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Analysis of relations between attitude towards mathematics, prior knowledge, self-efficacy, expected and actual grades in mathematics

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The purpose of this study is to analyse relations between students' attitude towards mathematics, prior knowledge, self-efficacy, expected grades, and performance in mathematics among 115 first-year engineering students. We combine two statistical techniques to analyse the data we generated by questionnaires and two tests. First, item-level modeling, in terms of confirmatory factor analysis, which we use to compute the factor scores of construct-validated measures, and to control for measurement errors. Second, composite modeling, in terms of path analysis, which we use to test the research hypotheses. The findings show that both self-efficacy and expected grades have substantial effects on students' performance. Prior knowledge has a non-trivial effect on self-efficacy which, in turn, plays a significant role in students' grade expectations. All other hypothesised relations are not significant. We argue that these findings confirm some basic tenets of social cognitive theory.

Keywords: Affect, higher education, item-level modeling, path analysis, self-efficacy.

Introduction

Affect in mathematics education

Students' affect is critical not only to their well-being but also to their performance in mathematics. Following the lines of thought proposed by Hannula (2012), we define affect in mathematics education research as a general concept that encapsulates factors, other than purely cognitive ones, such as attitude, beliefs, emotions, feelings, goals, moods, motivation, norms, values, and self-efficacy. Thus, each factor that constitutes a unit of mathematics-related affect is regarded as an overlap between cognition, emotion, and motivation of varying stability with psychological, physiological, and social dimensions (Hannula, 2012). Prominent among the mathematics-related affect factors are attitude towards mathematics and self-efficacy. It is arguable that the former is prominent for its incoherent conceptualisations within mathematics research community (Di Martino & Zan, 2010) while the latter is prominent for its high predictive power of performance and its causal relation with students' mathematics performance (Roick & Ringeisen, 2018; Zakariya, 2021a).

Attitude towards mathematics

In line with the theoretical framework proposed by Hannula (2012), attitude of students towards mathematics (henceforth, attitude) shares a boundary between cognition (e.g., knowledge), emotion (e.g., likes and dislikes), and motivation (e.g., internal and external drives to approach or refrain from mathematics activities). It can be operationalised and measured using self-report psychometric research measures. Empirical evidence shows attitude is predicted by prior mathematics knowledge (henceforth, prior knowledge) and it predicts students' subsequent performance on mathematics tasks (Chen et al., 2018; Lipnevich et al., 2016). Students that belief in their mathematics ability, those that like mathematics, and those that approach mathematics with its pre-conceived utility for future aspirations are usually successful in mathematics tasks. On the flip side, students that do not belief in their mathematics knowledge, those that dislike mathematics, and approach mathematics with ill-conceived utility of mathematics for future aspirations are usually unsuccessful in mathematics tasks. Some researchers (e.g., Kiwanuka et al., 2020) have also shown that there is a reciprocal effect between attitude and performance in mathematics. That is, high achievers tend to develop positive attitude. In return, students with positive attitude tend perform well on mathematics tasks. Thus, attitude plays a crucial role in students' success on mathematics tasks.

Mathematics self-efficacy

Self-efficacy has its roots in social cognitive theory as propagated in decades of work by Albert Bandura. It entails "beliefs in one's capabilities to organize and execute the courses of action required to produce given attainments" (Bandura, 1997, p. 3). As it relates to mathematics learning, self-efficacy is students' self-evaluation of competence to proffer correct solutions to mathematics tasks (Zakariya, Nilsen, et al., 2020b). It is a combination of confidence in ones' capacity and an estimation of outcome that follows ones' effort. There are four sources of self-efficacy – mastery experience, vicarious experience, social persuasion, and affective states – among which mastery experience i.e., self-interpretation of previous attainment has the highest influence on self-efficacy (Zientek et al., 2019). An accumulation of evidence suggests that self-efficacy is one of the best predictors of mathematics performance. Students with high sense of self-efficacy have low mathematics anxiety, adopt deep approaches to learning, and perform well on mathematics tasks (Rozgonjuk et al., 2020; Zakariya, Nilsen, et al., 2020b). Evidence supports consistency of a model of reciprocal effect between self-efficacy and mathematics performance with generated data across 24 countries (Williams & Williams, 2010). Moreover, Zakariya (2021a) provides a tentative evidence for causal effect of self-efficacy and performance in mathematics. Thus, self-efficacy is a critical factor for students' success in mathematics.

Relations between attitude and self-efficacy

The relationship between attitude and self-efficacy coupled with their combined effect on performance has been sparingly studied. Yet, the results of the available studies are promising. Randhawa et al. (1993) using structural equation modeling show that attitude significantly predicts self-efficacy which in turn predicts students' performance in mathematics. More so, the effect of attitude on performance is partially mediated by self-efficacy in a non-trivial way. However, the study by Randhawa et al. (1993) is relatively old and focuses on high school students whose findings may not be of direct relevance to undergraduate students. In a more recent study involving seventh graders, Recber et al. (2018) show that there is a non-trivial positive correlation between attitude and self-efficacy. Further, both constructs are significant predictors of performance in mathematics (Recber et al., 2018). Regrettably, correlation between two variables has limited value in terms of inferential deductions and tangible conclusions. A similar limitation can also be ascribed to the study by Öztürk et al. (2019) who report a correlational analysis between attitude and self-efficacy, and their predictive effect on performance of middle school students in mathematics.

The present study

The intention of the present study is to provide an evidence-based model of relationship between attitude, self-efficacy, prior knowledge, and undergraduate students' performance (expected and actual) in mathematics. This study differs from the previous attempts in many ways. First, we approach the analysis from structural equation modeling (SEM) perspective, instead of correctional analysis, which avails us the opportunity to test theory-based hypotheses. Second, we focus on undergraduate engineering students, who have mathematics as a core subject but whose affect (i.e., relations between attitude and self-efficacy) appears not to be given much attention. The inclusion of other factors such as prior knowledge and expected grades in our model constitutes another difference from the previous attempts. In specific terms, the present study addresses the following research question: To what extent do attitude, self-efficacy, prior knowledge, and expected grades predict each other and undergraduate students' performance in mathematics?

To address the research question, we draw on theoretical foundations and some insights from literature to hypothesise that attitude, expected grades, and self-efficacy predict performance and are predicted by prior knowledge. We admit that there is a possibility of reciprocal effect between attitude and self-efficacy. However, we ensure temporary precedence by collecting attitude data eight weeks before collecting data on self-efficacy. As such, we hypothesise that attitude has a non-trivial positive effect on self-efficacy. Given that outcome expectancy is an integral part of self-efficacy, we hypothesise positive effect of self-efficacy on students' expected grades.

Methods

Participants and measures

One hundred and fifteen undergraduate students (90 males) voluntarily gave consent and took part in the study that lasted for a semester. These students, average age between 21 – 25 years, followed a first-year calculus course in a Norwegian university. They completed attitude towards mathematics questionnaire (AtMQ) and sat for a test of prior mathematics knowledge (TPMK) in the third week of the semester. On the one hand, the AtMQ is a five-item measure (sample item: I'm interested in what I learn in math) designed to expose a single construct on a four-point Likert scale from strongly disagree to strongly agree. On the other hand, TPMK is a 16 – item (22 subitems) test of basic algebra, functions, and geometry. Previous studies show that both AtMQ and TPMK demonstrate appropriate validity and have acceptable indices of reliability of .92 and .78, respectively (Zakariya, Nilsen, Bjørkestøl, et al., 2020; Zakariya, Nilsen, et al., 2020a). Further, we administered calculus self-efficacy questionnaire (CSEI) at the end of the semester. The CSEI is 13-item measure on which students are to rate their competence to solving presented exam-like first-year calculus tasks on a scale of 100 points, whose validity and reliability indices have been previously studied with promising results (Zakariya et al., 2019). As a measure of expected grades, an item was appended to CSEI that asks students to report their expected grades in forthcoming calculus exam, at the time. The students' final exam scores in the calculus course serve as a measure of performance in mathematics. The full measures are available as appendices in the referenced validation studies.

Data analysis

We analysed the generated data using some techniques of SEM in two stages. Stage one involves evaluating measurement models for AtMQ, CSEI and TPMK, using confirmatory factor analysis. The rationale for this analysis is to detect and correct misspecification errors in the measurement models prior to hypothesis testing. Simultaneously, we confirm construct validity of each of the measures and compute the factor scores. The second stage of the analysis involves testing the hypothesised relations between the research constructs. We evaluate the structural models using robust maximum likelihood (MLR) estimator. The models are assessed for their global fits of the generated data using a combination of criteria. The model exhibits an exact global fit of the data if the chi-square value is not significant. There is an excellent fit of the data if the comparative fit index (CFI), Tucker-Lewis index (TLI) are greater than or equal to .95, and root mean square error of approximation (RMSEA) is either ≤ .06 or its 90% confidence interval (C.I.) contains 0.06 (Chen, 2007). The model exhibits an appropriate global fit of the data if the ratio of chi-square value to the degree of freedom is less than 3, CFI and TLI are close to or greater than .90, and RMSEA is less than .08 (Bentler, 1990; MacCallum et al., 1996). Significant parameter estimates show that the model exhibits a local fit of the data. We run all the analyses in Mplus 8.5 software.

Results and discussions

Measurement models

The first set of results concern evaluations of measurement models for each of the measures. For both AtMQ and CSEI, we evaluated a one-factor model each using MLR estimator. Following the recommendation by Zakariya (2021b), we correlated disturbances of item 2 and item 4 of AtMQ to improve the model fit. In a similar manner, we correlated disturbances of item 9 with item 11 and of item 12 with item 13 to achieve a model fit as recommended by Zakariya (2021a). Further, we evaluated a one-factor model of TPMK using robust unweighted least squares (ULSMV) estimator. This estimator takes care of the categorical scoring of the TPMK. The best 17 out of the 22 subitems of the TPMK are used for this analysis as recommended by Zakariya, Nilsen, et al. (2020a). The results are presented in Table 1.

Table 1: Goodness of fit statistics of measurement models of the research measures

Global fit indices	AtMQ model	CSEI model	TPMK model
Chi-square estimate (χ^2)	7.846	99.151	129.769
Degrees of freedom (df)	4	63	119
p – value	.097	.003	.236
$\chi^2 \,/\, df$	1.962	1.574	1.090
RMSEA [90% C. I.]	.091 [<.001 - .186]	.071 [.042 - .096]	.028 [.042 - .056]
CFI	.982	.905	.968
TLI	.954	.882	.964

The presented results in Table 1 show that there are exact fits of both AtMQ and TPMK models with the generated data. The non-significant chi-square values coupled with RMSEA, CFI, and TLI values that are within recommended ranges support the claim of exact fits of both AtMQ and TPMK models. That is, both AtMQ and TPMK measure the constructs (attitude and prior knowledge, respectively) they are purported to measure. More so, Table 1 reveals that CSEI model exhibits an appropriate model fit of the generated data. The chi-square value is significant but its ratio to the degree of freedom is less than 3. More so, the RMSEA value is less than 0.08 and both CFI and TLI are close to .90. These values support the appropriate fit of the CSEI model. That is, the CSEI measures the calculus self-efficacy of students it is supposed to measure.

Hypothesis testing (Addressing the research question)

After the evaluation of the measurement models of all the measures, we compute the factor scores of both AtMQ and CSEI using the default maximum of the posteriori distribution in Mplus because of the continuous nature of their datasets. On the other hand, Mplus uses maximum a posteriori method to compute the factor scores of TPMK because of the categorical nature of the dataset. Then, we saved the scores and use them to evaluate the hypothesised structural model of relationships between the research constructs. This evaluation avails the opportunity to test the research hypotheses and address the research questions. Figure 1 presents the goodness of fits statistics and the final evaluated model.

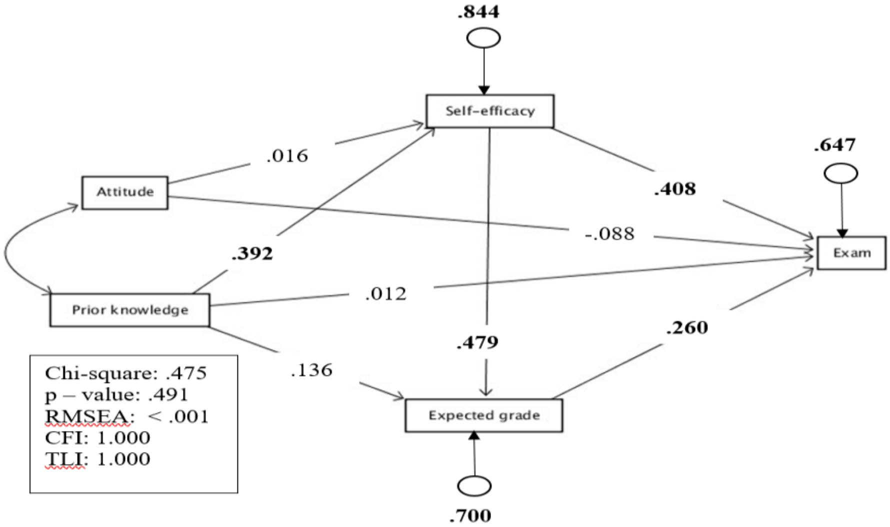

Figure 1: Evaluated hypothesised model of relationships between the research constructs with significant parameter estimates in bold faces

The presented results in Figure 1 shows some interesting findings. From the model fit perspective, Figure 1 shows that there is an exact model fit. That is, there is consistency between the hypothesised relationships and the generated data. In line with the postulations of the present study, Figure 1 confirms that self-efficacy and expected grades are significant predictors of students' performance in mathematics. That is, both high sense of self-efficacy and high students' expectations in the exams lead to high performance in mathematics. These findings corroborate previous studies (Rozgonjuk et al., 2020; Zakariya, 2021a) that have shown non-trivial relationships between self-efficacy and performance in mathematics. In support of the hypothesis of the present study, prior mathematics knowledge significantly predicts self-efficacy. This finding confirms a tenet of social cognitive theory that says that mastery experience (students' prior attainments) is an integral source of self-efficacy (Bandura, 1997). Figure 1 also provide empirical support for the non-trivial effect of self-efficacy on students' expected grades. This finding confirms the postulation of social cognitive theory that theorised outcome expectation as an integral part of self-efficacy (Bandura, 2012). Admittedly, it is logical that expected grade has a reverse effect on self-efficacy. However, we acknowledge this fact and take care of it by ensuring temporary presence with self-efficacy measure coming before the item on expected grade during the questionnaire administration. We recommend that future studies should be designed with this intention.

Contrary to the postulations of the present study, attitude fails to predict both self-efficacy and students' performance in mathematics. This assertion is deduced from Figure 1 that shows that the path coefficients (.016 and -.088) between attitude and the two variables (self-efficacy and exam) are not statistically significant. This finding that attitude does not predict mathematics achievement is aligned with some previous studies (e.g., Fernández-Cézar et al., 2021) although it does not support other studies that have reported substantial relationships between attitude and both self-efficacy and performance (Chen et al., 2018; Öztürk et al., 2019). It is possible that the findings of previous studies are not generalisable to our context. Another explanation for these unexpected findings could be a defect from the measure of attitude. Perhaps, the students had a different interpretation of AtMQ items from what the researchers intended. A future study may be designed to explore students' interpretations of these items. More so, Figure 1 shows that there is no evidence in the present study to substantiate non-trivial effects of prior knowledge on both the students' expected grades and performance in mathematics because the path coefficients (.012 and .136) are not significant. These findings are unexpected as well. A possible explanation could be a lack alignment between the knowledge assessed by PKMT and that of the current course. This observation requires further investigation. In sum, the results of the hypothesis testing address the research question by showing the extent to which attitude, self-efficacy, prior knowledge, and expected grades predict each other and undergraduate students' performance in mathematics.

Conclusion

We made some attempts in the present study to disentangle the complex relations between attitude, prior knowledge, self-efficacy, expected grades, and performance in mathematics among engineering first-year students. We combined item-level structural equation modeling techniques with composite modeling by using confirmatory factor analysis to compute factor scores which we further used in path analysis. This combined analytical procedure offers two advantages. First, we minimize biases from measurement errors by incorporating them in the item-level analysis. Second, we evaluate a complex model using a relatively small sample which would not have been possible, otherwise. The findings provide empirical support for substantial effect of self-efficacy and expected grades on students' performance in mathematics. They also confirm some theoretical postulations such as the crucial role of self-efficacy in students' outcome expectations on mathematics tasks. By implication, the findings support interventions on self-efficacy as a proxy to improve students' performance in mathematics.

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