

# Permanent deformation and fatigue damage interaction in asphalt concrete using energy approach

M. Alamnie & E. Tadesse

*Department of Engineering Sciences, Civil and Structural Engineering, University of Agder, Grimstad, Norway*

I. Hoff

*Department of Civil and Transport Engineering, Norwegian University of Science and Technology, Trondheim, Norway*

**ABSTRACT:** The interaction of fatigue and permanent deformation is very complex phenomenon and little attempt is made on this topic. This paper presents an investigation on the interaction of the two damages using the energy approach. Laboratory tests were conducted for both fatigue and permanent deformation on two different asphalt concrete in a sequential test procedure. A new failure criterion is proposed based on the dissipated energy ratio (DER). The proposed criterion for permanent deformation gives more damage indicator (inflection points) than the conventional flow number/strain rate criterion. The sequential test procedure is also found economical that can be standardized. From the study, strain hardening (pre-deformed) accelerates fatigue cracking susceptible. Similarly, high fatigued samples are more prone to permanent deformation than new samples. The energy approach is convenient for potential coupling of fatigue and permanent deformation interactions.

**Keywords:** Permanent deformation, fatigue, dissipated energy ratio, sequential test

## 1 INTRODUCTION

As asphalt concrete pavement is subjected to repetitive loads, different damage modes develop over its lifetime. Fatigue cracking and permanent deformation are the two dominant pavement damage mechanisms. One of the distinguishing features of fatigue and permanent deformation is the loading mode. In fatigue damage, cracks grow perpendicular to the tensile loading direction, whereas in permanent deformation, wing cracks can develop parallel to the load direction (Dyskin et al., 2003). The influence of temperature is another crucial factor in distinctive damage evolution. Fatigue is critical at intermediate and low temperatures. On the other hand, permanent deformation is high-temperature damage. At elevated temperatures, the critical energy threshold that causes fatigue cracking increases (or the mixture relaxes faster). Consequently, more energy is needed to initiate crack (Onifade et al., 2015, Sangpetngam et al., 2003). On the other hand, plastic flow and aggregate re-orientation dominate at high temperatures. Traditionally, bottom-up cracking of the asphalt concrete layer was the primary fatigue damage mechanism. However, experimental and field observations showed that top-down cracking due to tire compression is also the mechanism for fatigue damage (Pellinen et al., 2004, Roque et al., 2004). Crack initiation in compression occurs when the viscoplastic strain hardening reaches saturation at the flow number (FN).

The damages in asphalt concrete evolve through energy dissipation due to viscous flow leading to *fatigue cracking* and plastic flow for *permanent deformation* (Widyatmoko et al., 1999), and some part of the energy is transferred into heat (Di Benedetto et al., 2011). The energy dissipation caused material ductility exhaustion, hardening, and viscoplastic flow. Different energy-based failure criteria were proposed using the classic energy balance principle (Ghuzlan and Carpenter, 2000, Anderson et al., 2001, Shen et al., 2006, Korsunsky et al., 2007). The existing models treat fatigue and permanent deformation damage independently, and the complex interaction between the two damage modes is still not well researched. One of the limitations is the lack of an integrated testing protocol for fatigue-permanent deformation interaction. Some attempts can be found in literature, such as an Indirect Tension (IDT) testing for deformation and fracture (Bahadori et al., 2015) and a haversine loading waveform in fatigue tests (Liu et al., 2020, Gupta and Atul Narayan, 2019). The haversine load is used because it can be decomposed into pure creep (for permanent deformation part) and pure sinusoidal (fatigue part) components. However, the creep-recovery behavior cannot be captured with such approaches.

Moreover, field and laboratory observations have shown that fatigue cracking accompany permanent deformation (rutting) (Lundstrom et al., 2007, Pellinen et al., 2004). In addition, the same load causes both damages, and strain-hardening and cracking can develop simultaneously at intermediate temperatures. Therefore, the independent treatment of the two damages is an oversimplification and far from the actual condition. The pavement is more susceptible to fatigue cracking in the cold seasons, and micro-cracks can initiate. These pre-existing cracks can significantly accelerate the permanent deformation at high pavement temperatures in hot seasons. The inverse is true that viscoplastic strain hardened during the hot season is more susceptible to fatigue cracking in cold seasons. Both damages are critical at intermediate temperatures. Hence, the independent treatment of the two damages sets clear limitations for the mechanistic pavement design.

In this paper, an attempt is made to investigate the effect of fatigue on permanent deformation and vice versa using an energy approach. Several asphalt concrete samples were tested for fatigue (F) and permanent deformation (PD) in a Sequential Test (ST) procedure. The tests were performed in two different orders: the F-PD and PD-F sequences. A new energy-based failure criterion is proposed and validated using experimental data from two different asphalt mixtures.

## 2 ENERGY-BASED MODELS

The dissipated energy in a cyclic load is the area under the stress-strain hysteresis loop. In a cyclic creep-recovery test, material hardening grows, irrecoverable viscoplastic strain accumulates, and the hysteresis loops shift horizontally (Figure 1a). On the other hand, the idealized hysteresis loop in the cyclic fatigue test causes stiffness reduction and phase angle increment with negligible viscoplastic strain (hysteresis loops do not shift horizontally) (Figure 1b). Nevertheless, the stress-strain hysteresis in simultaneous damage conditions could evolve in a complex manner due to hardening-softening, healing, etc. The dissipated energy (DE) is expressed by the following integral, where  $\sigma(t)$  is the stress function and  $\frac{\partial \varepsilon(t)}{\partial \tau}$  is the strain rate.

$$DE = \int \sigma(t) \frac{\partial \varepsilon(\tau)}{\partial \tau} d\tau \quad (1)$$

The DE in fatigue and permanent deformation damages can be computed using the appropriate stress and strain rate functions in Equation 1.

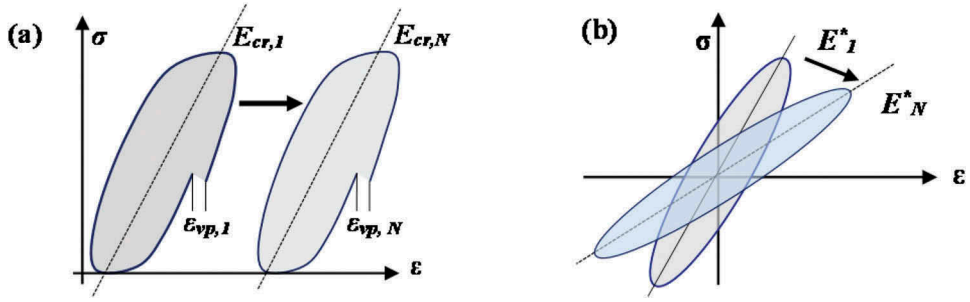


Figure 1. Schematic – stress-strain hysteresis loops (a) creep-recovery (b) tension-compression fatigue.

### 2.1 Fatigue failure criterion

For a strain-controlled fatigue test with sinusoidal strain wave, the dissipated energy due to fatigue ( $DE_f$ ) is obtained by substituting the sinusoidal stress ( $\sigma_s = \sigma_i \sin \omega t$ ) and strain ( $\epsilon_s = \epsilon_i \sin(\omega t - \varphi_i)$ ) signals into Equation 1 and integrating ( $\sigma_i$ ,  $\epsilon_i$ ,  $\omega$  and  $\varphi_i$  – stress amplitude, strain amplitude, angular frequency, and phase angle measured at cycle,  $i$ ).

$$DE_f = 2\pi\sigma_i\epsilon_i \sin(\varphi_i) \quad (2)$$

The dissipated energy ratio (DER) criterion is used (Ghuzlan and Carpenter, 2000) as a damage indicator.

$$DER = n \times \frac{DE_1}{DE_n} \quad (3)$$

Where  $DE_1$  –dissipated energy at initial cycle,  $DE_n$  – dissipated energy at  $n^{\text{th}}$  load cycle. For a sinusoidal loading, the DER can be expressed as,

$$DER = n \times \frac{\pi E^*_1 \epsilon_1^2 \sin \varphi_1}{\pi E^*_n \epsilon_n^2 \sin \varphi_n} \quad (4)$$

For a test in controlled-strain mode,  $\epsilon_1 = \epsilon_n$  and with approximation  $\frac{\sin \varphi_1}{\sin \varphi_n} \approx \frac{\varphi_1}{\varphi_n}$ , then DER is found as;

$$DER = \left( \frac{n}{E^*_n} \right) \left( \frac{E^*_1 \sin \varphi_1}{\sin \varphi_n} \right) = n \left( \frac{E^*_1}{E^*_n} \right) \left( \frac{\varphi_1}{\varphi_n} \right) \quad (5)$$

### 2.2 Permanent deformation failure criterion

The flow number (FN) has been used as a general failure criterion for permanent deformation (Biligiri et al., 2007). Flow number is a cycle where the viscoplastic strain rate start ascending (or the second derivative of the strain function changes from negative to a positive value, i.e.,  $\epsilon''_{vp} > 0$ ). The well-known Francken model (Francken and Clauwaert, 1987) is used to fit the permanent deformation test data (where A, B, C, D – are model coefficients, N – number of cycles).

$$\epsilon_{vp} = AN^B + C(e^{DN} - 1) \quad (6)$$

$$\dot{\epsilon}_{vp} = ABN^{B-1} + CDe^{DN} \quad (7)$$

The dissipated energy due to permanent deformation ( $DE_{pd}$ ) is obtained by substituting a haversine stress pulse  $\sigma_h = \sigma_0 \sin^2\left(\frac{\omega t}{2}\right)$  where  $\sigma_0$  is stress amplitude and strain rate (Equation 7) into Equation 1.

$$DE_{PD} = \frac{\pi \sigma_0}{\omega} \dot{\varepsilon}_{vp} \quad (8)$$

The permanent deformation failure criterion using DER takes the following form.

$$DER_{PD} = n \frac{DE_{PD1}}{DE_{PDN}} = n \frac{AB + CDe^D}{ABN^{B-1} + CDe^{DN}} \quad (9)$$

$n$  is current cycle number,  $DE_{PD1}$  is dissipated energy at the first cycle. The quantity  $AB + CDe^D$  is a material constant dependent on compressive stress level, temperature, and initial damage of the material. The exponent component ( $CDe^D$ ) is found very small compared to the first part and can be approximated as  $K \cong AB$ . The denominator ( $ABN^{B-1} + CDe^{DN}$ ) is the strain rate ( $\dot{\varepsilon}_{vp}$ ). Thus, the DER (for  $0 < K < 1$ ) is expressed as;

$$DER_{PD} = n \left( \frac{k}{\dot{\varepsilon}_{vp}} \right) \quad (10)$$

### 3 TEST METHOD

#### 3.1 Materials

Two different asphalt mixtures (AB11 and SKA11) sampled from asphalt mixing plants were used for tests in this research. The cylindrical specimens (150 mm by 180 mm height) were produced using a gyratory compactor, and the final specimens (100 mm diameter by 150 mm height) were produced by coring and cutting. In Table 1, the aggregate gradation of the mixtures is given where AB11 mixture contains a polymer-modified binder (PMB 65/105-60) and SKA11 is made of a 70/100 neat binder.

Table 1. Aggregate gradation.

Mix	Sieve size [mm]						Content [%]	
	16	11.2	8	4	2	0.25		0.063
	Percent Passing [%]							
AB11	100	95	70	48	36	15.5	10	5.6
SKA11	100	91.2	53.6	35.7	21.7	12.8	8.4	5.83

#### 3.2 Test procedure

A Sequential Test (ST) procedure is suggested in this paper. In this procedure, fatigue and permanent deformation tests were conducted sequentially on the same specimen. The sequences are referred as PD-F and F-PD sequences (F – for fatigue, PD – for permanent deformation), as shown in Figure 2. In the PD-F sequence, permanent deformation test is conducted first with in the steady-state region and the same sample is tested in Tension-compression fatigue test. Similarly, for F-PD sequence, the new sample is tested in Tension-compression fatigue first and then under-vent permanent deformation test. The test parameters are summarized in Table 2.

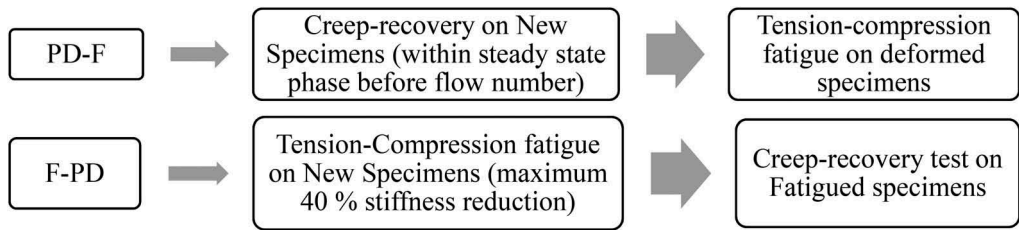


Figure 2. Sequential Test Procedure.

Table 2. Main characterization tests, parameters, and conditions.

Test	Temperature (°C)	Load waveform	Mode	Frequency/Time
T-C fatigue	10, 15, 21, 30	Sinusoidal	Control-strain (at 100, 150, 200, 300, 400 $\mu\epsilon$ )	10 Hz
Creep-recovery	21, 30, 40	Haversine	Control-stress (0.5 to 2 MPa)	Loading/rest time: 0.4 / 1.6 sec

## 4 RESULTS AND DISCUSSION

Several asphalt concrete specimens were tested in tension-compression fatigue and cyclic creep-recovery to investigate the interaction of fatigue and permanent deformation damages according to the sequential test procedure. The first part of the tests in the sequential procedure was conducted to induce the respective pre-damage on the specimen. For example, in the F-PD procedure, the fatigue part of the test is performed to induce a certain level of fatigue cracking (maximum of 40 % stiffness reduction) on the specimen. Then fatigued specimens are tested for permanent deformation until failure. The same is true for the PD-F procedure, where the first part of the test (i.e., PD) is conducted to induce a sufficient level of strain hardening to the specimen before the flow number (within the steady-state region not to failure). Then PD specimens were tested in T-C fatigue until failure.

### 4.1 The F-PD procedure

In the F-PD sequence, the specimens were tested in T-C fatigue in a controlled-strain mode at temperatures of 10, 15, 21, and 30 °C and at different on-specimen target strains (150, 200, 300, 400  $\mu\epsilon$ ). The maximum percentage of stiffness reduction during fatigue was no more than 40 %. Then the fatigued specimens were conditioned and tested at 30 and 40 °C in a controlled-stress creep-recovery test. Examples of test results in the F-PD sequence are shown in Figure 3. As can be seen (Figure 3 a and b), samples with high fatigue damage (high on-specimen strain like 400  $\mu\epsilon$ ) at 10 °C have a rapid rate of permanent deformation and a small flow number (FN). However, samples fatigued at 21 and 30 °C have little effect on the permanent deformation growth. This property is related to the need of high energy to initial fatigue cracking at elevated temperatures due to viscosity.

The flow number (FN) is a classic permanent deformation failure criterion for asphalt concrete. It marks the commencement of the tertiary stage, which is characterized by a high rate of deformation and micro-crack formation. The proposed DER criterion (Equation 10) gives an additional inflection point using the DER curve. It is referred to as the peak value (PV). As shown in Figure 4, four distinct creep stages can be visualized using the DER criterion; (I) An increasing rate in the first stage, (II) a steady-state

increase in the secondary stage, (III) increment at a decreasing rate in the third stage, and (IV) descending phase after the peak point in the fourth stage. The tertiary (III) phase is dominated by micro-crack formation and propagation, resulting in macro-crack formation at PV. It is believed that the PV indicates the formation of macro-cracks and unstable deformation. The DER starts descending rapidly in the fourth stage due to excessive deformation or high energy dissipation. For the F-PD sequence (samples with pre-existing cracks due to fatigue), wing cracks can grow parallel to the load direction. At PV, the accumulated wing cracks and micro-cracks coalesce and form macro-cracks. Therefore, the FN in the DER curve corresponds to a cycle where the steady-state DER start decreasing or the dissipated energy starts increasing. Moreover, Figure 5 shows that fatigued samples have higher DE or lower DER value than new samples. Since the fatigue test is conducted at lower temperatures, healing can occur during the rest period and during conditioning at a higher temperature (40 °C) for permanent deformation.

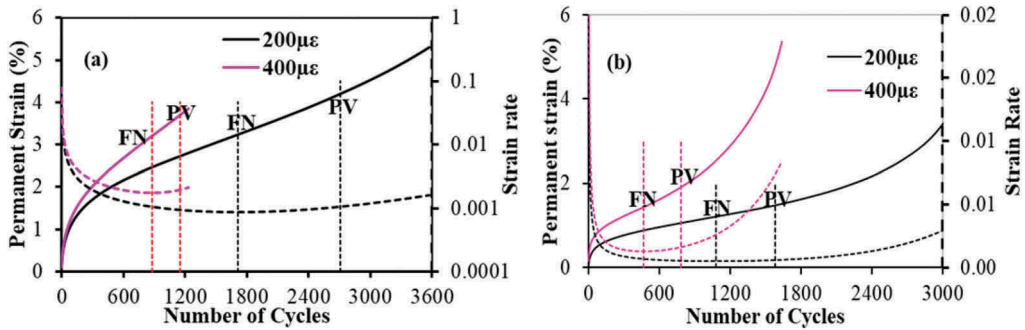


Figure 3. Permanent deformation of fatigued samples (F-PD) at 10°C and 200 and 400 µε target strain (a) AB11 – PD at 30°C,  $\sigma = 2\text{MPa}$  (b) SKA11 – PD at 40°C,  $\sigma = 0.65\text{MPa}$ .

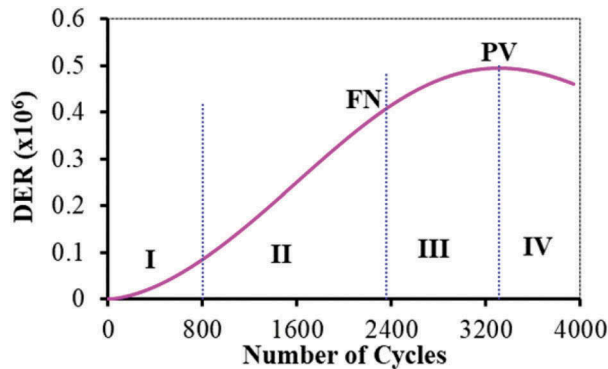


Figure 4. The typical four phases of DER Curve (F-PD - Fatigued at 10°C and 300µε, PD - 30°C and  $\sigma = 2\text{MPa}$ ).

Furthermore, the proposed failure criterion is verified using test data on a new specimen with confinement and a longer recovery period of 16 sec (Figure 6). Confinement played a role in the energy dissipation evolution, and DE is directly proportional to axial stress and temperature. The DER shows the level of damage (viscoplastic) of the sample more clearly than the DE quantity. Large DER means the material is less damaged, and the decreasing rate of DER indicates micro-crack is initiated and flow number identified.

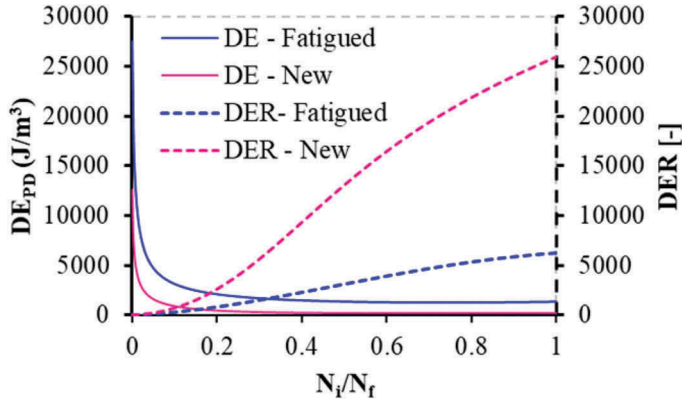


Figure 5. DE and DER for Permanent deformation (at  $\sigma = 0.5\text{MPa}$ ,  $40^\circ\text{C}$ ) on new and fatigued ( $10^\circ\text{C}$  and  $150\mu\epsilon$ ) samples.

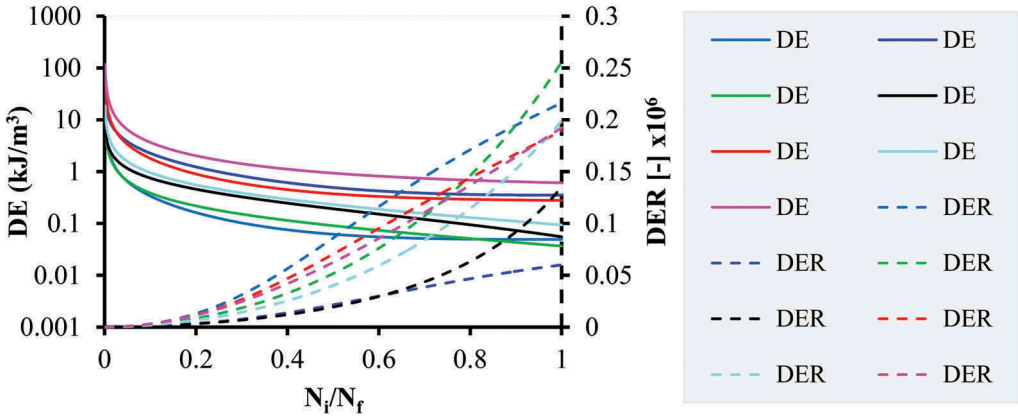


Figure 6. DE and DER due to permanent deformation with confinement on new specimens (at different axial stresses and temperatures) – AB11.

#### 4.2 The PD-F procedure

In the PD-F procedure, permanent deformation is induced on new specimens and the deformed specimen undergoes tension-compression fatigue. Figure 7 (a and b) shows that strain hardening due to permanent deformation increases asphalt concrete's apparent dynamic modulus (stiffness). This increase in stiffness makes the mixtures prone to rapid fatigue damage rate (D).

$$D = 1 - \frac{E_i^*}{E_o^*} \quad (11)$$

where  $E_i^*$  is dynamic modulus at cycle  $i$ , and  $E_o^*$  is the initial dynamic modulus. Hence, the effect of strain hardening has a crucial role in the fatigue damage evolution. In most cases, permanent deformation can accumulate before visible fatigue cracks are seen during pavement service life. In such conditions, the PD-F sequence is a realistic and accurate way to investigate the fatigue life of asphalt concrete mixtures experimentally. However, fatigue cracks can also develop before permanent deformation, especially in cold climates, due to thermal cracking as well as fatigue cracking at intermediate temperatures.

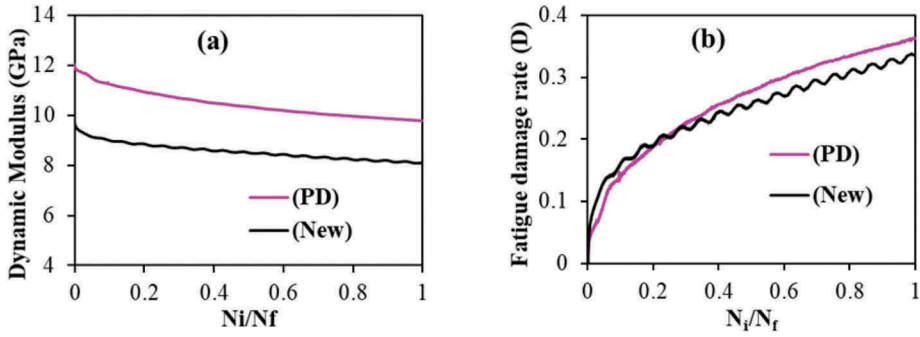


Figure 7. T-C fatigue at 10 °C and 150  $\mu\epsilon$  on new and deformed SKA11 samples (PD at 40°C,  $\sigma = 0.65\text{MPa}$ ) (a) apparent dynamic modulus (b) damage rate, D.

### 4.3 Coupling the dissipated energy

The total dissipated energy ( $DE_T$ ) on a specimen tested according to the sequential tests (F-PD and PD-F) can be approximated by the linear sum of dissipated energy. Thus,  $DE_T$  is expressed by adding Equation 2 and Equation 8 as follows.

$$DE_T = 2\pi\sigma_i\epsilon_i\sin(\varphi_i) + \frac{\pi\sigma_0}{\omega} \epsilon_{vp}^g \quad (12)$$

It is also important to underline that some creep deformation can be accumulated during fatigue tests that contribute to the hardening of the specimen. However, the amount of creep energy dissipation during cyclic fatigue tests is insignificant compared to that of cyclic.

Examples of dissipated energies during fatigue ( $DE_F$ ), permanent deformation ( $DE_{PD}$ ) and total dissipated energy ( $DE_T$ ) are shown in Figure 8. The DEs in permanent deformation damage are cumulative sum upto the flow number  $DE_{PD} = \sum_i^{FN} DE_i$ . The fatigue dissipated energy ( $DE_F$ ) is dependent on the target strain, temperature and initial level of hardening (deformation) as shown in Table 3. Generally,  $DE_F$  decreases as temperature increases due to viscoplastic heat loss instead of viscoplastic flow (deformation) at higher temperatures.

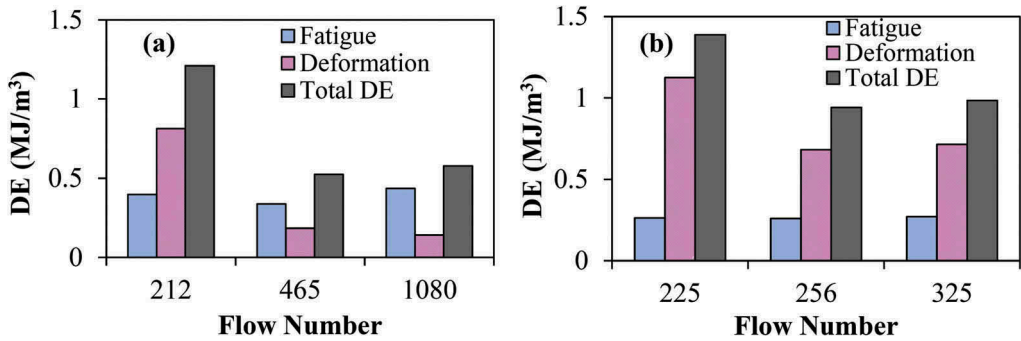


Figure 8. Dissipated energies ( $DE_F$ ,  $DE_{PD}$ ,  $DE_{Tot}$ ) for fatigued SKA11 samples (PD at 40°C,  $\sigma = 0.65\text{MPa}$ ) (a) 10 °C (b) 15 °C .



Table 3. Cumulative Dissipated Eenergy [J/m<sup>3</sup>] during fatigue test on new SKA11 samples.

Strain [ $\mu\epsilon$ ]	Fatigue test Temperature [ $^{\circ}\text{C}$ ]		
	10	15	21
150	1.97E+05	-	1.7E+05
200	3.68E+05	6.74E+05	2.4E+05
300	7.91E+05	5.54E+05	2.3E+05
400	8.00E+05	1.65E+06	-

#### 4.4 Summary

Permanent deformation and fatigue cracking are the most dominant pavement damages. These damages have been studied independently. There is also an assumption that permanent deformation occurs before fatigue cracking develops. However, such an assumption depends on the climatic condition and mixture properties. For instance, top-down cracks can develop in stiff and thick pavements due to tire compression before permanent deformation accumulates and the pre-existing crack can accelerate permanent deformation in later life of the pavement. The study presented in this paper indicates that fatigue and permanent deformation pre-damages have a significant role in the evolution of permanent deformation and fatigue cracking, respectively. Moreover, the sequential test procedure can be standardized to investigate the interaction between the two damages for a more realistic performance prediction. The F-PD procedure is more deterministic and easy to control the desired level of fatigue damage. However, the PD-F sequence should be conducted with care in order not to break the sample beyond the steady-state zone. The energy approach applied in this paper is simple and convenient to analyze the interaction of fatigue and permanent deformation.

## 5 CONCLUSION

The interaction of fatigue and permanent deformation is a very complex phenomenon and little attempt is made on asphalt concrete. In this paper, an investigation is made on the effect of fatigue on permanent deformation and vice versa in a sequential test procedure using an energy approach. The main findings are summarized as follows.

- The energy approach is convenient to analyze the interaction between permanent deformation and fatigue cracking.
- New failure criterion based on the dissipated energy ratio (DER) is proposed. The criterion is more comprehensive than the classic flow number (strain rate) for permanent deformation damage.
- The post flow number micro-crack growth rate can be visualized easily, and the macro-crack formation can be seen at the peak point of DER curve.
- It is observed that permanently deformed samples are more susceptible to fatigue cracking than new samples because of strain hardening. Fatigue damage at high temperatures (21 and 30 $^{\circ}\text{C}$ ) has a marginal effect on permanent deformation.
- The proposed sequential test method can make significant saving for both testing and material production.

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