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Master

Teachers' selection and enactment of mathematical tasks for instruction in the classroom

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Preface

This paper is part of a two-year master's degree program in mathematics at the University of Agder, Kristiansand Norway, specializing in mathematics Didactics. The thesis was completed during the second, third, and fourth semesters of the master's studies, at the Department of Mathematical Sciences of the Faculty of Engineering and Science.

This was my first time writing a voluminous paper like this, and I faced some difficulties in the beginning, but I was inspired by the saying that 'there is always a first time'. A lot of time and effort were tremendously invested towards the completion of this thesis. I was fortunate to have met a lot of experienced researchers who offered advice in many areas towards the completion of this paper. The most profound appreciation goes to God for being my source of vitality and direction. I am grateful for the study chance as well as the encounters I had in Norway.

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Abstracts

Researchers have revealed that most of the hours spent in mathematics lessons are used for solving mathematical tasks. A mathematical task is defined as an individual problem or a set of related problems that direct students' attention toward a particular mathematical thought. There have been significant changes in the mathematics school curriculum in many countries with Norway not being an exception. For example, fewer mathematical concepts are being taught deeply in Norway to stimulate conceptual understanding rather than just a shallow treatment of mathematical concepts. Also, the importance of teaching and learning approaches to motivating students to be active learners is emphasized in the curricula. The importance of students acting independently and taking responsibility for their learning is also highlighted. These changes have prompted the adoption of proper methods in the teaching and learning of mathematics, and the use of good mathematical tasks has been identified as one of the vehicles needed to drive these changes.

This research aims to investigate how mathematical teachers select and enact mathematical tasks in their classrooms by looking at the sources of the tasks they use, the cognitive demand of the tasks, the factors they consider when they are choosing the tasks, modifications they make to the tasks, if any, and why they make modifications to the tasks, if any, and finally the interactions in the classrooms that may change the cognitive demand of the tasks. In this study, a total of twenty-three teachers from the southern part of Norway were selected to participate in the study with three participants selected to participate in the case study and twenty others in the survey. The sources of empirical data for the study were interviews, classroom observations, curriculum documents (for the case study), and survey questionnaires (for the survey).

The findings of the study suggest that even though there is a plethora of resources/sources that the teachers get tasks from, the predominant of them is the textbook. Other major sources of tasks are mathematikk.no, and ndla.no, which are internet resources. Aside from the use of the resources as sources of tasks, other dominating uses of the resources include lesson planning and making instructional explanations. The findings indicate that most of the tasks used by the teachers are at the lower level of cognitively demanding tasks as per the task analysis guide of Stein and Smith (pg.16). The findings again show that the major factors that teachers consider when selecting tasks include tasks that stress on mathematical ideas, focuses on the students, and have applications to other topics. Furthermore, the findings revealed that even though the tasks selected by the teachers were lower-level tasks, however, the teachers did not modify the individual tasks to make them more challenging for the students. According to the teachers, the tasks were many and good, and served as a guide for the exams are some of the reasons why they did not modify the individual tasks. Moreover, the set of tasks was sometimes modified to reduce the time needed to complete them and make them suitable for the students. Lastly, the findings indicate that the instructional habits and dispositions of the teachers and the students in the classroom reduced the cognitive demands of the tasks.

As a didactical implication, Teachers are advised to re-design or modify textbook tasks to offer the opportunity for students to develop deeper mathematical thinking. Furthermore, teachers must set goals they want to achieve with the tasks and stick with them. If the goal is for the students to work with the tasks at the highest level of difficulty, then they are advised to use indirect ways to participate in the students thinking rather than getting directly involved. Future research must explore the relationship between students and mathematical tasks.

Keywords: mathematical tasks, mathematical task selection, tasks modifications, cognitive demand of mathematical tasks.

Sammendrag

Forskere har avslørt at de fleste timene brukt i matematikkundervisningen brukes til å løse matematiske oppgaver. En matematisk oppgave er definert som et individuelt problem eller et sett med relaterte problemer som retter elevenes oppmerksomhet mot en bestemt matematisk tanke. Det har skjedd betydelige endringer i læreplanen for matematikkskoler i mange land, og Norge er ikke et unntak. For eksempel undervises det i færre matematiske begreper dypt i Norge for å stimulere konseptuell forståelse i stedet for bare en overfladisk behandling av matematiske begreper. Også betydningen av undervisnings- og læringstilnærminger for å motivere elevene til å være aktive elever er understreket i læreplanene. Betydningen av at elevene opptrer selvstendig og tar ansvar for egen læring fremheves også. Disse endringene har ført til vedtak av riktige metoder i undervisning og læring av matematikk, og bruk av gode matematiske oppgaver har blitt identifisert som et av kjøretøyene som trengs for å drive disse endringene.

Denne forskningen tar sikte på å undersøke hvordan matematiske lærere velger og vedtar matematiske oppgaver i klasserommene sine ved å se på kildene til oppgavene de bruker, den kognitive etterspørselen av oppgavene, faktorene de vurderer når de velger oppgavene, modifikasjoner de gjør i oppgavene, hvis noen, og hvorfor de gjør endringer i oppgavene, Hvis noen, og til slutt samspillet i klasserommene som kan endre den kognitive etterspørselen etter oppgavene. I denne studien ble totalt tjuetre lærere fra Sør-Norge valgt ut til å delta i studien med tre deltakere valgt ut til å delta i casestudien og tjue andre i undersøkelsen. Kildene til empiri for studien var intervjuer, klasseromsobservasjoner, læreplandokumenter (for casestudien) og spørreskjemaer (for undersøkelsen).

Funnene i studien tyder på at selv om det er en mengde ressurser/kilder som lærerne får oppgaver fra, er læreboka den dominerende av dem. Andre viktige kilder til oppgaver er mathematikk.no, og ndla.no, som er internettressurser. Bortsett fra bruken av ressursene som kilder til oppgaver, inkluderer annen dominerende bruk av ressursene leksjonsplanlegging og instruksjonsforklaringer. Funnene indikerer at de fleste oppgavene lærerne bruker, er på lavere nivå av kognitivt krevende oppgaver i henhold til oppgaveanalyseveiledningen til Stein og Smith (s.16). Funnene viser igjen at de viktigste faktorene som lærere vurderer når de velger oppgaver, inkluderer oppgaver som legger vekt på matematiske ideer, fokuserer på elevene og har applikasjoner til andre emner. Videre viste funnene at selv om oppgavene lærerne valgte var oppgaver på lavere nivå, endret lærerne imidlertid ikke de enkelte oppgavene for å gjøre dem mer utfordrende for elevene. Ifølge lærerne var oppgavene mange og gode, og det å fungere som en rettesnor for eksamen er noen av grunnene til at de ikke endret de enkelte oppgavene. Videre ble settet med oppgaver noen ganger endret for å redusere tiden som trengs for å fullføre dem og gjøre dem egnet for studentene. Til slutt indikerte funnene at instruksjonsvanene og disposisjonene til lærerne og elevene i klasserommet reduserte de kognitive kravene til oppgavene.

Som en didaktisk implikasjon anbefales lærere å re-designe eller endre lærebokoppgaver for å gi studentene mulighet til å utvikle dypere matematisk tenkning. Videre må lærerne sette mål de ønsker å oppnå med oppgavene og holde seg til dem. Hvis målet er at studentene skal jobbe med oppgavene på høyeste vanskelighetsgrad, anbefales de å bruke indirekte måter å delta i studentenes tenkning i stedet for å bli direkte involvert. Fremtidig forskning må være rettet mot å utforske forholdet mellom studenter og matematiske oppgaver.

Nøkkelord: Matematiske oppgaver, matematisk oppgavevalg, oppgaveendringer, kognitiv etterspørsel av matematiske oppgaver.

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INTRODUCTION

Researchers have revealed that most of the hours spent in mathematics lessons are used for solving mathematical tasks (Roth & Givvin, 2008). A mathematical task is defined as an individual problem or a set of related problems that direct students' attention toward a particular mathematical thought (Stein et al., 1996). In many countries, and Norway is not an exception, the types of mathematical tasks and how these tasks are to be used for instruction in all levels of the classroom are made available in curriculum documents. They serve as a guide that helps teachers in the selection of mathematical tasks for instructional use. The education system makes these tasks available through textbooks, syllabi, websites, and software applications.

Many factors influence teachers' selection and enactment of tasks for use in the classroom. Some teachers consider the tasks in the curriculum documents to have a lower cognitive demand or a higher cognitive demand than their students can handle, and therefore, they modify these tasks to adjust to their students' needs (Stein et al., 2009). Curriculum documents and materials have been a vital input to enhancing mathematics teaching in a manner that bridges teachers' instruction and the designed outcome of the instruction (Askew et al. 2010; Cohen and Ball 1990; Kauffman et al. 2002; Pepin et al. 2001). According to Stein et al. (2009), the curriculum is made to contain several less cognitively demanding tasks with the potency to develop skills needed for a high-level thinking task. Also, the cognitive demand of a task as provided in the curriculum can change during the enactment of the task (Bogaart-Agterberg et al., 2021). Henningsen & Stein (1997) asserted that it is very demanding to hold the cognitive demand of a task throughout its implementation. During the implementation of tasks, teachers may lower the cognitive demand of tasks that require high cognitive demand (Davis et al., 2016).

1.1 Background of the study

There have been continual changes in classroom mathematics in Norwegian schools, not only in the Norwegian mathematics curriculum but also in the mathematical curricula of schools worldwide. For example, fewer mathematical concepts are being taught deeply in Norway to stimulate conceptual understanding rather than just a shallow treatment of mathematical concepts. Also, the importance of teaching and learning approaches to motivating students to be active learners is emphasized in the curricula. The importance of students acting independently and taking responsibility for their learning is also highlighted (Utdanningsdirektoratet, 2020).

These changes have prompted the adoption of new approaches to teaching, and the use of good mathematical tasks has been stipulated to be a major ingredient in achieving these goals. For example, Bergqvist (2007) investigated activities in early calculus courses and found a link between mathematical tasks and the transition in the curriculum. Roh and Lee (2016) also discussed activities that "bridge a gap between students' intuition and mathematical rigor" (Roh & Lee, 2016, p. 34), implying a link between how tasks are formulated and presented in upper secondary and university, and the transition in the curriculum. A focus on the nature of mathematical tasks and their usage in education has been a central theme in the Nordic nations (e.g., Bergqvist, 2007; Lithner, 2017), along with the concept of competence (Haavold, 2011; Lithner, 2017). This has informed both policymaking and discussions about students' mathematical achievement in Norway over the last few decades (Botten-Verboven et al., 2010). There are several resources (tasks) available in curriculum documents such as textbooks, syllabi, online sources, etc., and educational technologies such as computer software and applications that a mathematics teacher can choose for instruction with his/her students. But an important question is "How does a mathematics teacher consider these mathematical tasks as good tasks for instruction with his students, and based on what criteria does he or she select tasks for instruction with his students?"

One of the core responsibilities of teachers is to select or create "good tasks" for their students. In this context, a good task matches the capabilities and experiences of the students the teacher is dealing with (Stein and Smith 1998) and satisfies the teacher and the curriculum's goals and expectations. Stein and Smith (1998) suggest that a task on its own might be good but may not be so good during its implementation for a particular group of students if it does not consider the prior knowledge and experiences of the students with whom it is implemented. For instance, a task that can be classified as a highly cognitive demanding task for grade one pupils might appear as a low cognitive demanding task when it is implemented with six-graders (Araujo, 2012).

1.2 Significance of The Study

Every curriculum within an education system aims to ensure that the students are well-equipped with knowledge and understanding of the mathematical concepts they are been taught. Considering this, better policies are put in place to achieve this goal. One of the areas which should not be taken lightly to make this objective a reality is the teacher selection and enactment of the mathematical task for their students to solve. The influence that mathematical tasks have on the mathematical development of students has been given attention and thoroughly studied for a long time ago (e.g., Hiebert and Wearne, 1993; Marx and Walsh, 1988; Stein & Lane, 1996). These studies concluded that the cognitive demand required to solve a mathematical task and how the students develop an understanding of the mathematical concept being thought have a direct relationship (Hiebert and Wearne, 1993; Marx and Walsh, 1988; Stein & Lane, 1996).

The type of tasks that a teacher chooses for instruction in his/her classroom has an impact on the mathematical development and reasoning of his students. In other words, the level of mathematical understanding and thinking that students develop during instruction mainly depends on the level of the cognitive demands of the mathematical tasks they complete. The higher the cognitive demand of the mathematical task, the better the students develop a higher comprehension of mathematics, and the reverse is true. Stein and Smith (1998) indicated that there is a relationship between the kind of mathematical task students are assigned to work on and how students think mathematically:

> Tasks that ask students to perform a memorized procedure in a routine manner lead to one type of opportunity for student thinking; tasks that require students to think conceptually and that stimulate students to make connections lead to a different set of opportunities for student thinking. (p. 269)

Even though several studies have been conducted around teachers' task selection for instruction, there is still a need to further investigate the teachers' selection and enactment of mathematical tasks. This is because of the important role it plays in the development of students' mathematical understanding and reasoning skills and assists students in seeing mathematics as a constructive enterprise (Mason and Johnson-Wilder, 2004). Moreover, most of these available articles about teachers' selection of mathematics tasks for instruction were carried out outside the Nordic Area, advocating the need to carry out a study that is tailored to the setting of Norway and its surroundings. This study will add to the already existing research on teachers' selection and enactment of mathematical tasks for instruction and present a document in the field of task selection in the Nordic Area.

1.3 Motivation for the Study

There were many possibilities regarding choosing a topic for my master's thesis. But the researcher finally decided to work on teachers' selection and enactment of mathematical tasks for their students. The motivation for choosing this research area stems from a course (Working Methods in Mathematics) taken at the beginning of the researcher's program. In that course, they were given sets of mathematical tasks to work on ten (10) of them and present them at the end of the semester. But unfortunately, the researcher found it difficult to complete those tasks the researcher was not familiar with their ways and methods. In completing those tasks, they were asked to first specialize in the task, then generalize, and make a conjecture and prove the conjecture. Also, they were asked to reframe the problems, find an alternate solution, and provide a detailed explanation of their solution for the task. In the modeling part of the course, they were to refine the problems as the original tasks presented were sometimes ambiguous and vague. Examples of some of the tasks they worked on were "How far can you walk in a year?" and "Does it rain more in Bergen than in Kristiansand?".

While the researcher was finding it difficult to solve these tasks, the other Norwegian students in the class were finding it easier to work on the tasks. The researcher once approached the lecturer who taught the course to complain to him about how he/she was finding it difficult and that the tasks used in the course were not familiar to him/her. The response from the teacher was that "In this course, we are not doing anything new. It is just the mathematics we know that we are applying in this course." The most interesting part was that most of the students who took part in this course were bachelor's students who had just completed high school and were now studying at a university, but the researcher was a master's student. The problem was that mathematics is not done this way in the home country (Ghana). They only provide one solution, but maybe different approaches may be used, but in the end, the same answer is provided for the task and no explanation is expected when the task is completed. In fact, the way of mathematizing in the researcher's country is a bit different from the experienced during this course in Norway. So, the researcher wanted to know the nature of the mathematical tasks that were being used in the Norwegian classrooms that fostered such mathematical thinking in the Norwegian students in the course.

Moreover, the researcher was a mathematics teacher back in his home country. As a core subject (mathematics) teacher, the researcher was assigned to different classes with students with varying understandings of mathematics. For instance, in the sciences and technical classes, there were students with a high understanding of prerequisite mathematical knowledge, whilst, for other students in the Arts classes, it is the opposite. This sometimes makes the selection and enactment of tasks for the students at the same level but in different classes very challenging, even though the tasks for instructions are given in the curriculum documents provided by the Education Service. The researcher mostly selected the task as it appeared in the curriculum documents, but varied the cognitive demand of the task by developing strategies to assist the students whilst they worked on it. The researcher chooses mathematical tasks that required a higher cognitive demand from the curriculum documents for the students with an in-depth understanding of mathematics and less cognitively demanding tasks for the students with a lower understanding of mathematics. When it was required that each set of students work on the same task, verbal clues were used to reduce the cognitive demand of the task when enacting with the Arts classes. This research will not only unearth how teachers select and enact mathematical tasks with their students but will also help adopt some of the strategies that the teachers use when they encounter task selection challenges. It will give a view of how mathematics is about in Norway, which the researcher can incorporate into mathematics teaching.

1.4 The Research Questions

The aim of this research study is to answer the following questions:

1. How do teachers select mathematical tasks for their students?
    a. What is/are the source(s) of the tasks?
    b. What is the nature of the task they choose?
    c. What factors do teachers consider when choosing a task?
2. What modifications, if any, do teachers make to mathematical tasks before using them with their students?
    a. Why do teachers modify the tasks?
    b. How does this modification, if any, influence the characteristics of the tasks?
3. What interactions occur in the classroom that helps maintain or change the characteristics of the task during enactment?

After selecting the topic to study, the researcher read a lot of literatures that relates with the study and then discussed with the supervisor what was discovered from the literatures. The researcher with the assistance of the supervisor came out with the above-mentioned researcher questions which are similar to that of Araujo (Araujo, 2012). According to Stein and Smith (1998), the implementation of a task goes through three phases: the task as it appears in the curriculum, the teachers' selection and enactment of the task based on his/her expectations or the learning outcome, and how the students solve the task with or without the teachers' interference. The research questions were formulated based on the three phases of the task implementation phases proposed by Stein and Smith (1998).

The first research question deals with the strategies teachers consider before deeming a task a good task for their students. According to Stein and Smith (1998), a task is a good task if, first, it considers the age, grade level, prior knowledge and experiences of the students, and the learning expectations in the classroom; secondly, the task considers the four categories of cognitive demand. First, the first research question looks at from which sources and resources teachers select their mathematical tasks for instruction. There is a plethora of resources both in print and non-print that the mathematics teacher may select a task from. Research question 1a seeks to ascertain how teachers make use of these resources when it comes to selecting tasks for their students. Research question 1b seeks to determine the cognitive demand or the characteristics of the task(s) that teachers select and implement with their students. It looks at the content and the level of mathematical competence that the students are required to possess to solve those tasks. Again, research question 1c looks at the factors that teachers consider when selecting the task(s). These factors may vary from content, aesthetics, the teacher's goal, and the quality of the tasks to build on the mathematical competence of the students.

Moreover, the second research question is to investigate whether the teachers make modifications or changes to the questions in the curriculum documents that they use before enacting them with their students. Teachers might sometimes find the characteristics of the task as it appears in the curriculum documents to be inappropriate or unsuitable to the needs of their students or the expectations of him classroom. It could be that the cognitive demand is too high or too low to meet the learning expectations. The teachers may therefore modify the question to address the cognitive demand and to match the expectations of the classroom with the learning outcome. Sometimes also, the teacher may deem the tasks as they appear in the curriculum documents as good tasks which need no changes to be effected on them before enacting with their students. Research question 2a is in two folds; 1. Does the teacher make modifications to the tasks in the curriculum documents? 2. Do the teacher maintain the tasks just as they appear in the curriculum documents? Research question 2b then probes further to investigate the motives behind the teachers' modifying the task if any, and how these changes affect the characteristics of the task.

The third research question is to explore the teacher-student interaction during the enactment of the task. During the implementation of the task, the teacher may make comments that might further change the characteristics of the task. For instance, a teacher might make a comment about the previous knowledge of a subject, which might be pivotal foreknowledge required to solve the task the teacher is enacting with the students. This research question seeks to enquire about how the teacher interferes when that occurs in the classroom, which changes the cognitive demand of the task. These interactions may also include the culture in the classroom. That is the non-mathematical aspect of the classroom which may also affect the completion of the task. These may include but are not limited to the classroom norm, classroom arrangement, the classroom environment, teachers' motivation towards the students in completing the tasks, etc.

1.5 Structure of the thesis

This study is made up of six chapters. The chapter one which ends with this section talks about the introduction to the topic of the study, brief background information, the significance of the study, the motivation of the study, and the research questions.

Chapter two presents the theoretical frameworks that backs the study. The chapter gives a literature review on topics which underpins this study. The frameworks include the mathematical task design, cognitive demand of mathematical tasks, and teachers' knowledge. The chapter further presented literature on teachers' interaction with tasks after selection from curriculum documents and the teachers' responsibilities in the classroom during tasks enactment.

The methodology of the study is presented in the chapter three. The researcher deployed both a case study and a survey as the traditional methods of inquiry into this subject. The case study strategy is first described, followed by the survey. The research design is then presented, including the research method, participant selection, data collection, data management, and analysis strategy. Subsequently, the study's ethical considerations, validity, and trustworthiness are addressed.

The chapter four of the study presents the data analysis and findings of the study. First, a summary of the findings from all the data sets were presented and then the analysis of the results from the various data sets were presented.

Chapter five of this report include the discussion of the study's findings. First, the research questions of the study were addressed and the then the link between the researcher's interpretations of the findings and the literatures of the study were presented. The last chapter of the study entails the conclusion where the summary of the results, the limitations for the study, implications for teachings, and the recommendations for future studies were presented in the report.

The next chapter of the study look at the presentations of the literature that make the theoretical framework for the study.

THEORETICAL FRAMEWORKS

In this chapter, the researcher considered some of the theoretical frameworks that is fit for this research. In all, three theoretical frameworks were suitable for this research. First, the Mathematical Task Design, which is the general theoretical framework, is discussed. Moreover, other theoretical perspectives that are part of the general theoretical framework (Mathematical Task Design) are also discussed. They include the Cognitive Demand of Mathematical Tasks, which talks about the levels of difficulties or thinking required to solve a mathematical task, and the Teachers' Knowledge (the competence of the teachers to understand the goal of mathematical tasks as they appear in the curriculum, the pedagogical content, and knowledge of their students). In this chapter, these theories are stated, explained, and discussed why they are relevant to this research work.

2.1 The Mathematical Task Design

During a lesson, the aim of the mathematics teacher is to help his or her students understand the concepts he or she is teaching and develop the mathematical competency of students in general. One way of attaining this aim is to select tasks from various sources and appropriate (when necessary) the tasks for enactment with his or her students. The research questions were designed to investigate how teachers select tasks that they deem good tasks that meet the needs of their students. The motive was to explore the tasks: what are the nature (e.g., cognitive demand) of the tasks the teachers select, how the teachers interact with the tasks, the teacher and the students (teachers' instructional and students' learning habits and dispositions), and among the students (collaborative or individualistic approach to completing tasks), which may lead to the maintenance or the decline of the characteristics of the tasks as originally intended in the tasks the teachers selected from the available resources.

The Mathematical Task Design prescribes the contents or the ingredients that should be embedded into a task and what the task requires learners to do to enhance the student's capabilities for developing mathematical thinking (Breen and O'Shea, 2010). Simply put, it describes the nature of a mathematical task and the set of activities the students must perform to foster mathematical thinking. Kilpatrick et al. (2001) asserted that students' ability to have mathematical proficiency (i.e., the ability to think and understand mathematics) can be measured by five components: "conceptual understanding, procedural fluency, strategic competence (the ability to formulate and solve mathematical problems), adaptive reasoning (the capacity for logical thought, reflection, and justification), and productive disposition (seeing mathematics as worthwhile and being confident in one's own abilities)" (p. 116). If one needs to understand how students think mathematically, these strands need to be studied. This was not the focus of this study, and nothing was measured using this theory, but just mentioned to understand what manner of tasks need to be selected if teachers want to improve mathematical reasoning in their students.

The nature of mathematical tasks should not be routine, as this may be tedious for students to work with, but rather one that challenges them to think. Polya (1945), in the preface of his book "How to Solve It," stated that,

> Thus, a teacher of mathematics has a great opportunity. If he fills his allotted time with drilling his students in routine operations he kills their interest, hampers their intellectual development, and misuses his opportunity. But if he challenges the curiosity of his students by setting them problems proportionate to their knowledge and helps them to solve their problems with stimulating questions, he may give them a taste for, and some means of, independent thinking. (p. v).

But how can a mathematical task be designed in such a way that Polya's suggestions are reflected? Stein et al. (1996) stressed the relevance of the nature of tasks teachers assign to students to work on, as the tasks may influence the way students involve themselves in solving the tasks, learn to think mathematically and develop mathematical competence. They suggested that students should be given the opportunity to engage in mathematical tasks that are cognitively demanding, useful, and relevant. They averred that students should be given mathematical tasks that are at doing mathematical task level of cognitive demand as appears in Stein and Smith, 1998 (see p.16). They further explained that these sorts of tasks are symbolized by their ability to be presented in several ways, their ability to have multiple solution sets, and their expectations of students to justify their methods and comprehension verbally or in writing.

Also, Swan (2008) avowed that teaching becomes more fruitful when quality tasks are utilized in the classroom for instruction. He claimed that "the tasks we use should be accessible, extendable, encourage decision-making, promote discussion, encourage creativity, and encourage "what if" and "what if not?" questions" (p. 8). He designed a framework made of five kinds of tasks (in order of difficulty), which he thinks encourages students' conceptual understanding of mathematical concepts. The five order of tasks that he trusts stimulate students' conceptual understanding are,

- Tasks that ask students to identify mathematical objects,
- Tasks that demand students to interpret multiple representations,
- Tasks that assess students' ability to evaluate mathematical statements,
- Tasks that evaluate students' potential to formulate a problem; and
- Tasks that appraise students' capacity to analyze reasoning and solutions.

Furthermore, Mason and Johnson-Wilder (2004) asserted that students may see mathematics as a constructive enterprise if they (students) are made to engage in mathematical tasks that entail a couple of choices and provide students with the chance to deliberate on ideas. They also argue that mathematical tasks should be chosen to help students experience important mathematical concepts and ideas, and they should be moderately difficult but not too challenging for their experience and capabilities. According to them, the aim of mathematical tasks is "to initiate mathematically fruitful activity" (p. 25).

Again, Krainer (1993) opted for the shift away from the method of teaching where teachers perform most of the problem-solving activities to one where students are encouraged to actively participate (take the lead) in solving tasks in the classroom so that they (students) can form their own ideas and thinking. He claimed,

> "...learners should be seen not only as consumers but also producers of knowledge. The teacher's task is to organize an active confrontation of the pupils with mathematics. Powerful tasks are important points of contact between the actions of the teacher and those of the learner" (p. 68).

He further listed some properties which make mathematical tasks powerful tasks. He said quality tasks,

- should be interrelated to each other,
- should promote the creation of other relevant tasks,
- should initiate activities that produce concept formation, and
- should be designed in a way so that acting and interpreting are connected.

The primary source of instructional tasks influencing teachers' instruction is mathematics textbooks (Kaur & Lam, 2012). When there is a chasm between curriculum changes and textbooks, a teacher's duty as an interpreter or active user of textbook tasks becomes more important in meeting the curriculum's needs in their instruction. Teachers should be able to use appropriate instructional tasks by creating new ones or modifying existing ones while keeping current with curriculum changes (Lee et., al., 2017). However, despite the need to modify textbook tasks to encourage students' inquiry, mathematics teachers do not believe that task modification was necessary (Kim & Kim, 2014). Teachers must develop positive attitudes and orientations toward inquiry-based instruction, as well as identify the affordances and constraints of textbook tasks in terms of students' inquiry, in order to recognize the need for task modification (Lee et., al., 2017). They also asserted that to use appropriate tasks during class to achieve intended goals or curriculum standards, textbook tasks must be modified, or new tasks must be designed.

The Mathematical Task Design framework according to Breen and O'Shea (2010) gives an overview of the qualities of mathematical tasks teachers should consider for instruction in their classroom depending on the impact they want to have on the mathematical proficiency of their students. If they aim to inculcate mathematical thinking in their students, then the teachers should utilize tasks that require students to make conjectures, reason and prove, abstract, generalize, and specialize (Breen & O'Shea, 2010). On the other hand, if teachers are expecting their students to provide just the right answers to tasks that sometimes only produce procedural knowledge rather than a conceptual understanding of mathematical concepts and ideas, then memorization and routine algorithm tasks are the best. It is in the light of this framework that other frameworks used in this research are built. For example, the cognitive demand of a task looks at the level of thinking that students need to apply to be able to solve a task. The higher the cognitive demand of the task, the more likely it will promote mathematical thinking within the students. Also, the teacher's knowledge and experience (particularly about his/her students) assist him/her in designing tasks that build on students' prior knowledge.

2.2 The cognitive demand of mathematical tasks

The cognitive demand of the mathematical task framework is a section of the framework that was created through the QUASAR project, which used the criteria developed by Doyle (1983, 1988) and Hiebert and Wearne (1993) as its theoretical support (Arbaugh, F. & Brown, C. A., 2005). The framework observes the connections between the tasks as they go through the phases of being written (as the task appears in the curricula documents), set up (how the task is presented by the teacher in the classroom), implemented (how the task is completed by the students) and the students' learning (how the students develop the conceptual understanding of the mathematics concept being taught).

The aim of the first research question was to find the nature of the task teachers select. To answer that question, the tasks as they were in the curriculum materials that the teachers were using for instruction will be observed using Stein et al.'s (1996) framework (see p. 17). Burkhardt et al. (1990) referred to this first phase of Stein et al.'s (1996) framework as the available resources. At this stage, the researcher will collect all the resources that are available to the teachers (including what they have written in text and what is available on software). The researcher will then group the tasks based on the Task Analysis Guide (see Figure 2, p. 16) designed by Stein and Smith (1998). Cognitive demand refers to the level of thinking that is required by students to solve a particular task. The Task Analysis Guide by Stein and Smith (1998, see p. 16) categorized all mathematical tasks into four levels of cognitive demand. The first two categories are classified as low-cognitive-demanding tasks, whilst the last two categories are higher-thinking tasks.

The first group of tasks that are classified as low-cognitive-demanding tasks is termed "memorization." These sorts of tasks require students to reproduce facts, rules, formulas, or definitions they have already learned (Stein & Smith, 1998). An example of these kinds of tasks is when a teacher asks a student to mention an example of triangles (see Figure 1, p. 15). Stein and Smith (1998) suggested that these tasks are devoid of ambiguity and do not challenge students to refine (extract meaning out of abstract statements) the requirements of the task before solving them, but rather recall and reproduce what they already know. The second group of tasks under low-cognitive-demanding task is a procedure without connection. Stein and Smith (1998) also asserted that procedure without connection tasks is devoid of any ambiguity or uncertainty (i.e., they are straightforward tasks). Stein and Smith (1998) described procedure without connection tasks as tasks that require students to use a precise step-by-step plan (algorithm) to solve the task. Procedures without connection tasks do not pay attention to the students' development of mathematical understanding; instead, they only expect the students to produce the right answers for them (Stein and Smith, 1998). These tasks also have little uncertainty as to what and how they are to be completed and require little thinking from students (Stein and Smith, 1998). An example of such a task is a task that asks students to multiply two fractions.

The next two categories of tasks, which are described as highly cognitively demanding tasks, are the procedure with connection tasks and the doing mathematics tasks. The procedure with connection tasks utilizes prevalent procedures that have a relationship with underlying mathematical concepts and ideas which aim at assisting students to discover an intensive mathematical understanding (Stein and Smith, 1998). Unlike procedures without connection tasks, where students complete procedures but neglect to link the procedure to the underlying mathematical concept, procedures with connection tasks further connect the procedure with the underlying concept and thus enhance students' ability to penetrate intensively into why the procedure was used and how it connects with other mathematical concepts. An example of such a task is a task that asks students to use pattern blocks to find 1/6 of 12 by sketching their answer and explaining it. The other group of mathematical tasks under higher cognitive demanding tasks is doing mathematics. According to Stein and Smith (1998), doing mathematics tasks are non-algorithmic in nature (they do not follow any specific procedures or steps) and require students to discover from their own processes, rely on their experiences, and make appropriate use of them to complete the task. Stein and Smith (1998), as an example of Doing Mathematics task, considered the following:

> Shade 6 small squares in a 4 x 10 rectangle. Using the rectangle, explain how to determine each of the following: a) the percent of the area that is shaded, b) the decimal part of the area that is shaded, and c) the fractional part of the area that is shaded. (Stein & Smith, 1998, p. 3).

Also, it must be noted that the cognitive demand of mathematical tasks differs from student to student, grade level to level, and student's experience. A task that may seem like a memorization task for grade 9 students may for example be seen as a procedure with connection tasks when it is enacted with grade 6 pupils (Araujo, 2012).

Figures 1 and 2 show examples of tasks at each of the four levels of cognitive demand and characteristics of tasks at different levels of cognitive demand respectively.

Lower-Level Demands

Memorization

What is the rule for multiplying fractions?

Expected student response:

You multiply the numerator times the numerator and the denominator times the denominator.

or

You multiply the two top numbers and then the two bottom numbers.

Procedures without Connections

Multiply:

$$\frac{2}{3}\times\frac{3}{4}$$

$$\frac{5}{6}\times\frac{7}{8}$$

$$\frac{4}{9}\times\frac{3}{5}$$

Expected student response:

$$\frac{2}{3}\times\frac{3}{4}=\frac{2\times 3}{3\times 4}=\frac{6}{12}$$

$$\frac{5}{6}\times\frac{7}{8}=\frac{5\times 7}{6\times 8}=\frac{35}{48}$$

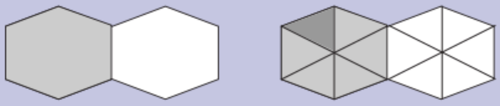
$$\frac{4}{9}\times\frac{3}{5}=\frac{4\times 3}{9\times 5}=\frac{12}{45}$$

Higher-Level Demands

Procedures with Connections

Find 1/6 of 1/2. Use pattern blocks. Draw your answer and explain your solution.

Expected student response:



First you take half of the whole, which would be one hexagon. Then you take one-sixth of that half. So I divided the hexagon into six pieces, which would be six triangles. I only needed one-sixth, so that would be one triangle. Then I needed to figure out what part of the two hexagons one triangle was, and it was 1 out of 12. So 1/6 of 1/2 is 1/12.

Doing Mathematics

Create a real-world situation for the following problem:

$$\frac{2}{3}\times\frac{3}{4}.$$

Solve the problem you have created without using the rule, and explain your solution.

One possible student response:

For lunch Mom gave me three-fourths of a pizza that we ordered. I could only finish two-thirds of what she gave me. How much of the whole pizza did I eat?

I drew a rectangle to show the whole pizza. Then I cut it into fourths and shaded three of them to show the part Mom gave me. Since I only ate two-thirds of what she gave me, that would be only two of the shaded sections.

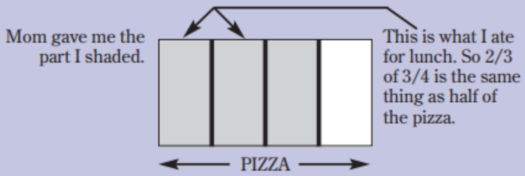


Figure 1: Examples of tasks at each of the four levels of cognitive demand (Stein and Smith, 1998)

Levels of Demands

Lower-level demands

(memorization):

- Involve either reproducing previously learned facts, rules, formulas, or definitions or committing facts, rules, formulas, or definitions to memory.
- Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure.
- Are not ambiguous. Such tasks involve the exact reproduction of previously seen material, and what is to be reproduced is clearly and directly stated.
- Have no connection to the concepts or meaning that underlie the facts, rules, formulas, or definitions being learned or reproduced Lower-level demands.

(Procedures without connections):

- Are algorithmic. Use of the procedure either is specifically called for or is evident from prior instruction, experience, or placement of the task.
- Require limited cognitive demand for successful completion. Little ambiguity exists about what needs to be done and how to do it.
- Have no connection to the concepts or meaning that underlie the procedure being used.
- Are focused on producing correct answers instead of on developing mathematical understanding.
- Require no explanations or explanations that focus solely on describing the procedure that was used.

Higher-level demands

(Procedures with connections):

- Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas.
- Suggest explicitly or implicit pathways to follow that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts.
- Usually are represented in multiple ways, such as visual diagrams, manipulatives, symbols, and problem situations. Making connections among multiple representations helps develop meaning.
- Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with conceptual ideas that underlie the procedures to complete the task successfully and that develop understanding. Higher-level demands

(Doing mathematics):

- Require complex and nonalgorithmic thinking—a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instructions, or a worked-out example.
- Require students to explore and understand the nature of mathematical concepts, processes, or relationships.
- Demand self-monitoring or self-regulation of one's own cognitive processes.
- Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task.
- Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions.
- Require considerable cognitive effort and may involve some level of anxiety for the student because of the unpredictable nature of the solution process required.

Figure 2: Characteristics of tasks at different levels of cognitive demand (Tasks Analysis Guide). Smith and Stein (1998)

The cognitive demand of mathematical tasks can change during the various stages of task setup and implementation developed by Stein, Grover, and Henningsen (1996). Next, the researcher examined the factors that may affect the cognitive demand of the mathematical tasks at the various phases. Figure 3 represents connections between various task-related variables and student learning (Stein et al., 1996). The three rectangles represent the three phases of the task setup and the implementation phases, whilst the circles demonstrate the factors which may influence the cognitive demand of the tasks at the various implementation phase.

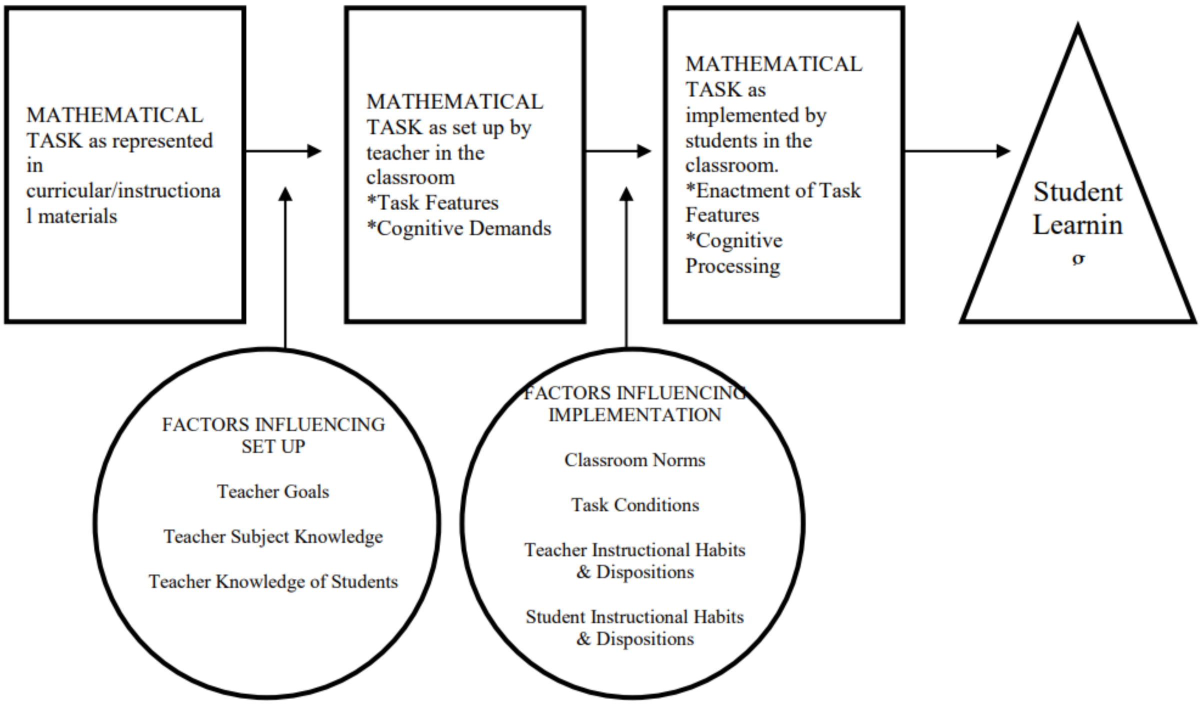


Figure 3: Relationship among task-related variables and student learning. Stein, Grover, and Henningsen, (1996).

2.2.1 The cognitive demands of mathematical tasks as set up by the teacher

The mathematical task setup compasses several activities from the moment the teacher transfers the task from his curricula document to the time he asks the students to start working on the problem. Stein et al (1996) explained task setup as:

> Task setup is defined as the task that is announced by the teacher. It can be quite elaborate, including verbal directions, distribution of various materials and tools, and lengthy

> discussions of what is expected. Task setup can also be as short and simple as telling the students to begin work on a set of problems displayed on the blackboard (p. 460).

Stein et al. (1996) also identified some factors which can lead to the change or the maintenance of the cognitive demand as it appears in the teachers' curriculum documents, which was the aim of research question 2(b) of this study. In Figure 3, Stein et al. (1996) described the circle on the left side as a factor that may cause the cognitive demand of a mathematical task to change or be kept as intended in the curriculum. These factors include the teacher's knowledge of the subject, what the teacher wants to achieve or his goal, and the teacher's perception of his or her students (Stein et al. 1996).

Teacher knowledge refers to how the teacher's understanding of the subject, the pedagogical objectives, and his knowledge of his or her students affect his or her methods of instruction. The concept of teacher knowledge is a framework used in this study and will be explained later in this chapter. When it comes to the teacher's goal, the teacher decides on the goal he wants to achieve with a task (Brown, 2009). The goal set by the teacher will then influence the tasks he or she chooses to enact with the students to achieve the set goal. If the task that appears in the curriculum documents does not match the goal set by the teacher, the teacher may alter the question to suit his or her goal, which may eventually change the cognitive demand of the original task as intended in the curriculum document. For instance, if a particular task in a curriculum document used by a teacher is aimed at assessing students' understanding of circles but the goal of the teacher is for his or her students to find the general equation of a circle when given three points on the circumference of a circle, then the teacher may modify the task to limit the students' understanding of anything that goes beyond the students' ability to find the equation of a circle from three points on the circumference, which may eventually change the cognitive demand of the task as intended in the curriculum. Therefore, if the goal of the teacher does not align with the task as it appears in the curriculum documents, teachers may modify the task and may cause maintenance or change in the cognitive demand of the task (Araujo, 2012).

2.2.2 Cognitive Demand of Mathematical task during implementation

The cognitive demand of mathematical tasks may also change during task implementation. The third research question of this study looks at what interactions go on in the classroom which may result in the maintenance or change in the cognitive demand of the mathematical task. The implementation of a task starts when the students begin working on the task until they move to another task (Araujo, 2012). It looks at the way in which students work on the task, whether they complete the task as intended in the setup or whether they change the task whilst they complete it (Stein et al., 1996), or in other words, the way in which students actually work on the task (Stein & Smith, 1998). In Figure 3, the circle on the right represents the factors that may affect the implementation phase of the task, which are the classroom norms, task conditions, the teacher's instructional habits and disposition, and the student's learning habits and dispositions.

The classroom norms refer to governing expectations with respect to what academic work will be done and by whom, and what the expectation is for quality and accountability. The task conditions refer to characteristics existing in the task for which it is deemed an appropriate task that will build on the prior knowledge and abilities of students and the appropriateness of time allocated to students to accomplish the task. The teacher's instructional habits and dispositions refer to the extent to which the teacher is willing to intervene and assist the students when they have difficulties solving the task and how long it will take before he intervenes. And the last, which is the student learning habits and dispositions, refers to how long students are willing to endure their struggle when they work on challenging mathematical tasks and the extent of self-monitoring, they engage themselves in (Stein et al., 1996).

Moreover, in addition to these factors, Stein and Smith (1998), outlined additional factors which may lead to the maintenance or decline of the cognitive demand of the mathematical task. Notably, among the cognitively declining factors is the routinization of the tasks, where students constantly force the teacher to simplify the task by clarifying the algorithms that they (students) need to complete the task. This results in the teacher taking over the thinking and thus decreasing the cognitive demand of the task. This can be linked with the case of Gaël as described by Brousseau and Warfield (Brousseau & Warfield, 1999). The study is about a group of students who repeatedly failed in mathematics and only mathematics. One of these students was eight and half-year Gaël, who upon further investigation was discovered that he had signed a contract of which he was not, or anyone was aware of. His part of the bargain was to never risk failure by actually attempting to understand the mathematics, but to always maintain his calmness and acquiescence; the adult's part was to abstain from criticizing him or making him uncomfortable and to supply the answers to the questions she had posed.

Apart from the fact that Gael was learning nothing, it was a very unpleasant arrangement all around (Brousseau & Warfield, 1999). The researcher believes these factors affect the implementation stage of the task and have linked it to the teacher and student habits and dispositions. Other factors which, according to Stein and Smith, reduce the cognitive demand of the task include the inappropriateness of the task to meet the student's abilities; insufficient apportionment of time for the completeness of the task (both linked with task conditions); students not being held accountable for their performances (teachers accepting wrong responses from students and students' perception that such a response may not count in grading); and classroom-management problems (which is related to classroom norms). Regarding maintenance of the cognitive demand of the task, Stein and Smith stated that the use of scaffolding by the teachers, the student's commitment to self-monitoring, the teachers' choice of tasks that build on students' prior knowledge, and the allocation of sufficient time to complete the tasks are vital.

2.3 The Teacher's Knowledge

The foundation for teachers' teaching methodologies mainly depends on the teacher's knowledge and experience. The teacher's knowledge includes both subject matter knowledge (SMK) and pedagogical content knowledge (PCK). SMK has been defined as "emphasizing knowledge and understanding of facts, concepts, and principles and the ways in which they are organized, as well as knowledge about the discipline" (Even, 1993, p. 94). Previous studies have revealed that students end up developing misconceptions, misunderstandings, and misinterpretations concerning mathematical concepts when the teacher has deficient knowledge about the subject matter (e.g., Valanides, 2000), and teachers who have higher SMK have a more assured opinion about their teaching methodologies than those who have insufficient SMK (Barlow & Cates, 2006; Quinn, 1997). According to Ozden (2008), the results from the studies of teachers' subject knowledge do not only influence their teaching methodologies but also their pedagogical content knowledge. Researchers have discovered that teachers' PCK plays a major role in students' academic performance (Darling-Hammond, 2000), and mathematics teachers must endeavour to enhance a wide SMK for the improvement of their students' mathematical development (NCTM, 2000; Rizvi, 2004; Schmidt et al., 2009).

Figure 4: A theoretical framework of teacher knowledge. Adapted from Lee et al (2018).

The concept of teachers' PCK was formalized by Schulman (1986), and he defined it as a "special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding" (p. 227). Since its introduction, the study of teacher knowledge has been more focused on the PCK than the SMK (Ball et al., 2008), which is because of the emergence of the various categories of the PCK (Lee et al., 2018). Several studies have tried to establish how teachers' knowledge contributes to students' mathematics academic performance (Wilson, Floden, & Ferrini-Mundy, 2001, 2002) and have linked the teachers' PCK to the student's performance (Ball, Hill, & Bass, 2005; Ball et al., 2008).

Since the emergence of the PCK, several researchers have grouped its sub-category differently. For instance, Hill et al. (2008) also classified the PCK into the knowledge of the curriculum, knowledge of mathematical content, and of students. Lannin et al. (2013) subscale the teachers' PCK into four categories: curriculum for mathematics, assessment for mathematics, instructional strategies for mathematics, and student understanding within mathematics. According to Hauk et al. (2014), the subscale involves knowledge of discourse, curricular thinking, anticipatory thinking, and implementation thinking. Although there have been several sub-categorizations for PCK, for the purpose of this study, we looked at teacher knowledge divided into Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK), and the PCK with the branches: Knowledge of Content and Teaching (KCT) and Knowledge of Content and Students (KCS) as it appears in the study conducted by Hill et al. (2008). But the question is how does teacher knowledge affect teachers' selection and enactment of mathematical tasks?

According to Stein et al. (1996), both teacher subject knowledge and teachers' knowledge of students affect the tasks set up by the teacher. If the teacher's knowledge or understanding of the goal of the mathematical task as presented in the curriculum is limited, he or she may modify the task and change the goal of the task. Moreover, insufficient teacher knowledge about a task can cause the teacher to omit some of the mathematical connections embedded in the task as presented in the curriculum document. The inability of the teacher to fully understand the task as presented in the mathematical document will imply that his or her pedagogical judgment, such as the time allocated to students to complete a task and the required output he or she expects from his or her students, may be affected. This will affect whether the teachers' attention is on students providing correct answers or helping students develop mathematical thinking.

The teacher's knowledge of his or her students affects task selection in many senses. First, if the teacher is aware of the cultural backgrounds of the students, he or she may include cultural contextualization in the task as stated by Remillard and Cahnmann (2005). Cultural contextualization is the use of background or setting in mathematical tasks that are directly related to the culture of the students, whether within or outside the classroom (Araujo, 2012). Furthermore, teachers' knowledge of the student's abilities and prior mathematical knowledge may affect the teacher's ability to choose tasks that are suitable for the student's abilities and improve their mathematical thinking and reasoning.

2.4 Teacher's interaction with tasks after selection from curriculum documents

One major activity of teachers is to select curriculum materials (particularly tasks) for enactment with their students (Araujo, 2012). Teachers provide a lot of learning opportunities for their students through the curriculum materials they use in their classrooms. According to Ben-Peretz (1990), the classroom learning processes, to some considerable measure, are determined by how teachers approach the content in a curriculum. But using curriculum materials as they appear in curriculum documents alone is unlikely to result in the growth of the capacity to constantly maintain or improve students' opportunities to engage in complex mathematical tasks. According to Mason (2002), experience alone does not imply learning; rather, learning necessitates people participating in structural investigation and the ability to reflect on their practice. Prior research on teachers who have developed transformative knowledge about teaching for understanding has involved sustained reflection on how student thinking develops in relation to specific instructional patterns. Due to the vital role teachers play in selecting and enacting mathematical tasks that foster a good learning process in the classroom, this research looks at the relationship between teachers and tasks. The study first looks at how other researchers have defined mathematical tasks and how the researcher has chosen to define a mathematical task as far as this study is concerned.

The word task has been used in several ways. Stein and Smith (1998) defined a task as a period of teaching process dedicated to the exploration of a specific mathematical concept. A mathematical task has also been defined as something a teacher deploys to demonstrate mathematics, to undertake cooperatively with students, or instruct the students to do something (Watson et al., 2015). A task may include a variety of linked problems or extensive work, up to the whole teaching session on a single complicated problem. A mathematical task has also been defined by Stein et. al., (1996) as a single problem or a set of problems that is aimed at directing students' attention to a particular mathematical concept. A task is an activity that is part of an instructional sequence. A tool/device used in the mathematics curriculum to introduce, develop, practice, consolidate, connect, and assess specific learning progressions (Umameh, 2020). In my opinion and as far as this research is concerned, a mathematical task is an activity or a set of activities, which forms part of the mathematics curriculum, devoted to ascertaining students understanding of a particular mathematical concept and enhancing students thinking and willingness to participate in mathematics. These sets of instructions or activities could be done by individual students, a group of students working cooperatively, or with the involvement and assistance of the teacher.

The mathematics education research community and policymakers have consistently argued for a student-centered instruction method instead of the conventional teacher-directed model that emphasizes methodologies and operations (e.g., Common Core Standards Initiative [CCSI], 2020). The student-centered strategy emphasizes the importance of reasoning, problem-solving, and communication (Ball & Bass, 2003). Students in such a classroom deal with complicated issues that can be resolved in a variety of ways and with various representations, consider different strategies, and evaluate each other's suggestions (Kazemi & Stipek, 2001; Sun and van Es, 2015). The Professional Standards for Teaching Mathematics (NCTM, 1991) contends that it is the core mandate for the teacher to choose and create meaningful and rewarding tasks and materials that allow students to develop mathematical understandings, expertise, interests, and attitudes.

To realize such an aspiration, teachers must develop instructional practices based on challenging mathematical tasks and cultivate a classroom atmosphere conducive to productive mathematical discourse. Teachers could therefore make sense of their students' mathematical knowledge and make additional curriculum choices that encourage learning based on this (Ball & Forzani, 2011; Kazemi, Gibbons, Lomax, & Franke, 2016). It is critical for mathematics teacher educators to probe how they can assist prospects and practicing teachers in gaining knowledge to enforce these techniques in the mathematics classroom (Grossman, Hammerness, & McDonald, 2009; Remillard & Geist, 2002).

Teachers need to modify existing tasks to be mathematically rich and challenging for the students. According to Jaworski (1992), mathematical challenging tasks refer to the problems presented to students to spur their mathematical thinking. Many researchers have proposed several frameworks for classifying mathematically challenging tasks (e.g., Evans & Swan, 2014; Schoenfeld, 2007; Schwartz et. al., 1995). In contrast to conventional teaching connected with rote memorization and methods, these frameworks seek to promote a vibrant perspective on what it means to promote students' direct engagement in mathematizing in the classroom (Boesen, Lithner, & Palm, 2018). One of these frameworks is the balanced assessment proposed by Schwartz and his colleagues (1995; Schwartz, & Kenney, 2008). This framework proposes that for teachers' tasks to be considered rich, they must certify four main assessment criteria which reflect the values of the mathematics reform movement. These assessment criteria demand students create a plan, decide on the strategy to use, solve a problem, and then justify their thinking (Schwartz, & Kenney, 2008).

According to Cuban (1993), mathematics curriculum implementations frequently fail to provide the expected benefits for students. One reason could be that a written set of guidelines for teachers is understood and implemented differently than policymakers and curriculum designers originally meant (Remillard & Bryans, 2004; Remillard & Heck, 2014). Mathematics tasks in the classroom, for example, are usually described, both casually and in research journals and documents, using a wide variety of descriptions such as 'authentic,' 'rich,' and 'complex' (Shimizu, Kaur, Huang, & Clarke, 2010). However, how do teachers interpret concepts like "rich" tasks? Is there a mutual understanding among teachers of what these adjectives imply when imposed on mathematics tasks? Academics, policymakers, and curriculum designers who utilize such words to express their motives will only be successful if teachers perceive the words in the planned contexts (Foster & Inglis, 2017). The mathematics tasks provided to students are critical to their learning (Watson & Ohtani, 2015); indeed, Sullivan, Clarke, as well as Clarke (2013) contended that the kind of teaching and what students learn is generally determined by the tasks that serve as the foundation for their actions. As a result, assisting teachers with the creation and choice of elevated mathematics tasks for use in the classroom is essential for enhancing students' mathematical learning (Jones & Pepin, 2016).

2.5 Mathematical Teachers' Classroom Responsibilities During Task Enactment

During the implementation of the tasks, the teacher is also tasked to manage the learning in the classroom which contributes to students' ability to reason mathematically. Kennewell et al. (2008) averred that the teacher's role in the classroom goes beyond managing the tasks for instruction. They stated that the teacher is more than just a facilitator of the activity; their role can be described as organizing the features so that the activity moves fruitfully toward the achievement of the intended instructional goals as well as task completion. Kennewell and colleagues explained teachers' role toward students' development of mathematical thinking as something far above just creating tasks for the students to complete but also, other activities.

According to Jaworski (1992), management of learning describes the teacher's responsibility for structuring the classroom environment for learning, developing social standards within the classroom situation, and creating opportunities for learners to participate in mathematics. It involves classroom arrangements, task and event management, and prevailing attitude setting, among others (Potari & Jaworski, 2002).

The teacher should be sensitive to his students. According to Ayalon et. al., (2020), sensitivity to students refers to a teacher's understanding of her students and awareness of their wants, as well as how she communicates with individuals and guidebooks group activity. For the mathematical student, sensitivity to students is split into two contexts: cognitive and affective (Potari & Jaworski, 2002). Cognitive sensitivity deals with the teacher's acceptance and acknowledgment of students' thinking. Teachers, for instance, may concentrate on what they need to rectify in students' thinking, such as a procedural error or a deep-seated misperception (Crespo, 2000; Empson & Jacobs, 2008). Furthermore, teachers may concentrate on how a student understands mathematics, such as determining the approach a student is employing (Empson & Jacobs, 2008), or the significance of what a student said or wrote (Crespo, 2000). Cognitive sensitivity also refers to the proper reaction to students' thoughts (Potari & Jaworski, 2002). Teachers, for example, may push a student to clarify essential mathematics in his or her involvement (Kazemi & Stipek, 2001) or promote team evaluation of a student's assertion (Ayalon & Even, 2016). Teachers may also acknowledge but not utilize a student's suggestion (Stockero & Van Zoest, 2013). Affective sensitivity entails cultivating students' personal opinions in, and respecting, their potential to do mathematics and think mathematically; it is concerned with students' well-being and positive mentality in the classroom setting, and it does not always relate to academic achievement (Potari & Jaworski, 2002). This research does not view the task used in the classroom as the only source of helping students develop mathematical understanding but also other interactions in the classroom foster mathematical thinking in the students.

METHODOLOGY

The methodology of the study is presented in this chapter. The researcher deployed both a case study and a survey as the traditional methods of inquiry into this subject. The case study strategy is first described, followed by the survey. The research design is then presented, including the research method, participant selection, data collection, data management, and analysis strategy. Subsequently, the study's ethical considerations, validity, and trustworthiness are addressed.

3.1 The case study and the short survey

The researcher re-examined the study's goals and then decided on a particular tradition of inquiry with which to conduct the research. The researcher made the decision that a case study coupled with a survey constituted the best technique since the goal of a case study is to obtain an in-depth assessment of a case in a specific context (Patton, 2002) and a survey makes it possible to generate data for scientific generalization. Yin (2009, p. 18) defines a case study as “an empirical inquiry that - investigates a contemporary phenomenon in depth and within its real-life context, especially when - the boundaries between phenomenon and context are not clearly evident”. According to Demarrias and Lapan (2004), case studies attempt to respond to focused questions by developing in-depth explanations and interpretations in a relatively short time. A case study is defined by Bogden and Biklen (1982, p. 58) as "a detailed examination of one setting, or a single subject, or a single depository of a document, or a particular event". Furthermore, Stake (1995), explained that a case study is the examination of the peculiarities and depth of a single situation, with the goal of understanding its activity within compelling circumstances. The case may be about a single organization; a single community; a single event; a single school; a single family; or a single person. A case study can be single or multiple. The goal of this study was to obtain a detailed understanding of the mathematical tasks used in the classroom of the teachers involved as the tasks go through the three phases described by Stein, Grover, and Henningsen, (1996). The researcher made use of multiple-case studies in this research. The three IB program teachers selected from the upper secondary school in the southern part of Norway with tasks in three phases served as multiple case studies on the kind of mathematical tasks teachers in Norway and the Nordic countries use in their classrooms. The multiple case study design allowed the researcher to gain a deeper understanding as well as more rigorous findings. According to Berg (2007), the use of multiple case studies allowed for a more in-depth understanding of a specific topic as well as theorizing in a broader context.

The use of a case study makes it possible to grasp and comprehend context (Paré and Elam, 1997). According to Zanal (2007), case study, with their thorough accounts, can help to explain the complex nature of real-life situations that experimental or survey studies cannot. Besides the advantages that are associated with the use of a case study, it comes with its own disadvantages. Cohen et al. (2007) asserted that researchers must exercise extreme caution because case study research is particularly susceptible to biased reporting: choosing only data that backs up a specific conclusion and thus misconstruing the authenticity of a case. Again, according to Zainal (2007), case studies use a relatively small number of subjects, and thus provide less basis for scientific generalization. Thus, the use of case studies has been criticized on the grounds of a lack of statistical data for generalization.

Even though case studies have been criticized on several grounds but also have some advantages when used for an inquiry. Because some of the research questions in this study required the observation of the classroom of some of the participants, the researcher deemed the use of a multiple case study as one the most appropriate way to enquire into the subject. For the most part of the research question, a survey design could be more effective to ascertain data for generalization. The researcher then thought it necessary to combine a survey with multiple case studies in order to obtain wider responses for generalizing his result.

It's difficult to come out with a precise definition for a survey (Bell, 2010) because every survey is distinct on its own (Aldridge and Levine, 2001). Aldridge and Levine further explained that a survey in itself is a collection of several case studies where the same question is being administered to all the individual cases involved, and a case study in itself may also include a survey. It is on this threshold that the researcher decided to deploy a survey alongside the case study. The researcher’s plan at the beginning of the study was to deploy only a case study as the approach for the data collection to address the research questions in the study but later realized that adding a short survey to the study will produce a result that - not to fully serve as a scientific outcome on the kind of tasks teachers use in the classroom – may provide a wider appreciation of the outcome of the study. The survey in this study was carried out using a questionnaire as the instrument for data collection.

Surveys give a result that can serve as a scientific generalization. However, surveys have been criticized on several grounds. Unlike experiments, where the researchers exercise full constraints in controlling the effect of extraneous variables that may influence the independent variable, people believe the variables in surveys are not properly managed and so its outcome cannot be scientific (Aldridge and Levine, 2001).

3.2 The Research Method

The goal of this study was to examine how mathematics teachers select mathematical tasks for enactment with their students in the classroom. To accomplish this goal, I used a qualitative research methodology. The use of the qualitative research methodology permitted for an in-depth investigation of how the teachers interact with mathematical tasks before deeming the tasks as appropriate for their students before enacting them with their students in the classroom. It also allowed the exploration of the relationship between the teachers and the students when it comes to their disposition towards task completion and how these dispositions affect the nature or level of difficulty of the tasks. The qualitative approach was used in this research because the researcher believed that different teachers have different approaches to selecting mathematical tasks for enactment in the classroom. There is no scientific approach to teachers' selection and enactment of mathematical tasks. According to Bell (2010), the qualitative approach is the best when the researcher questions the existence of social facts and hence doubts the use of a scientific approach when dealing with human beings. The qualitative approach to research is adopted when the researcher wants to understand an individual's perception of certain matters in the world (Bell, 2010).

Bryman (2016) described the qualitative research method as a research method that emphasizes words instead of numbers in data collection and analysis. Qualitative research has features that are suitable for small datasets, but its results are not quantitative in nature. According to Maxwell (2005), the use of qualitative research methodology provides several benefits which include understanding the meaning of events, experiences, or actions for study participants, understanding the participants' contexts and the potential influence of that context on the participants' actions, understanding unexpected events or influences, and understanding the processes that lead to events or actions. In this study, the use of a qualitative approach permitted the researcher to gain a thorough understanding of the individual participants through their words and their actions in the classroom.

3.3 The participants and setting.

As I have earlier stated in this chapter, this study combined both case studies and a survey as the approach for gathering the information. The participants for this study are put into two categories: the participants for the survey and the participants for the case studies. Both participants in the survey and the case studies were selected from the same school based in the southern part of Norway.

3.3.1 Participants in the survey

A total of 20 teachers were selected for the survey. The teachers involved are all secondary school teachers and all of them are mathematics teachers. Out of the 20 teachers selected for the survey, three (3) of them are teachers of the International Baccalaureate Program whilst the remaining seventeen (17) teach in the normal Norwegian Program. The researcher had no personal relationship with these teachers and did not know any of these teachers prior to the study and during the study. In selecting these participants for the study, the researcher first contacted the head of the mathematics department in the school and explained to her what this study was about and how he intended to collect data for the study. The head of the department then directed the researcher to one of the senior teachers in the school to explain further to him and provide him with any assistance the researcher may need. The researcher explained the details of the study to the senior teacher and the teacher promised to offer him the necessary assistance the researcher may need. The researcher then administered the confidential form to the senior teacher to be given to the teachers who were willingly to participate in the study. Later on, the researcher gave the survey questionnaires to the senior teacher who then submitted them to the teachers on a platform where all the teachers are. After some time, the researcher received a response from the teacher that the survey questionnaires have been completed by the targeted number of teachers. The researcher had no idea which teacher filled out which form and anonymity and confidentiality was fully captured during the survey.

3.3.2 Participants in the case studies

In the same setting and out of the 20 participants, the researcher through the assistance of the senior teacher was able to get three of them to participate in the case study. These three teachers were the only teachers in the International Baccalaureate Program who took part in the survey. The researcher chose only the IB program teachers to make the mode of communication in the classroom very affable and easy for him. The main language of instruction in the settings where the participants were chosen is Norwegian and the researcher has no command of the language, but the IB program teachers use English as the medium of instruction in their classroom which the researcher is conversant with.

The researcher then met the three participants individually and explained to them his purpose, his responsibility toward them, and how he intended to go about the whole process. All three participants agreed, and a date was scheduled for the collection of the data. The three teachers all teach at the same level but with different kinds of students based on academic performance. For anonymity, the researcher calls the three teachers involved; teacher 1, teacher 2, and teacher 3 respectively based on the level of achieving students he or she has in his or her classroom from the highest to the lowest.

The first teacher had a classroom constituted of higher-achieving students with a student population of five (5). During my four days of observing the classroom, the researcher recorded an average attendance of four (4) students. Furthermore, the researcher observed the classroom at the time when the teacher was going through vectors with the students. The second teacher’s classroom is made up of medium-achieving students with a student population of twenty (20). The researcher recorded an average attendance of eighteen (18) during my four-day classroom observation. And the last teacher, teacher 3, has a student population of twelve (12) but recorded an average attendance of eleven (11) during the days of the classroom observation. At the time of the classroom observation, both teacher 2 and teacher 2 were teaching statistics in their classroom but at differing levels and sub-topics.

3.4 The data collection

The use of multiple data sources and methods of data collection is one of the defining features of case study research, which is contended to improve rationality, accuracy, and data credibility (Patton, 2002). Patton further extolled the virtues of triangulation, claiming that by combining data sources, researchers can conquer much of the suspicion that emerges from the use of a single approach. In an attempt to address the following research questions:

1. How do teachers select mathematical tasks for their students?
    a. What is/are the source(s) of the tasks?

    b. What is the nature of the task they choose?
    c. What factors do teachers consider when choosing a task?
2. What modifications, if any, do teachers make to mathematical tasks before using them with their students?
    a. Why do teachers modify the tasks?
    b. How does this modification, if any, influence the characteristics of the tasks?
3. What interactions occur in the classroom that helps maintain or change the characteristics of the task during enactment?

several data collection methods were deployed as the main sources of data. As mentioned earlier, a survey questionnaire was administered to 20 teachers. Again, three methods of data collection were used in the case study to attain in-depth information from the three teachers to answer the questions. These data collection techniques are interviews (pre-classroom and post-classroom observation interviews), classroom observations, and the collection of teachers' task sheets.

For research question 1a, the researcher used sections 2 and 3 of the survey and the pre-interview responses from the participants as the primary sources of data to address the question. For 1b, the primary source of data that was available to answer the question was obtained from the task sheet that the researcher collected from the case studies. For the last part of the research question 1- 1c – the researcher again resolved to section 4 of the survey questionnaire, the pre-interview responses, and the post-interview responses from the three participants in the case studies.

Moreover, the main source of empirical data for research questions 2a and 2b is the pre-interview and post-interview responses from the participants in the case studies. Lastly, the researcher deployed the classroom observation data and the post-interview data as the main source of sufficient data to address question 3. A summary of the sources of data to address the various questions is provided below.

	Data source	Comments
RQ1a	- Survey - Pre-interview -	Sections 2 and 3 of the survey (discussed later in this chapter)
RQ1b	- Task collection	Analyse tasks for their design characteristics: cognitive demand, context, visual elements, etc.
RQ1c	- Survey - Pre-interview - Post-interview -	Section 4 of the survey (discussed later in this chapter)
RQ2a	- Pre-interview - Post-interview -	
RQ2b	- Pre-interview - Post-interview -	
RQ3	- Observations - Post-interview	comments regarding interactions that change the tasks

Table 3.1: Summary of sources of data to address the various research questions.

In the sections following this chapter, the pilot stud is discussed, and detailed explanations on how the various instruments were designed and the purpose for which they were designed are given.

3.4.1 The pilot study.

Before the commencement of the main study, a brief pilot study was conducted to evaluate the viability of research procedures, data collection methods, and questions, modify or reconfigure research methodology, try out sampling strategies, gain mastery and confidence, and refine other research techniques. This was carried out in May of 2022. A colleague teacher, who teaches mathematics in one of the lower secondary schools in the Adger region was contacted. The was already a plan for the methodology that was to be used to conduct the study, but I needed to develop mastery of how to use these tools very well before the commencement of the study. This called for a pilot study before the main study.

During the pilot study, I observed the teacher's classroom once, where the teacher had some tasks to be completed by the students. I also conducted two interviews; before and after the classroom observation to test the interview questions. Furthermore, the curriculum documents (tasks sheets) were also collected from the teacher and the teacher was also given the survey questionnaire to complete. The purpose of the case study was first to ensure that the questionnaires and other data collection methods, their wordings, and intended meaning, are well communicated. Secondly, its objective was to help the researcher to acquire research skills and a direct experience and understanding of the practical aspects involved in conducting interviews and observing lessons. Again, the pilot study was conducted to refine and finalize the research questions, theoretical frameworks, and methodology for the study.

After the studies, the interviews were transcribed and analyzed. The analysis of the tasks in the tasks sheet, the interviews, the survey questionnaire, and the classroom observations revealed the following:

- The main sources of tasks used by the teacher were the textbook, the internet (mathematikk.no, tes.com, teacherpay, and corbettmath), and exam questions. But on the tasks sheet collected, it was revealed that the tasks were sourced from the textbook and the internet.
- The tasks used were mostly of the lower level of tasks (i.e., procedures without connection tasks), according to the Tasks Analysis Guide provided by Stein and Smith, provided in the cognitive demand of mathematical tasks in the theoretical framework (15 out of 17).
- The tasks found on the internet were modified by the teacher. What triggered the modification according to the teacher was that tasks found on the internet are not tailored to suit the students of the class he is handling (IB students). So, he always tries to modify some things to make them inquiry-based and not take everything from the internet. Another reason for his modification was the high range of academic performance among the students. He modifies the tasks to meet the need of both the weak and the strong students.
- It was also observed in the classroom that the students asked a lot of questions, and the teacher gave a lot of explanations which in the teacher’s view changed the difficulties of the tasks.

3.4.2 The survey questionnaires

The survey questionnaire was the first instrument used in this research. It consists of four parts. It was expected that the participants use 12-15 minutes to complete the questionnaire. The first section of the questionnaire was to obtain demographic information about the participants. It looks at how long the respondent has been involved in the service, the grade or the class level he or she teaches, the number of periods he or she teaches in a week, and the track or program of his or her students whether it is a vocational program, International Baccalaureate Program, academic track program or any other program he or she has to specify. The responses to this part of the survey helped the researcher to compare and find out how the experience of the teacher, the level of students he or she handles, and the kind of track of students he or she teaches influence task selection.

The second section of the questionnaire aimed at collecting data on the role of textbooks in the teaching and learning of the participants in their profession. It looked at the role of textbooks in lesson planning, and how they utilize textbooks in making instructional explanations, as sources of exercises and homework, among others. The third section of the survey was about how the teachers use other sources or resources such as free internet sources, internet sources that require a subscription, other teachers or colleagues, auxiliary textbooks, or other books in their pedagogical processes. Section four of the survey questionnaire was on the factors that the participants consider when choosing tasks for instruction. The survey questionnaire was structured under the guidance of my supervisor. It includes both Likert-scale and open-ended questions. Section 4 contained a list of Likert-scale questions that the researcher wanted to observe to which extent they may affect teachers' task selection. Furthermore, the opened ended questions (sections 2 and 3) were added to explore more about the resources used by the teachers. A copy of the survey questionnaire that was used for this research has been provided in Appendix A for the perusal of the reader.

3.4.3 Teachers' task sheet

The collection of teachers' task sheets was one of the four data collection methods used in the case study. The main interest of this paper was to look at the tasks that the participants use for instruction. Before the researcher started with the daily pre-classroom observation interview, the researcher asked each teacher to give him a copy of the task sheet he or she has prepared to use for instruction in the classroom. In total, four task sheets were collected from each of the three teachers during the data collection period. The researcher spent some time looking into the tasks before the interview. Having acquired the task sheets, the researcher had to compare the tasks as they appear in the teachers' task sheets with the tasks as it appears in the original curriculum documents where the tasks were selected to observe the modification that has taken place on the tasks. Almost all the tasks were from the textbooks they use. Again, having access to the tasks sheet helped the researcher to analyze the cognitive demand and the other characteristics of the tasks the participants selected for their students. It also assisted the researcher to observe the changes in the cognitive demand of the tasks when modifications have taken place on the tasks.

3.4.4 Interviews

To fully attain the goal of this study, the use of interviews was key. The interview section of data collection was divided into two separate parts: the pre-classroom observation interview and the post-classroom observation interview. According to Cannell and Kahn (1968), an interview is a two-person conversation that the interviewer initiates with the explicit goal of gathering research-relevant data that is centered on content specified by research objectives of systematic description, forecasting, or explanation. But in this study, the interview was directed by the researcher's desire to obtain specific information towards addressing the research questions.

Both interviews were semi-structured interviews. Ayres (2008), asserted that a researcher has more control over the important subjects of the interview in a semi-structured interview than in an unstructured interview, but unlike structured interviews - typically used in quantitative research - that use closed questions, there is no fixed choice of responses to each question. Ayres then defined a semi-structured interview as a qualitative data collection approach in which the researcher poses a series of predefined but open-ended questions to respondents. Ayres again clarified that a researcher who uses a semi-structured interview has already prepared an interviewing guide in advance that will facilitate the interview. However, according to Bryman (2016), the researcher aims to find rich and detailed answers in semi-structured interviews, and thus the interviewer can change substantially from any schedule or guide that is being used and ask new questions that serve as follow-up questions to the responses of the interviewee in order to address the desire of getting the rich and detailed information. Furthermore, Bryman outlined that the following steps are involved in the formulation of questions for an interview guide:

- Identifying the general research area.
- Consider the specific research questions.
- Interview topics.
- Formulation of interview questions
- Review/revision of interview questions
- Make a pilot guide with the interview question.
- Determine new issues.
- Interview questions should be revised if needed.
- Complete or finalize the guide.

In this study, predetermined open-ended interview questions were formulated for both the pre-classroom and post-classroom observation interviews. This was done in order to have full control of the important issues in the study. Even though the questions were pre-determined, a room was also created for follow-up questions to ascertain detailed and concrete responses from the participants. This led to the fixation of extra questions which were originally not in the interview guide. In the preparation of the interview guide for this study, the overall study area of this study was considered first, and then, the research questions were also used as a guide. The interview questions were then formulated and with the support of my supervisor, they were reviewed to match the goals of the study. A pilot study was then carried out to ascertain the credibility and reliability of the data collection instruments. Furthermore, the interview questions were further reviewed after the pilot when it was realized that some of the questions were not essential and easily understood by the participant in the pilot study.

The interviews were one-to-one interviews where the researcher acted as the interviewer and each teacher acted as the interviewee. The interviews were all done orally. Clarity were provided to the respondents when needed to facilitate the flow of the interviews. The interviews were carried out smoothly as there were cooperation and understanding from both the interviewer and the interviewees. There were a couple of times that there was noise from the background as the interview was initially not done in a dedicated place. However, as time went on, an office was set apart for both the before and after interviews. Details on how the two different interviews and the questions and rationale behind each question are provided below.

3.4.4.1 Pre-classroom observation interviews

The researcher conducted a daily pre-classroom observation interview with each of the three teachers before that day's classroom observation to gain knowledge more about the teachers' selection of mathematics tasks. These interviews enabled the researcher to inquire about the tasks chosen by the teachers, the rationale or motive for selecting the tasks, the teachers' goals, as well as other instructional decisions. These interviews helped the researcher to ascertain information on the sources of the tasks, the modifications that have been affected on the tasks, if any, and the motive behind the modifications if any. The interview also helped the interviewer gain knowledge about the previous lesson that the teacher has taught which may influence the kind of tasks that may be used in the classroom. The issue of the teacher's knowledge of his or her students and how that affected the selection of tasks for instruction and the goals and expectations of the teacher from his or her students were also addressed in this interview.

The goal of the researcher was to interview each teacher four times with respect to this area. Teacher 1 and teacher 3 were interviewed four times each as was earlier stipulated. However, teacher 2 was interviewed three times due to a reason beyond the control of the researcher. The interview was also scheduled to have taken place within a period of 4 weeks starting from the 10th of September to the 10th of October 2022. However, the interview took eight (8) weeks to be completed. The first interview took place on the 13th of September with teacher 1 and the last interview ended on the 8th of November with teacher 3. The date for the interview was extended beyond the October ending which was earlier scheduled. This was due to the researcher's inability to seek the consent of the third participating teacher on time. However, this delay had a minimal effect on the timeline for the completion of the study. The timelines for the interviews are tabulated below.

	Interview 1	Interview 2	Interview 3	Interview 4
Teacher 1	13th Sept. 2022 09:30 – 09:39	16th Sept. 2022 12:00 – 12:07	20th Sept. 2022 09:30 – 09:37	23rd Sept. 2022 12:01 – 12:06
Teacher 2	13th Sept. 2022 12:02 – 12:10	16th Sept. 2022 09:31 – 09:37	20th Sept. 2022 12:04 – 12:11	

Teacher 3	28th Octo. 2022 09:30 – 12:38	1st Nov. 2022 12:05 – 12:11	4th Nov. 2022 9:33 – 09:38	8th Nov. 2022 12:01 – 12:05

Table 3.2: Summary of the interview schedule with the various participants.

The following were the questions that constituted the interview questions and the motive behind each question inclusive and the frequency at which they were asked within the interview schedule:

Which level or class are you currently teaching?

This question was asked once to each of the three participating respondents. This question sought to acquire information from the participants regarding the level or grade of the high school students that the teacher is handling. The justification for the inclusion of this question in the interview is that the level of students or the grade at which a teacher teaches affects the kind of instructional methods that he adopts. This consequently affects the kind of tasks he or she uses for instruction in his or her classroom.

Which mathematics topics have you taught this semester?

This subject of the interview aimed at gathering information on the previous topics that the teacher has already gone through with the students from the start of the semester. The interviews were conducted in consecutive classes of each teacher without a skip. As a result, this question was administered once to each teacher on the first day of the interview as the researcher had knowledge of the topics that has been taught on the subsequent days of the interview. The rationale behind the involvement of this question in the interview was that the students' knowledge of the previous topic affects the kind of tasks that are used for instruction in the classroom. The researcher wanted to know what previous topics have been taught, and how these topics are related to the current topic the teacher is teaching and investigate how this will have an influence on task selection.

Which mathematical topic are you currently teaching? What have you planned to teach today?

This question sought to find out the current mathematical topic the teacher is going through with his or her students. The aim was to enable the researcher to have a picture of what to expect in the day. The question was always asked during the pre-classroom observation interviews.

What mathematical task(s) have you chosen to use?

The aim of the study was to investigate the nature and the source(s) of the mathematical tasks that the teacher uses for instruction in the classroom. As such, this question played a pivotal role in the quest to find answers to address the research topic and the research questions. This question was indirectly a follow-up question to the responses to the previous question. If you have planned to teach this topic today, what mathematical tasks if any, do you plan to use for this plan that you have? This question was a recurring question in all the pre-classroom observation interviews.

Where is the source of the task?

This subject was a direct answer to research question 1a. there are several curriculum materials that are available for the teacher to select tasks from, but more often, a teacher may stick to just one or a few of these resources. These resources include but are not limited to textbooks, syllabi, software applications, online resources, and past questions. The question sought to first find out the sources of the tasks that the teachers had prepared for use in the classroom, and secondly to explore how teachers make use of the plethora of resources that are available for their instructional needs. The question was usually followed up with the question of why you selected the tasks from this source (intended to seek justification and reason for the use of the source).

What do you intend to achieve with the task(s)?

This question probes to obtain the goal or the intent with which the teacher had selected those tasks. It looked at what the teacher wanted to achieve with the tasks. The justification for the inclusion of this question in the interviews was to find out how the goals a teacher wants to achieve with his students affect task selection. This was also one of the recurring questions in the interview.

What modifications have you made to the task(s), if any?

This topic was introduced into the interview guide to unearth the changes that have taken place on the tasks (i.e., how the tasks in the original curriculum document have been altered by the teacher when he or she was transferring it into his curriculum documents). This question was administered in each of the pre-classroom interviews organized by the interviewer. This topic was introduced in the interview with the research question in mind. Research question 2a looks at what modifications have been affected on the tasks. The search for information to address this research question was the anchor for the involvement of this topic in the research question. The question sought to gather information on whether the tasks have been modified or it the same as it appears in the original curriculum document. The response to the question also prompted the researcher to observe the changes in the cognitive demand of the tasks after the modifications have been made.

Why have you chosen (or not) to modify this task?

This is a follow-up question to the previous question. If the response to the previous question is there have been some changes, then this question probes more about why the teacher has made changes to the tasks. If no alteration has been made, it also wants to know why there have been no changes. This question was also frequent throughout the interviews.

What is your perception of the mathematical knowledge and experience of your students?

This question was inserted to ascertain information about the knowledge of the students the teacher is handling. A teacher's knowledge of the students he or she is handling has been investigated to know that it affects the teacher's disposition in his or her instruction. The question probed to understand the level of mathematical competence that the teacher's students have (whether they are high-achieving, low-achieving or the class has a higher range of students in terms of their mathematical thinking and competence). During the interview, the responses to this question were probed further with the question "did it have an influence on the nature of tasks you selected for them?".

How will you introduce the task(s)?

This looked at how the teacher intended to introduce the tasks to his or her students, whether orally or printed out. Presenting orally may change the wording or different students may have different words for the same question which may change the task.

How will students work on the task(s)?

The aim of this question was to know whether the students will be working in groups, individually, or a combination of both.

What products do you expect the students to produce?

This question addresses the teacher's expectations from the students as far as task completion is concerned. Does the teacher expect the students to understand the tasks or just they been able to solve the tasks as presented to them was also one of the concerns of this question.

A total of twelve (12) questions were used as the interviewing guide for the pre-classroom observation interviews in this study. All the data from these interviews were transcribed within a period of one month alongside the post-classroom interviews. The interview guide for this section is presented in Appendix C.

3.4.4.2 The post-classroom observation interview

The researcher conducted interviews after the classroom observation each day. The main purpose of this interview was to gather more information about the activities that took place in the room. The researcher asked questions about how effective the class was and how it matched the expectations of the teacher. The researcher also enquired more about the dispositions of the teachers towards teaching and the dispositions of the students towards the completion of the tasks. Comments, hints, and students' arrangements were also probed during this interview. The interviews were carried out shortly after the classroom observations. The interviews took an average of 4 minutes to be completed. Each of the three (3) teachers was interviewed each day after the classroom. The subjects or questions which were involved in this post-classroom observation and the justification for their inclusions are provided below:

Did the lesson go as you expected it to?

This question was used to find out how the outcome of the teacher's lesson matched his or her expectations. How he or she expected his or her students to grasp the concept and use it to solve the tasks that were given afterward. The question sought from the teacher's perspective how he or she believes his or her students were able to complete the tasks to his or her utmost satisfaction. Did the students find the tasks too difficult or too easy to complete was the intention of this question.

Which task produced the most successful outcome?

Which task produced the least successful outcome?

This subject looked at the teacher's perspective on the effectiveness of the tasks that the teacher used for instruction in his or her classroom. By effectiveness, the researcher meant which tasks were the students able to understand and completed within the stipulated time period allocated to the tasks. Again, by effectiveness, the researcher was looking at the level of difficulty or the mathematical thinking that the students had to invest in completing the tasks. Furthermore, the least effective means the students found it difficult to solve the tasks on their own within the time frame allocated, and they had to rely on support from the teacher. The justification for the inclusion of this topic was that it helped the researcher to judge from both the teacher's and their students' points of view the cognitive thinking needed to complete the tasks at hand.

What would you do differently next time?

What would you do the same?

These two sets of questions were follow-up questions to the previous question. If the tasks were not effectively completed by the students what then does, he or she (teacher) intends to do better next time so that the tasks will be effectively completed? Moreover, if the tasks were effectively completed, what does he or she intend to repeat in his or her task selection techniques so that the tasks really match the capabilities of the students? It could also be that the ability or inability of the students to complete the tasks may be due to the reason that the tasks do not match their capabilities or competence. It could be that the cognitive demands of the tasks were too low or too high for their level of mathematical thinking. If this is so, what does the teacher intend to do better next time when choosing tasks for his or her students? The rationale for the question was to get more information about the technique the teacher uses for selecting tasks.

During the enactment you made some comments: a) what was the intent of the comment(s) b) How did those comments change the nature of the task?

The researcher forecasted that during the teaching, the teachers may make comments, give hints, and answer students' questions that may have an impact on the level of difficulty of the tasks. In lieu of this, the researcher added this question to the post-classroom observation interview guide to ask after the class.

In all, there were 6 guiding questions for the post-classroom observation interview guide. A summary of these questions has been provided in Appendix D.

3.4.5 The Classroom Observation

To answer the third research question in this study, the researcher had to add classroom observation as one of the data collection tools. The classroom observation helped the researcher to obtain sufficient data needed to address the question. This tool was deployed in the classroom of the three teachers selected for the case study. The classroom observation was a structured one. Bryman (2012) defined Structured observation as "a method for systematically observing the behaviour of individuals in terms of a schedule of categories. It is a technique in which the researcher employs explicitly formulated rules for the observation and recording of behaviour" (Bryman 2012, p. 270). In this study, the participants' classrooms were observed using a guide that helped the researcher to collect specific data he needed during the observation. The researcher gathered information about the activities that took place during the observation and the date and time within which every activity was carried out. Particularly, the researcher observed and obtained information on the grade (class or level) of the students, the number of students in the classroom, the topic for the day, how the teacher starts the lesson, and how the lesson progressed with specific attention to certain information.

As the lesson progressed, the researcher paid special attention to the order of events, by the minutes (time specific) in which they happened and how long it took to complete one activity. He also took note of the explanations to the tasks, when there were any before the students start working on them, and the explanation the teacher made whilst the students were working on the tasks. The tasks used and the order in which they were used were also considered and remarked on by the researcher as the lesson progressed. Other areas where the researcher devoted to obtaining information whilst the lesson progressed was how the students work (e.g., listening to the teacher and taking notes, individual seat work, group work, class discussion, pair work, etc.), homework or other external exercises that were given to the students, and quizzes or other formative assessments. The guide the researcher used for the classroom observation is provided in Appendix E.

The classroom observations were carried out on the same day the interviews were carried out. On the first day of visiting each teacher's classroom, the teacher introduced the researcher to his or her students and explained the researcher's purpose for visiting the classroom to the students. They clarified to the students that the researcher was not there to observe them (the students) but his concern is about them (the teachers). Each classroom observation was done in between the two interviews of the day for a teacher. Each of the three teachers' classrooms was observed four times on four different days and different topics or sub-topics except for teacher 2, whose class the researcher observed three times. The table below summarizes the schedule (date) and topic that the researcher observed in each teacher's classroom.

	Observation 1	Observation 2	Observation 3	Observation 4
Teacher 1	13th Sept. 2022 Vector equation of a line	16th Sept. 2022 Vector products and properties	20th Sept. 2022 Perpendicular and parallel vectors.	23rd Sept. 2022 Angles between plane
Teacher 2	13th Sept. 2022 Binomial Theorem	16th Sept. 2022 Mathematical Proofs	20th Sept. 2022 functions	
Teacher 3	28th Octo. 2022 Representing and describing data	1st Nov. 2022 Bivariate Data	4th Nov. 2022 Correlation (measuring correlation)	8th Nov. 2022 Least square regression Line

Table 3.3: Summary of time and topic observed during the classroom observation.

3.5 Data management

Because of the use of a qualitative approach in this study, a lot of data was collected during the study. This data includes the interviews, the teachers' curriculum documents which were collected during the study, the filled survey questionnaires, and the researcher's notes taken during the classroom observation. Van den Eynden et al. (2011) claimed that digital storage media are incredibly unreliable, as a result, a data storage strategy is very essential. To ensure the safety of the data, the researcher stored the audio-recorded interviews on both his computer and his mobile phone. The researcher created a password on both the folder on the computer and the mobile phone to prevent the interviews from being intruded on by second parties. The curriculum documents collected from the teachers, the filled survey questionnaires, and the field note of the researcher were kept in a locker and locked with a passcode which is only known by the researcher.

A month after the interviews, the researcher started transcribing the interviews. There were twenty-two (22) interviews in all with an average length of approximately four (4) minutes. The transcribed data were kept together with the file containing the audio-recorded interviews on the researcher's computer. To ensure anonymity, fictitious names were used to represent the participants. The researcher intends to keep all the data for a period of one year after the completion of the study and then finally destroy them. The strategy used for analyzing the data is provided in the sections below.

3.6 Strategy for data analysis

In qualitative research analysis, the researcher is tasked to organize and handle enormous quantities of data into smaller portions (Araujo, 2012). Data analysis is essential for conducting credible qualitative research. Indeed, the qualitative researcher is frequently referred to as the research instrument because his or her ability to comprehend, describe, and interpret experiences and perceptions is critical to uncovering meaning in specific circumstances and contexts (Maguire & Delahunt, 2017). Because of the vastness of my data corpus (i.e., all of the data collected for this study), open coding was my first step in the data organization and data reduction or condensation process. This is done to ensure that there is an order, structure, and interpretation to the huge amount of data collected (Denscombe, 2014). He (Denscombe) elaborated again that the analysis of qualitative data can take a variety of forms (i.e., there is no one specific way to analyze qualitative data) relating to the type of data and the purpose for which they are studied. Even though there is no specific way of analyzing qualitative data, the researcher of any study must make it known the method he or she deployed in the analysis of the data in his or her study to assist other researchers to evaluate the study and to compare with other work on the topic (Braun and Clarke, 2006).

The analysis of this study was done with the research questions as a focal point. A thematic analysis was deployed as the tool for analyzing the interview data. Thematic analysis is defined as the approach of identifying patterns or themes within qualitative data (Maguire & Delahunt, 2017; Braun & Clarke, 2006). Thematic analysis can be done in several ways (e.g., Alhojailan, 2012; Boyatzis,1998), however, because of this variety, there is some misunderstanding about the nature of the thematic analysis, including how it differs from qualitative content analysis1 (Vaismoradi, Turunen, & Bonda, 2013). In this analysis, the six steps thematic analysis of Braun and Clarke (2006) was deployed. The six steps thematic analysis of Braun and Clarke (written below in this section) is one of the most influential approaches to thematic analysis, at least in the social sciences, owing to the fact that it provides such a concise and workable framework for conducting thematic analysis (Maguire & Delahunt, 2017).

According to Braun and Clarke (2006), thematic analysis is the first qualitative method that should be learned because "it provides core skills that will be useful for conducting many other types of analysis" (p.78). They also clarified that a theme conveys something significant about the data in connection with the research question(s), as well as some level of patterned response or meaning within the data set. The objective of thematic analysis is to identify themes, or significant or captivating patterns in data, and then use these themes to address the research or say something about an issue (Maguire & Delahunt, 2017). A good thematic analysis does much more than simply summarize the data; it interprets and makes sense of it. A prevalent blunder is using the principal interview questions as the themes (Clarke & Braun, 2013). The six phases of Braun and Clarke’s thematic analysis are as follows:

- becoming acquainted with your data,
- creating initial codes,
- looking for patterns,
- reviewing themes,
- defining and naming themes, and
- producing the report.

The first step that the researcher took in the analysis was to read the transcribed interview over and over again to become familiarized with the data set available. The researcher took note of key things whilst reading through the data and underlined some other key points. In the consequent readings, the researcher used keywords (codes) in every sentence and paragraph which have a relation to answering the research questions. After assigning those codes to the relevant part of the interview, the researcher then organized the data by collecting all the parts of the interviews that point to the same information together (i.e., patterns were identified in the data set). The organized data were then transferred into a new word document to make it easy for further analysis. The codes which were been organized and placed in another file were further analyzed to develop themes that point out something interesting to the research questions. Finally, the initial themes were reviewed and developed into main themes, sub-themes, and sub-subthemes. A summary of how the various data set was analyzed and the themes that were developed from them are as follows:

3.6.1 The survey

After collecting the survey questionnaires which have been filled (20 in total), the researcher first read through them to observe if they have been properly filled. All twenty scripts were read through thoroughly. A word document was then created to tally the response for each question on the questionnaire. Section one of the survey did not provide any relevant information related to the research questions, as a result, not much emphasis was placed on that section. The sections of the survey that carried information necessary or that points out to the research questions were sections two, three, and four.

The survey was analyzed section by section. The second section of the survey looked at the role of textbooks as a resource for teachers. Section 2 stated seven (7) possible purposes for which a teacher may use a textbook, and another space for the respondents to specify if they have other purposes for which they use textbooks. After reading through the responses, the number of each purpose of textbook usage was collated to ascertain the number of teachers that use textbooks for the various purposes stated. It was analyzed by looking at the various ways the teachers use the textbook and the commonalities between the teachers when it comes to the variety of purposes for using the textbook. The responses were analyzed by looking at the various purposes the teachers use textbooks for. The table below indicates the outcome of the collation of the various purposes for section two.

Purposes of use	**Number of teachers**
Lesson planning	20
To prepare for making instructional explanations	15
To give student reading tasks	14
Source of exercises	18
To give homework from	14
To trigger student interest or curiosity	9
To provide a history of mathematics	5
Other	0

Table 3.4: Summary of primary results from section 2 of the survey questionnaire.

Section three looked at other resources rather than the textbook the teachers use for instruction, it was analyzed by recording and comparing all the resources that the teachers have listed on the questionnaires to identify similarities (the number of teachers who use a particular resource) and differences in the resources that the teachers use. Again, the outcome of section three is tabulated below.

Sources	No. Of teachers	Purpose of usage
Skolestudio	5	Sources of videos Instructional explanation 11 Source of exercises 2 Lesson planning
Mathematikk.no	6	Sources of Exercises 5
Ndla.no	8	Sources of Exercises 8 Give homework 1 Lesson planning 2 Instructional explanation 1 Trigger student interest 1
Andre Lorboker	4	Sources of exercises 2 Lesson Planning 1
Kahoot	3	Trigger interest Competition 2 Repetition
Gyldendal.no	2	Show the tasks 1 Source of exercises 1
Revision village	1	Lesson Planning 1 To give homework 1 Trigger student interest 1 Exams preparation 1
Question beinto	1	Lesson planning Source of Exercise 1
Udir.no	6	Lesson planning 2 Exams task 2 Source of exercises 2
Teams	1	Source of Exercise Lesson Planning
ITS	1	Source of Exercise Lesson Planning
Han	1	To make instructional explanations Reading tasks
Phet collerado	1	To make instructional explanations Trigger student interest
The textbook's site	1	Extra exercises
colleagues	2	Lesson Planning 11 Sources of exercises 11 Instructional explanations
Facebook	1	Source of funny exercises to criticize
GeoGebra	1	Did not specify
Python	1	Did not specify
Youtube	1	Trigger interest explanations

Table 3.5: Primary results from section 3 of the survey questionnaire.

The last section of the survey looks at the factors teachers consider when selecting tasks. Using a tally, the number of responses related to each Likert Scale for the theme was recorded in accord. The summary of those responses is below.

Task selection consideration	Un. Imp.	Sl. Imp.	Mo. Imp.	Imp.	very imp.
selecting tasks that contain **an important mathematical idea**	1	4	4	7	4
selecting tasks that promote **understanding** of a mathematical idea			2	10	8
selecting tasks that give another chance **to apply a known mathematical procedure**			4	12	4
selecting tasks that require **students to work in small groups**		4	13	3	
selecting tasks that **can create a classroom discussion**	1	2	13	4	
selecting tasks that require students **to explain and reflect on** their thinking		1	8	10	1
selecting tasks that have the potential to raise **student interest and motivation**		3	7	8	2
selecting tasks that have a context and making **real-life connections to other school subjects**		3	9	6	2
selecting tasks that make use of **visual drawings or illustrations of mathematical ideas**		1	12	5	2
selecting tasks that are quite **unique, interesting, and unlike** "textbook" tasks		6	9	4	
selecting tasks that are NOT **too difficult**	1		8	11	
selecting tasks that are NOT **too easy**		3	9	8	
selecting tasks that **do NOT** have a **complicated storyline or have long text** to read		6	7	4	3
selecting tasks that are given in **steps from easy to more difficult**		1	7	9	3
selecting tasks that sometimes **do NOT have all the information needed for a solution**, and students either need to make assumptions or do research to find the information	1	7	9	3	
selecting tasks that have **unexpected or surprising** answers	2	6	11		1
selecting tasks that students **do NOT have the knowledge to solve yet**, and learning new knowledge while working on the task	2	10	6	2	
selecting tasks that **do NOT seem to have "keywords" hinting** at **what mathematical procedure should be used** in the solution	1	4	12	2	1
selecting tasks **to introduce a new topic**		1	6	10	3
selecting tasks that **apply a mathematical topic or procedure after I teach it**			4	12	4

Table 3.6: Primary findings from section 4 of the survey questionnaire. ***NB: Un means unimportant, Sl means slightly, Mo means moderately, and Imp means important.***

All the responses to the survey were then analyzed, organized, and developed into codes and themes. The table below describes examples of how some of the codes were generated.

Codes	Themes	Main theme
Lesson planning	Purposes of textbooks	Resources used by the teachers
To prepare for making instructional explanations		
Source of exercises	Purposes of other sources	
To give student reading tasks		
Important mathematical idea	Stress mathematical idea	Task selection considerations
Understanding a mathematical idea		
Apply a mathematical idea		
Students work in small groups.	Theme: focus on the students	
Create classroom discussions.		
Explain and reflect on students thinking.		

Table 3.7: Examples of how the codes and themes were developed from the survey.

3.6.2 The interviews

The interviews that were conducted during the study were first transcribed into text. The researcher read and re-read through the transcribed interviews several times. Whilst he read through, relevant information in the data that points to the study's research questions was first highlighted. The researcher assigned initial codes (keywords) to the sections of the interview which were relevant to answering the research questions. The coded part of the interviews was then organized into a different file. The section was then read further to look for patterns which were then developed into themes.

Finally, the initial themes were developed and organized into main themes, sub-themes, and sub-subthemes. The main themes that were identified and developed in the interviews include the sources of the tasks, the task selection considerations, the modifications on the tasks as they appear in the textbook, and classroom activities that had an impact on the nature of the tasks. Examples of how the codes and patterns were extracted from the interview are discussed below.

Data set	Initial codes	potential themes
I have chosen formal exams questions because it is at the end of or halfway through vectors and the next part is vectors as a plane, which is much more difficult and now, we get a lot of old exam questions that go on the easy part of vectors on a line and the perpendicular and all these sorts of aspects.	Other sources	sources of task
… they are all from the textbook. With other sources, not today. I used to sometimes use something called revision village, it's an online thing that I subscribe to…	Internet, Textbook questions	
.. they have a very good textbook and instead of making a lot of work for myself, I use them because I of course gone through them and seen that these are good exercises …	Exercises are good.	No modification
Because they are good and there are a lot of them. So we have more than enough to then than to try to find new ways when we think they are good. If they weren't that good, I would of course make a lot of changes and search for more exercises from other places. But I think it's a good choice, the book we have	The tasks are many.	
The aim was to make it suitable for where my students are at the moment.	Make tasks suitable for students.	Tasks modified
So that they get closer to the exam questions. So that is my target, so they are going to have an exam, the exam tasks we have to practice on and work on them. So, I don't change them, but I change some in the book or other books to get closer to the exam	Bridge the gap with the exam questions.	
… I'm hoping that because I don't think they are too good to do that alone. They should have discussed more together so trying to, so they can do more and more together.	Allow students to complete tasks in groups.	Teachers' disposition during task completion
… I just see what issues they have with the tasks and then I just take it from their point of view. Sometimes I just try to see if they know what to find, you know how to do it, you know what this means because if I just come in and say oh this is, you have to do this and this, then they	Teacher students' discussion of a task	

forget about the rest.		

Table 3.8: Examples of how the codes and themes were developed from the interviews.

3.6.3 The tasks on the tasks sheet

The tasks on the task sheets which were collected during the research were analyzed using the Task Analysis Guide (TAG – detail of it in Figure 2 in the theoretical framework). The tasks were analyzed to obtain their cognitive demand. The TAG classified mathematical tasks into four categories according to the level of thinking required to solve them: memorization tasks, procedures without connection, procedures with connection, and doing mathematics. The classification of the tasks was done using the characteristics of the various tasks as they appear in Smith and Stein (Smith and Stein, 1998). To make things easier for the reader, the key characteristics of the four main cognitive demands of mathematical tasks according to Smith and Stein which were used to analyze the task on the task sheets have been highlighted below:

- **Memorization tasks**. Involve either reproducing previously learned facts, rules, formulas, or definitions or committing facts, rules, formulas, or definitions to memory. No procedure is required to solve the tasks, solved within a limited time frame. No ambiguity in the question, just a reproduction of materials which has already been seen.
- **Procedure without connection**. These tasks are algorithmic in nature. The procedure is either explicitly required or is obvious from prior instruction, experience, or task placement. They contain little ambiguity and require little thinking in completing them. They Have no relationship to the concepts or meanings underlying the procedure being used. They are focused on producing correct answers rather than developing mathematical thinking and they do not require explanations for the answers.
- **Procedure with connections**. They focus students' attention on using procedures to produce deeper levels of understanding of mathematical concepts and ideas. They propose explicit or implicit pathways to follow that are broad general procedures with

close connections to underlying conceptual ideas rather than narrow algorithms with opaque connections to underlying concepts. Answers to such questions can be generated in several ways and these tasks required a higher level of thinking to be completed.

- **Doing mathematics**. These kinds of tasks are considered the highest and the most difficult to complete. They are nonalgorithmic and require complex thinking to be completed. They require students to investigate and comprehend the nature of mathematical concepts, processes, or relationships. In order to complete the task, students must access relevant knowledge and experiences and apply them appropriately. Such tasks demand students to analyze them (task) and actively explore task constraints that may constrain possible solution strategies and solutions. They require substantial cognitive effort and may cause anxiety in the student due to the unpredictable nature of the solution process required (Smith & Stein, 1996). Examples of tasks relating to all the four levels of cognitive demand of tasks have been provided in Figure 1 (see p. 15).

These characteristics of tasks suggested by Stein and Smith were carefully followed and used to analyze the tasks that were received. The names of these tasks were used as the codes as illustrated in the table below.

Codes	Theme
Memorization tasks	The cognitive demand of mathematical tasks
Procedure without connection	
Procedure with connection	
Doing mathematics	

Table 3.9: Codes and themes developed from the task on the task sheets.

3.6.4 The Classroom Observations

During the classroom observation, the researcher took note of the exercises that the teachers used with the students in the classroom, how the tasks were introduced to the students, the time allocated for the completion of the tasks, similar examples the teachers themselves completed with the students, how the teachers expected the students to work on the tasks that have been assigned to them, among many other things. The analysis of this data was done using the factors in the second circle of the relationship between task-related variables and student learning by Stein, Grover, and Henningsen (1996), which have been already discussed in the theoretical framework (figure 3, see p. 17).

In this framework, as already explained, Stein and his colleagues looked at factors such as classroom norms, task conditions, teachers' instructional habits and dispositions, and students' instructional habits and dispositions as possible factors that may affect the cognitive demand of mathematical tasks as the tasks go through implementation in the classroom (more details provided in the theoretical frameworks). Furthermore, examples of the codes and themes that were generated from the classroom observations are in the table below.

Initial codes	Themes
Solving similar examples	Teachers' disposition toward task completion
Going around and assisting students to complete the tasks.	
Students share ideas.	Students' disposition toward tasks completion
Ask questions during task completion.	

Table 3.10: Examples of how the codes were generated from the classroom observations.

All the initial codes and themes that were generated from all the various data sets were then organized and tabulated below.

Theme: The course textbook. Codes: Source of task Lesson planning Make instructional explanations. Give reading task. Give homework. Trigger student interest. Provide a history of mathematics.	Theme: The cognitive demand of the tasks Codes: Memorization tasks Procedure without a connection Procedure with connection Doing mathematics Too difficult Too easy	Theme: Stress mathematical idea Codes: Important mathematical idea Understanding a mathematical idea. Apply a mathematical idea. Covers all that has been taught. Develop vocabulary
Theme: Tasks from the textbook. Codes: Good task A lot of exercises Matches students' abilities. Teachers' goal Progresses the tasks. Demands of the curriculum.	Theme: the content of the tasks Codes: Real-life connections Visual drawings and illustrations Unique and interesting.	Theme: focus on the students Codes: Students work in small groups. Create classroom discussions. Explain and reflect on students thinking. Students' interest and motivation
	Theme: design of the task Codes:	

Theme: Internet and other sources Codes: Source of task Lesson planning Source of video Create competition. Make instructional explanations. Give reading task. Exam Preparation Give homework. Trigger student interest. Provide a history of mathematics. Theme: modification of the tasks Codes: Time constraint Make tasks suitable for students. Tasks demand a lot of work. Tasks repeat themselves. Bridge the gap with the exam questions.	Students not having the knowledge to solve yet. No keywords hinting at what mathematical procedure to use. Steps from easy to more difficult Not having all information needed for the solution. Unexpected or surprising answers Complicated storyline. Theme: no modification to the tasks Codes: Exercises are good. Prevent extra work. The tasks are many. The topic is new. Guide for the exam Theme: Sources of tasks Codes: Textbook Exam tasks from the internet Self-made task	The mathematical knowledge and level of students. Theme: Application to other topics. Codes: To introduce a new topic Apply mathematics to another subject area. Theme: teachers' disposition toward task completion. Codes: Solving similar examples Going around and assisting students to complete the tasks. Allow students to complete tasks in groups. Allow students to ask questions during task completion. Theme: students' disposition toward task completion. Codes: Students share ideas. Ask questions during task completion. Students inactive during task completion

Table 3.11: Summary of the initial codes and themes developed from the data sets.

After generating the initial codes or themes, the next step taken was to identify the importance of the themes, what each theme is pointing to, and how the themes relate to each other. After a thorough analysis of these primary themes, they were grouped into the main themes, sub-themes, and sub-subthemes. The table below represents the final themes developed from the analysis.

Theme: Resources used by the teachers.	Theme: task selection considerations	Theme: Modification of the tasks
Subtheme: Sources of tasks Textbook = TTB Exam tasks from the internet = ETI Self-made task = SMT Subtheme: uses of textbook. Source of task = SOT Lesson planning = LP Make instructional explanations = MIE Give reading task = GRT Give homework =GHW Trigger student interest = TSI Provide history of mathematics = PHM Subtheme: Internet and other sources Source of task = SOT Lesson planning = LP Sources of videos = SOV Create competition = CC Repetition = RPT Make instructional explanations = MIE Give reading task = GRT Exam preparation = EXP Give homework =GHW Trigger student interest = TSI Provide history of mathematics = PHM Sub-subtheme: Tasks from the textbook. Good task = GT A lot of exercises = LOE Matches students' abilities = MSA Teachers' goal = TG Progresses the tasks = PTT Demands of the curriculum = DOC	Subtheme: The cognitive demand of the task Memorization tasks = MD Procedure without a connection =POC Procedure with connection = PWD Doing mathematics = MD Too difficult = TD Too easy = TE Subtheme: the content of the tasks Real-life connections = RLC Visual drawings and illustrations = VDI Unique and interesting = UI Subtheme: design of the task Students not having the knowledge to solve yet = SNK No keywords hinting at what mathematical procedure to use = KHP Easy to more difficult = ETD Not having all information needed for the solution. = INS Unexpected or surprising answers = USA Complicated storyline = CSL Subtheme: Stress mathematical idea Important mathematical idea = IMI Understanding a mathematical idea = UMI	subtheme: modification of the tasks Time constraint = TC Make tasks suitable for students = MTS Demand a lot of work = DLW Tasks repeat themselves = TRT Bridge the gap with the exam questions = BGE subtheme: no modification to the tasks Exercises are good = EAG Prevent extra work = PEW The tasks are many = TAM The topic is new = TIS Guide for the exam = GFE Theme: classroom activities on the tasks Subtheme: students' disposition toward task completion. Students share ideas = SSI Ask questions during task completion = SAQ Students inactive during task completion = SIC Theme: teachers' disposition toward task completion. Solving similar examples = SSE Going around and assisting students to complete the tasks = GAS Allow students to complete

	Apply a mathematical idea = AMI Covers all that has been taught = CAT Develop vocabulary = DV Subtheme: focus on the students. Students work in small groups = SSG Create classroom discussions = CCD Explain and reflect on students thinking = ERST Students' interest and motivation = SIM The mathematical knowledge and level of students = MKL Subtheme: Application to other topics. To introduce a new topic = INT Apply mathematics to another subject area = AMT	tasks in groups = ASG Allow students to ask questions during task completion = ASAQ

Table 3.12: Summary of final codes and themes developed from the data sets.

3.7 Ethical considerations

Issues relating to ethics were highly esteemed in this study. Before the start of the study, a letter was sent to the Norwegian Center for Research Data (NSD) to seek approval of the research topic, and the methodology, and be permitted to seek and dialogue with potential participants for this research project. Upon approval from the NSD, the head of the mathematics department of the upper secondary school where the study was conducted was contacted. The researcher explained to her what the study was about, and the number and track of teachers (teachers who teach in the International Baccalaureate Program) he needed for the case study. The head of the department then connected the researcher with one of the key teachers of the IB program, who then introduced the researcher to the other participating teachers individually.

After the meeting with the teachers, the chief ethical issues relating to this study were discussed. The issues of informed consent, participant anonymity and confidentiality, negotiating access, and privacy were discussed with the three participants for the case study. With the participants in the survey, a consent form was sent to them through their emails by one of the IB teachers. The researcher had no personal or face-to-face meetings with them, thereby ensuring anonymity. A copy of the consent form approved by NSD and presented to the participants for the survey is provided in Appendix F. The consent letter contains the purpose of the study and the intended method for collecting data. The letter also stated their willingness to withdraw from the study with or without any reason.

The three IB teachers who participated in the study were also presented with a different consent form approved by NSD. Prior to the data collection, these teachers were met one-on-one to further discuss the details of the study's methodology. The researcher also promised them on ensuring the confidentiality and anonymity of the data gathered from them. The participants were informed also of their right to withdraw from the research at any point in the research. The participants were allowed to make a fully voluntary decision to participate in the study. Furthermore, Pseudonyms (Teacher 1, Teacher 2, and Teacher 3) were used to represent the three teachers. The study was conducted in a comfortable atmosphere for the participants as there was no toucher, physical or psychological during the study. The interviews and all other data collected during the study are intended to be destroyed six months after the study.

3.8 Validity and Reliability

It must be stated emphatically that the findings from this study are not the sole results that could be generated from the data set. Other researchers may interpret the data set and come out with but not completely different results. Again, the researcher of the study from period to period and through learning may investigate the data set and come out with something not completely different. As a result of this, the researcher deployed intra-coder reliability to ascertain how different times and several observations in the data set may yield consistency and accuracy in the data analysis. Intra-coder reliability refers to the consistent manner of coding by the same researcher (Lavrakas, 2008). A second go-through of the data set by the researcher produced the same codes and themes for the findings in this study. The first coding was done between January and February 2023 and the latter within May-June 2023.

Furthermore, the findings of this study cannot be generalized as there were only three cases in the study. Even though a survey was used in this study, the number of participants was too small to produce a generalized result. Again, the study could have been more enlightening if the students were also involved in the study. This could have produced a better assessment of what a good task is from both the teachers and the students. Also, the activities in the classrooms could have been audio-recorded or video-recorded. This could have given more detailed information on the interactions between the teachers and the students in the classroom during task completion.

DATA ANALYSIS AND FINDINGS

The analysis of the study regarding the research questions of this study is presented in this chapter. The aim of this study was to examine teachers' selection of mathematical tasks for instruction in the classroom: the sources of the tasks they use, the cognitive demand of the tasks, the factors they consider before choosing the tasks, the modifications they make to the tasks if any, why they modify the tasks, how these modifications if any, affect the cognitive demand of the tasks, and the classroom interactions that have an influence on the cognitive demand of the tasks. The interview responses from the teachers, the tasks on the task sheets collected from the teachers, the survey questionnaire, and the data from the classroom observations were all analyzed in this chapter.

This chapter is made up of five sections. First, the summary of the results from the survey questionnaire, the interviews, the task sheets, and the classroom observations are presented in tables. The chapter proceeds (section 2) with the analysis of the resources used by the teachers which include the teachers' use of textbooks and the use of the internet and other resources in their pedagogical activities. Section 2 of the chapter again looked at the sources of tasks used by the teachers during the study, which include the tasks from the textbooks, the tasks from the exams which are found on the IB program website (internet), and the tasks made by the teachers themselves. Section 3 of the chapter looked at the analysis of factors teachers consider when choosing tasks for use with their students in the classroom. The fourth section of this chapter looked at the analysis of the modifications that teachers make to the tasks in the textbooks when they transfer those tasks into their curriculum documents, the reason for the modifications made, if any, and how these modifications, if any, affect the cognitive demand of the tasks. The last section of this chapter looks at the analysis of the classroom activities by the teachers and the students that had an effect on the tasks.

The names of the participants for the study have been presented by a pseudonym (Teacher 1, Teacher 2, and Teacher 3) to ensure anonymity and confidentiality. Extracts from the interviews which represent specific responses to the subject under discussion were used to explain the findings.

4.1 Summary of Results

The summary of results from all the various data collection tools – the interviews, classroom observations, the curriculum documents (task on the task sheet), and the survey questionnaires are all presented in the tables 4.1–4.4 below. Table 4.1 is explained after its presentation.

4.1.1 Summary of Results from the Survey

Section				
2 (uses of the textbook)	Uses of textbook		Number of respondents	
	SOT		18	
	LP		20	
	MIE		15	
	GRT		14	
	GHW		14	
	TSI		9	
	PHM		5	
3 (internet and other sources)	Internet and other sources	number of respondents (Uses the source)	Purpose of use	Number of respondents (Purpose of use)
	Skolestudio (school studio)	5	SOV	1
			MIE	2
			SOT	2
			LP	1
	Mathematikk.no	6	SOT	6
	Ndla.no	8	SOT	8
			GHW	1
			LP	2
			MIE	1
			TSI	1
	Andre Lorboker	4	SOT	2
			LP	1
	Kahoot	3	TSI	1
			CC	2
			RPT	1
	Gyldendal.no	2	SOT	2
	Revision village	1	LP	1
			GHW	1
			TSI	1
			EXP	1
	Udir.no	6	SOT	4
			LP	2
	Colleagues	2	SOT	2

			LP	2
			MIE	1
	Question beinto	1	SOT	1
			LP	1
	Teams	1	SOT	1
			LP	1
	ITS	1	SOT	1
			LP	1
	HAN	1	MIE	1
	Phet collerado	1	MIE	1
			TSI	1
	Facebook	1	SOT	1
	Youtube	1	MIE	1
			TSI	1
	Python, GeoGebra	1 each	Did not specify	

4 (Task selection consideration)	Task selection considerations	Degree of consideration				
		Un. Imp.	Sl. Imp.	Mo. Imp.	Imp.	V. imp.
	Stress mathematic idea					
	IMI	1	4	4	7	4
	UMI			2	10	8
	AMI			4	12	4
	Focus on students					
	SSG		4	13	3	
	CCD	1	2	13	4	
	ERST		1	8	10	1
	SIM		3	7	8	2
	Content centered					
	RLC		3	9	6	2
	VDI		1	12	5	2
	UI		6	9	4	
	Cognitive demand					
	TD	1		8	11	
	TE		3	9	8	
	The task Design					
	CSL		6	7	4	3
	ETD		1	7	9	3
	INS	1	7	9	3	
	USA	2	6	11		1
	SNK	2	10	6	2	
	KPH	1	4	12	2	1
	Application to other topics					
	INT		1	6	10	3
	AMT			4	12	4

Table 4.1: Summary of results from the survey. Explanations of codes and themes are defined in Table 3.12, (see p. 58).

Table 4.1 represents the summary of the results from the survey. The table has three sections: sections 2, 3, and 4. Section 2 has two columns with the results, where column one represents the purpose of the use of the textbook and column two, the corresponding number of responses related to each purpose of use of the textbook. Furthermore, section 3 has four columns. The first column contains the resources that the teachers use. Column 2 shows the number of teachers who use the resource. Columns 3 and 4 respectively display the purpose(s) of use of the resources and the number of respondents who use the resources for the corresponding purpose.

Section 4 is about task selection considerations. The questions that made up the task selection considerations were categorized into six main themes – stress mathematical ideas, focus on students, content-centered, cognitive demand, the task design, and application to other topics - depending on how the questions were related to each other. Each category has a set of codes (codes defined in Table 3.12) under them. The responses to the various codes were on a Likert scale ranging from unimportant (un imp) to very important (V. Imp). The Likert scale is represented on the table as the degree of consideration and represented by abbreviations as un. Imp – very important, sl. Imp – slightly important, mo. Imp – moderately important, imp – important, and v. imp – very important.

4.1.2 Summary of results from the task sheet

The summary of the results from the task sheets comprises the sources of the tasks observed, and the cognitive demand of the tasks observed.

4.1.2.1 Sources of the observed tasks

	Teacher 1	Teacher 2	Teacher 3	Total
Sources of tasks				
TTB	10	24	28	62
ETI	28	0	0	28
SMT	0	1	0	1
Total	38	25	28	91

Table 4. 2: Summary of the sources of the observed tasks used by the teachers. Codes are defined in Table 3.12, (see p. 58).

4.1.2.2 Summary of the cognitive demand of the tasks used by the teachers

	Teacher 1	Teacher 2	Teacher 3	Total
Cognitive Demand				
MM	0	4	4	8
PWOC	38	21	24	83
PWC	0	0	0	0
DM	0	0	0	0
Total	38	25	28	91

Table 4.3: Summary of results on the cognitive demands of the tasks from the original curriculum documents. The codes are defined in Table 3.12, (see p. 58).

4.1.3 Summary of results from the classroom observations

	Teacher 1	Teacher 2	Teacher 3	Total
Teachers' dispositions toward task completion	SSE	SSE	SSE	3
	GAS	GAS	GAS	3
	ASG	ASG	ASG	3
	ASAG	ASAG	ASAG	3
Students' dispositions toward task completion	SSI	SSI	SSI	3
	SAQ	SAQ	SAQ	3
		SIC		1

Table 4.4: Summary of results from the classroom observations. Codes and themes are explained in Table 3.12, (see p. 58).

4.1.4 Summary of results from the interviews

	Teacher 1	Teacher 2	Teacher 3	Total
Sources of tasks	TTB	TTB	TTB	3
	ETI	ETI	ETI	3
		SMT		1
Task selection considerations	DV	DV	DV	3
	MKL	MKL	MKL	3
Modifications	MTS			1
	TRT			1
	BGE	BGE		2
	DLW			1
	TC		TC	2
No modifications	TIS			1
	EAG	EAG	EAG	3
		GFE		1
			TAM	1
			PEW	1

Table 4.5: Summary of results from the interview. The definition for the codes and themes is found in Table 3.12, (see p. 58).

4.2 Sources of tasks/ Resources used by the teachers

The main aim of this study was to observe the tasks teachers select and enact with their students in the classroom. One way to achieve this was to investigate the resources the teachers use in their pedagogical activities and the purpose for which the teachers use the various resources. A survey questionnaire was designed by the researcher to collate information for this investigation. First, sections 2 and 3 of the survey questionnaires were analyzed to find empirical data for the resources used by the teachers in their teaching process. These sections of the survey deal with the teachers' use of textbooks and other resources use the teachers respectively. A summary of the results pertaining to teachers' use of resources and the purpose can be found in Table 4.1 (sections 2 and 3, see p. 63). Furthermore, the interviews were analyzed and provided adequate information for addressing the sources of tasks used by the teachers during the study. A detailed analysis of sections 2 and 3 of the results from the survey questionnaires and the interview responses relating to this part of the research has been provided in the proceeding subsections of this section.

4.2.1 Resources Used by Teachers in their teaching process

The responses provided in section 2 and section 3 of the survey revealed that there are three (3) main resources used by the teachers for teaching. First, the responses from the questionnaire provided that all the teachers use textbooks for their teaching, and for various related and varying purposes. Moreover, most of the respondents answered that they used various Internet resources in their teachings, and for various purposes. The last resources indicated by some teachers for use for their teaching and termed as "others" include but are not limited to other books and colleagues. In a nutshell, the resources used by the teachers are textbooks, internet resources, and other resources which include colleagues and other books.

4.2.1.1 The Use of Textbooks

Regarding the textbooks, as already indicated, all the respondents answered yes to the use of textbooks. That is, they all agreed that textbook is one of the resources they use in their pedagogical activities. There were seven (7) purposes for which the researcher forecasted might be the purposes for which the teachers use textbooks, and further asked the respondents to provide other usages of the textbook if they (their use of textbook) are not indicated in the list already provided in the questionnaire. The seven purposes provide in the research questionnaires are,

1. Source of task (SOT)
2. Lesson planning (LP)
3. Make instructional explanations (MIE)
4. Give a reading task (GRT)
5. Give homework (GHW)
6. Trigger student interest (TSI)
7. Provide a history of mathematics (PHM)

With regard to the source of task (SOT), eighteen (18) out of the twenty (20) respondents indicated that textbooks serve as the source of tasks for their teaching. Two respondents did not see textbooks as their source of tasks for instruction in the classroom. Furthermore, all the respondents (twenty out of twenty) indicated that they use the textbook for their lesson planning (LP), giving a perfect score (from the respondents' point of view) for the use of the textbook by the teachers for lesson planning. Another area where most of the respondents selected to be their major use of textbooks is to make instructional explanations (MIE). Fifteen (15) of the respondents indicated that they use the textbooks for instructional explanations, whilst five of the respondents did not indicate that to be their use of the textbook.

Moreover, equal responses were indicated for the use of the textbook to give a reading task (GRT) and to give homework (GHW) (fourteen (14) of the responses). Fourteen (14) out of the twenty (20) respondents recorded that they use the textbooks to give their students reading tasks and also use the textbooks to give homework to their students. two purposes on the list that received less recognition of use were the use of the textbook to trigger students' interest (TSI) and to provide a history of mathematics (PHM). Only nine (9) and five (5) of the respondents indicated that they use the textbooks to trigger students' interest and to provide a history of mathematics respectively. There were no other purposes indicated for the use of the textbooks by the respondents other than those that were already indicated on the survey questionnaires. Again, all the respondents indicated at least one of the seven purposes indicated above to be the purpose for which they use textbooks for their teaching activities. A summary of this is provided in section 2 of Table 4.1 (p. 63).

4.2.1.2 The Use of the Internet and other resources

Another major resource used by teachers in their teaching activities is the internet and other books or resources. Section three of the survey questionnaire explored the various internet site and webpages (that needs subscription or no subscription), and other resources the teachers use in their pedagogical activities. A plethora of internet resources was indicated by the respondents as their sources of information for their profession. Furthermore, other resources that are not Internet resources were stated by the respondents as their source of information for their teaching. Popular (in terms of the number of respondents who use them) among the internet website or resources used were skolestudio (school studio) mathematikk.no, ndla.no, glydendal.no, udir.no, and Kahoot. Other internet sources and other resources indicated by the respondents includes andre lorboker (other books), Revision Village, colleagues, Question Beinto, Teams, Han, ITS, Facebook, Phet Collerado, YouTube, Python, and GeoGebra (see Section 3 of Table 4.1, p. 63).

The analysis of the data received relating to this section indicated that there are five (5) respondents who use the skolestudio as a resource for their teaching. Furthermore, six (6) of the participants use mathematikk.no, eight (8) use ndla.no, two (2) uses glydendal.no, four (4) use udir.no, and 3 respondents also use Kahoot as their resources for teaching and learning. Moreover, four (4) other participants also responded that they use the andre lorboker (other books) whilst two (2) others indicated that they use their other colleagues as their resources for information to enhance their teaching. Lastly, one (1) participant each indicated that they use Revision Village, Question Beinto, Teams, Han, ITS, Phet Collerado, Facebook, YouTube, Python, and GeoGebra as their resources for teaching. Not all the participants listed that they use any of the resources stated in this sub-section as their resource for teaching. Some indicated the textbook as the only resource used for their pedagogical process.

All these resources are used by the teachers for various purposes which are related. The researcher forecasted seven main purposes for which these resources may be used by the teachers (as did for textbooks), but the analysis of the responses revealed that there are eleven (11) purposes for which these resources are used.

1. Source of task (SOT)
2. Lesson planning (LP)
3. Sources of videos (SOV)
4. Create competition (CC)

5. Repetition (RPT)
6. Make instructional explanations (MIE)
7. Give a reading task (GRT)
8. Exam preparation (EXP)
9. Give homework (GHW)
10. Trigger student interest (TSI)
11. Provide a history of mathematics (PHM)

The analysis of the data revealed that all the resources mentioned were used for at least one of the eleven purposes with the exception of two (Python and GeoGebra) for which the respondents did not specify what they used them for in their pedagogical activities. Of the five respondents who indicated that they use skolestudio, one (1) pointed out that he or she uses it as a source of video (SOV) for his lesson in the classroom and two participants provided that they use it to make instructional explanations (MIE). The skolestudio serves as a source of tasks (SOT) for two (2) of the respondents and one (1) of the participants uses it for lesson planning (LP). I want to state emphatically that the responses relating to the purpose of the use of a resource are not mutually exclusive. That is, some participants indicated that they used certain resources for more than one of the purposes.

Mathematikk.no was basically used as a source of mathematical tasks. All six (6) respondents relating to this mentioned that they used the resource as a source of tasks (SOT). Eight (8) out of eight (8) of the participants who use ndla.no signaled that they use the resource as a source of tasks (SOT). In addition, two (2) of them indicated that they use ndla.no for lesson planning. Also, ndla.no served as a resource used by one of the respondents to give homework (GHW), one participant to trigger students' interest (TSI) in mathematics, and one participant to make instructional explanations (MIE). Moreover, Kahoot was used by the teachers to trigger students' interest (TSI) in mathematics (one (1) participant), to create competition (CC) among the students (two (2) responses), and to repeat (RPT) lessons. Glydendal.no was also used by both participants who used them, as a source of tasks (SOT) for instruction. Udir.no was also indicated by all respondents as a source of task (SOT) (four (4) out of four (4)), and two of the respondents mentioned that they use Glydendal.no for lesson planning (LP).

The analysis of the other resources used by the respondents uncovered that most of them are used for the same purposes. For example, the responses corresponding to the use of Question Beinto, Teams, and ITS showed that the respondents who use these resources all use them as a source of tasks (SOT) and for lesson planning (LP). Furthermore, the respondents who indicated that they use YouTube and Phet Collerado displayed that these are used to make instructional explanations (MIE) and to trigger students' interest (TSI) in mathematics. Whilst Facebook was indicated to be used as a source of funny tasks (SOT), Han is used by the respondent to make instructional explanations (MIE). Andre lorboker is used as a source of tasks (SOT) by two of the respondents and one respondent uses it for lesson planning (LP). The one teacher who indicated he or she uses Revision Village uses it for four purposes: lesson planning (LP), giving homework (GHW), triggering students' interests (TSI), and Exam preparation (EXP). The last resource which is colleagues is used by two (2) respondents as a source of tasks (SOT), two (2) for lesson planning, and one (1) to make instructional explanations (MIE). A summary of this is provided in section 3 of Table 4.1 (p. 63).

4.2.2 Sources of tasks

These findings correspond to the tasks used by the three teachers who were used for the case study. The findings relate to the resources where the tasks used by those teachers were taken from. The source of empirical data for these findings was the interviews that were conducted before the classroom observation. The researcher asked the participants to provide copies of tasks that they had selected for enactment in the classroom that day. The participants are interviewed on the resources that those tasks were taken from. The resources which these teachers selected their tasks from are limited to the course textbook, exam questions from the internet, and self-made tasks. Even though some of them indicated that they sometimes select tasks from other resources, for the period of the study, the tasks they used were from these three sources indicated above. There were ninety-one (91) tasks in total on the task sheets that were collected from the three teachers, thirty-eight (38) from Teacher 1, twenty-five (25) from Teacher 2, and twenty-eight (28) from Teacher 3 (p. 64).

4.2.2.1 Tasks from the Textbooks (TTB)

The IB program has designated textbooks for teachers and students which the teachers are to use for instruction in the classroom, even though the teachers are allowed to use other resources. Each of these textbooks contains tasks on every sub-topic that is been taught, and more often than not, most of the tasks used in the classroom come from these textbooks. The textbooks also contain tasks at the end of each chapter relating to all the sub-topics that have been taught. Furthermore, the textbooks have a plethora of tasks at the end of the books pertaining to all the chapters in the textbooks (also deemed as likely exam questions). The textbooks contain a lot of tasks. This was known when the researcher received a copy of the textbooks from teacher 3 to go through it for perusal. During the interviews too, the participants confirmed that there are far too many exercises in the textbooks.

> Researcher: Are all these tasks from this book (textbook)? Do you have any other source for your tasks?
>
> Teacher 3: *Yes, they are all from the textbook.* With other sources, not today. I used to sometimes use something called Revision Village, it's an online thing that I subscribe to. If I need something in addition, *but there are a lot of exercises here and they are, for this curriculum good questions within the text, it's a lot of exercises and also at the end of the chapter, there are chapter review questions, and after that exams start questions.* So, most of those we use when we are practicing full tests.

The researcher observed the tasks that were on the task sheet provided on that day and asked the participant if all those tasks were from the textbook. The response indicated that 1. The tasks were from no other source either than the textbook, 2. There are a lot of exercises in this textbook (keywords in italics).

The data analyzed from the interviews indicated which specific questions the teachers selected from the textbooks. It was discovered during the analysis of the interviews that all the teachers used tasks that they selected for use in the classroom during the study.

> Researcher: Where did you get the tasks from?
>
> Teacher 1: These tasks came from *the book (textbook)* that we use.
>
> Teacher 2: The tasks are normally I start with *the exercises from the textbook*, and when we do that afterward, not this section but another section perhaps, that is Friday or next week we will do tasks from the exam, ...
>
> Teacher 3: Yes, they are all from *the textbook*.

As already mentioned, and revealed through the interview scripts, all three teachers had at least one question from the textbook. So, the textbooks provided by the IB program served as a primary source of tasks for the teacher for use in the classroom.

After collecting all the task sheets from the teachers, the researcher indicated with ink the tasks which were sourced from the textbook to separate them from the others. The tasks were later analyzed and separated into various sources from which the teachers mentioned they were taken. The analysis unearthed that there were sixty-two (62) out of the ninety-one (91) tasks that were selected from the textbook. This constituted ten (10) tasks from Teacher 1, twenty-four (24) from Teacher 2, and twenty-eight (28) from Teacher 3.

4.2.2.2 Task from the exam/internet

Another source where the observed tasks came from was exam questions which are primarily found on the internet (IB website where past questions for various topics are kept). During the analysis of the interviews, it was discovered that all the teachers used the exam tasks stored on the IB website when the need arose, even though only one of the teachers used some tasks from the website during the period of the study. The following are some responses from the teachers corresponding to the exam tasks.

> Researcher: What mathematical tasks have you chosen to use today?
>
> Teacher 1: *I have chosen formal exams questions* because it is at the end of or halfway through vectors and the next part is vectors as a plane, which is much more difficult and now, *we get a lot of old exam questions* that go on the easy part of vectors on a line and the perpendicular and all these sorts of aspects.
>
> Researcher: Can you tell me where you got these tasks from?
>
> Teacher 1*: These ones are old exam questions so the IB system has a website where you can go and search for instance vectors and then you get all the exam questions about vectors then I read through the tasks and then I singled out the ones that fit where we are at the moment.*
>
> Teacher 2: …, I start with the exercises from the textbook, and when we do that afterward, not this section but another section perhaps, that is Friday or next week we will do *tasks from the exam,* so they will know where the level for the exam is. …
>
> Researcher: I'm talking about the tasks that you will give them after the lesson today. Where are the sources of these tasks?
>
> Teacher 2: *It's online through the IB* so I just log in and I mark today, I'm going to have some exercises from, then I press functions and what chapters in functions and if there are any exams within, I can get it from my screen and I just select those tasks that I want.

In the teachers' responses, they made it clear that sometimes they use tasks from the exams which can be found on the IB program website. The tasks on the task sheets I collected and analyzed revealed that there were twenty-eight (28) of the tasks from the IB past exam website. Teacher 2 and Teacher 3 did not use any of the exam questions during the study. All twenty-eight tasks were found on the task sheets collected from teacher 1.

4.2.2.3 Self-made tasks (SMT)

The analysis of the interview data again displayed that some teachers create tasks by themselves. That is, they reason and bring tasks out of their own mind. This was revealed when the researcher asked one of the teachers where he or she got the tasks he or she will be using today. The response from the teacher is as follows.

> Researcher: Do you have some tasks for this lesson?
>
> Teacher 2: Yeah, the tasks are just, in the beginning, to see what they can recognize, and what they think of when they hear the word function. I will draw some functions on the blackboard and some non-functions, and we have to discuss which one is a function and which one is not a function. So that they can see, and I will do not typical functions, but I will draw some strange functions to see if we can get to the point of what is function.
>
> Researcher: Where did you get those tasks from?
>
> Teacher 2: *My head*. But it is just some pictures, that I would like to see if this is a function, yes, it's a strange one, I haven't seen this, but this is a function. Some are like sines, some are almost exponential, and some are like the absolute value of some things, but they haven't seen that yet.

The teacher indicated that the tasks were made in his own head. The task was about using sketches to identify functions and non-functions. There was only one of that questions that has many sub-questions. That is, several sketches were drawn and the students were asked to identify which ones are functions and which are non-functions. Only one of the teachers created answered during the interview that he or she created tasks from his head. A summary of the sources of tasks from the task sheets has been provided in Table 4.2.

4.3 Task Selection Considerations

This section looked at the analysis of factors teachers consider before choosing tasks for their students. The source of data to address this section is derived from the survey questionnaire, the interview, and the curriculum documents (task sheets) collected from the teachers. This section is further divided into six sub-sections. The analysis provided that some responses are related to one bigger umbrella and as such the responses were further developed into six themes including the cognitive demand of the tasks, the content of the tasks, the design of the tasks, stress mathematical ideas, focus on the students, and application to other topics. These subsections are discussed below with evidence from the data.

4.3.1 The Cognitive Demand of the Tasks

One of the research's goals was to find out the cognitive demands of mathematical tasks that are being used by teachers in the classroom. in view of this, the researcher included this as one of the major topics in the data collection tools. First, in the survey, the researcher included questions that looked at the extent to which teachers are likely to select tasks that are too difficult or too easy. These questions were aimed at collecting data on how teachers believe too difficult or too easy tasks are helpful in their activities with their students. Again, the tasks on the task sheets were also analyzed and placed into the four categories of the cognitive demand of mathematical tasks (Smith & Stein, 1998) to obtain information on the nature of tasks (cognitive demand) of tasks teachers used.

4.3.1.1 Too Difficult (TD) or Too Easy (TE) Task

Too difficult tasks as implied by Stein and Smith (1998) means tasks that require too much mathematical thinking or reasoning before they are completed. These kinds of tasks also require more time to be completed and do not specify any mathematical procedures or formulas that have to be followed to be completed. They are tasks that are at the third and fourth levels of the cognitive demand tasks of Smith and Stein. Moreover, too-easy tasks are also tasks in the memorization and procedure without connection as in Smith and Stein (1998). These kinds of tasks do not require so much mathematical thinking and as such less time is required to solve them. Again, the procedures for completing these tasks are clearly stated implicitly or explicitly.

The consideration relating to selecting tasks that are too difficult (TD) had all twenty participants answering. Their answers ranged from not important to important. Even though none of the participants selected that choosing tasks that are too difficult is very important to them, the majority of the participants – eleven (11) out of twenty (20) – responded that choosing tasks that are too difficult for their students is important to them. Furthermore, eight of the participants also asserted that choosing such tasks (TD tasks) is moderately important to them. One of them also answered that choosing too difficult (TD) tasks is unimportant to him or her and none of the participants chose slightly important as the response for selecting tasks that are too difficult. The consideration for choosing tasks that are too difficult from this study is seen as moderately important (8 responses) or important (9).

Again, selecting tasks that are too easy (TE) was neither seen as unimportant nor very important in this research. All the twenty responses to the question ranged between slightly important and important with many responses concentrated between moderately important and important. Most of the participants – nine (9) of them – responded that choosing tasks that are too easy (TE) is moderately important for them and eight of the participants answered that choosing tasks that are too easy (TE) is also considered when selecting tasks for enactment. Three (3) of the participants also asserted that they consider and choose tasks that are too easy (TE) for their students. in summary, the majority of the participants considered tasks that are too easy (TE) as important or moderately important.

4.3.1.2 The Cognitive Demand of the Tasks on the Task Sheets

The tasks on the task sheets that were collected during the study were again analyzed to look at the cognitive demands of the tasks. There were in total, ninety-one tasks from all three teachers at all the sections of observations. The cognitive demand of the tasks was ascertained using the task analysis guide provided by Smith and Stein in 1998. In their categorizing of tasks, they placed all mathematical tasks into four main categories according to the level of difficulty. I want to emphasize that some of the tasks that appeared on the tasks sheets may be very difficult and require high mathematical experience and thinking to complete them, however, these tasks were placed in the lower level tasks because they lack more of the key factors that qualify a task to be classified as higher level tasks. A higher-level task is not only about the level of difficulty but also demands that the students describe or justify their method of working. Higher-level tasks may also be ambiguous regarding the procedures that have to be followed to be completed. Let us take for example one of the difficult tasks on the task sheets that were collected from teacher 1 that he gave to his students after teaching on the topic vector equations of a line.

Exercise 9L1

There is a boat on the sea that needs help. Two boats are nearby, and they have received the Call. If you place the boat in need at the origin of the coordinate plane the other two boats called "Adrianne" and "Lilly" are at points A(-3,5) and L(7,9) respectively. The coordinates are given in kilometers. They move directly toward the boat in need, "Adrianne" with a speed of 4ms/s and "Lilly" with a speed of 6ms/s.

a. Show that the courses of these two boats have the following equations t seconds after starting.

   a = $(-3000i + 50000j)$ + t$\left(\frac{12}{\sqrt{34}}i - \frac{20}{\sqrt{34}}j\right)$ and

   l = $(7000i + 9000j)$ + t$\left(-\frac{42}{\sqrt{130}}i - \frac{54}{\sqrt{130}}j\right)$

b. Determine which boat will arrive first and how much longer it will take the other boat to reach the boat in need.

This question according to the interaction with the teacher after the classroom observation was a very difficult question for the students. only one of the students was able to complete the task.

Interviewer: There were some questions from the students during the class. How did that affect your goal for the tasks, especially 1d?

Teacher 1: They managed the arithmetic of it. So, what really was left was the interpretation, so therefore when I gave them a little logic in the right direction, they sweat and then it became a little easy. So, they didn't stop at the start, they really stopped at where they got -1=-1, "what is the interpretation of this" and I said the interpretation is that the line is in the plane. So, it's sort of every value is a right value. **And the last one, 9L1 is very difficult, so it's only one student who was able to manage that,** because there we are talking about the speed and you have to adjust the stretch factor to speed and that's high-level but when they got the hint of unit-factor, so not a very big hint but you can use it and suddenly it appears to make the task much very easy.

Initially, the students found task 9L1 very difficult to complete when the teacher had not intervened to give a clue on how to complete the tasks. Because in this question the best approach to use was the unit factor but the students had no clue of that until they had interacted with the teacher and he or she said so. Even though this question is difficult to complete, the researcher placed this task in the procedure without connection (PWOC) category of the Task Analysis Guide (TAG) of Smith and Stein (1996). The reason for categorizing this task in this category is as follows.

- The question did not require the students to interpret or explain how they were able to complete the tasks, which is one of the major differences between procedure without connection (PWOC) and procedure with connection (PWC) tasks.
- The procedure for completing the tasks was obvious just that the students found it difficult to see it.
- The focus was on producing correct answers rather than producing a deeper level of understanding of mathematical concepts and ideas.
- There was only one way not several ways to complete this task.

These strategies along with the TAG were the approach deployed in placing the tasks on the task sheets into the four categories of mathematical tasks described by Smith and Stein (1996).

In all, a total of ninety-one (91) tasks were presented on the task sheets of all three teachers. Out of these ninety-one (91) tasks, eight (8) of them were memorization (MM) tasks and the remaining (eighty-three (83)) were procedure without connection tasks (PWOC). Neither of the tasks was placed under the procedure with connection (PWC) nor Doing mathematics (DM) tasks. For emphasis, the grouping was done in accordance with the Task Analysis Guide of Smith and Stein (1996). Teacher 1 had a total task of thirty-eight (38) on the task sheet he or she presented. None of the tasks presented by Teacher 1 were memorization tasks (MM). All the tasks on the task sheet of teacher 1 were procedure without connection (PWOC) tasks. Again, teacher 2 had a total of twenty-five (25) tasks presented on his or her task sheets. Four of those tasks were memorization tasks (MM), and twenty-one of them were procedure without connection (PWOC) tasks. Moreover, there were four (4) tasks that were categorized under memorization (MM) tasks and twenty-four (24) procedure without connection (PWOC) tasks of the total of twenty-eight (28) tasks that were on the task sheet of teacher 3. A summary of these findings is tabulated in Table 4.3.

4.3.2 The Content of the Tasks

The study furthermore looked at the extent to which the content of a task can influence the choice of tasks by teachers. The content was about how the tasks are been presented (visual, text, etc.) and what they contain (the makeup or the storyline). The source of empirical data used to address this subsection was the survey questionnaire. Three codes were generated from the survey questions in relation to the content of the task: real-life connections (RLC), visual drawings and illustrations (VDI), and unique and interesting (UI).

Section 4 of the survey had three questions which the researcher teamed them up to form the task selection considerations relating to the content of the tasks. These were questions 8, 9, and 10 of section 4 of the survey questionnaire (see Appendix A). These were coded as real-life connections (RLC), visual drawings and illustrations (VDI), and unique and interesting (UI) respectively. The analysis of the data generated from these questions provided that when it comes to selecting tasks that have a context and making real-life connections to other school subjects (RLC), three (3) of the respondents deemed those tasks as slightly important, nine (9) of them provided that (RLC) were moderately important, six (6) deemed RLC as important, and two (2) of the participants recorded that RLC was very important. None of the respondents provided that RLC was unimportant. Furthermore, in relation to selecting tasks that make use of visual drawings or illustrations of mathematical ideas (VDI), six (6) participants responded that VDI was slightly important, twelve (12) participants responded that VDI was moderately important, and five (5) participants deemed VDI as important. Again, two (2) of the participants recorded that VDI was very important and none of the participants deemed VDI as unimportant. Last from the survey is selecting tasks that are quite unique, interesting, and unlike "textbook" tasks (UI). Nineteen (19) out of twenty (20) participants responded to this question. Six (6) of them saw UI as slightly important, nine (9) deemed UI as moderately important, and four (4) responded that UI was important. Neither of the participants stated that UI was unimportant or very important. A summary of this is provided in Table 4.1.

4.3.3 The Design of the Tasks

Regarding the task selection consideration based on the design of the tasks, the researcher grouped six of the survey questions to be related to this. These are questions 13 to 18 of section 4 of the survey questionnaire. They were coded as complicated storyline (CSL), steps from easy to more difficult (ETD), not having all information needed for the solution (INS), unexpected or surprising answers (USA), students not having the knowledge to solve yet (SNK), and no keywords hinting at what mathematical procedure to use (KPH) respectively.

Data collected from the participants in accordance with these questions revealed that zero (0), six (6), seven (7), four (4), and three (3) of the participants provided that CSL was unimportant, slightly important, moderately important, important, and very important respectively. Again, no participant recorded that ETD is important, as against one (1) that saw that it was slightly important, seven (7) that deemed it as moderately important, nine (9) as important, and three (3) as very important. One (1), seven (7), nine (9), three (3), and none of the participants recorded that INS was unimportant, slightly important, moderately important, important, and very important respectively.

Regarding USA, SNK, and KPH, two (2) each, and one (1) respondent saw them as unimportant respectively, whereas six (6), ten (10), and four (4) deemed them as slightly important respectively. Furthermore, eleven (11), six (6), and twelve responded that they were moderately important. None of the participants recorded that USA was important but one (1) recorded that USA is important. Two (2) participants each stated that SNK and KPH were important. Whilst one (1) saw KPH as very important, none of the participants recorded that SNK is very important. A summary of this is provided in Table 4.1**.**

4.3.4 Stress Mathematical Idea

Another area of task selection consideration that was looked at was tasks that stress students' understanding of mathematical ideas. Data used to address this topic was gathered from both the survey and the interviews. Questions 1, 2, and 3 of section 4 of the survey (see Appendix A) were teamed to answer this topic. Again, the analysis of the interviews provided data that was reliable to be categorized under this topic.

The survey questions that were categorized under this topic were coded as an important mathematical idea (IMI), understanding a mathematical idea (UMI), apply a mathematical idea (AMI). The results indicated that IMI was very important consideration to four (4) of the participants and an important consideration to seven (7) of them. Again, four of the participants each considered IMI as slightly or moderately important whilst one participant indicated that IMI was unimportant. None of the participants considered UMI and AMI as unimportant or slightly important. However, ten (10) of the participants responded that UMI was an important factor to consider when choosing a task, eight (8) deemed UMI as very important whilst two (2) saw UMI as moderately important factor. Moreover, a stunning number of the participants (twelve (12)) indicated that choosing AMI tasks was important to them, whereas four of the participants each considered AMI as moderately important or very important.

Moreover, during the interviews, the interviewer enquired more about the consideration or the goals with which the teachers selected the tasks they used for instruction. The analysis of the outcome of the data generated from this topic suggested there were some responses that were related to this sub-section. The researcher coded those responses as “develop vocabulary” (DV). That is, the teachers selected certain tasks for their students to complete because tasks were given to the students for the students to develop their mathematical vocabulary (understand the bigger picture of the mathematics being taught). The analysis showed that all three teachers considered selecting tasks that help students to develop their mathematical vocabulary. The following responses from the interview related to this:

> Researcher: What do you intend to achieve with these tasks you have selected for the students?
>
> Teacher 1: I intend to try to get my students to understand the bigger picture. So, in the classroom so far, we have looked at very specific what’s perpendicular, what’s parallel, and how do you write the line as a vector but now my goal is that with the task they should see how they can make two lines and if there is an unknown in there k, how can you define k so that they are all perpendicular. This is the sort of step up and see the bigger picture of the curriculum.
>
> Researcher: What is the goal for choosing these tasks?
>
> Teacher 1: The goal is for them to learn the arithmetic of how to find the vector product and hopefully they realize the idea of the area by cross product.

These were the responses from the teacher on his or her goals for selecting tasks for his students. The goal of the teacher was that the students would develop an understanding of the bigger picture (broaden their understanding of vectors – develop vocabulary). During his lecture, he taught very specific things such as perpendicular vectors, parallel vectors, and the line of a vector. Now the goal for choosing the tasks for the students was to help them to understand the bigger picture of these sub-topics. Again, the teacher selected the tasks to broaden the students’ understanding of the arithmetic of vector products, and through that knowledge, the students should be able to find the area using cross-product. Teacher 2 and Teacher 3 also selected tasks with the same motive in view. For example, Researcher: What did you consider before choosing those tasks for the students?

Teacher 3: That is, so they can understand and develop more understanding of what we are doing. When we go through something on the board, they are able to do it by themselves, not too difficult in the beginning, but then a little more advanced afterward. The main reason is to, make them do them themselves after I have gone through that lesson.

The teachers did not select tasks for the students to complete and produce correct answers, but they also considered selecting tasks that help students develop a deeper mathematical understanding – develop vocabulary.

4.3.5 Focus on Students

Again, another theme that was developed from the data set in relation to the main "tasks selection considerations" was selecting tasks that focus on the students. Questions 4, 5, 6, and 7 of section 4 of the survey questions were classified under this theme. Furthermore, analysis of the data from the interview also provided useful information related to this. So, the responses in line with these questions of the survey and the data from the interview were used as the data for this subject.

Questions 4, 5, 6, and 7 were assigned the codes students work in small groups (SSG), create classroom discussion (CCD), explain, and reflect on student thinking (ERST), and students' interest and motivation (SIM) respectively. A read-through of the responses related to these questions discovered that most of the participants (thirteen (13) each) indicated that SSG and CCD were moderately important when it comes to task selection. Four (4) and three (3) participants respectively deemed SSG as slightly important or important. Again, one (1), two (2), and four (4) participants indicated that CCD was unimportant, slightly important, or important. Regarding ERST one (1) participant indicated that it was slightly important, eight (8) indicated that it was moderately important, ten (10) recorded that it was very important, and one (1) participant stated that ERST was very important when selecting tasks. Furthermore, none of the participants recorded that SIM was unimportant. However, three (3) participants selected that SIM was slightly important as against seven (7) who indicated that SIM was moderately important. Again, eight (8) of the participants recorded that SIM was important whilst two (2) of the participants stated that SIM was very important.

Moreover, the analysis of the data from the interviews unearthed that there was a factor that the teachers considered when they were selecting the tasks for their students which the researcher believed is important to classify under this subject. The code provided for the factor was the mathematical knowledge and level of the students (MKL). This factor was obvious from the interview data taken from all the teachers. The following excerpt from the interviews relates to this subject.

> Researcher: How well do you know your students when it comes to their mathematical background?
>
> Teacher 1: Very well because I have been their teacher for one and a half years.
>
> Researcher: How do rank their mathematical performance?
>
> Teacher 1: High, all of them high. I said to you that it is only high-level students. They are the best of the students.
>
> Researcher: Did their performance affect the tasks you chose for them to enact?
>
> Teacher 1: Yes, I would have started much more simpler with the first ones if the class had a bigger range of students.

The interviewer asked Teacher 1 about his or her students' mathematical performance and how that affected the difficulty of tasks he or she selected to enact with them. Teacher 1 had full knowledge of his or her students' mathematical performance, and based on this knowledge, selected tasks that matched their capabilities. He or she again indicated that he would have started with simpler tasks if the students were not all that good. All the teachers asserted the fact that their students' mathematical thinking or level affected the kind of tasks they selected to enact with them. The excerpt below from the interview relates to Teacher 3 regarding how his or her understanding of the mathematical knowledge and performance of his or her students affected the tasks that he or she selected.

> Researcher: What is your perception of the mathematical knowledge and experience of your students?
>
> Teacher 3: They are low-achieving students. This is the easiest course in IB so those who don't need maths later are those who choose this.
>
> Researcher: Did that affect the tasks that you chose for them?

> Teacher 3: Oh yes. And it also affects what is in the book. The book is made for them and for that course. And the course is not for high-achieving students.

The response from Teacher 3 pointed out that the students' mathematical performance does not only affect the tasks teachers select for the students but also affects the kind of tasks that are presented in their (students') textbooks.

4.3.6 Application to Other Topics

The last code generated from the data set relating to the major theme "task selection consideration" was selecting tasks that have relations to other topics. The source of data for this theme was questions 19 and 20 of section 4 of the survey questionnaire. Question 19 was coded as "to introduce a new topic (INT)" whilst question 20 was coded as "apply mathematics to another subject area (AMT)".

The collation of the results from these questions indicated that ten (10) and twelve (12) of the participants recorded that INT and AMT were important factors they considered when choosing a task for their students. Again, four participants each selected that AMT was moderately important or very important. There was no selection for AMT as unimportant or slightly important. Furthermore, none, one (1), six (6), and three (3) of the participants asserted that INT was unimportant, slightly important, moderately important, and very important respectively. A summary is provided in section 4 of Table 4.1**.**

4.4 Modifications of the Tasks

One of the research's goals was to observe the modifications, if any, the teachers make to the tasks in the original curriculum before transferring them to their curriculum documents. For the purpose of emphasis, by modification, the researcher meant, changes or alterations made to the individual task or the sets of tasks (taking some tasks out of a group of tasks). The researcher further probed to look at the reasons behind the modifications, if any, and how these modifications, if any, affect the cognitive demand of the tasks. This section is divided into two sub-sections. First, the part that looked at the modifications to the tasks, and the reasons behind the modifications. The second sub-section looked at when the tasks were not modified and the reason why the tasks were not modified. The source of empirical data was the interviews and the task sheets.

4.4.1 Modifications to the Task

The researcher compared the tasks in the original document with the tasks that were presented by the teacher on their tasks sheet. The observation was that the teachers used the same task in the curriculum documents (textbooks and internet resources) with their students in the classroom. With textbooks, both the students and the teachers use the same textbook. After the teacher was done with what he or she had to teach for the day, he or she asked the students to open to a certain exercise in the textbook to complete them. Not many modifications were made to the individual tasks. Also, with the past exam questions that the teachers presented to the students to complete, the teachers logged into the website, printed the tasks that related to the topic they were teaching, and then presented them to the students. The modifications that were observed from the tasks were that the teachers selected some of the tasks (not all the tasks) and then left some out and asked the students to complete the selected ones. That is, the modification occurred to the group of tasks rather than the individual tasks. These modifications were observed from the task sheets of Teacher 1 and Teacher 3. Even though Teacher 2 asserted that he or she modifies the tasks, no such modifications were made during the study.

During the interviews, the researcher enquired from the teachers the reason they modified the tasks (selected only some tasks for the students to complete). From the responses of Teacher 1, the researcher assigned three codes as to why he or she modified the tasks. First, the tasks repeat themselves (TRT). Secondly, to make the tasks suitable for his or her students (MTS), and lastly, to bridge the gap with exam questions (BGE). Furthermore, one code was assigned to Teacher 3 regarding this modification. That is time constraint (TC). The excerpts below were the responses from Teacher 1 regarding the reason for the modification.

First conversation.

> Researcher: Have you made any modifications to the task as they appeared from the source?
>
> Teacher 1: The only thing I have done is to take out if they include planes because some exam questions start with easy ones with lines and so on, and then you get into planes at the end, so I have deleted the ones they haven’t done yet.
>
> Researcher: Why did you make those changes?

Teacher 1: The aim was to make it suitable for where my students are at the moment.

Second conversation.

Researcher: Have you made any modifications to any of these tasks?

Teacher 1: Not more than I have taken away, so for instance I chose only b because a is a lot of work and is on what we have before. So now I want to focus on just the angles, so then I pick on where the angles are, which is important.

Researcher: I want to know whether there have been changes to the tasks as they appear here.

Teacher 1: No, not more I said 3b and not 3a.

Researcher: Why did you intend to modify 3b but not 3a?

Teacher 1: No, I said I modify task 3 by asking them not to do the a, and not the task itself. But the task is about first finding where they intersect to find the point of intersection and then it's a find the angles between the two, but I don't want them to find the intersection because they work on that so it's just, it's going to take a long time and they learn anything new. So, I modify the task to only talk about the angle and not anything else. Modifications

Researcher: Why did you then choose not to modify the task?

Teacher 1: Because the task shows different ways they might get questions on the exam and generally on how vectors and planes work for instance 1d it is difficult to make without doing a lot of work I said, you end up with a line plane that you wonder whether the line is in the plane, it's durable but it takes a lot of time to find, but then they do the work and suddenly they just, they don't get any of the lambdas which is the one we use and the interpretation of what does it mean that the lambda disappears. And that will, that will mean it's not a point, it's anywhere.

From both conversations, the teacher made it clear that he or she had not made any modifications to the individual tasks, moreover what he or she did was that he or she took some of the tasks out from the plethora of tasks that were available for the students to complete. Again, from the conversations, the changes were driven by several reasons. From the first conversation, the teacher took out some tasks from the sets of tasks (tasks on planes), because his or her students were not yet taught about planes and the aim was to make the tasks suitable (MTS) for his or her students. In the second conversation, the teacher asked the students not to complete task a, because it demands a lot of work (DLW) and it related to what they had completed before – repetition (TRT). Again, the teacher stated that working on that tasks was going to take a lot of time (TC). Lastly, the teacher chose not to modify the tasks because the tasks showed different ways the students were likely to get the exam questions – bridge the gap with the exam questions (BGE).

Teacher 3 did omit some tasks from the set of tasks that were to be completed by the students after the lesson. This was done because of one reason – time constraint (TC). This was made known during the interview when the researcher asked him or her if she had some tasks for the day.

> Researcher: Do you have some tasks for this lesson?
>
> Teacher 3: Yes. It's from 3G and you the book and it's six different exercises there but I have chosen 4 & 5 because we don't have time for doing it and I hope to do 4 & 5 in class but I'm not sure if a, depending on how long it takes to go through the graph I'm going to show them. They are not familiar with this, and they are not that strong. Actually, I should have done that the last time, but I couldn't, I had to wait.

In exercise 3G, there were six tasks in all, but the teacher chose to complete only tasks 4 & 5 with the students due to insufficient time (TC). Again, the analysis of the tasks in exercise 3G showed that the tasks were almost the same (they were repetitive). Therefore, the teacher chose to modify the set of tasks by taking out four of the tasks. Even though during the observations, Teacher 2 did not make any modifications to the individual tasks or the set of tasks, in the interview, Teacher 2 asserted that he or she sometimes modify the tasks to make the tasks nearer to the exam questions – bridge gap with the exam (BGE). The excerpt below from the interview is related to this.

> Researcher: Why do you modify the textbook tasks?
>
> Teacher 2: So that they get closer to the exam questions. So that is my target, so they are going to have an exam, the exam tasks we have to practice on and work on them. So, I don't change them, but I change some in the book or other books to get closer to the exam.

4.4.2 No modifications to the tasks

As already mentioned, all the individual tasks used by the teachers were not modified. The modifications only occurred to the set of tasks (the teachers removed some tasks from the set of tasks). The researcher further probed to ascertain more information on why the individual tasks were not modified by the teacher. A number of reasons were given by the teachers as to why they did not modify the tasks, but the most predominant and common drive for no modification to the tasks was that the teachers' found the tasks in the original documents as good tasks – exercises are good (EAG) – for their students. Other themes developed from the interview in regard to why the teachers did not modify the tasks include, the topic is new (TIS), Prevent Extra Work (PEW), the Tasks are Many (TAM), and Guide for the Exam (GFE).

In relation to the first theme – exercises are good (EAG), all three teachers concorded that the one reason for not modifying the tasks was because they found the exercises in the original documents to be very good. They found the tasks in the original documents to be appropriate tasks that matched their goal, the goal of the curriculum, and the mathematical thinking and capabilities of their students. The following excerpts from the interview were the conversations that ensued between the researcher and Teacher 1 on this subject.

> Researcher: Have you made any modifications to the tasks?
>
> Teacher 1: Not more than the answers are wrong in the book, so I made the correct solutions.
>
> Researcher: So, you have made no modifications to the tasks?
>
> Teacher 1: No.
>
> Researcher: Why did you choose not to modify the tasks?
>
> Teacher 1: Why? Because I think the tasks are good. I think they give a level of steepness or difficulty where it starts off simple and ends up difficult. So, if they manage 6b, they've really understood what planes mean and I expect them not to manage it but some of them I speak with them and I feel that we are together but it's a difficult task so this time I didn't feel I needed different sources because the book has the progression that I like.

The response from the teacher indicated that the tasks in the original documents are good. Teacher 1 found the tasks to be good because of the steepness of the tasks. That is, the tasks started easy and then progressed to difficult as they went on (task design). This also prompted the teacher to stick only with the tasks from the textbook without looking for other resources for tasks. The teacher again expected the students to be able to manage (complete) the task which will mean that the students have been able to understand the concepts of planes.

Teacher 1 again highlighted that one of the reasons why he or she did not modify the tasks was that the topic he or she was teaching the students was a new topic (TIS). That is, according to the teacher, no modifications are made to the tasks when the topic is new. He or she uses the tasks with his or her students just as the tasks appear in the original curriculum document. The response from Teacher 1 in line with when the topic is new when he or she was asked about why he or she made no modifications to the tasks is below.

> Researcher: Have you made any modifications to them (the tasks)?
>
> Teacher 1: No modification.
>
> Researcher: Why did you choose not to make any modifications to these tasks?
>
> Teacher 1: Because we started something completely new, so they need to practice the basics and then the questions cannot be too complicated, so just get into vectors, find the cross product, and really you can do any directrices. So then using the book is fine.

So, no modifications are made by this teacher on the tasks when the topic is new. His or her aim for not modifying the tasks when the topic is new was that he or she wanted his students to practice the basics, and therefore no need to use difficult or complicated tasks.

Besides seeing the exercises in the textbooks or the original documents as good tasks, Teacher 2 again, did not modify the tasks for another reason. That is, the tasks he or she used were tasks from the past exam questions, there was no need to modify them so that the tasks would serve as a guide for the exam (GFE) which the students will be writing at the end of the season. The following excerpt from the interviews related to Teacher 2 not modifying the tasks because he or she wanted to use the tasks as a guide for the exam.

> Researcher: Have you made, or will you make any modifications to those tasks?
>
> Teacher 2: No, because these are the exam questions. So sometimes it is good to just train on the exercises that are given on the exams, so they can see how it is written, what you are supposed to find etc. So, for exams I don't modify some, but some examples or tasks in the I may modify it a bit. But for the exams, I don't.
>
> Researcher: Why don't you modify the exam questions?
>
> Teacher 2: Because the exam is the exam that is what they are going to manage so I don't think I need to change the exam because the exam is made by some external people, and they will make new ones.

First, the teacher provided that the tasks were past exam questions and as such there was no need to modify them. Second, the teacher did not modify the tasks because he or she wanted his or her students to observe how the exam questions are written so that they (the students) would be guided on how the exam questions are being presented by the external examiners.

Furthermore, there were two other reasons for Teacher 3 not to make any modifications to the tasks besides the tasks being good. The teacher did not modify the tasks because he or she found that modifying the tasks was too much work, so he or she wanted to prevent extra work (PEW). Again, he or she did not modify the tasks because the tasks were too many (TAM), and no need to modify them to create new tasks. The excerpt below his or her response on preventing extra work.

> Researcher: Have you made any modifications, or do you plan to make any modifications to the tasks?
>
> Teacher 3: No, not for these exercises. It is a lot of very good exercise.
>
> Researcher: Why have you chosen not to make any modifications?
>
> Teacher 3: Because I believe they are okay. They have a very good textbook and instead of making a lot of work for myself, I use them because I of course gone through them and seen that these are good exercises. Earlier when I was quite a fresh mathematics teacher, I make the examples myself, but I stopped doing that because I found out that was a lot of extra work, and I wasn't any better, it was actually much better to say that this example is on this page in the book because they are making if there were things that they didn't write down making finding it in the book also. But this book is made very quickly because it was just a couple of years there was a new curriculum in IB, so there are a lot of mistakes, especially in the answers but not in the book. They update it all the time, so it's getting better, but I will also talk about that with the students that, they should try to rely on themselves and not just the answers in the book, so they don't have it.

The teacher did not make any modifications to the tasks he or she selected from the curriculum document because the teacher believed they were a lot of good tasks. Upon further deliberations with him or her, it was observed that the teacher sees the tasks in the textbooks to be okay, and there was, therefore, no need to make a lot of work for him/herself by making any modifications to the tasks. The teacher had learned from experience from his or her earlier days in the professions that wasting a lot of time to modify tasks did not produce any better tasks than those that were already presented in the curriculum documents.

Again, the teacher notified that the textbook used to have mistakes in it in the early days of its publication, but not now. The textbook has been updated and it is now getting better. Furthermore, the conversations concerning the teacher seeing the tasks to be too many are also below.

> Researcher: Have you made any changes to the tasks as they appear in the textbook?
>
> Teacher 3: No
>
> Researcher: Why did you choose not to modify the tasks?
>
> Teacher 3: Because they are good and there are a lot of them. So, we have more than enough to do than to try to find new ways when we think they are good. If they weren't that good, I would of course make a lot of changes and search for more exercises from other places. But I think it's a good choice, the book we have.

Again, the teacher did not modify the tasks because there were a lot of tasks more than they could complete, and believed they were good tasks and there was no way to try other ways to make new tasks. A summary of the results relating to the modifications to the tasks has been provided in Table 4.5.

4.5 Classroom Activities on the Tasks

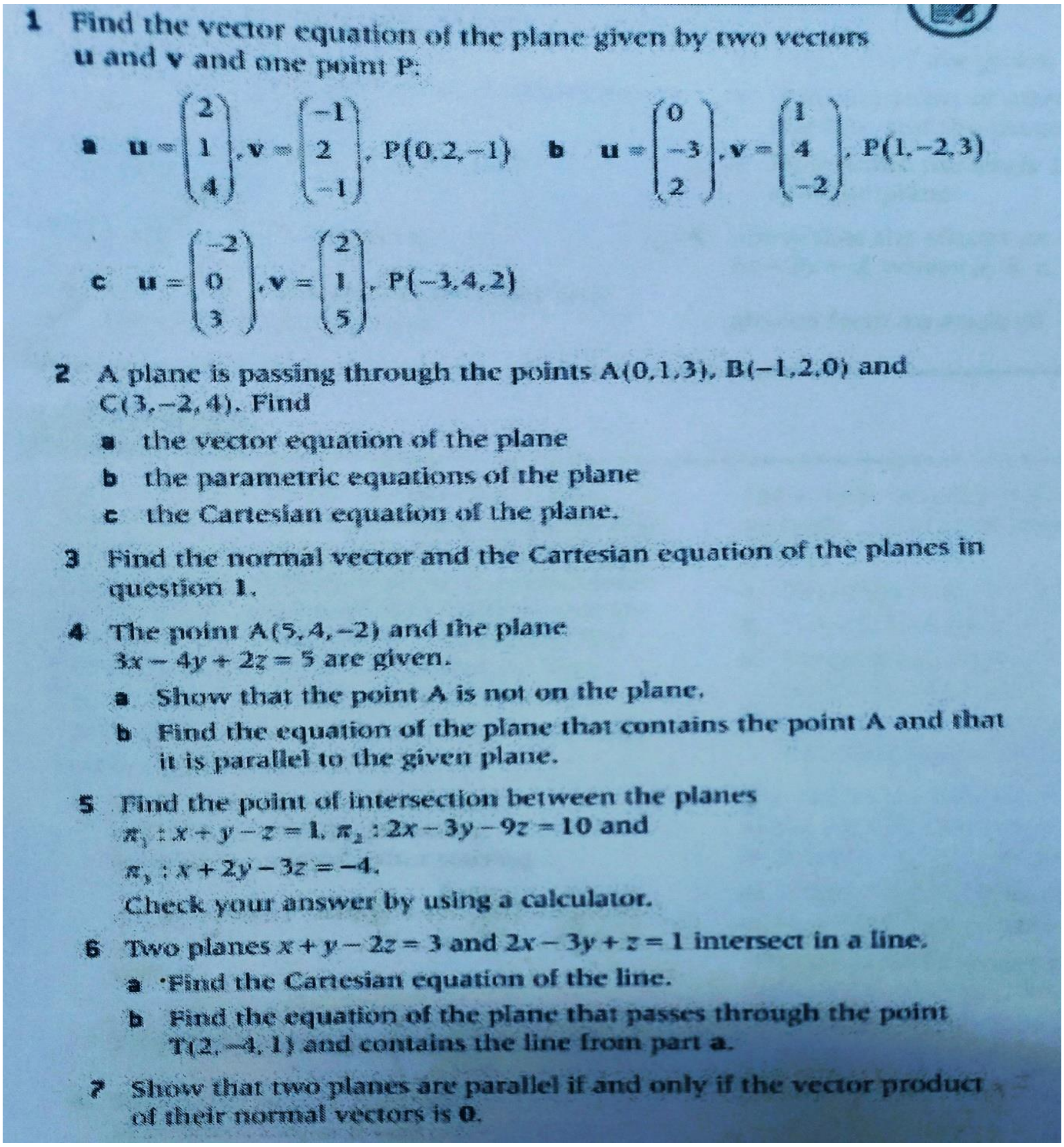
These findings are related to the final stage of task the Mathematical Task Framework (Stein et al., 1996), that is, the tasks as implemented in the classroom by the students. The source of data for these findings were classroom observations and interviews. Most of the things that the researcher observed in the classroom were confirmed by the teachers during the post-classroom observation interviews. The findings relating to the activities in the classroom are classified under two themes: teachers' instructional habits and dispositions, and students' instructional habits and dispositions.

4.5.1 Teachers' Instructional Habits and Dispositions

As already explained in the theoretical frameworks, the teacher's instructional habits and dispositions refer to the extent to which the teacher is willing to intervene and assist the students when they have difficulties solving the task and how long it will take before he or she intervenes (Stein et. al., 1996). During the classroom observations, the researcher looked at the various ways the teachers used to intervene in the task implementation process to help the students. Four themes were generated from the analysis of the data concerning this topic: Solving similar examples (SSE), Going around and assisting students to complete the tasks (GAS), Allow students to complete tasks in groups (ASG), and Allow students to ask questions during task completion (ASAQ).

4.5.1.1 Solved Similar Examples

The first observation that was seen was that the teachers solved similar examples (SSE) with the students in the classroom before giving the tasks for the day to the students to complete. This was observed in the classrooms of all the teachers in the study. The researcher took note of all the examples that were used by the teachers and compared them to the tasks that they (the teachers) gave to the students after the examples. The teachers also completed the examples by themselves during the lesson. The following are examples of the examples and the tasks that were presented to the students to complete during the classroom observations.



1 Find the vector equation of the plane given by two vectors **u** and **v** and one point P:

a $\mathbf{u}=\begin{pmatrix}2\\1\\4\end{pmatrix}, \mathbf{v}=\begin{pmatrix}-1\\2\\-1\end{pmatrix}$, $P(0,2,-1)$ b $\mathbf{u}=\begin{pmatrix}0\\-3\\2\end{pmatrix}, \mathbf{v}=\begin{pmatrix}1\\4\\-2\end{pmatrix}$, $P(1,-2,3)$

c $\mathbf{u}=\begin{pmatrix}-2\\0\\3\end{pmatrix}, \mathbf{v}=\begin{pmatrix}2\\1\\5\end{pmatrix}$, $P(-3,4,2)$

2 A plane is passing through the points $A(0,1,3)$, $B(-1,2,0)$ and $C(3,-2,4)$. Find

a the vector equation of the plane

b the parametric equations of the plane

c the Cartesian equation of the plane.

3 Find the normal vector and the Cartesian equation of the planes in question 1.

4 The point $A(5,4,-2)$ and the plane $3x-4y+2z=5$ are given.

a Show that the point A is not on the plane.

b Find the equation of the plane that contains the point A and that it is parallel to the given plane.

5 Find the point of intersection between the planes $\pi_1 : x+y-z=1$, $\pi_2 : 2x-3y-9z=10$ and $\pi_3 : x+2y-3z=-4$.

Check your answer by using a calculator.

6 Two planes $x+y-2z=3$ and $2x-3y+z=1$ intersect in a line.

a Find the Cartesian equation of the line.

b Find the equation of the plane that passes through the point $T(2,-4,1)$ and contains the line from part **a**.

7 Show that two planes are parallel if and only if the vector product of their normal vectors is **0**.

Figure 4.1: Tasks presented to the students to complete by Teacher 1.

Teacher 1: These examples and tasks were collected during the third day of the classroom observations when the teacher was teaching the topic “Perpendicular and parallel vectors”. First, the teacher introduced the topic, explained the key terms in the topic, and then used examples to explain further. After the explanations, the teacher gave the tasks to the students to complete. Figure 4.1 above was the set of tasks that the teacher gave to the students after the lesson. In this task, the students were asked to find the vector equations of planes, the parametric equations, the Cartesians equations, normal vectors, and points of intersections, and to show that the planes are parallel. But before the task was given to the students, the teacher had already gone through similar examples with the students. Figures 4.2 – 4.4 represent examples that the teacher went through with the students.

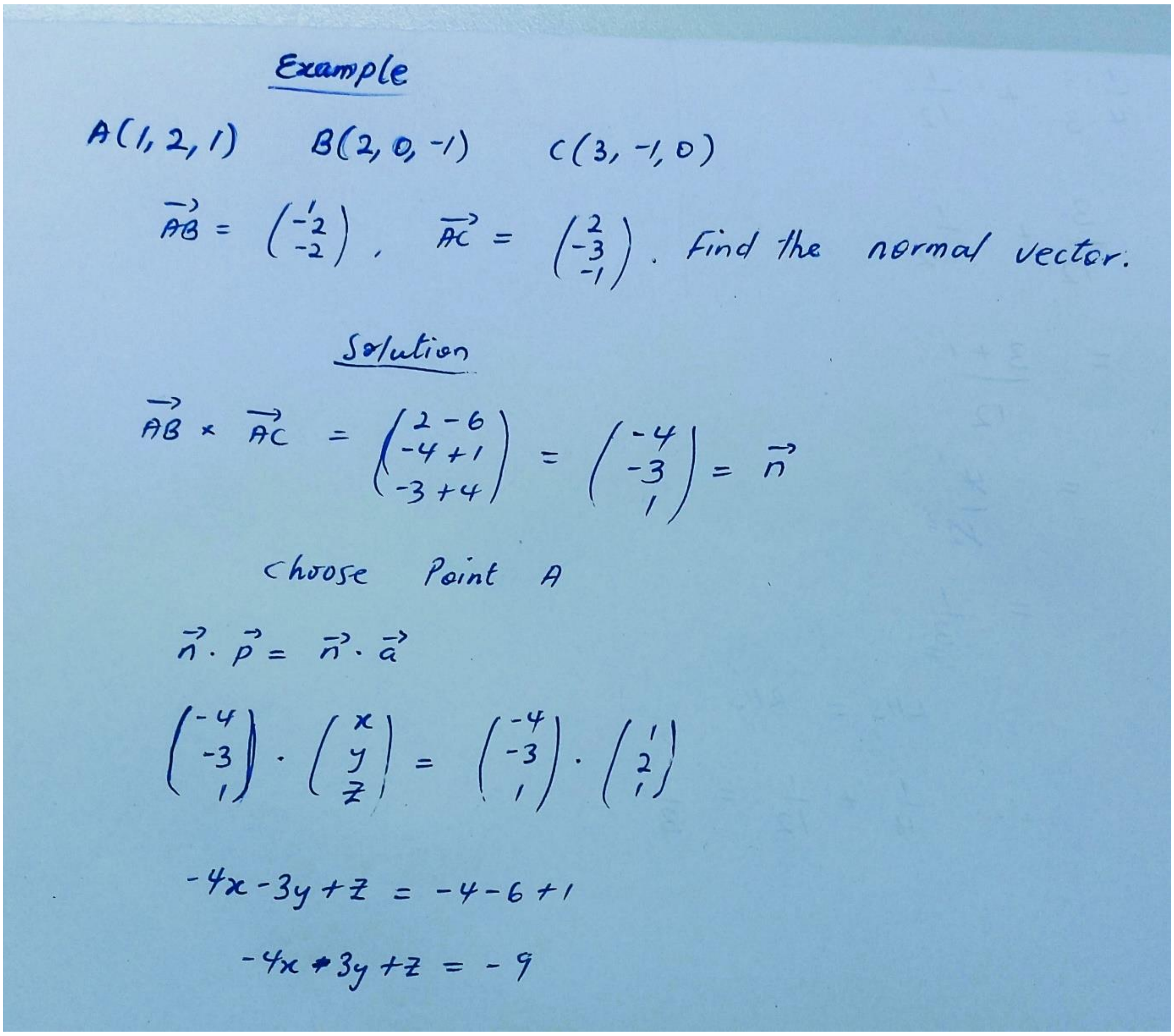
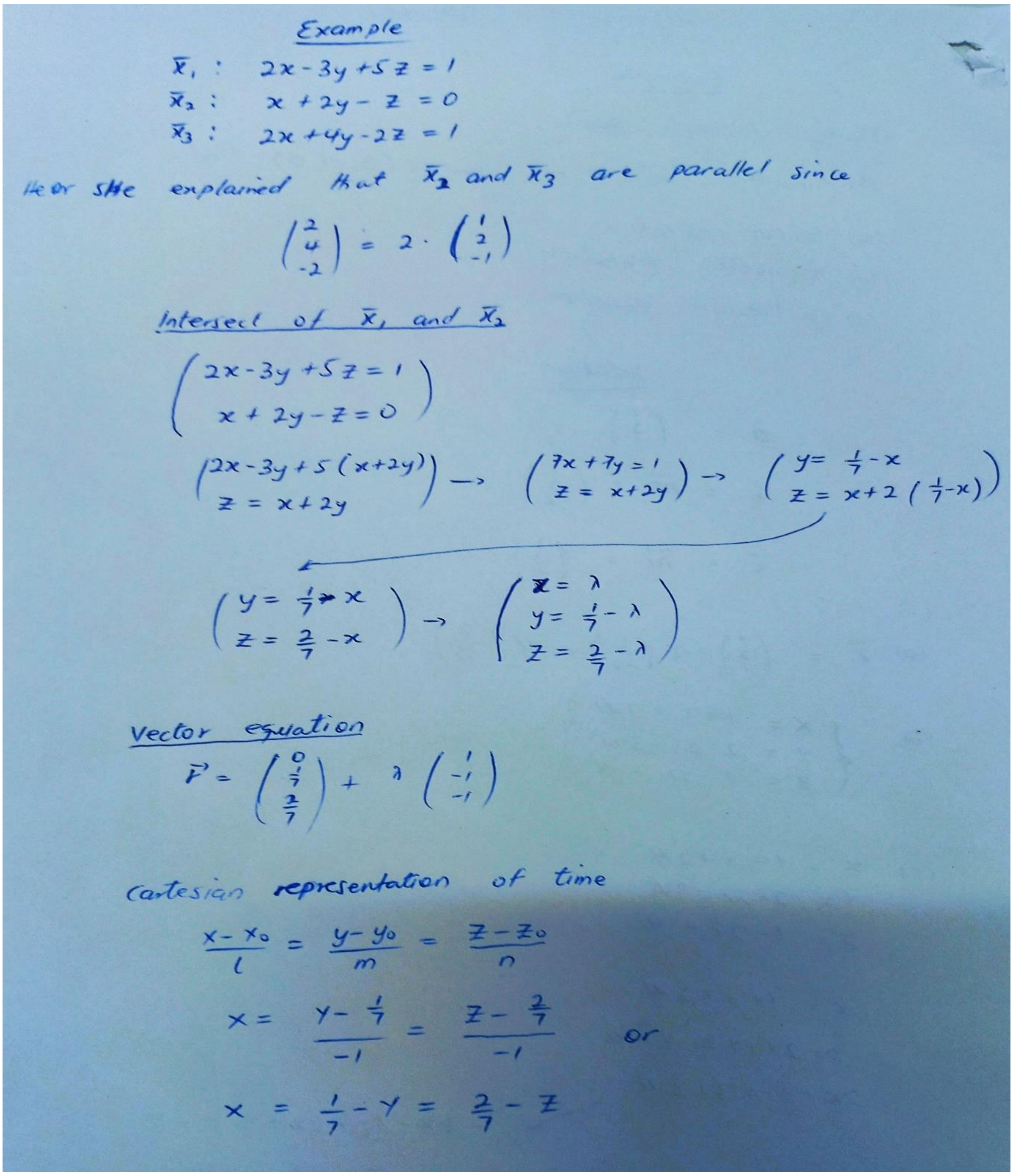


Figure 4.2: Similar task example solved by the Teacher 1(1).

In these examples, the teacher completed tasks that were about finding the normal vectors (figure 4.2), how to show that two planes are parallel, how to find the intersection of two planes, the vector equations, and the cartesian representations (Figure 4.3). Also in Figure 4, the teacher demonstrated how to find the vector equations, the parametric equations, and the Cartesian equations of the planes in three variables. By comparing the questions and the demands of the tasks that the teacher used as examples in the lesson and the tasks that he or she gave to the students to complete after the lesson, the researcher observed that the tasks that were given to the students after the lesson and the demand of the tasks were similar to those of the tasks that were used as examples by the teacher during the lesson.


Example

$\bar{x}_1 : 2x - 3y + 5z = 1$

$\bar{x}_2 : x + 2y - z = 0$

$\bar{x}_3 : 2x + 4y - 2z = 1$

He or she explained that $\bar{x}_2$ and $\bar{x}_3$ are parallel since

$$\begin{pmatrix} 2 \\ 4 \\ -2 \end{pmatrix} = 2 \cdot \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

Intersect of $\bar{x}_1$ and $\bar{x}_2$

$$\begin{pmatrix} 2x - 3y + 5z = 1 \\ x + 2y - z = 0 \end{pmatrix}$$

$$\begin{pmatrix} 2x - 3y + 5(x+2y) \\ z = x + 2y \end{pmatrix} \rightarrow \begin{pmatrix} 7x + 7y = 1 \\ z = x + 2y \end{pmatrix} \rightarrow \begin{pmatrix} y = \frac{1}{7} - x \\ z = x + 2\left(\frac{1}{7} - x\right) \end{pmatrix}$$

$$\begin{pmatrix} y = \frac{1}{7} - x \\ z = \frac{2}{7} - x \end{pmatrix} \rightarrow \begin{pmatrix} x = \lambda \\ y = \frac{1}{7} - \lambda \\ z = \frac{2}{7} - \lambda \end{pmatrix}$$

Vector equation

$$\vec{r} = \begin{pmatrix} 0 \\ \frac{1}{7} \\ \frac{2}{7} \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$$

Cartesian representation of time

$$\frac{x - x_0}{l} = \frac{y - y_0}{m} = \frac{z - z_0}{n}$$

$$x = \frac{y - \frac{1}{7}}{-1} = \frac{z - \frac{2}{7}}{-1} \quad \text{or}$$

$$x = \frac{1}{7} - y = \frac{2}{7} - z$$

Figure 4.3: Similar task example solved by the Teacher 1(2).

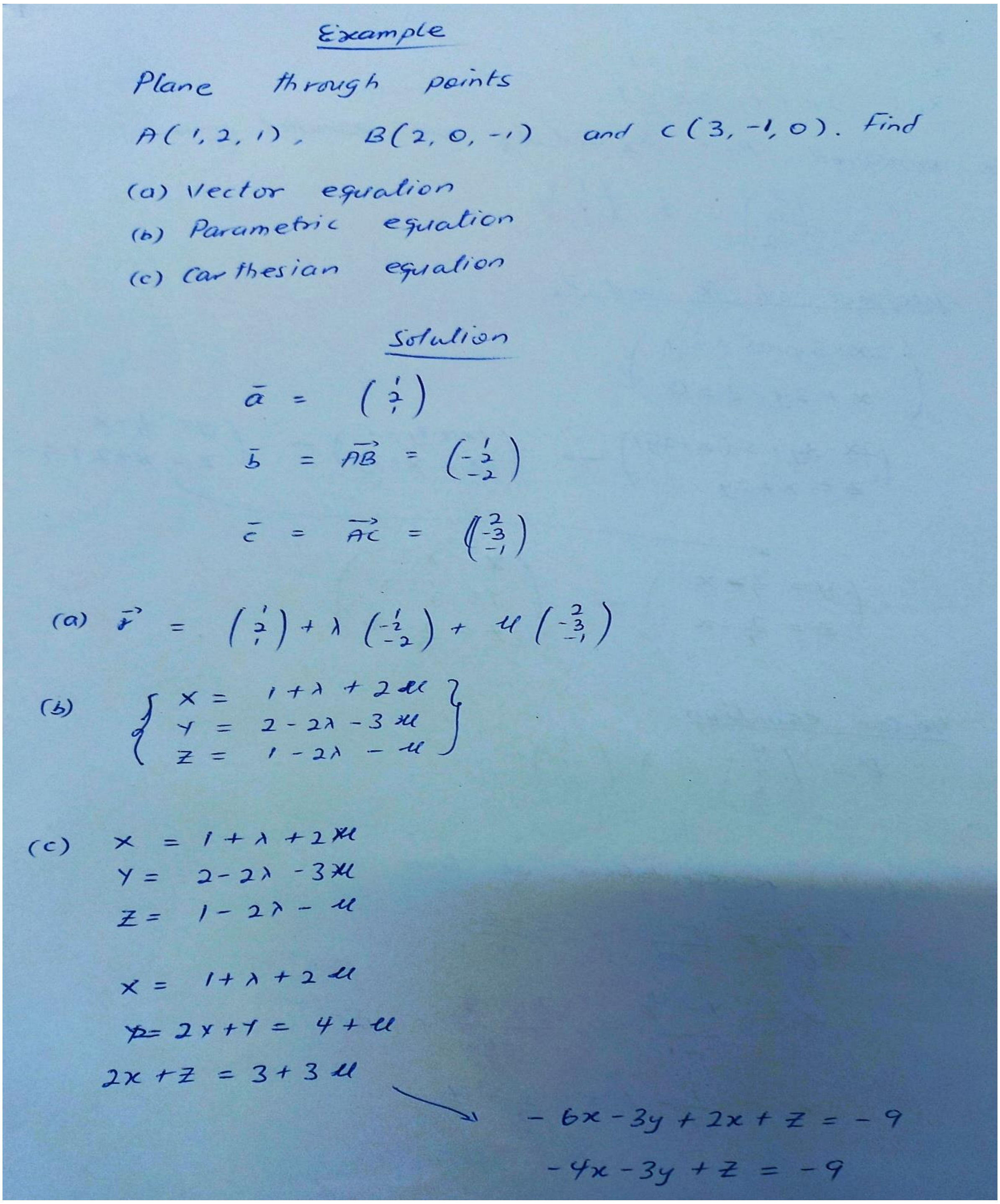
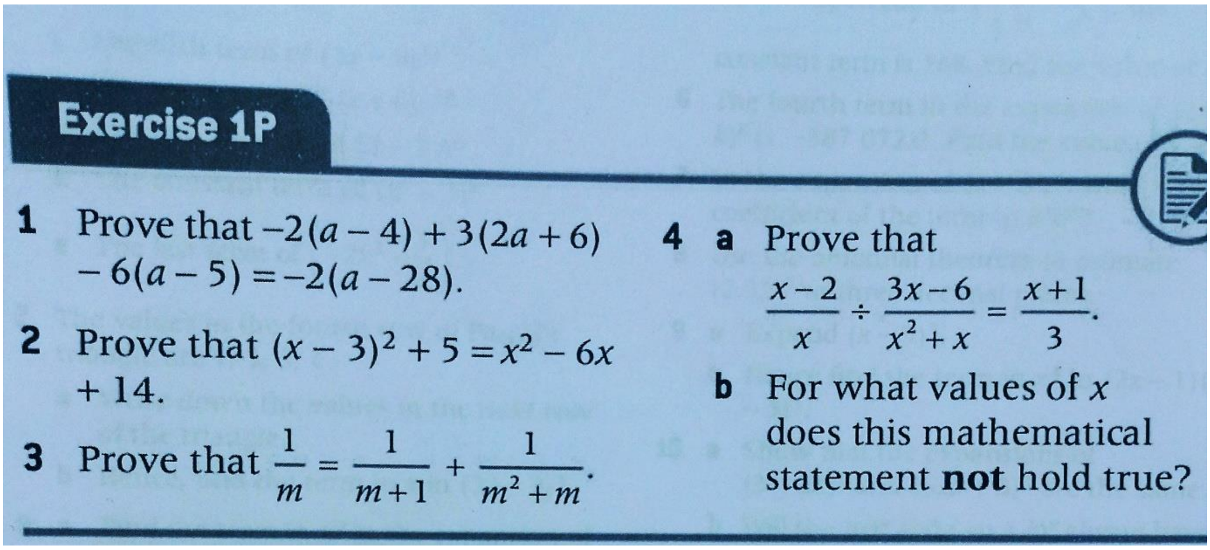


Figure 4.4: Similar task example solved by the Teacher 1(3).

Teacher 2. The data that was depended upon to make proof to this subject in relation to Teacher 2 was drawn from the second day of the observation when he or she was teaching the topic “mathematical proofs”. Even though the question he or she used was not the same as the questions on the tasks that were given to the students, the approach used to solve the example could be used to complete the other tasks. Figures 4.5 and 4.6 represent the tasks and the example used respectively.



Exercise 1P

1 Prove that $-2(a-4)+3(2a+6)-6(a-5)=-2(a-28)$.

2 Prove that $(x-3)^2+5=x^2-6x+14$.

3 Prove that $\frac{1}{m}=\frac{1}{m+1}+\frac{1}{m^2+m}$.

4 **a** Prove that $\frac{x-2}{x}\div\frac{3x-6}{x^2+x}=\frac{x+1}{3}$.

b For what values of x does this mathematical statement **not** hold true?

Figure 4.5: Task example presented to the students by Teacher 2.

In the example (figure 4.6), the teacher showed the process of tackling mathematical proofs. That is you hold one side constant and perform arithmetic on the other before it looks like the side held constant. Using this approach, it becomes easier for the students to complete the tasks that were presented on the task sheet (figure 4.5).

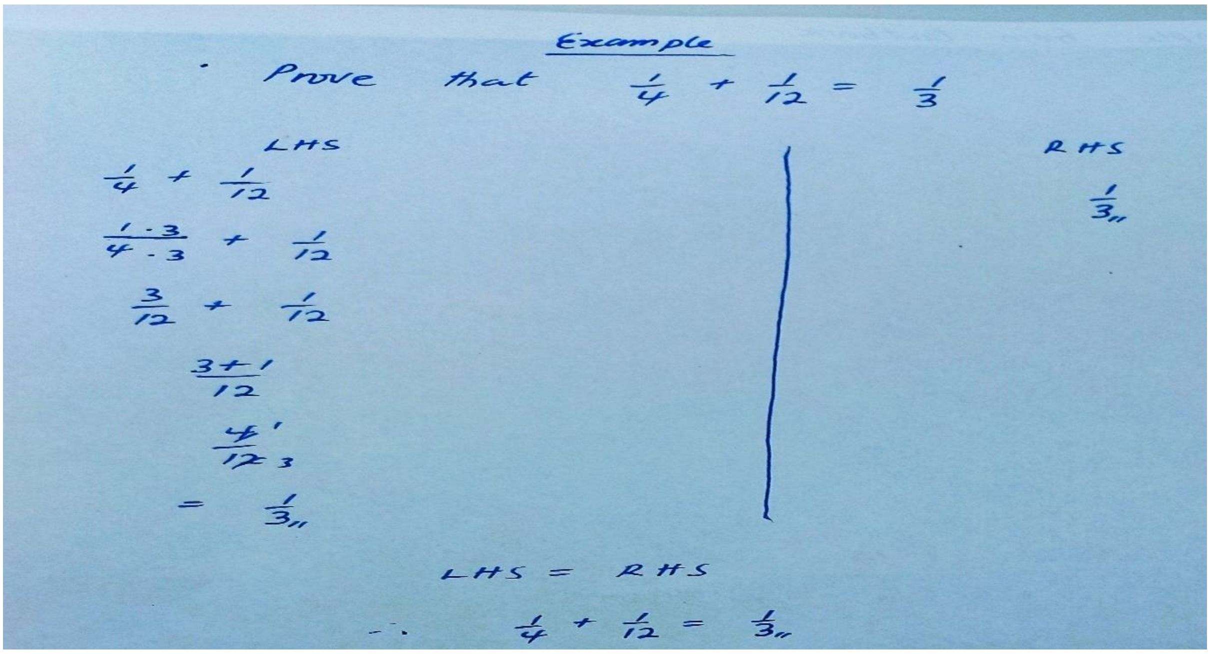
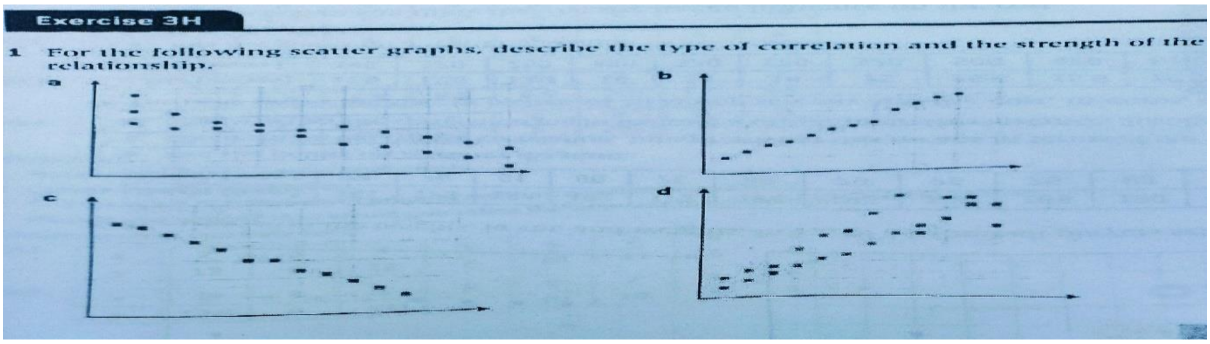


Figure 4.6: Similar task example solved by the Teacher 2.

Teacher 3. His or her findings related to the data collected from the first day of observing the classroom on the topic of "representing and describing data". The teacher solved similar examples in the lesson as the tasks that the students were to complete afterward. The tasks were about scatter diagrams and describing the correlation that exists between the plotted values. Figure 4.7 below was the tasks that the students were to complete and Figure 4.8 was the example the teacher completed.



Exercise 3H

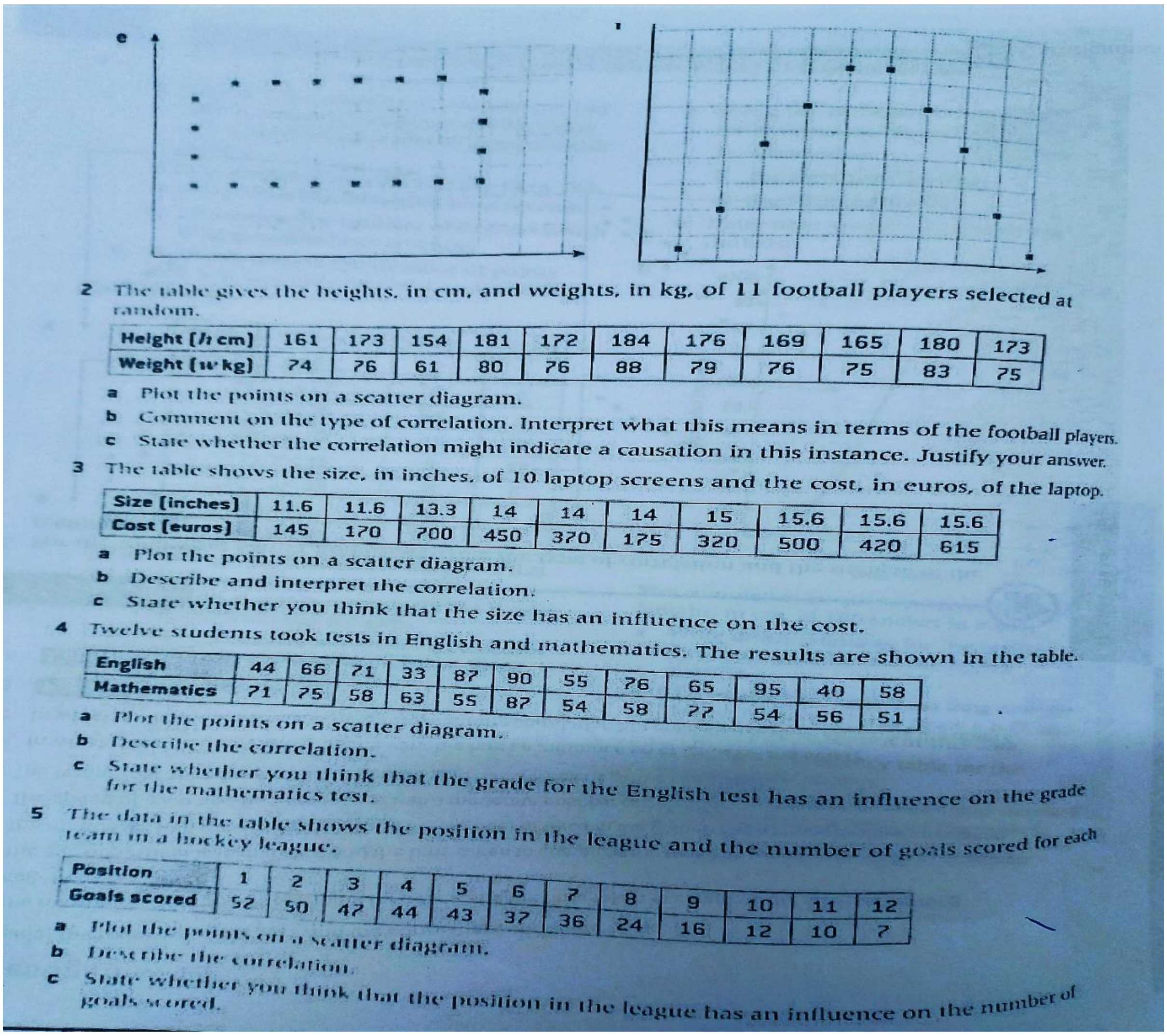
1 For the following scatter graphs, describe the type of correlation and the strength of the relationship.

a

b

c

d



e

f

2 The table gives the heights, in cm, and weights, in kg, of 11 football players selected at random.

Height (*h* cm)	161	173	154	181	172	184	176	169	165	180	173
Weight (*w* kg)	74	76	61	80	76	88	79	76	75	83	75

a Plot the points on a scatter diagram.

b Comment on the type of correlation. Interpret what this means in terms of the football players.

c State whether the correlation might indicate a causation in this instance. Justify your answer.

3 The table shows the size, in inches, of 10 laptop screens and the cost, in euros, of the laptop.

Size (inches)	11.6	11.6	13.3	14	14	14	15	15.6	15.6	15.6
Cost (euros)	145	170	200	450	370	175	320	500	420	615

a Plot the points on a scatter diagram.

b Describe and interpret the correlation.

c State whether you think that the size has an influence on the cost.

4 Twelve students took tests in English and mathematics. The results are shown in the table.

English	44	66	71	33	87	90	55	76	65	95	40	58
Mathematics	71	75	58	63	55	87	54	58	77	54	56	51

a Plot the points on a scatter diagram.

b Describe the correlation.

c State whether you think that the grade for the English test has an influence on the grade for the mathematics test.

5 The data in the table shows the position in the league and the number of goals scored for each team in a hockey league.

Position	1	2	3	4	5	6	7	8	9	10	11	12
Goals scored	52	50	47	44	43	37	36	24	16	12	10	7

a Plot the points on a scatter diagram.

b Describe the correlation.

c State whether you think that the position in the league has an influence on the number of goals scored.

Figure 4.7: Tasks presented to the students by Teacher 3.

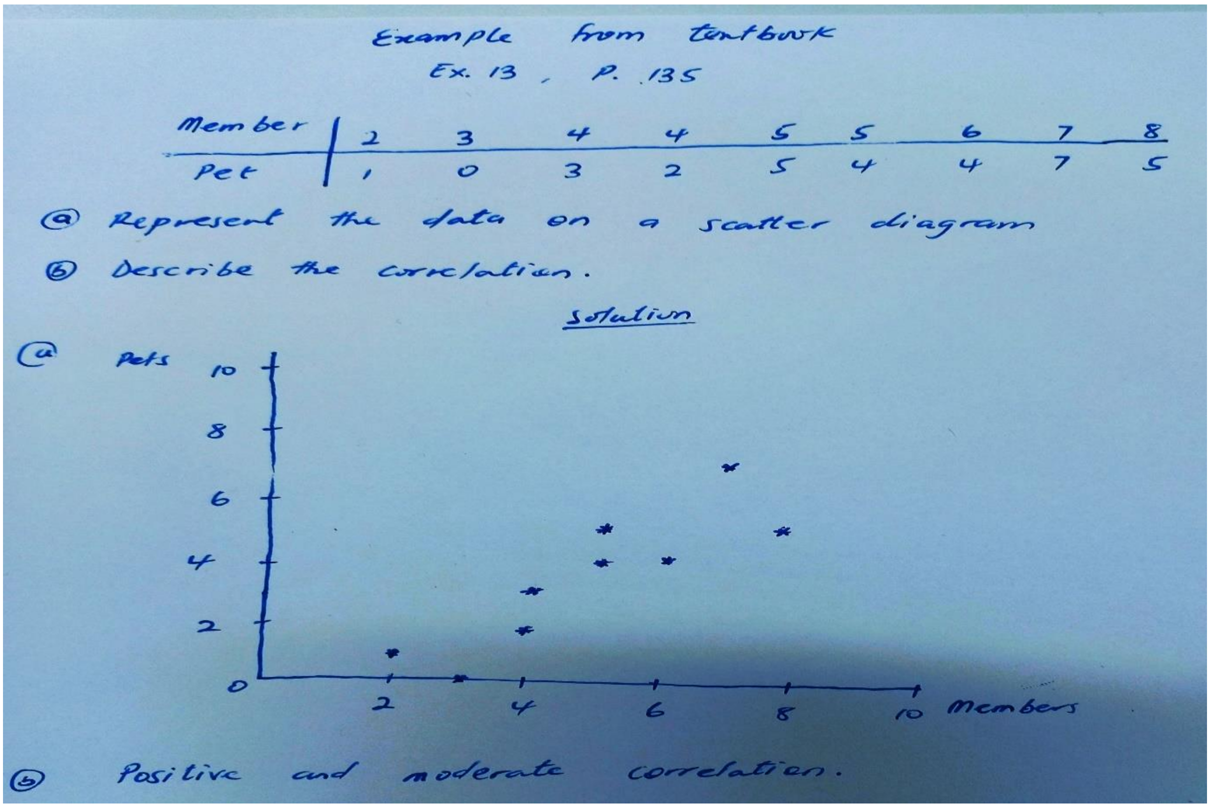


Figure 4.8: Similar task example solved by the teacher 3.

On the task sheet (figure 4.7), all the tasks asked the students to plot the data on a scatter diagram, describe the correlation that exists between the variables as plotted on the diagram, and then ask the students to think whether one of the variables influenced the other variable. The example done by the teacher was also about plotting data sets on a scatter diagram and describing the correlation that exists between the variables. By comparing, the tasks on the tasks sheet and the example the teacher solved were the same just that the task on the tasks sheet went further to ask the students to do other work which was not in the example.

4.5.1.2 Going Around and Assisting Students to Complete the Tasks (GAS)

Another thing that was observed and common in the classrooms of all the teachers was that the teachers went around the classroom whilst the students were completing the tasks. Whilst they went around the students, they interacted with the students about the tasks being completed. They assisted the students with the tasks as they went around. The researcher during the post-classroom observations asked the teachers about these interactions with the students and how these affected the difficulty of the tasks being completed. The response from Teacher 2 is as follows.

Researcher: I observed that you were discussing some things with the students whilst they were completing the tasks. How did those discussions assist the students in completing the tasks?

Teacher 2: I think most of them that I observed were about the structure and they were asking "is this correct structure", when I have done the statement somewhere, and yet still some in the exercises that I did on the blackboard when they calculate something it's the algebra they struggle sometimes with. They don't factorize the denominator, they just multiply the fractions, then it becomes more complex and much easier to do something wrong because it is a high number and ugly expression with power 4 instead of power 1. So, that was the main thing today actually.

Researcher: How did you help them to understand when you went around?

Teacher 2: I try to take it from their side of view and see why did you do this. It could be done in a different way. And then I sometimes help them in a different way, and they are like oh, yeah. And sometimes it's not illuminating for them, it's more, ok, what did you do know. So, it depends on previous knowledge.

Researcher: Did those hints affect the cognitive demand of the tasks?

Teacher 2: I don't know yet because I haven't gotten any answer from them. They have just written, and I have this evidence that seems like it's helping them. But I don't know because I have to check them at some other point.

This conversation ensued between the researcher and Teacher 2 after observing that the teacher went around whilst the students completed the tasks. In the conversation, the teacher acknowledged that as he went around, he or she assisted the students with difficulties they encountered whilst completing the tasks. Moreover, the researcher wanted to know from the teacher's perspective how these interactions affected the difficulty of the tasks, but the teacher was not certain whether the interactions affected the tasks or not.

4.5.1.3 Allow Students to Complete Tasks in Groups (ASG)

Another disposition that the teachers used in the classroom was that they allowed the students to complete the tasks in groups. This was mostly observed in the classrooms of Teacher 2 and Teacher 3. The excerpts below from the interviews of Teacher 2 are connected to this subject.

Researcher: I again observed interactions that existed between the students themselves. Did that affect the difficulty of the tasks?

Teacher 2: I think that is good. Because they talk on the same level and have sometimes, I would say the easier way to save the problem is when they say it in their own words what their levels are. Sometimes, I think it helps that they communicate with each other and sometimes when I am occupied with other people, you can't wait five minutes to get help, so they ask each other, and they can move on. So that is a good thing.

From his or her response, the teacher believed that it was a good practice that the students communicated with each other. He believed that was the easiest way to complete the tasks because the students explained or communicated with each other according to their levels. He again believed it saved time waiting for the teacher to help each student individually when they spoke with each other.

4.5.1.4 Allow Students to Ask Questions During Tasks Completion (ASAQ)

During the classroom observations, the researcher observed that one common practice among all the teachers was that they allowed their students to ask questions when the students were completing the tasks. When the students get stuck with the tasks, they usually ask the teachers for assistance. This practice was observed in all the classrooms of the teachers. The researcher after the classroom observations asked the teachers during the post-classroom interviews about the questions that the students asked and how these questions affected the cognitive demand or the difficulty of the tasks. The response from one of the teachers (Teacher 1) is below.

Researcher: Can you give me an overview of what happened in the classroom today?

Teacher 1: First, one student wasn't here on Friday so there was a repetition, and that wasn't quite long, and then surprisingly many asked about 1b, what perpendicular means, then I gave them a hint, very short, they remembered dot product zero and then they could work on it. And then the number 2 no one asked about it, because it's just like 1b but a bit difficult. Number 3 none asked about, but a couple of students asked about the 4c and that was one of the things I wanted them to think about because we haven't talked much about what the link between the point and the vector equation. So, when they get the point k=-12, what does that mean for my equation I found in 4b, so there were some questions there but also in one stage realized that the R vector is also x, y, and z, then I can take x equal to that part, y=12, and z=-k. One student was confused with 4d and thinks more should have been confused, they just guessed because in 4d you find the dot product, and then you get the d=0 and then you find the angle OBA and it's 90. But OB and AB are not what makes the angle OBA because that will be BO, BA, so realizing that it's 90, it either you shift them upright down you get the same, therefore it's 90. I don't think everyone understood, they just answered them. Almost everyone asked about 5, which I expected because we haven't talked about points or distance between 2 vectors but once they got the hint, I drew a sort of an origin, RA, RB, as well the distance between the endpoints of these 2 vectors, then they solved it very nicely, we used the calculator which is a TI so you can graph things. So, I can sort of they get this graph, so, if it's a paper too, they do all the mess with getting a root of and a lot of things, then they can just plug it in and use minimum values and find that. A few asked about 6a where they were to show that it's not perpendicular because usually, the question is to show that they are perpendicular and you end up with the dot product being -1, $-n^2$. And since n^2 is always positive, the product is always negative and hence never 90 or perpendicular. They really see it, but they want to confirm that it is correct. 7b too made the same arithmetic mistake with the root and forgot the two. They did the right thing. And of the students struggled with the cartesian of a line which we looked at last time but it's sort of if you don't remember that $x-x_0$ over direction, $y-y_0$ direction, $z-z_0$ direction. Then it's difficult to pick up but once you realize it, making vector equation is -1, 0, 3 and then 2,1,-1, so you can just, it's really quick to just pick but if you don't remember, it's a mess. And then 8c, they asked at the end mainly to confirm they were on the right path because 8 marks mean you should spend 8 minutes on it so it's a big task, and then if you have spent 4 minutes on it you feel like you have come to nothing.

Researcher: Do you believe those explanations you made to the questions affected the difficulty of the tasks?

Teacher 1: It made the next questions easier for the students especially maybe on this side. I don't think they were in the right mindset. A good example is 1b and 2 because 2 is more difficult than 1b. But if you understand 1b by explanation, then 2 becomes quite easy, because it is the same idea and the same theory behind that.

The concern of this study was to study the teachers and not the students. The activities of the teachers were the ones the researcher paid much attention to. From the conversation with Teacher 1, on the overview of the class that day, Teacher 1 highlighted the various questions that his or her students asked him during the task completion. This activity was also evident in the other two teachers' classrooms. Again, the researcher further asked the teacher about his or her thoughts on how those questions affected the difficulty of the tasks, and he or she replied that "It made the next questions easier for the students".

4.5.2 Students' Dispositions and Habits Toward Tasks Completion

The study mainly focused on teachers and the tasks that they used in the classrooms and not the students. However, the dispositions of the students in the classroom may influence the tasks. In this sub-section, the findings related to how the students behaved during the completion of the task and how it affected the difficulty of the tasks. Not much emphasis was placed here. The findings related to this were grouped into three teams. First, students share ideas (SSI), second, the students ask questions during task completion (SAQ), and lastly, students become inactive during task completion (SIC).

It was observed in all the classrooms that the students used to talk to each other for ideas during the task completion. That is, the students engaged in group work and depended on one another during the completion of the tasks. The teachers also encouraged the students to engage in this way of completing tasks. Another observation about the students was that they asked a lot of questions during the completion of the task. This can be seen in 4.5.1.4 of this chapter when the researcher commented on the teachers' dispositions towards task completion (Allow Students to Ask Questions During Tasks Completion). This again was done by all the students in the various classrooms. The last concern of this subject was only observed in the classroom of Teacher 2 and Teacher 3. The students most of the time did not take active involvement in the completion of the tasks. The teacher was always helping them complete all the tasks in the classroom. The researcher asked Teacher 3 why the students were ideal during the completion of the task. The excerpts below were the response from Teacher 3.

> Researcher: I observed that you solved almost all the questions for them. Was it because they found it difficult to do it?
>
> Teacher 3: Yeah. Because I feel that sometimes they are not able to get started the way I want them. I try to start and do something, and they can take a look if they want to, those who are better they don't have to look, they can work by themselves, but especially the weak ones, they need more help. Even though they are not that many, it takes time to talk to everyone so if some of them can get help from others, I think that is beneficial.

In the classroom, the researcher observed that the students were not active during the completion of the task. The teacher was always solving the tasks that were presented to them (students).

In a nutshell, this chapter presented the summary and analysis of the results that were obtained from the various data collection instruments that relate to the study's goal. In the next chapter, the discussion of the main findings that relate to the goal of the study is presented.

DISCUSSION

This chapter provides a discussion of the findings of the study. It has two sections. In the first section, the research questions are addressed. In the second section the findings of this study are discussed in relation to literature relating the sources of tasks/resources used by the teachers, the task selection considerations, the modifications to the tasks, and the classroom activities on the tasks.

5.1 Addressing the Research Questions

For emphasis and to make things easier for the readers, the research questions are repeated here.

1. How do teachers select mathematical tasks for their students?
   a. What is/are the source(s) of the tasks?
   b. What is the nature of the task they choose?
   c. What factors do teachers consider when choosing a task?
2. What modifications, if any, do teachers make to mathematical tasks before using them with their students?
   a. Why do teachers modify the tasks?
   b. How does this modification, if any, influence the characteristics of the tasks?
3. What interactions occur in the classroom that help maintain or change the characteristics of the task during enactment?

The aim of this study was to observe the tasks that teachers use in their classrooms, where the teachers get the tasks from, what is the nature of those tasks, and what factors do teachers consider before choosing the tasks to use with their students. Furthermore, do the teachers make modifications (changes) to the tasks, and how do these modifications, if any, affect the nature of the tasks? Lastly, the research looked at what activities take place in the classroom that affect the nature or cognitive demand of the tasks. To address this aim, data were collected through interviews, classroom observations, surveys, and curriculum documents (task sheets).

The survey was used to address questions 1a, 1b, and 1c. The data from the interview was used to address questions 1a, 1c, 2a, 2b, and 3. Also, the task sheets were used to answer questions 1b and 2a. and finally, the data from the classroom observations were used to address question 3. How the data from various sources helped to address these questions is described in Table 3.1.

5.1.1 Addressing Research Question 1

Research question 1 is about how teachers select mathematical tasks for their students. To answer this question, the researcher divided the question into three sub-questions. First where the teachers get the tasks were looked at. Secondly, the nature (cognitive demand) of the tasks the teachers selected was also observed. And lastly, the factors that teachers consider before choosing the tasks were also considered.

To address the first sub-question, the researcher ascertained the various resources used by the teachers and the purposes for which each resource is used by the teacher in their pedagogical process. Data collected from the interview and the survey formed the source of empirical data needed to address the question. The findings from these sets of data sources are reported in section 4.2 of this report/paper. In section 4.2, it was ascertained that there were two main resources used by the teachers in their teaching activities: 1. textbooks and 2. the internet and other resources. The textbooks were used for many purposes including source of tasks, for lesson planning, to make instructional explanations, to give a reading task, to give homework, to trigger student interest, and to provide a history of mathematics (section 4.2.1.1). The most predominant use of the textbook discovered in section 4.2.1.1 was for lesson planning (20 out of 20) and as a source of tasks (18 out of 20 participants). The textbook served as a source of tasks for 90% of the participants in the survey. With the internet and other resources used by the teachers (Section 4.2.1.2), several websites and other books were mentioned and were used for the same but not limited to that of textbooks. Most of these resources were used as a source for tasks and notable among them were mathematik.no, udir.no, and ndla.no.

Moreover, during the analysis of the data from the interviews, it was discovered that all three teachers selected all or some of their tasks from the textbook (designated for the course). Also, one of the teachers asserted that he or she sometimes does make tasks from his or her own mind, and one of these was seen during the study. The other source of tasks that were discovered during the interview was tasks from past exam questions which are made available on internet sources (Section 4.2.2).

On the second sub-question, the cognitive demand of the tasks on the task sheets that were collected from the teachers was assessed. The analysis revealed that all the tasks used by these teachers are at the lower level of mathematical tasks according to the Task Analysis

Guide (Figure 2) provided by Smith and Stein (1996). Out of the 91 tasks that were presented on the task sheets, 8 of them were memorization tasks and 83 of them were procedure without connection tasks (the two categories of the lower-level mathematical tasks). Even though some of the questions were difficult, they (tasks) did not fall into the higher-level mathematical tasks according to the TAG (figure 2). A summary and detailed analysis of these findings are recorded in Table 4.3 and Section 4.3.1.2 respectively.

The last sub-question was addressed using the analysis from Section 4.3. This section talked about the considerations teachers make when selecting tasks. The considerations were classified into six main themes with sources of data from the survey and the interviews. These themes were about cognitive demand, context, content, stress on mathematical ideas, focus on other topics, and student-centered. The teachers were asked various questions relating to these themes, and the level of consideration they make when selecting tasks based on these questions and themes. The analysis indicated that most of the teachers viewed tasks that are based on the theme "stress mathematical ideas" as the most important factor they consider when selecting tasks for enactment with their students. The questions under the theme recorded an average response of important on the Likert scale (most participants chose important). The other themes arranged in order of importance according to the responses from the participants are as follows: application to other topics, focus on students, cognitive demand, content, the task design. Other factors that the teachers consider before choosing tasks according to the interview data were tasks that help students understand mathematical concepts and tasks that match the mathematical knowledge of their students.

5.1.2 Addressing Research Question 2

Research question 2 is about the modifications that teachers make to the tasks when transferring the tasks from the original curriculum documents into their own notebooks or task sheets. The question first asks about what modifications, if any, do teachers make to mathematical tasks before using them with their students. It then probes further to ask the reason for modifications and how these modifications affect the cognitive demand of the tasks. Details are provided in Section 4.4.

The modifications to the tasks were observed on two different levels. First, the modification to the individual tasks on the tasks sheet, and second, the modifications to the set of tasks. The analysis showed that no modifications were made to the individual tasks that were used by the teachers, however, the teachers made modifications to the set of tasks that they presented in their task sheets. That is, the individual tasks appeared in the curriculum documents of the teachers the way they were in the original curriculum documents, however, some of the tasks in the original documents were taken out (the teacher asked the students not to complete them) during the enactment in the classroom.

The teachers did not modify the individual tasks for several reasons including that the teachers found the tasks in the original document as good tasks for their students and there was no need to modify the tasks. This reason for not modifying the tasks was common with all the teachers. Again, the teachers did not modify the tasks when the topic is new. Other reasons for not modifying the tasks were to prevent extra work, there were many tasks, and to serve as a guide for the exam. On the other hand, the set of tasks was also sometimes modified, and the reason for the modification was as follows. First, the tasks repeat themselves. The teachers found out that the tasks in the task sheets were repetitive and thus excused the students from completing some of the tasks on the task sheets. Furthermore, the teachers modified the set of tasks to make the tasks suitable for their students. They omitted some of the tasks that related to topics that they had not treated in the classroom. Another reason for modification was to bridge the gap between the tasks completed in the classroom and the exam questions. The last reason was due to insufficient time to complete all the tasks (time constraint).

The researcher sought from the teachers' point of view how the modifications to the set of tasks affected the cognitive demand of the tasks. The response from the teachers indicated that these changes did not affect the difficulties of the tasks that the students completed. The modifications did not take place on the individual tasks, so each individual task possessed the same level of difficulty needed to complete them. Moreover, the modifications did affect the time allocated to the students to complete the set of tasks.

5.1.3 Addressing Research Question 3

The last research question is about how the activities in the classroom affect the cognitive demand of the tasks during enactment. The analysis and findings in Section 4.5 were used to address this research question. Section 4.5 is related to the activities of both the teachers and the students in the classroom which had an influence on the difficulty of the tasks completed in the classroom.

On the part of the teachers, it was discovered that the teachers solve similar tasks as examples during the lecture as the tasks they later give the students to complete after the lecture. This made the tasks given to the students a mere formality as they had already seen the teacher solve similar tasks in the classroom. It was again discovered that whilst the students completed the tasks, the teachers went around and assisted the students when they were stuck with something. Furthermore, another discovery that was made was that the teachers allowed the students to complete the tasks in groups. That is, they allowed the academically strong students to mingle with the academically weak students whilst completing the tasks. Lastly, the teachers allowed the students to ask several questions during the completion of the tasks.

Moreover, with regard to students' instructional habits and dispositions during the completion of tasks, it was discovered that the students share ideas whilst completing the tasks. Another thing that was observed was that the students ask questions during task completion and finally, the students do not actively participate in the task completion. All these instructional habits and dispositions from both the students and the teachers diminished the cognitive demand of the tasks that were completed according to the teachers (revealed through the post-classroom interviews).

5.2 Discussion of the Study

This section contains a general discussion of the findings in relation to the literature review. The discussion considers the cognitive demands of the tasks, the modifications made to the tasks, and the activities in the classroom that had an influence on the tasks.

5.2.1 The cognitive demand of the tasks

The study looked at the nature/cognitive demand of the mathematical tasks as they go through three phases. First, the cognitive demand of the tasks as they appeared in the curriculum documents, second the cognitive demand as set up by the teacher, and lastly, the cognitive demand of the tasks during implementation. For emphasis, cognitive demand refers to the level of thinking needed to complete a task. The first phase is termed the cognitive demand for the available resources (Burkhardt, 1990). At this stage, the researcher grouped the tasks presented in the original curriculum document into the level of thinking needed to complete them using the Task Analysis Guide (Figure 2, p. 16). The researcher observed that the cognitive demand of the tasks presented in the curriculum documents was at a lower level in accordance with Figure 2, even though the tasks were very difficult to complete for some of the students. This agrees with Stein et al. (2009) when they asserted that the curriculum documents were made to contain several less cognitively demanding tasks with the potency to develop skills needed for a high-level thinking task.

The next phase is how the teacher sets up the tasks in the classroom. To make things easier for the reader, task setup refers to how the teacher introduces the tasks to the students and comprises several activities from the period the teacher transfers the tasks to his or her curriculum document to the time he or she asks the students to complete the tasks. In the findings, it was discovered that the teachers and the students all use the same textbooks. It was also discovered that the teachers give the same tasks in the textbook to the students to complete in class (see 4.2.2.1). The only difference between the tasks in the curriculum documents and the tasks set up by the teachers was that the teachers sometimes asked the students to complete some tasks and leave some tasks out. This change was driven by the teachers' knowledge of their students (to make tasks suitable for their students) as identified by Stein et al., (1996) when they claimed that the teacher's knowledge of his or her students affect the tasks set up by the teacher. This change did not have any effect on the cognitive demand of the individual tasks that were completed by the students as perceived by the teachers, even though there is no literature to support this.

The last phase of the cognitive demand of mathematical tasks is the implementation phase. This refers to how the tasks were completed in the classroom. In the study, it was revealed that a lot of activities took place in the classroom during the implementation of the tasks. These include teachers' solving similar examples of the tasks to be completed, allowing the students to ask questions, allowing the students to work in groups, and assisting the students when they get stuck which fall under teachers' instructional habits and disposition described by Stein et al. (1996) (see p. 17). Stein and Smith argue that teachers' instructional dispositions and habits reduce the cognitive demand of the tasks, and a notable factor is when the students constantly force the teacher to simplify the tasks by clarifying the algorithms that they (students) need to complete the tasks. This was also in the case of Gaël (Brousseau & Warfield, 1999), where the poor classroom arrangement warranted the student (Gaël) not understanding anything and kept on failing in mathematics. The teachers themselves agreed that these interactions affected the cognitive demands of the tasks.

Besides these, there are other ingredients or ways tasks should be designed to improve the cognitive demand to enhance students developing higher mathematical thinking. Most of such ingredients were not observed in the tasks observed in this study. For example, the tasks should be arranged in order, from less difficult to more difficult as the tasks progresses as suggested by Swan, (Swan, 2008). Furthermore, some of the tasks should have been designed to have several or multiple solutions as averred by Mason and Johnson-Wilder (Mason and Johnson-Wilder, 2004) and compel the students to make conjecture, reason and prove, abstract, generalize, and specialize (Breen & O'Shea, 2010).

5.2.2 Discussion of the Modifications to the Tasks

Curriculum documents (textbooks and resources from the internet) are the basic sources of tasks for instruction in the classroom. However, these documents sometimes contain only low-level tasks. This necessitates re-design or modifications to the tasks (when found inappropriate for developing students' mathematical thinking). According to Stein et al. (2009), the curriculum is made to contain several less cognitively demanding tasks with the potency to develop skills needed for a high-level thinking task which was the nature of the tasks used by the teachers in this study. In this study, it was discovered that all the tasks that were used by the teachers were of lower cognitive demands. Moreover, the teachers did not modify the individual tasks they found in the curriculum documents they used for instruction in the classroom.

Kaur & Lam (2012) agreed that the primary source of instructional tasks influencing teachers' instruction is mathematics textbooks (curriculum document). They further stated that when there is a chasm or break between curriculum changes and textbooks, a teacher's role as an interpreter or active user of textbook tasks becomes more important in meeting the curriculum's needs in their instruction. This assertion can be extended to when the tasks do not meet the current classroom goals as mentioned by Lee et, al., (2017) that teachers should be able to use appropriate instructional tasks by creating new ones or modifying existing ones while keeping current with curriculum changes.

In the findings, the main reason that facilitated the teachers not to modify the tasks was that all the teachers found the tasks in the textbooks to be appropriate for their students (even though the tasks were lower-level tasks) and found no need to modify them. This is consistent with Kim & Kim (2014) when they said despite the need to modify textbook tasks to encourage students' inquiry, mathematics teachers did not believe that task modification was necessary. Lee et, al., (2017) also advised that teachers must develop positive attitudes and orientations toward inquiry-based instruction and identify the affordances and constraints of textbook tasks in terms of students' inquiry to recognize the need for task modification. They continued by saying that to use appropriate tasks during class to achieve intended goals or curriculum standards, textbook tasks must be modified, or new tasks must be designed.

The literature provided information about the modifications of individual tasks and the benefits they provide in the mathematics classroom; however, nothing was said about the modification of the sets of tasks (taking some tasks out of the many tasks) to be completed by the students.

5.2.3 Discussion of Classroom Activities on the Tasks

The classroom activities on the tasks refer to the actions of the teachers and the students in the classroom during the completion of the tasks. Aside from assigning tasks to the students to complete in the classrooms, the teachers performed a variety of roles in the classroom during the completion of the tasks that made the students capable or successful in doing the assigned tasks. This is consistent with Kennewell et., al. (2008) who averred that the teacher's role in the classroom goes beyond managing the tasks for instruction. They stated that the teacher is more than just a facilitator of the activity; their role can be described as organizing the features so that the activity moves fruitfully toward the achievement of the intended instructional goals as well as task completion. Kennewell and colleagues explained teachers' role toward students' development of mathematical thinking as something far above just creating tasks for the students to complete but also, other activities.

These varieties of activities performed by the teachers included going around to assist the students when the students found something difficult with the tasks. Whilst assisting the students, the teachers do not impose their thoughts on the topic to the students, but the teachers first enquired from the students' understanding of the topic and then try to push the students towards developing a core understanding of the topic. The teachers were cognitively sensitive to the student's needs. According to Potari & Jaworski (2002), cognitive sensitivity deals with the teacher's acceptance and acknowledgment of students' thinking. They further stated that cognitive sensitivity deals with how a teacher properly reacts to students' thoughts. Cognitive sensitivity is about how a teacher may concentrate on how a student understands mathematics, such as determining the approach a student is employing (Empson & Jacobs, 2008), or the significance of what a student said or wrote (Crespo, 2000).

Other activities performed by the teachers in the classroom whilst the tasks were completed were that the teachers solved similar tasks themselves to serve as a guide for the students when completing other tasks, allowing the students to ask questions, and answer them, and allowing the students to work in groups which may fall under other activities described by Kennewell and his colleagues (2008).

The teachers could have performed other activities on the tasks in the classroom that could improve the tasks and give the students great opportunities for individual thinking. For example, the teachers could have stimulated students' curiosity by improving the tasks to their knowledge and assisting them in solving their problems with *stimulating questions* (Polya, 1945) and used instructional methods that would ensure that the students were active participants rather than consumers in the learning of mathematics as asserted by Krainer (Krainer, 1993).

CONCLUSION

The summary of results and discussion are presented in this final chapter. Following that, the study's limitations are discussed, and the chapter concludes with implications for teaching and suggestions for future research.

6.1 Summary of Results

This study was about how teachers select and use mathematical tasks in the classroom, the nature of the tasks, the factors they consider before choosing the tasks, the re-design or modifications made to the tasks, the interactions in the classroom with the tasks and how they affect the cognitive demand of the tasks. The sources of data for this study were interviews, classroom observations, surveys, and curriculum documents (task sheets).

The most common sources of tasks used by the teachers were tasks from textbooks and past exam questions that were made available on the program's (IB) website (Section 4.2.2.2). The teachers also used various resources for various reasons, notable among the use were for lesson planning and to make instructional explanations. Other popular resources the teachers selected tasks from were mathematikk.no, ndla.no, and udir.no.

Several factors were considered by the teachers when they selected the tasks for their students to complete. In the interview, it was discovered that there were two main factors that influenced the choice of mathematical tasks for their students; first, the teachers selected tasks that help the students to develop a higher mathematical competence (build mathematical vocabulary), and secondly, the teachers selected tasks based on their knowledge of their students' mathematical competence. In the survey, pre-determined factors were presented to the teachers to select from. These factors were then grouped into six themes (see Section 4.4). Most of the teachers agreed that selecting tasks that assist their students' developing mathematical ideas was the most important factor they consider when selecting tasks.

All the tasks that were presented on the tasks sheets by the teachers were lower-level tasks as per the Task Analysis Guide (see Figure 2). However, the teachers did not modify the tasks to provide more opportunities for the students. The omission to modify the tasks was that the teachers deemed the tasks to be good tasks and challenging enough for the level of their students (even though they were lower-level tasks). However, sometimes, the teachers modified the set of tasks (taking some tasks out), but this did not have any effect on the cognitive demand of the other tasks the students completed.

A lot of interactions took place during the completion of the tasks in the classroom. Both the students and the teachers displayed some instructional habits and dispositions (see Section 4.5) that declined the level of difficulty needed to complete the tasks. Teachers completed similar tasks as those presented to the students, allowed the students to work in groups, students asked a lot of questions during the task completion, students mostly did not take active involvement in solving the tasks, etc. All these, from the teachers' perspective, reduced the difficulty of the task and even made the next task easier.

6.2 Limitations of the Study

This study is limited by several factors, and this should serve as the focus through which the interpretations of the findings of the study should be seen. The first limitation is the sample size. The main findings of the study were related to the results from the case studies (3 cases), and even though there was a survey attached to this study, the participants involved were too small (twenty) to provide a statistical generalization. The use of three cases did not affect the validity of the study but limited the generalization of the study.

The second limitation of the study was the selection of the participants. The participants were selected based on a biased sampling technique. That is, the participants were selected based on their availability and their language of instruction in the classroom. The participants, especially for the three cases, were teachers who use the English language as the medium of instruction in their classrooms. Again, the participants selected were teachers who teach in the IB program. This also limited the ability to generalize the results.

The third limitation of the study was the research's sole focus on the teachers and their use of resources rather than perceiving from the students their thoughts on what resources or tasks they prefer to work with. This could have provided balanced findings from both the teachers and the students on what a good task or resource for instruction in the classroom.

6.3 Implications for Teaching

As already mentioned, there is a link between the tasks used in the classroom and how to achieve the goals in the new curriculum (Section 1.1, p.2). The findings of this study will serve to bridge the gap between available literature and the implementation of mathematics curriculum in the classroom. This study unearthed several findings that can be helpful for mathematics education. The research unearthed various resources available for mathematical pedagogical activities, the cognitive demand of the tasks in the original documents, and how the design and redesign of those tasks will provide several opportunities for students to learn, and the teacher's responsibilities in the classroom that help maintain or decline the cognitive demands of the tasks.

One of the chiefly limiting factors for teachers finding good instructional tasks is the unavailability of resources (Araujo, 2012). Practically, this research has increased recognition of a diverse set of resources that are freely available to mathematics teachers in schools and online, and what these resources could mean for their pedagogical practices. This knowledge may lead mathematics teachers in Norway and other places to develop a composed mentality and quality assessment standards in their resource selection, which could create an attractive possibility for students to learn quality mathematics. The awareness created in this study about the various resources available for the teachers will help reduce the time and energy teachers use to create their own tasks and help teachers select and compare tasks that are compelling and enhance students' participation and development of interest in mathematics in the classroom.

Again, this study confirmed as stated by Stein et al., (2009) that original curriculum documents are made to contain tasks that are of a lower cognitive demand. Lower cognitive demanding tasks do not push students to think deeply before completing them. These kinds of tasks do not help students to develop in-depth knowledge of mathematics. Teachers are advised to re-design or modify these tasks to offer the opportunity for students to develop deeper mathematical thinking (Stein et al., 2009: Lee et, al., 2017). In this study, it was discovered that none of the participants used any tasks that were of the higher cognitive demand category. These tasks must be used most often in the classrooms if the aim of the curriculum is to produce students who think deeply about mathematical concepts. In lieu of this, curriculum policymakers are recommended to clarify the kind of goals they want to achieve in mathematics and work toward improving the curriculum documents to offer these goals they stated. Moreover, pre-service teachers and in-service teachers must also be trained on the modification and re-designing of mathematical tasks and the opportunities that this activity provides for students to learn mathematics.

Furthermore, there were habits and dispositions from the teachers that influenced the cognitive demand of the tasks at the implementation stage. Teachers deployed several habits of assisting the students to complete the tasks when they (students) get stuck. These interactions reduced the level of difficulties of the tasks and made the task easier to complete. Teachers must set goals they want to achieve with the tasks and stick with them. If the goal is for the students to work with the tasks at the highest level of difficulty, then they are advised to use indirect ways to participate in the students thinking rather than getting directly involved.

6.4 Future Research

This study suggests several avenues for future research. For example, touching on the teacher's knowledge, this paper only looked at how the teacher's knowledge of his or her students affected the nature of tasks he or she uses in the classroom. Moreover, there is more to the teacher's knowledge that can affect the task selected by the teacher than what is measured in this study. Future researchers in this area can look at how the teacher's knowledge about the subject matter and the pedagogical content, and knowledge of content and teaching may affect the task selection process.

Again, this study was limited to teachers who teach students in the IB program. The IB program is mostly specific (according to the teachers) on the content and knowledge that must be taught in the classroom. The kind of tasks that must be used in the classroom are again provided in the curriculum documents and thus give the teachers little room for adjusting to using tasks from different sources other than those specified in the textbooks and the program's website. Future researchers can look beyond the IB program and extend their study to mathematics teachers who teach in the general Norwegian curriculum.

Furthermore, another area future researchers can explore is the relationship between students and mathematical tasks. They can look at the kind of mathematical tasks students deem as good mathematical tasks and how students' involvement in the mathematizing in the classroom contributes to their mathematical knowledge development. This study was only focused on the relationship between the mathematics teachers and the mathematics tasks and had nothing or little to do with tasks from the students' perspective. Future researchers can look at this to strike a balance for both parties (teachers and students) on what a good mathematics task is.

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APPENDICES

Appendix A: survey questionnaires

Section 1: Demographic information

1. How long have you been teaching mathematics: ___________ years.

2. Which grade level(s) do you teach mathematics this year?

   Grade levels: _______ and ________

   Number of periods per week you teach them: _________

3. Please tell about the track or program of your students (e.g., academic track, vocational track, or other tracks, if any).

   ____

Section 2: The role of the textbook in lesson planning and teaching,

1. Are you using a mathematics textbook? Yes ____ No ____ If yes, answer question 2, please.

2. What are you most likely to use the mathematics textbook for?

Purposes of use	**Yes-No,** if yes how often? => Rarely, frequently, very frequently: Please indicate below.
Lesson planning	
To prepare for making instructional explanations	
To give student reading tasks	
Source of exercises	
To give homework from	
To trigger student interest or curiosity	
To provide a history of mathematics	
Other	

Section 3: Other sources or resources you use, other than the textbook (such as free internet sources, internet sources that require a subscription, other teachers or colleagues, auxiliary textbooks, or other books)

Internet A	**Internet B** (if any)	**Other sources 1**
(please specify: ________________________)	(please specify: ________________________ _)	(please specify:_ ________________________)
How do you use this resource for instruction (please indicate below)	How do you use this resource for instruction (please indicate below)	How do you use this resource for instruction (please indicate below)

Other sources 2	**Other sources 3**
(please specify: ________________________)	(please specify: ________________________ _)
How do you use this resource for instruction (please indicate below)	How do you use this resource for instruction (please indicate below)

Examples of use:

a. Lesson planning
b. To make instructional explanations
c. To give student reading tasks
d. To provide a history of mathematics
e. Source of exercises
f. To give homework from
g. To trigger student interest or curiosity
h. Other (please specify)

Section 4: Task selection considerations

As teachers, we use a **task** to develop or assess a mathematical idea in teaching. The tasks may be for example, exercises from textbooks, word problems, larger tasks that guide students' work in steps, a modeling task, or an activity.

Teachers can find mathematical tasks from various sources. Yet, teachers select those tasks that are appropriate to their students and that will help them teach or assess a mathematical idea. Different task features may be important for different teachers. In this section, we ask you to identify the desirable task features, as you see them. Please mark how important are the following mathematical task features for you to use in the classroom.

1. selecting tasks that contain **an important mathematical idea** is

unimportant ( ) slightly important ( ) moderately important ( ) important ( ) very important ( )

2. selecting tasks that promote **understanding** of a mathematical idea is

unimportant (   ) slightly important (   ) moderately important (   )   important (   ) very important (   )

3. selecting tasks that give another chance **to apply a known mathematical procedure** is

unimportant (   ) slightly important (   ) moderately important (   )   important (   ) very important (   )

4. selecting tasks that require **students to work in small groups** is

unimportant (   ) slightly important (   ) moderately important (   )   important (   ) very important (   )

5. selecting tasks that **can create a classroom discussion** is

unimportant (   ) slightly important (   ) moderately important (   )   important (   ) very important (   )

6. selecting tasks that require students **to explain and reflect on** their thinking is

unimportant (   ) slightly important (   ) moderately important (   )   important (   ) very important (   )

7. selecting tasks that have the potential to raise **student interest and motivation** is

unimportant (   ) slightly important (   ) moderately important (   )   important (   ) very important (   )

8. selecting tasks that have a context and making **real-life connections to other school subjects** is

unimportant (   ) slightly important (   ) moderately important (   )   important (   ) very important (   )

9. selecting tasks that make use of **visual drawings or illustrations of mathematical ideas** is

unimportant (   ) slightly important (   ) moderately important (   )   important (   ) very important (   )

10. selecting tasks that are quite **unique, interesting, and unlike** "textbook" tasks are

unimportant (   ) slightly important (   ) moderately important (   )   important (   ) very important (   )

11. selecting tasks that are NOT **too difficult** is

unimportant (   ) slightly important (   ) moderately important (   )   important (   ) very important (   )

12. selecting tasks that are NOT **too easy** is

unimportant (   ) slightly important (   ) moderately important (   )   important (   ) very important (   )

13. selecting tasks that **do NOT** have a **complicated storyline or have long text** to read is

unimportant (   ) slightly important (   ) moderately important (   )   important (   ) very important (   )

14. selecting tasks that are given in **steps from easy to more difficult** is

unimportant ( ) slightly important ( ) moderately important ( ) important ( ) very important ( )

15. selecting tasks that sometimes **do NOT have all the information needed for a solution**, and students either need to make assumptions or do research to find the information is

unimportant ( ) slightly important ( ) moderately important ( ) important ( ) very important ( )

16. selecting tasks that have **unexpected or surprising** answers are

unimportant ( ) slightly important ( ) moderately important ( ) important ( ) very important ( )

17. selecting tasks that students **do NOT have the knowledge to solve yet**, and learning new knowledge while working on the task is

unimportant ( ) slightly important ( ) moderately important ( ) important ( ) very important ( )

18. selecting tasks that **do NOT seem to have "keywords" hinting what mathematical procedure should be used** in the solution is

unimportant ( ) slightly important ( ) moderately important ( ) important ( ) very important ( )

19. selecting tasks **to introduce a new topic** is

unimportant ( ) slightly important ( ) moderately important ( ) important ( ) very important ( )

20. selecting tasks that **apply a mathematical topic or procedure after I teach it** is

unimportant ( ) slightly important ( ) moderately important ( ) important ( ) very important ( )

21. If there are other considerations that are important for you, please write them in the box below.

This is the end of the survey. THANK YOU for your time.

Appendix B: Transcription from the Interviews

Teacher 1

Pre-interview 1

Which mathematical topics have you taught this semester?

This semester I have taught mathematical applications, no sorry analysis, and approaches which is an IB course, the hardest one. And in the Norwegian curriculum have taught 1T which is the hardest one in the first grade.

Which mathematical topics are you currently teaching?

1T in Norwegian and mathematical AA in IB

And what lessons have you planned to teach today?

Today it is mathematical AA where we summarise vectors up until the vector equation of a line.

What mathematical tasks have you chosen to use today?

I have chosen formal exams questions because it is at the end of or halfway through vectors and the next part is vectors as a plane, which is much more difficult and now, we get a lot of old exam questions that go on the easy part of vectors on a line and the perpendicular and all these sorts of aspects.

Can you tell me where you got these tasks from?

These ones are old exam questions so the IB system has a website where you can go and search for instance vectors and then you get all the exam questions about vectors then I read through the tasks and then I singled out the ones that fit where we are at the moment

Will you use the task in order as they appear in your curriculum document?

Yeah, I try to take a few simple ones first where you just introduce the idea of parallel and perpendicular and dot product, and then increasing becomes harder with the idea of how to find the speed of vector.

What other resources will you add to these tasks to teach?

Today nothing.

What do you intend to achieve with these tasks you have selected for the students?

I intend to try to get my students to understand the bigger picture. So, in the classroom so far we have looked at very specific what's perpendicular, what's parallel, and how do you write the line as a vector but now my goal is that with the task they should see how they can make two lines and if there is an unknown in there k, how can you define k so that they are all perpendicular. This is the sort of step up and see the bigger picture of the curriculum.

What does the curriculum intend for students to learn after this topic?

Basically, the idea of vectors, what are vectors, how do we treat them, and why are they important because they haven't touched vectors before, it is the first time they are seeing them, so they are mainly an introduction to why they are important.

Have you made any modifications to the task as they appeared from the source?

The only thing I have done is to take out if they include planes because some exam questions start with easy ones with lines and so on, and then you get into planes at the end, so I have deleted the ones they haven't done yet.

Why did you make those changes?

The aim was to make it suitable for where my students are at the moment.

How well do you know your students when it comes to their mathematical background?

Very well because I have been their teacher for one and a half years.

How do rank their mathematical performance?

High, all of them high. I said to you that it is only high-level students. They are the best of the students.

Did their performance affect the tasks you chose for them to enact?

Yes, I would have started much more simpler with the first ones if the class had a bigger range of students.

How are you going to present the tasks to them today?

Put it in front of them and say have fun. That is because it is a, they do it to revise rather than it is a new thing. Because if it is a new thing, I would have done more.

How do you expect the students to complete the tasks? As a group or individually?

Individually.

How are you going to assess the students?

The assessment is mainly a text at the end of the part of the vector, but IB is such that I don't give a grade, it is the exam at the end, somehow like the university where the grade comes at the end, so the sort of my assessment is also when I walk around. They get the feedback that they should know this, so this is how we do it and I see you where the level is.

What product do you expect students to produce from the tasks?

Nothing else than knowledge. So, the goal is that they should see the bigger picture.

Pre-interview 2

Where did you get the tasks from?

These tasks came from the book that we use.

Have you made any modifications to them?

No modification.

Why did you choose not to make any modifications to these tasks?

Because we started something completely new, so they need to practice the basics and then the questions cannot be too complicated, so just get into vectors, find the cross product and really you can do any directrices. So then using the book is fine.

What is the goal for choosing these tasks?

The goal is for them to learn the arithmetic of how to find the vector product and hopefully they realize the idea of the area by cross product.

What does the curriculum expect students to learn at the end of the lesson?

That they know what a cross product is, and that the area of a parallelogram can be given by the length of the cross product and how to operate it the long-term goal is that they should be able to use it to find the vector equation of a plane.

Did you consider their mathematical experiences when you chose those tasks?

I thought of that, that's why there is an extra task. I think maybe the three first today, and then the extra task if they do it really very quickly but since it is completely new, I put quite a little low, and therefore I expect them to do it quite quickly because they catch up quite quickly. But they also need some basic training in order to do that. So today, I don't expect them to ask many questions, especially the first three tasks there shouldn't be many questions about them

What do you expect them to produce from the tasks?

I expect them to be able to find the vector product and the area spanned by the two vectors or the area of the parallelogram made by the two vectors. That is the goal

Pre-interview 3

Which topic are you teaching today?

Today we make the planes more difficult so we start talking about what can happen when you have several planes and if they intersect or they are parallel and those things. So, while Friday was more just about learning how to do it, now it is to get this cognitive idea of what is a plane we were talking about the last time

Do you have some tasks for this topic?

Yes, as I wrote the 9I which you got the last time and I said if someone gets time for it, they start. They never got time for it the last time. So now we start with those, the first ones, numbers 4 and 5 is mainly do the work, they were not very difficult but time-consuming and the 6th one I hope they really have to think what this means.

Where did you get the tasks from?

Those tasks are from the textbook. It is not always that the book is good but with this one, it is because they progressed in, first, how do we do to *reform* and then there are three questions of three unknowns which always is difficult and then is this idea, the start from ec where you should find the intersections on, then the last question where you want the plane going through a different point but being parallel to the, no sorry, but containing the line they have only made, that is a mind ripping question

Do you have other tasks from other sources?

No, not today but on Friday, I am going to introduce exam questions.

Have you made any modifications to the tasks?

Not more than the answers are wrong in the book, so I made the correct solutions.

So, you have made no modifications to the tasks?

No.

Why did you choose not to modify the tasks?

Why? Because I think the tasks are good. I think they give a level of steepness or difficulty where it starts off simple and ends up difficult. So, if they manage 6b, they've really understood what planes mean and I expect them not to manage it but some of them I speak with them and I feel that we are together but it's a difficult task so this time I didn't feel I needed different sources because the book has the progression that I like

What do you expect students to produce today?

I expect them to go back and remember how they did the three questions of the three unknowns and hopefully, they will have the geometrical idea of a plane while last time it was mostly about the arithmetical, how do I find the numbers, now I hope they more end up with what it means that you have two planes

Pre-interview 4

What do you plan to teach today?

Today it's the angles between planes and vectors.

Do you have some tasks for the lesson?

Yes

Can you tell me the source of those tasks?

The tasks are from the book and one we work on it yesterday that I may ask them about it was from previous examples.

Have you made any modifications to any of these tasks?

Not more than I have taken away, so for instance I chose only b because a is a lot of work and is on what we have before. So now I want to focus on just the angles, so then I pick on where the angles are, which is important.

I want to know whether there have been changes to the tasks as they appear here.

No, not more I said 3b and not 3a.

Why did you intend to modify 3b but not 3a?

No, I said I modify task 3 by asking them not to do the a, and not the task itself. But the task is about first finding where they intersect to find the point of intersection and then it's a find the angles between the two, but I don't want them to find the intersection because they work on that so it's just, it's going to take a long time and they learn anything new. So, I modify the task to only talk about the angle and not anything else.

Why did you then choose not to modify the task?

Because the task shows different ways they might get questions on the exam and generally on how vectors and planes work for instance 1d it is difficult to make without doing a lot of work I said, you end up with a line plane that you wonder whether the line is in the plane, it's durable but it takes a lot of time to find, but then they do the work and suddenly they just, they don't get any of the lambdas which is the one we use and the interpretation of what does it means that the lambda disappears. And that will, that will mean it's not a point, it's anywhere.

Did you consider your students' mathematical knowledge and experience before choosing the task?

Not more than usual, but I know that they are quite strong, therefore, I feel like skipping a few tasks, therefore I have just chosen a few of them not all of them because it's a lot of repetition when there are many tasks

Do you believe the task can match the capabilities of your students?

Yes, but I expect for instance d several of them will get stuck because they won't understand how to interpret the answer and they will need help with it but the one we talk about it, I expect that they will understand the reasoning once they get there.

Does the cultural background of your students affect your task selection?

Yes, since I know they are quite strong, I choose to progress quickly. So, if you have not a strong student maybe you need a sort of task asked 4 times just to get in the working of how we do such a task but with my student, I give 1 or 2 and now I expect them, and now they have seen it and they are ok.

How do you expect your students to complete the task?

Individual to and to.

What do you expect from them at the end of the day?

Knowledge of how to find the angles.

Teacher 1 – Post classroom interview

Post-interview 1

Can you give me an overview of what happened in the classroom today?

First, one student wasn't here on Friday so there was a repetition, and that wasn't quite long, and then surprisingly many asked about 1b, what perpendicular means, then I gave them a hint, very short, they remembered dot product zero and then they could work on it. And then the number 2 no one asked about it, because it's just like 1b but a bit difficult. Number 3 none asked about but a couple of students asked about the 4c and that was one of the things I wanted them to think about because we haven't talked much about what the link between the point and the vector equation. So, when they get the point k=-12, what does that mean for my equation I found in 4b, so there were some questions there but also in one stage realized that the R vector is also x, y, and z, then I can take x equal to that part, y=12, and z=-k. One student was confused with 4d and thinks more should have been confused, they just guessed because in 4d you find the dot product, and then you get the d=0 and then you find the angle OBA and it's 90. But OB and AB are not what makes the angle OBA because that will be BO, BA, so realizing that it's 90, it either you shift them upright down you get the same, therefore it's 90. I don't think everyone understood, they just answered them. Almost everyone asked about 5, which I expected because we haven't talked about points or distance between 2 vectors but once they got the hint, I drew a sort of an origin, RA, RB, as will the distance between the endpoints of these 2 vectors, then they solved it very nicely, we used the calculator which is a TI so you can graph things. So, I can sort of they get this graph, so, if it's a paper too, they do all the mess with getting a root of and a lot of things, then they can just plug it in and use minimum values and find that. A few asked about 6a where they were to show that it's not perpendicular because usually, the question is show that they are perpendicular and you end up with the dot product being -1, $-n^2$. And since n^2 is always positive, the product is always negative and hence never 90 or perpendicular. They really see it, but they want to confirm that it is correct. 7b too made the same arithmetic mistake with the root and forgot the two. They did the right thing. And of the students struggled with the cartesian of a line which we looked at last time but it's sort of if you don't remember that x-x_0 over direction, y-y_0 direction, z-z_0 direction. Then it's difficult to pick up but once you realize it, making vector equation is -1, 0, 3 and then 2,1,-1, so you can just, it's really quick to just pick but if you don't remember, it's a mess. And then 8c, they asked at the end mainly to confirm they were on the right path because 8 marks mean you should spend 8 minutes on it so it's a big task, and then if you have spent 4 minutes on it you feel like you have come to nothing

Did the lesson go as you planned?

Yes, I will say so. I think they had a bigger understanding of the links between things, that is, the line and the perpendicular, and how we treat things.

Which of the task do you believe was completed as you planned it?

Basically, it was number 1 that I expected them to solve without me saying anything but other than that they felt, they managed it as I expected, and often the problem that they had is sloppy in arithmetic rather than understanding the lectures, they forgot the two or they did a minor mistake. 5 was a lot of questions from them, but I expected it because we haven't had that type of task.

Which of the task did you felt it was problematic for them to answer?

5, which was a difficult task, and then 8.

What changes do you intend to make so that they can answer those questions very well next time?

Not very much, because even though they were difficult, my feeling is that when they got to the tasks, they understood them. When I am going to do is that we have more tasks that look like 5, so we are going to start up with a new task that is really the same task and do that together and talk it through. Other than that, I am going to do much.

Do you believe those explanations you made to the questions affected the difficulty of the tasks?

It made the next questions easier for the students especially maybe on this side. I don't think they were in the right mindset. A good example is 1b and 2 because 2 is more difficult than 1b. But if you understand 1b by explanation, then 2 becomes quite easy, because it is the same idea and the same theory behind that.

Post-interview 2

Did the class go as you planned?

Yes, I will say so.

Which part do you believe you achieved the maximum outcome?

Yes, I felt it went as I expected and as I hoped.

Were there any difficulties you believe the students faced with the tasks?

That they didn't quite gloss off the question said. At the first ones and at the last ones. It was too simple for them because the one I did was subtracting and everything because we got points. And the first one should be a bit easy, you get this vector, and this vector, and this point and you should just say a point lambda vector, lambda vector. So, it takes just 15 seconds. But they didn't realize that it was so simple, so they overcomplicated. But that was ok.

Did the questions from the students change the level of difficulty of the tasks?

No, I felt it met the problem I wanted them to meet but I was an introduction and therefore they weren't that big cognitive problems. That ones are coming next week. So now it's mainly doing the work and learning and they are quite good students, so now it's just sit down and work and they managed it. But next week, suddenly it's going to be three planes and you should find the intersection of them and then they really must twist their mind and work.

Post-interview 3

Did the class go as you planned?

Yes, I will say so.

Which task produced the most effective outcome?

I like the fourth, that is question 4 because it is really simple but they must understand this idea that they show that the point is not in the plane, so they can just plug in the x, y, and z values and see that they get a contradiction with haven't really worked with it but when I went round and looked they all struggled a bit but they realized it without asking me. And that is the best task because then they learn better than if I sit down and say you have to do this.

Which of the task did the students found it difficult?

The last one 6b which I expected because it is a difficult one to visualize.

How did the interactions between the students affect the difficulty of the tasks?

I hope so. The best and I encourage them, as much as possible help each other and not asked me, because I have already told them once on the whiteboard and they learn more both the one that tries to explain it and the one that gets it explained off and better off as student-student rather than teachers do.

I also observed that you were explaining some concepts to some students in the middle of the completion of the task, how did it affect the task's difficulty?

I think she only had a problem realizing "what do they want me to do here" then I tried to sort of visualize what is the question and once she got this visualized idea "oh you are right it was two planes in a line" then the task became much easier to solve because she knew she has to do.

Post-interview 4

How did the lesson go today?

Yes, almost better, I expected them to have problems somehow more than they did.

Which task produced the most effective outcome?

Almost everything especially 1d, which I said is difficult because it's a special case it's hard to interpret so most of them struggled with that one, but then they got for instance 9k4, which has no numbers whatsoever, it's only a_s, b_s, and they cancelled, but they had trust issues in the middle of it and also without me doing anything. So, they did well.

There were some questions from the students during the class. How did that affect your goal for the tasks, especially 1d?

They managed the arithmetic of it. So, what really was left was the interpretation, so therefore when I gave them a little logic in the right direction, they sweat and then it became a little easy. So, they didn't stop at the start, they really stopped at where they got -1=-1, "what is the interpretation of this" and I said the interpretation is that the line is in the plane. So, it's sort of every value is a right value. And the last one, 9L1 is very difficult, so it's only one student who was able to manage that, because there we are talking about the speed and you have to adjust the stretch factor to speed and that's high-level but when they got the hint of unit-factor, so not very big hint but you can use it and suddenly it appears to make the task much very easy.

Teacher 2

Pre-interview 1

What topics have you taught this semester?

We have done sequences and series and signal notations. That is how far with have been. So today we are going to start on the binomial theorem.

What topics are you currently teaching?

Yeah, today it's a binomial theorem and we use the pascal triangle. That is the topic for today also. But first, the section will start with an exercise that they want me to go through because I have some difficulties with it.

What tasks have you chosen for today's lesson?

Yes, it is a primary exercise directly to the binomial theorem. The IB is very specific sometimes; this is what we want to do, and we have to do it. So, it's very external and precise to the point.

From which resources did you get those tasks?

The tasks are normally I start with the exercises from the textbook, and when we do that afterward, not this section but another section perhaps, that is Friday or next week we will do tasks from the exam, so they will know where the level for the exam is. Today not many other sources because they have to try to do as many exercises as possible and see the connection and then in other sections, I will do from other books, I think.

What do you intend to achieve with these tasks that you have chosen?

Today it's just the binomial theorem, so that's the very concrete thing and that is to the point, and later perhaps we should see further use in other subjects

Have you made any modifications or adjustments to the tasks as they appeared in the original curriculum document?

Not the exercises in general but the things that I will be presenting may be different from the book.

Why have you decided to make no modifications to the tasks?

The tasks in the books start quite easily and then it continues to be more difficult as long as it goes on. Today I'm just going for *one point* actually, to have to try to do as many exercises as possible. Sometimes that is good to get the basics of the new curriculum, and afterward, we try to integrate it into something else.

How do you assess their mathematical performance when you are teaching?

When they do the exercises, I just walk around to see if they have managed to do them. I also tell my students to check the answers if it is correct and if they are not, they should ask me

What's the level of your students in terms of their mathematical performance?

My students are middle and upward and high. There are many good, not all are at the top, but many are good.

Did the range in their mathematical performance affect the tasks you chose?

Many of them are from different countries so they have different curriculum backgrounds. Some know this, some don't know so it's quite difficult to see the whole class.

How do you intend to introduce the tasks to them today?

I will start with a riddle, two riddles, and no one riddle on a thinking exercise.

How do you expect the students to work on the tasks?

As a group whole, because I'm thinking out loud first, I give them a few seconds so they think about it individually, then they can do something and take it out loud in the class.

How are you going to assess how the students have answered those questions?

I will just take it as they have seen it. We think together so, that's my point. Sometimes, I just write the exercise on the board, and they think, and try to solve it, but I try to explain the final solution.

What do you expect the students to do today?

Section 3 I think will be quite good, very precise, and very logical and you can also see it virtually.

Pre-interview 2

What lesson are you teaching today? Is it still on the pascal triangle?

Not that much but I will take a little bit of repetition because last time many didn't have the same purpose as normal. That was strange actually.

I can see your class is made up of students from diverse countries. How does that affect your task selection?

It doesn't affect it that much because we have a syllabus that we are going through, so we have to do that, but some of these people have done this before so they think it's a lot easier than somebody else and I think that affects the work in the class. I think there are two students I think that has done this before.

The tasks you will be using today where did you select them from?

Some are from our book, but some are from a different book, and I also use some exercises from the Norwegian exam but that is in a different course that they are still …, so today, I picked something from different books.

Have you made any adjustments or modifications to these tasks, or you are going to present them just as they appear in these books?

Some modifications but not much. I just want to see that they can see that they are almost the same but just a little bit different.

Why did you make those modifications to the tasks?

I have done that for a long time. So, it’s just I see it, oh it’s very similar but just near little bit modification and some are, it seems easy but it’s very difficult to write the solution. That is why I picked some exercises so that it’s really easy to show and argue mathematical and the other ones are a little step further, so I have to write something more but just not mathematics, but just use normal language.

Did those changes affect the cognitive demand of the tasks?

I don’t know; I have to see.

Pre-interview 3

What are you planning to teach today?

Today we are going to start the chapter with functions and introduce functions to see what a function is and how we can represent it in many ways and then just to calculate how if you have the inlet, what is the outlet. So that is the focus today.

Do you have some tasks for this lesson?

Yeah, the tasks are just, in the beginning, to see what they can recognize, and what they think of when they hear the word function. I will draw some functions on the blackboard and some non-functions, and we have to discuss which one is a function and which one is not a function. So that they can see and I will do not typical functions but I will draw some strange functions to see if we can get to the point of what is function.

Where did you get those tasks from?

My head. But it is just some pictures, that I would like to see if this is a function, yes, it’s a strange one, I haven’t seen this, but this is a function. Some are like sines, some are almost exponential, and some are like the absolute value of some things, but they haven’t seen that yet.

So after these illustrations, will you have some tasks for the students to complete themselves?

Yeah, they will just follow the books so that I get the basic in, perhaps next week or after the autumn break, they will get some tasks near the exam, along the exam’s tasks, so they see that ok I’m up to the level that I should be.

Where did you get those tasks from?

It’s a database with all the exams.

All talking about the tasks that you will give them after the lesson today. Where are the sources of these tasks?

It’s online through the IB so I just log in and I mark today, I’m going to have some exercises from, then I press functions and what chapters in functions and if there are any exams within, I can get it from my screen and I just select those tasks that I want.

Have you made or will you make any modifications to those tasks?

No, because these are the exam questions. So sometimes it is good to just train on the exercises that are given on the exams, so they can see how it is written, what you are supposed to find etc. so, exams I don’t modify, but some examples or tasks in the I may modify it a bit. But for the exams, I don’t.

Why don’t you modify the exam questions?

Because the exam is the exam that is what they are going to manage so I don't think I need to change the exam because the exam is made by some external people and they will make new ones.

Why do you modify the textbook tasks?

So that they get closer to the exam questions. So that is my target, so they are going to have an exam, the exam tasks we have to practice on and work on them. So, I don't change them, but I change some in the book or other books to get closer to the exam.

Post-interview 1

Did the lesson go as you wanted it?

Yeah, I think so, because, some of them were introduced to a new theorem as a method and many of them took it well when I walked around and observed the pupils. When I walked through afterward, it was just the structure of the mathematics on the paper that needed to be corrected. But they got the mathematical and the algebraic expressions quite good.

Which of the tasks were the students able to work on well to your satisfaction?

The odd and the even calculations. They got that one quite fast I think, and that was good.

And which of the tasks do you think they find quite difficult?

The cross sum. I think many of them found it difficult and the three consecutive numbers. That is a whole new way of thinking. One of the people asked how do we write this on paper because that is more difficult than the previous one that they had because that is, "this is even, this is odd" so you have to actually write the mathematical thinking that you had because that is not a typical way of doing it just with an algebraic expression. So, you have to write the words and that is sometimes more difficult.

What do you plan to do better next time?

On this, I do not know. I'm actually hoping to get a little bit more time to calculate some things and do more examples, and the next time, I will show them the exam, the exercises on the exam. Because it's like the book sometimes they feel like the book is a little bit lower than the exam but we have to walk first before we can run. So next week we will do some exam training in this exercise. I think that is very important so then, oh, here is the list, that's what we have to achieve.

I observed that you were discussing some things with the students whilst they were completing the tasks. How did those discussions assist the students in completing the tasks?

I think most of them that I observed were about the structure and they were asking "is this correct structure", when I have done the statement somewhere, and yet still some in the exercises that I did on the blackboard when they calculate something it's the algebra they struggle sometimes with. They don't factorize the denominator, they just multiply the fractions, then it becomes more complex and much easier to do something wrong because it is a high number and ugly expression with power 4 instead of power 1. So, that was the main thing today actually.

How did you help them to understand when you went around?

I try to take it from their side of view and see why did you do this. It could be done in a different way. And then I sometimes help them in a different way and they are like oh, yeah. And sometimes it's not illuminating for them, it's more, ok, what did you do know. So, it depends on previous knowledge.

Did those hints affect the cognitive demand of the tasks?

I don't know yet because I haven't gotten any answer from them. They have just written, and I have this evidence that seems like it's helping them. But I don't know because I have to check them at some other point.

Post-interview 2

Did the lesson go as you planned?

Yeah, I think so. Many had a good representation of what a function is. They managed to solve it, but some, trouble occurred as I thought it could be and that's when they calculated the algebraic expression. So when you get, if -x in the power of 2 and you insert a negative number, that is always it is a positive or a negative number, so that is just arithmetic, and that is always the problem. But I think many of them are quite good. They had today a better pace at the tasks.

I observed you explain with illustrations what a function is. Did those explanations help the students to solve the tasks you gave them afterward?

I hope so. Sometimes many just need a formal explanation up going to know what they are actually going to find and when I walked around the classroom I could observe to see have gotten the point, and if I see they don't have, they sit down and discuss them, so that is a, they can see ok this is the point, this is what we have to do and why.

I observed during the task completion there were a lot of interactions between you and the students. How did those interactions affect the cognitive demand of the tasks?

It depends on the question. If they don't know the method or they don't know the symbolism or something, I just see what issues they have with the tasks and then I just take it from their point of view. Sometimes I just try to see if they know what to find, you know how to do it, you know what this means because if I just come in and say oh this is, you have to do this and this, then they forget about the rest.

I again observed interactions that existed between the students themselves. Did that affect the difficulty of the tasks?

I think that is good. Because they talk on the same level and have sometimes, I would say the easier way to save the problem is when they say it in their own words that their levels are. Sometimes, I think it helps that they communicate with each other and sometimes when I am occupied with other people, you can't wait five minutes to get help, so they ask each other and they can move on. So that is a good thing.

Teacher 3

Pre-interview 1

I want to know which mathematics topic have you taught this semester.

This semester we started with statistics but in the beginning, we had all kinds of rounding and some geometry and now we are going to do statistics for quite a long period.

Which mathematical topic are you teaching now?

Statistics, we are on how to show how they represent or describe a different type of data. So, it is a box and whisker plot, histograms, and today it's a cumulative frequency graph.

Do you have some tasks for this lesson?

Yes. It's from 3G and you the book and it's six different exercises there but I have chosen 4 & 5 because we don't have time for doing it and I hope to do 4 & 5 in class but I'm not sure if a, depending on how long it takes to go through the graph I'm going to show them. They are not familiar with this, and they are not that strong. Actually, I should have done that the last time but I couldn't, I had to wait.

Are all these tasks from this book? Do you have other any other source for your tasks?

Yes, they are all from the textbook. With other sources, not today. I used to sometimes use something called revision village, it's an online thing that I subscribe to. If I need something in addition, but there are a lot of exercises here and they are, for this curriculum good questions within the text, it's a lot of exercises and also at the end of the chapter is there is a chapter review, and after that exams *start* questions. So, most of those we use when we are practicing full tests.

What are your goals for using these tasks?

What I have shown them on the board. They are to see that they are able to do what I was hoping that they could do.

Have you made any changes to the tasks?

No

Why did you choose not to make any changes to the tasks?

Because they are, I think the exercises in the book are very good

How are you going to introduce the tasks?

After I have shown them on the board like we've been talking, I'm going to do an example, then list these exercises on the board and then I go around and help them and I also want them to talk together, to help each other to understand what we are talking about.

What is your perception of the mathematical knowledge and experience of your students?

They are low-achieving students. This is the easiest course in IB so those who don't need maths later are those who choose this.

Did that affect the tasks that you chose for them?

Oh yes. And it also affects what is in the book. The book is made for them and for that course. And the course is not for high-achieving students.

How do expect the students to work on the tasks?

Together, talk.

What do you expect from them?

I expect them to ask me when they don't understand what to do and I hope that at the end they grasp what I want them to, so they understand how useful this type of curve is and all the things that they can, all the information they can get from a cumulative frequency graph.

Pre-interview 2

So which class are you teaching today?

It's still 2IBa. We will be using the class always

What topic are you teaching today?

Yeah, we are continuing in what we did last time, the statistics chapter, and today we are going to talk about bivariate data and correlation. Not calculating the correlation, but just plotting to see the feeling. Later we will do the calculations.

Do you have some tasks or exercises for this topic you will be teaching today?

Yes, I have an example that I will show on the board first and then I have some exercises that they will continue to do then after that we will do more exercises for the whole chapter because this is the last part of this chapter.

Where did you get those examples and the exercises from?

From the book, from the textbook. No other sources, no, not for this, I use a revision village when I need something in additional but it's more than enough so far.

What influenced your choice of these tasks?

I just think about what they should learn, and then there are a lot of exercises which are almost the same, I try to take exercises so that they can try on everything we have talked about without using too much time because they have other subjects that they need to acquire also.

Have you made any modifications, or do you plan to make any modifications to the tasks?

No, not for these exercises. It is a lot of very good exercise.

Why have you chosen not to make any modifications?

Because I believe they are okay. They have a very good textbook and instead of making a lot of work for myself, I use them because I of course gone through them and seen that these are good exercises. Earlier when I was quite a fresh mathematics teacher, I make the examples myself, but I stopped doing that because I found out that was a lot of extra work, and I wasn't any better, it was actually much better to say that this example is on this page in the book because they are making if there were things that they didn't write down making finding it in the book also. But this book is made very quickly because it was just a couple of years there was a new curriculum in IB, so there are a lot of mistakes, especially in the answers but not in the book. They update it all the time, so it's getting better, but I will also talk about that with the students that, they should try to rely on themselves and not just the answers in the book, so they don't have it

What do you want to achieve with these tasks?

I want them to learn more statistics. I don't think I have a better answer to that.

Pre-interview 3

What do you intend to teach this morning?

We are going to start with chapter 6. We were earlier in chapter 3 and we skipped 4 & 5 because we want to continue with statistics. It is more of a correlation today. We can do, to calculate the correlation on the GDC which this calculator. So that is the main thing today.

Do you have some tasks that you are going to use for the lesson today?

Yes, I found some tasks in the book, especially in the earlier kind of learning on this topic. Also, there is a lot in the book, and I will also copy the example that I am going to go through on the board because the book has also digital resources on how to do it on the calculator. Because there are a lot of new things that fall in there, I will go through it with them but they will give this paper also so they can go back and use it to practice later.

Apart from the tasks from the textbook, do you have other supplementary tasks from another source?

No, no. I'm not able to use all the tasks in the book especially now at this early stage. When we are revising later, then we will do more especially from the revision list, but still, we haven't done any exercises in chapter 6 yet and the exercises in the book are good.

What did you consider before choosing those tasks for the students?

That is, so they can understand and develop more understanding of what we are doing. When we go through something on the board, they are able to do it by themselves, not too difficult in the beginning, but then a little more advanced afterward. The main reason is to, make them do them themselves after I have gone through that lesson.

Have you made any changes to the tasks as they appear in the textbook?

No.

Why did you choose not to modify the tasks?

Because they are good and there are a lot of them. So we have more than enough to then than to try to find new ways when we think they are good. If they weren't that good, I would of course make a lot of changes and search for more exercises from other places. But I think it's a good choice, the book we have.

Do you believe the tasks match the mathematical competence of your students?

I just see if they can manage them or what kind of questions they have and I go around, when I pick the exercises, I try to find the exercises that they do several things, not just one thing that connects to the topic and if they are able to do it without my help, then I am very pleased, but normally they ask, but sometimes they don't understand the thing I will do almost everything but just slightly something that they want to ask about, so it's a wide range of competence in the class.

Pre-interview 4

What topic are you teaching today?

I'm continuing with statistics and today it is linear regression finding the line that fits the scatter diagram

Do you have some tasks that you will be using today?

Yes, as normal I try to do something to show them on the board, an example, and talk about them, the main things and then we do the tasks that cover what we have been talking about.

Where is/are the sources of those tasks?

From the textbook

What influenced your restriction of taking the tasks only from the textbook and no other source?

Because there are a lot of exercises in the textbook, and they are good. This is the first time we have been talking about linear regression and we have used the exercises so maybe later for revision, I will go to, for example, the revision village or somewhere else. Mostly revision village and in the textbook, they have a video exercise for every chapter and in addition exam questions for that. But for the test, we will soon have a test now also, and then I will pick exam questions, all the exam questions because that's what we need to practice and prepare for the final exams.

Has there been any modifications, or do you intend to modify the tasks?

No.

Why have you chosen not to modify the tasks?

Because they are good. Of course, I have gone through them, and I see that they are suitable, and hopefully, those who have made the textbook also have considered that while making it, so I think the book is a good choice for the curriculum. So, we are actually very lucky that we have such a lot of good exercises to work with.

Did you consider the level of mathematical thinking of your students before choosing those tasks?

Oh yes of course. I wanted to give them tasks that they are able to solve, not too difficult but then I can challenge them to get more understanding of the topic. That is what I am always aiming for.

Do you believe these tasks you have chosen are good enough to build on their mathematical knowledge?

Yes, especially in the beginning when it is a new thing for them, then it doesn't have to be too sophisticated. They will feel that it's too difficult and they get lost. So, it is important for me to, so they can keep the spirit, so they want to continue even though they don't get it in the beginning, that they continue and ask and talk with the neighbour. I'm hoping that because I don't think they are too good to do that alone. They should have discussed more together so trying to, so they can do more and more together.

Post-interview 1

Did the lesson go as you planned?

Kind of. They were all struggling, I know. So, they took much more time than did the previous years. We need to take the time and this is quite complex, so they are not that logically good. So, then, I think it was ok, but I would have of course wanted to grasp it quicker but that was not this group. It is the weakest group I have had for a lot of years, but they are very nice, they try, so that's why I'm sure they will make it.

Which task produced the most effective outcome?

I think task number 4. And that was why I also want them to do homework on something that looks approximately the same hopefully to, because what I know about exam questions then they can get both exercise 4 and exercise 5, but if they don't know how to do it as we did it 4, then they have trouble.

I observed that you solved almost all the questions for them. Was it because they found it difficult to do it?

Yeah. Because I feel that sometimes they are not able to get started the way I want them. I try to start and do something, and they can take a look if they want to, those who are better they don't have to look, they can work by themselves, but especially the weak ones, they need more help. Even though they are not that many, it takes time to talk to everyone so if some of them can get help from others, I think that is beneficial.

I again observed that you engaged in conversations with some of the students when they were completing the tasks. Do you believe that interaction changed the difficulty of the tasks?

Yes, I hope, I really hope so. I think it did. I hope it increased their understanding and reduced the difficulty of the tasks. Because sometimes they are new to the IB, so it is a special way of asking questions, so they are not that used to it and that's why they need help to find out what this is really about.

What do you plan to do differently next time you meet them?

I don't think I will do anything different. I need to look at the plan. I think it will be just more exercise next time because they really need it.

Post-interview 2

Can you give me an overview of how the class went?

I think it was quite good today. Because we had time to do so many exercises. Not that many but to this group it's quite a lot. So we didn't have to do too many theories and it was easy and then they did some exercises then first, for what we went through and then we did some more that is a repetition from one week ago and that was very very good because I thought they will struggle so hard. Even though they were tired now and all that, I am in good mood now, much better than in the beginning.

I observed you added other exercises during the class. Where did you get those tasks from?

Yeah, from the book. I thought it was maybe too little. In the beginning, I did want to get them too much because then they will be too much, but I saw that they were working quite well, then I can just add a couple more.

Which of the task did they worked on it to your utmost satisfaction?

I think it was the exams type question and maybe that was the first one. And number 9 and the next one too because they had to, in the beginning I didn't remember what it was and we just talked about most of it and then I didn't have to tell them, they remembered. So, for example there was one guy who was wondering about what IQR inter-quartile range what is it, and I just told them and said something in the middle, "oh yes yes, now I remember and I will always remember", so more of that.

I observed you were explaining somethings to them in Norwegian. Can you tell some of those things?

Yes, that was also about the when it's a class, what do I say, we talked about the modal class because have talked about modal earlier but not the modal class, and also how they calculate the mean when it is in classes, but they were actually 8 values between 0 and 10 for example and then I said they could be, all of them equals zero, we don't know, we guess that they are spread evenly out, so we use the middle number. I didn't talk so much about classes earlier because I thought it was always standing and talking and talking, so now we could do that, and then the class starts too, hopefully, most of them.

How did these explanations help the students to complete the tasks?

Oh yes, I'm sure because then they understood more about what they these are actually about. I think when I talked to them directly, they remembered much more than when I am on the board because then someone always forces when we talk directly then they, and that's also good because it was the greatest that they did, and it is easier to learn and make them understand. These are not mathematicians, so they need the help they can get, and I tried to give them that so they can understand more about what we doing.

Which of the tasks did they find difficult to answer?

There is always a,b,c,d,.. and the last ones are always difficult to answer. I am quite happy that they managed to do the first part of it because the IB system is also, to get the best grade you don't have to have everything correct because you have a little time, so you have to practice skipping things you don't know and be sure that you have done all things you know and then it's the easiest thing and then they can get quite even though they are not quite good in maths they can do quite well and for these students, they are not going to use maths later not for studying but they will use it in their daily life of course so we don't prove some things but do more practical things. And when they are able to do not too difficult stuff, then they manage quite well.

Post-interview 3

Did the class go as you planned?

Yes, or as I hoped. They are two different things. I thought it was also quite good today because we had the time to do exercises and that is what I feel that at some times we struggle with the time. I think this time was quite ok.

Did the goal for selecting the tasks met?

Yes, I did a, maybe it just one, one more, that was because I saw that they were doing quite well and also it is important that they have a good practice with the calculator because it is so important for the exam and the exams are important for the students.

I observed that the calculator was used most often in doing the tasks.

Yeah, also because they are not, it's not in the curriculum to do the calculation by hand for the correlation coefficient and also this exercises could show them the difference between strong and not strong correlation. They could get some kind of feeling of what they did, even though they are not doing it by hand, they know exactly what kind of number is it, but then they can see the connection between the scatter diagrams and the value, direction is easy but sometimes, especially with the outliers, if it is one far away and it disturbs the correlation a lot and if you take it away and it looks quite nice.

Did any of the tasks changed during the enactment?

No, it wasn't. it was more than enough to, it's a lot with the GDC to remember all these buttons and all that. I was also quite pleased with the, I felt that because we had the time, they were able to do it more than ones, so it was quite okay.

There were a lot of comments from you whiles the students were completing the tasks. What are some of those comments that changed the difficulty of the tasks?

I don't know. Actually, it's not difficult things but it's difficult for them. Maybe this last one to find out how to check for outliers when it was bivariate data and not just one variable. I haven't taught that much that, then I saw that could cause a trouble that's why I tried to explain it for them so they could a, because some had done it and it didn't much the answers back in the book and with all those mistakes that normally they seem not the same and you have to do it twice and then you probably trust yourself but the problem was that they haven't done it properly on the GDC because it was two variables and I could have shown them how to do it with two variables on the GDC, but I thought that was too much. So, I was afraid showing even more but maybe that wasn't a good thing to do, maybe I show that much that was just a, I chose to do it that way and hope it will work.

Post-interview 4

How were you impressed with the class today?

I'm always pleased with the class but maybe I am not that pleased with myself because I wasn't that aware that people so important for them in the coming test, especially with the last example, but again that part isn't that important. You must go through because it might come but this is much more important with what we are doing linear regression and all that. And I feel that they are, well, they are not there yet but they are on their way, so I think it was not that bad for the test, so when I could completely see what they had, I have understood but some of the questions sounds like they don't understand but they understand more than, because I have experienced that now, so I think the exercises are ok. I don't think I will change anything but will work more with them. It's not that much new on Friday so we have this Tuesday just to do exercises and of course pick much more of the kind of the same types that are on how to interpret correlation coefficient, what the equation exactly is, they felt that troubled because they are not that strong in maths in solely understanding what a straight line might be. But then again with most of the regression lines exercises are in contest and that may help more than I was aware of.

Are they allowed to use the GDC in their final exams?

Yes, that's why I am working so much with it because it's two papers and this class has the GDC in both papers. Other classes have that only in the paper 2. That's why it's so important whether you like it or not that is the way it is.

Does the use of the GDC makes the tasks easy for them?

I'm not sure, but it is very important that they are really familiar with the GDC for the exams. It's always a discussion, when I learn maths I didn't have it all because of my love. And I feel that that is the best way, but then again, the reality is that they can use it on the exam and it's limited time so that's why I'm really stressing about how to use it.

But do you believe they would be able to the tasks without the GDC?

They are not able to do linear regression for example, not at all. Actually, they struggle a bit when they had the code to translate it to an equation, but when I tell them then they are like oh yes it might be that, but they haven't gotten their answer truly like that. Even though they actually should be able to do it, they think about that is the way they should do it,

So, the GDC reduced the difficulty of the tasks?

Yes, so they can do a lot more of the exercises and hopefully that is also for in contest understand more.

Appendix C: Pre-classroom Interview Questions

1. Which level or class are you currently teaching?

2. Which mathematics topics have you taught this semester?
3. Which mathematical topic are you currently teaching? What have you planned to teach today?
4. What mathematical task(s) have you chosen to use?
5. Where is the source of the task?
6. What do you intend to achieve with the task(s)?
7. What modifications have you made to the task(s), if any?
8. Why have you chosen (or not) to modify this task?
9. What is your perception of the mathematical knowledge and experience of your students?
10. How will you introduce the task(s)?
11. How will students work on the task(s)?
12. What products do you expect the students to produce?

Appendix D: Post-classroom interview questions

1. Did the lesson go as planned?
2. Which task produced the most successful outcome?
3. Which task produced the least successful outcome?
4. What would you do differently next time?
5. What would you do the same?
6. During the enactment you made some comments: a) what was the intent of the comment(s) b) How did those comments change the nature of the task?

Appendix E: Classroom Observation Guide

TAKE NOTES ON:

Date and time

Grade level,

Number of students in the classroom

Topic

How did the teacher start the lesson?

How did the lesson progress, with specific attention to,

- Order of events (by minutes as they happened)
- Explanations made,
- Question and answers (if any),
- Tasks used in the lesson, and the order in which they are used,
- How did the students work (e.g., listening to the teacher and taking notes, individual seat work, group work, class discussion, pair work, etc.),

- What did students produce,
- Any homework is given,
- Any quiz or other formative assessments.

Appendix F: consent form

Do you want to participate in the research project *"teachers' selection of mathematical tasks for instruction"*?

Purpose

The aim of the research is to explore how teachers select mathematical tasks from various resources and interact with these tasks to make them good tasks that meet his or her goal and builds on the mathematical thinking of his or her students. I want to find answers to the following questions:

- How do teachers select tasks, what is the nature of the task and what influence the teacher's choice of the task?
- What modifications teachers make on tasks, why they modify the task, and how these modifications affect the tasks?
- What interactions occur in the classroom that help maintain or change the characteristic of the task during enactment?

In this master's study, your personal information (e.g. name or location) will not be linked to the data collection, and the data will not be used for purposes other than this study.

Who is responsible for the research project?

The Department of Mathematical Sciences at the University of Agder is responsible for the project.

Why are you asked to participate?

The selection for this study will be mathematics teachers who teach in high school. You have been chosen for this program as you teach mathematics in high school. What does it mean for you to participate?

If you choose to participate in this research, it will mean that you will be interviewed. The interview will be divided into three:

1. You will be engaged in a short interview before the lesson about your goals and expectations for the lesson. It will take about 5-8 minutes.
2. You will be interviewed after the lesson about how the tasks you chose influenced your goals and expectations.
3. I will keep a copy of the tasks you use for instruction during the observation period,
4. I will be observing the mathematics classes of the selected classrooms for two weeks,
5. You will be asked to fill a short survey. It will take about 10 minutes.

I take audio recordings and notes from the interview, which will later be transcribed.

To strengthen the validity of this research, I will conduct a classroom observation whilst the lesson is ongoing. I will observe and note down what is observed.

It is voluntary to participate

It is voluntary to participate in the project. If you choose not to participate, you can withdraw your consent at any time without giving any reason. All your personal data will then be deleted. It will have no negative consequences for you if you do not want to participate or later choose to withdraw.

Your privacy – how we store and use your information

We will only use your information for the purposes we have disclosed in this writing. We treat the data confidentially and in accordance with the data protection regulations. I will only have access to the data collection. But I want to replace names and contact information with codes. List of names, contact information and the respective codes will be stored separated from the rest of the data. I want to store my data on a secure server. Participants will not be recognized in publication, and in the publication, names or personal information of the participants will not be referenced.

What happens to your data when we finish the research project?

The information is anonymized when the project is completed/thesis is approved, which is scheduled for June 2023. The data will be retained for one year after the research is completed, after which all data including electronic recording will be permanently deleted.

What gives us the right to process personal data about you?
We process information about you based on your consent.

On behalf of the Department of Mathematical Sciences, University of Agder, NSD – Norwegian Centre for Research Data AS has assessed that the processing of personal data in this project is in accordance with the data protection regulations.

Your rights

As long as you can be identified in the data material, you have the right to:

- access to what information we process about you and to obtain a copy of the information
- to correct information about you that is incorrect or misleading
- to have personal data about you deleted
- to lodge a complaint with the Norwegian Data Protection Authority about the processing of your personal data

If you have any questions about the study, or would like to know more about or take advantage of your rights, please contact:

- Supervisor Professor Cengiz Alacaci, Department of Mathematical Sciences, University of Agder (cengiz.alacaci@uia.no)
- Project Owner: Alex Arhin
- Our Data Protection Officer: Ina Danielsen, personvernombud@uia.no

If you have any questions related to NSD's assessment of the project, please contact:

- NSD – Norwegian Centre for Research Data AS by email (personverntjenester@nsd.no) or by phone: +47 53 21 15 00.

Sincerely,

Alex Arhin
(Researcher/supervisor)

Declaration of consent

I have received and understood information about the project *"what is a good mathematics task"*, and have been given the opportunity to ask questions. I agree to:

- ☐ to participate in an individual interview
- ☐ to participate in being observed in a classroom situation
- ☐ to participate in the survey

I agree that my information will be processed until the project is completed

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(Signed by project participant, date)