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RESEARCH ARTICLE

Capacity Driven Small Cell Deployment in Heterogeneous Cellular Networks: A Performance Analysis

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Abstract

Heterogeneous Cellular Networks (HCNets) are one of the key enabling technologies to improve performance gain of future cellular networks. Stochastic geometry is considered a promising tool to model and analyze HCNets. Users and base stations are generally distributed uniformly using a Homogeneous Poisson Point Process (HPPP). The assumption of uniformly distributed users is not suitable in HCNets because of the existence of clustered users in hotspots. In order to consider the correlation between the users and base stations, deployment of small base stations in these areas are of great concern to increase the performance of HCNets. In this paper, we assume the notion of mixed user distribution, wherein the network users are the superposition of clustered and uniform users, modeled through HPPP and Poisson Cluster Process (PCP), respectively. We evaluate outage probability and rate coverage of the proposed HCNet model. We compare the network performance of the proposed mixed user distribution model with the conventional uniformly distributed user model. The analytical results are validated using Monte-Carlo simulations. Our results show that the proposed HCNet model of mixed user distribution outperforms the uniformly distributed user model in terms of outage probability and rate coverage.

KEYWORDS:
Capacity driven small cell deployment, Heterogeneous cellular networks, Matern cluster process, Non-uniform user distribution, Poisson cluster process, Stochastic geometry.

1 | INTRODUCTION

The heterogeneity of cellular networks leads to increase in capacity due to spectrum reuse and densification via deployment of low power Small Base Stations (SBSs) in a Macro Base Station (MBS) coverage region [1,2,3]. Leveraging SBS deployment, it is assumed that Heterogeneous Cellular Networks (HCNets) contribute 56× gain to the 1000× traffic demand in the enhancement of Fifth-Generation (5G) cellular networks [4]. Keeping in view the network performance, cost, and energy consumption, SBSs need to be optimally deployed in the hotspots (area of interest, e.g., shopping malls, cafeteria, airports etc.), where the user density is high. For the deployment of SBSs and distribution of users, researchers consider stochastic models to analyze the network performance gain tractably and accurately [5,6]. Homogeneous Poisson Point Process (HPPP), a tool from stochastic geometry, is used to deploy different tiers of Base Stations (BSs) as well as users randomly throughout the network [7,8]. However,

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user deployment according to HPPP may not reflect the real scenario of user distribution in HCNet because all users are not uniformly distributed as a high fraction of users is located at the hotspots. Hence, Poisson Cluster Process (PCP) is used to model closely packed daughter points throughout the area. For the deployment of daughter points in the proximity of uniformly deployed parent points, special cases of PCP, such as Thomas Cluster Process (TCP)[9] and Matern Cluster Process (MCP)[10,11] are used in HCNets. Using the idea of PCP, in this paper, we develop and analyze the proposed HCNet model in which users are assumed to be distributed according to two different point processes, i.e., Pedestrian and high way users are assumed to be deployed uniformly according to HPPP, and closely packed users are considered to be deployed according to PCP. The SBSs are deployed through HPPP, which act as parent points for clustered users, and are assumed to be located at the center of clusters.

1.1 | Related Work and Contributions

Using stochastic geometry framework, single-tier and multi-tier cellular networks are analyzed for coverage probability and rate coverage in[12,13]. Similarly, HCNets are modeled for load analysis and interference management in[14] and their traffic offloading and throughput analyses are studied in[15,16] considering uniform deployment of users according with HPPP. HCNets are analyzed for coverage and rate under urban and suburban scenario in[?]. Interference mitigation in orthogonal frequency multiple access based HCNets is studied in[?]. Furthermore, the non-uniform users model in HCNet is investigated in[17], where the users are deployed according to the conditional thinning property of HPPP. In[18], the authors analyzed the performance of HCNets by assuming non-uniform distribution of SBSs modeled with a Poisson Hole Process (PHP). To improve the uplink performance, the authors in[19] use decoupled association to analyze coverage probability in millimeter wave hybrid HCNets. In most of the literature, the analysis of HCNets is performed with uniformly distributed users using HPPP. However, this assumption is not suitable for capacity-driven and user-centric SBS deployment. This is due to the fact that operators are usually interested in deploying low power SBSs in the area where users are highly populated. To capture the crowded users, PCP is more suitable to model clustered users in HCNets. The authors in[20,21] analyze coverage probability in HCNets using PCP to model clustered users, while SBSs and MBSs are distributed using PPP. The work in[22] studies the effect of distance deviation of BS from the cluster center on coverage probability, where the users are assumed to be clustered around HPPP-deployed social attractors (hotspots) and SBSs are deployed at fixed distances from the social attractors. The authors analyze uplink performance of millimeter-wave HCNet in[23] with user-centric small cells deployment. A recent work[24] considers non-uniform users distribution in downlink millimeter-wave HCNets with line-of-sight (LOS) and non line-of-sight (NLOS) transmissions.

Our work is different from the presented state-of-the-art in a sense that we consider joint clustered and uniform users throughout the network to capture the correlation between users and BSs and its effect on the performance of HCNets. We also extend the analysis to analyze the impact of BS density on outage probability, rate coverage, and cell load in such deployment while the presented literature focuses on coverage analysis only. To bridge this gap, in our proposed model, PCP[25] and HPPP are used to deploy users, which is an accurate and realistic approach as compared with HPPP based distribution model in HCNets[26].

Note that this study is not intended to optimize network performance by adjusting user distribution. Instead, we model the network and study its performance with a more realistic user distribution, and thereafter deploy SBSs accordingly to cover more users for better performance. More specifically, we consider non-uniform user distribution in which users are more closely packed at some points in the network, and we call these closely packed users a cluster (e.g., users at shopping malls, offices etc.). The center location of these clusters is modeled as PPP while users around the center are modeled using MCP. In order to avoid coverage holes, we deploy small SBSs at the center of these clusters, hence, the locations of SBSs also become PPP. The performance improves because of the correlation between users and SBSs. By considering the nature of user distribution in the network and deploying SBSs in cluster centers, we aim to improve the system performance eventually. It is worth noting that we do not deploy SBS first and gather users around the SBS to get better coverage. Instead, once the location of the cluster is identified, SBSs are then deployed to provide coverage to clustered users. The authors in[20,21,22,24] approximate the uniform users by assuming infinite cluster radius. In contrast, for tractability, our model exploits the approach already presented in the literature to analyze outage probability of uniform users. The main contributions of this paper are:

1. We consider non-uniform user distribution[†] throughout the network, which is more realistic as compared to the uniform user distribution. For non-uniform user distribution modeling, we use MCP (a special case of PCP). The users are distributed around their parent points, i.e., SBSs, within a circular region of fixed radius. Employing this scheme, the performance of the network is significantly improved as compared to conventional uniform user distribution.

[†]The terms non-uniform user distribution and clustered based user distribution are used interchangeably in this paper.

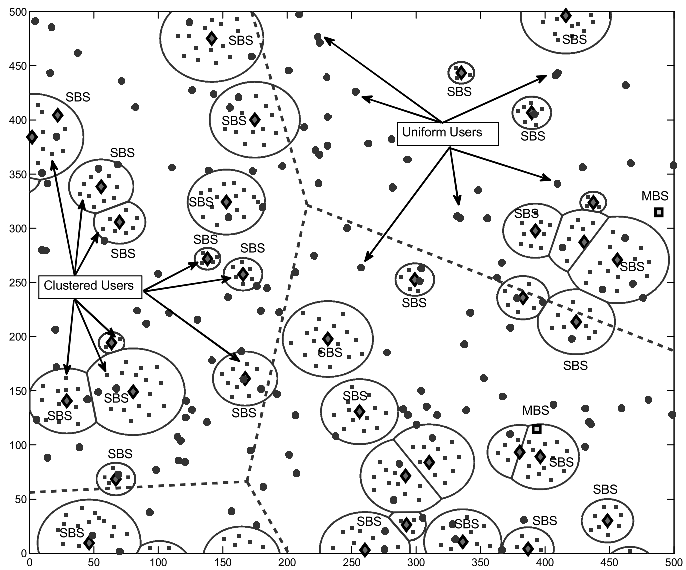


FIGURE 1 A two-tier HCNet model: MBSs (represented by squares) and SBSs (represented by diamonds) are distributed according to two independent HPPPs. Dash-dotted lines and circles represent the coverage area of MBSs and SBSs, respectively. A mixed user distribution (uniform users plus clustered users) is used with dots representing clustered users around SBSs and stars representing uniformly distributed users.

2. Besides non-uniform user distribution, we also consider that a fraction of uniformly deployed users are randomly located in the network in a less populated area. Furthermore, we develop the proposed model and characterize the network performance parameters for the randomly deployed users based on the conventional approach already used in the literature. We also derive the network performance parameters for the clustered users separately. The overall network performance of mixed users, containing a set of clustered as well as uniformly distributed users, is also derived and compared with uniform and clustered users only model.
3. The load characterization of a tagged BS is a crucial part in the derivation of per-tier rate coverage in case of non-uniformly distributed users based HCNets. To the best of our knowledge, this is the first attempt to characterize the load of each tier BS while considering non-uniform user distribution model.
4. Expressions for the outage probability and the rate coverage are derived while assuming both uniform and clustered user distributions. We also compare the outage probability and rate coverage of the proposed mixed users model with uniformly deployed user model and clustered user model. In addition, it is demonstrated that the proposed user-centric HCNet model best fits the simulation results.

The rest of the paper is organized as follows. The system model comprising of BS distribution and user distribution models is discussed in Section 2. Association probabilities, followed by outage probability and rate coverage analyses of uniform users are briefly presented in Section 3. Section 4 focuses on the analysis of non-uniform users (clustered users) and mixed user distribution in HCNet. Numerical results are discussed in Section 5 and the paper is concluded in Section 6.

2 | SYSTEM MODEL

In this section, the proposed HCNet model is presented in which SBSs are deployed according to HPPP and are assumed to be the parent points of the clustered users.

2.1 | BS Distribution

We consider a two-tier HCNet model consisting of MBSs and SBSs deployed via two independent HPPPs, Φ_m and Φ_s, with densities λ_m and λ_s, respectively, as shown in Fig. 1 . The MBS coverage area is overlaid with a large number of low power SBSs. The deployment density and transmit power of each tier BSs are different from other tiers while all BSs in a tier are assumed to have the same Signal-to-Interference Ratio[‡] (SIR) thresholds and transmit power levels. The BSs in each tier consist of a set of open access BSs and closed access BSs (to which only licensed users can connect). The open access MBSs and SBSs are distributed via independent HPPPs, Φ_m^{oa} and Φ_s^{oa}, with densities $\lambda_m^{oa} > 0$ and $\lambda_s^{oa} > 0$, respectively. Similarly, the closed access MBSs and SBSs are distributed via independent HPPPs, Φ_m^{ca} and Φ_s^{ca}, with densities $\lambda_m^{ca} > 0$ and $\lambda_s^{ca} > 0$, respectively. The total density of BSs in the ith tier is $\lambda_i = \sum_{j \in \{oa,ca\}} \lambda_i^j, \forall i \in \{m, s\}$, where m and s represent MBS and SBS, respectively. The transmit power levels of the ith tier open access and closed access BSs are denoted by P_i and p_i , respectively, $\forall i \in \{m, s\}$, such that $1 \leq m \leq M$ and $1 \leq s \leq S$. Here M and S denote the total number of MBSs and SBSs, respectively.

2.2 | User Distribution

Unlike uniformly distributed users where no correlation between users and BSs are considered, this paper focuses on the setup in which users and BSs are correlated because the BSs are deployed in the area where the user density is high. The users, located in these areas form clusters, resulting in non-uniform user distribution throughout the network. This non-uniform cluster based user distribution results in a correlation between users and SBSs because users are assumed to be gathered around SBSs. Each SBS is assumed to be the parent point located at the center of cluster and users are uniformly distributed around the SBS in a circular region in the form of a cluster as daughter points. These clustered users are assumed to be distributed over $\mathbb{R}^2$ through MCP (a special case of PCP) Φ_u, with density $\lambda_u^{\text{PCP}} = N\lambda_s^{oa}$, where N denotes the number of users distributed in each cluster with radius R. The correlation between SBSs and clustered users reflects the scenario of SBS deployment in the hotspots to provide maximum coverage. The Probability Density Function (PDF) of clustered user distribution around SBS located at the cluster center is given by

$$f_{D_i}(d_i) = \begin{cases} \frac{1}{\pi R_i^2}, & \text{if } ||d_i|| \leq R_i \\ 0, & \text{otherwise} \end{cases}, \tag{1}$$

where D_i is the random distance between users and ith tier serving BSs, and R_i denotes the ith cluster radius. In a real network scenario, all users in the network are not located in a cluster form. Hence, a fraction of uniform users (like pedestrian and highway users) is deployed according to HPPP, Φ_u^{HPPP}, with density λ_u^{HPPP}.

The downlink SIR analysis is performed for a randomly selected user from a randomly selected cluster termed as representative cluster. For simplicity, we consider a single BS as a subset of SBS tier to analyze the performance of the SBS located at the center of the cluster separately. The SBS tier is the union of representative cluster center SBS $\{\text{SBS}_0\}$ and a set of SBSs $\{\text{SBS}_j\}$ located outside the representative cluster. For stationary HPPP, the origin can be shifted to user location without changing the statistics of locations via HPPP according to Slivnyaks Theorem[27]. The downlink SIR for the user located at the origin from all the BSs in the Euclidean space, assuming d_j^* is the distance between a typical user and the serving BS, is given by

$$\text{SIR}(d_j^*) = \frac{P_j^* h_{d_j^*} ||d_j^*||^{-\alpha_j}}{I}. \tag{2}$$

Here h_j^*, P_j^*, and α_j are the Rayleigh fading channel gain, transmit power, and path loss exponent , respectively. $I = \sum_{i \in \mathcal{B}} \sum_{j \in \Phi_i \setminus d_j^*} P_{ij} h_{d_{ij}} ||d_{ij}||^{-\alpha_{ij}}$ is the total interference from the jth BSs in the ith tier $\forall i \in \mathcal{B}$ in the network, where $\mathcal{B} \in \{m, s\}$. The index i represents the ith tier in the network while index j denotes the jth BS in the ith tier. Note that we consider the interference power from all the BSs in the network including serving tier and cross-tier. The first summation describes interference power from all the BSs in each tier except the serving BS, while the second summation adds received interferer power from the co-tier BSs as well as cross-tier BSs. The analysis is performed following the maximum received power association strategy, in which a user connects to the BS from which it receives maximum power. The location of the ith tier serving BS, d_i^*, from which the user receives maximum power is given as

$$d_i^* = arg \max_{d_i \in \Phi_i} P_i h_{d_i} ||d_i||^{-\alpha_i}.$$

‡The network is assumed to be interference-limited with no shadowing.

TABLE 1 Notation summary

Notation	Description
Φ_i^{oa}, Φ_i^{ca}	ith tier open access and closed access HPPPs , respectively
$\lambda_m^{oa}(\lambda_m^{ca}), \lambda_s^{oa}(\lambda_s^{ca})$	MBS, SBS tier open access (closed access) BS densities, respectively
R	Cluster radius
$\mathcal{B}$	Number of tiers in the network
N	Number of users per cluster
ζ_i	SIR threshold of ith tier BS $\forall i \in \{m, s\}$
ψ_i	Rate threshold of ith tier BS $\forall i \in \{m, s\}$
$\mathbb{P}(\mathcal{A}_i)$	ith tier association probability $\forall i \in \{m, s\}$
$\mathbb{O}_i^{\text{PPP}}$,$\mathbb{O}_i^{\text{PCP}}$	ith tier outage probability of uniformly distributed and clustered users $\forall i \in \{m, s\}$
$\mathbb{R}_i^{\text{PPP}}$,$\mathbb{R}_i^{\text{PCP}}$	ith tier rate coverage of uniformly distributed and clustered users $\forall i \in \{m, s\}$
$\mathbb{P}$(HPPP)	Probability that a user is randomly selected from the uniformly distributed users
$\mathbb{P}$(PCP)	Probability that a user is randomly selected from clustered users
W	Availiable bandwidth

Based on the proposed setup and assumptions, the association probability, outage probability, and rate coverage for uniform user distribution are presented in Section 3.

3 | ANALYSIS OF UNIFORMLY DISTRIBUTED USERS

Before going to the detailed description of non-uniformly distributed users model, we first consider HCNet model in which users and SBSs are both uniformly distributed according to two independent HPPPs in the entire 2D plane. The derived expressions of outage and rate analysis, following the same approach as used in [8,28], are briefly discussed here. These expressions are used to capture the performance of uniformly distributed users in the proposed system model presented in Fig. 1.

3.1 | Association Probability and Cell Load of Uniformly Distributed Users

The probability that a randomly selected uniformly distributed user associates with the ith tier BS, located at z_i in two tier HCNet, $\mathbb{P}(\mathcal{A}_i^{\text{HPPP}})$, [§] is given as [8]

$$\mathbb{P}(\mathcal{A}_i^{\text{HPPP}}) = 2\pi\lambda_i \int_0^\infty z_i \exp\left\{ -\pi \sum_{\substack{j\in\mathcal{B} \\ j\neq i}} \lambda_i \left(\frac{P_j}{P_i}\right)^{2/\alpha_i} z_i^{2/\alpha_i} \right\} dz_i. \tag{3}$$

According to the mean load approximation [14], the average cell load of the ith tier BSs, $\hat{L}_i^{\text{HPPP}}$, can be written as

$$\hat{L}_i^{\text{HPPP}} = 1 + \frac{1.28\lambda_u^{\text{HPPP}}\mathbb{P}(\mathcal{A}_i^{\text{HPPP}})}{\lambda_i}. \tag{4}$$

3.2 | Outage Probability Analysis of Uniformly Distributed Users

Based on the association probability expression in Subsection 3.1, per-tier ouatge probability is defined as the probability that the typical user is out of coverage given that the user is served by the associated tier BS. For an interference limited network, following the maximum received power strategy [8], the outage probability of the typical user being served by the ith tier BS

[§] $\mathcal{A}_i^{\text{HPPP}}$ is an association event of HPPP distributed users with the ith tier BSs.

located at distance z_i for the uniformly distributed users, $\mathbb{O}_i^{\text{HPPP}}$, is given as

$$\begin{aligned}\mathbb{O}_i^{\text{HPPP}} &= \mathbb{P}\big\{\text{SIR}(z_i) < \zeta_i\big\},\\ &= 1 - \int_0^{\infty} \mathbb{P}\big\{\text{SIR}(z_i) \geq \zeta_i\big\} f_{Z_i^{\text{HPPP}}}(z_i) dz_i,\end{aligned} \tag{5}$$

where ζ_i is the predefined SIR threshold of the ith tier BS. The outage probability expression in[8] is extended for HCNet with a set of open access and closed access BSs following the procedure used in[14]. Per-tier outage probability, $\mathbb{O}_i^{\text{HPPP}}$, of a typical user (for which the analysis is performed) in HCNet with the set of open access and closed access BSs in the ith tier containing HPPP distributed user can be written as

$$\mathbb{O}_i^{\text{HPPP}} = 1 - \frac{2\pi\lambda_i}{\mathbb{P}(\mathcal{A}_i^{\text{HPPP}})} \int_0^{\infty} z_i \exp\Bigg\{ -\frac{\zeta_i z_i^{\alpha_i}}{P_i} - \pi \sum_{\substack{j\in\mathcal{B}\\ j\neq i}} \left(\frac{P_j}{P_i}\right)^{2/\alpha_i} \Big(\lambda_j^{oa}\Big[1 + \mathcal{H}\big(\zeta_i, \alpha_i\big)\Big] + \lambda_j^{ca}\mathcal{T}\big(\zeta_i, \alpha_i\big)\Big) z_i^2 \Bigg\} dz_i, \tag{6}$$

where $\mathcal{H}(\zeta_i, \alpha_i) = \frac{2\zeta_i}{\alpha_i - 2}\, {}_2F_1\left[1, 1 - \frac{2}{\alpha_i}; 2 - \frac{2}{\alpha_i}; -\zeta_i\right]$. Here ${}_2F_1$ [.] is Hypergeometric function and $\mathcal{T}(\zeta_i, \alpha_i) = \zeta_i^{2/\alpha_i} \frac{2\pi\csc(\frac{2\pi}{\alpha_i})}{\alpha_i}$.

The total outage probability of users throughout the network with uniformly distributed users in two tiers HCNet, $\mathbb{O}_{\text{total}}^{\text{HPPP}}$, is now given as

$$\mathbb{O}_{\text{total}}^{\text{HPPP}} = \sum_{i\in\mathcal{B}} \mathbb{P}(\mathcal{A}_i^{\text{HPPP}})\mathbb{O}_i^{\text{HPPP}}. \tag{7}$$

3.3 | Rate Coverage of Uniformly Distributed Users

Rate coverage is the probability that the rate achieved by a user associated with the ith tier BSs is greater than the rate threshold. Following the mean load approximation (4), per-tier rate coverage of a randomly located user, given that the user connects with the ith tier BSs, is given as

$$\mathbb{R}_i^{\text{HPPP}} = \sum_{n>0} \mathcal{P}_i(n) \frac{2\pi\lambda_i}{\mathbb{P}(\mathcal{A}_i^{\text{HPPP}})} \int_0^{\infty} z_i\ \exp\Bigg\{ -\frac{\big(2^{\frac{\psi_i \hat{L}_i}{W}} - 1\big) z_i^{\alpha_i}}{P_i} - \pi \sum_{\substack{j\in\mathcal{B}\\ j\neq i}} \left(\frac{P_j}{P_i}\right)^{2/\alpha_i} \Big(\lambda_j^{oa}\Big[1 + \mathcal{H}(\zeta_i, \alpha_i)\Big] + \lambda_j^{ca}\mathcal{T}\big(\zeta_i, \alpha_i\big) z_i^2 \Bigg\} d_{z_i}, \tag{8}$$

where ψ_i denotes the predefined rate threshold, $\hat{L}_i$ is the average cell load of the ith tier BS, and W is the available resource bandwidth. $\mathcal{H}(\psi_i, \alpha_i)$ and $\mathcal{T}(\psi_i, \alpha_i)$ can be obtained by setting $\zeta_i = (2^{\frac{\psi_i \hat{L}_i}{W}} - 1)$ in $\mathcal{H}(.)$ and $\mathcal{T}(.)$ in (8). Similarly, the total rate coverage of uniformly distributed users, $\mathbb{R}_{\text{total}}^{\text{HPPP}}$, is given as by

$$\mathbb{R}_{\text{total}}^{\text{HPPP}} = \sum_{i\in\mathcal{B}} \mathbb{P}(\mathcal{A}_i^{\text{HPPP}})\mathbb{R}_i^{\text{HPPP}}. \tag{9}$$

4 | ANALYSIS OF HCNET WITH NON-UNIFORM USERS

In this section, we develop and analyze the non-uniform user distribution based HCNet model presented in Section 2, with user-centric SBS deployment focusing on the areas where users are closely located in the form of clusters. The SBSs are assumed to be deployed at the center of each cluster. The locations of SBSs throughout the network are modeled according to HPPP. The user distribution follows MCP around SBS at the hotspots in a circular disc with radius R. The analysis is performed for a randomly selected user from the randomly selected cluster (representative cluster). In this section, expressions for association probability, outage probability, and rate coverage of non-uniform users based HCNet are derived.

4.1 | Association Probability of Clustered Users

Here we derive the association probability expression for non-uniform clustered users in the proposed system model. This probability is defined as the probability of successful connection of the user to a particular tier BS. Based on the maximum received power association strategy, let the BS from which the user receives maximum power be located at random distance Z_i, the PDF and Complementary Cumulative Distribution Function (CCDF) of Z_i are given, respectively, as

$$f_{Z_i}(z_i) = 2\pi\lambda_i z_i e^{-\pi\lambda_i z_i^2}, z_i \geq 0, \tag{10}$$

$$F_{Z_i}(z_i) = e^{-\pi\lambda_i z_i^2}, \qquad z_i \geq 0, \tag{11}$$

where $i \in \{m, s\}$ s.t., the BS $\in \Phi_i$. Based on the locations of the users in representative cluster, the PDF and CCDF of the distance between user and nearest cluster center BS at z_0 is given by[21]

$$f_{Z_0}(z_0) = \frac{2z_0}{R^2}, \quad 0 \leq z_0 \leq R, \tag{12}$$

$$F_{Z_0}(z_0) = \frac{R^2 - z_0^2}{R^2}, \quad 0 \leq z_0 \leq R. \tag{13}$$

Now, by using the definition of association probability of the ith tier, $\mathbb{P}(\mathcal{A}_i)$, we can rewrite $\mathbb{P}(\mathcal{A}_i)$ as

$$\mathbb{P}(\mathcal{A}_i) = \mathbb{P}\left\{ \bigcap_{j\in\mathcal{B}} 1\left(Z_i > \left(\frac{P_j}{P_i}\right)^{1/\alpha_i} Z_j \right) \right\}.$$

Lemma 1. Association probability of a randomly selected user distributed via MCP with the ith tier BSs where $i \in \{0, \mathcal{B}\}$, $\mathbb{P}(\mathcal{A}_i^{\text{PCP}})$, can be written as

$$\mathbb{P}(\mathcal{A}_i^{\text{PCP}}) = \begin{cases} \frac{2}{R^2} \int_0^R z_0 \exp\left\{ -\pi \sum_{\substack{j\in\mathcal{B}\\ j\neq i}} \lambda_i^{oa} \left(\frac{P_j}{P_0}\right)^{2/\alpha_i} z_0^2 \right\} dz_0, \text{ if } i = 0 \\ \frac{2\pi\lambda_i^{oa}}{R^2} \int_0^{UL} z_i \left\{ R^2 - \left(\frac{P_j}{P_i}\right)^{2/\alpha_i} z_i^2 \right\} \exp\left\{ -\pi \sum_{\substack{j\in\mathcal{B}\\ j\neq i}} \lambda_i^{oa} \left(\frac{P_j}{P_i}\right)^{2/\alpha_i} z_i^2 \right\} dz_i, \text{ if } i \in \mathcal{B} \end{cases}, \tag{14}$$

where $UL = \frac{R}{\left(\frac{P_j}{P_i}\right)^{1/\alpha_i}}$.

Proof. See Appendix A. □

It is worth noting that 0th tier contains cluster center SBSs only and is assumed to be a subset of SBS tier, therefore, $P_0 = P_s$. The typical user in the representative cluster is assumed to be associated only with the open access BSs while the closed access BSs act as interferers.

4.2 | Distribution of Serving Distances of Clustered Users

Here we derive the PDF of serving distances of non-uniform users such that the typical user connects with the ith tier BS. On the basis of an association event[¶], serving distance is the distance between the nearest BS and the user. Assuming the serving BS to be located at distance z_i from the user, the PDF of the serving distance Z_i, given that the clustered user is served by the ith tier BSs such that $i \in \{0, \mathcal{B}\}$ where 0th tier represents the cluster center BS, is given in Lemma 2.

Lemma 2. The PDF of serving distance of a typical user, given that users are distributed via MCP (PCP), $f_{Z_i^{\text{PCP}}}(z_i)$, can be written as

$$f_{Z_i^{\text{PCP}}}(z_i) = \begin{cases} \frac{2z_0}{\mathbb{P}(\mathcal{A}_0)R^2} \exp\left\{ -\pi \sum_{\substack{j\in\mathcal{B}\\ j\neq i}} \lambda_i^{oa} \left(\frac{P_j}{P_0}\right)^{2/\alpha_i} z_0^2 \right\}, \text{ if } i = 0 \\ \frac{2\pi\lambda_i}{\mathbb{P}(\mathcal{A}_i)R^2} z_i \exp\left\{ -\pi \sum_{\substack{j\in\mathcal{B}\\ j\neq i}} \lambda_i^{oa} \left(\frac{P_j}{P_i}\right)^{2/\alpha_i} z_i^2 - \pi\lambda_i z_i^2 \right\} \left\{ R^2 - \left(\frac{P_j}{P_i}\right)^{2/\alpha_i} z_i^2 \right\}, \text{ if } i \in \mathcal{B} \end{cases}. \tag{15}$$

Proof. See Appendix B. □

[¶] An association event, $\mathcal{A}_i$, is dependent on random distance vector which specifies that the received power from the jth BS in the ith tier is greater than from any of the remaining BSs.

$$\mathbb{O}_i^{\text{PCP}} = \begin{cases} 1 - \frac{1}{\mathbb{P}(\mathcal{A}_0^{\text{PCP}})} \int\limits_{z_i>0} \exp\left\{ -\frac{\zeta_i z_0^{\alpha_i}}{P_0} - \pi \sum\limits_{\substack{j\in\mathcal{B}\\ j\neq i}} \left(\frac{P_j}{P_0}\right)^{2/\alpha_i} \left(\lambda_j^{oa}\left[1+\mathcal{H}(\zeta_i,\alpha_i)\right] + \lambda_j^{ca}\mathcal{T}(\zeta_i,\alpha_i)\right) z_i^2 \right\} \frac{2z_0}{R^2}\, dz_0, & \text{if } i=0 \\ 1 - \frac{2\pi\lambda_i}{\mathbb{P}(\mathcal{A}_i^{\text{PCP}})} \int\limits_{z_i>0} z_i \exp\left\{ -\frac{\zeta_i z_i^{\alpha_i}}{P_i} - \pi \sum\limits_{\substack{j\in\mathcal{B}\\ j\neq i}} \left(\frac{P_j}{P_i}\right)^{2/\alpha_i} \left(\lambda_j^{oa}\left[1+\mathcal{H}(\zeta_i,\alpha_i)\right] + \lambda_j^{ca}\mathcal{T}(\zeta_i,\alpha_i)\right) z_i^2 \right\} \mathcal{L}_{\mathcal{I}^{oa}_{(i,0)}}\left(\frac{\zeta z_i^{\alpha}}{P_i}\right) dz_i, & \text{if } i\in\mathcal{B} \end{cases} \tag{18}$$

Using Lemma 1 and Lemma 2, the outage probability expressions are derived in Subsection 4.3.

4.3 | Outage Probability of Clustered Users

Per-tier outage probability of the non-uniformly distributed clustered users is the probability that the SIR achieved by a user is less than the pre-defined SIR threshold, given that the user is associated with ith tier BS such that $i \in \{0, \mathcal{B}\}$. Hence, per-tier outage probability of clustered users, $\mathbb{O}_i^{\text{PCP}}$, can be expressed as

$$\mathbb{O}_i^{\text{PCP}} = 1 - \int\limits_0^\infty \mathbb{P}\left\{ \text{SIR}(z_i) \geq \zeta_i \right\} f_{Z_i^{\text{PCP}}}(z_i) dz_i. \tag{16}$$

Based on the proposed system model in Section 2, as each tier comprises of open access as well as closed access BSs, the downlink interference $I(.)$ in (2) can be rewritten as a sum of interferences from the open access and the closed access BSs, i.e.,

$$I = \sum_{\substack{i\in\mathcal{B}\\ j\in\Phi_i}} \mathcal{I}^{oa}_{(i,j)} + \sum_{\substack{i\in\mathcal{B}\\ j\in\Phi_i}} \mathcal{I}^{ca}_{(i,j)}. \tag{17}$$

Using Lemma 1 and Lemma 2, per-tier outage probability for clustered users, given that the typical user associates with the ith tier BS, is stated in the following theorem.

Theorem 1. *Per-tier outage probability of a typical user from Φ_i^u where $u \in \{PCP\}$, given that the user associates with ith tier BS, is given by (18) (on the top of the page).*

Here, $\mathcal{L}_{\mathcal{I}_{oa(i,0)}}(\frac{\zeta z_i^{\alpha}}{P_i})$ is the Laplace transform of the interference from the representative cluster BSs belonging to the ith tier open access BS. $i = 0$ is the case when the user is served by the BS located at the center of representative cluster in the 0th tier and $i \in \{m, s\}$ is the case when the user connects to the BS $\in \mathcal{B}$, such that $i \neq 0$.

Proof. See Appendix C. □

The total outage probability of clustered users in the network, $\mathbb{O}_{\text{total}}^{\text{PCP}}$, can be written as

$$\mathbb{O}_{\text{total}}^{\text{PCP}} = \mathbb{P}(\mathcal{A}_0^{\text{PCP}})\mathbb{O}_0^{\text{PCP}} + \sum_{i\in\mathcal{B}} \mathbb{P}(\mathcal{A}_i^{\text{PCP}})\mathbb{O}_i^{\text{PCP}}. \tag{19}$$

4.4 | Rate Coverage of Clustered Users

The rate coverage of the clustered users, $\mathbb{R}_i^{\text{PCP}}$, is defined as the probability that the rate of the ith tier user is greater than or equal to certain pre-defined rate threshold ψ_i. Mathematically,

$$\mathbb{R}_i^{\text{PCP}} = \mathbb{P}\left\{ \frac{W}{L_i^{\text{PCP}}} \log_2 \left\{ 1 + \text{SIR}_i(z) \right\} \geq \psi_i \right\}, \tag{20}$$

where L_i^{PCP} is the load of the tagged BS in the ith tier and B is the available bandwidth. To evaluate per-tier rate coverage, the load of the tagged BSs, given that the users are deployed via MCP, is presented next.

4.4.1 | Per-Tier Load Characterization with Non-uniform User Distribution in HCNet

The load of the tagged BS in an HCNet with non-uniform user distribution plays a crucial part in rate coverage derivation. The load characterization of non-uniform users in HCNets is one of the main technical contribution of this paper, which is given in the next Lemma.

Lemma 3. The load, L_i^{PCP}, of the ith tier BS, is defined as the average number of non-uniform users associated with the ith tier BSs and can be written as

$$L_i^{\text{PCP}} = \begin{cases} N_i \int\limits_0^R \exp\left\{ -\pi \sum\limits_{\substack{j\in\mathcal{B}\\ j\neq i}} \lambda_i^{oa} \left(\frac{P_j}{P_0}\right)^{2/\alpha_i} \right\} \frac{2z_0}{R^2} dz_0, & \text{if } i = 0 \\ 2\pi\lambda_u^{\text{PCP}} \int\limits_0^{UL} z_i \exp\left\{ -\pi \sum\limits_{\substack{j\in\mathcal{B}\\ j\neq i}} \lambda_i^{oa} \left(\frac{P_j}{P_i}\right)^{2/\alpha_i} z_i^{2/\alpha_i} \right\} \frac{R^2 - \left(\left(\frac{P_j}{P_i}\right)^{1/\alpha_i} z_i\right)^2}{R^2} dz_i, & \text{if } i \in \mathcal{B} \end{cases}, \tag{21}$$

where the upper limit of the second integral, i.e., UL, is the same as used in Lemma 1, N_i denotes the cluster size, and $\lambda_u^{\text{PCP}} = N_i \lambda_s^{oa}$ represents the non-uniform user density in the network.

Proof. The average number of users in the ith tier is the ratio between the average number of the ith tier users and the average number of ith tier BSs. Let the area of the network be represented by $\mathbb{A}$, total number of BSs by $\mathbb{U}_b$, and the number of users in the ith tier by $\mathbb{U}_i$. Then, the load of the ith tier BS, L_i^{PCP} is given as

$$L_i^{\text{PCP}} = \frac{\mathbb{U}_i}{\mathbb{U}_b} = \frac{\mathbb{P}(\mathcal{A}_i^{\text{PCP}})\lambda_u^{\text{PCP}}\mathbb{A}}{\lambda_i \mathbb{A}} = \mathbb{P}(\mathcal{A}_i^{\text{PCP}})\left(\frac{\lambda_u^{\text{PCP}}}{\lambda_i}\right).$$

Combining with association probability of the clustered users from (15) and setting $\lambda_u^{\text{PCP}} = N\lambda_s^{oa}$ completes the proof. □

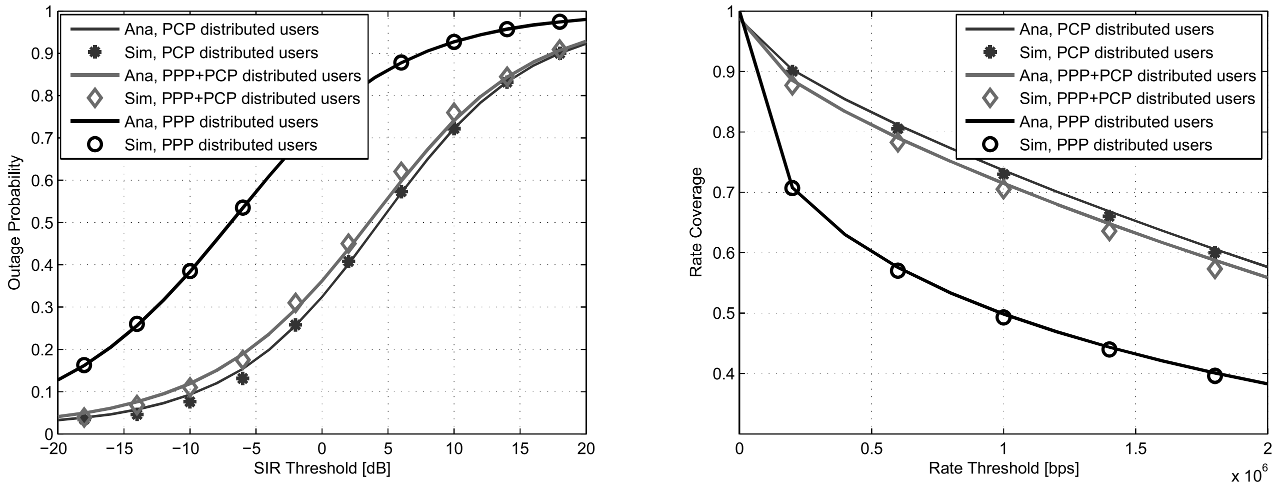
4.4.2 | Per-Tier Rate Coverage of Clustered Users

Theorem 2. *The per-tier rate coverage of a typical user from cluster deployed user set Φ_i^u, conditioned that the typical user is served by the ith tier BS, $\mathbb{R}_i^{PCP}$, is given as*

$$\mathbb{R}_i^{PCP} = \begin{cases} \frac{1}{\mathbb{P}(\mathcal{A}_0^{PCP})} \int\limits_{z_0>0} \exp\Bigg\{ -\frac{\Psi(\psi_i, L_i^{PCP}) z_0^{\alpha_i}}{P_i} - \pi \sum\limits_{\substack{j\in\mathcal{B}\\ j\neq i}} \left(\frac{P_j}{P_0}\right)^{2/\alpha_i} \left(\lambda_j^{oa}\left[1 + \mathcal{H}\{\Psi(\psi_i, L_i^{PCP}), \alpha_i\}\right] + \lambda_j^{ca}\mathcal{T}\left\{\Psi(\psi_i, L_i^{PCP}), \alpha_i\right\}\right) z_0^2 \Bigg\} \frac{2z_0}{R^2}\, d_{z_0}, & \textit{if } i = 0 \\ \frac{2\pi\lambda_i}{\mathbb{P}(\mathcal{A}_i^{PCP})} \int\limits_{z_i>0} \exp\Bigg\{ -\frac{\Psi(\psi_i, L_i^{PCP}) z_i^{\alpha_i}}{P_i} - \pi \sum\limits_{\substack{j\in\mathcal{B}\\ j\neq i}} \left(\frac{P_j}{P_i}\right)^{2/\alpha_i} \left(\lambda_j^{oa}\left[1 + \mathcal{H}\left\{\Psi(\psi_i, L_i^{PCP}), \alpha_i\right\}\right] + \lambda_j^{ca}\mathcal{T}\left\{\Psi(\psi_i, L_i^{PCP}), \alpha_i\right\}\right) z_i^2 \Bigg\} \mathcal{L}_{\mathcal{I}_{(i,0)}^{oa}}\left(-\frac{\Psi(\psi_i, L_i^{PCP}) z_i^{\alpha_i}}{P_i}\right) d_{z_i}, & \textit{if } i \in \mathcal{B} \end{cases}, \tag{22}$$

where $\mathcal{L}_{\mathcal{I}_{oa(i,0)}}\left(\frac{\Psi(\psi_i, L_i^{PCP}) z_i^{\alpha_i}}{P_i}\right)$ is the Laplace transform of interference from the cluster center BSs belonging to open access ith tier, $\Psi\left(\psi_i, L_i^{PCP}\right) = \left(2^{\frac{\psi_i L_i^{PCP}}{W}} - 1\right)$, and $\mathcal{H}(.)$ and $\mathcal{T}(.)$ are the same as already defined in Theorem 1 by replacing the argument.

Proof. See Appendix D. □



(a) **(b)**

FIGURE 2 Comparison of PPP, PCP and mixed (PPP+PCP) user distributions for $R = 50$ m and $N = 10$: (a) Outage probability versus SIR threshold. (b) Rate coverage versus rate threshold. PPP represent conventional uniform user model.

Total rate coverage of a randomly selected user from the clustered user set, $\mathbb{R}_{\text{total}}^{\text{PCP}}$, can now be written as

$$\mathbb{R}_{\text{total}}^{\text{PCP}} = \mathbb{P}(\mathcal{A}_0^{\text{PCP}})\mathbb{R}_0^{\text{PCP}} + \sum_{i\in\mathcal{B}} \mathbb{P}(\mathcal{A}_i^{\text{PCP}})\mathbb{R}_i^{\text{PCP}}. \tag{23}$$

4.5 | Outage Probability and Rate Coverage of Mixed Users

Based on the outage probability and rate coverage expressions derived in Subsections 4.3 and 4.4, respectively, we derive the outage probability and rate coverage of the proposed mixed user distribution,[#] which is a combined set of both clustered users and uniformly distributed users. The analysis is performed for a randomly selected user from any of the two users sets.

The outage probability and rate coverage of mixed users is the combination of the outage probabilities and rate coverages of the clustered and unifrom users conditioned that the randomly selected user belongs to uniformly distributed or clustered user sets. Hence, based on the average number of uniform and clustered users, the probabilities that the selected user belongs to the uniformly distributed users or the clustered users[18] are given, respectively, by

$$\mathbb{P}(\text{HPPP}) = \frac{\lambda_u^{\text{HPPP}}}{\lambda_u^{\text{HPPP}} + \sum_{i\in\mathcal{B}} N\lambda_i^{oa}}, \tag{24}$$

$$\mathbb{P}(\text{PCP}) = \frac{N\lambda_i^{oa}}{\lambda_u^{\text{HPPP}} + \sum_{i\in\mathcal{B}} N\lambda_i^{oa}}, \tag{25}$$

where $\mathcal{B} \in \{m, s\}$ and λ_u^{HPPP} is the density of uniformly distributed users in the network. The total outage probability of mixed users, $\mathbb{O}_{\text{total}}^{\text{MIXED}}$, can be written as

$$\mathbb{O}_{\text{total}}^{\text{MIXED}} = \mathbb{P}(\text{HPPP})\mathbb{O}_{\text{total}}^{\text{HPPP}} + \mathbb{P}(\text{PCP})\mathbb{O}_{\text{total}}^{\text{PCP}}. \tag{26}$$

Similarly, the total rate coverage of mixed users, $\mathbb{R}_{\text{total}}^{\text{MIXED}}$, is written as

$$\mathbb{R}_{\text{total}}^{\text{MIXED}} = \mathbb{P}(\text{HPPP})\mathbb{R}_{\text{total}}^{\text{HPPP}} + \mathbb{P}(\text{PCP})\mathbb{R}_{\text{total}}^{\text{PCP}}, \tag{27}$$

where $\mathbb{P}(\text{HPPP})$ and $\mathbb{P}(\text{PCP})$ are given in (24) and (25), respectively.

[#] As mixed user distribution is the superposition of PPP distributed user set and PCP distributed user set, therfore, the term 'mixed user' is used for PPP distributed users plus PCP distributed users.

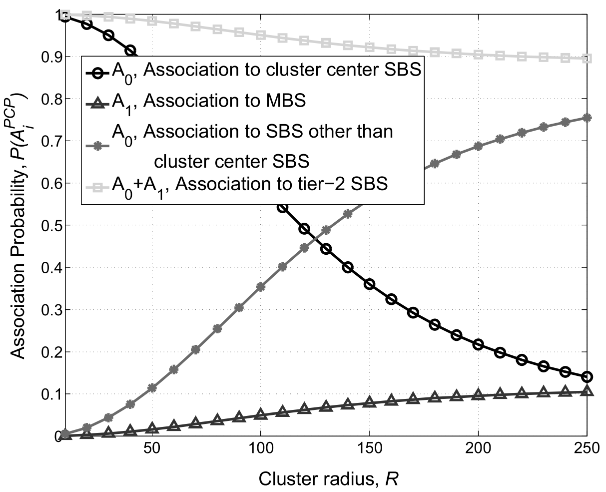


FIGURE 3 Per-tier association probabilities versus cluster radius R (in meters) where users are distributed according to PCP.

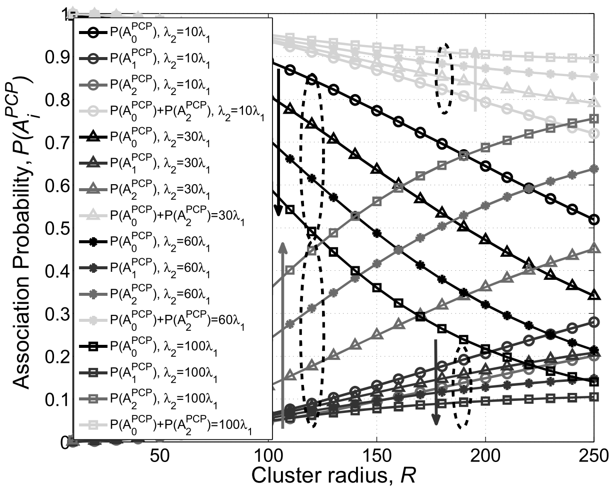


FIGURE 4 Comparison of per-tier association probability versus cluster radius R (in meters) for different SBS densities, in case of PCP distributed users.

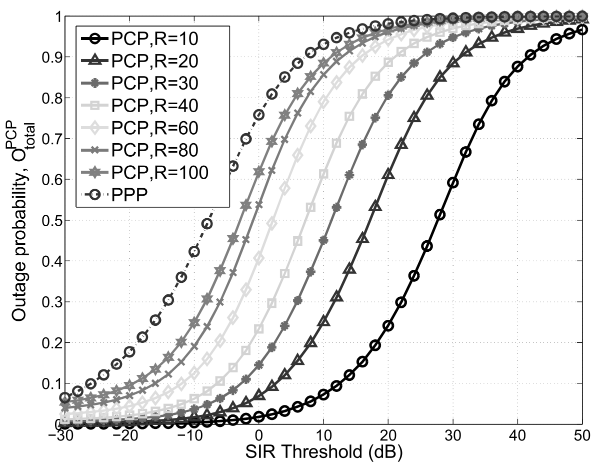


FIGURE 5 Comparison of outage probability versus SIR threshold for different cluster radius R (in meters), in case of PCP distributed users.

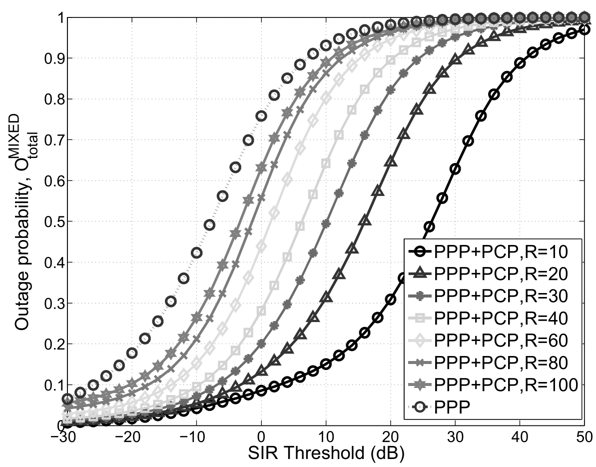


FIGURE 6 Comparison of outage probability versus SIR threshold for different cluster radius R (in meters), in case of mixed (PCP+HPPP) user distribution.

5 | NUMERICAL RESULTS AND DISCUSSIONS

In this section, we discuss the results for outage probability and rate coverage of the proposed HCNet model. Simulations are performed for two-tier network while assuming a simulation area of radius = 1 km. The density of open access MBSs is $\lambda_m^{oa} = 1/(\text{simulation area})$, and SBS tier consists of open access and closed access BSs with densities $\lambda_s^{oa} = \lambda_s^{ca} = 100 \times \lambda_m^{oa}$. The values of transmit power levels of MBSs and SBSs are considered to be 53 dBm and 33 dBm, respectively. Moreover, $\alpha_i = \alpha_s = \alpha_m = 3.5$.

5.1 | Model Validation and Effect of Users Distribution on Outage Probability

The model is validated using Monte Carlo simulations, by averaging 10000 iterations. Outage probability and rate coverage is evaluated while considering three different user deployment strategies. In the first case the users are uniformly distributed

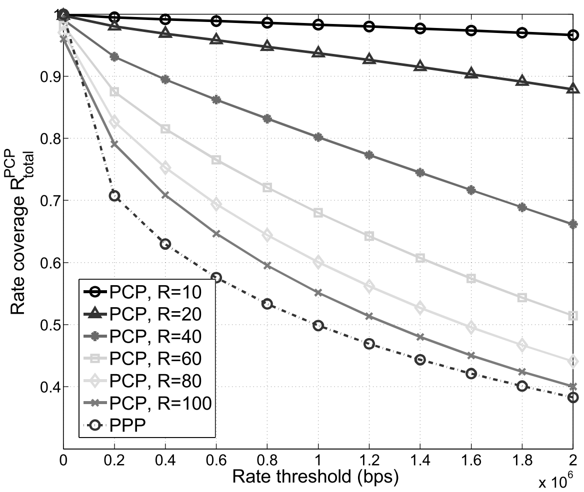


FIGURE 7 Comparison of rate coverage for different cluster radii R (in meters) with fixed cluster size $N = 10$, in PCP distributed users model.

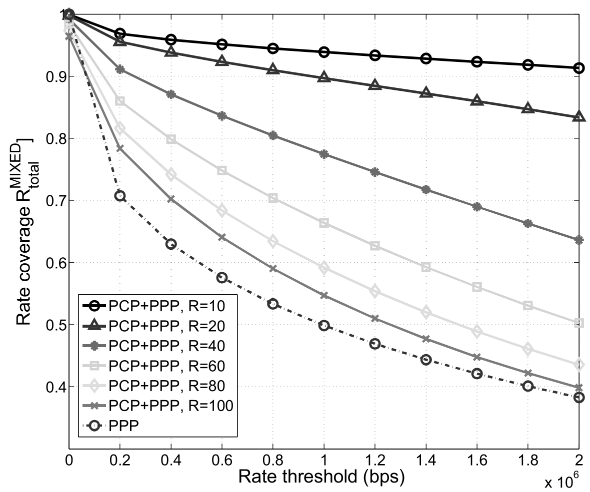


FIGURE 8 Comparison of rate coverage for different cluster radii R (in meters) with fixed cluster size $N = 10$, in mixed (PCP+HPPP) user distributions model.

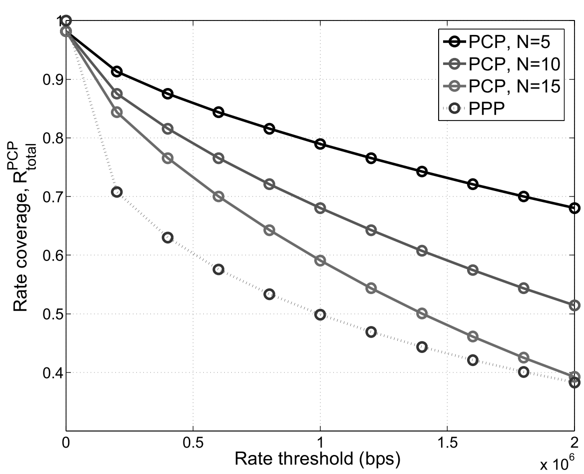


FIGURE 9 Comparison of rate coverage for different cluster sizes N with fixed cluster radius $R = 60$ m, in PCP distributed users model.

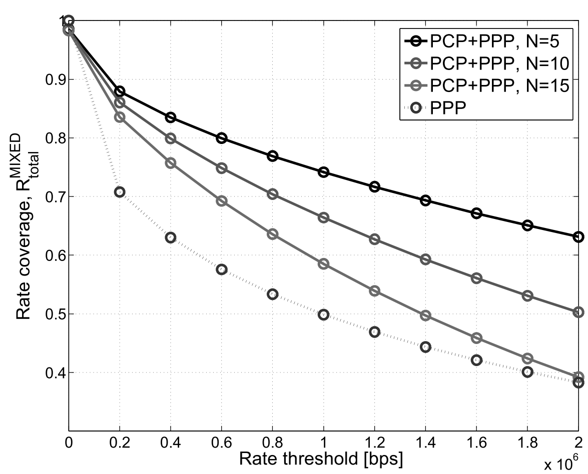


FIGURE 10 Comparison of rate coverage for various cluster sizes N with fixed cluster radius $R = 60$ m, in mixed (HPPP+PCP) distributed users model.

according to HPPP throughout the network, in the second case the users are distributed according to PCP with SBS located at the center of the cluster with fixed radius, and in the third case users are clustered around SBSs and a fraction of users are uniformly distributed in the network. The comparisons of outage probability versus SIR threshold, and rate coverage versus rate threshold are investigated for the three aforementioned user deployment strategies.

It is evident from Fig. 2 that the analytical results closely match the simulation results. Outage probability of HPPP based distributed user model is higher than that of the mixed user distribution with fixed cluster radius $R = 50$ m and cluster size of 10, while the rate coverage is smaller compared with PCP and mixed users HCNets. This is because, in the clustered case, users are more closely located to SBSs while in mixed user distribution the probability of random users is smaller as compared with clustered users. The outage probability of the clustered user model is smaller compared with the mixed user distribution because the probability of a user of interest located in the clustered user set is higher than in the uniformly deployed users set.

5.2 | Effect of Cluster Density on Per-tier User Association

In Fig. 3, the association probability is compared with various values of cluster radius. For fixed cluster size, user association with the cluster center SBS decreases, however, user association with SBSs (other than the cluster center SBS) increases with increase in the cluster radius. This is because the users become farther from the cluster center SBS as the cluster size increases. Similarly, per-tier association probability of SBSs (located outside the representative cluster) is higher than the MBS because the users still get higher SIR from SBSs compared with the MBS.

In Fig. 4, per-tier association probability of the cluster based users versus cluster radius with various values of cluster density is studied. The association probability is mainly dependent on cluster radius. It can be observed that the association probability of the users with cluster center SBS decreases with increase in the SBS density. The association probability with SBSs (located outside the cluster center) and MBS increases with increase in the SBS density. This is due to the fact that the chance of cluster overlapping increases which results in highest received power from the BSs located outside the cluster.

5.3 | Effect of Cluster Radius on Outage Probability

The outage probability versus various values of SIR threshold for cluster based user deployment model is presented in Fig. 5. It can be observed from the figure that the outage probability of the users increases with increase in the cluster radius (from 10 m to 100 m). For a smaller cluster radius with fixed cluster size, the cluster is more dense, hence, the probability of a user being in outage is lower. By increasing the cluster radius, the outage probability increases and converges to uniformly distributed user model. Similarly, the outage probability of uniformly distributed users model is also investigated, as dipicted in Fig. 5. It is a clear evidence that the outage probability for the clustered user model with fixed cluster radius of 100 m is still lower than for the uniformly distributed users model.

In Fig. 6, the outage probability is compared with cluster radius assuming mixed users distribution. The plots show increasing behavior of the outage probability with increasing SIR threshold for different cluster radii. However, the outage probability in mixed user model is slightly higher than for the clustered user model only. This is because a fraction of users (uniformly distributed users) are located farther from the cluster center SBS.

5.4 | Effect of Cluster Radius on Rate Coverage

The rate coverages of clustered user distribution are presented in Fig. 7. As evident from the figure, the rate coverage decreases with increase in cluster radius. This effect is due to the reason that users get further apart from the SBS located at representative cluster and the chances of more users from other clusters associating with this SBS increases. Due to this higher user association the BS load increases and hence rate coverage decreases. Moreover, in Fig. 7, the rate coverage is compared with uniformly distributed users case. It is also clear from the figure that the rate coverage for clustered users with larger cluster radius of 100 m is still higher than the uniformly distributed user case.

Fig. 8 shows that the rate coverage of mixed user distribution is smaller as compared with the clustered users distribution. However, it is still higher than the uniformly distributed users model. The reason for this effect is that the mixed user distribution includes the uniform users set also, which increase the cell load. For the rate threshold of 1 Mbps and a cluster radius of 20 m, the average user rate in case of mixed users is improved by 40% compared with the uniformly distributed users case. Similarly, the user rate for clustered users is higher by 5% than for the mixed users model.

5.5 | Effect of Cluster Size on Rate Coverage

In Fig. 9 rate coverages of clustered (PCP based only) users deployment are investigated for different number of users per cluster, keeping $R = 60$ m. The rate coverage of the clustered users decreases for higher number of users per cluster. For a rate threshold of less than 2 Mbps and $N = 15$, the user rate is higher than for the uniform users deployment only.

The rate coverages of mixed distributed users deployment are investigated for different number of users per cluster, keeping $R = 60$ m in Fig. 10. For the rate threshold of 1 Mpbs, the user rate decreases by 8% with increase in the cluster size from 10 to 15 users per cluster. The results show that rate coverages of both user models decrease for larger cluster size because the higher number of users per cluster results in increased load on the cluster center SBS.

6 | CONCLUSIONS

In this paper, we developed and analyzed non-uniform user distribution in two tier HCNets, which reflects more realistic user distribution scenario where all users are not supposed to be uniformly distributed. Hence, the users deployed via PCP are assumed to be the daughter points of HPPP distributed SBSs. We analyzed the outage probability and rate coverage of the proposed HCNet model. Our results show that the SBSs at hotspots increases the network performance. The performance of the network for a denser cluster is higher. Furthermore, the outage probability of the proposed model is smaller than for the uniformly distributed user model. The rate coverage in the proposed model is higher because fewer users are associated with each BS, which results in decreased cell load. It is concluded that the proposed model best fits in case of dense HCNets where users are closely located near SBSs.

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APPENDIX

A PROOF OF LEMMA 1

Association probability of the clustered users can be written as

$$\begin{aligned}\mathbb{P}(\mathcal{A}_i^{\text{PCP}}) &= \mathbb{E}_{Z_i}\left\{\bigcap_{i\in\mathcal{B}}\left(Z_i > \left(\frac{P_j}{P_i}\right)^{1/\alpha_j} Z_j\right)\right\}\\ &= \mathbb{E}_{Z_i}\left\{\prod_{\substack{j\in\mathcal{B}\\ j\neq i}}\mathbb{P}\left(Z_i > \left(\frac{P_j}{P_i}\right)^{1/\alpha_j} Z_j\right)\right\}\\ &= \mathbb{E}_{Z_i}\left\{\prod_{\substack{j\in\mathcal{B}\\ j\neq i}} F_{Z_i}\left(\left(\frac{P_j}{P_i}\right)^{1/\alpha_j} Z_i\right)\right\}\\ &= \int_0^\infty \prod_{\substack{j\in\mathcal{B}\\ j\neq i}} F_{Z_j}\left(\left(\frac{P_j}{P_i}\right)^{1/\alpha_j} Z_j\right) f_{Z_j}(z_j)dz_j. \end{aligned} \tag{A1}$$

Similarly, we can write the association probability of the user with its cluster center SBS and with other BSs in the ith tier by setting $i = 0$ and $i \in \mathcal{B}$, respectively, as:

For 0th tier $(i = 0)$:

$$\mathbb{P}(\mathcal{A}_0^{\text{PCP}}) = \int_0^R \prod_{\substack{j\in\mathcal{B}\\ j\neq i}} F_{Z_0}\left(\left(\frac{P_j}{P_0}\right)^{1/\alpha_j} Z_0\right) f_{Z_0}(z_0)dz_0. \tag{A2}$$

For ith tier where $i \in \mathcal{B}$ and $i \neq 0$:

$$\begin{aligned}\mathbb{P}(\mathcal{A}_i^{\text{PCP}}) &= \mathbb{E}_{Z_i}\left\{\prod_{\substack{j\in\mathcal{B}\\ j\neq i}}\mathbb{P}(Z_i > \left(\frac{P_j}{P_i}\right)^{1/\alpha_i} Z_i)\right\}\\ &= \mathbb{E}_{Z_i}\left\{\mathbb{P}\left(Z_i > \left(\frac{P_j}{P_0}\right)^{1/\alpha_i} Z_j\right)\prod_{\substack{j\in\mathcal{B}\\ j\neq i}}\mathbb{P}\left(Z_j > \left(\frac{P_j}{P_i}\right)^{1/\alpha_i} Z_j\right)\right\}\\ &= \int_0^\infty F_{Z_0}\left(\left(\frac{P_j}{P_0}\right)^{1/\alpha_i} Z_i\right)\prod_{\substack{j\in\mathcal{B}\\ j\neq i}} F_{Z_i}\left(\left(\frac{P_j}{P_i}\right)^{1/\alpha_i} Z_i\right) f_{Z_i^{\text{PCP}}}(z_j)dz_j. \end{aligned} \tag{A3}$$

Substituting the values of CCDF and PDF from (10)-(13) in (A2) and (A3) completes the proof of Lemma 1.

B PROOF OF LEMMA 2

The PDF of serving distances from ith tier BS is given by

$$\begin{aligned} f_{Z_i^{\text{PCP}}}(z_i) &= \frac{d}{dz_i}\left\{\mathbb{P}\left\{Z_i > z|\mathbb{P}(\mathcal{A}_i^{\text{PCP}}\right\}\right\}\\ &= \frac{d}{dz_i}\left\{\frac{\mathbb{P}\left(Z_i > \left(\frac{P_j}{P_i}\right)^{1/\alpha_j} Z_j\right)}{\mathbb{P}(\mathcal{A}_i^{\text{PCP}})}\right\}\\ &= \frac{d}{dz_i}\left\{\frac{1}{\mathbb{P}(\mathcal{A}_i^{\text{PCP}})}\int_z^\infty \prod_{\substack{j\in\mathcal{B}\\ j\neq i}} F_{Z_i}\left(\left(\frac{P_j}{P_i}\right)^{1/\alpha_i} Z_j\right) f_{Z_i}(z_i)dz_i\right\} \end{aligned}$$

$$=\frac{1}{\mathbb{P}(\mathcal{A}_i^{\text{PCP}})}\prod_{\substack{j\in\mathcal{B}\\ j\neq i}} F_{Z_i}\left(\left(\frac{P_j}{P_i}\right)^{1/\alpha_i} Z_i\right) f_{Z_i}(z_i). \tag{B4}$$

For 0th tier ($i = 0$) :

$$\begin{aligned} f_{Z_0^{\text{PCP}}}(z_0) &= \frac{1}{\mathbb{P}(\mathcal{A}_0^{\text{PCP}})}\prod_{\substack{j\in\mathcal{B}\\ j\neq i}} F_{Z_i}\left(\left(\frac{P_j}{P_0}\right)^{1/\alpha_i} Z_i\right) f_{Z_0}(z_0) \\ &= \frac{1}{\mathbb{P}(\mathcal{A}_0^{\text{PCP}})}\exp\left(-\pi\sum_{\substack{j\in\mathcal{B}\\ j\neq i}}\lambda_j^{oa}\left(\frac{P_j}{P_0}\right)^{2/\alpha_i} Z_0^2\right)\frac{2z_0}{R^2}. \end{aligned} \tag{B5}$$

For ith tier where $i \in \mathcal{B}$ and $i \neq 0$:

$$\begin{aligned} f_{Z_i^{\text{PCP}}}(z_i) &= \frac{1}{\mathbb{P}(\mathcal{A}_i^{\text{PCP}})} F_{Z_0}\left(\left(\frac{P_j}{P_0}\right)^{1/\alpha_i} Z_0\right)\prod_{\substack{j\in\mathcal{B}\\ j\neq i}} F_{Z_i}\left(\left(\frac{P_j}{P_0}\right)^{1/\alpha_i} Z_i\right) f_{Z_i}(z_i) \\ &= \frac{2\pi\lambda_i^{\text{oa}}}{\mathbb{P}(\mathcal{A}_i^{\text{PCP}})} z_j \exp\left(-\pi\sum_{\substack{j\in\mathcal{B}\\ j\neq i}}\lambda_j^{\text{oa}}\left(\frac{P_j}{P_i}\right)^{1/\alpha_j} Z_j^2 - \pi\lambda_j z_j^2\right)\frac{R^2-\left(\frac{P_j}{P_i}\right)^{2/\alpha_j} z_j^2}{R^2} \\ &= \frac{2\pi\lambda_i^{\text{oa}}}{\mathbb{P}(\mathcal{A}_i^{\text{PCP}})} z_j \exp\left(-\pi\sum_{j\in\mathcal{B}}\lambda_j^{\text{oa}}\left(\frac{P_j}{P_i}\right)^{1/\alpha_j} Z_j^2\right)\frac{R^2-\left(\frac{P_j}{P_i}\right)^{2/\alpha_j} z_j^2}{R^2}. \end{aligned} \tag{B6}$$

Hence, (B5) and (B6) are obtained by substituting in (B4), the CCDF and PDF as in (10)-(13).

C PROOF OF THEOREM 1

The outage probability according to (16) is given by

$$\mathbb{O}_i^{\text{PCP}} = 1 - \int_0^\infty \left\{ P\Big(\text{SIR}\big(z_i\big) \geq \zeta_i\Big)\right\} f_{Z_j^{\text{PCP}}}(z_i)dz_i, \tag{C7}$$

where the probability that SIR is greater than ζ_i is given as

$$\begin{aligned} \mathbb{P}\big(\text{SIR}(d_i) > \zeta_i\big) &= \mathbb{P}\left(\frac{P_i h_i z_i^{-\alpha_i}}{\sum_{j\in\mathcal{B}} I_{(i,j)}} > \zeta_i\right) \\ &= \mathbb{P}\left(h_i > \frac{\zeta_i z_i^{\alpha_i}}{P_i}\left(\sum_{j\in\mathcal{B}} I_{(i,j)}\right)\right) \\ &= \mathbb{P}\left(h_i > \frac{\zeta_i z_i^{\alpha_i}}{P_i}\left(\sum_{i\in\mathcal{B}} \mathcal{I}_{(i,j)}^{oa} + \sum_{i\in\mathcal{B}} \mathcal{I}_{(i,j)}^{ca}\right)\right) \\ &= \exp\left(\frac{\zeta_i z_i^{\alpha_i}}{P_i}\right)\prod_{j\in\mathcal{B}}\mathcal{L}_{\mathcal{I}_{(i,j)}^{oa}}\left(\frac{\zeta_i z_i^{\alpha_i}}{P_i}\right)\prod_{j\in\mathcal{B}}\mathcal{L}_{\mathcal{I}_{(i,j)}^{ca}}\left(\frac{\zeta_i z_i^{\alpha_i}}{P_i}\right). \end{aligned} \tag{C8}$$

Here, $\mathcal{L}_{\mathcal{I}^{oa}_{(i,j)}}(.)$ and $\mathcal{L}_{\mathcal{I}^{ca}_{(i,j)}}(.)$ represent the Laplace transforms of the interference from open access and closed access BSs, respectively. $\mathcal{L}_{\mathcal{I}^{oa}_{(i,j)}}(.)$ can be written as

$$\begin{aligned}\mathcal{L}_{\mathcal{I}^{oa}_{(i,j)}}\left(\frac{\zeta_i z_i^{\alpha_i}}{P_i}\right) &= \mathbb{E}_{\mathcal{I}^{oa}_{(i,j)}}\left[\exp\left\{-\frac{\zeta_i z_i^{\alpha_i}\mathcal{I}^{oa}_{(i,j)}}{P_i}\right\}\right]\\ &= \mathbb{E}_{\Phi_i}\left[\exp\left\{-\frac{\zeta_i z_i^{\alpha_i}}{P_i}\sum_{i\in\mathcal{B}} P_j h_i ||z_i||^{-\alpha_i}\right\}\right]\\ &= \mathbb{E}_{\Phi_i}\left[\prod_{z_i\in\Phi_i}\mathbb{E}_{h_i}\left\{\exp\left(-\frac{\zeta_i z_i^{\alpha_i}}{P_i}P_j h_i||z_i||^{-\alpha_i}\right)\right\}\right]\\ &\overset{(a)}{=} \mathbb{E}_{\Phi_i}\left[\prod_{z_i\in\Phi_i}\frac{1}{1+\frac{\zeta_i z_i^{\alpha_i}}{P_i}P_i||z_i||^{-\alpha_i}}\right]\\ &\overset{(b)}{=} \exp\left\{-2\pi\lambda_i\int_{(\frac{P_j}{P_i})^{1/\alpha_i} z_i^{-\alpha_i}}^{\infty}\frac{z_i}{1+\frac{\zeta_i^{-1} z_i^{-\alpha_i}}{P_j}P_i||z_i||^{-\alpha_i}}d_{z_i}\right\}\\ &\overset{(c)}{=} \exp\left\{-\pi\lambda_i^{oa}\left(\frac{P_j}{P_i}\right)^{2/\alpha_i}\mathcal{H}(\zeta_i,\alpha_i)z_i^{2/\alpha_i}\right\}.\end{aligned} \tag{C9}$$

In (C9), Step (a) follows Rayleigh fading assumption of channel gain and independence from HPPP, Step (b) is obtained by using probability generating functional of PPP, and Step (c) is obtained by using the procedure of employing change in variable and integrating over the limits.

As the Laplace transform of interference from closed access BSs is independent of z_i, it can simply be obtained by substituting the lower limit of integral in (C9) Step (b) equal to zero for the case of open access, and is given as

$$\mathcal{L}_{\mathcal{I}^{ca}_{(i,j)}}\left(\frac{\zeta_i z_i^{\alpha_i}}{P_i}\right) = \exp\left\{-\pi\lambda_i^{ca}\mathcal{T}(\zeta_i,\alpha_i)\left(\frac{P_j}{P_i}\right)^{2/\alpha_i} z_i^2\right\}. \tag{C10}$$

Now the outage probability, when a user connects to SBS located at the center of representative cluster, is given by:

For $i = 0$:

Using (C7) and (C8), the outage probability of the clustered users when the user is connected to the cluster center SBS can be written as

$$\mathbb{O}_0^{\mathrm{PCP}} = 1 - \int_0^\infty \exp\left(\frac{\zeta_i z_i^{\alpha_i}}{P_i}\right)\prod_{j\in\mathcal{B}}\mathcal{L}_{\mathcal{I}^{oa}_{(i,j)}}\left(\frac{\zeta_i z_i^{\alpha_i}}{P_i}\right)\prod_{j\in\mathcal{B}}\mathcal{L}_{\mathcal{I}^{ca}_{(i,j)}}\left(\frac{\zeta_i z_i^{\alpha_i}}{P_i}\right) f_{Z_0^{\mathrm{PCP}}}(z_0)dz_i. \tag{C11}$$

For $i = \mathcal{B}; i \neq 0$:

The per-tier outage probability, when the user is connected to a BS other than the BS located at the center of representative cluster, can be rewritten as

$$\mathbb{O}_i^{\mathrm{PCP}} = 1 - \int_0^\infty \exp\left(\frac{\zeta_i z_i^{\alpha_i}}{P_i}\right)\mathcal{L}_{\mathcal{I}^{oa}_{(i,0)}}\left(\frac{\zeta_i z_i^{\alpha_i}}{P_i}\right)\prod_{j\in\mathcal{B}}\mathcal{L}_{\mathcal{I}^{oa}_{(i,j)}}\left(\frac{\zeta_i z_i^{\alpha_i}}{P_i}\right)\prod_{j\in\mathcal{B}}\mathcal{L}_{\mathcal{I}^{ca}_{(i,j)}}\left(\frac{\zeta_i z_i^{\alpha_i}}{P_i}\right) f_{Z_i}(z_i)dz_i, \tag{C12}$$

where $\mathcal{L}_{\mathcal{I}^{oa}_{(i,0)}}\left(\frac{\zeta_i z_i^{\alpha_i}}{P_i}\right)$ is the interference received from the BS located at the center of the representative cluster.

If the serving BS of the user is other than the cluster center SBS and is located at distance Z_i, then the cluster center SBS also contributes to the total interference. In this case the conditional PDF of z_0 can be written as

$$f_{Z_0}\left(z_0 | Z_0 > \left(\frac{P_i}{P_j}\right)^{1/\alpha_i} z_i\right) = \frac{f_{z_0}(Z_0)}{F_{Z_0}\left(\left(\frac{P_i}{P_j}\right)^{1/\alpha_i} z_i\right)}.$$

The Laplace transform of interference from the cluster center SBS can be written as

$$\begin{aligned}\mathcal{L}_{\mathcal{I}^{oa}_{(i,0)}}\left(\frac{\zeta_i z_i^{\alpha_i}}{P_i}\right) &= \mathbb{E}_{Z_0}\left[\mathbb{E}_{h_i}\left\{\exp\left(-\frac{\zeta_i z_0^{\alpha_i}}{P_i}P_j h_i||z_0||^{-\alpha_i}\right)\right\}|Z_0 > \left(\frac{P_i}{P_j}\right)^{1/\alpha_i} z_i\right]\\ &= \mathbb{E}_{Z_0}\left[\frac{1}{1+\frac{\zeta_i z_i^{\alpha_i}}{P_i}P_i||z_0||^{-\alpha_i}}|Z_0 > \left(\frac{P_i}{P_j}\right)^{1/\alpha_i} z_i\right]\\ &= \int_l^\infty \frac{1}{1+\frac{\zeta_i z_i^{\alpha_i}}{P_i}P_i||z_0||^{-\alpha_i}} f_{Z_0}\left(z_0|Z_0 > \left(\frac{P_i}{P_j}\right)^{1/\alpha_i} z_i\right) d_{z_0},\end{aligned} \tag{C13}$$

where $l = \left(\frac{P_j}{P_i}\right)^{1/\alpha_i} z_i^{-\alpha_i}$. Substituting $\mathcal{L}_{\mathcal{I}^{oa}_{(i,0)}}(.)$, $\mathcal{L}_{\mathcal{I}^{oa}_{(i,j)}}(.)$, $\mathcal{L}_{\mathcal{I}^{ca}_{(i,j)}}(.)$ using (C9), (C10), (C13), and the PDFs of serving distances using (B5) and (B6) into (C11) and C(12) completes the proof of Theorem 2.

D PROOF OF THEOREM 2

The per-tier rate coverage for non-uniform users can be written as

$$\mathbb{R}_i^{\mathrm{PCP}} = \mathbb{P}\left\{\mathrm{SIR}_i(z) \geq 2^{\psi_i L_i/W} - 1\right\}. \tag{D14}$$

Similarly, the rate coverage in case of the clustered users can be derived by deriving proof similar to Theorem 1 and replacing $\zeta_i = \Psi_i(\zeta_i, L_i) = 2^{\psi_i L_i/W} - 1$.

For $i = 0$:

When the typical user connects with the cluster center SBS, the rate coverage of the typical user can be approximated as

$$\mathbb{R}_0^{\mathrm{PCP}} = \frac{2}{\mathbb{P}(\mathcal{A}_0^{\mathrm{PCP}})R^2}\int_{z_i>0} z_i \exp\left\{-\frac{\Psi(\zeta_i, L_i)z_i^{\alpha_i}}{P_0} - \pi\sum_{j\in\mathcal{B}}\left(\frac{P_j}{P_i}\right)^{2/\alpha_i}\left(\lambda_j^{oa}\left[1+\mathcal{H}\left(\Psi(\zeta_i, L_i), \alpha_i\right)\right] + \lambda_j^{ca}\mathcal{T}\left(\Psi(\zeta_i, L_i), \alpha_i\right)\right)z_i^2\right\} dz_i. \tag{D15}$$

For $(i = j, i \neq 0)$:

Similar to Theorem 1, the rate coverage of ith tier BS other than the cluster center SBS is given as

$$\mathbb{R}_i^{\mathrm{PCP}} = \frac{2\pi\lambda_i}{\mathbb{P}(\mathcal{A}_i^{\mathrm{PCP}})}\int_{z_i>0} \exp\left\{-\frac{\Psi(\zeta_i, L_i)z_i^{\alpha_i}}{P_i} - \pi\sum_{j\in\mathcal{B}}\left(\frac{P_j}{P_i}\right)^{2/\alpha_i} bigg(\lambda_j^{oa}\left[1+\mathcal{H}\left(\Psi(\zeta_i, L_i), \alpha_i\right)\right] + \lambda_j^{ca}\mathcal{T}\left(\Psi(\zeta_i, L_i), \alpha_i\right)z_i^2\right\} \mathcal{L}_{\mathcal{I}^{oa}_{(i,0)}}\left(\frac{\Psi(\zeta_i, L_i)z_i^{\alpha}}{P_i}\right) dz_i. \tag{D16}$$

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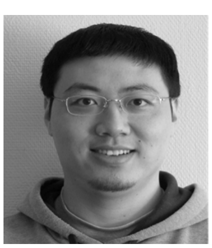

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