On stiffness and damping of vibro-impact dynamics of backlash

Michael Ruderman

***Abstract*— We consider the stiffness and damping properties of the vibro-impact in a backlash pair. Opposed to the existing and mostly used models of the backlash, we address the problem of contact and separation, and the associated force propagation within a mechanical pair, from a viewpoint of vibroimpact dynamics. We discuss the impact forces with the coefficient of restitution as a principal factor which shapes the transient backlash response. We show that a common approach to modeling the backlash by means of a dead-zone in a restoring force is unsuitable for correctly capturing the mechanical impact. We exemplary demonstrate a qualitative accord between an experimental backlash response and the postulated modeling approach. Backlash related energy losses of the vibroimpact damping are also addressed in brief.**

I. INTRODUCING REMARKS AND PROBLEM DESCRIPTION

Backlash effect, known also as a mechanical play in which the displacement of one mechanical part produces an equal displacement of another mechanical part first after taking up a defined clearance in the direction of drive, has already been in focus of earlier studies, i.e. [1], on stability of the closed-cycle (in the modern terminology *closed-loop*) control systems. Since there, the research (and the correspondingly published literature) in the mechanical engineering and machines, equally as in the system and control theory, have time and again addressed the backlash phenomenon. From a viewpoint of the principal mechanical structure, the backlash pair is nothing but a stiff arrangement of two interconnected masses, cf. Fig. 1, where a parameterizable clearance (with the size denoted by 2β) allows for a transiently decoupled motion in the relative $(x_1, \dot{x}_1)$ and $(x_2, \dot{x}_2)$ coordinates. Often, one of the masses is also associated with an external excitation force u, so that the m_1 inertial body is sometimes referred to as the driving and the m_2 inertial body as the driven part; but of course that is a matter of definition only. No need to say in details that the forces and relative displacements are in the generalized coordinates, so that the translational and rotational mechanisms can be understood in a same manner. However, we will keep in mind the schematic structure as in Fig. 1, while the most common applications are yet associated with the rotary joints, couplings, and gearing, just like an example that we will show qualitatively by the end of the paper.

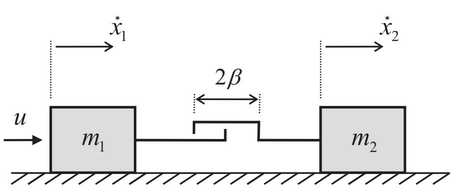

Fig. 1. Schematic representation of a backlash pair.

Novel aspects in the backlash modeling and, especially, in the analysis of a provoked dynamic response of the entire system with backlash came up from quite different perspectives and methodologies. A kinematic backlash pair, in the most simple case, was thought as a (static) input-output nonlinearity, for which a standard describing-function analysis (see e.g. [2], [3] for basics of describing functions) was often performed. This allowed ad-hoc approximating the backlash behavior within the loop of a dynamic subsystem. An associated (inherent) disadvantage is to omit the dynamic behavior of the backlash itself and the force propagation through it. A more advanced study of the so-called dynamic backlash, with explicit consideration of two colliding bodies and associated energy, correspondingly power, losses in the spectral distribution, was proposed in [4] for the describing function approach. Also an explicit consideration of the colliding pair of the backlash has gained popularity by allowing for a more physical interpretation of the impact, correspondingly engagement. Such approaches were based on the Newton's collision law e.g. [5] and hybrid dynamics modeling with a variable structure e.g. [6]. Although those approaches agree with the main laws of conservation of momentum, they often reveal difficulties with an associated jump mapping, corresponding with switchings of the variable structure dynamics. In such cases, the displacement rate undergoes an instantaneous jump at each backlash contact, while the forward and backward couplings of both impacting rigid-bodies are governed by a discrete impulse (reset) map. Also the structural damping aspects remained under developed. Those issues of an impulse-type switching at impact were recognized already in [7] and considered to be less physically justified, once the development and then the reduction of contact forces at impact and, correspondingly, separation are considered. Even when assuming a suitable switching model, correspondingly hybrid model, of the relative motion with backlash, an appropriate force transmission upon the impact and separation remain the most crucial for capturing the behavior of a backlash pair. This becomes

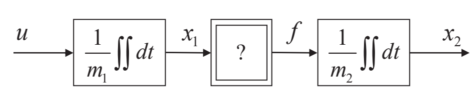

Fig. 2. One-way signals propagation through the backlash pair.

M Ruderman is with the Faculty of Engineering and Science, University of Agder (UiA). Postal address: P.B. 422, Kristiansand,4604, Norway. Email: michael.ruderman@uia.no

particularly evident when looking at the simplified signals flow as depicted in Fig. 2. Note that here, only a one-directional force propagation is shown, and the relative motion of both inertial bodies is simplified to solely the double-integrator case. Although any disturbances, like for example the kinetic friction and others, are omitted in Fig. 2, the question of properly capturing the state-varying transmission force f remains non-trivial. Here, the *state-varying* refers to the instantaneous state (in other words operational mode) of the backlash, which can be either within the *gap*, or in *engagement*, or at the *impact* or *separation*, cf. [8]. Moreover, the uncertainties in modeling the *restitution coefficient*, from the one side, and the numerical issues of a jump mapping, from the other side, can lead to a backlash pair violates the contact constraints. All these by-effects can render the mentioned hybrid modeling as less robust and reliable for practical engineering studies of the systems with backlash.

Another, and actually more frequently chosen, way to capture the backlash behavior in a coupled mechanical pair with clearance [9] is based on the dead-zone nonlinearity, while that case no damping at all is taken into account in association with the backlash itself. It has been, however, recognized that if one incorporates a nonzero viscous damping of the (more specifically) shaft of a gear system [10], or of the (more generally) *vibroimpact* system, the contact forces experience some spurious transitions. Note that these spurious step-wise force transitions, during the engagement and disengagement of a mechanical pair, were already analyzed in [7]. As an evasive way, an extension of the dead-zone model was proposed in [10], while allowing for a linear damping in the shaft and introducing an internal backlash state with the case-difference dynamics. TThis approach, which experienced certain dissemination in the several afterwards following researches, does not however take into account the vibroimpact damping. As a result, the introduced dynamic state of backlash becomes redundant and superfluous for the shafts with high stiffness and low damping coefficients. At the same time, a dead-zone based modeling does not require discontinuous jumps and can be, therefore, favorable in several cases. In the recent work, we will demonstrate the operational conditions, where the dead-zone based approximation of a backlash pair will, however, fail to capture the system dynamics.

In view of the above elucidated issues, it is worth noting that the contact forces of backlash, with the correspondingly varying stiffness and damping, belong to the not entirely solved research questions. One can notice that an associated modeling and identification are essential for various engineering applications, for an illustrative example see e.g. [11]. The rest of this note is as follows. In section II, we will recall the dead-zone based approximation of a backlash pair and analyze why it is less suitable for capturing the backlash transitions. In section III, the vibroimpact dynamics of a backlash is introduced based on the seminal work [7]. In section IV, we will (i) briefly address the energy losses associated with backlash and (ii) exemplary compare the measured [12] and modeled backlash response.

II. DEAD-ZONE BASED BACKLASH APPROXIMATION

An often used modeling approach for describing a two-inertia system, which is interconnected with a clearance, is utilizing the dead-zone function

$$\Upsilon(z)=\begin{cases} z-\beta, & z\geq\beta,\\ 0, & |z|<\beta,\\ z+\beta, & z\leq-\beta. \end{cases} \quad (1)$$

The function (1) is then used prior to a high-valued stiffness K, thus capturing the force transmission (through backlash) between both moving masses. When assuming, additionally, some linear damping $b_1\dot{x}_1$ and $b_2\dot{x}_2$ of the first and second inertial term, correspondingly, the overall dynamics of a mechanical backlash-pair, cf. Fig. 1, results to a model shown below as a block diagram. Note that the transfer function

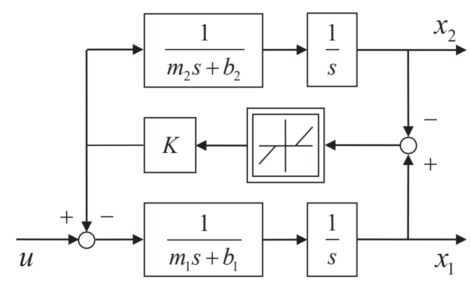

Fig. 3. Block diagram of a two-inertia system with backlash, which is captured by the dead-zone nonlinearity in feedback.

blocks, as these used in Fig. 3, assume the dynamic model written in the Laplace domain (i.e. with s to be the complex variable), while the dead-zone nonlinearity (1) is acting in a feedback channel. Here we recall that a single-valued static nonlinearity can be always isolated into a single feedback loop, this way yielding a classical Lur'e type system, e.g. [2], [3]. This backlash pair model (Fig. 3) became particularly popular in the motion control communities due to its simplicity and ease of representation and analysis in both, the time and frequency domains. Apart from an uncertain, and mostly high-valued, stiffness K which can cause difficulties of a numerical implementation, the drawbacks of this model, i.e. of (1) with $z=x_1-x_2$, become particularly visible in the relative (x_1,x_2) and (z,f) coordinates. Here we recall that apart from the stiffness force $K\cdot\Upsilon(z)$, the overall contact force f is often including an additional linear damping term $\sim\dot{z}$; that case is however not shown in Fig. 3.

An exemplary backlash map, in the relative (x_1,x_2)-coordinates, is shown in Fig. 4. The response of the dead-zone based model is plotted in (a), and an excerpt from the experimental measurements [12] is plotted in (b). One can recognize an appearance of the spurious transient waves in the shape of the modeled backlash map. On the contrary, the measured backlash is heavily damped upon few oscillating waves. The spurious oscillations of the backlash state z are also exemplary shown in Fig. 5. A typical response of the model (i.e. Fig. 3) is depicted versus that one of the vibroimpact model (i.e. f as eq. (5)), both using the same slow periodic excitation $u(t)$. If the frequency of u-excitation

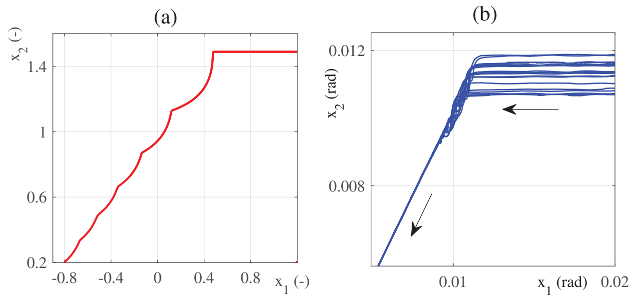

Fig. 4. Dead-zone based modeling and measured [12] backlash response.

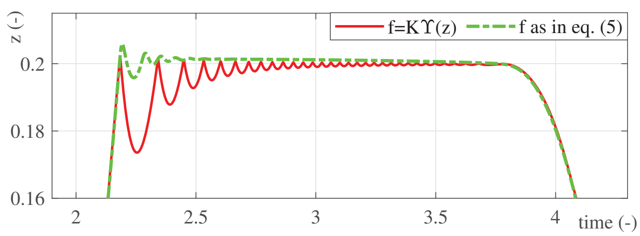

Fig. 5. Typical response of the backlash state z when using the dead-zone based model with $f = K\Upsilon(z)$ and vibroimpact model with f as eq. (5).

increases, the backlash pattern of the dead-zone based model becomes even more oscillating and does not converge to the coupling state, correspondingly engagement mode, of the backlash. In the worth case, the modeled backlash becomes chaotically oscillating between both $\pm\beta$ boundaries and the state trajectories become largely unpredictable.

III. Vibroimpact Dynamics of Backlash System

Throughout the rest of the paper we will further deal with a mechanical system of two rigid bodies, with the lumped inertial masses m_1 and m_2, cf. Fig. 1. The system dynamic equations, coupled through the contact force f, are given by

$$m_1\ddot{x}_1 = u_1 - b_1\dot{x}_1 - f, \qquad (2)$$

$$m_2\ddot{x}_2 = u_2 - b_2\dot{x}_2 + f. \qquad (3)$$

Both rigid bodies, moving in the generalized coordinates x_1 and x_2, are driven by the generalized forces u_1 and u_2 and allow for an additional viscous damping with the coefficients b_1 and b_2. In the following, for the sake of simplicity and without losing generality, we assume $u_2 = 0$, thus meaning the first body of a backlash pair is the driving and the second is the driven one. The dead-zone, as a clearance between both bodies with the size 2β, is captured (in a same and usual way) by the piecewise smooth nonlinear function

$$z = \begin{cases} x_1 - x_2 - \beta, & (x_1 - x_2) \geq \beta, \\ 0, & |x_1 - x_2| < \beta, \\ x_1 - x_2 + \beta, & (x_1 - x_2) \leq -\beta. \end{cases} \qquad (4)$$

For the vibroimpact system can deal with not only ideal elastic contacts and, therefore, allow also for structural damping, Hunt and Crossley [7] proposed a nonlinear damping term. This one is well in accord with the restitution coefficient e which is driven by $e = 1 - \alpha\dot{z}_i$. Here we recall that the vibroimpact dynamics assumes $\dot{z}_o = -e\dot{z}_i$, where the relative displacement rates are denoted by the subscripts $\{i, o\}$ – for the before ("in") and, correspondingly, after ("out") collision. The restitution coefficient itself is usually $0 \leq e \leq 1$, with the left and right boundaries representing an absolute plastic and absolute elastic impact, respectively. According to [7], the overall contact force can be captured by

$$f = \lambda z^n \dot{z} + k z^n, \qquad (5)$$

while through choosing $\lambda = 1.5\alpha k$ one can approach the above introduced coefficient of restitution. For the construction materials (like e.g. steel or bronze) α will have a relatively small value, thus keeping impact in a predominantly elastic region, that for a limited range of the vibroimpact velocities. It is also worth noting that (5) reveals a certain physical sense, since the structural damping increases with the depth of penetration, i.e. with the growing $|z|$. The contact force is namely zero immediately at the instant of an engagement or disengagement and, then, evolves continuously upon the contact. Note that this is independent of the velocity magnitude $|\dot{z}_i|$ at impact, – the feature which cannot be provided by a linear damping of the vibroimpact.

It is obvious that the transient dynamics of vibroimpact and, thus, the contact response of a backlash pair depend on the choice of free parameters in (5). Here it is worth recalling that for $n = 3/2$ it is consistent with the Herzian theory of contacting spheres under static conditions, while for $n = 1$ it captures the most simple case of two flat surfaces under impacting, cf. [7]. While the stiffness k will (naturally) determine the oscillating transient upon the contact, the restitution shaping factor α is predominant for the damping and, correspondingly, energy losses during the penetration, i.e. for $|z| > \beta$, see Fig. 6. Here both, the backlash contact force f and the penetration state $|z| > \beta$, are exemplary shown and plotted as unitless, while the k-variation is by the factor 5 and α-variation is by the factor 10, these for the sake of a better comparison.

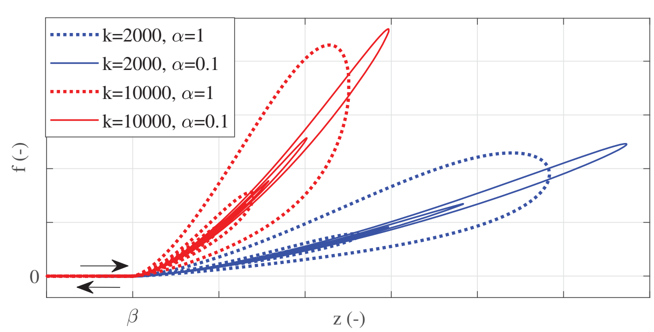

Fig. 6. Exemplary map of the backlash contact force f over the penetration state $|z| > \beta$, for different stiffness k and restitution shaping factor α.

IV. Contact Energy Losses and Experimental Example

The above discussion and contact modeling, which follows [7], allows for assuming the backlash force (5) which captures both, the nonlinear stiffness and structural damping of the impact and engagement modes. In the following, we will briefly discuss the energy losses which are associated with backlash dynamics. Afterwards, an experimental example of the measured backlash response is qualitatively compared with the modeling, but without an explicit identification of the free parameters.

A. Energy losses of backlash

Often, a mechanical pair $\{m_1, m_2\}$ is subject to the transient or even steady oscillations, so that the dynamic state $z(t)$ of the backlash is under a periodic impact. This corresponds to a classical vibroimpact dynamics, where the oscillation cycles are also associated with the damping energy losses. In engineering practice, it is well known that the undesired clearances in the multi-body structures can reduce the nominal eigenfrequency to some extent, while the overall system damping increases. The structural dissipation of a kinetic energy of the moving pair $\{m_1, m_2\}$ is mostly not of a pure viscoelastic nature, and is rather of a hysteresis-type, when being expressed in the relative (z, f) coordinates.

Since the energy losses, denoted by ΔE, upon one full and symmetric backlash cycle Γ are twice the area of the corresponding closed hysteresis loop, cf. Fig. 6, one can find the analytic solution for the dissipated energy. Integrating (5), with respect to z, and taking it twice one obtains

$$\Delta E = 3\alpha k \oint_{\Gamma} z^n \dot{z}\, \mathrm{d}z, \qquad (6)$$

for which evaluation the knowledge of z and its rate are only required. For the bounded system forces and, therefore, relative velocities $|\dot{x}_1|, |\dot{x}_2| < \mathrm{const}$ and penetrations $|z| < \mathrm{const}$, one can estimate an upper bound $\widehat{\Delta E} \approx \mathrm{const}$ for the cyclic energy dissipation through the vibroimpact. Then, for the known oscillation frequency ω_0, it is possible to predict the energy losses since these are proportional to $\omega_0 \widehat{\Delta E}$.

B. Experimental example

For qualitatively evaluating the above modeling, we compare the numerical simulation of (2)-(5), made for the varying α, with the measured transient backlash response, i.e. in terms of the $z(t)$ time series. The measured (see [12] for details) z-response is shown in Fig. 7 (a). Its modeled counterpart is shown in Fig. 7 (b) for different α values. Note that since no explicit parameters identification is performed, the simulated z value is without units. This is mainly due to the fact of double-integrators, cf. (2), (3), which accurate prediction is impeded by inherent integration errors (u is the only used input) and weakly known initial conditions of the backlash state. At the same time, one can recognize that an appropriately chosen α-factor, which primarily determines the restitution coefficient, allows a good agreement with the measurement in the form of the transient oscillations of $z(t)$. One can also recognize that further decreasing of α (see green line in Fig. 7 (b)), results in a more elastic impact, with multiple 'bouncing ball'-type oscillations. When the α-factor is increased (see red line in Fig. 7 (b)), the impact is close to be 'critically damped' and no transient z-oscillations occur.

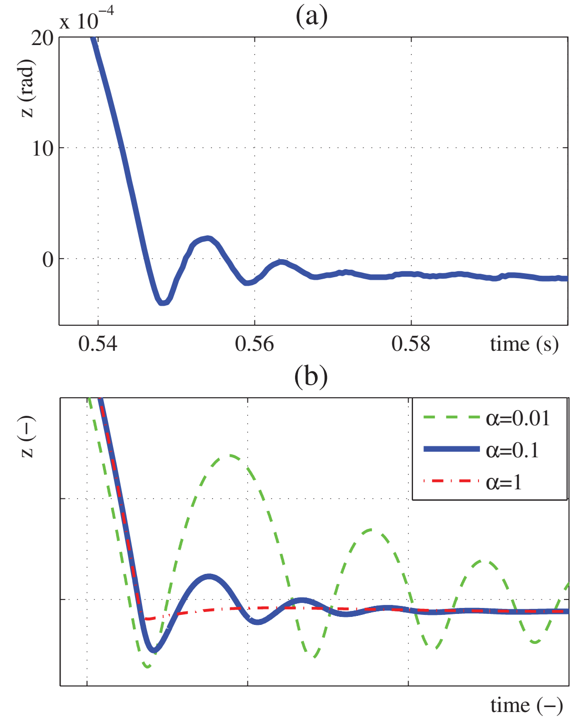

Fig. 7. Fragments from the experimentally measured [12] backlash response z in (a), versus the computed z for varying α-factor in (b)

Acknowledgment

This work has received funding from the European Union Horizon 2020 research and innovation programme H2020-MSCA-RISE-2016 under the grant agreement No 734832.

References

[1] A. Tustin, "The effects of backlash and of speed-dependent friction on the stability of closed-cycle control systems," *Journal of the Institution of Electrical Engineers-Part IIA: Automatic Regulators and Servo Mechanisms*, vol. 94, no. 1, pp. 143–151, 1947.

[2] J. Slotine and W. Li, *Applied nonlinear control*. Prentice Hall, 1991.

[3] H. Khalil, *Nonlinear Systems*, 3rd ed. Prentice Hall, 2002.

[4] R. Barbosa and J. Machado, "Describing function analysis of systems with impacts and backlash," *Nonlinear Dynamics*, vol. 29, pp. 235–250, 2002.

[5] M. Mata-Jimenez and B. Brogliato, "Analysis of pd and nonlinear control of mechanical systems with dynamic backlash," *Journal of Vibration and Control*, vol. 9, pp. 119–155, 2003.

[6] P. Rostalski, T. Besselmann, M. Baric, F. Van Belzen, and M. Morari, "A hybrid approach to modelling, control and state estimation of mechanical systems with backlash," *International Journal of Control*, vol. 80, no. 11, pp. 1729–1740, 2007.

[7] K. Hunt and F. Crossley, "Coefficient of restitution interpreted as damping in vibroimpact," *J. of Applied Mechanics*, vol. 42, 1975.

[8] M. Ruderman and L. Fridman, "Model-free sliding-mode-based detection and estimation of backlash in drives with single encoder," *IEEE Tran. on Control Systems Technology*, vol. 29, pp. 812–817, 2021.

[9] T. Goodman, "Dynamic effects of backlash," *Machine Design*, vol. 35, no. 12, pp. 150–157, 1963.

[10] M. Nordin, J. Galic, and P.-O. Gutman, "New models for backlash and gear play," *International journal of adaptive control and signal processing*, vol. 11, no. 1, pp. 49–63, 1997.

[11] A. Lagerberg and B. Egardt, "Backlash estimation with application to automotive powertrains," *IEEE Transactions on Control Systems Technology*, vol. 15, no. 3, pp. 483–493, 2007.

[12] M. Ruderman, S. Yamada, and H. Fujimoto, "Backlash identification in two-mass systems by delayed relay feedback," *Journal of Dynamic Systems, Measurement, and Control*, vol. 141, no. 6, p. 061007, 2019.