The modelling cycle as analytic research tool and how it can be enriched beyond the cognitive dimension

Pauline Vos, Peter Frejd

The modelling cycle as analytic research tool and how it can be enriched beyond the cognitive dimension

Pauline Vos[1] and Peter Frejd[2]

[1]University of Agder, Norway; pauline.vos@uia.no

[2]University of Linköping, Sweden; peter.frejd@liu.se

The modelling cycle is a theoretical construct frequently applied in research studies on mathematical modelling. On the one hand, the modelling cycle highlights essential aspects of modelling, which makes it a tool for conceptualizing. On the other hand, the modeling cycle can be used as a research tool for analysis of students' work. In the latter case, it has the limitation of primarily yielding results of a cognitive nature. We sought ways to include other aspects to analyze, such as metacognitive strategies, tool use, and social norms. These aspects support and change the cognitive activities involved in mathematical modelling practice. Rather than the standard modelling cycle, we propose an enriched modelling cycle with overarching layers for analysis of results. The enriched modelling cycle is a wider theoretical framework with interacting dimensions that affect the phases in the modelling cycle. We discuss potentials and challenges of this framework for new research studies.

Keywords: cognition, mathematical modelling, modelling cycle, social norms, theoretical frame.

Introduction

In this theoretical paper, we focus on theoretical constructs applied in many research studies on the teaching, learning, and assessing of mathematical modelling. Review studies on research on mathematical modelling education have been published by Cevikbas et al. (2021), Geiger and Frejd (2015), Kaiser and Brand (2015), Schukajlow et al. (2018) and Stillman (2019). These reviews show that a considerable number of studies apply a modelling cycle (MC) as theoretical framework for analysis of data in their research. With this paper, we aim at opening a discussion on benefits and limitations of MCs as a tool for analysis. This leads us to present possibilities to enrich the MC as theoretical framework, so it can assist researchers in further analyzing and theorizing mathematical modelling education[1].

This paper arose from a discussion on the importance of collaboration in mathematical modelling, as also noted by Blum (2002): how can aspects of groupwork, such as *agency* and *accountability* be analyzed in research applying an MC? Also, we discussed that research using a MC-based framework observed students having *blockages* in the modelling process, and that these were of a *cognitive* nature, that is, in students' minds (e.g., Galbraith & Stillman, 2006). However, students' problems could also be *blockages* caused by the norms of the didactical contract (Brousseau, 2002), where students are taught, for example, to avoid using extra-mathematical knowledge in mathematics task. When analyzed as blockages caused by the environment, the research results shed light on underlying mechanisms that hinder change; blockages in the educational environments may be more persistent

[1] This paper extends a short paragraph in Frejd and Vos (2021), where we presented the enriched modelling cycle. In the present paper we have more room for backgrounds and elaborations.

and harder to remove than cognitive blockages in students. So, why does research applying a MC-based framework yield cognitive blockages and not *socio-cultural* blockages?

Our discussion led to studying the essence of MC-based frameworks, regarding their benefits and limitations in analyzing aspects in modelling activities. This inspired us to develop an enrichment perspective on MC-based frameworks, so these can zoom out and yield analytic results that could be, for example, *social* in nature. This enrichment perspective should assist researchers of modelling education to create new angles in their data analysis, and thus reach new research results.

The modelling cycle as tool for conceptualizing and analyzing

Much research on mathematical modelling describes mathematical modelling through a *modelling cycle* (Niss & Blum, 2020; Geiger & Frejd, 2015). A MC is a schematic diagram showing mathematical modelling as a cyclic process, which consists of subsequent phases. See Figure 1 for an often-used example from Blum (2015), which shows seven phases in the modelling process; other MCs may have fewer or more phases and other wordings (Perrenet & Zwaneveld, 2012).

The MC in Figure 1 builds on an earlier version by Blum and Leiβ (2007), in which the 1st phase was named *understanding*, to indicate that the modeling process starts from a problem situation that needs to be understood. In the new version of this MC (Blum, 2015), it is written *constructing* to indicate that a modeller needs to create a mental model of the problem and the task ahead. After this start, the modeller goes through different phases by structuring and simplifying the problem context (e.g., making a rough drawing of the problem situation), which is mathematizable (e.g., by creating algebraic formulas), and which can be worked on mathematically (e.g., by manipulating the algebraic formulas). The mathematical results thereof can be interpreted and validated considering the original problem. In case the results are considered inadequate for the real situation, the entire modelling process is repeated. If the modeller is 'ready', the results can be *exposed*, that is: presented to others.

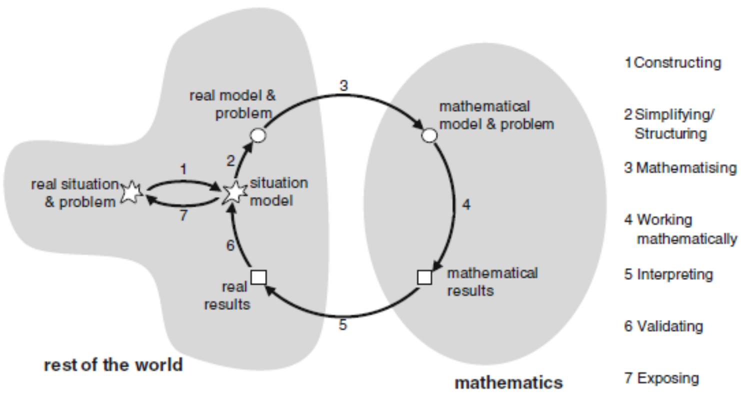

Figure 1: Modelling cycle from Blum (2015)

When students are given a modelling task, students follow other routes than what is described in a MC, 'jumping' back-and-forth between phases (Borromeo Ferri, 2006; Ärlebäck, 2009). However, most phases are somewhere observed in students' activities. Thus, a MC does neither show what a modeller does step-by-step, nor is it a recipe to be strictly followed. Niss and Blum (2020) explain that a MC "should be understood as an *analytic* (ideal-type) *reconstruction* of the steps of modelling necessarily present, explicitly or implicitly, as an instrument for capturing and understanding the principal processes in mathematical modelling" (p. 14, italics by the authors). Thus, a MC is a *tool* for researchers and teachers to apprehend, comprehend, recognize, explain, and analyze important aspects in modelling, independent of whether it is done by an expert or a novice. Thus, a MC does not offer a definition, that is, it does not offer an explicit statement clarifying what mathematical modelling is. Also, a MC does not characterize mathematical modelling; that is, it does not offer qualities of modelling. Rather, a MC *conceptualizes* mathematical modelling; that is, it offers an abstract and structured idea of essential aspects, which is simplified so it is practical for use in teacher education, in educational-political discussions, and in research. In other words, a MC is a *model*.

The advantages of conceptualizing mathematical modelling through a MC are manifold. For instance, MCs show that modelling is complex, and that each phase affects others dynamically. Also, MCs show that modelling starts from real life and returns to it, and that mathematics is a useful toolbox in the solution process. Also, MCs show that modelling is not a purely mathematical activity, yet that mathematical activities play a central role. Also, MCs show that modelling differs from 'applying mathematics', which starts from a mathematical object, concept or algorithm that subsequently is used in a non-mathematical context, regardless of whether then a problem will be solved.

Apart from using MCs as conceptualization tool, researchers use MCs as an *analytic tool* to analyze their data in light of the different phases that a MC distinguishes. For example, we see that MCs are used to analyze students' activities regarding when they are in which phase (e.g., Ärlebäck, 2009), to analyze students' modelling competences regarding whether students are able to 'pass' a certain phase (e.g., Haines, Crouch, & Davis, 2000), to analyze mathematics tasks for certain emphases of modelling (Frejd, 2011), or to analyze classroom culture for an emphasis on certain modelling phases (Brady & Jung, 2021). The use of MCs as analytic tool yields a rich body of knowledge.

The modelling cycle with other dimensions than the cognitive dimension

When MCs are used as analytic tool in research of mathematical modelling, the results will be framed by it. The standard MC describes *cognitive* activities, which are activities that a researcher can observe in, or deduct from, a modeller's speech, gestures, writings, reactions and other explicit or implicit expressions. More generally, cognitive activities involve mental efforts to use and make sense of information. Activities such as speaking, listening, reading, remembering, non-routine problem solving, decision making, and sense making are mentioned as examples of cognitive activities. Cognitive activities can be learnt through experience or by being taught.

When an analytic framework has a cognitive focus, the research results will accordingly be primarily of a cognitive nature. This means that these have an individual's or a group's mental activities as unit of analysis. With an emphasis on cognitive aspects, the research may not capture other aspects that also play a role in mathematical modelling. Below, we give a few aspects that are not immediately captured by a theoretical framework based on the cognitive activities in a MC.

A dimension for metacognitive strategies

Successful mathematical modelling involves *metacognitive strategies* (e.g., Maaβ, 2006; Stillman, 1998, Vorhölter, 2018). These are needed for regulating and coordinating the many processes in modelling, both individual and group processes. During the modelling work, aims and outcomes need to be coordinated and regulated considering (1) goals in the task, (2) resources present, (3) the didactical contract from the teacher, and so forth. Different metacognitive strategies can be linked to each of the different phases in a MC, see Table 1. For instance, when starting, students need to 'read' the intentions into a task description and anticipate what they can do to reach a satisfying answer. In each of the phases in the MC, they can expect unexpected situations and may reflectively change the initial plans. They need to anticipate, reflect, plan, monitor, etc. From a research point of view, to analyze metacognitive strategies, one needs a different theoretical framework than for cognitive activities. Yet, metacognitive strategies and cognitive activities are intertwined. So, one can perceive the metacognitive strategies as an overarching layer over the standard MC, whereby the metacognitive strategies and the cognitive activities are two dimensions in one theoretical framework.

A dimension for tool use

Another aspect in mathematical modelling not captured in the standard MC is the use of *tools*. Therefore, Greefrath (2011) drew an alternative MC describing functions of digital tools in each phase of the MC. We want to extend this idea, building on Vygotskian theory (Williams & Goos, 2013), which explains that any cognitive activity is always *mediated* by tools, such as pens, blackboards, or digital tools. Mediation entails that the tool changes both the results of the activity (e.g., a mathematical answer becomes more precise), but also changes the cognitive activities (e.g., writing down intermediate steps off-loads memory demands). When starting on a modelling task, a modeller can try to understand the problem by using *Wikipedia* as inquiry tool. Another tool at the start of a modelling process is the *task sheet*, which offers students the information to be used and the guidelines to follow. Important tools in modelling are paper and pencil for making notes and sketches. At the very end of the modelling process, a modeller will present the results of the activity, possibly in written form or in an oral presentation to an audience. Thus, tool-use can be another analytic dimension that can be an overarching layer over the standard MC, see Table 1 and Figure 2.

A dimension for social norms

Another analytic dimension for research on mathematical modelling can be *social norms*. These are socially shared, implicit or explicit standards of acceptable behavior. As Blum's (2015) MC shows, modelling takes place in two worlds: the 'mathematical world' and the 'rest of the world', in which there are different social norms. For instance, in the 'rest of the world', number answers can be estimations and, hence, not so mathematically precise. Yet, when presenting the final answer of the problem to the *client*, a modeller will abide to presentation norms (e.g., correct spelling, attractive lay-out). Regarding norms in the mathematical world, Yackel and Cobb (1996) described *socio-mathematical norms*, such as the use of preferred symbols (e.g., x and y) rather than creative inventions (e.g., ʒ and ʗ), and the specific way to justify claims (by giving a proof rather than a few examples). Also, there are classroom norms, also known as the didactical contract (Brousseau, 2002). Also, in groupwork, there may be competing norms, with some students making the effort because they consider the activity relevant, whereas others do it to pass the exam (Hernandez-Martinez & Vos, 2018). Thus, social norms will impact any modelling activity in many ways, and these may differ between the phases. We put some norms indicatively in Table 1 without claim of completeness, since research on this theme is still scarce and recent (e.g., Bonotto, 2020; Dede, 2019). Table 1 shows the phases in the MC with analytic dimensions for metacognition, tool use, and social norms that all differently interact and modify the cognitive modelling activities.

Table 1: Phases in the modelling process with indicative dimensions for cognitive activities, metacognitive strategies, tool use and social norms

	Cognitive activities	**Metacognitive strategies**	**Tool use**	**Social norms**
1	Con-structing	strategies to understand and reformulate the problem, to use additional information	Interpret task sheet, investigate resources (e.g. Wikipedia)	Norms within the team, in the classroom, norms of the *owner/client* of the problem
2	Simpli-fying/ structuring	strategies to select and organize information, develop plans, anticipate later actions, to monitor progress	Experiment with pen-and-paper (p&p), sketch & drawing tools, spreadsheets, etc.	Norms within the team, in the classroom, norms about what aspects to choose and the extent to which creativity is permitted
3	Mathema-tizing	strategies to organize infor-mation, develop & implement plans, to monitor progress	Visualize and organize with p&p, spreadsheet, plotter	Norms within the team, in the classroom, norms on using standard methods and being creative
4	Working mathema-tically	strategies to implement plans, to monitor progress	Calculate & simulate with pen-and-paper, Geogebra, CAS, etc,	Norms within the team, socio-mathematical norms on rigor, accuracy, use of common sense
5	Inter-preting	strategies to interpret results, to face unplanned outcomes	Visualize with p&p, presentation tools	Norms within the team, in the classroom and with the client in the process of interpreting
6	Validating	strategies to verify results, to invite critique, to evaluate the process and products	Control using p&p, information resources	Norms within the team, in the classroom and with the client on what can be regarded as ‘validating’
7	Exposing	strategies to present results, to communicate and convince	Present using p&p or digital tools	Norms in the team, classroom, with the *client*, focusing on convincing

Conclusion

In this paper, we have looked at MC-based frameworks for analyzing aspects of modelling education and observed that such research has yielded a rich body of results, but that these are primarily of a cognitive nature and may obscure other aspects that play a role in modelling. Therefore, we suggest enriching the MC with overarching dimensions, such as metacognition, tool use or social norms, see Figure 2. The overarching dimensions can be supplemented or replaced by other dimensions that also affect the MC phases in different ways. Examples of alternative dimensions are *creativity* (Lu &

Kaiser, 2021), *flexibility* (Andresen, 2007) or *language* (Vorhölter et al., 2013). We suggest that also students' *attitudes* may differ between the phases; so far, research connecting modelling to attitudes has focused on how modelling activities relate to students' attitudes in mathematics in general (Chamberlin & Sriraman, 2019), and not on affect in different phases of modelling

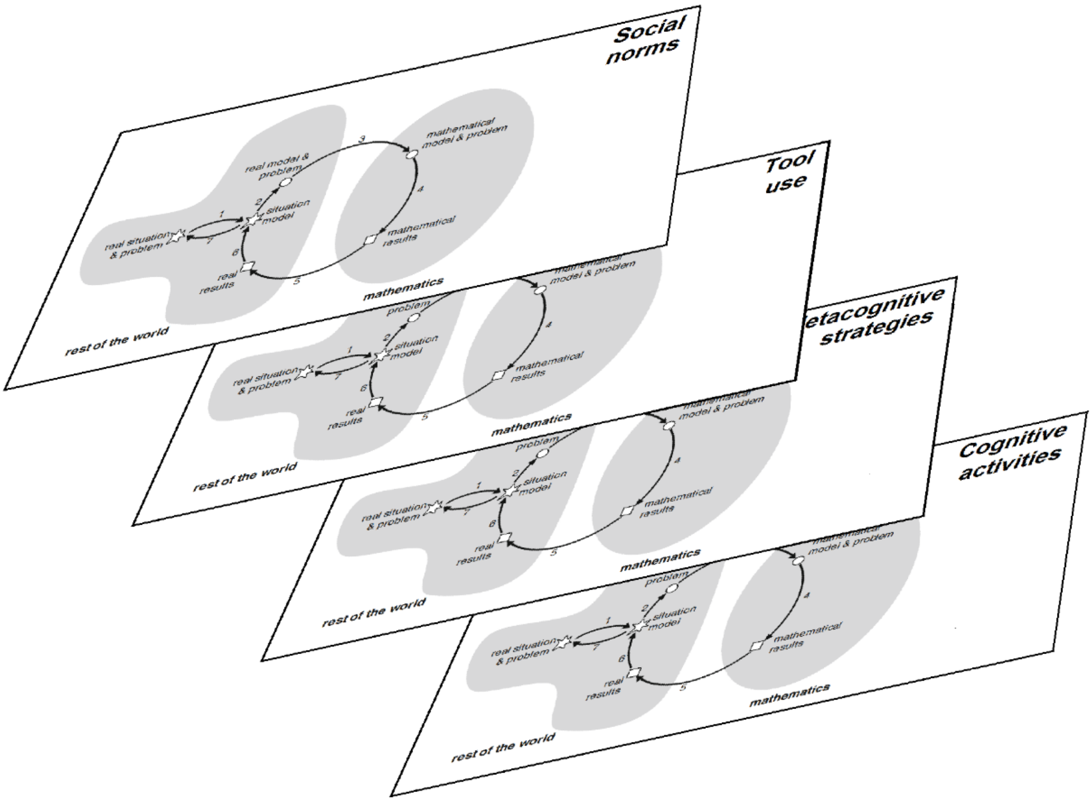

Figure 2: The enriched modelling cycle with four dimensions for analyzing modelling activities

Mathematical modelling is a complex and dynamic activity, and because of that it deserves to be studied from different perspectives. An enriched MC with a variety of overarching dimensions over the standard MC may enable modelling researchers to extend and deepen their research. This should give theoretical insights into how different dimensions interact and may reveal how students' cognitive modelling competencies are affected by various aspects that haven been obscured so far.

Of course, any theoretical frame has its limitations. The enriched MC is an analytic tool for students' modelling activities, and not a tool to design modelling tasks or a modelling curriculum. Also, it may be overwhelming to novices, and therefore, in teacher education or in educational-political discussions, the standard MC is more practical to keep a focus on aspects of mathematical modelling. However, the enriched MC should enable researchers to zoom out beyond the cognitive dimension and to further theorize the learning, teaching and assessing of mathematical modelling.

References

Andresen, M. (2007). Introduction of a new construct: The conceptual tool "flexibility". *The Mathematics Enthusiast, 4*(2), 230–250.

Ärlebäck, J. B. (2009). On the use of realistic Fermi problems for introducing mathematical modelling in school. *The Montana Mathematics Enthusiast, 6*(3), 331-364.

Blum, W. (2002). ICMI Study 14: Applications and modelling in mathematics education–Discussion document. *Educational Studies in Mathematics, 51*(1), 149–171.

Blum, W. (2015). Quality teaching of mathematical modelling: What do we know, what can we do? In S. J. Cho (Ed.), *The Proceedings of the 12th International Congress on Mathematical Education* (pp. 73–96). Springer.

Blum, W., & Leiβ, D. (2007). How do students and teachers deal with modelling problems? In C. Haines, P. Galbraith, & S. Khan (Eds.), *Mathematical modelling: Education, engineering and economics* (pp. 222–231). Woodhead. https://doi.org/10.1533/9780857099419.5.221

Bonotto, C. (2010). Engaging students in mathematical modelling and problem posing activities. *Journal of Mathematical Modelling and Application, 1*(3), 18–32.

Borromeo Ferri, R. (2006). Theoretical and empirical differentiations of phases in the modelling process. *ZDM, 38*(2), 86-95. https://doi.org/10.1007/BF02655883

Brady, C., & Jung, H. (2021). Modeling presentations: toward an assessment of emerging classroom cultures of modeling. *Educational Studies in Mathematics, 109,* 237–261. https://doi.org/10.1007/s10649-021-10056-x

Brousseau, G. (2002). *Theory of didactical situations in mathematics: Didactique des mathématiques, 1970–1990*. Springer.

Cevikbas, M., Kaiser, G., & Schukaljow, S. (2021). A systematic literature review on the current discussion on mathematical modelling competencies: state-of-the-art and developments in conceptualizing, measuring and fostering. *Educational Studies in Mathematics, 109*, 205–236. https://doi.org/10.1007/s10649-021-10104-6

Chamberlin, S. A., & Sriraman, B. (Eds.). (2019). *Affect in mathematical modeling*. Springer.

Dede, A. T. (2019). Arguments constructed within the mathematical modelling cycle. *International Journal of Mathematical Education in Science and Technology, 50*(2), 292–314. https://doi.org/10.1080/0020739X.2018.1501825

Frejd, P. (2011). An investigation of mathematical modelling in the Swedish national course tests in mathematics. In M. Pytlak et al. (Eds.), *Proceedings of the 7th Congress of the European Society for Research in Mathematics Education* (pp. 947–956). University of Rzeszów.

Frejd, P., & Vos, P. (2021). A commentary on the Special Issue “Innovations in measuring and fostering mathematical modelling competencies”. *Educational Studies in Mathematics, 109,* 455–468. https://doi.org/10.1007/s10649-021-10122-4

Galbraith, P., & Stillman, G. (2006). A framework for identifying student blockages during transitions in the modelling process. *ZDM, 38*(2), 143–162. https://doi.org/10.1007/BF02655886

Geiger, V., & Frejd, P. (2015). A reflection on mathematical modelling and applications as a field of research: Theoretical orientation and diversity. In G. A. Stillman et al. (Eds.), *Mathematical Modelling in Education Research and Practice: Cultural, Social and Cognitive Influences* (pp. 161–171). Springer. https://doi.org/10.1007/978-3-319-18272-8_12

Greefrath, G. (2011). Using technologies: New possibilities of teaching and learning modelling–Overview. In G. Kaiser et al. (Eds.), *Trends in teaching and learning of mathematical modelling* (pp. 301–304). Springer. https://doi.org/10.1007/978-94-007-0910-2_30

Haines, C., Crouch, R., & Davis, J. (2000). *Mathematical modelling skills: A research instrument (Technical Report No. 55).* University of Hertfordshire, Department of Mathematics.

Hernandez-Martinez, P., & Vos, P. (2018). “Why do I have to learn this?” - A study from mathematical modelling education about the relevance of mathematics. ZDM Mathematics Education, 50(1-2), 245–257. https://doi.org/10.1007/s11858-017-0904-2

Kaiser, G., & Brand, S. (2015). Modelling competencies: Past development and further perspectives. In G. A. Stillman, W. Blum & M. Salett Biembengut (Eds.), *Mathematical Modelling in Education Research and Practice* (pp. 129–147). Springer. https://doi.org/10.1007/978-3-319-18272-8_10

Lu, X., & Kaiser, G. (2021). Creativity in students’ modelling competencies: conceptualisation and measurement. *Educational Studies in Mathematics*, *109*, 287–311. https://doi.org/10.1007/s10649-021-10055-y

Maaβ, K. (2006). What are modelling competencies? *ZDM, 38*(2), 113–142. https://doi.org/10.1007/BF02655885

Niss, M., & Blum, W. (2020). *The learning and teaching of mathematical modelling* (1st ed.). Routledge. https://doi.org/10.4324/9781315189314

Perrenet, J., & Zwaneveld, B. (2012). The many faces of the mathematical modeling cycle. *Journal of Mathematical Modelling and Application, 1*(6), 3–21.

Schukajlow, S., Kaiser, G., & Stillman, G. (2018). Empirical research on teaching and learning of mathematical modelling: a survey on the current state-of-the-art. *ZDM Mathematics Education, 50*(1-2), 5–18. https://doi.org/10.1007/s11858-018-0933-5

Stillman, G. (1998). The emperor’s new clothes? Teaching and assessment of mathematical applications at the senior secondary level. In P. Galbraith et al. (Eds.), *Mathematical modelling: Teaching and assessment in a technology-rich world* (pp. 243–253). Horwood.

Stillman, G. A. (2019). State of the art on modelling in mathematics education—Lines of inquiry. In G. Stillman & J. Brown (Eds.), *Lines of Inquiry in Mathematical Modelling Research in Education* (pp. 1–20). Springer. https://doi.org/10.1007/978-3-030-14931-4_1

Vorhölter, K. (2018). Conceptualization and measuring of metacognitive modelling competencies: Empirical verification of theoretical assumptions. *ZDM Mathematics Education, 50*(1), 343–354. https://doi.org/10.1007/s11858-017-0909-x

Vorhölter, K., Greefrath, G., Borromeo Ferri, R., Leiß, D., & Schukajlow, S. (2019). Mathematical Modelling. In H. N. Jahnke and L. Hefendehl-Hebeker (Eds.), *Traditions in German-Speaking Mathematics Education Research*. Springer. https://doi.org/10.1007/978-3-030-11069-7_4

Williams, J., & Goos, M. (2013). Modelling with mathematics and technologies. In M.A. Clements et al. (Eds.), *Third International Handbook of Mathematics Education* (pp. 549-569). Springer.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education, 27*(4), 458-477. https://doi.org/10.2307/749877