Master

A Wavelet Analysis of the Bitcoin-Hashrate Nexus Accounting for the Effects of Energy Commodities

HERMAN SCHIRMER FARSTAD

AXEL WILLIAM PATRICKSSON

SUPERVISOR

Jochen Jungeilges

University of Agder, 2023

Faculty of Business and Law

Department of Economics and Finance

Preface

This thesis serves as the culmination of five years of study and represents the product of five months of diligent research. Our appreciation and gratitude go out to all those who have supported us through the challenging academic landscape of pursuing an MSc in Business and Administration. We would like to express our sincere gratitude to Professor Jochen Jungeilges, our supervisor and sparring partner, for his invaluable guidance and support throughout the project. His topic and methodology proposal, as well as his extensive knowledge, proved crucial to the success of this thesis.

We recognize and express our appreciation to our families, friends, colleagues, and fellow students, whose unwavering support and love have been crucial to our success. Though their contributions may not have been explicitly stated, they were fundamental to our progress, and we are immensely grateful for their understanding and assistance.

We take great pride in our research and the significant effort we have invested in it. This academic journey has been a formidable challenge. Not only has it been a test of our intellectual abilities but also a source of personal growth and development. As we conclude this chapter in our academic lives, we do so with a sense of accomplishment and fulfillment.

Abstract

This study investigates the relationship between the growth rates of Bitcoin and Bitcoin hashrate while controlling for the effect of energy commodities, specifically two-month futures on Brent crude oil, coal, and natural gas. Based on daily data from January 2013 until December 2022, we utilize the wavelet methodology to analyze dynamics both in time and frequency. Building on the previous work of Rehman and Kang (2021), this study extends the sample period and improves the replicability of their findings. Controlling for the effect of energy commodities, our analysis reveals several interesting results, highlighting the temporal and dynamic nature of these relationships. Our most significant observation that was discovered in both bi- and multivariate forms of the wavelet methodology is the low-frequency in-phase coherence between bitcoin's returns and hashrate growth rates, which persists from the beginning of 2020 until the end of our sample period in 2023, with hashrate growth rates leading bitcoin returns. These findings suggest that the link between the returns on bitcoin and hashrate growth rates while considering the impact of the energy commodities is complex and context-dependent, and further research is needed to fully understand the underlying mechanisms driving these relationships. Our study contributes to the existing literature on the Bitcoin-hashrate nexus by providing a more comprehensive analysis that accounts for the dynamic nature of these relationships, and by improving the replicability of previous research.

Keywords: Bitcoin price, Bitcoin hashrate, energy commodities, wavelet analysis, wavelet coherence, time-frequency comovement

Table of content

List of figures and tables

1. Introduction

Bitcoin is a digital currency that operates outside the realm of traditional financial institutions. It was created by an unknown person or group of people using the pseudonym Satoshi Nakamoto (Adams & Curry, 2023). It operates on a decentralized blockchain, where transactions are securely and transparently recorded. The question is, is it a revolutionary innovation that will change the way we think about money, or is it a dangerous and volatile currency that threatens to upend the global financial system? The truth is, no one knows for sure. Despite its growing popularity, Bitcoin remains shrouded in mystery. It has been associated with everything from underground marketplaces to international espionage (Eckel, 2019). It has been lauded as a tool for financial freedom and criticized as a tool for money laundering and other illegal activities (Eckel, 2019). Yet despite the controversies and uncertainties, Bitcoin continues to gain acceptance and adoption among businesses and individuals around the world. With its decentralized nature and potential for anonymity, it offers a level of financial freedom that was previously unimaginable. Conducting research on Bitcoin is therefore crucial for promoting responsible and informed use of this innovative financial technology, particularly given the lack of regulatory oversight and the potential for illicit activity associated with cryptocurrencies.

Bitcoin mining is the process by which new bitcoins are created and transactions are verified on the blockchain. This process involves solving complex mathematical equations using powerful computers, which are rewarded with newly created bitcoins for each block of verified transactions (Houy, 2014). This process not only verifies transactions, but also creates new bitcoins and maintains the security of the network by preventing fraud and double-spending and is therefore important for the ecosystem of cryptocurrencies. The majority of Bitcoin mining takes place in regions with cheap electricity, such as China, where coal-fired power plants are used to generate electricity at a low cost. This has led to concerns about the environmental impact, which eventually resulted in China banning Bitcoin mining (Browne, 2022).

Bitcoin hashrate is a measure of the computational power dedicated to the Bitcoin network, which is used to maintain the security of the network and process transactions (Schinckus et al., 2022). It refers to the speed at which a miner or group of miners can perform the mathematical calculations required to add a new block of transactions to the Bitcoin blockchain. Bitcoin hashrate is measured in hashes per second (H/s), with one hash representing one mathematical calculation (Wade, 2023). The higher the hashrate, the more secure and stable the network is, as it becomes increasingly difficult for an attacker to manipulate the blockchain and control the network. Consequently, the Bitcoin hashrate serves as a measure of the total computational power engaged in the mining of Bitcoin and thus, the intensity of Bitcoin mining.

When analyzing the time-frequency comovement between relative changes of Bitcoin price and hashrate, it is interesting to observe the impact of energy commodities. This consideration can help to distinguish the influence of energy prices from other fundamental factors that drive the supply and demand of Bitcoin. As Bitcoin mining requires a significant amount of energy, changes in energy prices can have a notable impact on the profitability of mining and, as a result, the overall hashrate of the network. Consequently, fluctuations in energy prices may also affect the price of Bitcoin by influencing the cost of producing new units of the currency. However, the drivers of Bitcoin price are diverse, and not only driven by the price of energy. Therefore, it is necessary to control for the effect of energy commodities to isolate the impact of these fundamental factors on the variously driven time-frequency comovement between Bitcoin price and hashrate. This paper aims to investigate the time-frequency comovement between Bitcoin price and hashrate, while controlling for the effect of two-month futures on coal, Brent crude oil, and natural gas. These energy commodities are chosen because of the accessibility of relevant price data and their dominant share as energy sources in the mining process, as noted by de Vries et al. (2022). While oil is not extensively utilized as an energy source for electricity, it remains one of the primary sources of energy globally, rendering it relevant for this analysis. Furthermore, examining the potential disparities between the effects of different energy commodities on the relationship between Bitcoin price and hashrate is of interest.

Our thesis is motivated by Rehman and Kang's (2021) original paper on the relationship between Bitcoin hashrate and Bitcoin prices in the presence of energy commodity futures. Their research builds on three main hypotheses. The first hypothesis centers on the relationship between Bitcoin hashrate and Bitcoin price. Garcia et al. (2014) argue that the fundamental value of bitcoin should be no less than the cost of producing one bitcoin, which can be linked to the hashrate. Given this theoretical foundation, Rehman and Kang posit that a relationship between Bitcoin price and hashrate appears sensible. The second hypothesis relates to the mining process and, correspondingly, hashrate, and energy commodity prices, which serve as input factors in mining operations. As the mining of Bitcoin is an energy-intensive operation, its profitability is likely to be sensitive to the prices of the energy sources used. Consequently, energy prices may affect Bitcoin prices indirectly through its effect on mining profitability. Lastly, Rehman and Kang hypothesize around Bitcoin´s position as a financial asset, and its potential uses in portfolio management. In combination with the energy commodity market as an investment option, they claim that Bitcoin can serve as a diversifier and hedge in a portfolio.

Other researchers have previously studied factors affecting the price of Bitcoin and hashrate, and how they are related to one another. According to Fantazzini and Kolodin (2020), the causality is unidirectional from Bitcoin price to hashrate, with lags ranging from 1 to 6 weeks. Additional research also suggests that an increase in hashrate indicates that miners are confident in the development of the cryptocurrency, therefore increasing its price. To deepen the understanding of the dynamics of Bitcoin, researchers such as Lin and An (2021), Bhambhwani et al. (2019), Kristoufek (2015), and Moussa et al. (2021) have also been looking into the factors effecting the price. To reach a better understanding of the complex price process, which most likely depends on a broad range of factors, further research is needed.

We believe that the hypotheses and topics covered by Rehman and Kang (2021) are still highly relevant today, and as a result, we chose to focus our thesis on replicating and extending their research. As the sample used in the original paper only covers data from the beginning of 2013 until October 2018, we believe that there could be interesting findings when extending their research until the end of 2022. This extension will then cover periods of greater financial uncertainty and instability caused by the COVID-19 pandemic and Russia´s invasion of Ukraine. We have adapted the wavelet analysis methodology used by Rehman and Kang, as this technology provides the opportunity to analyze non-linear time-frequency relations between two processes in time. Beyond that, the approach can be extended to a multivariate setting in a straightforward way. Within the wavelet analysis toolbox, the wavelet coherence method is highly effective in measuring comovement between processes in time for a range of frequencies. For multivariate analysis with wavelet technology, Mihanović et al. (2009) introduced extensions to the wavelet coherence with partial wavelet coherence and multiple wavelet coherence. This is implemented in our analysis to assess the results of factoring out, or in, the effects of the energy commodities on the coherence between Bitcoin price and Bitcoin hashrate.

In our effort to replicate Rehman and Kang´s original research, we aimed to compile their dataset and extend it to our desired sample period. As a result, the goal was to obtain daily data on Bitcoin prices, Bitcoin hashrate, and two-month futures on Brent crude oil, natural gas, and coal from January 1st, 2013, until December 31st, 2022. However, the lack of disclosure on data collection and the actual data sources used by Rehman and Kang makes it challenging to copy their exact trading products. Consequently, we constructed what we believe to be a correct sample of the daily observations for the variables included in the research over the past 10 years.

Our study, which builds upon the work of Rehman and Kang, uncovers certain similarities as well as minor divergences. Nonetheless, the most intriguing outcomes emerge subsequent to the time frame studied by Rehman and Kang. Our analysis demonstrates the existence of high wavelet coherence between the returns on bitcoin and hashrate growth rates, particularly in the 128–256-day frequency range from 2020 until 2023, with hashrate growth rates leading bitcoin returns. However, when the influence of returns on two-month coal and gas futures is filtered out, the correlations between the returns on bitcoin and hashrate growth rates tend to weaken. Conversely, when the effect of returns on oil futures is removed, the correlations remain relatively stable. The inclusion of returns on energy commodities renders the correlations more frequent across all times and frequencies. Moreover, the incorporation of returns on the oil futures results in a sustained correlation between bitcoin returns and hashrate growth rates throughout the entire period under observation (2013–2023) at a 256–512-day frequency.

From a financial perspective, we discuss possible implications from the findings of our research. Both diversification opportunities for portfolio management and potential investment strategies are argued for. Our hypothesis builds on the similarities between outputs in the wavelet analysis and traditional financial parameters such as correlation and cross-correlation. Since wavelet coherence is analogous to correlation, situated in a time and frequency space, and the phase relation between the analyzed variables reveals a lead/lag relation similar to a cross-correlation, we believe in its potential usefulness for sophisticated investors. Results of the wavelet analysis show during which periods and at what frequency there has been a correlation between variables. This information may enable increased precision for building diversified portfolios when low or negative correlation actually occurs. Further, a lead or lag relationship between two variables can be used to invest in the lagging variable based on the information gained from the leading variable. A strategy like this would still require some proof of causality to have any substantiation.

Our thesis continues by presenting the current literature prevailing on the topic of the relationship between the price of Bitcoin and factors affecting it, such as the hashrate and energy commodities. The literature review is presented in Chapter 2. Chapter 3 covers the data collection process and the extent of our data, a description of the historical events in our data, an explanation of our data modifications, and a part covering the descriptive statistics. The research methodology is presented in Chapter 4. After covering the basics of the wavelet approach, we give a detailed account of the various types of wavelet analysis utilized in our research. In Chapter 5, we present the results of our study, consisting mainly of various wavelet coherence plots revealing patterns of comovement between the variables in our research. A discussion of our findings is provided in Chapter 6. Both attempts at explaining the relationships we observe from the plots as well as possible implications from a financial perspective are covered there. Chapter 7 contains the conclusion of the paper, accompanied by suggestions towards future research.

2. Literature review

As the concept of Bitcoin and cryptocurrencies is still a relatively new topic in the financial sphere, with history only dating back to the late 2000s, research remains limited to a certain degree. Especially the literature on the connection between energy commodities and the Bitcoin-hashrate nexus appears to be rather sparse. However, several studies have covered the relationship between Bitcoin price and hashrate, and Bitcoin price and various commodities separately. Overall, many of the studies point towards similar findings. We will present some of the contributions we find relevant to the literature on this topic in the following section, beginning with papers covering the Bitcoin and hashrate relationship before looking at papers researching the Bitcoin relationship with different commodities. Lastly, we present the paper by Rehman and Kang that covers the combined relationships also included in our study.

Kristoufek (2015) made an early effort to examine potential drivers for the price of Bitcoin. The findings of his research revealed that standard fundamental factors, such as usage in trade, money supply, and price level across exchange rates are the main drivers of Bitcoin's price over the long term. The incentive for more users to become Bitcoin miners due to rising prices is offset by specialized mining hardware that has resulted in higher hash rates and difficulty levels over time. Through the use of continuous wavelet analysis, the author is able to comment on the interconnection between Bitcoin prices and the potential drivers both over time and on differences prevailing between short-term and long-term connections. The author leaves no open problems for future research.

The research paper by Bhambhwani et al. (2019) investigated whether hashrate and network related to blockchain affect cryptocurrency prices. The researchers discover a significant long-run relationship between these characteristics and the prices of Bitcoin as well as other cryptocurrencies. The methodologies used to conduct the research are Dynamic Ordinary Least Square (DOLS) regression, factor analysis, and out-of-sample tests for robustness purposes. Suggestions for future research imply analyzing regulation and political risk which might have an effect on cryptocurrency returns.

Fantazzini and Kolodin (2020) investigated the relationship between Bitcoin price and hashrate by disentangling the effects of the energy efficiency of Bitcoin mining equipment, Bitcoin halving, and of structural breaks in the price dynamics. The findings suggest that a unidirectional causality is going from Bitcoin price to hashrate with lags ranging from 1 to 6 weeks. To model the dynamics of the Bitcoin network's energy efficiency, the researchers proposed a new methodology based on exponential smoothing. According to the authors, incorporating both modern empirical finance asset-pricing models and theory-driven valuation models would be an interesting avenue for future research.

Kubal and Kristoufek (2022) conducted research on the Bitcoin-hashrate nexus within the endogenous system. The findings reveal that the whole system is well structured and delivers economically and logically sound results when it comes to the network security narrative. The authors argue that increases in the security of the network, represented by the hashrate, are to be reflected in the increase of Bitcoin prices. A system of equations was created with the goal of including explanatory variables capable of capturing the dynamics of the investigated relationship. Because of the assumed and later confirmed endogeneity, they used the two-stage least squares estimation with all of its complexities. The researchers suggest that alternative sources of retail investors' interests in the market should be explored in more detail, as well as transaction costs of trading cryptocurrencies.

In the paper by Yuan et al. (2022), an effort is made to analyze the information spillover between factors in the Bitcoin market. A quantile Vector Auto Regression (VAR) connectedness network framework is applied to assess the static and dynamic spillover effects between Bitcoin price, hashrate, mining difficulty, electricity demand, and energy consumption. With data spanning from February 10th, 2017, to March 29th, 2022, the authors are able to show that both electricity demand and hashrate act as primary sources of risk in the Bitcoin network. For further understanding of the Bitcoin network and its comovements and causal connections, the paper suggests applying methods based on quantile VAR and multiple wavelet coherence in future research.

The study by Bouri et al. (2017) researched the relationship between Bitcoin and energy commodities by assessing the ability of Bitcoin to act as a hedge, or diversifier, against daily movements in the commodities. The results indicate that Bitcoin acted as a hedge against both synthetic commodity indices in the pre-crash period before December 2013, whereas the effect was absent post-crash. The researchers not only unveiled the time-dependent nature of Bitcoin's role, but they also drew attention to the dissimilarity in the dynamic correlations exhibited by the extreme downward and extreme upward price movements. The empirical investigation was carried out in two phases. The initial phase entailed the estimation of the dynamic conditional correlations (DCCs) between Bitcoin and each of the three commodity indices, employing a pairwise approach. The subsequent stage involved evaluating the hedging and safe-haven characteristics of Bitcoin in relation to these commodities. This was achieved by regressing the previously mentioned pairwise DCCs on dummy variables that represented the extreme downward movements in the return distribution. The authors suggest that future researchers should take Bitcoin liquidity into consideration.

Lin and An (2021) conduct research to test the asymmetric relationship between Bitcoin price and futures prices on commodities both in the short- and long-run. A nonlinear autoregressive distributed lag, or NARDL, approach on weekly data for Bitcoin, silver, gold, natural gas, and Brent oil from January 2014 to December 2020 is used for the empirical work in order to assess the research question. Lin and An conclude that for all four commodities futures, there exists an asymmetric long-run relationship. However, on a short-run basis, only the gold and silver futures show an asymmetric relationship with the Bitcoin price. Acknowledging the rapid development in cryptocurrency markets, suggestions for future research from this paper include extensions to research focusing on different economic conditions and the inclusion of other nonlinear econometric models.

The paper by Moussa et al. (2021) aims to study the underlying dynamic relationship between Bitcoin and energy commodities. According to the research, both logarithmic prices on gold and Brent crude oil have an impact on bitcoin logarithmic prices. To describe the relationship, the researchers used a smooth transition error correction model to account for the presence of nonlinearity and asymmetry in Bitcoin's adjustment process towards long-term equilibrium levels. This paper makes no recommendations for future research.

Othman and Bendob (2022) conducted research on Bitcoin mining’s energy consumption and global carbon dioxide emissions. The wavelet coherence methodology was used to capture time and frequency relationships. The findings reveal that from 2012 to 2013, there were in-phase associations between Bitcoin mining’s energy consumption and the global carbon emissions index at various frequencies and within different time frames. However, in 2018, the association was anti-phase with a frequency between 16 and 32 weeks. The researchers argue that their results emphasize the importance of considering the environmental side of cryptocurrency mining operations, and consequently, miners should consider wind and solar power as alternative power sources for future operations within mining.

The paper by Rehman and Kang (2021) analyzes the time-frequency comovement and causality between Bitcoin price and hashrate, controlling for the effect of the energy commodity futures of Brent crude oil, natural gas, and coal. They utilize a wavelet coherency method to analyze time-frequency relations for bivariate data in addition to partial and multiple wavelet techniques for multivariate analyses. Multivariate analyses were incorporated to address the role of the energy commodities on the Bitcoin hashrate nexus. Findings suggest that Bitcoin exhibits significant correlation with oil and gas over various periods and frequencies, while coal futures appear to have an absence of significant correlation with Bitcoin. Rehman and Kang state that the partial wavelet analysis shows negligible effects of the energy commodities on the relationship between Bitcoin and hashrate. In contrast, the multiple wavelet analysis appears to show significant areas of correlation for the Bitcoin and hashrate relationship for the inclusion of all commodities separately. This finding highlights the importance of considering the energy usage aspect in the relationship between Bitcoin and hashrate. Interestingly, they claim that the correlation between Bitcoin and hashrate remains negligible in the bivariate model, implying that increased mining activity does not necessarily influence the Bitcoin price, and vice versa. Their study points the way forward for future research by incorporating country-specific mining processes and bitcoin returns, as well as returns on local energy commodity markets.

In general, all papers presented in our literature review studying the relationships between Bitcoin and potential influential factors reveal some degree of connectedness. The research conducted on the relation between Bitcoin and hashrate shows bidirectional connections, as results point to both hashrate driving the price of Bitcoin and the price of Bitcoin driving the hashrate. Hashrate and electricity demand are also found to be sources of risk in the Bitcoin network, as pointed out by Yuan et al. (2022). Among the papers studying the influence of energy commodities on Bitcoin pricing, all results show significant relationships between the Bitcoin price and energy commodities, especially in the long run. Othman and Bendob (2022) show through their study on the connection between Bitcoin mining's energy consumption and global carbon emissions the importance of considering the environmental aspect of mining processes. Ultimately, the Rehman and Kang paper makes an effort to connect all factors in their wavelet analysis and reveals the importance of including energy commodities when assessing the connectedness between Bitcoin price and hashrate.

With the current strand of literature established, we believe that our paper will contribute to this topic of the relationship between Bitcoin, hashrate, and energy commodities by studying the complex connectedness of all three aspects. We approach this task by using the sophisticated wavelet methodology, in line with Rehman and Kang (2021), with the novelty of including a period in the sample where the economic situation worldwide was characterized by pronounced uncertainties and a persisting decline in economic activity.

3. Data

3.1. Data gathering

Aiming at analyzing the time-frequency comovement between Bitcoin and hashrate while controlling for the effect of certain energy commodities, we gathered daily data on Bitcoin prices, Bitcoin hashrate, and two-month futures for Brent crude oil, natural gas, and coal. As our study is motivated by the Rehman and Kang (2021) paper, and serves as a replication with extensions to their study, efforts were made to replicate their sample and add to that. As a result, we aim to use the same variables, with similar operationalizations as they have, ranging from the beginning of 2013 with the extension to recent data; more specifically, data until the end of 2022. The task of replicating Rehman and Kang's data turned out to be a bigger challenge than initially expected. Their paper reveals limited insight into the data gathering process and specific information regarding their actual data. The only information stated by Rehman and Kang is the range of their sample, and that the Bitcoin hashrate data is sourced from blockchain.com, and the Bitcoin price and the energy commodity prices are extracted from Thomson Reuters Data Stream (Rehman & Kang, 2021). Our repeated attempts at reaching out to Rehman and Kang for clarification around their data remained unsuccessful. Despite various attempts, we were not successful in converting the data available from blockchain.com into an applicable format for our analysis. Therefore, we obtained the Bitcoin hashrate data from an alternative source. The Nasdaq database turned out to provide satisfactory data on the Bitcoin hashrate, similar to blockchain.com. Consequently, we retrieved our Bitcoin hashrate data from their "Blockchain" data products list.

Further, for the data on the assets extracted from Thomson Reuters Data Stream by Rehman and Kang, our attempts to replicate their data also failed. For neither of the assets used in their research, a ticker, or the relevant specification of the actual trading product, is reported. In the Thomson Reuters Data Stream, the only reasonable Bitcoin price source available has a limited time span, ranging only from July 2014 until the present day. This limitation excludes the extraction of data from the Thomson Reuters Data Stream for us in order to replicate Rehman and Kang. As a solution to sourcing Bitcoin price data for our entire desired sample period, we then resorted to the Coinmetrics database, which provides daily price data on Bitcoin since mid-2010. Regarding the commodity price data, reportedly extracted from Thomson Reuters Data Stream, replicability proved equally challenging. Rehman and Kang communicate that they utilize commodity futures; however, nothing is mentioned beyond that regarding the nature of these futures. Within the Thomson Reuters Data Stream, a wide selection of commodity futures product data is available. It was anyway not straightforward to match any of these products with the price plots presented in Rehman and Kang (2021). Therefore, we were not able to replicate the data from our motivational paper for the energy commodities either. For accessibility and possible future replication, we then chose to sample our commodity future price data from the New York Mercantile Exchange, available through Yahoo Finance. These are two-month futures identified by the tickers CL=F, NG=F, and MTF=F for oil, gas, and coal, respectively. By using Yahoo Finance for sourcing data, we could utilize the *getSymbols* function in R to directly import the data into the computational software.

More specifically, Bitcoin prices are extracted in USD, hashrates in terahashes per second, and energy commodity futures in USD. The data ranges from January 1st, 2013, until December 31st, 2022, as mentioned earlier. Both data on the Bitcoin price and the Bitcoin hashrate are available every day, all year, without exception. This is not the case for the energy commodity futures, which are only traded Monday through Friday and not on certain public holidays. In order to have a matching dataset for all variables, we chose to omit trading days for all assets where one or more assets were missing values. This adjustment to the data was conducted by adding the missing calendar days to the energy commodity futures data, with NA values for missing data. Further, data for all five assets was merged into one matrix, sorted by date. From the full matrix, all rows containing NA values were omitted to make sure all variables in the dataset had valid values for all observations. As a result, our sample contains 2411 observations for all five assets over the ten-year sample period.

3.2. Description of data

Figure 1 - Bitcoin Price in USD for the sample period (2013-01-01, 2022-12-31)

Bitcoin's price began around $13 in 2013, and it reached a high of $260 in April. However, due to a massive sell-off, the price fell significantly in the same month. For a few months, the price hovered around $100 before falling again in November 2013. At the end of the year, the price recovered slightly, closing at around $730. Since the beginning of 2014, the price of Bitcoin has steadily declined. The price dropped from around $730 in January to around $315 in October. Several factors contributed to the decline, including regulatory concerns, security breaches at exchanges, and negative media coverage. By the end of the year, the price had recovered slightly, closing at around $320.

In 2015, Bitcoin's price was trading at around $250, after experiencing a significant drop from its, at that time, all-time high of $1,200 in late 2013. Bitcoin's price rose steadily in 2015 and 2016, with only minor fluctuations, reaching a high of $1,000 in early 2017. The cryptocurrency's steady growth during this time period was driven by increased adoption by businesses and individuals, as well as advancements in the underlying technology. Bitcoin experienced a sudden and rapid increase in price in 2017, reaching an all-time high of nearly $20,000 in December of that year. The increase was fueled by a surge in investor interest and media attention, as well as possible market manipulation by traders (Griffin & Shams, 2020). However, the bubble burst soon after, with Bitcoin's price plummeting to around $3,000 by the end of 2018. According to Rooney (2018) this dramatic price drop was arguably due to a combination of factors, including increased regulatory scrutiny and market manipulation.

After the bubble burst, Bitcoin's price entered a period of consolidation in 2018 and 2019. During this period, the cryptocurrency market as a whole experienced a downturn, with prices for most cryptocurrencies falling significantly. However, Bitcoin's price remained relatively stable compared to other cryptocurrencies, hovering around $6,000 - $8,000 during this time. In 2020, Bitcoin's price began to surge once again, reaching a high of nearly $65,000 in April of 2021. This resurgence was fueled by increased institutional adoption, with major companies like Tesla and Square investing in Bitcoin (Browne, 2021). Additionally, the COVID-19 pandemic and consequent economic uncertainty contributed to the increase in Bitcoin's price as investors sought a safe haven asset. However, the price fell sharply in May 2021 due to increased regulatory scrutiny in China and concerns about the environmental impact of Bitcoin (Pound, 2021). In 2022 and 2023, Bitcoin's price continued to be volatile, with fluctuations ranging from $30,000 to $60,000.

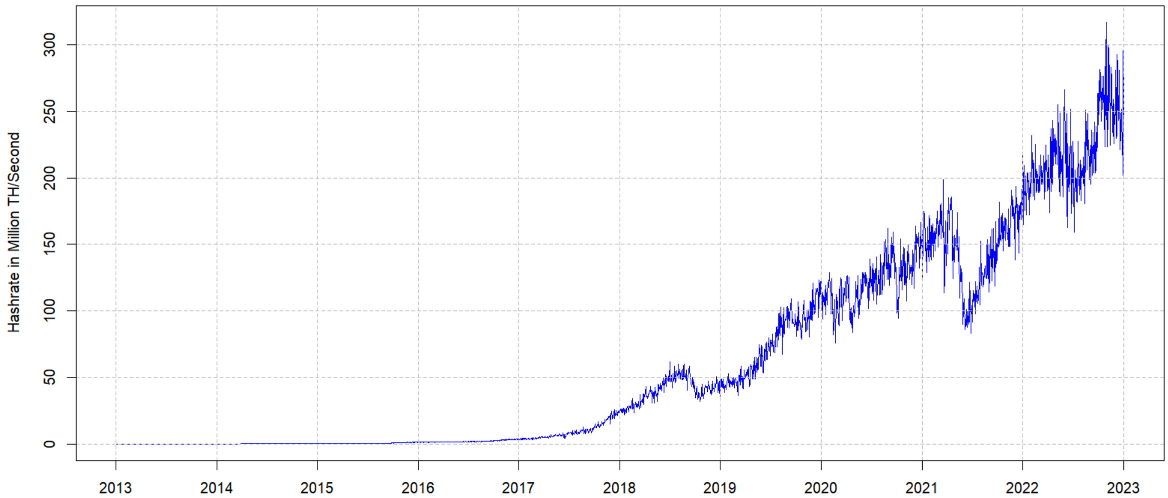


Figure 2 - Development of the Bitcoin Hashrate for the sample period (2013-01-01, 2022-12-31) in millions of terahashes per second

Bitcoin's hashrate was approximately 300 petahashes per second (PH/s) going into 2015. Throughout 2015 and 2016, Bitcoin's hashrate saw steady growth, reaching 1,500 PH/s around the summer of 2016. This increase in hashrate was most likely due to the introduction of more efficient mining hardware and an increase in the number of miners joining the network. Bitcoin's hashrate increased dramatically in late 2017, reaching a peak of over 14,000 PH/s in December. The activation of the Segregated Witness protocol, which increased the block size limit and allowed for more transactions to be processed, contributed to the increase in hashrate (BitMEX, 2018). Furthermore, the rise in Bitcoin's price coincided with an increase in mining profitability and a sharp rise in miner participation. After peaking in mid-2018, Bitcoin's hashrate experienced a significant decline, dropping from around 55,000 PH/s to 35,000 PH/s by the end of 2018. This drop was caused partly by the market downturn that affected the entire cryptocurrency market at the time. During 2019, however, the hashrate increased, hovering from around 40,000 and peaking above 100,000 PH/s.

Bitcoin's hashrate began to rise again in 2020, reaching an all-time high of more than 180,000 PH/s in May 2021. Beijing’s restrictions on Bitcoin mining caused hashrates to plummet in 2021 following the all-time high (Worldcoin, 2022). Consequently, Chinese crypto exchanges left the country, causing uncertainty among Bitcoin supporters about the prospects for industry recovery following Beijing's regulatory ruling. The departure of most of China’s Bitcoin miners to countries such as the U.S. and Kazakhstan underscores the resilience of the cryptocurrency industry. This observation highlights the challenge of effectively outlawing cryptocurrencies, given the decentralized nature of the technology and the ability of miners to relocate to other regions where regulations may be more favorable. This revival was also fueled in part by increased competition among miners as new and more efficient mining hardware was introduced. Bitcoin's hashrate remained volatile in 2022 and 2023, fluctuating between 150,000 and 250,000 PH/s. However, the increase in hashrate has raised concerns about the environmental impact of Bitcoin mining, as mining hardware consumes a significant amount of energy (Jones et al., 2022). As a result, there has been a significant increase in interest in developing more environmentally friendly mining solutions, such as using renewable energy sources. Renewable energy sources such as wind and solar power are becoming increasingly cost-competitive, however, miners are usually incentivized to consume inexpensive energy to optimize profit margins (Anderson & Reddy, 2023).

According to Agur et al. (2022), the energy consumption of cryptocurrency assets can vary depending on the design elements of the underlying Distributed Ledger Technology (DLT) network, such as the consensus mechanism and level of control. Non-proof-of-work permissioned networks can offer significant energy efficiency gains over legacy systems that are used for credit card payments, and central bank digital currencies (CBDC) could also be designed to use less energy-intensive infrastructures. However, potential environmental benefits will depend on additional factors, including regulation and compliance costs, and whether additional features are required for CBDCs. While methodologies and data for assessing the full payment chain are still in progress, there is potential for digital means of payments to improve energy efficiency.

Figure 3 - Oil futures prices in USD for the sample period (2013-01-01, 2022-12-31) – Ticker from YahooFinance: CL=F

Crude oil futures experienced a period of relative stability, with prices around $90 per barrel during most of 2013. However, towards the end of 2014, a combination of oversupply in the global oil market and weaker demand, especially from China and Europe, led to a sharp decline in prices (Stocker et al., 2018), with crude oil falling to below $50 per barrel by early 2015. This trend continued through the year, with prices reaching a low of $26 per barrel around the beginning of 2016. From June of 2016 until around June of 2018, the price of crude oil recovered somewhat, driven by a combination of OPEC production cuts and increased demand from emerging markets (DiChristoffer & Meredith, 2018). However, by late 2018, crude oil fell below $50 per barrel again. Many believe concerns about a global economic slowdown and the impact of U.S. sanctions on Iran led to a renewed decline in the future price of crude oil (DiChristoffer & Meredith, 2018). The first half of 2019 saw a modest recovery in the price of crude oil; however, the second half of the year was marked by increased concerns about global economic growth and a trade war between the U.S. and China, which led to a decline in oil prices (Kumar, 2019). By the end of 2019, crude oil was trading at around $61 per barrel.

The COVID-19 pandemic of 2020 had a profound impact on the global oil market, as travel restrictions and reduced economic activity led to a significant decline in demand. Crude oil futures experienced their largest one-day decline on April 20, 2020, when prices briefly turned negative due to a lack of storage capacity (Reed & Krauss, 2020). By the end of the year, prices had recovered somewhat but were still trading at around $48 per barrel. Prices reached a high of $82 per barrel in October 2021, before declining towards the end of the year. However, this drop was followed by an increase in price until June 2022, when crude oil futures peaked above $120 per barrel as supplies were tight following the war in Ukraine (Kearney, 2022). Prices fell in the second half of the year as central banks raised interest rates, fueling fears of a recession (Kearney, 2022).

Figure 4 - Natural Gas futures prices for the sample period (2013-01-01, 2022-12-31) - Ticker from YahooFinance: NG=F

The price of natural gas futures has experienced significant volatility over the past decade, with highs and lows reflecting fluctuations in demand and supply. In 2013, the price of natural gas started the year at around $3.4/MMBtu (Metric Million British Thermal Unit) and remained relatively stable until late November, when it began to climb, reaching a high of approximately $4.5/MMBtu in December. In 2014, the price of natural gas continued to rise, peaking at $6.1/MMBtu in February. However, the price fell sharply in the second half of the year, dropping to a low of $2.5/MMBtu in December. During 2015, the price of natural gas remained low at about $2.5/MMBtu, as oversupply continued to weigh on the market (Mills, 2016). However, in 2016, the price began to recover from $2/MMBtu in January and climb steadily throughout the year, reaching a high of around $3.8/MMBtu towards the end of the year. The rise was likely due to a decline in production, increased demand from the power sector, and an increase in exports due to cold weather forecasts (Partners, 2016).

Natural gas futures remained rather stable in 2017, unlike in 2018. Natural gas experienced an increase in price, from $3/MMBtu in January and reaching a high of about $4.9/MMBtu in November. During this period, the weather was cold and supplies were low, which are likely the reasons behind the increase in natural gas futures (Domm, 2018). The price remained rather stable around $2.5/MMBtu during 2019, before experiencing another drop in the beginning of 2020 due to the uncertainties around the COVID-19 pandemic (Stevens, 2020). The price dropped to around $1.9/MMBtu as the pandemic led to a sharp drop in demand from the power and industrial sectors, exacerbating oversupply concerns (Stevens, 2020). However, prices increased to around $3/MMBtu during November of 2020. In January of 2021, the price stabilized around $2.8/MMBtu, before reaching a 10-year high of $6.2/MMBtu in November of the same year. This period of high prices lasted only a short period of time. Already in December, the prices were back at around $3.7/MMBtu.

However, during 2022, the natural gas futures price experienced a sudden rise in price, likely during the war in Ukraine and the tense situation between Russia and the rest of the western world, reaching $9.3/MMBtu in August. Consequently, Russia's gas exports to Europe were constrained. However, by the 31st of December 2022, the price had dropped to around $3.7/MMBtu. The drop is likely driven by dynamics in European markets (Agnolucci et al., 2023).

Figure 5 - Coal futures prices for the sample period (2013-01-01, 2022-12-31) - Ticker from YahooFinance: MTF=F

From 2013 to 2016, the Coal price experienced a stable but marginal decrease during the entire period, from about \$85 to \$50 per metric ton. However, in 2016, the price started to rise abruptly, likely due to a combination of factors, such as supply disruptions in China (Cornot-Gandolphe, 2017) and the closing down of coal mines (Stanway & Sarkar, 2016). It wasn't until mid 2018 when the price per metric ton passed \$100, before declining to around \$40 in March of 2020 when the outbreak of COVID-19 impacted demand for coal. From March of 2020 and the next 12 months coal exhibited steady growth. From March until August of 2021, the growth rate increased rapidly, peaking at around \$270 per metric ton around August, the highest level since 2011. This was soon followed by a drop in the price of coal futures in the last quarter of 2021. However, in early 2022, the price rose again due to supply disruptions caused by heavy rains in Indonesia and Australia, coupled with strong demand from China and India. The price for coal futures peaked at \$433 in late February of 2022. When Russia invaded Ukraine, the coal future prices became more volatile than ever before (IEA, 2022). The high in late February was followed by a drop to \$265 in March, before rising again in the following months. From July 2022 until 2023 the price of coal futures dropped from around \$310 to \$172 per metric ton due to a push for more competitive coal prices (Frangoul, 2022).

3.3. Logarithmic growth rate transformation

When examining the relationships between Bitcoin and hashrate, while controlling for energy commodity future prices, we convert all prices and terahashes per second into growth rates by calculating their respective logarithmic differences. For the financial price data, this corresponds to the returns on the assets, or typically, log returns. However, we cannot yield returns on a hashrate, thus it is just referred to as the hashrate logarithmic growth rates. The growth rates are calculated by equation (1)

$$R_t = \log\left(\frac{Value_t}{Value_{t-1}}\right). \tag{1}$$

In order to have unit-free measurements for all variables in the research, this transformation of prices and TH/s is necessary. Logarithmic growth rates transformed variables facilitate the possibility of comparing variables that originally had different units and studying their relationships. Additionally, a property of logarithmic growth rates is that they are thought of as continuously compounded rates, which gives them a time-additive attribute. As our study contains time series, the logarithmic growth rate transformation is therefore considered crucial.

Further in the thesis, we will just refer to it as returns and growth rates. As this transformation is performed by calculating logarithmic differences, we lose one observation of the total sample. Consequently, we are left with a sample of 2410 observations.

To be able to properly implement the Brent crude oil futures data we have at hand into our dataset, a manual growth rate calculation was made for the observations surrounding the negative price of April 20th, 2020. These rates were then substituted for the NA values resulting from taking the natural logarithm to a negative number. As these rates are highly extreme, with a daily loss of 306% followed by a daily gain of 127%, we chose to exclude them from our descriptive statistics presented later, since this was a singular event that occurred due to extreme circumstances.

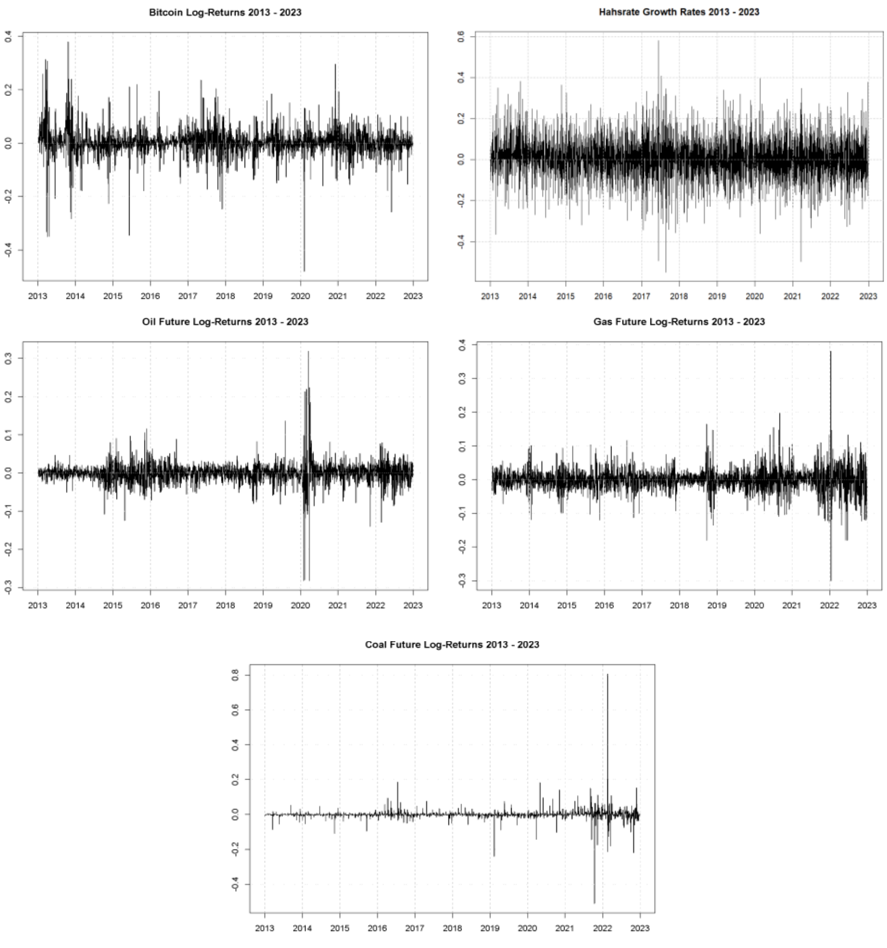


Figure 6 – Logarithmic growth rates of Bitcoin price, Bitcoin Hashrate, Oil Futures, Gas Futures, and Coal Futures for the sample period (2013-01-01, 2022-12-31)

Figure 6 shows the daily growth rates plotted against time from 2013 until 2023. The plot shows that hashrate growth rates have been the most volatile, while returns on the coal futures have been the least volatile. With the exception of hashrate growth rates, all assets have experienced a significant return shock in the aftermath of the COVID-19 pandemic. Particularly, energy commodity futures have exhibited intensified fluctuations over the past 18 months, accompanied by notable signs of special observations. This was evident in March 2020 for oil and March 2022 for coal and gas, likely attributable to the COVID-19 pandemic and the invasion of Ukraine, respectively.

3.4. Descriptive data analysis

Table 1 and Table 2 present the descriptive statistics for growth rates on all assets in our research. For our full sample, ranging from the beginning of 2013 until the end of 2022, the statistical measures of the minimum and maximum daily growth rates, the arithmetic daily mean growth rate, the standard deviation of daily growth rates, and the skewness and kurtosis of growth rates are presented in Table 1. The largest observed daily losses are 54.86%, 48.09%, and 51.13%, respectively, for Bitcoin hashrate, Bitcoin price, and the coal future, whereas the oil and gas futures have experienced daily losses of 28.22% and 30.05%. During the sample period, a maximum gain of 80.72% was observed for the coal future and 58.03% for the Bitcoin hashrate, while the Bitcoin price, the oil future, and the gas future saw maximum daily gains of 38.05%, 31.96%, and 38.17%, respectively. All of the daily mean growth rates lie close to zero on the positive side. The mean returns on the commodity futures hardly deviate from zero, with all of them having means less than a tenth of a percent. For Bitcoin hashrate and price, the daily mean growth rates are somewhat higher at 0.68% and 0.30%, respectively. Further, for the standard deviation of the daily growth rates, we see higher levels for the Bitcoin hashrate and price than for the commodity futures. At 11.82% standard deviation for the hashrate and 5.42% for the Bitcoin price, compared to 2.89%, 3.58%, and 2.89% for the commodity futures. The Bitcoin variables are consequently more volatile than the energy commodities.

	Bitcoin Hashrate	**Bitcoin Price**	**Oil**	**Natural Gas**	**Coal**
Min	-0.5486	-0.4809	-0.2822	-0.3005	-0.5113
Max	0.5803	0.3805	0.3196	0.3817	0.8072
Mean	0.0068	0.0030	0.0002	0.0001	0.0004
Std.dev	0.1182	0.0542	0.0289	0.0358	0.0282
Skewness	-0.0192	-0.4125	0.0974	0.1916	6.8914
Kurtosis	0.7312	10.3577	24.2717	9.7931	331.6338

Table 1 - Descriptive statistics of the full sample from January 1st, 2013 – December 31st, 2022

	Bitcoin Hashrate	Bitcoin Price	Oil	Natural Gas	Coal
Min	-0.5486	-0.3506	-0.1241	-0.1201	-0.1087
Max	0.5803	0.3805	0.1162	0.1163	0.1870
Mean	0.0108	0.0045	-0.0002	0.0000	0.0001
Std.dev	0.1177	0.0589	0.0213	0.0270	0.0123
Skewness	-0.0795	-0.2130	0.0915	-0.0396	2.1959
Kurtosis	1.1817	8.6078	3.6337	2.3093	56.4647

Table 2 - Descriptive statistics of the attempted replication of the Rehman and Kang sample from January 1st, 2013 – October 12th, 2018

	Bitcoin Hashrate	Bitcoin Price	Oil	Natural Gas	Coal
Min	-0.5486	-0.3454	-0.9358	-0.8017	-0.5970
Max	0.5943	0.3083	0.9012	0.6975	0.5900
Mean	0.0070	0.0029	-0.0002	0.0001	0.0002
Std.dev	0.1233	0.0472	0.1934	0.1386	0.1180
Skewness	0.0258	-0.4256	-0.0406	-0.0560	-0.0113
Kurtosis	1.0318	8.4092	3.2475	6.5017	10.9190

Table 3 - Rehman and Kang´s descriptive statistics, copied from Rehman and Kang (2021)

Bitcoin returns and hashrate growth rates both have a negative skew, whereas the three energy commodities have a positive skew. The positive (negative) skew means that the growth rates distribution has a longer/fatter tail on the right (left) hand side, indicating a sample where you should expect to observe fewer major gains (losses) and more small losses (gains). As all skewness estimates are non-zero, one might suggest a lack of symmetry in the distribution of all variables. However, performing statistical tests of the hypothesis of a zero skewness[1], we find that the skewness of hashrate growth rates and the returns on oil futures fail to meet the criteria of significantly deviating from zero. As a result, we cannot reject the hypothesis of symmetric distributions for those two variables. Returns on the three remaining variables all meet the criteria to significantly deviate from zero such that we can reject the hypothesis of symmetric distributions in those cases. Except for the Bitcoin hashrate variable, all variables are associated with leptokurtic distributions. All kurtosis estimates exceed 3, which indicates

[1] The ‘stat.desc’ function of the R package “pastecs” provides the option to return *normal distribution statistics,* where the measure *skew.2SE* offer an option for testing the hypothesis of the skewness significantly deviating from zero. *skew.2SE* is derived from dividing the skewness measure by two times the standard error of the skewness ($\sqrt{6/n}$). If |*skew.2SE*| > *1*, then the skewness significantly deviates from zero. From our analysis, we calculated the following *skew.2SE* for hashrate growth rates and the returns on bitcoin price, oil, gas, and coal respectively; <u>-0.1928</u>, -4.1360, <u>0.9762</u>, 1.9208, 69.1002. The underlined values, corresponding to the hashrate growth rates and returns on the oil futures, do not exceed the threshold value of the significant criterium.

tail which are fatter than those of a Gaussian normal distribution. In such distributions, we would expect to observe more of the extreme growth rates. A kurtosis less than 3 - as found in the case of the Bitcoin hashrate - indicates a platykurtic distribution, with opposite characteristics to the leptokurtic distribution.

Table 2 represents the attempted replication of the sample used in our motivational paper by Rehman and Kang (2021) in their research. Compared to their descriptive statistics, we observe considerable differences despite our efforts to replicate their sample. As mentioned previously, this is mainly a result of their limited description of the data collection process and actual data used in their research, leading to an unattainable sample replication. Most notable are the differences related to the energy commodities. Rehman and Kang´s minimum and maximum daily returns for the commodities seem rather extreme, with all three seeing returns well below and above -55% and 55%, respectively. According to Rehman and Kang´s statistics, the oil futures used in their research had an unbelievable maximum daily drop in return of 93.85% and a maximum gain of 90.12%. According to an article by Fisher (2020), the largest one-day drop in the oil price since the first Gulf War occurred on the 9th of March 2020, at around -30%. This strongly contradicts the value observed by Rehman and Kang.

All of our commodity futures observations for the same sample period are far less extreme, with oil having the highest daily drop of -12.41% and coal having the highest daily gain of 18.70%. For Bitcoin price and hashrate, our values appear to be reasonably similar to Rehman and Kang´s. In our statistics, the mean values for the commodities closely align with those reported in the Rehman and Kang statistics. Likely, they have more of their extreme observations on both the negative and positive sides, leaving the mean unchanged. With more of the extreme observations, higher levels of standard deviation can also be found in the Rehman and Kang statistics than in ours. The unavailability of the data used for energy commodities in the study conducted by Rehman and Kang has hindered us from replicating their findings. Our repeated attempts to communicate with the authors, including outreach from our supervisor, have remained unanswered. Consequently, our inability to replicate their data might lead to divergent findings.

4. Methodology

This chapter will provide a basic background on the theory behind Wavelets.

4.1. Wavelet background

4.1.1. Fourier transform

The concept of the Fourier transform was first introduced by the French mathematician and physicist Jean-Baptiste Joseph Fourier in the early 19th century (Bracewell, 1989). Fourier's work on the transform, which was published in a series of papers between 1807 and 1822, established the foundation for the study of Fourier series and the Fourier transform, which are now fundamental tools in the field of mathematics and signal processing (Bracewell, 1989).

The Fourier Transform is a mathematical tool used to transform signals from the time domain to the frequency domain, and is the main tool of Fourier analysis (Chui, 1992, p. 23). The framework enables us to represent signals as the sum of their individual frequencies, providing insights into the signal's spectral content. It is widely used in engineering and among mathematicians, including signal processing, image processing, and communication systems (Chui, 1992, p. 23).

One of the key limitations of the Fourier transform is that it assumes the signal being analyzed is periodic, which is not always the case. Additionally, the Fourier transform can be sensitive to transient events, such as spikes or sharp transitions that occur in the time domain, as they use sinusoidal waves to capture the characteristics of a signal (Bracewell, 1989). In such cases, the Fourier transform may not accurately represent the frequency content of the signal, making it difficult to analyze the signal precisely (Oppenheim et al., 1999).

4.1.2. Wavelets

Wavelets, in contrast to Fourier analysis, have only been used for about three decades (Colldeforns Papiol et al., 2018). Because of the framework's ability to process signals, it has become a research topic in finance as well as other fields. Wavelet analysis includes a transformation that decomposes a function into a set of wavelets. The goal is to determine how much of a wavelet is present in a signal at a given scale and location (Torrence & Compo, 1998). This is referred to as convolutions. Because wavelet analysis employs wavelets rather than sinusoidal waves, the framework can extract more information from a signal, particularly in high-frequency spikes (Lassaline et al., 2020). Wavelet analysis, as opposed to Fourier analysis, allow researchers to select a wavelet family, or basis, based on the type of function they want to approximate. The wavelet's goal is to compress information from a complex signal into simple wavelets that vary in time and frequency.

Instead of decomposing a signal into constituent harmonic functions as in Fourier analysis, wavelet analysis transforms it into scaled and translated versions of an original (mother) wavelet (Rathinasamy et al., 2017). The wavelet transforms can be both discrete and continuous, depending on the number of scales and locations one considers in the transform.

4.1.3. Use

Wavelets offer a powerful alternative to traditional Fourier-based methods, providing a time-frequency representation of signals and images that is well-suited to many real-world applications. Wavelet analysis is useful for signals that have non-stationary properties, meaning that their frequency content changes over time, as it provides a multi-resolution view allowing for a more accurate analysis of the temporal and frequency characteristics (Torrence & Compo, 1998). The formula of Fourier transform alone is inadequate for these purposes (Chui, 1992, p. 49). As mentioned previously, wavelets have recently become a tool used to conduct financial analysis, enabling researchers to analyze comovement and causality in the time-frequency domain simultaneously. Separating local from global, ephemeral from permanent, is a source of concern in economics and finance. The wavelet methodology has proven to be an excellent tool for isolating these effects (Ramsey, 1999).

4.2. Wavelet transformation

To analyze a signal in the time and frequency domain by use of wavelets, a wavelet transformation of the signal is required. This operation transforms a signal into a set of wavelets (Torrence & Compo, 1998). Where the Fourier transformation uses sine and cosine functions in its transformation process, a wavelet transformation utilizes wavelet functions in its transformation. Wavelets oscillate similar to sine or cosine functions; however, they assume non-zero values for a finite interval of the x-axis. As mentioned earlier, wavelets are part of a family of functions. They all share some of the same attributes needed to qualify as a wavelet function. The function must have a mean of zero and be localized in time and frequency space, as a minimum (Torrence & Compo, 1998). Every different wavelet has characteristics to it that makes it more or less suited to represent specific signals for the time and frequency analysis. For every wavelet transformation, a "mother wavelet" is chosen and the signal function of the time-series is then transformed into a scaled and time localized version of the mother wavelet.

There are two types of wavelet transformations, the Discrete Wavelet Transformation (DWT) and the Continuous Wavelet Transformation (CWT) (Daubechies, 1992). For this paper, only the CWT method will be used. However, a short description of the DWT will be given as well.

4.2.1. Discrete wavelet transformation

The discrete wavelet transformation (DWT) transforms the signal of a time-series into wavelets using a discrete set of scale and location parameters. From the set of scale and locations, the resulting output will be a finite number of wavelets. For simplicity, we can say that the DWT operates on a dyadic scale, which requires it to have a sample size to the power of 2 to achieve a full transformation of the sample. A DWT can be facilitated with both an orthogonal and a non-orthogonal wavelet function, whereas the CWT can only operate with a non-orthogonal wavelet function (Daubechies, 1992).

4.2.2. Continuous wavelet transformation

The continuous wavelet transformation (CWT) is highly useful for bivariate and multivariate cases, where the aim is to study relationships between two or more processes in time and frequency space. In contrast to the DWT, the CWT considers all possible scales and locations within a given time and frequency set. The continuous nature of the transformation results in an infinite set of wavelets.

$$W(s,u) = \frac{1}{\sqrt{|s|}} \int_{-\infty}^{\infty} f(t)\psi\left(\frac{t-u}{s}\right) dt \tag{2}$$

Equation (2) is the formula for the CWT (Daubechies, 1992). Here, s and u represent the scale and location parameters respectively, while t stands for time. ψ denotes the wavelet function of the transformation. The transformation is then obtained by projecting the wavelet function at scale s and location u onto the signal of the time series, $f(t)$ at time t. From the projection of the wavelet function onto the signal function, a convolution of the two functions will provide basis for the transformation. A normalization, given by one over the square root of the absolute value of the scale, is introduced to ensure the normality of the coefficient. The wavelet coefficient $W(s,u)$ is a representation of the time-series signal both in time and frequency.

In our analysis, the mother wavelet function utilized is the Morlet wavelet. Equation (3) presents the Morlet function, where ω_0 denotes the central frequency component of the wavelet. ω_0 is typically set equal to 6 in order to meet the requirement of a wavelet (Farge, 1992).

$$\psi_0(\eta) = \pi^{-1/4} e^{i\omega_0\eta} e^{-\eta^2/2} \tag{3}$$

This complex valued wavelet function is a popular function for use in time series analysis, and based on its unique properties that suit a number of applications, we chose the Morlet. The Morlet wavelet demonstrates remarkable efficacy and effectiveness due to several key features such as its time-frequency localization, its complex valued nature, the Gaussian shape it possesses, its adaptability, and the fact that there already exists an established use of it in existing literature. As the Morlet wavelet is localized both in time and frequency, it is capable of capturing information on both aspects of a signal simultaneously (Mallat, 1999). This property can also facilitate the analysis of non-stationary processes where there is a change in frequency for the signal over time. To capture potential amplitude modulations and phase shifts of a signal possibly occurring in a process with both oscillatory and non-oscillatory components, the complex-valued nature of the Morlet wavelet is useful (Grinsted et al., 2004). Because the Morlet wavelet function is modulated by a Gaussian function, it also has a Gaussian shape, which makes it suited for analyzing signals containing narrowband frequencies. This is highly beneficial for capturing oscillatory components with well-defined peaks in frequency (Torrence & Compo, 1998). Further, according to Percival and Walden (2000), the Morlet wavelet is easily adaptable, meaning it can be scaled and translated to fit the characteristics of the analyzed signal without trouble. Altogether, these characteristics make the Morlet wavelet function a well-suited mother wavelet for analyzing our time series data.

4.3. Cross wavelet transform

Where a single wavelet transformation only provides information about the characteristics of a process in the time-frequency domain, the cross-wavelet transform (XWT) enables the possibility to reveal interactions between two processes (Torrence & Compo, 1998). This transformation combines the wavelet coefficients of two signals to generate the combined cross-wavelet power of the two-time series. With the cross-wavelet power, areas in the time-frequency space with high levels of common power for the two time-series can be identified. Common power in this regard can be interpreted as local covariance between time series at various scales (Barunik et al., 2011).

The cross-wavelet transform represents the wavelet coefficients of two signals, x and y, in the time-frequency domain, and can be written as:

$$W^{xy}(s,u) = W^{x}(s,u) * W^{y*}(s,u), \tag{4}$$

where $W^{x}(s,u)$ and $W^{y}(s,u)$ are the wavelet transforms of signals x and y, respectively, at scale s and time u, and the * symbol denotes the complex conjugate. In the comovement analysis of two time series, we are not only interested in the areas where the series show high common power, but also areas with comovement and lower common power. As the XWT fails to identify areas of the comovement with low common power, a further method for analyzing will be presented (Barunik et al., 2011).

4.4. Wavelet power spectra

The wavelet power spectrum is a measure of the power or amplitude of each frequency component of a time series at each scale (Torrence & Compo, 1998). It is calculated by applying a wavelet transform to the time series, which decomposes the signal into a set of wavelets at different scales, as described in the CWT. The wavelet coefficients are then squared and summed over all scales to obtain the wavelet power spectrum defined by $|W_{x}(s,u)|^{2}$ (Barunik et al., 2011).

4.5. Wavelet coherence

As an extension to the XWT, wavelet coherence is a useful method to overcome the shortcomings mentioned above. Wavelet coherence can be described as a measure showcasing the local correlation in time and scale for two time series. The wavelet coherence can be defined as the squared absolute value of the smoothed cross wavelet spectra normalized by the product of the smoothed individual wavelet power spectra of each series (Barunik et al., 2011)

$$R^2(s,u) = \frac{\left|S\left(s^{-1}W_{xy}(s,u)\right)\right|^2}{S\left(s^{-1}|W_x(s,u)|^2\right)S\left(s^{-1}\left|W_y(s,u)\right|^2\right)}. \tag{5}$$

In equation (5), $R^2(s,u)$ represents the square of the wavelet coherence between two signals at a specific scale (s) and time (u). The numerator is the square of the magnitude of the cross-wavelet transform between the two signals at a given scale (s) and time (u), denoted as $W_{xy}(s,u)$. The denominator of the formula is the product of the magnitudes of the individual wavelet transforms of each signal at the same scale (s) and time (u). These individual wavelet transforms are denoted as $W_x(s,u)$ and $W_y(s,u)$, respectively, and are the power spectral density matrices of the two signals. $|\cdot|$ denotes the absolute value of the complex-valued function. Additionally, we have that S is the smoothing parameter. Smoothing of the wavelet coherence is required to prevent a result of one for all scales in the analysis (Barunik et al., 2011). The coefficient resulting from a wavelet coherence given by $R^2(s,u)$ is a quantity that ranges between 0 and 1, where values closer to 1 indicate that the signals are highly similar at a particular time-frequency location, and values closer to 0 indicate that the signals are dissimilar (Barunik et al., 2011).

In addition to quantifying the linear dependence between two signals, the wavelet coherence also provides information about their phase relationship. The wavelet coherence phase provides information about the delay between the oscillations of the two time-series being examined. This information can be used to better understand the relationship between the time series, such as the degree to which they are synchronized or out of phase. The phase relationship between two signals can be used to determine the lead or lag between the signals at a particular frequency and scale (Barunik et al., 2011).

The phase relationship between signals x and y at a given scale and time can be quantified using the phase angle of the cross-wavelet transform (Torrence & Compo, 1998):

$$\phi_{xy}(u,s) = \tan^{-1}\left(\frac{\mathfrak{I}\left\{S\left(s^{-1}W_{xy}(u,s)\right)\right\}}{\mathfrak{R}\left\{S\left(s^{-1}W_{xy}(u,s)\right)\right\}}\right). \tag{6}$$

From Equation (6), $\mathfrak{I}$ and $\mathfrak{R}$ represent the imaginary and real parts of the wavelet coefficients at two different scales, respectively. The smoothing operator denoted by S applies a low-pass filter to the wavelet coefficients, effectively removing high-frequency components that may be the result of noise or other artifacts (Barclay et al., 1997). The wavelet function at the given translation and scale, denoted by $W_{xy}(u,s)$, is smoothed before computing the phase angle $\phi_{xy}(u,s)$ between the wavelet coefficients using the arctangent function. The choice of S depends on the specific application and the desired level of noise reduction. s^{-1} in the phase formula for wavelets refers to a scaling factor that is used to adjust the scale of the wavelet function $W_{xy}(u,s)$ at the given translation u and scale s. This scaling process is repeated multiple times to obtain wavelet coefficients at different scales, which are used to compute the phase angle $\phi_{xy}(u,s)$ between two wavelets at different scales. By adjusting the scale of the wavelet function, the wavelet transform can capture both local and global features of the signal being analyzed (Torrence & Compo, 1998).

The formula returns the phase angle of the complex-valued cross-wavelet transform as a real number in radians, which indicates the relative phase between the two signals. The phase angle represents the temporal relationship between different frequency components of two time series and can be used to characterize periodicity in signal relationships. The phase angle $\phi_{xy}(u,s)$ ranges from -π to π radians, where a positive phase angle indicates a leading relationship (i.e., signal x leads signal y) and a negative phase angle indicates a lagging relationship (i.e., signal y leads signal x) (Oppenheim et al., 1996). A phase angle of 0 or π indicates that the two signals are perfectly in-phase or out-of-phase, respectively, at that specific scale and time. The value of π is used to represent the range of possible phase angles between -π and π radians, where π radians is equivalent to 180 degrees (Oppenheim et al., 1996). The phase angle of a complex number is measured as the angle between the positive real axis and the vector representing the complex number in the complex plane. A phase angle of π radians corresponds to a phase shift of half a cycle or 180 degrees, which represents a complete inversion of the signal's polarity.

Monte Carlo simulations are used to provide significance levels of the coherences. This approach is particularly useful when dealing with non-stationary signals (Torrence & Compo, 1998).

4.6. Partial wavelet coherence

When analyzing the comovement between two time series by using wavelet coherence, it might be relevant to factor out the effects on the coherence from other time series. This is effectively achieved by conducting a partial wavelet coherence (PWC) analysis. PWC computes the wavelet coherence between time series y and x_1 when the influence of time series x_2 is eliminated (Torrence & Compo, 1998). Listed below are the squared wavelet coherences of the three time-series (y, x_1, x_2) in equation (7)-(12).

$$R(y, x_1) = \frac{S[W(y, x_1)]}{\sqrt{S[W(y)] \cdot S[W(x_1)]}} \tag{7}$$

$$R^2(y, x_1) = R(y, x_1) \cdot R(y, x_1)^* \tag{8}$$

$$R(y, x_2) = \frac{S[W(y, x_2)]}{\sqrt{S[W(y)] \cdot S[W(x_2)]}} \tag{9}$$

$$R^2(y, x_2) = R(y, x_2) \cdot R(y, x_2)^* \tag{10}$$

$$R(x_2, x_1) = \frac{S[W(x_2, x_1)]}{\sqrt{S[W(x_2)] \cdot S[W(x_1)]}} \tag{11}$$

$$R^2(x_2, x_1) = R(x_2, x_1) \cdot R(x_2, x_1)^* \tag{12}$$

Similarly to the partial correlation squared, a partial wavelet coherence squared can be computed on the basis of (7)-(12) to form the following equation (Ng & Chan, 2012):

$$R_p^2(y, x_1, x_2) = \frac{|R(y, x_1) - R(y, x_2) \cdot R(y, x_1)^*|^2}{[1 - R(y, x_2)]^2 [1 - R(x_2, x_1)]^2}. \tag{13}$$

A low value of the PWC squared revealed at a high wavelet coherence squared indicates that time series x_1 has no significant influence on time series y in that particular time-frequency space, while time series x_2 dominates the effect on y 's variance (Ng & Chan, 2012). For the opposite case, the opposite relation of the coefficients applies. If both $R_p^2(y, x_1, x_2)$, and $R_p^2(y, x_2, x_1)$ are significant, then both time-series (processes) x_1 and x_2 affect y in a systematic and significant way. As with the wavelet coherence, a Monte Carlo method is used to estimate the significance levels of the coefficients, in line with Mihanović et al. (2009).

4.7. Multiple wavelet coherence

Contrary to the PWC, a multiple wavelet coherence (MWC) aims to measure the effect of multiple independent variables on a dependent variable. Mihanović et al. (2009) point out that the existing equivalence between multiple regression analysis and cross-spectral analysis can be extended directly to cross-wavelet analysis, with for instance a close analogy between the squared wavelet coherence and traditional correlation coefficients. Keeping this analogy in mind, extending the wavelet coherence to a multiple wavelet coherence is uncomplicated. By utilizing the WTC between y and x_1, y and x_2, and x_1 and x_2 shown in the PWC calculations we get the following equation for the application of MWC:

$$R_m^2(y, x_1, x_2) = \frac{R^2(y, x_1) + R^2(y, x_2) - 2[R(y, x_1) \cdot R(y, x_2)^* \cdot R(x_2, x_1)^*]}{1 - R^2(x_2, x_1)}. \tag{14}$$

Equation (14) yields the resulting wavelet coherence squared that computes the proportion of wavelet power of the dependent time series y which is explainable by the two independent time series x_1 and x_2 at a given time and frequency (Mihanović et al., 2009). Originally, the equation contains a component, ´Re´, in the last object of the numerator that accounts for the complex values originally seen in cross-spectral analysis. However, after engaging in communication with Mihanović et al., we adopted this specific version of the equation without ´Re´. In a response to our request for a further explanation of the component, they concluded that it is redundant for the purpose of our thesis. By their own consideration, equation (14) is still valid after removing the ´Re´ component, as all terms of the equation already are real. To calculate significance sets for the MWC, Monte Carlo simulation is used, as with the other coherence measures (Mihanović et al., 2009).

4.8. Drawbacks of the wavelet methodology

Despite wavelets ability to compress information and detect comovement, the framework has its limitations. As described by Graps (1995), the calculation of wavelet transforms can be computationally intensive, especially for large signals and images. This computational complexity can be a drawback for applications that require real-time processing.

Additionally, wavelets are not always orthogonal, which means that there can be cross-talk and interference between different scale components (Nason, 2008). Consequently, this may lead to inaccurate representation, or introduction of artifacts, in the signals generated by the wavelet transforms. For financial applications, this might be problematic as it can lead to false spikes or oscillations in volatility estimates. This will in turn create an opportunity for over or under estimation of risk. Furthermore, the interference between scale components can result in a loss of resolution at the high frequency components, which makes it difficult to identify possible short-term fluctuations, or trends. Another issue that may occur due to this, is false correlation or comovement of the time-processes. Lastly, as an example, in the case of high-frequency trading, cross-talk may exist between the high-frequency components at one scale and the low-frequency components at the next scale. The accuracy of trading signals generated from wavelet analysis may therefore be erroneous, leading to false positives or false negatives in trading decisions.

While wavelets can provide a multiscale representation of signals and images, this can also make it difficult to determine the optimal scale for a particular signal or image (Addison, 2002). This can impact the accuracy and interpretability of the wavelet representation. Additionally, the number of available basis functions is limited, and some signals or images may not be accurately represented using the available wavelets. This can limit the versatility of wavelets for representing a wide range of signals and images (Mallat, 1999).

Despite these limitations, wavelets are still widely used in many applications due to their ability to provide a multiscale representation of signals and images. This relates to finance in the sense that time series may exhibit volatility clustering, where periods of high volatility are followed by other periods of high or low volatility, and vice versa. These volatility patterns may occur at different time scales, and wavelet analysis can help to identify and characterize them. According to Chui (2014), careful consideration of these limitations can help practitioners to choose the most appropriate wavelets for a particular application and to optimize the use of wavelets to achieve the best results (Chui, 2014).

4.9. Computational framework and software

In this project, we used RStudio (version 2022.07.2+576) as the primary software and the following packages: ‘wavelets’, ‘biwavelet’, and ‘vectorwavelet’. These packages allow for a comprehensive set of functions for wavelet analysis. They additionally provide visualizations enabling interpretation of the findings.

The ´wavelets´ package provides functions for performing continuous wavelet transforms, discrete wavelet transforms and wavelet coherence analysis such as DWT and MODWT (Aldrich, 2020). The ´biwavelet´ package extends the capabilities of the Wavelet package by enabling bivariate wavelet analysis of two time series simultaneously. This package can be used to analyze the phase relationship between two signals at different scales and frequencies (Gouhier, 2021).

The ´vectorwavelet package provides functions for analyzing multivariate signals using wavelets. This package can be used to perform vector wavelet decomposition and reconstruction, which is useful for analyzing the frequency components of a multivariate signal. The "mwc" function was used to analyze multiple wavelet coherence (Oygur et al., 2021).

Overall, the combination of R and these packages presents a powerful toolkit for conducting wavelet analysis and interpreting the results. The visualization functions in these packages were particularly useful for interpreting complex wavelet transforms and identifying meaningful frequency components in our data. The reproducibility of the analysis was ensured by including the specific versions of R and packages used, as well as the script used for the analysis.

5. Results

In this section, we apply the wavelet coherence approach to investigate the relationships between bitcoin returns, hashrate growth rates, and the returns on the energy commodities across a range of time-frequency domains, including short-, mid-, and long-term intervals. The wavelet coherence approach is a powerful analytical tool that can help identify specific time periods and frequency ranges where co-movements between bitcoin returns and hashrate growth rates may occur. In the wavelet coherence plots, the time component is presented on the horizontal axis, while the vertical axis displays frequency values represented in days. Further, the phase relationships between variables are indicated by the arrows. The black outlines on the plots indicate regions that reach the 5% significance level. This significance level is estimated by utilizing Monte Carlo simulations with phase-randomized surrogate series. The solid curve known as the cone of influence indicates the extent to which the edge effect[2] impacts a particular area. This area is shaded. The power spectrum is illustrated using a color gradient, with dark blue representing 0 and dark red representing 1.

The frequencies involved in the coherence analysis we use are equivalent to days. Consequently, a correlation at, for instance, frequency 64 can be interpreted as a 64-day lagged correlation. As our full sample spans 10 years with 2410 observations, a full year would roughly be comparable to the 256-day frequency, a half-year to the 128-day frequency, and so on. When comparing our results with the results of Rehman and Kang (2021), there might be deviations in the frequency aspect resulting from a different number of observations per year in our samples. As mentioned before, Rehman and Kang disclose limited information regarding their data. Thus, we have no knowledge of the number of observations in their sample. As Bitcoin data is available on a daily basis, it might be that their sample contains 365 observations per year, facilitated by interpolation of commodity futures data for no-trading days. If this is the case, a full year in Rehman and Kang's results would be equal to a frequency of 365.

On wavelet coherence plots, arrows indicate the phase. As detailed in Section 4.5, a phase difference of zero implies that the analyzed time series move together at a specific scale

[2] Since wavelets are based on sliding windows, there is no data outside the endpoints of the time series to include in the analysis, and as a result, the wavelet coefficients near the edges of the series may be affected by this lack of data. The cone of influence indicates the area where the edge effect may occur and where the wavelet coefficients should be treated with caution.

(Barunik et al., 2011). If the time series are positively (negatively) correlated, arrows point to the right (left) when they are in-phase (anti-phase) (Gouhier, 2021). An arrow pointing upwards suggests that the second time series leads the first, while an arrow pointing downwards indicates that the first time series leads the second. The second and first time series refer to their positions in the list of variables given in the caption of our plots. Returns on the Bitcoin price are first in all our cases, while the rest of the studied variables alternate as the second variable in our result plots. Typically, there is a combination of positions, such as an arrow pointing down and to the right, indicating that the time series are in-phase, with the second time series leading the first one.

5.1. Wavelet coherence analysis

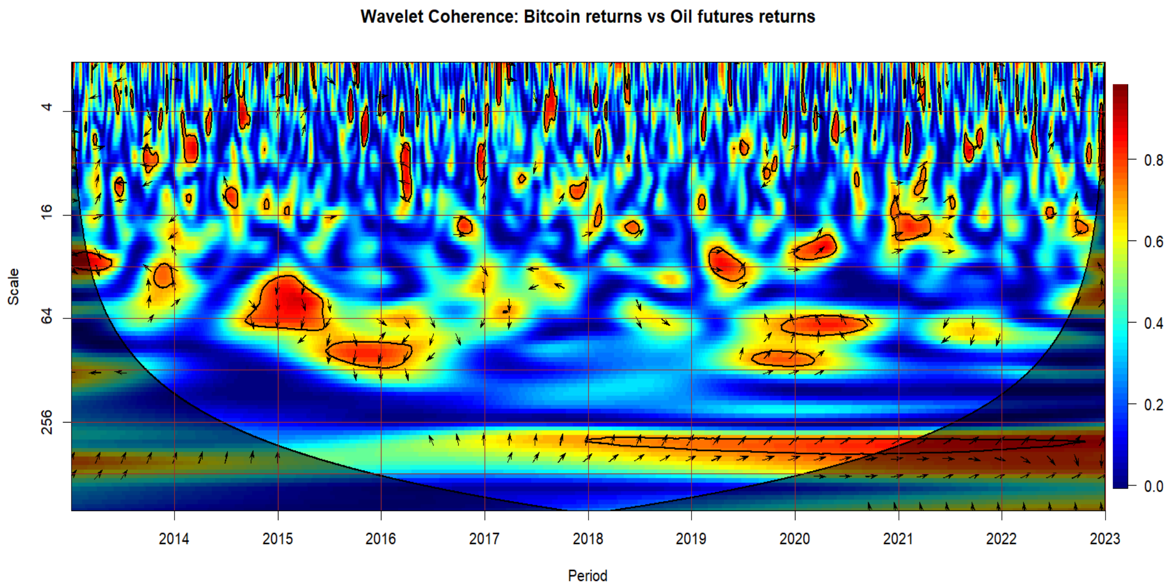


Figure 7 - Wavelet coherence analysis between the bitcoin returns and the returns on oil futures for the sample period (2013-01-01, 2022-12-31

Based on the wavelet coherence plot presented in Figure 7, it is evident that the returns on bitcoin and oil exhibit various degrees of correlation across the time-frequency space, ranging from 2013 until 2023. Notably, a distinctive pattern of 32-128 days is observed from the beginning of 2015 until mid-2016. This pattern indicates a significant correlation between the returns on bitcoin and oil, signaling that bitcoin returns lead oil returns over the specified time scales.

Moreover, the plot suggests that from 2017 until 2021, there is a noteworthy shift in the relationship between bitcoin and oil returns. Specifically, there is a clear indication that oil returns led bitcoin returns from 2017-2019. From 2019-2021, the relationship shifts towards phase synchrony at a frequency of 256 days. Although the relationship shifts towards in-phase, bitcoin returns are still trailing oil returns during 2019 to 2021. This suggests that the returns on bitcoin and oil have become increasingly synchronized over time. There are additionally sporadic significant signs of correlation between bitcoin and oil returns ranging from 16-64-day frequency between 2019 and mid-2022.

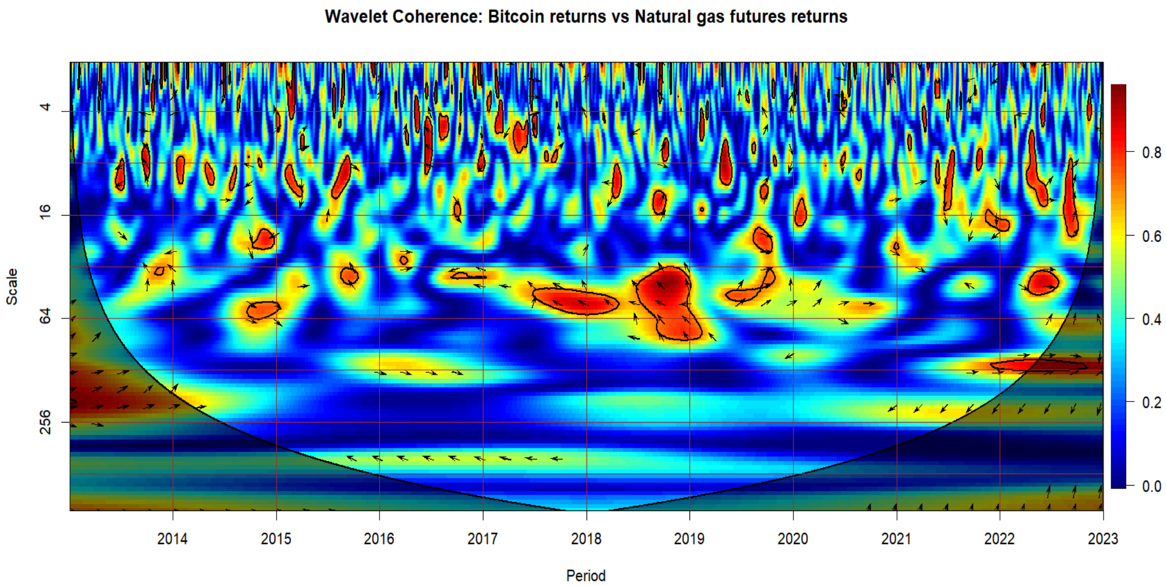


Figure 8 - Wavelet coherence analysis between the bitcoin returns and the returns on natural gas futures for the sample period (2013-01-01, 2022-12-31)

From mid-2017 until 2019, Figure 8 shows that there existed an inverse relationship between the returns on bitcoin and natural gas, with bitcoin returns trailing behind natural gas returns. There are various, temporal regions of significant correlation between the variables on a 4-16-day frequency during the entire sample period. Other than that, there are no obvious patterns of correlation between bitcoin returns and the returns on 2-month natural gas futures.

In Figure 9, there is a pattern of an anti-phase relationship between bitcoin returns and coal future returns, with coal future returns leading bitcoin returns from 2016 until mid-2018. The same pattern continues until 2021; however, the relationships are not significant at the 95% level from 2018 until 2021. Bitcoin and coal returns are in-phase from late 2014 until mid-2015, with a frequency of 64 days. From mid-2015 until 2016, bitcoin and coal returns were in anti-phase, with coal leading bitcoin returns between 16- and 64-days frequency. There is a similar indication of the same relationship from mid-2017 until 2018 on a 32–64-day frequency.

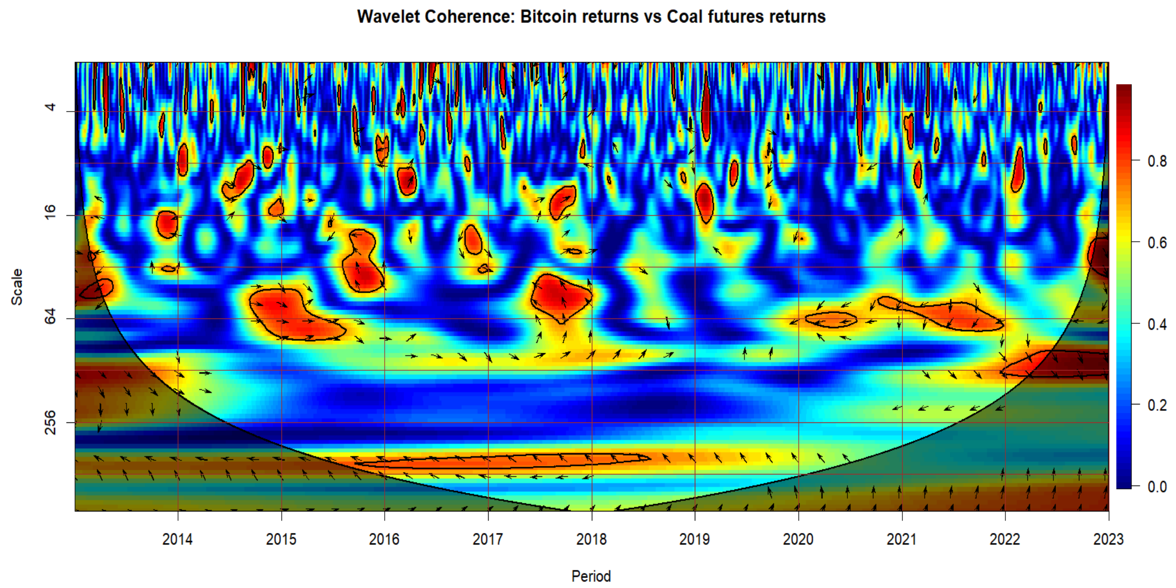


Figure 9 - Wavelet coherence analysis between the bitcoin returns and the returns on coal futures for the sample period (2013-01-01, 2022-12-31)

In Figure 10, we see the wavelet coherence between bitcoin returns and hashrate growth rates. In the first half of 2014, the pair was in anti-phase with 16-32 days frequency. From the middle of 2015 until 2017, hashrate growth rates led bitcoin returns with a 128-day frequency. In the end of 2018, the pair shift to have an in-phase relationship on a 32-64-day frequency until the beginning of 2019. From 2020 until mid-2022, there is a significant region with a 128-256-day frequency indicating that hashrate growth rates led bitcoin returns and that the pair was in-phase. This is the most obvious pattern during the period ranging from 2013 until 2023.

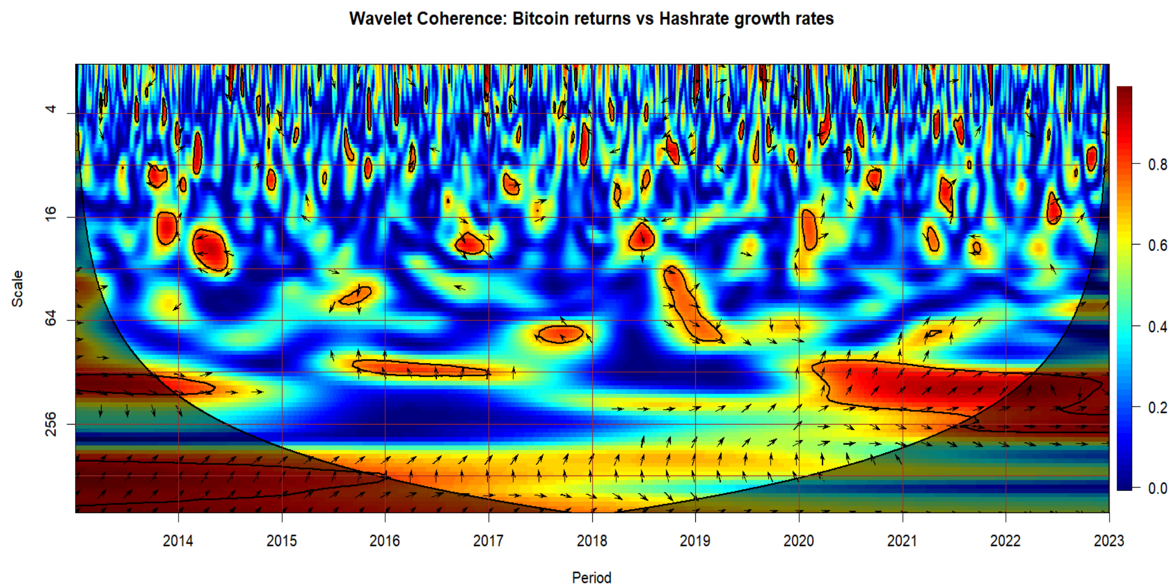


Figure 10 - Wavelet coherence analysis between the bitcoin returns and the hashrate growth rates for the sample period (2013-01-01, 2022-12-31)

5.2. Partial wavelet coherence

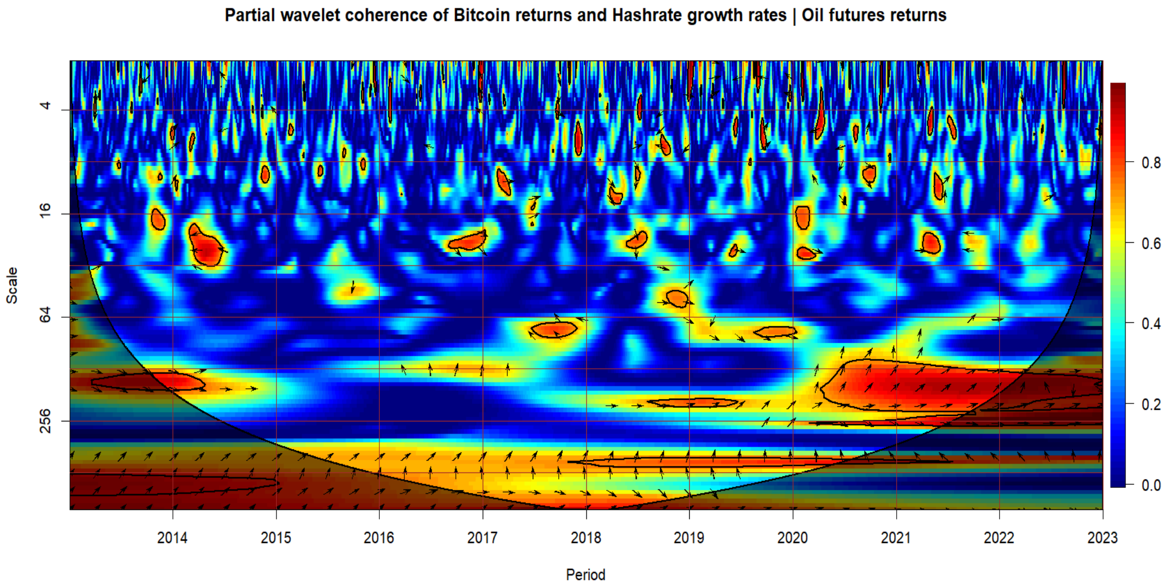


Figure 11 - PWC analysis of the effect of the hashrate growth rates on the bitcoin returns when controlling for the effects of returns on the oil futures for the sample period (2013-01-01, 2022-12-31)

Figure 11 displays the partial wavelet coherence between bitcoin returns and hashrate growth rates while controlling for the effect of the returns on two-month oil futures. Specifically, in early 2014, a co-movement between bitcoin returns and hashrate growth rates was observed at the frequencies of 16-32 days and 128-256 days. However, until late 2016, after mid-2014, no significant co-movements were detected. Subsequently, sporadic regions indicating co-movement between bitcoin returns and hashrate growth rates were observed at various frequencies between late 2016 and 2018, although these regions exhibit inconsistency over time. From 2018 onwards, a relationship between bitcoin returns and hashrate growth rates is found at frequencies of 256-512 days when controlling for the impact of oil returns. Furthermore, a significant co-movement between bitcoin returns and hashrate growth rates is observed at frequencies of 128-256 days, ranging from mid-2020 until mid-2022, which appears to be the most stable and consistent region throughout the analyzed period.

The presented partial wavelet coherence plot in Figure 12, which controls for the influence of returns on natural gas futures, reveals a comparable pattern to the previously discussed plot. The plot exhibits intermittent regions that indicate temporal comovement between the bitcoin returns and the hashrate growth rates across all frequencies from 2013 to 2020. Specifically, the region in early 2014, with a frequency of 16-32 days, displays similar patterns as the one observed in the earlier plot. Additionally, the region observed in the latter part of 2017 shows analogous features to the previous plot when controlling for the effect of oil on a 64-day frequency.

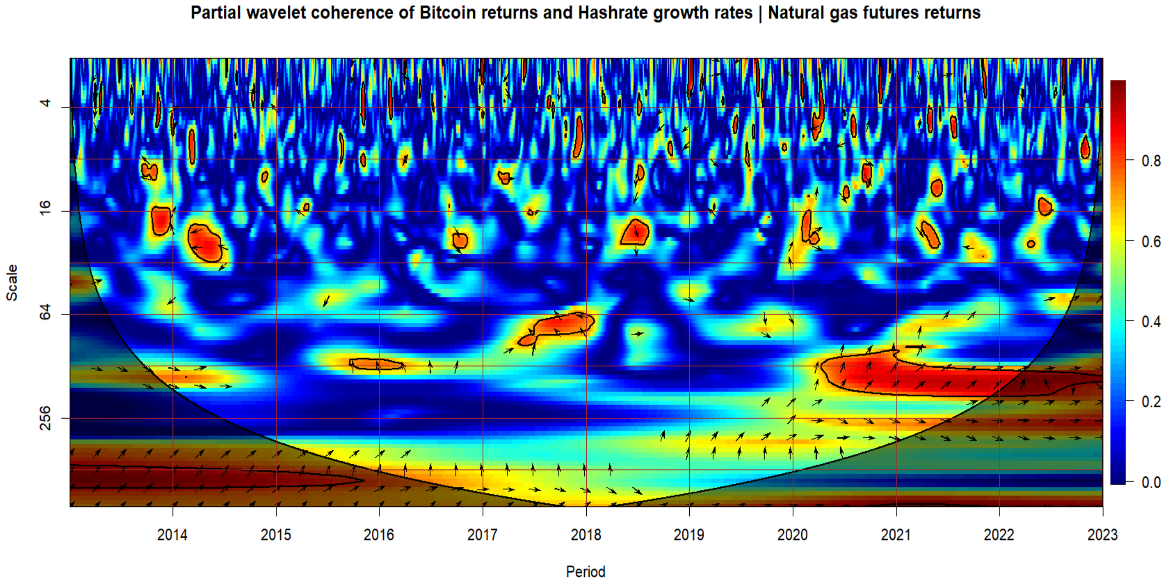


Figure 12 - PWC analysis of the effect of the hashrate growth rates on the bitcoin returns when controlling for the effects of returns on the natural gas futures for the sample period (2013-01-01, 2022-12-31)

Of particular note is the prominent region from mid-2020 until mid-2022 that appears to persist up to the current time frame. This specific pattern of comovement is also present when accounting for the effect of oil rather than gas, and is significant over a 2-year duration on a 128-256-day frequency. This finding suggests a robust correlation between the bitcoin returns and the hashrate growth rates, even after controlling for external factors.

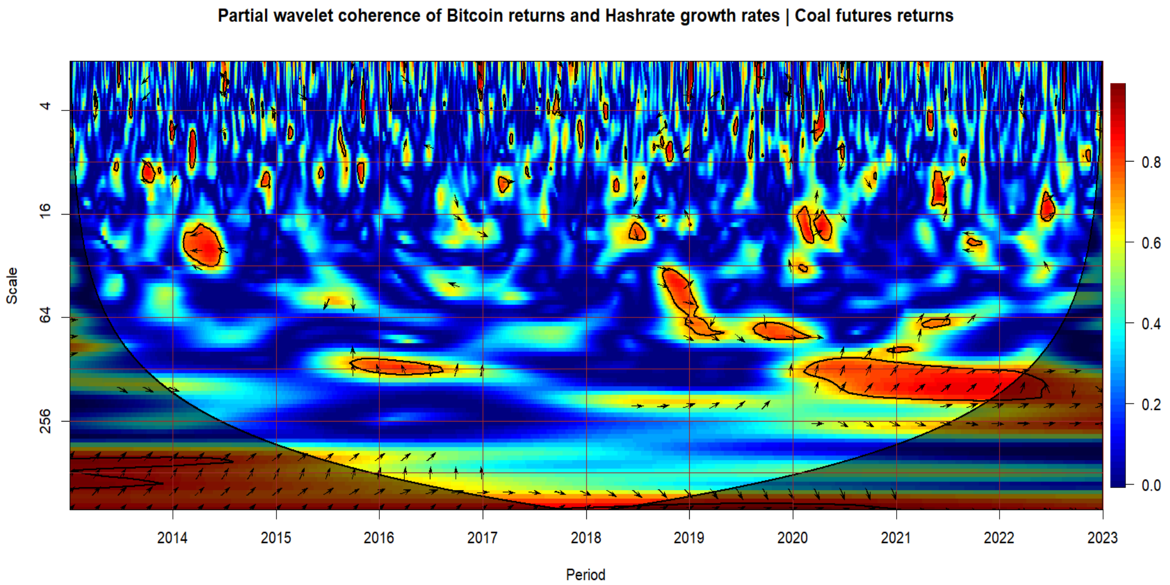


Figure 13 - PWC analysis of the effect of the hashrate growth rates on the bitcoin returns when controlling for the effects of returns on the coal futures for the sample period (2013-01-01, 2022-12-31)

When analyzing the correlation between the bitcoin returns and the hashrate growth rates, while controlling for the effects of returns on the coal futures, the PWC analysis presented in Figure 13 shows hardly any notable areas of significant correlation. Similar to the analyses controlling for returns on the oil and gas futures, the only major area of significant correlation appears at the beginning of 2020 and stretches well into 2022. This area lies within the period of 128-256 days. Otherwise, just a few smaller areas of significant correlation are apparent. In the period of 16-32 days, significant correlation occurred for half a year from the beginning of 2014. Moreover, during the period around 2016, a notable correlation pattern emerged at the 128-day frequency, and from 2019 to 2020, smaller areas of correlation within the 32-64-day range were observed.

5.3. Multiple wavelet coherence

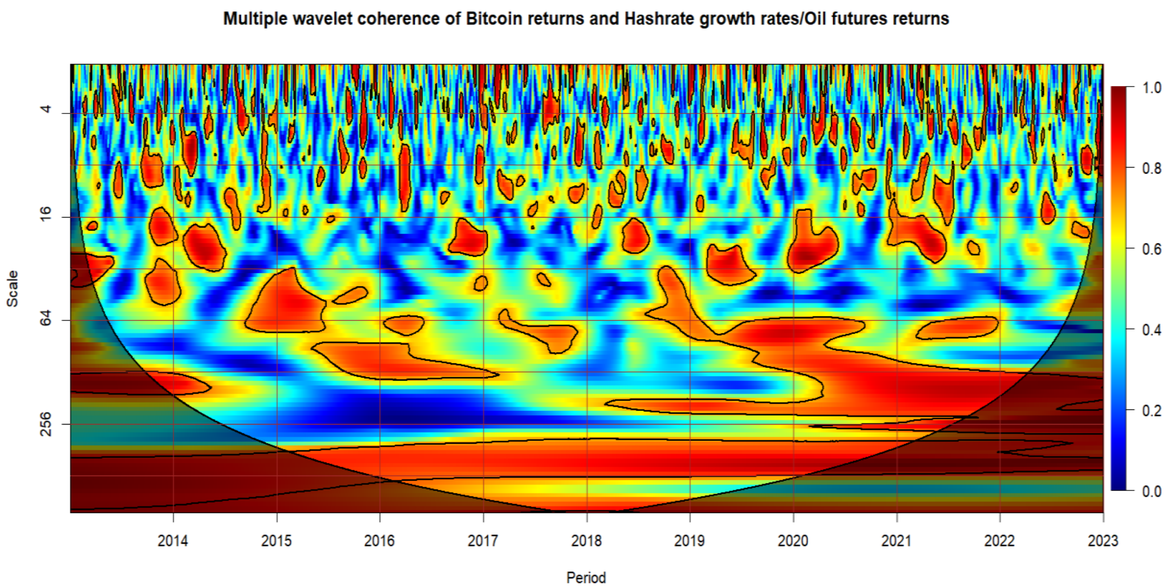


Figure 14 - MWC analysis of the combined effect of hashrate growth rates and returns on oil futures on bitcoin returns for the sample period (2013-01-01, 2022-12-31)

By factoring in, instead of out, the effect of both hashrate growth rates and the returns on the energy commodities, it is apparent that significant correlation is much more present throughout the time and frequency spectrum for the MWC analyses than the PWC analyses. From Figure 14, showing the MWC analysis on bitcoin returns and the combined effect of hashrate growth rates and returns on the oil futures, we see considerable areas of significant correlation across several parts of the timeline for all frequencies between 16 and 512 days. The only major exception without significant correlation is a period between 2015 and 2017 for the 128–256-day frequency. For higher frequencies than 16 days, significant correlation is found across the timeline in small areas. Overall, we find that there are four prominent main areas of significant correlation for the bitcoin returns and the combined effect of the hashrate growth rates and the returns on the oil futures. From mid-2014 until mid-2015, we detect a significant correlation in the 32–64-day period. Throughout 2015-2017, with parts of 2018, there was also significant correlation in the period of 64-128 days. Perhaps the most interesting part of the plot is the band of significant correlation spanning across the whole sample for the period of 256-512 days. Additionally, we find an extensive region of significant correlation toward the end of the sample, beginning in 2019 and extending to the end of 2022, with a presence in the period of 64-256 days.

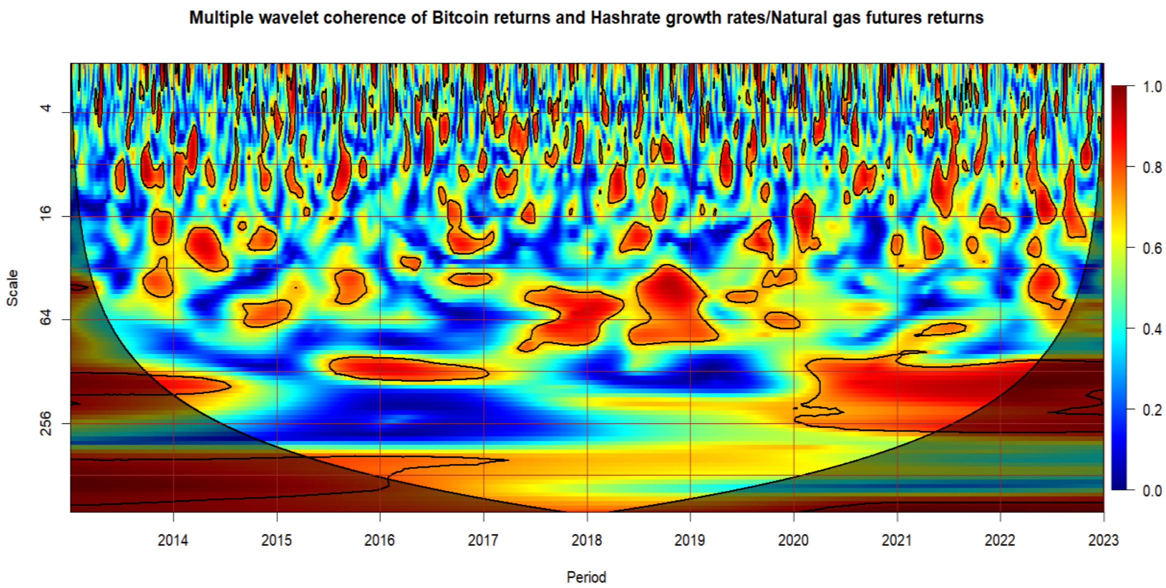


Figure 15 - MWC analysis of the combined effect of hashrate growth rates and returns on natural gas futures on bitcoin returns for the sample period (2013-01-01, 2022-12-31)

Similar to the MWC analysis covering the combined effect of the hashrate growth rate and returns on the oil futures on bitcoin returns, the MWC analysis for the hashrate growth rate and returns on the natural gas futures shows several areas with significant correlation across the whole spectrum. From Figure 15, we observe one major area of significant correlation and three additional noteworthy areas. From 2020-2023, significant correlation is present continuously for a period of 128-256 days. Halfway through 2015, an area of significant correlation that ends in the beginning of 2017 emerges at the 128-day period. In addition to this region, two other areas of significant correlation can be observed between mid-2017 and slightly into 2019 for periods ranging from 32 days to between 64 and 128 days. As with the MWC results for the combined effect of hashrate growth rate and returns on the oil futures on bitcoin returns, this analysis also shows small areas of significant correlation scattered all over the higher frequency region.

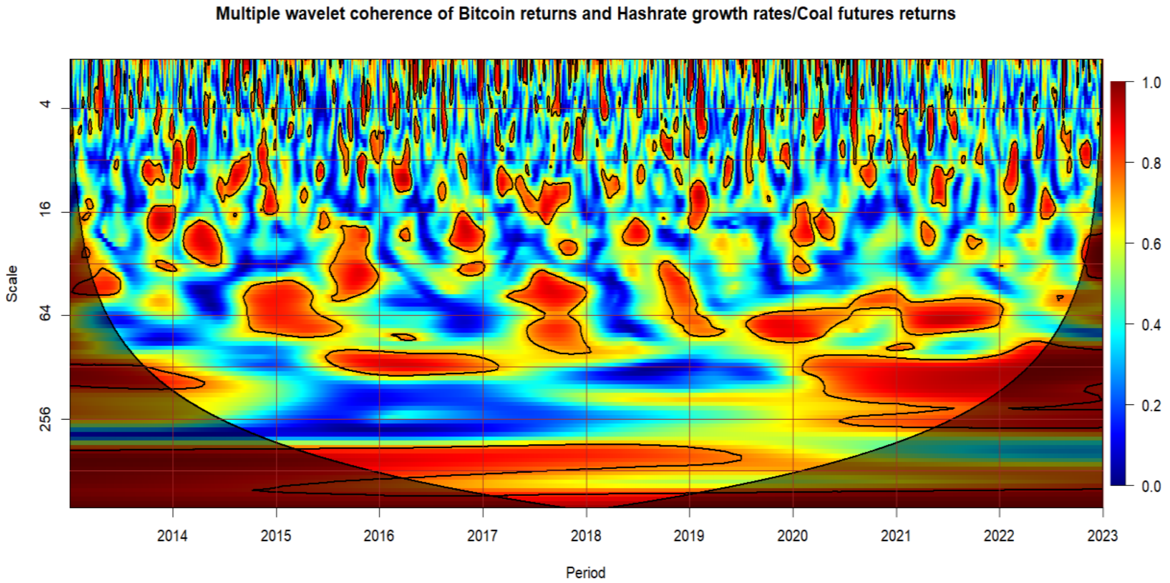


Figure 16 - MWC analysis of the combined effect of hashrate growth rates and returns on coal futures on bitcoin returns for the sample period (2013-01-01, 2022-12-31)

The results of analyzing the MWC for the combined effect of hashrate growth rate and returns on the coal futures on the bitcoin returns are presented in Figure 16. These results share similarities with both previous MWC analyses presented in Figure 14 and Figure 15. All three MWC analyses have a prominent area of significant correlation at the end of the sample period, beginning around 2020 and covering a period of 128–256 days. Additionally, the hashrate growth rate and coal returns MWC analysis identifies a region around the 256–512-day period quite similar to the band of significant correlation also found in Figure 15 for the hashrate growth rate and oil returns MWC analysis. However, for this analysis, the band extends only until the end of 2019. Further, the hashrate growth rate and coal returns combined show several areas of significant correlation to the bitcoin returns in the region of 32–128-day periods from 2015 until 2022, which is between half a year and one and a half years. Again, for the higher frequency periods, small areas of significant correlation are found all across the timeline.

5.4. Summary

Our wavelet coherence analysis, both in the bi- and multivariate form, renders various interesting findings. An outstanding feature that surfaces repeatedly throughout our analysis is the low frequency connectedness at 128-256 days between the bitcoin returns and the hashrate growth rate from the beginning of 2020 until the end of our sample (2023). This observation is present in all analyses including the hashrate growth rates, substantiating the claim of a low-frequency connectedness between the bitcoin returns and the hashrate growth rates from the beginning of 2020. The regions of comovement are enlarged when including the effect of the returns on two-month oil futures, as done in the MWC. We know from the wavelet coherence plot in Figure 7 that there are areas of high coherence between bitcoin returns and returns on the oil futures in the mid- to low-frequency region from 2019 onwards. Therefore, it makes sense that the areas of comovement are more substantial when including returns on the oil futures in the MWC between bitcoin returns and hashrate growth rate.

By considering the three different types of analyses; wavelet coherence, PWC, and MWC, some patterns emerge. Most obvious is the difference between including and excluding the effects of the returns on the energy commodity futures on the coherence between bitcoin returns and hashrate growth rates. It is clear that when we control for the effects of the returns on the commodities, far less coherence is observed between the returns on bitcoin and hashrate growth rates than when we include the returns on the commodities into the coherence analysis. Intuitively, this makes sense, as the MWC is a result of the combined effect of two variables on the dependent variable, as opposed to just one variable's effect alone. Visually, it also appears from the basic wavelet coherence plots that returns on the oil futures and hashrate growth rates are the two variables with the highest amount of coherence with the bitcoin returns. Further, it seems that oil futures returns, followed by coal futures returns, have the biggest impact on the combined effect together with the hashrate growth rates towards the bitcoin returns, as shown by the MWC results. Lastly, by factoring out the returns on the commodity futures, all three resulting plots share a resemblance to each other. As the PWC in practice is a coherence analysis between bitcoin returns and a more isolated effect of the hashrate growth rates for all cases, the outcome is not entirely unexpected.

Our findings exhibit some similarities and differences with the results of the study conducted by Rehman and Kang. By examining their coherence plots, included in the appendix Section 9.1, we specifically observe that our research reveals comparable wavelet coherence between the bitcoin returns and the returns on the oil futures. Additionally, the wavelet coherence between bitcoin returns and hashrate growth rates appears to be similar in both studies. However, discrepancies are observed in the wavelet coherence between bitcoin returns and coal returns, and bitcoin returns and natural gas returns. These disparities align with the discrepancies in the descriptive statistics we have outlined in Section 3.4.

The partial wavelet coherence plots, controlling for the effects of returns on the oil futures, coal futures, and gas futures, yield analogous results when examining the relationship between bitcoin returns and hashrate growth rates. As the data related to bitcoin price and hashrate aligns with the dataset employed in the study by Rehman and Kang, we anticipated similar findings, given the elimination of the effects of energy commodities where data discrepancies exist. The multiple wavelet coherence analysis also produces certain discrepant outcomes, as predicted. The MWC of bitcoin returns and hashrate growth rates when including the influence of oil futures returns aligns with the results obtained in the study by Rehman and Kang. Surprisingly, the MWC of bitcoin returns and hashrate growth rates combined with the impact of natural gas futures returns reveals similar outcomes in both studies. This may stem primarily from the comovement between bitcoin returns and hashrate growth rates, as the addition of natural gas futures returns does not produce any dissimilarities in our findings relative to those of Rehman and Kang. The inclusion of returns on the coal futures produces some resemblances; however, some deviations are observed.

To summarize, our analysis reveals similar results to the study conducted by Rehman and Kang regarding bitcoin returns, hashrate growth rates, and returns on two-month oil futures. However, it is worth noting that while our analysis generally diverges from the results obtained by Rehman and Kang in relation to the analyses containing returns on two-month futures on natural gas and coal, there were some instances where similarities were observed.

6. Discussion

This section of our thesis will discuss the findings from our analysis, trying to both understand the underlying events causing the results and explain the implications they have from a financial perspective. The price dynamics of Bitcoin have been a subject of debate since the cryptocurrency's adoption as a digital currency. Our study has focused on exploring the underlying relationships between the Bitcoin price and hashrate nexus. Considering the complexity of this relationship, we believe it is interesting to examine how the impact of two-month futures on traditional commodities, such as oil, coal, and natural gas, may affect their relationship. By exploring the effects of these commodities, we can better understand the extent to which external market factors may influence the Bitcoin market.

6.1. Interpretation of the results

From our analysis, we observed that a region of significant coherence is present between the bitcoin returns and hashrate growth rates throughout the results from 2020-2023 at a 128-256-day frequency. As the hashrate growth rates lead the returns on bitcoin for the whole region, this implies a low-frequency, or long-run, correlation where the returns on bitcoin follow the relative changes in the hashrate growth rates. If we put this period in perspective of events that happened in the economy, it is reasonable to assume that it is linked with the effects of the COVID-19 pandemic and then subsequently the European energy crisis caused by Russia’s invasion of Ukraine. Intuitively, it is logical that during this period, where there has been greater economic uncertainty than the decade prior to it, the pricing of Bitcoin is more affected by fundamental factors, such as the input cost of mining bitcoins. As investors generally have less resources to speculatively invest in Bitcoin, it is more sensible that an estimate of the fundamental value of Bitcoin plays a greater role in the pricing. Grounded in the paper by Garcia et al. (2014), we have previously argued that the cost of mining, measured by hashing, can serve as a lower bound estimate of Bitcoin’s fundamental value. Following that connection, the major coherence observed between the returns on bitcoin and hashrate growth rates from 2020-2023 is quite reasonable. In plain words, this coherence shows that hashrate growth rates are leading the returns on bitcoin with roughly a half to a year’s lag from the beginning of 2020 until the end of 2022, indicating that the bitcoin returns, and therein the price of Bitcoin, are affected by aspects of mining that are reflected in the hashrate. Additionally, the return-on-bitcoin’s sensitivity to input costs of mining during the period of higher economic uncertainty is further highlighted by the results of the MWC analysis. Specifically, the inclusion of returns in the energy commodities on the coherence between bitcoin returns and hashrate growth rates generates more pronounced coherence between the combined effects of the hashrate growth rates together with returns on each of the commodities and the bitcoin returns.

A key takeaway from the results of our analysis is the differences in how much the commodity variables affect the movement of the bitcoin returns. Based on the energy mix used in the Bitcoin network, it could be expected that the gas and coal futures would have the greatest effect on the bitcoin returns and hashrate relationship, as they are the major sources of energy used in Bitcoin mining (de Vries et al., 2022). In the partial wavelet coherence analysis, we observed that the coherence between bitcoin returns and hashrate growth rates diminished in certain regions when the effect of energy commodities was filtered out. Specifically, controlling for the effect of two-month futures on coal and natural gas led to a more pronounced reduction in coherence compared to controlling for two-month oil futures. This finding is expected, given that coal and natural gas are the primary input commodities used in electricity generation for Bitcoin mining. Therefore, ignoring their effects would result in a loss of coherence in some regions. Conversely, oil is not a typical input factor in the production of electricity and consequently not in Bitcoin mining; hence, its effect on the coherence between bitcoin returns and hashrate growth rates is arguably less significant. This assumption is strengthened by the results obtained from the partial wavelet coherence analysis, as most regions of coherence remained rather similar to the wavelet coherence between bitcoin returns and hashrate growth rates.

An example is the notable temporal correlation between the returns on bitcoin and hashrate growth rates at the end of 2018 and the beginning of 2019. This coherence was observed across frequencies spanning from 32 to slightly beyond 64 days. Similarly, coherence between the returns on bitcoin and gas futures was also witnessed at the same frequencies and time. It could be deduced that gas futures may account for the observed coherence between the returns on bitcoin and hashrate growth rates based on these observations. This assertion was confirmed by partial wavelet coherence analysis, which filtered out the impact of natural gas futures. The abovementioned region of coherence diminished considerably, with only a negligible region of coherence remaining. These findings suggest that the relationship between the returns on bitcoin and hashrate growth rates could be significantly impacted by the returns of the energy commodities, specifically natural gas and coal, but also oil.

On the other hand, the returns on bitcoin and oil futures have some regions of coherence, especially in the low frequencies from 2018 all the way until 2023. These regions of coherence intensified after early 2020, indicating that events such as the COVID-19 pandemic might have influenced their relationship. Notably, the coherence between the returns on bitcoin and two-month oil futures exhibited an in-phase relationship during this period, as the price of both assets increased considerably from March 2020 to mid-2021, a period that lies within the cone of influence in the wavelet coherence plot. It is plausible to argue that the leading relationship between oil returns and bitcoin returns suggests that a rise in oil prices may be a symptom of economy-wide inflationary processes. Consequently, this could result in increased returns on bitcoin and other assets. On the other side, Bilušić (2022) argues that people who spend more money buying necessities in times of economic distress spend less money investing in cryptocurrencies, suggesting that there might be a vague macro-relationship between the returns on two-month oil futures and bitcoin. This could be an avenue for future research.

The multiple wavelet coherence analysis has revealed that the returns on bitcoin and hashrate growth rates exhibit temporal coherence with all energy commodity futures. The analysis shows a higher degree of coherence as the resulting output effectively captures the interrelated relationships among three variables simultaneously, surpassing the limited scope of pairwise coherences. The most notable finding is the sustained coherence observed when considering the combined effect of hashrate growth rates and the impact of two-month oil futures. This coherence exists at a frequency range of 256 to 512 days throughout the entire sample period, suggesting that returns on oil futures could have a continuous effect on the relationship between bitcoin returns and hashrate growth rates in the long term. However, the results obtained from the partial wavelet coherence analysis indicate that this enduring relationship may be spurious. Specifically, the returns on oil futures exhibit coherence at the same frequency range from 2018 until the end of the sample period, whereas the coherence between the returns on bitcoin and hashrate growth rates is observed at that same frequency range from the start of the sample period until 2018. These findings demonstrate the potential value of using a combination of analyses to gain a deeper understanding of the underlying mechanisms driving the observed relationships. Specifically, the use of multiple analyses may function as a means of checking the robustness of the findings and can enable researchers to better discern the validity and significance of their findings.

6.2. Financial perspective

6.2.1. Diversification potential

As our analysis serves the purpose of gaining a better understanding of the factors affecting the returns, and in turn the price, of Bitcoin, it also provides insight into potential implications for its use from a financial perspective. We explore the use of wavelet coherence as a dynamic measure of correlation in frequency and time, which has potential applications in portfolio theory and portfolio construction. Regions with low wavelet coherence between two assets indicate that they have low correlation at that given time and frequency, suggesting an opportunity for portfolio diversification during that period with an investment horizon corresponding to the given frequency. Additionally, regions of high coherence that are out of phase correspond to negative correlation and will therefore also serve well as potential diversification opportunities. By employing these principles, we argue that the energy commodities used in our analysis have diversification potential in a portfolio together with Bitcoin for long-term investments. They all exhibit low to zero coherence with the bitcoin returns at the 128-256-day frequency for the whole period contained in the cone of influence, roughly spanning from early 2014 until the beginning of 2022. Moreover, we observe that returns on gas futures exhibit an out-of-phase coherence with bitcoin returns at the 32-64-day frequency from mid-2017 until the beginning of 2019, suggesting high diversification potential for portfolios including Bitcoin and gas futures with medium-term investment horizons during this period. This area stands out as the only notable out-of-phase area of coherence in the wavelet coherence analysis between the returns on bitcoin and the energy commodities. Additionally, there are various other areas of low to zero coherence between returns on bitcoin and the energy commodities that could possibly imply diversification opportunities if an investor is able to capture the patterns of low coherence.

6.2.2. Investment opportunities

Considering that out-of-phase and in-phase relationships correspond to a negative and positive correlation, respectively, the phase relationship also reveals information regarding the lead or lag dependency of the two variables analyzed. With information regarding lead or lag dependency, opportunities to invest based on this could arise. If a coherence analysis reveals that one variable is lagging the other in an in-phase relationship at a given frequency, it implies that an increase (decrease) in the leading variable today will lead to an increase (decrease) in the lagging variable X number of days in the future, corresponding to the frequency where the coherence is present. The opposite relation would apply for an out-of-phase relation. An investor must be aware that wavelet analysis only looks back in time, so it would therefore be necessary to forecast patterns of coherence in order to exploit future investment opportunities. Based on our findings, there are a handful of these relationships worth considering among our variables. Firstly, we observe that the returns on oil futures lead the returns on bitcoin from 2018 to 2022 at a 256-day frequency with an in-phase relation. This implies that positive (negative) returns for oil futures at any point during 2018-2022 should lead to positive (negative) returns for bitcoin approximately one year later. In contrast, the bitcoin-oil pair exhibits an opposite relationship from 2015-2016, where returns on bitcoin lead returns on oil at a 32-128-day frequency during an out-of-phase relationship. Here, positive (negative) returns on bitcoin between 2015 and 2016 should lead to negative (positive) returns for oil futures between one month and six months later.

Further, both returns on gas and coal futures lead returns on bitcoin in out-of-phase relations. The relation between gas and bitcoin returns is present from mid-2017 to early 2019 at a 32-64-day frequency, while returns on coal and bitcoin have relations both from 2016 to mid-2018 at a 256-512-day frequency and from 2017-2018 at a 32-64-day frequency. In both cases, positive (negative) returns on either gas or coal futures during the relevant periods of coherence should lead to negative (positive) returns on bitcoin at a later point in time corresponding to the frequencies of their relationships.

Our most significant finding is the area of coherence between returns on bitcoin and the hashrate growth rates from 2020 to well into 2022, which remains significant at a 128-256-day frequency for all coherence analyses between returns on bitcoin and the hashrate, including both bivariate and multivariate cases. This relationship is consistently in phase, with the hashrate growth rates leading returns on bitcoin. Thus, we expect a positive (negative) growth rate for the hashrate to lead to a positive (negative) return on bitcoin half a year to a year later during this period. Put simply, increased mining activity during this period should result in higher Bitcoin prices six months to a year later. If these lead/lag relationships hold, investors could potentially trade on the assets in our study based on this information.

6.2.3. Causality

However, a cross-correlation by itself, which is what the lagged coherence represents, is not enough to determine one variable's movement in the future based on its correlated variable's movement today. In order to substantiate a hypothesis like this, proof of a causal relationship between the assets is essential. Since there is no causality testing in our research, no causal relationships can be established. Having said that, we do conduct analyses that may hint towards signs of causality. The PWC and MWC analyses enable us to assess the coherence between bitcoin returns and hashrate growth rates both when factoring out the effects of each energy commodity separately and including the effects of these as well. By factoring out the effects of the energy commodities, we can observe which areas of coherence remain between the returns of bitcoin and the hashrate growth rates when the influence of oil, gas, and coal futures is accounted for. Naturally, there are several other factors that may influence the bitcoin returns and hashrate growth rate nexus. However, despite factoring out the influence of each of the energy commodities, the significant low-frequency coherence from 2020 extending into 2022 remains relatively constant. Additionally, when including the effects of each energy commodity in the coherence analysis between returns on bitcoin and hashrate growth rates, as done in the MWC, the one apparent area of significant coherence that prevails is the same area at low frequency from 2020 into 2022. The fact that this area continues to exist even when influence from other factors is counted out or included may hint towards a causal relation between the returns on bitcoin and the hashrate growth rates.

6.2.4. Remarks on financial modelling

More generally, we find it interesting to examine the resulting plots of our analysis and observe that practically no stable relations exist between the researched variables. From a financial perspective, this could imply that non-dynamic modeling of financial time series data may not suffice to fully comprehend the relationship between different processes. Based on our own results, a potential outcome of said shortcomings could be that a non-dynamic model shows no correlation between the returns on bitcoin and the hashrate growth rates, thus resulting in investors overlooking the influence of hashrate growth rates on bitcoin returns in recent years, as uncovered by our wavelet analysis. This underscores the importance of being cautious when relying solely on the results of a single model to analyze financial data. As advancements in technology and financial modeling continue to occur, investors may derive immense benefits from adopting more sophisticated techniques to uncover potentially concealed relationships.

6.3. Overview of findings

Conclusively, the analysis reveals mostly temporal relationships, although we have an ongoing pattern of coherence between bitcoin returns and hashrate growth rates from 2020 until the end of the sample period. While there are indications outside the cone of influence that the relationship may continue, the certainty of this occurrence remains speculative, as the model lacks the ability to forecast whether this association will endure or if it is merely temporary. Regarding our analyses' link to real-world happenings, it is surprising that the returns on the coal futures affect the relationship between the returns on bitcoin and hashrate growth rates even though Bitcoin mining was banned in China. However, as the major participants of the mining network relocated their operations to countries such as Kazakhstan, where they were also able to power their operations by coal, the overall consumption of coal to produce electricity for mining remained rather stable, and thus we were not able to observe significant differences as a consequence of these regulatory implementations. The major influencers of the Bitcoin-hashrate relationship are two-month futures on coal and natural gas, with occasional links to oil futures at certain times and frequencies. The financial implications of our findings include hedging and diversification opportunities for portfolios containing Bitcoin and two-month futures of Brent crude oil, natural gas, and coal. It is essential to note that investors would need to forecast future coherences and would likely need to base that investment decision on other frameworks beyond solely wavelet analysis. Nonetheless, this research sheds light on some of the relationships affecting the Bitcoin-hashrate nexus while controlling for the effect of two-month futures on Brent crude oil, natural gas, and coal, providing a better understanding of the dynamics in these markets.

7. Conclusion

Motivated by the research of Rehman and Kang (2021), this thesis studies the time and frequency connectedness between the Bitcoin price, hashrate, and energy commodity futures in both time and frequency domains. The study utilizes data on the prices of Bitcoin, two-month futures on Brent crude oil, natural gas, and coal, as well as the Bitcoin hashrate from January 1st, 2013, until December 31st, 2022, which is an extension of the dataset used in Rehman and Kang's research. This extension provides an opportunity to examine the possible relationships among the variables during periods of uncertainty in the global economy that were not studied in previous research. To enable comparability, all variables were transformed into logarithmic growth rates. Furthermore, we utilized a wavelet analysis framework to capture bivariate and multivariate correlation in the time and frequency domains. Specifically, we employed the wavelet coherence method for bivariate analysis and partial and multiple wavelet coherence methods for multivariate analyses.

Our study is consistent with the existing literature on the topic, as discussed in the literature review presented in Chapter 2, which demonstrates the existence of relationships between Bitcoin price and hashrate, as well as Bitcoin price and energy commodities. Notably, we observe significant areas of coherence primarily at frequencies ranging from 32-256 days. These findings are consistent with those of prior studies, including Bhambhwani et al. (2019), Lin and An (2021), Moussa et al. (2021), and Rehman and Kang (2021), which have identified long-term relationships between Bitcoin price and either hashrate or energy commodities. Our analysis is particularly aligned with the results of Rehman and Kang (2021), which highlight the importance of accounting for the effects of energy usage and commodities when examining the link between Bitcoin prices and hashrate. Overall, our findings contribute to the growing body of literature on this topic and reinforce the need for further research to fully understand the mechanisms driving the relationship between Bitcoin prices and Bitcoin mining, represented by the hashrate.

As anticipated, the returns on gas and coal futures, being the primary sources of electricity used in Bitcoin mining, exert the greatest influence on the relationship between bitcoin returns and hashrate growth rates. This observation is supported by the partial wavelet analysis, where eliminating the impact of returns on gas and coal futures results in a more significant loss of areas of coherence between bitcoin returns and hashrate growth rates compared to eliminating the effect of oil returns. This finding suggests that certain areas of coherence between bitcoin returns and hashrate growth rates mainly arise from the influence of the returns on gas or coal futures on the hashrate growth rate. Thus, the loss of coherence after removing the effects of gas and coal signifies their impact on the Bitcoin price-hashrate relationship. Further, we observe that when including the effects of the energy commodities in the multivariate wavelet analysis, returns on oil futures, in combination with the returns on bitcoin and hashrate growth rates, cause the greatest degree of areas with significant coherence. While returns on oil futures have a relatively weaker influence on the Bitcoin price-hashrate nexus than the other commodities, including the effects of oil returns into the relationship between bitcoin returns and hashrate growth rates naturally increases coherence. Even though the long-term coherence at a 256-512-day frequency that lasts throughout the sample period seems to be persistent, it could be coincidental, as argued in Chapter 6. Other than that, all results from the MWC analysis exhibit greater coherence than those from the wavelet coherence and PWC analyses, which is unsurprising given the inclusion of two variables influencing a dependent one.

We believe that our contribution to the literature brings new, interesting findings to the topic. By including data until the end of 2022, we have been able to capture a prominent relationship between the Bitcoin price and hashrate from 2020 that may reveal new characteristics of the Bitcoin dynamics. During this period involving more economic uncertainty due to COVID-19 and the subsequent situation with Russia and Ukraine, we observe an undisputable correlation where the hashrate growth rates drive the returns on bitcoin at a 126-258-day frequency. According to Bilušić (2022), people spend less money investing in cryptocurrencies during financial distress. Our analysis suggests that such situations result in the pricing of Bitcoin becoming increasingly reliant on its fundamental value, which is based on mining costs. Our findings thus lend support to the theory of Garcia et al. (2014), who argue that the fundamental value of Bitcoin should, as a minimum, cover the cost of producing one bitcoin through mining. As such, we believe there is support for the hypothesis that the pricing of Bitcoin is increasingly influenced by mining operations during times of greater economic uncertainty or decline.

As an implication of our findings, we argue that regions of low or out-of-phase coherence between variables in our results may provide opportunities for diversification in portfolio management. We observe these regions for returns on all commodity futures and the hashrate growth rate against the returns on bitcoin, at various periods and frequencies. Additionally, the information on phase relationships from the wavelet coherence analyses can indicate cross-correlative relationships that provide trading signals for investors. If causality can be established, an investor may then invest in a lagging asset X number of days, corresponding to the frequency of the coherence, after the leading asset yielded positive returns. Again, all variables in our study are associated with regions of either a lead or lag relationship against the returns on bitcoin at various frequencies and time periods. It is important to note that investors would need to forecast these regions of coherence. This may require the use of statistical modeling or simulation techniques beyond wavelet analysis alone. Lastly, we find it relevant to briefly mention our observations on the lack of stable relationships among the studied variables throughout our results. We cannot draw any conclusions based on the observations in our study alone. However, by assessing additional studies employing wavelet techniques, we recognize that this observation is a recurring feature of the relationships between variables resulting from the wavelet coherence analyses. Thus, we do think it sheds a light on potential shortcomings of traditional static modeling.

While wavelet coherence analysis can be a powerful tool for identifying cross-correlative relationships between financial assets, it is important for investors to consider its limitations. As outlined in Section 4.8, wavelet analysis may not always accurately represent the underlying data due to cross-talk and interference between different scale components. This can lead to false correlation or comovement of the time processes, and the accuracy of trading signals generated from wavelet analysis may be erroneous. Additionally, the loss of resolution at high-frequency components makes it difficult to identify possible short-term fluctuations or trends, potentially leading to an over- or underestimation of risk. Also, practitioners can choose the most appropriate wavelets for a particular application and optimize their use to achieve the best results; however, choosing the wrong wavelet may lead to misinterpretation of the signals, which could result in inaccurate analysis and flawed investment decisions. Therefore, it is crucial for practitioners to have a solid understanding of the strengths and limitations of wavelet analysis and to carefully consider their choice of wavelets in order to minimize the risks associated with using this technique in financial applications. Despite these limitations, wavelets remain useful in identifying and characterizing patterns of coherence between time series at different times and frequencies.

Future studies in this field to further explore the importance of energy prices on the Bitcoin or cryptocurrency price-mining relationship could research more isolated sections of Bitcoin or cryptocurrency markets with a high proportion of mining. By looking at country-specific cases, for instance, where the actual energy prices included in the mining process can be sourced, this provides a direct link between the price of energy consumption and the price of cryptocurrencies. Another interesting endeavor on this topic would be to do back-testing on the hypothesized financial implications presented in this thesis. Additionally, as mentioned in the discussion, the inclusion of macro factors when studying the relationship between Bitcoin price and energy commodities may uncover some underlying aspects of their relationships. Lastly, cryptocurrencies have demonstrated their viability as enduring solutions in various domains. Nevertheless, addressing the environmental impact associated with their usage necessitates a crucial shift towards incorporating more renewable energy sources. In light of this imperative, it is crucial to commence studies that explore the integration of renewable energy sources within the cryptocurrency-mining nexus. Such investigations are instrumental in acquiring an initial understanding of the intricate dynamics underlying this intersection, facilitating the formulation of informed strategies and policies for a sustainable and environmentally conscious future.

8. References

Adams, M., & Curry, B. (2023, 18.03.2023). *Who Is Satoshi Nakamoto?* Forbes. Retrieved 24.04.2023 from https://www.forbes.com/advisor/investing/cryptocurrency/who-is-satoshi-nakamoto/

Addison, P. S. (2002). *The Illustrated Wavelet Transform Handbook: Introductory Theory and Applications in Science, Engineering, Medicine and Finance*. Taylor & Francis. https://books.google.no/books?id=RUSjIMQACQQC

Agnolucci, P., Nagle, P., & Temaj, K. (2023, 09.03.2023). Bubble trouble: what's behind the highs and lows of natural gas markets? *World Bank Blogs*. https://blogs.worldbank.org/opendata/bubble-trouble-whats-behind-highs-and-lows-natural-gas-markets

Agur, I., Deodoro, J., Lavayssière, X., Martinez Peria, S., Sandri, D., Tourpe, H., & Villegas Bauer, G. (2022). Front Matter. *FinTech Notes*, *2022*(006), 30. https://doi.org/10.5089/9798400208249.063

Aldrich, E. (2020). *Functions for Computing Wavelet Filters, Wavelet Transforms and Multiresolution Analyses*. In (Version 0.3-0.2) [R Package]. CRAN. https://cran.r-project.org/web/packages/wavelets/wavelets.pdf

Anderson, E., & Reddy, R. (2023, 17.03.2023). *Bitcoin Mining Is Set to Turn Greener*. GLOBAL X. Retrieved 09.05.2023 from https://www.globalxetfs.com/bitcoin-mining-is-set-to-turn-greener/

Barclay, V. J., Bonner, R. F., & Hamilton, I. P. (1997). Application of Wavelet Transforms to Experimental Spectra: Smoothing, Denoising, and Data Set Compression. *Analytical Chemistry*, *69*(1), 78-90. https://doi.org/10.1021/ac960638m

Barunik, J., Vacha, L., & Kristoufek, L. (2011). Comovement of Central European stock markets using wavelet coherence: Evidence from high-frequency data. *Charles University Prague, Faculty of Social Sciences, Institute of Economic Studies*(2011/22). https://www.econstor.eu/handle/10419/83377

Bhambhwani, S., Delikouras, S., & Korniotis, G. (2019). Do Fundamentals Drive Cryptocurrency Prices? *SSRN Electronic Journal*. https://doi.org/10.2139/ssrn.3342842

Bilušić, M. (2022, 08.03.22). *Is There Correlation Between Bitcoin and Oil?* ALTCOINBUZZ. Retrieved 20.04.23 from https://www.altcoinbuzz.io/cryptocurrency-news/is-there-correlation-between-bitcoin-and-oil/

BitMEX. (2018, 22.03.2018). *Update: SegWit transaction capacity increase compared to Bitcoin Cash*. BitMEX. Retrieved 09.03.2023 from https://blog.bitmex.com/the-segwit-transaction-capacity-increase-update/

Bouri, E., Jalkh, N., Molnár, P., & Roubaud, D. (2017). Bitcoin for energy commodities before and after the December 2013 crash: diversifier, hedge or safe haven? *Applied Economics*, *49*(50), 5063-5073. https://doi.org/10.1080/00036846.2017.1299102

Bracewell, R. N. (1989). The Fourier Transform. *Scientific American*, *260*(6), 86-95. http://www.jstor.org/stable/24987290

Browne, R. (2021, 13.04.2021). *Bitcoin hits new all-time high above $63,000 ahead of Coinbase debut*. CNBC. Retrieved 09.03.2023 from https://www.cnbc.com/2021/04/13/bitcoin-hits-new-all-time-high-above-62000-ahead-of-coinbase-debut.html

Browne, R. (2022, 19.05.2022). *Bitcoin production roars back in China despite Beijing's ban on crypto mining*. CNBC. Retrieved 21.03.2023 from https://www.cnbc.com/2022/05/18/china-is-second-biggest-bitcoin-mining-hub-as-miners-go-underground.html

Chui, C. K. (1992). *An introduction to wavelets* (2nd ed.). Harcourt Brace Jovanovich.

Chui, C. K. (2014). *An Introduction to Wavelets*. Elsevier Science. https://books.google.no/books?id=_jmeBQAAQBAJ

Colldeforns Papiol, G., Ortiz Gracia, L., Oosterlee, C. W., & Corral Cano, Á. (2018). *Wavelet approach in computational finance / by Gemma Colldeforns Papiol ; supervised by Dr. L. Ortiz Gracia ; co-supervised by Prof. C.W. Oosterlee and tutored by Dr. A. Corral Cano* Universitat Autònoma de Barcelona. Departament de Matemàtiques].

Cornot-Gandolphe, S. (2017, March 2017). *The Steam Coal Market in 2016: the Supply Shock*. French institute of international relations. Retrieved 14.03.2023 from https://www.ifri.org/en/publications/notes-de-lifri/steam-coal-market-2016-supply-shock
Daubechies, I. (1992). *Ten lectures on wavelets*. Philadelphia, PA : Society for Industrial and Applied Mathematics, 1992. https://search.library.wisc.edu/catalog/999685462102121
de Vries, A., Gallersdörfer, U., Klaaßen, L., & Stoll, C. (2022). Revisiting Bitcoin's carbon footprint. *Joule*, *6*(3), 498-502. https://doi.org/https://doi.org/10.1016/j.joule.2022.02.005
DiChristoffer, T., & Meredith, S. (2018, 07.12.2023). *OPEC and allies agree to cut oil production by 1.2 million barrels per day*. CNBC. Retrieved 09.03.2023 from https://www.cnbc.com/2018/12/07/opec-meeting-saudi-arabia-and-russia-look-to-impose-production-cuts.html
Domm, P. (2018, 19.11.2018). *Natural gas jumps to 4-year high in panicky trading as snow, cold push across US*. CNBC. Retrieved 14.03.2023 from https://www.cnbc.com/2018/11/14/natural-gas-prices-surge-jumping-as-much-20-percent-in-wild-trading.html
Eckel, M. (2019, 28.11.2019). *How Much Did Russian Spy Agencies Rely On Bitcoin? New Hints In Leaked Recordings*. RadioFreeEurope RadioLiberty. Retrieved 20.03.2023 from https://www.rferl.org/a/how-much-did-russian-spy-agencies-rely-on-bitcoin-new-hints-in-leaked-recordings-/30297083.html
Fantazzini, D., & Kolodin, N. (2020). Does the Hashrate Affect the Bitcoin Price? *Journal of Risk and Financial Management*, *13*(11), 263. https://www.mdpi.com/1911-8074/13/11/263
Farge, M. (1992). WAVELET TRANSFORMS AND THEIR APPLICATIONS TO TURBULENCE. *Annual Review of Fluid Mechanics*, *24*(1), 395-458. https://doi.org/10.1146/annurev.fl.24.010192.002143
Fisher, T. (2020, 09.03.2020). *Oil prices experience largest one-day drop since first Gulf War*. Retrieved 13.03.2023 from https://landline.media/oil-prices-experience-largest-one-day-drop-since-first-gulf-war/
Frangoul, A. (2022, 19.12.2022). *Global coal use is on course to hit all-time high this year, IEA says*. CNBC. Retrieved 14.03.2023 from https://www.cnbc.com/2022/12/19/the-planets-use-of-coal-slated-to-hit-all-time-high-this-year-iea.html
Garcia, D., Tessone, C. J., Mavrodiev, P., & Perony, N. (2014). The digital traces of bubbles: feedback cycles between socio-economic signals in the Bitcoin economy. *J R Soc Interface*, *11*(99). https://doi.org/10.1098/rsif.2014.0623
Gouhier, T. C. G., Aslak; Simko, Viliam. (2021). *Conduct Univariate and Bivariate Wavelet Analyses*. In (Version 0.20.21) [R Package]. CRAN. https://github.com/tgouhier/biwavelet
Graps, A. (1995). An introduction to wavelets. *IEEE Computational Science and Engineering*, *2*(2), 50-61. https://doi.org/10.1109/99.388960
Griffin, J., & Shams, A. (2020). Is Bitcoin Really Untethered? *The Journal of Finance*, *75*. https://doi.org/10.1111/jofi.12903
Grinsted, A., Moore, J. C., & Jevrejeva, S. (2004). Application of the cross wavelet transform and wavelet coherence to geophysical time series. *Nonlin. Processes Geophys.*, *11*(5/6), 561-566. https://doi.org/10.5194/npg-11-561-2004
Houy, N. (2014). *The Bitcoin mining games*. https://EconPapers.repec.org/RePEc:gat:wpaper:1412
IEA. (2022). *Coal 2022 - Analysis and forecast to 2025* [Fuel Report]. https://www.iea.org/reports/coal-2022
Jones, B. A., Goodkind, A. L., & Berrens, R. P. (2022). Economic estimation of Bitcoin mining's climate damages demonstrates closer resemblance to digital crude than digital gold. *Scientific Reports*, *12*(1), 14512. https://doi.org/10.1038/s41598-022-18686-8
Kearney, L. (2022, 30.12.2022). *Oil ends year of wild price swings with 2nd straight annual gain*. Reuters. Retrieved 09.03.2023 from https://www.reuters.com/business/energy/oil-set-close-higher-2022-turbulent-year-marked-by-tight-supplies-2022-12-30/
Kristoufek, L. (2015). What Are the Main Drivers of the Bitcoin Price? Evidence from Wavelet Coherence Analysis. *PLOS ONE*, *10*(4), e0123923. https://doi.org/10.1371/journal.pone.0123923

Kubal, J., & Kristoufek, L. (2022). Exploring the relationship between Bitcoin price and network's hashrate within endogenous system. *International Review of Financial Analysis*, *84*, 102375. https://doi.org/https://doi.org/10.1016/j.irfa.2022.102375
Kumar, D. K. (2019). *Oil spills into U.S.-China trade war, prices slump*. Reuters. Retrieved 09.03.2023 from https://www.reuters.com/article/us-usa-trade-china-oil-idUSKCN1VD22D
Lassaline, N., Brechbühler, R., Vonk, S. J. W., Ridderbeek, K., Spieser, M., Bisig, S., le Feber, B., Rabouw, F. T., & Norris, D. J. (2020). Optical Fourier surfaces. *Nature*, *582*(7813), 506-510. https://doi.org/10.1038/s41586-020-2390-x
Lin, M.-Y., & An, C.-L. (2021). The relationship between Bitcoin and resource commodity futures: Evidence from NARDL approach. *Resources Policy*, *74*, 102383. https://doi.org/https://doi.org/10.1016/j.resourpol.2021.102383
Mallat, S. (1999). A Wavelet Tour of Signal Processing. In *123Library* (2 ed.). Academic Press. https://www.123library.org/ebook/isbn/9780080520834/
Mihanović, H., Orlić, M., & Pasarić, Z. (2009). Diurnal thermocline oscillations driven by tidal flow around an island in the Middle Adriatic. *Journal of Marine Systems*, *78*, S157-S168. https://doi.org/https://doi.org/10.1016/j.jmarsys.2009.01.021
Mills, A. (2016, 31.01.2016). *Oversupply of oil and gas creates decline in industry in 2015*. Times Record News. Retrieved 14.03.2023 from https://archive.timesrecordnews.com/columnists/alex-mills/oversupply-of-oil-and-gas-creates-decline-in-industry-in-2015-280b9594-d732-43d6-e053-0100007f3b6a-367127871.html/
Moussa, W., Mgadmi, N., Béjaoui, A., & Regaieg, R. (2021). Exploring the dynamic relationship between Bitcoin and commodities: New insights through STECM model. *Resources Policy*, *74*, 102416. https://doi.org/https://doi.org/10.1016/j.resourpol.2021.102416
Nason, G. (2008). *Wavelet Methods in Statistics with R*. Springer. https://books.google.no/books?id=xQBDeoZTxWQC
Ng, E. K. W., & Chan, J. C. L. (2012). Geophysical Applications of Partial Wavelet Coherence and Multiple Wavelet Coherence. *Journal of Atmospheric and Oceanic Technology*, *29*(12), 1845-1853. https://doi.org/10.1175/JTECH-D-12-00056.1
Oppenheim, A. V., Schafer, R. W., & Buck, J. R. (1999). *Discrete-time signal processing (2nd ed.)*. Prentice-Hall, Inc.
Oppenheim, A. V., Willsky, A. S., & Nawab, S. H. (1996). *Signals & systems (2nd ed.)*. Prentice-Hall, Inc.
Othman, D. A., & Bendob, D. A. (2022). *Bitcoin Mining's Energy Consumption And Global Carbon Dioxide Emissions: Wavelet Coherence Analysis*. www.amf.org.ae: Arab Monetary Fund Retrieved from https://www.amf.org.ae/en/publications/work-papers/bitcoin-minings-energy-consumption-and-global-carbon-dioxide-emissions
Oygur, T., Unal, G., Gouhier, T. C., Grinsted, A., & Simko, V. (2021). *Vector Wavelet Coherence for Multiple Time Series*. In (Version 0.1.0) [R Package]. CRAN. https://github.com/toygur/vectorwavelet
Partners, F. D. (2016). *GAS PRICES WERE MIXED IN 2016. WHAT WILL HAPPEN IN 2017?* F&D Partners. Retrieved 14.03.2023 from https://fanddpartners.com/gas-prices-were-mixed-in-2016-what-will-happen-in-2017/
Percival, D. B., & Walden, A. T. (2000). *Wavelet Methods for Time Series Analysis*. Cambridge University Press. https://books.google.no/books?id=XF4urPW362AC
Pound, J. (2021, 21.05.2021). *The crypto collapse: Here's what's behind bitcoin's sudden drop*. CNBC. Retrieved 09.03.2023 from https://www.cnbc.com/2021/05/19/the-crypto-collapse-heres-whats-behind-bitcoins-sudden-drop.html
Ramsey, J. B. (1999). The Contribution of Wavelets to the Analysis of Economic and Financial Data. *Philosophical Transactions: Mathematical, Physical and Engineering Sciences*, *357*(1760), 2593-2606. http://www.jstor.org/stable/55181
Rathinasamy, M., Agarwal, A., Parmar, V., Khosa, R., & Bairwa, A. (2017). Partial wavelet coherence analysis for understanding the standalone relationship between Indian Precipitation and Teleconnection patterns. *arXiv preprint arXiv:1702.06568*.

Reed, S., & Krauss, C. (2020, 28.09.2021). *Too Much Oil: How a Barrel Came to Be Worth Less Than Nothing*. The New York Times. Retrieved 09.03.2023 from https://www.nytimes.com/2020/04/20/business/oil-prices.html

Rehman, M. U., & Kang, S. H. (2021). A time–frequency comovement and causality relationship between Bitcoin hashrate and energy commodity markets. *Global Finance Journal*, *49*, 100576. https://doi.org/https://doi.org/10.1016/j.gfj.2020.100576

Rooney, K. (2018, 13.06.2018). *Much of bitcoin's 2017 boom was market manipulation, research says*. CNBC. Retrieved 09.03.2023 from https://www.cnbc.com/2018/06/13/much-of-bitcoins-2017-boom-was-market-manipulation-researcher-says.html

Schinckus, C., Nguyen, C. P., & Chong, F. H. L. (2022). Cryptocurrencies' hashrate and electricity consumption: evidence from mining activities. *Studies in Economics and Finance*, *39*(3), 524-546. https://doi.org/10.1108/SEF-08-2021-0342

Stanway, D., & Sarkar, H. (2016, 22.02.2016). *China to close more than 1,000 coal mines in 2016: energy bureau*. REUTERS. Retrieved 14.03.2023 from https://www.reuters.com/article/us-china-energy-coal-idUSKCN0VV0U5

Stevens, P. (2020, 30.04.2020). *Energy demand, hit by coronavirus crisis, is set to see record drop this year, IEA says*. CNBC. Retrieved 14.03.2023 from https://www.cnbc.com/2020/04/30/energy-demand-set-to-fall-the-most-on-record-this-year-amid-coronavirus-pandemic-iea-says.html

Stocker, M., Baffes, J., & Vorisek, D. (2018, 09.03.2023). What triggered the oil price plunge of 2014-2016 and why it failed to deliver an economic impetus in eight charts. *World Bank Blogs*. https://blogs.worldbank.org/developmenttalk/what-triggered-oil-price-plunge-2014-2016-and-why-it-failed-deliver-economic-impetus-eight-charts

Torrence, C., & Compo, G. P. (1998). A Practical Guide to Wavelet Analysis. *Bulletin of the American Meteorological Society*, *79*(1), 61-78. https://doi.org/10.1175/1520-0477(1998)079<0061:APGTWA>2.0.CO;2

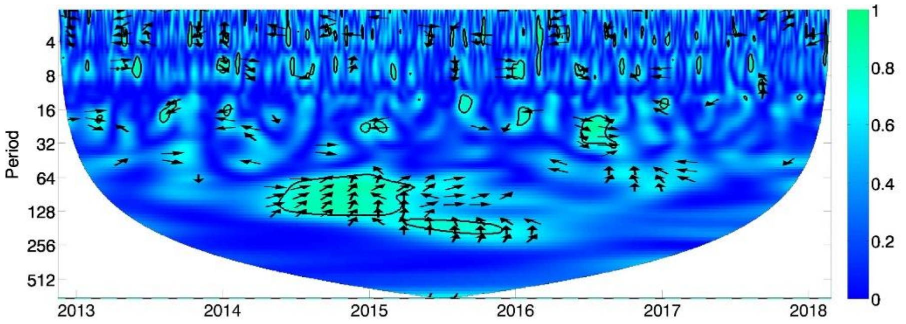
Wade, J. (2023, 28.02.2023). *Hash Rate*. Investopedia. Retrieved 21.03.2023 from https://www.investopedia.com/hash-rate-6746261

Worldcoin. (2022, 14.12.2022). *All You Need to Know About China's Crypto Ban*. WORLDCOIN. Retrieved 18.04.23 from https://worldcoin.org/articles/china-crypto-ban

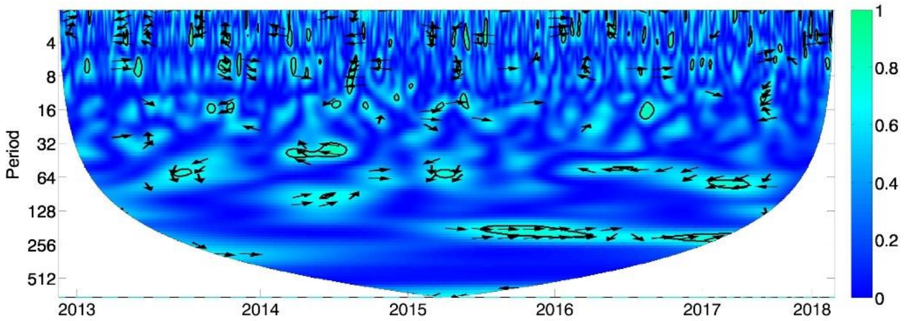
Yuan, X., Su, C.-W., & Peculea, A. D. (2022). Dynamic linkage of the bitcoin market and energy consumption:An analysis across time. *Energy Strategy Reviews*, *44*, 100976. https://doi.org/https://doi.org/10.1016/j.esr.2022.100976

9. Appendix

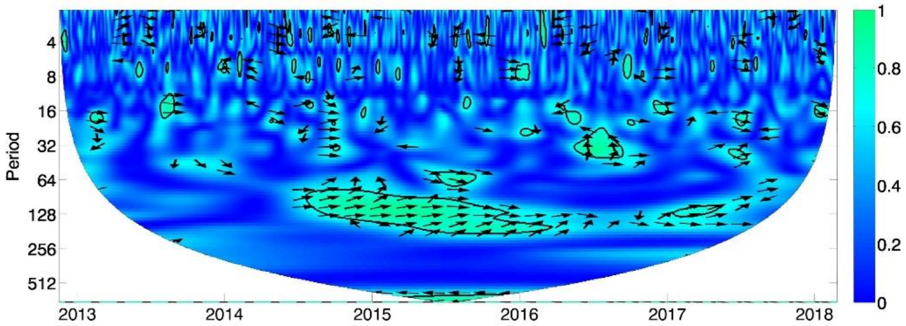
9.1. Wavelet coherence analysis results from Rehman and Kang (2021)



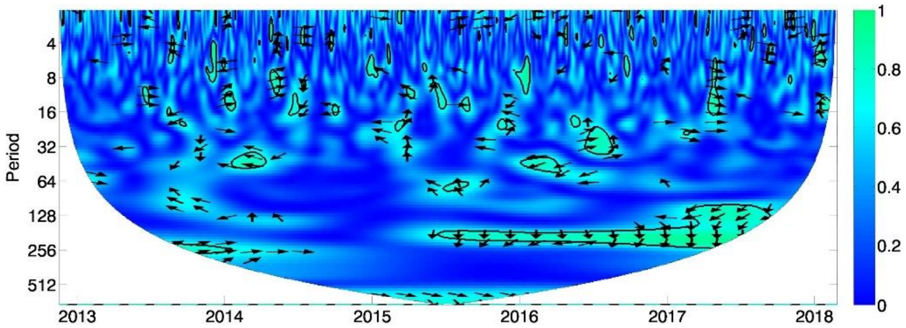
(a) **BTC–OIL**



(b) **BTC–COAL**



(c) **BTC–GAS**



(d) **BTC–HASHRATE**

Figure 17 - Wavelet coherence plots from Rehman and Kang (2021)

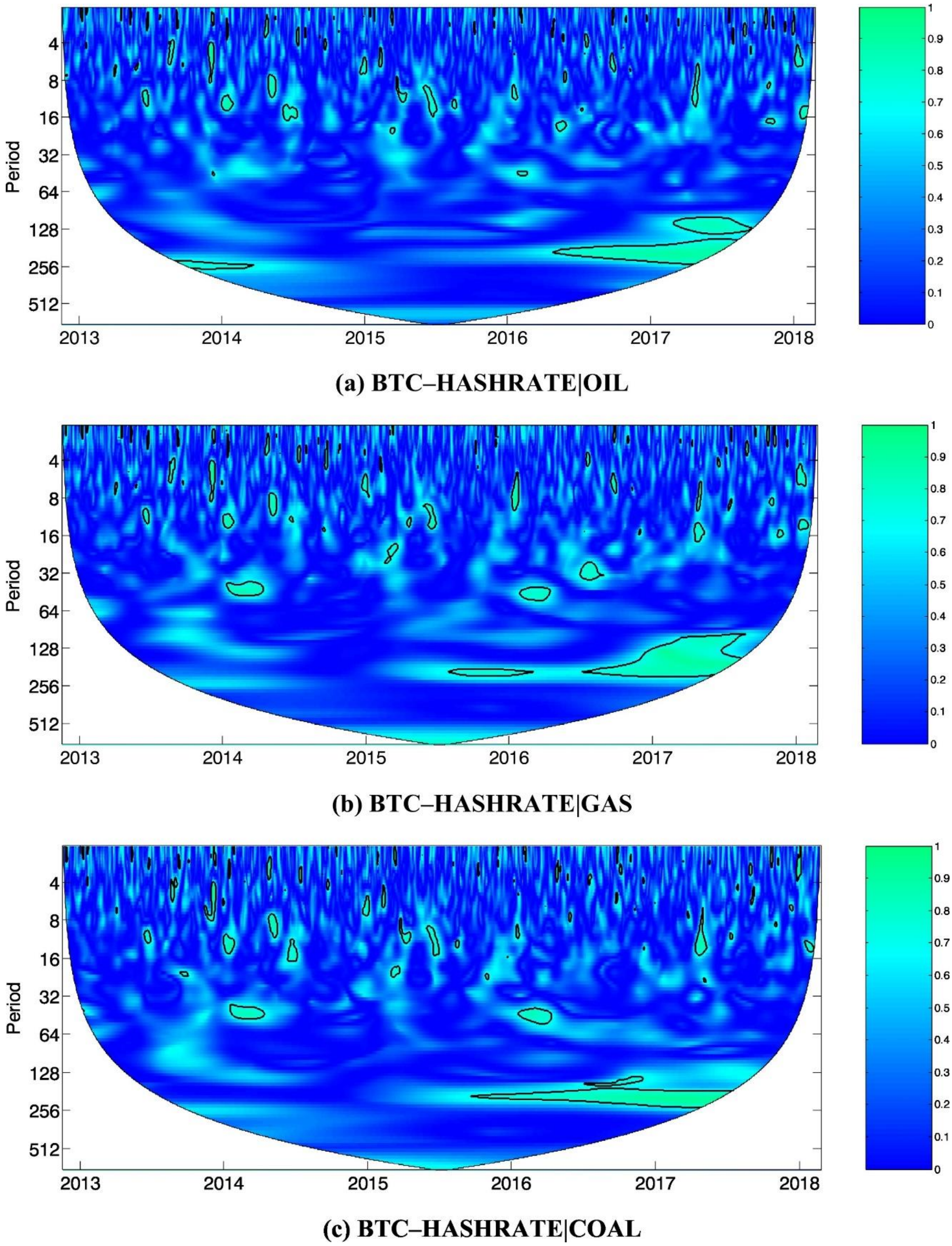
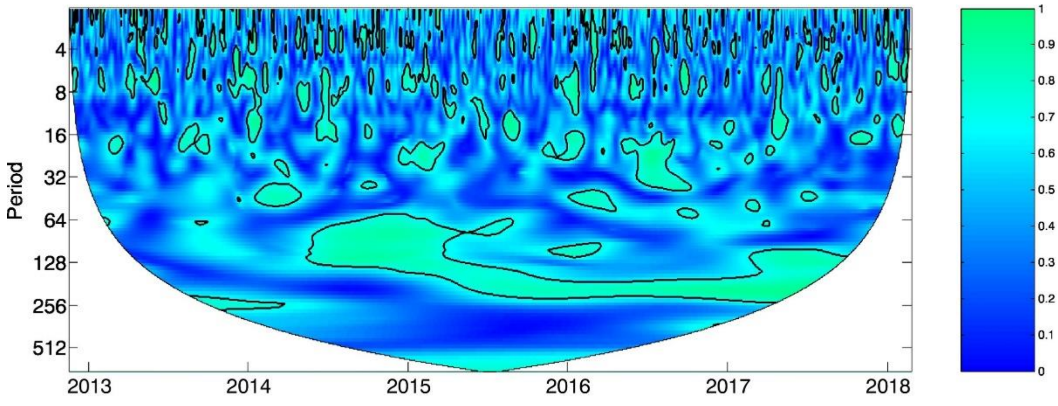
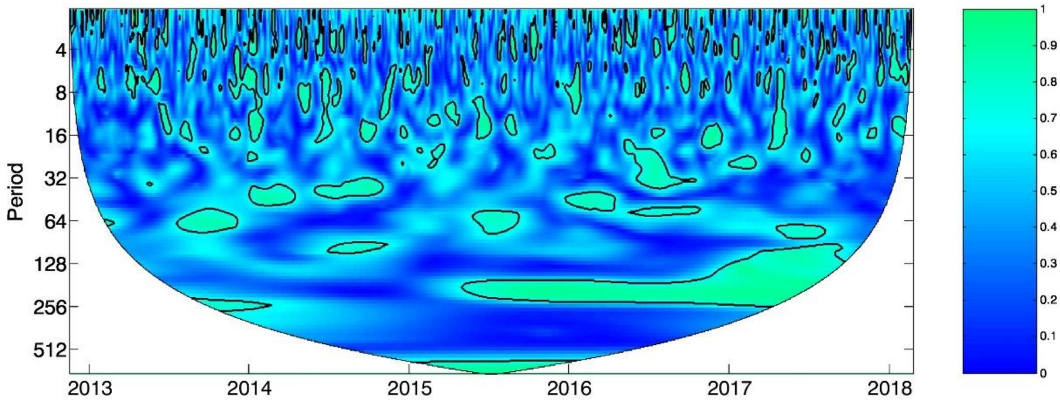


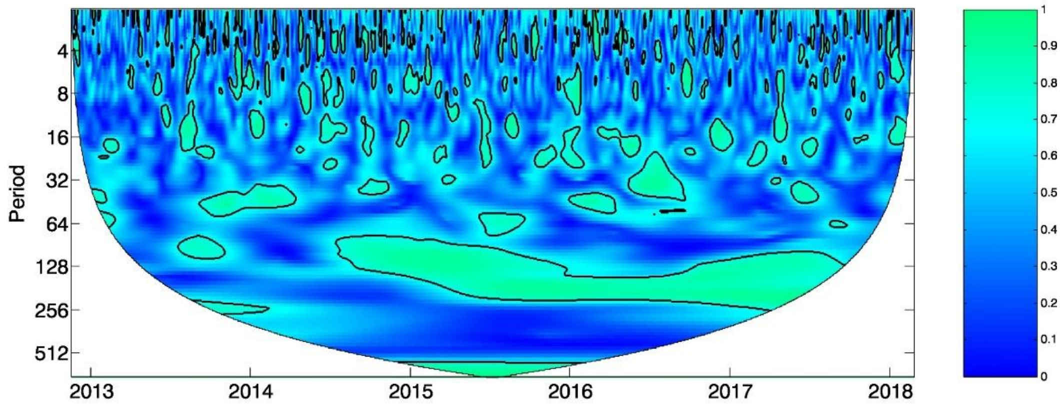
Figure 18 - Partial wavelet coherence plots from Rehman and Kang (2021)



(a) BTC–HASHRATE|OIL



(b) BTC–HASHRATE|GAS



(c) BTC–HASHRATE|COAL

Figure 19 - Multiple wavelet coherence plots from Rehman and Kang (2021)

9.2. R – script

```
rm(list=ls(all=TRUE))

library(quantmod)

library(lubridate)

library(pastecs)

library("astsa")

library(MTS)

library(wavelets)

library(biwavelet)

library(rjson)

library(dplyr)

library(vectorwavelet)

library(corrplot)


###### Data import from Yahoo Finance ################################################


setSymbolLookup(QQQ='yahoo',SPY='yahoo')

getSymbols(c('QQQ','SPY'))


#getSymbols('BTC-USD',src = 'yahoo',return.class = 'xts',from = '2015-01-01',to = '2022-12-31')

getSymbols('CL=F',src = 'yahoo',return.class = 'xts',from = '2013-01-01',to = '2022-12-31')

getSymbols('NG=F',src = 'yahoo',return.class = 'xts',from = '2013-01-01',to = '2022-12-31')

getSymbols('MTF=F',src = 'yahoo',return.class = 'xts',from = '2013-01-01',to = '2022-12-31')


BTCHASH = read.csv("BTCHash_data2.csv")

index = seq(3652,1,-1)

BTCHASH = cbind(BTCHASH,index)

BTCHASH = BTCHASH%>%arrange(index)


###### OIL ############################################################################
```

```
start = as.Date('2013-01-01')
end = as.Date('2022-12-31')
regind = seq(from = start,to = end, by = "day")

regular_xts <- xts(seq_along(regind), order.by = regind)

OIL = merge(`CL=F`,regular_xts)

####### BTC ###########################################################################

#BTCHASH = merge(BTCHASH,regular_xts)

####### GAS ###########################################################################

GAS = merge(`NG=F`,regular_xts)

####### COAL ##########################################################################

COAL = merge(`MTF=F`,regular_xts)

###### Data-Matrix Generation #############################################################

datmat = cbind(BTCHASH[,2:3],OIL[,4],GAS[,4],COAL[,4])
colnames(datmat) = c("BTC","HASH","OIL","GAS","COAL")
datmat = na.omit(datmat)
#datmat$BTC = as.numeric(gsub(",","",datmat$BTC))

###### Price Plots ####################################################################

plot(datmat[,2],type="l", col="blue", ylab = "Hashrate in Million TH/Second",xlab = "", xaxt ='n', yaxt
= 'n')
```

```
grid(nx = NA, ny = NULL, lty = 2, col = "gray", lwd = 1)
abline(v = seq(0, 2411, 241), lty = 2, col = "gray", lwd = 1)
axis(side = 1, at = c(seq(0, 2411, 241)), labels = c(seq(2013, 2023, 1)))
axis(side = 2, at = c(seq(0, 350000000, 50000000)), labels = c(seq(0, 350, 50)))

plot(datmat[,1],type="l", col="blue", ylab = "BTC/USD",xlab = "", xaxt ='n')
grid(nx = NA, ny = NULL, lty = 2, col = "gray", lwd = 1)
abline(v = seq(0, 2411, 241), lty = 2, col = "gray", lwd = 1)
axis(side = 1, at = c(seq(0, 2411, 241)), labels = c(seq(2013, 2023, 1)))

plot(datmat[,3],type="l", col="blue", ylab = "OIL/USD",xlab = "", xaxt ='n', yaxt = 'n')
abline(h = seq(-40, 140, 20), v = seq(0, 2411, 241), lty = 2, col = "gray", lwd = 1)
axis(side = 1, at = c(seq(0, 2411, 241)), labels = c(seq(2013, 2023, 1)))
axis(side = 2, at = c(seq(-40, 120, 20)), labels = c(seq(-40, 120, 20)))

plot(datmat[,4],type="l", col="blue", ylab = "GAS/USD",xlab = "", xaxt ='n')
abline(h = seq(0, 10, 2), v = seq(0, 2411, 241), lty = 2, col = "gray", lwd = 1)
axis(side = 1, at = c(seq(0, 2411, 241)), labels = c(seq(2013, 2023, 1)))

plot(datmat[,5],type="l", col="blue", ylab = "COAL/USD",xlab = "", xaxt ='n')
abline(h = seq(0, 500, 50), v = seq(0, 2411, 241), lty = 2, col = "gray", lwd = 1)
axis(side = 1, at = c(seq(0, 2411, 241)), labels = c(seq(2013, 2023, 1)))
axis(side = 2, at = c(seq(0, 450, 50)), labels = c(seq(0, 450, 50)))

###### Log-Return Transformation ######################################################

btc.log = diff(log(datmat$BTC))
hash.log = diff(log(datmat$HASH))
oil.log = diff(log(datmat$OIL))
gas.log = diff(log(datmat$GAS))
coal.log = diff(log(datmat$COAL))
```

```
datmat.log = cbind(btc.log,hash.log,oil.log,gas.log,coal.log)

desStatRet = format(round((stat.desc(datmat.log, norm = TRUE)),digits = 4),scientific = FALSE)
desStatRet

###### Return Plots ######################################################################

plot(hash.log, type = "l", ylab = "", xlab = "", xaxt = 'n', main = "Hahsrate Growth Rates 2013 - 2023")
grid(nx = NA, ny = NULL, lty = 2, col = "gray", lwd = 1)
abline(v = seq(0, 2411, 241), lty = 2, col = "gray", lwd = 1)
axis(side = 1, at = c(seq(0, 2411, 241)), labels = c(seq(2013, 2023, 1)))

plot(btc.log, type = "l", ylab = "", xlab = "", xaxt = 'n', main = "Bitcoin Returns 2013 - 2023")
grid(nx = NA, ny = NULL, lty = 2, col = "gray", lwd = 1)
abline(v = seq(0, 2411, 241), lty = 2, col = "gray", lwd = 1)
axis(side = 1, at = c(seq(0, 2411, 241)), labels = c(seq(2013, 2023, 1)))

plot(oil.log, type = "l", ylab = "", xlab = "", xaxt = 'n', main = "Oil Futures Returns 2013 - 2023")
grid(nx = NA, ny = NULL, lty = 2, col = "gray", lwd = 1)
abline(v = seq(0, 2411, 241), lty = 2, col = "gray", lwd = 1)
axis(side = 1, at = c(seq(0, 2411, 241)), labels = c(seq(2013, 2023, 1)))

plot(gas.log, type = "l", ylab = "", xlab = "", xaxt = 'n', main = "Gas Futures Returns 2013 - 2023")
grid(nx = NA, ny = NULL, lty = 2, col = "gray", lwd = 1)
abline(v = seq(0, 2411, 241), lty = 2, col = "gray", lwd = 1)
axis(side = 1, at = c(seq(0, 2411, 241)), labels = c(seq(2013, 2023, 1)))

plot(coal.log, type = "l", ylab = "", xlab = "", xaxt = 'n', main = "Coal Futures Returns 2013 - 2023")
grid(nx = NA, ny = NULL, lty = 2, col = "gray", lwd = 1)
abline(v = seq(0, 2411, 241), lty = 2, col = "gray", lwd = 1)
```

```
axis(side = 1, at = c(seq(0, 2411, 241)), labels = c(seq(2013, 2023, 1)))

###### Coherence analysis facilitation ###################################################

datmat.log[1737,3]=(datmat[1738,3]-datmat[1737,3])/datmat[1737,3]
datmat.log[1738,3]=(datmat[1739,3]-datmat[1738,3])/(-datmat[1738,3])

oil.log[1737]=(datmat[1738,3]-datmat[1737,3])/datmat[1737,3]
oil.log[1738]=(datmat[1739,3]-datmat[1738,3])/(-datmat[1738,3])


corr = cor(datmat.log)

ind = 1:2410

btc = cbind(ind,btc.log)
hash = cbind(ind,hash.log)
oil = cbind(ind,oil.log)
gas = cbind(ind,gas.log)
coal = cbind(ind,coal.log)


btchash = cbind(as.numeric(datmat.log[,1]),as.numeric(datmat.log[,2]))

nrands = 5000

###### Wavelet Coherence  ############################################################

wtc.btchash = wtc(btc, hash, nrands = nrands)
wtc.btcoil = wtc(btc, oil, nrands = nrands)
wtc.btcgas = wtc(btc, gas, nrands = nrands)
```

```
wtc.btccoal = wtc(btc, coal, nrands = nrands)

par(oma = c(0, 0, 0, 1), mar = c(5, 4, 5, 5) + 0.1)
n = length(btc[, 1])

### BTC-Hash Plot ###

plot(wtc.btchash, fill.cols= NULL, plot.phase = TRUE,xaxt = 'n', lty.coi = 1,  col.coi = "black", lwd.coi = 
2,
    lwd.sig = 2, arrow.cutoff = 0.5, arrow.lwd = 0.03, arrow.len = 0.10, ylab = "Scale", xlab = "Period",
    plot.cb = TRUE, main = "Wavelet Coherence: Bitcoin returns vs Hashrate growth rates")

abline(v = seq(241, n, 241), h = 1:16, col = "brown", lty = 1, lwd = 1)
axis(side = 1, at = c(seq(0, n, 241)), labels = c(seq(2013, 2023, 1)))

### BTC-Oil Plot ###

plot(wtc.btcoil, fill.cols= NULL, plot.phase = TRUE,xaxt = 'n', lty.coi = 1,  col.coi = "black", lwd.coi = 2,
    lwd.sig = 2, arrow.cutoff = 0.5, arrow.lwd = 0.03, arrow.len = 0.10, ylab = "Scale", xlab = "Period",
    plot.cb = TRUE, main = "Wavelet Coherence: Bitcoin returns vs Oil futures returns")

abline(v = seq(241, n, 241), h = 1:16, col = "brown", lty = 1, lwd = 1)
axis(side = 1, at = c(seq(0, n, 241)), labels = c(seq(2013, 2023, 1)))

### BTC-Gas Plot ###

plot(wtc.btcgas, fill.cols= NULL, plot.phase = TRUE,xaxt = 'n', lty.coi = 1,  col.coi = "black", lwd.coi = 2,
    lwd.sig = 2, arrow.cutoff = 0.5, arrow.lwd = 0.03, arrow.len = 0.10, ylab = "Scale", xlab = "Period",
    plot.cb = TRUE, main = "Wavelet Coherence: Bitcoin returns vs Natural gas futures returns")

abline(v = seq(241, n, 241), h = 1:16, col = "brown", lty = 1, lwd = 1)
```

```
axis(side = 1, at = c(seq(0, n, 241)), labels = c(seq(2013, 2023, 1)))


### BTC-Coal Plot ###


plot(wtc.btccoal, fill.cols= NULL, plot.phase = TRUE,xaxt = 'n', lty.coi = 1,  col.coi = "black", lwd.coi =
2,
    lwd.sig = 2, arrow.cutoff = 0.5, arrow.lwd = 0.03, arrow.len = 0.10, ylab = "Scale", xlab = "Period",
    plot.cb = TRUE, main = "Wavelet Coherence: Bitcoin returns vs Coal futures returns")


abline(v = seq(241, n, 241), h = 1:16, col = "brown", lty = 1, lwd = 1)
axis(side = 1, at = c(seq(0, n, 241)), labels = c(seq(2013, 2023, 1)))



###### Partial Wavelet Coherence  ##########################################################



pwtc.bho <- pwtc(btc, hash, oil, nrands = nrands)
pwtc.bhg <- pwtc(btc, hash, gas, nrands = nrands)
pwtc.bhc <- pwtc(btc, hash, coal, nrands = nrands)




### BTC-Hash|Oil Plot ###


plot(pwtc.bho, fill.cols= NULL, xaxt = 'n', lty.coi = 1,  col.coi = "black", lwd.coi = 2,
    lwd.sig = 2, ylab = "Scale", xlab = "Period", plot.cb = TRUE, plot.phase = TRUE, arrow.cutoff = 0.5,
arrow.lwd = 0.03, arrow.len = 0.10,
    main = "Partial wavelet coherence of Bitcoin returns and Hashrate growth rates | Oil futures
returns")


abline(v = seq(241, n, 241), h = 1:16, col = "brown", lty = 1, lwd = 1)
axis(side = 1, at = c(seq(0, n, 241)), labels = c(seq(2013, 2023, 1)))
```

BTC-Hash|Gas Plot

```
plot(pwtc.bhg, fill.cols= NULL, xaxt = 'n', lty.coi = 1,  col.coi = "black", lwd.coi = 2,
    lwd.sig = 2, ylab = "Scale", xlab = "Period", plot.cb = TRUE, plot.phase = TRUE, arrow.cutoff = 0.5,
arrow.lwd = 0.03, arrow.len = 0.10,
    main = "Partial wavelet coherence of Bitcoin returns and Hashrate growth rates | Natural gas
futures returns")

abline(v = seq(241, n, 241), h = 1:16, col = "brown", lty = 1, lwd = 1)
axis(side = 1, at = c(seq(0, n, 241)), labels = c(seq(2013, 2023, 1)))
```

BTC-Hash|Coal Plot

```
plot(pwtc.bhc, fill.cols= NULL, xaxt = 'n', lty.coi = 1,  col.coi = "black", lwd.coi = 2,
    lwd.sig = 2, ylab = "Scale", xlab = "Period", plot.cb = TRUE, plot.phase = TRUE, arrow.cutoff = 0.5,
arrow.lwd = 0.03, arrow.len = 0.10,
    main = "Partial wavelet coherence of Bitcoin returns and Hashrate growth rates | Coal futures
returns")

abline(v = seq(241, n, 241), h = 1:16, col = "brown", lty = 1, lwd = 1)
axis(side = 1, at = c(seq(0, n, 241)), labels = c(seq(2013, 2023, 1)))
```

Multiple Wavelet Coherence

```
mwc.bho <- mwc(btc, hash, oil, nrands = nrands)
mwc.bhg <- mwc(btc, hash, gas, nrands = nrands)
mwc.bhc <- mwc(btc, hash, coal, nrands = nrands)
```

BTC-Hash/Oil Plot

```
plot.vectorwavelet(mwc.bho, fill.cols= NULL, xaxt = 'n', lty.coi = 1,  col.coi = "black", lwd.coi = 2,
                lwd.sig = 2, ylab = "Scale", xlab = "Period", plot.cb = TRUE, main = "Multiple wavelet coherence of Bitcoin returns and Hashrate growth rates/Oil futures returns")

abline(v = seq(241, n, 241), h = 1:16, col = "brown", lty = 1, lwd = 1)
axis(side = 1, at = c(seq(0, n, 241)), labels = c(seq(2013, 2023, 1)))

### BTC-Hash/Gas Plot ###

plot.vectorwavelet(mwc.bhg, fill.cols= NULL, xaxt = 'n', lty.coi = 1,  col.coi = "black", lwd.coi = 2,
                lwd.sig = 2, ylab = "Scale", xlab = "Period", plot.cb = TRUE, main = "Multiple wavelet coherence of Bitcoin returns and Hashrate growth rates/Natural gas futures returns")

abline(v = seq(241, n, 241), h = 1:16, col = "brown", lty = 1, lwd = 1)
axis(side = 1, at = c(seq(0, n, 241)), labels = c(seq(2013, 2023, 1)))

### BTC-Hash/Coal Plot ###

plot.vectorwavelet(mwc.bhc, fill.cols= NULL, xaxt = 'n', lty.coi = 1,  col.coi = "black", lwd.coi = 2,
                lwd.sig = 2, ylab = "Scale", xlab = "Period", plot.cb = TRUE, main = "Multiple wavelet coherence of Bitcoin returns and Hashrate growth rates/Coal futures returns")

abline(v = seq(241, n, 241), h = 1:16, col = "brown", lty = 1, lwd = 1)
axis(side = 1, at = c(seq(0, n, 241)), labels = c(seq(2013, 2023, 1)))
```

9.3. Discussion paper – Herman Farstad: Responsible

Bitcoin is a digital currency that has gained widespread attention in recent years. It is a decentralized currency that operates on a peer-to-peer network, allowing users to transact without the need for a central authority. However, Bitcoin's emergence as a new form of currency has also raised questions about its ethical implications and responsibilities. As a digital currency that operates outside the traditional financial system, bitcoin can potentially be used in illegal activities as it cannot be monitored the same way as traditional currencies. Additionally, the energy-intensive process of Bitcoin mining has sparked debates around the environmental impact of its use. Therefore, "responsibility" is a term of great relevance in terms of understanding cryptocurrencies' unregulated nature.

In this paper, we study the time-frequency comovement between the returns of bitcoin and hashrate, while simultaneously controlling for the effect of selected energy commodities, namely, two-month futures contracts for Brent crude oil, coal, and natural gas. Our primary objective is to enhance our understanding of the intricate dynamics of the cryptocurrency system. Given the association of cryptocurrencies with unethical practices, it motivates us to conduct thorough research on this topic.

The first area of which Bitcoin challenges ethical standards is within the mining domain.
Bitcoin mining is a process of verifying and adding transactions to the blockchain network using computational power (Houy, 2014). It involves solving complex mathematical problems, which requires a significant amount of energy consumption and contributes to carbon emissions, as mining rigs are often powered by coal. Bitcoin mining therefore poses ethical challenges, particularly in terms of its environmental impact and responsible use of energy resources. The energy-intensive nature of Bitcoin mining has been criticized for its carbon footprint, which contributes to climate change. Furthermore, the concentration of mining power in the hands of a few individuals or groups poses ethical concerns related to the decentralization and democratization of cryptocurrency systems. As such, responsible and ethical Bitcoin mining practices are crucial to ensuring that the use of energy resources is sustainable and equitable, and that the integrity of the cryptocurrency system is maintained. This research will shed a light on the Bitcoin mining process to spread awareness of this ethically challenging concept.

Despite China’s recent ban on Bitcoin mining, miners in the country are still operating through underground operations (Sigalos, 2022). In fact, new research from the Cambridge Centre for Alternative Finance shows that Chinese Bitcoin mining activity has quickly rebounded, making up just over 22% of the total Bitcoin mining market by September 2021 (Browne, 2022). However, this resurgence of Bitcoin mining in China raises ethical and responsible questions. As previously mentioned, Bitcoin mining is a process that requires a significant amount of energy and can have a significant environmental impact. China's heavy reliance on coal and other fossil fuels for energy production exacerbates these concerns.

The underground nature of these operations may raise questions about potential non-compliance with labor laws and regulations, leading to concerns that workers might be subjected to unsafe and unhealthy working conditions. The lack of regulations and oversight in the industry could contribute to these issues, which may challenge ethical norms and the concept of responsibility. From my perspective, it is important for responsible Bitcoin mining practices to prioritize the ethical treatment of workers, including the provision of safe and healthy working conditions, as well as fair compensation. Implementing regulatory frameworks that ensure compliance with labor laws and regulations could be instrumental in addressing these concerns. Furthermore, the Chinese government's crackdown on Bitcoin mining highlights the importance of regulatory frameworks that balance the need for energy efficiency and environmental sustainability with the benefits of the technology (Sigalos, 2021). Transparency and accountability measures, such as audits and reporting requirements, can also promote responsible mining practices.

Cryptocurrencies are decentralized and operate through blockchain technology, which allows for anonymity and confidentiality in transactions. While anonymity offers privacy and security benefits, it also poses ethical challenges related to responsible use and compliance with legal and regulatory frameworks. The anonymity of bitcoin transactions can be exploited for illegal activities, such as money laundering, tax evasion, trade of illegal goods, and so on. Additionally, the lack of transparency and accountability in bitcoin transactions can lead to market manipulation and price volatility, which can affect the financial stability of individuals and institutions. Therefore, responsible, and ethical use of bitcoin as currency must take into account the potential risks and comply with legal and regulatory frameworks to ensure that the integrity and stability of the cryptocurrency system is maintained.

We overcome these challenges by responsibly shining a light on the dynamics in the cryptocurrency systems. The findings of this research can also present ethical challenges, particularly if they are used for speculative or manipulative purposes. The volatile nature of cryptocurrency prices makes them susceptible to manipulation, and findings that may contribute to such activities must be dealt with responsibly. Our responsibility is to accurately present our findings, in an unbiased manner, and to advise readers not to contribute to activities that are illegal or unethical.

There are certain traits of the digital currencies that are ethical. For instance, Bitcoin as digital currency is decentralised. One of the key features of Bitcoin is its decentralized nature. This means that there is no central authority that controls the currency, making it more resistant to corruption. Decentralization can promote financial freedom and independence from centralized institutions, particularly in countries where the government has a history of financial oppression. Even though the public is unaware of who is behind the creation of Bitcoin, no central authority is in control as far as we know.

Bitcoin also offers some degree of transparency, although anonymous. Transactions are recorded on a public ledger called the blockchain, which is transparent and verifiable. This means that transactions cannot be manipulated, and all participants can see the details of each transaction. This level of transparency can promote trust and accountability. Bitcoin additionally uses advanced cryptographic techniques to secure transactions and prevent fraud. While it is true that cryptocurrencies have been used for illegal activities, such as money laundering, it is important to note that traditional currencies are also used for these purposes. In fact, according to the United Nations Office on Drugs and Crime, the majority of money laundering still occurs in cash (Worldcoin, n.d.).

Another important consideration in assessing the responsibility of Bitcoin is its potential role in illicit activities. The anonymity and lack of regulation inherent in cryptocurrency transactions have led some to raise concerns about its use in money laundering, terrorism financing, and other criminal activities. While it is true that traditional financial systems are also vulnerable to such abuses, the decentralized and anonymous nature of Bitcoin may create additional challenges in detecting and preventing illicit activities. As such, responsible use of Bitcoin will require a collaborative effort between governments, law enforcement, and industry stakeholders to establish and enforce appropriate regulations.

The rapid evolution and adaptation of cryptocurrencies have raised important questions about their impact on global economic systems and financial stability. The potential for cryptocurrency markets to be highly volatile and prone to speculation and manipulation has led some to question whether these technologies pose a systemic risk to the broader economy. Additionally, the emergence of new and untested financial instruments such as Initial Coin Offerings (ICOs) has led to concerns about the potential for fraud and abuse in the crypto space (Hornuf, Kück, & Schwienbacher, 2022). As such, ensuring the responsible integration of cryptocurrencies into the wider financial system will require ongoing investigation and risk management by policymakers as well as industry participants.

The relationship between Bitcoin and energy commodities is an interesting area of study for several reasons. Firstly, as mentioned earlier, bitcoin mining is an energy-intensive process that requires a significant amount of electricity to maintain the network and validate transactions. This makes bitcoin's energy consumption a crucial factor in its cost and value, and it creates a unique interdependence between Bitcoin and energy commodities such as oil, natural gas, and oil. Even though natural gas and coal are arguably the most common energy sources to bitcoin mining, we also find it interesting to analyse the effect of oil futures as it is a much-traded product. The cost of energy is a major factor in the cost of Bitcoin mining, and fluctuations in energy prices can have a significant impact on the profitability of mining operations. This means that the price of Bitcoin is likely affected not just by market demand, but also by changes in energy prices.

If Bitcoin mining were to shift to renewable energy sources, it could help drive down the cost of renewable energy and accelerate the transition away from fossil fuels. I do believe that Bitcoin as a concept will have a hard time growing significantly as long as the currency's impact on the environment is as it is today, even though it has many valuable features such as being decentralized and so on. For the time being, it does not seem likely that Bitcoin's transition to renewable energy sources is coming, and it is therefore important to shed light on the dynamics of the bitcoin-hashrate-energy commodities nexus.

A plausible strategy to mitigate the environmental impact of the highly energy-intensive process of Bitcoin mining involves active participation in the flexibility market. The integration of Bitcoin mining with the flexibility market can offer a viable solution for environmentally friendly mining operations, particularly by leveraging renewable energy sources like wind or solar power (Statnett, 2022). The volatile nature of renewable energy production necessitates efficient utilization and management of electricity generation. By participating in the flexibility market, Bitcoin miners can adjust their energy consumption patterns to align with the availability of renewable energy. They can strategically increase their mining activities during periods of high renewable energy production, effectively utilizing excess capacity that would otherwise be curtailed. Conversely, during periods of low renewable energy generation, miners can reduce their energy consumption or temporarily pause mining operations, thus reducing reliance on fossil fuel-based grid electricity. This integration not only enables more sustainable and greener mining practices but also enhances the overall efficiency and stability of the electricity grid by providing a demand response mechanism that balances intermittent renewable energy generation with mining energy requirements.

In summary, the question of whether Bitcoin is a responsible concept is a complex and multifaceted one that requires careful consideration of a range of factors. While its decentralized nature and potential to promote financial inclusion may be viewed as responsible, the environmental impact of Bitcoin mining, the potential for illicit activities, and the broader implications for global economic systems must also be taken into account. Moving forward, it will be essential to balance the benefits and costs associated with cryptocurrencies and establish appropriate regulatory frameworks and risk management practices to ensure that these technologies are used responsibly and sustainably. The flexibility market may be a means of reaching the goal of sustainable mining practices.

The course on sustainable capitalism is highly relevant to the topic of responsibility concerning Bitcoin. The course taught us the importance of balancing profit and growth with long-term social and environmental sustainability. It emphasized the need to conserve natural resources and minimize environmental impacts, which directly relates to the ethical challenges associated with Bitcoin. As Bitcoin mining consumes a vast amount of energy and generates a significant carbon footprint, it raises questions about the responsibility of individuals and organizations that engage with the technology. By considering the principles of sustainable capitalism to the topic of Bitcoin, we can find ways to evaluate economic growth and usage of the cryptocurrency with environmental sustainability and responsibility.

References

Bowne, R. (2022, 18. May) *Bitcoin production roars back in China despite Beijing's ban on crypto mining.* CNBC. https://www.cnbc.com/2022/05/18/china-is-second-biggest-bitcoin-mining-hub-as-miners-go-underground.html

Houy, N. (2014). The bitcoin mining game. Available at SSRN 2407834.

Hornuf, L., Kück, T., & Schwienbacher, A. (2022). Initial coin offerings, information disclosure, and fraud. Small Business Economics, 58(4), 1741-1759.

Sigalos, M. (2021, 19. December) *Inside China's underground crypto mining operations, where people are risking it all to make bitcoins.* CNBC. https://www.cnbc.com/2021/12/18/chinas-underground-bitcoin-miners-.html

Statnett. (2022, 07. December) *Fast frequency reserve – FFR*. Statnett. https://www.statnett.no/for-aktorer-i-kraftbransjen/systemansvaret/kraftmarkedet/reservemarkeder/ffr/?fbclid=IwAR0Rwc49z3DKbokQ-yUl4RERq4CphCRKExJdFuzp1uPku1ndR_bUXgKJjn4

Worldcoin. (n.d). *Understanding money laundering: how common is it in crypto?* Worldcoin. https://worldcoin.org/articles/cryptocurrency-money-laundering

9.4. Discussion paper – Axel William Patricksson: International

This discussion paper serves as my concluding remarks upon finalizing a chapter in life. Through years of education, my journey as a student culminates in the work presented in our master's thesis. Together with Herman Farstad, we have embarked on a thesis that completes our degree at the master's program in business and administration at the University of Agder, specializing in "analytical finance". I found the process of writing this thesis in cooperation with Herman to be highly successful and believe we have complemented each other well throughout our period of working together. As a final prerequisite, we are required to write a discussion paper reflecting on our thesis in the light of an overarching topic. In my case, this overarching topic is "International". As such, the goal of this paper is to discuss how our thesis relates to concept of "international". Firstly, a short presentation of our thesis will be given, before everything is linked to the topic of international, and then briefly summarized and concluded.

Our thesis focuses on the relationship between the price of Bitcoin and the mining of Bitcoin, while also including the effects of three prominent energy commodities on this relationship. More specifically, we employ a set of wavelet techniques to analyze the time-frequency comovement of bitcoin returns and Bitcoin hashrate growth rates, while controlling for the effects of returns on two-month futures on Brent crude oil, natural gas, and coal. As the process of Bitcoin mining is highly energy intensive, it is associated with massive electricity consumption, thus making it prone to movements in the energy marked. Therefore, we found it interesting to do a study where our objective was to gain an improved understanding of the complex nature of the relationship between Bitcoin price, hashrate, and energy commodities. Our results proved that, indeed, there is influence from the energy commodities on the Bitcoin price-hashrate nexus that should be accounted for when dealing with the relationship between prices and mining of Bitcoin. Furthermore, we observed that the price of Bitcoin appears to be considerably more linked to the hashrate, or mining, during periods of greater economic uncertainty or decline.

In the objective of relating our thesis to the overarching topic of "international", I believe it is highly relevant to focus on the concept of Bitcoin. From an international perspective, Bitcoin is highly interesting. Firstly, Bitcoin's decentralized nature is a key factor in its international appeal. It works on a blockchain network with no central authority or government oversight.

This decentralized structure provides an alternative financial system for individuals and businesses globally. Bitcoin, particularly in areas with limited access to traditional banking services or unstable economies, delivers a sense of autonomy, security, and trust. It gives users complete control over their funds without relying on intermediaries or centralized entities. This characteristic of decentralization makes Bitcoin an appealing alternative for people seeking financial sovereignty and independence, leading to its global popularity.

Further, the ability to facilitate borderless transactions makes Bitcoin a game-changer in the global context. Leveraging blockchain technology, individuals and businesses can transfer value across international borders quickly and cost-effectively. Unlike traditional financial systems that involve complex intermediaries, Bitcoin allows for direct peer-to-peer transactions without geographical limitations. This has the potential to revolutionize cross-border payments, remittances, and international trade by eliminating the need for middlemen and reducing transaction costs. The borderless nature of Bitcoin empowers individuals and businesses to engage in global commerce without the barriers of traditional financial systems, promoting efficiency, accessibility, and financial inclusion on an international scale.

In addition to borderless usage, Bitcoin's accessibility is a driving force behind its international significance. With just a smartphone and internet access, anyone can participate in the Bitcoin network, regardless of their location or socio-economic background. This characteristic of inclusivity breaks down barriers and offers financial opportunities for individuals who lack access to traditional banking services. In regions where traditional banking infrastructure is limited or unreliable, Bitcoin provides an alternative avenue for financial inclusion. It allows unbanked individuals to store value, make transactions, and engage in economic activities independently. Bitcoin's decentralized and permissionless nature creates a level playing field where anyone can be part of the global financial ecosystem, promoting financial empowerment and opening doors to economic opportunities on an international scale.

Bitcoin's store of value properties and its potential as an inflation hedge make it a compelling asset from an international financial perspective. With a limited supply capped at 21 million coins, Bitcoin is designed to be deflationary over time. This characteristic differentiates it from traditional fiat currencies that are subject to inflationary pressures. In countries facing hyperinflation or economic instability, Bitcoin can serve as a reliable store of value, preserving wealth and protecting individuals and businesses from currency devaluation. The decentralized nature of Bitcoin also shields it from political and economic uncertainties that can impact traditional financial systems. As a result, Bitcoin has gained popularity as a hedge against inflation, offering individuals and institutions an alternative avenue for wealth preservation on an international scale.

From a financial view, not only is the store of value attribute of Bitcoin appealing, also its price volatility has attracted significant attention from global investors and traders. Its potential for high returns, coupled with the emergence of cryptocurrency exchanges and investment platforms, has opened up investment and speculative opportunities on an international level. Bitcoin transcends traditional borders and markets, enabling individuals and institutions worldwide to participate in this emerging asset class. The global accessibility of Bitcoin allows for diversification of investment portfolios and exposure to a decentralized, non-correlated asset. While Bitcoin's volatility carries risks, it also presents opportunities for profit-making and capital appreciation. As a result, Bitcoin has garnered interest from both retail and institutional investors seeking exposure to the potential growth and innovation in the cryptocurrency market.

Another key point of Bitcoin's appeal is its underlying technology, the blockchain, which has far-reaching implications beyond digital currency. Its potential to revolutionize various industries, including supply chain management, identity verification, decentralized finance, and more, makes it an intriguing subject from an international perspective. Governments, businesses, and entrepreneurs worldwide are exploring blockchain applications and investing in research and development. The technology's ability to provide transparent, secure, and efficient solutions has the potential to disrupt existing systems and processes on a global scale. By embracing blockchain technology, countries can drive innovation, enhance efficiency, and foster economic growth. The international interest in Bitcoin extends beyond its monetary value and encompasses the transformative power of its underlying technology.

A major challenge, and big question regarding Bitcoin is the regulatory and legal landscape surrounding cryptocurrencies. Governments and regulatory bodies across the globe have taken varied approaches to address the challenges and opportunities presented by Bitcoin. Some countries have embraced Bitcoin, recognizing its potential for economic growth and innovation. Others have implemented regulations to ensure consumer protection, financial stability, and anti-money laundering measures. Additionally, there are jurisdictions that have imposed restrictions or bans on cryptocurrencies due to concerns over financial risks or illicit activities. The evolving regulatory environment creates a complex international landscape that attracts attention from governments, financial institutions, and legal experts worldwide. Discussions, policy developments, and international cooperation are crucial to shaping a balanced and effective regulatory framework that promotes responsible innovation while mitigating potential risks.

The nature of Bitcoin, and its global prominence has fostered collaboration and cooperation among governments, financial institutions, and industry players. International organizations, such as the Financial Stability Board and the International Monetary Fund, are actively studying and discussing the implications of cryptocurrencies. The global community is engaged in dialogue, knowledge-sharing, and efforts to develop consistent frameworks and standards. Collaborative initiatives aim to promote responsible innovation, ensure financial stability, and safeguard against risks associated with cryptocurrencies. Cross-border cooperation is crucial in addressing challenges related to money laundering, terrorist financing, and regulatory harmonization. By fostering collaboration, the international community can collectively leverage the potential of Bitcoin and other cryptocurrencies while establishing safeguards and promoting transparency, integrity, and stability in the global financial ecosystem.

While Bitcoin certainly provides multiple opportunities for improved financial solutions, international cooperations, seamless financial interactions worldwide and so on, one major concern of the concept is its environmental impact. Bitcoin mining, the process by which new bitcoins are created and transactions are verified, has raised concerns about its significant energy consumption and environmental impact. The mining process involves solving complex mathematical problems that require substantial computational power, resulting in high energy demands. As Bitcoin mining operations have expanded globally, so too have concerns about the carbon footprint associated with this energy-intensive process. The reliance on fossil fuels for electricity generation in certain regions further exacerbates the environmental impact. From an international perspective, addressing the environmental challenges posed by Bitcoin mining is crucial for sustainability. It calls for exploring energy-efficient mining techniques, renewable energy sources, and incentivizing environmentally responsible practices within the Bitcoin mining community. Striking a balance between the potential benefits of Bitcoin and its environmental impact is essential for a more sustainable future.

In summary, Bitcoin holds international appeal due to a range of opportunities and possibilities enabled by its technological innovation and decentralized nature. It has the potential to make financial markets accessible for societies without access to established, stable traditional banking systems, and also bring a higher degree of autonomy over user's own funds. While Bitcoin has a lot of potential, it also has challenges. The environmental aspect of the Bitcoin network is one major challenge. Due to the high energy intensity of its mining process, electricity consumption is a concern which only will increase with increased use of Bitcoin. In a world where we are already facing life threatening environmental changes, the added stress on our global environment from the implementation of Bitcoin could pose a threat to its viability. As studied in our thesis, the relationship between Bitcoin price and hashrate is clearly influenced by energy commodities. This result emphasizes Bitcoin's dependance on energy sources to keep running. As suggested in the concluding phase of our paper, we believe that increased focus on implementation of renewable energy solutions for Bitcoin mining is critical for further success of the concept.