Research Article

Robust Decentralized Adaptive Neural Control for a Class of Nonaffine Nonlinear Large-Scale Systems with Unknown Dead Zones

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The problem of robust decentralized adaptive neural stabilization control is investigated for a class of nonaffine nonlinear interconnected large-scale systems with unknown dead zones. In the controller design procedure, radical basis function (RBF) neural networks are applied to approximate packaged unknown nonlinearities and then an adaptive neural decentralized controller is systematically derived without requiring any information on the boundedness of dead zone parameters (slopes and break points). It is proven that the developed control scheme can ensure that all the signals in the closed-loop system are semiglobally uniformly ultimately bounded in the sense of mean square. Simulation study is provided to further demonstrate the effectiveness of the developed control scheme.

1. Introduction

During the past several decades, a large number of research results have been obtained on the problems of stability analysis and control design for nonlinear systems because of the extensive existence of nonlinearity in the practical systems. So far, there are many control methods proposed to control design of nonlinear systems, such as adaptive backstepping control [1–4], fault-tolerant control [5–9], H_∞ control [10–16], and fuzzy control [17–21]. In the control of uncertain complex nonlinear systems, backstepping-based neural networks or fuzzy adaptive control technique is an efficient and practical strategy, and many interesting results have been obtained for a class of uncertain nonlinear strict-feedback systems; for example, see [22–37]. In addition, adaptive neural or fuzzy backstepping control approach can also be extended to control a class of nonaffine pure-feedback nonlinear systems representing a class of more general lower triangular systems, in which no affine appearance of the state variables can be used as virtual control signals. This

makes it quite difficult and more meaningful to control the pure-feedback systems. By using small-gain theorem, in [38], an adaptive neural control scheme is presented for a class of completely nonaffine pure-feedback systems. Afterwards, some researchers further consider other types of pure-feedback nonlinear systems, such as pure-feedback systems with time-delay [39] and with dead-zone [40].

Large-scale system is considered as a dynamical system which is composed of some lower-order subsystems with interconnections and often exists in many practical applications such as electric power systems, computer network systems, and aerospace systems. Decentralized adaptive control of unknown nonlinear interconnected systems has attracted much research attention because this problem is important both theoretically and practically. Up to now, many interesting adaptive decentralized control approaches for large-scale nonlinear systems have been proposed in [41, 42] and the references therein. By combining backstepping technique together with adaptive neural or fuzzy control, many research papers have been published in [28–32] for

affine nonlinear large-scale interconnected systems and in [43, 44] for nonaffine nonlinear large-scale systems. However, these researches have not considered the effect of the system input signal with dead zone. The existence of dead zone input nonlinearity severely degrades system performance. So, dead zone nonlinearity has to be considered when controller is designed in many industrial processes, such as valves, DC servo motors, and other devices. In [45], an adaptive robust control scheme is proposed for a class of nonlinear systems, in which the dead zone nonlinearity is expressed by a combination of a line and a disturbance-like term and its parameters are tuned by using adaptive technique. Then, some backstepping-based adaptive control schemes are developed for nonlinear systems with unknown dead zone input [46–50].

Based on the above observations, this paper focuses on the problem of adaptive neural decentralized control for a class of nonaffine large-scale nonlinear systems with unknown dead zones. The proposed adaptive neural controller guarantees that all the signals in the closed-loop systems remain semiglobally uniformly ultimately bounded in the sense of mean square, and the error signals eventually converge to small neighborhood around the origin. The main advantages of this research lie in that (i) the dead-zone inverse as well as the prior knowledge of bounds of dead-zone parameters (slopes and break-points) is not required; (ii) only one adaptive parameter is involved in the developed controller for each subsystem. As a result, the computational burden is significantly alleviated. In this way, the proposed control law could be easily implemented in practical applications. Simulation results are provided to further illustrate the effectiveness of the proposed control approach.

The paper is organized as follows. Section 2 provides the problem formulation and preliminaries. The control design and analysis of state feedback controller are given in Section 3, following a simulation example in Section 4. Section 5 concludes this paper.

2. Problem Formulation and Preliminaries

Consider a class of pure-feedback nonlinear interconnected large-scale systems with dead zones and N subsystems, and the ith ($i = 1, 2, \ldots, N$) subsystem is in the following form:

$$\dot{x}_{i,j} = f_{i,j}\left(\overline{x}_{i,j}, x_{i,j+1}\right) + \varphi_{i,j}\left(\overline{y}\right), \quad 1 \le j \le n_i - 1,$$

$$\dot{x}_{i,n_i} = f_{i,n_i}\left(\overline{x}_{i,n_i}, u_i\right) + \varphi_{i,n_i}\left(\overline{y}\right), \quad (1)$$

$$y_i = x_{i,1},$$

where $\overline{x}_{i,j} = [x_{i,1}, x_{i,2}, \ldots, x_{i,j}]^T$ and $\overline{y} = [y_1, y_2, \ldots, y_N]^T$. $x_i = [x_{i,1}, x_{i,2}, \ldots, x_{i,n_i}]^T \in R^{n_i}$ and $y_i \in R$ are the state variables and the output of the ith subsystem, respectively. $f_{i,j}(\cdot) : R^{j+1} \to R$, ($j = 1, 2, \ldots, n_i$) are unknown smooth nonlinear functions; $\varphi_{i,j}(\cdot) : R^N \to R$ ($j = 1, 2, \ldots, n_i$) are unknown interconnections between the ith subsystem and

other subsystems, with $f_{i,j}(0) = \varphi_{i,j}(0) = 0$; $u_i \in R$ is the output of an unknown dead zone and defined as

$$u_i = D\left(v_i\right) = \begin{cases} g_{ir}\left(v_i\right), & v_i \ge b_{ir}, \\ 0, & b_{il} < v_i < b_{ir}, \\ g_{il}\left(v_i\right), & v_i \le b_{il}, \end{cases} \quad (2)$$

where $b_{il} < 0$ and $b_{ir} > 0$ are the unknown parameters and $v_i(t) \in R$ is the input of the dead zone.

For the unknown dead zone input, the following assumption is required.

Assumption 1 (see [40]). The functions, $g_{il}(v_i)$ and $g_{ir}(v_i)$, are smooth and there exist unknown positive constants, $k_{il0}, k_{il1}, k_{ir0}$, and k_{ir1}, such that

$$0 < k_{il0} \le g'_{il}\left(v_i\right) \le k_{il1}, \quad \forall v_i \in \left(-\infty, b_{il}\right],$$

$$0 < k_{ir0} \le g'_{ir}\left(v_i\right) \le k_{ir1}, \quad \forall v_i \in \left[b_{ir}, +\infty\right), \quad (3)$$

and $\beta_{i0} \le \min\{k_{il0}, k_{ir0}\}$ is an unknown positive constant, where $g'_{il}(v_i) = (dg_{il}(z)/dz)|_{z=v_i}$ and $g'_{ir}(v_i) = (dg_{ir}(z)/dz)|_{z=v_i}$.

Remark 2. Assumption 1 is similar to the assumption in [40] where β_{i0} is a known constant. However, Assumption 1 does not require it to be known. So, Assumption 1 relaxes the limitation in [40].

Based on Assumption 1, the dead zone (2) can be rewritten as [40]

$$u_i = D\left(v_i\right) = K_i^T\left(t\right) \Phi_i\left(t\right) v_i + d_i\left(v_i\right), \quad (4)$$

where

$$\Phi_i\left(t\right) = \left[\varphi_{ir}\left(t\right), \varphi_{il}\left(t\right)\right]^T,$$

$$\varphi_{ir}\left(t\right) = \begin{cases} 1, & v_i\left(t\right) > b_{il}, \\ 0, & v_i\left(t\right) \le b_{il}, \end{cases}$$

$$\varphi_{il}\left(t\right) = \begin{cases} 1, & v_i\left(t\right) < b_{ir}, \\ 0, & v_i\left(t\right) \ge b_{ir}, \end{cases}$$

$$K_i\left(t\right) = \left[K_{ir}\left(v_i\left(t\right)\right), K_{il}\left(v_i\left(t\right)\right)\right]^T,$$

$$K_{ir}\left(v_i\left(t\right)\right) = \begin{cases} 0, & v_i\left(t\right) \le b_{il}, \\ g'_{ir}\left(\xi_{ir}\left(v_i\left(t\right)\right)\right), & b_{il} < v_i\left(t\right) < +\infty, \end{cases}$$

$$K_{il}\left(v_i\left(t\right)\right) = \begin{cases} g'_{il}\left(\xi_{il}\left(v_i\left(t\right)\right)\right), & -\infty < v_i\left(t\right) < b_{ir}, \\ 0, & v_i\left(t\right) \ge b_{ir}, \end{cases}$$

$$d_i\left(v_i\right) = \begin{cases} -g'_{ir}\left(\xi_{ir}\left(v_i\right)\right) b_{ir}, & v_i \ge b_{ir}, \\ -\left[g'_{il}\left(\xi_{il}\left(v_i\right)\right) + g'_{ir}\left(\xi_{ir}\left(v_i\right)\right)\right] v_i, & b_{il} < v_i < b_{ir}, \\ -g'_{il}\left(\xi_{il}\left(v_i\right)\right) b_{il}, & v_i \le b_{il}, \end{cases} \quad (5)$$

where $\xi_{il}(v_i) \in (v_i, b_{il})$, if $v_i < b_{il}$; $\xi_{il}(v_i) \in (b_{il}, v_i)$, if $b_{il} \le v_i < b_{ir}$; $\xi_{ir}(v_i) \in (b_{ir}, v_i)$, if $b_{ir} < v_i$; $\xi_{ir}(v_i) \in (v_i, b_{ir})$, and if $b_{il} < v_i \le b_{ir}$, and $|d_i(v_i)| \le p_i^*$, p_i^* is an unknown positive constant with $p^* = (k_{ir1} + k_{il1}) \max\{b_{ir}, -b_{il}\}$.

Remark 3. As shown in [40], there exist some other expressions for the case of linear dead zone outside the deadband, but (4) includes the most actual cases and $K_i^T(t)\Phi_i(t) \in [\beta_{i0}, k_{il1} + k_{ir1}] \subset (0, +\infty)$.

Based on mean value theorem [51], $f_{i,j}(\cdot)$ in (1) can be described as

$$f_{i,j}\left(\overline{x}_{i,j}, x_{i,j+1}\right) = f_{i,j}\left(\overline{x}_{i,j}, x_{i,j+1}^0\right) + h_{\mu_{i,j}}\left(x_{i,j+1} - x_{i,j+1}^0\right),$$

$$f_{i,n_i}\left(\overline{x}_{i,n_i}, u_i\right) = f_{i,n_i}\left(\overline{x}_{i,n_i}, u_i^0\right) + h_{\mu_{i,n_i}}\left(u_i - u_i^0\right),$$

(6)

where smooth function $f_{i,j}(\cdot)$ is explicitly analyzed between $f_{i,j}(\overline{x}_{i,j}, x_{i,j+1})$ and $f_{i,j}(\overline{x}_{i,j}, x_{i,j+1}^0)$, $h_{\mu_{i,j}} := h_{i,j}(\overline{x}_{i,j}, x_{\mu_{i,j}}) = (\partial f_{i,j}(\overline{x}_{i,j}, x_{i,j+1})/\partial x_{i,j+1})|_{x_{i,j+1}=x_{\mu_{i,j}}}, x_{i,n_i+1} = u_i, x_{\mu_{i,j}} = \mu_{i,j}x_{i,j+1} + (1 - \mu_{i,j})x_{i,j+1}^0, 0 < \mu_{i,j} < 1, i = 1, 2, \ldots, N, j = 1, 2, \ldots, n_i$.

Next, by substituting (6) and (4) into (1), and choosing $x_{i,j+1}^0 = 0, u_i^0 = 0$, we obtain

$$\dot{x}_{i,j} = h_{\mu_{i,j}}x_{i,j+1} + f_{i,j}\left(\overline{x}_{i,j}, 0\right) + \varphi_{i,j}\left(\overline{y}\right), \quad 1 \leq j \leq n_i - 1,$$

$$\dot{x}_{i,n_i} = h_{\mu_{i,n_i}} K_i^T(t)\Phi_i(t)v_i + h_{\mu_{i,n_i}}d_i(v_i) + f_{i,n_i}\left(\overline{x}_{i,n_i}, 0\right) + \varphi_{i,n_i}\left(\overline{y}\right),$$

$$y_i = x_{i,1}.$$

(7)

Assumption 4 (see [40]). The signs of $h_{\mu_{i,j}}$, $1 \leq i \leq N$, $1 \leq j \leq n_i$, do not change, and there exist unknown constants b_m and c such that

$$0 < b_m \leq \left|h_{\mu_{i,j}}\right| \leq c < \infty.$$

(8)

Remark 5. Assumption 4 means that $h_{\mu_{i,j}}$ are strictly either positive or negative. Without loss of generality, it is assumed that $0 < b_m \leq h_{\mu_{i,j}}$. In addition, by means of Assumptions 1 and 4, it can be further supposed that

$$0 < b \leq h_{\mu_{i,j}}, \qquad 1 \leq i \leq N, \qquad 1 \leq j \leq n_i - 1,$$

$$0 < b \leq h_{\mu_{i,n_i}} K_i^T(t)\Phi_i(t),$$

(9)

where $b = \min\{b_m, b_m\beta_{i0}\}$ is an unknown constant.

Assumption 6 (see [31]). For uncertain nonlinear functions $\varphi_{i,j}(\overline{y})$ in (1), there exist unknown smooth functions $\varphi_{i,j,l}(y_l)$ such that for $1 \leq i \leq N$, $1 \leq j \leq n_i$,

$$\left|\varphi_{i,j}\left(\overline{y}\right)\right|^2 \leq \sum_{l=1}^{N}\varphi_{i,j,l}^2(y_l),$$

(10)

where $\varphi_{i,j,l}(0) = 0$, $l = 1, 2, \ldots, N$.

Noting that $\varphi_{i,j,l}(y_l)$ in (10) is smooth function with $\varphi_{i,j,l}(0) = 0$, so there exist unknown smooth functions $\overline{\varphi}_{i,j,l}(y_l)$ such that

$$\left|\varphi_{i,j}\left(\overline{y}\right)\right|^2 \leq \sum_{l=1}^{N}y_l^2\overline{\varphi}_{i,j,l}^2(y_l).$$

(11)

In what follows, RBF neural networks are applied to approximate any continuous function $f(X): R^n \rightarrow R$,

$$f_{nn}(X) = W^T\phi(X),$$

(12)

where $X \in \Omega_Z \subset R^q$ is the input vector with q being the neural networks input dimension, weight vector $W = [w_1, w_2, \ldots, w_l]^T \in R^l$, $l > 1$ is the neural networks node number, and $\phi(X) = [\phi_1(X), \phi_2(X), \ldots, \phi_l(X)]^T$ means the basis function vector with $\phi_i(X)$ being chosen as the commonly used Gaussian function of the form

$$\phi_i(X) = \exp\left[-\frac{(X - \mu_i)^T(X - \mu_i)}{\eta_i^2}\right], \quad i = 1, 2, \ldots, l,$$

(13)

where $\mu_i = [\mu_{i1}, \mu_{i2}, \ldots, \mu_{iq}]^T$ is the center of the receptive field and η_i is the width of the Gaussian function. In [52], it has been indicated that with sufficiently large node number l, the RBF neural networks (12) can approximate any continuous function $f(X)$ over a compact set $\Omega_Z \subset R^q$ to arbitrary any accuracy $\varepsilon > 0$ as

$$f(X) = W^{*T}\phi(X) + \delta(X), \quad \forall z \in \Omega_z \in R^q,$$

(14)

where W^* is the ideal constant weight vector and defined as

$$W^* := \arg\min_{W \in R^l}\left\{\sup_{Z \in \Omega_Z}\left|f(X) - W^T\phi(X)\right|\right\},$$

(15)

and $\delta(X)$ denotes the approximation error and satisfies $|\delta(X)| \leq \varepsilon$.

Lemma 7 (see [53]). *Consider the Gaussian RBF networks (12) and (2). Let $\rho := (1/2)\min_{i \neq j}\|\mu_i - \mu_j\|$; then an upper bound of $\|\phi(X)\|$ is taken as*

$$\|\phi(X)\| \leq \sum_{k=0}^{\infty}3q(k + 2)^{q-1}e^{-2\rho^2k^2/\eta^2} := s.$$

(16)

It has been shown in [38] that the constant s in Lemma 7 has a limited value and is independent of the variable X and the dimension of neural weights l.

3. Adaptive Neural Control Design

In this section, an adaptive neural backstepping control scheme will be developed. During the controller design, for the ith subsystem, RBF neural network $W_{i,j}^T\phi(X_{i,j})$ will be utilized to approximate the packaged unknown function $F_{i,j}(X_{i,j})$ at step j. Both the virtual control signals $\alpha_{i,j}(X_{i,j})$

and adaption laws $\dot{\widehat{\theta}}_i$ will be constructed in the following forms:

$$\alpha_{i,j}\left(X_{i,j}\right) = -k_{i,j}z_{i,j} - \frac{1}{2a_{i,j}^2}z_{i,j}\widehat{\theta}_i\phi_{i,j}^T\left(X_{i,j}\right)\phi_{i,j}\left(X_{i,j}\right), \quad (17)$$

$$\dot{\widehat{\theta}}_i = \sum_{j=1}^{n_i}\frac{\lambda_i}{2a_{i,j}^2}z_{i,j}^2\phi_{i,j}^T\left(X_{i,j}\right)\phi_{i,j}\left(X_{i,j}\right) - \gamma_i\widehat{\theta}_i, \quad (18)$$

where $i = 1, 2, \ldots, N$, $j = 1, 2, \ldots, n_i$, $k_{i,j}$, $a_{i,j}$, λ_i, and γ_i are positive design parameters, $X_{i,1} = x_{i,1}, X_{i,j} = [\overline{x}_{i,j}^T, \widehat{\theta}_i]^T$, $(j = 2, \ldots, n_i)$ with $\overline{x}_{i,j} = [x_{i,1}, x_{i,2}, \ldots, x_{i,j}]^T$, and $z_{i,j}$ satisfy the following coordinate transformation:

$$z_{i,j} = x_{i,j} - \alpha_{i,j-1} \quad (19)$$

with $\alpha_{i,0} = 0$. $\widehat{\theta}_i$ is used to estimate an unknown constant θ_i which will be defined as

$$\theta_i = \max\left\{\frac{1}{b}\left\|W_{i,j}\right\|^2; j = 1, 2, \ldots, n_i\right\}, \quad (20)$$

where b is specified in Remark 5, and $\|W_{i,j}\|$ will be defined at the jth step. Specially, α_{i,n_i} is the actual control input signal u_i.

For simplicity, in the following, the time variable t and the state vector $\overline{x}_{i,j}$ will be omitted from the corresponding functions and let $\phi_{i,j}(X_{i,j}) = \phi_{i,j}$.

Step 1. Based on $z_{i,1} = x_{i,1}$, $z_{i,2} = x_{i,2} - \alpha_{i,1}$, the error dynamic $z_{i,1}$ satisfies

$$\dot{z}_{i,1} = h_{\mu_{i,1}}z_{i,2} + h_{\mu_{i,1}}\alpha_{i,1} + f_{i,j}\left(\overline{x}_{i,j}, 0\right) + \varphi_{i,1}\left(\overline{y}\right). \quad (21)$$

Consider a Lyapunov function candidate as $V_{i,1} = (1/2)z_{i,1}^2 = (1/2)y_i^2$. Then, the time derivative of V_1 along (21) satisfies

$$\dot{V}_{i,1} = y_i\left(h_{\mu_{i,1}}z_{i,2} + h_{\mu_{i,1}}\alpha_{i,1} + f_{i,1}\left(\overline{x}_{i,1}, 0\right) + \varphi_{i,1}\left(\overline{y}\right)\right). \quad (22)$$

By using (11) and the completion of squares, we get

$$y_i\varphi_{i,1}\left(\overline{y}\right) \le \frac{1}{2}y_i^2 + \frac{1}{2}\sum_{l=1}^{N}y_l^2\overline{\varphi}_{i,1,l}^2\left(y_l\right). \quad (23)$$

Further, (22) can be rewritten as

$$\dot{V}_{i,1} = y_i\left(h_{\mu_{i,1}}z_{i,2} + h_{\mu_{i,1}}\alpha_{i,1} + f_{i,1}\left(\overline{x}_{i,1}, 0\right) + \frac{1}{2}y_i\right)$$
$$+ \frac{1}{2}\sum_{l=1}^{N}y_l^2\overline{\varphi}_{i,1,l}^2\left(y_l\right). \quad (24)$$

Step j ($2 \le j \le n_i - 1$). According to (19), one has

$$\dot{z}_{i,j} = h_{\mu_{i,j}}z_{i,j+1} + h_{\mu_{i,j}}\alpha_{i,j} + f_{i,j}\left(\overline{x}_{i,j}, 0\right) + \varphi_{i,j}\left(\overline{y}\right) - \dot{\alpha}_{i,j-1}, \quad (25)$$

where

$$\dot{\alpha}_{i,j-1} = \sum_{k=1}^{j-1}\frac{\partial\alpha_{i,j-1}}{\partial x_{i,k}}\left(f_{i,k}\left(\overline{x}_{i,k+1}\right) + \varphi_{i,k}\left(\overline{y}\right)\right) + \frac{\partial\alpha_{i,j-1}}{\partial\widehat{\theta}_i}\dot{\widehat{\theta}}_i. \quad (26)$$

Take Lyapunov function $V_{i,j} = (1/2)z_{i,j}^2$; then we have

$$\dot{V}_{i,j} = z_{i,j}\left(g_{\mu_{i,j}}z_{i,j+1} + g_{\mu_{i,j}}\alpha_{i,j} + f_{i,j}\left(\overline{x}_{i,j}, 0\right) + \varphi_{i,j}\left(\overline{y}\right)\right.$$
$$\left. - \sum_{k=1}^{j-1}\frac{\partial\alpha_{i,j-1}}{\partial x_{i,k}}\left(f_{i,k}\left(\overline{x}_{i,k+1}\right) + \varphi_{i,k}\left(\overline{y}\right)\right) - \frac{\partial\alpha_{i,j-1}}{\partial\widehat{\theta}_i}\dot{\widehat{\theta}}_i\right). \quad (27)$$

Following the same line as the procedures used in (23), one has

$$-z_{i,j}\sum_{k=1}^{j-1}\frac{\partial\alpha_{i,j-1}}{\partial x_{i,k}}\varphi_{i,k}\left(\overline{y}\right)$$
$$\le \frac{1}{2}z_{i,j}^2\sum_{k=1}^{j-1}\left(\frac{\partial\alpha_{i,j-1}}{\partial x_{i,k}}\right)^2 + \frac{1}{2}\sum_{k=1}^{j-1}\sum_{l=1}^{N}y_l^2\overline{\varphi}_{i,k,l}^2\left(y_l\right), \quad (28)$$

$$z_{i,j}\varphi_{i,j}\left(\overline{y}\right) \le \frac{1}{2}z_{i,j}^2 + \frac{1}{2}\sum_{l=1}^{N}y_l^2\overline{\varphi}_{i,j,l}^2\left(y_l\right). \quad (29)$$

It can be easily verified from (27) to (29) that

$$\dot{V}_{i,j} \le z_{i,j}\left(h_{\mu_{i,j}}z_{i,j+1} + h_{\mu_{i,j}}\alpha_{i,j} + f_{i,j}\left(\overline{x}_{i,j}, 0\right)\right.$$
$$- \sum_{k=1}^{j-1}\frac{\partial\alpha_{i,j-1}}{\partial x_{i,k}}f_{i,k}\left(\overline{x}_{i,k+1}\right) + \frac{1}{2}z_{i,j}$$
$$\left. + \frac{1}{2}z_{i,j}\sum_{k=1}^{j-1}\left(\frac{\partial\alpha_{i,j-1}}{\partial x_{i,k}}\right)^2 - \frac{\partial\alpha_{i,j-1}}{\partial\widehat{\theta}_i}\dot{\widehat{\theta}}_i\right)$$
$$+ \frac{1}{2}\sum_{k=1}^{j}\sum_{l=1}^{N}y_l^2\overline{\varphi}_{i,k,l}^2\left(y_l\right). \quad (30)$$

Step n_i. Similar to (25), the following equation can be obtained:

$$\dot{z}_{i,n_i} = h_{\mu_{i,n_i}}K_i^T(t)\Phi_i(t)v_i + h_{\mu_{i,n_i}}d_i(v_i) + f_{i,n_i}\left(\overline{x}_{i,n_i}, 0\right)$$
$$+ \varphi_{i,n_i}\left(\overline{y}\right) - \dot{\alpha}_{i,n_i-1}, \quad (31)$$

where $\dot{\alpha}_{i,n_i-1}$ is shown in (26) with $j = n_i$. Choose a Lyapunov function as

$$V_{i,n_i} = \frac{1}{2}z_{i,n_i}^2 + \frac{b}{2\lambda_i}\widetilde{\theta}_i^2, \quad (32)$$

where $\widetilde{\theta}_i = \theta_i - \widehat{\theta}_i$ is the parameter error and $\lambda_i > 0$ is a design parameter.

Then, the following result holds.

$$
\begin{aligned}
\dot{V}_{i,n_i} = z_{i,n_i} &\left(h_{\mu_{i,n_i}} K_i^T(t) \Phi_i(t) v_i + h_{\mu_{i,n_i}} d_i(v_i) \right. \\
&+ f_{i,n_i}(\overline{x}_{i,n_i}, 0) + \varphi_{i,n_i}(\overline{y}) - \frac{\partial \alpha_{i,n_i-1}}{\partial \widehat{\theta}_i} \dot{\widehat{\theta}}_i \\
&\left. - \sum_{k=1}^{n_i-1} \frac{\partial \alpha_{i,n_i-1}}{\partial x_{i,k}} \left(f_{i,k}(\overline{x}_{i,k+1}) + \varphi_{i,k}(\overline{y}) \right) \right) \\
&- \frac{b}{\lambda_i} \widetilde{\theta}_i \dot{\widehat{\theta}}_i.
\end{aligned}
\tag{33}
$$

Repeating the same derivations as (28)–(30) produces

$$
\begin{aligned}
\dot{V}_{i,n_i} \leq z_{i,n_i} &\left(h_{\mu_{i,n_i}} K_i^T(t) \Phi_i(t) v_i + h_{\mu_{i,n_i}} d_i(v_i) \right. \\
&+ f_{i,n_i}(\overline{x}_{i,n_i}, 0) - \sum_{k=1}^{n_i-1} \frac{\partial \alpha_{i,n_i-1}}{\partial x_{i,k}} f_{i,k}(\overline{x}_{i,k+1}) \\
&\left. + \frac{1}{2} z_{i,n_i} \sum_{k=1}^{n_i-1} \left(\frac{\partial \alpha_{i,n_i-1}}{\partial x_{i,k}} \right)^2 + \frac{1}{2} z_{i,n_i} - \frac{\partial \alpha_{i,n_i-1}}{\partial \widehat{\theta}_i} \dot{\widehat{\theta}}_i \right) \\
&+ \frac{1}{2} \sum_{k=1}^{n_i} \sum_{l=1}^{N} y_l^2 \overline{\varphi}_{i,k,l}^2(y_l) - \frac{b}{\lambda_i} \widetilde{\theta}_i \dot{\widehat{\theta}}_i.
\end{aligned}
\tag{34}
$$

Now, consider a Lyapunov function for the whole system as

$$
V = \sum_{i=1}^{N} \sum_{j=1}^{n_i} V_{i,j} = \sum_{i=1}^{N} \left(\frac{1}{2} y_i^2 + \sum_{j=2}^{n_i} \frac{1}{2} z_{i,j}^2 + \frac{b}{2\lambda_i} \widetilde{\theta}_i^2 \right).
\tag{35}
$$

Then, combining (24) together with (30) and (34), one has

$$
\begin{aligned}
\dot{V} \leq \sum_{i=1}^{N} y_i &\left\{ h_{\mu_{i,1}} \alpha_{i,1} + f_{i,1}(\overline{x}_{i,1}, 0) + \frac{1}{2} y_i \right. \\
&\left. + \frac{1}{2} y_i \sum_{l=1}^{N} \sum_{s=1}^{n_l} \sum_{k=1}^{s} \overline{\varphi}_{l,k,i}^2(y_i) \right\} \\
+ \sum_{i=1}^{N} \sum_{j=2}^{n_i-1} z_{i,j} &\left\{ h_{\mu_{i,j}} \alpha_{i,j} + h_{\mu_{i,j}} z_{i,j-1} \right. \\
&+ f_{i,j}(\overline{x}_{i,j}, 0) - \sum_{k=1}^{j-1} \frac{\partial \alpha_{i,j-1}}{\partial x_{i,k}} f_{i,k}(\overline{x}_{i,k+1}) \\
&\left. + \frac{1}{2} z_{i,j} + \frac{1}{2} z_{i,j} \sum_{k=1}^{j-1} \left(\frac{\partial \alpha_{i,j-1}}{\partial x_{i,k}} \right)^2 \right\}
\end{aligned}
$$

$$
\begin{aligned}
+ \sum_{i=1}^{N} z_{i,n_i} &\left\{ h_{\mu_{i,n_i}} K_i^T(t) \Phi_i(t) v_i + h_{\mu_{i,n_i}} d_i(v_i) \right. \\
&+ f_{i,n_i}(\overline{x}_{i,n_i}, 0) - \sum_{k=1}^{n_i-1} \frac{\partial \alpha_{i,n_i-1}}{\partial x_{i,k}} f_{i,k}(\overline{x}_{i,k+1}) \\
&\left. + \frac{1}{2} z_{i,n_i} \sum_{k=1}^{n_i-1} \left(\frac{\partial \alpha_{i,n_i-1}}{\partial x_{i,k}} \right)^2 + \frac{1}{2} z_{i,n_i} \right\} \\
- \sum_{i=1}^{N} \sum_{j=2}^{n_i} z_{i,j} &\frac{\partial \alpha_{i,j-1}}{\partial \widehat{\theta}_i} \dot{\widehat{\theta}}_i - \sum_{i=1}^{N} \frac{b}{\lambda_i} \widetilde{\theta}_i \dot{\widehat{\theta}}_i,
\end{aligned}
\tag{36}
$$

where the fact of

$$
\begin{aligned}
\frac{1}{2} \sum_{i=1}^{N} \sum_{s=1}^{n_i} \sum_{k=1}^{s} \sum_{l=1}^{N} y_l^2 \overline{\varphi}_{i,k,l}^2(y_l) \\
= \frac{1}{2} \sum_{i=1}^{N} \sum_{l=1}^{N} \sum_{s=1}^{n_i} \sum_{k=1}^{s} y_i^2 \overline{\varphi}_{l,k,i}^2(y_i)
\end{aligned}
\tag{37}
$$

has been used in above inequality.

By using adaptive laws defined in (18) and rearranging sequence, we can obtain

$$
\begin{aligned}
- \sum_{i=1}^{N} \sum_{j=2}^{n_i} & z_{i,j} \frac{\partial \alpha_{i,j-1}}{\partial \widehat{\theta}_i} \dot{\widehat{\theta}}_i \\
= - \sum_{i=1}^{N} \sum_{j=2}^{n_i} & z_{i,j} \frac{\partial \alpha_{i,j-1}}{\partial \widehat{\theta}_i} \left(\sum_{k=1}^{n_i} \frac{\lambda_i}{2a_{i,k}^2} z_{i,k}^2 \phi_{i,k}^T \phi_{i,k} - \gamma_i \widehat{\theta}_i \right) \\
= \sum_{i=1}^{N} \sum_{j=2}^{n_i} & z_{i,j} \frac{\partial \alpha_{i,j-1}}{\partial \widehat{\theta}_i} \gamma_i \widehat{\theta}_i \\
- \sum_{i=1}^{N} \sum_{j=2}^{n_i} & z_{i,j} \frac{\partial \alpha_{i,j-1}}{\partial \widehat{\theta}_i} \sum_{k=1}^{j-1} \frac{\lambda_i}{2a_{i,k}^2} z_{i,k}^2 \phi_{i,k}^T \phi_{i,k} \\
- \sum_{i=1}^{N} \sum_{j=2}^{n_i} & z_{i,j} \frac{\partial \alpha_{i,j-1}}{\partial \widehat{\theta}_i} \sum_{k=j}^{n_i} \frac{\lambda_i}{2a_{i,k}^2} z_{i,k}^2 \phi_{i,k}^T \phi_{i,k} \\
\leq \sum_{i=1}^{N} \sum_{j=2}^{n_i} & z_{i,j} \frac{\partial \alpha_{i,j-1}}{\partial \widehat{\theta}_i} \gamma_i \widehat{\theta}_i \\
- \sum_{i=1}^{N} \sum_{j=2}^{n_i} & z_{i,j} \frac{\partial \alpha_{i,j-1}}{\partial \widehat{\theta}_i} \sum_{k=1}^{j-1} \frac{\lambda_i}{2a_{i,k}^2} z_{i,k}^2 \phi_{i,k}^T \phi_{i,k} \\
+ \sum_{i=1}^{N} \sum_{j=2}^{n_i} & \frac{\lambda_i}{2a_{i,j}^2} z_{i,j}^2 \left(\sum_{k=2}^{j} \left| z_{i,k} \frac{\partial \alpha_{i,k-1}}{\partial \widehat{\theta}_i} \right| \right).
\end{aligned}
\tag{38}
$$

Taking $|d_i(v_i)| \leq p_i^*$ into account, one has

$$
z_{i,n_i} h_{\mu_{i,n_i}} d_i(v_i) \leq \frac{1}{2} z_{i,n_i}^2 h_{\mu_{i,n_i}}^2 + \frac{1}{2} p_i^{*2}.
\tag{39}
$$

By substituting (38) and (39) into (36), it follows

$$\dot{V} \leq \sum_{i=1}^{N} y_i \left(h_{\mu_{i,1}} \alpha_{i,1} + F_{i,1}\left(X_{i,1}\right) \right)$$

$$+ \sum_{i=1}^{N} \sum_{j=2}^{n_i-1} z_{i,j} \left(h_{\mu_{i,j}} \alpha_{i,j} + F_{i,j}\left(X_{i,j}\right) \right)$$

$$+ \sum_{i=1}^{N} z_{i,n_i} \left(h_{\mu_{i,n_i}} K_i^T(t) \Phi_i(t) v_i + F_{i,n_i}\left(X_{i,n_i}\right) \right) \tag{40}$$

$$+ \frac{1}{2} \sum_{i=1}^{N} p_i^{*2} - \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{n_i} z_{i,j}^2 - \sum_{i=1}^{N} \frac{b}{\lambda_i} \widetilde{\theta}_i \dot{\widehat{\theta}}_i,$$

where the packaged functions $F_{i,j}(X_{i,j})$, $i = 1, 2, \ldots, N$, are specified as

$$F_{i,1}\left(X_{i,1}\right) = f_{i,1}\left(\overline{x}_{i,1}, 0\right) + y_i + \frac{1}{2} y_i \sum_{l=1}^{N} \sum_{s=1}^{n_l} \sum_{k=1}^{s} \overline{\varphi}_{l,k,i}^2\left(y_i\right),$$

$$F_{i,j}\left(X_{i,j}\right) = h_{\mu_{i,j}} z_{i,j-1} + f_{i,j}\left(\overline{x}_{i,j}, 0\right) - \sum_{k=1}^{j-1} \frac{\partial \alpha_{i,j-1}}{\partial x_{i,k}} f_{i,k}\left(\overline{x}_{i,k+1}\right)$$

$$+ z_{i,j} + \frac{1}{2} z_{i,j} \sum_{k=1}^{j-1} \left(\frac{\partial \alpha_{i,j-1}}{\partial x_{i,k}} \right)^2 + \frac{\partial \alpha_{i,j-1}}{\partial \widehat{\theta}_i} \gamma_i \widehat{\theta}_i$$

$$- \frac{\partial \alpha_{i,j-1}}{\partial \widehat{\theta}_i} \sum_{k=1}^{j-1} \frac{\lambda_i}{2a_{i,k}^2} z_{i,k}^2 \phi_{i,k}^T \phi_{i,k}$$

$$+ \frac{\lambda_i}{2a_{i,j}^2} z_{i,j} \left(\sum_{k=2}^{j} \left| z_{i,k} \frac{\partial \alpha_{i,k-1}}{\partial \widehat{\theta}_i} \right| \right),$$

$$j = 2, \ldots, n_i - 1,$$

$$F_{i,n_i}\left(X_{i,n_i}\right) = f_{i,n_i}\left(\overline{x}_{i,n_i}, 0\right) - \sum_{k=1}^{n_i-1} \frac{\partial \alpha_{i,n_i-1}}{\partial x_{i,k}} f_{i,k}\left(\overline{x}_{i,k+1}\right)$$

$$+ \frac{1}{2} z_{i,n_i} \sum_{k=1}^{n_i-1} \left(\frac{\partial \alpha_{i,n_i-1}}{\partial x_{i,k}} \right)^2 + \frac{1}{2} z_{i,n_i} h_{\mu_{i,n_i}}^2 + z_{i,n_i}$$

$$+ \frac{\partial \alpha_{i,n_i-1}}{\partial \widehat{\theta}_i} \gamma_i \widehat{\theta}_i - \frac{\partial \alpha_{i,n_i-1}}{\partial \widehat{\theta}_i} \sum_{k=1}^{n_i-1} \frac{\lambda_i}{2a_{i,k}^2} z_{i,k}^2 \phi_{i,k}^T \phi_{i,k}$$

$$+ \frac{\lambda_i}{2a_{i,n_i}^2} z_{i,n_i} \left(\sum_{k=2}^{n_i} \left| z_{i,k} \frac{\partial \alpha_{i,k-1}}{\partial \widehat{\theta}_i} \right| \right). \tag{41}$$

Because $f_{i,j}$, $h_{\mu_{i,j}}$, and $\overline{\varphi}_{l,k,i}$ are unknown smooth functions, $F_{i,j}(X_{i,j})$, $i = 1, 2, \ldots, N$, $j = 1, 2, \ldots, n_i$ cannot be used to define the virtual control input $\alpha_{i,j}$ and the real controller u_i. Then, RBF neural network $W_{i,j}^T \phi_{i,j}(X_{i,j})$ is utilized to model $F_{i,j}(X_{i,j})$, such that for a given positive constant $\varepsilon_{i,j}$,

$$F_{i,j}\left(X_{i,j}\right) = W_{i,j}^T \phi_{i,j}\left(X_{i,j}\right) + \delta_{i,j}\left(X_{i,j}\right), \tag{42}$$

where $\delta_{i,j}(X_{i,j})$ is approximation error and satisfies $|\delta_{i,j}(X_{i,j})| < \varepsilon_{i,j}$. Subsequently, by Young's inequality, one has

$$z_{i,j} F_{i,j}\left(X_{i,j}\right)$$

$$= z_{i,j} \frac{W_{i,j}^T}{\|W_{i,j}\|} \phi_{i,j} \|W_{i,j}\| + z_{i,j} \delta_{i,j}\left(X_{i,j}\right) \tag{43}$$

$$\leq \frac{b}{2a_{i,j}^2} z_{i,j}^2 \theta_i \phi_{i,j}^T \phi_{i,j} + \frac{1}{2} a_{i,j}^2 + \frac{1}{2} z_{i,j}^2 + \frac{1}{2} \varepsilon_{i,j}^2,$$

where $i = 1, 2, \ldots, N$, $j = 1, 2, \ldots, n_i$ and the unknown constant $\theta_i = \max\{(1/b)\|W_{i,j}\|^2; j = 1, 2, \ldots, n_i\}$.

Substituting (42) into (40) and using (43) produces

$$\dot{V} \leq \sum_{i=1}^{N} z_{i,1} \left(h_{\mu_{i,1}} \alpha_{i,1} + \frac{b}{2a_{i,1}^2} z_{i,1} \theta_i \phi_{i,1}^T \phi_{i,1} \right)$$

$$+ \sum_{i=1}^{N} \sum_{j=2}^{n_i-1} z_{i,j} \left(h_{\mu_{i,j}} \alpha_{i,j} + \frac{b}{2a_{i,j}^2} z_{i,j} \theta_i \phi_{i,j}^T \phi_{i,j} \right)$$

$$+ \sum_{i=1}^{N} z_{i,n_i} \left(h_{\mu_{i,n_i}} K_i^T(t) \Phi_i(t) v_i + \frac{b}{2a_{i,n_i}^2} z_{i,n_i} \theta_i \phi_{i,n_i}^T \phi_{i,n_i} \right)$$

$$+ \frac{1}{2} \sum_{i=1}^{N} p_i^{*2} + \sum_{i=1}^{N} \sum_{j=1}^{n_i} \left(\frac{1}{2} a_{i,j}^2 + \frac{1}{2} \varepsilon_{i,j}^2 \right) - \sum_{i=1}^{N} \frac{b}{\lambda_i} \widetilde{\theta}_i \dot{\widehat{\theta}}_i. \tag{44}$$

Now, construct the virtual control signals $\alpha_{i,j} = -k_{i,j} z_{i,j} - (1/2a_{i,j}^2) z_{i,j} \widehat{\theta}_i \phi_{i,j}^T \phi_{i,j}$, where $i = 1, 2, \ldots, N$, $j = 1, 2, \ldots, n_i$, and $\alpha_{i,n_i} = u_i$. Then, by using (9), we can obtain

$$z_{i,j} h_{\mu_{i,j}} \alpha_{i,j} \leq -k_{i,j} b z_{i,j}^2 - \frac{b}{2a_{i,j}^2} z_{i,j}^2 \widehat{\theta}_i \phi_{i,j}^T \phi_{i,j},$$

$$j = 1, \ldots, n_i - 1,$$

$$z_{i,n_i} h_{\mu_{i,n_i}} K_i^T(t) \Phi_i(t) v_i \leq -k_{i,n_i} b z_{i,n_i}^2 - \frac{b}{2a_{i,n_i}^2} z_{i,n_i}^2 \widehat{\theta}_i \phi_{i,n_i}^T \phi_{i,n_i}, \tag{45}$$

where $i = 1, 2, \ldots, N$.

Substituting (45) into (44) and using adaptive laws $\dot{\theta}_i$ in (18) result in

$$\dot{V} \leq \sum_{i=1}^{N} \sum_{j=1}^{n_i} z_{i,j} \left(-k_{i,j} b z_{i,j} + \frac{b}{2a_{i,j}^2} z_{i,j} \widetilde{\theta}_i \phi_{i,j}^T \phi_{i,j} \right)$$

$$+ \sum_{i=1}^{N} \sum_{j=1}^{n_i} \left(\frac{1}{2} a_{i,j}^2 + \frac{1}{2} \varepsilon_{i,j}^2 \right) + \frac{1}{2} \sum_{i=1}^{N} p_i^{*2}$$

$$- \sum_{i=1}^{N} \frac{b}{\lambda_i} \widetilde{\theta}_i \left(\sum_{j=1}^{n_i} \frac{\lambda_i}{2a_{i,j}^2} z_{i,j}^2 \phi_{i,j}^T \phi_{i,j} - \gamma_i \widehat{\theta}_i \right)$$

$$\leq -\sum_{i=1}^{N}\sum_{j=1}^{n_i} k_{i,j} b z_{i,j}^2 + \sum_{i=1}^{N} \frac{\gamma_i b}{\lambda_i} \widetilde{\theta}_i \widehat{\theta}_i$$

$$+ \sum_{i=1}^{N}\sum_{j=1}^{n_i}\left(\frac{1}{2}a_{i,j}^2 + \frac{1}{2}\varepsilon_{i,j}^2\right) + \frac{1}{2}\sum_{i=1}^{N} p_i^{*2}$$

$$\leq -\sum_{i=1}^{N}\left(\sum_{j=1}^{n_i} k_{i,j} b z_{i,j}^2 + \frac{\gamma_i b}{2\lambda_i}\widetilde{\theta}_i^2\right)$$

$$+ \sum_{i=1}^{N}\sum_{j=1}^{n_i}\left(\frac{1}{2}a_{i,j}^2 + \frac{1}{2}\varepsilon_{i,j}^2 + \frac{\gamma_i b}{2\lambda_i}\theta_i^2\right) + \frac{1}{2}\sum_{i=1}^{N} p_i^{*2},$$

$$(46)$$

where $\widetilde{\theta}_i \widehat{\theta}_i \leq -(1/2)\widetilde{\theta}_i^2 + (1/2)\theta_i^2$ has been used in the above inequality.

Define

$$a_0 = \min\left\{2k_{i,j} b, \gamma_i, i = 1, 2, \ldots, N, \ j = 1, 2, \ldots, n_i\right\};$$

$$(47)$$

$$b_0 = \sum_{i=1}^{N}\sum_{j=1}^{n_i}\left(\frac{1}{2}a_{i,j}^2 + \frac{1}{2}\varepsilon_{i,j}^2 + \frac{\gamma_i b}{2\lambda_i}\theta_i^2\right) + \frac{1}{2}\sum_{i=1}^{N} p_i^{*2},$$

we have

$$\dot{V} \leq -a_0 V + b_0, \quad t \geq 0. \tag{48}$$

Furthermore, multiplying (48) by $e^{a_0 t}$ yields

$$\frac{d}{dt}\left(V e^{a_0 t}\right) \leq b_0 e^{a_0 t}. \tag{49}$$

Integrating (49) over $[0, t]$ gives

$$0 \leq V(t) \leq \frac{b_0}{a_0} + \left(V(0) - \frac{b_0}{a_0}\right) e^{-a_0 t}, \tag{50}$$

which implies that

$$V(t) \leq V(0) + \frac{b_0}{a_0}, \quad \forall t > 0, \tag{51}$$

which means that all the signals in the closed-loop system are semiglobally uniformly ultimately bounded.

Furthermore, it is easily obtained that

$$V(t) \leq \frac{b_0}{a_0}, \quad t \longrightarrow +\infty. \tag{52}$$

Therefore, based on the definition of V in (50), the error signals $z_{i,j}$ and $\widetilde{\theta}_i$ eventually converge to the compact set Ω_s, which is specified as

$$\Omega_s = \left\{ z_{i,j}, \widetilde{\theta}_i \mid \left|z_{i,j}\right| \leq \sqrt{2\frac{b_0}{a_0}}, \left|\widetilde{\theta}_i\right| \leq \sqrt{\frac{2\lambda_i}{b}\frac{b_0}{a_0}}, \right.$$

$$(53)$$

$$\left. 1 \leq i \leq N, 1 \leq j \leq n_i \right\}.$$

At the present stage, adaptive decentralized control procedure has been completed via backstepping technique. The above design procedures and stable analysis are summarized in the following theorem.

Theorem 8. *Under Assumptions 1–6, consider the closed-loop nonlinear system consisted of* (1), *unknown dead zone nonlinearities* (2), *controller* (17), *and adaptive law* (18). *Under the action of controller* (17), *for any initial conditions* $[z_i^T(0), \widehat{\theta}_i(0)]^T \in \Omega_0$ *(where Ω_0 is an appropriately chosen compact set), all the signals in the closed-loop system are semiglobally uniformly ultimately bounded in the sense of mean square, and the error signals $z_{i,j}$ and $\widetilde{\theta}_i$ eventually converge to the compact set Ω_s in* (53).

4. Simulation Example

In this section, in order to illustrate the effectiveness of the proposed control scheme, consider the nonaffine interconnected nonlinear system with dead zones as

$$\dot{x}_{1,1} = \left(1 + x_{1,1}^2\right) x_{1,2} + x_{1,2}^3 + y_2 \ln\left(1 + y_1^2\right),$$

$$\dot{x}_{1,2} = \left(5 + \sin\left(x_{1,1} x_{1,2}\right)\right) u_1 + 0.2 u_1^5 + \sin\left(y_1^2\right) y_2,$$

$$y_1 = x_{1,1},$$

$$\dot{x}_{2,1} = \left(3 + \sin\left(x_{2,1}\right)\right) x_{2,2} + 0.3 x_{2,2}^5 + y_1 \cos\left(y_2^2\right), \tag{54}$$

$$\dot{x}_{2,2} = \left(2 + x_{2,1}^2\right) u_2 + \frac{1}{5}\sin\left(u_2\right) + y_2^3 y_1,$$

$$y_2 = x_{2,1},$$

where $u_i = D(v_i)$ is defined as

$$u_1 = D(v_1) = \begin{cases} 1.5\left(v_1 - 2.5\right), & v_1 \geq 2.5, \\ 0, & -1 < v_1 < 2.5, \\ 0.5\left(v_1 + 1\right), & v_1 \leq -1, \end{cases}$$

$$(55)$$

$$u_2 = D(v_2) = \begin{cases} 0.8\left(v_2 - 2\right), & v_2 \geq 2, \\ 0, & -1.8 < v_2 < 2, \\ 1.2\left(v_2 + 1.8\right), & v_2 \leq -1.8, \end{cases}$$

where $x_{1,1}$, $x_{1,2}$, $x_{2,1}$, and $x_{2,2}$ are the state variables and y_i is the system output. u_i and v_i are the output and input of the dead zone nonlinearity, respectively. It is obvious that the system is of nonaffine structure and satisfies Assumptions 1, 4, and 6. By using Theorem 8, the virtual control law, the actual controller, and the adaptive laws are constructed as

$$\alpha_{i,1} = -k_{i,1} z_{i,1} - \frac{1}{2a_{i,1}^2} z_{i,1} \widehat{\theta}_i \phi_{i,1}^T \phi_{i,1},$$

$$u_i = -k_{i,2} z_{i,2} - \frac{1}{2a_{i,2}^2} z_{i,2} \widehat{\theta}_i \phi_{i,2}^T \phi_{i,2}, \tag{56}$$

$$\dot{\widehat{\theta}}_i = \sum_{j=1}^{2} \frac{\lambda_i}{2a_{i,j}^2} z_{i,j}^2 \phi_{i,j}^T \phi_{i,j} - \gamma_i \widehat{\theta}_i,$$
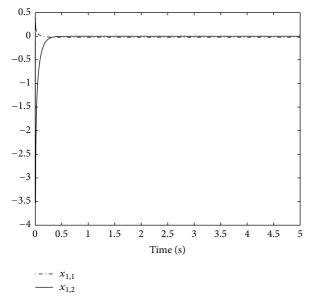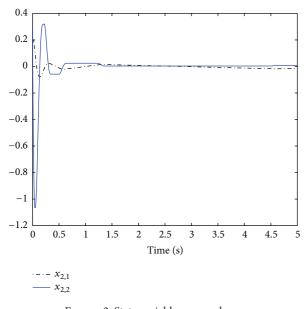
FIGURE 1: State variables $x_{1,1}$ and $x_{1,2}$.

- - - $x_{1,1}$
——— $x_{1,2}$

FIGURE 2: State variables $x_{2,1}$ and $x_{2,2}$.

- - - $x_{2,1}$
——— $x_{2,2}$

FIGURE 3: The control signals v_1 and v_2.

——— v_1
- - - v_2
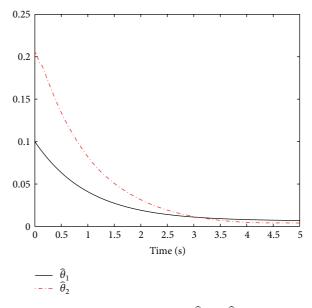

FIGURE 4: Adaptive laws $\hat{\theta}_1$ and $\hat{\theta}_2$.

——— $\hat{\theta}_1$
- - - $\hat{\theta}_2$

where $z_{i,1} = x_{i,1}$, $z_{i,2} = x_{i,2} - \alpha_{i,1}$, and $i = 1, 2$. The simulation is run under the initial conditions $[x_{1,1}(0), x_{1,2}(0), x_{2,1}(0), x_{2,2}(0)]^T = [0.4, -0.1, 0.2, 0.3]^T$, and $[\hat{\theta}_1(0), \hat{\theta}_2(0)]^T = [0.1, 0.2]^T$. In the simulation, design parameters are taken as follows: $k_{1,1} = k_{1,2} = k_{2,1} = k_{2,2} = 11$, $a_{1,1} = a_{1,2} = a_{2,1} = a_{2,2} = 3$, $\gamma_1 = \gamma_2 = 1$, and $\lambda_1 = \lambda_2 = 2$.

The simulation results are indicated by Figures 1–4. Figure 1 shows the state variables $x_{1,1}$ and $x_{1,2}$ of the first subsystems. Figure 2 shows the second subsystems state variables $x_{2,1}$ and $x_{2,2}$. Figure 3 displays the control signals v_1 and v_2, and Figure 4 shows the response curve of the adaptive parameters $\hat{\theta}_1$ and $\hat{\theta}_2$. Apparently, simulation results show

that good convergence performances are achieved and all the signals in the closed-loop system are bounded.

5. Conclusion

In this paper, a robust decentralized adaptive neural control approach has been developed for a class of large-scale nonaffine nonlinear systems with dead zones. The proposed decentralized controller can guarantee that all the signals in the closed-loop systems are semiglobally uniformly ultimately bounded in the sense of mean square, and the error signals eventually converge to small neighborhood around the origin. The main advantage of the proposed controller

is that the prior knowledge of bounds of dead zone slopes is not required. In addition, only one adaptive parameter needs to be updated online for each n-order subsystem. In this way, the computational burden is significantly alleviated. Simulation results have been provided to further illustrate the effectiveness of our results.

There are some problems remaining to be considered, for example, how to generalize the result in this paper to stochastic pure-feedback nonlinear systems with dead zones and how to design an output-feedback adaptive neural controller for original system (1).

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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