Adaptive Backstepping Control of Uncertain Nonlinear Systems with Input and State Quantization

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***Abstract*—Though it is common in network control systems that the sensor and control signals are transmitted via a common communication network, no result is available in investigating the stabilization problem for uncertain nonlinear systems with both input and state quantization. The issue is solved in this paper, by presenting an adaptive backstepping based control algorithm for the systems with sector bounded input/state quantizers. In addition to overcome the difficulty to proceed recursive design of virtual controls with quantized states, the relation between the input signal and error state need be well established to handle the effects due to input quantization. It is shown that all closed-loop signals are ensured uniformly bounded and all states will converge to a compact set. Experimental results are provided to validate the effectiveness of the proposed control scheme.**

Keywords: Adaptive control, Backstepping, state/input quantization, nonlinear systems.

I. Introduction

Quantized control has attracted considerable attention in recent years, due to its theoretical and practical importance in network control systems [1], [2]. A great number of representative results have been reported on analysis and control of quantized feedback systems, as can be observed in [3]–[11], [18], [19]. However, most of the results are concerned with either input quantization or state quantization.

For example, the stabilization problem of systems with exact dynamics and input quantization has been studied in [3], [12]–[14]. As we know, system uncertainties and nonlinearities inevitably exist in physical systems. Adaptive approaches [4], [6], [8], [9], [15]–[17], [20] are usually employed to investigate the control problem for uncertain systems with input quantization. In [4] and [15], adaptive quantized control schemes for uncertain systems are proposed, where the hysteresis quantizer is originally designed. The system stability condition in [4] and [15] is closely dependent on the designed control signal, however it is not easy to check whether the condition is satisfied or not beforehand. This limitation is removed in [6], where an adaptive backstepping control scheme is proposed for uncertain strict-feedback nonlinear systems. However, the nonlinear functions are assumed to satisfy global Lipschitz conditions. Such restrictive conditions are relaxed in [16] and [17]. In [16], an implicit adaptive controller is developed for the system, where unknown parameters only appear in the last differential equation of the dynamic model. Nevertheless, it is not easy to obtain explicit controller since an equation related to a hyperbolic tangent function needs to be solved. In [17], a hyperbolic function is introduced into the adaptive controller to compensate the effect of input quantization. Similarly, a novel smooth function is adopted in [8] to generate the controller which can eliminate the effects of input quantization and actuator faults. In [9], a control algorithm for stochastic strict-feedback nonlinear systems with input quantization is developed based on fuzzy adaptive technique. In [20], the finite-time tracking control problem for quantized nonlinear systems is studied from the output feedback perspective.

Lately, the control problem of a system with state quantization is also deeply studied in [5], [10], [22], [23]. Note that the system dynamics in these works are precisely known. For uncertain linear systems, an adaptive supervisory control scheme has been presented in [24]. In [25], the control problem for linear systems with quantized measurements and bounded disturbances is considered, where an adaptive controller is designed. Adaptive backstepping technique has proven to be an effective tool to treat high-order systems with parametric uncertainties [26]–[28]. Fruitful results have also been reported on adaptive backstepping control of uncertain nonlinear systems with input quantization [6], [8], [16], [17]. However, adaptive backstepping control results to address uncertain systems with state quantization are very limited. The major challenge lies in that the differentiability of virtual control inputs is required. When quantized states are adopted directly to design virtual control in previous step, the signal is non-differentiable which makes the subsequent backstepping design steps be difficult to follow. Recently, an effective adaptive backstepping control scheme is proposed for uncertain nonlinear system with state quantization in [11], where the quantization errors of transmitted state are bounded by certain constants.

This paper is concerned with the adaptive backstepping control problem for uncertain nonlinear systems with both state and input quantization. So far, only a few works have been reported to handle the issue with simultaneous existence of quantizers in both uplink and downlink communication channels of networked control systems. References [29]–[32] are some examples, however only linear systems are considered. Such a problem is important as it is common in

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general networked systems that the sensor and the controller are connected via a shared communication network. In such networked control system, the state measurements and control signals are often processed both by quantizers. The main contributions of this paper are summarized as follows.

- The stabilization problem for a class of uncertain high-order nonlinear systems with both input and state quantization is investigated and a backstepping based adaptive control solution is provided. Compared with the existing results investigating only input quantization, the main challenge is that only quantized states can be utilized to construct the virtual control signal in each recursive step. Hence the virtual controls are discontinuous, of which the derivatives cannot be computed as often done in standard backstepping design procedure [26]. To overcome the difficulty, differentiable virtual controls are firstly designed by assuming the states are not quantized. Their partial derivatives multiplied by the quantized states are then utilized to complete the design of virtual controls for the case of state quantization.
- Note that some techniques are also presented in [11] to handle the effects of state quantization. However, only bounded error quantizers are considered in [11]. By contrast, a more general type of input/state sector bounded quantizers are considered in this paper. Since the quantization errors depend on the inputs of quantizers, they cannot be ensured bounded automatically. This constitutes as the main challenge to handle the effects of input and state quantizers in stability analysis. By well establishing the relation between the input signal and error state, the closed-loop system stability can be achieved by choosing proper design parameters.

For easier reading, we provide a description of the symbols used in this paper.

TABLE I
LIST OF THE SYMBOLS USED IN THIS PAPER

Symbols	Description
$x_i, i = 1, \dots, n$	states of the system
$\psi(\cdot)$ and $\phi(\cdot)$	known nonlinear functions
θ	unknown system parameter
$u(t)$	control input
L_ψ and L_ϕ	globally Lipschitz constants
L_θ	upper bound of system parameter θ
$Q_i(\cdot)$, $i = 1, 2$	generalized quantizer
$x^q = Q_i(x)$	quantized signal of variable x
δ_i and Δ_i, $i = 1, 2$	quantization parameters
$c_i, i = 1, \dots, n$ and Γ	design parameters
$L_{(\cdot)}$, $\Delta^1_{(\cdot)}$ and $\Delta^2_{(\cdot)}$	some constants related to design parameters, quantization parameters and system parameters L_ψ, L_ϕ, L_θ

II. Problem Statement

In this paper, a quantized feedback system is considered, as shown in Fig. 1. The system under consideration is represented by

$$x^{(n)}(t) = \psi\left(x(t), \dot{x}(t), \dots, x^{(n-1)}(t)\right) + \phi^T\left(x(t), \dot{x}(t), \dots, x^{(n-1)}(t)\right)\theta + u(t), \quad (1)$$

where $x^{(i)}(t) \in \Re^1$, $i = 0, 1, \dots, n-1$ are the states of the system. $u(t) \in \Re^1$ is the control input. $\psi \in \Re^1$ and $\phi \in \Re^r$ are known nonlinear functions. $\theta \in \Re^r$ denotes the vector of constant parameters, which are assumed to be unknown. This class of uncertain nonlinear systems have been widely investigated in [27], [33], [34].

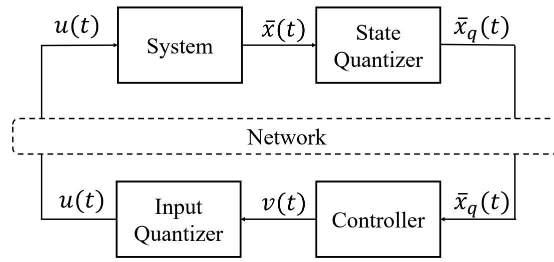


Fig. 1. Input and state quantized control systems

The states $\bar{x}(t) = [x(t), \dot{x}(t), \dots, x^{(n-1)}(t)]^T$ and the designed control signal $v(t)$ are quantized at the encoder side. The two quantizers are modeled as follows.

$$\bar{x}^q(t) = Q_1(\bar{x}(t)) = [Q_1(x), Q_1(\dot{x}), ..., Q_1(x^{(n-1)})]^T \quad (2)$$
$$u(t) = Q_2(v(t)) \quad (3)$$

For system (1), the conditions of the existence and uniqueness of solution are assumed to be satisfied. To generate control laws, we impose the following assumptions.

Assumption 1. *The functions ϕ and ψ are globally Lipschitz continuous. That is,*

$$|\psi(y_1) - \psi(y_2)| \leq L_\psi \parallel y_1 - y_2 \parallel \quad (4)$$
$$\parallel \phi(y_1) - \phi(y_2) \parallel \leq L_\phi \parallel y_1 - y_2 \parallel \quad (5)$$

where L_ψ and L_ϕ are positive constants, $y_1, y_2 \in \Re^n$ are real vectors.

Assumption 2. *The unknown parameter vector θ falls within a known compact convex set C_θ such that $\parallel \theta \parallel \leq L_\theta$ for any $\theta \in C_\theta$ and a positive constant L_θ.*

The control objective is to design a quantized controller $u = Q_2(v)$ with an appropriate adaptive control law for $v(t)$ in system (1) by using only quantized states $\bar{x}^q = Q_1(\bar{x})$ such that all the closed-loop signals are uniformly bounded.

In this paper, sector bounded quantizers are considered, which have the following property.

$$|Q_i(y) - y| \leq \delta_i \parallel y \parallel + \Delta_i, \quad i = 1, 2 \quad (6)$$

where y, $Q_i(y)$ represent the input and output of the quantizer Q_i, respectively. $0 < \delta_i < 1$ and $\Delta_i > 0$ are quantization parameters. It can be shown that logarithmic quantizer and hysteresis quantizer in [6], [35] satisfy the property (6).

III. Design of Adaptive Backstepping Controller

Define a group of new variables as $x_1 = x$, $x_i = x^{(i-1)}$, $i = 2, 3, \ldots, n$. System (1) can be rewritten as

$$\begin{aligned}\dot{x}_i &= x_{i+1}, \quad i = 1, ..., n-1 \\ \dot{x}_n &= u(t) + \psi(x_1, ..., x_n) + \theta^T \phi(x_1, ..., x_n) \\ &= u(t) + \psi(\bar{x}) + \theta^T \phi(\bar{x}) \end{aligned} \tag{7}$$

where $\bar{x} = [x_1, x_2, ..., x_n]^T$. $u(t)$ is the quantized input with $u(t) = Q_2(v(t))$, where $v(t)$ denotes the control input to be designed by utilizing only quantized states $\bar{x}^q = Q_1(\bar{x})$.

- If the states $\bar{x}$ and input v are not quantized, v_0 and $\hat{\theta}_0$ are adopted to denote the final control input and parameter estimator introduced for the unknown vector θ. The adaptive controller can be designed by applying standard backstepping design technique in [26].

Introduce the change of coordinates as

$$z_1 = x_1 \tag{8}$$
$$z_i = x_i - \alpha_{i-1}, \; i = 2, \ldots, n. \tag{9}$$

α_{i-1} is the virtual control function chosen as follows for each step i, which is a function of $x_1, .., x_{i-1}$.

$$\alpha_1 = -c_1 z_1 \tag{10}$$
$$\alpha_i = -c_i z_i - z_{i-1} + \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} x_{k+1}, \; i = 2, \ldots, n \tag{11}$$

where c_i, $1 \le i \le n$, are positive constant design parameters.

Remark 1. *It can be shown by induction proof that $\frac{\partial \alpha_i}{\partial x_k}$, $k = 1, \ldots, i$ are constants depending on $c_1, \ldots, c_i$. For $i = 1$, $\frac{\partial \alpha_1}{\partial x_1} = -c_1$. For $i = 2$, $\frac{\partial \alpha_2}{\partial x_1} = -1 - c_1 c_2$ and $\frac{\partial \alpha_2}{\partial x_2} = -c_1 - c_2$. Suppose that $\frac{\partial \alpha_{i-1}}{\partial x_k}$, $k = 1, \ldots, i-1$ and $\frac{\partial \alpha_{i-2}}{\partial x_k}$, $k = 1, \ldots, i-2$ are constants depending on $c_1, \ldots, c_{i-1}$ and $c_1, \ldots, c_{i-2}$, respectively. Then for $\alpha_i = -c_i(x_i - \alpha_{i-1}) - (x_{i-1} - \alpha_{i-2}) + \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} x_{k+1}$, $i = 3, \ldots, n$, we have*

$$\begin{aligned}\frac{\partial \alpha_i}{\partial x_1} &= c_i \frac{\partial \alpha_{i-1}}{\partial x_1} - \frac{\partial \alpha_{i-2}}{\partial x_1} \\ \frac{\partial \alpha_i}{\partial x_k} &= c_i \frac{\partial \alpha_{i-1}}{\partial x_k} - \frac{\partial \alpha_{i-2}}{\partial x_k} + \frac{\partial \alpha_{i-1}}{\partial x_{k-1}}, \quad k = 2, \ldots, i-1 \\ \frac{\partial \alpha_i}{\partial x_i} &= -c_i + \frac{\partial \alpha_{i-1}}{\partial x_{i-1}}.\end{aligned}$$

Clearly, $\frac{\partial \alpha_i}{\partial x_k}$, $k = 1, \ldots, i$ are constants depending on $c_1, \ldots, c_i$.

The final control v_0 is chosen as

$$v_0 = \alpha_n - \psi(\bar{x}) - \hat{\theta}_0^T \phi(\bar{x}) \tag{12}$$
$$\dot{\hat{\theta}}_0 = \Gamma \phi(\bar{x}) z_n \tag{13}$$

where Γ is a positive definite matrix.

Define an estimation error as $\tilde{\theta}_0 = \theta - \hat{\theta}_0$. Considering the Lyapunov function

$$V_0 = \sum_{i=1}^{n} \frac{1}{2} z_i^2 + \frac{1}{2} \tilde{\theta}_0^T \Gamma^{-1} \tilde{\theta}_0 \tag{14}$$

whose derivative can be computed as

$$\begin{aligned}\dot{V}_0 &= -\sum_{i=1}^{n-1} c_i z_i^2 + z_n \left(\alpha_n + \tilde{\theta}_0^T \phi(\bar{x}) + z_{n-1} - \dot{\alpha}_{n-1} \right) \\ &\quad -\tilde{\theta}_0^T \Gamma^{-1} \dot{\hat{\theta}}_0 \\ &= -\sum_{i=1}^{n} c_i z_i^2 \end{aligned} \tag{15}$$

Thus based on [26], it implies that all the signals are uniformly bounded.

- When state $x_i(t)$ is quantized with the quantizer $Q_1(x_i)$, the quantized state is defined as

$$x_i^q = Q_1(x_i), \; i = 1, 2, ..., n \tag{16}$$

Thus $\bar{x}^q(t)$ in (2) satisfies that $\bar{x}^q(t) = [x_1^q(t), \ldots, x_n^q(t)]^T$. To facilitate the adaptive controller design, the quantized input $u(t)$ is factored into the following form.

$$u(t) = Q_2(v(t)) = v(t) + d_u(t) \tag{17}$$

where $d_u = u(t) - v(t) \in \Re^1$. $v(t)$ denotes the control input to be designed by using the quantized states $\bar{x}^q$. Due to the property of considered quantizer in (6), the nonlinear part $d_u(t)$ satisfies the following property.

$$|d_u(t)| \le \delta_2 |v(t)| + \Delta_2 \tag{18}$$

Define the error variables as

$$z_1^q = x_1^q \tag{19}$$
$$z_i^q = x_i^q - \alpha_{i-1}^q, \; i = 2, ..., n \tag{20}$$

where

$$\alpha_1^q = -c_1 z_1^q \tag{21}$$
$$\alpha_i^q = -c_i z_i^q - z_{i-1}^q + \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} x_{k+1}^q, i = 2, ..., n \tag{22}$$

where c_i are positive constants. The adaptive controller is designed as

$$u(t) = Q_2(v) \tag{23}$$
$$\begin{aligned} v(t) &= \alpha_n^q - \psi(\bar{x}^q) - \hat{\theta}^T \phi(\bar{x}^q) \\ &= -c_n z_n^q - z_{n-1}^q + \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_k} x_{k+1}^q \\ &\quad -\psi(\bar{x}^q) - \hat{\theta}^T \phi(\bar{x}^q) \end{aligned} \tag{24}$$
$$\dot{\hat{\theta}} = Proj\{\Gamma \phi(\bar{x}^q) z_n^q\} \tag{25}$$

where $\hat{\theta}$ is the parameter estimator introduced for unknown vector θ, Γ is a positive definite matrix, $Proj\{\cdot\}$ is the projector operator given in [26]. Note that similar to [11], the partial derivatives $\frac{\partial \alpha_{n-i}}{\partial x_k}$, $i = 1, \ldots, n-1$, which are calculated from functions α_i designed as (11) for the case if states and input are not quantized, are adopted to design α_i^q, $i = 2, \ldots, n$ in (22).

Remark 2. *Note that only the quantized measurable states $x_i^q = Q_1(x_i)$, $i = 1, ..., n$ can be utilized to generate the virtual control α_i^q in (22), final control signal v in (24) and* *the parameter updating law in (25). If we follow standard backstepping design procedure in [26], α_i^q in (22) will be designed in the form of $-c_i z_i^q - z_{i-1}^q + \dot{\alpha}_{i-1}^q$. However, since α_{i-1}^q involves quantized states x_{i-1}^q, α_{i-1}^q becomes discontinuous and its derivative cannot be computed. This obstacle is removed by not differentiating α_i^q in the process of adaptive control design. Instead, α_i^q in (22) is constructed by utilizing the derivative terms $\sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} x_{k+1}^q$. As seen from Remark 1, the partial derivatives $\frac{\partial \alpha_{i-1}}{\partial x_k}$ can be calculated because the virtual control α_i, designed for the case if the states are not quantized, is differentiable in x_k.*

Remark 3. *Note that the projector operator $Proj\{\cdot\}$ is used in the parameter estimator (25) to ensure that $\| \hat{\theta} \| \leq L_\theta$. Let $\tilde{\theta} = \theta - \hat{\theta}$. It is also ensured that $\| \tilde{\theta} \| \leq L_\theta$. It is worth mentioning that the boundedness of θ, $\hat{\theta}$ and $\tilde{\theta}$ and the following property*

$$-\tilde{\theta}^T \Gamma^{-1} Proj\{\tau\} \leq -\tilde{\theta}^T \Gamma^{-1} \tau, \forall \hat{\theta} \in C_\theta,\ \theta \in C_\theta. \tag{26}$$

are helpful to guarantee the closed-loop system stability.

IV. Stability Analysis

To analyze the closed-loop system stability, we first establish some preliminary results in the form of the following lemmas. The proofs are given in Appendix A and Appendix B, respectively.

Lemma 1. *The state $\bar{x} = [x_1, ..., x_n]^T$, $\bar{x}^q = [x_1^q, \ldots, x_n^q]^T$, and the control $v(t)$ in (24) satisfy the following inequalities:*

$$\begin{aligned} \| \bar{x} \| &\leq L_x \| z \| && (27)\\ \| \bar{x}^q \| &\leq (1+\delta_1) L_x \| z \| + \Delta_1 && (28)\\ |v| &\leq L_v \| z^q \| && (29) \end{aligned}$$

where $z = [z_1, \ldots, z_n]^T$, $z^q = [z_1^q, \ldots, z_n^q]^T$. L_x and L_v are positive constants defined as follows, which relate to the design parameters $c_1, ..., c_n$.

$$\begin{aligned} L_x &\triangleq \left(\sum_{i=1}^{n} L_{x_i}^2 \right)^{\frac{1}{2}} && (30)\\ L_v &\triangleq L_{\alpha_n} + L_\psi L_x + L_\theta L_\phi L_x, && (31) \end{aligned}$$

where

$$\begin{aligned} L_{\alpha_1} &\triangleq c_1, \quad L_{x_1} \triangleq 1 && (32)\\ L_{x_i} &\triangleq 1 + L_{\alpha_{i-1}}, \quad i = 2, \ldots, n && (33)\\ L_{\alpha_i} &\triangleq c_i + 1 + \sum_{k=1}^{i-1} \left| \frac{\partial \alpha_{i-1}}{\partial x_k} \right| L_{x_i}, i = 2, \ldots, n. && (34) \end{aligned}$$

Lemma 2. *The effects of state quantization are bounded by functions of z as follows:*

$$|\psi(\bar{x}^q) - \psi(\bar{x})| \leq \Delta_1 \Delta_\psi^1 + \delta_1 \Delta_\psi^2 \| z \| \tag{35}$$

$$\| \phi(\bar{x}^q) - \phi(\bar{x}) \| \leq \Delta_1 \Delta_\phi^1 + \delta_1 \Delta_\phi^2 \| z \| \tag{36}$$

$$|z_i^q - z_i| \leq \Delta_1 \Delta_{z_i}^1 + \delta_1 \Delta_{z_i}^2 \| \bar{z}_i \|, \quad i = 1, \ldots, n \tag{37}$$

$$|\alpha_i^q - \alpha_i| \leq \Delta_1 \Delta_{\alpha_i}^1 + \delta_1 \Delta_{\alpha_i}^2 \| \bar{z}_i \|, \quad i = 1, \ldots, n \tag{38}$$

$$\| z^q - z \| \leq \Delta_1 \Delta_z^1 + \delta_1 \Delta_z^2 \| z \| \tag{39}$$

where $\bar{z}_i = [z_1, \ldots, z_i]^T$, and

$$\Delta_\psi^1 \triangleq L_\psi, \Delta_\psi^2 \triangleq L_\psi L_x \tag{40}$$

$$\Delta_\phi^1 \triangleq L_\phi, \Delta_\phi^2 \triangleq L_\phi L_x \tag{41}$$

$$\Delta_{z_1}^1 = \Delta_{z_1}^2 \triangleq 1 \tag{42}$$

$$\Delta_{\alpha_1}^1 = \Delta_{\alpha_1}^2 \triangleq c_1 \tag{43}$$

$$\Delta_{z_i}^1 \triangleq 1 + \Delta_{\alpha_{i-1}}^1, \ i = 2, \ldots, n \tag{44}$$

$$\Delta_{z_i}^2 \triangleq L_{x_i} + \Delta_{\alpha_{i-1}}^2, \ i = 2, \ldots, n \tag{45}$$

$$\Delta_{\alpha_i}^1 \triangleq c_i \Delta_{z_i}^1 + \Delta_{z_{i-1}}^1 + \sum_{k=1}^{i-1} \left| \frac{\partial \alpha_{i-1}}{\partial x_k} \right|, i = 2, \ldots, n \tag{46}$$

$$\Delta_{\alpha_i}^2 \triangleq c_i \Delta_{z_i}^2 + \Delta_{z_{i-1}}^2 + \sum_{k=1}^{i-1} \left| \frac{\partial \alpha_{i-1}}{\partial x_k} \right| L_{x_k}, i = 2, \ldots, n \tag{47}$$

$$\Delta_z^j \triangleq \left(\sum_{i=1}^{n} \left(\Delta_{z_i}^j \right)^2 \right)^{1/2}, \ j = 1, 2 \tag{48}$$

Based on Lemmas 1 and 2, the main results of this paper can be formally stated in the following theorem.

Theorem 1. *Consider the closed-loop system consisting of system (1) with state quantization (2) and input quantization (3) satisfying the property (6), the designed adaptive controller (23)-(24) and parameter estimator update law (25). If the design and quantization parameters satisfy*

$$c - \delta_2 \beta_1 - \delta_1 \beta_2 - \sum_{i=1}^{5} r_i \geq \epsilon > 0, \tag{49}$$

where

$$\begin{aligned} c &\triangleq \min\{c_1, c_2, ... c_{n-1}, c_n\} && (50)\\ \beta_1 &\triangleq L_v (1 + \delta_1 \Delta_z^2) && (51)\\ \beta_2 &\triangleq \Delta_{\alpha_n}^2 + \Delta_\psi^2 + B_2, && (52) \end{aligned}$$

ϵ, $r_i (i = 1, \ldots, 5)$ are positive constants. The following results can be guaranteed.

1) *All the closed-loop signals are uniformly bounded.*
2) *The stabilization error $\| z(t) \|$ is ultimately bounded as follows*

$$\| z(t) \| \leq \sqrt{\frac{M}{\epsilon}} \tag{53}$$

where

$$M \triangleq \frac{(\Delta_2)^2}{4r_1} + \frac{(\delta_2 L_v \Delta_1 \Delta_z^1)^2}{4r_2} + \frac{(\Delta_1 \Delta_{\alpha_n}^1)^2}{4r_3} + \frac{(\Delta_1 \Delta_\psi^1)^2}{4r_4} + \frac{(\Delta_1)^2 (B_1)^2}{4r_5} + (\Delta_1)^2 B_0 \tag{54}$$

with

$$B_0 \triangleq L_\theta \Delta_\phi^1 \Delta_{z_n}^1 \tag{55}$$

$$B_1 \triangleq L_\theta \Delta_\phi^1 + L_\theta L_\phi \delta_1 \Delta_{z_n}^2 + L_\theta L_\phi (1 + \delta_1) L_x \Delta_{z_n}^1 \tag{56}$$

$$B_2 \triangleq L_\theta \Delta_\phi^2 + L_\theta L_\phi (1 + \delta_1) L_x \Delta_{z_n}^2. \tag{57}$$

Proof. The Lyapunov function for the entire closed-loop system is defined as

$$V = \sum_{i=1}^{n} \frac{1}{2} z_i^2 + \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta}, \tag{58}$$

where z_i is given in (8)-(9) and $\tilde{\theta} = \theta - \hat{\theta}$. From (23)-(25), the derivative of V is calculated as

$$\begin{aligned}
\dot{V} &= -\sum_{i=1}^{n-1} c_i z_i^2 + z_{n-1} z_n - \tilde{\theta}^T \Gamma^{-1} \dot{\hat{\theta}} \\
&\quad + z_n \Big(v(t) + d_u - \alpha_n + \alpha_n + \psi + \phi^T \theta - \dot{\alpha}_{n-1} \Big) \\
&= -\sum_{i=1}^{n-1} c_i z_i^2 - \tilde{\theta}^T \Gamma^{-1} \dot{\hat{\theta}} + z_n \Big(\alpha_n^q - \psi(\bar{x}^q) - \hat{\theta}^T \phi(\bar{x}^q) \\
&\quad - \alpha_n + \alpha_n + \psi + \theta^T \phi - \dot{\alpha}_{n-1} + z_{n-1} + d_u \Big) \\
&= -\sum_{i=1}^{n-1} c_i z_i^2 + z_n \Big(\alpha_n - \dot{\alpha}_{n-1} + z_{n-1} \Big) \\
&\quad + z_n d_u + z_n \Big(\alpha_n^q - \alpha_n \Big) + z_n \Big(\psi(\bar{x}) - \psi(\bar{x}^q) \Big) \\
&\quad + z_n \Big(\theta^T \phi(\bar{x}) - \hat{\theta} \phi(\bar{x}^q) \Big) - \tilde{\theta}^T \Gamma^{-1} \dot{\hat{\theta}} \\
&\leq -\sum_{i=1}^{n} c_i z_i^2 + |z_n||\alpha_n^q - \alpha_n| + |z_n||\psi(\bar{x}) - \psi(\bar{x}^q)| \\
&\quad + |z_n d_u| + \Big(\theta^T \phi(\bar{x}) z_n - \hat{\theta}^T \phi(\bar{x}^q) z_n - \tilde{\theta}^T \phi(\bar{x}^q) z_n^q \Big)
\end{aligned} \tag{59}$$

Using the properties (5), (6), (28) in Lemma 1, and (36) and (37) in Lemma 2, the last three terms in (59) satisfies the following inequality

$$\begin{aligned}
& \theta^T \phi(\bar{x}) z_n - \hat{\theta}^T \phi(\bar{x}^q) z_n - \tilde{\theta}^T \phi(\bar{x}^q) z_n^q \\
= \; & \theta^T \phi(\bar{x}) z_n - \theta^T \phi(\bar{x}^q) z_n + \tilde{\theta} \phi(\bar{x}^q) z_n - \tilde{\theta} \phi(\bar{x}^q) z_n^q \\
\leq \; & \| \theta \| \, |z_n| \, \| \phi(\bar{x}) - \phi(\bar{x}^q) \| + \| \tilde{\theta} \| \| \phi(\bar{x}^q) \| \, |z_n - z_n^q| \\
\leq \; & L_\theta \|z\| (\Delta_1 \Delta_\phi^1 + \delta_1 \Delta_\phi^2 \|z\|) \\
& + L_\theta L_\phi \|\bar{x}^q\| (\Delta_1 \Delta_{z_n}^1 + \delta_1 \Delta_{z_n}^2 \|z\|) \\
\leq \; & (\Delta_1)^2 B_0 + \Delta_1 B_1 \| z \| + \delta_1 B_2 \| z \|^2
\end{aligned} \tag{60}$$

where B_j, $j = 0, 1, 2$, are defined in (55)-(57).

Using (18) and the properties (29), (39), the term $|z_n d_u|$ in (59) satisfies

$$\begin{aligned}
|z_n d_u| &\leq \Delta_2 |z_n| + \delta_2 |z_n||v| \\
&\leq \Delta_2 |z_n| + \delta_2 |z_n| L_v \| z^q \| \\
&\leq \Delta_2 |z_n| + \delta_2 |z_n| L_v (\| z \| + \Delta_1 \Delta_z^1 + \delta_1 \Delta_z^2 \| z \|) \\
&\leq \delta_2 L_v (1 + \delta_1 \Delta_z^2) \| z \|^2 + (\Delta_2 + \delta_2 L_v \Delta_1 \Delta_z^1) \|z\|
\end{aligned} \tag{61}$$

Using the properties (35), (38), (60), (61) and the Young's inequality with positive parameter r (i.e. $|ab| \leq ra^2 + \frac{b^2}{4r}$), (59) is further computed as

$$\begin{aligned}
\dot{V} &\leq -\sum_{i=1}^{n} c_i z_i^2 + \delta_2 L_v (1 + \delta_1 \Delta_z^2) \| z \|^2 \\
&\quad + (\Delta_2 + \delta_2 L_v \Delta_1 \Delta_z^1) \| z \| + \Delta_1 \Delta_{\alpha_n}^1 |z_n| \\
&\quad + \delta_1 \Delta_{\alpha_n}^2 \| z \|^2 + \Delta_1 \Delta_\psi^1 |z_n| + \delta_1 \Delta_\psi^2 \| z \|^2 \\
&\quad + (\Delta_1)^2 B_0 + \Delta_1 B_1 \| z \| + \delta_1 B_2 \| z \|^2 \\
&\leq -\left(c - \delta_2 \beta_1 - \delta_1 \beta_2 - \sum_{i=1}^{5} r_i \right) \| z(t) \|^2 + M \\
&\leq -\epsilon \| z(t) \|^2 + M
\end{aligned} \tag{62}$$

where the inequality $c - \delta_2 \beta_1 - \delta_1 \beta_2 - \sum_{i=1}^{5} r_i \geq \epsilon > 0$ in (49) has been used. ϵ, $r_i (i = 1, \ldots, 5)$ are positive constants and c, β_1, β_2, M are defined in (50), (51), (52), (54), respectively. It is shown from (62) that $\dot{V} < 0$, $\forall \| z(t) \| > \sqrt{\frac{M}{\epsilon}}$. Thus the ultimate bound of $z(t)$ satisfies (53).

From (39) and the boundedness of z, z^q is bounded. Thus x_1^q and α_1^q in (21) is bounded. From (20), x_2^q is bounded. Thus α_2^q is bounded. By the same token, the boundedness of x_i^q and α_i^q for $i = 3, \ldots, n$ can be shown. The boundedness of $\hat{\theta}$ is ensured by the projection operator (25) as discussed in Remark 3. Based on Assumption 1, it implies that $v(t)$ in (24) is bounded. Therefore, the boundedness of all the closed-loop signals can be ensured. □

Remark 4. *Though it is naturally motivated by the fact that measurement and control signals are transmitted via a common network in networked systems, only a few results have been developed for linear systems with both quantizations [30]–[32]. So far, no result is available for uncertain nonlinear systems. The compensation is a non-trivial work compared to the exsiting results investigating input quantization solely including [6], [8], [9], [16], [17], [20], [21]. The main challenge and key technique to design adaptive backstepping controller [26] for uncertain higher-order systems with quantized states are discussed in Remark 2.*

Note that similar technique of using derivatives $\sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} x_{k+1}^q$ in the design with quantized states is also presented in [11]. However, input quantization is not considered in [11], and the state quantizer satisfies the bounded error property, i.e. $|Q_1(x_i) - x_i| \leq \Delta_1$. It refers a special case that $\delta_i = 0$ for (6) in this paper. As emphasized in [17], compared to the bounded error quantizers, the sector bounded quantizers have unequal quantization levels and are the coarsest quantizers which minimize the average rate of communication instances and are easy to implement.

Since the quantization errors depend on the inputs of quantizers, they cannot be ensured bounded automatically. This constitutes as the main challenge to handle the effects of both state and input quantization in stability analysis. This results in the input quantization error depending on the control signal v as in (18), hence the effect of quantized input cannot be simply treated as bounded disturbance term. The key step to handle the effect of input quantization $Q_2(v)$ is to establish the relation between the control signal v and the system state z by showing the property (29) in Lemma 1 and (39) in Lemma 2.

Since only quantized states $[x_1^q, ..., x_n^q]^T$ can be adopted in control design, the key step to compensate for the effect of state quantization in stability analysis is to compensate for the effects of the terms $z_n(\alpha_n^q - \alpha_n)$, $z_n(\psi(\bar{x}) - \psi(\bar{x}^q))$

and $(\theta^T\phi(\bar{x})z_n - \hat{\theta}^T\phi(\bar{x}^q)z_n - \tilde{\theta}^T\phi(\bar{x}^q)z_n^q)$ in (59). By establishing the properties (27), (28) in Lemma 1 and (35)-(38) in Lemma 2, these terms can be bounded by the functions related to the state z. Thus all these effects of the last six terms in (59) can be compensated as shown in (62) if the condition (49) is satisfied.

Remark 5. *Note that the inequality (49) provides some insights on how to choose the quantization parameters δ_1 and δ_2. It implies that the number of quantization levels for δ_i is finite since all the closed-loop signals are bounded. Besides, β_1 and β_2 are computable from the definitions (51) and (52), which depends on the design parameters c_i, the system parameters in Lemma 1 and Lemma 2, and L_ψ, L_ϕ which are assumed to be known in Assumption 1.*

Remark 6. *Another important difference between current paper and [11] lies in the adoption of projection technique in the design of adaptive law (25). Thus the boundedness of $\|\tilde{\theta}\|$ can be ensured. Considering the case with both input and state quantizers satisfying bounded error property as mentioned in Remark 4, by following similar analysis in Section IV, the closed-loop system stability can be shown. In particular, all the terms involving $\|z\|^2$ as derived in (60) and (61) will be absent since $\delta_i = 0$ in this case. Then the result in the form of $\dot{V} \leq -\epsilon\|z\|^2 + M$ can be obtained without the need of a sufficient condition related to the control and quantization parameters. This also constitutes as one major improvement achieved in this paper with comparison to [11] in the case of bounded error quantizers.*

Remark 7. *It can be observed from (54)-(56) that the upper bound of the stabilization error in the sense of (53) can be decreased if the quantization parameters δ_i and Δ_i are decreased while all design parameters c_i are kept unchanged. One limitation of this paper is that only sufficient condition of closed-loop system stability is provided, whereas no explicit supremum of the stabilization error accurately characterizing its value range is derived. Hence, further investigation is required to make the theoretical results be less conservative in practical application.*

V. Experimental Results

To validate the effectiveness of the presented adaptive control scheme, experimental studies have been conducted based on a hardware-in-the-loop platform established with a two-wheeled robot (Quanser Qbot 2e) and positioning system as illustrated in Fig. 2.

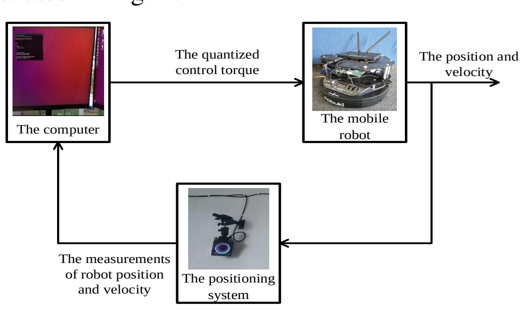


Fig. 2. The hardware-in-the-loop experimental platform.

The kinematic model of the mobile robot is given below [36].

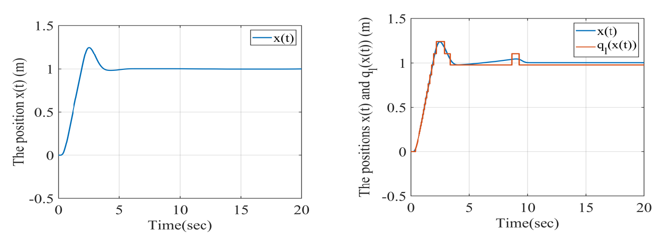
$$\dot{\eta} = \begin{bmatrix} \cos\psi & 0 \\ \sin\psi & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \tag{63}$$

where v and ω are the linear and angular velocities applied to the robot, respectively. $\eta = [x, y, \psi]^T$ represents the vector of position and orientation. To validate the theoretical results, we set the angular velocity at $\omega(t) = 0$ and the initial value of $\psi(t)$ at $\psi(0) = 0$. Thus $\psi(t) = 0$, $y(t) = 0$, $\forall t \geq 0$. Besides, a virtual dynamic system is constructed in the computer for v as $\dot{v} + \theta v^2 = q_l(u(t))$, where θ is the damping coefficient, $u(t)$ is the control torque, $q_l(\cdot)$ represents the input quantization function. Combining the dynamics of the states x and v, the simplified model characterizing one-dimensional movement of the robot along x axis is given as follows.

$$\dot{x} = v \tag{64}$$

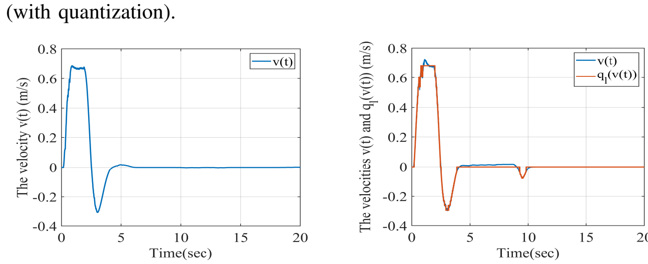
$$\dot{v} = -\theta v^2 + q_l(u(t)) \tag{65}$$

In the experiment, the control objective is to regulate $x(t)$ at a desired fixed point $x_s = 1$. $\theta = 0.1$ is supposed to be unknown. Besides, only the quantized states $q_l(x)$ and $q_l(v)$ can be adopted to design the control torque $u(t)$. All the initial conditions are chosen at 0, i.e. $x(0) = v(0) = \hat{\theta}(0) = 0$. The designed control parameters are chosen as $c_1 = 1.5$, $c_2 = 0.9$, $\Gamma = 0.1$ and $L_\theta = 0.2$. The logarithmic quantizers introduced in [6] with quanzation parameters $\delta = 0.06$, $y_{\min,state}$ = 0.04 and $y_{\min,input}$ = 0.08 are adopted to quantize the states $(x(t), v(t))$ and control torque $(u(t))$. It can be checked that condition (28) is satisfied with the designed control and quantization parameters. In Fig. 3, 4 and 5, the performance of position $x(t)$, velocity $v(t)$ and control torque $u(t)$ are given with comparison to the case without quantization. Clearly, all the observed states and control torque are bounded in both cases. Moreover, desired regulation performance is achieved in the case with both input and state quantization, while the transmitting cost can be reduced.



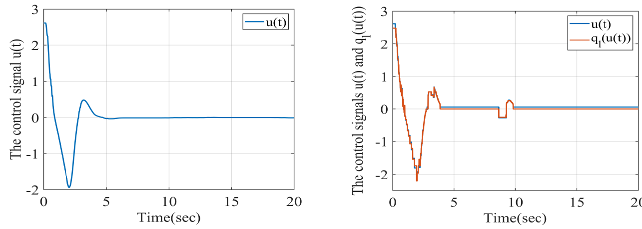
(a) $x(t)$ in Case I (b) $x(t)$ and $q_l(x(t))$ in Case II

Fig. 3. The comparison of $x(t)$ for Case I (without quantization) and Case II (with quantization).



(a) $v(t)$ in Case I (b) $v(t)$ and $q_l(v(t))$ in Case II

Fig. 4. The comparison of $v(t)$ for Case I (without quantization) and Case II (with quantization).



(a) $u(t)$ in Case I (b) $u(t)$ and $q_l(u(t))$ in Case II

Fig. 5. The comparison of $u(t)$ for Case I (without quantization) and Case II (with quantization).

VI. Conclusion

In this paper, adaptive backstepping controllers have been designed for uncertain nonlinear systems with both state and input quantization. Compared to the existing result on adaptive backstepping control with quantized states, a more general sector bounded quantizers are considered. Besides, projection technique is adopted to modify the design of adaptive law. Thus the closed-loop system stability can be shown without the need of sufficient condition dependent on the designed control and quantization parameters. Note that the nonlinear functions involved in the system's dynamics need satisfy the Lipschitz condition, which is required in many related references. Moreover, the proposed control method is only applicable when all the system's states are measurable. Hence, it will be an interesting topic to remove the Lipschitz condition and investigate adaptive quantized control with unmeasurable states.

Appendix A
Proof of Lemma 1

Proof. From the definitions z_i in (8)-(9) and α_i in (10)-(11), it can be derived that

$$|x_1| = |z_1| \tag{66}$$

$$|\alpha_1| \leq c_1|z_1| \triangleq L_{\alpha_1}|z_1| \tag{67}$$

$$\begin{aligned} |x_2| &\leq |z_2 + \alpha_1| \leq |z_2| + L_{\alpha_1}|z_1| \\ &\leq (1 + L_{\alpha_1}) \parallel [z_1, z_2]^T \parallel \triangleq L_{x2} \parallel [z_1, z_2]^T \parallel \end{aligned} \tag{68}$$

$$\begin{aligned} |\alpha_2| &\leq c_2|z_2| + |z_1| + \left|\frac{\partial \alpha_1}{\partial x_1} x_2\right| \\ &\leq \left(c_2 + 1 + \left|\frac{\partial \alpha_1}{\partial x_1}\right| L_{x_2}\right) \parallel [z_1, z_2]^T \parallel \\ &\triangleq L_{\alpha_2} \parallel [z_1, z_2]^T \parallel \end{aligned} \tag{69}$$

Following the similar procedure for $i = 1, 2, .., n-1$, we have

$$\begin{aligned} |x_i| &\leq |z_i + \alpha_{i-1}| \leq |z_i| + L_{\alpha_{i-1}} \parallel [z_1, ..., z_{i-1}]^T \parallel \\ &\leq (1 + L_{\alpha_{i-1}}) \parallel [z_1, z_2, ..., z_i]^T \parallel \\ &\triangleq L_{x_i} \parallel [z_1, z_2, ..., z_i]^T \parallel \end{aligned} \tag{70}$$

$$\begin{aligned} |\alpha_i| &\leq c_i|z_i| + |z_{i-1}| + \left|\sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} x_{k+1}\right| \\ &\leq \left(c_i + 1 + \sum_{k=1}^{i-1} \left|\frac{\partial \alpha_{i-1}}{\partial x_k}\right| L_{x_i}\right) \parallel [z_1, z_2, ..., z_i]^T \parallel \\ &\triangleq L_{\alpha_i} \parallel [z_1, z_2, ..., z_i]^T \parallel \end{aligned} \tag{71}$$

$$\begin{aligned} \parallel \bar{x} \parallel &= \left(\sum_{i=1}^{n} x_i^2\right)^{1/2} \leq \left(\sum_{i=1}^{n} L_{xi}^2 \parallel [z_1, z_2, ..., z_i]^T \parallel^2\right)^{1/2} \\ &\leq \left(\sum_{i=1}^{n} L_{xi}^2\right)^{\frac{1}{2}} \parallel z(t) \parallel \triangleq L_x \parallel z(t) \parallel \end{aligned} \tag{72}$$

In view of (6), $\bar{x}^q$ satisfies

$$\begin{aligned} \parallel \bar{x}^q \parallel \leq & \parallel \bar{x} \parallel + \delta_1 \parallel \bar{x} \parallel + \Delta_1 \\ &\leq (1 + \delta_1) L_x \parallel z \parallel + \Delta_1 \end{aligned} \tag{73}$$

From (19)-(22), by following the analysis in (66)-(72), we obtain that

$$|x_i^q| \leq L_{x_i} \|[z_1^q, z_2^q, \ldots, z_i^q]^T\| \tag{74}$$

$$|\alpha_i^q| \leq L_{\alpha_i} \|[z_1^q, z_2^q, \ldots, z_i^q]^T\| \tag{75}$$

$$\|\bar{x}^q\| \leq L_x \|z^q\| \tag{76}$$

From (24) and (74)-(76), we have

$$\begin{aligned} |v| &\leq |\alpha_n^q| + |\psi(\bar{x}^q)| + \parallel \hat{\theta} \parallel \parallel \phi(\bar{x}^q) \parallel \\ &\leq L_{\alpha_n} \parallel z^q \parallel + L_\psi \parallel \bar{x}^q \parallel + L_\theta L_\phi \parallel \bar{x}^q \parallel \\ &\leq (L_{\alpha_n} + L_\psi L_x + L_\theta L_\phi L_x) \parallel z^q \parallel \triangleq L_v \parallel z^q \parallel \end{aligned} \tag{77}$$

□

Appendix B
Proof of Lemma 2

Proof. Using Lipschitz conditions of ψ and ϕ in Assumption 2, the following expressions can be derived.

$$\begin{aligned} & |\psi(x_1^q, .., x_n^q) - \psi(x_1, .., x_n)| \\ &\leq L_\psi \parallel \bar{x}^q - \bar{x} \parallel \leq L_\psi \Delta_1 + L_\psi \delta_1 \parallel \bar{x} \parallel \\ &\leq L_\psi \Delta_1 + L_\psi \delta_1 L_x \parallel z \parallel \triangleq \Delta_1 \Delta_\psi^1 + \delta_1 \Delta_\psi^2 \parallel z \parallel \end{aligned} \tag{78}$$

$$\begin{aligned} & \parallel \phi(x_1^q, .., x_n^q) - \phi(x_1, .., x_n) \parallel \\ &\leq L_\phi \parallel \bar{x}^q - \bar{x} \parallel \\ &\leq L_\phi \Delta_1 + L_\phi \delta_1 L_x \parallel z \parallel \triangleq \Delta_1 \Delta_\phi^1 + \delta_1 \Delta_\phi^2 \parallel z \parallel \end{aligned} \tag{79}$$

From (9)-(10), and (22)-(23), it is shown that

$$\begin{aligned} |z_1^q - z_1| &= |x_1^q - x_1| \leq \Delta_1 + \delta_1 |x_1| \\ &\triangleq \Delta_1 \Delta_{z_1}^1 + \delta_1 \Delta_{z_1}^2 \parallel z_1 \parallel \end{aligned} \tag{80}$$

$$\begin{aligned} |\alpha_1^q - \alpha_1| &= |-c_1(z_1^q - z_1)| \leq c_1 \Delta_1 \Delta_{z_1}^1 + c_1 \delta_1 \Delta_{z_1}^2 \parallel z_1 \parallel \\ &\triangleq \Delta_1 \Delta_{\alpha_1}^1 + \delta_1 \Delta_{\alpha_1}^2 \parallel z_1 \parallel \end{aligned} \tag{81}$$

$$\begin{aligned} |z_2^q - z_2| &= |(x_2^q - x_2) - (\alpha_1^q - \alpha_1)| \\ &\leq \Delta_1 + \delta_1 L_{x_2} \|\bar{z}_2\| + \Delta_1 \Delta_{\alpha_1}^1 + \delta_1 \Delta_{\alpha_1}^2 \parallel z_1 \parallel \\ &\leq \Delta_1 \Delta_{z_2}^1 + \delta_1 \Delta_{z_2}^2 \parallel \bar{z}_2 \parallel \end{aligned} \tag{82}$$

$$\begin{aligned} |\alpha_2^q - \alpha_2| &= \left|-c_2(z_2^q - z_2) - (z_1^q - z_1) + \frac{\partial \alpha_1}{\partial x_1}(x_2^q - x_2)\right| \\ &\leq \Delta_1 \left(c_2 \Delta_{z_2}^1 + \Delta_{z_1}^1 + \left|\frac{\partial \alpha_1}{\partial x_1}\right|\right) \\ &\quad + \delta_1 \left(c_2 \Delta_{z_2}^2 + \Delta_{z_1}^2 + \left|\frac{\partial \alpha_1}{\partial x_1}\right| L_{x_2}\right) \parallel \bar{z}_2 \parallel \\ &\triangleq \Delta_1 \Delta_{\alpha_2}^1 + \delta_1 \Delta_{\alpha_2}^2 \parallel \bar{z}_2 \parallel \end{aligned} \tag{83}$$

where $\bar{z}_i = [z_1, \ldots, z_i]^T$. Along the analysis lines of z_i in (9), α_i in (11), z_i^q in (20), α_i^q in (22), we have

$$\begin{aligned}
& |z_i^q(x_1^q,..,x_i^q) - z_i(x_1,..,x_i)| \\
= & \left|(x_i^q - x_i) - (\alpha_{i-1}^q - \alpha_{i-1})\right| \\
\leq & \Delta_1 + \delta_1 L_{x_i} \parallel \bar{z}_i \parallel + \Delta_1\Delta_{\alpha_{i-1}}^1 + \delta_1\Delta_{\alpha_{i-1}}^2 \parallel \bar{z}_{i-1} \parallel \\
\leq & \Delta_1\Delta_{z_i}^1 + \delta_1\Delta_{z_i}^2 \parallel \bar{z}_i \parallel
\end{aligned} \tag{84}$$

$$\begin{aligned}
& |\alpha_i^q(x_1^q,..,x_i^q) - \alpha_i(x_1,..,x_i)| \\
\leq & \left|-c_i(z_i - z_i^q) - (z_{i-1}^q - z_{i-1})\right| \\
& + \left|\sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k}\left(x_{k+1}^q - x_{k+1}\right)\right| \\
\leq & \Delta_1 \left(c_i\Delta_{z_i}^1 + \Delta_{z_{i-1}}^1 + \sum_{k=1}^{i-1}\left|\frac{\partial \alpha_{i-1}}{\partial x_k}\right|\right) \\
& + \delta_1 \left(c_i\Delta_{z_i}^2 + \Delta_{z_{i-1}}^2 + \sum_{k=1}^{i-1}\left|\frac{\partial \alpha_{i-1}}{\partial x_k}\right| L_{x_k}\right) \parallel \bar{z}_i \parallel \\
\leq & \Delta_1\Delta_{\alpha_i}^1 + \delta_1\Delta_{\alpha_i}^2 \parallel \bar{z}_i \parallel
\end{aligned} \tag{85}$$

$$\begin{aligned}
& \parallel z^q - z \parallel \\
= & \left(\sum_{i=1}^n |z_i^q - z_i|^2\right)^{1/2} \\
\leq & \left(\sum_{i=1}^n \left(\Delta_1\Delta_{z_i}^1\right)^2\right)^{1/2} + \left(\sum_{i=1}^n \left(\delta_1\Delta_{z_i}^2\right)^2\right)^{1/2} \parallel z \parallel \\
\triangleq & \Delta_1\Delta_z^1 + \delta_1\Delta_z^2 \parallel z \parallel
\end{aligned} \tag{86}$$

□

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