Opportunities for reasoning and argumentation

Norwegian mathematics textbooks for grades 5-7

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Preface

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When I was younger and got tired during long hikes with my parents, there was always one thing they would do to keep me walking. They would quiz me. And there were two things in particular that I loved being quizzed on. Capital cities, and mathematics. I have always been eager to learn, to understand, and to ask questions. And my favorite question was, and still is: Why? This question is one of the things that led me to the subject of this thesis. The need to know and understand why. The goal of becoming a teacher that can share the joy of understanding and knowing why. And the wish for a mathematics education that does not ignore the importance of understanding and knowing why.

This thesis marks the end of my teacher education, five beautiful, challenging, and inspiring years in a city far from home. I am forever grateful for all the experiences this journey has provided, which have helped form me into the person I am today. I would like to thank all professors, teachers, pupils, classmates, and friends that have helped me along the way and been a part of the journey. I would also like to thank Yusuke Shinno, Takeshi Miyakawa, Ryoto Hakamata, Hiroki Otani, Christine Knipping, Fiene Bredow, and David Alexander Reid for including me in their international research project. A double thanks to David, as my supervisor, for suggesting this subject, for being extremely helpful and understanding, and for including me in the community of mathematics education researchers. Thank you to my family for engaging conversations and questions regarding my thesis, and to my partner, Stian, for being patient and understanding, supporting and loving.

Summary

Utdanningsdirektoratet (2019a; 2019b) states that reasoning and argumentation shall permeate the entire mathematics education, and as one of six core elements in the curriculum it is among the most important contents in mathematics education. Reasoning and argumentation in mathematics is about understanding that mathematical rules and results have obvious reasons and are not random, and to give reasons and prove that approaches, reasons, and solutions are valid (Kunnskapsdepartementet, 2019a). Several researchers refer to textbooks as central in teaching, both in planning and completing lessons (Wong & Sutherland, 2018; Pepin et al. 2013). The goal of the present study is to understand what opportunities there are for reasoning and argumentation in Norwegian mathematics textbooks for grades 5-7, and the research questions are: 1) To what extent do Norwegian mathematics textbooks use words related to reasoning and argumentation?; and 2) How are the words related to reasoning and argumentation used in the textbooks, and what usages offer opportunities for reasoning and argumentation? Reasoning and argumentation as defined in the curriculum must be deductive, and proof is included as a central part of the core element *Reasoning and argumentation* (Valenta & Enge, 2020). Proof is defined by Stylianides (2007) as a mathematical argument for or against a mathematical claim expressed by statements, forms of reasoning, and forms of representations that are known to a classroom community. I have in this study used the method of Document analysis, consisting of a superficial examination of the textbooks to examine the extent of the words related to reasoning and argumentation, followed by a closer analysis. I have adapted the aspects *structure*, *language*, and *function* as defined by Miyakawa and Shinno (2021) in the closer analysis, to examine the usage of the words related to reasoning and argumentation, and to consider what usages offer opportunities for reasoning and argumentation. The findings of the superficial examination show that the words occur rarely or not at all in the textbooks analyzed. The words *resonnere* and *bevise* do not occur, *argumentere* occurs 9 times, *begrunne* occurs 169 times, *forklare* occurs 271 times, and *vise* occurs 609 times in the 15 analyzed textbooks. Several usages are found not to offer opportunities for reasoning and argumentation, but some usages are found to offer such opportunities. The textbooks do offer opportunities for reasoning and argumentation; however, the overall understanding of such opportunities is that they are few, which contrasts both with the curriculum and the call from mathematics educational researchers for reasoning and argumentation to be a central part of mathematics education. This study could be beneficial for understanding what opportunities there are for reasoning and argumentation in Norwegian mathematics textbooks and for understanding the difficulties teachers and pupils might face when searching for opportunities to teach, guide, support, learn and understand reasoning and argumentation in mathematics.

Sammendrag

Utdanningsdirektoratet (2019a; 2019b) hevder at resonnering og argumentasjon skal gjennomsyre opplæringsløpet i matematikk, og er som et av seks kjerneelementer regnet som det viktigste innholdet i matematikkutdanningen. Resonnering og argumentasjon i matematikk handler om å forstå at matematiske regler og resultater har klare begrunnelser og ikke er tilfeldige, samt å begrunne og bevise at fremgangsmåter, resonnementer og løsninger er gyldige (Kunnskapsdepartementet, 2019a). Flere forskere peker på lærebøker som sentrale i undervisning, både i planlegging og gjennomføring (Wong & Sutherland, 2018; Pepin et al. 2013). Målet med denne studien er å forstå hvilke muligheter for resonnering og argumentasjon som tilbys i norske matematikklærebøker for 5.-7. trinn, og forskningsspørsmålene som stilles er: 1) I hvilken grad bruker lærebøkene ord relatert til resonnering og argumentasjon?; og 2) Hvordan benyttes ord relatert til resonnering og argumentasjon i lærebøkene, og hva slags bruk tilbyr muligheter for resonnering og argumentasjon? Resonnering og argumentasjon som definert i læreplanen må være deduktiv og bevis inngår som en sentral del av kjerneelementet *Resonnering og argumentasjon* (Valenta & Enge, 2020). Bevis defineres av A. Stylianides (2007) som et matematisk argument for eller mot en matematisk påstand uttrykt med uttalelser, former for resonnering og representasjonsformer som er kjent for et klassesamfunn. Jeg har i denne studien benyttet metoden Dokumentanalyse, bestående av en overfladisk undersøkelse av lærebøkene for å undersøke omfanget av ordene relatert til resonnering og argumentering, etterfulgt av nærmere analyse. I den nærmere analysen har jeg tilpasset aspektene *struktur*, *språk* og *funksjon* definert av Miyakawa og Shinno (2021) for å undersøke bruken av ordene relatert til resonnering og argumentasjon og for å vurdere hvilke bruksområder som tilbyr muligheter for resonnering og argumentasjon. Funnene av den overfladiske undersøkelsen viser at ordene sjeldent eller aldri forekommer i lærebøkene som er analysert. Ordene *resonnere* og *bevise* forekommer aldri, *argumentere* forekommer 9 ganger, *begrunne* forekommer 169 ganger, *forklare* forekommer 271 ganger og *vise* forekommer 609 ganger i de 15 lærebøkene analysert. Flere bruksmåter viser seg å ikke tilby muligheter for resonnering og argumentasjon, men det er også bruksmåter som viser seg å tilby slike muligheter. Lærebøkene tilbyr muligheter for resonnering og argumentasjon, men den helhetlige forståelsen av slike muligheter er imidlertid at de er få, noe som står i kontrast både til læreplanen og matematikkutdanningsforskeres etterlysning av resonnering og argumentasjon som sentralt i matematikkutdanningen. Denne studien kan være gunstig for å forstå hvilke muligheter som finnes for resonnering og argumentasjon i norske lærebøker i matematikk, og for å forstå hvilke utfordringer lærere og elever møter når de søker etter muligheter for å undervise, guide, støtte, lære og forstå resonnering og argumentasjon i matematikk.

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1. Introduction

The Ministry of Education and Research (Kunnskapsdepartementet) added six core elements to the new Norwegian curriculum in mathematics which was implemented in Norwegian schools August 1st, 2020. The Norwegian Directory of Education and Training (Utdanningsdirektoratet, 2019a) states that the core elements are the most important academic content that the pupils are supposed to work with during their education. The core elements are supposed to characterize the content and the progression of the subject (Utdanningsdirektoratet, 2019b, 01:13) and the focus in this study is based on one of these six core elements: *Reasoning and argumentation*.

> Reasoning in mathematics means the ability to follow, assess and understand mathematical chains of thought. It means that the pupils shall understand that mathematical rules and results are not random, but have clear reasons. The pupils shall formulate their own reasoning in order to both understand and [solve] problems. Argumentation in mathematics means that the pupils give reasons for their approaches, reasonings and solutions, and prove that these are valid. (Kunnskapsdepartementet, 2019b, p. 3)

Reasoning and/or argumentation is also mentioned in two of the other core elements, in *Representation and communication* where the pupils shall use the mathematical language in argumentation and reasoning, and in *Abstraction and generalization* where abstraction is about developing a formal language in mathematics to give formal reasoning (Kunnskapsdepartementet, 2019b, p. 3). In other words, the curriculum states that the pupils shall get to reason and argue when learning mathematics, and they shall also use, and develop a formal, mathematical language for said reasoning and argumentation. Utdanningsdirektoratet (2019b, 01:13) states that the pupils need time to work on the core elements to get an understanding of the subject. The goal of the present study is to understand what opportunities there are for reasoning and argumentation in Norwegian mathematics textbooks for grades 5-7.

Research show that mathematical reasoning is infrequent in mathematics classrooms (Stacey & Vincent, 2009), and based on research that shows that Norwegian teachers frequently use textbooks both for lesson preparations and in class (Pepin et al., 2013), I want to investigate opportunities for reasoning and argumentation in Norwegian mathematics textbooks. Bakken & Bakken (2021) point out that textbooks are a major source of tasks and activities used for both classroom activities and for homework and it is reasonable to assume that textbooks are used as a main resource for teaching (Pepin et al., 2013). Fujita and Jones (2014) speak of the challenges mathematics educators face in their efforts to develop a classroom community that foster mathematics as reasoning, and state that textbooks are one important source of support to those challenges. The goal of the present study is to understand what opportunities there are for reasoning and argumentation in

Norwegian mathematics textbooks for grades 5-7. The textbooks studied were written or revised after the implementation of the new Norwegian curriculum and published by three Norwegian textbook publishers that are assumed to be the largest (Tømmerdal, 2021).

The definition of reasoning and argumentation as given in the curriculum gives the impression that proof and proving are key components in this core element, although the word *prove* is only used once (Valenta & Enge, 2020). Valenta and Enge (2020) point out that proof is a central part of mathematical reasoning, and for that reason, they view reasoning (and other proof-related competencies) as significant when working with proofs. Stylianides' (2007) definition of proof explains that a proof is a mathematical argument with some given characteristics. The mathematical argument must be deductive to be called a proof, and the core element's definition of both reasoning and argumentation also gives the impression that the accepted form of reasoning and argumentation is deductive (Valenta & Enge, 2020). This is also supported by Reid (2022) who points out that the core element of *Reasoning and argumentation* only includes processes related to validating. I will be using Stylianides' (2007) definition of proof and the curriculum's definitions of reasoning and argumentation in the present study.

Ball et al. (2002, p. 907) state that,

> Proof is central to mathematics and as such should be a key component of mathematics education. This emphasis can be justified not only because proof is at the heart of mathematical practice, but also because it is an essential tool for promoting mathematical understanding.

The statement is supported by Stylianides and Stylianides (2009, p. 238) "proof is at the core of doing and knowing mathematics". The Norwegian curriculum in mathematics seems to acknowledge the importance of proof as reasoning and argumentation are implemented into the curriculum as a core element. In fact, Utdanningsdirektoratet states that reasoning and argumentation shall permeate mathematics education (2019b, 03:17). The core element and the stated importance reasoning and argumentation have in mathematics education, both in the curriculum and by mathematics education researchers, form the basis of the present study.

Godino and Recio (1997) remind us that proving activities can be recognized through different terms, such as *justify*, *argue*, and *demonstrate*, and that the words related to argumentation, reasoning, and proof, which are neither of the three, all have "a common idea, - that of *justifying* or *validating* a statement (thesis) by providing *reasons* or *arguments* "(p. 314 [italics added]). Identifying how words related to reasoning and argumentation are used in the textbooks could give an impression of the opportunities for the pupils to engage in reasoning and argumentation activities, and the research questions I would like to explore in the present study are:

1. To what extent do the textbooks use words related to reasoning and argumentation?
2. How are words related to reasoning and argumentation used in the textbooks, and what usages offer opportunities for reasoning and argumentation?

In Chapter 2 I will explain relevant concepts and prior research will be presented. I will also explain developed frameworks for analyzing reasoning and proof in school textbooks and the classroom. In Chapter 3 I will present the methodology of the present study. Chapter 4 will present the analysis and results of the present study. I will discuss the results in light of related research in Chapter 5. Chapter 6 will include a summary of the present study, and a final comment.

2. Literature review

The goal of the present study is to understand what opportunities there are for reasoning and argumentation in mathematics textbooks for Grades 5-7. This chapter will introduce related research and concepts related to reasoning and argumentation. In section 2.1. I will present prior research on reasoning, argumentation, and proof in curricula. The first research presented is by Valenta and Enge (2020) who investigated proof-related competencies in the Norwegian curriculum, and then I will present research on the role of proof in mathematics curricula by Hanna (1995). Section 2.2. will introduce a definition of proof by A. Stylianides (2007) to justify the choice of viewing proof in connection with reasoning and argumentation as defined in the curriculum. In section 2.3. I will present relevant frameworks applied in several of the studies that will be presented in section 2.3., and a framework by Miyakawa and Shinno for analyzing proving activities. In section 2.4. I will present previous research on reasoning, argumentation, and proof in textbooks.

2.1. Reasoning, argumentation, and proof in the curriculum

Valenta and Enge (2020) investigated proof-related competencies in the Norwegian curriculum in mathematics. Their interpretation of the curriculum's definition of argumentation in its core elements is the one I will be using in the present study. Reasoning and argumentation in the way the curriculum views it has to be deductive (2020, p. 3). Valenta and Enge justify their connection between the core element *Reasoning and argumentation* and proof further by stating that "[i]n order to clarify the significance that the various proof-related competencies may have for working with proof, we see proof in connection with mathematical reasoning since proof is a central part of mathematical reasoning" (p. 6, my translation). Building their analysis on the framework developed by Hemmi et al. (2013) for categorizing proof-related competencies they found that no competence aims for the grades 1-10 included goals of the pupils' developing proofs or proving and there was little focus on the development of logical thinking and work connected to definitions. However, they found that the curriculum in their competence aims facilitated pupils working with proof-related competencies like exploring and non-deductive argumentation in mathematics and working with different representations and connections. They conclude that the lack of emphasis on proof and proving, the lack of focus on logic and formal aspects of mathematics, and the lack of emphasis on the role of mathematical definitions, show that the curriculum does not facilitate for development of proofs from the argumentations that are produced. (Valenta & Enge, 2020, p. 15).

Hanna (1995) asks the question of what role proof should have in mathematics education. She says that if the goal of a curriculum is to reflect mathematics itself, proof should be a part of the said curriculum and that the main purpose of proof in the classroom should be to promote understanding, as this is a key function of proofs in the mathematics community (p.

42). This is not to say that the mathematics education should use the proofs only as they are used in the mathematics community, which often is to justify or clarify, but that its main function should be that of explanation.

> To say that proof should be explanatory is not to say it cannot take different forms. It might be a calculation, a visual demonstration, a guided discussion observing proper rules of argumentation, a preformal proof, an informal proof, or even a proof that conforms to strict norms of rigour, all depending on the grade level and the context of the institution. (Hanna, 1995, p. 47)

The pupils shall not only understand that the mathematics they learn in school is true, but also why it is true, and this is where the proofs play a significant role in mathematics education (Hanna, 1995). In the Norwegian curriculum this is supported in the core element *Reasoning and argumentation*, when an explanation of mathematical reasoning is provided, "the pupil shall understand that mathematical rules and results are not random, but have clear reasons" (Kunnskapsdepartementet, 2019b, p. 3).

Both of these articles help explain why I have extended my research beyond the terms reasoning and argumentation to involve proof. Valenta and Enge (2020) point out that reasoning and argumentation in the way it is defined by the curriculum's core element has to be deductive, and that proof is a central part of mathematical reasoning. Supported by Hanna (1995) that explains why proof should play a significant role in mathematics education, in an equivalent way to how Kunnskapsdepartementet (2019b) explains why mathematical reasoning is at the core of Norwegian mathematics education.

2.2. A definition of proof

That proof, argumentation, and reasoning is important in the mathematics education is internationally agreed upon (Ball et al., 2002; Hanna, 1995; Stylianides, 2007; Stylianides, 2009; Szücs, 2022), but the definition of proof and proving is still unclear (Balacheff, 2008; Reid 2005; Stylianides, 2007). In the present study I will be using the definition from Andreas Stylianides' (2007, p. 291):

> *Proof* is a *mathematical argument*, a connected sequence of assertions for or against a mathematical claim, with the following characteristics:
>
> 1. It uses statements accepted by the classroom community (*set of accepted statements*) that are true and available without further justification;
> 2. It employs forms of reasoning (*modes of argumentation*) that are valid and known to, or within the conceptual reach of, the classroom community; and

3. It is communicated with forms of expression (*modes of representation*) that are appropriate and known to, or within the conceptual reach of, the classroom community.

Examples of the Three Components of a Mathematical Argument Mentioned in the Definition of Proof

Component of an argument	Examples
Set of accepted statements	Definitions, axioms, theorems, etc.
Modes of argumentation	Application of logical rules of inference (such as modus ponens and modus tollens), use of definitions to derive general statements, systematic enumeration of all cases to which a statement is reduced (given that their number is finite), construction of counterexamples, development of a reasoning that shows that acceptance of a statement leads to a contradiction, etc.
Modes of argument representation	Linguistic (e.g., oral language), physical, diagrammatic/pictorial, tabular, symbolic/algebraic, etc.

Figure 2.1 Examples of the Three Components of a Mathematical Argument Mentioned in the Definition of Proof (Stylianides, 2007, p. 292).

Stylianides (2007, p. 292) notes that there are other classes of arguments that can lead to valid conclusions, such as arguments by analogy, but the definition describes a class of arguments that qualify as proofs and that typically represents the end of a mathematical exploration, deductive argumentation. The definition does not devalue the importance of other mathematical reasoning and argumentation activities that could support the development of proofs, such as empirical arguments. Stylianides emphasizes,

> I do not suggest that we rid school mathematics from empirical ways of thinking. Rather, I suggest that we do not call mathematically unqualified arguments "proofs", and that we help students understand the distinction between arguments that do and do not qualify as proofs. (2007, p. 298)

The classroom community in Stylianides' definition is considered to consist primarily of the pupils, where the teacher has a special role as the person trying to connect their pupils with a piece of broader mathematical knowledge. This is reflected in the curriculum where Kunnskapsdepartementet (2019b, p. 10) states in the formative assessments section for grade 5 that the teacher shall provide guidance and adapt their teaching so the pupils can both use the guidance they are offered and develop their competencies for using mathematical concepts when arguing. Stylianides (2007) notes that the *conceptual reach of the classroom community* does not make up for individual differences in understanding or knowledge but views the community as a whole. It is not necessary that every individual in the community understands, or accepts, the proof, but that the statements, or arguments, can be assumed as within the reach of the individuals under guidance by the teacher.

According to Krummheuer (1995), an argument is the product of the process of argumentation, and although he does not necessarily speak of mathematical arguments and mathematical argumentation, his description shows that he is interested in the process leading up to the verification of a statement (Reid & Knipping, 2010). “Usually, these techniques or methods of establishing the claim of a statement are called an argumentation. Thus a successful argumentation refurbishes such a challenged claim into a consensual or acceptable one for all participants” (Krummheuer, 1995, p. 232). In addition, Krummheuer views argumentation as essential for learning mathematics, and his approach does not interfere with learning proof (Krummheuer, 2007, p. 62).

Viewing argumentation like Krummheuer (1995; 2007), as an essential process for learning mathematics, and viewing argumentation as the process leading up to the verification of a statement, supports viewing proof as relevant for the present study. Stylianides’ (2007) definition of proof also supports the choice of seeing proof as a central part of my investigation of reasoning and argumentation in middle school textbooks, based on the definitions given in the curriculum. The modes of argumentation (forms of reasoning) that would be identified as valid by Stylianides are the same that would fit in the definitions of argumentation and reasoning given by Kunnskapsdepartementet (2019b), deductive arguments (Valenta & Enge, 2020).

2.3. Frameworks for analyzing reasoning, argumentation, and proving

In 2008 and 2012 two frameworks were developed for analyzing reasoning and proof in mathematics textbooks and mathematics curricula (Stylianides, 2008; Thompson et al., 2012) that have been included in a lot of the research done on the field in the following years. In this section, I will go more into depth about these frameworks to better explain the chosen methods used by several authors (e.g., Bieda et al., 2014; Wong and Sutherland, 2018; Tømmerdal, 2021), and to later explain why I have chosen to not use them in my research. Finally, this section will include a framework by Miyakawa and Shinno (2021) for characterizing proof and proving activities from a cultural perspective.

Gabriel Stylianides (2008) proposed an “analytic framework that can serve as a useful platform for conducting different kinds of investigations with a focus on RP [reasoning-and-proving]” (p. 9). Reasoning-and-proving is a hyphenated term used to describe four major activities involved when establishing mathematical knowledge. I will use the notion RP when referring to the term in the present study. The four activities identified in RP are: identifying patterns; making conjectures; providing non-proof arguments; and providing proofs. The last two activities are captured "under the notion of 'providing support to mathematical claims' and the first two activities under the notion of 'making mathematical generalizations' (p. 9). The analytic framework includes a mathematical, a psychological, and a pedagogical component, set by the conceptualization of RP.

The mathematical component includes the four activities, with its primary feature being the integration of well-known activities related to the engagement with proof. G. Stylianides refers to two types of patterns within the first activity: definite patterns and plausible patterns. When encountering a definite pattern, it is mathematically possible to provide conclusive evidence for the selection of the pattern. The plausible pattern is not mathematically possible to provide conclusive evidence to, as there might be more than one pattern that would fit the given data. For the second activity, making a conjecture, G. Stylianides defines a conjecture as a "reasoned hypothesis about a general mathematical relation based on incomplete evidence" (2008, p. 11), and notes two crucial differences between making conjectures and identifying patterns. The first difference is that when conjecturing, one formulates a hypothesis that has a domain of reference that could extend beyond the cases which gave rise to the hypothesis, but when identifying a pattern, the statement does not necessarily extend beyond the given cases. Secondly, the hypothesis set forth when conjecturing is neither considered true nor false and is subject to testing, whereas a statement representing a pattern based on given data is not open to doubts about its truth.

The third activity, providing non-proof arguments, is defined as arguments for or against mathematical claims that do not qualify as proofs, and G. Stylianides' framework distinguishes between two kinds of these non-proof arguments. Empirical arguments provide inconclusive evidence for the truth of mathematical claims, whereas a rationale is an argument that can neither be qualified as proof nor an empirical argument. Using A. Stylianides' (2007) definition of proof for the final activity, providing a proof, G. Stylianides (2008) distinguishes between two kinds of proof. Generic examples are proofs that use a particular case as the representative of a general case. An example of a generic proof is shown in Figure 2.2. The example shows that 4 x 3 = 3 x 4, this being the particular case, used to represent the general case a x b = b x a. Demonstrations are proofs that do not use particular cases, and instead use variables to stand for all possible cases (e.g., contradiction, counterexample, mathematical induction).

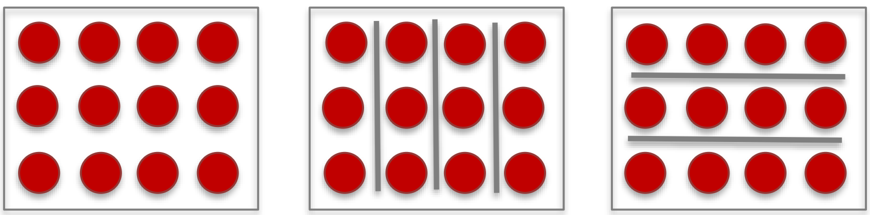


Figure 2.2 Generic example.

The psychological component focuses on the learner and could be used to investigate the learner's subjective meaning of terms such as proof and conjecture. When a learner is given a task to prove a mathematical claim, an empirical argument (according to a researcher or a teacher) could be perceived as a proof in the eye of the learner. The final component of the framework uses both mathematical and psychological components. The pedagogical perspective to the research of RP would, according to G. Stylianides, focus on two primary and interrelated issues. Firstly, how the learner's nature of a RP-related mathematical object compares with the mathematical nature of the said mathematical object, and secondly, (when such comparisons have been done and differences are identified) how one can instruct the learners to help them gradually refine their perceptions towards the conventional understandings.

Thompson et al. (2012) investigated opportunities to learn reasoning and proof in High School mathematics textbooks and developed a framework for analyzing proof-related reasoning in textbooks. Their framework was initially guided by The Reasoning and Proof Standard in *Principles and Standards for School Mathematics* by the National Council of Teachers of Mathematics (NCTM, 2000), and a framework developed by Third International Mathematics and Science Study (TIMSS) for analyzing curriculum (Valverde et al. 2002). The former recommended that the students were given opportunities to: recognize that reasoning and proof are fundamental aspects of mathematics; that they investigate and make mathematical conjectures; that they evaluate and develop mathematical proofs and arguments; and that they use various forms of reasoning and methods for proofs (Thompson et al., 2012, p. 256). Thompson et al. chose to address the latter three recommendations in their initial framework. "The second and third bullets capture the dual aspect of exploration and confirmations, (…) [t]he fourth bullet captures technical aspects of constructing proof not captured by the previous two" (p. 256). The framework developed by TIMSS consisted of codes for content, performance expectation, and perspective (Thompson et al. 2012, p. 256). Mathematical reasoning was among the performance expectations, with six subcategories (developing notation and vocabulary, developing algorithms, generalizing, conjecturing, justifying, and proving, and axiomatizing). Thompson et al. noted:

> Although our thinking about a framework evolved over time, certain fundamental assumptions never changed. 1. The framework should be broad enough to capture various forms of both proof and disproof in different content areas. 2. The framework should be useable to analyze any high school mathematics textbook, whether aligned more with goals of reform or traditional approaches to teaching. 3. The framework should be useable to analyze fundamental characteristics of both the narrative and the exercise sets (e.g., homework tasks) of textbooks. 4. The framework should build upon existing related research. (2012, p. 257)

Using the broad categories identified in the Principles and Standards for School Mathematics and the TIMSS curriculum analysis framework Thompson et al. (2012) did a pilot study on several textbooks not included in the final sample. After the pilot study, they developed a framework built on the collective work of the TIMSS framework, the Principles and Standards for School Mathematics, the frameworks by Balacheff (1988), Sowder and Harel (1998), and Miyazaki (2000). The term proof-related reasoning includes "making and investigating conjectures, developing and evaluating deductive arguments, and other experiences, such as finding counterexamples or correcting mistakes in logical arguments, that are fundamental elements of mathematical reasoning" (Thompson et al., 2012, p. 258). The framework consists of codes for both the exercises within lessons and the narratives.

Miyakawa and Shinno (2021) proposed that proving activities can be characterized by three aspects. Their goal was to provide a theoretical perspective to capture the cultural specificities of proof and proving in given institutions. The authors understand the word *cultural* as *institutional*, and the cultural factors determine the way students relate to proof and proving.

> [P]roving activities […] can be characterized by three aspects: *Structure*, *language*, and *function* […] *Structure* denotes here the organization of reasoning or arguments showing how different statements consisting a proof are connected […] *Language* is the semiotic representation […] used in a given institution to express the arguments and structure of reasoning […] *Function* […] attributed to the proof differs according to the institution and is not reserved to the ones often mentioned in the literature (verification, illumination, communication, systematization, and discovery). (p. 251-252)

The structure of the tasks includes the type of reasoning and argumentation structures that correspond with Stylianides' (2007) modes of argumentation, shown in Figure 2.1. In everyday life, arguments are often used to convince others of the validity of a statement, whereas in proof in school mathematics often requires a basic step as well as a chain of steps in the propositional logic. The language aspect of proving activities corresponds with Stylianides' (2007) modes of argument representation. Miyakawa and Shinno (2021, p. 252) mention representations "such as gestures, oral and/or written discourse, diagrams". The function of the proof varies depending on the institution they are used in, and what is called a proof in one institution may not be called a proof in a different institution based on the function of the justification given.

Miyakawa and Shinno (2021) provides two cases of classroom activities, showing how their theoretical perspective can be applied to analyze the cultural specificities of proof and proving. They analyze the classroom activities by identifying "the *structure* of reasoning given in these cases, the *language* used to describe the structure, and the *function* played by proof and proving in the classroom" (p. 254). Based on the teachers' explanation of the proof, they identify the *structure* in the first case as based on propositional logic. Because the teacher never used the words *any* or *all* while explaining, and because the statements were connected from the hypotheses to the conclusions. In the second case, the process of proving went backwards, from conclusion to hypothesis. When identifying the *language*, Miyakawa and Shinno found that the teacher used a combination of oral discourse, diagrams, and gestures, and when referring to points or angles the teacher used demonstrative words in place of labels. They point out that this could have implied that the teacher viewed the gestures and diagrams as part of the proving. In the second case, the proof was written in Japanese with many symbols, but not in proper Japanese sentences. It consisted of a list of symbolized statements with properties. The students were informed by the teacher to write the conclusion of the proof in proper Japanese. The *function* of proving in both cases was not to convince the students, as they were familiar with the statement from previous lessons, and as they took the statements for granted. In the first case, the *functions* identified by Miyakawa and Shinno were to provide a logical value, to allow the students to reuse the statement in other proofs, and to systematize previous knowledge. In the second case Miyakawa and Shinno identified another *function*, that of explaining why a statement holds. They note, "the main function of proving was to create a deductive chain reaching to the conclusion from the hypotheses and show the logical structure" (2021, p. 255).

The four activities identified in Stylianides (2008) as activities for RP, can all be found in the Norwegian curriculum's core elements. Kunnskapsdepartementet (2019) emphasizes that searching for patterns, finding relationships, and getting to a shared understanding are essential elements of exploration and problem solving in mathematics, as described in the core element *Exploration and problem solving*. These activities are what Stylianides (2008) calls reasoning, but the activity of providing non-proof arguments can be part of the discussion leading to a shared understanding. The activity of providing proofs is found in the core element *Reasoning and argumentation*. The term RP is a broader term that includes activities not found in *Reasoning and argumentation*. The term proof-related reasoning as defined by Thompson et al. (2012) also includes activities that can be found in other core elements in the curriculum. The process of making and investigating conjectures is by Thompson et al. seen as a proof-related activity and as previously stated, this activity is found in the core element *Exploration and problem solving*. For that reason, I have chosen against using the framework for analyzing RP activities, and the framework for analyzing proof-related reasoning. I will not be identifying opportunities for discovering patterns or creating or investigating conjectures as opportunities for argumentation and reasoning unless further instructions provide opportunities for reasoning or argumentation as described in the curriculum. I will be adapting aspects of the framework developed by Miyakawa and Shinno (2021), and I explain this further in Chapter 3.

2.4. Reasoning, argumentation, and proof in textbooks

Stylianides and Harel (2018, p. v) state that there is a rapidly expanding body of research in the field of reasoning, argumentation, proof, and proving. This section will give insight to some previous findings from earlier studies related to reasoning, argumentation, and proof in textbooks.

2.4.1. Previous research on forms of reasoning

In 2009 Stacey and Vincent examined the types of reasoning used in explanatory texts in textbooks for eight-grade pupils in Australia. Through content analysis, they examined all explanations given in explanatory sections for seven chosen topics, a total of 69 separate explanations in nine different textbooks, to identify the nature of reasonings. They discovered 7 different modes of reasoning; 1) Deduction using a general case; 2) Deduction using a specific case; 3) Deduction using a model; 4) Concordance of a rule with a model; 5) Experimental demonstration; 6) Appeal to authority; and 7) Qualitative analogy. The first three modes were associated with a deductive class in Harel and Sowder's (2007) proof scheme class and the proportion of topics with deductive explanations varied between 50% and 100%.

The first three modes of reasoning found in Stacey and Vincent's (2009) study correspond with the curriculum's reasoning and argumentation. These deductive modes of reasoning are arguments or reasonings that support (or oppose) a mathematical statement consisting of the characteristics described by Stylianides (2007).

2.4.2. Previous research on the occurrence of proof

Nordstöm and Löfwall (2005) investigated the occurrence of proofs and items related to constructing proofs in Swedish textbooks for five levels of difficulty in upper secondary school mathematics. Their findings showed that there were few occurrences of proofs and discussions on proofs and that the textbooks rarely made proof explicit in their explanatory sections. Due to the sparse number of occurrences, they discarded their initial quantitative method and instead chose a qualitative approach, counting the percentage of tasks offering opportunities for proving and studying the items that had significance for the learning of proof. In the studied textbooks, Nordström and Löfwall found that about 2% of all tasks gave opportunities for proving and that these tasks were more common in the domain of geometry, about 10% of tasks found in that domain. They also found that words like *proof*, *definition*, and *assumption* were avoided in the studied textbooks, and one of the textbooks first gave a definition of *proof* ("logical reasoning without gaps" (p. 453)) and then used the word *explanation* instead of *proof* later in the textbook. In a different textbook, the word *justification* was used for the derivations of formulas for areas of polygons instead of the word proof.

2.4.3. Previous research on opportunities for reasoning-and-proving and proof-related reasoning

Thompson et al. (2012) analyzed the nature and extent of reasoning and proof in samples from 20 textbooks, most intended for students in grades 9-12 while some were used also in grades 7-9, using their developed framework. They found that 50% of narratives were justified in the three content areas examined (in algebra: exponents, logarithms, and polynomial expressions, equations, and functions), 10% were left for the students to justify, and the remainder had no justification at all. Less than 6% of exercises included proof-related reasoning in exercise sets. However, the opportunities to evaluate or develop arguments only occurred about 2-3% of the time, either for a specific or a general case. They concluded that "without additional instruction, students may have limited experiences with proof-related reasoning" (p. 282).

Jon Davis (2012) examined reasoning and proof opportunities in secondary mathematics textbook units in the United States, using Stylianides' (2009) framework for examining the nature of RP. In the textbook expositions, the frequency of sentences devoted to RP was 27, 49, and 21%, and the student tasks in the textbooks devoted to RP were 4, 9 and, 21% in the three examined textbooks. Findings also suggested that students were more likely to engage in one of the four categories within the framework, the identification of definite patterns, a total of 197 tasks from the total of 400 student tasks.

Bieda et al. (2014) studied opportunities for RP in elementary mathematics textbooks in the US. They adapted the frameworks developed by Stylianides (2009) and Thompson et al. (2012) and employed that "*reasoning* involves engaging in processes to generalize mathematical phenomena and/or conjecturing about mathematical relationships, whereas *proving* involves justifying a mathematical claim to be true for the domain to which the claim applies, using logically valid reasoning". Using a method of content analysis, a total of 28,210 problems were coded. Their first step in the coding process was to identify problems that contained a list of keywords (explain, describe, predict, show, write a rule, tell why, tell how, justify, and prove), followed by assigning the identified tasks with their *Type of Problem* code. The next steps were to interpret the problems and determine a *Purpose of RP Problem* code, assign codes for *Type of Argument Elicited* and *Intended Outcome of RP Problem*, and finally applying definitions of sub-codes for *Type of Argument Elicited*. Their findings showed that a small percentage (3.7%) of the 28,210 problems included opportunities for RP. In the seven studied text materials, this percentage varied slightly, but at most 4.6% of the problems in one given text gave opportunities for students to engage in RP.

In 2014, McCrory and Stylianides investigated RP in mathematics textbooks for prospective elementary teachers (McCrory & Stylianides, 2014). They collected data from 16 mathematics textbooks in print in the U.S. and the first step in their analysis was to locate potential opportunities for RP using the tables of contents and indexes. Their reasoning for this choice was to take the perspective of the users of the textbooks wanting to locate RP without reading every chapter of the books. McCrory and Stylianides sought out the locations where the authors explicitly addressed issues related to RP using a developed list of terms that could be logical indicators of said issues. When they had identified relevant pages, the pages were inspected using the following three questions: "1. Is the referenced page about reasoning-and-proving? 2. On the referenced page, how is the term used or presented? 3. Does the referenced page indicate how the term is connected to, or used in, other parts of the textbook?" (2014, p. 121). Their findings showed that students might have difficulties locating RP when using the textbooks independently, that instructors often would need additional materials related to RP, and that how the textbook authors present RP varies. McCrory and Stylianides point out three divergent examples of the latter, from not using related terms in the tables of contents or the indexes but providing numerous proofs, to RP referenced both in the tables of contents and indexes as well as interwoven through the textbooks, and numerous index references to the chapter of problem solving where RP was explained.

Otten et al. (2014) investigated the mathematical nature of RP opportunities in geometry textbooks in the United States. The study involved six textbooks designed for students between 13 and 16 years old in stand-alone geometry courses. By selecting the lessons (numbered sections with expository text and student exercises) using stratified random sampling, a total of 37% (212 of 580) of the lessons in the six textbooks were analyzed. They also included one chapter review from each textbook in their analysis. The framework used was based on the framework by Thompson et al. (2012) but with some modifications to capture the nature of mathematical statements around which the RP opportunities took place, and to distinguish between exercises that asked students to *explain* and *prove* drawing on Stylianides' (2009) framework. When examining the student exercises, they followed Hanna and de Bruyn (1999) in using keywords as a primary means to distinguish between the two categories of developing arguments, *construct a proof*, and *develop a rationale or other non-proof argument*. Their findings include that RP activities were relatively numerous but that the opportunities for constructing proofs and for thinking about RP were very few. They also found that general mathematical statements constituted the majority of RP items in the textbook's expositions. RP activities were most commonly asking students to determine the truth value of a mathematical claim and asking students to provide non-proof justifications for mathematical statements.

Wong and Sutherland (2018) examined RP in algebra in mathematics textbooks in Hong Kong. They used A. Stylianides' (2007) definition of *proof* and adapted the analytic framework for analyzing RP activities by G. Stylianides (2009). Their findings showed that the opportunities for students to learn RP in the algebraic chapters in the chosen textbook series were limited to 410 out of 3241 tasks (13%). Of the 410 tasks that offered opportunities for RP, nearly 89% were categorized as *Providing support to mathematical claims*, and there were few opportunities for conjecturing, showing that *making mathematical generalizations* and providing support to mathematical claims often were treated in isolation from each other. The curriculum only mentioned proof in the learning targets of geometry, so the results were close to what was expected. However, they point out that this contrasts with the international call for reasoning and proof to permeate school mathematics.

Tømmerdal (2021) investigated opportunities for RP in chapters on fractions in Norwegian mathematics textbooks for fifth grade. The textbooks analyzed were Multi 5A Elevbok, Matematikk 5 Grunnbok, Matemagisk 5A Grunnbok, and Matemagisk 5B Grunnbok, all written or revised after the implementation of the new Norwegian curriculum. Using A. Stylianides' (2007) definition of *proof* and G. Stylianides' (2009) framework for analyzing RP activities, he found that 3% of the total 1548 exercises offered opportunities for RP activities. Out of the 47 tasks offering these opportunities, 70% included opportunities for developing hypotheses, of which 85% contains qualities that could make them precursors of proof, and 30% out of the 47 offered opportunities for developing proofs. More than 99% of the RP activities discovered included opportunities for pupils to work with singular cases, whereas opportunities for working with a finite number of cases, and an infinite number of cases were 0.07% and 0.78% respectively. Tømmerdal states that although his findings might not be reflected in the remaining chapters of the examined textbooks, the findings suggest that RP has not found its place in the examined textbooks.

A common finding for several of the summarized articles is that there are few opportunities for students engaging in activities including deductive argumentation and constructing proofs (e.g., Bieda et al., 2014; Davis 2012; Tømmerdal, 2021; Wong & Sutherland, 2018). Tømmerdal (2021) and Bieda et al. (2014) found that as few as (roughly) 3% of analyzed activities offered opportunities for RP using the term as defined by Stylianides (2008), and Nordström and Löfwall (2005) found in their research that only 2% of all tasks in the analyzed textbooks provided opportunities for proving. The goal of the present study is to understand what opportunities there are for reasoning and argumentation, as defined in the curriculum's core element, in textbooks for grades 5-7, and the research questions are, 1) To what extent to the textbooks use words related to reasoning and argumentation, and 2) How are words related to reasoning and argumentation used in the textbooks, and what usages offer opportunities for reasoning and argumentation. In the next chapter, I will explain methodological decisions taken for collecting and analyzing data and discuss potential opportunities and limitations resulting from those decisions.

3. Methodology

In this chapter I will explain the method chosen for the present study. I will begin with presenting the textbooks examined and then present the method of document analysis, combining elements from methods used in some of the prior research presented in the chapter 2 and elements of the framework by Miyakawa and Shinno (2021). The final two sections of the chapter will present some ethical reflections and reflections of the validity of the present research.

3.1. The textbooks

The data was collected from 15 textbooks from three Norwegian textbook series in mathematics for grades 5-7, Matematikk, Matemagisk, and Multi, all from different publishers, namely Cappelen Damm (CD), Aschehoug Undervisning (A), and Gyldendal (G) respectively. The publishers provide two textbooks for each grade, although in slightly diverse ways. Both Matemagisk (A) and Multi (G) consist of one book for each semester of the school year, Primary/Pupil Textbook (NO: Grunnbok/Elevbok) A and Primary/Pupil Textbook B. Matematikk (CD) consists of one primary textbook and one complimentary task book (NO: Oppgavebok) for the entire school year. The reason I have chosen to include the complimentary task books provided by CD and their Matematikk-series is that it is, in the same sense as the primary books, meant to be used by the pupils, both in class and in homework. It is also meant to be used alongside the primary textbooks. Digitalized versions of the textbooks were chosen because of their accessibility and search functions. The digital versions are identical to the physical textbooks, in content and page span. The publishers provide different additional materials such as additional digital resources and teacher guides. No data were collected from these sources, as I wanted to examine the opportunities for reasoning and argumentation made directly available to the pupils.

Textbook	Publisher	Number of pages
Matematikk 5 Grunnbok	Cappelen Damm	221
Matematikk 5 Oppgavebok	Cappelen Damm	141
Matematikk 6 Grunnbok	Cappelen Damm	243
Matematikk 6 Oppgavebok	Cappelen Damm	177
Matematikk 7 Grunnbok	Cappelen Damm	223
Matematikk 7 Oppgavebok	Cappelen Damm	165
Matemagisk 5A Grunnbok	Aschehoug Undervisning	141
Matemagisk 5B Grunnbok	Aschehoug Undervisning	167
Matemagisk 6A Grunnbok	Aschehoug Undervisning	121
Matemagisk 6B Grunnbok	Aschehoug Undervisning	205
Matemagisk 7A Grunnbok	Aschehoug Undervisning	155
Multi 5A Elevbok	Gyldendal	135
Multi 5B Elevbok	Gyldendal	135
Multi 6A Elevbok	Gyldendal	143
Multi 6B Elevbok	Gyldendal	135

Table 3.1 Textbooks examined in the present study.

From the year 2000, the governmental quality control of teaching aids such as textbooks, to ensure that the textbooks were in line with the goals of the curriculum, was revoked to ensure that teachers had the opportunity to focus on the content of the curricula rather than the textbooks and that the teachers had the opportunity to execute their professional autonomy (Tømmerdal, 2021). Utdanningsdirektoratet provide an article, written by Svingen and Gilje (2018), on how one can identify and evaluate the quality of teaching aids in mathematics. They provide research-based information on the characteristics of what makes a good teaching resource, and some quality criteria that one can follow when evaluating teaching aids.

3.2. Document analysis

For this research, the approach chosen is the one of document analysis, using Bowen's (2009) definition. The document analysis is a qualitative approach that has elements from both content analysis, and thematic analysis. It is "a systematic procedure for reviewing or evaluating documents – both printed and electronic (computer-based and Internet-transmitted) material" (p. 27). Bowen continues, "Like other analytical methods in qualitative research, document analysis requires that data be examined and interpreted in order to elicit meaning, gain understanding, and develop empirical knowledge" (p. 27). Bowen lists several advantages of the document analysis as a qualitative research method: it is an *efficient* method; many documents are available to the public (*availability*); the documents are unaffected by the research process (*lack of obtrusiveness and reactivity*); the documents are stable (*stability*); *exactness*, meaning that exact references easily can be included; and that the documents can provide broad *coverage* of e.g., a subject (p. 31).

The first step of the analysis was to organize the information given in the textbooks. Looking for opportunities for reasoning and argumentation I first had to locate potential opportunities. My methods incorporate the method of sampling used by Bieda et al. (2014) and McCrory and Stylianides (2014) and the list of words by Bieda et al. When locating words related to reasoning and argumentation (see words with English translations p. 22), I created an overview of the number of times they were used in the different textbooks, and where they could be found when examining them at a later stage. This is typical of content analysis, "the process of organising information into categories related to the central question of the research" (Bowen, 2009, p. 32).

The second step was to go back to the pages and tasks where the relevant words were found, re-read them, and examine whether they gave opportunities for reasoning and argumentation. This process of closer examination is where the thematic analysis happens when trying to recognize patterns within the collected data from the earlier stage, and the themes emerging from this create the categories for analysis (Bowen, 2009, p. 32). In this process I used the different modes of argumentation listed by Stylianides (2007), the curriculum's definition of reasoning and argumentation (Kunnskapsdepartementet, 2019b), and the three aspects defined in Miyakawa and Shinno's (2021) framework.

3.2.1. Superficial examination

The first step of the document analysis, what Bowen (2009) calls the superficial examination, was to create a list of keywords related to reasoning and argumentation to locate potential opportunities for pupils engaging with the activities of reasoning and argumentation. Godino and Recio (1997) state that one can recognize proving activities through different terms, and such can also be said about reasoning and argumentation. Bieda et al. (2014) used a list of keywords when identifying activities of RP which included the words *explain*, *justify*, *show*, and *prove*. Similarly, McCrory and A. Stylianides (2014) developed a list of terms that related to the two major activities in G. Stylianides (2008) term RP, generating mathematical generalizations, and justifying mathematical generalizations. Their list of terms related to the development of mathematical arguments included, *assumption*, *reasoning*, *deductive* and *indirect reasoning*, *compound* or *conditional statements*, and *definition*. Related to distinct functions that an argument or a proof can serve, their list included, *explanation*, *falsification*, and *justification*, and terms related to different proof methods included, *counterexample*, *contradiction*, *mathematical induction*, and *proof* or *proving*. In the present study, I have adopted some of the terms listed by Bieda et al. (2014) and McCrory and Stylianides (2014). The words examined in the present study are, *explain*, *justify*, *show*, *prove*, as well as the terms *reason* and *argue* themselves. The common idea of these terms is that they are all related to reasoning and argumentation in that they, by providing reasons or arguments, seek to justify or validate a statement (Godino & Recio, 1997).

Hemmi et al. (2013, p. 359) state that "[e]xplanation in mathematics often refers to making mathematical connections explicit" and that it is part of "a critical process of accepting mathematical arguments and should be involved in mathematical reasoning at all levels". In addition to the process of accepting mathematical arguments, I consider *explaining* as a potential process of providing mathematical arguments. Supported by the statement by Hemmi et al., the word *explain* is considered related to reasoning and argumentation in the present study. Staples et al. (2016, p. 448) define a *justification* as "*an argument that demonstrates (or refutes) the truth of a claim that uses accepted statements and mathematical forms of reasoning*" and a definition of a justification provided by Bergwall and Hemmi (2017) is: "any mathematical argument designed to verify, explain, or convince the reader about the validity of a statement" (p. 4). The two definitions support viewing the activity of justifying as related to reasoning and argumentation. As in the English language, the Norwegian *vis* can mean more than one thing. Depending on the context it can mean simply showing something or describing or explaining something. To identify opportunities for explaining as previously defined, the word *show* is viewed as relevant to reasoning and argumentation. Using Stylianides' (2007) definition of proof supports including the words *prove* and *proof* in the list of keywords related to reasoning and argumentation.

The words in Table 3.2 are written in the present infinitive and the base noun form for simplicity, but all conjugations and other forms of the words are viewed as pertinent.

Norwegian		**English**	
Verb	**Noun**	**Verb**	**Noun**
Argumentere	Argument	Argue	Argument, argumentation
Resonnere	Resonnering/Resonnement	Reason	Reasoning (as a process/as a product)
Bevise	Bevis	Prove	Proof
Forklar	Forklaring	Explain	Explanation
Begrunn	Begrunnelse	Justify	Justification
Vise		Show	

Table 3.2 Keywords in base verb and noun form with Norwegian-English translations.

Reid (2022) notes that there are two Norwegian words for the English *reasoning*, *resonnering* and *resonnement*. "Both are nouns, but 'resonnering' is closer to the verb form and is used to refer to the process of reasoning, while 'resonnement' refers to the product of reasoning" (p. 3).

The keywords were counted manually in the examined textbooks, due to the possibility of the words being used in figures and images, or in a way that would not show up when searching the digital documents. Pages including the words were noted with the number of times the words were used on those pages (see Table 3.3). When identifying and counting words I did not differentiate between the narratives and the pupil tasks in the textbooks. Every occurrence of the word was counted and noted, no matter whether it was used in the introduction of the textbook, in an example, in a statement, in the goals of chapters, or in an activity. This was done to identify not only where the pupils themselves would be providing reasoning or argumentation, but also to locate potential usages of the keywords that could provide information about how the textbooks expected the words to be understood or interpreted by the pupils. If the word *forklare* was found in an example task and the textbook provided an example of how such tasks could be answered, the pupils would get an idea of what the textbook meant by *forklare* and apply the same, or a similar, strategy when solving similar tasks themselves. To not rule out such examples or other potential opportunities for pupils to get information, to learn, or to apply reasoning and argumentation, all occurrences of the keywords were counted.

	Explain	**Justify**	**Show**	**Prove**	**Argue**	**Reason**
Page; (occurrences>1)	49;2	27	3		2	
	79;2	44	36		3	
	84	74	41		139	
	90	98	43			
Total	12	8	41	0	3	0

Table 3.3 Excerpt of the keyword count from Matematikk 5 Grunnbok, Cappelen Damm. The numbers following semicolons refer to the number of occurrences on the given page.

3.2.2. Closer analysis

The next step of the analysis was to investigate the locations of the keywords and examine how the words were used. This process included trying to recognize patterns of what usages that would offer opportunities for reasoning and argumentation both within one textbook, one textbook series, and between the different textbook series. Similar to the first step of this closer analysis, McCrory and Stylianides (2014) asked the question: “On the referenced page, how is the term used or presented?” (p. 121) in their analysis of textbooks using a list of terms that would be logical indicators of RP to locate the potential opportunities for such activities.

When interpreting potential locations of opportunities for reasoning and argumentation, the work was built on Miyakawa and Shinno’s (2021) framework for analyzing proving activities. They proposed a triplet of actions that would constitute proving activities, *structure*,

language, and *function*. Although the framework is intended to be used to examine proving in the classroom, it is interesting to observe and examine aspects of it in textbooks as well. The *structure* in their framework denotes the organization of the arguments or reasonings, whereas the structure I will be examining in the present study denotes the organization of the activities found including the keywords related to reasoning and argumentation.
To investigate how the words related to reasoning and argumentation are used, it is not only the specific sentence using the keyword that is of interest, but the statements or questions following or prior to it. Examining the surrounding text or figures could offer more insight to whether opportunities for reasoning and argumentation are provided. If a keyword is found in a subtask, I will examine the other subtasks the activity includes as well as the one where the keyword is used. Figure 3.1 gives an example of such usage that would lead to examining the prior tasks. Figure 3.2 shows the entire task, and the use of the keyword *forklar* in this case does not offer such opportunities.

d Forklar regnemetodene dine til en annen elev. Regner dere på samme måte?

Figure 3.1 Multi 5A Elevbok, Gyldendal, p. 121. d – Explain your methods for calculating to another pupil. Do you calculate in the same way?

4.59 Regn ut. Legg merke til hvordan du regner.

a 6,0 – 5,7	**b** 2,3 – 2,0	**c** 7,5 – 0,5
7,0 – 6,7	3,3 – 3,0	7,5 – 1,5
8,0 – 7,7	4,3 – 4,0	7,5 – 2,5

d Forklar regnemetodene dine til en annen elev. Regner dere på samme måte?

Figure 3.2 Multi 5A Elevbok, Gyldendal, p. 121. a – Calculate. Pay attention to how you calculate.

The *language* aspect in Miyakawa and Shinno’s (2021) framework refers to semiotic representations and correspond with Stylianides’ (2007) modes of argument representation. Ball et al. (2002, p. 909) remind us that “language is essential for mathematical reasoning and for communication about mathematical ideas, claims, explanations, and proofs”. The language used in the textbooks varies, and the language used to present activities can be presented in a written form, using either words, symbols, figures, or a combination of these. Depending on the sentence structure, the language in the activity could imply different expectations from the pupils. An example of this could be a task where the pupil is asked to *explain how* they think while solving an equation. Although the word *explain* is listed as a keyword that could offer an opportunity for a deductive argument, the rest of the words in the task asks for a description of their thoughts, not a deductive argument or deductive reasoning. On the other hand, if a task asks the pupils to *explain why* the sum of the interior angles in triangles always is 180°, one can imagine that a deductive argument is expected. At the same time one can find tasks using nearly the same language but with different expectations (see Figure 4.17 and Figure 4.18).

A second example of how the language plays part in examining the opportunities for reasoning and argumentation would be a task that asks the pupils to justify using a method described previously in the textbook. An example of this can be seen in Figure 4.9. The textbook asks the students to answer using one of four semiotic representations shown on the previous page, varying from concrete materials to a number line and symbolic representations. In the present study I will be examining not only what language can be found directly in the activities, but also what language might be expected by the pupils when doing the activities. What language is expected of the pupils may vary from generic examples, drawings, and symbolic algebra, and this will be influenced by the knowledge the pupils have of the mathematical language and the norms in the classroom community (Stylianides, 2007). It will also be influenced by what the activity tells the pupils to do, like in the example referred to above.

The *function* in Miyakawa and Shinno's (2021) framework refers to the purpose of the argument. In the present study, the keywords often clarify the function of the activity, like explaining, justifying, or showing, but I will be analyzing whether there are less obvious functions of the activities, such as verifying, describing, communicating, etc. The definitions of reasoning and argumentation used in the present study led to the investigation of whether the function of the activities would provide opportunities for deductive reasoning and argumentation. If any less obvious function is found, it will be specified in Chapter 4.

3.3. Ethical reflections

When conducting research there are always ethical considerations to keep in mind. Research ethics consist of a set of norms that have been developed through time and that is rooted in the international research society (Den nasjonale forskningsetiske komité for samfunnsvitenskap og humaniora [NESH], 2021, p. 6). The truth norm speaks of the search for the truth, the understanding of the truth, honesty, and integrity. Methodological norms, such as objectivity, clarity, accountability, and verifiability, ensure that scientific methods are followed in responsible ways. Research is regulated by institutional norms that contribute to an open, collective, independent, and critical research.

In the present study the goal is to understand what opportunities there are for reasoning and argumentation in Norwegian mathematics textbooks for the middle school grades, and I have, to the best of my ability, shared the research in its entirety to disseminate the research in a way that illuminate the search for truth, integrity, and honesty. NESH (2021) reminds us that the research should be replicable, and for that to be possible the researcher must stay true to their scientific integrity, meaning to share the research as it was conducted without being misleading or distorting information (Høgheim, 2020). The methodology used in the present study has been shared in its entirety to facilitate for verifiability. To recognize the work of others and to refer to it in proper ways are also part of the ethical considerations that must be taken when conducting research (NESH, 2021), and all previous research and work done by others used in the present study is referred to where it is used and in the reference list.

My intention with the present study has not been to rank the textbooks examined. The different use of words related to reasoning and argumentation is expected as the textbook series are written by different authors, and to examine the different use of these keywords, both within one textbook or one textbook series, or between the textbook series, has been central to the study. The tasks and figures from the textbooks used to present findings were chosen to show the different usages of the keywords, not to favor or highlight one textbook or one series over the others.

3.4. Validity

Postholm and Jacobsen (2021) state that the researcher must systematically reflect on two relations when conducting research. The first relation is the limitations connected to the research and it refers to the validity of the research. There are limitations to every research, and one limitation in the current study is the list of keywords. There might be other words used to present activities related to reasoning and argumentation, and I do not claim that the keywords listed are the only words related to these activities. A word that was located in one of the textbooks at a later point in the study, which exemplifies a different word used, was the word *overbevise* (*convince*). The fact that I did not locate the word sooner could imply that it was not used often, but because it was not included in the first step of my analysis, it could very well have occurred several times without being noticed. In other words, I do not claim to have found all opportunities for reasoning and argumentation, and neither is that the goal of the present study. Rather, I want to investigate how the textbooks use the keywords and what usages of these words might offer opportunities for reasoning and argumentation. A second limitation of the present study is that the textbook series often provide teacher guides to complement their textbooks. These guides often give information about the activities which is not always clear in the textbooks and can help the teacher in guiding the pupils through activities. In the present study I did not examine the teacher guides, and the results could have turned out differently if I did. Examining the textbooks' use of the keywords with the explanations and guides given in the teacher guides for the identified activities could illuminate more opportunities for reasoning and argumentation than found in the textbooks alone.

The second relation stated by Postholm and Jacobsen (2021) is about the possible implications the chosen methods could have on the results of the research. This refers to the reliability of the research. An implication of the method used in the present study is the fact that the keywords are located and counted manually. The decision for doing it this way has been explained previously, a digital document search did not include every occurrence, but the human factor means that words could have been overlooked or missed, or the count could be wrong. A way of limiting the implications of the manual count could be to have someone else redo the process and compare the findings. A second implication can be the analysis of the occurrences of the keywords and the interpretations of the activities. This part of the research is based on my interpretations of the activities and my understanding of the terms reasoning and argumentation. A different researcher with a different understanding of what makes an opportunity for reasoning and argumentation might find other results during the closer analysis of the occurrences of the keywords. To limit this implication, I have described the definitions of the terms and the analysis, so that if someone were to reconstruct the present study, they would have the opportunity to understand and interpret in a comparable way to what I have done.

Lincoln and Guba (1985) propose a set of criteria for assessing the trustworthiness of a qualitative research. The first criterium is credibility, and credibility is about how believable the findings are. To support the credibility of the present study I have accurately shared the methodology and the results. The material where the data has been collected is available to the public, and this is one of the positive sides of a document analysis like in the present study. The second criterium proposed by Lincoln and Guba (1985) is transferability, which refers to whether the findings could apply to other contexts. In Chapter 2.4 I have shared summaries of previous research done to the field of reasoning and argumentation in mathematics textbooks. Some these show findings of similar nature to what I have found in the present study, which support the belief that this study is transferable.

Dependability is about whether the findings are likely to appear at other times. As this research is conducted in documents the chances of the results changing over time are small when referring to the first part of the analysis, the superficial examination. There might be human errors as the keywords were counted manually, but other than counting a word twice or missing a word, the results will be similar at a later point in time. When referring to the closer analysis of the occurrences of the keywords, there might be a different outcome of the study. Lincoln and Cuba (1985) argue that they disagree with the presupposition that there is a single absolute account of social reality and that there can be more than one such account (Bryman, 2016). As previously mentioned, what I categorize as opportunities for reasoning and argumentation is based on my understanding of the curriculum's definition of the terms, and if that understanding changes or someone with a different understanding replicates this study, the opportunities found in the present study might not be considered opportunities at a later point in time.

4. Results from the research and data collections.

In this chapter the analysis and results from the 15 textbooks will be presented. The research questions for the present study are 1) *to what extent do the textbooks use words related to reasoning and argumentation*, and 2) *how are words related to reasoning and argumentation used in the textbooks, and what usages offer opportunities for reasoning and argumentation?*

As explained in chapter 3.2 the textbook analysis started with a superficial examination followed by a closer analysis of the keywords' occurrences. Section 4.1 will present results from the superficial examination. To show to what extent the different textbook series have used the keywords I will present the results both generally and specifically. I would like to specify that this is not to highlight or disfavor textbooks, as this study does not aim to find all opportunities for reasoning and argumentation, and there are potentially opportunities not using the keywords the present study focuses on. Section 4.2 will present results from the closer analysis. The examples and excerpts used to present the different usages of the keywords are selected independently from what textbook they are found in, as the goal of the study is to understand how the keywords are used and what usages offer opportunities for reasoning and argumentation, not to disfavor or favor textbooks.

4.1. Superficial examination

As shown in Table 3.1 the total number of pages examined in the superficial examination was 2507. Table 4.1 shows how many times the keywords are used in total in the textbooks examined. More detailed numbers from the different textbook series can be found in the appendix (Table I.1, Table I.2, and Table I.3). The words *resonnere* and *bevis* are intentionally left out of the tables due to the fact that those words do not occur in any of the examined textbooks. Table 4.2 shows how many pages include the keywords in total from the examined textbooks. Complete lists of pages including the keywords in the textbooks are provided in Appendix C.

	Forklare	Begrunne	Vise	Argumentere
Total occurrences	271	169	609	9

Table 4.1 Total occurrences of keywords.

	Forklare	Begrunne	Vise	Argumentere
Total pages	186	131	362	9

Table 4.2 Total pages including keywords.

Figure 4.1 includes a diagram that shows how many percent of the total pages of the textbook series for each grade that include the words *forklare* and *begrunne*. The diagram does not show whether the keywords occurred once or more than once on any given page. The blue bars represent Matematikk from Cappelen Damm (CD), the green bars represent Matemagisk from Aschehoug Undervisning (A), and the red bars represent Multi from Gyldendal (G). The textbook series are represented with totals from the textbook pairs for each grade, the A and B books from Matemagisk (A) and Multi (G), and the Primary and Task books from Matematikk (CD).

Begrunne occurs on 2.8% (CD), 6.5% (A), and 2,6% (G) of the pages in the textbook pairs from the three series for grade 5. For grade 6 the word occurs on 3.1% (CD), 9.8% (A), and 2.2% (G) of the pages, and for grade 7 it occurs on 5.2% (CD) and 17.4% (A) of the pages. Both textbooks for grade 7 from Gyldendal, and the textbook for the second semester (Book B) from Aschehoug Undervisning, had not yet been published at the time of analysis.

The word *forklare* typically occurs more often than *begrunne* in the same textbook series for each grade, except from in Matematikk 7 (CD) where *begrunne* is found more commonly than *forklare*. The word *forklare* occurs 3.9% (grade 5), 4.0% (grade 6), and 1.0% (grade 7) in the textbook pairs from Matematikk (CD). It occurs on 10.1%, 18.4% and 23.9% of the pages in the pairs from Matemagisk (A) for grade s 5,6, and 7 respectively, and in the textbook pairs from Multi (G) the word *forklare* occurs 6.3% (grade 5) and 5.8% (grade 6).

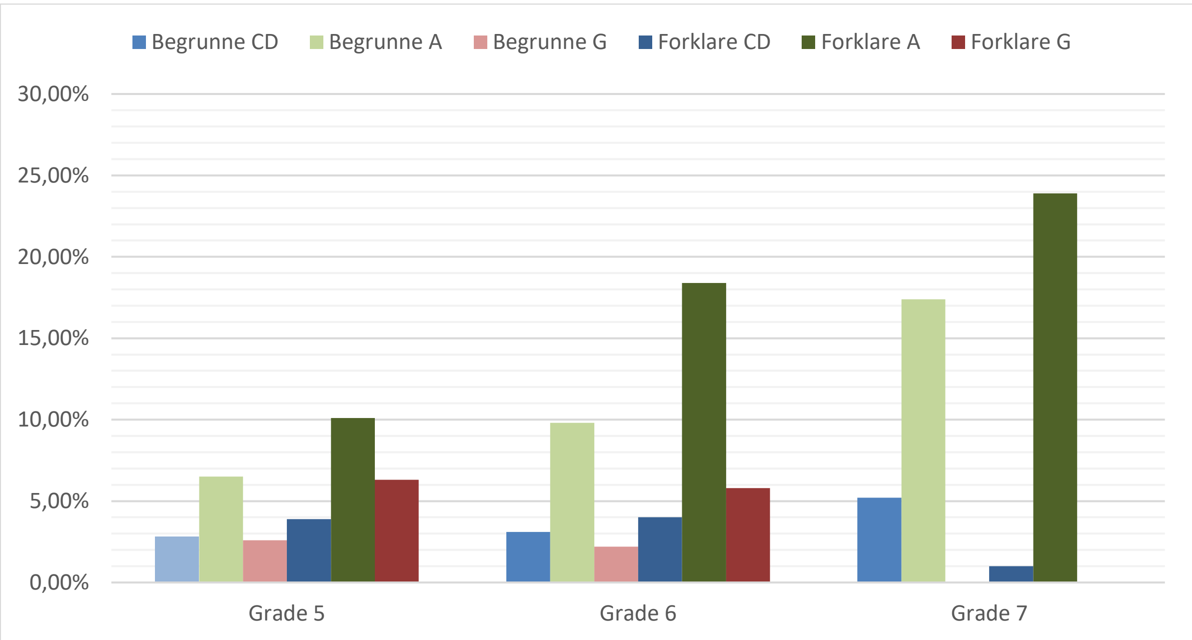


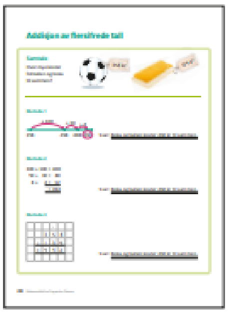
Figure 4.1 Percentage of pages in textbook pairs that include the keywords forklar and begrunn.

4.2. Closer analysis

This section is divided into subsections with closer analysis of some occurrences of *argumentere* (4.2.1.), *begrunne* (4.2.2.), *forklare* (4.2.3.), and *vise* (4.2.4.). I will present a selection of occurrences that reflect how the different usages may, or may not, offer opportunities for reasoning and argumentation as defined by the core element *Reasoning and argumentation*.

4.2.1. Argumentere

The word *argumentere* is only used in one of the textbook series, the Matematikk series from Cappelen Damm. It is used twice in the introductory pages of each textbook, when explaining the different activities in the textbook's chapters. The descriptions give clear instructions of what is expected by the pupils in these activities. The first mentioning of the word happens when the book introduces the Conversation activities found in the chapters (Figure 4.2). In these activities the pupils shall reflect and *argue* for different solutions. On the next page of the introductory pages in the True or False activities (Figure 4.3), the pupils are expected to consider and *argue* whether a list of statements are true or false. When looking at the True or False activities throughout the textbooks, the word *argumentere* is never used, and instead the activities use the word *begrunne* when asking the pupils to assess a list of statements (Figure 4.10). I will come back to these activities in sub-section 4.2.2.



Samtale

Alle kapitlene har samtaleruter i ramme. Ruta er delt med en strek. Problemstillingen som står over streken, skal dere snakke om og forsøke å løse. Dere skal reflektere og argumentere for ulike løsninger. Under streken presenterer vi ett eller flere løsningsforslag eller metoder som dere kan reflektere over og drøfte.

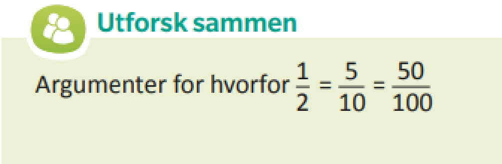
Figure 4.2 Matematikk 5 Grunnbok, Cappelen Damm, p. 2. Conversation. All chapters have conversation boxes in a frame. The box is divided with a line. You are supposed to discuss and try to solve the problem over the line. You are supposed to reflect and argue for different solutions. Under the line we present one or more possible solutions or methods that you can reflect on and discuss.

Sant eller usant?

Sant eller usant? er en samling utsagn som dere skal vurdere og argumentere for om er sanne eller usanne.

Figure 4.3 Matematikk 5 Grunnbok, Cappelen Damm, p. 3. True or false? True or false? Is a collection of statements that you are supposed to consider and argue whether they are true or false.

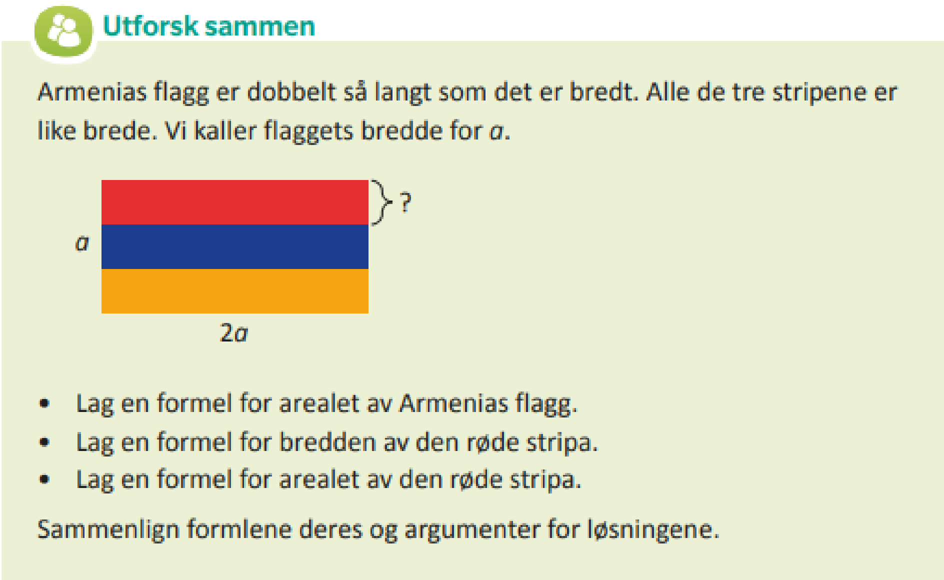
The word *argumentere* appears once more in each grade's textbook, in "utforsk sammen" (*explore* together) activities. In grade 5 (Figure 4.4) the pupils are asked to *argue* why $\frac{1}{2} = \frac{5}{10} = \frac{50}{100}$. The pupils do not get any additional information in the task itself, but the task is found after the pupils are introduced to the connections between decimal numbers, fractions, and percentages. To argue why one-half is five-tenths could offer an opportunity for applying accepted forms of argumentation, based on the pupils understanding of fractions.


Utforsk sammen

Argumenter for hvorfor $\frac{1}{2} = \frac{5}{10} = \frac{50}{100}$

Figure 4.4 Matematikk 5 Grunnbok, Cappelen Damm, p. 139. Argue for why 1/2 = 5/10 = 50/100.

In grade 6 (Figure 4.5) the pupils are asked to *argue* for their solutions. The pupils are informed that the flag is twice as long as it is wide, and that the width is called *a*. The three stripes in the flag are equally wide. The pupils are then asked to create a formula for the area of the flag, a formula for the width of the red stripe, and a formula for the area of the stripe. Finally, they are asked to compare their formulas with someone else's and argue for their solutions. This activity could potentially provide an opportunity for deductive reasoning and argumentation. When arguing for the solutions to the tasks on the area of the flag, deductive argumentation from the diagram and their knowledge of the area formulae could be offered, but so could argumentation about how the tasks were interpreted.


Utforsk sammen

Armenias flagg er dobbelt så langt som det er bredt. Alle de tre stripene er like brede. Vi kaller flaggets bredde for *a*.

- Lag en formel for arealet av Armenias flagg.
- Lag en formel for bredden av den røde stripa.
- Lag en formel for arealet av den røde stripa.

Sammenlign formlene deres og argumenter for løsningene.

Figure 4.5 Matematikk 6 Grunnbok, Cappelen Damm, p. 195. Explore together. Armenia's flag is twice as long as it is wide. All three stripes are of equal width. We call the width of the flag for a. – Create a formula for the area of Armenia's flag. – Create a formula for the width of the red stripe. – Create a formula for the area of the red stripe. Compare your formulas and argue for the solutions.

In grade 7 (Figure 4.6) the pupils are asked to *argue* for which mental calculation strategy they prefer applying when solving 1452+190 and 1452-398. To argue for which mental calculation strategy they prefer does not offer opportunities for accepted modes of reasoning or forms of argumentation, and one cannot deduce a correct answer.

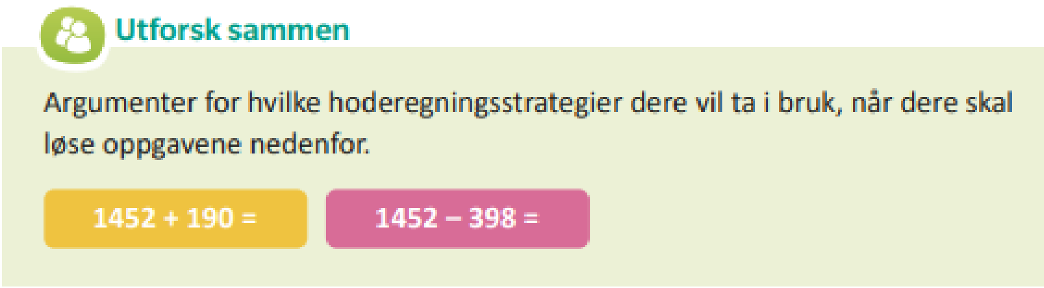


Figure 4.6 Matematikk 7 Grunnbok, Cappelen Damm, p. 9. Argue for which mental calculation strategy you would prefer using when solving the problems below.

4.2.2. Begrunne

The word *begrunne* appears a total of 169 times in the textbooks. Table 4.3 shows the diverse ways the word is used in sentences throughout the examined textbooks, and Figure 4.7 shows how many occurrences of each structure that is found. The most common use of the word *begrunne* is *begrunn svaret/svarene*, making up 86% of the 169 occurrences.

Norwegian	**English**
Gi/finn/skriv en begrunnelse	Give/find/write a justification
Begrunn at	Justify that
Begrunn valget	Justify your choice
Begrunn hvordan	Justify how
Begrunn hvorfor	Justify why
Begrunn svaret/svarene	Justify your answer(s)

Table 4.3 Translation of the usages of begrunne.

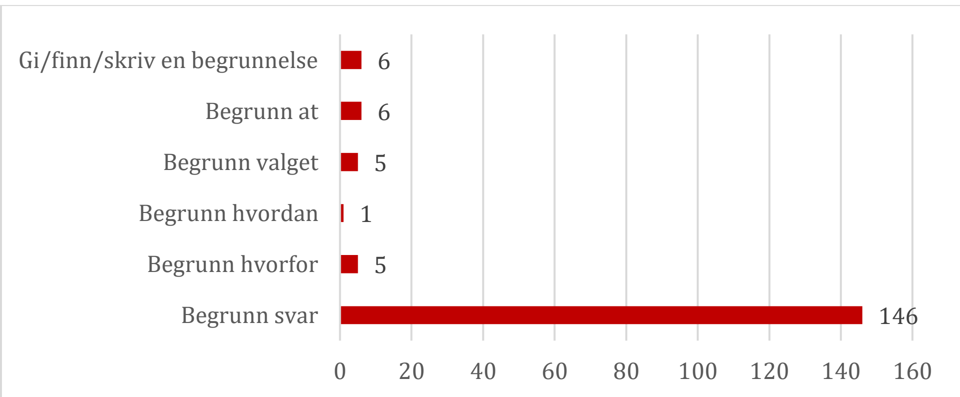


Figure 4.7 Distribution of the diverse ways the word begrunne is used.

Figure 4.8 and Figure 4.9 show excerpts from two different textbooks where *begrunn svaret* occur. Task 1.44 in Figure 4.8 asks the pupils if it is possible to add two prime numbers and get a prime number. The pupils are then asked to show examples that justifies their answer. This first task can easily be proven correct by providing a single example. The following two tasks ask whether it is possible to get an odd number by adding either two odd numbers, or two even numbers. The answer to both questions is no, and the only way to show this is by deductive argumentation. However, the pupils are asked to provide examples to justify their answers. As there is no way to show that it is impossible to get an odd number in both cases using examples, it seems that the textbook does not expect the pupils to reason or argue deductively, and no such opportunity is offered.

1.44 Kan du addere to primtall og få et primtall som svar? Vis eksempler som begrunner svaret ditt.

1.45 Kan du addere to oddetall og få et oddetall som svar? Vis eksempler som begrunner svaret ditt.

1.46 Kan du addere to partall og få et oddetall som svar? Vis eksempler som begrunner svaret ditt.

Figure 4.8 Matematikk 7 Oppgavebok, Cappelen Damm, p. 14. 1.44 – Can you add two prime numbers and get a prime number? Show examples that justify your answer. 1.45 – Can you add two odd numbers and get an odd number? Show examples that justify your answer. 1.46 – Can you add two even numbers and get an odd number? Show examples that justify your answer.

In Figure 4.8 the tasks specifically ask for examples that justifies the answers, but inFigure 4.9, task 15 ask the pupils to *justify* their answers by thinking like the four cartoon figures on the previous page do.

The first cartoon figure (Hiyanna) justifies using drawing, or concrete materials, to show parts of a whole. The visual form of the argument representation allows for a visual comparison of the two fractions. However, as no further justification is provided, it could also show an experiment done to find the correct answer, which is not argumentation as defined by the curriculum. Henrik's justification is a combination of visual and verbal argumentation. The verbal argument uses logical reasoning, stating that 1 is half as much as 2, whereas 7 is more than half as much as 12, therefore, $7/12 > 1/2$. Tuva also uses a visual form of argumentation in the first part of her justification. The second part of her justification shows a comparison of the fractions using common denominators, referring to the visual representation showing fractions as parts of a set. The final cartoon figure, Yonas, uses a number-line model, which this textbook refers to as elevators in an apartment building (Figure I.5). Referring to Matemagiskhuset, he uses verbal argumentation to show that Floor I is higher up in the apartment building than Floor H, which could justify his answer. However, like Hiyanna's justification, this could also be an experiment done to find the correct answer to the problem, not to justify. All of these argument representations are known to, or within the conceptual reach of, the classroom community and the modes of argumentation are at least potentially deductive. As the pupils are asked to justify their answers like the cartoon figures have done, task 15 offers at least potential opportunities for reasoning and argumentation.

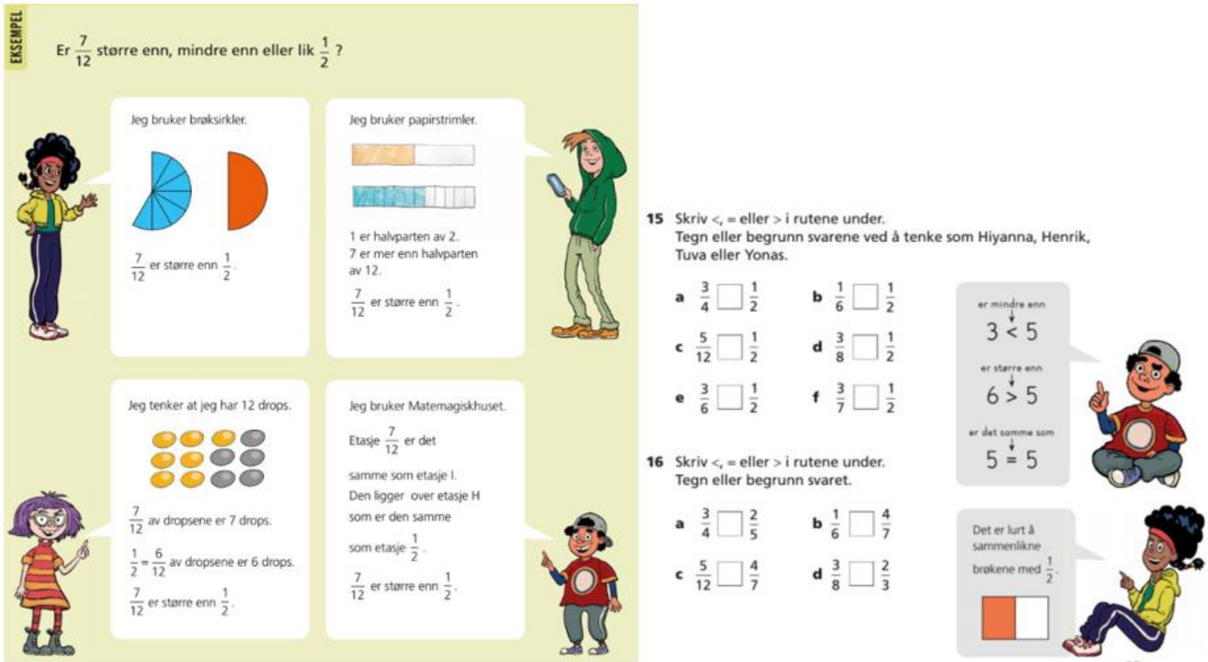


Figure 4.9 Matemagisk 5A Grunnbok, Aschehoug Undervisning, p. 68-69. (p. 68) Example. Is 7/12 bigger than, smaller than or equal to 1/2? (Hiyanna) I use fraction circles. 7/12 is bigger than 1/2. (Henrik) I use paper strips. 1 is half as much as 2. 7 is more than half as much as 12. 7/12 is bigger than 1/2. (Tuva) I imagine having 12 pieces of candy. 7/12 of the pieces is 7 pieces of candy. 1/2 = 6/12 of the pieces is 6 pieces of candy. 7/12 is bigger than 1/2. (Yonas) I use the Matemagisk house. Floor 7/12 is the same as floor I. Floor I is above floor H which is the same as floor 1/2. 7/12 is bigger than 1/2. (Page 69) 15 – Write <, = or > in the squares below. Draw or justify the answers by thinking like Hiyanna, Henrik, Tuva, or Jonas. 16 – Write <, = or > in the squares below. Draw or justify the answer. (Yonas) < means smaller than, > means bigger than, = means equal to. (Hiyanna) It is smart to compare the fractions with 1/2.

When the Matematikk (CD) series introduces their true-or-false activities they state that the pupils shall *argue* whether statements are true or false. When looking closer at these tasks, which the textbooks identify as opportunities for argumentation, it is interesting to consider what kinds of arguments are expected. First of all, the textbooks never use the word *argumentere* in these activities outside of the introduction, and instead they use *begrunn* (justify). Figure 4.10 shows how the activities are presented in the textbook series. In Figure 4.10 one can see that some of the statements potentially could offer an opportunity for deductive argumentation (bullet point 3, 4, 5, 7 and 8). Other statements offer arguments a single step from a remembered definition or procedure. To argue whether the decimal system consists of ten numbers offer no opportunity for deductive reasoning, but the statement that the biggest digit consisting of five numbers is 90 000 could offer opportunities for applying accepted modes of reasoning and forms of argumentation, e.g., by using counterexamples to prove the statement wrong. This is common throughout these activities. Some of the statements offer opportunities for reasoning and argumentation, others do not.

Sant eller usant?

Begrunn svarene

- Titallssystemet har ti sifre: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.
- Når sifrene settes sammen, kan vi lage uendelig mange tall.
- Det største femsifrede tallet vi kan lage, er 90 000.
- Det dobbelte av 24 er 50.
- 13 + 9 = 13 + 10 – 1.
- Subtraksjon er det samme som addisjon.
- Halvparten av 100 er 50.
- Tiervennen til 7 er 4.

Figure 4.10 Matematikk 5 Grunnbok, Cappelen Damm, p. 44. True or false? Justify your answers. – The decimal system has ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. – When the digits are put next to each other, we can make indefinite numbers. – The largest five-digit number we can create is 90 000. – 50 is twice as much as 24. – 13 + 9 = 13 + 10 – 1. – Subtraction is the same as addition. – 50 is half as much as 100. – The tens compliment of 7 is 4.

The type of justification the textbook activities expect from the pupils is not often made explicit in the activities. The activities shown in Figures 4.7 and Figure 4.8 are both examples of where the pupils are provided with instructions on how they are expected to justify. Figure 4.11 shows a third example of how the textbooks chose to instruct the pupils to the expected method for justification. This excerpt is a comment provided by a cartoon figure on the bottom of a page (Figure 4.12). The page includes a set of statements about four-sided polygons that the pupils shall assess whether is true or false and they are asked to justify their answers using a figure shown and a table providing information about some characteristics of the geometric figures. The table is shown in the appendix, Figure I.1, with translations in Table I.4Table I.4. The comment on the bottom of the page instructs the pupils to use accepted forms of argumentation and reasoning when justifying why a statement would be false. The first two statements (subtasks h and i) provide opportunities for using counterexamples to prove the statement to be false, whereas the final two statements (subtasks j and k) provide no such opportunities. The textbook offers no other comments about how to justify a statement to be true, except from the comment in the task that states that the pupils shall justify using the provided information from the table and the figure.

Figure 4.11 Matemagisk 6B Grunnbok, Aschehoug Undervisning, p. 73. It can be smart to make a counterexample when justifying that something is not true.

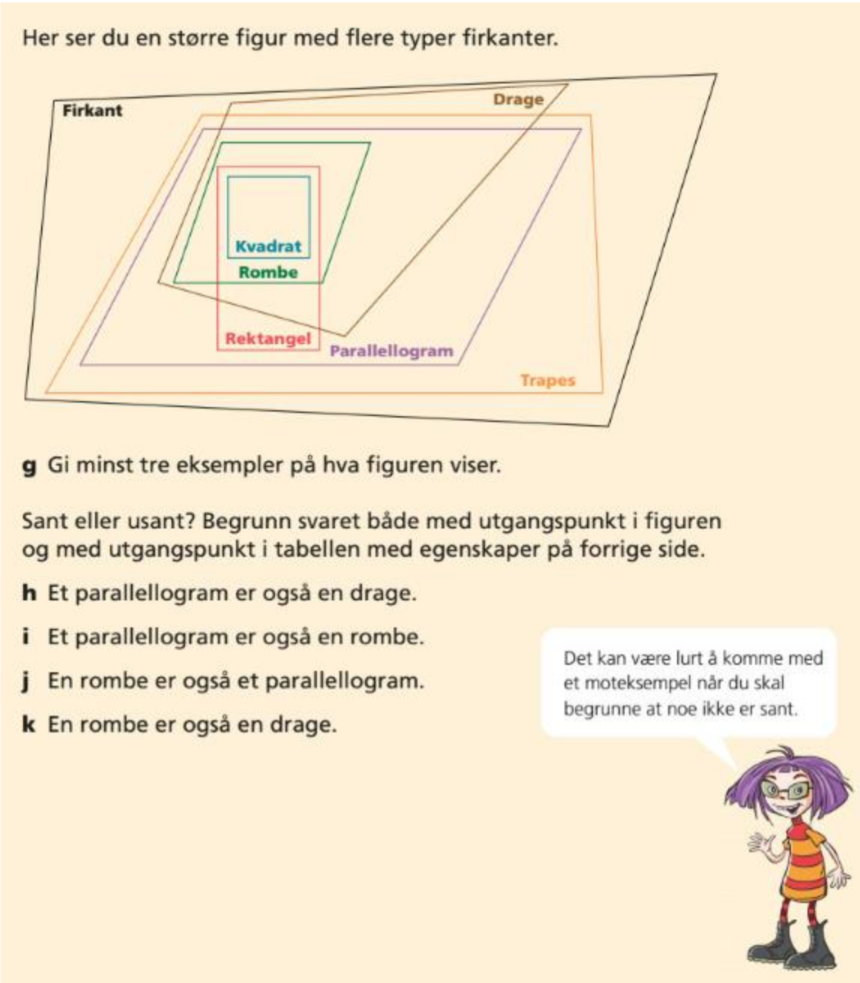


Figure 4.12 Matemagisk 6B Grunnbok, Aschehoug Undervisning, p. 73. Here you can see a larger figure with several types of four-sided polygons. g – Give at least three examples of what the figure shows. True or false? Justify the answer by using both the figure above and the table with the characteristics on the previous page. h – A parallelogram is also a kite. i – A parallelogram is also a rhombus. j – A rhombus is also a parallelogram. k – A rhombus is also a kite.

4.2.3. Forklare

The word *forklare* occurs a total of 271 times in the 15 examined textbooks. Table 4.4Table 4.4 shows the diverse ways the word is used in the textbooks with English translations, and Figure 4.13 shows how many times the different usages occur.

Norwegian	**English**
Å kunne forklare	To be able to explain
Forklar	Explain
Forklar hva/hvordan du tenker	Explain what/how you think
Forklar sammenhenger/forskjeller	Explain connections/differences
Forklar et mønster/figur/utregning/begrep/osv.	Explain a pattern/ figure/calculation/term/etc.
Forklar hvordan	Explain how
Forklar at	Explain that
Forklar hvorfor	Explain why
Forklar med egne ord/tegninger/modeller	Explain using your own words/drawings/models
Forklar hvert steg av en utregning	Explain every step of a calculation
Forklar ved hjelp av figur/bilde/tabell	Explain using a figure/picture/table
Forklar hva/hvordan noe viser/gjør	Explain what/how something shows/does
Forklar hva du tror	Explain what you think/believe

Table 4.4 Translation of the usages of forklare.

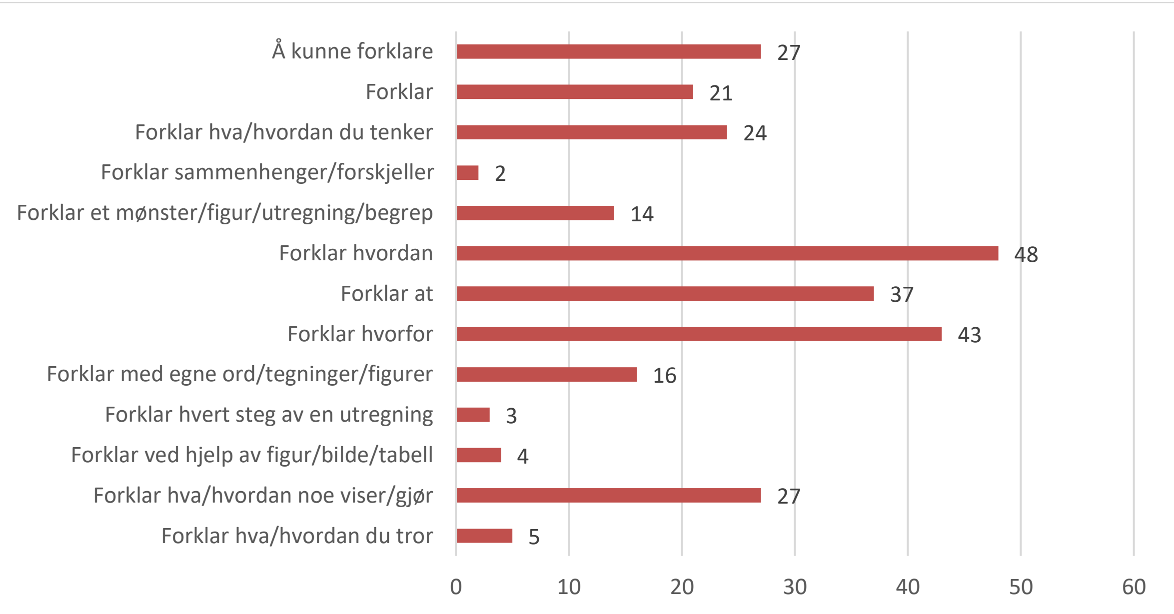


Figure 4.13 Distribution of the diverse ways the word forklare is used.

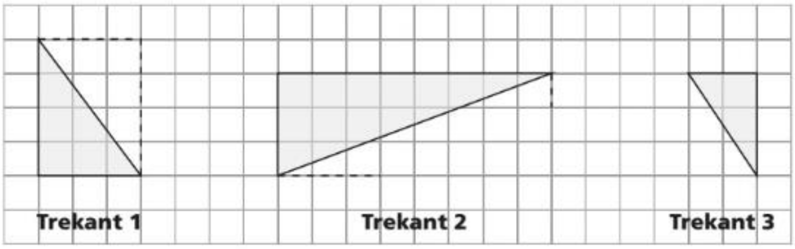
One of the occurrences found more often is *forklar hvordan*, a total of 17.7% out of the 271 times the word *forklar* occurs. Figure 4.14 shows an example of a task asking the pupils to *explain how* we can calculate the area of a right-angle triangle. The task is found following a task where the pupils are asked to calculate the area of four right-angle triangles drawn on a squared paper with dotted lines making up rectangles, and a second task where the pupils are asked to calculate the area of two right-angle triangles with known lengths of the two sides creating the right angle. On the previous page the pupils are provided with an explanation about how one can use parallelograms to calculate the area of a triangle. This explanation is shown in Figure I.2 with a translation. The task shown in Figure 4.14 offers a potential opportunity for reasoning and argumentation based on the pupils' previous knowledge of the properties of right-angle triangles or based on their previous knowledge about the area of triangles in general.

23 Forklar hvordan vi kan regne ut arealet av en rettvinklet trekant.

Figure 4.14 Matemagisk 6B Grunnbok, Aschehoug Undervisning, p. 143. Explain how we can calculate the area of a right-angle triangle.

Forklar hvorfor makes up 15.9% of the total occurrences of *forklar*. Figure 4.15 and Figure 4.16 show two activities that ask the pupils to explain why. Figure 4.15 is found a few pages behind Figure 4.14 and show how similar tasks use the keywords slightly differently from one another. Subtask (a) asks the pupils to explain why the triangles are right-angle triangles. The task is following an exploration activity where the pupils are asked to draw two identical right-angle triangles, cut them out, and then explore what geometric shapes they can create by laying the triangles next to each other. Next to this activity is a cartoon figure reminding the pupils that one of the angles in a right-angle triangle has to be 90° (Figure I.3). The subtasks following the one shown in Figure 4.15 is introduced with a statement saying that one can expand the triangles into rectangles. An explanation of why the triangles are right-angle triangles would be describing the characteristics of right-angle triangles, and repeating the comment made by the cartoon figure a bit further up the page, thereby not offering opportunities for reasoning and argumentation as defined by the core element *Reasoning and argumentation* in the curriculum. *Forklar hvorfor* occurs once more in the same activity, in one of the following subtasks, subtask (d), which asks the pupils to *explain why* the areas of the triangles are half as big as the areas of the rectangles. This subtask could potentially offer opportunities for reasoning and argumentation based on the pupils' understanding of the previous subtasks and activities or their previous knowledge about the properties of rectangles, triangles in general or right-angle triangles.

16 **a** Forklar hvorfor trekantene er rettvinklede.



Vi kan utvide trekantene til rektangler.

b Tegn av trekant 2 og trekant 3 og utvid dem til rektangler.

c Regn ut arealet av rektanglene. Oppgi svaret i antall ruter.

d Forklar hvorfor arealet av trekantene er halvparten av arealet av rektanglene.

Figure 4.15 Matemagisk 6B Grunnbok, Aschehoug Undervisning, p. 138. a – Explain why the triangles are right-angle triangles. We can expand the triangles into rectangles. b – Copy triangle 2 and triangle 3 and expand them into rectangles. c – Calculate the area of the rectangles. Give the answer in number of squares. d – Explain why the area of the triangles are half the area of the rectangles.

The second activity (Figure 4.16, subtask b) asks the pupils to explain why Hermann's addition procedure is wrong. The subtask follows a description of Hermann's calculation of adding two fractions with different denominators where he did not get the correct answer. This subtask offers an opportunity for reasoning and argumentation where the pupils can explain based on their prior knowledge of addition procedures of fractions with different denominators.

Hermann har regnet ut $\frac{1}{3}+\frac{1}{2}$, men han har fått feil svar. $\frac{1}{3}+\frac{1}{2} = \frac{1+1}{3+2} = \frac{2}{5}$

a Hvordan tror dere Hermann har tenkt?

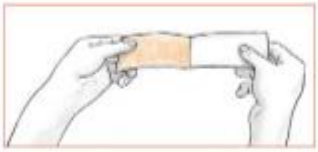
b Forklar hvorfor svaret til Hermann er feil.

Figure 4.16 Matemagisk 7A Grunnbok, Aschehoug Undervisning, p. 66. Hermann has calculated 1/3 + 1/2 but have gotten an incorrect answer. a – How do you think Hermann thought? b – Explain why Hermann's answer is wrong.

As shown when analyzing the three different occurrences of *forklar hvorfor*, it is not necessarily possible to say if one usage, such as *forklar hvorfor*, always offers, or never offers opportunities for reasoning and argumentation. Figure 4.17 and Figure 4.18 show how *forklar at* also can be found to both offer and not offer such opportunities. Figure 4.17 shows an activity that instructs the pupils to fold a paper strip in half. The pupils are then asked to unfold the strip and color one of the sides next to the crease the fold created. In subtask (3), the pupils are asked to *explain that* they have colored ½ of the paper strip. The function of explaining in this task is that of justifying a presentation, rather than reason or argue as defined by the core element *Reasoning and argumentation*.

(1) Brett papirstrimmelen i to like store deler.

(2) Brett ut og fargelegg én av de to delene.



(3) Forklar at du har fargelagt $\frac{1}{2}$ av papirstrimmelen.

Figure 4.17 Matemagisk 5A Grunnbok, Aschehoug Undervisning, p. 21. 1 – Fold the paper in two equal parts. 2 – Unfold and color one of the two parts. 3 – Explain that you have colored 1/2 of the paper strip.

Figure 4.18 shows a subtask (e) that asks the pupils to *explain that* $\frac{4}{12} = \frac{2}{6} = \frac{1}{3}$. Figure I.4 shows the activity in its entirety, and the pictures the pupils are asked to refer to in subtasks (a)-(d). Subtask (a) asks them to explain that the first picture show that $\frac{4}{12}$ of the eggs are dotted, subtask (b) asks the pupils to explain that the second picture shows that $\frac{2}{6}$ of the eggs are dotted, subtask (c) asks them to explain that the third picture show that $\frac{1}{3}$ of the eggs are dotted, and subtask (d) asks what is similar or different about the three pictures. The final subtask, shown in Figure 4.18, seems to offer an opportunity for reasoning and argumentation based on the pictures and the previous subtasks, as well as previous knowledge about fractions.

e Forklar at $\frac{4}{12} = \frac{2}{6} = \frac{1}{3}$.

Figure 4.18 Matemagisk 5A Grunnbok, Aschehoug Undervisning, p. 60. Explain that 4/12 = 2/6 = 1/3.

The textbooks sometimes use the word *forklar* without any following words. These usages are often found to be *Vis og forklar*, *Forklar!*, or *Forklar.* Figure 4.19 shows an example of how this usage occur in the textbooks. In this task, the pupils are asked to explain which calculations are done correctly and which are done incorrectly. Following this task is an explanation of how one can add and subtract fractions with common denominators (Figure I.6). Based on the pupils' understanding of fractions with common denominators, and their understanding of fractions in general, this task could offer opportunities for reasoning and argumentation.

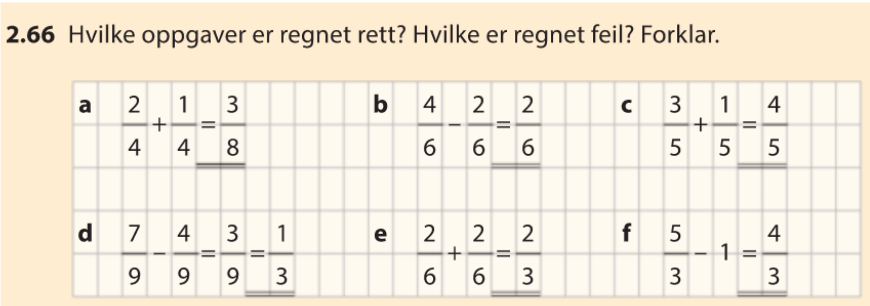


Figure 4.19 Multi 5A Elevbok, Gyldendal, p. 66. Which problems are calculated correctly? Which are calculated incorrectly? Explain.

4.2.4. Vise

From the quantitative data one can clearly see that the word *vise* is used the most in total, but the count does not differentiate between whether the text tells that a figure shows something, or if the pupils are asked to show something themselves. Figure 4.20 shows examples of both usages mentioned above. Neither of these usages of the word *vis* offer an opportunity for argumentation, it is first used to communicate what the table shows, and then it asks the pupils to use the given data and show it using a different representation. The majority of the occurrences are of similar nature to the ones shown in Figure 4.20.

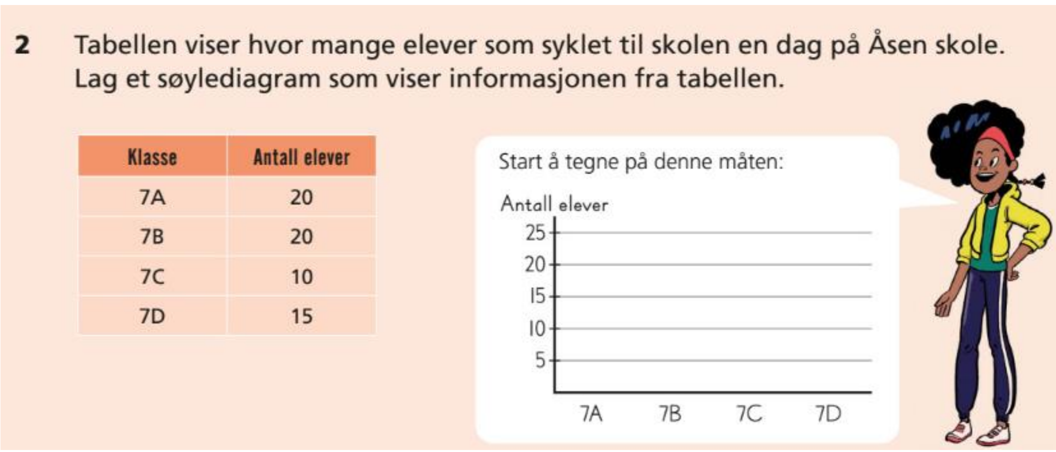


Figure 4.20 Matemagisk 7A Grunnbok, Aschehoug Undervisning, p. 17. The table shows how many pupils used their bikes to school one day at Åsen school. Create a bar chart that shows the information in the table.

In the activity shown in Figure 4.21 the word *vis* is used to ask the pupils to show their thoughts (subtask d). In the tasks prior to the ones shown in the figure, the pupils are told that Henrik reads five pages in a book every evening. On a Sunday he starts reading on page 162. Subtask (c) asks what day of the week Henrik will finish the book, and subtask (d) asks the pupils to show how they think when solving the prior subtask. This activity is found in a chapter on number sequences, and prior to this activity the pupils are introduced to some methods that can be helpful for discovering the next *n* numbers in a sequence, using a number line, or using a table (Figure 4.22). The function of the word *vis* in subtask (d) is to describe a process applied or the thought process that went into solving the prior subtask. This specific usage of the keyword does not offer opportunities for reasoning and argumentation.

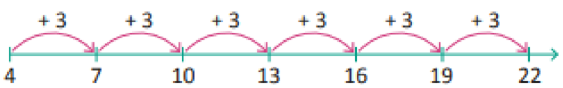
Boka har 196 sider.

c) Hvilken ukedag er han ferdig med boka?

d) Vis hvordan du tenker.

Figure 4.21 Matematikk 5 Grunnbok, Cappelen Damm, p. 81. The book has 196 pages. c – On what day will he finish the book? d – Show how you think.

Metode 1



Metode 2

Figur nr	1	2	3	4	5	6	7
Antall fyrstikker	4	7	10	13	16	19	22

Figure 4.22 Matematikk 5 Grunnbok, Cappelen Damm, p. 81.

5. Discussion

The goal of the present study is to understand what opportunities there are for reasoning and argumentation in Norwegian mathematics textbooks for grades 5-7, and the research questions asked are: 1) to what extent do the textbooks use words related to reasoning and argumentation; and 2) how are words related to reasoning and argumentation used in the textbooks, and what usages offer opportunities for reasoning and argumentation? So far, I have presented relevant theory and previous research. I have explained the methods used for examining and analyzing the textbooks, and I have presented the results of the examination and analysis. In this chapter I will discuss the findings of the present study and explain connections to previous research and findings.

The main findings of the present study revealed in the previous chapter are:

a. The keywords occur rarely or not at all in the examined textbooks.
b. The usages of the keywords located in the superficial examination does not always offer opportunities for reasoning and argumentations.
c. Although there might not be many, there are opportunities for reasoning and argumentation in the examined textbooks.

In section 5.1. I will discuss the findings related to the few occurrences of the keywords in connection with previous research and related theory. I will also discuss the usages of the words and what the findings suggest about the opportunities for reasoning and argumentation related to the usages. Finally, section 5.2. will include suggestions for further research and what the results of the present study offer to the mathematics educational society.

5.1. General discussion

I have in the present study located a total of 1058 usages of the keywords *argumentere*, *forklare*, *begrunne*, and *vise*. These occurrences are identified as potential opportunities for reasoning and argumentation during the superficial examination. The word *forklare* occurs on 7.4% of the total 2507 pages in the textbooks, the word *begrunne* occurs on 5.2%, and *vise* occurs on 14.4%. *Argumentere* only occurs on 0.36% of the pages.

Two of the six words listed as related to reasoning and argumentation (Table 3.2Table 3.2) did not occur in any of the 15 examined textbooks, the words *resonnere* and *bevise*. A third word was used very rarely, the word *argumentere*. The words *resonnere* and *argumentere* occur several times in the curriculum, and one of six core elements is called *Reasoning and argumentation*. Utdanningsdirektoratet state that reasoning and argumentation is supposed to permeate the subject, as the core elements are the most important contents in mathematics, and they are supposed to characterize the content and progression of the mathematics education (Utdanningsdirektoratet, 2019a; 2019b). The word *bevise* does not occur in the textbooks, but in contrast to *resonnere* and *argumentere* it has not been given the same amount of obvious attention in the curriculum. The word *prove* is mentioned once in the final sentence of the core element *Reasoning and argumentation* in the definition of argumentation. “Argumentation in mathematics means that the pupils give reasons for their approaches, reasonings and solutions, and prove that these are valid” (Kunnskapsdepartementet, 2019b). However, Valenta and Enge (2020) argue that proof and proving is central to mathematical reasoning and argumentation as defined in the core element. To find that the words *bevise*, *resonnere*, and *argumentere* are used very rarely or not at all in any of the examined textbooks does not necessarily mean that there are no proofs or opportunities for the pupils to engage in reasoning and argumentation in the textbooks, but it does give rise to the question whether it has been given the attention called for by the curriculum and by educational researchers. Bakken & Bakken’s (2021) claim that “textbooks fail to meet the demands set in the curriculum and the recommendations of educational research”, which could be supported by the absence of the words *resonnere*, *argumentere*, and *bevise*. These findings suggests that there is a contrast between the contents of the textbooks and the curriculum, and the recommendations of educational research (e.g., Ball et al. 2002; Hanna, 1995), which calls for reasoning and argumentation as a central part of mathematics education. Such contrast is also found by Wong and Sutherland (2018), but in their case the curriculum did not provide attention to reasoning and argumentation outside of the learning targets of geometry.

Nordström and Löfwall (2005) found that words like *proof*, *definition*, and *assumption* were avoided in the textbooks examined in their research, not unlike what my findings show about the three words, *resonnere*, *argumentere*, and *bevise*. Nordström and Löfwall (2005) state that the fact that the words are avoided could have some significance to the pupils’ access to such activities. That the words were avoided in the present study could imply that both teachers and pupils might have difficulties trying to locate activities that offer opportunities for reasoning and argumentation, especially if they are unfamiliar with other terms connected to these activities. McCrory and Stylianides (2014) came to a similar conclusion, based on their analysis of textbook indexes and tables of content, and stated that these difficulties could occur when the pupils independently tried to search for opportunities for RP. *Argumentere* is used twice in the introductory pages of the primary textbooks in the series from Cappelen Damm, which could help point to activities that are said to provide opportunities for argumentation. However, although the word never occurs in those activities throughout the textbooks, one could imagine that the user of the textbook would believe that these activities always provide such opportunities based on the description, which is not the case. The statements in the True or false activities do in some cases offer opportunities for reasoning and argumentation as defined by the curriculum, but in other cases they do not.

Begrunne is used in place of *argumentere* in the True or false activities, and the pupils are asked to *justify their answer(s)*. This could imply that the textbook authors believe that the words *argumentere* and *begrunne* have the same meaning, and that the pupils might be more comfortable with the word *begrunne* as this is chosen to present the activities outside the introductory pages. A similar finding is presented in Nordström and Löfwall (2005), who found that the word *proof* occurred in the beginning of one textbook, but then was replaced with *explanation* later in the same textbook. In a second textbook, the word *justification* was also used in place of *proof*. If the words are used synonymously or if words like *resonnere*, *argumentere*, and *proof* are avoided in the textbooks, the pupils might experience difficulties in distinguishing between the concepts of justification, explanation, reasoning, and argumentation later in their education, as it can lead to a confusion about notions formed in the middle school grades (Nordström & Löfwall, 2005).

The word *begrunne* occurs in every textbook across the textbook series examined. Bergwall and Hemmi (2017) state that a justification is a mathematical argument that is designed to verify, convince, or explain the validity of a statement. 86.4% of the 169 identified occurrences is the usage *begrunn svaret*, and as shown in Chapter 4, some of these occurrences do offer opportunities for reasoning and argumentation, while others do not offer such opportunities. In some cases, the opportunities for reasoning and argumentation when justifying are influenced by the textbook's direct instructions to the pupils. As shown in Figure 4.8, Figure 4.9, and Figure 4.11, the textbooks offer direct instructions as to what type of justification is expected. In Figure 4.9 the pupils are provided with four different argument representations, and the pupils are asked to justify their answers following four cartoon figures. Two of the cartoon figures, Hiyanna and Yonas both justifies using visual representations. However, these visual justifications might not reflect the cartoon figures' thinking, and they might be experiments. Stacey and Vincent (2009) refer to this as *experimental demonstrations* and place these activities in the empirical class in Harel and Sowder's (2007) proof scheme. If the cartoon figures do experiment, the pupils following those experiments might believe that they are justifying, but they do not get opportunities for reasoning and argumentation. The expected forms of argumentation shown are both deductive (Figure 4.11) and non-deductive (Figure 4.8). Otten et al. (2014) found that the opportunities for RP more often included providing non-proof justifications than deductive modes of argumentation. Non-proof justifications are often asked for also in the textbooks in the present study, as shown by the activities in Figure 4.8, which asks specifically for non-deductive argumentation and some of the statements in Figure 4.10, which offer argumentation only a single step from given definitions. Otten et al. (2014) suggest that a result of the lack of opportunities for deductive forms of reasoning could be that pupils may continue to rely on non-deductive forms of reasoning, such as empirical reasoning, and therefore may experience difficulties with mathematical reasoning and argumentation and producing valid proofs later in their education.

The results of the closer analysis show that the usages of the keywords do in some cases offer opportunities for reasoning and argumentation, while in other cases they do not. *Forklar hvorfor*, and *forklar at* are two examples of usages that are found to both offer, and not to offer opportunities for reasoning and argumentation. The overall understanding of the opportunities is that they are few, and that several of the usages of the keywords are found to not offer opportunities for reasoning and argumentation. The usage *å kunne forklare* is an example of one occurrence that never offer such opportunities, as it is used to inform the textbook users of the goals of the textbooks or chapters, and the two usages of *vise* shown in Figure 4.20 are also found to not offer opportunities for reasoning and argumentation. Thompson et al. (2012) state, in regard to their findings, that students might have limited experiences with proof-related reasoning. Without additional support the pupils using the textbooks in the present study might also experience limited opportunities for reasoning and argumentation. It is important to note that there are opportunities for reasoning and argumentation in the examined textbooks, although they might be few. Reasoning and argumentation are central to mathematics and the mathematics education as the pupils are supposed to know that the rules they learn, and the results they get, are not random but that they have obvious reasons (Kunnskapsdepartementet, 2019b). However, if the pupils are supposed to develop a better knowledge about reasoning and argumentation, the findings of the present study suggest that they need additional support.

Tømmerdal (2021) investigated opportunities for RP in tasks found in the fraction chapters of four of the fifth-grade textbooks examined in the present study. Tømmerdal states that although his study does not show if his findings are reflected in the other chapters in the textbooks, his findings do suggest that RP has not found its place in the textbooks (2021, p. 59). The results of the present study support this statement, as the findings indicate that reasoning and argumentation have not found its place in the textbooks examined. Tømmerdal (2021) found that only 3% of the tasks found in the chapters on fractions offer opportunities for engaging with activities within the broader term RP, which in addition to activities included in reasoning and argumentation also includes activities that are found in other core elements. Based on his findings, it is imaginable that the same tasks would provide a similar, or even smaller, number of opportunities for reasoning and argumentation as defined by the curriculum. More research is needed to further explore the opportunities for reasoning and argumentation in Norwegian mathematics textbooks, and I will in the following section share two ideas for future research.

5.2. Implications and further research

Bieda et al. (2014) and Tømmerdal (2021) found that the opportunities for RP were as few as 3-4% of the examined data. As previously stated, the term RP as defined by Stylianides (2008) includes activities such as identifying patterns and conjecturing, which is not found in the core element *Reasoning and argumentation*. The findings presented by Bieda et al. (2014) and Tømmerdal (2021) using the broader term RP, could imply that the opportunities for reasoning and argumentation in the examined data would be even fewer than 3%. It would be interesting to examine the same data but with regards to the definitions of reasoning and argumentation in the curriculum, and to compare the number of pupil tasks that provide opportunities for reasoning and argumentation with the findings in Bieda et al. (2014) and Tømmerdal (2021).

As previously stated, one of the possible implications of the findings in the present study is that the pupils might experience difficulties locating activities providing opportunities for reasoning and argumentation and that they might have limited opportunities for reasoning and argumentation. The role of the teacher in the classroom community is to provide guidance so that the pupils can develop their mathematical reasoning and argumentation (Stylianides, 2007). It would be interesting to examine how teachers guide their pupils during activities found offering opportunities for reasoning and argumentation in the textbooks examined in the present study. It would also be interesting to examine how, or even if, the teachers use the textbooks as a resource for teaching reasoning and argumentation.

Bakken & Bakken (2021) state that textbook researchers have a crucial task of sharing their findings with textbook authors, publishers, teachers, pupils, and legislators. The findings in the present study could imply that reasoning and argumentation have not been provided the attention called for in the curriculum, and to share this with the community could lead to some implications.

First of all, if the goal of the textbook authors is to reflect the curriculum in their textbooks, they have to be informed about potential contrasts between the curriculum and the textbooks, such as the contrast suggested by the findings in the present study. Educational researchers, textbook researchers, textbook authors, textbook publishers, and teachers can work together to supply the schools with textbooks that reflect both the call for reasoning and argumentation as of the most important content of mathematics education by the curriculum, and the international call for more attention to reasoning and argumentation by educational researchers.

Secondly, if the pupils using the textbooks are supposed to learn and practice mathematical reasoning and argumentation, teachers have to be aware that these opportunities are few in the examined textbooks, and often difficult to locate if not being familiar with related words.

Knowing this, the teachers can provide the additional information necessary to achieve opportunities for reasoning and argumentation, and they can supply the textbooks with additional resources. Teacher educators aware of this contrast could supply teacher students with the knowledge necessary to provide their future pupils with opportunities for reasoning and argumentation without relying on the textbooks.

6. Conclusion

The goal of the present study has been to understand the opportunities for reasoning and argumentation in Norwegian mathematics textbooks for grades 5-7, and the research questions were: 1) to what extent do the textbooks use words related to reasoning and argumentation; and 2) how are words related to reasoning and argumentation used in the textbooks, and what usages offer opportunities for reasoning and argumentation? I have chosen to include the term *proof* as defined by Stylianides (2007) as the definition of reasoning and argumentation implies that *proving* is central and that the accepted forms of reasoning and modes of argumentation coincide with Stylianides' accepted forms of reasoning. This is supported by Reid (2022) and Valenta and Enge (2020). Further, I have presented related theory and prior research. Most of the related prior research have examined reasoning-and-proving (RP) or proof-related reasoning, and those terms and frameworks have also been described. The method of data analysis of the present study was a document analysis, as defined by Bowen (2009), and the analyzed documents were 15 Norwegian mathematics textbooks from three different publishers. The superficial examination examined the occurrence of words found related to reasoning and argumentation (*begrunne*, *forklare*, *vise*, *bevise*, *argumentere*, and *resonnere*) to locate potential opportunities for reasoning and argumentation. The closer analysis included revisiting and examining the usage of the keywords in the located occurrences. This analysis adapted aspects of the framework developed by Miyakawa and Shinno (2021), to examine the *structure*, the *language*, and the *function* of the activities located including the keywords. The results presented showed that the keywords were used rarely or not at all in the examined textbooks. The different usages of those words related to reasoning and argumentation that did occur, did not always provide opportunities for reasoning and argumentation as defined by the curriculum. I have discussed what implications may be derived from the findings. The textbooks contrast the call for reasoning and argumentation as of the most important contents of mathematics education by both the curriculum (Kunnskapsdepartementet, 2019b; Utdanningsdirektoratet, 2019a) and in educational research (e.g., Hanna, 1995; Ball et al. 2002). Although there are some opportunities for reasoning and argumentation, these opportunities are few, and might limit the pupils understanding of reasoning and argumentation. In addition, they might experience difficulties trying to locate such opportunities as the words are used rarely or not at all. To fully understand the opportunities for reasoning and argumentation provided by the examined textbook, I suggest further research of such opportunities in the textbooks. I also suggest researching the usage of the textbooks in schools as support when teaching reasoning and argumentation to understand as to which degree the indicated lack of opportunities affect the pupils learning or the teachers teaching reasoning and argumentation.

Finally, I want to make some personal reflections on the study presented in this thesis. Like I mention in the preface, I have always been interested in understanding and asking why in mathematics. Understanding why we use certain rules and understanding why applying certain rules always give the right answer have brought many challenges throughout my personal mathematics education. I also recall the frustration of not understanding why when investigating procedures, rules, formulas, etc. in the mathematics textbooks, and that the textbooks rarely asked for reasoning, argumentation, proof, justification or explanations of these procedures, rules, formulas, etc. Luckily, there is a greater focus on reasoning and argumentation in the curriculum today than when I was in elementary, secondary, and high school. However, the findings I have presented in this thesis indicate that these opportunities are still few, 12 to 15 years later. I have to some extent achieved my goal of understanding the opportunities for reasoning and argumentation in the textbooks studied, mostly in regard to the words found related to reasoning and argumentation. The textbooks rarely or never use the words related to reasoning and argumentation, and the usages of these words do in some cases offer opportunities for reasoning and argumentation, while in other cases they do not.

Understanding *why* in mathematics may seem difficult, but it is far from impossible with proper tools, guidance, and support.

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I. Appendices

Appendix A: Number of keyword occurrences

	Argumentere	Begrunne	Forklare	Vise	Number of pages in the textbook pairs
Cappelen Damm	3	10	14	56	362
Aschehoug Undervisning	0	34	44	118	308
Gyldendal	0	8	18	37	270
Total	3	52	76	211	940

Table I.1 Number of keyword occurrences by textbook pairs, fifth grade.

	Argumentere	Begrunne	Forklare	Vise	Number of pages in the textbook pairs
Cappelen Damm	3	13	17	44	420
Aschehoug Undervisning	0	42	101	48	326
Gyldendal	0	7	18	76	278
Total	3	62	136	167	1024

Table I.2 Number of keyword occurrences by textbook pairs, sixth grade.

	Argumentere	Begrunne	Forklare	Vise	Number of pages in the textbook pairs
Cappelen Damm	3	20	4	116	388
Aschehoug Undervisning	0	35	55	117	155
Gyldendal	NDA	NDA	NDA	NDA	NDA
Total	3	55	59	231	543

Table I.3 Number of keyword occurrences by textbook pairs, seventh grade.

Appendix B: Additional figures and translations from the textbooks

Egenskap	Kvadrat	Rektangel	Rombe	Parallellogram	Trapes	Drage
Alle vinklene er 90°.	X	X				
Motstående vinkler er like store.	X	X	X	X		
Alle sidene er like lange.	X		X			
To og to sider er like lange.	X	X	X	X		X
To og to sider er parallelle.	X	X	X	X		
Diagonalene er like lange.	X	X				
Vinkelen mellom diagonalene er 90°.	X		X			X

Figure I.1 Matemagisk 6B Grunnbok, Aschehoug Undervisning, p. 72.

Characteristic	Square	Rectangle	Rhombus	Parallelogram	Trapezoid	Kite
All angles are 90°.	X	X				
Opposite angles are of equal size.	X	X	X	X		
All sides are of equal length.	X		X			
Two and two sides are of equal lengths.	X	X	X	X		X
Two and two sides are parallel.	X	X	X	X		
The diagonals are of equal length.	X	X				
The angle between the diagonals is 90°.	X		X			X

Table I.4 Translation of Figure I.1.

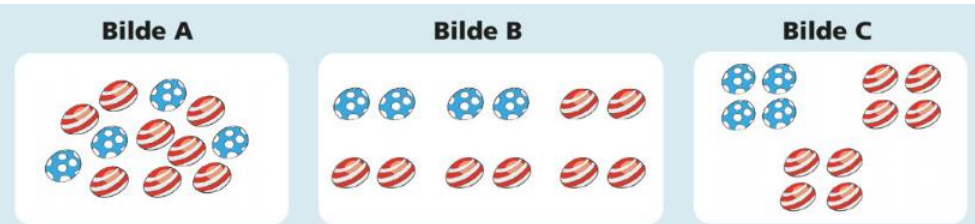
Et parallellogram kan alltid gjøres om til et rektangel med samme areal. Derfor er arealet av et parallellogram $A = g \cdot h$, der g er grunnlinja og h høyden i parallellogrammet.

Vi kan alltid utvide en trekant til et parallellogram med dobbelt så stort areal. Derfor er arealet av trekanten halvparten av arealet av parallellogrammet. Vi kan skrive det som en formel:

$$A = \frac{g \cdot h}{2}$$

Figure I.2 Matemagisk 6B Grunnbok, Aschehoug Undervisning, p. 142. A parallelogram can always be converted into a rectangle with equal area. That is why the area of a parallelogram is $A = g \times h$, where g is the baseline and h is the height of the parallelogram. We can always expand a triangle to a parallelogram with double the area. That is why the area of the triangle is half the area of the parallelogram. We can write it as a formula: A = (g · h)/2.

Figure I.3 Matemagisk 6B Grunnbok, Aschehoug Undervisning, p. 138. In a right-angle triangle one of the angles is 90°.



a Forklar at bilde A viser at $\frac{4}{12}$ av eggene er prikkete.

b Forklar at bilde B viser at $\frac{2}{6}$ av eggene er prikkete.

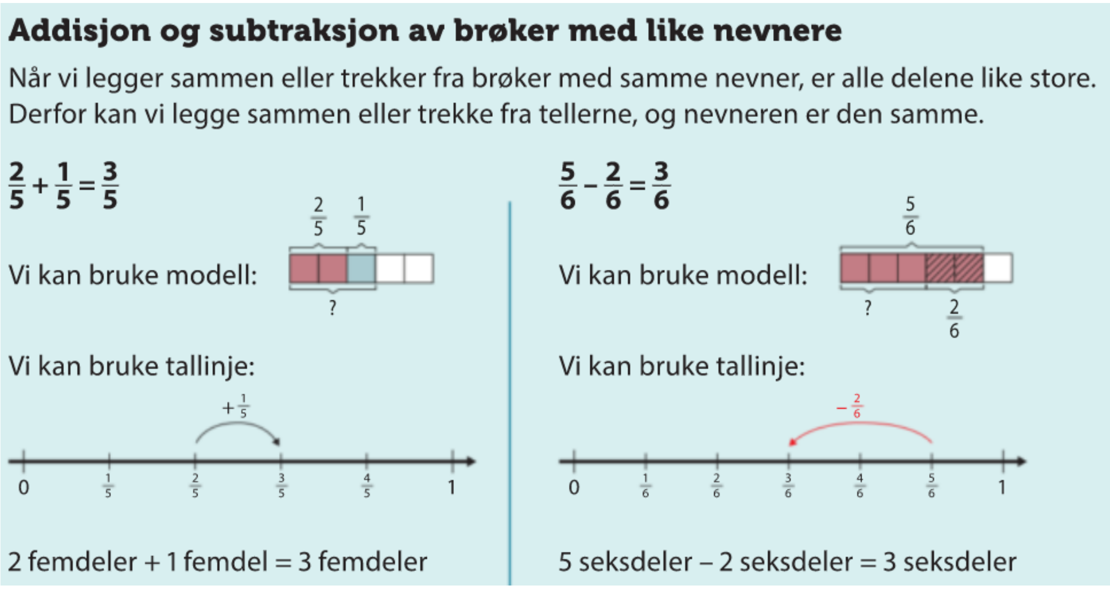
c Forklar at bilde C viser at $\frac{1}{3}$ av eggene er prikkete.

d Hva er likt, og hva er ulikt på bilde A, B og C?

e Forklar at $\frac{4}{12} = \frac{2}{6} = \frac{1}{3}$.

Figure I.4 Matemagisk 5A Grunnbok, Aschehoug Undervisning, p. 60. a – Explain that picture A shows that 4/12 of the eggs are dotted. b – Explain that picture B shows that 2/6 of the eggs are dotted. c – Explain that picture C shows that 1/3 of the eggs are dotted. d – What is similar, and what is different between pictures A, B, and C? e – Explain that 4/12 = 2/6 = 1/3.

Figure I.5 Matemagisk 5A Grunnbok, Aschehoug Undervisning, p. 64.



Addisjon og subtraksjon av brøker med like nevnere

Når vi legger sammen eller trekker fra brøker med samme nevner, er alle delene like store. Derfor kan vi legge sammen eller trekke fra tellerne, og nevneren er den samme.

$\frac{2}{5} + \frac{1}{5} = \frac{3}{5}$

Vi kan bruke modell:

Vi kan bruke tallinje:

2 femdeler + 1 femdel = 3 femdeler

$\frac{5}{6} - \frac{2}{6} = \frac{3}{6}$

Vi kan bruke modell:

Vi kan bruke tallinje:

5 seksdeler – 2 seksdeler = 3 seksdeler

Figure I.6 Multi 5A Elevbok, Gyldendal, p. 66. Addition and subtraction of fractions with common denominators. When we add or subtract fractions with common denominators, all parts are of equal size. For that reason, we can add or subtract the numerators, and the denominator is the same. We can use a model. We can Use a number-line.

Appendix C: Pages including keywords in the textbooks

C.1. Matematikk 5 Grunnbok, Cappelen Damm.

	Argumentere	Begrunne	Forklare	Vise
Page	2	27	49	3
	3	44	79	36
	139	74	84	41
		98	90	43
		147	100	54
		150	113	67
		184	157	68
		216	162	69
			191	70
				81
				82
				84
				93
				95
				96
				97
				105
				119
				150
				153
				155
				193
				195
				197
				203
				215
				216
				217

Table I.5 Pages including keywords Matematikk 5 Grunnbok, Cappelen Damm

C.2. Matematikk 5 Oppgavebok, Cappelen Damm

	Argumentere	Begrunne	Forklare	Vise
Page		89	49	15
		129	114	42
				50
				62
				64
				71
				84
				90
				120
				121
				130

Table I.6 Pages including keywords Matematikk 5 Oppgavebok, Cappelen Damm

C.3. Matematikk 6 Grunnbok, Cappelen Damm

	Argumentere	Begrunne	Forklare	Vise
Page	2	38	9	3
	3	74	17	31
	195	117	45	35
		160	108	40
		204	116	47
		219	125	51
		226	136	70
		235	176	83
		238	177	89
		239		110
				123
				156
				157
				184
				185
				187
				206
				228
				243

Table I.7 Pages including keywords Matematikk 6 Grunnbok, Cappelen Damm

C.4. Matematikk 6 Oppgavebok, Cappelen Damm

	Argumentere	Begrunne	Forklare	Vise
Page		161	144	15
		172		16
				19
				23
				42
				44
				53
				132
				133

Table I.8 Pages including keywords Matematikk 6 Oppgavebok, Cappelen Damm

C.5. Matematikk 7 Grunnbok, Cappelen Damm

	Argumentere	Begrunne	Forklare	Vise
Page	2	39	117	3
	3	71	143	31
	9	82	145	35
		112	161	53
		154		90
		183		101
		192		102
		199		103
		205		113
		216		114
		220		115
		222		163
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				220
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Table I.9 Pages including keywords Matematikk 7 Grunnbok, Cappelen Damm

C.6. Matematikk 7 Oppgavebok, Cappelen Damm

	Argumentere	Begrunne	Forklare	Vise
Page		14		12
		43		14
		143		44
		154		64
		158		68
				69
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				122
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				160
				161
				162
				163

Table I.10 Pages including keywords Matematikk 7 Oppgavebok, Cappelen Damm

C.7. Matemagisk 5A Grunnbok, Aschehoug Undervisning

	Argumentere	Begrunne	Forklare	Vise
Page		69	2	2
		72	4	7
		83	21	21
		101	29	25
		112	60	26
		123	66	27
		126	90	29
		133	96	32
		139	112	39
			116	40
			132	42
			139	45
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				129
				131
				135
				136

Table I.11 Pages including keywords Matemagisk 5A Grunnbok, Aschehoug Undervisning

C.8. Matemagisk 5B Grunnbok, Aschehoug Undervisning

	Argumentere	Begrunne	Forklare	Vise
Page		35	2	2
		37	4	8
		47	9	9
		49	12	10
		51	15	13
		53	16	14
		57	21	15
		63	22	19
		71	25	20
		88	37	27
		89	50	30
			59	34
			65	38
			95	42
			99	61
			107	62
			109	74
			114	90
			122	123
				129
				143
				145
				147
				149
				155
				159
				160
				161
				162

Table I.12 Pages including keywords Matemagisk 5B Grunnbok, Aschehoug Undervisning

C.9. Matemagisk 6A Grunnbok, Aschehoug Undervisning

	Argumentere	Begrunne	Forklare	Vise
Page		9	2	2
		21	4	10
		25	16	20
		34	17	25
		49	18	39
		52	24	43
		54	29	55
		65	30	57
		85	34	85
		112	35	86
			39	87
			48	96
			50	99
			54	104
			56	108
			57	111
			71	113
			88	
			97	
			99	
			109	
			110	
			117	

Table I.13 Pages including keywords Matemagisk 6A Grunnbok, Aschehoug Undervisning

C.10. Matemagisk 6B Grunnbok, Aschehoug Undervisning

	Argumentere	Begrunne	Forklare	Vise
Page		16	2	2
		17	4	17
		23	8	35
		31	23	38
		38	24	72
		48	26	78
		50	29	79
		59	35	90
		65	41	97
		70	55	113
		72	58	115
		73	67	125
		78	71	131
		79	72	145
		110	78	154
		125	79	172
		129	90	178
		157	97	184
		161	104	187
		172	116	190
		183	124	191
		193	125	
			132	
			133	
			138	
			142	
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			167	
			170	
			172	
			175	
			194	
			195	

Table I.14 Pages including keywords Matemagisk 6B Grunnbok, Aschehoug Undervisning

C.11. Matemagisk 7A Grunnbok, Aschehoug Undervisning

	Argumentere	Begrunne	Forklare	Vise
Page		13	2	2
		16	4	13
		19	9	15
		20	16	16
		21	29	17
		24	51	18
		27	55	19
		29	58	20
		30	61	21
		32	62	22
		43	65	24
		44	66	25
		45	67	26
		47	72	27
		49	74	28
		50	75	29
		54	78	30
		74	80	31
		77	81	32
		79	83	34
		80	87	39
		102	93	40
		107	102	45
		115	108	46
		118	111	47
		121	112	48
		145	114	49
			115	50
			117	52
			121	53
			125	58
			135	62
			136	63
			137	64
			138	65
			143	67
			153	71
				74
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				78
				80
				81
				83

Cont.	Argumentere	Begrunne	Forklare	Vise
				85
				94
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				98
				107
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				113
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				144
				150
				152

Table I.15 Pages including keywords Matemagisk 7A Grunnbok, Aschehoug Undervisning

C.12. Multi 5A Elevbok, Gyldendal

	Argumentere	Begrunne	Forklare	Vise
Page		59	2	8
		64	5	18
		93	46	32
		111	50	33
			56	47
			62	50
			66	57
			103	59
			110	61
			119	91
			121	116

Table I.16 Pages including keywords Multi 5A Elevbok, Gyldendal

C.13. Multi 5B Elevbok, Gyldendal

	Argumentere	Begrunne	Forklare	Vise
Page		8	2	5
		34	30	8
		67	32	10
			34	14
			84	16
			93	24
				29
				32
				100
				103
				105
				106
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				113
				127
				128
				130
				131

Table I.17 Pages including keywords Multi 5B Elevbok, Gyldendal

C.14. Multi 6A Elevbok, Gyldendal

	Argumentere	Begrunne	Forklare	Vise
Page		13	2	5
		48	5	7
		126	61	10
			62	13
			66	17
			80	
			113	

Table I.18 Pages including keywords Multi 6A Elevbok, Gyldendal

C.15. Multi 6B Elevbok, Gyldendal

	Argumentere	Begrunne	Forklare	Vise
Page		26	2	7
		38	5	10
		43	18	31
			32	36
			56	44
			71	49
			91	53
			105	75
			116	76
				77
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Table I.19 Pages including keywords Multi 6B Elevbok, Gyldendal