Adaptive Control Design for Underactuated Cranes with Guaranteed Transient Performance: Theoretical Design and Experimental Verification

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***Abstract*—For antiswing control of underactuated cranes, how to guarantee the converging speed of cranes through control design is essential but still remains unsolved. In this paper, the adaptive antiswing control for underactuated gantry cranes with guaranteed transient performance under unmodeled dynamics and external disturbances is investigated. To solve this problem, a set of filters are proposed to make the backstepping technique applicable for the control of crane systems. Then through variable transformation the position error and swing angel could be guaranteed converging to the origin with a given exponential speed. Hardware experiments are conducted to show that the proposed scheme achieves better control performance over existing methods, and it is illustrated that the proposed control scheme possesses strong robustness to unmodeled uncertainties and external disturbances.**

***Index Terms*—Adaptive control; Robust Control; Underactuated cranes; Transient performance.**

I. Introduction

The crane systems, including tower cranes, gantry cranes, bridge cranes etc. which could be used in various industrial applications[1], [2], [8], [9], are one of the most important mechanical systems in industry. The cranes belong to the family of underactuated mechanical systems, where the number of available actuator is less than the degree of freedom (DOF), such as nonholonomic mobile robots[25], underactuated planar robots[26], pendubot[27] and helicopters[29], etc. The gantry cranes are consisted of rails, motor-driven carts and rope suspended to the cart. The cart moves on the railway to transport the load to the desired position. Therefore the main control objective for gantry cranes is to design effective control schemes to achieve both position of the cart and antioscillation of the rope while admitting strong robustness to external disturbances[36]-[38]. The control of the cranes with guaranteed transient performance, which means the cart moves to target position and the swing of the rope is suppressed as soon as possible, is desirable in industrial application, since it will increase transportation efficiency and ensure the operations safety under various environments to avoid serious disasters such as collisions.

Many control schemes have been proposed for the control of underactuated crane systems based on various control techniques in literature, such as adaptive control, sliding-mode control, model predictive control. For example, in [2] an antiswing control scheme for underactuated gantry cranes is proposed by defining a sliding-like surface and based on which the control is designed to keep the error variables staying on the surface. In [3], an adaptive robust control scheme is proposed for tower cranes which simultaneously rotates and moves the cart by using sliding mode technique and adaptive model-reference control approach. In [13] a kind of neural network-based adaptive control method is proposed which can provide control for both actuated and unactuated state variables based on the original nonlinear ship-mounted crane dynamics without any linearizing operations. In [8] a control design method including path planning and tracking control is proposed for underactuated crane systems by combining theoretical analysis with empirical path planning methods. Online update law is introduced to guarantee that the controller is valid under different working conditions. In [17] an efficient control scheme which captures the movement of a three-dimensional overhead crane is presented. The control scheme keeps the crane system states stay on the manifold. In [21], a kind of anti-swing control method is proposed for 3-dimensional (3-D) underactuated overhead crane systems by partially feedback linearization. In [22] a kind of quasi-proportional integral derivative control method is proposed to control the underactuated double-pendulum crane systems. In [23], adaptive control for the cranes is investigated, where the update law is designed to achieve accurate identifications of unknown parameters and exact compensation of the gravity-related lumped term. An improved feedback controller with an integral term is proposed in [24] for 3-D tower cranes without linearization, which can achieve both antiswing and positioning control while being able to reduce steady errors under inaccurate friction compensation. In [18] an integral barrier Lyapunov function based control method is proposed for the underactuated crane systems to suppress the undesirable swing of the flexible crane system with boundary output constraints. In [19], the control framework is established by total energy shaping, and an additional term is proposed into the adaptive control law to prevent the trolley from running out of the permitted range for underactuated crane systems. A model predictive control scheme which guarantees swing of the rope is proposed in [20] for a class of 2-D overhead crane systems. In [11] a control system for rubber-tired gantry (RTG) cranes to track three actuated outputs and stabilize two unactuated outputs is proposed. In [12] the dynamic model of double-pendulum shipboard cranes is proposed and then a nonlinear antiswing feedback controller to achieve stable cargo transportation is provided.

From the above the literature reviews, it is fair to conclude that the position and anti-swing control of underactuated cranes, including tower cranes, overhead cranes and gantry cranes, have been extensive investigated and various control schemes have been proposed. However, how to guarantee that the position error and the swing angel converge to the origin with a pre-defined speed under parametric uncertainties and external disturbances, which is essential for industrial application, still remains unsolved. The control of nonlinear systems with guaranteed transient performance is a hot research topic and many results have been obtained. For example, in [39] robust adaptive control schemes for SISO strict feedback nonlinear systems are proposed which are capable of guaranteeing prescribed performance bounds by the stabilization of the unconstrained system through variable exchange. By following this idea,

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many similar schemes are proposed. In [6] adaptive backstepping control schemes for parametric strict feedback systems are proposed to accommodate actuator failures based on a prescribed performance bound. In [33] a PI control with adaptively adjusting gains is proposed for uncertain nonlinear systems in Brunovsky form to guarantee transient and steady-state performance. In [34], [35] the transient performance of nonlinear systems are guaranteed through a set of speed functions. However, none of these control schemes could be applied to the underactuated crane systems directly. This is mainly because the crane systems are a class of underactuated high-order nonlinear system in a non-strict-feedback form, hence backstepping control technique could not be applied directly.

In this paper, we aim to solve this problem by proposing a filter based control scheme for the non-strict-feedback crane system which renders the error dynamic system in a low-triangular form. Meanwhile through transient bound functions the control objective becomes stabilizing an unconstrained nonlinear system with backstepping control technique. The position error and swing angle will converge to the neighbor of the origin with a given exponential speed. Some parameters of the crane and the friction are not required to be known in the control design. Through constraint variable exchange, the swing angle will be guaranteed within an upper bound, therefore a common assumption that swing angle is less than a given constant is no longer needed.

With aforementioned features, the main contributions of this paper could be summarized as follows:

- As far as we are concerned, this is the first work which solves the adaptive control problem for underactuated crane systems with unknown parameters and external disturbances, meanwhile guaranteeing that the position error and the swing angel will converge to a ball of the origin whose radius could be arbitrarily small. Besides, the position error and the swing angel will converge with a given exponential speed.
- To make the backstepping control technique be applicable to the crane systems, a filter based control scheme is proposed for the non-strict-feedback system. Through transient bound functions, the control objective is equivalently transformed to stabilizing an unconstrained nonlinear system with backstepping control technique. Instead of offline experiment tests and data fitting to obtain the friction force, adaptive control is applied to estimate the unknown parameters online.

A series of hardware experiments is used to verify the performance of our method, which documents that it can achieve better performance than linear quadratic regulator[31] and sliding mode control[30], and it is robust against parameter uncertainties, initial swing perturbation, and external disturbances during the transferring stage, which illustrates its promising application prospect.

II. Problem Formulation

A. Underactuated Crane Model

We consider the underactauted crane systems where the following equations could be obtained:

$$(M+m_p)\ddot{x}+m_pl\ddot{\theta}\cos\theta-m_pl\dot{\theta}\sin\theta=u+F_r+d(t) \quad (1)$$

$$m_pl^2\ddot{\theta}+m_pl\cos\theta\ddot{x}+m_pgl\sin\theta=0 \quad (2)$$

where M denotes the weight of the crane system, m_p denotes the weight of the load. l denotes the length of the rope, and g is the gravitational constant, $g=9.8m/s^2$. $x(t)$ and $\theta(t)$ represent the position of the cart and the swing angle of the rope, respectively. $u(t)$ is the control input, F_r denotes friction and viscous damping forces which is approximately modeled as

$$F_r=-f_1\tanh(\dot{x}/\upsilon)+f_2|\dot{x}|\dot{x} \quad (3)$$

where f_1, f_2 and υ are the friction parameters. The aim of crane control is to make the cart moves to the desired potion meanwhile suppress the swing of the rope at initial stage. To proceed, the following assumptions are considered.

Assumption 1: The length of the rope l is available for control design.

Assumption 2: The measurement noise of the sensor is not considered.

Assumption 3: The initial value of swing angel satisfies $\theta(0)\in(-\pi/2,\pi/2)$.

Remark 1: *Assumption 1 is needed since the dynamics of $\dot{x}$ and $\dot{\theta}$ should be decoupled through a variable exchange due to the reason that it is an underactuated system, and it is commonly required[2], [19]. In practice, the length of the rope easily could be obtained since in the model (1)-(2) l is a constant. The other parameters include M, m_p, f_1, f_2 and υ are not required to be known.*

Remark 2: *Note that it is commonly assumed that swing angle $\theta(t)$ satisfies $\theta(t)\in(-\pi/2,\pi/2)$ in almost all existing results, for example, [2], [19], [20], [23], [24] and many references therein. In this paper, through variable constraint control, swing angle $\theta(t)$ will be guaranteed within the interval above if $\theta(0)\in(-\pi/2,\pi/2)$. Therefore such an assumption is no longer needed.*

To facilitate the control design and stability analysis, some variable transformations are made firstly. Equations (2) can be organized by dividing both sides with m_pl as follows:

$$\ddot{x}=-g\tan\theta-\frac{l\ddot{\theta}}{\cos\theta} \quad (4)$$

Then substitute (4) into (1) and make some arrangements to obtain

$$\begin{aligned}&-\frac{(M+m_p\sin^2\theta)l}{\cos\theta}\ddot{\theta}-m_pl\dot{\theta}^2\sin\theta\\&\quad-(M+m_p)g\tan\theta=u+F_r+d(t)\end{aligned} \quad (5)$$

where $d(t)=-\frac{d(t)\cos\theta}{M+m_p\sin^2\theta}$. Equation (5) could be further rearranged as

$$\begin{aligned}\ddot{\theta}=&-\frac{m_p\dot{\theta}^2\sin\theta\cos\theta}{M+m_p\sin^2\theta}-\frac{(M+m_p)g\sin\theta}{(M+m_p\sin^2\theta)l}\\&-\frac{\cos\theta}{(M+m_p\sin^2\theta)l}\Big(u+F_r+d(t)\Big)\end{aligned} \quad (6)$$

By letting $x_1=x$, $x_2=\dot{x}$, $\theta_1=\theta$, $\theta_2=\dot{\theta}$, then the original crane dynamics can be rearranged into

$$\begin{aligned}\dot{x}_1&=x_2\\\dot{x}_2&=-g\tan\theta_1-\frac{l}{\cos\theta_1}\dot{\theta}_2\\\dot{\theta}_1&=\theta_2\\\dot{\theta}_2&=\varsigma_1u+\bar{f}(x_1,x_2,\theta_1,\theta_2)+d_1(t)\end{aligned} \quad (7)$$

where $\varsigma_1=-\frac{\cos\theta_1}{(M+m_p\sin^2\theta_1)l}$, $\bar{f}=-\frac{m_p\theta_2^2\sin\theta_1\cos\theta_1}{M+m_p\sin^2\theta_1}-\frac{(M+m_p)g\sin\theta_1}{(M+m_p\sin^2\theta_1)l}-\frac{\cos\theta_1}{(M+m_p\sin^2\theta_1)l}F_r$, $d_1(t)=-\frac{\cos\theta_1 d(t)}{(M+m_p\sin^2\theta_1)l}$. To eliminate $\dot{\theta}_2$ in the dynamic of x_2 to facilitate the control design, let a new variable η_1 defined as

$$\eta_1(t)=x_1-x_d+\mu(\theta_1) \quad (8)$$

where $\mu(\theta_1)$ is a function of θ_1 to be introduced later[2]. Then the time derivative of $\eta_1(t)$ is

$$\dot{\eta}_1 = x_2 + \frac{\partial \mu}{\partial \theta_1}\theta_2 \tag{9}$$

and

$$\ddot{\eta}_1 = -g\tan\theta_1 - \frac{l}{\cos\theta_1}\dot{\theta}_2 + \frac{\partial \mu}{\partial \theta_1}\dot{\theta}_2 + \frac{\partial^2 \mu}{\partial \theta_1^2}\theta_2^2 \tag{10}$$

From (10) we know if let $\frac{\partial \mu}{\partial \theta_1}\dot{\theta}_2 = \frac{l}{\cos\theta_1}\dot{\theta}_2$, then $\dot{\theta}_2$ is eliminated. Therefore we obtain

$$\mu(\theta_1) = l\ln(\frac{1}{\cos\theta_1} + \tan\theta_1) \tag{11}$$

The closed-loop system is transformed as

$$\begin{aligned} \dot{\eta}_1 &= x_2 + \frac{l}{\cos\theta_1}\theta_2 \\ \dot{x}_2 &= -g\tan\theta_1 + \frac{l\tan\theta_1}{\cos\theta_1}\theta_2^2 \\ \dot{\theta}_1 &= \theta_2 \\ \dot{\theta}_2 &= \varsigma_1 u + \bar{f}(x_1, x_2, \theta_1, \theta_2) + d_1(t) \end{aligned} \tag{12}$$

Let

$$\begin{aligned} \eta_2 &= x_2 + \frac{l}{\cos\theta_1}\theta_2 \\ x_3 &= -g\tan\theta_1 \\ x_4 &= -\frac{g}{\cos^2\theta_1}\theta_2 \end{aligned} \tag{13}$$

and taking time-derivative of η_1, η_2, x_3 and x_4, it yields

$$\begin{aligned} \dot{\eta}_1 &= \eta_2 \\ \dot{\eta}_2 &= x_3 - \frac{l x_3 x_4^2}{(g^2 + x_3^2)^{1.5}} \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= \varsigma u + \vartheta^T(t) f + \psi + \bar{d}(t) \end{aligned} \tag{14}$$

where $\psi = 2\theta_2^2\tan\theta_1$, $\varsigma = \frac{g}{(M+m_p\sin^2\theta_1)l\cos\theta_1}$, $\bar{d}(t) = \frac{g d_1(t)}{(M+m_p\sin^2\theta_1)l\cos\theta_1}$, $\vartheta(t) = [\frac{m_p}{M+m_p\sin^2\theta_1}, \frac{M+m_p}{(M+m_p\sin^2\theta_1)l}, \frac{-f_1\tanh(\dot{x}/\varsigma)}{(M+m_p\sin^2\theta_1)l}, \frac{f_2}{(M+m_p\sin^2\theta_1)l}]^T$, and $f = [\frac{g\theta_2^2\sin\theta_1}{\cos\theta_1}, \frac{g^2\sin\theta_1}{\cos^2\theta_1}, \frac{g}{\cos\theta_1}, \frac{g|\dot{x}|\dot{x}}{\cos\theta_1}]^T$, It is clear that (14) is not a strict-feedback system, thus the control schemes which are proposed to guarantee the transient performance of strict-feedback systems[6], [14], [15], [16], [33], [34], [39] could not be applied for (14). Also the backstepping control scheme could not be used directly.

The control objective of this paper is to design an adaptive control law for u such that position stabilization error $\tilde{x}_1 = x_1 - x_d$ and swing angel θ_1 converge to an arbitrarily small ball with a given exponential converging rate, i.e.,

$$\begin{aligned} |\tilde{x}_1(t)| &\le (x_0 - \beta)e^{-\alpha t} + \beta \\ |\theta_1(t)| &\le (\theta_0 - \beta)e^{-\alpha t} + \beta \end{aligned} \tag{15}$$

where $\alpha > 0$ is a given converging rate and $\beta > 0$ could be arbitrarily small, $x_0 > \max\{\beta, x_1(0)\}$ and $\theta_0 > \max\{\beta, \theta_1(0)\}$ are positive constants. From (12) it can be observed that the system is un-defined at $\theta_1 = \pm\frac{\pi}{2}$. To avoid this, θ_0 is set as $|\theta_0| = \frac{\pi}{2} - \sigma_1$ with σ_1 being a small positive constant.

To proceed, firstly a robust adaptive law for u will be designed such that η_1 and x_3 will converge at an exponential speed. Secondly it will be proved that $\tilde{x}_1$ and θ_1 will converge satisfying (15) by properly designing the control parameters.

Remark 3: *As shown in (14), the crane system is underactuated, since η_1 and x_3 should be stabilized at the same time while the only control input is u. It can be also observed that (14) is not a strict-feedback nonlinear systems. The control of nonlinear systems with guaranteed transient performance is a hot research topic and many results have been obtained. However, all existing control schemes could not be applied to the underactuated crane ystems directly. This is mainly because that the crane systems are a class of underactuated high-order nonlinear systems in non-strict-feedback form, hence backstepping control technique could not be applied directly. In this paper, we will solve this problem by proposing a filter-based backstepping control scheme, which allows the backstepping control to be applied.*

III. Control design

One of the major difficulties of achieving (15) is that (14) is not a strict-feedback system, therefore control design methods such as backstepping [5] and control design skills which can guarantee transient performance such as prescribed performance bounds [6] could not be directly applied for (14). To proceed, we modify (14) as

$$\begin{aligned} \dot{\eta}_1 &= \eta_2 \\ \dot{\eta}_2 &= k_\eta x_3 \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= \varsigma u + \vartheta^T(t) f + \psi + \bar{d}(t) \end{aligned} \tag{16}$$

where $k_\eta = 1 - \frac{l x_4^2}{(g^2+x_3^2)^{1.5}}$ is a control gain, which means control law u also needs to guarantee

$$|x_4| \le \varpi_0 \tag{17}$$

where $\varpi_0 = \sqrt{\frac{g^3}{l}}$ to make $k_\eta > 0$. To accomplish (15) and (17), firstly introduce a smooth performance function $\kappa(t)$ as follows

$$\kappa(t) = (\kappa_0 - \kappa_\infty)e^{-at} + \kappa_\infty,$$

where $a > 0$ and $\kappa_0 \ge \kappa_\infty > 0$ which will be designed later. Now define new variables

$$\begin{aligned} e_1 &= S_1^{-1}(\frac{\eta_1}{\kappa(t)}),\ e_2 = \eta_2 \\ e_3 &= S_2^{-1}(\frac{x_3}{\kappa(t)}),\ e_4 = S_3^{-1}(x_4) \end{aligned} \tag{18}$$

where

$$S_i(x) = \frac{b_i e^{(x+v_i)} - a_i e^{-(x+v_i)}}{e^{(x+v_i)} + e^{-(x+v_i)}},\ i = 1, 2, 3. \tag{19}$$

with a_i and b_i being positive constants to be designed, $v_i = \frac{1}{2}\ln\frac{a_i}{b_i}$. $S_i(x)$ is a strict monotonic function with

$$(i) \quad -a_i < S_i(x) < b_i \tag{20}$$

$$\begin{aligned} (ii) \quad & \lim_{\gamma\to+\infty} S_i(x) = b_i, \\ & \lim_{x\to-\infty} S_i(x) = -a_i \end{aligned} \tag{21}$$

Taking time-derivative of e_i, $i = 1, ..., 4$, yields

$$\begin{aligned} \dot{e}_1 &= \gamma_1 e_2 - \lambda_1 e_1 \\ \dot{e}_2 &= \gamma_2 e_3 \\ \dot{e}_3 &= \gamma_3 e_4 - \lambda_2 e_3 \\ \dot{e}_4 &= \gamma_4 u + \lambda_3\vartheta^T(t) f + \lambda_3\psi + \lambda_3\bar{d}(t) \end{aligned} \tag{22}$$

where $\gamma_1 = \frac{1}{2}\Big[\frac{1}{\eta_1+\alpha_1\kappa} - \frac{1}{\eta_1-\beta_1\kappa}\Big]$, $\lambda_1 = \gamma_1\dot{\kappa}\frac{S_1(e_1)}{e_1}$, $\gamma_2 = k_\eta\kappa\frac{S_2(e_3)}{e_3}$, $\gamma_3 = \frac{1}{2}\frac{S_3(e_4)}{e_4}\Big[\frac{1}{x_3+\alpha_2\kappa} - \frac{1}{x_3-\beta_2\kappa}\Big]$, $\lambda_2 = \gamma_3\dot{\kappa}\frac{S_2(e_3)}{e_3}$, $\lambda_3 = \frac{1}{2}\Big[\frac{1}{x_4+\alpha_3} - \frac{1}{x_4-\beta_3}\Big]$, $\gamma_4 = \lambda_3\varsigma$. Note that (22) is still not a strict feedback system due to existence of γ_1, γ_2 and γ_3. Therefore γ_1, γ_2 and γ_3 should be tackled to facilitate the application of backstepping technique. It is easy to check that $\gamma_1 \neq 0$, $\gamma_2 \neq 0$ and $\gamma_3 \neq 0$ if all signals in the closed-loop system are bounded.

Now we are going to propose a set of filters to handle γ_i. Let α_i, $i = 1, 2, 3$ be virtual control for e_{i+1} and $\alpha_{i,f}$ is the filtered signal of $\frac{\alpha_i}{\gamma_i}$

$$\epsilon_i\dot{\alpha}_{i,f} + \alpha_{i,f} = \frac{\alpha_i}{\gamma_i} \tag{23}$$

where ϵ_i is a small positive constant to be designed. Furthermore, define

$$z_j = e_j - \alpha_{j-1,f} \tag{24}$$

where $j = 1, ..., 4$, $\alpha_{0,f} = 0$ and let y_i, $i = 1, 2, 3$, be new variables as

$$y_i = \alpha_{i,f} - \frac{\alpha_i}{\gamma_i} \tag{25}$$

Now the control design is finished step by step. Firstly let

$$V_z = \frac{1}{2}\sum_{i=1}^{4} z_i^2 + \frac{1}{2}\sum_{k=1}^{3} y_k^2$$

and define a set

$$\Xi = \{(z_i, y_k) : V_z \leq p_1\} \tag{26}$$

where p_1 is a positive constant such that Ξ encloses the initial points $z_i(0)$ and $y_i(0)$.

Step 1: From (24) and (25) it can be obtained that

$$e_2 = z_2 + y_1 + \frac{\alpha_1}{\gamma_1} \tag{27}$$

Taking (27) into the first equation of (22) yields

$$\dot{z}_1 = \gamma_1 z_2 + \gamma_1 y_1 + \alpha_1 - \lambda_1 z_1$$

Considered the first Lyapunov function

$$V_1 = \frac{1}{2}z_1^2 + \frac{1}{2}y_1^2$$

whose time-derivative is

$$\begin{aligned}\dot{V}_1 &= z_1(\gamma_1 z_2 + \gamma_1 y_1 + \alpha_1 - \lambda_1 z_1) + y_1\Big(-\frac{y_1}{\epsilon_1} + (\frac{\alpha_1}{\gamma_1})'\Big)\\ &\leq z_1\gamma_1 z_2 + \varepsilon_1 + \frac{z_1^2\gamma_1^2 y_1^2}{4\varepsilon_1} + z_1(\alpha_1 - \lambda_1 z_1)\\ &+ y_1\Big(-\frac{y_1}{\epsilon_1} + (\frac{\alpha_1}{\gamma_1})'\Big)\end{aligned}$$

The virtual controller α_1 is designed as

$$\alpha_1 = -k_1 z_1 + \lambda_1 z_1$$

where k_1 is a positive constant. Clearly $(\frac{\alpha_1}{\gamma_1})' = h_1(z_1, y_1, t)$ is bounded in Ξ and let ω_1 and ν_1 be upper bounds of h_1 and $z_1^2\gamma_1^2$ in Ξ, then

$$y_1(\frac{\alpha_1}{\gamma_1})' \leq \varepsilon_1 + \frac{\omega_1^2 y_1^2}{4\varepsilon_1}$$

where ε_1 is a small positive constant, which means

$$\dot{V}_1 \leq \gamma_1 z_1 z_2 - k_1 z_1^2 - \Big(\frac{1}{\epsilon_1} - \frac{\nu_1 + \omega_1^2}{4\varepsilon_1}\Big)y_1^2 + 2\varepsilon_1$$

Let $\epsilon_1 = \frac{4\varepsilon_1}{2(\nu_1+\omega_1^2)}$, then

$$\dot{V}_1 \leq \gamma_1 z_1 z_2 - k_1 z_1^2 - \frac{1}{2\epsilon_1}y_1^2 + 2\varepsilon_1$$

Step 2: Taking time-derivative of z_2 yields

$$\dot{z}_2 = \gamma_2 e_3 + \frac{\alpha_{1,f}}{\epsilon_1} - \frac{\alpha_1}{\gamma_1\epsilon_1} \tag{28}$$

Taking $e_3 = z_3 + y_2 + \frac{\alpha_2}{\gamma_2}$ into (28) yields

$$\dot{z}_2 = \gamma_2 z_3 + \gamma_2 y_2 + \alpha_2 + \frac{\alpha_{1,f}}{\epsilon_1} - \frac{\alpha_1}{\gamma_1\epsilon_1} \tag{29}$$

Considered the following Lyapunov function

$$V_2 = V_1 + \frac{1}{2}z_2^2 + \frac{1}{2}y_2^2 \tag{30}$$

whose time-derivative is

$$\begin{aligned}\dot{V}_2 &= -k_1 z_1^2 - \frac{1}{2\epsilon_1}y_1^2 + 2\varepsilon_1 + z_2(\gamma_1 z_1 + \gamma_2 z_3 + \gamma_2 y_2\\ &+ \alpha_2 + \frac{\alpha_{1,f}}{\epsilon_1} - \frac{\alpha_1}{\gamma_1\epsilon_1}) + y_2\Big(-\frac{y_2}{\epsilon_2} + (\frac{\alpha_2}{\gamma_2})'\Big)\\ &\leq -k_1 z_1^2 - \frac{1}{2\epsilon_1}y_1^2 + 2\varepsilon_1 + \gamma_2 z_2 z_3 + \varepsilon_2 + \frac{z_2^2\gamma_2^2 y_2^2}{4\varepsilon_2}\\ &+ z_2(\alpha_2 + \lambda_1 z_1 + \frac{\alpha_{1,f}}{\epsilon_1} - \frac{\alpha_1}{\gamma_1\epsilon_1})\\ &+ y_2\Big(-\frac{y_2}{\epsilon_2} + (\frac{\alpha_2}{\gamma_2})'\Big)\end{aligned}$$

The virtual controller α_2 is designed as

$$\alpha_2 = -k_2 z_2 - \gamma_1 z_1 - \frac{\alpha_{1,f}}{\epsilon_1} + \frac{\alpha_1}{\gamma_1\epsilon_1} \tag{31}$$

where k_2 is a positive constant. Clearly $(\frac{\alpha_2}{\gamma_2})' = h_2(z_i, y_i, 1/\epsilon_1, t)$, $i = 1, ..., 4$ is bounded in Ξ and let ω_2 and ν_2 be upper bounds of h_2 and $z_2^2\gamma_2^2$ in Ξ, then

$$y_2(\frac{\alpha_2}{\gamma_2})' \leq \varepsilon_2 + \frac{\omega_2^2 y_2^2}{4\varepsilon_2}$$

which means

$$\begin{aligned}\dot{V}_2 &\leq \gamma_2 z_2 z_3 - k_1 z_1^2 - k_2 z_2^2 - \frac{1}{2\epsilon_1}y_1^2 + 2\varepsilon_1 + 2\varepsilon_2\\ &- \Big(\frac{1}{\epsilon_2} - \frac{\nu_2 + \omega_2^2}{4\varepsilon_2}\Big)y_2^2\end{aligned}$$

Let $\epsilon_2 = \frac{4\varepsilon_2}{2(\nu_2+\omega_2^2)}$, then

$$\begin{aligned}\dot{V}_2 &\leq \gamma_2 z_2 z_3 - k_1 z_1^2 - k_2 z_2^2 - \frac{1}{2\epsilon_1}y_1^2 - \frac{1}{2\epsilon_2}y_2^2\\ &+ 2\varepsilon_1 + 2\varepsilon_2\end{aligned}$$

Step 3: Taking time-derivative of z_3 yields

$$\dot{z}_3 = \gamma_3 e_4 - \lambda_2 e_3 + \frac{\alpha_{2,f}}{\epsilon_2} - \frac{\alpha_2}{\gamma_2\epsilon_2} \tag{32}$$

Taking $e_4 = z_4 + y_3 + \frac{\alpha_3}{\gamma_3}$ into (35) yields

$$\dot{z}_3 = \gamma_3 z_4 + \gamma_3 y_3 + \alpha_3 - \lambda_2 e_3 + \frac{\alpha_{2,f}}{\epsilon_2} - \frac{\alpha_2}{\gamma_2\epsilon_2} \tag{33}$$

Considered the following Lyapunov function

$$V_3 = V_2 + \frac{1}{2}z_3^2 + \frac{1}{2}y_3^2$$

whose time-derivative is

$$\begin{aligned}\dot{V}_3 \le & -k_1 z_1^2 - k_2 z_2^2 - \frac{1}{2\epsilon_1}y_1^2 - \frac{1}{2\epsilon_2}y_2^2 + 2\varepsilon_1 + 2\varepsilon_2 + \varepsilon_3 \\ & + \frac{z_3^2\gamma_3^2 y_3^2}{4\varepsilon_3} + z_3(\alpha_3 - \lambda_2 e_3 + \lambda_2 z_2 + \frac{\alpha_{2,f}}{\epsilon_2} - \frac{\alpha_2}{\gamma_2\epsilon_2}) \\ & + y_3\Big(-\frac{y_3}{\epsilon_3} + (\frac{\alpha_3}{\gamma_3})'\Big) + \gamma_3 z_4 z_3\end{aligned}$$

The virtual controller α_3 is designed as

$$\alpha_3 = -k_3 z_3 - \gamma_2 z_2 + \lambda_2 e_3 - \frac{\alpha_{2,f}}{\epsilon_2} + \frac{\alpha_2}{\gamma_2\epsilon_2}$$

where k_3 is a positive constant. Clearly $(\frac{\alpha_3}{\gamma_3})' = h_3(z_i, y_i, 1/\epsilon_1, 1/\epsilon_2, t)$, $i = 1, ..., 4$ is bounded in Ξ and let ω_3 and ν_3 be upper bounds of h_3 and $z_3^2\gamma_3^2$ in Ξ, then

$$y_3(\frac{\alpha_3}{\gamma_3})' \le \varepsilon_3 + \frac{\omega_3^2 y_3^2}{4\varepsilon_3}$$

which means

$$\begin{aligned}\dot{V}_3 \le & \gamma_3 z_3 z_4 - k_1 z_1^2 - k_2 z_2^2 - k_3 z_3^2 - \frac{1}{2\epsilon_1}y_1^2 - \frac{1}{2\epsilon_2}y_2^2 + 2\varepsilon_1 \\ & + 2\varepsilon_2 + 2\varepsilon_3 - \Big(\frac{1}{\epsilon_3} - \frac{\nu_3 + \omega_3^2}{4\varepsilon_3}\Big)y_3^2 \end{aligned} \tag{34}$$

Let $\epsilon_3 = \frac{4\varepsilon_3}{2(\nu_3+\omega_3^2)}$, then

$$\begin{aligned}\dot{V}_3 \le & \gamma_3 z_3 z_4 - k_1 z_1^2 - k_2 z_2^2 - k_3 z_3^2 - \frac{1}{2\epsilon_1}y_1^2 - \frac{1}{2\epsilon_2}y_2^2 + 2\varepsilon_1 \\ & + 2\varepsilon_2 + 2\varepsilon_3 - \frac{1}{2\epsilon_3}y_3^2\end{aligned}$$

Step 4: Taking time-derivative of z_4 yields

$$\dot{z}_4 = \gamma_4 u + \lambda_3 \vartheta^T(t) f + \lambda_3\psi + \lambda_3\bar{d}(t) + \frac{\alpha_{3,f}}{\epsilon_3} - \frac{\alpha_3}{\gamma_3\epsilon_3} \tag{35}$$

Let

$$\theta = \sup_{t>0}\|\vartheta(t)\|, D = \sup_{t>0}\|\bar{d}(t)\|$$

and

$$V_4 = V_3 + \frac{1}{2}z_4^2 + \frac{1}{2}\tilde{\theta}^2 + \frac{1}{2}\tilde{D}^2 + \frac{1}{2\bar{\varsigma}}\tilde{p}^2$$

where $\hat{\theta}$ and $\hat{D}$ are estimate of θ and D, $\tilde{\theta} = \theta - \hat{\theta}$, $\tilde{D} = D - \hat{D}$, $\bar{\varsigma}$ and $\tilde{p}$ will be defiend later, then time-derivative of V_4 is

$$\begin{aligned}\dot{V}_4 = & -k_1 z_1^2 - k_2 z_2^2 - k_3 z_3^2 - \frac{1}{2\epsilon_1}y_1^2 - \frac{1}{2\epsilon_2}y_2^2 - \frac{1}{2\epsilon_3}y_3^2 + 2\varepsilon_1 \\ & + 2\varepsilon_2 + 2\varepsilon_3 + \gamma_3 z_3 z_4 + z_4(\gamma_4 u + \lambda_3\vartheta^T(t) f + \lambda_3\psi \\ & + \lambda_3\bar{d}(t) + \frac{\alpha_{3,f}}{\epsilon_3} - \frac{\alpha_3}{\gamma_3\epsilon_3}) - \tilde{\theta}\dot{\hat{\theta}} - \tilde{D}\dot{\hat{D}} - \tilde{p}\dot{\hat{p}}\end{aligned}$$

Clearly from Lemma 3 of [7],

$$\begin{aligned}z_4\lambda_3\vartheta^T(t) f &\le \theta\frac{z_4^2\lambda_3^2 f^T f}{\sqrt{z_4^2\lambda_3^2 f^T f + \chi^2}} + \theta\chi \\ z_4\lambda_3\bar{d}(t) &\le D\frac{z_4^2\lambda_3^2}{\sqrt{z_4^2\lambda_3^2 + \chi^2}} + D\chi\end{aligned}$$

where $\chi = e^{-at}$, $a > 0$, then

$$\begin{aligned}\dot{V}_4 \le & -k_1 z_1^2 - k_2 z_2^2 - k_3 z_3^2 - \frac{1}{2\epsilon_1}y_1^2 - \frac{1}{2\epsilon_2}y_2^2 - \frac{1}{2\epsilon_3}y_3^2 + 2\varepsilon_1 \\ & + 2\varepsilon_2 + 2\varepsilon_3 + \gamma_3 z_3 z_4 + z_4(\lambda_3\varsigma u + \frac{z_4\hat{\theta}\lambda_3^2 f^T f}{\sqrt{z_4^2\lambda_3^2 f^T f + \chi^2}} + \lambda_3\psi \\ & + \frac{z_4\hat{D}\lambda_3^2}{\sqrt{z_4^2\lambda_3^2 + \chi^2}} + \frac{\alpha_{3,f}}{\epsilon_3} - \frac{\alpha_3}{\gamma_3\epsilon_3}) - \tilde{\theta}(\dot{\hat{\theta}} - \frac{z_4^2\lambda_3^2 f^T f}{\sqrt{z_4^2\lambda_3^2 f^T f + \chi^2}}) \\ & - \tilde{D}(\dot{\hat{D}} - \frac{z_4^2\lambda_3^2}{\sqrt{z_4^2\lambda_3^2 + \chi^2}}) - \tilde{p}\dot{\hat{p}} + D\chi + \theta\chi\end{aligned}$$

Furthermore, from the definition of ς in (14), $|\varsigma| > \bar{\varsigma} := \frac{g}{(M+m_p)l}$, let

$$\begin{aligned}u &= \frac{\bar{u}}{\lambda_3} \\ \bar{u} &= -\frac{z_4\hat{p}^2\alpha_u^2}{\sqrt{z_4^2\hat{p}^2\alpha_u^2 + \chi^2}}\end{aligned} \tag{36}$$

where $p = \frac{1}{\bar{\varsigma}}$ and p is the estimate of p, then

$$\begin{aligned}z_4\lambda_3\varsigma u &= -\varsigma\frac{z_4^2\hat{p}^2\alpha_u^2}{\sqrt{z_4^2\hat{p}^2\alpha_u^2 + \chi^2}} \\ &\le -\bar{\varsigma}\frac{z_4^2\hat{p}^2\alpha_u^2}{\sqrt{z_4^2\hat{p}^2\alpha_u^2 + \chi^2}} \\ &\le \bar{\varsigma}\chi - \bar{\varsigma}z_4\hat{p}\alpha_u \\ &= \bar{\varsigma}\chi - z_4\alpha_u + \bar{\varsigma}z_4\tilde{p}\alpha_u\end{aligned}$$

Then

$$\begin{aligned}\dot{V}_4 \le & -k_1 z_1^2 - k_2 z_2^2 - k_3 z_3^2 - \frac{1}{2\epsilon_1}y_1^2 - \frac{1}{2\epsilon_2}y_2^2 - \frac{1}{2\epsilon_3}y_3^2 + 2\varepsilon_1 \\ & + 2\varepsilon_2 + 2\varepsilon_3 + z_4(-\alpha_u + \gamma_3 z_3 + \frac{z_4\hat{\theta}\lambda_3^2 f^T f}{\sqrt{z_4^2\lambda_3^2 f^T f + \chi^2}} + \lambda_3\psi \\ & + \frac{z_4\hat{D}\lambda_3^2}{\sqrt{z_4^2\lambda_3^2 + \chi^2}} + \frac{\alpha_{3,f}}{\epsilon_3} - \frac{\alpha_3}{\gamma_3\epsilon_3}) - \tilde{\theta}(\dot{\hat{\theta}} - \frac{z_4^2\lambda_3^2 f^T f}{\sqrt{z_4^2\lambda_3^2 f^T f + \chi^2}}) \\ & - \tilde{D}(\dot{\hat{D}} - \frac{z_4^2\lambda_3^2}{\sqrt{z_4^2\lambda_3^2 + \chi^2}}) - \tilde{p}(\dot{\hat{p}} - z_4\alpha_u) + D\chi + \theta\chi + \bar{\varsigma}\chi\end{aligned}$$

The control input and parameter estimator are designed as

$$\begin{aligned}\alpha_u = & k_4 z_4 + \gamma_3 z_3 + \frac{z_4\hat{\theta}\lambda_3^2 f^T f}{\sqrt{z_4^2\lambda_3^2 f^T f + \chi^2}} + \lambda_3\psi \\ & + \frac{z_4\hat{D}\lambda_3^2}{\sqrt{z_4^2\lambda_3^2 + \chi^2}} + \frac{\alpha_{3,f}}{\epsilon_3} - \frac{\alpha_3}{\gamma_3\epsilon_3}\end{aligned} \tag{37}$$

and

$$\begin{aligned}\dot{\hat{D}} &= -k_d\hat{D} + \frac{z_4^2\lambda_3^2}{\sqrt{z_4^2\lambda_3^2 + \chi^2}} \\ \dot{\hat{\theta}} &= -k_\theta\hat{\theta} + \frac{z_4^2\lambda_3^2 f^T f}{\sqrt{z_4^2\lambda_3^2 f^T f + \chi^2}} \\ \dot{\hat{p}} &= -k_p\hat{p} + z_4\alpha_u\end{aligned} \tag{38}$$

then

$$\begin{aligned}\dot{V}_4 \le & -k_1 z_1^2 - k_2 z_2^2 - k_3 z_3^2 - k_4 z_4^2 - \frac{1}{2\epsilon_1}y_1^2 - \frac{1}{2\epsilon_2}y_2^2 \\ & - \frac{1}{2\epsilon_3}y_3^2 - \frac{k_\theta}{2}\tilde{\theta}^2 - \frac{k_d}{2}\tilde{D}^2 - \frac{k_p}{2}\tilde{p}^2 + 2\varepsilon_1 + 2\varepsilon_2 \\ & + 2\varepsilon_3 + D\chi + \theta\chi + \bar{\varsigma}\chi + k_\theta\theta^2 + k_d D^2 + k_p p^2\end{aligned} \tag{39}$$

Remark 4: *As stated previously, in terms of control design, the main difficulty of stabilizing the underactuated crane system lies on the fact that k_η in (16) contains x_3 and x_4, therefore backstepping control design could not be applied directly to (16), so as to the γ_i in (22). To solve this problem, we propose a filter based backstepping control with introducing a set of filters (23). With these filters, the virtual control law in ith step only involves signals e_1,...,e_i as shown in (22). Therefore transient bounds technique could be applied to guarantee the transient performance of the crane system.*

Remark 5: *ϵ_i, $i = 1, 2, 3$ are control parameters to be chosen in the designing. Smaller ϵ_i mean the radius of the ball which the position error and swing anle converge into will be smaller. However, the magnitude if control signal will be larger. Therefore this is a trade-off problem between the control cost and transient performance.*

IV. Stability and Transient Performance Analysis

Now we are at the position of establishing the main result of this paper in the following theorem.

Theorem 1: Consider the closed-loop system including the underactuated crane system (1)-(2) with parametric uncertainties and external disturbance, control input (36) and parameter estimators (38). All signals in the closed-loop systems are bounded. Furthermore, the position error and swing angel will converge to an arbitrarily small region of the origin with a given exponential converging speed satisfying (15).

Proof: From (39) one could obtain that

$$\dot{V}_4 \leq -kV_4 + \sigma_1 \tag{40}$$

where $k = 2\min\{k_1, k_2, k_3, k_4, \frac{1}{2\epsilon_1}, \frac{1}{2\epsilon_2}, \frac{1}{2\epsilon_3}, \frac{k_\theta}{2}, \frac{k_D}{2}, k_p\bar{\varsigma}\}$ and $\sigma_1 = 2\varepsilon_1 + 2\varepsilon_2 + 2\varepsilon_3 + D\chi + \theta\chi + \bar{\varsigma}\chi + k_\theta\theta^2 + k_D D^2 + k_p p^2$. Therefore $\dot{V}_4 \leq 0$ when $V_4 \geq \frac{\sigma_1}{k}$. By properly choosing the control parameters and p_1 in (26), it is shown that Ξ is an invariant set. Therefore z_1, z_2, z_3, z_4, y_1, y_2, y_3, $\tilde{\theta}$, $\tilde{D}$ and $\tilde{p}$ are bounded, which from (24) further implies that e_1, e_2, e_3 and e_4 are bounded. From (18) we know η_1, η_2, x_3 and x_4 are bounded. Therefore all signals in the closed-loop system are bounded. Thus from (18)

$$-\alpha_3 < x_4 < \beta_3 \tag{41}$$

Therefore by choosing $\alpha_3 = \beta_3 = \varpi_0$, (17) is satisfied. Also from (18)

$$\begin{aligned} |\eta_1| &< \beta_1\kappa(t) \\ |x_3| &< \beta_2\kappa(t) \end{aligned} \tag{42}$$

where $\alpha_1 = \beta_1$ and $\alpha_2 = \beta_2$. From (13) we know

$$\tan\theta_1 < \frac{\beta_2}{g}(\kappa_0 - \kappa_\infty)e^{-at} + \frac{\beta_2}{g}\kappa_\infty \tag{43}$$

Therefore one can obtain

$$\theta_1 < \frac{\beta_2\kappa_\theta}{g}(\kappa_0 - \kappa_\infty)e^{-at} + \frac{\beta_2\kappa_\theta}{g}\kappa_\infty$$

where

$$\kappa_\theta = \sec^2(\arctan(\frac{\beta_2\kappa_0}{g}))$$

From (8) and (11) we know

$$\left| x_1 - x_d + l\ln(\frac{1}{\cos\theta_1} + \tan\theta_1) \right| < \beta_1\kappa(t)$$

then one can obtain

$$|x_1 - x_d| < (\beta_1 + l\kappa_x\beta_2\kappa_\theta)\kappa(t)$$

where

$$\kappa_x = \sec(\arctan(\frac{\beta_2\kappa_0}{g})).$$

Therefore by properly choosing β_1, β_2, κ_0 and κ_∞ such that

$$\begin{aligned} &\beta_2\kappa_\theta\kappa_\infty \leq g\beta, \ (\beta_1 + l\kappa_x\beta_2\kappa_\theta)\kappa_\infty \leq \beta \\ &\beta_2\kappa_\theta\kappa_0 \leq gx_\theta, \ (\beta_1 + l\kappa_x\beta_2\kappa_\theta)\kappa_0 \leq x_0 \end{aligned}$$

then

$$\begin{aligned} |\tilde{x}_1(t)| &\leq (x_0 - \beta)e^{-\alpha t} + \beta \\ |\theta_1(t)| &\leq (\theta_0 - \beta)e^{-\alpha t} + \beta \end{aligned}$$

This ends the proof of theorem 1. □

V. Experimental Results

To verify the effectiveness of the control scheme, a series of experimental tests are carried out on a gantry crane system, which is shown in Fig.1. The physical parameters for the gantry crane system are given as $M = 1.6kg$, $m_p = 1.2kg$, $l = 0.8m$, $g = 9.8m/s^2$. Throughout the experiments the control gains are designed as follow: $k_1 = 1$, $k_2 = 1.8$, $k_3 = 2$, $k_4 = 1.5$, $k_d = 2$, $k_\theta = 2$, $k_p = 3$, $\epsilon_1 = 0.08$, $\epsilon_2 = 0.03$, $\epsilon_3 = 0.01$, $\chi = e^{-0.5t}$. The control parameters given for the exponential speed are: $\alpha = 0.4$, $\beta = 0.05$, $x_0 = 0.4$, $\theta_0 = 2$. The initial value of the states are $x(0) = 0$ and $\theta(0) = 0$.

The trolley position signal is measured by the encoder embedded in the servo motor SGW7J-02AFC6S, and the swing angel is detected by angle sensors that are equipped under the trolley. For digital computation, the sampling period is set as 1 ms, and the control algorithm runs in the environment of Microsoft Visual Studio 2018 under Windows 10. A GTS-400-PV(G)-PCI motion control board is applied to collect data from the sensors and convey the control commands generated by the computer to the servo actuators controlling the motor.

A. Experiment 1

Firstly, the proposed control scheme will be compared with existing controllers to show its effectiveness and superiority. The LQR control method [31] and SMC control method [30] are used for comparison. A LQR controller with an expression

$$u = -K_1(x - x_d) - K_2\dot{x} - K_3\theta - K_4\dot{\theta} \tag{44}$$

where $J = \int_0^\infty (X^TQX + Ru^2)dt$ is designed as the cost function for LQR where $X = [x - x_d, \dot{x}, \theta, \dot{\theta}]^T$ denotes the state, $Q = diag\{10, 10, 50, 0.1\}$ and $R = 0.1$. The control parameters for LQR are chosen as follows: $K_1 = 3.1623$, $K_2 = 4.3898$, $K_3 = -5.7902$, $K_4 = -2.7238$. The sliding mode control is given by

$$\begin{aligned} u = &\frac{(M + m_p\sin 62\theta)l}{l - \alpha_{21}\cos\theta}k_s sgn(s) - m_p\sin\theta(g\cos\theta + l\dot{\theta}^2) \\ &- \frac{(M + m_p\sin^2\theta)l}{l - \alpha_{21}\cos\theta}\Big(\lambda_{11}\dot{x} + \lambda_{21}\dot{\theta} - \frac{\alpha_{21}g}{l}\sin\theta\Big) \end{aligned} \tag{45}$$

where $s = \dot{x} + \lambda_{11}(x - x_d) + \alpha_{21}\dot{\theta} + \lambda_{21}\theta$ is the sliding surface with $\lambda_{11} = 1.2$, $\lambda_{21} = -2$, $\alpha_{21} = 0.2$, $k_s = 1.2$. To avoid chattering phenomenon, $sng(s)$ is replaced with $\tanh(10s)$ in the experiment.

Fig.2-Fig.4 show the position of the cart x, swing angel θ and control input $u(t)$ of LQR control method, sliding mode control method and the control scheme proposed in this paper respectively.

Fig. 1. Experimental setup of the crane system.

As shown in the experiment, the consumed control time is 2.78s for LQR control method, 4.61s for the SMC controller, and 2.12s for the control scheme proposed in this paper, where the ultimate positioning errors are all within 3mm. As for the swing angel, the LQR controller makes the maximum amplitude 3.32° and the residual amplitude 0.96°. The SMC controller makes the maximum amplitude 1.85° and the residual amplitude 0.16°. Our proposed control scheme makes the maximum amplitude 1.69° and the residual amplitude 0.03°. The ultimate position stabilization error of our control scheme is $0.3cm$. Therefore in terms of position stabilization and anti-swing control, the control scheme proposed in this paper outperforms existing control schemes such as LQR and SMC schemes.

Fig.5 shows the new defined signal $\xi = \sqrt{(x_1 - x_d)^2 + \theta_1^2}$, which also verify the conclusion that our control scheme outperforms existing control schemes such as LQR and SMC schemes. To verify (15), Fig.6 and Fig.7 shows $\tilde{x}_1$ and θ_1 against $(x_0 - \beta)e^{-\alpha t} + \beta$ and $(\theta_0 - \beta)e^{-\alpha t} + \beta$. Therefore the position error and swing angel will converge with a given exponential speed compared with the existing control schemes.

B. Experiment 2

Next, we want to illustrate the robustness of the presented method under external disturbances. Specifically, the payload is externally perturbed at a certain point and with which the controller should act efficiently to suppress the swing of the payload.

Fig.8-Fig.10 show the position of the cart x, the swing angel θ and the control input $u(t)$ of LQR control method, sliding mode control method and the control scheme proposed in this paper respectively under external disturbance. It could be shown that all control schemes preserve certain robustness under external disturbance, but in terms of anti-swing control and control performance, the control scheme proposed in this paper outperforms LQR and SMC controllers.

VI. Conclusion

In this paper, the adaptive antiswing control for underactuated gantry cranes with guaranteed transient performance under unmodeled dynamics and external disturbances is investigated. To sovle this problem, a set of filters are proposed to make the backstepping technique applicable for the control of crane systems. Then through variable transformation the position error and swing angel could be guaranteed to be able to converge to the origin with a given exponential speed. Hardware experiments are conducted to show that the proposed scheme achieves better control performance over existing methods, and it is illustrated that the proposed control scheme possesses strong robustness to unmodeled uncertainties and external disturbances. Some possible future works include extending

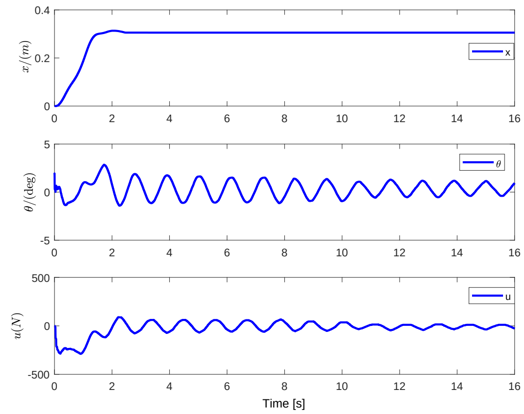

Fig. 2. Experiment 1: the LQR control method: position of the cart x, payload swing $\theta(t)$, and control input $u(t)$.

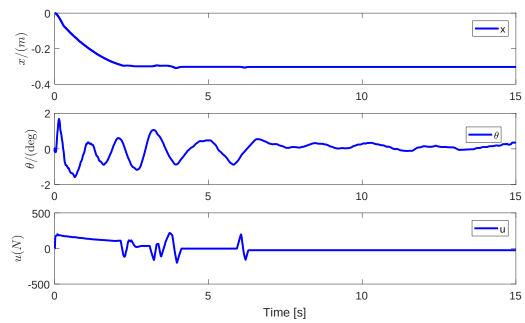

Fig. 3. Experiment 1: the SMC control method: position of the cart x, payload swing $\theta(t)$, and control input $u(t)$.

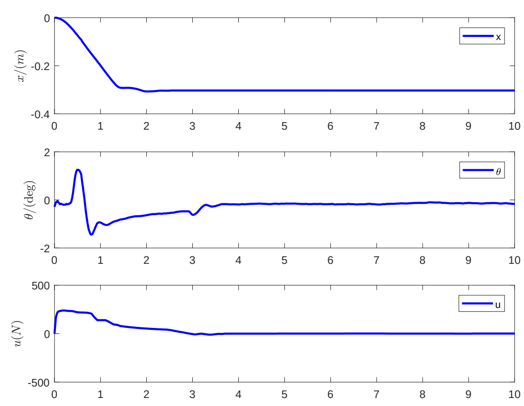

Fig. 4. Experiment 1: the control method proposed in this paper: position of the cart x, payload swing $\theta(t)$, and control input $u(t)$.

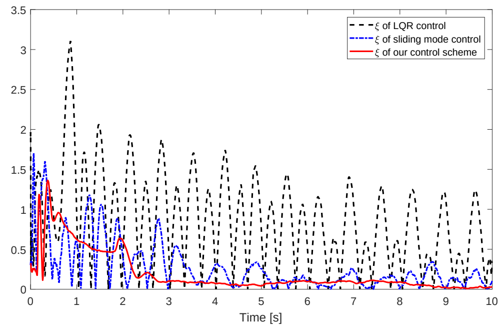

Fig. 5. Experiment 1: signal $\xi = \|(\tilde{x}_1, \theta_1)\|$ for $t \in (0, 10]$.

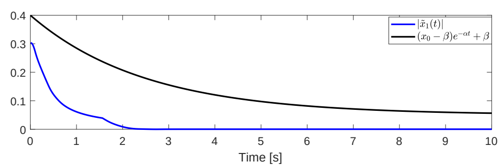

Fig. 6. Experiment 1: the control method proposed in this paper: position of the cart x, payload swing $\theta(t)$, and control input $u(t)$.

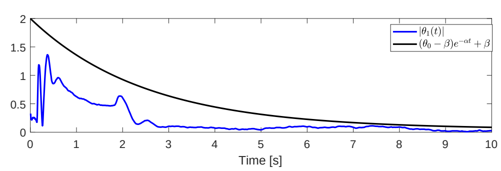

Fig. 7. Experiment 1: the control method proposed in this paper: position of the cart x, payload swing $\theta(t)$, and control input $u(t)$.

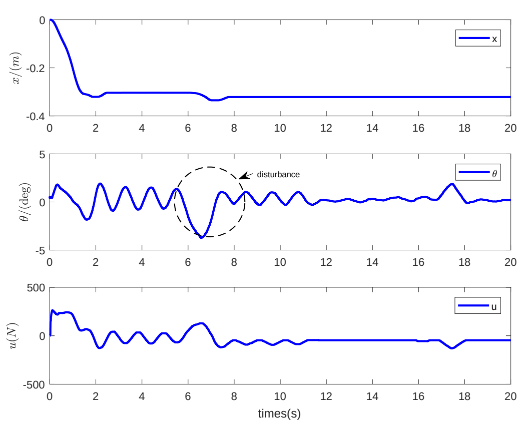

Fig. 8. Experiment 2: the LQR control method under external disturbance: positioning of the cart x, payload swing $\theta(t)$, and control input $u(t)$.

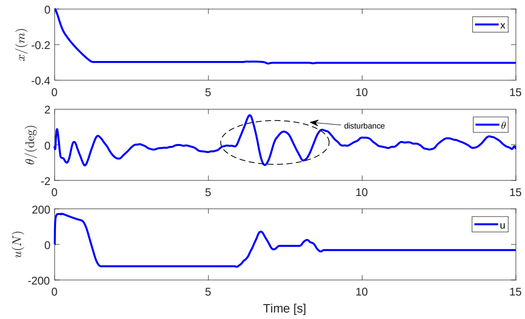

Fig. 9. Experiment 2: the SMC control method under external disturbance: positioning of the cart x, payload swing $\theta(t)$, and control input $u(t)$.

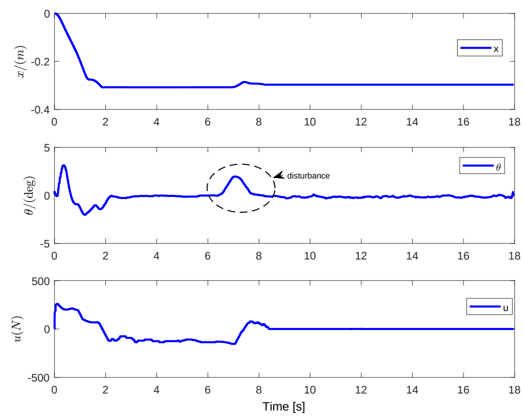

Fig. 10. Experiment 2: the control method proposed in this paper under external disturbance: positioning of the cart x, payload swing $\theta(t)$, and control input $u(t)$.

the control scheme to three-dimensional crane systems, considering the issues of output feedback, event-triggering control and measurement noise.

References

[1] E. M. Abdel-Rahman, A. H. Nayfeh, and Z. N. Masoud, "Dynamics and control of cranes: A review", *J. Vib. Control*, vol. 9, no. 7, 863-908, 2003.

[2] N. Sun, Y. Fang and H. Chen, "A New Antiswing Control Method for Underactuated Cranes With Unmodeled Uncertainties: Theoretical Design and Hardware Experiments", *IEEE Trans. Industrial Electronics*, 62(1), 453-465, 2015.

[3] A. Le , S. Lee , "3D cooperative control of tower cranes using robust adaptive techniques", *J. Frankl. Inst.* 354 (18):8333-8357, 2017.

[4] L. Praly, "Asymptotic Stabilization via Output Feedback for Lower Triangular Systems With Output Dependent Incremental Rate", *IEEE Trans. Automatic Control*, 48(6), 1103-1109, 2003.

[5] M. Krstic, I. Kanellakopoulos, & P. Kokotovic, *Nonlinear and Adaptive Control Design*. John Wiley and Sons, 1995.

[6] W. Wang, C. Wen. "Adaptive actuator failure compensation control of uncertain nonlinear systems with guaranteed transient performance". *Automatica* 46(12): 2082-2091, 2010.

[7] Z. Zuo, C. Wang, "Adaptive trajectory tracking control of output constrained multi-rotors systems", *IET Control Theory and Application*, 8(13):1163-1174, 2014.

[8] Y. Fang, B. Ma, P. Wang, X. Zhang. "A Motion Planning-Based Adaptive Control Method for an Underactuated Crane System", *IEEE Transactions on Control Systems Technology*, 20(1), 241-248, 2012.

[9] J. Gao, L. Wang, R. Gao and J. Huang, "Adaptive control of uncertain underactuated cranes with a non-recursive control scheme", *Journal of the Franklin Institute*, 356(18):11305-11317, 2019.

[10] W. Wang, J. Huang, C. Wen. "Prescribed performance bound-based adaptive path-following control of uncertain nonholonomic mobile robots". *International Journal of Adaptive Control and Signal Processing*, 31(5), 805-822, 2017.

[11] L. A. Tuan, "Neural Observer and Adaptive Fractional-Order Backstepping Fast-Terminal Sliding-Mode Control of RTG Cranes", *IEEE Transactions on Industrial Electronics*, 68(1):434-442, 2021.

[12] N. Sun, Y. Wu, L. Xiao, "Nonlinear Stable Transportation Control for Double-Pendulum Shipboard Cranes With Ship-Motion-Induced Disturbances", *IEEE Transactions on Industrial Electronics*, 66(12):9467-9479, 2019.

[13] T. Yang , N. Sun, H. Chen and Y. Fang, "Neural network-based adaptive antiswing control of an underactuated ship-Mounted crane with roll motions and input dead zones", *IEEE Trans. Neural Netw. Learn. Syst*., 31(3):901-914, 2020.

[14] J. Huang, W. Wang, C. Wen, J. Zhou and G. Li, "Distributed adaptive leader-follower and leaderless consensus control of a class of strict-feedback nonlinear systems: a unified approach", *Automatica*, DOI: 10.1016/j.automatica.2020.109021, 2020.

[15] J. Huang, W. Wang, C. Wen and G. Li, "Adaptive Event-Triggered Control of Nonlinear Systems with Controller and Parameter Estimator Triggering", *IEEE Transactions on Automatic Control*, 65(1):318-324, 2020.

[16] W. Wang, C. Wen, J. Huang et al. "Adaptive consensus of uncertain nonlinear

systems with event triggered communication and intermittent actuator faults", *Automatica*, DOI: 10.1016/j.automatica.2019.108667, 2020.
[17] L. Lee, C. Huang, S. Ku, Z. Yang, C. Chang. " Efficient visual feedback method to control a three-dimensional overhead crane". *IEEE Transactions on Industrial Electronics*, 61(8), 4073-4083, 2014.
[18] W. He, S. Zhang and S. S. Ge. "Adaptive Control of a Flexible Crane System With the Boundary Output Constraint", *IEEE Transactions on Industrial Electronics*, vol. 61, no. 8, 4126-4133, 2014.
[19] N. Sun, Y. Fang, H. Chen, "Adaptive antiswing control for cranes in the presence of rail length constraints and uncertainties", *Nonlinear Dyn.*, 81 41-51, 2015.
[20] H. Chen, Y. Fang, and N. Sun, "A Swing Constraint Guaranteed MPC Algorithm for Underactuated Overhead Cranes", *IEEE/ASME Trans. Mechatron*. 21: 2543-2555, 2016.
[21] X. Wu, X. He, "Partial feedback linearization control for 3-D underactuated overhead crane systems", *ISA Transactions*, 65:361-370, 2016.
[22] N. Sun, T. Yang, Y. Fang, Y. Wu, H. Chen, "Transportation Control of Double-Pendulum Cranes With a Nonlinear Quasi-PID Scheme: Design and Experiments", *IEEE Transactions on Systems, Man, and Cybernetics: Systems* 49:1408-1418, 2019.
[23] N. Sun, T. Yang, H. Chen, Y. Fang, Y. Qian, "Adaptive anti-awing and positioning control for 4-DOF rotary cranes subject to uncertain/unknown parameters with hardware experiments", *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 49:1309-1321, 2019.
[24] N. Sun, Y. Wu, H. Chen, Y. Fang, "Antiswing cargo transportation of underactuated tower crane systems by a nonlinear controller embedded with an integral term", *IEEE Transactions on Automation Science and Engineering*, 16: 1387-1398, 2019.
[25] J. Huang, C. Wen, W. Wang, Z.-P. Jiang, "Adaptive output feedback tracking control of a nonholonomic mobile robot", *Automatica*, 50:821-831, 2014.
[26] X. Xin and Y. Liu, "Reduced-order stable controllers for two-link underactuated planar robots", *Automatica*, vol. 49, no. 7, pp. 2176-2183, 2013.
[27] D. Xia, L. Wang, and T. Chai, "Neural-network friction compensation based energy swing-up control of Pendubot," *IEEE Trans. Ind. Electron*., vol. 61, no. 3, pp. 1411-1423, 2014.
[28] K.-S. Hong, "An open-loop control for underactuated manipulators using oscillatory inputs: Steering capability of an unactuated joint," *IEEE Trans. Control Syst. Technol*., vol. 10, no. 3, pp. 469-480, 2002.
[29] D. Song, J. Han, and G. Liu, "Active model-based predictive control and experimental investigation on unmanned helicopters in full flight envelope," *IEEE Trans. Control Syst. Technol.*, vol. 21, no. 4, pp. 1502-1509, 2013.
[30] N. B. Almutairi and M. Zribi, "Sliding mode control of a threedimensional overhead crane," *J. Vib. Control*, vol. 15, no. 11, 1679-1730, 2009.
[31] M. Nazemizadeh. "A LQR Optimal Method to Control the Position of an Overhead Crane", *International Journal of Robotics and Automation*, 3(4):252-258, 2014.
[32] N. Sun, Y. Fang, and X. Zhang, "Energy coupling output feedback control of 4-DOF underactuated cranes with saturated inputs," *Automatica*, vol. 49, no. 5, pp. 1318-1325, 2013.
[33] Y. D. Song, Y. Wang, and C. Wen, "Adaptive fault-tolerant PI tracking control with guaranteed transient and steady-state performance," *IEEE Trans. Autom. Control*, vol. 62, no. 1, pp. 481-487, Jan. 2017.
[34] K. Zhao, Y. D. Song, T. D. Ma, and L. He, "Prescribed performance control of uncertain Euler-Lagrange systems subject to full state constraints," *IEEE Trans. Neural Netw. Learn. Syst.*, 29(8):3478-3489, 2018.
[35] K. Zhao, Y. D. Song, "Removing the feasibility conditions imposed on tracking control designs for state-constrained strict-feedback systems", *IEEE Trans. Autom. Control*, 64(3):1265-1271, 2019.
[36] Z. Zhao, X. He, Z. Ren, G. Wen. "Boundary Adaptive Robust Control of a Flexible Riser System with Input Nonlinearities". *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol.49, no.10, pp. 1971-1980, 2019.
[37] Z. Zhao, C. K. Ahn, H. Li. "Deadzone Compensation and Adaptive Vibration Control of Uncertain Spatial Flexible Riser Systems". *IEEE/ASME Transactions on Mechatronics*, vol. 25, no. 3, pp. 1398-1408, 2020.
[38] Z. Zhao, C. K. Ahn. "Boundary Output Constrained Control for a Flexible Beam System with Prescribed Performance". *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, DOI: 10.1109/TSMC.2019.2944900, 2019.
[39] C. P. Bechlioulis and G. A. Rovithakis, "Adaptive control with guaranteed transient and steady state tracking error bounds for strict feedback systems," *Automatica*, vol. 45, no. 2, pp. 532-538, 2009.

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