International Journal of Crashworthiness

ISSN: (Print) (Online) Journal homepage: https://www.tandfonline.com/loi/tcrs20

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To cite this article: Gulshan Noorsumar, Svitlana Rogovchenko, Kjell G. Robbersmyr & Dmitry Vysochinskiy (2021): Mathematical models for assessment of vehicle crashworthiness: a review, International Journal of Crashworthiness, DOI: 10.1080/13588265.2021.1929760

To link to this article: https://doi.org/10.1080/13588265.2021.1929760

INTERNATIONAL JOURNAL OF CRASHWORTHINESS
https://doi.org/10.1080/13588265.2021.1929760

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Mathematical models for assessment of vehicle crashworthiness: a review

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ABSTRACT
This article reviews approaches to mathematical modeling of a vehicle crash. The growing focus on vehicle and occupant safety in car crashes has triggered the need to study vehicle crashworthiness in the initial stages of vehicle development. The major motivation for this work is to support vehicle crashworthiness design during the product development process.The article is divided into two parts; the first one overviews existing mathematical models used to solve engineering problems. The second part describes modeling strategies applied for replicating non-linear vehicle crash event and occupant kinematics in an occupant protection loadcase. We also highlight alternative modeling strategies using hybrid modeling techniques aimed at the improvement of the vehicle development process.

Abbreviations: AI: Artificial Intelligence; ANN: Artificial Neural Network; MBS: Multi Body Systems; FEM: Finite Element Methods; FEA: Finite Element Analysis; FE: Finite Element; CAD: Computer Aided Design; CAE: Computer-Aided Engineering; LMS Models: Lumped Mass Spring Models; DOF: Degrees of Freedom; UHSS: Ultra-High Strength Steel; VDCS: Vehicle Dynamics Control Systems; NHTSA: National Highway Transport Safety Administration; FMVSS: Federal Motor Vehicle Safety Standards; V2V: Vehicle–Vehicle; RSM: Response Surface Methodology; BEV: Barrier Equivalent Velocity; PDE: Partial Differential Equation

ARTICLE HISTORY
Received 4 February 2021
Accepted 3 May 2021

KEYWORDS
vehicle crashes; injuries; occupant protection; mathematical modeling; lumped parameter modeling; finite element methods; multi-body systems; response surface methodology; crash response models

1. Introduction

The notion of ‘crashworthiness’, first used in the aerospace industry in the early 1950s, introduced the measure of the ability of the structure to protect its occupants in survivable crashes [1]. In the automotive industry, the term refers to vehicle’s structural abilities to plastically deform and absorb sudden impact loads maintaining enough survival space for the occupants. Crashworthy vehicle structures should be stiff in bending and torsion for proper ride and handling and should minimize fore-aft vibrations that give rise to harshness.

According to Du Bois et al. [1], the vehicle should be able to: (i) deform plastically in the front end and absorb crash energy in case of a frontal crash and prevent intrusions in the driver compartment; (ii) deform plastically in the rear end to protect occupants in case of a rear impact; and (iii) have well-designed side structures preventing intrusion into passenger compartment and opening of doors due to loading in a crash.

Most safety regulations require crash testing at a specialized facility to determine the crashworthiness parameters. Car manufacturers conduct full vehicle or sled tests to ensure that the car design meets the regulations. Usually, crash-testing is time consuming and costly. Mathematical models are employed to represent crash dynamics, for example, in the case of a car impacting a barrier or another car. These models involve differential equations of motion describing the deformation of the parts in the vehicle. The occupants in the car can also be included in a mathematical model to predict injury values during a crash. Construction of an appropriate model involves the elimination and minimization of effects deemed to be negligible. The quantities that are modeled are expressed as functions depending on independent and controllable variables. Non-linear physical systems very often are modeled by ordinary and partial differential equations. To find specific solutions of such differential equations one needs initial and/or boundary conditions. Solutions can be validated with empirical data from the physical experiment, see, for instance, Shier and Wallenius [2].

The classes of differential equations to which the analytical solutions exist are very limited; therefore, numerical methods are being employed. In this case computational inaccuracies add up to the inherent inaccuracies of the model and the result must be compared with the experimental data. As suggested by Marion and Lawson [3], one of possible approaches to mathematical modeling involves the following steps: (a) building; (b) studying; (c) testing; and (d) use of the model. Vehicle crashes are highly non-linear transient dynamic phenomena. In an impact, a non-linear relation holds between applied force and displacements; it appears due to geometrical non-linearity (non-linear behaviour of highly deformable bodies leading

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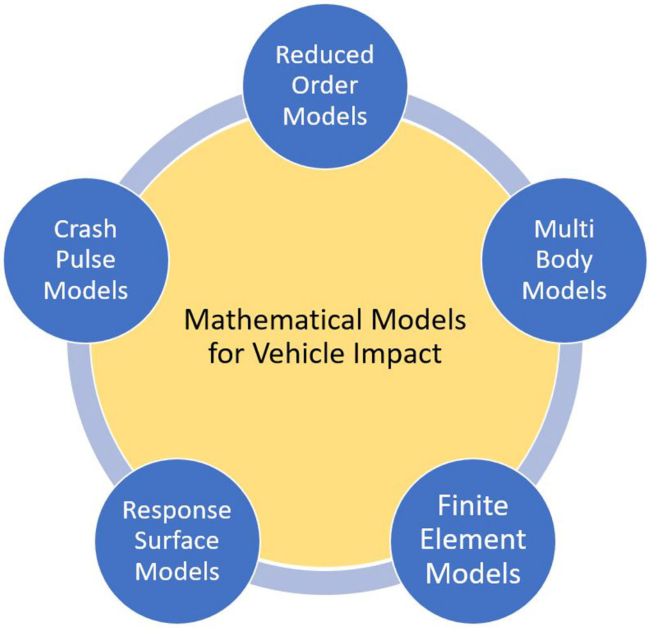

Figure 1. Common used models for vehicle crash.

to non-linear strain-displacement relations), material nonlinearity (elasto-plastic material) and combinations of these two types of non-linearities. Material nonlinearity depends on a number of factors: rate of deformation, temperature, pressure, humidity, age of the material and the deformation history [4]. In case of vehicle impacts, it has a significant influence on the deformation and it is important to replicate material and geometrical non-linearity in vehicles while modeling the crash phenomenon. To deal with such non-linearities, Finite Element Method (FEM) is often employed. It has higher accuracy but includes manual efforts to mesh the parts along with increased computational efforts. In contrast, simplified mathematical models are less resource-consuming yet they have lower prediction levels. In several studies models which replicate the collision mechanics with considerable confidence were developed, however a compromise between computational time and accuracy is always present.

This article reviews the existing approaches to mathematical modeling of car crashes. Although the use of models helps to reduce the dependence of automotive design on physical crash test data for determining crash parameters and injury values to occupants, they do not fully replace real time tests due to certain inevitable assumptions which restrict the analysis of the kinematics of the event in detail. The research reported in the literature indicates the need to further improve the predictive power of existing models for efficient application in a vehicle design development (Figure 1).

2. Methodology for crash modeling

2.1. Reduced order dynamic models

These models have reduced complexity yet capture the kinematics of the crash with the load paths and components. The methodology includes the use of lumped parameter models, fine-grained lumped models and coarse mesh finite element models. One of the major challenges is that the accuracy of these models is affected by the simplifications and reduced number of degrees of freedom (DOF). Lumped parameter models are the most commonly used reduced order models; they include spring-damper systems replicating a deformable part and a concentrated mass representing the undeformed structures like engine and transmissions. Passenger compartment integrity is essential for vehicle structural loadcases; for simplicity, it is often assumed that the passenger compartment is integrated with the chassis as a lumped mass. However, occupant protection models need to accommodate for compartment deformations in order to understand the cabin intrusions and better predict possible crash scenarios. Lumped parameter models are also used to predict occupant movements and possible injuries in a car crash. The use of lumped masses for head, torso and legs, all connected by springs replicating joints, helps to understand the head and neck deflections and torso movements in a crash.

The idea behind a reduced order model is to depict the rigid components as masses connected by springs and determine the forces acting on the masses from the external impact and the spring forces. These forces and energy conversion laws are used to determine the governing equations of motion which are set up using one of the following formulations.

Newtonian mechanics

The Newtonian approach relies on three Newton’s laws of motion [5]. The mechanics of particles can be described by the Newton’s laws of motion which which describe the relationship between an object’s motion and the forces acting on it.

Lagrangian mechanics

The Lagrangian approach uses energies rather than forces to define the dynamics of a system. The Lagrangian is the central quantity in Lagrangian mechanics, it obeys the following equations:

$$\frac{d}{dt}\frac{\partial L}{\partial q_{i}^{\cdot}} - \frac{\partial L}{\partial q_i} = Q_i,$$

where, in general case, $L = T - V$, T is the total kinetic energy of the system equal to the sum of the kinetic energies of the particles, $q_i, i = 1, .., n$ are generalized coordinates and V is the potential energy of the system.

Hamiltonian mechanics

In Hamiltonian mechanics, the time evolution is obtained by computing the Hamiltonian of the system in the generalized coordinates. The Hamiltonian principle describes the motion of those mechanical systems for which all forces are derivable from a generalized scalar potential that can be a function of the coordinates, velocities and time [5].

Lagrangian and Hamiltonian principles together form a compact invariant way of obtaining the mechanical equations of motion.

Reduced-order models allow prediction of large deformation structures, help in analyzing component-level simulations during the early vehicle development process and assist in developing new vehicle architectures for automotive applications. They distinguish themselves from other methodologies by including design dimensions in the system; users are able to develop a predictive model which may not depend on vehicle crash data besides the validation phase of modeling.

2.2. Multi-body models

A Multi Body System (MBS) is a system that consists of rigid bodies, or links, that are connected by joints which restrict relative motion of the parts. The study of MBS distingushes forward dynamics which analyzes the motion of mechanical systems under forces, whereas the inverse dynamics deals with the analysis of forces causing the motion of bodies [6]. Multi-body models are used for both dynamic and kinetic analysis [7]. Lagrange devised the formulation for the dynamics of multi body systems in 1788 in *Mecanique Analytique* [8] and since then is recognized as the father of multi-body dynamics. Important additions to this methodology include application of friction (by Coulomb [9]), beam elasticity (by Euler [10]), contact compliance (by Hertz [11]) and lubrication (by Reynolds [12]). Two hundred years after the formulation was proposed by Lagrange, the methodology gained further impetus with the introduction of improved matrix manipulation techniques by Denavit and Hartenberg [13]. During the past century, the improvements in solution methods and their computational efficiency supported applications of this methodology in different aspects of machine design including vehicle design analysis [7]. The analysis of linkage mechanisms developed by Wittenbauer [14] was followed by the use of rigid body dynamics for the analysis of human gait by Fischer [15]. Segel [16] studied the motion of a vehicle on a flat road in response to steering control. Orlandea et al. [17] proposed a practical solution methodology for large rigid MBS based on the Lagrangian dynamics for constrained systems; this led to the development of ADAMS (automatic dynamic analysis of mechanical systems), the driving force behind many advancements in the automotive industry (Figure 2).

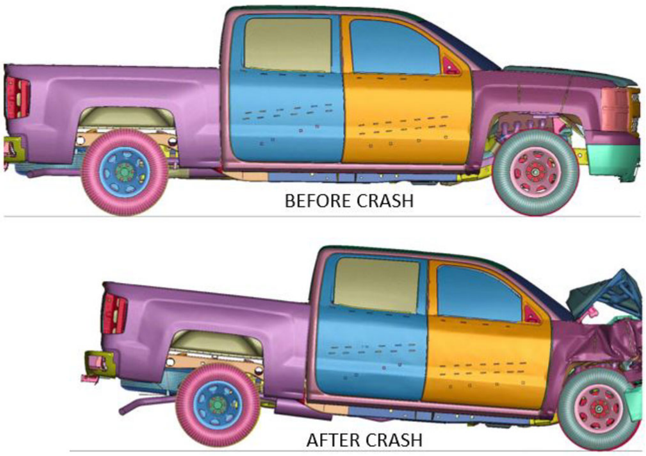

Figure 2. LS Dyna FE model of a full vehicle crash.

Constructing the governing equations for MBS is challenging; one of the classical approaches is based on the Lagrange method for setting up the equations which are solved numerically afterwards. However, this approach is time consuming, especially with systems having large number of components. Nikravesh [18] has proposed a new methodology for constructing equations of motion for an MBS based on a body-coordinate formulation using Newton-Euler equations and a joint-coordinate formulation employing relative coordinates. The study also describes systematic transformation from the former to the latter formulation. The complexity of dynamic equations of motion makes such models challenging computationally; this stimulated the development of the software for computer simulation since 70’s. The programming codes support different functionalities ranging from the generation of equations of motion to numerical simulations for solving the equations [6]. Examples of computer code guidelines can be found in the articles of Barley and Cripps [19] and Dopker [20].

Multi-body models are applied in vehicle development process for several decades to design vehicle handling and suspension systems [21]. One of the studies in this context is due to Hegazy et al. [22] where the vehicle structure is represented by rigid bodies connected by springs, dampers and joints. Recently MBSs have been also used to develop generic models for the study of crashworthiness in vehicles and for the prediction of the impact of crashes on vehicles during the development process [23]. Lower accuracy and cumbersome process required for developing the model are the limitations of this methodology; although it is quite useful for early development phases of vehicle design. MBSs are used for the development of occupant and pedestrian models in crash analysis where one of the main challenges is related to the replication of the anthropometry of the human body. The representation of different joints of the body has been implemented in several commercial software programs like MADYMO. Similarly to reduced order models, this methodology has insufficient accuracy and less detailed modeling of the system. For instance, the occupant and pedestrian human body models lack details like skin and ligaments which might be critical for determining certain loadcase parameters in occupant and pedestrian protection.

2.3. Non-linear finite element models

Finite element modeling uses finite element method (FEM) to solve boundary value problems (BVPs) for partial differential equations (PDEs) arising in many physical and engineering problems. The solution of such problems for PDEs can be considered in two forms: strong and weak. A strong form of the governing equations states that the solution must satisfy the problem at every point of the domain along with boundary conditions; it assumes that the classical solution to the problem exists. A weak form states that the solution must satisfy the problem in an integral sense and is used when the classical solution to a problem cannot be established. FEM is a special method which subdivides the original BVP into smaller problems called finite elements in order to approximate PDEs. The solution is derived using numerical methods for solving systems of algebraic equations and systems of ordinary differential equations. The basic steps of an FEM are [25]: establishing the strong formulation, obtaining the weak formulation, choosing approximations for the unknown functions, choosing the weight functions, and solving the system.

The finite element models are developed by discretizing the CAD surface into elements and nodes which cover the geometry of the vehicle (mesh) and the finite element BVPs are developed from the discretization. These problems are solved in order to determine the nodal displacements. The elemental stresses and strains can be derived from the explicit finite element method. In order to get a better approximation it is preferable to have a higher mesh size with more nodes covering the domain.

The FEM approach in engineering was developed in the early 1940s when Hrennikoff [26] and Courant [27] used mesh discretization for elasticity and structural analysis problems. Clough published the first article on FEM in 1960 suggesting that two-dimensional elements connected to more than two nodes can be used to solve problems in continuum mechanics [28]. In 1965, NASA Structural Analysis software (Nastran) was developed to solve structural analysis problems; this paved the way to simulation of engineering stress strain problems with software codes. In the following decade Alberto Peano developed the first professional FEM p-version code which was used by Szabo in an industrial implementation PROBE in 1982. The qualitative research of Spethmann et al. based on expert interviews analyses the impact of the use of finite element methods in vehicle crash simulations on productivity and problem-solving [29]. The authors argue that since the 1960s, when the explicit FEM was developed and applied to crash events, it became not only an alternative to physical destructive testing but also a method for solving problems which formerly had been impossible to solve. Even though automotive industry gained trust in crash simulations, the lack of appropriate software and hardware brought them to a standstill in the late 1970s to early 1980s. The article highlights the emergence of supercomputers in the late 1980s which aided research to improve the performance of passive safety systems in a crash. Since then the dependence of engineers on computer software programs and computer power has been constantly growing. The FEM approach is widely used by automakers to simulate crash although the process is time-consuming and requires skills to develop the full size finite element models. Another shortcoming of the FEM in crash simulations is the dependence of the results on CAD data for the structure and non-linear material properties of vehicle structure. The stiffness and dimensions of each component need to be defined before the solver is used to determine the acceleration and deformation in the crash event. The process of detailed intrinsic meshing is cumbersome and requires training to represent the entire CAD surface with a discretized mesh. This calls for research and predictive simulations at early design stages thus possibly reducing the number of re-design stages since the timescales tend to become shorter in automotive industry. Improvements can be achieved through the collaboration of car manufacturers with academic institutions in multidisciplinary research.

One of the major challenges in using software programs like LS-Dyna or PAMCRASH for engineers transitioning into automotive industries is the extensive training required to understand the solver codes and assumptions made during the analysis. These complex programs are not a part of curriculum for engineering students or academic research and there is a need to bridge this gap between academia and specific requirements of the industry.

2.4. Response surface models

The Response Surface Models (RSM) are statistical approximation models which do not rely on the physical description of the objects but explore the relationship between the input (predictor, or design variable) and output response (dependent variable) using a number of experiments in which the predictor variables are changing. In automotive industry, RSM can be employed to measure the performance of the system and, in combination with numerical simulation methods, they are used to improve or optimize a product and its performance [30]. The methodology was developed by Box and Wilson who used the sequential method in chemical process design [31]. The motivation for their work was the problem of planning and analyzing experiments in search of desirable conditions on a set of controllable, or design, variables [32]. The response surface analysis can be viewed as analysis that deals with a fitted function and accommodates a large collection of techniques. RSM uses linear and quadratic models to fit a sequence of local regression models with experimental data.

The RSM algorithm consists of the four steps: (a) perform a statistically designed experiment, (b) estimate coefficients in the response surface equation, (c) check on the adequacy of the equation (*via* lack-of-fit test), and (d) study the response surface in the region of interest [32].

For engineering applications, the process of constructing models often includes the following three steps:

- design of experiments: this involves setting the factors at different levels for proper experiments and ensuring that the boundary values as well as the entire area of the interest of the model is tested for different combinations of variables;
- data collection: the process involves running the experiments to collect the data including FE simulations or real time crash tests.

- data fitting: this is the final step which involves using algorithms to fit the sample data matching specific requirements. The feasible design solution is obtained at this step and used for design recommendations or relevant changes aimed at meeting the crash loadcase requirements.

The RSM methodology was used in non-linear finite element models where accurate response surface models are constructed and evaluated for repeated replacement of the finite element model at each time step of the analysis [33]. In comparison with the modeling based on sensitivity analysis, the RSMs provide considerably more accurate predictions reducing dependence on FE models [34]. One of the shortcomings of the RSM technique is the dependence on real crash test/simulation data. Such models are unable to predict new scenarios in crash loadcase and have been found to be less accurate for non-linear impacts. It is crucial to know the algorithm behind the RSMs, otherwise it becomes a “black box” approach and finding the magnitude of approximation errors is difficult [35]. Another limitation of this technique is that the developed response surface is invalid for regions other than those set in the problem. The RS methodology fits the data to a second order polynomial, in which case the technique gives accurate prediction but may fail for problems with higher order polynomial approximations.

The RSM methodology is also useful in parameter identification models which help predict the stiffness and damping values for vehicle deformation; such models find extensive applications in accident reconstruction.

2.5. Crash Pulse models

Crash pulses represent the dynamic response of a vehicle in a crash event and serve as a validation for most algorithms developed to predict crash responses. These models also help to explain the energy conversions in vehicle structure during the impact; structural optimizations are also based on crash pulses [34]. Furthermore, crash pulses are used in validation of crash simulations where most validation algorithms compare model simulations with real time crash data [36].

The crash pulses are modeled using the function representing the vehicle acceleration and the crash process. If $x(t)$ stands for the acceleration, then the crash pulse model F_θ should ensure that

$$\begin{gathered} r_a(t_0) = x(t_0) - F_\theta(t_0) \approx 0, \\ r_v(t_0) = \int_0^{t_0} r_a(t)\mathrm{d}t \approx 0, \\ r_d(t_0) = \int_0^{t_0} r_v(t)dt = \int_0^{t_0}\int_0^{t_0} r_a(t)\mathrm{d}t\mathrm{d}t \approx 0 \end{gathered}$$

at all times $t_0 \geq 0$ where $r_a(t), r_v(t), r_d(t)$ are the residual signals of acceleration, velocity and displacement, respectively [36]. In the past, the crash pulse was represented using different pulse shapes including square, triangular, half-sine and even polynomial functions. In general, a crash pulse is defined only for a specified crash scenario and may not be applicable for different loadcases. There could be numerous factors influencing crash pulses such as velocity of impact, crash model and other collision conditions. However, researchers developed efficient schemes to overcome this problem in crash modeling. For instance, Wei has proposed a crash pulse model to determine crashworthiness of vehicles [36]. This methodology resembles reduced order modeling, however these models find applications in accident reconstruction and depend on crash pulses or crash data for model development and validation.

3. Applications of modeling strategies

3.1. Reduced order models

The standard approach for lumped mass spring (LMS) models is that bodies are represented by concentrated point masses which are connected by linear/non-linear springs. The springs are defined by force-deformation and force–velocity curves and deform due to the application of a force. This approach was first introduced in automotive suspension design in the early 1900s and has been extensively used in automotive development since then.

The article by Kamal [37] is one of the earliest studies in modeling of crash events using lumped parameter models. The model developed in this article includes three mass components and eight resistances representing the deformable structures of the vehicle. The lumped masses represent the body chassis mass, the engine transmission and the vehicle bumper. The non-linear resistances along with the inertial components (lumped masses) are used to solve the basic equations of motion numerically. The dynamic force acting on the resistances is approximated using static forces acting on the vehicle during the crash event, where the constant factor is assumed to be independent of the geometry of the structure. It is assumed that the structure is two-dimensional with a closed rigid frame. This implies that the model may not predict the vehicle behaviour out of plane forces experienced by the structure in a crash. The study correlates well with physical test data for displacement while the acceleration peaks are not well correlated. However, the trend for the acceleration curves is similar which indicates that the model predicts the event’s kinematics to a reasonable extent. The static and dynamics force-deformation curves show a lower peak for the static curve which is expected because the model does not account for the impact loading acting on the structure in a dynamic crash event. The study also includes elastic body analysis for the vehicle passenger compartment and calculates the forces exerted on the members in case when the occupant compartment is not considered a rigid lumped mass. A parameter study on the elastic passenger compartment indicates that the structure’s capability to withstand crash increases with increasing metal thickness. This observation is in line with the basic understanding of bending forces, that is, the thickness of the structure contributes to the crashworthiness of the body (Figure 3). 1 shows the model developed by Kamal.

Identification of parameters involves a range of approaches, for instance, a piece-wise linear approach where

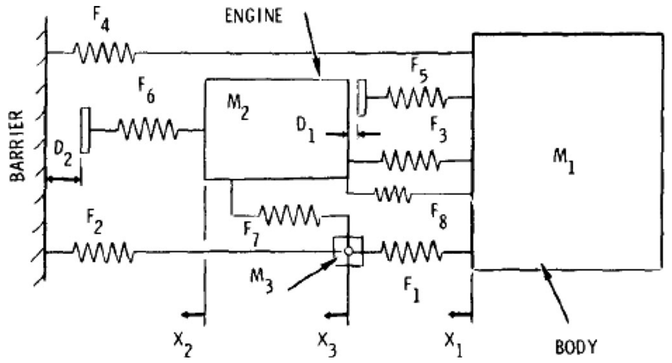

Figure 3. Vehicle impact simulation model in [37].

the force deformation characteristics are represented by the hat functions or Chebyshev polynomials. The studies conducted in [38, 39] used optimization approaches to estimate crash parameters. The algorithm developed in [38] helps optimize the acceleration data for a full frontal crash using the force deformation curves for a few components in the vehicle. In [39], the solution space approach is used to develop an algorithm which is applied to three engineering vehicle crash scenarios. The algorithm determines the force deformation curves used for frontal crash components. This approach is illustrated with an example where the algorithm is used to determine whether submarining occurs in the rear passenger seat and to design the car seat-belt and rear occupant structure which ensure the absence of submarining during a crash event. The optimization problem has a solution meeting the design constraints.

Sharp et al. [40] used Lagrangian method to simulate vehicle motion. The equations of motion take into consideration external forces acting on the vehicle and incorporate pitching, rolling and yawing effects on the car. The equations also include the sprung mass of the vehicle and unsprung masses per wheel along with the moment of inertia in the x, y and z axes. The numerical model predicts the body roll, pitch and yaw angles and the tire forces in the longitudinal and lateral directions. This mathematical model replicates the motion of an ideal vehicle with inertial forces and the coupling between pitch and bounce. The limitations of the model include the lack of non-linear springs and anti-roll bar to represent the suspension system in more detail.

A method for finding the parameter values for spring elongations was developed by Mentzer et al. [41] who used real time crash test data to determine the mass of the components from acceleration and wall contact forces. They obtained the force-deformation curves for the springs from the load paths under the condition that the system should have comparable motions of its masses so that the force and acceleration curves match the test data. This condition is difficult to achieve as the number of load paths could be higher than the mass elements. This is the reason why the least square method is used for the parameter identification in a full crash test data. Some of the drawbacks of this approach are: the energy absorption by the honeycomb structure during deformation was neglected; it is assumed that no rotational energy is lost in offset impacts. The rotational energy losses were accounted for in the SISAME 3 D model adopted later by NHTSA where the masses were no longer considered as point masses which improved the model's reliability.

The early approaches to parameter determination in LMS models proved to be efficient and were further developed to improve agreement between model outputs and data sets; a number of parameter identification techniques used by researchers in vehicle modeling, will be discussed in our future research article.

Cheva et al. [42] developed a lumped parameter model to replicate a zero degree frontal crash test and a 40% offset deformable barrier crash test. The barrier is defined as a large lumped mass as well as the firewall which represents the passenger compartment. The deformation of the firewall indicates the intrusion in the occupant compartment. The left and right sides of the vehicle were modeled separately so that the same model can be used with minor modifications for an offset crash event. The model includes mass components representing several parts in the deformable zone like the engine assembly, radiator, suspension components, and front rails. The crash was simulated at 48 and 56 kmph and the results were validated against physical crash test data. The same model was used for 40% offset deformable barrier loadcase with the barrier imparting more load on one side. Then the upper rails have higher load from the deformable barrier causing higher deformation on the impacted side. The event kinematics are observed to be different in an offset crash scenario compared to a full frontal loading case.

3.1.1. Discrete time domain simulations

The crash behaviour can also be described using discrete time domain simulation in lumped parameter models. The approach allows to predict and understand the crash response in terms of deformation, acceleration, velocity and rotation angles during the entire span of the crash event.

The research by Elkady et al. [43, 44] focuses on developing mathematical models for replicating a vehicle crash using non-linear springs for the vehicle bumper. The lumped parameter model developed in [43] and [45] uses a lumped mass representing the vehicle body and four spring damper units to replicate the suspension and wheels. It is assumed that the vehicle is moving on a flat asphalted road and the vertical motion of the tyres is neglected. The model is designed to explore the effects of Vehicle Dynamics Control Systems (VDCS) on the crash mitigation for an offset impact with a rigid barrier. The effect of ABS (anti-lock braking system) is also simulated by using a braking force component in the equation of motions. The front deformable members are presented by non-linear springs with force deformation characteristics and the forces on the springs during the crash are calculated using numerical methods. The model is validated by comparing the acceleration and deformation of the front end structures to the physical test data. The study concludes that the values of the post impact speed of the vehicle in the mathematical model and in the

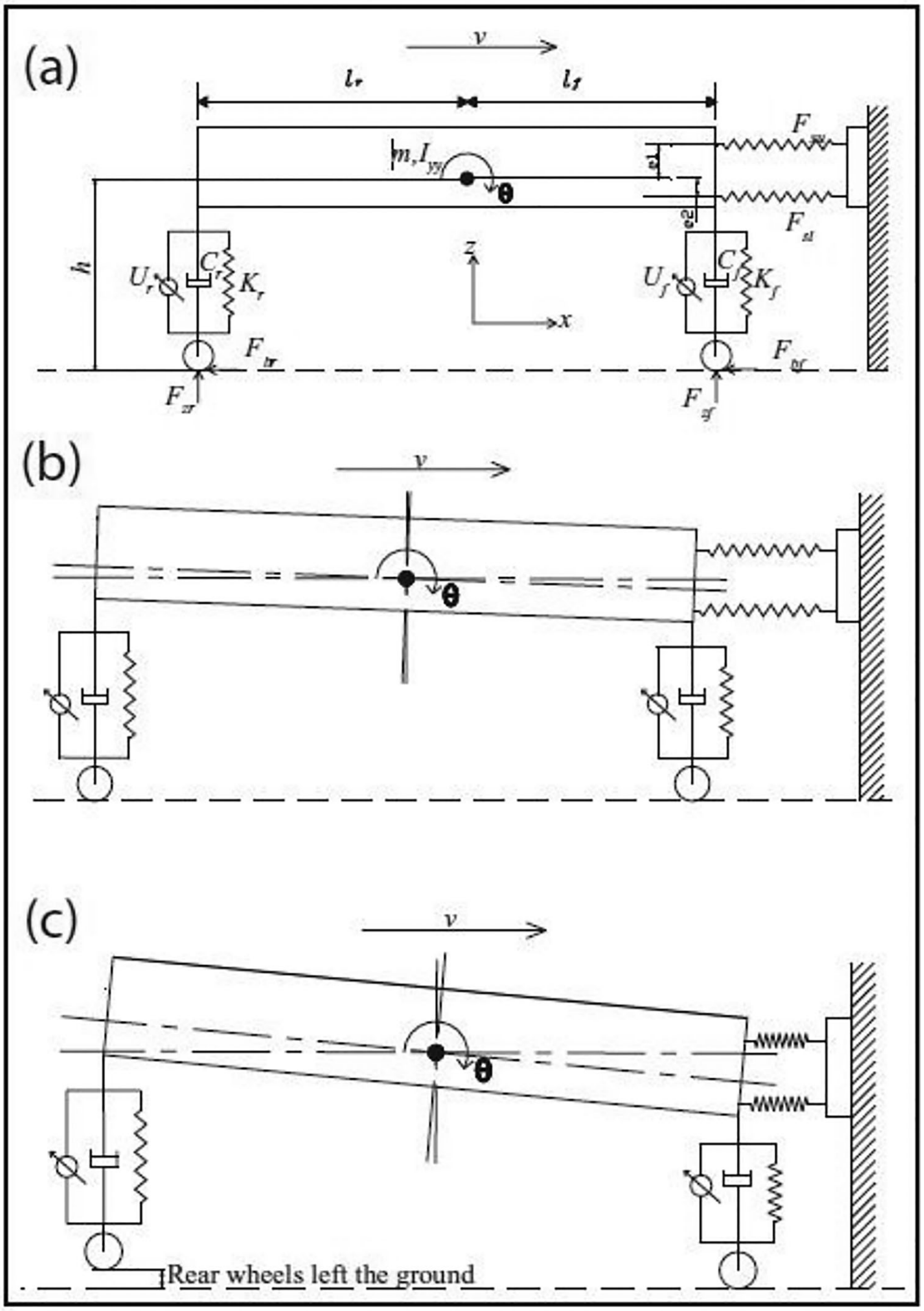

Figure 4. Barrier impact simulation model in [44].

physical test agree well. The variation in the curves for the front end deformation suggests shortcomings of the model due to the inaccurate values of the system parameters. The article also discusses the effects of VDCS on the collision response for a 50 percent offset impact (Figure 4).

The same 6 DOF mathematical model is employed to solve the equations of motion using Euler's method for full frontal and offset impact [46] (Figure 5).

It is demonstrated that in the case of the vehicle deformation and deceleration during the crash the effect of the active VDCS is negligible. However, the vehicle pitch angles show an improved vehicle behaviour with an active VDCS in the car. The model in this this study does not include the front bumper mass or a rigid mass like an engine or battery which may contribute to the deceleration and deformation of the vehicle. In Elkady et al. [47], the vehicle model is modified by adding a lumped mass for a front bumper which connects the front end members represented by springs.

An offset impact with another identical vehicle is studied to understand the crash response of the vehicle and how it differs from the case of rigid barrier impact. The simulations are performed for the impact speed of 55 kmph with different car masses. The study could be extended to understand the deceleration in the vehicles for different impact speeds and vehicle masses.

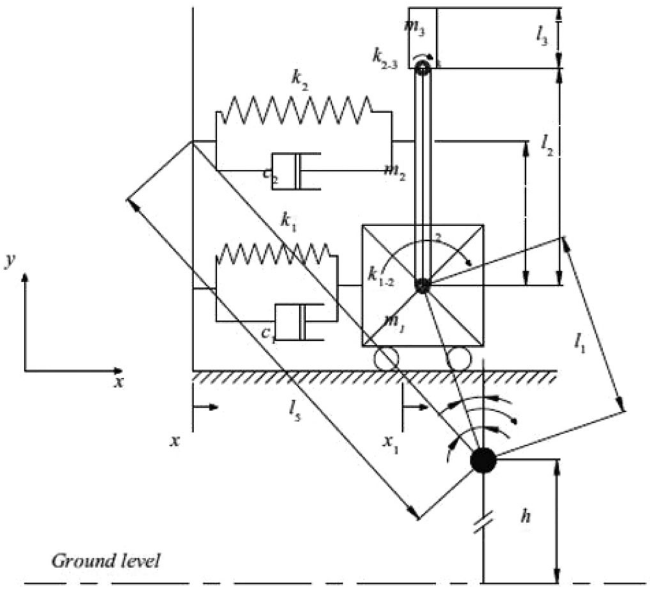

Figure 5. 3-DOF occupant multi-body model in [44].

Elmarakbi et al. [48] developed a mathematical model for smart structures which improves the crashworthiness response of a vehicle in a barrier impact. The smart structures are represented by spring mass damper systems for vehicle and occupant and are simulated numerically with the help of an optimization algorithm which minimizes the intrusion in the occupant compartment and the deceleration injury for the occupant. The injury curves obtained from the simulation are compared to the vehicle model without smart structures.

Ionut et al. [49] developed a 2D mathematical model which includes 2 vehicles with 2 occupants to analyse the occupant kinematics in a frontal collision with another vehicle.

They use Lagrange's generalized formulation to obtain the system of five equations. The numerical solution provides the displacement and velocity of each of the vehicle bodies and the velocities of the occupant's head and thorax. The model was validated against real test data demonstrating good correlation. The parameterization of the stiffness of the seat belt springs is used to understand the influence of the spring stiffness on the occupant deceleration and displacement.

A National Highway Transport Safety Administration (NHTSA) Lumped Parameter Model was developed by Deb et al. [50] for a side impact crash scenario. The authors identified lumped masses which were then added to the existing model based on finite element analysis of two passenger vehicles. The validation of this model was conducted with simulations of two vehicles Dodge Neon and Dodge Intrepid. The authors suggested the methodology of determining the spring characteristics from the FE model using contact introduced between two components. This gives the force displacement characteristics for the spring members.

The deformation characteristics of a vehicle under front-to-side impact were calculated by Prochowski et al. [51] using experimental and analytical equations. The combined deformation of both vehicle bodies due to the force was plotted for the impact duration. The stiffness of each vehicle was predicted based on equal force experienced by both vehicles, suggesting that for a medium size car the average side impact stiffness is a quarter of the front side stiffness. The authors challenge the existing method of calculating the side stiffness from force deformation curves asserting that it overestimates the side stiffness of the car body at a front-to-side collision. They argue that using only the central part of the deformation zone for calculating the stiffness is only a few percent lower than using the whole deformation zone for the measurements. The use of the central portion for the measurements does not only simplify them but also provides a higher accuracy of data for the measurements.

Jonsen et al. [52] proposed a lumped parameter model to represent a bumper in a crash.

The system uses an optimization software INVSYS where an unconstrained subspace-searching subplex method is implemented. The algorithm identifies the local minima and can be applied for optimizing noisy objective functions. The objective function is defined to minimize the error between the calculated and measured displacements; constraints include masses, damping and stiffness constants along with total mass of the vehicle. The authors claim that if the DOF of the system is increased to two, the error is reduced. This result is validated using FE bumper system connected with a 2 DOF spring mass damper system allowing only longitudinal motion.

The research on LMS models for vehicle crash has progressed slowly from simple spring mass models to more complex multiple DOF models with spring-mass-damper systems and non-linear springs. We remark that the governing equations of motion usually use Newton-Euler formulation but in the models including occupants Lagrangian formulation has been employed.

3.2. Multi-body models

Ambrosio et al. [53] developed a full vehicle crash model using an MBS with plastic hinge deformation. The entire vehicle is represented by kinematic joints, data for hinge deformation were derived from CAD data and finite element simulations. Sousa et al. [23] suggested a generic car model containing different parts including suspensions, tires, occupants and structural components contributing to load path. The representation of the structural components uses the plastic hinge approach. The model was validated against a completely known finite element vehicle model and can be fine-tuned to have the same crash responses as in the crash tests without the knowledge of the structure of the tested vehicle. The study emphasizes the need for simple mathematical models in early stages of vehicle development process. Carvalho et al. [54] use the plastic deformation methodology to develop an optimization algorithm for identifying multibody models for crash analysis. The solution to the problem is obtained through sequential application of genetic and gradient based optimization methods. This methodology has been also employed to define an MBS for a large family car for the case of front and side crashes.

King et al. [55] developed a mathematical model for an airbag which, in conjunction with a three DOF occupant model, can predict the effects of an airbag deployment on the occupant. The authors impose the following requirements to this model: the airbag is spherical and mounted on a steering wheel; the airbag is already inflated when the simulation starts but with a low pressure and is expected to expand radially due to gas filling in the bag; the pressure is distributed uniformly and the deformation of the wall of the airbag is linearly elastic. The three governing equations in this model are the elasticity equation, continuity equation and the equation for state of the gas. The equations describing what happens after the contact of the occupant with the airbag are proposed and the configuration of the deformed airbag is discussed. The mathematical model is implemented in a computer program written in FORTRAN IV where it is merged with the 3 DOF model of an occupant. The model describes the contact of the airbag with the occupant and the code reads contact information based on the occupant's position at any given time. The results of the simulation were validated at the sled facility at the Wayne State University using anthropometric dummies. The model's curves exhibit good correlation with the experimental data.

Elkady et al. [44] developed a 3 DOF multi body mathematical model to simulate a crash event of a car with an occupant (Figure 5).

Three masses representing the lower body replicate the legs and pelvic area of the occupant who can perform translation and rotation motion about the center of gravity (CG) of the body. The model replicates a seat belt with 2 spring damper systems and mitigates the impact for the occupant. The MBS is integrated with the vehicle model developed in the article. Under the full frontal barrier crash the lower part of the body moves forward while the middle and upper body rotate slowly; the spring forces in the seat belt are introduced to reduce the rotation and movement of the body. Lagrange's method is employed to derive the equations of motion. The system of equations is solved numerically to compute the occupant body deceleration. The results from the vehicle crash model are used in the simulation of the crash impact on the occupant. The results highlight the importance of using seat belts, emphasizing that in crash events seat belts are the primary restraints in the vehicle safety system. The rotation angle of the middle body is similar to the pitch of the vehicle in the crash; the crash causes a neck rotation which could be fatal for an occupant. Remarkably, the change in the seat belts' spring stiffness positively affects the neck rotation and deceleration of the occupant. This study demonstrates that the use of a hybrid technique mixing LMS with MBS models improves the overall crash response prediction. Euler and Lagrangian equations are employed for vehicle and occupant models respectively; the advantages of using each of the approaches are discussed.

Hassan et al. [56] and Shi et al. [57] presented a multi body model of the cervical spine of a 50th percentile male occupant in a crash event performing FE simulations of two generic compact sedan cars in front and rear impact collisions. The single-DOF model included only rotational visco-elastic joints, and the two-DOF model allowed axial extension. It is shown that in a frontal collision, the highest risk of injury was for the lower cervical spine, and in a rear collision the most serious injury occurs in the upper to mid cervical spine. The MBS models were validated against FE data and are in agreement with the simulation data generated from FE tests.

Portal et al. [24] developed an accident reconstruction model using 3 D rigid body mechanics. The rigid car body is modelled with nine rigid bodies and eight kinematic joints representing different vehicle components; the study includes also a motorcycle model and a human biomechanical model. The human biomechanical model features eight rigid bodies and thirteen kinematic joints which replicate different parts of the body. These models were used to study a frontal collision between a car and a motorcycle, an offset collision between two cars and a pedestrian impact.

3.3. Finite element models

Finite element models have applications in many engineering problems; a non-linear transient impact analysis of vehicle crash is one of the areas where these models produce reliable results. Thomke et al. [29] presented the evolution of crash simulations which originated in the military domains in the late 1960s. The automotive industry embraced this technique in the early 1970s, however the first full body vehicle crash simulation was conducted only in the mid-1980s. The authors highlight the importance of FEM simulations for predicting vehicle crashworthiness. Benson et al. [58] presented the calculations for crashworthiness design for automotive structures. This work laid the foundations for future FE models using different types of elements and mesh size for capturing the vehicle geometry and employing various techniques to measure the stress and strain from the simulations. Pifko and Winter [59] provided an overview of the theory behind FE, methods used to set up the governing equations based on Lagrangian equations and establish the failure criterion. They also draw parallels with the aircraft simulations to understand the application of FEM in the field of automotive safety pointing out the need for computational scientists to describe physical systems in detail prior to the solution of the associated differential equations.

Böttcher et al. [60] described the progress with the use of FE models in automotive industry acknowledging that virtual simulations developed rapidly over the last 20 years. Virtual simulations have come a long way into supporting the vehicle development process from a smaller model size and lower accuracy to computationally intensive simulation models which capture almost every part of the vehicle geometry and achieve improved prediction levels. The authors point out that along with the standard loadcases, simulations nowadays feature even active sensing techniques like airbag deployment. Airbag sensing calibration technique using virtual simulations has been demonstrated by Kiefer *et al* [61] who developed the algorithm for airbag deployment and discussed the advantages of using a virtual calibration technique for airbag sensing. The study shows that the model does not need to be too complex unlike the one for full vehicle loadcases, which reduces the computational costs. Recently, FEM has been used by the authors for determining the crash response in welded vehicles which contributed to the development of more stringent norms for improper repairs on UHSS structural members [62].

FEM have also been extensively used for developing simulation models to determine injuries to occupants in a crash. Kirkpatrick et al. [63] employed the software LS Dyna to develop and validate biofidelic models of varying degrees representing an occupant in a crash. In the automotive industry these virtual models replace real time tests with dummies or cadavers. The dummy modeling developed in this article differs from the rigid body kinematics modeling of body parts like head, neck and abdomen because it accounts for the reflexes and joints in a human body during collisions. Putra [64] presented a head-neck FE model for an average female occupant utilizing an optimization strategy. The model employs an active neck muscle controller to represent human reflexes during whiplash induced rear-impact. The FEM was also used by several authors to develop pedestrian humanoid models which simulate the behaviour of pedestrian-vehicle crashes, see Howard et al. [65], Pak et al. [66] and Meng et al. [67]. Detailed FE models of a pedestrian replicate the anthropometry of a human head and legs and proved to be useful for predicting head and leg injuries in pedestrian collision scenarios.

Design of complex elastic and inelastic material models for simulation in crash loadcases has been a challenge for engineers since the accuracy of a finite element model is highly influenced by the replication of the behaviour of non-linear inelastic material in crash simulations. Ramaswamy et al. [68] highlight the need for the development and validation of material models for the simulations of loadcases identifying the parameters that influence the robustness of quasi-static bending simulation for the evaluation and performance of material model in out-of-plane loading scenarios.

Several researchers have used FEM to validate accident reconstruction models in the recent past, see, for instance, Numata et al. [69], Yu et al. [70] and Xueyan et al. [71]. Accident scenarios can be reproduced successfully in finite element models and reconstruction models can be validated in the absence of real time crash data.

It is worth mentioning that during the last decade there have been only small advancements in the finite element methodology; however applications of computer simulations for analysing crash scenarios have increased significantly. Researchers and industry experts rely on virtual crash simulation data for a big part of the product development process because this allows to reduce the product development timeline.

3.4. Response surface models

One of the approaches to the modeling of a vehicle crash which can address the drawbacks associated with LMS models employs Artificial Neural Networks (ANN). The new approach needs training on existing crash test data so that it can be used to predict crash scenarios. The data can be generated using finite element models as well, which makes it easier to collect necessary sets of curves for different crash scenarios. However, this approach is not very efficient for developing new car models or for the optimization and design of structures because it relies on existing data and predicts the impact characteristics only by using available crash test data. Omar et al. [72] use a recurrent neural network to predict the crashworthiness of a vehicle in a frontal crash demonstrating that ANN can be trained for non-linear impact models and produce satisfying results with good confidence levels.

Several researchers used identification of parameters for developing predictive models for crash loadcases. Joseph et al. [73] suggested a parameter identification method for a thoracic impact model predicting the chest injuries. The method minimizes the error between results from the mathematical model and experimental data using an optimization algorithm demonstrating a reasonable correlation between the curves which agrees with the known results. The use of the chest injury metrics for the validation of the mathematical model instead of real time acceleration data suggests that these models could also support occupant protection loadcases.

Ghannam et al. [74] present a mathematical model to determine the initial impact velocity of full frontal vehicle-to-vehicle test modes using the Barrier Equivalent Velocity (BEV) concept. The model is based on a basic mass-spring damper; it determines the velocity of a vehicle impacting another vehicle by calculating the crush energy of both vehicles and using the conservation of energy principles to define the initial velocity of the car. Two major assumptions require that the lateral and rotational energies are negligible compared to the initial kinetic energy of the bullet vehicle and the force-deformation curves in the vehicle front end for both vehicles are linear. The authors introduce a scaling factor to account for the non-linear force deformation characteristics, the lateral and rotational energies, thus ensuring that the model predicts correctly the real test velocity. The curves are validated with physical test data and scaling factors are added if necessary to adjust the graphs. It is concluded that the rotational and lateral energies have small influence on the initial velocity.

Several studies include optimization strategies to predict crash kinematics. The methodology uses a combination of LMS and FEM to define the system and then curve fitting techniques to determine parameters. Munyazikwiye et al. [75] used a double spring mass damper model with two masses representing the front rail and the driver compartment respectively representing a car hitting a rigid barrier. The equations of motion are derived and solved with the help of a real time test crash pulse inputted into the MATLAB model. The spring stiffnesses and damper constants are derived by converting the state-space realization to transfer function. The mass distribution of the

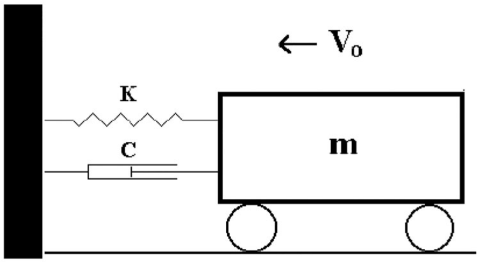

Figure 6. Vehicle impact (Kelvin) model in [78].

vehicle is verified by comparing the curve generated by the model with the physical test data to select the most feasible mass distribution based on the dynamic crush of the passenger compartment. The data from the four test cases is checked against the physical tests. The model does not account for material non-linearities and vehicle geometry for predicting the vehicle crashworthiness. However, the study gives an insight into the use of transfer functions for predicting crash injury values. Munyazikwiye et al. [76] used genetic algorithm for parameter optimization to estimate the front deformation characteristics in case of a vehicle-barrier impact and a vehicle-to-vehicle impact. Physical crash test data are used to fit the curves and determine piecewise linear spring deformation and damper characteristics. Usta et al. [77] used a genetic algorithm and RSM to design crashworthy concentric circular tubes which crush on impact absorbing the impact energy (Figure 6).

3.5. Crash pulse model

Crash pulse models have been used to represent acceleration, velocity and displacement wave forms of a structure undergoing crash. The first step is to generate crash pulse data by running physical tests or FE simulations to gather an understanding on the type of impact. Signal pre-processing is an important step in the crash pulse methodology; it includes filtering, re-sampling, synchronizing and trimming the pulse [36]. This is followed by studying the crash stages and dividing the crash pulse into regions which better represent the deformation and intrusion behaviour. Woolley [79] proposed a crash pulse model which could be divided into two regimes: the dynamic compression and rebound phases. The compression phase is defined by the maximum dynamic crush in a vehicle impacting a barrier and its velocity becoming zero. The rebound phase in a vehicle-to-vehicle crash is the time when the two vehicles start moving away from each other which leads to their separation. The solutions to the differential equations derived in this article can have varying periodic characteristics (like sine or cosine) in the compression phase, and behave like polynomial functions in the rebound phase. The author introduced a transitional trigonometric function to model a crash pulse and validated the model against real time crash.

Cheng [80] analysed crash response using wavelets and wavelet packets decomposing stationary and transient crash signals into piece-wise stationary signals. The decomposed signals can undergo decomposition analysis if the signals from a non-stationary source become stationary after decomposition. The study uses a 1997 Honda Accord crash test data and the fifth-order Daubechies wavelet (db5) to represent the motion of the structural components. The signal is compressed so that the time series contains a small number of coefficients for estimating body injuries during a crash. The authors also highlight another possible application of their methodology to predict best and worst performance in a sled test based on the impact pulse and for determining the range of performance using optimization techniques.

Crash pulse data with Haversine pulse were employed to study structural response of vehicle to impact [81]. The crash pulse was used for different speeds and it was observed that the energy absorption had a linear relationship with the displacement for a range of velocities. Similar behaviour is observed in the plots of absorbed energy vs deflection. When the data from sine model were plotted and compared with the real time crash data, acceleration curves showed good correlation. It was observed that the sine wave performed well for the full frontal barrier test while triangle pulse model showed good correlation for the offset model. The study does not explain why different models show good correlation to different loadcases; this indicates the need for more work on the loadcase comparison. Wei et al. [82] proposed a model using piecewise linear functions to describe the crash impulse based on CAE simulation data. They conclude that the model can be used to describe well the crash process exactly and can be used to predict crash under different conditions by varying the model parameters.

Prediction of crash pulses is an interesting area of research where different techniques including convolution methods [78] where a transfer function is employed for providing the output to the linear system. The vehicle crashing against a barrier can be represented as a spring damper system which is inputted with an excitation and an output response is expected; the process which transforms this input to an output in the time domain is described by the transfer function.

We recognize that this is a relatively new field of vehicle impact modeling and the opportunities to continue research in this domain should be further explored.

4. Discussion and conclusions

Each of the modelling strategies discussed in this review have been applied across different engineering domains to solve complex non-linear dynamic problems. The research focused on the improvement of these methodologies to address problems which were difficult or impossible to solve. We observe the tremendous growth of application areas whereas the development of alternative modeling strategies was strongly influenced by the availability of increased computational power. The parallel growth in computational power from supercomputers to parallel CPUs helps solve complex equations with high level of accuracy and saves time.

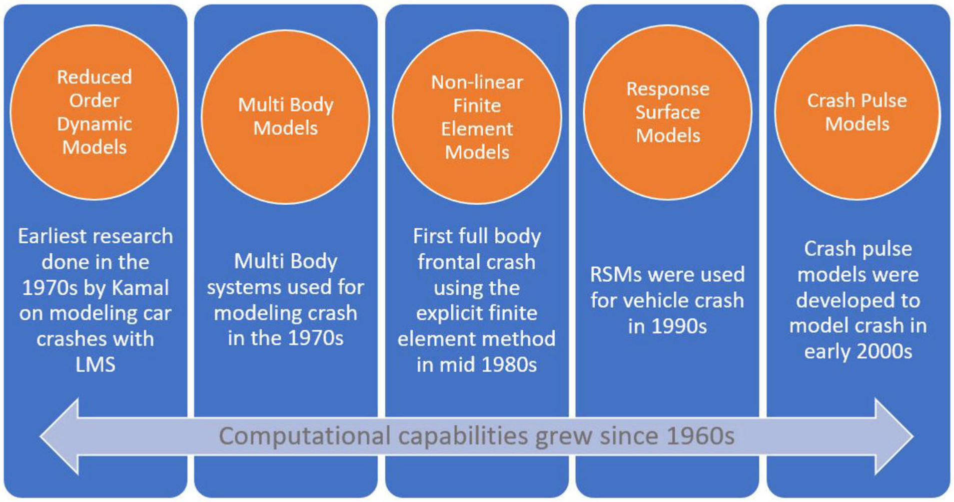

Figure 7. Evolution of vehicle crash simulations.

Although mathematical modeling of vehicle crash started to develop since the 1970s, the confidence in these models has significantly grown over the years. This is a positive trend reducing the dependence on physical crash tests. Mathematical models serve as a starting point for vehicle architecture development process providing recommendations to the studio and design teams; they are also employed during component design or for making changes in the existing components. Lumped parameter models show reasonable prediction power for frontal and side impacts. The major challenge faced in this field is the parameter identification which is partially resolved now using several identification strategies which however still have certain shortcomings. This hindered the use of LMS models in automotive industry during the development stages due to concerns related to new stringent safety regulations. The development of LMS models have slowly progressed from simple Kelvin models to complex spring-mass models with multiple springs and dampers representing the vehicle deformable features. The integration of occupant models in the car structure implies the addition of higher number of variables in the models but yields far greater understanding of the loadpaths in a crash event (Figure 7).

Response surface methods have gained momentum in the recent past as well but their application is limited due to the fact that they cannot be used for new vehicle architectures or for changing structural and occupant protection regulations with new crash scenarios. However, reinforcement learning methods can be employed to overcome these limitations. RSMs have proved to be highly effective in modifying existing designs of vehicle structures and decision making has been easier without running virtual or physical tests based on data collection and using algorithms to interpret the feasible design space. This has helped determine feasible and non-feasible design regions for many component level loadcases and makes engineering judgements easier for design teams. The emergence of efficient machine learning tools and algorithms is a promising trend in the automotive industry which can increase confidence in the reliability of the analysis of non-linear transient impacts without physical tests.

Quantitative methods, although less significant for understanding the impact mechanics in detail, provide valuable observations on the crashworthiness of a vehicle, like the available crush space or coefficient of restitution. These methods are a backbone of most mathematical models which analyse the load paths of the vehicle impact.

It is imperative for engineers and academicians to be aware of important modeling strategies and carefully access the advantages and shortcomings of each of these methods in order to apply the most appropriate one based on the considerations of accuracy and efficiency required in the solution. The automotive industry is quite fast paced in terms of developing new products and improving existing architectures, the short product development cycle triggers the need for reliable virtual modeling methodologies which predict crashworthiness performance as close as within 5-10% of the physical tests. In addition, the vehicle safety regulations have become more stringent over time as the focus on vehicle safety has gained momentum during the recent years. This in turn puts pressure on vehicle manufacturers who have to fulfill these regulations developing new products. This implies that industry experts resort to processes which are time consuming or computationally intensive to get the satisfactory confidence levels of their results; this sets a constraint on the adoption of new strategies or mathematical models for the development cycle which should be less complex yet explain the dynamics of the problem equally well. The experts look for methodologies which solve engineering problems with software automation or data science and help to come up with new products for the competitive automotive market. On the other hand, the academic community is equipped with the opportunity to explore different strategies but sometimes lacks the infrastructure and computational power to resolve complex modelling problems. There is a strong need to bridge this knowledge gap between research groups and engineering applications to ensure the improvement of the product development process.

The non-linearity of dynamic impact in a vehicle crash along with the need for energy absorbing features to establish structural integrity in the vehicle is one part of a larger problem which also involves replication of anthropometric data of a human body model under the crash impact for the analysis of the injury values for different body parts. There is a need to explore hybrid modeling strategies which could combine methodologies reviewed in this article to achieve the right balance of accuracy and efficiency in the solution; several relevant studies combining different modeling strategies aimed to overcome existing limitations.

This article provides the concise overview of the existing research and challenges arising in the mathematical modeling of vehicle crashes. We identify possible areas of improvements in this domain and emphasize a strong need to build more confidence towards replacing physical tests with simplified but accurate mathematical models. The literature review conducted in this article also highlights opportunities for improving mathematical models with vehicle structure and occupants to understand the impact dynamics under different crash scenarios. There is also a need to implement parameter identification strategies which incorporate the non-linear material properties of the front end members in the LMS models and validate them against physical test data. The growing need for infrastructural developments which allow to run finite element simulations on hundreds of parallel CPUs instead of running multiple physical tests to determine crashworthiness requirements calls for research in the area of reliable reduced order FEM models which are computationally less intensive. We also recognize the remarkable advancements in the field of machine learning and data science and the opportunities they bring for the development of robust models for predicting crash responses.

Disclosure statement

No potential conflict of interest was reported by the author(s).

Funding

The authors thank to University of Agder (Norway) for support during the research and helping us with the resources.

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