correspond to RQ1, RQ2a & RQ2b and RQ3 respectively. I will first start the report on the students' (Group C's) mathematical modelling activities.

6.5.1 Students' mathematical modelling activities

Details about Group C are presented in Sub-Section 3.4.3. I will present each of the components of the activity system (see Sub-Section 4.1.3 and Section 4.2).

Subject of the activity

Three students (Nils, Anna and Jørn) volunteered as the focus group (Group C). The teacher describes this group as a same-achievement group (see Table 5.1 for their assigned grades). Table 5.1 also shows the approximate time the students used in solving Tasks 1 and 2. The three students adopted different roles concerning tool usage while working on both tasks (see Table 5.2 on page 130).

Community

The community of the activity was made up of students. Five groups worked together in the classroom (see the group reports of all the groups in Appendix D.3), of which particular attention was paid to the focus group (Group C). The community was formed spontaneously to solve Tasks 1 and 2 and then dissolved.

Object of the activity

The researcher assumes that the object of the activity is to solve the modelling tasks with digital technologies and write a report. The students confirmed this objective at some points in their engagement with both tasks. There are also instances where the students have to re-read the problem and remind themselves of the objective/goal of solving the task; for example, see Excerpt 6.5.1 below:

Nils:	Yep, you can check the populations.
Jørn:	[reads a part of the second task] so that the needs of the three towns are served in a fair way. So, what's fair?
Nils:	I would say is a place closer to the more densely populated area.

Excerpt 6.5.1

From Excerpt 6.5.1, Nils suggests the group checks the population of three cities as they discuss the optimal position. Jørn then re-read a part of the problem text, reminding the group of the goal of the task (raising the question about fairness).

Mediating artefacts/tools for the subject-object interaction

The mediating artefacts or tools mediating the students' activities are physical (digital technologies) and non-physical (group work) tools. I will first present a report on the digital technologies that Group C used while working on both tasks.

Digital technologies

Group C used Excel/spreadsheet while working on Task 1 and used Google Maps, Google Search and GeoGebra while working on Task 2 (see Table 6.1). Table 6.32

below shows a part of the transcription aligned with codes, showing examples of different times Excel/spreadsheet mediated the interactions between the students and solving Task 1. The recorded dialogue in Table 6.32 is partitioned into two sections/parts (separated by ……). The first part shows the students using Excel/spreadsheet to present their data after analyzing the task and knowing what to do. At one point, one student suggested using GeoGebra, but the students continued with the initial tool. The second part of the dialogue shows the students generating their data in Excel/spreadsheet. Thus, the students drag down the selected few points or data to generate complete data representing the problem.

Code	Digital technology (Excel/spreadsheet)
Task	Task 1
Context	The students first put the information gathered into Excel. The students then generated their entire data by dragging down the selected points or initial data.
Recorded dialogue	*Nils*: [Reads the first task aloud]. *Anna*: Erm… we could make a graph, *Nils*: Yeah. *Anna*: Or we could make an Excel [opens Excel on the computer, see Image 1]. We kind of make a function in Excel… because it would have to be recursive. Right? *Nils*: Erm, not necessary, because the thing is that we need to… erm because 5000 euros and 100 people will buy, if it is 5100 euros 99 people will buy it. That means that you lost… you lost on customers gain erm like *Anna*: Ok *Nils*: So, basically you need to find the function where that doesn't reward anymore, so then we need to figure out *Anna*: I don't really understand how you would be able to like make a function in GeoGebra though [opens GeoGebra on the computer]. *Nils*: We could try… try Excel… because then it will be… …….. *Anna*: And you could just drag that one. *Nils*: Yeah [drags C2 down, see Image 2]. You can drag that down until the final break point where it doesn't make money anymore. *Anna*: [selects A3, B3, and C3 and drags them down, see Image 2], so as you see… I need to drag it all the way down [selects A24, B24, and C24 and drags them down, see Image 3]. Erm, I did… erm it would be 75 customers for 7500 euros per car. *Nils*: Yeah, so that is *Anna*: And beyond that the revenue starts declining. *Nils*: Yep. So, again this is recursive function, so we can't do it in GeoGebra.

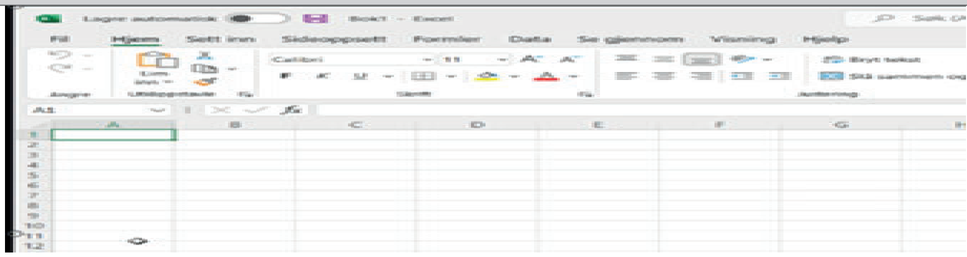
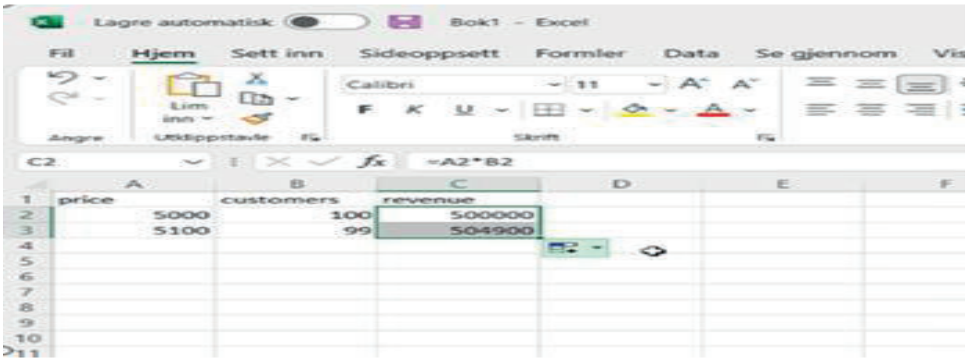
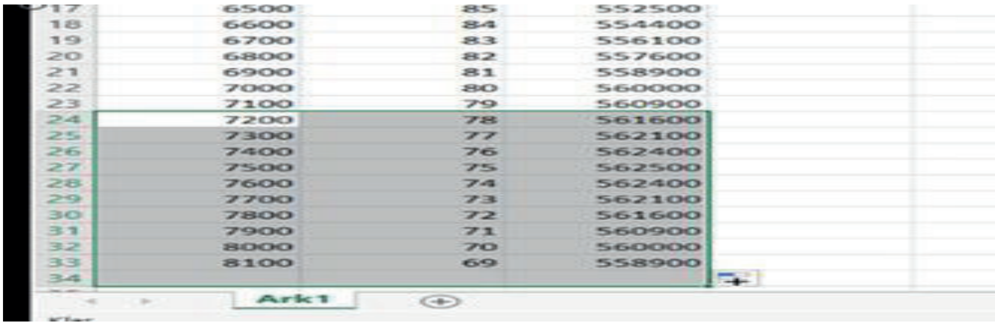
	Anna: Erm.
Image	 **Image 1**  **Image 2**  **Image 3**

Table 6.32: Sample Data Aligned with Codes (Digital Technology—Excel): Group C—Task 1.

Concerning Task 2, Tables 6.33 and 6.34 below show a part of the transcription aligned with codes, showing examples of different times Google Maps, Google Search, and GeoGebra mediated the interactions between the students and solving Task 2. The recorded dialogue in Table 6.33 is partitioned into two sections/parts (separated by ……). The first part shows the students searching for the three cities and saving them on Google Maps. The second part of the dialogue shows the students searching for the population of the three cities using Google Search. Table 6.34 shows the students searching for a theoretical optimal position using GeoGebra. Thus, the students imported the saved location from Google Maps into GeoGebra and constructed the middle point representing the theoretical optimal position.

Code	Digital technology (**Google Maps** & **Google Search**)
Task	Task 2
Context	The students first searched for the positions of the three cities on Google Maps. The students also searched for the population of each city with Google Search.
Recorded dialogue	*Anna*: [Reads the second task aloud]. *Nils*: You have to pull up the Google Maps then *Anna*: Erm we can go to Google Maps [opens Google Maps, and search for Kristiansand, see Image 1] and then Kristiansand. *Nils*: Lillesand *Anna*: Erm Lillesand [search for both Lillesand and Vennesla and save them on the map, see Image 1] ….. ………. *Jørn*: [reads a part of the second task] so that the needs of the three towns are served in a fair way. So, what's fair? *Nils*: I would say is a place closer to the denser populated area. *Anna*: [search with Google Search, the population of Vennesla] it will be 14000. *Nils*: And then erm Lillesand. Anna: [search with Google Search, the population of Lillesand]. *Nils*: 10000, Lillesand is smaller. *Anna*: Yep, but if we do that [search with Google Search, the population of Kristiansand, see Image 2] *Jørn*: Kristiansand is not that big. *Nils*: It's not that many but it feels 10 times as much as the others [laughs]. *Jørn*: Yeah.

Image

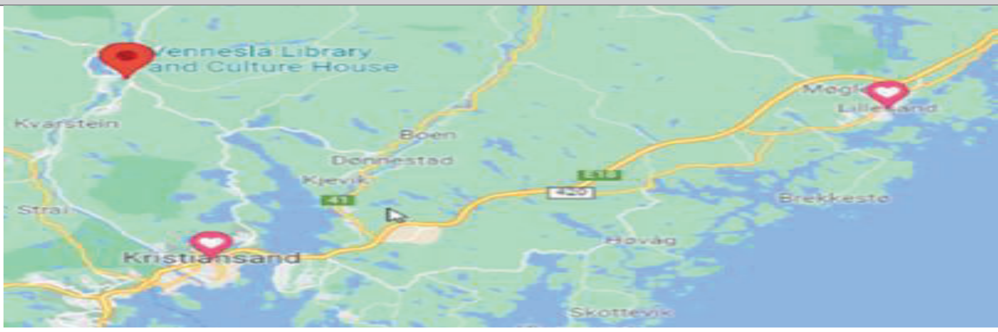


Image 1

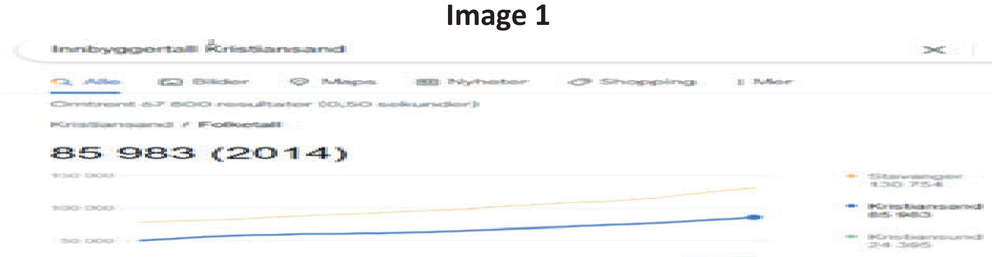


Image 2

Table 6.33: Sample Data Aligned with Codes (Digital Technology—Google Maps & Search): Group C—Task 2.

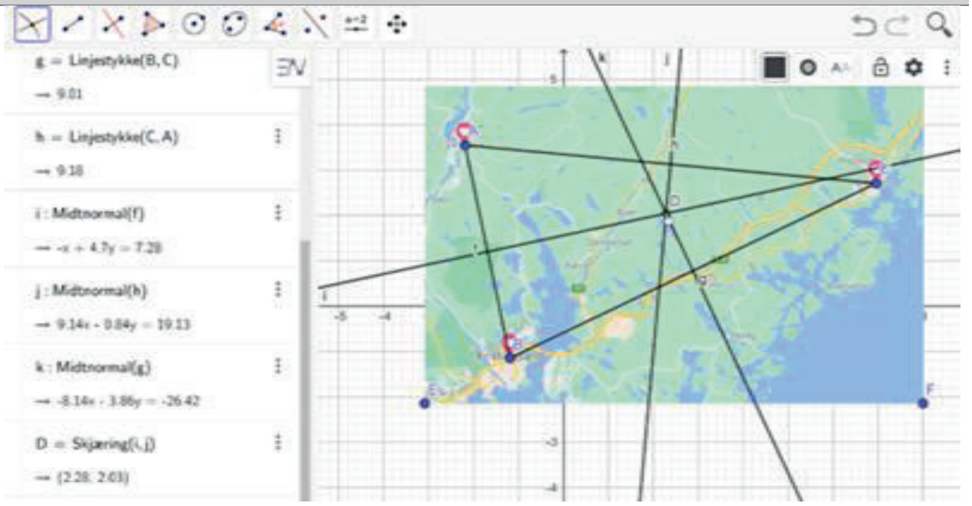
Code	Digital technology (GeoGebra)
Task	Task 2
Context	The students finally constructed a theoretical optimal position with GeoGebra.
Recorded dialogue	*Nils*: Yeah, but the thing is that if you just have a hypothetical optimal place. *Anna*: Erm *Nils*: So right now, we have one based on pre-existing road networks erm pre-existing locations but if you just put the map erm into GeoGebra we can make a theoretical best location. *Anna*: [marks the three cities on Google Maps and exports the image into GeoGebra and find the center of the three cities (with the help from Nils), see Image 3]. *Nils*: That will be the [laughs] ideal location if you do just based on… *Anna*: If we do not factor in population or roads. *Nils*: Yeah, if you don't factor in population and roads, that will be the ideal location. *Anna*: If everyone can just fly [writes the final statement of the report with the help of peers].
Image	

Table 6.34: Sample Data Aligned with Codes (Digital Technology—GeoGebra): Group C—Task 2.

Group Work

Table 6.2 (on page 158) presents the number of times the different interaction categories or sequences occurred in the episodes of the students' activities. From Table 6.2, 0 out of 2 episodes describes asymmetrical contingency in the activities relating to Task 1, whilst 1 out of 5 episodes describes this contingency in the activities relating to Task 2. Again, 0 out of 2 and 2 out of 2 episodes describe respectively reactive and mutual contingencies in the activities relating to Task 1, and 2 out of 5 and 2 out of 5 episodes describe respectively reactive and mutual contingencies in the activities relating to Task 2. Thus, the interactions were more critical and demonstrated tentative ideas being offered. This might result from the nature of the students (high-performing students).

Rules for the subject-community interaction

Table 6.3 (on page 158) presents the explicit and implicit rules observed.

Explicit rule

The students worked in a group, and the technology used was not imposed on them; as such, Table 6.1 shows the different digital technologies used by Group C as they worked on both tasks. Another explicit rule was time constraints. From Table 5.1, Group C spent less time on both tasks. Thus, the students used 8 minutes on Task 1 and 31 minutes on Task 2 (because they finished early on the first task and used the remaining time on the next task).

Implicit rule

No implicit rules were observed or recorded in the activities of Group C. This might result from the fact that this group is made up of high-performing students working together.

Division of labour for the community-object interaction

The students had roles that were constant throughout the activities (see Table 5.2). Other roles in the form of leading, opposing, suggesting, supporting, non-contributing, and questioning and challenging changed at different times during the students' activities. Table 6.4 (on page 160) gives the frequency of roles taken by each member of Group C as they worked on both tasks.

Roles adopted by students

From Table 6.4, relating to Tasks 1 and 2, all members of Group C took suggesting and supporting roles at some time. None of the students took the opposing role in either of the activities, and Nils only took the leading role in one of the episodes relating to Task 2. This might result from the characteristics of the students (high-performing students). There were a few instances where Anna (in one episode) and Jørn (in three episodes) took the questioning and challenging role in the activities relating to Task 2. Jørn was the only member who, on some occasions, took the non-contributing role in both activities, although he is a high-performing student (like the others).

I will now present a report on the modelling actions of Group C that emerged, and the role digital technologies played in these actions. This report corresponds to RQ2a and RQ2b.

6.5.2 Emergence of modelling actions and the role of digital technologies

The student's actions are divided into categories (coded in Table 9.6 in Appendix E.2). I will present the analysis of the emerging modelling actions of Group C according to the categories in Table 6.35 and 6.36 below. The description of Tables 6.35 and 6.36 is similar to Tables 6.12 and 6.13 (see the second paragraph on page 174). I will further explain Tables 6.28 and 6.29 with some excerpts from the video recording transcriptions and screen recordings as evidence.

Modelling actions	Episodes of the students' activity	
	1	2
Breaking the task into manageable parts	X	-
Searching for a model	X	-
Finding a solution for the model	-	X
Explaining the results in real terms	-	X
Checking the results for adequacy	-	-
Role of technological tools		
Data entry and generation (Excel/Spreadsheet)	X	X

Table 6.35: The modelling actions that emerged in Group C's activities (regarding Task 1) and the role of the digital technologies.

Modelling actions	Episodes of the students' activity				
	1	2	3	4	5
Breaking the task into manageable parts	X	X	X	-	-
Searching for a model	X	-	-	-	-
Finding a solution for the model	-	X	X	X	X
Explaining the results in real terms	-	-	-	X	X
Checking the results for adequacy	-	-	-	X	-
Role of technological tools					
Researching (Google Search, Google Maps)	X	-	X	-	-
Measuring (Google Maps)	-	X	X	-	-
Geometric construction (Geogebra)	-	-	-	-	X

Table 6.36: The modelling actions that emerged in Group C's activities (regarding Task 2) and the role of the digital technologies.

I will present each category in Tables 6.35 and 6.36 and the role of digital technology.

Breaking the task into manageable parts

Under the category "breaking the task into manageable parts" in Table 6.35 (relating to Task 1) and 6.36 (relating to Task 2), there were different times in the episodes where the students performed an action of breaking the task into manageable parts. In Table 6.36, it was counted only in the first episode (out of 2 episodes), while in Table 6.36, it was counted in episodes 1, 2 and 3 (out of 5 episodes). This shows that Group C started their work on Tasks 1 and 2 by breaking the task into manageable parts. However, relating to Task 2, the students perform this action in the next two episodes. In this case, the students searched for extra information to understand the task demands better. For example, in the second part of the dialogue in Table 6.33 (on page 219), the students searched for the population of the three cities as extra information for their analysis. Thus, the students searched for extra information by re-reading a part of the problem text.

Role of digital technology

Regarding Task 1, Group C did not use any digital technology under this category; the students only discussed the task and agreed on the variables needed to make a model (see the first part of the dialogue in Table 6.32 on page 218). Regarding Task 2, Group C used Google Maps and Google Search to seek information about the identified variables (see Table 6.33).

I will now present the next category, 'searching for a model', that emerged in the activities of Group C.

Searching for a model

The students performed the action of searching for a model in one of the episodes in the activities. In Table 6.35 (relating to Task 1), it was counted in the first episode (out of 2 episodes), while in Table 6.36 (relating to Task 2), it was only counted in the first episode (out of 5 episodes). From these results, the students searched for a model after breaking the task into manageable parts. The students also did not come back to search for a new model or update their model but went ahead with the initial model they searched for. For instance, Group C presented their initial variables in Excel/spreadsheet and later looked for the optimal location by generating all the data from the initial variables (see the first part of the dialogue in Table 6.32). On the other hand, the model that the students set up for Task 2 is described in Table 6.33. In Table 6.33, the students used Google Maps to set up their model by researching the positions of the three cities (which form a triangle). In the subsequent episodes, the students later analyzed the optimal place to build the shopping mall based on the positions of these cities.

Role of digital technology

Regarding Task 1, Group C inserted their initial variables in Excel/spreadsheet and later generated the entire data. Relating to Task 2, Group C used Google Maps to seek information about the positions of the three cities.

The following presentation is on the report on the category 'finding a solution for the model' that emerged in Group C's activities.

Finding a solution for the model

In the activities, the students performed this action in several of the episodes (compared to other categories of modelling actions). In Table 6.35 (relating to Task 1), it was counted in the second episode (out of 2 episodes), while in Table 6.36 (relating to Task 2), it was counted in 4 episodes (out of 5 episodes). This shows that many students' actions were about finding a solution for their model. Regarding Task 1, the students generated their entire data set from the initial data (or variables) using Excel/spreadsheet (see the second part of the dialogue in Table 6.32). In working on Task 2, the students measured the travel distances between the three cities and the proposed optimal location. The students also factored in the population of the cities in their analysis (see Table 6.33). Again, the students constructed a circumcircle/circumcenter of a triangle (with the three cities as the triangle's vertices) without considering the roads and other factors (see Table 6.34 on page 220). Thus, the students constructed a perpendicular bisector of each side of the triangle (where the corner points represent the three cities) and then searched for the point of intersection (optimal point) of these perpendicular bisectors. The students constructed a circle with the point of intersection as the centre point, and the circle passes through all the triangle's corner points (or vertices).

Role of digital technology

Regarding Task 1, Group C used Excel/spreadsheet to generate the entire data (through which they found their answer). Regarding Task 2, Group C used Google Maps to seek information about the positions of the three cities and to measure the distances between these cities. They also used Google Search to seek information about the population of each city for their analysis. The students also used GeoGebra to construct a geometric shape representing the cities and looked for the geometric middle point.

The following presentation is on the report on the category 'explaining the results in real terms' that emerged in Group C's activities.

Explaining the results in real terms

In Table 6.35 (relating to Task 1), the category 'explaining the results in real terms' was counted in the second episode (out of 2 episodes), while in Table 6.36 (relating to Task 2), this category was counted in the last two episodes (out of 5 episodes) where Group C performed the action of explaining the results in real terms. This shows that the students performed this action when they were sure about their mathematical results. In finalizing their results, Group C wrote a report where they interpreted the mathematical results in real terms (see Group C's report in Appendix D.3). From this report (relating to Task 1), Group C explained that the total revenue would be at its highest at 75 customers buying the car for 7500 euros. Regarding Task 2, Group C concluded that the shopping centre should be located in Kjevik (the hypothetical best location), which is fairly placed based on the population of the three cities. In this case, the travel time is shorter for the larger populated city (which might result in less carbon dioxide—CO_2 emissions). Again, Group C constructed a general optimal location (theoretical best location) without considering the roads and population of the cities (see Table 6.34).

Role of digital technology

Related to both Tasks 1 and 2, Group C wrote their final report on a Word Document. The other groups that worked alongside Group C also wrote their final report in a Word Document (see Appendix D.3 for the solution reports of all the groups in School C). Group C also used GeoGebra to construct the theoretical best location without considering other factors, as they explain their results.

I will now present the final category, 'checking the results for adequacy', that emerged in the activities of Group C.

Checking the results for adequacy

The students checked the results for adequacy in one of the episodes concerning Task 2. In Table 6.35 (relating to Task 1), this category was not counted in any of the episodes, while in Table 6.36 (relating to Task 2), it was counted in episode 4 (out of 5 episodes). This shows that the students were satisfied with the results (relating to Task 1) from the generated data in Excel/spreadsheet without further analysis. Concerning Task 2, the students performed this action close to the end of the activities.

Role of digital technology

Regarding Task 1 and 2, Group C did not use any digital technology under this category; the students only discussed the results they had.

I will now present a report on the affordances and constraints of the digital technologies that emerged in the activities of Group C, and this report corresponds to RQ3.

6.5.3 Emergence of technological, mathematical and socio-cultural affordances and constraints

The frequency of actualized technological, mathematical, and socio-cultural affordances and constraints that emerged in Group C's activities regarding Tasks 1 and 2 are presented in Table 6.37 and 6.38, respectively. I will present the analysis of the emerging affordances and constraints of Group C according to the categories of affordances and constraints listed in Tables 6.37 and 6.38. The description of Tables 6.37 and 6.38 is similar to Tables 6.20 and 6.21. I will further explain Tables 6.37 and 6.38 with some excerpts from the video recording transcriptions and screen recordings as evidence.

Affordances & Constraints	Digital Technologies				
	GeoGebra	Excel	Calculator	Google Search	Google Maps
Technological Affordances					
Data entry and generation	-	2	-	-	-
Mathematical Affordances					
Arithmetic and statistics	-	2	-	-	-
Socio-cultural Affordances					
Common focus	-	2	-	-	-

Table 6.37: The frequency of actualized technological, mathematical, and socio-cultural affordances that emerged in Group C's activities regarding Task 1.

Affordances & Constraints	Digital Technologies				
	GeoGebra	Excel	Calculator	Google Search	Google Maps
Technological Affordances					
Researching	-	-	-	3	2
Measuring	-	-	-	-	5

Geometric construction	**2**	**-**	**-**	**-**	**-**
Mathematical Affordances					
Analyzing	**-**	**-**	**-**	**3**	**7**
Linking representations	**2**	**-**	**-**	**-**	**-**
Socio-cultural Affordances					
Common focus	**1**	**-**	**-**	**1**	**2**

Table 6.38: The frequency of actualized technological, mathematical, and socio-cultural affordances that emerged in Group C's activities regarding Task 2.

I will now start with a presentation of technological affordances and constraints emerging in the activities of Group C.

Technological affordances and constraints

The digital technology Group C used in Task 1 was Excel/spreadsheet. Group C also used Google Maps, Google Search and GeoGebra while working on Task 2 (see Table 6.1). From Tables 6.37 and 6.38, the technological affordances and constraints recorded are researching, measuring, visualizing (ø), geometric construction, experimenting/changing (ø), data entry and generation, and calculating (ø).

Researching. Regarding Task 2, Google Maps and Google Search provided information about the cities' position/location and population as the students interacted with these technologies (see Table 6.33).

Measuring. Relating to Task 2, Google Maps affords the measure of travel time between the cities and the optimal place for the shopping centre.

Visualizing. This category was not identified in the student's activities in both tasks.

Geometric construction. Relating to Task 2, GeoGebra afforded the construction of a figure (triangular shape) that represents the positions of the three cities. GeoGebra also afforded the construction of the triangle's geometric middle point, which connects the three cities (see Table 6.34).

Experimenting/changing. This category was not identified in the student's activities in both tasks.

Data entry and generation. Relating to Task 1, Excel/spreadsheet allowed entering the set of values (or variables) in the spreadsheet and generating the entire data set. Thus, the students keyed in a few data sets and then selected and dragged these data to obtain the entire data (see Table 6.32).

Calculating. This category was not identified in the student's activities in both tasks.

Mathematical affordances and constraints

From Tables 6.37 and 6.38, the mathematical affordances and constraints recorded are clarification (ø), analyzing, simulating and visualizing (ø), linking representations, regularity and variations (ø), and arithmetic and statistics.

Clarification. This category was not identified in the student's activities in both tasks.

Analyzing. Group C reconciled their model with reality (relating to Task 2). They did so by factoring the roads (measuring the time of travel) and the population of each city in their discussions, and based on these factors, they found a hypothetical, optimal location for the shopping centre. The students also constructed a theoretical optimal location with GeoGebra, which they compared with the hypothetical optimal location (see Table 6.33 and 6.34).

Simulating and visualizing. This category was not identified in the student's activities in both tasks.

Linking representations. Regarding Task 2, the students could represent the coordinates (from Google Maps) of the three cities in a graphical view in GeoGebra (see Table 6.34).

Regularity and variations. This category was not identified in the student's activities in both tasks.

Arithmetic and statistics. At the mathematical task level, Excel/spreadsheet affords numerical computations such as multiplication (see Table 6.32).

Socio-cultural affordances and constraints

The socio-cultural affordances and constraints that emerged or actualized were common focus (in both Task 1 and 2; see both Table 6.37 and 6.38), observing and improving strategies (ø), and authority of the digital technology (ø).

Common focus. The students shared the same computer and had the facility to look at the same thing and point at what was presented on the computer.

Observing and improving strategies. This category was not identified in the student's activities in both tasks.

Authority of the digital technology. This category was not identified in the student's activities in both tasks.

In summary, I have reported on the activities of Group C along three themes (listed in Section 6.2). In the next section, I will present the case study report of Group D. As stated earlier (in previous sections), I noted that I will only present issues that are different from the already presented group reports while presenting the report of the current group. In the same way, I will only present issues different from Group A, B and C while writing the report of Group D.

6.6 Case study report: Group D (Olga, Hege and Lena)

This section offers a description of the case study report of the third school (Group D; see Sub-Section 3.4.4). The narrative is presented in an order as highlighted in Section 6.2. Group D are second-year lower secondary school students (Grade 9). Group C comprises three students aged between 14 – 15 years. The reports in Sub-Sections 6.6.1, 6.6.2 and 6.6.3 correspond to RQ1, RQ2a & RQ2b and RQ3 respectively. I will first start the report on the students' (Group D's) mathematical modelling activities.

6.6.1 Students' mathematical modelling activities

Details about Group D are presented in Sub-Section 3.4.4. I will present each of the components of the activity system (see Sub-Section 4.1.3 and Section 4.2).

Subject of the activity

Three students (Olga, Hege and Lena) volunteered as the focus group (Group D). The teacher describes this group as mixed-achievement (see Table 5.1 on page 129 for the grades assigned to each student). Table 5.1 also shows the approximate time the students used in solving both tasks. The three students adopted different roles concerning tool usage while working on both tasks (see Table 5.2 on page 130).

Community

The community of the activity was made up of students. Six groups worked together in the classroom (see the group reports of all the groups in Appendix D.4), of which particular attention was paid to the focus group (Group D). The community was formed spontaneously to solve both tasks and then dissolved.

Object of the activity

The researcher assumes that the object of the activity is to solve Tasks 1 and 2 with digital technologies and write a report. The students confirmed this objective at some points in their engagement with both tasks.

Code	Digital technology (**Calculator**, **GeoGebra** & **Excel/spreadsheet**)
Task	Task 1
Context	The students first use a calculator on the computer to solve the task. The students then use GeoGebra after using the calculator for a few computations. The students finally used Excel/spreadsheet to generate their data.
Recorded dialogue	*Hege*: We have to use the computer. *Lena*: Oh yes *Olga*: Erm [Open calculator on the computer]. *Lena*: Erm take erm *Olga*: Say we have 5500, then it will be 95 times it, multiply 95 [Enters 5500 times 95 on the calculator, see Image 1]. *Hege*: Yes. *Lena*: Ok, we must write it there [Pointing to the question paper], write it here… …….. *Hege*: What if we write the points in… *Olga*: 50. *Lena*: Yeah. *Hege*: In GeoGebra, and… *Olga*: Yes yes yes *Hege*: And then just look at the graph. *Lena*: Oh yeah, you are so smart. *Olga*: [Open GeoGebra on the computer, see Image 2]. So, it generates it for us, erm … ……… *Hege*: What if we use regneark (spreadsheet)? *Olga*: Actually, we should use regneark (spreadsheet) for this

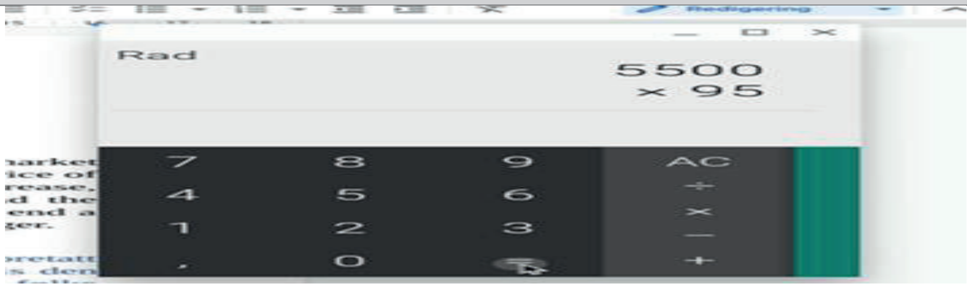
<table>
<tr><td></td><td>Hege: Okey.
Lena: Go to regneark (spreadsheet).
Olga: Yes, I try, Okey [Goes to Google platform and select Excel/spreadsheet]. Actually, we should have gone into the classroom thing.
Hege: Okey, we can just throw it in afterwards.
Lena: Oh yes. Can we…. where we….
Olga: New regneark (spreadsheet) [Opens spreadsheet on the computer, see Image 3]</td></tr>
<tr><td>Image</td><td>


Image 1

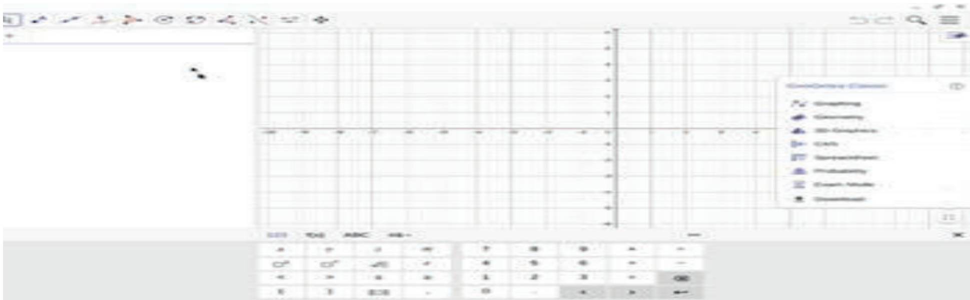



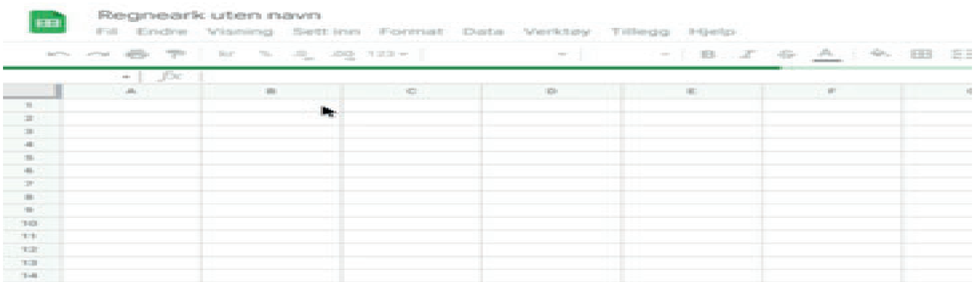
Image 2


Image 3
</td></tr>
</table>

Table 6.39: Sample Data Aligned with Codes (Digital Technology—Calculator, GeoGebra & Excel): Group C—Task 1.

Mediating artefacts/tools for the subject-object interaction

The mediating artefacts or tools mediating the students' activities are physical (digital technologies) and non-physical (group work) tools. I will first present a report on the digital technologies that Group D used while working on both tasks.

Digital technologies

Group D used GeoGebra, a calculator on the computer and Excel/spreadsheet while working on Task 1 and used Google Maps and GeoGebra while working on

Task 2 (see Table 6.1). Table 6.39 above shows a part of the transcription aligned with codes showing examples of different times GeoGebra, calculator and Excel/spreadsheet mediated the interactions between the students and solving Task 1. The recorded dialogue in Table 6.39 is partitioned into three sections/parts (separated by ……). The first part shows the students using a calculator on the computer to compute the product of the number of people that buy the car and the price of the car. The students repeated this procedure for some data sets and then moved on to a new tool. The second part of the dialogue shows the students using GeoGebra to generate their data set. Thus, they plotted some points in the graphic view in GeoGebra and tried searching for a method that would help in generating their data (but they were unsuccessful). Finally, the third part of the dialogue shows the students generating their data in Excel/spreadsheet.

Code	Digital technology (**Google Maps** & **GeoGebra**)
Task	Task 2
Context	The students first searched for the positions of the three cities on Google Maps. They then inserted the picture from Google Maps into GeoGebra and connected the three points with the line segment. The students finally searched for the middle point of the triangle, representing the optimal position.
Recorded dialogue	*Hege*: This one is like the helicopter one yesterday. *Olga*: Yes, which is easy. I know how to do that. Google … [Opens Google maps] *Hege*: I read it, [Reads the second question aloud]. *Olga*: Do we count Kristiansand as Lund, or do we count Kristiansand as Kristiansand town? *Lena*: Kristiansand town. *Olga*: We take Kristiansand Kommune [Searches for Kristiansand Kommune and saves it on Google maps, see Image 1] ………. *Olga*: No, it is a triangle. Shall we have a triangle? *Hege*: [Selects polygon from the menu bar and tries to join the three cities in the form of polygon in GeoGebra]. *Olga*: No, don't touch in the circle, have to take it in the middle. No, you are not supposed to have a point there [Moves the computer to herself]. You have the point on the wrong place. It is actually very annoying. You have to have a point there, at that thing. Okey? There, yes [Joins the three cities together in a triangular polygon form, see Image 2]. *Hege*: Okey. ………. *Olga*: Yes, but if we first have a point in the middle.

	Lena: Okey. *Olga*: We have to make that circle [Selects circle with center through point from the menu bar]. How big a circle do we want? *Lena*: You can’t take it so big. *Olga*: But look at that [Make a circle through the three cities, see Image 3]. Like this. *Lena*: Say, the … *Olga*: Now it is equally long, is it not? *Hege*: Yes, just put it in.

Image

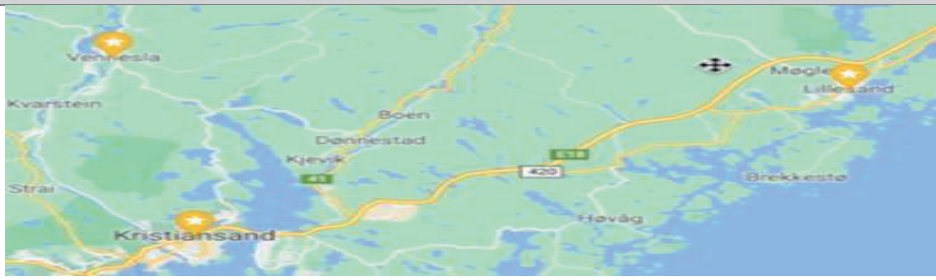


Image 1

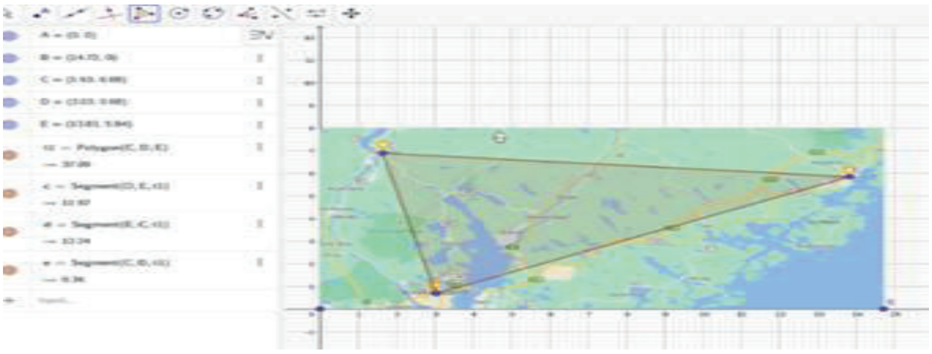


Image 2

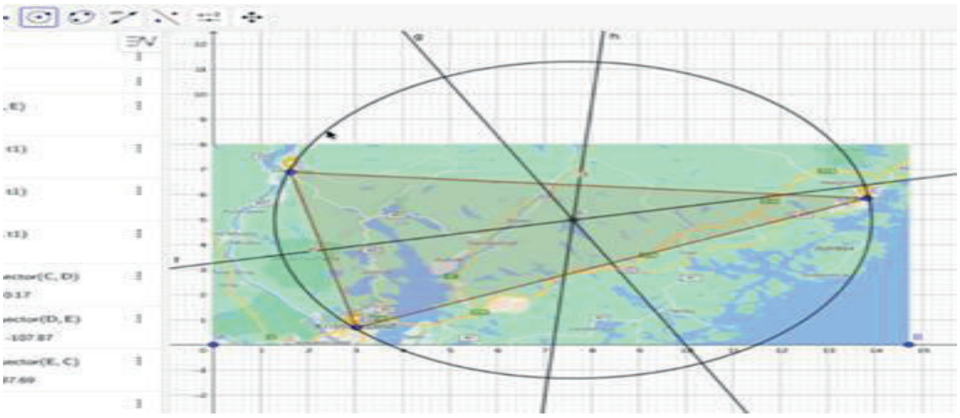


Image 3

Table 6.40: Sample Data Aligned with Codes (Digital Technology—Google Maps & GeoGebra): Group C—Task 2.

Concerning Task 2, Table 6.40 above shows a part of the transcription aligned with codes showing examples of different times Google Maps and GeoGebra mediated the interactions between the students and solving Task 2. The recorded dialogue in Table 6.40 is partitioned into three sections/parts (separated by ……).

The first part shows the students searching for the three cities and saving them on Google Maps. The second part of the dialogue shows the students connecting the three points (representing the three cities) with line segments, forming a triangle. The students did this by taking a screenshot on Google Maps and inserting it into GeoGebra for further analysis. The third and final part of the dialogue shows the students constructing a geometric middle point for the triangle.

Group Work

Table 6.2 (on page 158) presents the number of times the different interaction categories or sequences occurred in the episodes of the students' activities. From Table 6.2, 1 out of 6 episodes describes asymmetrical contingency in the activities relating to Task 1, whilst 1 out of 4 episodes describes this contingency in the activities relating to Task 2. Again, 3 out of 6 and 2 out of 4 episodes describe respectively reactive and mutual contingencies in the activities relating to Task 1, and 0 out of 4 and 3 out of 4 episodes describe respectively reactive and mutual contingencies in the activities relating to Task 2. Thus, the interactions in the students' activities were more critical and demonstrated tentative ideas being offered. This might result from the level of the students (lower secondary level) instead of their assigned grades or performance (mixed achievement group).

Rules for the subject-community interaction

Table 6.3 (on page 158) presents the explicit and implicit rules observed.

Explicit rule

The students worked in a group, and the technology used was not imposed on them; as such, Table 6.1 shows the different digital technologies used by Group C as they worked on both tasks. Another explicit rule was time constraints. Group D's activities were not affected by time constraints as they solved both tasks within the expected time (see Table 5.1).

Implicit rule

No implicit rules were observed or recorded in the activities of Group D.

Division of labour for the community-object interaction

The students had roles that were constant throughout the activities (see Table 5.2). Other roles in the form of leading, opposing, suggesting, supporting, non-contributing, and questioning and challenging changed at different times during the students' activities. Table 6.4 (on page 160) shows the number of roles each member of Group D took as they worked on both tasks.

Roles adopted by students

From Table 6.4, relating to Tasks 1 and 2, all members of Group D took suggesting and supporting roles at some time. Olga was the only member of Group D who took the leading role in one episode of each activity. This might result from the level of the students (lower secondary level) as these students were actively involved throughout the activities. As such, none of the students took the non-contributing role in both activities. Hege took the opposing role once in the activities relating to Task 2, whilst Olga took this same role once in the activities relating to Task 1. On the other hand, Lena took the opposing role twice in the activities relating to Task 1 and once in the activities relating to Task 2. Hege was the only member of Group D who took the questioning and challenging role twice in the activities relating to Task 1.

I will now present a report on the modelling actions of Group D that emerged, and the role digital technologies played in these actions. This report corresponds to RQ2a and RQ2b.

6.6.2 Emergence of modelling actions and the role of digital technologies

The student's actions are divided into categories (coded in Table 9.6 in Appendix E.2). I will present the analysis of the emerging modelling actions of Group C according to the categories in Tables 6.41 and 6.42 below. The description of Tables 6.41 and 6.42 is similar to Tables 6.12 and 6.13. I will further explain Tables 6.41 and 6.42 with some excerpts from the video recording transcriptions and screen recordings as evidence.

Modelling actions	Episodes of the students' activity					
	1	2	3	4	5	6
Breaking the task into manageable parts	X	-	-	-	-	-
Searching for a model	X	X	X	X	-	-
Finding a solution for the model	-	X	X	X	X	-
Explaining the results in real terms	-	-	-	-	-	X
Checking the results for adequacy	-	-	-	-	-	X
Role of technological tools						
Calculating (Computer calculator, GeoGebra, Excel)	-	X	X	X	-	-
Geometric construction (GeoGebra)	-	-	X	-	-	-
Data entry and generation (Excel/Spreadsheet)	-	-	-	X	X	

Table 6.41: The modelling actions that emerged in Group D's activities (regarding Task 1) and the role of the digital technologies.

Modelling actions	Episodes of the students' activity			
	1	2	3	4
Breaking the task into manageable parts	X	-	-	-
Searching for a model	X	X	-	-
Finding a solution for the model	-	X	X	X
Explaining the results in real terms	-	-	X	X
Checking the results for adequacy	-	-	X	X
Role of technological tools				
Researching (Google maps)	X	-	-	-
Measuring (GeoGebra)	-	-	X	-
Geometric construction (Geogebra)	-	X	X	-

Table 6.42: The modelling actions that emerged in Group D's activities (regarding Task 2) and the role of the digital technologies.

I will present each category in Tables 6.41 and 6.42 and the role of digital technology.

Breaking the task into manageable parts

Under the category "breaking the task into manageable parts" in Table 6.41 (relating to Task 1) and 6.42 (relating to Task 2), there were different times in the episodes where the students performed an action of breaking the task into manageable parts. In Table 6.41, it was counted only in the first episode (out of 6 episodes), while in Table 6.42, it was counted only in the first episode (out of 4 episodes). This shows that Group D started their work on both tasks by breaking the task into manageable parts. Thus, the students identified their variables in the first episode and worked with these initial variables in the subsequent episodes.

Role of digital technology

Regarding Task 1, Group D did not use any digital technology under this category; the students only discussed the task and agreed on the variables needed to make a model. Regarding Task 2, Group D used Google Maps to search for the positions of the three cities (see the first part of the dialogue in Table 6.40).

I will now present the next category, 'searching for a model', that emerged in the activities of Group D.

Searching for a model

The students performed the action of searching for a model in some of the episodes in the activities. In Table 6.41 (relating to Task 1), the category 'searching for a model' was counted in the first four episodes (out of 6 episodes), while in Table 6.42 (relating to Task 2), this category was counted in the first two episodes (out of 4 episodes). From these results, the students searched for a model after breaking the task into manageable parts. Regarding Task 1, the students tried different methods (or solution strategies) as they searched for a model. Thus, the students used a calculator to compute their initial variables and then used another tool to generate their data (for which they were unsuccessful). The students finally used Excel/spreadsheet as they searched for a model (see Table 6.39). Regarding Task 2, the students took a screenshot on Google Maps (capturing the positions of the three cities) and inserted it into GeoGebra for further analysis (see the first and second part of the dialogues in Table 6.40).

Role of digital technology

Regarding Task 1, Group D used three different tools to search for the model. They started with a calculator (for multiplication operations) and then moved on to GeoGebra (for constructing a function and multiplication operation). They finally used Excel/spreadsheet to compute the product between the number of people and the car price and generate the entire data set (see Table 6.39). Relating to Task 2, Group D used Google Maps to search positions of the cities and GeoGebra to construct a line segment connecting these cities (see Table 6.40).

The following presentation is on the report on the category 'finding a solution for the model' that emerged in Group D's activities.

Finding a solution for the model

The students performed this action in several of the episodes (compared to other categories of modelling actions). In Table 6.41 (relating to Task 1), it was counted in 4 episodes (out of 6 episodes), while in Table 6.42 (relating to Task 2) it was counted in 3 episodes (out of 4 episodes). This shows that many students' actions were about finding a solution for their model. Relating to Task 1, the students started with the computation of their initial variables using a calculator. The students opted for another strategy as they thought the initial strategy might take longer. As such, the students went back to the action of 'searching for a model' by plotting their initial variables in GeoGebra. The students tried to find another point: the product of the number of people buying the car and the price at which they bought the car, but were unsuccessful after some attempts (unsuccessful action of finding a solution for the model). The students then inserted their initial variables in Excel/spreadsheet. With this tool, the students could generate all the data representing the problem situation (see Table 6.39). In working on Task 2, Group

D constructed a figure (a triangle) representing the three cities and constructed a circumcircle/circumcenter of this triangle. Thus, the students constructed a perpendicular bisector of each side of the triangle (where the corner points represent the three cities) and then searched for the point of intersection (optimal point) of these perpendicular bisectors. The students constructed a circle with the point of intersection as the centre point, and the circle passed through all the corner points of the triangle (see Table 6.40). The students also measured the distances between the cities (vertices of the triangle) and the optimal point to ensure their answer (see Table 6.43 on page 240).

Role of digital technology

Regarding Task 1, Group D used a calculator on the computer for computations and GeoGebra for geometric construction (although not successful) and calculations. Again, Group D used Excel/spreadsheet to calculate and generate their entire data set. Relating to Task 2, Group D used GeoGebra for a geometric construction of a figure (in this case, a triangle) that represents the positions of the three cities. The students used GeoGebra to locate the geometric middle point of this figure and again measured the distances between this point and the three cities.

The following presentation is on the report on the category 'explaining the results in real terms' that emerged in Group D's activities.

Explaining the results in real terms

In Table 6.41 (relating to Task 1), the category 'explaining the results in real terms' was counted in the last episode (out of 6 episodes), while in Table 6.42 (relating to Task 2), this category was counted in the last two episodes (out of 4 episodes). This shows that the students performed this action at the end of the activities relating to Task 1 and then closer to the end of the activities relating to Task 2. It also shows that the students performed this action when they were sure about their mathematical results. In finalizing their results, Group D wrote a report where they interpreted the mathematical results in real terms (see Group D's report in Appendix D.4). From this report (relating to Task 1), Group D explained that the optimal price of the car is 7500 euros when 75 people buy the car. The maximum revenue will be 562500 euros. Regarding Task 2, Group D concluded that the optimal location has the same distance to all three cities (they did not consider actual roads). The students also searched for this optimal location on Google Maps (which happens to be Dragsholtvatnet – a lake close to Tveit).

Role of digital technology

Related to both Tasks 1 and 2, Group D wrote their final report on a Word Document. The other groups that worked alongside Group D also wrote their final report in a Word Document (see Appendix D.4 for the solution reports of all the groups in School D).

I will now present the final category, 'checking the results for adequacy', that emerged in the activities of Group D.

Checking the results for adequacy

The students checked the results for adequacy in one of the episodes concerning Task 1. In Table 6.41 (relating to Task 1), the category 'checking the results for adequacy' was counted in the last episode (out of 6 episodes), while in Table 6.42 (relating to Task 2), this category was counted in the last two episodes (out of 4 episodes). This shows that the students performed this action at the end of the activities relating to Task 1 and then closer to the end of the activities relating to Task 2. Excerpt 6.6.1 shows a part of the transcription where the students discussed going down with the price after finding the price that generates the maximum revenue (relating to Task 1):

Hege: Now we are going to explain why that price, is the best price.

Olga: But we have not tried to go down. I cannot understand how we can go down. That is what I wondered.

Hege: What do you mean by down?

Olga: You started in the beginning to go down instead of going up.

Hege: Yes, I think more people who buy it. I don't think it means anything.

Lena: But it is not allowed, it is not allowed.

Olga: I don't believe it is allowed either.

Excerpt 6.6.1

From Excerpt 6.6.1, Olga suggested they try to go down with the price, but the peers agreed on the final results instead of discussing this new idea further. The outcome could be exciting if the students tried out this new idea and compared it to their initial strategy (or final solution). Table 6.43 below shows a part of the transcription aligned with codes, showing examples of Group D checking the results for adequacy while working on Task 2. From Table 6.43, Lena insisted the group measures the distances between the optimal location and the three cities, even though Olga explained that the distances are equal since the circle goes through all three points.

Role of digital technology

Regarding Task 1, Group D did not use any digital technology under this category; the students only discussed the results they had. Regarding Task 2, Group D used GeoGebra to measure the distances between the optimal location and the three cities (see Table 6.43).

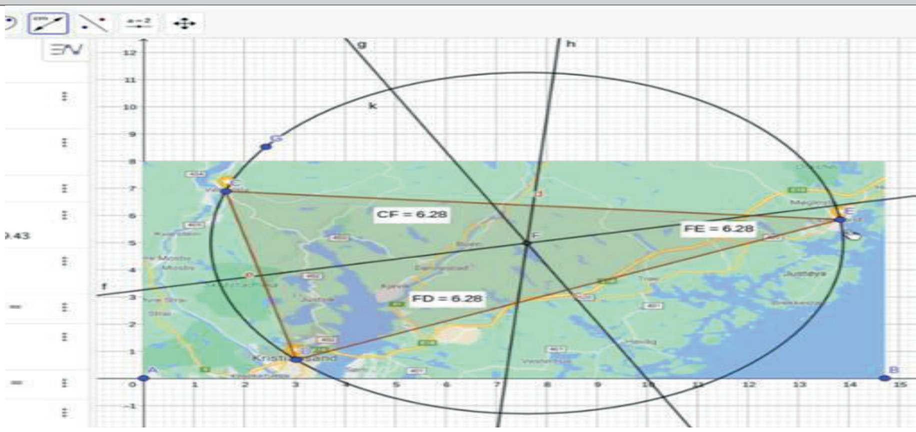
Code	Checking the results for adequacy
Task	Task 2
Context	The students measured the distances between the optimal point and the three cities (in GeoGebra) to be sure these distances were equal.
Recorded dialogue	*Olga*: Now you can see it is equally long between all points. *Lena*: We can measure. *Hege*: [Searches for the distance between each city and the middle point, see the image below]. *Olga*: Why are you measuring? It is the same length. Aha! It is not as long as that, is it? That is the fairest. *Hege*: Oh, yeah [Finished measuring the lengths, see the image below]. *Lena*: Yes. *Olga*: To have it there, are we certain? *Hege*: Yeah.
Image	

Table 6.43: Sample Data Aligned with Codes (Checking the results for adequacy): Group D.

I will now present a report on the affordances and constraints of the digital technologies that emerged in the activities of Group D (corresponds to RQ3).

6.6.3 Emergence of technological, mathematical and socio-cultural affordances and constraints

The frequency of actualized technological, mathematical, and socio-cultural affordances and constraints that emerged in Group D's activities regarding Tasks 1 and 2 are presented in Table 6.44 and 6.45, respectively. I will present the analysis of Group D's emerging affordances and constraints according to the categories listed in Tables 6.44 and 6.45. The description of Tables 6.44 and 6.45 is similar to Tables 6.20 and 6.21. I will further explain Tables 6.44 and 6.45 with some excerpts from the video recording transcriptions and screen recordings as evidence.

Affordances & Constraints	Digital Technologies				
	GeoGebra	Excel	Calculator	Google search	Google maps
Technological Affordances					
Calculating	1	1	2	-	-
Geometric construction	2	-	-	-	-
Data entry and generation	-	2	-	-	-
Mathematical Affordances					
Linking representations	2	-	-	-	-
Arithmetic and statistics	1	2	2	-	-
Socio-cultural Affordances					
Common focus	1	2	1	-	-

Table 6.44: The frequency of actualized technological, mathematical, and socio-cultural affordances that emerged in Group D's activities regarding Task 1.

Affordances & Constraints	Digital Technologies				
	GeoGebra	Excel	Calculator	Google search	Google maps
Technological Affordances					
Researching	-	-	-	-	3
Measuring	1	-	-	-	-
Geometric construction	2	-	-	-	-
Mathematical Affordances					
Analyzing	-	-	-	-	3
Linking representations	2	-	-	-	-

Arithmetic and statistics	**1**	**-**	**-**	**-**	**-**
Socio-cultural Affordances					
Common focus	**2**	**-**	**-**	**-**	**1**
Authority of the digital technology	**1**				

Table 6.45: The frequency of actualized technological, mathematical, and socio-cultural affordances that emerged in Group D's activities regarding Task 2.

Technological affordances and constraints

The digital technologies Group D used while working on Task 1 were GeoGebra, a calculator on the computer and Excel/spreadsheet. Group D also used Google Maps and GeoGebra while working on Task 2 (see Table 6.1). From Tables 6.44 and 6.45, the technological affordances and constraints recorded are researching, measuring, visualizing (ø), geometric construction, experimenting/changing (ø), data entry and generation, and calculating.

Researching. Regarding Task 2, Google Maps allows retrieval of information about the position/location of the cities as the students interact with this technology (see the first part of the dialogue in Table 6.40).

Measuring. Relating to Task 2, GeoGebra affords the measure of distances between the cities and the optimal place for the shopping centre. Thus, the students measured these distances to be sure they were equal (see Table 6.43).

Visualizing. This category was not identified in the student's activities in both tasks.

Geometric construction. Regarding Task 1, the students used GeoGebra to construct a function that would represent their initial variables and again to generate the rest of the data through this function (but they were unsuccessful). For example, Table 6.46 below shows a part of the transcription aligned with codes showing examples of Group D constructing a function with GeoGebra while working on Task 1. That is, the students perceived GeoGebra could afford them the drawing of a function or generate their entire data if they key in their initial variables. The students could not achieve this, as they needed the corresponding value to have a pair of points for C (i.e., $C = (5000, 500000)$ representing the price of the car and the total revenue at that price), which is a constraint. Relating to Task 2, GeoGebra afforded the construction of a figure (triangular shape) that represents the positions of the three cities. GeoGebra also afforded the construction of the triangle's geometric middle point, which connects the three cities (see the second and third part of the dialogues in Table 6.40).

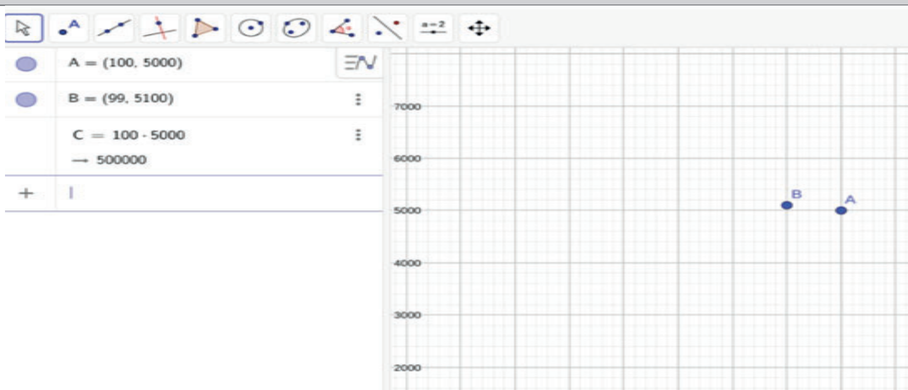
Code	Technological affordances and constraints (**Geometric construction**)
Task	Task 1
Context	The students in this episode made lines to connect the three points representing the three cities in GeoGebra.
Recorded dialogue	*Olga*: Shall we see how much we earn in total or how much we are going to charge for it? *Hege*: I will try something. *Olga*: Now, look at me. *Lena*: Are we going to write 98? *Hege*: Erm *Olga*: Write 100 *Hege*: [writes the product of 100 and 5000 in the algebra section, see the image below]. *Olga*: 100 and 5000 are, yeah. *Hege*: 100 is not correct *Lena*: Yes, it is good, it's good. *Hege*: There will be no point *Olga*: There is no graph. It is only to add one zero, two zeros [moves the computer to herself]. *Hege*: Ouch *Olga*: Damn
Image	

Table 6.46: Sample Data Aligned with Codes (Technological affordances and constraints): Group D.

Experimenting/changing. This category was not identified in the student's activities in both tasks.

Data entry and generation. Relating to Task 1, Excel/spreadsheet allowed the students to enter the set of values (or variables) in the spreadsheet and generate the entire data set. Thus, the students keyed in a few data sets and then selected and dragged these data to obtain the entire data set (see the third part of the dialogue in Table 6.39).

Calculating. Relating to Task 1, the calculator on the computer and Excel/spreadsheet afforded the calculation of the product of the number of people and the corresponding price of the car (see the first and third part of the dialogues in Table 6.39). Similarly, GeoGebra also afforded this sort of calculation (see Table 6.46).

Mathematical affordances and constraints

From Tables 6.44 and 6.45, the mathematical affordances and constraints recorded are clarification (ø), analyzing, simulating and visualizing (ø), linking representations, regularity and variations (ø), and arithmetic and statistics.

Clarification. This category was not identified in the student's activities in both tasks.

Analyzing. Group D reconciled their model with reality (relating to Task 2). They did so by locating the optimal point (constructed in GeoGebra) in Google Maps. Thus, after constructing it in GeoGebra, the students returned to Google Maps to locate the optimal location on the map (see Group D's report in Appendix D.4).

Simulating and visualizing. This category was not identified in the student's activities in both tasks.

Link representations. Regarding Task 1, the students moved between numeric and graphical representation but were unsuccessful in their approach (see Table 6.46). Relating to Task 2, the students could represent the coordinates (from Google Maps) of the three cities in a graphical view in GeoGebra (see the second and third part of the recorded dialogue in Table 6.40).

Regularity and variations. This category was not identified in the student's activities in both tasks.

Arithmetic and statistics. At the mathematical task level, Excel/spreadsheet, a calculator, and GeoGebra allow numerical computations such as multiplication (see Table 6.39).

Socio-cultural affordances and constraints

The socio-cultural affordances and constraints that emerged or actualized were common focus (in both Task 1 and 2), observing and improving strategies (ø), and authority of the digital technology (only in Task 2) (see Table 6.44 and 6.45).

Common focus. The students shared the same computer and had the facility to look at the same thing and point at what was presented on the computer.

Observing and improving strategies. This category was not identified in the student's activities in both tasks.

Authority of the digital technology. From the recorded dialogue in Table 6.43 (on page 240), Olga pointed out that the distances between the middle point (optimal location) and the three cities are equal since the circle passes through all the points. Lena suggested that they still measure these distances. In this case, Lena would rather accept the answer (outcome) from the digital technology than her peers or go through the operation of measuring these distances to ensure their final results.

In summary, I have reported on the activities of Group D along three themes (listed in Section 6.2). In the next section, I will present the cross-case analysis of all the groups together.

6.7 Cross-case analysis

This section presents the cross-case analysis of the cases (or reports) in Sections 6.3, 6.4, 6.5 and 6.6. The section highlights significant similarities and differences between the case study reports. Khan and VanWynsberghe (2008) argue that cross-case analysis is a research method that facilitates the comparison of commonalities and differences in the events, activities and processes of the different cases. For instance, themes appearing in each case were tabulated (see Section 6.1) to facilitate a cross-case comparison. The case study reports of each case (or group) follow a particular structure (see Section 6.2) that helps address the research questions, but I will follow a different structure for this section. In this case, I will only present the significant similarities and differences in the activities of the groups. The discussion of the cross-case analysis follows a structure that helps to facilitate the discussion of the significant issues arising from the research study (see Section 7.2). The similarities and differences in the activities of the groups presented in this section follow the structure, interaction sequences, roles adopted by the students and students' solution strategies. Although the structure above will help to facilitate the discussion of significant issues arising from the research, there are instances where I will refer to aspects of this structure while addressing the research questions. For instance, aspects that concern the solution strategies of the groups. I will first present the cross-case analysis of the interaction sequences followed by roles adopted by the students and students' solution strategies. These items are interrelated, but I present them in a linear manner.

6.7.1 Interaction sequences

Table 6.2 presents the interaction sequences recorded in Groups A, B, C (upper secondary school students) and D (lower secondary school students) activities relating to Tasks 1 and 2. From Table 6.2, the interaction sequences recorded were asymmetrical contingency, reactive contingency, mutual contingency, and the combination of asymmetrical and pseudocontingency:

Asymmetrical contingency

In this contingency, the response of *Student A* to *Student B*'s comments/ideas within a group interaction is affirmative, non-critical and non-challenging, and in this contingency, a high-achiever or performing student often dominates in the activities (Peter-Koop, 2002; Esmonde, 2009). This type of contingency was recorded in several episodes (especially Groups A and B) relating to the activities of Task 1 as compared to Task 2 across the groups. That is (see Table 6.2), 7 out of 12 episodes describe asymmetrical contingencies in the activities of Group A relating to Task 1, while 2 out of 9 episodes describe asymmetrical contingencies concerning Task 2. Similarly, 6 out of 7 episodes describe an asymmetrical contingency in the activities of Group B relating to Task 1, while 1 out of 8 episodes describes an asymmetrical contingency concerning Task 2. In the case of Groups C and D (about Task 1), 0 out of 2 and 1 out of 6 episodes describe an asymmetrical contingency, respectively, and about Task 2, 1 out of 5 and 1 out of 4 episodes describe asymmetrical contingencies, respectively.

The nature of both tasks might be a contributing factor to this development. Thus, Task 1 contains more information in connection with given numbers in the task, which must be added by applying mathematical formulas. For instance, the students might develop a function from the given information. In such a situation, the student who dominates the group might propose a strategy for developing a function (which the other members might follow). If the other members follow the dominant student's proposed function, then the interaction sequence might likely be an asymmetrical contingency. Task 2, conversely, does not have numerical values in its presentation but requires extra-mathematical knowledge (the result of experience, see Sub-Section 5.5.2). In this case, much of the discussion depends on the student's experiences and not just on applying mathematical formulas (as in Task 1). Again, taking a closer look at the group dynamics (as presented above), asymmetrical contingency was more prevalent in the episodes of Group A and B relating to Task 1, as compared to Group C and D. If we relate this to the characteristics of the students, then it is reasonable that this type of contingency is diminished in the activities of Group C (a group of same-achievement students); as the students in this group are high performing students. On the contrary, the contingency discussed here is also diminished among Group D, although this group is made up of mixed-achievement students. Regarding the activities of Task 1, Groups A and B (made up of mixed-achievement students) record asymmetrical contingencies in most of their respective episodes compared to Group D (also made up of mixed-achievement students). The differences between Groups A, B and D, in this case, might be the level of the students. Thus, Group A and B are in the upper secondary school, whilst Group D is in the lower secondary school.

Reactive contingency

In this contingency, a group member considers and critically reviews another member's ideas, which leads to improved decision-making or content. Each individual's response is almost entirely dependent on the preceding response of the other. Reactive contingency was recorded in several episodes regarding Task 2 compared to Task 1 across the groups. That is (see Table 6.2), 1 out of 12 episodes describes reactive contingency in the activities of Group A relating to Task 1, while 4 out of 9 episodes describe reactive contingencies concerning Task 2. In the activities of Group B, 0 out of 7 episodes describe a reactive contingency relating to Task 1, while 5 out of 8 episodes describe a reactive contingency concerning Task 2. In the case of Groups C and D (about Task 1), 0 out of 2 and 3 out of 6 episodes describe a reactive contingency, respectively. Concerning Task

2, 2 out of 5 and 0 out of 4 episodes describe reactive contingencies, respectively. From the description above, if we take a critical look at Task 1, fewer reactive contingencies were recorded in the episodes of Groups A and B. On the contrary, there are more reactive contingencies relating to Task 2 in Groups A and B's episodes. However, these groups are made up of mixed-achievement students (yet still, the students were able to review each other's ideas critically). Comparing Groups A and B to Group D (another mixed-achievement group), reactive contingency was recorded more in the episodes of Group D regarding Task 1. No reactive contingency was recorded in the episodes of Group D regarding Task 2. However, more mutual contingencies (3 out of 6, see Table 6.2) were recorded.

Mutual contingency

In this contingency, different perspectives are acknowledged and synthesized into a collective response. That is, sense-making and conversations are mutually driven. This contingency was recorded in several episodes (especially for Groups A and B) regarding Task 2 compared to Task 1 across the groups. That is (see Table 6.2), 1 out of 12 episodes describes mutual contingency in the activities of Group A regarding Task 1, while 3 out of 9 episodes describe mutual contingency regarding Task 2. In the activities of Group B, 1 out of 7 episodes describes a mutual contingency regarding Task 1, while 2 out of 8 episodes describe a mutual contingency regarding Task 2. In the case of Groups C and D (about Task 1), 2 out of 2 and 2 out of 6 episodes describe a mutual contingency, respectively, and about Task 2, 2 out of 5 and 3 out of 4 episodes describe mutual contingencies, respectively. From the description above, Group C (same-achievement and high-performing students) recorded more mutual contingencies (similarly reactive contingency) in its episodes. Groups A and B recorded more mutual contingencies in the episodes relating to Task 2 than in Task 1, although these groups are made up of mixed-achievement students. Conversely, Group D is also made up of mixed-achievement students (just like Groups A and B). However, many mutual contingencies were recorded in both Task 1 and 2.

Pseudocontingency

In this contingency, students' responses are individualized, and group members might be unwilling to consider other suggestions for improvement or change. This contingency was recorded only in Group A's activities regarding Task 1. From Table 6.2, 3 out of 12 episodes of Group A's activities regarding Task 1 show the combination of asymmetrical contingency and pseudocontingency. In these three occasions (see Table 6.10 for an example), three students working on Task 1 with one computer, the interaction between two of the students depicts asymmetrical contingency, whilst the interaction between the other student (interacting with the computer alone) and these two students describe a pseudocontingency. Thus, the two students might discuss a particular strategy for solving the task while the other student might try a different strategy on the computer.

I will present a cross-case analysis of the roles adopted by the students in the following subsection.

6.7.2 Roles adopted by students

Table 6.4 presents the frequency of roles adopted by each member of the groups. From Table 6.4, the roles adopted by the students were leading, opposing, suggesting, questioning and challenging, supporting and non-contributing. The teacher or researcher did not assign these roles but are characterizations of the observed patterns of students' participation in the group activity (see Section 2.4).

Leading

This role describes a student dominating the communications within group interactions, leading the group through his/her ideas. From Table 6.4, the activities of Group A (the first block in grey) regarding Task 1 and under the 'leading' role, Thea led the group 11 times while Rolf led the group only once in the episodes. Thea was assigned the same grade as Rolf, but their teacher emphasized that Thea performs higher than Rolf within the attainment band (see the last paragraph in Sub-Section 3.4.1). The number of leading roles regarding Task 2 reduces in the activities of Group A. Thus, Thea led the group 5 times as the group worked on Task 2. Similarly, Thor led Group B 5 times while Emil led the same group a single time in the episodes (see in Table 6.4, the activities of Group B in the second block in white regarding Task 1—under the leading role); even though Emil was assigned grade 5 and Thor was assigned grade 4. Thus, not all high-achieving students necessarily dominate in group interactions. The number of leading roles regarding Task 2 was reduced in Group B. Specifically, Thor only led the group once as the group worked on Task 2. In the narrative so far (considering Groups A and B—a mixed-achievement group), high-achieving students are often dominant in group interactions regarding Task 1 and less often dominant in the activities regarding Task 2. Group D is a mixed-achievement group. However, it was recorded once in the episodes in both Task 1 and 2, where a student (Olga) leads the group (see in Table 6.4, the activities of Group D in the last block in white relating to Task 1—under the leading role).

Questioning and challenging

This role describes a student questioning another student's idea(s) to understand his/her thinking, and the response to the question might also be seen as an attempt to clarify, elaborate, evaluate, or justify one's thinking. From Table 6.4 (see 'questioning & challenging' column for Group A and Task 1), Kåre (the student with the lowest score within the group) questioned and challenged the idea(s) of the peers three times within the episodes. In the case of Group B, Emil questioned and challenged his peers' idea(s) twice in the episodes, although he scored the highest within the group. Considering Group D, Hege (the student assigned the average grade between the other two students) questioned and challenged the idea(s) of the peers two times in the episodes. All the groups mentioned above are mixed-achievement groups, and it was recorded in the episodes that a member (one that was assigned either a low, average or high grade) within the group questioned and challenged the idea(s) of peers concerning Task 1. However, none of Group C members (same-achievement) was seen questioning and challenging the idea(s) of peers in the episodes regarding Task 1 (see 'questioning & challenging' column for Group C and Task 1 in Table 6.4). Against this, some members of Group A (Kåre & Rolf), Group B (Thor, Emil & Ella) and Group C (Anna & Jørn) questioned and challenged the idea(s) of peers in the episodes corresponding to Task 2 (see 'questioning & challenging' column for Group A, B & C and Task 1 in Table 6.3). However, none of the Group D members was seen questioning and challenging the idea(s) of peers in the episodes regarding Task 2.

Opposing

In this role (unlike questioning and challenging), students do not agree to or accept their peers' comments (or idea/s) and/or introduce their ideas/solutions while solving a common task. There are instances where students opposed peers' idea(s) while working on Tasks 1 and 2. For instance (see 'opposing' column concerning Group A and Task 1 in Table 6.4), Thea and Kåre in Group A opposed (once within the episodes) the idea(s) of peers as they worked on Task 1. Meanwhile, in Task 2, Thea, Kore, and Rolf opposed (3 times for Thea and a single time for the others within the episodes) the idea(s) of peers as they collaboratively worked together. In Group B, only Ella opposed (once within the episodes) the idea(s) of others while working on Task 1. However, in Task 2, Thor, Emil and Ella (2 times each within the episodes) opposed the idea(s) of peers. The least performing student (or the student who was assigned the lowest grade) within Group B (Tore) did not oppose the idea(s) of peers while working on either Task 1 or 2. In Group D, Olga and Lena opposed the idea(s) of peers while working on Task 1, whereas Hege and Lena opposed the ideas of peers while working on Task 2. Considering Group A, B and C (mixed-achievement students), it was recorded in the episodes that one/two student/s opposed the idea(s) of peers regarding Task 1, while two/three students opposed the idea(s) of peers as they worked on Task 2. Against this, it was recorded in the episodes that none of Group C members (same-achievement students) opposed the idea(s) of peers while working on both Task 1 and 2 (see 'opposing' column for Group C and Task 1 & 2 in Table 6.4). In the next paragraph, I will present both supporting and suggesting roles and within these roles, I will compare the frequency of students' roles within these two roles.

Supporting and suggesting

A student only agrees to comments in the supporting role without adding anything new or critically assessing it. In contrast, in the suggesting role, a student typically recommends an idea to assist (or add up to) the idea(s) of the other student. From Table 6.4, the 'supporting' and 'suggesting' columns for Group A (mixed-achievement students) and both Task 1 and 2, Thea did not adopt the supporting role in Task 1 but adopted this role once in the episodes while working on Task 2. Again, Thea adopted a suggesting role once in both Task 1 and 2 episodes. The frequency here was low compared to her peers. For instance, it was recorded in the episodes that Kåre adopted supporting roles 9 and 6 times in Task 1 and 2, respectively. Regarding the 'suggesting' role, Kåre once adopted this role in both Task 1 and 2. Thus, Kåre mostly supported the idea(s) of peers in both tasks and only added up to the idea(s) of peers in very few instances. One possible explanation of this pattern is that Thea performed higher (or was assigned a higher grade) than Kåre and led the group. On the other hand, it was recorded in the episodes that Rolf adopted supporting roles 3 and 5 times in Task 1 and 2, respectively. Rolf adopted a suggesting role 4 times and once in Task 1 and 2, respectively. Although Rolf adopted supporting roles in several episodes (regarding Task 1 and 2), he was assigned the same grade as Thea (although there are differences between the attainment bands).

From Table 6.4, 'supporting' and 'suggesting' columns for Group B (mixed-achievement students) and both Task 1 and 2, Thor (the student who was assigned grade 4) and Emil (the student who was assigned grade 5) adopted a supporting role 2 and 5 times in Task 1, respectively. It was recorded that Thor and Emil adopted a suggesting role 7 times each in the episodes of Task 2. Considering the 'suggesting' role, Thor and Emil adopted this role 4 and 2 times within the episodes of Task 2. Emil (and not Thor) adopted a suggesting role 4 times in the episodes of Task 1. This might result from Thor taking the leading role in Task 1. The two other students (Ella and Tore) mostly supported the idea(s) of peers rather than adding up to the idea(s) of peers. Thus, Tore (the student assigned grade 1) adopted the supporting role 7 times in both the episodes of Task 1 and 2 and the 'suggesting' role only once in the episodes of Task 2. Similarly, Ella (the student assigned grade 3) adopted a supporting role 5 times in Task 1 and 2 and 'suggesting' roles 2 and 3 times in Task 1 and 2, respectively.

From Table 6.4, the 'supporting' and 'suggesting' columns for Group C (same-achievement students) and D (mixed-achievement students) and both Task 1 and 2, there is a balance between these adopted roles. In Group C (specifically Nils), it was recorded in the episodes that Nils adopted a supporting role 2 and 4 times in Task 1 and 2, respectively. Similarly, Nils adopted a suggesting role 2 and 4 times within the episodes of Task 1 and 2, respectively. For example, in Group D (specifically Hege), it was recorded in the episodes that Hege adopted a supporting role 5 and 4 times in Task 1 and 2, respectively. Similarly, Hege adopted a suggesting role 5 and 3 times within the episodes of Task 1 and 2, respectively. Although there is a balance between the adopted roles in Group D, a student in the group primarily supported rather than suggested within the group interactions. That is, it was recorded in the episodes that Lena (the student assigned the lowest grade in Group D) adopted a supporting role 6 and 4 times in Task 1 and 2, respectively, and a suggesting role 2 and 1 times in Task 1 and 2, respectively. This pattern is similar to the group members assigned the lowest grades in Groups A and B. To explain the balance (in general) between the supporting and suggesting roles adopted by Group C, this balance occurred as Group C was made up of high-performing students. On the other hand, there is a balance between the two adopted roles in Group D, although it was a group of mixed-achievement students. However, these students were in the lower secondary school compared to the other groups (upper secondary school).

Non-contributing

In this role, the student does not contribute entirely to the group work or contributes at some point but remains silent most of the time. This role was recorded in some of the episodes of Groups A, B and C (students at the upper secondary school). From Table 6.4 (see 'non-contributing' column for Group A and in both Tasks 1 and 2), it was recorded in the episodes that Rolf was non-contributing 5 and 2 times in Tasks 1 and 2, respectively. Rolf was assigned the same grade as the student who took the leading role (Thea); however, Rolf did not contribute to the discussions in some episodes. The teacher corresponding to Group A described Rolf as one who has little motivation within the group (see the last paragraph in Sub-Section 3.4.1), which could contribute to the non-contributing role. In the case of Group B, it was recorded in the episodes that Ella (an average-performing student) adopted a non-contributing role once in both tasks. Lastly, in the case of Group C (same-achievement students), it was recorded in the episodes that Jørn adopted a non-contributing role 2 times in Task 1 and once in Task 2. That is, Jørn did not participate in the activities relating to Task 1, as Task 1 only had 2 episodes (see Table 9.3). This also shows that a student who does not contribute to group discussions is not necessarily a low-performing student. How do we then address the issue that Jørn is a high-performing student among a same-achievement group and non-contributing in Task 1? Group C (among other groups) was randomly formed (see Section 5.2), so a student might not actively involve him/herself in the activities if the student is not comfortable with the other group members. In the case of Group C, the corresponding teacher pointed out that the students have known each other since the start of Autumn 2021 (see the second paragraph in Sub-Section 3.4.3), and the study was conducted in October/November 2021 (see Table 5.3). Contrarily to this, none of the students in Group D (lower secondary students) adopted a non-contributing role in the episodes of both tasks.

I will present a cross-case analysis of the students' solution strategies in the following subsection.

6.7.3 Students' solution strategies

In this subsection, I will present the students' solutions for Tasks 1 and 2. From Table 6.1, the groups used several digital technologies while working on both tasks. The groups' solution strategies shared some similarities and differences.

Task 1

Groups A and B used the trial-and-error method by analyzing patterns of numbers after searching for a function that represents the number of people buying the car and the price at which they buy it. In the case of Group B, the students drew the function $f(x) = -0.01x + 150$ with a set of points in GeoGebra, which represents the number of people who will buy the car at a specific price (see Table 6.25). Afterwards, Group B inserted some numbers in the function and found the corresponding values, for which they computed the product of these two values using a calculator device. Group B continued with this procedure until they arrived at an answer. Similarly, Group A drew the function $f(x) = -x + 100$ with GeoGebra, representing the number of people buying the car at a specific price (see Table 6.6). Group A inserted some numbers in the function and found the corresponding values, for which they computed the product of these two values using a calculator device (see Table 6.16). At one point, Group A made sliders to find the total revenue instead of inserting numbers (see Table 6.11). Group A made a slider ($a = 100$), but it had no link with the function ($f(x) = -x + 100$), and as such, they returned to their initial strategy. This could have been effective if the students had inserted the function $f(x) = -x + 100$ in the algebra view in GeoGebra with $x = a$ (forming a slide $a = 1$ and an equation $\text{eq1}: x = 1$). Then, intersecting the function $f(x)$ and the equation eq1 with the intersection point A (i.e., $\text{A} = \text{Intersect}\,(f, \text{eq1}, 1)$) might help to regulate the number of people buying the car and the price at which they buy the car (using the slider). At another point, Group A keyed in the function $f(g) = x * y$ which provided an 'illegal function' feedback (meaning there should be a proper definition for g) (see Table 6.10). In this case, the students should have keyed in the function g to obtain a result. Again, Group A keyed in the correct variables for the equation $y = 100x + 5000$ of the car's selling price. However, they still did not achieve the desired results (see Table 6.23). Since the students could not combine $f(x) = -x + 100$ and $y = 100x + 5000$. In the end, Group A returned to the initial strategy they began with. Group C decided to make a function in Excel/spreadsheet instead of GeoGebra as they thought the generated data would be recursive (see Table 6.32). Group C generated their data with Excel/spreadsheet and looked for the maximum revenue. Group D first started solving the problem with a calculator on the computer and later realized it would take them longer to compute all the values (see Table 6.39). Group D decided to plot some points and find the graph with GeoGebra; as such, they plotted two points and searched for the product of one of the points. However, no corresponding graph appeared on the graphical view in GeoGebra (see Table 6.46). Group D finally used Excel/spreadsheet to generate their data and found the best-selling price (see Table 6.39).

Task 2

Groups A, B and C used Google Maps to locate the positions of the three cities and Google Search to find the population and travel time between the three cities. In the case of Group A, using Google Search and Google Maps, the students found, analyzed and discussed the optimal position for building the shopping centre (see

Tables 6.9, 6.15 and 6.17). In the case of Group B, the students looked for the positions of the three cities on Google Maps and, transferred the coordinates of these positions into GeoGebra (forming a triangle) and searched for the middle point (using the median of a triangle or centroid approach) (see Tables 6.26 and 6.27). Group B transferred the coordinates of the found middle point back to Google Maps to locate the optimal position. Similar to Groups A and B, Group C also used Google Maps to locate the three cities and Google Search to search the population of cities (see Table 6.33). Later, Group C used GeoGebra to find the best theoretical location without considering the roads or population. Thus, Group C took a screenshot from Google Maps, inserted it in GeoGebra and searched for the middle point (using the circumcircle/circumcenter of a triangle approach) (see Table 6.34). On the other hand, Group D started their work by locating the positions of the three cities using Google Maps. However, Group D did not consider other factors, such as the roads and population of the cities. Like Group C, Group D also found the best theoretical location using GeoGebra without considering the roads or population. Thus, Group D also took a screenshot from Google Maps, inserted it in GeoGebra and searched for the middle point (using the circumcircle/circumcenter of a triangle approach) (see Table 6.40). Group D further measured the distances between the cities (vertices of the triangle) and the optimal point (circumcenter) to be sure of their answer (see Table 6.43).

6.8 Summary of the chapter

I have in this chapter presented the analysis of results that helps in addressing the research questions in the next chapter. I first presented an overview of data analysis in tabular form. That is, a tabular presentation on different digital technologies the students used while working on Task 1 and 2, the interaction sequences and the roles adopted by the students as they worked in a group, the emergence of modelling actions and the role digital technology played, and the emergence of technological, mathematical, and socio-cultural affordances and constraints recorded in the students' activities. The presentation in this tabular form helped to facilitate a cross case comparison among the cases (or groups). Secondly, I presented a structure for reporting the case study reports. This structure was designed in such a way that helps in addressing the research questions. Thirdly, I presented the case study reports of each group following the same structure as mentioned above. Finally, I presented the cross case analysis of the groups. This cross-case analysis follows a structure that will facilitate the discussion of significant issues arising from the research study (which I will discuss in the next chapter).

Having presented the results of this research study, I will, in the next chapter (Chapter 7), present the discussion of the study. In the discussion, I will address the research questions and discuss significant issues arising from the research that I consider having the potential to contribute to mathematics education research.

7 Discussion

In this chapter, I address the research questions and discuss significant issues arising from the research that I consider to have the potential to contribute to mathematics education research. The research questions are addressed in Section 7.1, where I organize the research questions around the three main themes of the research. This is followed by a discussion of significant issues arising from the research in Section 7.2. In Section 7.3, I reflect on theoretical perspectives and their link with the findings. The chapter ends with a summary in Section 7.4.

7.1 Addressing the research questions

The three research questions are organized around the themes, students' mathematical modelling activities, the emergence of modelling actions and the role of digital technologies, and the emergence of technological, mathematical, and socio-cultural affordances and constraints. These themes formed the structure of the group reports in Section 6.2. In the first theme, I discuss the components of the activity system (concerning the sub-questions of the first research question) interacting with each other in the context of the students' modelling activities. In the second theme, I discuss five categories of modelling actions that emerged in the student's activities and the role digital technologies played within the modelling actions that emerged. Finally, in the third theme, I discuss three categories of affordances and constraints of the digital technologies that emerged in the students' activities. The categories in the second and third themes are just names used to report or describe the students' modelling processes and their use of digital technologies. I have respectively defined and explained these categories in Sub-Sections 2.1.3 and 4.4.2 (concerning modelling actions) and in Section 4.3 and Sub-Section 4.4.1 (concerning affordances and constraints).

7.1.1 Students' mathematical modelling activities

I repeat, for the reader, the first research question:

RQ1: *How do students solve mathematical modelling tasks with the aid of digital technologies?*

RQ1a: *What digital technologies did the students use in solving the two mathematical modelling tasks?*

RQ1b: *What contingencies were shown in the student interactions when solving the two mathematical modelling tasks with the aid of digital technologies?*

RQ1c: *What are the rules that mediate students' mathematical modelling activities when solving the two mathematical modelling tasks with the aid of digital technologies?*
RQ1d: *What roles did the students adopt in the student interactions when solving the two mathematical modelling tasks with the aid of digital technologies?*

From a CHAT perspective, the first research question explores how secondary students solve mathematical modelling tasks using digital technologies. Empirical data were collected through recorded conversations (video recordings) and computer activities (screen capture software) to address the research questions above. I analyzed the research questions from CHAT perspective (activity system). The activity system (or unit of analysis) in this study is a group of students solving mathematical modelling tasks with the aid of digital technologies. The components of the activity system are subject, community, object, mediating artefacts/tools (RQ1a & RQ1b), rules (RQ1c) and division of labour (RQ1d). The subject and community were framed by the methodology, and these are not results. As such, I will only discuss the components with the results emerging from the data.

Object of the activity

The researcher assumes that the object of the activity is to solve the mathematical modelling tasks with the aid of digital technologies and write a report. The students ratified this objective at different points while working on both tasks through their utterances and engagement with each other. Klang et al. (2021) argue that "solving a problem is a matter of goal-oriented reasoning". This goal-oriented reasoning starts "from understanding the problem to devising its solution by using known mathematical" strategies (ibid., p. 4). In this study, the student's objectives in solving both tasks guided them throughout the activities. In some instances, the students had to re-read the problem text and remind themselves of the objective or goal of solving the task when they were unsure of their strategy while working on the task. For instance, from Excerpt 6.5.1 (on page 216), Nils suggests the group checks the population of the three cities as they discuss the optimal location, while Jørn re-reads a part of the problem text to remind the group of the goal of the task. DiNapoli (2019) argues that students re-read the problem multiple times to figure out what they need to know, and re-reading might be classified as a cognitive learning strategy (Di Leo et al., 2019).

Digital technologies (RQ1a)

The tasks given to the students allow the use of several digital technologies. Thus, the students' mathematical modelling activities were technology-enabled, which included using multiple digital technologies to solve a single mathematical task (Abramovich, 2022). For instance, Monaghan (2016a) demonstrates how several tools could be used in working on a single mathematics task (see Sub-Section 2.2.2). In this study, all the groups used several digital technologies while working on both tasks. Table 6.1 (on page 157) presents the digital technologies the groups used while working on Tasks 1 and 2. Hegedus et al. (2017) argue that the features of digital technology might support individual preferences and approaches while working on a task. From Sub-Section 6.7.3 (regarding Task 1), the students used the calculator (Groups A, B and D) for the calculation of large numbers and GeoGebra for visual representation and drawing of functions (Groups A, B and D) (Flehantov & Ovsiienko, 2019; Flehantov et al., 2022). In the case of Groups A and B, the calculator complimented GeoGebra as the students used the calculator to calculate large values/numbers produced from the graph in GeoGebra (e.g., see Group A's activities in Table 6.16). On the other hand, the students used an Excel/spreadsheet for numerical calculations and representation of numerical results in tables (Groups C and D) (Flehantov & Ovsiienko, 2019; Flehantov et al., 2022). Before Group D used an Excel/spreadsheet, they switched between tools when the initial tool did not give them the desired results (see Table 6.39).

Regarding Task 2, the students used Google Maps (all groups), Google Search (Groups A, B and C) and GeoGebra (Groups B, C and D) (see Table 6.1). There are several reasons and factors behind students' selection or switching between digital technologies whilst working on mathematical tasks (Geiger et al., 2002; Owens-Hartman, 2015; Anastasakis et al., 2017; Hillesund, 2020), which I will further discuss in Sub-Section 7.2.4. Another thing to note is that digital technologies were not imposed on the students; they were allowed to choose which technology suits them best. This brought about some dynamics in the students' solution processes (see Sub-Section 6.7.3). Jacinto and Carreira (2017) argue that different ways of tackling a problem might be revealed if students can choose digital technologies. The nature of the digital technologies gave the students options while working on both tasks. In this case, the tool allowed mathematics to be explored in diverse ways from different perspectives (Hoyles, 2018). For instance, Group A drew a direct function in GeoGebra whilst Group B plotted some points and then made a graph representing these points (see Sub-Section 6.7.3). Hoyles (2018) points out that students might easily be aware of what varies and what does not through reflection and manipulating a sketch on a graph. For example, in Group A's activities, the students noticed that making a slider might help them reach the desired answer quickly and were aware that the slider does not affect the function as they dragged it (see Table 6.11). Drawing students' attention to manipulating the sliders to fit with the function might help their learning (as GeoGebra offers the platform for such mathematical relations to be expressed).

In summary, the students used several tools while working on Tasks 1 and 2. Their use of tools depends on factors such as one tool complimenting the other or switching to another tool when the first one does not give the desired results. Students also select one tool over the other based on the demands of the tasks. I will discuss why students select or switch to another tool in Sub-Section 7.2.4.

Group interactions (RQ1b)

Different opinions and ideas emerge as the students work on the tasks together. These different opinions and ideas help to improve the students' solution strategies. For instance, from Table 6.34, one student suggests that the group consider a generalized model that does not factor the population and roads in their analysis. Goos et al. (2002, p. 218) argue that interactions in group work shape problem-solving outcomes as challenges eliciting clarification and justification of strategies stimulate further monitoring, which might lead to errors being noticed or fruitful strategies being endorsed. Different interaction sequences emerged in the groups' activities (see Table 6.2). Table 6.2 (on page 158) presents the interaction sequences that emerged within the students' activities. These interaction sequences were pseudo, asymmetrical, reactive and mutual contingencies. I have presented an analysis of these contingencies in Sub-Section 6.7.1. There are several reasons for the type of contingency observed, which could be the strategy adopted by the students, the nature of the task, and the characteristics of the students, among others. For instance, asymmetrical contingency was counted in most of the episodes of the activities of upper secondary and mixed-achievement groups (Groups A and B) regarding Task 1 compared to Task 2. However (concerning these same groups), reactive and mutual contingencies were counted in most episodes regarding Task 2 compared to Task 1. On the other hand, regarding both tasks, reactive and mutual contingencies were counted in most episodes of the activities of Group C (upper secondary and high-performing students) and Group D (lower secondary and mixed-achievement students). One possible reason for these dynamics could be the task's nature and the students' characteristics, which I will discuss in Sub-Section 7.2.1. On the nature of the task, Clark et al. (2014) suggest that the problem type might lead to effective group interaction and activity, whilst Brady and Jung

(2022) add that the choice of tasks stimulates students' interest in solving the problem. Concerning the characteristics of students, Peter-Koop (2002) asserts that predominantly in mixed-ability groups, the high-performing students frequently dominate the group (asymmetrical contingency). Esmonde (2009) adds that high-achieving students tend to dominate group interactions. Lowrie (2011) argues that in such situations, the responses of others are generally influenced by the ideas and strategies that the dominant person has formulated. Another contingency observed in the data is the combination of pseudo and asymmetrical contingency, which occurs when the interaction between student—student and student—computer is a pseudocontingency, whilst the student—student interaction is an asymmetrical contingency (see Table 6.10).

Rules (RQ1c)

The rules for the activity are both explicit and implicit. These rules influence how/why students might act within the activity. Table 6.3 (on page 158) presents the explicit and implicit rules observed in the students' activities.

Explicit rule

There was no restriction on digital technologies in the student's activities; the groups used different digital technologies while working on Tasks 1 and 2 (see Table 6.1). The students first analyzed the problem and then selected a particular digital technology to solve the problem. For instance, Group C selected Excel/spreadsheet instead of GeoGebra as they thought their data was recursive (see the first part of the dialogue in Table 6.32). Santos-Trigo (2019) points out that using digital technologies might demand that students analyze and discuss what problem-solving strategies appear essential while working on a task. Thus, the students know what they are looking for and how digital technology can help them achieve that. Jacinto and Carreira (2017) emphasize that choosing a particular digital technology does not only involve students' skills but also the interplay between mathematical skills and the perception of the affordances and constraints of the digital technology (I will further discuss the affordances and constraints of digital technologies in Sub-Section 7.1.3). Another explicit rule recorded in Table 6.3 is time constraint. This was recorded in Groups A (regarding Task 1) and B (regarding Task 2). Concerning Task 1, there were instances within Group A's activity where new ideas were dismissed due to time constraints. For instance, a student dismissed another student's suggestions of using the spreadsheet to generate their data because they were already close to finding the answer (see Excerpt 6.3.1 on page 173). This incident might result from time constraints or the student's preference for the solution strategy. Concerning Task 2, there were instances where conclusions drawn were accepted without further analysis. For instance, members of Group B went on to write the final report, although one member argued that they needed to consider the roads before making the final decision (see Excerpt 6.4.1 on page 203). Caviola et al. (2017) argue that time constraints interfere with decision-making as they alter the selection of a strategy in problem-solving (ibid., p. 7). Again, time constraints in a problem-solving situation might affect performance. Heinze et al. (2009) point out that open problems might give room for flexible or adaptive use of strategies in mathematics, and Rieskamp and Hoffrage (2008) emphasize that leaving a considerable amount of time to complete a task enables students to get a more significant amount of information. Caviola et al. (2017) add that students get enough information (given sufficient time) by focusing attention on essential task features, ultimately selecting an optimal strategy. In the case of Group B, the students spent 40 minutes (20 minutes more than the expected time) but could not draw the desired conclusion. The students might have spent much time on other parts of the task.

Implicit rule

From Table 6.3, the only implicit rule observed was dismissing comments or suggestions. This was mainly observed in Groups A (relating to Task 1) and B (Relating to Task 1 and 2). In the case of Group A, the students first accepted the new idea and rejected it after trying it out, and it did not yield any better results (see Table 6.11 on page 171). Thus, students dismiss comments or suggestions when they do not fit the current strategy. Group B, on the other hand, rejected the new idea without trying it out (see Excerpt 6.4.2 on page 204). Thus, the students only listened to the explanation of the one that suggested this idea and continued with their initial strategy without trying out this new idea. In these two scenarios, there is a possibility that students (in a group activity) might accept new ideas and work on them (and when they are unsuccessful, they return to the initial strategy) or reject them without working on them. It is usual for a group member to suggest new ideas or strategies in problem-solving, which might be evaluated by other members (Goos & Galbraith, 1996). Esmonde (2009) argues that when group members critically examine new suggested ideas/strategies, and these ideas/strategies are not quickly accepted or rejected, it shows effective collaboration in group work. Hence, Levenson and Molad (2022) point out that rejecting an idea might constrain fluency in collaborative group work. On the other hand, Hernandez-Martinez and Harth (2015) argue that new ideas are of little use if they are not specific or have no connection with the current groups' understanding of the problem in group interactions. Hernandez-Martinez and Harth (2015) further gave a reason for the rejection, which is because these new ideas are communicated without confidence or they are not sufficiently clear to connect with the group's current thinking, even though the new ideas might have steered the group's thinking in the right direction.

Division of labour (RQ1d)

In the modelling activities, students had different roles. The roles concerning tool usage were constant throughout the activities (see Table 5.2 on page 130). From Table 5.2, students assigned the highest grade (high-performing) were mainly responsible for the computer activities. Other roles, such as leading, opposing, suggesting, questioning and challenging, supporting and non-contributing, changed during students' activities (see Table 6.4). Table 6.4 (on page 160) presents the roles taken by each group member while working on Tasks 1 and 2. From Table 6.4 (and Sub-Section 6.7.2), the high-performing students in the mixed-achievement group (see Table 5.1) often took the leading role while working on Task 1 (Groups A and B). However, this role was counted less in the activities relating to Task 2. The leading role was also counted less in the activities of same-achievement (high-performing students—Group C) and mixed-achievement (lower secondary—Group D) groups relating to both tasks. This suggests that the student's characteristics and the nature of the task could determine the frequency of the leading role counted in the episodes. Questioning and challenging roles are mostly seen among upper secondary students compared to lower secondary students. The supporting and suggesting roles were counted more in the activities of all the groups. The opposing role was seen among the mixed-achievement groups (Groups A, B and D). However, this role was not counted among the high-achievement group (Group C). There were also a few instances where some students took non-contributing roles in the activities of Group A (Rolf), B (Ella) and C (Jørn). Some reasons for the non-contributing role might be that: the student in Group A was described as one with the highest mathematical understanding and nonetheless appeared to have a slightly lower motivation among the peers; the student in Group B was the only female among three male students in the group; and the student in Group C does not see the others as best friends as the group was formed spontaneously and dissolved after the activities (I will further discuss students' roles in Sub-Section 7.2.2).

In summary, I have discussed how CHAT helps describe the interactions in the students' activity in new ways. The elements of CHAT, the subject (characteristics of each student), the community (group of secondary school students), the object (solving Task 1 and 2), mediating artefacts/tools (digital technologies—RQ1a and group interactions—RQ1b), the rules (time constraints, availability of digital technologies, implicit rules—RQ1c), and the division of labour (roles adopted by the students—RQ1d) are seen as a whole, or as collective system interacting with each other, in contrast to cognitive approaches focusing on heuristics and modelling processes. Again, from CHAT perspective, the student-student interactions are intertwined with (or directed by) the individual's engagement with the digital technology, influencing the activity's outcome. In the following subsection, I will address the second research question (RQ2a & RQ2b).

7.1.2 Emergence of modelling actions and the role of digital technologies

In this subsection, I assume modelling actions exist, and I have discussed this in detail in Sub-Sections 2.1.3 and 4.4.2. From a CHAT perspective, I consider activities in modelling to be performing actions and operations towards an object (solving mathematical modelling tasks/ developing a technology-based model/solution). These actions and operations are specific to the context of the task. The interpretations of the goals behind the students' actions are only from the researcher's observations (since the students were not asked for the reasons for their actions). Again, I subscribe to Niss and Blum's (2020) top-down approach, which is an overarching entity (called the modelling competency) in the singular, for which sub-competencies are derived (secondary objects) (see Sub-Section 2.1.3). The actions and operations of the students are viewed through the lens of the modelling process (see Table 9.6 in Appendix E.2 for categories of modelling actions used in this study).

I repeat, for the reader, the second research question:

RQ2a *What modelling actions emerge during the mathematical modelling activities of the students?*

RQ2b *What part do the uses of digital technologies play within the modelling actions that emerge?*

From a CHAT perspective, the second research question explores the modelling actions that emerge within the students' activities and the role of digital technologies in these emerging actions. Empirical data were collected through recorded conversations (video recordings) and computer activities (screen capture software) to address RQ2a and RQ2b. The discussion is centred around the second structure (or theme) for the case study reports of the groups (see Section 6.2). Thus, the findings regarding RQ2a and RQ2b are discussed around the categories: breaking the task into manageable parts; searching for a model; finding a solution for the model; explaining the results in real terms; and checking the results for adequacy. The sequence above does not mean the students solve the tasks linearly (following the same structure order). Recall that the unit of analysis is a group of students solving mathematical modelling tasks with the aid of digital technologies (see Sub-Section 5.1.3). The focus of RQ2a and RQ2b is on how group modelling actions develop and the role of digital technologies in these group actions. I do not discuss individual actions at each stage of the categories but group actions to gain a broader picture of modelling actions emerging in the students' activities. Although individual actions affect the interactions taking place in the group activities, I will discuss these individual actions at the collective level. Below is a discussion of each of the categories of modelling actions emerging, and the discussion relates to both Tasks 1 and 2, where significant aspects that deal with each task are highlighted.

Breaking the task into manageable parts

Breaking the task or problem into manageable parts is seen as an action where students perform operations such as making assumptions and simplification, constructing relations, seeking information, and recognizing quantities that influence the problem situation. Breaking the task into manageable parts is done to understand the problem or have a clearer view of the task demands. The actions are done on a conscious level, whilst the operations are done on a more subconscious level (not always the case, as some operations can be done at a conscious level). Furthermore, some of the operations were completed with digital technologies.

Regarding Task 1 (see Tables 6.12, 6.28, 6.35 and 6.41) and 2 (see Tables 6.13, 6.29, 6.36 and 6.42), it was observed that the first action the students performed was to break the task or problem into manageable parts, and they did that to have a clearer view of what the task demands (from the researcher's perspective). Regarding the initial solution process of Task 1, the students recognized variables such as the number of people buying the car, the price at which they buy the car, and the maximum revenue of the car-selling company. Similarly, the students recognized variables regarding Task 2, such as the positions of the three cities and discussed the issue of fairness. Again, except Group D (lower secondary school students), the other groups (upper secondary school students) recognize variables such as the population of the cities and the distances and time of travel between the cities (see Task 2 in Sub-Section 6.7.3). In an example of the 'lunch problem', Garfunkel and Montgomery (2016) argue that students in the lower grades might only count the number of items whilst those in the higher grades might take more sophisticated information into account. Working on a task in group activities, students read the problem text together, with one member reading it out loud (Albarracin et al., 2019). This activity is easy to identify by researchers in the recordings; for instance, see the dialogue in Table 6.15 (on page 178), where Kåre (a member of Group A) reads the second task aloud. The students then discussed the problem (after reading) to understand or have a clearer view of the task's demands. Polya (2004) argues that the first step to problem-solving is understanding the problem. Thus, the problem text must be understood so that the students can point out the problem's principal parts, the unknown, and the available conditions, amongst other factors. Yimer and Ellerton (2010) adds that understanding the problem (engagement) can be described as the initial confrontation and making sense of the problem (initial understanding, analysis of information, and reflecting on the problem) (Rott et al., 2021). The argument above relates to problem-solving but can also be used to explain the initial activities in the modelling process. Understanding the problem text is subjective as it might depend on the nature of the task and the characteristics of the students, among others. Students sometimes solve problems correctly without understanding them (Reusser, 1988); for instance, they might focus on what to do with the numbers in the problem situation if the problem text has some numeric elements (Verschaffel et al., 2000). An example in this study was Group D's activities relating to Task 2. From the first part of the dialogue in Table 6.40 (on page 233), Hege pointed out that the task is similar to the helicopter task (see Task A in Appendix B); they worked on a day before the current/main activity. In this case, Group D did not consider other factors (e.g., actual travel time or distance) but instead considered the air distance in their analysis (which makes sense in the case of the helicopter but not with cars). From this scenario, it appears that the students' modelling actions (or so to say, modelling competences) might not be carried on from one task to the other. Thus, these modelling actions (or modelling competences) are not seen as a general manifestation, but rather, these actions are specific to the task context. Regarding Task 1, almost all the first two episodes of the students' activities show the students breaking the task into manageable parts. Concerning Task 2, it was counted sometimes in other episodes (aside from the first two) that the students in Groups A, B and C (upper secondary students) sought extra information to understand better what the task demanded. These groups sometimes seek extra information (when they feel the information they have is not enough) by re-reading the problem text (Albarracin et al., 2019; DiNapoli, 2019). For instance, the second part of the dialogue in Table 6.33 (on page 219) shows a member of Group C re-reading a part of the problem text.

Role of digital technology

What role do digital technologies play under the category 'breaking the task into manageable parts' that emerged in the students' activities? Different digital technologies were used by the students in the process of breaking the task into manageable parts. At a certain point where the members of Group A were looking for variables, the conditions available, and constructing a relation for the identified variables in the problem situation, the students used Google Search (translate) to seek information about a word (konstantledd) in their discussions (see Table 6.14). This happened as the students are Norwegians and have English as their second language. Klock and Siller (2020) report that students might struggle to understand the problem text written in a foreign language. Working on Task 2, Groups A, B and C used Google Maps and Google Search to seek information about the identified variables, whilst Group D only used Google Maps (see Table 6.1). This could be because the students in Groups A, B, and C are upper secondary school students and might take more sophisticated information into account while working on Task 2 (compared to Group D, lower secondary school students). Greefrath and Siller (2017) emphasize that the role of digital technologies, as described in the activities of the groups above, is 'researching' (that is, researching information on the internet).

Searching for a model

The students perform the modelling action "searching for a model" after breaking the task or problem into manageable parts. In this case, the students put the pieces together (the manageable parts) to form a model, which they later solved. Searching for a model is seen as an action where students perform operations such as translating the real problem into a mathematical problem, representing the mathematical problem in the technological world (digital technology), and simplifying the model, among others. In the students' activity, they first put their identified variables together to form a mathematical problem and then represented the mathematical form in the technological world (digital technology). The action of searching for a model was done towards the goal of setting up a mathematical model. Albarracin et al. (2019) point out that the activity 'developing the model' is a category that collects all instances regarding the creation of a mathematical model, either when approaching the problem more generally by discussing aspects related to a potential real-life model or when elaborating the actual mathematical model (ibid., p. 219). The category 'searching for a model' was counted in at least the first, second or third episodes of the students' activities (see Tables 6.12, 6.13, 6.28, 6.29, 6.35, 6.36 and 6.42), except Group D's activities relating to Task 1 (see Table 6.41). The students generally searched for a model and later looked for a solution. Again, the groups identified the same variables but set up different models (see Sub-Section 6.7.3). Setting up the model for both tasks depended on the students' information after breaking the task into manageable parts. The action of finding a solution to the model shows how efficient the model is. For instance, in the case of Group D, the students searched for another model when they encountered a problem in finding a solution for the model (see Table 6.39).

Role of digital technology

What role do digital technologies play under the category 'searching for a model' that emerged in the students' activities? In Task 1, to translate the problem situation into a mathematical problem, Groups A and B used GeoGebra to construct or create the function representing their data (see Table 6.12 and 6.28, respectively). Groups C and D also used Excel/spreadsheet to set up their model (see Table 6.35 and 6.41, respectively). However, before Group D settled on using Excel/spreadsheet to set up the model, they tried using a calculator on their computer and GeoGebra. In summary, GeoGebra was used for geometric construction, and Excel/spreadsheet was used for data entry and generation. Regarding Task 2, the groups used Google Maps to seek information about the positions of the three cities and GeoGebra (only Group B, C and D) to construct a geometrical shape representing the position of these cities (see Tables 6.13, 6.29, 6.36 and 6.42). The results above reinforce the findings by Greefrath and Siller (2017), which point out (in a diagram about modelling paths and digital technology use) that digital technologies might be used to investigate (researching or seeking information), construct, and draw in the translation between the real model or the problem situation and the mathematical model.

Finding a solution for the model

Finding a solution for the model is seen as an action directed towards solving the mathematical questions within the model. The students performed this action in several of the episodes in the activities (see Tables 6.12, 6.13, 6.28, 6.29, 6.35, 6.36, 6.41 and 6.42). Thus, many students' actions were about finding a solution for their model. To solve the mathematical questions within the model, the students worked mathematically as they applied heuristics strategies in the form of operations like observing the effect of parameters on the graph, mathematical manipulations and computations, and analyzing, amongst others. The students found a mathematical solution to their model using different strategies. Under this category, I will address the question, 'How do the students solve the mathematical questions in the model they set up?'. Using the Modelling Activity Diagram (MAD, see 'modelling frameworks/cycle' in Sub-Section 2.1.1), Albarracin et al. (2019) put the category above into two different activities: estimating and calculating. An estimating activity describes the moments when the students made estimates as they discussed what values to assign to specific quantities needed to complete a calculation, whilst a calculating activity describes situations where the students perform calculations either with a device or metal calculations (ibid.). In this study, I consider the activities of estimating and calculating to be operations under the category 'finding a solution for the model'. In Sub-Section 6.7.3, I showed how the groups solved Tasks 1 and 2. Groups A and B used the trial-and-error method by analyzing patterns of numbers after searching for a function that represents the number of people buying the car and the price at which they buy it. At some point, the student tried other methods (e.g., introduced a new equation—see Table 6.23 on page 191) but was unsuccessful. The difficulty here was the manipulation of two functions/equations, that is, putting the two functions together to represent the total revenue of the car-selling company. This difficulty is consistent with the report by Pedersen (2015), which points out that Norwegian upper secondary school students do not perform well enough on items that place high demands on symbol manipulation. He further emphasizes that these students' strengths are in items requiring text comprehension in which the students formulate the mathematical expressions needed to find the solution. In the case of Group A, the students formulated mathematical expressions for the situations described in Task 1 but had difficulties manipulating these mathematical expressions. Groups C and D used Excel/spreadsheet to generate their data. Concerning Task 2, I further introduced a sub-category, *analyzing* (defined in Table 9.6 in Appendix E.2), to describe the students' activities. This sub-category characterizes how the students solve the mathematical questions within the model. This sub-category was observed in the activities when the students reconciled their model with reality, which was done by performing operations such as comparing the population of different cities, distances and time of travel between cities, discussing the positions of the cities on Google Maps, importing of images from Google Maps or other images into GeoGebra and manipulating the geometric figure/shape of the image, among others. All groups (except Group D—lower secondary students) factored the roads, time of travel and population of the cities in their analysis. Garfunkel and Montgomery (2016) emphasize that students in higher grades might take more sophisticated information into account while solving mathematical modelling tasks compared with lower grades. The case of Group D could also be their prior experience with the previous task, as they worked on the 'rescue helicopter task' (see Task A in Appendix B), where they considered the air distance and not the road distance or time of travel on Google Maps.

Role of digital technology

What role do digital technologies play under the category 'finding a solution for the model' that emerged in the students' activities? Relating to Task 1, to solve the mathematical questions within the model, it was observed that some students (Group A and B) used GeoGebra to change the values of the function and the calculator for calculating the product of larger values (see Table 6.12 and 6.28 respectively). Other students (Groups C and D) also used Excel/spreadsheet to generate their data (see Table 6.35 and 6.41, respectively). In Task 2, it was observed that the students used Google Maps to seek information about the positions, distance, and travel time between the cities. The students used Google Search to seek information about the population of the three cities. Again, the students used GeoGebra to experiment/change and visualize the geometrical shape of the three cities (I will further discuss this in Sub-Section 7.1.3). Greefrath and Siller (2017) point out in a diagram about modelling paths and digital technology use that digital technologies might be used to investigate (researching or seeking information), construct/draw, measure, and experimentalize the process between the mathematical model and the mathematical results.

Explaining the results in real terms

Students performed an action of explaining the results in real teams after finding an answer to the mathematical questions in the model. Before the students interpreted their results, they sometimes checked their results and then found another solution to the model if there was a problem with the answer they checked. Explaining the results in real terms is seen as an action directed towards the goal of interpreting the mathematical results in real situations. In this case, the students perform operations such as highlighting the meaning of the results and generalizing the model to fit other situations, among others (see the category 'explaining the results in real terms' in Table 9.6 in Appendix E.2). These operations are done on a more subconscious level as Boromeo Ferri (2006) asserts that the transition between mathematical results and real results 'is often not done with awareness by the' students (ibid.). At the end of the activities, the students linked the mathematical results obtained in the model to the real problem (and this became the real result). The students wrote a report and explained their mathematical results (see Appendix D for the solution reports for Groups A, B, C and D and the other groups in the same classroom with them). Regarding Task 1, all the groups settled on selling the car for 7500 euros for 75 people and generating a maximum revenue of 562500 Euros. Although the groups had the correct answer, they did not construct a representation for the maximum revenue in their interpretations. Thus, a representation that fits the situation described in Task 1 (which can also be used in other situations). The students might have seen no point in going further if they had an answer to the problem. Regarding Task 2, the students wrote a report explaining their choice of the optimal position for the shopping centre. That is, the students reconcile their mathematical results with reality. Especially the students in the upper secondary school (Group A, B and C) did not only choose the geometric middle (either a circumcircle/circumcenter or the centroid of a triangle) of the three cities but also considered the roads, distance between the cities, time of travel, the population, the environment, among others (see Task 2 in Sub-Section 6.7.3). To solve a task such as Task 2, the students' opinions influence the kind of answer they give, and this is seen in the interpretations of the students' solutions. In this case, linking reality and the mathematics the students did. Garfunkel and Montgomery (2016) view the computations of a modelling task as one aspect and comment that students have to think about making decisions in the face of uncertainty, which is doing the mathematics and reconciling the results with reality, making the mathematics more relevant and exciting. Interpreting the results involves considering how the model could be adapted for other situations. Group C acknowledged this aspect by using GeoGebra to construct the geometric middle of the three cities (circumcircle/circumcenter) when roads and other factors are not considered (see Table 6.34).

Role of digital technology

What role do digital technologies play under the category 'explaining the results in real terms' that emerged in the students' activities? Relating to Task 2, in an action of explaining the mathematical results in real terms, at one point, the students (Group C) used GeoGebra to construct the geometric middle (circumcircle/circumcenter) of the three cities, to generalize their model (while not considering factors that affect the model in reality). Although the example of modelling path and digital tool use described by Greefrath and Siller (2017) did not describe the use of digital technologies in the process between the mathematical results and the real results/situation. However, Greefrath and Siller (2017) point out that digital technology might be used to construct and experiment for the purposes described in the activities of Group C above.

Checking the results for adequacy

Validation of a mathematical modelling solution involves checking the real results in the situation model for adequacy, sometimes done within (partial results) or near the end (final results) of the students' activities. Checking the results is seen as an action in which the students reflect on other ways of solving the problem and critically examine the results to validate their solution. In the activities regarding Task 1, it was observed that most students (Groups A, B, and C) reflected on and criticized their results and the strategies used in solving the task (see Tables 6.12, 6.28, 6.35 and 6.41). In one example, a member of Group A suggested the group use an Excel/spreadsheet to generate their data instead of changing the values of the function, one after the other (see Excerpt 6.3.1 on page 173). In another example, Group A (and Group B) chose a number below and above the ideal number of people that maximizes the company's revenue to verify their answer (see Table 6.18 on page 183). In this case, the students were aware of their actions, which Boromeo Ferri (2006, p. 93) describes as 'knowledge-based validation'. Thus, the students agree or disagree with their results based on their extra-mathematical knowledge. In this study, Groups A and B, with their extra-mathematical knowledge, agreed on the results after verifying with a number above and below the ideal number. Group C did not check their results for adequacy (see Table 6.35). Thus, the students in Group C were convinced of their results after they looked through the data set they generated with Excel/spreadsheet. Group D also generated their data set with Excel/spreadsheet, but they discussed their final results further. In this case, the students in Group D discussed going down with the price of the car to see what happens (but that suggestion was later rejected) (see Excerpt 6.6.1 on page 239). The outcome could be exciting if the students tried and compared this new idea to their initial strategy (or final solution). However, the group agreed on their final solution instead of going further with the task. Albarracin et al. (2019) point out validations can happen when the group agrees. In the activities regarding Task 2, it was observed that the students reflected on and/or criticized their results and the strategies used in solving the task in some of the episodes (see Tables 6.13, 6.29, 6.36 and 6.42). Task 2 is an open task and does not have a single answer. However, the students had to reconcile their answers with reality. In the activities of Groups A, B and C regarding Task 2, the students did not directly verify their results (compared to the activities of Task 1) (see Task 2 in Sub-Section 6.7.3). They verified their results by arguing about factors like distance, time of travel, population, and the environment, among others, that affect the results. Another explanation is that the students were not quite sure of the results as they were with Task 1, although they considered some factors in reality. Boromeo Ferri (2006, p. 93) describes another validation called 'intuitive validation' (opposite to the earlier validation described), which is a situation where students might find out that their results could be wrong for reasons they cannot explain. In the case of Groups A, B and C, I would say that the validation is both knowledge-based and intuitive in the sense that the students have knowledge about the factors affecting their optimal choice and at the same time, they felt that their results were not adequate (not confident in the results). It might be the case that the students usually solve tasks with a single straight answer, affecting their judgment of the results of Task 2. Group D only searched for the theoretical optimal location, not the hypothetical one (see Table 6.40). Group D verified their results of Task 2 by measuring the distances between the middle point and the three cities (see Table 6.43). Furthermore, with this measurement, they were sure that the optimal location has an equidistance to the cities.

Role of digital technology

What role do digital technologies play under the category 'checking the results for adequacy' that emerged in the students' activities? Relating to Task 1, to validate the results, some students (Group A and B) used GeoGebra to change the values of the function (that is, choosing numbers above and below the ideal number) and the calculator for calculating the product of these selected numbers/values (see Table 6.12 and 6.28). Regarding Task 2, it was observed that some students (Group D) used GeoGebra to measure the distances between the optimal location and the cities. Greefrath and Siller (2017) point out that digital technology might be used to experiment, calculate, and measure within the above activities.

In summary, the discussion in this subsection shows how CHAT helps describe the students' activities, particularly their mathematical modelling processes. That is, I analyzed the modelling processes the students engaged in by identifying the modelling activities in terms of their actions. Most modelling processes follow descriptions of the modelling cycle from a cognitive perspective (Boromeo Ferri, 2006). In this case, much attention is given to the transitions between the modelling phases or nodes of the modelling cycle. The normative description of the modelling process is considered ideal; however, the empirical descriptions differ from the normative description (see Sub-Section 2.1.1). Cai et al. (2014) argue that a modelling cycle might not show most students' actual work in a mathematical activity. Thus, there might be some difficulties in the qualitative identification of the stages of the modelling process corresponding to each episode of students' work (Ärlebäck, 2009; Czocher, 2016; Albarracin et al., 2019). As such, Albarracin et al. (2019) use the components of Modelling Activity Diagrams (MAD: reading, making model/modelling, estimating, calculating, validating and writing) to characterize the modelling processes of students. These activities in MAD might help qualitatively identify the stages of the modelling processes. However, it does not include the role played by digital technologies in these activities. This research study (in a different way) presents students' modelling processes as emerging actions (from a CHAT perspective) and considers the role of digital technologies in these emerging actions.

The empirical description of the modelling processes discussed in this subsection is considered in view of the ontology of CHAT, which emphasizes that modelling processes are not ideal but instead develop within an activity. For instance, the categories of modelling actions discussed above are different concerning the nature of the task and the characteristics of each group, among others. That is, the student's approach towards Task 1 differs from their approach towards Task 2. Again, the approach towards a particular task differs among the groups. Concerning the characteristics of the students, the different roles adopted by the students or the forms of interactions taking place within the student's modelling activities (as addressed in Sub-Section 7.1.1) might also help in understanding the differences in the modelling actions that emerged across the groups. The categories of modelling actions discussed above clearly show the work pattern of the students. From a methodological point of view, I argue that characterizing the different activities the students engage in when solving a mathematical modelling task is more straightforward and more apparent in terms of the categories of the modelling actions. In conclusion, from a CHAT perspective, I consider students' activities in modelling as performing actions and operations towards an object (solving a mathematical modelling task/developing a technology-based model/solution). The categories of modelling actions help qualitatively identify the stages of the modelling processes. The role digital technologies played in the emerging modelling actions was also observed in the students' activities. That is, some of the operations done by the students were completed with digital technologies. The tendency to use specific digital technologies for a particular operation depends on the student's approach towards the tasks and others (more of students' tendency of using particular digital technologies to come in Sub-Section 7.2.4).

I will address the third research question (RQ3) in the following subsection.

7.1.3 Emergence of affordances and constraints of digital technologies in mathematical modelling activities

In this section, I assume technological, mathematical, and socio-cultural affordances and constraints exist, and I have discussed these in Section 4.3 and Sub-Section 4.4.1. I also acknowledge that affordances perceived and actualized are two distinct things; nonetheless, in most instances, affordances are perceived before being actualized. However, for this discussion, I will focus on the actualized affordances (the action itself) emerging in the students' interactions with digital technologies. One should note that affordances arise from the students—digital technology relation and not just the digital technology. For instance, GeoGebra can afford the bisection of an angle, which is meaningful only if the students can use the 'Angle Bisector' feature in GeoGebra.

I repeat, for the reader, the third research question:

RQ3 *What affordances and constraints of the digital technologies emerge as the students develop a technology-based model/solution?*

From a CHAT and Affordance Theory perspective, RQ3 explores the affordances and constraints that emerge within the students' activities as the students interact with digital technologies. Empirical data were collected through recorded conversations (video recordings) and computer activities (screen capture software) to address RQ3. The findings regarding RQ3 are discussed considering the categories of technological, mathematical, and socio-cultural affordances (see Sub-Section 4.3.4). Recall that the unit of analysis is a group of students solving mathematical modelling tasks with the aid of digital technologies (see Sub-Section

5.1.3). As such, I will centre the discussion on the affordances and constraints of the digital technologies emerging in group activities around shared affordances and not focusing on individual or collective affordances (see Sub-Section 4.3.4). Shared affordances might come into play when the students share one common working space (using only one computer). At the same time, collective affordances might come into play when the students use their individual computers while working on the task. In one of the principles for using Affordance Theory, Volkoff and Strong (2017) state that one should recognize social forces that affect affordance actualization. That is the social context in which these affordances are actualized. Bloomfield et al. (2010) argue that the affordances of digital technologies, practically, cannot be easily separated from the arrangement through which they are actualized, as Volkoff and Strong (2017) point out that one needs to consider how the presence of other people using the same digital technology for similar or related purposes might affect an actor's behaviour. As such, I discuss the shared affordances of the groups in the social context. That is, the combination of CHAT and Affordance Theory helps address the affordances of digital technologies and the activities mediated by digital technologies. This combination does not separate the affordances of digital technologies from the activities mediated by these digital technologies. I will discuss the emerging categories of affordances and constraints below (where the discussion is situated around Tasks 1 and 2); however, significant aspects that deal with each task are highlighted.

Technological affordances and constraints

The operations performed by the groups are characterized by the usability features (see Sub-Section 4.3.4) of the digital technologies. Digital technology provided the students with specific functional opportunities while working on Tasks 1 and 2. Furthermore, these functional opportunities become meaningful if the students can use/reach them. Technological affordances are properties of digital technology linked to usability in such a way that the digital technology allows for the accomplishment of a set of tasks efficiently and effectively that satisfies the user (Kirschner et al., 2004). Table 6.1 presents the digital technologies each group used while working on both tasks. Thus, the students used at least one of the digital technologies (GeoGebra, calculator, Google Search, and Excel/spreadsheet) while working on Task 1, and at least two of the digital technologies (GeoGebra, calculator, Google Search and Google Maps) while working on Task 2. These digital technologies supported students' perceived ease of researching, measuring, visualizing, geometric construction, experimenting/ changing, data entry and generation, and calculating (codes for analyzing technological affordances and constraints, and well documented in Table 9.7 in Appendix E.3). Table 6.20, 6.21, 6.30, 6.31, 6.37, 6.38, 6.44 and 6.45 presents the counts of the actualized technological affordances and constraints that emerged in the episodes of the activities of the groups. The students used different approaches while working on both tasks (see Sub-Section 6.7.3). As such, there were differences in the affordances and constraints that emerged. The technological affordances and constraints are the same as the role of digital technologies in the emerging modelling actions presented in Sub-Section 7.1.2. I will now discuss each of the technological affordances and constraints in the order presented in Section 6.2:

Researching

Regarding Task 1, Google Search afforded the retrieving of information on the internet of one of the variables in the proposed equation of the students (only in the activities of Group A, see Table 6.14). Regarding Task 2, Google Search afforded the retrieving of information about the population of the three cities as the students (Groups A, B and C) analyzed the optimal location where the shopping centre would be built. On the other hand, Group D (students at the lower secondary) could not use/reach this affordance of Google Search as they did not perceive it. One possible explanation could be the students' strategies for solving Task 2. The characteristics of Group D also played a part. Thus, students at the higher level might take more sophisticated information into account while solving a modelling task compared with students at the lower level (Garfunkel & Montgomery, 2016). Again, Google Maps allowed the groups to retrieve information about the cities' positions.

Measuring

Regarding Task 1, the category 'measuring' was not identified in the student's activities. Regarding Task 2, Google Maps afforded Groups A, B, and C the measuring of distances between the cities. These groups did not only look for the cities' positions but also the actual distances between these cities (and the time of travel). These measurements influenced the students' choice of the optimal position for the shopping centre. Similarly, as discussed under 'researching' above, Group D could not use/reach the affordance 'measuring' as they did not perceive it even though they could retrieve information about the positions of the three cities on Google Maps. GeoGebra afforded Group D the measure of distances between points when the students verified the equidistance between the optimal location and the three cities (see Table 6.43).

Visualizing

Regarding Task 1, the category 'visualizing' was not identified in the students' activities. Regarding Task 2, Google Maps afforded Group A the visualization (see Table 6.17) of the location of the three cities as they turned on the satellite in Google Maps. With this functionality of Google Maps, the students could access the environment surrounding their choice of the optimal place for the shopping centre. GeoGebra afforded Group B the visualization of a point in the graphic view. In this case, Group B made a duplicate (Point M in Image 2 in Table 6.27) of the middle point (Point D in Image 1 in Table 6.27). The students then made the coordinates of the duplicate point visible in the graphic view of GeoGebra.

Geometric construction

Regarding Task 1 (see Task 1 in Sub-Section 6.7.3), GeoGebra afforded Groups A (see Table 6.6) and B (see Table 6.25) the drawing of a function. The students also perceived that GeoGebra could afford them the drawing of a function, but they could not reach that. For instance, a member in Group A perceived that GeoGebra could afford an efficient way of finding the maximum revenue but could not reach it (see Table 6.10 on page 170). Thus, the function keyed in GeoGebra needed to be correctly defined. GeoGebra has a constraint that it is impossible to draw an undefined function, and this constraint might help the students find ways to define a function properly to attain the desired result. GeoGebra could afford the drawing of a function if the students could key in the correct variables. In another instance, in Group A's activity, the students keyed in the correct variables for the equation representing the car's selling price; however, they still did not achieve the desired results (see Table 6.23 on page 191). One reason could be that the students could not combine the initial and new functions. On the other hand, Group D perceived that GeoGebra could afford the drawing of the function/graph if they inserted some points, but they could not reach it (see Table 6.46 on page 243). Regarding Task 2, GeoGebra afforded Group B (see Table 6.27), C (see Table 6.34) and D (see Table 6.40) the construction of geometric shapes or figures (see Task 2 in Sub-Section 6.7.3). Thus, the students constructed a geometric middle point (circumcircle/circumcenter or centroid) of a triangle that connects the three cities.

Experimenting/Changing

Regarding Task 1, GeoGebra afforded Groups A and B experimenting/changing of parameters in the functions they drew. Thus, the students used the trial-and-error approach with different numbers while finding the maximum revenue of the car-selling company (see Task 1 in Sub-Section 6.7.3). At one instant, a member of Group A perceived that GeoGebra could afford them an efficient way of finding the maximum revenue instead of trying out numbers, one after the other. This influenced another member to suggest they use sliders, but the slider they made did not affect the function since it had no link with the function (see Table 6.11). Drawing students' attention to such constraints might help them improve their problem-solving strategies. Regarding Task 2, GeoGebra afforded Group B experimenting/changing in the process where the students were looking for the geographic middle point of the triangular shape (see Table 6.27). The students arrived at the middle (optimal) point by experimenting or changing the coordinate points several times until they were satisfied with the optimal point.

Data entry and generation

Regarding Task 1, Excel/spreadsheet afforded Groups C and D entry and generation of data. In the case of Group C, the students perceived that Excel/spreadsheet could afford data entry and generation because their data set is recursive, and they thought they could not use GeoGebra (see Table 6.32). Practically, the spreadsheet view in GeoGebra could afford them data entry and generation, but this was not meaningful as the students could not use it. In the case of Group D (see Table 6.39), the students first used a calculator on the computer to compute the values and then shifted to GeoGebra for efficiency. The students did not actualize what they perceived GeoGebra could afford them. Group D finally perceived that Excel/spreadsheet could afford them data entry and generation, and in the end, they were able to actualize that. Regarding Task 2, the category 'data entry and generation' was not identified in the students' activities.

Calculating

Regarding Task 1, the use of a calculator afforded Groups A (see Table 6.16), B (see the third part of the recorded dialogue in Table 6.25) and D (see the first part of the dialogue in Table 6.39) numeric calculations. The students used the calculator to calculate larger values (thus, the product of the number of people and the corresponding price of the car). Regarding Task 2, only Group B used a calculator in their working processes. In this case, the calculator device afforded the calculation of the median of the distances between the optimal point and the three cities (see the second part of the recorded dialogue in Table 6.27).

I will now discuss the emergence of mathematical affordances and constraints in the students' activities. The discussion below is linked to the discussion above, as technological affordances and constraints are prerequisites for mathematical affordances and constraints. I will use the same examples under technological affordances and constraints in the discussion below.

Mathematical affordances and constraints

The technological affordances and constraints of digital technologies at the operational level support the mathematical affordances and constraints emerging at the action level. That is, students' actions in the solution processes are conditioned by the features of digital technologies at the operational level. Several mathematical affordances and constraints emerge as students interact with digital technologies while working on mathematical modelling tasks. The mathematical affordances and constraints of the digital technology that emerged were clarification, analyzing, simulating and visualizing, linking representations, regularity and variations, and arithmetic and statistics (codes for analyzing mathematical affordances and constraints, and well documented in Table 9.7 in Appendix E.3). Table 6.20, 6.21, 6.30, 6.31, 6.37, 6.38, 6.44 and 6.45 presents the counts of the actualized mathematical affordances and constraints that emerged in the episodes of the activities of the groups. I will now discuss each of the mathematical affordances and constraints in the order as presented in Section 6.2 (with the corresponding technological affordances and constraints in brackets):

***Clarification* (researching)**

Regarding Task 1, Google Search afforded Group A clarification (or meaning) of a mathematical term during a mathematics discourse (see Table 6.14). Retrieving information or searching for the meaning of a mathematical term during a group interaction might help bring out the understanding of a mathematical concept in a mathematics discourse. Regarding Task 2, the category 'clarification' was not identified in the student's activities.

***Analyzing* (researching, measuring and visualizing)**

Regarding Task 1, the category 'analyzing' was not identified in the student's activities. Regarding Task 2, one primary mathematical affordance and constraint that emerged was analyzing. In creating a model, the students (especially Groups A, B and C) used Google Search to retrieve information about the population of the three cities, as they thought about fairness. Furthermore, with this information, they were able to analyze the situation. The features of Google Maps afforded all the groups several technological opportunities (as highlighted above), and these opportunities created the platform for the students to analyze their solutions. That is, by locating the cities' positions (all groups), distances between the cities, and the time of travel between the cities (only Group A, B and C). With these facts and figures retrieved from Google Maps, the students could make a mathematical comparison as they looked for the optimal position for the shopping centre.

Simulating and visualizing **(visualizing)**

Regarding Task 1, the category 'simulating and visualizing' was not identified in the students' activities. Regarding Task 2, GeoGebra afforded Group B simulation and visualizing as the students explored the behaviour of the geometric object to see what happens. Thus, the students first made duplicates of points and moved (or manipulated) these points dynamically to see the changes in the distances between the optimal location and the three cities. This activity echoes previous research highlighting that specific dynamic geometry simulations might allow students to visualize some mathematical problems (for instance, area optimization problems) (Pierce & Stacey, 2010).

Linking representations **(geometric construction)**

Regarding Task 1, GeoGebra afforded Group A (see Table 6.6) the movement between function/equation and graphical representation. GeoGebra also afforded Groups B (see Table 6.25) and D (see Table 6.46) the movement between numeric and graphical representation (but Group D were not successful in their approach). GeoGebra enhanced students learning by linking representations, and the choice of representations could also result from the strategy the students adopted (I will discuss this point in detail in Sub-Section 7.2.3 and 7.2.4). Zbiek et al. (2007) draw attention to the role that cognitive tools (see Section 2.2), in our case GeoGebra, play in mathematical activity through externalizing representations. Externalizing representations describe the display on the screen's surface that is visible to the students, which can be shared and discussed with others. Digital technology acts as a mediator between the user and the outcome of the mathematical representation (desired by the user). In this case, the digital technology performs specific mathematical actions (such as creating external mathematical representation) at the students' command. Pierce and Stacey (2010, p. 8) point out that "theories of multiple representations propose that a key to students' understanding is their ability to link representations and gain representational fluency, where they can interpret mathematical ideas in distinct representations". Regarding Task 2, Group B (see Table 6.27), C (see Table 6.34) and D (see Table 6.40) were able to represent the coordinates (from Google Maps) of the three cities in the graphical view in GeoGebra. These groups constructed a geometric figure (i.e., a triangular shape). Pierce and Stacey (2010) argue that incorporating dynamic geometry into some digital technologies might add to the possibility of linking geometric representations. The students discussing the positions of the three cities on Google

Maps could be a discussion resulting from their internal mental representation, and displaying this idea on the surface of the screen (concerning GeoGebra) could form an externalized representation (Zbiek et al., 2007) easing the sharing and discussion of ideas in a mathematics discourse. These students went on to find the middle point of the constructed triangle by dragging the corners of the triangle, using the perpendicular bisector, and drawing a circle (done by Groups C and D) that passes through all three cities.

Regularity and variations **(experimenting/changing)**

Regarding Task 1, GeoGebra allowed Groups A and B to explore the regularity and variations in the solution model as they changed the values in the constructed function. That is, the students observed the effect of the changed parameters on the graph as they searched for the maximum revenue of the car-selling company, and this step was repeated until they found the desired result. At some point, Group A tried making a slider to vary the parameter to see the effect on the graph of their function, but that was not actualized as the slider did not have a clear definition and subsequently had no effect on the graph (see Table 6.11). Regarding Task 2, the students in Group B manipulated or changed the coordinate points in the graphical view in GeoGebra several times until they were satisfied with the optimal point. For instance, Group B first created a centre point (initial optimal point) using the median of a triangle or centroid approach and later searched for another point by changing the position of the initial optimal point (see Table 6.27).

Arithmetic and statistics **(calculating, data entry and generation)**

Regarding Task 1, the calculator afforded Groups A and B numerical computation, such as multiplication. Thus, the students used the calculator to compute the product of larger values. Using a calculator improves speed and accuracy, as Pierce and Stacey (2010) argue that calculations done with paper-and-pencil could be error-prone or time-consuming. However, depending on the nature of the task, using a calculator could also be time-consuming. For instance, in the case of Group D, the students used a calculator on the computer to compute the product of the number of people and the price at which they bought the car (see Table 6.39). This approach was inefficient for Group D; hence, the students shifted to GeoGebra and later to Excel/spreadsheet. On the other hand, Excel/spreadsheet afforded the students (Groups C and D) technology-generated statistical data sets (Pierce & Stacey, 2010). In this case, the students could drag and extend their data after entering a few data sets derived from the problem. As stated earlier, calculations done by hand are error-prone and time-consuming (calculator use could also be time-consuming, as seen in Group D's activities); using Excel/spreadsheet could resolve this issue. Another thing is that Excel/spreadsheet could afford the students the representation of the data set through the generation of histograms, functions, and regression analysis, among others. However, the students did not perceive these, so they did not actualize them. A possible reason might be that the students were only interested in the answer and did not need some form of representation (e.g., a function or equation) to report their answer. Regarding Task 2, the calculator afforded Group B numerical computation, such as the median of the distances between the ideal optimal location and the three cities.

I will now discuss the emergence of socio-cultural affordances and constraints in the students' activities.

Socio-cultural affordances and constraints

At the activity level, socio-cultural affordances and constraints emerge in joint mathematical discourse or interactions that occur through collaboration within group activities. Socio-cultural affordances and constraints, in our case, are social affordances described as the properties of digital technology acting as social-contextual facilitators relevant to the student's social interaction (Kirschner et al., 2004). An example could be that when a group member steps onto the social stage and solves a task with a unique strategy, the properties of the digital technology might invite, allow, encourage, or even guide another member to initiate or suggest another strategy to either repair divergences or improve previous strategy in the course of social interaction. The digital technologies stimulate cooperation between the students as they work on mathematical modelling tasks. The interaction between the students and the digital technologies induces affordances at a collective level (my use of collective here is not the same as 'collective affordances' as defined in Sub-Section 4.3.4). Tables 6.20, 6.21, 6.30, 6.31, 6.37, 6.38, 6.44 and 6.45 present the counts of the actualized socio-cultural affordances and constraints that emerged in the groups' activity episodes. The socio-cultural affordances that emerged or actualized in the students' activities were *common focus* (in both Task 1 and 2), *observing and improving strategies* (only in Task 1), and *authority of the digital technology* (in both Task 1 and 2). These categories were codes for analyzing socio-cultural affordances and constraints and are well documented in Table 9.7 in Appendix E.3. I will now discuss each of the socio-cultural affordances and constraints in the order as presented in Section 6.2:

Common focus

The students shared the same computer and had the facility to look at the same thing and point at what was presented on the computer (except those that used a calculator device or their mobile phone), and this helped the students to create a shared goal as they had the facility to look at and follow the same thing in their interactions. Regarding Task 1, the affordance *common focus* emerged in Groups A, B, C and D's activities as they agreed on a shared goal through a flow of turn-taking, dialogue, and action. In Tables 6.20, 6.30, 6.37 and 6.44, the affordance *common focus* emerged as the groups interacted with GeoGebra, Excel/spreadsheet and a calculator (software on the computer). Several examples of *common focus* emerged in the groups, which were similar but in a different context; as such, I will only discuss an example from Group A's activity. Thus, to visually demonstrate an individual's reasoning to another member, the students used GeoGebra as a reference tool by looking at the coordinate axis and sketching with paper-and-pencil ($f(x) = 100x$) in relation to the coordinate axis (see Table 6.6 on page 166). In responding to one another, the students had to interpret and evaluate the visualized ideas and then give a response. In this situation, Thea (a member of Group A) responded to the initial function proposed by Kåre (another member of Group A). She used GeoGebra for reference in anchoring her proposition by suggesting the function $f(x) = -x + 100$. Again, *common focus* only emerged in the activities of the group (Group D) that used a calculator on the computer but not the groups that used a calculator on a device or mobile phone. Thus, there was no social affordance in the interaction of the students who used a calculator device, as the individuals had the device to themselves. Regarding Task 2, the affordance *common focus* emerged in Groups A, B, C and D activities as they agreed on a shared goal through a flow of turn-taking, dialogue and action while engaging with Google Maps, Google Search and GeoGebra. Several examples of *common focus* emerged in the group, which were similar but in a different context; as such, I will only discuss an example from Group A's activity. Thus, in an example of Group A's activities, Thea was searching for the position of the cities on the map, Kåre pointing at the cities on the screen, and Rolf visualizing his reasoning by joining the points of the cities by hand and concluding that it will form a triangle (see Table 6.15 on page 178). This kind of interaction might not be possible if the students were using their individual computers within a group setting, and collective affordances might instead come into play in this setting.

Observing and improving strategies

Regarding Task 1, the affordance *observing and improving strategies* emerged only in the activities of Group A (see Table 6.20). In this case, Group A did not get into a situation marked by uncertainties or diverging from their initial strategy in the solution process. Instead, they needed to improve their strategy for efficiency (see Table 6.11). Thus, a member of Group A perceived that GeoGebra could afford them sliders; however, it was not actualized as the tool has a constraint that the slider must be well-defined to affect the function. The students had similar dialogue in other instances where they tried to make a new function (but it was not well defined). At a time when the function was well defined, they were not able to manipulate or put together the two functions (see Table 6.23). This narrative shows that digital technologies might be used to maintain and improve shared knowledge and ideas at a collective level (although the students might not always be successful with the new idea/s introduced). Regarding Task 2, the category *observing and improving strategies* was not identified in the student's activities.

Authority of the digital technology

Regarding Task 1, the affordance *authority of the digital technology* emerged only in Groups A and B activities (see Table 6.20 and 6.30, respectively). In Group A's situation, a member (Rolf), after observing the solution strategy, suggests using a spreadsheet to efficiently generate the data as another member (Thea) maintains that the group should continue with the previous or existing strategy as she had personalized the problem-solving strategy (see Except 6.3.1 on page 173). Thea's strategy starts with a graphical representation and later analyzes patterns of numbers and observes the car-selling company's revenue increment. Thea dismisses Rolf's comments and returns to the strategy described above, thinking they are already close to the answer (which might also be a time factor). However, subscribing to Rolf's suggestions might have helped the group generate their data with the spreadsheet and find a function representing the data. That is, the features of GeoGebra allow multiple problem-solving strategies. However, the approach used by the group depends on the representational choice of the students taking the leading role, especially when they think they are close to finding the answer. Similar issues occurred as Group B worked on Task 1 but in a different context (see Excerpt 6.4.4 on page 215). From Excerpt 6.4.4, Emil came up with the idea of making a function representing the revenue. However, Ella insisted they continue with the ongoing strategy (although Thor, taking the leading role, was interested in this new idea). There might be a possibility of Ella insisting the group continue with the ongoing strategy if she thinks they are close to finding the answer. Regarding Task 2, the affordance *authority of the digital technology* emerged only in the activities of Group D (see Table 6.45). In this case, a member of Group D pointed out that the distances between the middle point (optimal location) and the three cities are equal since the circle passes through all the points (with the middle point as the centre point). Another member suggested that they still measure these distances. Thus, this member would rather accept the answer (outcome) from the digital technology than her peers or perform the operation of measuring these distances to ensure their final results (see Table 6.43 on page 240).

In summary, affordances and constraints of digital technologies emerge as students develop a technology-based model/solution. These affordances and constraints depend on the characteristics of each group, the nature of the task, and what is perceived in the use of digital technology, among others. Again, affordances and constraints emerge at all three levels in Leont'ev's model of activity. At the operational (technological affordances) level, digital technologies provided the students with specific functional opportunities such as calculating, researching, measuring, geometric construction, visualizing, data entry and generation, and experimenting/changing while working on the tasks. The technological affordances of digital technologies at the operational level support the mathematical affordances emerging at the action level. Thus, at the action level, digital technologies afford clarification in a mathematics discourse, analyzing, linking mathematical representations, simulation and visualizing, regularity and variations, and arithmetic and statistics. Finally, at the activity or collective level, digital technologies induce specific social affordances as the students engage with the technology in social interactions. These social affordances could be the common focus, observing and improving strategies and authority of digital technology (more of this to come in Sub-Section 7.2.3).

Some relevant issues must be discussed further after addressing the research questions; in the next section, I will discuss four significant issues arising from the study with existing literature.

7.2 Discussion of significant issues arising from the research

In this section, I discuss four significant issues from the research with existing literature. The findings discussed in this section have the potential to contribute to mathematics education research and suggest new insights into students' mathematical modelling with the aid of digital technologies. The study's results discussed in this section (and elsewhere) are only meant to be suggestive and cannot be generalized. Furthermore, the evidence provided are counts; thus, the number of times this evidence was counted/recorded in the student's activities. In addition, these counts are episodes (a category created by the researcher and has no intrinsic mathematical property; see Sub-Section 5.7.2) describing the students' activities concerning Task 1 and 2. This section is divided into four subsections that discuss each significant issue. Although these four significant issues (or subsections) are interrelated, I linearly present them. In Sub-Section 7.2.1, I explore the distinctions between the different interaction sequences concerning Tasks 1 and 2. I further explore in Sub-Section 7.2.2 the consistency of students' roles within group interactions. In Sub-Section 7.2.3, I identify and discuss the influence of digital technologies on group interactions. Lastly, in Sub-Section 7.2.4, I discuss students' tendency to select or use a particular digital technology.

7.2.1 Interaction sequences and task design

While a substantial number of studies have investigated how group activities connect with mathematical task design or the dynamics within group interactions with the tasks the students work with, fewer have sought to develop categorizations of interaction sequences about the type of mathematical modelling tasks. Sahlberg and Berry (2003) proposed an example of such a categorization. Sahlberg and Berry presented types of mathematical tasks and their corresponding types of exchange models (see Sub-Section 2.4.2). In the case of mathematical modelling tasks (in general), they argued that there are several opportunities for rich, equal exchange within collaborative student group work. Alternatively, Peter-Koop (2002) and Lowrie (2011) categorized the interaction sequences of a group of students (based on the framework by Jones and Gerard (1967)) as they work on mathematical tasks (see Sub-Section 2.4.1). Peter-Koop (2002) analyzed the interaction process of a group of students as they worked on open real-world problems. Similarly, Lowrie (2011) also analyzed the interaction process of a group of students as they worked on real-world problems using genuine artefacts. In the two scenarios, Sahlberg and Berry's (2003) work gives a connection between a mathematical modelling task and its corresponding type of exchange in a general sense, while both the work by Peter-Koop (2002) and Lowrie (2011) focused on the types of interaction sequences emerging and not the distinction between the tasks. These categorizations provide helpful descriptions of the type of exchange model associated with a mathematical modelling task and the dynamics of interaction sequences as a group of students solve a real-world problem. However, neither of these studies appears to acknowledge the distinctions within interaction sequences concerning the type of mathematical modelling task and the characteristics of the students. This research study addresses this gap. Analysis of students' activities provides evidence for developing a categorization that addresses this research gap. My main claim in this subsection is:

Not all mathematical modelling tasks produce rich equal exchange within a collaborative students' group work.

The backing I provide for this claim is the frequency of interaction sequences recorded from the students' activities involving Tasks 1 and 2, as well as the characteristics of the students. To discuss the claim (in italics) above in the subsequent paragraphs, I will first briefly describe the nature of Tasks 1 and 2 (see Section 5.5) and the characteristics of the students (see Table 5.1) in this study (I repeat this for the reader). Task 1 is a closed-ended task (or an open-ended task, depending on the approach the students take) with a realistic context (see Sub-Section 2.3.1) and contains more information in connection with given numbers in the task, which has to be added through inner-mathematical knowledge (e.g., application of some mathematical formulas). At the same time, Task 2 is an open-ended task with an authentic context (see Sub-Section 2.3.1) but has no number in its presentation and might require extra-mathematical knowledge (resulting from experience). Both tasks link school mathematics tasks with real-life experiences. The characteristics of the students about the frequency of interaction sequences are based on their performance or attainment in the exam. That is, Groups A, B (Grade 12 and 11, respectively) and D (Grade 9) consist of students with different attainments (mixed-achievement). In contrast, Group C (Grade 11) consists of students with the same attainment (same-achievement) or a group of high-performing students (see Table 5.1 on page 129). Consistent with the methodological approach outlined in Chapter 5, data collection and analysis were conducted, and video and audio recordings, screen capture software, and fieldnotes provided data that supported the processes of category creation, confirmation, and refinement concerning distinctions between the interaction sequences recorded in the student's activities. Table 6.2 (on page 158) presents the frequency of the different group interactions recorded concerning Task 1 and 2: asymmetrical contingency, reactive contingency, mutual contingency and pseudocontingency. I have presented an analysis of Table 6.2 in Sub-Section 6.7.1.

Asymmetrical contingency. From Sub-Section 6.7.1, I presented that asymmetrical contingency was recorded in most of the episodes regarding Task 1 compared to Task 2, and the nature of the task could be a factor. Thus, this contingency was recorded less in Task 2, irrespective of the student's characteristics. Again, this contingency was recorded mostly among mixed-achievement students (Groups A and B) compared to same-achievement and high-performing students (Group C) concerning Task 1. What could be the differences between Groups A, B and D (mixed-achievement groups)? The differences in levels in these groups might suggest that how students are treated in these levels might contribute to the increase of asymmetrical contingency among students at the upper secondary level. For instance, Markussen et al. (2011) argue that students (transiting from the lower secondary school) are treated as if they could cope with the demands of upper secondary education. Moreover, this treatment could also result from teachers' beliefs and practices in mathematics teaching at both levels (Nilsen, 2010). Another interpretation of the differences between the levels (Groups A, B and D) could be competition among the students. Thirty-five years ago, Feldlaufer et al. (1988) conducted a longitudinal study investigating the relation between changes in classroom and family environment across four domains (mathematics, English, social interactions and physical activities) and highlighted 'competition among students' as one of the differences in the classroom environment in a transition between two levels (for instance the transition between elementary school and junior high school). Their results showed that students see their pre-transition classrooms as more competitive than their post-transition classrooms, although observer reports indicate no difference in competition (ibid.). To explain this further, Feldlaufer and colleagues pointed out that as students become more self-conscious, they might avoid overt competition, or there could also be a possibility that students are more likely to compete with students they come to know well across academic and social domains (ibid., p. 152). The study by Feldlaufer and colleagues was in the American context (with students in Grade 6-7), but this might also apply in the Norwegian context (although further research is needed). In the Norwegian context (specifically, this study), students in Group A (Grade12) and B (Grade 11) in this study have known each other for $1\frac{1}{2}$ years and $\frac{1}{2}$ year (or one semester) respectively. In contrast, students in Group D (Grade 9 students) have known each other since Grade 5 (see the field notes on the teacher's report in the second paragraphs in Sub-Sections 3.4.1, 3.4.2 and 3.4.4). This might explain students' engagement with each other. From the discussion above, it could be conjectured that once students reach Grade 11 (upper secondary school), they tend to be treated differently than they were in

Grade 9 (lower secondary), and they start to act differently as well. Again, students at the lower secondary are competitive with one another compared to students at the upper secondary level. That is, students in the upper secondary might be more supportive of one another within group interactions.

Reactive contingency. In Sub-Section 6.7.1, I presented that reactive contingency was recorded in most of the episodes regarding Task 2 compared to Task 1 across the groups. In Peter-Koop's (2002) studies, most interaction sequences found throughout their entire data set (in both mixed ability and among low achievers) were characterized by reactive contingency. However, a different pattern was recorded in this study based on the description of reactive contingency (see Sub-Section 2.4.1). Again, reactive contingency was recorded in more episodes of Group D (lower secondary and mixed-achievement) compared to Groups A and B (upper secondary and mixed-achievement) concerning Task 1. The explanation under asymmetrical contingency might also apply here.

Mutual contingency. In Sub-Section 6.7.1, I presented that mutual contingency was overall counted in more of the episodes in the activities of Groups A and B regarding Task 2 compared to Task 1. On the other hand, the count of this contingency was much similar in the activities of Group C (same-achievement and high-performing students), irrespective of the task. This supports Peter-Koop's argument that mutual contingencies are infrequent in students' activities and tend to occur when high-achievement students work together. Contrarily to this, Groups A and B record more mutual contingencies in the episodes regarding Task 2 than in Task 1, although these groups consist of mixed-achievement students. Again, Group D consists of mixed-achievement students, but many mutual contingencies were recorded in the episodes concerning Tasks 1 and 2. We could explain the dynamics in the activities of Group D compared to Groups A and B from the perspective of 'competition among students' (as described under asymmetrical contingency). Again, mutual contingency was primarily recorded in the activities relating to Task 2 (compared to Task 1), irrespective of the characteristics of the students. Thus, the nature of the task plays a vital role in the mutual contingency observed in the students' activities and not only the characteristics of the students. These results expand Sahlberg and Berry's (2003) results, adding that not all mathematical modelling tasks produce rich equal exchange (e.g., mutual and reactive contingencies) within a collaborative group work.

Pseudocontingency. In Sub-Section 6.7.1, I presented that pseudocontingency was only counted in the episodes of Group A regarding Task

1. This contingency is a limited case of social interaction (Jones & Gerard, 1967). However, it was recorded (in this study) together with other contingencies within the episodes. Thus, the interaction between *Student A* and *B* depicts asymmetrical contingency, while the interaction between *Student C* (interacting with the computer alone) and *Students A* and *B* describes a pseudocontingency. To achieve a mutual (or reactive) contingency form of interaction in such a situation, the students must discuss one strategy together at a time (before discussing another strategy). Panselinas and Komis (2009) argue that students (focusing on one object) can come together with different skills and scaffold each other. Thus, the students might complement each other based on their unique skills (ibid.). In such situations, students might stimulate each other with ideas at the edge of their knowledge, understanding and skills (Sahlberg & Berry, 2003), and this might not happen if students turn to concentrate on their ideas, neglecting others.

In summary, there is evidence (though inconclusive) that not all mathematical modelling tasks produce rich equal exchange within a collaborative students' group work, which adds to Sahlberg and Berry's (2003) earlier suggestion. Considering the dynamics of interaction sequences within the students' activities, both reactive and mutual contingencies were primarily recorded in the episodes relating to Task 2 compared to Task 1 (although they are both mathematical modelling tasks). On the other hand, asymmetrical contingencies were recorded more in the episodes of the activities relating to Task 1, where the groups (Group A and B) were made up of mixed-achievement students or high-performing students dominating in the activities (as suggested by Peter-Koop (2002) and Lowrie (2011)). On the contrary, asymmetrical contingency was recorded less in the episodes (relating to Task 1) in Group D's (also consists of mixed-achievement students) activities. However, more reactive and mutual contingencies were recorded. This might result from 'competition among students', which is more between students at the lower secondary level (Group D) and less between students at the upper secondary level (Group A and B). The nature of the task also plays a vital role in the dynamics of the interaction sequences recorded in this study. The results discussed here are only meant to be suggestive but not conclusive.

I have so far discussed, in general, the dynamics within interaction sequences as the students worked on Task 1 and 2, and a further discussion on the roles students adopt within group interactions might help to understand these dynamics within the interaction sequences. In the following subsection, I will discuss the consistencies of students' roles within group interactions.

7.2.2 Consistency of students' roles within group interactions

In the previous subsection, I discussed the dynamics within the interaction sequences recorded within the students' activities; that is, looking at the interaction sequences without the roles the students adopted. In this subsection, I zoom into the interaction sequences and discuss the roles adopted by the students (or the consistencies of the students' roles) while working on Tasks 1 and 2. Table 5.1 presents the characteristics of each group member, which is helpful in the discussion in this sub-section. Several studies have investigated the roles students adopt, either assigned by the teacher (Smith, 1996; Rosser, 1998; Crawford, 2001) or by the students themselves (Radinsky, 2008), depending on their characteristics, in both collaborative and cooperative group work; while fewer have sought to develop categorizations of the role students adopt in both collaborative and cooperative group work as students work on mathematical modelling tasks. I will limit the discussion to collaborative group work as this study's group activity is described as collaborative, see Section 2.4. For instance, much of the discussion is centred around the students who adopt the *leading* or *questioning and challenging* roles (Goos et al., 2002; Peter-Koop, 2002; Esmonde, 2009). However, alongside these roles, other roles emerged in this study (*opposing*, *supporting*, *suggesting* and *non-contributing*). Table 6.4 (on page 160) presents the analysis of the frequency of roles adopted by each member of the groups recorded about Tasks 1 and 2. I have presented an analysis of Table 6.4 in Sub-Section 6.7.2.

Leading. The leading role was more prevalent among members of Groups A and B (mixed-achievement students) compared to Group C (same-achievement and high-performing students) (see Sub-Section 6.7.2). A number of literature emphasize that high-achieving students often dominate group interactions (Peter-Koop, 2002; Esmonde, 2009). The leading role was counted less in Group D's episodes, even though the group is a mixed-achievement group. Applying a similar argument from the discussion under asymmetrical contingency (see Sub-Section 7.2.1), I argue that the competition among students (Feldlaufer et al., 1988) might play a role in the dynamics of the leading role adopted concerning Groups A, B and D. That is, as the competition among students is higher at the lower secondary than in the upper secondary, there might be a possibility that all students within the group would want to talk through their ideas instead of one/two individual/s leading. The nature of the task also played a role, as the number of leading roles adopted decreased in the activities involving Task 2 (compared to Task 1).

Questioning and challenging. This role is seen as one of the factors in group interaction success (Goos et al., 2002; Paterson & Watt, 2014). Challenging each other's thinking might help develop new concepts within group interactions (Sahlberg & Berry, 2003). In Sub-Section 6.7.2, I presented that questioning and challenging the idea(s) of peers within group interactions is seen among mixed-achievement groups (both upper and lower secondary level) concerning Task 1. Again, concerning Task 2, questioning and challenging the idea(s) of peers is seen among upper secondary school students (either mixed- or same-achievement group) compared to students at the lower secondary school.

Opposing, **supporting, suggesting and non-contributing.** In Sub-Section 6.7.2, I presented the dynamics of opposing, supporting, suggesting and non-contributing roles. These roles might further help categorize students' roles while working on mathematical modelling tasks aside from leading and questioning and challenging roles. In Sub-Section 6.7.2, I presented that opposing ideas were not recorded among same-achievement and high-performing students (Group C) regarding Tasks 1 and 2. The students assigned the lowest grade (among the mixed-achievement students) usually adopted the supporting role. In contrast, the others assigned the highest grade mostly adopted the suggesting role (Groups A, B and C). A non-contributing role was recorded among upper secondary students compared to lower secondary students; this might be due to the competition among students, as Feldlaufer et al. (1988) suggested (or other factors).

These characterizations above contribute to understanding students' adopted roles in mathematical modelling activities related to Tasks 1 and 2. So far, I have discussed, in general, the roles students adopt within group interactions while working on Task 1 and 2; I have not discussed the technologies the students used in these adopted roles and interaction sequences. This does not mean the technology is separated from the students' activities; it is an integral part (Borba & Villarreal, 2006). As such, in the following subsection, I will discuss the influence of digital technologies on group interactions.

7.2.3 Influence of digital technologies on group interactions

An extensive number of studies have investigated the use of digital technologies in learning mathematics (Goos et al., 2003). Different frameworks (instrumental approach, co-action with digital technologies, co-construction of tools, and others) describe processes for the formation of a tool and students' use of tools in a mathematics classroom environment (Trouche, 2005; Moreno-Armella et al., 2008; Doerr and Zangor, 2000. See Section 2.2). These frameworks do not outline the role of digital technologies as mediating tools for social interaction among peers (in transforming an artefact into a tool). The availability of digital technologies alone does not ensure the development of collaborative practices (such as small-group interactions) in the learning environment (Goos et al., 2003; Geiger et al., 2010). However, the interactions between students (group interactions) in collaborative learning depend on the digital technology used and the mathematical task (Geiger & Goos, 1996; Geiger et al., 2010). In Sub-Section 7.2.1 and 7.2.2, I discussed the interactions recorded in the students' activities regarding Tasks 1 and 2 and the consistency of students' roles within group interactions. In these subsections, I pointed out how the interaction sequences recorded are linked to the task's nature and the students' characteristics. Geiger et al. (2010) argue that a task focusing on process (rather than product) might encourage collaborative discussion. Although both Tasks 1 and 2 focus on the process (producing a form of collaborative discussion) there is still a distinction in the interaction sequences emerging as the students work on these tasks.

Significant bodies of literature support the premise that digital technologies play a role in mediating collaborative learning processes (Geiger et al., 2010). For instance, Goos et al. (2003) investigated the 'perspective on technology-mediated learning in secondary school mathematics classroom', and their result showed how technology might facilitate collaborative inquiry in small group interactions and whole class discussions. Thus, Goos and colleagues illustrate four roles (master, servant, partner and extension of self, see Section 2.2) of technology concerning teaching and learning interactions (ibid.). Again, Geiger et al. (2010, p. 54) addressed the question, "How can CAS (Computer Algebra Systems) mediate and support productive social interactions between students?" in a mathematical modelling activity. Their results showed that CAS-enabled technologies have a role to play as provocateurs of productive student-student-teacher interaction. That is, unexpected output on the CAS-equipped devices (e.g., Nspire handhelds) influenced the students' activities by confronting students with an unanticipated result, which (in turn) provoked the rethinking of their original assumption (ibid.). These examples focus on teachers and students in a secondary school mathematics classroom. In contrast, in this study, I only focus on the students (i.e., student-student-technology interactions) (see Section 1.3). The findings of this study expand on previous studies (e.g., Goos et al., 2003; Geiger et al., 2010) about the influence of digital technologies in group interactions. That is, looking at the influence of digital technologies in group interactions in a mathematical modelling activity from an Affordance Theory perspective.

In Sub-Section 7.1.3, I addressed the issue of affordances and constraints of digital technologies emerging as groups of students develop a technology-based model/solution in a mathematical modelling activity. These affordances and constraints were grouped under technological, mathematical and socio-cultural. However, for this subsection, I will discuss socio-cultural affordances and constraints (in an attempt) to address the influence of digital technologies in group interactions. The presence of other students using the same digital technology for similar or related purposes might affect a student's behaviour (Volkoff & Strong, 2017). In this case, how a student uses digital technologies within group interactions (in mathematical modelling activity) might affect another student's behaviour. The students used different digital technologies in solving both tasks (see Table 6.1 on page 157), and the socio-cultural affordances and constraints were categorized as *common focus* (creating a shared goal), *observing and improving strategies* (Roschelle & Teasley, 1995; Granberg & Olsson, 2015) and *authority of the digital technology* (personalizing of problems in group situations) (Lowrie, 2001) (see the last paragraph on page 108). I have already discussed these categories in Sub-Section 7.1.3, but for this subsection, I will highlight certain instances addressing the influence of digital technologies in group interactions.

Common focus describes the situation where the students have the facility to look at the same thing within group interactions as they share the same source. For instance, students in Group A agreed on a shared goal through a flow of turn-taking, dialogue and action (see Table 6.6). In this case, GeoGebra was used as a reference tool to visualize one's reasoning during a mathematical discourse (Granberg & Olsson, 2015). Thus, to visually demonstrate their reasoning to one another, one student used GeoGebra as a reference tool by pointing to the coordinate axis and sketching with paper-and-pencil ($f(x) = 100x$) in relation to the coordinate axis, while another student responding to this used GeoGebra to demonstrate her suggested function $f(x) = -x + 100$ visually.

Observing and improving strategies describes the use of digital technology in maintaining shared knowledge and ideas through verifying ideas or settling disagreements by performing tests, among others. For instance, GeoGebra could be used to maintain and improve shared ideas in group interactions (Granberg & Olsson, 2015). In this study (especially in the activities of Group A—see Sub-Section 6.7.3), one student observed the solution strategy and felt there was the fastest way to find the maximum revenue but could not visually demonstrate his ideas (see Table 6.11). This could be that the student was not confident enough to express his thoughts. However, this triggered another student to come up with the idea of making slides. Even though this student made a slider, it did not affect the graph as the slider has no link with the function. The group then returned to the initial strategy they began with. There was no divergence in the strategy they began with, as the strategy adopted only needed improvement to be more efficient. In this situation, GeoGebra might offer the possibility of observing and repairing divergences or improving solution strategies in group interactions if the function and the slider under construction are mathematically linked.

Authority of the digital technology describes situations where students only accept an answer from the digital technology as the correct answer and/or personalize the problem based on one's interest, which can be the type of mathematical representation offered by the digital technology. For instance, a member of Group D would only accept an answer from the digital technology than her peers (see Table 6.43). In another instance, a student in Group A comes up with an idea to generate their data efficiently after observing the group's initial strategy (see Excerpt 6.3.1). However, the student who took the leading role had personalized the problem-solving strategy. Thus, she dismissed the comments and returned to the existing idea, thinking they were already close to the answer. Subscribing to the suggestions might have helped the group generate their data with the spreadsheet and find a function representing the data. From this narrative, I argue that the features of GeoGebra allow multiple problem-solving strategies, and the approach used by the group depends on the representational choice of the member taking the leading role (see Sub-Section 7.2.2), especially when they think they are close to finding the answer. This echoes previous research that points out that personalizing problems might hinder the potential for sophisticated sense-making (in a group activity/interaction) that could lead to a better activity outcome (Lowrie, 2011). Thus, the more personalized the students might want the problem to be, the more likely these students might complete aspects of the problem individually (and not consider the ideas of others) (Lowrie, 2011). Drawing students' attention to not personalizing the problem-solving strategy and considering input from peers might benefit students' learning and achievement in mathematical modelling with digital technologies. Hernandez-Martinez and Harth (2015) argue that new ideas introduced in group interaction are of little use if they are not specific and/or have no connection with the groups' understanding of the problem; one of the reasons for rejecting a new idea could be that the new ideas are either communicated without confidence or not clear enough to connect with the group's current thinking. Alternatively, as in one of the cases in this study (see Excerpt 6.3.1), a new idea would involve restarting when a solution was imminent.

From previous research in mathematical modelling activities, digital technologies have a role to play as provocateurs of productive student-student-teacher interaction (Goos et al., 2003; Geiger et al., 2010), and this current study adds to this body of work that digital technologies might also serve as a reference tool to visually demonstrate one's ideas while creating a shared goal (*common focus*). Granberg and Olsson (2015) point out that digital technologies might be used for observing and repairing divergences and misconceptions in group activities. Thus, at some point in the students' activities, they might find themselves in a situation marked by uncertainty, divergence or misconception, where these students might use digital technology to verify knowledge or settle disagreements by performing tests. However, it was found in this study that digital technologies might provide a platform for *observing and improving strategies* within group interactions in a mathematical modelling activity. Thus, the students were sure about their solution processes, and their strategies were not divergent. However, these students needed a more efficient way of solving the problem, and digital technology provided the platform for observing and improving strategies. Another influence of digital technologies in group interactions is personalizing a problem (*authority of the digital technology*) resulting from the type of mathematical representation offered by the digital tool and the characteristics of the students (e.g., working style or choice of mathematical representation).

In the following subsection, I will discuss students' tendency to select or use a particular digital technology, having discussed the influence of digital technologies on group interactions.

7.2.4 Students' tendency to select or use a particular digital technology

Generally, there are several reasons behind students' tendency to select or use a particular digital tool regularly (Margaryan et al., 2011) in a mathematical activity (see Sub-Section 2.2.2). These reasons could be the experience of the students with digital technology (Owens-Hartman, 2015; Jacinto & Carreira, 2017; Gueudet & Pepin, 2018) or familiarity with the operations of digital technology (Geiger et al., 2002), goal-oriented based on the demands of the task (Anastasakis et al., 2017), simplicity and efficiency of the digital technology (Hillesund, 2020), among others. The studies above (and others elsewhere) mostly touch on digital technologies in mathematical problem-solving activities and not necessarily in modelling activities. Furthermore, in the activities of the studies above, the students were either allowed or instructed to use specific digital technologies in classroom observation, and empirical data were collected through video recordings, interviews, and others. In general, there is a limited literature base related to the issue of students' selection and/or switching between digital technologies when learning or doing mathematics (Geiger et al., 2002). This current study seeks to extend knowledge in this domain through the analysis of video recordings and screen capture software that targeted students' selection and/or switching between different digital technologies in a mathematical modelling activity, where the students are allowed to use any tool (no restriction of digital technology use). Allowing the students to choose freely brought about some dynamics in the students' solution process (see Sub-Section 6.7.3). Thus, the students use different technologies while working on Tasks 1 and 2 (see Table 6.1). The discussion in this subsection is based on the observation (video recording and screen capture software) of the students as they work on both tasks. I will discuss the reasons behind students' tendency to select or switch between digital technologies while solving Tasks 1 and 2, respectively:

Task 1. From Sub-Section 6.7.3, the problem-solving strategies of Groups A and B could be described as a function representation where the students start with a graphical representation (in GeoGebra) and analyze patterns of numbers (using a calculator for larger values) (Yerushalmy, 2000). Thus, the students used a calculator for numerical calculations and GeoGebra for visual representation and drawing of a function (Flehantov & Ovsiienko, 2019; Flehantov et al., 2022). The problem-solving strategy here might have influenced the students to use GeoGebra and the calculator while working on Task 1. In one instance, a member of Group A suggested another strategy that would involve using another tool (Excel/spreadsheet), but the other students rejected this idea. The reason could be that a new idea would involve restarting when a solution is imminent. In this situation, the students will prefer continuing with the first tool (GeoGebra) instead of using another tool (Excel/spreadsheet), which would involve restarting. Group A also used Google Search to find the meaning of specific terms during their mathematical discourse, as the student's first language is Norwegian. Group C selected Excel/spreadsheet instead of GeoGebra as they thought their generated data would form a recursive function and that they could not use GeoGebra. The students might prefer Excel/spreadsheet over GeoGebra, maybe because they use Excel/spreadsheet often or are more familiar with its operations (Geiger et al., 2002) in dealing with situations described in Task 1. The students' problem-solving strategy might have also influenced their choice of digital technology. For instance, Flehantov and Ovsiienko (2019) and Flehantov et al. (2022) argue that students use Excel/spreadsheet for numerical calculations and represent numerical results in tabular form. It could also be the simplicity and efficiency (Hillesund, 2020) of using Excel/spreadsheet over GeoGebra in this situation. Group D switched between a calculator, GeoGebra and Excel/spreadsheet. Firstly, Group D's selection of a calculator might be due to the nature (or demands) of the task (Anastasakis et al., 2017). Thus, they saw some numbers in the task and decided to compute them, but they later saw that it would take them some time. Secondly, it might also be the inefficiency of using a calculator that made Group D choose GeoGebra (Hillesund, 2020). The working style of the students also played a role here. Thus, they moved from a numerical (solving the task arithmetically) to a graphical representation (plotting a set of points). Group D switched to Excel/spreadsheet when they could not find the graph with GeoGebra. The simplicity of Excel might have influenced their choice. Thus, the students switched back to a numerical representation when they struggled with the graphical representation. Switching back to the numerical representation, the students did not go back to using a calculator but rather Excel for simplicity and efficiency.

Task 2. From Sub-Section 6.7.3, Groups A, B and C used Google Maps to locate the positions of the three cities and Google Search to find the population and the travel time between the three cities. The students might have chosen these tools due to the nature (or demands) of Task 2 (Anastasakis et al., 2017). Group B also used GeoGebra, and one reason could be the manipulation of points. Thus, transferring the location points from Google Maps into GeoGebra and performing certain operations like inserting new points, dragging points, and measuring distances between points, among others. Again, Group C used GeoGebra to find the theoretical best location without considering the roads or population. In this case, the students might have chosen GeoGebra to generalize the optimal position without considering some factors. Group D also used GeoGebra while searching for the theoretical best location (after locating the positions of the three cities on Google Maps). The students might have chosen Google Maps and GeoGebra due to their prior experience (Owens-Hartman, 2015; Jacinto & Carreira, 2017; Gueudet & Pepin, 2018). For instance, Group D's approach to solving Task A, “Rescue helicopters” (see Appendix B), is similar to how they solved Task 2. Thus, the students only considered the air distance, not the actual roads.

In summary, previous research has shown that students might choose certain digital tools based on their experience with the digital technology, familiarity with a particular digital tool, the demands of the tasks, simplicity and efficiency of the digital tool and others. This current study adds to this body of work that students might choose certain digital tools while working on Task 1 based on the adopted problem-solving strategy, familiarity with the operations of the digital tool, the nature of the task, rejecting other tools and keeping the current tool when the solution is imminent and a new idea/s (using other tools) would involve restarting, switching between tools for simplicity and efficiency. Again, students might choose certain digital tool while working on Task 2 based on the nature of the task, ease of manipulating a set of points, generalization of a solution and prior experience. Affordance Theory helped detect or capture the students’ activities with tools. That is, the narrative in this subsection is based on the affordances and constraints of the digital tools that emerged during the students’ mathematical modelling activities (see Sub-Section 7.1.3).

So far, I have discussed four significant issues arising from the research with existing literature. In the next section, I will present some reflections on using theoretical perspectives and their link with the research findings. That is, I will reflect on the possible consequences of the different frameworks adopted and the research methods used concerning the findings of this research study.

7.3 Reflection on the use of theoretical perspectives and their link with the research findings

In this section, I present a discussion of my reflection on the consequences of the different theoretical frameworks adopted and the methodology used concerning the findings of this study. CHAT was adopted as an overarching theoretical framework with Affordance Theory as a supplementary theory. CHAT also provided an analytical framework through the expanded mediational triangle (Engeström, 2001) and the three-level hierarchy of an activity (Leont’ev, 1977). At the same time, Affordance Theory offered an analytical framework through three levels of affordances and constraints developed in mathematics education (Hadjerrouit, 2019). Combining these frameworks helped focus on different aspects of the student’s activities. The combination of CHAT and Affordance Theory pointed towards the necessary dimensions to consider for conducting a research study on students' mathematical modelling with the aid of digital technologies. These theories also allow the adoption of multiple methods to research a particular phenomenon (Nardi, 1996). As such, my choice of methodology for this study corresponded to the adopted theories (Radford, 2008a). Adopting CHAT (expanded mediational triangle) allowed me to take a socio-cultural perspective where I considered multiple data sources that added to this research study's epistemological foundations. For instance, I had data from video recordings, screen capture software, fieldnotes and students' worksheets in my findings regarding the first research question (RQ1). If I had not considered these data sources, I could not have come up with the present conclusions about relationships among the aspects of the student's activity. One example is the relationship between the characteristics of the students (grades assigned to them and their level—either lower or upper secondary) and the interaction sequences recorded, with data sources from fieldnotes and video recordings, respectively. In this case, the individual data sources only partially viewed the students' activity. Thus, using one data source, such as video recordings, helped describe the students' actions and discourse without a proper account of their computer activities (from the viewpoint of the collective activity system).

To study the students' mathematical modelling processes, I zoomed into the students' activity and considered the mathematical aspect of the student's activity. CHAT (in this case, the expanded mediational triangle) does not offer a characterization of students' mathematical modelling processes. However, with Leont'ev's three-level hierarchy of an activity, I redefined the students' activity in mathematical modelling as modelling actions, operations, and activity. I then used the modelling competence framework to characterize the modelling actions and operations within the students' activity. To achieve this purpose, I first viewed mathematical modelling as a mathematical activity in the sense of CHAT. Secondly, I viewed modelling competence as a process and not a product by presenting the ontology and epistemology of modelling competence from a CHAT perspective. This framework complemented existing research (mainly done from a cognitive perspective) in the sense that it added digital tools dimensions and environmental dimensions to the analysis of the modelling actions and operations (characterized by the modelling competence framework). I also considered multiple data sources while looking at this aspect of the study. That is, in my findings relating to the second research questions (RQ2a & RQ2b), I had data from video recordings, screen capture software, fieldnotes and students' worksheets.

I further zoomed into the students' activity and considered the relationship between the students and the digital technology used. The three levels of affordances and constraints allowed me to characterize students' relations with digital technologies in mathematical modelling activities. Affordance Theory helped in understanding how students within the group utilized (or took advantage of) the affordances and constraints of digital technologies to perform their contribution in a group activity. This did not consider the social aspect of the student's activity. As such, using CHAT (Leont'ev's three-level hierarchy of an activity) in combination with Affordance Theory allowed me to view the socially mediated aspects of group work (e.g., group activities mediated with digital technologies) and how the student(s) within the group took advantage of the environment (affordances and constraints of digital technologies) to perform their activities (Kuswara et al., 2008). Again, I considered multiple data sources while looking at this aspect of the study. Thus, in my findings relating to the third research question (RQ3), I had data from video recordings, screen capture software and fieldnotes. Despite the advantages of the adopted theoretical perspectives discussed above, I have addressed some associated limitations in Section 8.3.

7.4 Summary of the chapter

In this chapter, I revisited the research questions organized around three themes. These themes were students' mathematical modelling activities, emergence of modelling actions and the role of digital technologies, and emergence of affordances and constraints of digital technologies in mathematical modelling activities. All the themes combined helped to view students' mathematical modelling with digital technologies holistically. The chapter also addresses four significant issues arising from the research regarding existing literature. These issues are the relation between interaction sequences and task design, the consistency of students' roles within group interactions, the influence of digital technologies on group interactions, and students' tendency to select or use a particular digital technology while working on Tasks 1 and 2. The findings helped understand students' mathematical modelling with digital technologies from different perspectives. The findings also had the potential to contribute new insights into these research areas and add to the growing literature on mathematical modelling with digital technologies from the student's perspective. To end the chapter, I reflected on the consequences of the different theoretical frameworks adopted and the methodology used concerning the findings of this study.

8 Conclusion and Implications

This chapter is the final chapter of this research study, where the themes pursued in each of the preceding chapters and associated findings are synthesized. This chapter is divided into five sections. Section 8.1 brings together the whole dissertation and addresses the research questions. Section 8.2 presents a reflection on the quality of the thesis. Section 8.3 presents the limitations of this research. In Sections 8.4 and 8.5, I highlight the implications of the research to the field of mathematics education and propose issues for future research, respectively.

8.1 Summary

Digital technologies in mathematical modelling activities are receiving increased attention in curriculum and policy documents in Norway and internationally. Implementing pedagogical reforms associated with these areas raises many questions and issues to explore in Norwegian and international contexts (Bakken & Andersson-Bakken, 2021; Berget, 2022). An issue that has emerged from different interests in technology and students' mathematical modelling activities is the different forms of interaction within students' mathematical modelling activities with the aid of digital technologies in group settings. While most of the research literature in technology-mediated activities in mathematical modelling is now significant, much of the research in this area is often done from a cognitive perspective focusing on heuristics and modelling processes. Aside from the cognitive aspects, other dimensions or aspects are needed in the ongoing discussions in this area (Vos & Frejd, 2022). This study has taken a socio-cultural perspective that places interaction and activity between students and digital technologies in mathematical modelling activities at the centre.

The main goal and sub-goals of this research study are listed as follows:

- To explore how secondary school students solve mathematical modelling tasks with the aid of digital technologies.
    - To examine the various forms of interactions taking place within the students' activity.
    - To investigate the students' working processes in the students' activity.
    - To examine students' interactions with digital technologies in the students' activity.

The main goal and sub-goals were stated previously in Section 1.3 (on page 6). To research into these goals, three research questions were formulated (see Section 1.4 on page 7):

RQ1: *How do students solve mathematical modelling tasks with the aid of digital technologies?*
RQ1a: *What digital technologies did the students use in solving the two mathematical modelling tasks?*
RQ1b: *What contingencies were shown in the student interactions when solving the two mathematical modelling tasks with the aid of digital technologies?*
RQ1c: *What are the rules that mediate students' mathematical modelling activities when solving the two mathematical modelling tasks with the aid of digital technologies?*
RQ1d: *What roles did the students adopt in the student interactions when solving the two mathematical modelling tasks with the aid of digital technologies?*

RQ2a: *What modelling actions emerge during the mathematical modelling activities of the students?*
RQ2b: *What part do the uses of digital technologies play within the modelling actions that emerge?*

RQ3: *What affordances and constraints of the digital technologies emerge as the students develop a technology-based model/solution?*

The direction of this thesis has been guided by the research questions stated above. The theoretical framework shaped these questions and operationalized them via the research design. This study was framed within a qualitative research paradigm based on an activity theorist epistemology and ontology. Two theories were combined to form the theoretical framework for justifying the appropriateness and usefulness of chosen constructs under investigation: Cultural-Historical Activity Theory (CHAT) and Affordance Theory. The combination of CHAT and Affordance Theory highlighted the necessary dimensions to consider for conducting a research study on students' mathematical modelling using digital technologies.

Adopting CHAT (Engeström's expanded mediational triangle) allowed me to take a socio-cultural perspective where I considered multiple data sources that add to the epistemological foundations of this research study. To study the students' mathematical modelling processes, I zoomed in on the activity system and considered the mathematical aspect of the student's activity. In this zooming-in, CHAT (in this case, Engeström's expanded mediational triangle) does not offer a characterization of students' mathematical modelling processes. However, with Leont'ev's three-level hierarchy of an activity, I redefined the students' activity in mathematical modelling as modelling actions, operations, and activity. I further zoomed into the activity system and considered the relationship between the students and the digital technology used. The three levels of affordances and constraints (technological, mathematical and socio-cultural) allowed me to characterize students' relation with digital technologies in mathematical modelling activities. Affordance Theory helps in understanding how students within the group utilize (or take advantage of) the affordances and constraints of the digital technologies to perform their contribution in a group activity. However, it does not consider the social aspect of the students' activity. As such, I used CHAT (Leont'ev's three-level hierarchy of an activity) in combination with Affordance Theory, which allowed me to view the socially mediated aspects of group work (e.g., group activities mediated with digital technologies), and how student(s) within the group took advantage of the environment (affordances and constraints of digital technologies) to perform their activities.

An ethnographical case study research design was adopted in this research study. The study involved four secondary schools in southern Norway, and in each school, participants were randomly selected from among the students who volunteered. Empirical data were collected in Autumn 2021 through video recordings (recorded conversations and actions of students), screen capture software (computer activities), fieldnotes, and students' worksheets. A combination of inductive and deductive approaches was used in the data analysis. I will now discuss the main findings of this doctoral research study.

Firstly, the findings reveal that the elements of CHAT, that is subject (characteristics of each secondary school student), the community (group of secondary school students), the object (solving mathematical modelling tasks), mediating artefacts/tools (digital technologies—RQ1a and group work/interaction—RQ1b), the rules (time constraints, availability of digital technologies, implicit rules—RQ1c), and the division of labour (roles adopted by the students within the group—RQ1d) are seen as a collective system interacting with each other, in contrast to cognitive approaches focusing on heuristics and modelling processes. From a CHAT perspective, the student-student interactions are directed by individuals' engagement with the digital technology, which influences the outcome of the activity.

Secondly, the findings show that students' activities in mathematical modelling can be considered as performing actions (modelling actions) and operations (modelling operations) towards an object (solving a mathematical modelling task) from a CHAT perspective, where digital technologies played an essential role within the modelling actions that emerged. In the action of breaking the task into manageable parts, digital technologies (e.g., Google Maps and Google Search) were used to seek information (researching) about identified variables. Breaking the task into manageable parts (which involves operations like reading the problem text, seeking information, and others) was recorded as the first action of the students while working on the tasks. Digital technologies (e.g., GeoGebra, Excel/spreadsheet and Google Maps) were also used for geometric construction, data entry and generation, and seeking information, among others, while searching for a model. In the action of finding a solution for the model, digital technologies (e.g., GeoGebra, Excel/spreadsheet, calculator, Google Maps and Google Search) were used for changing/experimenting, calculating, visualizing, data entry and generation and seeking information. The action of finding a solution for the model was the central part of the students' activity as the majority of the episodes of the students' activity describes this action. In the action of explaining the results in real terms, digital technologies (e.g., GeoGebra) were used for geometric construction (in the case of generalizing the solution). Finally, in the action of checking the results for adequacy, digital technologies (e.g., GeoGebra) were used for changing/experimenting and measuring. These modelling actions emerging are not a general manifestation in the students' activities but depend on the characteristics of each group, and the nature of the tasks, among others.

Thirdly, the findings highlight technological, mathematical and socio-cultural affordances and constraints of the digital technologies that emerged in students' mathematical modelling activities with the aid of digital technologies. These affordances and constraints that emerged depend on each group's characteristics, the nature of the task, and what is perceived of the digital technology, among others. At the operational (technological affordances) level, digital technologies provided the students with specific functional opportunities such as researching, measuring, visualizing, geometric construction, experimenting/changing, data entry and generation, and calculating while working on the tasks. The technological affordances of digital technologies at the operational level support the mathematical affordances emerging at the action level. Thus, at the action level, digital technologies afford clarification in a mathematics discourse, analyzing, linking mathematical representations, simulating and visualizing, regularity and variations, and arithmetic and statistics. Finally, at the activity or collective level, digital technologies induce specific social affordances as the students engage with the technology in social interaction. These social affordances could be a common focus, observing and improving strategies and authority of digital technology.

In the next section, I will discuss reflections on the quality of this doctoral dissertation.

8.2 Reflection on the quality of the thesis

Several studies have discussed issues on the quality of a research study (Simon, 2004; Niss, 2010b), and the common things that these studies share on what a researcher needs to provide in the stages of the thesis are as follows:

- Choice of research question
- Quality of the research design
- Justification of the methodology
- Justification of the analysis of data
- Justification of the conclusions

Simon (2004) points out that "a research study, from questions to conclusion, can be thought of as the construction and presentation of a warranted argument" (ibid., p. 159). In this case, the quality of a thesis could be evaluated based on how the researcher was able to meet these warrants throughout the study. Based on the structure above, I will attempt to provide an argument for this. I started this research study with clear initial research questions and methodology but not a clear theoretical perspective. Adapting a set of theoretical perspectives and my literature review ensured that my new research questions were founded on terminology that was operational and provided a basic starting point for advancing the research field. My choice of theoretical framework, CHAT and Affordance Theory, to study students' mathematical modelling with the aid of digital technologies was based on an interest in studying different forms of interactions within the students' activities (this was elaborated and justified in Chapter 4). In Chapter 5, I provided an argument for my choice of research design. To address the newly formulated research questions, an ethnographical case study design was used to make sense of the modelling actions, affordances, and constraints that emerged by observing students' interactions with digital technologies while working on mathematical modelling tasks. Again, I considered this study as explorative as I did not look for definite answers according to predefined criteria but rather sought to unveil new qualitative insights into students' mathematical modelling with the aid of digital technologies. There are some pitfalls associated with the explorative nature of this study, which I addressed in Sections 5.9 and 8.3. Thus, issues concerning validity and trustworthiness.

My choice of qualitative methodology follows from the ethnographical case study design chosen. Chapter 5 presents a rich context description of how the study was conducted. Radford (2008a) explains that a well-connected theoretical framework and methodology help distinguish between relevant and irrelevant data. As such, a researcher needs to consider the consistency between the chosen theoretical framework and the methods utilized to derive results from the data. The analysis in this study reflects these ideas. The data analyzed were authentic transcripts of events from classroom filming and screen capture, and the coding was theory-informed inductive coding (see Sub-Section 5.7.2). This coding strategy emerges as an interplay between the theoretical framework and the empirical data (an accepted methodology in qualitative research if the data is considered with a specific theoretical background in mind) (Patton, 2002; Braun & Clarke, 2006). From this perspective, I used a combined inductive and deductive thematic analysis technique to analyze the empirical data (see Sub-Section 5.7.3). In this case, connections between the codes that transpired in the data analysis were condensed into categories or themes that reflected the chosen theoretical background. The results were supported by referring to excerpts and images (taken from the screen capture software) from the transcripts, which adds to the trustworthiness of the analysis. In the end, the conclusion chapter indicated the study's contributions to knowledge in mathematics education (see Section 8.4).

In the next section, I will discuss the limitations of this research study.

8.3 Limitations

The issue of limitations in a research study is essential to address to evaluate the quality of the research conducted. As described in Section 5.1, this research has been conducted within a qualitative research paradigm, and the results must be critically evaluated from this perspective. In Section 5.9, I have elaborated on the measures undertaken throughout the research process to ensure the validity and trustworthiness of the results and conclusions. Despite these measures and the present study's contributions, some limitations are worth mentioning.

The research reported in this thesis arises from in-depth analyses of four cases involving specific contextual conditions. Each case involved a small number of participants (3-4 students in a group) in the selected contexts. Therefore, the findings drawn from this research have clear contextual influences such as the knowledge level of the students, the nature of Task 1 and 2, the characteristics of the students, prior experience with digital technologies, the mathematics courses taken, the mathematics curriculum, and others. The participant sampling was random, whereas the research context was purposefully selected in the case studies. For this reason, the results cannot be quantitatively (or statistically) generalized, for instance, to all secondary school students' activities. Thus, other participants might have tackled Tasks 1 and 2 differently, leading to different observations concerning their use of digital technologies in mathematical modelling activities. However, the conditions of the environment, which became evident in the students' activity, can be generalized (analytical generalization, see Section 5.9) in the sense that these point to the significant aspects of the student's mathematical modelling activity. In response to the criticism that case studies lack generalizability of results, Flyvbjerg (2006) points out that case studies provide a generalization through "the force of example". In this case, I argue that this research offers an example of secondary school students' mathematical modelling activities and the conditions which became evident through the research. A case study emphasizes the depth of study rather than the breadth and focuses on the particular, not the general. Accordingly, this research study aims to gain insight and seek to understand students' mathematical modelling with the aid of digital technologies rather than generalization.

My choice of research methods for analyzing students' modelling actions and the affordances and constraints that emerged have some limitations. Using video recordings, screen capture software, students' worksheets and fieldnotes revealed extensive insights into the students' activities. However, students' reasons for using a particular digital technology or performing some modelling actions were not recorded. In this case, the students did not explain why they 'broke the task into manageable parts', 'searched for a model', or used certain digital technologies and others. Nonetheless, the results of this research were based on classroom observations. I indicated in the study that the results were only meant to be suggestive as there were certain aspects where the empirical data collected was insufficient to draw conclusions about students' choices. For instance, when Group C claimed that the data is recursive and they cannot use GeoGebra, this could have been taken up in a stimulated recall interview. Stimulated recall interviews could have been conducted to investigate further the students' perspectives on the actions and operations they performed during or after the activities. In this manner, the analysis could be further enriched due to the additional information from the students. However, stimulated recall interviews after the students' activities might not guarantee that the students will give precisely what they were thinking at the time the particular incident happened.

Another limitation of my research is related to the introductory activity. The students only solved three example questions in the introductory activity before the main activity (see Section 5.3), which might not be enough in terms of their familiarity with the task. Engaging the students in several introductory activities might have influenced the outcome of this study, for instance, in the situation where Group D used the same approach of the 'helicopter task' for Task 2 (considering air distance instead of actual roads). The narrative might not be the same if Group D had worked on several examples in the introductory activity before the main activity. My limited acquaintance with the student's native language (Norwegian) was a limitation in this research. The students' activities would have been more beneficial and purposeful if they had used their native language. To reduce this effect, the students were allowed to communicate in Norwegian when they could not express themselves well in English. As such, there were instances where the students communicated in Norwegian (Group D, especially). Hence, I involved a native transcriber of the students' dialogues.

The theoretical framework adopted has some limitations. I have justified my choice of theoretical framework in Chapter 4 and reflected on using these theoretical perspectives and their link with the research findings in Section 7.3. However, some further limitations regarding the theoretical perspective are worth mentioning. The theoretical stance adopted in this study does not capture every aspect of the study. Addressing the social dimensions and digital technologies used, utilizing the categories of affordances and constraints has shown to be an appropriate methodological approach for exploring technological, mathematical and socio-cultural affordances and constraints that emerged. However, the approach outlined in this study is not intended to map all potential technological, mathematical and social affordances and constraints, but it is flexible enough to capture other affordances beyond the ones presented in this study. On the other hand, CHAT cannot provide a complete description of students' gains in conceptual understanding. Conceptual understanding is usually examined from a constructivist point of view (Badie, 2016). Radford (2008c) argues that the constructivist perspective focuses on cognition and puts the individual in the centre. CHAT in this study does not capture students' construction of mathematical knowledge through mathematical discourse, even though the concept of modelling actions helps in explaining the students' modelling processes. Commognition theory captures the mathematical discourse during the activities. CHAT, in this study, focuses on acting humans, whereas the commognition theory focuses on humans who communicate. Commognition is a theoretical framework developed by Sfard (2008) that focuses on social and individual aspects of thinking and learning (Ärlebäck & Frejd, 2013). In this framework, learning is seen as a change in discourses. According to Sfard (2008), discourse is characterized by the meaning and use of language. Ärlebäck and Frejd (2013) point out that to be engaged in the activity of modelling means to participate in a modelling discourse. Again, Ärlebäck and Frejd (2013) use Sfard's commognition to analyze and decern discursive objects in a dialogue between students engaged in a modelling activity, and the results show that a variety of signifiers from different discourses come into play during modelling.

Other theories capture the missing issues that cannot be captured with CHAT. Some of these issues are the institutional aspects of teaching and learning; didactical situations and the role of the milieu; body language and gestures; and others in the modelling context. Chevallard's (1992) Anthropological Theory of Didactics (ATD) captures institutional aspects and praxeologies; Brousseau's (1997) Theory of Didactical Situations (TDS) captures the milieu, didactical and a-didactical situations, and feedback during modelling activities with digital technologies; and the embodied cognition captures gestures and body language. Combining these alternative or supplementary theories raises the question of networking theories, which is a complex issue that cannot be addressed in sufficient detail in this study. I will further discuss this in the subsequent sections.

In the next section, I will present the implications of this research study. Thus, the contributions of this research study to mathematics education research might suggest new insights into students' mathematical modelling with the aid of digital technologies. I will also discuss the implications of this study for practice.

8.4 Implications

In this section, I will consider some possible implications of this study. Based on the analysis and results, theoretical, methodological and practical implications are suggested, and I will focus on aspects relevant to lower and upper secondary school.

8.4.1 Theoretical implications

This study explored how secondary school students solve mathematical modelling tasks with the aid of digital technologies. The study focused on the different forms of interactions within the students' activities; a fundamental aspect of this study has been exploring these different forms of interactions from a CHAT and Affordance Theory perspective (as described in Sections 4.2 and 4.4). From this perspective, I will present this study's theoretical implications below. The discussions of these implications are put under sub-headings: Students' mathematical modelling activities from a CHAT perspective; a combination of CHAT and Affordance Theory; an influence of digital technologies in group interactions; students' tendency to select or use particular digital technology; types of modelling tasks and associated interaction sequences; and students' roles in mathematical modelling activities.

Students' mathematical modelling activities from a CHAT perspective

While the majority of the research literature in the area of technology-mediated activities in mathematical modelling is now significant, much of the research in these areas is often done from a cognitive perspective with a focus on heuristics and modelling processes (Cevikbas et al., 2021). This study takes a socio-cultural perspective, CHAT, to explore the different interactions and modelling processes of the students' activities. The elements of CHAT (Engeström's expanded mediational triangle) are seen as a collective system interacting with each other in contrast to cognitive approaches. That is, we cannot separate the students' modelling processes from the digital technologies used, the nature of the tasks, the students' characteristics, and the activity rules (both implicit and explicit), among others. Analyzing students' mathematical modelling activities from a CHAT perspective gives a broader picture of how students solve mathematical modelling tasks with the aid of digital technologies.

From the cognitive perspective, much of the analyses of students' modelling activities are done by using a modelling cycle. Cai et al. (2014) argue that a modelling cycle might not show most of the actual work students do in a mathematical activity. That is, there might be some difficulties in the qualitative identification of the stages of the modelling process corresponding to each episode of students' work (Ärlebäck, 2009; Czocher, 2016; Albarracin et al., 2019). As such, Albarracin et al. (2019) use the components of Modelling Activity Diagrams (MAD: reading, making model/modelling, estimating, calculating, validating and writing) to characterize the modelling processes of students. These activities in MAD might help qualitatively identify the stages of the modelling processes. However, it does not include the role played by digital technologies in these activities. This study addresses this issue by presenting (in a different way) students' modelling processes as emerging actions (in view of CHAT) and provides the role that digital technology plays in these emerging actions. From a CHAT (Leont'ev's three-level hierarchy of an activity) perspective, I consider students' activities in modelling as performing actions (modelling actions) and operations (modelling operations) towards an object (solve a mathematical modelling task) (see Table 4.2 on page 114). The categories of modelling actions help qualitatively identify the stages of the modelling processes. These categories also show the role of digital technologies in these categories (see the results in Tables 6.12, 6.13, 6.28, 6.29, 6.35, 6.36, 6.41 and 6.42).

Combination of CHAT and Affordance Theory

There have been very few studies (Albrechtsen et al., 2001; Bærentsen & Trettvik, 2002; Martinovic et al., 2013; Fredriksen, 2021) that have combined CHAT and Affordance Theory to study students' activities with technology. Finding a connection between these theories in mathematical modelling with the aid of digital technologies is essential in contributing to the theories themselves. The combination of CHAT and Affordance Theory helps address the affordances and constraints of digital technologies and the activities mediated by these digital technologies (see Sub-Section 4.4.1). This combination does not separate the affordances and constraints of digital technologies from the activities mediated by these digital technologies. That is, the affordances and constraints of digital technologies emerging in this study depend on the characteristics of each student group, the nature of the task, and what is perceived of the digital technology, among others. Affordances and constraints emerge at all three levels in Leont'ev's model of activity. At the operational (technological affordances) level, digital technologies provided the students with specific functional opportunities such as calculating, researching, measuring, geometric construction, visualizing, data entry and generation, and experimenting/changing while working on the tasks.

These functional opportunities do not arise in isolation but connect with the activities of the students. That is, the approach the students adopt while solving a mathematical modelling task might determine the tool or the features of the tool to use. The technological affordances of digital technologies at the operational level provide support for the mathematical affordances that emerged at the action level. Thus, at the action level, digital technologies afforded clarification in mathematics discourse, analyzing, linking mathematical representations, simulation and visualizing, regularity and variations, and arithmetic and statistics. Finally, at the activity or collective level, digital technologies induce specific social affordances as the students engage with the technology in social interaction. These social affordances were common focus, observing and improving strategies, and authority of the digital technology. Although the combination of CHAT and Affordance Theory helps to address the affordances and constraints of digital technologies in the students' activities, there are still challenges to connecting CHAT and Affordance Theory (e.g., on the ontological basis—Albrechtsen et al., 2001). The connection in this study was based on the concept of emergence (see Sub-Sections 4.3.1 and 4.3.2). However, further research is needed in the ongoing discussion on the connection between CHAT and Affordance Theory.

Influence of digital technologies in group interactions

Different frameworks (the instrumental approach, co-action with digital technologies, and co-construction of tools) describe processes for the formation of a tool and students' use of tools in a mathematics classroom environment (Trouche, 2005; Moreno-Armella et al., 2008; Doerr and Zangor, 2000. See Section 2.2). These frameworks do not outline the role of digital technologies as mediating tools for social interaction among peers. Concerning the premise that digital technologies play a role in mediating collaborative learning processes, there are significant bodies of literature that offer support for this premise (Geiger et al., 2010). From previous research in mathematical modelling activities, digital technologies have a role to play as provocateurs of productive student-student-teacher interaction (Goos et al., 2003; Geiger et al., 2010), and this current study adds to this body of work that digital technologies might also serve as a reference tool to visually demonstrate one's ideas while creating a shared goal (*common focus*). Granberg and Olsson (2015) point out that digital technologies might be used for observing and repairing divergences and misconceptions in group activities. That is, at some point in the students' activities, they might find themselves in a situation marked by uncertainty, divergence, or misconception, where these students might use digital technology to verify knowledge or settle disagreements by performing tests. However, it was found in this study (see the discussion on socio-cultural affordances in constraints in Sub-Section 7.1.3) that digital technologies might provide a platform for *observing and improving strategies* within group interactions in a mathematical modelling activity. Thus, the students were sure about their solution processes, and their strategies were not divergent. However, these students needed a more efficient way of solving the problem, and the digital technology used provided the platform for observing and improving strategies. Another category of the influence of digital technologies in group interactions is the *authority of digital technology*. In this study, this category describes situations where students only accept an answer from the digital tool as the correct answer and/or personalizing the problem based on their interest, which can be the type of mathematical representation offered by the digital tool and the characteristics of the students. I will further discuss this in Section 8.5, where I connect Actor-Network-Theory to the authority of digital technology. In summary, the categories discussed above help identify the influence of digital technologies on group interactions (in mathematical modelling activities).

Students' tendency to select or use a particular digital technology

This study expands on previous research by identifying students' tendency to use specific digital technologies when they engage in a mathematical modelling activity. From previous research students might choose certain digital technologies based on their experience with the digital technology (Owens-Hartman, 2015; Jacinto & Carreira, 2017; Gueudet & Pepin, 2018) or familiarity with a particular digital technology (Geiger et al. (2002), goal-oriented based on the demands of the task (Anastasakis et al., 2017), simplicity and efficiency of the digital technology (Hillesund, 2020), among others. This current study adds to this body of work that students might choose certain digital technologies while working on mathematical modelling tasks based on the adopted problem-solving strategy, familiarity with the operations of the digital technology, the nature of the task, rejecting other tools and keeping the current tool when the solution is imminent and a new idea/s (using other tools) would involve restarting, switching between tools for simplicity and efficiency, for generalization of a solution, and prior experience. Affordance Theory helped detect or capture these students' tendencies to use specific digital technologies in their activities. Thus, the affordances and constraints of the digital technologies that emerged during the students' mathematical modelling activities.

In this case, the affordances of digital technologies determine students' choice of digital technologies in mathematical modelling activities.

Types of modelling tasks and associated interaction sequences

The findings reveal that not all mathematical modelling tasks produce rich equal exchange within a collaborative students' group work, which adds to Sahlberg and Berry's (2003) earlier suggestion. Considering the dynamics of interaction sequences within the students' activities in this study, both reactive and mutual contingencies are more associated with Task 2 than Task 1 (although they are both mathematical modelling tasks). Asymmetrical contingency is more associated with Task 1 than with Task 2. The findings in this study have revealed that not only the nature of the task that brings about these dynamics within interaction sequences but also the characteristics of the student groups contribute to this (see Sub-Section 7.2.1). For instance, asymmetrical contingency is more linked to a group made up of mixed-achievement students at the upper secondary school or high-performing students dominating within a group (as suggested by Peter-Koop (2002) and Lowrie (2011)) than a similar group at the lower secondary level.

Students' roles in mathematical modelling activities

A number of studies have investigated the roles students adopt, either assigned by the teacher (Smith, 1996; Rosser, 1998; Crawford, 2001) or by the students themselves (Radinsky, 2008), depending on their characteristics, in both collaborative and cooperative group work. Fewer studies have sought to develop categorizations of students' roles in collaborative and cooperative group work as students work on mathematical modelling tasks (Goos et al., 2002; Peter-Koop, 2002; Esmonde, 2009). Among these fewer studies, much of the discussion is centred around the students who adopt the leading or questioning and challenging roles. This study highlights the above roles and adds opposing, supporting, suggesting and non-contributing roles. The findings show that high-performing students in the mixed-achievement group usually adopt a leading role in the episodes. In contrast, within a same-achievement group (group of high-performing students), students do not often adopt a leading role (the same applies to the group at the lower secondary level, although a mixed-achievement group). Questioning and challenging roles were often adopted among mixed-achievement groups (both upper and lower secondary level); this role was adopted more often among upper secondary school students (either mixed- or same-achievement group) compared to students at the lower secondary school level. In the opposing role, none of the same-achievement group members opposed the idea(s) of peers, but this role was present in the mixed-achievement groups. There was also a balance between supporting and suggesting roles among students within same-achievement groups and students at the lower secondary level (who were mixed-achievement students), compared to mixed-achievement students at the upper secondary school. Lastly, the non-contributing role was only adopted by some students at the upper secondary school within the episodes. These characterizations above contribute to the understanding of roles adopted by students in mathematical modelling activities relating to Tasks 1 and 2.

8.4.2 Methodological implications

There have been several studies on students' mathematical modelling activities and the role of digital technologies (Goos et al., 2003; Geiger et al., 2010; Gallegos & Rivera, 2015; Greefrath & Siller, 2017; Greefrath et al., 2018). Other studies (Ärlebäck, 2009; Czocher, 2016; Albarracin et al., 2019) also present a framework (Modelling Activity Diagram-MAD) that might aid researchers in the analysis of mathematical modelling processes students engage (but the framework does not include the role played by digital technologies). Combining ideas from the above studies, this study provides a template that might aid researchers in analyzing students' mathematical modelling processes (modelling actions) and the roles of digital technologies in these processes. That is, the framework in this study might help qualitatively identify the stages of the students' modelling processes and the role that digital technologies play in these processes. To aid the identification of the interaction sequences in the students' activities, I developed an analytical scheme that combined the framework by Jones and Gerard (1967) and Mercer (1994) and also partitioned the students' activities into episodes. This analytical scheme might be employed in group interactions in other contexts (other than mathematical modelling activities).

8.4.3 Implications for practice

On the practical level, this research has raised awareness of the kinds of interaction sequences emerging from different mathematical modelling tasks, emerging modelling actions, technologies in students' mathematical modelling activities, and others. This awareness might suggest to mathematics teachers, educators, task designers, and others in Norway and elsewhere concerning students' mathematical modelling with the aid of digital technologies. The discussions that follow are put under the subheadings task design, modelling actions, and technologies in mathematical modelling activities.

Task Design

The findings of this thesis suggest that not all mathematical modelling tasks produce rich equal exchange within collaborative students' group work. In this regard, task designers and teachers must consider the kinds of mathematical modelling tasks given to the students. Suppose the purpose of giving the task is to generate reactive and mutual (contingencies) interactions within small group activities. In that case, the task should require extra-mathematical knowledge (e.g., Task 2 in this study). This applies if the target group is made up of mixed-attainment students (at the upper secondary level). With groups of both same-attainment students (high-performing and upper secondary students) and mixed-attainment students (lower secondary level), the interactions that emerge might be reactive and mutual irrespective of the mathematical modelling tasks (e.g., Task 1 and 2 in this study).

Modelling actions

Earlier in Section 3.3, I discussed one curricular issue about mathematical modelling tasks used in Norwegian schools. Thus, there are different perspectives on mathematical modelling in the curriculum, the textbook tasks, and the national exam (Berget, 2022). In this case, if we analyze the tasks in textbooks through the steps in a modelling cycle, there are some steps (such as constructing, simplifying, mathematizing, validating and exposing) that are missing whilst the steps present are working mathematically and interpreting (see for example an example task illustrated in Figure 3.1 on page 82). Students in this context mainly work mathematically on the given numbers in the task using digital technologies. However, the curriculum highlights all parts of the modelling process. This study assesses students' mathematical modelling processes via the categories of modelling actions as they work on mathematical modelling tasks (Task 1 and 2). Table 4.2 (on page 114) shows that the steps in the modelling cycle above are embedded in the actions and operations of the students as they engage with the tasks. In this case, not only a part of the modelling process but all parts of the process were evident in the students' activity (see the discussion in Sub-Section 7.1.2), as intended by the curriculum.

Technologies in mathematical modelling activities

Digital technologies in mathematical modelling activities are receiving increased attention in curriculum and policy documents in Norway and internationally. As such, there is a need to critically examine the affordances and constraints of digital technologies in mathematical modelling activities. These affordances and constraints might be opportunities for students' learning. For instance, how students can make sliders to manipulate their constructed function, what students need to do when they get feedback (e.g., illegal function) from the digital technology, and how to construct function with recursive data, among others. Again, the findings of this study revealed that one of the key factors to the modelling outcome resides in the interactions within group activities generated by digital technologies.

In the next section, I will present some suggestions for future research.

8.5 Future research

Opportunities for further research into a range of issues arise from this research study from both theoretical and methodological perspectives. The qualitative nature of my data did not allow me to statistically generalize the findings of this research. In this case, broadening the range of schools, classroom settings, and participants is warranted to investigate whether similar findings can be obtained in different contexts. Thus, a study that will include a broader range of attainment groupings, age groupings, classroom contexts, technological aracialities, among others. This is particularly relevant to the findings in this study that phenomena such as modelling actions, affordances and constraints emerge within students' activities in particular contexts, communities, cultures, institutions, and others (see Sub-Sections 4.3.1 and 4.3.2). During the classroom observations, regarding other groups that were not the focus group, some students remained silent throughout the activities (mostly doing their own thing with their computers). Extending this study into settings that include such students within a group is particularly important to understand how they interact with their peers in mathematical modelling activities using digital technologies. Feldlaufer and colleagues' (1988) study on 'competition among students' (for instance, the transition between elementary and junior high school) was conducted in the American context (over three decades ago). Similar research in the Norwegian context would be worthwhile in understanding the dynamics within the students' interactions.

As discussed in Section 8.3 above, one of the limitations of this study concerns the introductory activity, where the students only solved three examples in one lesson before the main activity. Future studies should include several periods of introductory activities to investigate how the interactions between students and digital technologies develop over time. It would be fascinating to study the interactions within the students' activities if some tasks are redesigned to have features of other tasks (for example, redesigning Task 1 to have features of Task 2). The findings of this study revealed some affordances and constraints of the digital technologies that emerged in the students' activities, which are essential in the students' mathematics learning. The National Council of Teachers of Mathematics (2000) points out that technology is essential in teaching and learning mathematics and might influence the mathematics taught, which might also enhance students' learning. In this case, monitoring how technology is used might benefit teaching and learning mathematics. Drijvers (2015) argues that the design of digital technology and the corresponding tasks and activities, the role of the teacher, and the educational context are some factors that need to be considered when integrating digital technologies into the education system. That is, the role of the teacher is critical in developing effective learning and teaching practices in mathematics classrooms (National Council of Teachers of Mathematics, 2000). Research is needed into how teachers deal with the affordances and constraints of digital technologies in students' mathematical modelling activities. For instance, how teachers engage with students as they make sliders to manipulate their constructed function, deal with feedback (e.g., illegal function) from digital technology, and construct a function with recursive data, among others.

Again, in Section 8.3, I raised concerns about the limitation of the theoretical framework adopted, as such. I will recommend some theoretical perspectives that may be used to further research mathematical modelling with digital technologies. An aspect that needs exploration is the concept of the "authority of digital technology". Actor-Network Theory (ANT) provides a framework that explains that everything exists in a network of interactive relationships, including people, technology and non-living or inanimate objects (Callon, 1984; Law, 1992; Latour, 2005). As such, ANT might help understand the emergence of new technologies in the interactions between humans and technologies. In this regard, we can further explore the concept of "authority of digital technology" from an ANT perspective. In the terminology of ANT, actors include human and non-human actors/agents. In this case, digital technology is not simply a tool or an artefact, but an active agent involved in producing knowledge. ANT proposes that human and non-human agents are equally important and can influence the development of social-ecological systems (Dwiartama & Rosin, 2014). Agency in ANT manifests only in the relation of actors to each other; in this case, agency is distributed between human and non-human agents.

Another thing to consider is tensions within the students' mathematical modelling activities with digital technologies. These tensions can be caused by conflicting relations with the teacher or among fellow peers (Gedera, 2016). In CHAT, such internally co-existing opposites are termed dialectical contradictions (Roth & Radford, 2011). Engeström (2001) considers contradictions within the activity system as sources of change and development, and the dialectical dimension of these contradictions might be essential to investigate, as they are inherent features of the activity system. That is, contradictions that emerge inside and between elements of the activity system are vital for understanding its dynamics. Earlier in the discussions under limitation (see Section 8.3), I mentioned other theories like ATD, TDS and commognition that might capture certain aspects of the students' activity that CHAT could not. Combining these alternatives or supplementary theories raises the question of networking theories, which can be dealt with in future studies. Bikner-Ahsbahs (2016) argues that network theories mean building relations among theories and that networking allows for explicitly working with different theories to benefit from their theoretical strengths (ibid., p. 34). Radford (2008a) argues that a connection between theories can happen at the level of principles or as a combination/coordination of these depending on their compatibility (e.g., CHAT and commognition) or incompatibility (e.g., CHAT and constructivism or TDS) of their theoretical premises (ibid.). Networking of CHAT and other theories can also happen at the methodologies and research questions level.

Bringing this doctoral thesis to a conclusion, I hope that a research partnership can be found for which I can share experiences and my growing expertise in studies involving students' activities with digital technologies. Again, I hope this study's findings might contribute to enhancing the ongoing discussions on students' mathematical modelling with the aid of digital technologies.

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Appendices

Appendix A: Consent form

Are you interested in taking part in the research project?
" (The Use of Digital Technologies for Modelling Realistic Problems at the Secondary School Level)"?

This is an inquiry about participation in a research project where the primary purpose is to investigate how secondary school students solve mathematical modelling tasks with the aid of digital technologies. In this letter, we will give you information about the purpose of the project and what your participation will involve.

Purpose of the project

The research project aims to examine the relation between the student and the digital technologies, and the modelling competences that will emerge during the modelling activities. The research study tends to address these central questions:

1. How do secondary school students mathematize a realistic problem with the aid of digital technologies?
2. What sort of modelling competence emerge during the modelling activities?
   a. What part do particular uses of digital technologies play as the students mathematize realistic problems?
3. What affordances and constraints emerge as the students develop a technology-based solution/model during the modelling activities?

The research project is part of a PhD study at the University of Agder.

Who is responsible for the research project?

The University of Agder (Mathematical Sciences Department) is responsible for the project. The researcher responsible is Obed Opoku Afram (a PhD student), but Professor Said Hadjerrouit and Professor John Monaghan supervise the project.

Why are you being asked to participate?

You have been selected because you are a student at a lower/upper secondary school in the Agder region. On the other hand, the program/department leader responsible for the mathematics subject has been contacted for his/her consent.

What does participation involve for you?

The methods employed for data collection are handwritten materials, screen capture software, stimulated-recall interviews, video recordings and classroom observation (field notes). The handwritten materials are mainly about the answer sheets provided by the students, whilst the screen capture software gives details or step-by-step solutions of the students on the computer. The video recordings will mainly focus on the communication between the students while working on the tasks. The interviews will be tape-recorded. The stimulated-recall approach will be used to get information from the students solely based on their solution to the tasks and the communications between the students during the modelling activities (All data will be completely anonymous; there will be no link between the registered signatures on the consent forms and the data collected).

- If you choose to participate in the project, you will solve a set of mathematical modelling tasks with the teacher's help. You will work in groups. It will take approximately 1 hour. This section allows you to familiarize yourself with the mathematical modelling tasks. I will take field notes during this section. No personal data is included in the field notes (I will only note down the processes you used in solving the tasks).
- You will again solve mathematical modelling tasks without the teacher's help. You will be working in groups. It will take approximately 45 minutes. I will take field notes during this section. No personal data is included in the field notes (I will only note down the processes you used in solving the tasks). There will be a video recording whilst you are solving the tasks. A screen capture software will also get details of your work on the computer.
- You will also participate in a 10-20-minute interview, where you talk about your solutions to the modelling task. A voice recorder will be used during the interview.
- I will observe the participants and take notes during the preliminary and main sections (no personal data will be registered on the individual students during the observation).
- I will also ask the teacher to give his/her reflections on the modelling activities, that is, the teacher’s opinion of the students’ solving the modelling tasks with the technological tool within the two sections. I will record the interview with a voice recorder.

Participation is voluntary

Participation in the project is voluntary. If you choose to participate, you can withdraw your consent without giving a reason. All information about you will then be made anonymous. You will have no negative consequences if you choose not to participate or later decide to withdraw. It will not affect your relationship with the teacher or the program leader. No information or data will be recorded on students who do not participate in the research, especially during the observations.

Your personal privacy – how we will store and use your personal data

We will only use your data for the purpose(s) specified in this information letter and will process it confidentially per data protection legislation (the General Data Protection Regulation and Personal Data Act).

- The only persons who can access the personal data are the researcher and supervisors at the University of Agder.
- All necessary precautions will be taken to ensure no unauthorized persons can access the personal data. I will use pseudonyms to ensure your identity is not revealed in any part of the research project. The personal data will be stored on an external hard drive and the University of Agder server.

The participants will not be recognizable in any form of publication.

What will happen to your personal data at the end of the research project?

The project is scheduled to end on May 24, 2023. After the research project is done, the personal data, including the voice recordings and video recordings, will be completely deleted.

Your rights

So long as you can be identified in the collected data, you have the right to:

- access the personal data that is being processed about you.
- request that your data be deleted.
- request that incorrect personal data about you be corrected/rectified.
- receive a copy of your data (data portability), and
- send a complaint to the Data Protection Officer or the Norwegian Data Protection Authority regarding processing your personal data.

What gives us the right to process your personal data?

We will process your data based on your consent. Based on an agreement with the University of Agder (Mathematical Sciences Department), NSD – The Norwegian Centre for Research Data AS has assessed that the processing of personal data in this project is per data protection legislation.

Where can I find out more?

If you have questions about the project or want to exercise your rights, contact:

- The University of Agder (Department of Mathematical Sciences) via
  - Professor Said Hadjerrouit (Supervisor), by email: said.hadjerrouit@uia.no
  - Professor John Monaghan (Supervisor), by email: john.monaghan@uia.no
  - Obed Opoku Afram (student), by email: obed.afram@uia.no or by telephone: +47 40384669.
- NSD – The Norwegian Centre for Research Data AS, by email: (personverntjenester@nsd.no) or by telephone: +47 55 58 21 17.

Yours sincerely,

Obed Opoku Afram (Researcher)	Prof. Said Hadjerrouit (Supervisor)	Prof. John Monaghan (Supervisor)

Consent form

I have received and understood information about the project 'The use of technological tools for modelling realistic problems at the secondary school level' and have been allowed to ask questions. I give consent:

- ☐ to participate in the group activities
- ☐ to participate in an interview
- ☐ to participate in the video recordings
- ☐ for notes to be taken about my involvement during the activities
- ☐ for the teacher to give his/her reflection about me during the modelling activities.

I consent to processing my personal data until the project's end date, approximately May 2023.

(Signed by participant, date. **NB! If you are under 16, your superior must also sign.**)

Appendix B: Mathematical modelling tasks

Task A: *Rescue helicopters*

In Southern Norway, one rescue helicopter is responsible for the four ski resorts (Bortelid Ski, Brokke Alpinsenter, Fidjeland Skitrekk As and Knaben Ski). Since rescue missions need to be fast and efficient, a strategic, reasonable location must be found for the base of the rescue helicopter. Where should the rescue helicopter be positioned? Explain your approach comprehensively.

Task B: *Chlorine*

You had a summer job at Scandic Sørlandet. As part of your duty, you are required to prepare for the summer season with a daily cleaning program, using chlorine. Initially, on day one, a 15-litre starting dose of chlorine is poured into the swimming pool. After 24 hours, 15% of the chlorine content disappeared. An extra litre of chlorine is poured into the pool every morning for the rest of the season. The hotel manager wants to buy extra chlorine and will need your suggestion. As the one in charge of the pool, explain to the manager what will happen with the chlorine in the pool over time.

Hints: How much chlorine will be in the pool one day after adding the extra daily litre? How much chlorine will there be after two days? After three days? Formulate a recursive function describing the chlorine amount in the pool.

Task C: *The cost of having your own car*

The cost of owning a car depends on the number of kilometres travelled per month. A car, on average, uses 0.5 litres of gas per 10 kilometres. The cost of owning a car changes linearly with distance and the cost of gas.

1. Create a model for the total cost per distance travelled.
2. Eirik lives in Lund and works at Vågsbygd. He drives to work from Monday to Friday. Assuming Eirik goes to work throughout the month, make an estimate of the monthly cost of the distance travelled to work.
3. Sunniva wants to drive from Kristiansand to Trondheim and has a budget of NOK 600 for her car expenses. Find the maximum distance she can travel with this budget. Explain.
4. Find the y-intercept in the graph of cost versus distance. How do you interpret it?

Task D: *Population growth*

A company in Norway needs to expand its production, considering the population growth in the next ten years. Figures from Statistisk Sentralbyrå show that in 2011, the Norwegian population was 4.92 million, and in 2012, the number was 4.99 million. Now, using data from the last ten years, explain to the manager the linear growth of the Norwegian population in the next ten years.

Task E: *Waste management*

Returkraft AS is located outside Kristiansand and operates an energy recovery plant that was put into operation in 2010. The plant receives and recovers residual waste from households and companies in the entire region. Approximately 130,000 tonnes of residual and special waste are handled annually, and Returkraft receives approximately 2,708.33 tonnes of residual and special waste weekly. Returkraft currently has six garbage truck drivers, and in one day, each of them delivers an average of k tonnes of waste.

1. Create a model for the total waste received by Returkraft daily.
2. Estimate the total waste received by Returkraft in the first three months of 2021.
3. Returkraft has a total waste of approximately 43,300 tonnes. Find the number of days required to get this amount of waste. Explain

Task F: *Which holiday job?*

The holidays are approaching, and your best friend Kristin would like to make some money to purchase gifts. She found one job that would pay 20kr/hr above the minimum wage. Another job offers to pay half the minimum wage plus a commission of 20kr per item she sells. Kristin asks for your help as a mathematician. Which job would you advise her to take?

Task G: *Which apartment?*

Mariann got a new job at Kristiansand Kommune for five years. She must move from Stavanger to Kristiansand for this new job. She found two cozy apartments in different places. The first apartment is 400 meters from her workplace, and the rent is 10000 NOK with electricity included each month. The second apartment is 6.5 km from her workplace, and the rent is 8000 NOK, excluding monthly electricity. Mariann does not have a car, and she only relies on public transport. Mariann asked for your help as a mathematician because she needed to save money within this period. Which apartment would you advise her to take?

Task H: *Illegal fishing task*

On a foggy November morning, the Norwegian police patrol boat sets sail from the safe harbour to track illegal, unregistered, and unreported fishing. The conditions for this are terrible because the estimated visibility is only about 500 m. Nevertheless, the police commander orders the boat to head Northeast. The boat leaves the port at 7 a.m. At the same time, an unregistered fishing ship with a mast height of about 45 m sets sail toward the Southeast. As the Norwegian police patrol boat left the port, the unregistered fishing ship was located 7 km to the north of the port and 2 km to the east of the port. The Norwegian police patrol boat is one and a half times as fast as the unregistered fishing ship. Will the unregistered fishing ship be spotted? Explain your opinion.

Appendix C: Description of episodes of the transcriptions

In this appendix, I present the description of episodes of the transcriptions of the four schools. The episodes are categories the researcher created and have no intrinsic mathematical property (see Sub-Section 5.7.2). These categories were created for analysis in this study. I will present these categories in table form for each group under the subsections below.

C.1: Group A

Task	Episode	Description of Group A's activities
1	1	The students identified and classified the variables in the mathematical modelling task. They also constructed relations between the variables identified with paper-and-pencil.
	2	The students transferred their ideas from paper-and-pencil to GeoGebra. They created a function representing the number of people buying the car at a given price.
	3	The students tested their graph or function with 50 people to find the maximum company sales. They looked for the price of the car when 50 people bought it.
	4	The students tested their graph or function with 70 people to find the maximum price. They had 8000 to be the price of the car with this number of people. Moreover, 8000 is not enough compared to the 10,000 they had with 50 people, so they decided to try something below 70.
	5	One student thought of the fastest way to find the values but was unsure what to do. This thought triggered another student to suggest a procedure, but that did not materialize. The students then reverted to the previous way of finding the maximum price.
	6	The students tested their graph or function with 51 people to find the maximum price. They still had a value under 10000 (the price for 50 people). One student draws the attention of the others to the fact that they had not factored in the number of people buying the car at that price to get the total revenue. The students then used a calculator on the mobile phone to compute the product.
	7	The students computed the product of 70 people and a price of 8000 with the calculator. One student tried finding the fastest way

		to find the product with GeoGebra, while the others computed the product with a calculator.
	8	The students computed the total revenue if 40 people bought the car. A student suggested they could find the total revenue between 50 and 100 people based on the earlier values. Another student tried finding the function of the car's price but could not find the function of the product of the people buying the car and the price of the car.
	9	The students computed the total revenue if 60, 80 and 85 people bought the car, respectively. With the values obtained, one student suggested that the maximum revenue would be between 70 and 80 people.
	10	One student suggested using the spreadsheet to find all the values. Another dismissed the idea because she thought they were close to finding the maximum value. The students then continued calculating the product of 81 and 6900.
	11	The students computed the total revenue if 75 (the mid value between 70 and 80) people bought the car after realizing the revenue was the same if 70 or 80 people bought it.
	12	The students verified their answers after they found that the company gets the maximum revenue if 75 people buy the car at 7500. They also wrote a report concerning their results and delivered it to the teacher at the end of the activities.
2	1	After reading the mathematical modelling task, the students suggested the optimal location and located the three cities on Google Maps.
	2	One student suggested that they should look at the population of each city. Another student suggested an optimal location on the map, but the others did not agree.
	3	The students discussed that some cities have shopping centres, affecting their optimal location choice.
	4	The students searched for the population of the three cities with Google Search.
	5	The students discussed the issue of fairness regarding the population and the fact that some cities have shopping centres.
	6	The students turned on the satellite on Google Maps as they looked at the driving time from Lillesand and Vennesla to Kristiansand.
	7	The students zoomed in on the area around Kjevik on the map as they argued about choosing the optimal place.
	8	The students argued about placing the centre close to IKEA.
	9	The students discussed their final ideas about the placement of the shopping centre. After agreeing on a location, they wrote a report and delivered it to the teacher at the end of the activity.

Table 9.1: A summary of Group A’s activities in Task 1 & 2 divided into episodes.

C.2: Group B

Task	Episode	Description of Group B's activities
1	1	The students identified and classified the variables in the mathematical modelling task.
	2	The students put the information gathered into the spreadsheet in GeoGebra. That is, one column is for the number of people buying the car, and another column is for the price of the car. They went on to draw a linear function from the data generated.
	3	The students searched for the best-selling price by analyzing the linear graph they drew from the generated data. They looked for the price of the car when 100 people bought it and when nobody bought it.
	4	The students looked for the maximum company's sales revenue. They multiplied the number of people buying the car by the price of the car, doing that for both 100 people and one person.
	5	The students used the same procedure to find the revenue if 50 people bought the car.
	6	The students looked for the total revenue for 75 people. They chose 75 because it is the middle number between 100 and 50 people.
	7	The students verified their answers with the revenue for 70 and 80 people after finding that the maximum revenue is attained when 75 people buy the car. They also wrote a report concerning their results and delivered it to the teacher at the end of the activities.
2	1	The students looked for the population of the three cities and the positions of the three cities using Google Search and Google Maps, respectively. The students moved the positions of the three cities from Google Maps into GeoGebra. That is, by copying the coordinates of the cities' positions on Google Maps and plotting them in GeoGebra.
	2	The students made lines to connect the three points representing the three cities in GeoGebra.
	3	The students deleted the lines they made in Episode 2 and connected the points representing the three cities with line segments (forming a triangle). They searched for the midpoint of each side of the triangle and, using the midpoints, constructed the median of the triangle (centroid). The students fixed a midpoint between the cities, claiming the centroid is the centre of the triangle.
	4	The students analyzed the newfound centre of the three cities.
	5	The students duplicated the centre and moved it to the desired point, which is fair for all the cities.
	6	The students computed the median of the distances between the centre and the cities. They also analysed the centre while considering the roads and time of travel.

	7	The students inserted the coordinates of the centre (optimal location) from GeoGebra into Google Maps, saved the position of the centre on Google Maps, and analyzed it while considering the time of travel.
	8	The students debated the position of the centre on Google Maps concerning the time of travel. They also debated the issue of fairness in choosing the optimal position for the shopping centre. They finally agreed to write a report after the argument and deliver it to the teacher at the end of the activities.

Table 9.2: A summary of Group B's activities in Task 1 & 2 divided into episodes.

C.3: Group C

Task	Episode	Description of Group C's activities
1	1	The students identified and classified the variables in the mathematical modelling task. The students chose Excel/spreadsheet over GeoGebra and inserted their variables in Excel/spreadsheet.
	2	The students generated their data and found the maximum revenue for the car-selling company. They finally wrote a report concerning their results and delivered it to the teacher at the end of the activities.
2	1	The students located the three cities on Google Maps and discussed the issue of fairness in the task.
	2	The students looked for the travel time between the three cities to the suggested optimal place and between the cities and another optimal place that already has a shopping centre.
	3	The students looked for the population of the three cities as they discussed the optimal place.
	4	The students wrote a report after their analysis.
	5	The students constructed an optimal place using GeoGebra without factoring in the population and roads. Thus, they took a screenshot of the cities' positions on Google Maps and inserted it into GeoGebra. The students represented the three cities with points in GeoGebra, forming a triangle. The students then constructed the circumcenter/circumcircle of this triangle. The students finally updated their report and delivered it to the teacher at the end of the activities.

Table 9.3: A summary of Group C's activities in Task 1 & 2 divided into episodes.

C.4: Group D

Task	Episode	Description of Group D's activities
	1	The students identified and classified the variables in the mathematical modelling task.
	2	The students first used a calculator (on the computer) to compute the product of the number of people buying the car and its price.

1	3	The students used GeoGebra because they realized using the calculator would take some time. They used GeoGebra to plot some points (the number of people on the x-axis and the price of the car on the y-axis). The students computed the product of the number of people and the price of the car in the 'algebraic view' in GeoGebra, but no representation of this computation was shown in the 'graphic view'.
	4	The students decided to use Excel/spreadsheet after they were unsuccessful in an attempt to use GeoGebra. They used Excel/Spreadsheet to find the company's maximum revenue. Thus, the students first computed the product of the number of people and the price of the car (in only one column in the spreadsheet), using a few data sets.
	5	The students decided to create two columns, with one column representing the number of people and the other column representing the price of the car. The students then created another new column representing the total revenue (product of people and car price). The students generated their data for the number of people buying the car, the price they bought the car, and the company's total revenue. The students then decided on the best price for the company to maximize revenue.
	6	The students discussed their results and finally wrote a report, which they delivered to the teacher at the end of the activities.
2	1	The students located the three cities on Google Maps.
	2	The students took a screenshot of the cities on Google Maps and inserted it into GeoGebra. The students represented the three cities with points in GeoGebra forming a triangle.
	3	The students constructed a circumcenter/circumcircle of the triangle representing the three cities and measured the distance between the circumcenter and the triangle's vertices.
	4	The students analyzed the circumcenter of the triangle. They then looked for the circumcenter's position on Google Maps and searched for the Kommune (municipality) to which the optimal place belonged. The students finally agreed on the optimal location and wrote a report concerning their results, which they later delivered to the teacher at the end of the activities.

Table 9.4: A summary of Group D's activities in Task 1 & 2 divided into episodes.

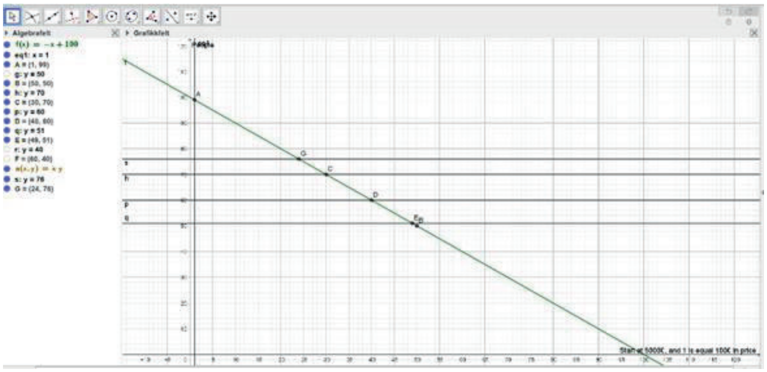
Appendix D: Solution reports of all groups in the different schools

In this appendix, I will present the solution report of each group within each school. Each school had other groups with the focus group (marked in red) that worked on Tasks 1 and 2. Empirical data (video recording, screen capture software, fieldnotes and students' worksheets) was collected from the activities of the focus group. However, all other groups (including the focus group) submitted their solution report after the activities. I will present the solution report of each group respectively under the subsections below.

D.1: School A

Group 1 (**Focus group: Group A**):

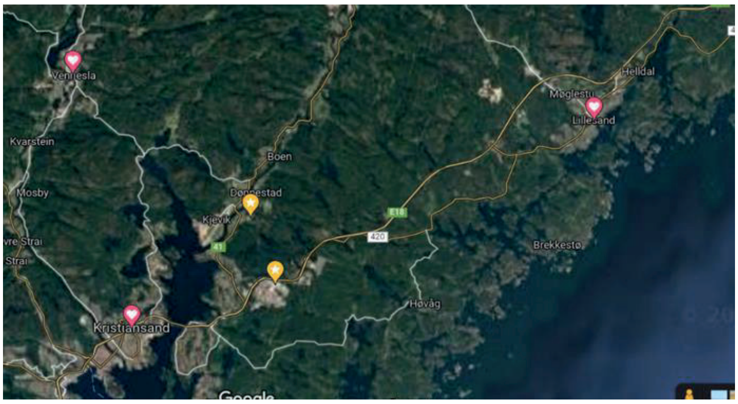
Task 1



The maximal revenue would be if the company sold each car for 7500£ for 75 people and the end income would be 562 500£.

We found out of this when we made a linear graph, f(x)=-x+100. On the x-axes was the 0, 5000£ and then when x is 1 it meant that we multiplied 1 by 100£ and then pluses 5000£ with 100£. And the y-axes were people.

Task 2



Our first location is on the mega center that already exist which is Sørlandssenteret. The center is located good in terms of population. It is easy to drive to the Sørlandssenteret from all the cities. It seems far away from Lillesand, but the highway (I mean E18) makes it much easier for drivers to come to our new mega center. If we don't have the opportunity to place it on a location where a mega center already exist, then we would have placed it near where Kjevik is. We located our mega center at the other side of Kjevik so that people will not get disturbed by the noises of the airplanes from the airport. There is also a camping place and a beach nearby. People from these camping places can visit the mega center easily which will help the tourism to grow faster in Agder. It is also easy to find where Kjevik is because of the airport which is popular.

Group 2

Task 1

100	5000	500000
99	5100	504900
98	5200	509800
97	5300	514700
96	5400	519600
95	5500	524500
94	5600	529400
93	5700	534300
92	5800	539200
91	5900	544100
90	6000	549000
89	6100	553900
88	6200	545600
87	6300	548100
86	6400	550400
85	6500	552500
84	6600	554400
83	6700	556100
82	6800	557600
81	6900	558900
80	7000	560000
79	7100	560900
78	7200	561600
77	7300	562100
76	7400	562400
75	7500	562500
74	7600	562400
73	7700	562100

Her ser vi at det stiger helt fra til bilen koster 7500kr som er den prisen de kan sette for å tjene mest mulig. Da er det 75 personer som fortsatt vil kjøpe bilen og da tjener ed 62500kr mer enn utgangspunktet.

Task 2

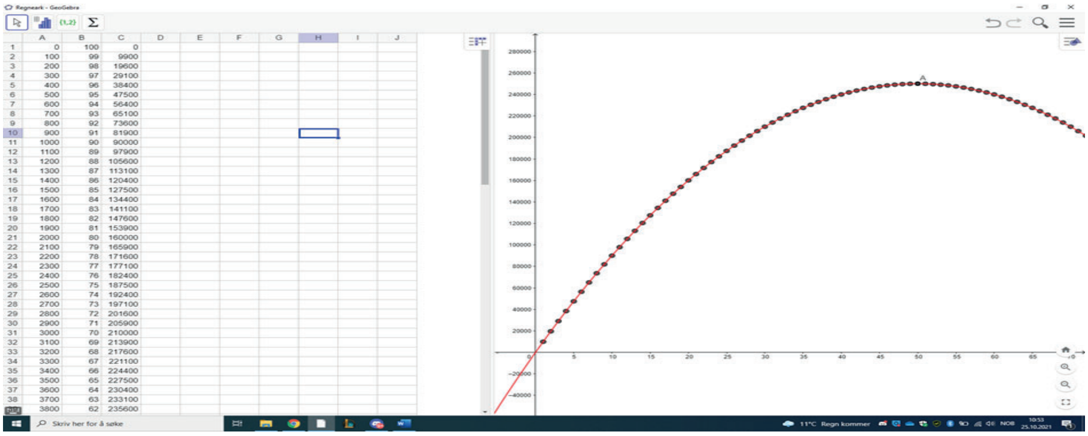
Bygg et kjøpesenter i Vennesla

Lillesand har et kjøpesenter har allerede et kjøpesenter og de er til og med færre enn oss med god rating og E18 vegen om de skal på sørlandssenteret. Det er veldig mange mennesker i Kristiansand, men de har allerede 3 kjøpesentre omtrent og god kollektiv transport. Så med tanke på sentrene som vi har fra før av så er det rettferdig om vi i Vennesla også får et som har en god befolkningsvekst.

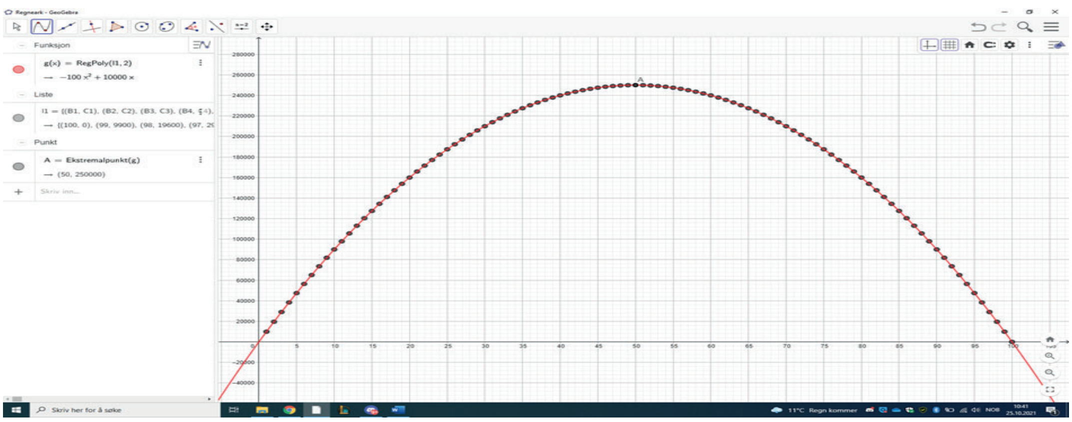
Hvis vi sier at det ikke finnes noen sentere fra før av tenker vi at der sørlandssenteret er plassert er veldig bra gjennomtenkt med tanke på at man kan kjøre E18 hele vegen fra Lillesand og Søgne. Vi kan kjøre Ålefjær selv om det kanskje er litt mer tungvint, men det er en fin veg fra alle plasser inn til sørlandssenteret med cirka like lang kjøretid, hvis en ikke tenker buss/kollektiv transport.

Group 3

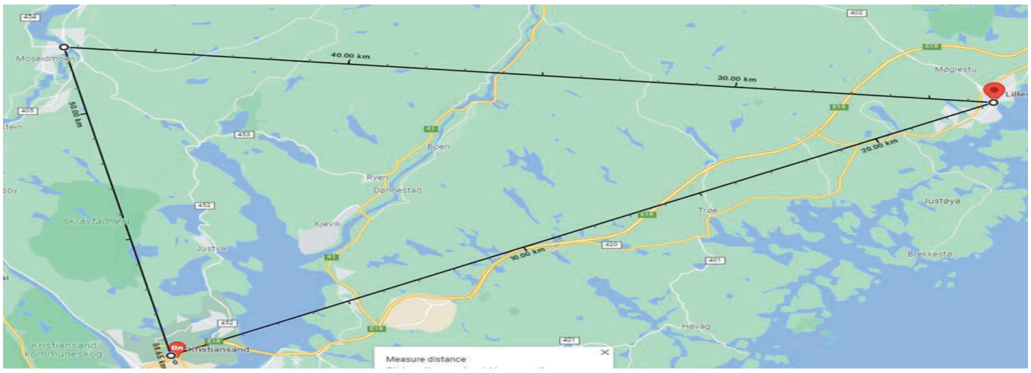
Task 1



We found this task similar to some of the other tasks we have worked with earlier. That's why we thought we could solve them the same way. We took it to the spreadsheet and wrote on the first column from 1 to 100, then on the second column 0 on the first row and then added 100 for each row. We made sure that the 0 was on the same row as 100 because for every hundred euros added to the car, there would be one less buyer. Then, on a third row, we multiplied the ones on the same row like 0*100, 100*99, 200*98 and so on. After doing this till row 100, we got the numbers we needed. In conclusion we found out that at 50 buyers and a price of 10000 euros, they would have a revenue sale at 250000 euros. This is the maximum total they can make by thinking there would be 1 less buyer for each 100 added.



Task 2



We measured the distance between the cities, the amount of people living in the cities and excluded sørlandsenteret. With this we came up with what we would call the most optimal location, which would be Boen. This would be because it is sort of in the middle of all the cities and we have located it closer to Kristiansand since there are more citizens there which would make the senter more profitable.

D.2: School B

Group 1

Task 1

	A	B	C
15	87	6300	548100
16	86	6400	550400
17	85	6500	552500
18	84	6600	554400
19	83	6700	556100
20	82	6800	557600
21	81	6900	558900
22	80	7000	560000
23	79	7100	560900
24	78	7200	561600
25	77	7300	562100
26	76	7400	562400
27	75	7500	562500
28	74	7600	562400
29	73	7700	562100
30	72	7800	561600
31	71	7900	560900
32	70	8000	560000
33	69	8100	558900
34	68	8200	557600
35	67	8300	556100
36	66	8400	554400
37	65	8500	552500

Kjære salgssjef

Den beste salgsprisen for bilen for å maksimere selskapets salgsinntekter er 562 500. (se blå markering)

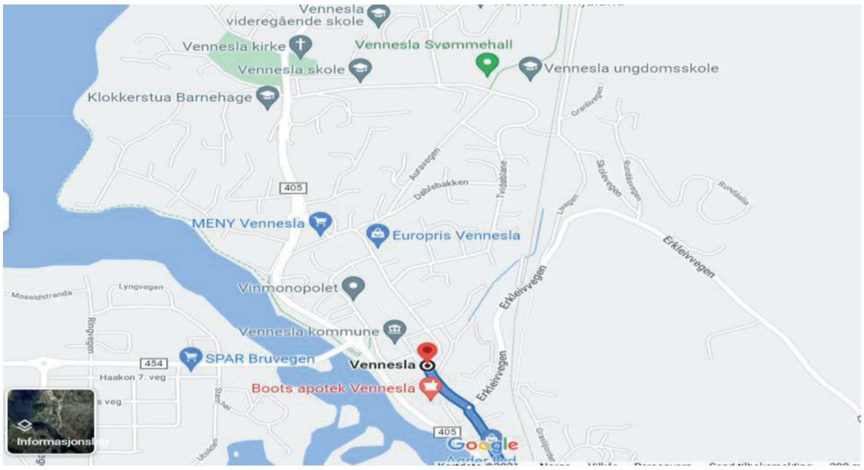
I et regneark, skrev vi ned fra 100 personer og under, nedover til person 1. I ruten ved siden i tabellen skrev vi ned 5000, og som da sank med 100 for hver person minus. Så ganga vi de to sammen og fant det største tallet.

Task 2

Kjære ordfører

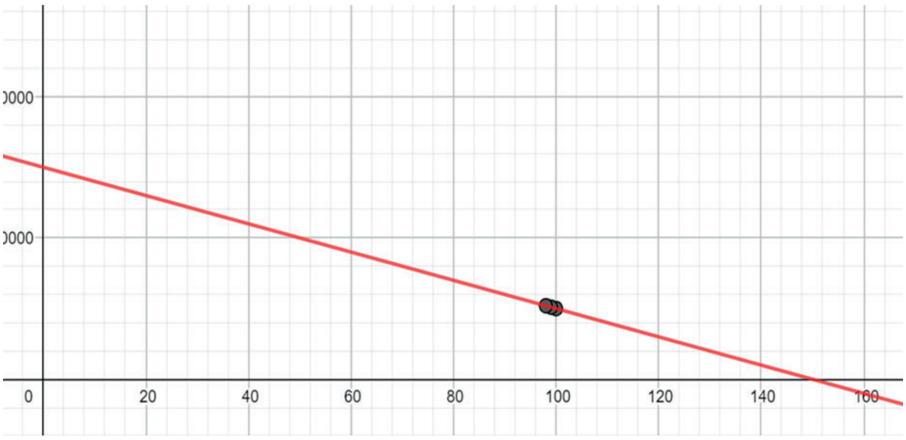
Det bor 85 000 i Kristiansand, 10 000 i Lillesand og 14 000 i Vennesla. Det er 19 min mellom Kristiansand og Vennesla, og 25 min mellom Kristiansand og Lillesand. Dette er bare 5 minutter forskjell, så det tar vi ikke hensyn til.

Lillesand er en ganske koselig, liten by, mens Vennesla er stygt og kjedelig. Derfor tenker vi at det nye kjøpesenteret kan bygges rundt området ved Vennesla. Da kan de ha noe spennende og moderne der.



Group 2

Task 1



By decreasing the price a bit, you increase how many people by the car thus increasing your income.

Task 2

Population of the three cities

Kristiansand 85000

Vennesla 12,816

Lillesand 10,106

Distance

Vennesla to Kristiansand-16.8km

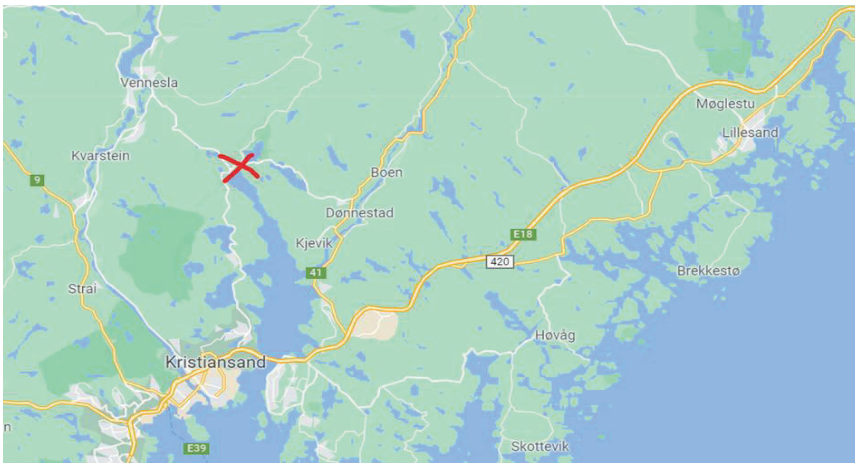
Lillesand to Kristiansand- 27.5km

Time to get from x to y

Vennesla to Kristiansand- 19 min

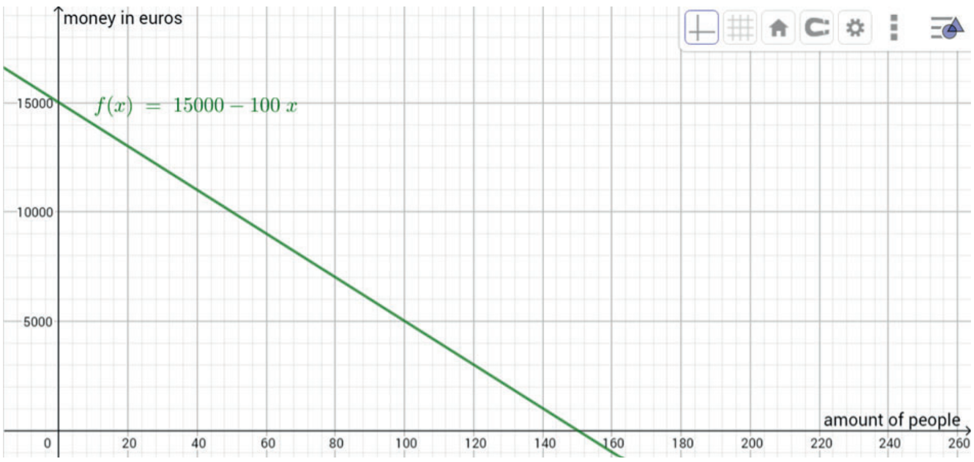
Lillesand to Kristiansand- 24 min

After looking at the data and the map we decided to put it on the X



Group 3

Task 1



The starting point on the y-axis is the highest amount of money that by the simple statistic in the task says will make no one buy it. From that you can subtract 100 euros for every person (every x) which will therefore show you how the increase of money will decrease the amount of people buying the car.

The optimal cost for the car is xxxxxx euros.

The reason of this is because xxx people*xxxx euros=xxxxx euros which is the highest amount of money they can get for selling that model.

Group 4

Task 1

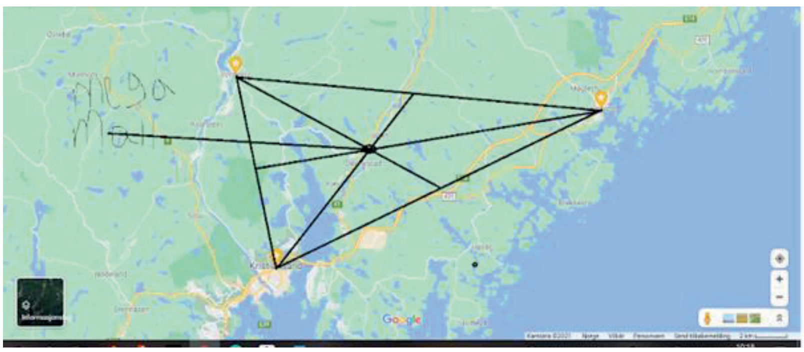
To solve the problem and show what the optimal price of the car was, we opened a document in Excel. We first put in the initial numbers, a price of 5000 and amount of buyers being 100. Then we set up a function to calculate how they interacted with each other. Doing this, we found that with a price of 10000, there would be 50 buyers. When we had this, we put in the calculation to find total revenue, that being price*buyers. With this, we found that the total revenue stopped increasing at 75 buyers before decreasing again,

8100	69	558900
8200	68	557600
8300	67	556100
8400	66	554400
8500	65	552500
8600	64	550400
8700	63	548100
8800	62	545600
8900	61	542900
9000	60	540000
9100	59	536900
9200	58	533600
9300	57	530100
9400	56	526400
9500	55	522500
9600	54	518400
9700	53	514100
9800	52	509600
9900	51	504900
10000	50	500000

6800	82	557600
6900	81	558900
7000	80	560000
7100	79	560900
7200	78	561600
7300	77	562100
7400	76	562400
7500	75	562500
7600	74	562400
7700	73	562100
7800	72	561600
7900	71	560900
8000	70	560000
8100	69	558900
8200	68	557600
8300	67	556100
8400	66	554400
8500	65	552500
8600	64	550400
8700	63	548100
8800	62	545600

As shown from Excel in these pictures, Price being on the left, amount of buyers predicted in the middle and total revenue on the right, the price stops increasing at 75 buyers, earning a total revenue of 562500 Euro, and then starts decreasing. With this, we can determine that the optimal price for the new car is 7500 Euro.

Task 2



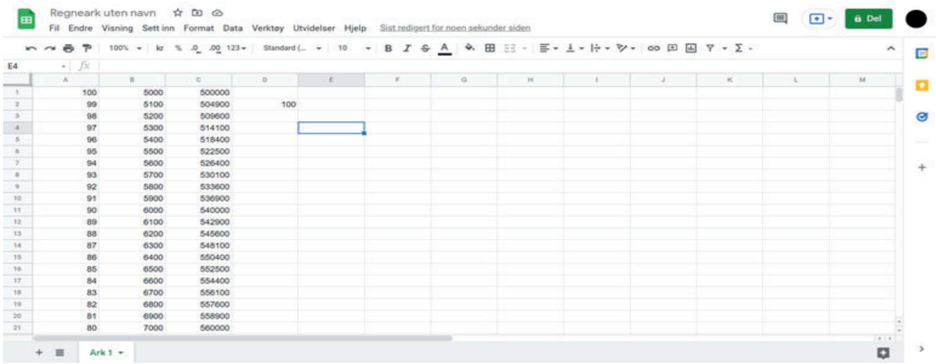
The first step we took was to locate the given locations on google maps. We then created a line connecting the three points together, which formed an isosceles triangle. We then find the middle of the triangle by drawing lines from each corner (or vertex) of a triangle to the midpoint of the opposite sides, then those three lines meet at a center, or centroid, of the triangle. The centroid is the triangle's center of gravity, where the triangle balances evenly.
When we found the midpoint we realised that the mega-mall is going to be on the point where all the lines cross each other.

Group 5

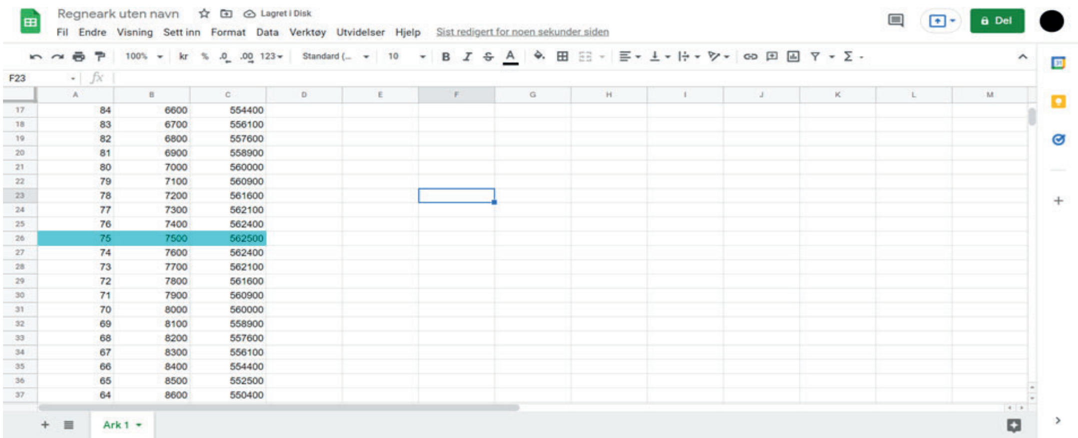
Task 1

Kjære selskapets salgssjef

Vi satte opp alt i et regneark. Vi startet med å dra ned alle tallene fra 100 - 0. Etter dette tok vi 5000 i celle B2 og plusset det med den låste cellen D3 i celle B3. Etter dette dro vi ned tallene slik at det ble likt som 100 - 0. Tilslutt multipliserte vi celle A1 - B1, A2 - B2, osv. Deretter fant vi det høyeste tallet som var i celle 75.



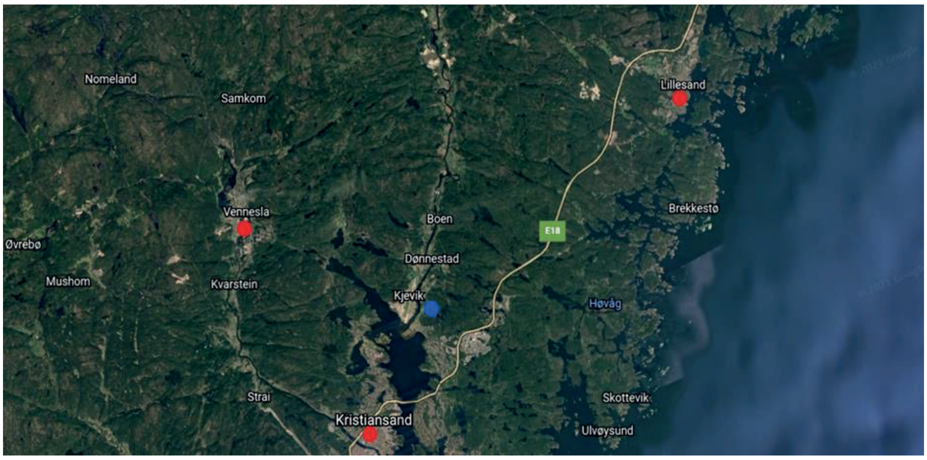
	A	B	C	D
1	100	5000	500000	
2	99	5100	504900	100
3	98	5200	509600	
4	97	5300	514100	
5	96	5400	518400	
6	95	5500	522500	
7	94	5600	526400	
8	93	5700	530100	
9	92	5800	533600	
10	91	5900	536900	
11	90	6000	540000	
12	89	6100	542900	
13	88	6200	545600	
14	87	6300	548100	
15	86	6400	550400	
16	85	6500	552500	
17	84	6600	554400	
18	83	6700	556100	
19	82	6800	557600	
20	81	6900	558900	
21	80	7000	560000	



	A	B	C
17	84	6600	554400
18	83	6700	556100
19	82	6800	557600
20	81	6900	558900
21	80	7000	560000
22	79	7100	560900
23	78	7200	561600
24	77	7300	562100
25	76	7400	562400
26	75	7500	562500
27	74	7600	562400
28	73	7700	562100
29	72	7800	561600
30	71	7900	560900
31	70	8000	560000
32	69	8100	558900
33	68	8200	557600
34	67	8300	556100
35	66	8400	554400
36	65	8500	552500
37	64	8600	550400

Svaret vårt er altså at den beste salgsprisen for bilen, for å oppnå maksimale salgsinntekter er: 7500 Euro

Task 2



Kjære departementet for etablering av senter for Kristiansand, Lillesand og Vennesla.

Våre medborgere og medmennesker, vi vil gjerne komme med en avgjørelse for vårt forslag av prosjektet angående et sentralt senter.

Etter god undersøkelse for hvor dette skal plasseres i forhold til rettferdighet, og hva som kan være mest aktuelt for alle tre plasser, har vi kommet frem til at den beste plassen å ha senteret ville ha vært på Sørøst siden av Kjevik, I enden av hamrevatnet

Grunnen for dette er, at dette er utrolig sentralt for alle disse byene, og ville derfor vært rettferdig med tanke på tidsbruk på veiene bort. Dette er også i nærheten av flyplassen Kjevik, og vil derfor være en god overgang for folk som kommer for å ta fly, til å kunne ha tilgang til et nærliggende senter der de kan kjøpe for sine behov. I nærheten er også Hamresanden, som er en stor og populær strand i Kristiansand. Derfra kan man sette grunnlag for en potensiell by, om dette hadde vært aktuelt, noe som vi absolutt synes.

Vennlig hilsen Gruppe 5

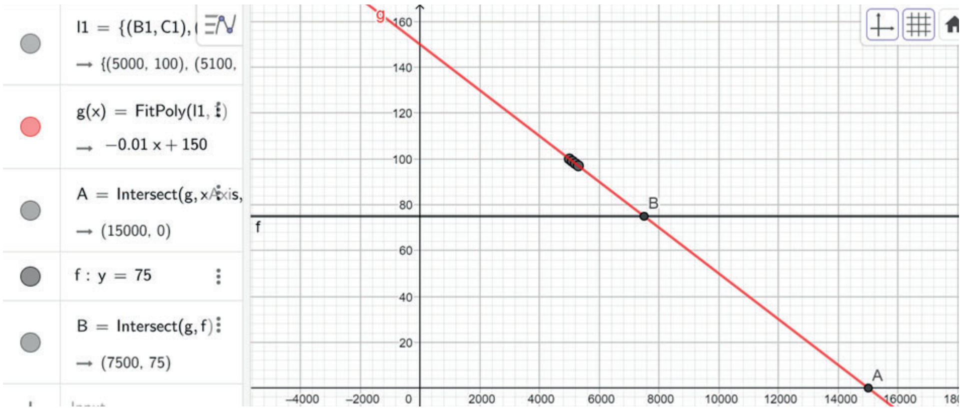
Group 6 (**Focus group: Group B**)

Task 1

Dear sales manager of solar cars,

We have looked through the data provided from your research, and we have found that selling the solar power car for 5000 euros is not maximizing the sales revenue for your company. The optimal selling price per car is 7500 euros. This will lead to the company’s sales revenue increasing by 62,500 euros if you manage to sell 75 cars. The amount of people buying is still sustainable in this plan.

Kind regards, your advisors

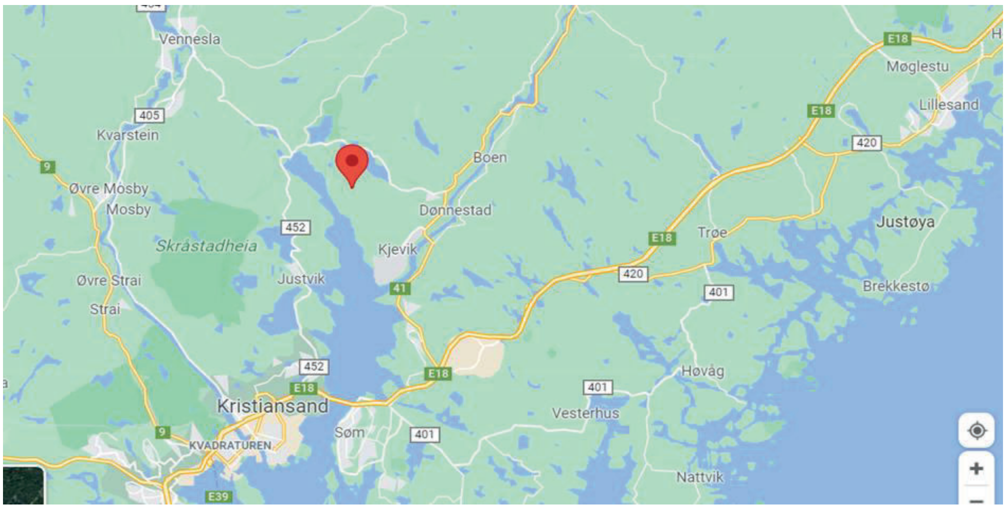


Task 2

Dear minister,

Dønnestad is the best place for the mall. This would be the mathimatical central area for the three cities Kristiansand, Lillesand and Vennesla. This is an easy access mall were everyone can drive to. There are roads that people from every city can drive on to go to the mall, Kristiansand and Lillesand uses E18 and Vennesla uses 453. This might not be the most optimal location, however with more resources and time we can find a fair place for all the cities.

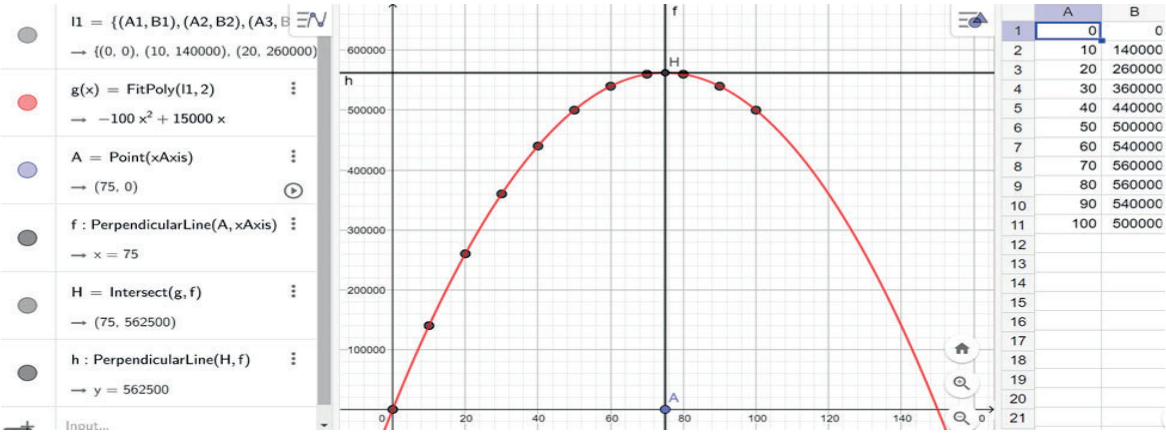
In kind regards, your advisors



Group 7

Task 1

Having 100 customers would give us 5000€ in revenue. Later we found out that increasing the price range at 100 decreases our audience by one customer. To find the formula we had to put different values into a spreadsheet and then we made a scatterplot out of it. Our conclusion is that the ideal selling price would be to have the vehicle cost 7,500€ for 75 customers.

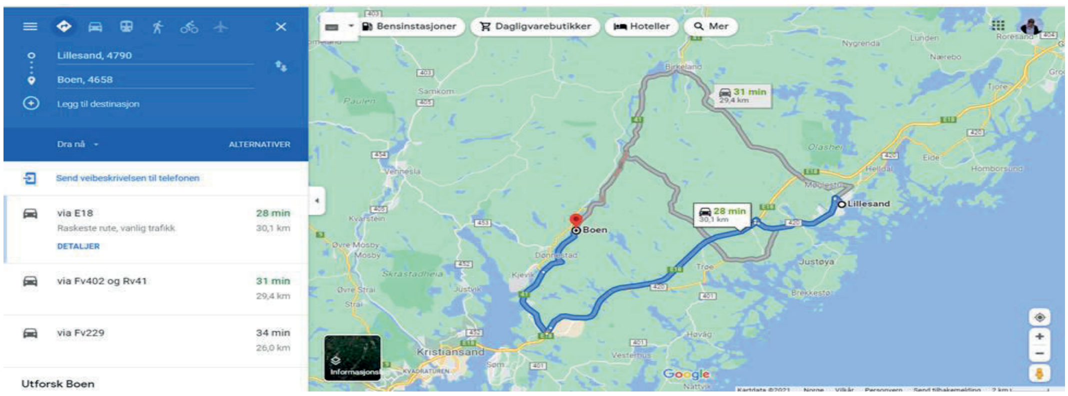


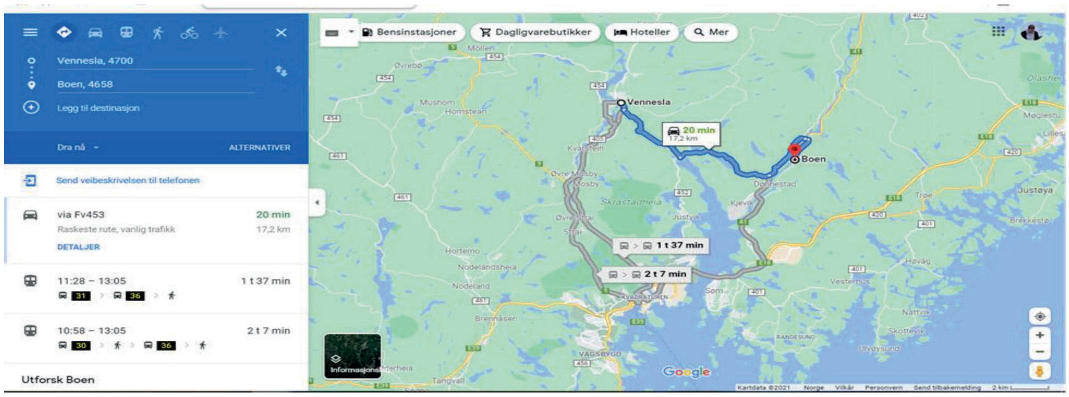
Formula: $-100x^2+15000x$

Task 2

close to tveit, Kristiansand- reason is stede er ca like avstand til det nye senteret. And Vennesla and Lillesand people are already used to coming to Kristiansand a lot already. problem- The place is a little selfish because it would be easier to get to the mall if you live in Kristiansand and a little harder if you live in Lillesand and Vennesla-

Boen-reason because from Vennesla til Boen is veien omtrent 20 min. Og fra Lristiansand til Boen is 18 min, og til Lillesand til Boen er 28 min. The problem with this is how Lillesand citizens will have to come all the way to Kristiansand and go off to boen from there.





D.3: School C

Group 1

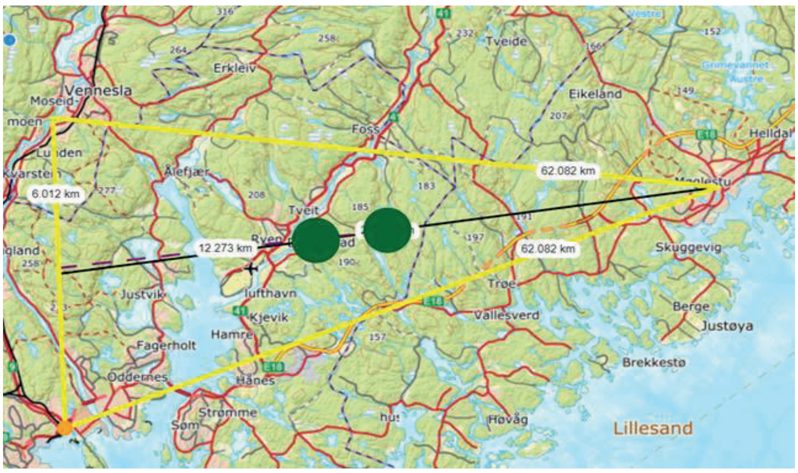
Task 1

pris i euro	personer	inntak i euro
5000	100	500000
5100	99	504900
5200	98	509600
5300	97	514100
5400	96	518400
5500	95	522500
5600	94	526400
5700	93	530100
5800	92	533600
5900	91	536900
6000	90	540000
6100	89	542900
6200	88	545600
6300	87	548100
6400	86	550400
6500	85	552500
6600	84	554400
6700	83	556100
6800	82	557600
6900	81	558900
7000	80	560000
7100	79	560900
7200	78	561600
7300	77	562100
7400	76	562400
7500	75	562500
7600	74	562400
7700	73	562100
7800	72	561600
7900	71	560900

Dear sales-manager. We have figured out that the best selling price is 7 500 euros. This means that the company revenue is maximized at 562 500 euros. After the price becomes higher than 7 500, people loose interest and the income will decrease. Sincerely Group 1.

Task 2

To the ministry in charge of the new shopping center. We believe that the best location for the center will be at Dønnestad. This is because it is located almost in the middle of the cities, however the exact middle would be in nowhere so we moved it out closer to the road, therefore Dønnestad. In addition, Dønnestad is closest to Kristiansand with the most inhabitants, second closest to Vennesla with the second largest population and a bit further away from Lillesand because they had the fewest inhabitants. Sincerely Group 1.



Group 2

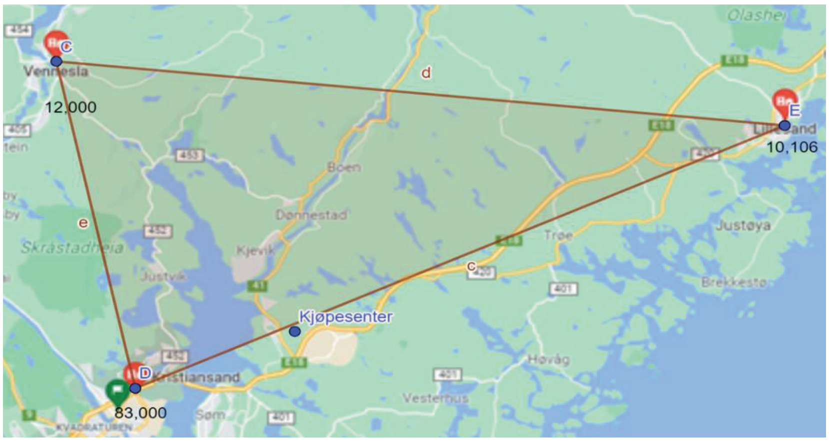
Task 1

	A	B	C	D	E	F	G	H	I	J	K	L
16		87	6300	548100								
17		86	6400	550400								
18		85	6500	552500								
19		84	6600	554400								
20		83	6700	556100								
21		82	6800	557600								
22		81	6900	558900								
23		80	7000	560000								
24		79	7100	560900								
25		78	7200	561600								
26		77	7300	562100								
27		76	7400	562400								
28		75	7500	562500								
29		74	7600	562400								
30		73	7700	562100								

We conclude that the company's maximal revenue would be 562500 euros, with a unit price of 7500 euros per unit and 75 buyers.
We came to this answer by using excel and making a table where we looked for the highest total price when summing up how much money the company would make for all the products sold.

Task 2

Firstly, we got a map of the region containing these cities.



Then we found the approximate number of people living in each area and used those numbers to determine where we would place the supermarket. We placed it along the motorway because we believe it would be the fastest way of travel between all cities, but also because there are other supermarkets close to where we put ours, and we believe that customers coming from those places would also take a look at our supermarket.

Group 3

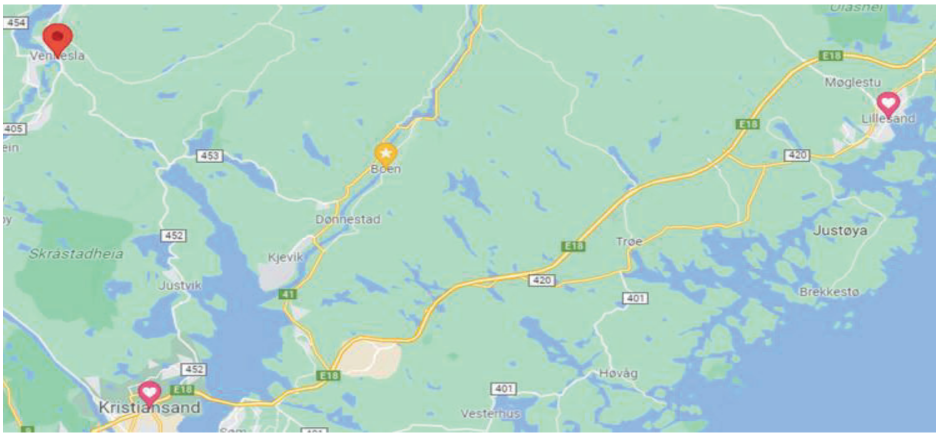
Task 1

price increase	100		100	2	200	5000	509600
Person decrease	1		100	3	300	5000	514100
			100	4	400	5000	518400
			100	5	500	5000	522500
			100	6	600	5000	526400
			100	7	700	5000	530100
			100	8	800	5000	533600
			100	9	900	5000	536900
			100	10	1000	5000	540000
			100	11	1100	5000	542900
			100	12	1200	5000	545600
			100	13	1300	5000	548100
			100	14	1400	5000	550400
			100	15	1500	5000	552500
			100	16	1600	5000	554400
			100	17	1700	5000	556100
			100	18	1800	5000	557600
			100	19	1900	5000	558900
			100	20	2000	5000	560000
			100	21	2100	5000	560900
			100	22	2200	5000	561600
			100	23	2300	5000	562100
			100	24	2400	5000	562400
			100	25	2500	5000	562500
			100	26	2600	5000	562400
			100	27	2700	5000	562100
			100	28	2800	5000	561600
			100	29	2900	5000	560900
			100	30	3000	5000	560000

The product has the highest profit when the price is increased by 2500euro, and the customer amount is 75people and decreased by 25customers. The maximized sales revenue is 62500eur.

Orginal price	5000		customers	customer decrease	price increase	original price	new price
Original amnount of customers	100		=B$2*1	1	=B3	5000	=(G2+F2)*(D2-E2)
price increase	100		=B$2*1	2	200	5000	=(G3+F3)*(D3-E3)
Person decrease	1		=B$2*1	3	300	5000	=(G4+F4)*(D4-E4)
			=B$2*1	4	400	5000	=(G5+F5)*(D5-E5)
			=B$2*1	5	500	5000	=(G6+F6)*(D6-E6)
			=B$2*1	6	600	5000	=(G7+F7)*(D7-E7)
			=B$2*1	7	700	5000	=(G8+F8)*(D8-E8)
			=B$2*1	8	800	5000	=(G9+F9)*(D9-E9)
			=B$2*1	9	900	5000	=(G10+F10)*(D10-E10)
			=B$2*1	10	1000	5000	=(G11+F11)*(D11-E11)
			=B$2*1	11	1100	5000	=(G12+F12)*(D12-E12)
			=B$2*1	12	1200	5000	=(G13+F13)*(D13-E13)
			=B$2*1	13	1300	5000	=(G14+F14)*(D14-E14)
			=B$2*1	14	1400	5000	=(G15+F15)*(D15-E15)
			=B$2*1	15	1500	5000	=(G16+F16)*(D16-E16)
			=B$2*1	16	1600	5000	=(G17+F17)*(D17-E17)
			=B$2*1	17	1700	5000	=(G18+F18)*(D18-E18)
			=B$2*1	18	1800	5000	=(G19+F19)*(D19-E19)
			=B$2*1	19	1900	5000	=(G20+F20)*(D20-E20)
			=B$2*1	20	2000	5000	=(G21+F21)*(D21-E21)
			=B$2*1	21	2100	5000	=(G22+F22)*(D22-E22)
			=B$2*1	22	2200	5000	=(G23+F23)*(D23-E23)
			=B$2*1	23	2300	5000	=(G24+F24)*(D24-E24)
			=B$2*1	24	2400	5000	=(G25+F25)*(D25-E25)
			=B$2*1	25	2500	5000	=(G26+F26)*(D26-E26)
			=B$2*1	26	2600	5000	=(G27+F27)*(D27-E27)
			=B$2*1	27	2700	5000	=(G28+F28)*(D28-E28)
			=B$2*1	28	2800	5000	=(G29+F29)*(D29-E29)
			=B$2*1	29	2900	5000	=(G30+F30)*(D30-E30)
			=B$2*1	30	3000	5000	=(G31+F31)*(D31-E31)

Task 2



The shopping center should be built in Boen. Since Boen is the closest place to the center of the three places. (Vennesla, Lillesand and Kristiansand)

Group 4

Task 1

We found out that if 75 people buy the car, the company would make the most income by earning 7500 euros each car, which in total equals 562500 euros.

folk	euro	inntak i euro
100	5000	500000
99	5100	504900
98	5200	509600
97	5300	514100
96	5400	518400
95	5500	522500
94	5600	526400
93	5700	530100
92	5800	533600
91	5900	536900
90	6000	540000
89	6100	542900
88	6200	545600
87	6300	548100
86	6400	550400
85	6500	552500
84	6600	554400
83	6700	556100
82	6800	557600
81	6900	558900
80	7000	560000
79	7100	560900
78	7200	561600
77	7300	562100
76	7400	562400
75	7500	562500
74	7600	562400
73	7700	562100

75 can buy the car so that the company could make the mist income. The income would then be at 562 500 euros.

Task 2

we looked a google maps and found out where it would be most smart to place the mall compared with distance, location and roads.

Group 5 (**Focus group: Group C**)

Task 1

Letter:

The total revenue would be at it's highest at 75 customers at €7500 per car, beyond that increasing the price wouldn't affect the total revenue in a positive way which is shown in this graph.

	A	B	C
1	price	customers	revenue
2	5000	100	500000
3	5100	99	504900
4	5200	98	509600
5	5300	97	514100
6	5400	96	518400
7	5500	95	522500
8	5600	94	526400
9	5700	93	530100
10	5800	92	533600
11	5900	91	536900
12	6000	90	540000
13	6100	89	542900
14	6200	88	545600
15	6300	87	548100
16	6400	86	550400
17	6500	85	552500
18	6600	84	554400
19	6700	83	556100
20	6800	82	557600
21	6900	81	558900
22	7000	80	560000
23	7100	79	560900
24	7200	78	561600
25	7300	77	562100
26	7400	76	562400
27	7500	75	562500
28	7600	74	562400

	A	B	C
1	price	customers	revenue
2	5000	100	=A2*B2
3	=A2+100	=B2-1	=A3*B3
4	=A3+100	=B3-1	=A4*B4
5	=A4+100	=B4-1	=A5*B5

We used a recursive function to figure out the optimal price for the car.

Task 2

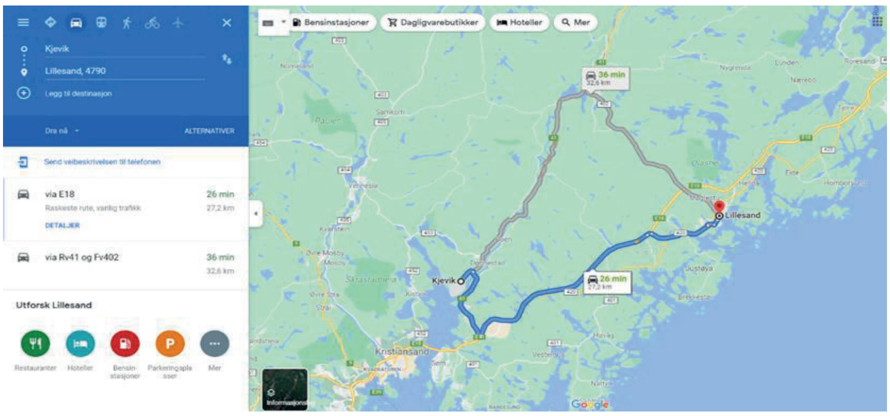
If we put the mega shopping center in Kjevik it would be fairly placed based on the population and travel time of the three cities. Kristiansand has the least amount of travel time because Kristiansand has a larger population which constitutes that having the shopping center closer to Kristiansand would result in less CO2 emissions. Vennesla also has a larger population than Lillesand which is why we placed the shopping center closer to Vennesla than Lillesand.

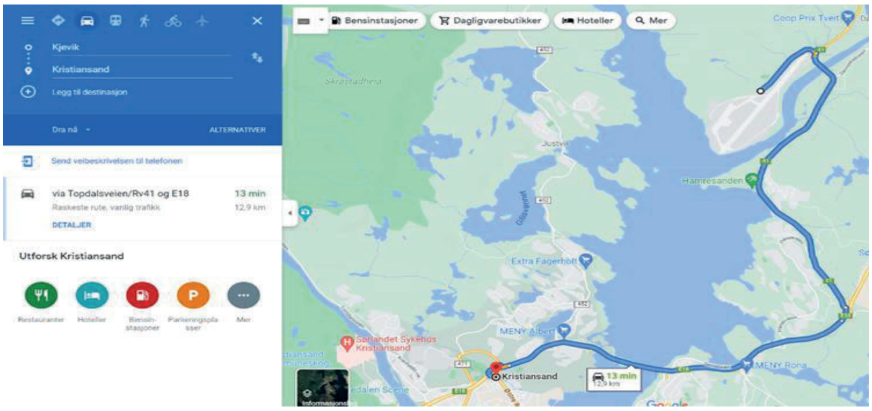
Population numbers:

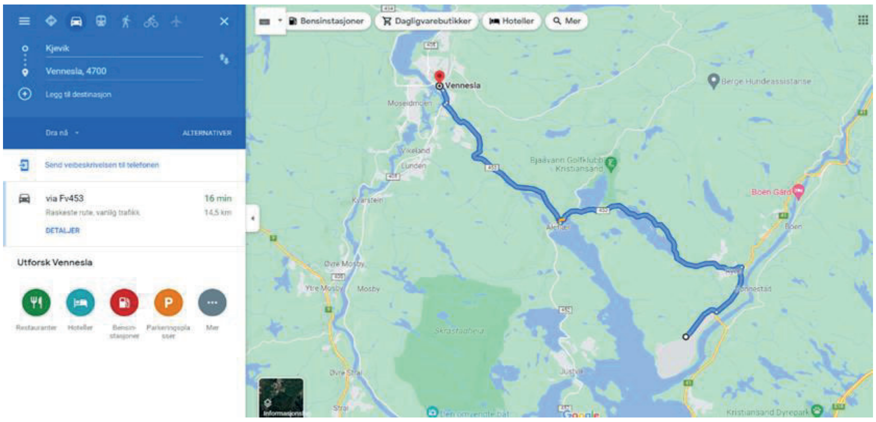
Kristiansand: 85 983 (2014)

Vennesla: 13 986 (2014)

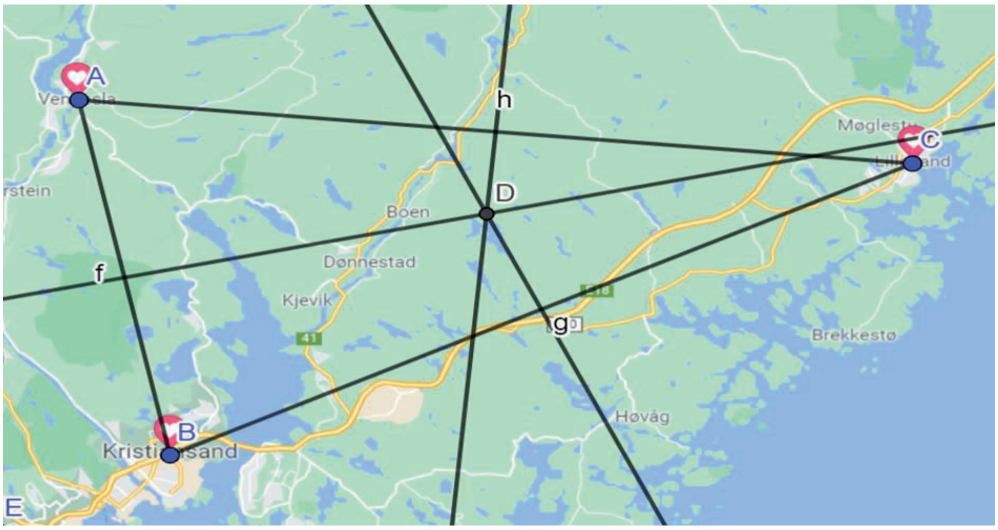
Lillesand: 10 106 (2014)







If we do not factor in population or pre-existing roads, then point D would be the ideal location for the shopping center.



D.4: School D

Group 1

Task 1

7500

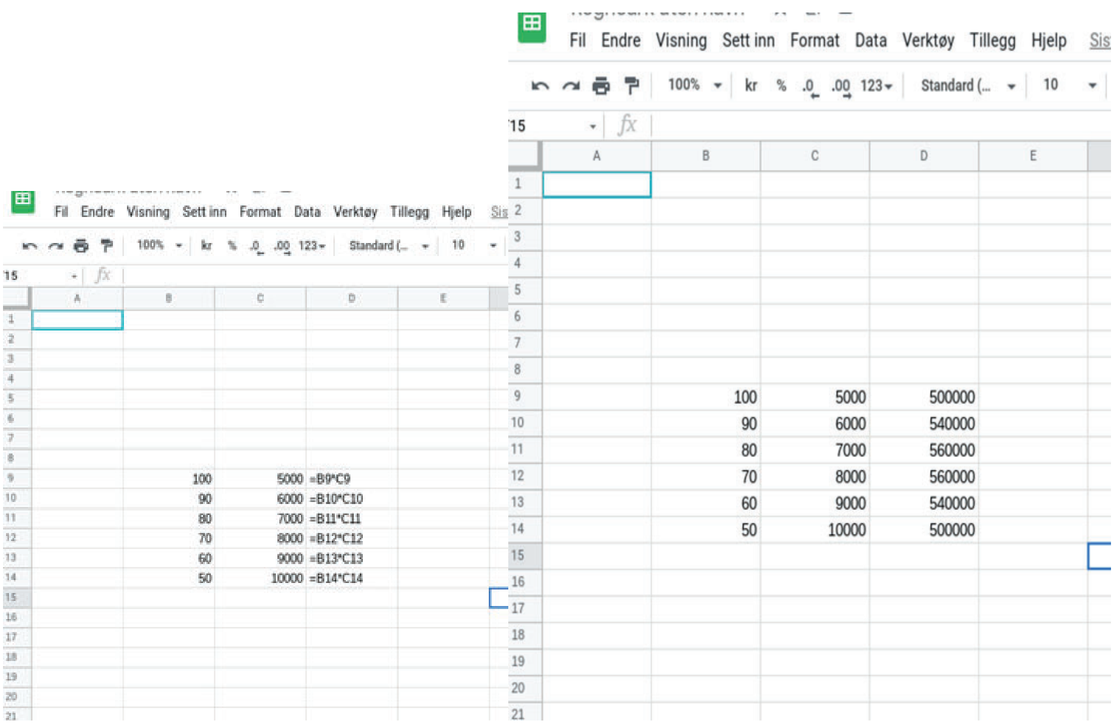
We lose 1 person every time we add an extra 100 euro, but we also gain more money from the people that still will buy. When you add 1000 euro you lose 10 people. So we took 15.000 euro to check how many customers we would have and we wouldn't have anyone left. Then we figured that if you lose all your customers at 15.000 euro you are gonna have the most customers at 7500 euro. You will also get the most money at 7500 euro.

Task 2

The shopping center should be placed where the lines are crossing, we used geogebra and google maps. We marked the different places then we found the middle point in geogebra.

Group 2

Task 1

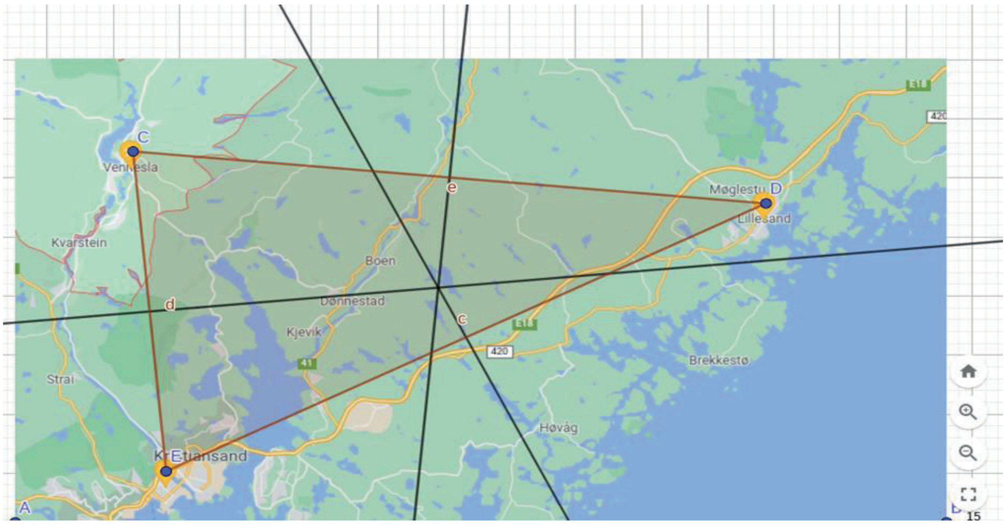


	A	B	C	D	E
9		100	5000	=B9*C9	
10		90	6000	=B10*C10	
11		80	7000	=B11*C11	
12		70	8000	=B12*C12	
13		60	9000	=B13*C13	
14		50	10000	=B14*C14	

	A	B	C	D	E
9		100	5000	500000	
10		90	6000	540000	
11		80	7000	560000	
12		70	8000	560000	
13		60	9000	540000	
14		50	10000	500000	

Du bør selge bilen for mellom 7000-8000 for det er da du tjener mest. Det kan du se på disse bildene.

Task 2



Kjøpesenteret bør ligge nærme det sorte krysset, men også nærme veien så det blir letter å komme seg dit. Det svarte punktet er i midten (i midten av alle punktene)

Group 3

Task 1

100 personer kjøper for 5000 = 500 000.

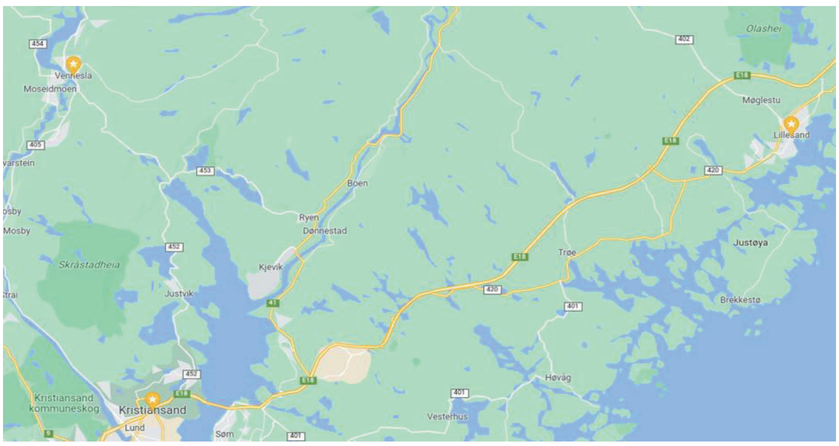
75 personer kjøper for 7500= 562 500.

50 personer kjøper for 10 000 = 500 000.

25 personer kjøper for 12 500 = 312 500.

75 personer kjøper for 7500= 562 500 svar

Task 2



Dette var så langt vi kom på denne oppgaven.

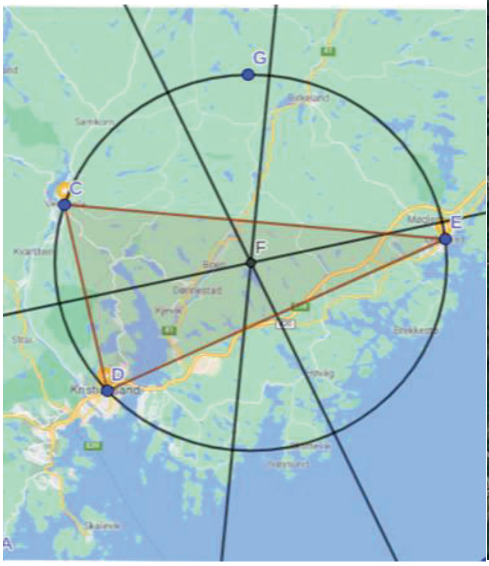
Group 4

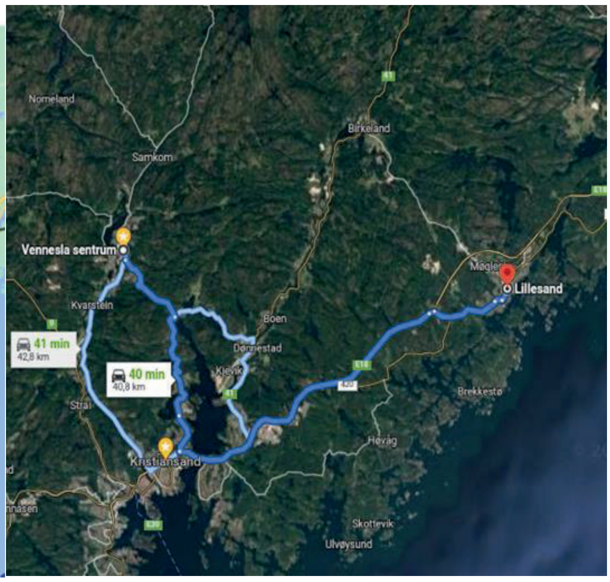
Task 1

89	6100	542900
88	6200	545600
87	6300	548100
86	6400	550400
85	6500	552500
84	6600	554400
83	6700	556100
82	6800	557600
81	6900	558900
80	7000	560000
79	7100	560900
78	7200	561600
77	7300	562100
76	7400	562400
75	7500	562500
74	7600	562400
73	7700	562100
72	7800	561600
71	7900	560900

Task 2

Vi fant ut at den beste plassen å plassere kjøpesenteret var i dønnestad, vi kom fram til dette ved at vi tok å fant midtpunktet mellom alle stedene i geogebra, men så var det en urealistisk plass å ha kjøpesenteret siden det var midt i skogen. Så derfor gikk vi inn på google maps og tok opp kjøreveiene og så hvordan man kunne kjøre og da fant vi ut at de som kom fra vennesla kunne kjøre en annen vei. Så vi plasserte det litt nærmere vennesla og litt lenger borte fra lillesand siden de er nærmere sørlandssenteret.





Group 5 (**Focus group: Group D**)

Task 1

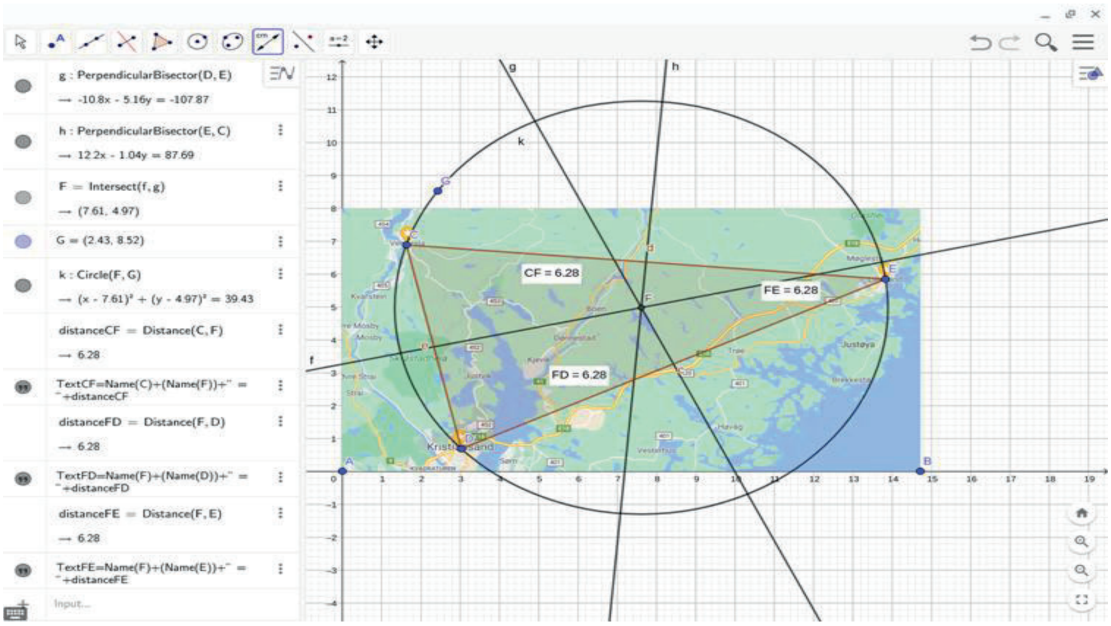
Hei. Vi fant ut at den optimale prisen på bilen er 7500 euro. Da er det 75 folk som vil kjøpe bilen din. Da får du i inntekt 562500 euro. Det er den høyeste prisen du kan tjene ved å øke prisen.

	A	B	C	D	E	F
8			94	5600	=C8*D8	
9			93	5700	=C9*D9	
10			92	5800	=C10*D10	
11			91	5900	=C11*D11	
12			90	6000	=C12*D12	
13			89	6100	=C13*D13	
14			88	6200	=C14*D14	
15			87	6300	=C15*D15	
16			86	6400	=C16*D16	
17			85	6500	=C17*D17	
18			84	6600	=C18*D18	
19			83	6700	=C19*D19	
20			82	6800	=C20*D20	
21			81	6900	=C21*D21	
22			80	7000	=C22*D22	
23			79	7100	=C23*D23	
24			78	7200	=C24*D24	
25			77	7300	=C25*D25	
26			76	7400	=C26*D26	
27			75	7500	=C27*D27	
28			74	7600	=C28*D28	
29			73	7700	=C29*D29	
30			72	7800	=C30*D30	
31			71	7900	=C31*D31	
32			70	8000	=C32*D32	
33			69	8100	=C33*D33	
34			68	8200	=C34*D34	
35			67	8300	=C35*D35	
36			66	8400	=C36*D36	
37			65	8500	=C37*D37	

	A	B	C	D	E	F
8			94	5600	526400	
9			93	5700	530100	
10			92	5800	533600	
11			91	5900	536900	
12			90	6000	540000	
13			89	6100	542900	
14			88	6200	545600	
15			87	6300	548100	
16			86	6400	550400	
17			85	6500	552500	
18			84	6600	554400	
19			83	6700	556100	
20			82	6800	557600	
21			81	6900	558900	
22			80	7000	560000	
23			79	7100	560900	
24			78	7200	561600	
25			77	7300	562100	
26			76	7400	562400	
27			75	7500	562500	
28			74	7600	562400	
29			73	7700	562100	
30			72	7800	561600	
31			71	7900	560900	
32			70	8000	560000	
33			69	8100	558900	
34			68	8200	557600	
35			67	8300	556100	
36			66	8400	554400	
37			65	8500	552500	

Task 2

Hei. Vi mener at det er mest rettferdig hvis kjøpesenteret ligger der vi har punktet f det på bildet. Da er det like langt fra kristiansand, lillesand og vennesla. Vi kom fram til dette svaret med å først finne hvor kommunene er. Så plott vi punkter på hver av stedene. Etter det fant vi ut av hvor det er like langt fra hver at de til midten. Stedet heter Dragsholtvatnet.



Group 6

Task 1

5000 euro = 100 people buying car

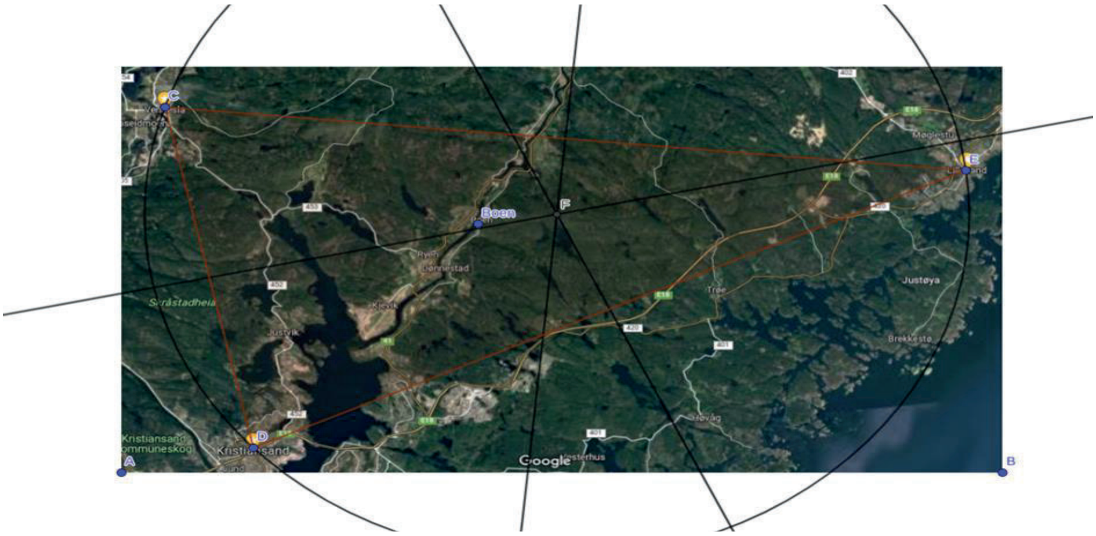
Then the company will get paid 5000 * 100 = 500000 euro

5100 euro = 99 people buying car

5100 * 99 = 504900 euro

Task 2

We think it should be placed in Boen, because the center of Vennesla, Lillesand and Kristiansand is a little east of Boen. And on the west side Vennesla and Kristiansand is placed so if it's placed a little more to west it's placed closer to most of the people.



Appendix E: Codes for data analysis

In this appendix, I present the codes for data analysis corresponding to the three research questions. This appendix is divided into three subsections, and each of the subsections presents the codes for data analysis in a table form. The first section presents the codes for data analysis corresponding to research question one (RQ1). The second subsection presents the codes for data analysis corresponding to research question two (RQ2a & RQ2b). The third section presents the codes for data analysis corresponding to research question three (RQ3).

E.1: Codes for RQ1

There are nine principal codes presented in Table 9.5 below. Each of the nine codes has subcodes. Seventeen of the sub-codes are theory-driven codes (the ones in **green**), and five sub-codes (the ones in **blue**) emerged inductively from the empirical data. The theory-driven codes were modified over time to define the students' activities clearly.

First Research Question (RQ1)	
Code: Object of the activity	
Sub Code	**Ratify the objective (RO)**
Definition/ Description	Members of the group ratify the objective of solving the mathematical modelling tasks. Students, at some point in the solution process, make known what they need to do or achieve, and this entails the objective of solving the task.
Example	Student A: So, what do we do here? Student B: Solve the tasks with GeoGebra and produce a report.
RQ1a	
Code: Digital technology	
Sub Code	**GeoGebra (DTG)**
Definition/ Description	Students use the GeoGebra software to solve the tasks by making tables, drawing graphs, and other things.
Example	Student A: Let's go to GeoGebra [opens GeoGebra]. Student B: Should we say the x-axis is the people? [draws a function in GeoGebra with the x and y-axis representing the number of people and the price of the car, respectively].
Sub Code	**Excel/ Spreadsheet (DTE)**
Definition/ Description	Students solve the tasks with Excel by making tables, drawing graphs, and other things.
Example	Student A: I am unfamiliar with GeoGebra; we could use Excel/spreadsheet [opens Excel/spreadsheet]. Student B: Yes, let us generate our data with Excel [enters a set of data points in Excel/spreadsheet].
Sub Code	**Calculator (DTC)**
Definition/ Description	Students use a calculator device, a calculator on the computer or a mobile phone for mathematical computations.
Example	Student A: Can you check the product of the two numbers with the calculator? Student B: That gives us 24823 [multiplication with a calculator device].
Sub Code	**Google Maps (DTM)**
Definition/ Description	Students search for places on maps and actual distances or travel time between cities.
Example	Student A: We should open Google Maps and locate the positions of the three cities [locates the three cities on Google Maps].
Sub Code	**Google Search (DTS)**
Definition/ Description	Students search for information on the internet with a Google search engine.

Example	Student A: We should also look for the population. Student B: Yes. Can you Google search the population of those cities? ….
RQ1b	
Code: Pseudocontingency	
Sub Code	**Individualized (PCI)**
Definition/ Description	Interaction that indicates possessiveness of own contribution. Unwilling to consider other's suggestions for improvement or change. In this situation, students can follow their process or line of thought without contributing to other students' ideas. A student can also work independently on a computer while others discuss different ideas or strategies.
Example	Student A: The distance between the two cities by car should be measured. Student B: No, we need the air distance instead. Student C: By car, it is 22 minutes and 18km. Student B: I would say 12km by air distance.
Code: Asymmetrical contingency	
Sub Code	**Agreement/ Affirmation (ACA)**
Definition/ Description	Interaction that is supportive and affirming. Non-critical. Agreement with what was suggested without cause to review or challenge.
Example	Student A: We need to consider the people living there. Student B: Yes, that is right. Student A: This city has more people, so it is here. Student B: Good choice.
Sub Code	**Consensus/ Clarification (ACC)**
Definition/ Description	Interaction that builds understanding of suggestions or ideas but in a non-critical, non-challenging and non-expansive way. The questions the students ask are for clarification and to build a consensus.
Example	Student A: We need to find the product of these two. Student B: Why the product? Student A: That gives us the maximum revenue.
Sub Code	**Elaboration (ACE)**
Definition/ Description	Questions are asked to seek further detail about how to do things or clarify why a partner suggests a particular course of action (interaction is more expansive). Elaboration is slightly different from consensus or clarification. In elaboration, students sometimes repeat the argument made by their peers in their understanding and, at times, ask questions for further details.
Example	Student A: This is how the linear graph will be.

	Student B: So, you mean the graph declines when people increase? Student A: No, we have this graph when the number of people decreases.
Code: Reactive contingency	
Sub Code	**Critical/ Constructive (RCC)**
Definition/ Description	Consideration and critical review of others' ideas in a way that leads to improved decision-making or content.
Example	Student A: We do not need to consider the people. Student B: It's included in the question. Student A: How does the number of people affect the optimal place? Student B: It is about fairness.
Sub Code	**Justification (RCJ)**
Definition/ Description	Interaction that seeks justification of perspectives or ideas being offered, with a focus on how they will improve decision-making or output quality. Reasons for suggestions are pursued through probing questioning or offering alternatives.
Example	Student A: We can use the calculator for each revenue. Student B: That is much work. Why not GeoGebra? Student A: OK, let us try that, but how do we find each revenue with that? Student B: By highlighting the two columns….
Code: Mutual contingency	
Sub Code	**Negotiation (MCN)**
Definition/ Description	Interaction that demonstrates tentative ideas being offered and debated. Different perspectives are acknowledged and synthesized into a collective response.
Example	Student A: How do we find the revenue? Student B: Use the table or spreadsheet in GeoGebra. Student A: The first two columns represent the people and car prices. Student B: Do we use the third column for the revenue? Student A: We can use any for the revenue.
RQ1c	
Code: Explicit rules	
Sub Code	**Time constraint (ERT)**
Definition/ Description	When students are under time pressure to complete a task, they either move on to the next task or quickly write a report for their answer.
Example	Student A: We now know the answer. Should we present our answer on the graph? Student B: No, the answer is enough. We only have a little time.

Sub Code	**No restrictions on digital tools (ERR)**
Definition/ Description	Students use one or several digital tools to solve the task.
Example	Student A: Let us start with the calculator. Student B: It is taking a longer time with the calculator. Student C: We can now use Excel. That will take less time.
Code: Implicit rules	
Sub Code	**Dismissing comments/suggestions (IRD)**
Definition/ Description	Students dismiss comments or suggestions from peers when they do not fit into the current strategy of solving the problem.
Example	Student A: Oh! I think we can solve the task with GeoGebra. We need a function for the data set. Student B: No, let us continue with Excel. We are almost there. Student C: Yes, I agree.
RQ1d	
Code: Roles of Students	
Sub Code	**Leading role (RSL)**
Definition/ Description	Students take the leading role by telling the other members what to do and also dominating the communications—the ones whose ideas are considered valuable or worthy of consideration at most times.
Example	Student A: Let us devise a plan for solving this problem. OK, let us first find the minimum wage. It is better with this strategy.
Sub Code	**Opposing role (RSO)**
Definition/ Description	Student(s) do not agree with the comments or ideas of others compared to theirs. They oppose the ideas/solutions of others and/or introduce their ideas/solutions.
Example	Student A: No, you cannot do that. Let us instead multiply the minimum wage and the number of items sold.
Sub Code	**Questioning and challenging role (RSQ)**
Definition/ Description	Students question and challenge the input of other members when they do not fully understand those ideas. The students do not bring in an alternative idea but only question the ideas of others for clarity.
Example	Student A: I think we let $f(x)=ax$. Student B: OK, but you know a is a constant and might affect the graph. Student A: Yeah, it is a constant. That is the gradient. Student B: And where is the intersection?
Sub Code	**Suggesting role (RSS)**
Definition/ Description	Students suggest or recommend an idea to support the idea of another group member. This happens when the student gets stuck or missing something. Another thing is that it happens also

	even though the student is not stuck, but an idea is suggested to add to existing idea(s) or improve existing idea(s). There are situations when a student tries to put their ideas on the computer after observing the input of peers and not necessarily engaging in the discussion. We can say he/she is suggesting an idea but not communicating it.
Example	Student A: This graph is not what we want. The numbers are too large. Student B: Maybe we have to rescale the numbers.
Sub Code	**Non-contributing role (RSN)**
Definition/ Description	When the students do not contribute to the group work, the student only observes the other members but does not put in any effort.
Example	Student A: I am pretty useless here, to be honest. I do not understand what is going on here [remains silent as the other members work on the task].
Sub Code	**Supporting role (RSX)**
Definition/ Description	When students assist other students by agreeing to their ideas whilst adding nothing to those ideas (different from the suggesting role), in this view, the students sometimes use the calculator to compute the values given by the other group members, using expressions like 'yeah, that's true'. When the discussion or interaction is mutual, and students agree to each other's idea(s), we can say that each group member is taking a 'supporting role'.
Example	Student A: We have to find the product of the number of people and the price of the car. So, 80 times 7000 will be … Student B: That will be 560000 [uses the calculator on his mobile phone].

Table 9. 5: A description of codes for analyzing the empirical data (in addressing the first research question—RQ1).

E.2: Codes for RQ2a & RQ2b

There are six principal codes presented in Table 9.6 below. Each of the six codes has subcodes. Twenty-three of the sub-codes are theory-driven codes (the ones in **green**), and one sub-code (the one in **blue**) emerged inductively from the empirical data. The theory-driven codes were modified over time to define the students' activities clearly.

Second Research Questions
RQ2a
Code: Breaking the task into manageable parts

Sub Code	**Assumption and simplification (BAS)**
Definition/ Description	Classification of the variables in the realistic problem. The dependent variables (thus, the variables the model seeks to explain) and independent variables (the remaining variables). Some independent variables might be neglected due to the relatively small effect on the model. Simplification here is also about knowing your variables and reducing the variables to the only important ones to work with.
Example	Student A: Let us assume the minimum wage is 100 NOK …. and that in every 1 hour, the items double. Student B: The key variables here are the wage and working time….
Sub Code	**Constructing relations (BCR)**
Definition/ Description	Interrelationships of variables, that is, identifying or constructing the relations between the variables. There might be an additional simplification to hypothesize relationships between the variables.
Example	Student A: The minimum wage should equal the number of items [writes w=2t on paper].
Sub Code	**Seeking information (BSI)**
Definition/ Description	When students look for available information and differentiate between relevant and irrelevant information, when the students identify relevant questions in the given realistic problem, and during the solution process, the students might still search for information to update the model with facts and figures. While working on the problem, the students might again look for the meaning of some words as they communicate their ideas to their peers.
Example	Student A: Let us Google Search and look up the minimum wage and total hours in Norway. Student B: The task does not require the total hours. Student C: Yes, only the minimum wage is essential.
Sub Code	**Recognizing quantities (BRQ)**
Definition/ Description	When the students recognize quantities that influence the situation described in the problem text by naming the quantities and identifying the key variables in the situation, the student identifies relevant questions in the given real-world situation.
Example	Student A: When we sell the car at 5000, a hundred people will buy it. Let x be the number of people that will buy the car.
Code: Searching for a model	
Sub Code	**Translating the real problem into a mathematical problem (SMT)**

Definition/ Description	When the students translate the realistic problem into a mathematical problem, that is, from the text form into a mathematical form (algebraic, graphic, numeric).
Example	Student A: The relation can be written as $C(x) = K + 10x$
Sub Code	**Representing the mathematical problem in the technological world (SMR)**
Definition/ Description	When the students move the mathematical problem (such as equation, function, figure, diagram, table, term, etc.) into the GeoGebra software, Google Maps, and others.
Example	Student A: Now that we have the relation, let us use the spreadsheet in GeoGebra to create a table and a graph.
Sub Code	**Simplified model (SMS)**
Definition/ Description	When the students combine the relevant quantities and their relations to form an equation, these quantities and their relations are simplified. The students simplify by reducing the number and complexities of the relevant quantities, leading to a more precise model. Two or more equations can be put together as one model. A simplified model could also be constructing geometric figures or shapes with information from the recognized quantities and, for instance, using the positions of some recognized cities on Google Maps to construct a triangle or rectangle, among others.
Example	Student A: If we factorize the equation, we will get a precise relation. Student B: Before we factorize, let us put these two equations together [puts $f(x)$ and $g(x)$ together as one equation].
Sub Code	**Appropriate notations (SMA)**
Definition/ Description	When the students choose appropriate mathematical notations and represent the mathematical situations graphically.
Example	Student A: Let us represent this function as $f(x)$.
Code: Finding a solution for the model	
Sub Code	**Apply mathematical knowledge (FK)**
Definition/ Description	When the students apply their mathematical knowledge to solve the problem. Application of mathematical concepts in solving the problem and utilizing mathematical concepts and procedures.
Example	Student A: The scattered points look like an exponential function. And so, we can use the relation $f(x) = a\,x + b$
Sub Code	**Analyzing (FA)**
Definition/ Description	When the students reconcile their model with reality or analyze it to arrive at a better one, this could be an analysis of points on a graph or places on Google Maps, among others. It could also be the comparison of the population of different cities. Again, it

	could be an analysis of distances while considering the travel time and roads. Analyzing could also be importing images from Google Maps (or other images) into GeoGebra and manipulating the image in the form of geometric figures or shapes.
Example	Student A: The four towns form a rectangular shape on the map. We can find the midpoint as the optimal place. Student B: Geographically, you are right, but that will be in the middle of the lake. This point could be the best place.
Sub Code	**Effect of parameters (FE)**
Definition/ Description	When the students observe and explore the parameters on the graph, they might use the sliders in GeoGebra to vary parameters to see the effects on the function(s) or diagram on the graph, new graphs or lines (for instance, x=1 or y=3) could be constructed, and their intersection with the main graph could be observed.
Example	Student A: If we show B2 (the amount of chlorine) on the graph, we can adjust the values. Student B: That is true; when we change the values of B2 using the slider, the amount of chlorine does not change over time.
Sub Code	**Mathematical manipulations and computations (FM)**
Definition/ Description	When the students dynamically manipulate mathematical figures and shapes to see what happens, this could also be the manipulation of a geometric figure in the process of finding the midpoint, among others. Computation here is about performing some calculations with the calculator. It could also be entering and generating data with the spreadsheet.
Example	Student A: I think we have to use "move" in GeoGebra to check if we keep the other points constant and move this point; the circle will still pass through all the points. Student B: The circle did not pass through the points when I did that.
Code: Explaining the results in real terms	
Sub Code	**Appropriate mathematical language (EA)**
Definition/ Description	The students might use appropriate mathematical language to communicate their solutions. Thus, the students might use the correct mathematical terms, words, expressions, etc., to communicate about the solution or model.
Example	Student A: In the end, we found that the distance between the three towns can be explained using Pythagoras' theorem. That is the distance between………
Sub Code	**Generalizing the model (EG)**

Definition/ Description	The students might generalize the model or solution to the realistic problem. They might generalize the solution to suit a different context.
Example	Student A: Our model is a model for the transition from old telephones to modern mobile phones and power consumption. However, this model can also be applied to the transition from old television to modern television and the power consumption involved.
Sub Code	**Meaning of results (EM)**
Definition/ Description	When the students present the results obtained in the model as the real solution, they achieve real results.
Example	Student A: We had $x = 5$, meaning the chlorine level remains at 5 litres after adding 1 litre daily.
Code: Checking the results for adequacy	
Sub Code	**Check and reflect (CR)**
Definition/ Description	When the students critically check and reflect on found solutions. That is, by reflecting on other ways of solving the problem and going through the modelling process if the solution does not fit the situation. It could also be a group member reflecting and/or criticizing founded answers suggested by another member, and the suggested answers or strategies are not always the final results.
Example	Student A: I think our final model will work. Student B: Let us apply an algebraic method and see if it will be the same as the graphical method we used. Student C: We could also review our process and see if the final model works.
RQ2b	
Code: Role of digital technologies	
The **yellow text** codes represent the 'technological affordances and constraints' codes in Table 9.7.	
Sub Code	**Calculating (Calculator) (RTC)** (**TC**)
Definition/ Description	Making calculations with a handheld device or a software-based computer.
Example	Student A: What is the value of 515 times 413? Student C: The answer is 212695; I used the calculator.
Sub Code	**Researching (Google Search, Google Maps) (RTR)** (**TR**)
Definition/ Description	Researching information on the internet about the meaning of some words in the problem text, the actual value of some variables, and finding places on the map.
Example	Student A: I do not understand the word 'knot'. Student B: I just googled it, and it is the unit of speed.
Sub Code	**Measuring (GeoGebra, Google Maps) (RTM)** (**TM**)

Definition/ Description	Finding the distances between points, the lengths of segments, the sizes of angles or the gradients of lines and segments. To find the distance between two cities on Google Maps or measure the travel time between two cities on the map.
Example	Student A: Should we measure the distance between Ship A and B? Student B: Yes, the distance is 5cm when the speed of Ship A is twice that of Ship B [measuring two points on GeoGebra].
Sub Code	**Experimenting/Changing (GeoGebra) (RTE)** (**TE**)
Definition/ Description	The students might change the parameters, conditions or assumptions of a drawing or functions and observe the effects. It could also be drawing new graphs or lines to find the intersection between the new graph or line with the previous one. To construct the midpoint of a triangle or other geometric figures.
Example	Student A: Let us put a set of new values in the function and see if it will still be constant [test the function with new values].
Sub Code	**Geometric construction (GeoGebra) (RTG)** (**TG**)
Definition/ Description	To draw graphs and functions. Drawing simple geometric objects into a coordinate system. Using points, lines, sections, circles, polygons, etc., in drawing geometric objects and diagrams. Drawing a function with a set of points or equations. Plotting of points to represent a data set. To take a screenshot of Google Maps, insert it into GeoGebra, and construct segments to link the marked places on the map.
Example	Student A: Let us represent the three towns as three points and draw a line linking the points.
Sub Code	**Visualizing (Geogebra, Google Maps) (RTV)** (**TV**)
Definition/ Description	Drawing in or moving segments in order to represent previously found values graphically. For instance, moving points to create a segment of previously determined length. Switching to satellite on Google Maps to visualize the reality of the environment. Movement of points and/or attaching labels to points to see the point's value.
Example	Student A: I think we should move one of the points further to increase the size of the triangle.
Sub Code	**Advanced geometric construction (GeoGebra) (RTA)** (**TAG**)
Definition/ Description	Drawing more complex geometric objects and configurations using intermediate steps or auxiliary lines. For instance, using angle bisectors to split an angle into two equal angles.
Example	Student A: By splitting this angle into two, we can have two equal sizes of this figure.

	Student B: That is true; if the sides are equal, then we will have a rectangle.
Sub Code	**Data entry and generation (GeoGebra, Excel/Spreadsheet) (RTD) (TD)**
Definition/ Description	Enter a set of values or variables in the spreadsheet and generate the rest of the data set by selecting a few values and dragging them down.
Example	Student A: Label A1 and B2 as customer and revenue respectively. Student B: Yeah; enter our values and generate the rest of the data [selects A2, A3, B2, and B3 and drags them down].

Table 9.6: A description of codes for analyzing the empirical data (in addressing the second research questions—RQ2a & RQ2b).

E.3: Codes for RQ3

There are two principal codes presented in Table 9.7 below. One of the three codes is already presented in Table 9.6. The rest of the two codes presented in Table 9.7 have subcodes. Eight of the sub-codes are theory-driven codes (the ones in **green**), and three sub-codes (the ones in **blue**) emerged inductively from the empirical data. The theory-driven codes were modified over time to define the students' activities clearly.

Third Research Question (RQ3)	
Code: Technological affordances and constraints	
The same codes as the codes under '*role of digital technology*' in Table 9.6 (**I label the codes in green text**)	
Code: Mathematical affordances and constraints	
Sub Code	**Use real data (MU)**
Definition/ Description	Work on real problems involving calculations that are error-prone (when done by hand) and time-consuming. In this case, the students use actual data or values for the variables they identify in the realistic problems in creating the model. For instance, students may collect accurate data on the number of cars crossing a particular road in a certain period and then use GeoGebra or Excel to analyze this data and create a mathematical model.
Example	Student A: We can use the Norwegian population for the past 20 years. Student B: Sure! Let us use the figures to create a linear model in GeoGebra.
Sub Code	**Clarification (MC)**

Definition/ Description	Retrieving information on the internet of some variables (this often happens when students have English as their second language). Using the search engine to find the meaning of a mathematical term during group interactions.
Example	Student A: That will be a fix, 'Konstantledd' or something? Student B: [searches for the meaning of Konstantledd], the last number is constantly articulated.
Sub Code	**Analyzing (MA)**
Definition/ Description	To use accurate information or data in creating the model. When students use information like the position of places on maps, the population of different cities and the distances between cities/towns to analyze their model. That is, comparing and evaluating places using the number of people living within these places and the travel time between the different places.
Example	Student A: The location needs to be closer to Town M. Student B: [searches for the population of Town M and N], it should be closer to Town N since more people live there than in Town M.
Sub Code	**Simulating and visualizing (MS)**
Definition/ Description	Simulating and visualizing the mathematical concepts. When the students simulate and visualize the mathematical concepts to better understand the concepts and uncertainties in the mathematical concepts. For instance, manipulating or simulating mathematical figures and shapes dynamically to see what happens.
Example	Student A: I think we should move one of the points further to increase the size of the triangle. Student B: Yes, maybe the circle will pass through all the points, or this only applies to equilateral triangles.
Sub Code	**Linking representations (ML)**
Definition/ Description	Connecting mathematical representations. When the students connect the mathematical representations, for instance, the movement between geometric, numeric or table, graphic, and symbolic or algebraic representations. Change one representation and see changes in the other representation. This could also be the translation of coordinate points from Google Maps (or importing maps) to Geogebra and constructing a geometric figure or shape using the points on the map.
Example	Student A: Now that we have the table, let us use the 'graphic view' to see the function resulting from the table.
Sub Code	**Regularity and variations (MR)**
Definition/ Description	To explore the regularity and variations in the solution model. The students might explore the regularity and variations, which

	is done by observing the effect of parameters on the graph. For instance, to use sliders in GeoGebra to vary parameters to see the effects on the function(s) on the graph. Drawing a new graph or line to observe the intersection point. Finding the midpoint of the translated coordinates (or the screenshot from Google Maps, which is in the form of a geometric figure or shape).
Example	Student A: If we show B2 (the amount of chlorine) on the graph, we can adjust the values. Student B: That is true; when we change the values of B2 using the slider, the amount of chlorine does not change over time.
Sub Code	**Arithmetic and statistics (MAS)**
Definition/ Description	Performing numerical computations such as addition, subtraction, multiplication, division, raising powers, and extraction of roots, among others. This is done using a calculator. Organizing, generating, and calculating data in a spreadsheet. For instance, the rows and columns in Excel can be used to organize data manipulations like arithmetic operations. The collection, analysis, interpretation, and presentation of data. Measuring distances between points in GeoGebra.
Example	Student A: Let us find the product of the number of people and the price of the car if 70 people buy it. Student B: That will be 70 multiplied by 6500 [uses the calculator].
Code: Socio-cultural affordances and constraints	
Sub Code	**Common focus (SC)**
Definition/ Description	When students (in a group) solve a problem using digital technology, they share the same source. They have the facility to look at the same thing and point at what is presented on digital technology. This helps the students in creating a shared goal. For instance, GeoGebra can be used as a reference tool to visualize one's reasoning during a mathematical discourse.
Example	Student A: Let us start with a straight square, like this (uses her hands to form a square parallel with GeoGebra's X- and Y- axis. Student B: Or we could make one with a slope [he draws a tilted square with the mouse]. Or is it possible to create an ordinary… I mean… one with no slope? Student A: Yes, you create two parallel lines like this… and two vertical [draws with her finger horizontal and vertical lines parallel with the X- and Y-axis]. Student B: Then, we can angle it…so that we have lines with a slope [draws a titled square with the mouse].
Sub Code	**Observing and repairing divergences (SOR)**

Definition/ Description	The digital technology is used as a way of maintaining shared knowledge and ideas. In some instances, the students might find themselves in a situation marked by uncertainty and divergences (among others), which might cause their solution process to cease. However, digital technology could be used to verify knowledge or settle disagreements by performing tests, referencing, etc.
Example	Student A: However, if *m* is negative, what way… I mean… is the slope up or down? Student B: I think down… let us try (writes y=-2x+3 at the algebra section of GeoGebra). Student A: Great. OK, downhill slope.
Sub Code	**Observing and improving strategies (SOI)**
Definition/ Description	The digital technology is used as a way of maintaining and improving shared knowledge and ideas. When students collectively solve a mathematical task with digital technology, a group member could observe the solution process and improve the strategy used due to the affordances of the digital technology perceived by him/her. This is an individual input for a collective purpose. In some situations, the students could observe and perceive an affordance but cannot actualize it, resulting from undefined or not well-defined functions and, for instance, making sliders that do not affect the graph.
Example	Student A: Let us draw the function $f(x)$=-x+100 [draws the function in GeoGebra] and erm put in the *x* values until we get the maximum price. Student B: I feel there is a faster way of doing it. We can find the sliders to control the *x* values so that we do not enter them one after the other [insert a slider x=a].
Sub Code	**Authority of the digital technology (SA)**
Definition/ Description	Students accept the answer from the digital tool as the correct answer. For instance, if students are working with GeoGebra and the tool gives a result opposite that of another student's results, the other student believes the result from GeoGebra than that of the peers. This affects group interactions. At the individual level, the representational choice could affect the features of the digital technology to be used, which, in the end, affects group interaction at the collective level. There are also situations where students uphold their strategy or results from digital technology and do not accept other strategies when they think they are close to finding the results.
Example	Student A: I think the population of Vennesla is 14,000.

	Student B: (Google search the population of Vennesla), it is 14935. Student C: Yeah, that is almost 15,000. Student B: But we need to use 14,935.

Table 9.7: A description of codes for analyzing the empirical data (in addressing the third research question—RQ3).