Opportunities and Challenges when Students Work with Vocationally Connected Mathematics Tasks

Trude Pedersen Sundtjønn

UiA University of Agder

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Abstract

Students in vocational education in Norway take a compulsory mathematics course in the first year of their programme. This course offers opportunities for engaging with mathematics which can be highly relevant to practice in many workplaces. Working with vocationally connected mathematics tasks, tasks designed to draw on students' future working contexts, is one way of trying to connect to students' possible future vocations. This study aims to understand how students interact with such tasks with a particular emphasis on the roles of norms, authenticity and students' positioning between the practices of school, the workplace and everyday life.

In this study, I observed students working with specially designed vocationally connected mathematics tasks in three different vocational education programmes in Norway: Design and Crafts, Media and Communication, and Technical and Industrial Production. The data comprise of video recordings and field notes of the students' interaction with the tasks. Grounded in a socio-cultural approach, with an emphasis on students as participants in multiple communities of practice, the analysis is framed in the theoretical concepts of norms, authenticity and boundary object representations. Working from the starting point that the students are experienced as participants in mathematical classrooms but are still newcomers in their future vocational practice, the analysis concentrates on enacted norms in the classroom and references to out-of-school routines, knowledge and practices.

My findings revealed that vocationally connected tasks enabled students to draw on routines and knowledge from the relevant vocational practice, and that this led to shifts in student and teacher roles in terms of who was regarded as an expert in the classroom. However, the norms of traditional classroom mathematics continued to dominate, leading to fluctuations in norms as students positioned themselves between different communities of practice. Consequently, even though students might identify authentic aspects in the tasks, they often disregarded routines and knowledge from out-of-school practice if these disrupted the solution strategies common to classroom practice.

The thesis makes a contribution to knowledge in the field of sociomathematical norms in vocational education. In particular, it examines how tasks and norms are mutually constitutive, and the importance of this relationship in a successful implementation of vocationally connected mathematics tasks. It

also draws attention to the difficulties in manipulating authenticity, and the force of norms when exploiting authenticity in task design.

The implications of this study are that vocationally connected tasks have the potential to generate disturbances in classroom norms, which can then create opportunities to loosen up or change traditional interaction patterns and make space for discussions of what mathematics is in different practices. To understand and use mathematics appropriately in a given situation is important aspects of intellectual autonomy in mathematics, and I argue that opportunities for change in student and teacher roles can provide possibilities for a greater degree of student autonomy.

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1 Vocationally Connected Mathematics Tasks in School-based Mathematics Classes

In this thesis, I present research on the opportunities and challenges when students in vocational education programmes work with mathematics tasks designed to be connected to vocational contexts. The aim of the study is to understand how students interact with classroom mathematics tasks which are designed to draw on the students' future vocational contexts. To meet this aim, I have studied the students' interactions with the tasks and the teacher, and investigated normative activity, task authenticity and the participants' positioning in and between practices.

This chapter begins with a short episode from the data collected in this study. The episode illustrates the opportunities and challenges that can arise when students work with mathematics tasks which are embedded in school practices and have the potential to be vocationally relevant. I then present the background, the relevance of the research and the research questions which this study addresses. Finally, I explain the structure of the thesis.

1.1 A Short Introductory Classroom Vignette

The following short vignette from the data illustrates some potential opportunities and challenges when students work with classroom mathematics tasks with a vocational context. The vignette is presented here to give the reader a reference point for how such a task can give rise to differences in interaction patterns and the positioning of the participants.

The vignette is from a mathematics class for students attending the Technical and Industrial Production vocational education programme. The students are in their first year of the programme and have not yet started their workplace placements. Their education during this first year is based in school, with training in the school workshop (see Section 2.1 for further explanations). The students worked on a task intended to be connected to their future vocational context, namely, to figure out the placement of the legs of a jack stand so that the three legs are evenly distributed. This task can be solved in different ways. The intention of the mathematics teacher for how the students could solve the task was that they would calculate the circumference of the circular pipe from its given diameter, and then divide the circumference into three equal parts. The students had met this problem some weeks earlier in their vocational training class when they welded a jack stand. Their teacher of the students' vocational training class had told the mathematics teacher that the students were unsure of how to find the proper placement of the legs and suggested that the relationship between circumference and diameter could be investigated during a mathematics lesson. Figure 1.1 shows a photograph of a commercially made jack stand, and one can see that it has a circular pipe as the middle part with three legs welded on.

Figure 1.1: An example of a jack stand with three legs.

First, the students worked individually with the task. Then Ingeborg, the teacher, led a class discussion about the solution to the task on the blackboard. One classical mathematical way of solving the question involves the following calculations (43 mm is the given diameter of the pipe):

$$O = 2 \cdot \pi \cdot r = \pi \cdot d$$

$$O = \pi \cdot 43 \text{ mm}$$

$$O = 135.09 \text{ mm}$$

Then divide the circumference in three equal parts

$$\frac{O}{3} = \frac{135.09 \text{ mm}}{3} = 45.03 \text{ mm}$$

The legs need to be 45 mm apart

The following episode occurred when Ingeborg asked the class for a solution and an explanation of their solution process. The students suggested a procedure that was the same as the solution presented above, and Ingeborg wrote this solution on the blackboard. Then Ingeborg and one of the students (Martin) had the conversation shown in Table 1.1.

Table 1.1: Conversation between about where to round off in calculations.

Turn	English translation	Norwegian	Comments
1	Ingeborg: 3.14 times 43 that is 145 (faint voice: no), 135.	Ingeborg: 3,14 gange 43 det blir 145 (svakt: nei) 135.	Ingeborg writes 135.02 on the blackboard while she talks.
2	Martin: It isn’t, damn what is it called, it’s millimetres. I think when you are supposed to measure, you get millimetres, and it isn’t as though we can measure down to zero point two, so then I just rounded off to just.	Martin: Det er jo ikke, faen hva heter det for noe, det er jo millimeter der. Tenker jo når du skal måle lissom, så har du jo millimeter. Det er ikke som vi kan måle heilt ned til null komma to så da runda æ av til bare.	Martin explains his calculation of $3.14 \cdot 43 = 135.02$, and that he rounds off the answer. Martin shows his knowledge of the vocational practice, and reacts to the mathematical solution, maybe because he knows that it is not possible (or necessary) to measure less than millimetres for this kind of work.
3	Ingeborg: So, we round it off?	Ingeborg: Så vi runder av?	Ingeborg asks a clarifying question about Martin’s statement.
4	Martin: Yes.	Martin: Ja.	
5	Ingeborg: Yes, but, it may be clever to keep it when we calculate, and then round it off when we have divided by three? Then we get as accurate as possible. So if we take 135.02 millimetres and divide by three, then we round off, what do we get?	Ingeborg: Ja, men, det kanskje være lurt å beholde det når vi regner, også runder vi av når vi har delt på tre? Så får vi det så nøyaktig som mulig ned til dit. Så viss vi da tar 135,02 millimeter og deler på tre, så runder vi det av, hva får vi?	While Ingeborg talks, she writes $\frac{135.02}{3}$ on the blackboard. Ingeborg refers indirectly here to the usual practice in school mathematics, of rounding off when all calculations are done.
6	Martin: We get 45.	Martin: Får vi 45.	

Here, one can note that in Turn 2, Martin referred to his experiences, and treated the question in relation to routines that he knew of about how to measure in real life. There is, then, a noticeable difference between Martin's suggestion, "it isn't as though we can measure down to zero point two, so then I just rounded off" (Turn 2) and the teacher's reply that "it may be clever to keep it when we calculate, and then round it off when we have divided by three" (Turn 5). The teacher suggested rounding off after one has done all calculations, while Martin suggested rounding off during the calculations.

I argue that in this vignette there are traces of different expectations of what the task is about, why the task is solved, what the solution should be used for, and how to work with the task. It seems as though the teacher and Martin have different expectations of how to approach the problem. The teacher is probably thinking of the task with regards to the norms of a mathematics classroom, where it can be important to remember not to round off too early. However, Martin may be looking at the task from a practical vocational viewpoint, doing the task according to the routines of a vocational practice, where the solution is accurate enough for the purpose of adding evenly space legs to the jack stand. The two participants interact and discuss the task with different expectations.

As I will discuss in Chapter 3, I argue that mathematical practices are qualitatively different in different practices. Therefore, to use a vocational context in the mathematics classroom is, as shown in this short episode, not straightforward. There are opportunities and challenges that can arise while working with mathematics tasks designed to draw on the students' future vocational contexts; this is the subject of this thesis.

1.2 Background and Relevance of the Research

This study is based on observations of mathematics classrooms for students that study vocational education programmes in Norway. In Chapter 2, I describe the structure of vocational education programmes in Norway, but I will first explain the national and international relevance of the research, and its interest for the mathematics education community in general.

1.2.1 National and International Relevance

Norway is in need of a well-educated population, both for participation in our democratic society and to secure future usefulness in employment (NOU 2019: 25, 2019). However, upper secondary education in Norway has problems with high rates of failure and early leaving. These statistics are especially bad in the vocational education programmes, where around 67 % of students finished within a six-year period, which is around 20 percentage points lower than the number of students that finish general education studies (Statistics Norway, 2020).

The high rate of early leaving in vocational education is the subject of discussion amongst teachers, researchers, politicians and the general public. There are no easy answers to the challenge that it presents, and a range of possible solutions have been suggested: to focus more on the apprenticeship period; to have fewer theoretical subjects in the first year of the vocational education programmes; to have more or better follow up of students while they are in transition phases in their education, and so on (Markussen, Frøseth, & Sandberg, 2011). My study is based upon the fact that one reason for the high failure rate in vocational education is that students frequently fail the compulsory mathematics course (Stene, Haugset, & Iversen, 2014).

Students who enrol in vocational education programmes have to complete a mathematics course, usually the mathematics course known as 1P-Y. This course covered at the time of my data collection a curriculum which was identical to about 60 % of the mathematics course 1P for students in general education programmes (see also Section 2.2), and all of the nine vocational education programmes had the same curriculum. Consequently, critical voices point to the lack of vocational relevance of the common core curricula, including the mathematics course (see for instance Hiim (2013)).

The Norwegian Directorate for Education and Training clarified the Common Core Curriculum with the instruction that mathematics in vocational education programmes should be connected to students’ future vocations (2010a). This is challenging, both because the mathematics curriculum overlaps with the 1P course in general education and because it has the same curriculum for all nine vocational education programmes. Additionally, the issue of vocational connections is challenging for teachers. According to Nicol (2002) teachers are often unfamiliar with the specific mathematical practices needed for their students’ future vocations. This is a challenge because mathematical activity at the workplace is different than mathematics in educational settings (FitzSimons, 2001; Wake, 2014).

Hence, there is a need for further research on how and in which ways vocationally connected mathematics can be implemented in Norwegian vocational education programmes. This issue is also of international relevance: while there are significant variations in how different countries plan and implement vocational education (Lillejord et al., 2015; Stene, Haugset, & Iversen, 2014; Sträßer, 2014), a common issue for vocational programmes is that students may not see the relevance of learning mathematics. One reason for this may be the issue of how mathematics and mathematical practices are mostly implicit in workplace and vocational settings (see Section 3.2 for elaboration in this issue).

1.2.2 Relevance to Mathematics Education Research

The research literature in mathematics education is dominated by theoretical and empirical material on how students learn mathematics in school settings. Bakker (2014) argues that most research in mathematics education deals with mathematics in a school setting, and that only a small subset of research focuses on workplace mathematics. He claims that it is important, but difficult, to study relationships between education and the world our education is supposed to prepare students for, and that only a few studies deal with the relationship between them. Furthermore, FitzSimons (2014) points to the complexities and tensions that are part of vocational mathematics education. She argues that translating observations of workplace mathematics directly into school mathematics curricula is problematic. Doing so means that much of the context-based knowledge in workplace or vocational practices is missing, making it necessary to study how to bring workplace and school practices together in order to learn from each other. Similarly, Wake (2014) argues that research needs to look closer into mathematics learning in combination with context (for example, workplace practices) to be able to better design mathematics curricula that will be relevant in the future.

In this study, I explore some of these issues with an emphasis on students in vocational education mathematics classes as they work with tasks designed to draw on the students' future vocational contexts. While my research takes place in a school context and its related practices, there are out-of-school practices which play important parts in the students' lives. The students are on the verge of becoming members of a vocational community but are still newcomers in this practice. As Lave and Wenger (1991) would say, they are on the verge of becoming legitimate peripheral participants in their chosen vocation. At the same time, they are members of a mathematics classroom community in their school, where they are given opportunities to work with vocationally connected mathematics tasks. Hence, this study's relevance for the mathematics education community lies in its potential to provide a research-based insight into what happens in the implementation when students' future vocational contexts are used to design classroom mathematics tasks.

1.3 Aim, Concepts and Research Questions

My research analyses how students work with tasks designed to draw on their future vocational contexts. In this section, I present the aim of this study, and define some of the fundamental concepts used in this thesis. I close this section by presenting the research questions.

In this research project I aim to provide a detailed study of the implementation of vocationally connected mathematics tasks as part of the mathematics course in three vocational education programmes in Norway: Design, Arts and Crafts; Media and Communication; and Technical and Industrial Production. Although the research is limited to these three programmes, it has relevance for mathematics teaching and learning in general throughout Norway.

As mentioned above, The Norwegian Directorate for Education and Training (2010a) requires that mathematics teaching should be connected to vocational pathways. While connections between mathematics and vocational practices can be achieved in several different ways, I will concentrate on the design and implementation of vocationally connected mathematics tasks. Research shows that most time in mathematics classrooms is spent on tasks (Alseth, Breiteig, & Brekke, 2003; Grønmo, Onstad, & Pedersen, 2010; Hiebert, 2003). Grønmo et al. (2010) found, in the Third International Math and Science Study (TIMSS), that Norwegian students report a strong focus in lessons on solving problems that are similar to problems in their textbooks. Students and teachers report that less time is spent on other ways of working with mathematics (such as discussing and choosing strategies for problem solving, solving problems, and explaining and accounting for reasoning), compared to the other countries represented in TIMSS. Time spent on mathematics tasks can be advantageous or disadvantageous, depending on how students and teachers use the opportunities for learning mathematics that tasks provide. However, I would argue that the kind of tasks that teachers assign, and the implantation of these, is an important factor in the quality of students' experiences of mathematics classes in Norway.

When the Norwegian government encourages, and requires vocational connections in mathematics courses, it is possible to infer that such mathematics teaching is intended to give something more, different or better than 'normal' mathematics courses. Dahlback, Haaland, Hansen, and Sylte (2011) suggest two different reasons for adding vocational relevance to common core subjects like mathematics. The first is to provide students with meaningful and relevant vocational training for their future careers, while the second is to motivate and engage them. Hiim (2013) points to the fact that vocational teachers say that modern vocational practices require knowledge of mathematics and other common core subjects, but that the specific needs differ within the various vocational practices. Williams and Wake (2007) have shown that identifying mathematics in workplaces is difficult, and I would argue that the other way around, drawing on vocational contexts in mathematics classrooms, is not straightforward either. As the vignette above illustrates, the teacher and student seem to differ in their expectations of the task. They therefore discuss the task from different points of view. This thesis investigates the opportunities and challenges that come up in the classroom implementation of vocationally connected tasks.

1.3.1 Concepts and Definitions in Connection to Vocationally Connected Mathematics

In this section, I clarify the definitions of some key words and concepts used in this thesis. These initial definitions are elaborated and explained further in my discussion of theoretical concepts in Chapter 4. In describing the mathematics tasks in this study, I chose to use the term **vocationally connected** as a translation of the Norwegian word "yrkesretting". There are a few instances in the international literature where the terms "vocationalism" or "vocationalization" are used (Bell & Donnelly, 2009), but these words do not capture the Norwegian "yrkesretting" well, so I chose to use "vocationally connected" instead. By vocationally connected mathematics, I am referring to classroom mathematics in which vocational applications are clear and relevant to students' vocational education programmes. The construction of vocationally connected mathematics is often related to **workplace mathematics**. By "workplace mathematics", I mean the mathematics that is in use in workplaces. Wasenden (2001) notes that vocationally connected mathematics courses are intended to give students the mathematical competencies that are needed for their vocational practices. Figure 1.2 depicts vocationally connected mathematics as a bridge between mathematics (as a school subject) and the vocational subjects.

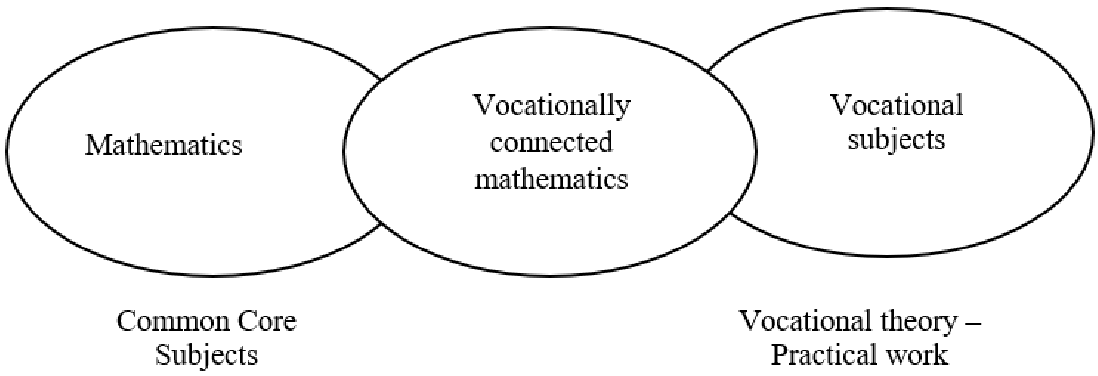

Figure 1.2: Vocational connected mathematics (Wasenden, 2001, p. 50, my translation of the figure). Wasenden (2001) points out that figure is meant as an illustration and should not be read as a Venn diagram.

I follow Mason and Johnston-Wilder's (2006) definition of a mathematics **task**: they define a task as "what learners are asked to do: the calculations to be performed, the mental images and diagrams to be discussed, or the symbols to be manipulated" (p. 4). This definition does not constrain a task to a specific length or size. For example, I will talk about "the hair salon budget task" as one task, even though it consists of several sub-questions. However, the questions that comprise one task are related to one common vocational practice, such as making jack stands or a creating a hair salon budget. Together, these concepts contribute to **vocationally connected mathematics tasks**[1], which I define as tasks in mathematics classes that are intended to draw on the students' future vocational contexts.

Norms are the written or unwritten rules of different practices (Yackel & Cobb, 1996). It is possible to break, challenge and change norms, and participants, like students and teachers, can have different perceptions of what the norms in the classroom are. Norms will be discussed further in the theory chapter (Section 4.4). **Enacted norms** are the norms that can be observed in a certain setting, for instance in a classroom (Levenson, Tirosh, and Tsamir, 2009). Such enacted norms can be different from the intended norms of the teacher.

[1] Norwegian: Yrkesrettede oppgaver.

1.3.2 Research Questions

Recognising the importance of good mathematics teaching and learning of mathematics in vocational education, my overarching research aim is to explore how students interact with classroom mathematics tasks which are designed to draw on the students' future vocational contexts. This aim generates the following research questions:

- RQ1: What characterises the enacted norms in the classroom when students work with vocationally connected mathematics tasks?
- RQ2: What connections do students make with workplace practices and out-of-school knowledge when engaged with vocationally connected tasks?
- RQ3: What opportunities and challenges arise in employing vocationally connected tasks in school-based mathematics classes?

With the first two research questions I analyse what happened during the classroom implementation of vocationally connected tasks. The third question is more overarching, and I look into opportunities and challenges with classroom mathematics tasks which are designed to draw on the students' future vocational contexts.

1.4 Overview of the Thesis

In Chapter 2, I explain the background and context of the Norwegian school system, in particular the structure of vocational education programmes. In Chapter 3, I present previous research on mathematics practices in and out of school, including research on vocational mathematics and workplace mathematics. I then look into research on how vocational and workplace mathematics has been introduced in school settings. In Chapter 4, I explain and justify my theoretical perspectives, which are based on Wenger's (2000) concept of Communities of Practice. Important issues are the concepts of boundary objects, authenticity and norms. In Chapter 5, I present my methodological choices, and data collection methods, data analyse and the trustworthiness of the research.

The task design and findings are presented in Chapters 6, 7 and 8. In Chapter 6, I present background for the task design, and three vocationally connected tasks, before I show and analyse the implementation of the tasks in Chapter 7. In Chapter 8, I conduct a cross-case analysis of my three cases, and discuss normative activity, task authenticity and membership across practices. In Chapter

9 I reflect over my research design and discuss opportunities and challenges when students work with vocationally connected tasks. Here I also discuss the study's contributions to mathematical education research and pedagogical implications.

2 Background and the Empirical Context

In this chapter, I first explain the Norwegian education system, with an emphasis on vocational education programmes. I then present some of the challenges that exist in these programmes, and I explain how mathematics is placed in the Norwegian vocational education. This background information is important, since vocational education systems differ across the world. Since the students in vocational education has participated in ten years of mathematics classes before they start their vocational studies, I begin with a short overview of the Norwegian education system. I then move on to the vocational education programmes where I highlight various aspects of the nature of mathematics classes in vocational upper secondary schools in Norway.

2.1 The Norwegian Education System

Norway has ten years of compulsory education. Pupils start in Grade 1 when they are six years old and continue until the end of lower secondary school (Grade 10). Only 4 % of pupils attend private schools (The Norwegian Directorate for Education and Training, 2019), hence I concentrate on state schools here. The basic structure of the education system is outlined in Table 2.1.

Table 2.1: Outline of the Norwegian Education System.

<table>
<tr><th colspan="2">Norwegian School System</th><th>Grade</th><th>Age</th></tr>
<tr><td colspan="2">Kindergarten (voluntary)</td><td></td><td>1-5</td></tr>
<tr><td colspan="2">Primary School</td><td>1-7</td><td>6-12</td></tr>
<tr><td colspan="2">Lower Secondary School</td><td>8-10</td><td>13-15</td></tr>
<tr><td colspan="2">Upper Secondary School (voluntary)</td><td rowspan="2">11 →</td><td rowspan="2">16 →</td></tr>
<tr><td>Vocational Education and Training Programmes
(2 years in school
2 years apprenticeship)</td><td>General
Study Programmes
(3 years)</td></tr>
</table>

The education system is based on the principles of education for everyone, equality and the provision of adapted education within the National Curriculum

(The Norwegian Ministry of Education and Research, 2006). Almost all pupils attend the school which is nearest to their home, and low, medium and high attainers study together. Most pupils with special needs are integrated into ordinary classes with additional support and aid[2] (The Norwegian Directorate for Education and Training, 2013b).

Pupils have no options regarding which subjects to study in the period of compulsory education to Grade 10, with the exception of Foreign Language and Elective Subjects[3]. All of these factors contribute to a compulsory education with a broad range of students in all subjects.

2.1.1 Upper Secondary Education

At the end of lower secondary school, pupils receive grades in all subjects, and in addition take one graded written exam and one graded oral exam. Pupils are entitled to admission to upper secondary school and have the right to start at one (of a ranked list of three) education programmes that they apply to. However, the admission is dependent on pupils' lower secondary school grades, which means that those with weak grades can end up in study programmes which they have little interest in. Even though upper secondary school in Norway is not compulsory, almost 98 % of pupils started upper secondary school in the school year 2012, the year in which this study took place (The Norwegian Directorate for Education and Training, 2014a). This indicates that not moving on to upper secondary school is not seen as a viable option in Norway today.

Upper secondary education is divided into General Studies Programmes, and Vocational Education and Training Programmes (see Table 2.1). Local counties are responsible for upper secondary education and decide how many places to offer in the different educational programmes. There are usually several upper secondary schools in each county. An upper secondary school will usually have two or three different educational programmes, and some have both General Studies Programmes and Vocational Education and Training Programmes.

When I conducted this study about 59 % of the students choose General Studies Programmes, and the rest choose Vocational Education and Training

[2] 8.6 % of pupils have special needs teaching to a lesser or larger extent. 92 % of these pupils are members of an ordinary class. In total, less than 1 % of the pupils are organized in other ways than an ordinary class.

[3] Norwegian: Valgfag. Elective subjects have a total of 171 hours (2 % of the total school time), and pupils can choose between subjects such as "Physical Activity and Health," "Design and Redesign," "International Cooperation," "Research in Practice," and "Living Heritage."

Programmes (The Norwegian Directorate for Education and Training, 2014a). The proportion of students in Programmes for General Studies has increased slightly in recent years. The available choices in Programmes for General Studies when my study was conducted (2012 - 2014) were Specialisation in General Studies, Sports and Physical Education, and Music, Dance and Drama. On completion of the three years of upper secondary school, students gain a general university and college admission certification, needed for acceptance into further study at the tertiary level.

Students who follow Vocational Education Programmes likewise have the opportunity to continue toward a general education certificate after their two first years of vocational education, if they take a supplementary year of general studies instead of doing an apprenticeship. This supplementary year is usually taken at the same upper secondary school as the student's first two years of vocational education, although this depends on how the county has chosen to organise its upper secondary schools. Students who take this extra year are qualified to continue to higher education with the same opportunities as students who have followed the general education studies path from the outset of upper secondary school. However, this supplementary year is generally regarded as quite difficult. Over half of the students who began a supplementary year in 2011-12 failed in one or more subjects, and therefore did not receive a general education certificate (The Norwegian Directorate for Education and Training, 2013a).

2.1.2 Vocational Education and Training Programmes

There are nine Vocational Education and Training programmes, and this study includes cases from three of them: Programme for Design, Arts, and Crafts; Programme for Media and Communication; and Programme for Technical and Industrial Production. These are broad entry programmes for vocational education; students choose a specialisation in their second year. For instance, a student in the Programme for Technical and Industrial Production will have the opportunity, depending on their choices in the second year, to take up an apprenticeship in a variety of vocations such as motor vehicle mechanic or laboratory technician. This structure has the organisational advantage that students can use their first year in upper secondary school to decide which of the many vocations, within their chosen educational programme, they will specialise in the three following years.

The main model for vocational training is the 2+2 model, illustrated in Table 2.2. This is a combination of two years in school, with education and training, and then two years paid in-service training in an enterprise. The first two years is mainly school based, with only a small amount of time spent in enterprises. Students in vocational education programmes are usually organised into classes of around 15 students according to their vocational education programme, and these 15 students have almost all their lessons together in the class.

Table 2.2: Outline of the 2+2 model for vocational education and training.

<table>
<tr><th>Year</th><th></th><th>Place</th></tr>
<tr><td>11</td><td>Vg1 (Vocational Education year 1)</td><td rowspan="2">Upper Secondary School</td></tr>
<tr><td>12</td><td>Vg2 (Vocational Education year 2)</td></tr>
<tr><td>13</td><td rowspan="2">Combined Training and Productive Work</td><td rowspan="2">Enterprise</td></tr>
<tr><td>14</td></tr>
<tr><td colspan="3">Craft certificate is obtained after a final examination</td></tr>
</table>

The subjects in the first two years of Vocational Education Programmes are divided into Common Core Subjects, Common Programme Subjects and an In-Depth Study Project[4]. In the two first years of Vocational Education Programmes, common core subjects have 588 hours; common programme subjects have 954 hours; and the In-depth Study Project 421 hours. The common core subjects are Norwegian, Mathematics, English, Natural Science, Social Science and Physical Education, all of which the students have met during their earlier schooling. The common programme subjects comprise students' chosen vocational subjects.

This means that about 30 % of allotted time in the two first years of vocational education is used on academic subjects that students have met in their previous education. An important aspect is that Common Core Subjects share the same curriculum across all Vocational Education and Training Programmes, following a curriculum which is similar to the equivalent subjects in General Studies Programmes. In mathematics, the curriculum goals in Vocational Education Programmes comprise around 60 % of the curriculum goals in General Studies Programmes (see Section 2.2).

[4] Norwegian: Prosjekt til fordypning.

Sharing the same curriculum across all Vocational Education and Training Programmes means that students who want to switch to another education programme have the opportunity to start again without the need to retake common core subjects (The Norwegian Ministry of Education and Research, 2013). While this is an advantage for students who realise that they have made a wrong educational choice, it constrains possibilities for adapting the curriculum to specific educational programmes.

A noted above, Mathematics is a common core subject, together with Norwegian, English, Natural Science, Social Science and Physical Education. The Norwegian Directorate for Education and Training (2010a) specified that "teaching [in the Common Core subjects] should be as relevant as possible, with adaptions for students' different education programmes" (my translation), and this has been regarded as an instruction to connect the common core subjects with students' vocational programme context.

Common Programme Subjects vary in accordance with individual education programmes; for example, students in the Technical and Industrial Productions education programme will learn about Production[5], Technical Services[6], and Documentation and Quality[7]. After two years in upper secondary school, students will undertake a two-year apprenticeship, before they take final examinations and obtain a craft certificate. During the first two years in school, students are supervised by teachers in workshops at the school, but during this time they will only have short visits to workplaces.

Vocational education is often regarded as different and somewhat separate from other kinds of school education. Learning practices in vocational education have been described by the Swedish researcher V. Lindberg (2003) as different from both learning practices in grades 1 to 10, and practices in the workplace. She characterises the vocational education model in the Nordic countries as a hybrid between practices, describing most of the school tasks that students do as "contextualised in the vocation" (p. 162), while much of the work they do in the school-based workshop is self-instructional and relies on being able to read and use books and manuals that are used in industry. Tasks in school-based workshops often have dual goals of producing an artefact and training students in

[5] Norwegian: Produksjon.
[6] Norwegian: Tekniske tjenester.
[7] Norwegian: Dokumentasjon og kvalitet.

vocation-related procedures. For example, students in Technical and Industrial Productions education will repair lawnmowers or do small car paint jobs in the school-based workshop, before they become apprentices in an enterprise in year three.

Jensen (2017) describes what it means to be competent is constructed differently for the same students in the mathematics subject and the vocational subjects in vocational education in Norway. She argues that in the mathematics class the students were regarded as competent for knowing the methods that the teacher earlier had introduced, but in the workshop the students had a high degree of responsibility for authoring solutions and making sure that their solutions were suitable to professional standards.

Høst (2012) reports that Norwegian students in his study expected vocational education to be different from usual classroom teaching, and they felt that they learned most from, and were most interested in, practical training. Dahlback et al. (2011) showed that vocationally connected practical tasks motivated students when the common core subjects and the common programme subjects were integrated. However, they also found, as did Hiim (2013), that implementing such integration was difficult because of the broad first year in vocational education programmes, reporting that this was problematic for both students and teachers. They point out that teachers often do not have competence in the whole range of potential vocational opportunities in the education programme, especially in the first year of the education; for instance, a teacher who has a background as a hairdresser might be required to teach students who want to become florists (Dahlback et al., 2011). In the next section, I elaborate on some issues in vocational education that are the centre of public debate.

2.1.3 Challenges within Vocational Education in Norway

In Norway, it is a governmental aim that as many students as possible in upper secondary education should complete their education and obtain either a general education certificate or a craft certificate (The Norwegian Ministry of Education and Research, 2013). But as already noted, drop-out and early school leaving is a problem in upper secondary education, and only about 67 % of students in vocational education programmes complete their studies within a six-year period, meaning over 30 % do not complete within this time (Statistics Norway, 2020). This situation has been investigated from different viewpoints, either with the spotlight on vocational education as a whole, or with a more subject-specific view.

Research has shown that background variables such as educational level and ethnicity of parents increase the likelihood of early school leaving and non-completion in upper secondary school (Markussen, 2010). Furthermore, low grades at lower secondary school level increase the likelihood of drop-out (Markussen, Frøseth, Lødding, & Sandberg, 2008; Markussen et al., 2011). Other reported problems concern students' transitions: Hernes (2010) recommends increased follow up in the critical periods if transition from lower to upper secondary school, and into the apprentice period. A major problem is that around 25 % of the vocational students each year do not get an apprenticeship, and instead have to do their last two years based in school (NOU 2019: 25, 2019).

Skogseid, Skogseid, and Kovač (2013) investigated student drop-out from vocational education at the beginning of the apprenticeship period, finding that several different factors were involved, ranging from problems with the work environment itself to too little support from the school and/or the training office when students encountered problems in their apprenticeship. Lillejord et al. (2015) state that some possible solutions for reducing early school leaving and non-completion can be to have actions directed towards attendance, guidance by older students, and more systematic introducing students to workplaces at an early stage in their education, so that they see that what they are studying at the moment have relevance for their future.

A nationwide project called "New Possibilities[8]" was established in 2010 by The Norwegian Ministry of Education and Research to address the problems with drop-out, aiming to increase successful completion of Upper Secondary Education and Training from 70 % to 75 % (The Norwegian Ministry of Education and Research, 2016). The New Possibilities project focused on training teachers with the aim of increasing low achievers' readiness for upper secondary education by organising intensive teaching for low achievers in tenth grade. The project lasted three years, from Autumn 2010 until 2013, and was intended to improve cooperation between state, counties (responsible for upper secondary schools) and municipalities (responsible for lower secondary schools) for reducing drop-out numbers. A particularly relevant part of the project was the Common Subjects, Vocationally Connections, Relevance (my translation,

[8] Norwegian: NyGiv.

shorten FYR[9] in Norwegian) project for the vocational education programmes, which established teacher networks with the intention of making teaching in the common core subjects more relevant and connected to students' future vocations.

Researchers evaluating New Possibilities found that intensive teaching for low achievers in tenth grade showed no significant effects with regards to pupils' grades in lower secondary school (Eielsen, Kirkebøen, Leuven, Rønning, & Raaum, 2013). Rønning, Hodgson, and Tomlinson (2013), and Lødding and Holen (2013), reported that the teaching was organized by withdrawing pupils from their ordinary classes into small groups, and that the pupils generally were positive about the effect of their participation in the project. However, Huitfeldt, Kirkebøen, Strømsvåg, Eielsen, and Rønning (2018) found that there were no significant effects of the intensive teaching on students' completion rates in the cohort of students five years after the students started upper secondary school.

In the governmental white paper NOU 2019: 25 (2019) it is argued that education will be more and more important in the future, and that Norway needs skilled workers. They have looked at possible different models for secondary education in Norway and suggest that it should be room for more variation in the length of the vocational educational programmes than the 2 + 2 model gives today. They argue for moving away from the possibility of switching educational programmes without too much loss of time and instead having more tailored curriculum goals in the common core subjects in the different vocational programmes. They propose striking a balance between the common knowledge that the student needs, and the fact that the common subjects should support the education in the vocational subjects.

Tessem (2013) suggested a change from the 2+2 model toward a four-year model, and closer integration between school and enterprise. There has also been discussion for increased awareness of the possibility for some students obtaining a certificate of skills on a lower level[10], instead of a craft certificate (The Norwegian Directorate for Education and Training, 2018). At the moment this is little used form of secondary education.

The challenges within vocational education in Norway are complex and not easily solved. In the next section, I present the particular problematic area that is of interest in this study: namely, the common core subject of mathematics.

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[9] Norwegian: Fellesfag, Yrkesretting, Relevans.
[10] Norwegian: Praksis-brev.

2.2 Mathematics in Vocational Education Programmes

Students enrolled in Programmes for Vocational Education and Training take a compulsory mathematics course in the first year of their vocational education. Mathematics is the subject in which most students in vocational education programmes struggle to complete (Stene, Haugset, & Iversen, 2014), making the mathematics course of central importance with respect to the problem of non-completion.

The curriculum[11] allows the students to choose between two mathematics courses, 1P-Y and 1T-Y. The course 1P-Y is regarded as the easiest mathematics course. In short, the 1Y-T curriculum is intended to be more theoretical, while 1P-Y is meant to be more practical. The mathematics course involves a total of 84 hours of teaching in the 11th school year, equivalent to about three 45-minute lessons a week. Almost all students (around 95 %) in vocational education programmes choose 1P-Y (The Norwegian Directorate for Education and Training, 2020a) at the time of the data collection. Students who choose 1Y-T are almost all from the Electricity and Electronics vocational programme.

In Norway mathematics is challenging for many students, and the number of students that get the two lowest grades, is around 20 % of the students after their 10 years of school (The Norwegian Directorate for Education and Training, 2020d). This unfortunate trend continues at upper secondary school, and in the written exam for 1P-Y in 2017/18 14 % of the students failed the exam (NOU 2019: 25, 2019). My study investigated students’ experiences in the mathematics course 1P-Y. 1P-Y follows the same curriculum across all of the nine vocational education programmes and is a shortened version of the competence aims of the mathematics course 1P followed by students in general education programmes. This way of setting up the curriculum originates from the 1994 education reform in Norway. Before this, mathematics in the vocational education programmes had (since the 1970s) been regarded as a tool intended to help learning in vocational subjects; it was tailored and connected to the various vocational education programmes (Wasenden, 2001). Although, as noted above, the common curriculum across programmes increases students’ options for changing their Education Programme without too much additional coursework, criticism of

[11] The curriculum valid at the time of the data collection was the Knowledge Promotion Reform - 2006.

the current system points to the lack of vocational relevance of the common core curricula (see for instance Hiim, 2013).

The competence aims in 1P-Y are divided into three main areas: Numbers and Algebra in Practice, Geometry, and Economics (The Norwegian Directorate for Education and Training, 2010b). Students in the general education programmes have additional competence aims in Probability and Functions. The competence aims[12] for 1P-Y are listed below:

Numbers and algebra

The aims of the course are that the pupils should be able to

- Estimate answers, calculate practical tasks, with and without technical aids, and evaluate how reasonable the results are
- Interpret, process, evaluate and discuss the mathematical content of written, oral and graphic presentations
- Interpret and use formulas that apply to everyday life, working life and the education programme area
- Calculate with proportions, percentages, percentage points and growth factors
- Deal with proportional and inversely proportional magnitudes in practical contexts

Geometry

The aims of the course are that the pupils should be able to

- Use similarity, scale and the Pythagorean Theorem in calculations and practical work
- Solve practical problems involving length, angle, area and volume
- Calculate using different measurement units, use different measuring tools and evaluate measurement accuracy
- Interpret and prepare working drawings, maps, sketches and perspective drawings related to working life, art and architecture

Economics

The aims of the course are that the pupils should be able to

- Calculate using price indexes, currencies, real wages and nominal wages

[12] These were the competence aims at the time I collected my data (The Norwegian Directorate for Education and Training, 2010b). The curriculum had a minor revision in 2013. The curriculum for 1P-Y after the revision was still the same for all vocational education programmes. A new curriculum is implemented in 2020 where the curriculum is no longer the same for the different vocational education programmes.

- Calculate wages, and compose budgets and accounts using various tools
- Calculate taxes
- Examine and evaluate consumption and various terms for loans and savings using web-based consumer calculators

(The Norwegian Directorate for Education and Training, 2010b, translated by me)

The mathematics curriculum does not specify potential connections with the different vocational education programmes. But formulations such as “calculate practical tasks”, “interpret and use formulas that apply to day-to-day life and working life”, “practical contexts,” “practical work”, “solve practical problems,” and “interpret, make, and use sketches and working drawings for problems from cultural and working life” point toward mathematics used outside of school. As noted above, the Norwegian Directorate for Education and Training (2010a) emphasises that common core subject teaching should be relevant and adapted to suit different education programmes[13]. In addition, the objectives of the mathematics curriculum point out that mathematics “can form the basis (…) for participation in working life and recreational activities” (The Norwegian Directorate for Education and Training, 2010b, p. 2).

In order to teach mathematics in upper secondary education, including the 1P-Y course, a teacher is required to have at least 60 ECTS points in mathematics (The Norwegian Ministry of Education and Research, 2006). The 1P-Y course is mostly taught by teachers who teach mathematics on several levels and education programmes in upper secondary education. Many teachers with the required formal competence in mathematics do not have experience of working in vocational education programmes or the students’ future workplaces, and as a result can find it difficult to utilise vocational contexts in their teaching.

Textbooks written for the 1P-Y course claim that they are vocationally connected. But the two major textbook series[14] at the time of the data collection (2011-2012) were written to cover all nine vocational education programmes (Engeseth, Heir, Moe, & Kielland, 2013; Oldervoll, Orskaug, Vaaje, Svorstøl, & Hals, 2014). In the New Possibility project an important part of the project was the production and collection of vocationally connected tasks (The Norwegian

[13] Norwegian: Opplæringsloven kapittel 1, andre ledd § 1-3 videregående opplæring: ‘‘Opplæringa i fellesfaga skal vere tilpassa dei ulike utdanningsprogramma’’.
[14] ‘Sinus 1P-Y’ and ‘Matematikk for yrkesfag’.

Directorate for Education and Training, 2017), signalising the need for education material for vocational relevance[15].

Students taking 1P-Y course are graded by their mathematics teacher. In addition, and in keeping with the assessment practice in Norway of randomly selecting students for written and/or oral examination, some students may be examined by an oral or written end of year exam in the course. The curriculum specifies that the written exam is written locally, but counties cooperate to make the exams. These examinations present an opportunity for connecting the mathematics course with vocational contexts, but even though questions have real world contexts, I would argue that they are in fact quite similar to the examinations taken by general education programme students in mathematics course 1P. Examples can be found here[16]. This suggests that there is little incentive for teachers to connect the 1P-Y mathematics course with vocational contexts in order to prepare for the examination.

However, the most important factor to consider in this course is that, when students start their vocational education programmes, they have already studied mathematics in school for 10 years. This means that they have a much experience with what mathematics is (as a school subject). They know how to act in a mathematics classroom, they have their identity as a mathematics student, and they know which norms apply in a mathematics course. During their years in vocational education, these students will gradually become participants in vocational practices, and will therefore have opportunities to use mathematics in ways which are different from the practices of school mathematics classrooms. In the next chapter, I discuss a range of research on mathematics practices in school, the workplace, and everyday life, highlighting the ontological issues which arise in and between practices. In Chapter 4 I return to a discussion of classroom norms and the development of a theoretical framework which understands students and teachers as members of communities of practices.

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[15] These are now available on the webpage https://fyr.ndla.no/

[16] https://www.vigoiks.no/content/view/full/2238

3 Mathematics Practices in School and the Workplace

My research is connected to many themes in mathematics education: vocational and workplace mathematics, daily life mathematics, vocational education, adult learning of mathematics, and transfer, boundaries and connections between school mathematics, vocational mathematics in school and workplace mathematics. I concentrate here on research on workplace mathematics and the implementation of vocational mathematics in education practices. I have also studied some seminal works in the field of daily life mathematics and mathematics use in out-of-school settings. The reason for this, as I will argue in this chapter, is that mathematics as seen in the mathematics classrooms is not the same as mathematics found in workplaces.

I searched the ERIC, Google Scholar, SpringerLink, and Bibsys databases for the terms "vocational mathematics", "workplace mathematics", "mathematical modelling + workplace", "context workplace mathematics", "everyday mathematics", "adults learning mathematics", and "word problems + context". In addition, relevant articles identified through reference lists in articles that I found in the initial searches has been included. This literature review is limited to peer-reviewed articles dating back to 2000, with an exception made to include some older seminal works in the field. I also included some Norwegian and Swedish doctoral and master theses. The following table presents an overview of the studies, and the thematic organisation of the literature review. I would place my own research in the row ''Introducing Vocational and Workplace Mathematics into School Contexts''.

Table 3.1: Overview of studies in the literature review.

Theme	Nordic Research	Rest of world research
Contrasting mathematics practices in and out of school	Mosvold (2005)	d'Ambrosio (1985), Barton (1996), Civil (2002), Cooper and Harries (2002), FitzSimons and Boistrup (2017), Greer, Verschaffel, and Mukhopadhyay (2007), Jurdak and Shahin (2001), Lave (1988), Lampert (1990), Millroy (1992), Noss (2002), Nunes, Schliemann, and Carraher (1993), Straesser (2015), Verschaffel,

		De Corte, and Lasure (1994), Verschaffel, Schukajlow, Star, Van Dooren (2020)
Workplace mathematics	Johansson (2014), Saló i Nevado, Holm and Pehkonen (2011), Saló i Nevado and Pehkonen (2018)	Brown (2002), Coben and Hutton (2013), FitzSimons (2001), Hoyles, Noss, and Pozzi (2001), Hutton (1998), Magajna and Monaghan (2003), Masingila (1994), Moreira and Pardal (2012), Nicol (2002), Roth (2014), Straesser (2015), Wake (2014), Williams and Wake (2007), Weeks, Lyne, and Torrance (2000), R. Zevenbergen and Zevenbergen (2009), R. J. Zevenbergen (2011)
Introducing Vocational and Workplace Mathematics into School Contexts	Aretorn (2012), Bekkeseth (2009), Bø (2013), Frejd and Muhrman (2020), Irebro (2014), Jensen (2017), Johannessen (2012), L. Lindberg and Grevholm (2011), Svanberg (2014), Særsland (2018), Utvik (2012)	Bonotto (2001, 2005, 2013), Dalby and Noyes (2015), De Bock, Verschaffel, Janssens, Van Dooren and Claes (2003), Hahn (2000), Hudson (2008), Kaiser and Schwarz (2010) , Lowrie (2011), Monaghan (2007), Vos, Devesse and Pinto (2007)
Mathematics Learning for the Workplace: Understanding Movement between Practices	Kilbrink and Bjurulf (2013)	Bakker and Akkerman (2014), Bakker, Groenveld, Wijers, Akkerman, and Gravemeijer (2014), Coben and Weeks (2014), LaCroix (2014), Martin and LaCroix (2008), Triantafillou and Potari (2014)

In this literature review, I first present studies of mathematical practices in different settings, both in and out of school. I look closer into workplace mathematics, before I present research from studies in which vocational and workplace mathematics have been implemented into school or educational practices.

3.1 Contrasting Mathematics Practices In and Out of School

The challenges and concerns about what mathematics in vocational programmes should look like indicates important differences between mathematical practices in different settings, and ontological claims about what mathematics is. Students meet a subject in school known as mathematics, where they learn how to calculate in different mathematical areas of arithmetic, measurement, geometry, probability and statistics. People who work as engineers describe mathematics as a tool, and mathematicians in universities may say that mathematics is about abstraction, proof and gradually working out new mathematical structures (Ernest, 1991/2004).

d’Ambrosio (1985) claims that there is a need to consider a broader understanding of what mathematics is, introducing the concept of ethnomathematics as “mathematics which is practised among identifiable cultural groups” (p. 45), for instance different professions. Ethnomathematics view mathematics as a cultural product, and Barton (1996) points out that ethnomathematics is the study of one group ‘‘attempting to understand particular practices and conceptions which are held by another group’’ (p. 220). For me, this means that the ontology of mathematics, what mathematics really is, will be different according to which cultural group you ask.

Ernest (1991/2004) discusses what mathematics is perceived as and states that one common point of view is that mathematics is pure, neutral, value-free and that mathematicians discover ‘god given’ structures. He compares this view with that what he calls social constructivism, in which one sees mathematics as embedded into cultural practices, and that mathematics is value laden, culture bound, corrigible and constructed between participants. For Ernest (1991/2004) there is a difference between academic knowledge and culturally embedded knowledge, and this stems from point that mathematics is different in different groups.

For students in school, mathematics is the subject they have been working on in mathematics classes, with its rules and routines for how to work with the subject and interact with fellow students and teachers. This is different from what mathematics is for mathematicians in universities, and different from what mathematics is in an engineering practice. And these kinds of mathematics are again different from mathematics which can be found in workplaces, often hidden into vocational practices, tools and routines. In this thesis, I take the ontological view that mathematics and practices cannot be separated, and therefore mathematics is different in different practices. I do this because, as I argue in Chapter 4, I utilise a sociocultural view of students as participants in different practices, and the students are a part of a practice of schooling in the mathematics classroom, but are at the same time starting to become members of a vocational practice.

Lampert (1990) points out that knowing mathematics in classrooms is connected with getting the right answer quickly, and following, remembering and applying rules given by the teacher – ‘‘mathematical *truth is determined* when the answer is ratified by the teacher’’ (Lampert, 1990, p.31). Our understanding of how to do mathematics and what it means to know mathematics in school is ‘‘acquired through years of watching, listening, and practicing’’ (Lampert, 1990, p. 31). She compares this to the mathematics done by mathematicians, where important aspects of their work are the ability to evaluate assumptions and discuss the foundations for arguments. Civil (2002) argues that ‘‘mathematics as a discipline deals with ill-defined problems; it requires time, persistence, and flexibility’’ (p. 42), and both Civil and Lampert assess that what is known as mathematics in school classrooms is different from the research discipline.

Likewise does school mathematics differ a great deal from the mathematics of everyday life and mathematics in workplaces. Lave (1988) studied how adults use mathematics in their daily lives, focusing on grocery shoppers and people starting on diets. Observing how they used mathematics in an everyday context, Lave found that as the dieters gained familiarity with the diet, they transformed their use of measuring, and invented their own units to cope with the diet restrictions and minimise time and effort used on mathematics. The grocery shoppers demonstrated many strategies for optimising “best buy” strategies when they shopped, but when they were tested in equivalent problems on a school test, they resorted to procedures that they likely learned in school, and made more mistakes than while shopping.

Nunes et al.’s (1993) classic study contrasted school mathematics and street mathematics. They regarded both as mathematics, but within different cultural practices. Nunes et al. found that young market sellers on the street were able to solve arithmetic problems connected to sales when the researchers posed as customers in the market, but when the children were asked similar questions about the same mathematics in a school setting they had problems getting the right answer. In the market setting the children used different strategies, however in the school setting they tried (and often failed) to use strategies learned in school.

Focusing on mathematics as seen in workplaces, it may be difficult to recognise mathematics in the way it is thought of in school settings. FitzSimons and Boistrup (2017) argue that workplace problems are embedded into contextual knowledge, and Noss (2002) points out that workers' mathematical knowledge is anchored in their knowledge of the context. Millroy (1992) studied the mathematics that is embedded into the work of carpenters in South Africa and showed that many mathematical notions are both present and needed in their work. But the mathematics is expressed differently in comparison with mathematics in school practices and relies more on tacit knowledge and framing questions into the carpentry context.

As argued, identifying mathematics use outside of school settings can be difficult, and many will recognise students' complaint in mathematics classes: "When will we use this?" In the Norwegian setting, Mosvold (2005) studied Norwegian teachers' beliefs about the implementation of mathematics in everyday life. He found that even though some teachers often emphasise real-life connections, pupils were not encouraged to formulate problems from their own experiences outside the mathematics classroom. The lack of such problem formulation is an indication of a cultural practice in which school mathematics is something other or different from mathematics in daily life or workplace settings.

Jurdak and Shahin (2001) asked one group of high school students, and one group of plumbers, to do the same task: make a cylinder which could contain one litre and be 20 cm high. For both groups this was an unfamiliar task, the students were not used to constructing cylinders from given dimensions, and the plumbers often constructed cylinders, but they would make them according to standard dimensions, which were different from the dimensions given in this task. They found that the students worked from the formula of a cylinder and had to be prompted to make a physical container, and did the task more or less as they would do in a mathematics classroom, without any empirical testing of their final product. In contrast the plumbers connected the given dimensions to another container they usually made, which was 1.2 litre, and reasoned what the changed height would mean for the difference in circumference. When the plumbers had constructed the container, they checked the size by adding one litre of liquid. The plumbers' attention was connected to the goal of the activity – producing the cylinder, while the students lost track of this goal.

In school settings pupils work in mathematics classrooms, on tasks given by the teacher. One way of working in mathematics is with word problems. Verschaffel et al. (2000) explain the characteristics of word problems as:

> Word problems can be defined as verbal descriptions of problem situations wherein one or more questions are raised the answer to which can be obtained by the application of mathematical operations to numerical data available in the problem statement (Verschaffel et al., 2000, p. ix).

In mathematics classrooms students work on word problems with an explicit context. However; word problems are often criticised for being "poorly disguised exercises in one of the four basic [mathematics] operations" (Gravemeijer, 1997, p. 390). Research shows that pupils have difficulties with word problems based on real-world situations (Greer et al., 2007; Verschaffel et al., 1994, Verschaffel, Schukajlow, Star, Van Dooren, 2020). However, Cooper and Harries (2002) found that it is possible to rewrite word problems in ways which support students to use more realistic considerations. If the students were encouraged to imagine situations in which the answer would be different from the result of the numerical calculation, they were willing to introduce realistic responses to the questions. In these responses, they utilised both arithmetic and everyday knowledge. These studies show that students have expectations of what mathematics is, and that mathematics that is done in school setting is something which has its own rules, routines and practices.

The studies referred to above all identify mathematics in people's everyday lives, but the mathematics is often wrapped into practices or procedures disguising the mathematics to a point that people are unaware of it. This invisibility of mathematics has been particularly identified in the workplace setting (Williams & Wake, 2007). Straesser (2015) points out that mathematics used in the workplace is so connected to vocational practices and constraints that it is almost impossible to separate it from those practices. The next section is about workplace mathematics, before I turn to how vocational and workplace mathematics has been introduced in school contexts.

3.2 Workplace Mathematics

In the research reports reviewed, it seems quite clear that workplace mathematics is different from mathematics in school. Wake (2014) states that "mathematical activity in workplaces, where and when it occurs, looks very different from that in educational settings" (p. 272). FitzSimons (2001) similarly says that each workplace "has its own discourses and, arguably, its own ethnomathematics" (p. 377) and Straesser (2015) points out that the different kinds of mathematics needed at the workplaces varies according to the domain of work. Such differences can be seen in the studies discussed below.

Masingila (1994) studied the mathematical practices used in carpet laying. She identified four mathematical concepts in use: measurement, geometry, computational algorithms, and ratio and proportion. She found that when measuring in the context of carpet laying, the issue of estimation and rounding off numbers was an important part of the work. The additional time and effort needed for accuracy meant that it would be more expensive to be completely precise than if the carpet layers estimated while measuring. They needed to be flexible and to adapt to the different constraints on the job sites, and therefore needed to be adaptive in their problem solving. Many of the procedures the workers used in the carpet laying had been captured in routines, without mathematical explanations.

Such rules or routines, referred to as ‘‘rules of thumb’’, were likewise found by Roth (2014). He showed that even though the mathematics that trainee electricians learned in college was contextualised in a vocational setting, the practical implementation of the mathematical methods was done "[by] following the rules of thumb that they have learned in the course of their apprenticeship" (p. 185). In college, the students had learned trigonometry to find angles and proper distance from the obstacles to bend electrical conduits. But when they worked in the field, they learned a set of unwritten rules, such as that the look of the finished bends was important, and that bending smaller angles is preferred in order to ease the pulling of wires. He concluded that no one used the trigonometry they had learned but used bending practices without knowing the underlying (mathematical) reasons for the unwritten rules.

Moreira and Pardal (2012) studied mathematics in masonry and found that the masons used mathematics knowledge in routine and intuitive ways. They would use Pythagorean triplets to make 90 degrees angles and figured out inclination of roofs without connecting it to mathematics. Likewise, in a study of boatbuilding apprentices, there were very different ways of working within the trade, compared to school mathematics practices (R. Zevenbergen & Zevenbergen, 2009). The apprentices needed skills in problem solving and estimation, but also “having the eye for what looks right” (p. 195). Tasks would be completed with trial-and-error methods because of unique features on different boats. They used mathematical skills such as measurement, estimation of numbers and space, but without the formality and precision of school mathematics. The importance of estimation is furthermore seen in a study of young workers in the retail industry, where the young workers would use estimation to check if the bill looked appropriate (R. J. Zevenbergen, 2011). The cashiers needed problem solving skills to be able to find errors made by the cash register, but they did not value being able to correctly add items by hand.

The mathematics needed and used in nursing practices has been widely studied (see for instance Brown, 2002; Coben & Hutton, 2013; Hutton, 1998; Weeks et al., 2000). Errors in nursing mathematics can have fatal outcomes, and it is important that calculations of dosages of drugs are correct. Hoyles et al. (2001) studied how proportional reasoning is used in nursing practice. They found that nurses used a wide range of strategies when calculating different drug dosages, but when they were asked how to calculate dosages, they referred to the “nursing rule,” a fixed calculation routine. The researchers observed that the nurses changed their strategies according to the different types of drugs and packaging. Their drug administration was error-free, and they used many flexible strategies. In addition, the nurses relied on what “felt okay” and their familiarity with the drugs. There was a gap between what they did in practice and the mathematical strategies they were taught in their training programmes. Johansson (2014) followed a nursing aide in her work and found that it was important to know mathematical concepts such as reading charts, comparing values, and adding values. However, the nursing aide valued knowledge that came from the “third eye,” the feeling that experienced nursing aides had for the patients’ possible problems. Johansson (2014) argues that such experience cannot be developed in school, even if one uses “so-called real world problems” (p. 80).

Saló i Nevado and Pehkonen (2018) studied what mathematics is needed by cabinetmakers, and how problem solving is interlocked in their daily work. They found that the cabinetmakers used mathematics in two different ways; basic mathematics as addition, subtraction, multiplication, measuring and geometry, but also used what can be thought of as mathematical problem solving to figure out many non-routine solutions during their work. Saló i Nevado, Holm, and Pehkonen (2011) found that farmers use buckets and so on as measuring devices, similar to Lave's study of dieters who invented their own units to minimise effort used on mathematics.

There have also been a number of studies of the mathematical practices used in workplaces which rely heavily on technology. When studying technicians who worked with computer aided design of moulds for glass factories, Magajna and Monaghan (2003) found that the technology shaped the mathematical procedures. Workers had different methods to calculate the volume of the shapes and did not reflect upon the validity of the different methods. If a problem or error situation arose, the problem was not analysed mathematically, but pragmatically solved, either with the use of a different method, or worked out manually. The important goal of the (mathematical) activity was the final product, not how the problem was solved.

As many of the above studies illustrate, there is a difference between mathematics in school settings and mathematics identified in workplaces. Both Nicol (2002) and Williams and Wake (2007) note that teachers, pupils, and workers find it difficult to identify mathematics in the workplace. Nicol (2002) reported from a study in which prospective teachers visited workplaces to connect mathematics with workplace practices. The prospective teachers were mostly positive about their workplace visits, but experienced difficulties in identifying mathematics in use in the workplaces. They found it difficult to design mathematics tasks on the basis of the workplace visits and would often lose the authentic context when they tried to make mathematically demanding tasks.

Williams and Wake (2007) argue that mathematics in workplaces is hidden in black boxes (i.e., that the workers do not know that it is mathematics they are using and doing). The mathematics is either concealed by technology or procedures that are not understood by their users. They argue that two kinds of black boxing processes exist: the mathematics can be hidden in instruments and routines such as the computer aided design tools in Magajna and Monaghan's (2003) study, the cash registers in R. J. Zevenbergen's (2011) study or in the unwritten rules for bending practices of electricians (Roth, 2014). The other kind of black boxing is the result of a division of labour, with workers being isolated or protected from mathematics by rules, norms and expectations. This is a more social black boxing, either conscious or unconscious, and can be seen, for example, when only some people in the workplace are able to de-code mathematical practices and routines (Williams & Wake, 2007).

The research on workplace mathematics reviewed here shows that recognising mathematics in the workplace is difficult for both workers and outsiders. These ontological considerations underline the complexity of the challenges involved in how to teach vocational and workplace mathematics; in the next section, I will present research on mathematics teaching and learning which is concerned with bringing out-of-school contexts into the mathematics classroom.

3.3 Introducing Vocational and Workplace Mathematics into School Contexts

Several studies consider attempts to make mathematics more accessible by linking it with everyday or workplace contexts. For example, Hudson (2008) studied a project where students in need of additional academic support were placed in an industry setting to learn and use mathematics. He found that the students reported a sense of lack of meaning in the school mathematics. But when the students worked in pairs with industry mentors in the factory, they were able to use appropriate mathematical skills with confidence.

Kaiser & Schwarz (2010) investigated high achieving upper secondary school students' responses to genuine problems faced by applied mathematicians working in industry. The students reported after a week-long mathematical modelling project that they had learned a lot, and the majority of the students said that they would like to have such problems occasionally included in their mathematics classes. A Norwegian master thesis studying affect in vocational education found that students felt more positive toward mathematics if the mathematics class was connected to their vocational education programme (Bekkeseth, 2009). This was also the case in two small Swedish studies of restaurant and food education programmes students' attitude to mathematics (Irebro, 2014; Svanberg, 2014).

However, most studies tend to highlight the issues raised above regarding ontology and mathematics. For example, in order to present mathematics as a means of understanding everyday activities, Bonotto introduced artefacts such as restaurant menus and advertising leaflets into classroom activities (Bonotto, 2001, 2005, 2013). In his study, students (of age 10-11) were able to utilise the artefacts to create mathematical problems. Some of the problems they created were open-ended or contained several potential solutions, unlike more traditional school mathematics problems (Bonotto, 2013). But in another study, he showed that such artefacts can bring about a tension between school mathematics and everyday knowledge (Bonotto, 2005). In this study, students used supermarket receipts and baking powder labels to introduce multiplication with decimal numbers. The students had relevant out-of-school experiences which predated the mathematics they were supposed to learn. This knowledge supported a movement between understanding the problem and reviewing the results. The students were aware, though, of the difference between reality and their school-based work, and some students resisted changes from their ordinary working practices in the school, in which a single solution to given problems were the norm.

In a French education context, similar to Norwegian vocational education, Hahn (2000) studied jewellery shop apprentices in their mathematics course, and the application of proportionality and calculations with percentages. She found that in some cases students rejected the realism of problems from the workplace, and one shop owner is cited in the paper as saying ‘‘the teacher’s solution was wrong because it never happened like that in the shop’’ (p. 161). When the students were given percentage questions, they demonstrated two different practices depending on which environment they solved the questions in. In the mathematics classroom, they used procedures they had learned there, and in their technology classroom, the students used procedures learned in the workshop. They only used school practices in their technology classroom if they did not know a professional practice.

In a Swedish setting, Frejd and Muhrman (2020) studied one mathematics teacher and one vocational educational teacher collaborated on teaching activities both in the mathematics classroom and the vocational classroom, a training hair salon. They found that tools, norms and how to work differed significantly in the two settings, and that in the training hair salon the students took responsibility for their own learning, helped each other, and behaved as they would in a business setting. Here the students discussed their findings and evaluated their answers. In comparison in the mathematics classroom the teacher was the one that showed what was supposed to be done, and the students asked the teacher instead of other students. Frejd and Muhrman (2020) conclude that the vocational classroom setting gave the students access to workplace-authentic tools, and norms of a workplace, and there were much more unexpected problem-solving in the training hair salon.

Aretorn (2012) reported likewise that teachers of electrical installations and maintenance and teachers of mathematics had different explanations for the same mathematical electricity tasks. She found that the mathematics teachers used more general mathematics-based explanations, while the electricity teachers were more context-specific in their explanations. A related study in Sweden reports on the impact of integrating mathematics and vocational subjects by giving mathematics teachers the opportunity to work closely with vocational subject teachers (L. Lindberg and Grevholm, 2011). The researchers found that students had improved outcomes in both mathematics and the vocational subjects if mathematics and vocational teaching were integrated.

Likewise, Dalby and Noyes (2015) found that students in vocational studies in United Kingdom experienced contrasting practices in the vocational education training, and in the mathematics classrooms. In vocational lessons students were expected to work as professionals, they asked each other questions and were supposed to take responsibility for decisions. In the lessons practical skills were valued, but the theoretical knowledge was embedded into the students work. While in many of the mathematics classes the students met a typical mathematics classroom, where the teacher controlled what to do and how to do it. They had little agency, were supposed to work individually and were judged by academic expectations. In addition, there were some mathematics classrooms which Dalby and Noyes found were more similar to the vocational classrooms. In these mathematics classes the teacher used situations which the students had experienced in the vocation and tried to synchronise their teaching of mathematical themes with the needs of the vocational programme. These classrooms also had flatter social structures, used peer learning and the teachers adapted a facilitation role. Dalby and Noyes (2015) argue that the students are in the middle of two different educational traditions, with contrasting values and traditions. This is also seen in Jensen's (2017) study where the students' competence is constructed differently in the mathematics classroom and the vocational education subjects. In the mathematics classroom the students had little room for making choices and sharing insights, but in the vocational education the students were accountable for finding solutions and being critical to their own and other's ideas.

Monaghan (2007) reported on a project in which students worked on a problem from a freight firm regarding how to track truck deliveries by GPS traces and geometrical considerations. He found that the students transformed the original task into a new, subtly different task. The students made the tasks about set of simpler shapes, instead of polygons, which was what the original task called for. The company director had stressed that because of a nearby highway the polygon shape was needed, but the students still transformed the task to something they were able to solve. Monaghan (2007) argues that this transformation is related to the students' understanding of the goal of the activity, mathematics and their everyday knowledge. He points out that when mathematics is used in workplaces it is with a goal of getting something done or produced, which differs from the goal of the activity in the mathematics class.

A study on modelling non-linear geometry problems, which utilised contextual and non-contextual problems, showed that students did better on problems *without* authentic contexts (De Bock et al., 2003). The context based information about the geometrical problems was intended to keep students from making mistakes while scaling up or down figures with volume or area. One group of students watched video clips of Gulliver's travels before the students were given the test in a school setting. However, the students which had seen the video did worse on the questions which utilised a context in the problem statement. De Bock et al. (2003) argue that this may be because of the weak operationalisation of authenticity, and that because of the authenticity, the questions may have been presented in a unfamiliar way in the context of a school test and that the students may have been more involved in the context than the resulting mathematics. This study shows that there is difficulty involved when trying to add context in a school context.

Lowrie (2011) used genuine artefacts such as a theme park map and menus when he challenged students to solve a realistic mathematics problem. He found that the students' knowledge of the theme park context gave them opportunities to use realistic considerations when working with the problem. However, the students became very engaged in discussions of issues that were not necessary for solving the task, drawing on personal knowledge, such as that it would be reasonable to budget for increased prices over the holidays. They had problems regulating and reconciling their personalised understandings of the context to develop a common mathematical solution in the group. Lowrie concludes that the genuine artefacts helped the students to access personal experiences that would aid making sense of the problem-solving activity but that the challenge lay in merging this potential with productive group dynamics. The use of artefacts is similar to what is seen in studies conducted by Vos et al. (2007) in Mozambique. Here, they used authentic resources such as baskets, fish traps, and newspaper clippings in classrooms to integrate students' daily life experiences with their academic mathematics. They found that the authentic resources did not automatically change the classroom dynamics, but together with a focus on open-ended questions and group work, their intervention was successful.

In Norway, Johannessen (2012) looked at secondary students' perceptions of mathematics and its relevance to their Building and Construction Technology education programme in her master thesis. She found that the students regarded tasks as more meaningful when they were related to their (future) workplace. Særsland (2018) likewise, in her master thesis, found that vocational students that took the mathematics course 1P-Y reported that experienced relevance were positive for their motivation. In another master thesis, Utvik (2012) found that students used several mathematical resources and practices in their programme subjects. The students needed to calculate with numbers, and use geometrical shapes such as area, the Pythagorean Theorem, and the use of correct units. However, Bø's (2013) master thesis reports that students are not particularly influenced by their experiences in the programme subject when they learned about scaling in mathematics.

Overall, a wide range of studies indicate that introducing every day or work-based contexts into school classrooms is not a straightforward or unproblematic way for students to learn mathematics. The ontological issues highlighted above indicate that the blending of practices is more complex than we might think. In the next section I review research on mathematics learning for the workplace, focusing on movement between practices.

3.4 Mathematics Learning for the Workplace: Understanding Movement between Practices

Several studies explore the issue of how workplace mathematics can be integrated into vocational educational settings, and the relationship between practices. For example, Bakker et al. (2014) designed a computer tool intended to develop better proficiency in proportional reasoning for students studying to become lab technicians, where diluting solutions will be a central skill in their future work life. They report significant improvement in the students' skills on proportional reasoning on a post-test. Coben and Weeks (2014) also report a positive impact on students' learning following use of a computer-based tool which simulated an authentic situation for nursing students. The students had to identify relevant information and calculate appropriate drug dosages in a nursing context. In a different (but related) research project, students in their last year of laboratory education were studied to find out how they integrated statistical and work-related knowledge (Bakker & Akkerman, 2014). One important issue in the (future) work of the students was to calibrate and compare different measurement methods and machines. The students were given a previous student's report on a similar calibration, and meetings were arranged between teachers, students and workplace supervisors to help the students to conduct their own research projects on calibrations. The study showed that the students responded positively to this project, and that they significantly increased their level of integration between workplace and school-related knowledge. Bakker and Akkerman (2014) point out that this increase could be due to the reflections with multiple perspectives conducted in and in-between the meetings.

LaCroix (2014) studied the mathematical activities of adult students in pipe trades pre-apprenticeship training. In the course, mathematical topics related to the pipe trade were generally introduced in a classroom setting and practiced on pencil-and-paper exercises. The students then applied this to fabrication tasks in a workshop. He found that students' difficulties with these tasks could be attributed to the students' novice levels of knowledge about objects in the pipe trade, rather than to mathematical difficulties. Similarly, in a study of apprentice ironworkers' performance on a workplace task involving technical plans and finding out the weight of a structure that would be lifted by a crane, knowledge of the work context was important (Martin & LaCroix, 2008). For example, one student reflected on the beams in the task as real objects and could identify that they had made a mistake in the calculations because the answer did not look correct. In vocational training, students work on the basis of a commitment to providing sensible answers in the context, rather than being concerned with purely mathematical calculations.

Triantafillou and Potari (2014) studied how apprentice engineering students in telecommunications transferred knowledge about the place value system from school into how a work setting. Students had to interpret authentic patch boards in a telecommunication closet. Triantafillou and Potari found that the five participants recognised the context, and that they recognised place value was a shared mathematical object between school and workplace. They found four factors that facilitated or constrained the students' possibility for transfer of their knowledge between the different contexts: the use of metaphors, context dependency, problem solving strategies, and the students' motivation.

Finally, in Sweden, Kilbrink and Bjurulf (2013) studied what teachers and supervisors in vocational upper secondary education thought was important regarding transfer between school and workplace. They emphasised four key issues as important: transfer of basic knowledge such as mathematics skills, transfer of principles and skills (for example, between different machines or materials), transfer of written material such as the ability to read CAD drawings, and the transfer of experience. Communication, financial resources and reflections were factors that facilitated possibilities for transfer of the students' knowledge between school and workplace. By communication they stress the importance of knowing what has been done in the different practices, if the teacher knows what has happened in the workplace, he can build on that in school, and the other way around. This is also connected to the issue of reflection, where the teachers and supervisors point out that the students need to have time to explore and make mistakes, and then reflect upon their experiences. All in all, these studies indicate that movement between practices is complex and relies on the nature of mathematical understanding in different contexts.

3.5 Summary

Many of the studies discussed in this review relate to the issue of transfer (of knowledge), that is, how something learned in one setting can be applied in a different setting (Säljö, 2003). However, as Wake (2014) writes, the mathematics education community has been plagued with problems on how to transfer what is learned and known from one setting to another. He points out that in school, mathematics is the object of study, while in workplace settings mathematics is used as a tool for productive outcomes for the workplace.

Similarly, Bakker and Akkerman (2014) reject the concept of transfer because of its assumption of a one-directional single process, preferring the notion of boundary crossing, because of its potential to capture the back-and-forth movement of participants between different practices, for instance between school and workplace settings. Vocational pathway students are in transition between school education and their future workplace practices, and they meet mathematics in different guises in this process. I explore the idea of boundary crossing in detail in the next chapter.

This literature review shows that although there appear to be a number of opportunities for recognising and learning mathematics when students are working between practices, it is not easy to integrate vocational mathematics into educational practices (FitzSimons, 2014). Although students often ignore context in mathematics problems (Greer et al., 2007; Hahn, 2000), careful consideration of task set-up can make students consider and use their out-of-school knowledge (Bonotto, 2005, 2013; Cooper & Harries, 2002; Lowrie, 2011). However, the ontological issue of what mathematics is in different settings, and how students, workers and teachers recognise it, leads to complex outcomes for attempts to support both school and vocational mathematics learning.

4 Theoretical Perspectives

The literature review indicates a number of issues: problems with the concept of transfer or boundary crossings between mathematics in different contexts, difficulties in researching movement between practices, and a recognition of how different practices are associated with different expectations by students and teachers. Therefore, I needed a theory which captures practice and movement between practices, since I aim to understand how students interact with vocationally connected tasks. Wenger's (1998) communities of practice provides a lens to understand the nature of practice and the competences, artefacts, and shared repertoire of a practice, the role of participants in practices, and the nature of boundary crossings. This chapter begins with a discussion of communities of practice, before moving to an exploration of work on boundary objects and boundary crossing. Consideration of boundary objects in educational settings leads to a discussion of authenticity in task design, and the nature of authenticity as a social construct. Finally, this chapter turns to a discussion of classroom norms as components of communities of practice and expectations of tasks as part of classroom practices. At the end of the chapter, I summarise the theoretical perspectives, and why they are relevant and useful for my research.

4.1 Students as Participants in Sociocultural Practices

In this study, I have chosen to utilise a sociocultural view, with an emphasis on students and teachers as participants belonging to multiple communities of practice. In the intersection between mathematics and work life which vocational education represents, students interact with both mathematics and vocational practices, and both practices have cultural, institutional and historical dimensions. Wenger (1998) argues that communities of practice evolve within the wider context of society, and the practices of a community are, to a greater or lesser extent, shaped by conditions which are outside of members' control. Schooling is clearly a practice shaped by many conditions outside of students' and teachers' control, such as education policy and popular discourses concerning the value of mathematics. Nevertheless, responses to these conditions by the participants of the community of practice are produced within, and mediated by, the constraints and affordances which exist within the practice setting.

4.1.1 What is a Community of Practice?

A community of practice is a particular kind of community, defined by Wenger (1998) in terms of three characteristics: mutual engagement, a joint enterprise and a shared repertoire. The concept of mutual engagement highlights the idea that participants in a community interact with each other in accordance with established norms and relationships. Mutual engagement requires that interactions take place, for instance through discussion and the exchange of information and beliefs. For Wenger (1998), mutual engagement concerns not just what an individual knows about a practice, but also knowing what they need to know, and how to ask for help.

Joint enterprise concerns the collective understanding of the purpose of the community (Wenger, 2000). It is negotiated in response to the situation that participants are a part of. The concept of 'joint' does not mean that everybody in the community of practice must be in agreement with each other, but that the enterprise is ''communally negotiated'' (p. 78). Participants are interconnected, and need to find ways to work, coordinate and live with their differences. Negotiating a joint enterprise leads to accountability for those who are involved in the community of practice: they must act in accordance with "what matters and what does not, what is important and what is not, […], what to talk about and what to leave unsaid, what to justify and what to take for granted'' (p. 81). The concept of joint enterprise is closely connected to the concept of norms, elaborated in Chapter 4.4.

A shared repertoire includes routines, histories, tools and working methods in the community of practice which are created or adapted and have become part of the practice. Wenger (1998) proposes that this shared repertoire is useful for two reasons: interpretations of actions and practices are well-established, but also a shared repertoire can be re-engaged with in new situations. For instance, students and teachers have implicitly agreed routines for how to act in a classroom. If a student were to move to another school, they would know how to act like a student, even though people and surroundings would be different.

Wenger (1998, p. 125-126) gives several indicators for the existence of a community of practice, including: sustained relationships, fast movement of information, knowing what others can do, shared jokes and stories, and the ability to judge which actions are appropriate. In a community of practice, a participant begins as a newcomer, and a peripheral member of the group (a ‘legitimate peripheral participant’), before gradually becoming an established member of the community of practice (Lave & Wenger, 1991). Being a competent or an established member of a community of practice entails being able to understand, support and add to its joint enterprise, being able to engage within its relationships and norms, and being able to draw on the shared repertoire in appropriate ways in the community (Wenger, 2000). The trajectory from newcomer to established member of the group involves “participation as a way of learning’’ (Lave & Wenger, 1991, p. 95). The students in my study are on a trajectory from newcomer to competent member in their chosen vocational practice and are established members in a classroom practice of learning mathematics in a school classroom.

Wenger (1998) argues that some groups of people - for example, speakers of the same language, or all students in a school - are too wide-ranging to be treated as a community of practice. They may lack mutual engagement, joint enterprise or a shared repertoire. However, such groups can be usefully viewed as constellations of interconnected practices comprised of communities of practice which are related by a common enterprise, or because they belong to an institution or have shared artefacts or members in common (Wenger, 1998).

4.1.2 Identity and Learning

Wenger (2000) points out that people define themselves by what communities they belong to, as well as which communities they do not belong to, and hence membership of communities of practice relates to identities. Everybody is part of several communities of practices at the same time; for instance, a person can be a mother and part of the community of practice of the family, and a doctor and part of the community of practice of the medical community. In addition, the same person can be part of a local union, a choir and the community of friends from her university studies. Wenger (2000) claims that ‘‘multimembership is an inherent aspect of our identities’’ (p. 239). Even though someone may act differently in the different communities that they are members of, they do not stop being a parent in their workplace community or being a doctor at home.

Wenger (1998) considers learning as social participation, where participation is a process of being or becoming an active participant in a community. This means that identity and learning are closely connected in terms of becoming a full participant in the community (Lave & Wenger, 1991). For an individual, learning will be ‘‘an issue of engaging in and contributing to the practices of their communities’’ (Wenger, 1998, p. 7). For a community, learning will be about giving room and space to new members in the community and changing and improving the shared repertoire. So learning is about gradually constructing an identity as a competent participant where one can negotiate meanings and actions within the practice and coordinate and adhere to the norms of a practice (Wenger, 1998).

4.1.3 Schools and Communities of Practice

In schools, both teachers and students are members of several communities of practice. Teachers are participants in teachers’ unions, the faculty at school, a subset community of the faculty that teaches the same subject, or the same grade level. They are part of the community of practice in the classroom, together with their students. Classrooms as communities of practice are different from the communities of practice associated with the subject being learned. For example, taking students and teachers in a physics classroom as an example of a community of practice, Lave and Wenger (1991) note that the developing community of practice is not the community of physicists, but rather a community of schooled adults.

As discussed in Chapter 2, a school class in secondary school in Norway consists of students that have chosen the same education programme, and most are the same age. They will study together, meet every day, maybe be friends in their spare time, but what makes them a community of practice is their mutual engagement in the mathematics class or other subjects. Wenger (1998) argues that a community of practice does not mean homogeneity; being together creates differences as well as similarities, and participants have unique identities and roles. In a mathematics class or a common programme subject class, the students and the teacher share a mutual engagement in the issue of schooling. They interact with each other and draw on each other’s competence and contributions. The joint enterprise of a mathematics class or a common programme subject class is multifaceted. It can, for some students, be just about getting a grade at the end of the semester; for others, the social life in the class might be their most important enterprise. But as Lave and Wenger (1991) propose, they are engaged in reproducing the community of schooled adults.

The indicators of a community of practice are recognisable in many classrooms; for instance, if students want to know an answer to a mathematics question, they would know which of their fellow students to ask. Information of many kinds spread quickly through a student group, and they know which actions are appropriate or not in a given situation. A shared repertoire in a mathematics class can reference earlier work and tasks, which words are used, and how one speaks about mathematics. In Wenger's terms, learning in this school setting will be about gradually becoming a full member of the community of practice through participation in its practices. As Jensen (2017) argued, what is regarded as being competent is different in mathematics classrooms and in vocational training settings. So the question arises, what is seen as necessary for being a full member in a classroom, and in particular a mathematics classroom, or a mathematics classroom in a vocational education pathway? Is this the same as what is necessary for being a full member of a vocational practice, with respect to the mathematics involved? And how does multimembership in different practices, the mathematics classroom and the vocational practice, shape the students' interactions with the vocationally connected tasks?

4.1.4 Boundaries

For Wenger (1998), communities of practice cannot be understood or be considered in isolation from the rest of the world. An important aspect of a community of practice is, therefore, the boundaries of the community: where the boundary is, and what happens when someone or something crosses the boundary of a community of practice. Boundaries trace the edges of a community of practice, and the mutual engagement, joint enterprise and shared repertoire of the community will distinguish between outsiders and members of the community.

In his later work, Wenger notes that the boundaries of communities of practices will often be fluid, and are often unspoken (Wenger, 2000). Such boundaries are important because they often offer learning opportunities which differ from those within a community of practice. At the boundary, the shared repertoire of the community may not match the competences required in the community of practice across the boundary. Mathematics, in its different guises, can be seen as being on the boundary between the mathematics classroom and the vocational practices, and the repertories of the participants in the different communities may differ from what is needed of competence. This will therefore spur a need for new or changed competences, which then can be taken up into the shared repertoire of the community.

For Wenger (2000), the boundaries of communities of practices are often shifting and malleable. He points out that boundaries are important sites for learning, where different perspectives meet, and new possibilities can come to the fore. However, as Star (2010) argues, a ‘‘boundary implies something like edge or periphery, as in the boundary of a state or a tumor. Here, however, it is used to mean a shared space, where exactly that sense of here and there are confounded’’ (Star, 2010, pp. 602-603). Boundaries can be found in workplaces, in everyday life or within schools. In workplaces, experts from different fields or practices interact and collaborate across their fields of expertise. In education, there can be boundaries between what is done in school and in work practice.

Akkerman and Bakker’s (2011a) definition of boundaries between different practices are helpful for understanding movement between the school and work practices of mathematics. They explain that boundaries are a “sociocultural difference leading to discontinuity in action or interaction” (p. 133). For example, a discontinuity in action is if knowledge experienced in a school setting is not possible to use by the participant in a workplace setting (or another setting out of the school practice). So there is for instance one practice, the school mathematics classroom, in which a participant knows the routines and how to act and in which ways one should solve problems. When the participant moves to another practice, for instance a workplace setting, the routines and ways to act do not seem the same, and the participant has to learn new routines in order to be able to cope in this new setting. In this case, according to Bakker and Akkerman (2014), a boundary between the two practices exists such that what one knows in one practice does not seem relevant or possible to use in the other practice.

Boundaries are defined by, and dependent on, researchers’ distinctions between different communities or practices (Akkerman, 2011). For example, I define the mathematics class as one practice, and the common programme subject class as another practice. It may have been possible to define all students’ school classes as a joint practice of schooling. But in this case the mutual engagement, joint enterprise and shared repertoire of all the students’ classes is probably too widely defined to be useful. Therefore, it is more fruitful to see these two practices as constellations of interconnected practices which have a common enterprise, and belong to the same institution and have members in common.

Being a full participant in the mathematics class is probably not the same as being a full participant in the common programme subject class. There will be different ways of engaging within the relationships and norms of routines, and differences in what constitutes appropriate use of the shared repertoire.

In their review of literature on boundary crossing and boundary objects, Akkerman and Bakker (2011a) argue that boundaries carry potential for learning, with identification, coordination, reflection, and transformation being four possible learning processes at the boundaries between practices. They use the concept of learning in a broad sense, encompassing both new understandings and institutional development. Identification refers to the way in which practitioners can understand their own practice in comparison with another practice. Coordination concerns how different practices cooperate to figure out different parts of work, and then coordinate across boundaries. Here, the goal is to facilitate movement between the different practices. Reflection concerns how, as a result of boundary crossing, one can identify different practices, and then reflect on one's own. An example of this would be that by visiting workplaces, teachers can become aware of and reflect on how mathematics plays another role in workplaces than it does in the school setting (Williams & Wake, 2007). Finally, transformation concerns changed or new practices. For a transformation to happen, Akkerman and Bakker argue, there needs to be a perceived problem or need that leads to confrontation between different practices, and a need for reorganisation of routines or new working methods within the communities of practice.

Wenger (2000) notes three types of bridges across boundaries: people (brokers), artefacts (boundary objects) and interactions between people from different communities of practice. Brokers are people who translate and coordinate perspectives between different communities of practice. For instance, a new employee could point out that they did a similar process differently in her last job and introduce a new way of carrying out the process in her new job. By making connections between communities of practice, brokers may make new connections and open new possibilities for meaning. Boundary objects are objects such as documents, routines, concepts or artifacts that can indirectly translate and coordinate practices between different communities of practice.

Boundary crossing and boundary objects are helpful theoretical concepts in order to explain movement across or between different communities. I expand on the concept of boundary objects in education in the Section 4.2 and explore the connection between boundary objects and authenticity in Section 4.3.

Participants in a mathematics classroom are also members of other communities of practices. Students in vocational education programmes are at this point in time where they study mathematics, newcomers into their chosen vocation, but they are in many ways old-timers when it comes to how to be a participant in a mathematics classroom. In the case of vocationally connected mathematics tasks, there may be points of contact between the community of practice of the mathematics class, the community of practice of the common programme subject classes, and the community of practice of the vocation. An important aspect of communities of practices are the joint enterprise and shared repertoire, which is connected to the issue of norms. This will be discussed in Section 4.4.

4.2 Boundary Objects in Education

Boundary crossing concerns how persons move across or back and forth between different communities. One example is student teachers, who are participants in a teacher education programme at university but become members of the teaching staff in school placements. Following school placement, they will again be participants in the teacher education programme at university. They will experience various values and norms in these different practices and will have to negotiate how to act in the different settings (Akkerman & Bakker, 2011a). When students in vocational education visit businesses on work placements, they cross boundaries between school and enterprise.

Boundary objects are “object[s] that cross boundaries” (Akkerman & Bakker, 2011b, p. 2), and were first introduced by Star and Griesemer (1989), who noted that “boundary objects are (…) both plastic enough to adapt to local needs and the constraints of several parties employing them, yet robust enough to maintain a common identity across sites” (p. 393).

Star and Griesemer (1989) argue that boundary objects can be either abstract mental models or concrete objects. They list specimens, field notes, museums and maps as boundary objects produced and used by theoretical researchers, amateur biologists and sponsors. These items have different meanings for participants in different practices. They use as an example that a mounted bird is used by participants in different ways. For a museum sponsor, the bird can be a way of attracting public interest, while for researchers, it is a start of a scientific inquiry (Star & Griesemer, 1989).

Another example of a boundary object is a practice journal that the student teachers can bring to both their teacher education programme and to their practice school (Akkerman & Bakker, 2011a). The practice journal can be read and discussed by participants in the teacher education programme (the teacher educators and students) and by participants in the practice schools (principals, mentor teachers and students). It therefore crosses the boundary between university and school and is of (different) use to the participants in the different practices. Jurdak (2016) argues that an item can be thought of as a boundary object depending on the situation it is used in. For example, ‘‘a car is not a boundary object, though it is made of material, but becomes one if used to bus students to racially mixed schools’’ (Jurdak, 2016, p. 67). Wenger (1998) likewise argues that not all objects are boundary objects and claims that for an object to be a boundary object, it needs to bring about and help to coordinate collaborations between different practices.

Tuomi-Gröhn (2003) claims that boundary objects are important because they facilitate and promote collaboration between participants from different practices. In mathematics education research, there are various examples of research on boundary objects. Hoyles, Noss, and Kent (2004) argue that in mathematics, a boundary object can help teachers and learners (two different communities) to come to an agreement that they are talking about the same (abstract) mathematical notion. For example, they see the output of computer algebra systems (CAS) software as a possible boundary object. Further examples of boundary objects are annual pension statements used to communicate between customers, technical experts and customer support (Kent, Noss, Guile, Hoyles, & Bakker, 2007), and boundary objects like graphs, charts, tables and figures for communication between sales agents and customers at a mortgage firm (Bakker, Kent, Hoyles, & Noss, 2011). Bakker et al. (2011) studied the role of boundary objects in making communication across different practices fail or succeed. Their study explored how mortgage sales agents communicated with customers with a graphical example of a future payment plan. The payment plan was generated by a computer programme as a graph, and the mortgage sellers had to interpret and explain the graph to customers. Here, the graph acts as a boundary object between the seller and the customer.

In the case of the use of vocationally connected tasks in a mathematics classroom setting, I used objects such as photographs of items which are in use in the vocational practice. Here these objects are constructed for an educational purpose, and such photographs or illustrations are representations of an object from the vocational practice but are not the object itself. I therefore argue that photographs or illustrations are *representations* of boundary objects and have a particular role to play in connecting the mathematics in each practice. Hence, these objects are not boundary objects in the classical sense, where boundary objects contribute to communication across boundaries between participants in different practices. I have chosen to name such objects *boundary object representations*. The boundary object representation does not directly facilitate communication between participants from different practices, but instead facilitates communication between participants in the classroom (see Figure 4.1). The boundary object representation gives an indication of potential boundary crossings between the mathematics classroom and the vocational practice.

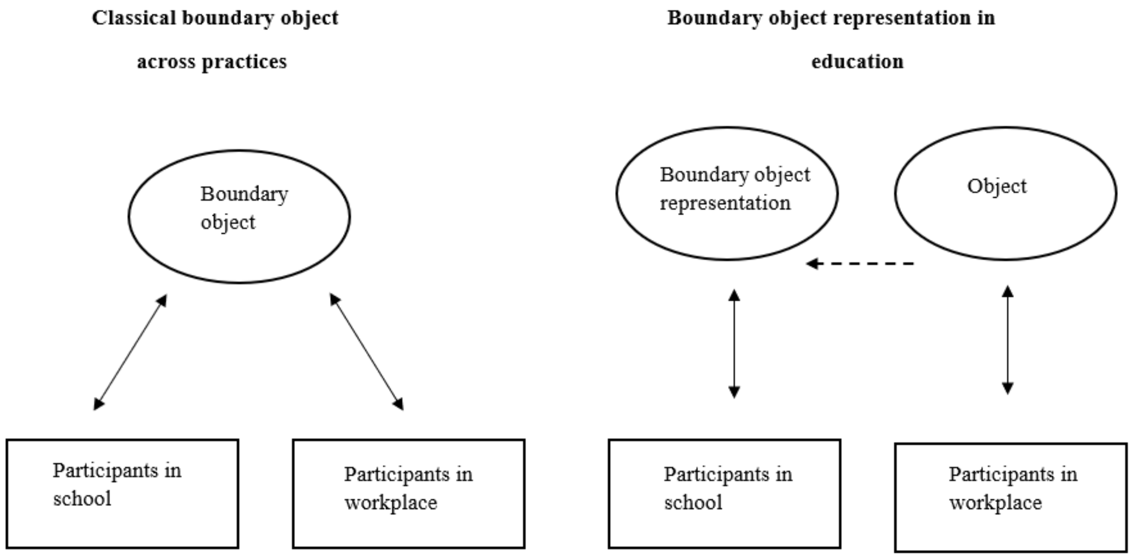

Figure 4.1: Boundary objects in education

The communication when using boundary object representation is no longer between participants of different practices (for instance school and workplace), but the participants in a practice (for instance school) communicate about the boundary object representation. I will explain Figure 4.1 by looking at the example of the jack stand. The left side of Figure 4.1 shows boundary objects as it facilitates communication between two practices. A real jack stand (as depicted in Figure 1.1) could be seen as a boundary object between students in a mathematics classroom and participants in a welding workshop. In the workplace the jack stand is a tool in use when working on cars. It may be premade, or it may be produced in a workshop when it is needed. In the school setting the students were asked to weld such a jack stand. This is a training task for welding, in which the participant needs to place the legs equally around the middle pipe before starting to weld. In these two different settings, the workshop at school and a workplace, participants could use real jack stands to communicate around.

However, on the right side of Figure 4.1 I refer to *boundary object representations*. In the mathematics class, the task included a photograph of the jack stand in order to support students' work on geometrical concepts such as diameter and circumference. It is not the jack stand itself. As a boundary object representation, it is intended to facilitate connections between the students in a mathematics classroom and the practice of the vocation. However; in the mathematics classroom the boundary object representation has meaning as a part of a task for learning and applying mathematics, and how to complete the task is regulated by routines and norms of the mathematics classroom. The students can use the boundary object representation to connect what they do in the mathematics classroom with what they know of in the vocational setting, however the boundary object representation is not used in the communication between two different practices.

As shown in Chapter 3, such connections are not easy to use in the mathematics classroom, and there are differences between what is regarded as valid reasoning, working methods and goals in the two practices. In the next section, I will expand on the concept of authenticity, which is important for further discussion of the nature of boundary object representations in the tasks used in my research.

4.3 Authenticity in Education

To describe connections to the vocational practices I have chosen to use the terms "authentic" and "authenticity" when I describe aspects of the tasks. Vos (2011) and Verschaffel, Greer, and de Corte (2000) point out that the term authenticity has been used in a range of meanings in mathematics learning and teaching.

I follow Vos (2011, 2015) in looking at authentic aspects in tasks. She suggested that it is best to take a "pragmatic" definition of authenticity for separate aspects of tasks (themes, resources, activities) if these are "clearly not created for educational purposes" (Vos, 2011, p. 713). Vos (2015) claims that authenticity is a social construct, i.e., something that is agreed upon through a social process. This point connects the understanding of authenticity to the concept of communities of practice, and I will claim that what is authentic can differ between different communities of practice. Vos (2011) argues that an understanding of authenticity does not hinge on just one community member's view, but ‘‘the term authenticity can be a qualification clear to all actors, even if the aspect has no meaning or relevance to them’’ (p. 720).

She defines an authentic aspect of a task as needing “(1) an out-of-school origin and (2) a certification of originality” (Vos, 2015, p. 105). The second criterion, certification of originality, can be satisfied in different ways. One possibility for this certification of originality is that participants from diverse practices vouch for the authenticity. Such certification does not need to be by people working on the issue every day, but the authenticity can be confirmed (or denied) by external ‘experts’ from the practice or other stakeholders. Other ways of certifying originality include being in a physical relevant environment, for instance going on a field trip to a car garage or a hair salon. Or the task could use the same software as would be used out of the school setting (Vos, 2015). In my study, external ‘experts’ could have been an established member of the community of practice of a relevant enterprise.

However, although it is possible to find authentic questions, authentic objects and so on, it is not possible to have a completely authentic task. This is because when the task is given as a part of an education setting, with the intention of bringing about learning in the students, the task is removed from its practice and inserted into the school education practice. This point is also made by Verschaffel et al. (2000) who point to the fact that all their studies took place within a scholastic setting, instead of being ‘‘involved in truly authentic problem situations’’ (p. 50).

Vos (2011) states that the concept of authenticity has a binary definition, in that an aspect of a task either is or is not authentic. This means that although an entire task cannot be authentic as Verschaffel et al. (2000) argue, aspects of the task can be authentic. She claims that all tasks in educational settings must have adaptations. For example, students might work with authentic data material, but will maybe use Excel, instead of a specially adapted software used in a workplace.

There are other ways of defining the concept of authenticity, and I will discuss Verschaffel et al.'s (2000) list of different aspects, and Palm's (2008) framework for authentic aspects. For identifying aspects to check with the definition of authenticity I have used these lists as starting points.

Verschaffel et al. (2000) give examples of what they call authenticity can vary with regard to different aspects of a problem, including the following:

- is the problem imposed by another or posed by the solver?
- is the goal of the solver to succeed in the classroom or solve an engaging problem?
- is the problem pre-formulated with the required data or can the solver formulate and find the necessary sources herself?
- is the problem to be solved alone or with the help of others or cultural tools?
- will the solution be judged by purely mathematical criteria, or also by non-mathematical (political, moral, social) criteria?
- what consequences will the activity have – good or bad marks from a teacher or a genuine reaction from the social or material environment?

They claim that if a task has more elements of the second alternative in each of these criteria, then it will be more authentic.

Palm (2008) made a framework for evaluation of aspects of authenticity. Here he divides into five main categories: event, question, purpose in the task context, language use, and information/data. The event component of a task concerns the task's position in the world. The situation that the task refers to should have taken place or have the possibility for taking place.

The question part of Palm's (2008) framework analyses the nature of the question(s) posed in the task: Are the questions asked the same way that one would ask them in the event in the practice? The purpose of the answer(s) to the question(s) posed is also important. The concept of language use concerns the words, length of text, use of pictures or objects, and the way the task is presented. He argues that one should use terms and information in relation to the event and the purpose of the task which would be appropriate in the workplace setting.

The information and data aspect concerns the information needed and given in a task (Palm, 2008). The first aspect of information concerns the existence of data and information in the event situation. This information should be the same in the task as in the real event situation, not more or less. The second aspect of information should be as correct as it would have been in the vocational practice, and as close to reality as possible. A third part aspect of information concerns the specificity of the information, which should be specific to the event situation and not general.

Unlike Vos, Palm claims that a task can be authentic if the different categories in his framework are fulfilled, but I follow Vos in arguing that while parts of a task can be authentic, the whole task itself cannot be, since its implementation in the educational setting of a mathematics classroom means that it cannot be completely authentic with regard to vocational practices.

Although, as already noted, Vos points out that not all aspects of a task can or will be authentic, she claims that tasks which have a number of authentic elements will be more engaging (Vos, 2011). Similarly, Palm (2008) showed that increased task authenticity had the effect that pupils used their real world knowledge more effectively in their solutions. Vos (2018) argues that several studies show that are the authentic questions in the tasks which are most important for students' motivation.

As the literature review in Chapter 3 showed, it is not easy to integrate vocational mathematics into educational practices (FitzSimons, 2014), and task authenticity clearly plays a part in this. In this study, I employ the concept of authenticity as defined by Vos (2015), namely, that for something to be authentic, it needs an out-of-school origin and a certification of originality. I use the concept of authenticity to discuss how boundary object representations can be regarded as objects that have out-of-school origins, and how their certification emerged during the implementation of the tasks. I analysed in what ways the certification emerged, and how the students interacted with authentic and non-authentic aspects. Authenticity is not only important with regards to boundary object representations; it is also important in the analysis of the students' interactions with other aspects within the tasks. For example, I explore the aspect of the questions posed by the tasks and if these given questions have an out-of-school origin, and if and how this aspect is certified. Likewise, I investigate if and how task themes, resources and data are authenticated by the students, and also if the purpose and consequences of the tasks are regarded as authentic.

Vos' (2015) argument that authenticity is a social construct, and its implications for community agreement on what constitutes authenticity, means that task authenticity is connected to normative aspects of mathematics classrooms. This is the subject of the next section.

4.4 Norms in Mathematics Classrooms

The three characteristics of a community of practice are mutual engagement, a joint enterprise and a shared repertoire (Wenger, 1998). All three characteristics concern appropriate ways to interact between the participants in the community. Mutual engagement implies that participants must interact with each other in accordance with established norms and relationships. Maintaining and participating in a joint enterprise means that participants need to find ways to coordinate how they act in accordance what matters in the community. A shared repertoire includes routines, histories, tools and working methods in the community of practice which are created or adapted and have become part of the practice. All three aspects of a community of practice draw on the idea of norms, and in this section, I examine norms in detail, and their role in different communities of practice.

As discussed in Chapter 3, mathematics in school settings is different from mathematics in workplaces. What it means to know and do mathematics in school is gradually established through children's experiences as students of mathematics. Students figure out how a school mathematics classroom operates, and what its routines and implicit and explicit norms (or rules) are. The students in this study met a new community of practice, namely their future vocation. There they will gradually figure out the shared repertoire, working methods, and how to act in ways which are appropriate to the practice.

In a social situation, norms may be described as shared meanings about the rules of the situation. Participants use these shared meanings to coordinate, interpret and evaluate their common activity, and ''share some common sense of 'What is it that's going on here?''' (Turpen & Finkelstein, 2010, p. 3). In other words, norms are unwritten rules behind our actions in social situations. Norms reflect what is considered to be expected and acceptable among the participants in a practice, what discourse is possible and meaningful, and which working methods and activities are acceptable and expected. The norms of a situation regulate what participants will regard as correct or incorrect routines, and what expectations they have of each other (Stephan, 2014).

Lopez and Allal (2007) describe norms as expectations between students and teachers which are established in the course of interactions in the classroom. McNeal and Simon (2000) argue that the negotiation of norms is an ongoing, often implicit, process. Sometimes it may seem as though there are no negotiations, but these become noticeable when divergent interpretations and evaluations of the situation arise. When this happens, there may be ‘‘not only instances of misunderstanding and miscommunication, but moments where previously implicit understandings must be made explicit’’ (p. 278).

In this study, inspired by Stephan (2014), I define norms in school settings as the expectations the students and teachers have to themselves and each other during their interactions in the classroom. Students and teachers are participants in multiple communities of practice: in the mathematics classroom, in the whole school, and with their peers and family outside the school setting. Students’ interaction with peers, teachers, and tasks provides opportunities to gradually establish norms of how to act in the mathematics classroom community.

According to Cobb, Yackel, and Wood (1993), norms can be constituted both explicitly and implicitly, and they are constantly renegotiated through all interactions in the classroom. For example, a norm can be that ‘‘others should be listened to when they are speaking’’ (Hofmann & Ruthven, 2018, p. 504). One explicit way of establishing this norm would be that the teacher asks students to be quiet and listen when other students explain. A more implicit way of constituting the norm could be for the teacher to ask students to retell what the previous student said. The teacher could acknowledge and praise students who demonstrated that they had paid attention to previous speakers. Such action on the part of the teacher is only one part of the story, however. In order to establish a norm, the various participants must participate in ways that support the norm. If the students repeatedly showed little interest in what others said, then the previously mentioned norm about listening to others would probably not be established in that classroom regardless of the teacher’s intentions.

Norms in a classroom are not individual constructs, but social constructions that are built and negotiated between the participants in a particular practice (Yackel, 2001). And norms are not universal constructs that are taken into the class by the teacher, nor unchanging over the school year. A sociocultural approach highlights that the students and the teacher in a classroom are not individual actors but interact and participate in ways which shape classroom processes (Bowers, Cobb, & McClain, 1999).

Norms emerge from the interactions between the teacher and the students, leading Cobb et al. (2001) to use the local classroom community as their reference point. This does not mean that the teacher and the students are a separate unit from the rest of the society. Students have previous experiences of mathematics classrooms, and a teacher will have his or her own experiences as a student at school, as a mathematics student, as a student teacher and as a teacher in previous schools. The teacher is a part of a community of teachers, both locally at the school, but also at the national level. Teachers will have to act in accordance with the curriculum and prepare students for an exam. All these experiences and previous interactions experienced by both the teacher and the students in a classroom are the background material to their current interactions, contributing to how similar many mathematics classrooms are across schools, levels and countries. As Yackel and Cobb (1996) note, "what becomes mathematically normative in a classroom is constrained by the current goals, beliefs, suppositions, and assumptions of the classroom participants" (p. 460).

Nevertheless, there are many similarities between different classrooms, both within a school, across the education system in a country, and maybe in many mathematics classes the world over. Students' (and teachers') views about mathematics and how mathematics classes are conducted is ‘‘constructed though years of socializing through schooling’’ (Ju & Kwon, 2007, p. 268). Even when small children start in school they have an idea of what it means to do mathematics (Franke & Carey, 1997), and during their years in school their beliefs about mathematics and mathematics classes will be shaped by their interactions with teachers, other students, parents, textbooks, the curriculum, examinations and expressed public opinion about mathematics (Franke, Kazemi, & Battey, 2007). All these interactions will be a part of what forms students' individual beliefs about mathematics. Franke et al. (2007) point out that ‘‘the ways norms are shaped influences which students learn, what they learn, and how they learn it’’ (p. 238). The individual beliefs of the participants about mathematics and mathematics classes will in turn be a part of the continuing negotiation of classroom norms.

Yackel and Cobb (1996) differentiate between classroom *social norms* and *sociomathematical norms*. They define classroom social norms as norms that would be equally valid in other subjects than mathematics, while sociomathematical norms "focus […] on normative aspects of mathematical discussions that are specific to students' mathematical activity" (p. 461). For example, a norm in a classroom could be that ‘students should explain their answers’. According to Yackel and Cobb (1996), this norm will be considered as a social norm, since it can be equally important in other school subjects, such as history or natural science. Other examples of social norms may be that one should explain solutions and reasoning, one should offer different solutions than the previous speakers, and one should challenge others' thinking or reasoning (Yackel & Cobb, 1996).

A sociomathematical norm concerns what is specific in mathematical activity (Yackel & Cobb, 1996). In contrast to the social norm that one should explain answers, Yackel & Cobb (1996) propose that a sociomathematical norm concerns what is an acceptable *mathematical* explanation. So, for example, the sociomathematical norm could be that 'an acceptable explanation will refer to a procedure shown by the teacher'. A slightly different sociomathematical norm may be 'an acceptable explanation will refer to the mathematical objects at stake'.

Yackel and Cobb (1996) use "what counts as mathematically different, mathematically sophisticated, mathematically efficient, and mathematically elegant" (p. 461) as examples of areas where sociomathematical norms can be found. Said another way, social norms concern the normative activity of participation in any practice, but sociomathematical norms concern the criteria for what is valued in mathematics classrooms (Stephan, 2014).

Thus, Yackel and Cobb (1996) distinguish social and sociomathematical norms in the following way:

> to further clarify the subtle distinction between social norms and sociomathematical norms we offer the following examples. The understanding that students are expected to explain their solutions and their ways of thinking is a social norm, whereas the understanding of what counts as an acceptable mathematical explanation is a sociomathematical norm. Likewise, the understanding that when discussing a problem students should offer solutions different from those already contributed is a social norm, whereas the understanding of what constitutes mathematical difference is a sociomathematical norm (p. 461).

Levenson, Tirosh, and Tsamir (2009) point out that the norms of a specific classroom can vary, according to which perspective is studied. They distinguish "three aspects of sociomathematical norms: teachers' endorsed norms, teachers' and students' enacted norms, and students' perceived norms" (p. 172). The meaning of "endorsed norms" is the norms that the teacher expresses as important in the classroom, for example, that the students should explain their answers. Enacted norms are norms as they appear in the interactions in the classroom, which can be different from, or the same as, the endorsed norms. The students' perceived norms are what the students perceive as important and valued in the classroom, which again can be different from the enacted and endorsed norms (Levenson et al., 2009). For instance, an endorsed norm can be the teacher says that students should share their mathematical ideas. However, if it turns out that students who share their mathematical ideas get unhelpful comments from their peers, the enacted norm in the classroom could then shift towards students *not* sharing their mathematical ideas. A perceived norm might be that students think that being fast when working with mathematics tasks is important and valued. If we consider Yackel and Cobb's account of norms within this framework, we can see that they focus on enacted norms and identified both social norms and sociomathematical norms by looking at what seemed to be regularities in the interactions in the classroom.

I have used the concept of social and sociomathematical norms in in my analysis. This is similar to Brousseau's (1997) concept of ‘‘didactical contract’’. Both of these concepts concern rules and routines and the relationship between students and teacher in the classroom: Brousseau and Warfield (1999) described the didactical contract as ‘‘the set of (specific) behaviors of the teacher which is expected by the student and the set of behaviors of the students which is expected by the teacher’’ (p. 47).

In parallel with Yackel and Cobb's distinction between social and sociomathematical norms, Brousseau (1997) emphasises that the didactical contract is not about general pedagogical rules in the classroom, but about the rules relating to the specific mathematical knowledge in the situation, although certain classroom practices will often be present as effects of the didactical contract (Brousseau, 2009). An important point is that didactical contracts vary according to the mathematical notion at stake (Brousseau & Warfield, 2014). Pierce, Stacey, and Wander (2010) point out that an important difference between the didactical contract and the concept of sociomathematical norms is that didactical contracts will break down because ‘‘the teacher cannot provide for the student all that the student is to learn, but can only provide situations from which it may be learned’’ (p. 685), and argue sociomathematical norms are similar to the permanent aspects of didactical contracts.

In the context of vocational mathematics, the concept of social and sociomathematical norms enable me to study how students interact with vocationally connected mathematics tasks. As diSessa and Cobb (2004) argue ‘‘although sociomathematical norms are specific to mathematics, they are relatively broad in that they cut across mathematical domains’’ (p. 96), which was what I needed.

Rules and routines, and the relationship between teacher and students, are a central part of what constitutes the community of practice in the mathematics classroom. Likewise, participants in vocational practices have their own rules and routines which may be different from the mathematics classroom. Norms regulate large or small issues and may differ from classroom to classroom, though there are often recognisable features of normative activity across mathematics classrooms. As discussed in Chapter 3, mathematics is intertwined with the practice it is a part of. As what mathematics is can be regarded as quite different depending on which practice one is part of, there is an important link between participants’ views about the nature of mathematics, and what norms can or will be constituted by that practice.

As explained in Section 3.1, when students work on word problems, they often only manipulate the given numbers in the word problems, instead of engaging with the given context, and using and questioning the problem statements. The students are “engaging in what might be called suspension of sense-making - suspending the requirement that the problem statements make sense” (Schoenfeld, 1991, p. 316). There can be several reasons for trying to solve problems uncritically and doing calculations without further reflection. These reasons can include factors such as students’ trust in teachers, their expectation that problems will be reasonable, and their belief that one should be able to answer questions from the given information in a problem statement (Schoenfeld, 1991). Such responses can be seen as rational given students’ experiences in the school mathematics: Wyndhamn and Säljö (1997) assert that “students learn to identify word problems as mathematical exercises in which an algorithm is hidden and is supposed to be identified” (p. 366). Suspension of sense-making when solving mathematics problems can be thought of as meaningful for students in a school setting. Schoenfeld (1991) claims that “there is reason to believe that such suspension of sense-making develops in school, as a result of schooling” (p. 317).

Yackel and Cobb (1996) point out that the teacher is a representative of the mathematical community, and thereby has an important role in constituting what is valued mathematically. In their research they had the goal that students would have intellectual autonomy. Yackel and Cobb (1996) show that in the classroom they studied, the teacher helped some of the children to get to the point that the explanations and the judgement of what counts as mathematical explanations over a school year became objects of reflections. With the guidance of the teacher the students were not only concerned with understanding the mathematics, but also with making sure that others understood the explanations.

They link this to intellectual autonomy and state that ‘‘autonomy is defined with respect to students’ participation in the practices of the classroom’’ (Yackel & Cobb, 1996, p. 473). Intellectual autonomy in mathematics means to them that students apply their own knowledge in mathematical decision making, instead of relying for instance on the teacher’s expertise and the teacher’s previously shown solutions. If the students have intellectual autonomy, they can for instance judge when and what to contribute for mathematical valid reasoning. Yackel and Cobb argue that the teacher can help the students to become validators and that the students themselves then can judge what is appropriate mathematically. This is seen as a positive feature of an inquiry classroom, in which the teacher has an important role in establishing norms and what mathematical understanding is.

To summarise, social norms concern normative rules of participation in any practice, but sociomathematical norms concern the criteria for what is valued in mathematics classrooms (Stephan, 2014). Enacted norms are norms as they appear in the classroom interactions, and social norms as norms that is norms which could be valid without mathematical relations and sociomathematical norms as norms specific for mathematical activity.

4.5 Summary: Theoretical Framework

This chapter has described my use of a sociocultural view of learning to investigate vocationally connected mathematics tasks, emphasising that students are participants in multiple communities of practice, including that of the mathematics classroom in vocational education programmes. In the mathematics classroom, the students are old-timers, they are experienced as students in the community of practice, and in the vocational practice they are newcomers into the community of practice. I have explored how this starting point leads to a consideration of the nature and role of boundary objects and what I call boundary object representations for educational purposes. The socially constructed nature of authenticity highlights how boundary objects representations and other aspects of tasks can be regarded as authentic (or not). I have also explored how an understanding of practice leads to a consideration of classroom norms, particularly sociomathematical norms (Yackel & Cobb, 1996).

Together with the literature reviewed in Chapter 3, these theoretical considerations lead to the research questions for this study, which concerns norms in the classrooms, the ways in which students make connections with workplace practices and out-of-school knowledge, and finally what opportunities and challenges exist in the use of vocationally connected tasks. The research questions are:

- RQ1: What characterises the enacted norms in the classroom when students work with vocationally connected mathematics tasks?
- RQ2: What connections do students make with workplace practices and out-of-school knowledge when engaged with vocationally connected tasks?
- RQ3: What opportunities and challenges arise in employing vocationally connected tasks in school-based mathematics classes?

In the next chapter I move on to how the theoretical perspectives are operationalised and discuss my methodological choices.

5 Methodology

In this chapter, I explain and justify my methodological choices and methods I applied in my research. The chapter is structured with an explanation of my methodological stance, and why case study research was appropriate, before I describe my methods of data gathering. I then continue with explanations of how the data are analysed. In the final sections of the chapter I discuss trustworthiness and ethical issues in my research.

5.1 Framing the Research: Epistemology and Ontology

My research can be situated in the interpretive paradigm, which is concerned with understanding and interpreting the nature of human action in the world (Ernest, 1994). The sociocultural framework employed to understand learning in this thesis entails understanding students' relationships with mathematics as situated within, and in interaction with, cultural and institutional practices. The specific aim of this study is therefore to understand how students interact with vocationally connected mathematics tasks in school practice.

Ontology - the object of study, or unit of analysis - in sociocultural accounts of learning is based on the idea that persons and the social world are mutually constitutive of each other (Packer & Goicoechea, 2000). This means that people shape the cultural world but are at the same time shaped and changed by the different practices they are part of. This is also the case in mathematics classrooms, where experiences and expectations of mathematics classrooms shape student action, and the interactions in the classroom again change and shape expectations. These interactions extend beyond the individual classroom, experiences and expectations exist in the communities of practices which include schools, families, popular media and workplaces. In education itself, there are multiple communities of practices, including those of being a mathematics teacher or a student in a class in vocational education.

For example, a mathematics teacher engages in pedagogic practices which are those held in common by the community of teachers to which he or she belongs (either locally, in a particular school, or more generally across the community). The teacher also affects the community through participation, for instance, when introducing different pedagogic practices as a result of professional development. Likewise, students are shaped by, and shape, mathematics classroom communities, for instance by their interaction with vocational communities of practice, and their everyday life outside school, which may include interaction with family members who have particular experiences or narratives of mathematics. This sensitivity to context and individual and collective narratives means that we cannot claim to be able to identify objective truths (Walker, 1983). Subjectivity is of importance in interpretative research (Lincoln & Guba, 2000), and my ontological stance and my methodological choices in this thesis reflects this.

Epistemology focuses on what counts as knowledge, and how humans acquire this knowledge (Ernest, 1994). As in the case of ontology, claims as to what knowledge is and how it develops are influenced by the situation of this study within a socio-cultural framework. This theoretical position means that it is not possible to find knowledge that is *not* socially constructed by the participants in a research study. This includes the researcher themselves; as Bassey (1999) and Walker (1983) argue, the research context can be changed by a researcher who asks questions and participates. In this study this is important to reflect upon. One issue is that while I have a background in mathematics and mathematics education, I have no experience of the students' future vocations, neither have I studied or taught on a vocational education programme. Hence my interpretations of the various episodes are likely to be different from the interpretations of people with experience of such practices, and this factor will need to be taken into account in the analysis and discussion of what happened in the mathematics classrooms.

5.2 Case Study Research

My research is an exploration of students' interactions with vocationally connected tasks. Because case studies can be valuable sources of information for observing phenomena in a natural environment and are characterised by a focus on individuals' perceptions of events and detailed descriptions of the relevant events to the case (Cohen, Manion, & Morrison, 2013), I found this would be a valuable way of exploring my research aim. My empirical data was gathered in three different vocational education programme classes (see Section 5.3) which can be seen as typical classes in vocational education programmes in Norway. A case is a bounded system; for example, it could be the study of a child, or a class, organisations or all schools in a district (Stake, 1995; Yin, 2014). Such bounded systems are an outcome of the researcher's stance of what to study.

I consider the three different classes as three cases delineated according to the vocational education programmes they fall into. The students, the teacher, and classroom activities (including the vocationally connected tasks) are three separate cases in terms of their respective roles in the Design, Arts, and Crafts programme, the Technical and Industrial Production programme, and the Media and Communication programme. The research is thus what Stake (1995) calls "a collective case study" or Yin (2014) defines as a "multiple-case study". This does not mean that the cases need to be representative of the population, but there should still be variation in characteristics of the cases, and good opportunities to learn from the different cases (Stake, 1995; Yin, 2014). The classes were selected as non-probability samples (Cohen et al., 2013), and as further elaborated in Section 5.3, I chose cases that differed in vocational education programme, geographical location of the school, and school size. There were also differences in the gender compositions in the classes. Nevertheless, the classes cannot provide a picture of the whole variety of different education practices.

Advantages of case study research are that a case can often be recognised as similar to, or different from, the reader's own experience, and that through a case, one can find events that are unanticipated by the researcher (Cohen et al., 2013). In a case study, one may capture details and unique experiences that may be lost in large-scale studies which are limited to questionnaires and surveys (Cohen et al., 2013). Cases can also be an efficient way of communicating research results to non-specialists by raising awareness and providing insight into a situation (Yin, 2014). Some problems with case study research are a perceived lack of generalisability, problems with cross-checking, validity, subjectivity and ethical risks (Bassey, 1999; Cohen et al., 2013; Stake, 1995). I will discuss cross-checking and ethical risks in Sections 5.6 and 5.7, and instead of discussing the validity, I will present the trustworthiness of my research in Section 5.6.

Walker (1983) gives three reasons why case studies are potentially problematic: case studies are interventions into the lives of others, they provide a subjective view, and are, by their nature, conservative – that means they report only a snapshot of practice. The first issue is that of intervention: the researcher's presence in classrooms and interactions with students and teachers can shape what happens in those classrooms. It is quite possible that the students and teachers acted differently while I was present, and if so, my results may be different from what one might obtain with other methodological approaches. It was therefore important that I was reflective about the influence of my presence, both with regards to the results, and ethical issues with my presence. I discuss these issues further in Section 5.6 and 5.7. The next issue is the inevitably biased view of what the researcher chooses to observe and analyse. Thus, I was likely to find results which reflected my perspective, and the issues and episodes that I noticed may not have been the same as those that would be noticed by other researchers, or experienced as equally important and worthwhile by the students and teachers. This concern stems from my stance that knowledge is socially constituted by participants. The last problem is that case studies are conservative; even though one reports diligently what happened in the case, practice can and will change (Walker, 1983). He stresses the point that "case studies tell *a* truth but not *the* truth" (p. 165). The practice I observed in the classrooms is a snapshot of a changing story, in which the practice changes over time.

One important reason for conducting case studies is to develop theory. One way of doing this is by generalisation, and a researcher needs to report cases meticulously so that readers can "recognize essential similarities to cases of interest to them, [and] establish the basis for naturalistic generalization" (Stake, 1978, p. 7). Such generalisation could be that the teachers recognise the issues reported in my research and can use the results in their own classes with adaptions, and that other researchers can find similarities in their research. Case studies are generalisable in terms of theoretical propositions, but not in terms of statistical generalisations (Yin, 2014). Such generalisations may apply to a variety of contexts, and not just cases which are similar to the original. After the analysis of the individual cases, one can generalise and build theory from a multiple-case study. One should then analyse across the cases, and indicate why certain cases had certain results, and if any of the other cases had contrasting results (Yin, 2014). Similarly, Eisenhardt (1989) divides research strategies on theory building into within-case analysis and cross-case pattern search, before one shapes the hypotheses by looking for iterative evidence across the cases and search for the "why" behind relationships. Afterwards, one compares the results with both conflicting and similar literature, before reaching closure (Eisenhardt, 1989). In Chapter 9 I reflect upon the research design, my theoretical contributions and pedagogical implications of my study.

5.3 The Three Cases

The mathematics course 1P-Y takes place in the first year of vocational education programmes as described in Section 2.1. The data were collected in three different schools, Bjørke, Gran and Osp secondary schools (pseudonyms). All three schools have programmes both for general studies and vocational education programmes. The three schools have between 200 and 1000 students and they are based in different geographical settings. Two of the schools are situated in a rural area and one in a town centre. The schools do not differ significantly with regard to their students' socioeconomical background and ethnicity. The schools (and classes) were chosen because of their different educational programmes, their different geographical placement, and most importantly, the teachers and students' willingness to let me conduct the research.

Because of the possibility of identification, I have not specified size and location of the different schools. I use pseudonyms for the teachers and students, and these pseudonyms are used consistently through the thesis. Students that I do not follow closely are only identified by "student" in the transcripts. I am identified by my own name, Trude, throughout the thesis.

The data were collected in spring of 2012 (Bjørke school), and during the school year 2012-13 for Gran and Osp schools. I first made initial contact with the schools, and then teachers and students were asked informally if they would be willing to participate in the research, before I settled on which classes I would observe. Students and teachers were told that I was interested in researching vocationally connected mathematics tasks, with an emphasis on what happens in the classroom setting. Teachers and students were then asked for formal consent. This process is described in detail in Section 5.7 on ethics.

5.3.1 The Design, Arts, and Crafts Class

In Bjørke secondary school the mathematics teacher, Alexander (male), taught students in the vocational education programme Design, Arts, and Crafts[17] in mathematics 1P-Y. After completing two years in school and their two-year apprenticeship period, the students in this education programme are qualified for work in various professions such as dressmaking, jewellery design, hairdressing, woodworking or child care and youth work (The counties' information service for applicants to upper secondary education and training, 2013b).

[17] Norwegian: Design og håndverk.

The class consisted of 15 female students, which is quite usual for this education programme. Within Norway, about 89 % of students in Design, Arts, and Crafts programmes are female (Norwegian Directorate for Education and Training, 2014b). One of the students declined to be a part of the research and worked apart from the class while I gathered my data. I followed the class for three days (in total five lessons of 45 minutes) in mathematics, and I visited the students in their Common Programme Subjects for two consecutive days. This was my first data gathering period; where three of the lessons was a trial of one vocationally connected task, the hair salon budget task, together with the teacher. This task is further discussed in Section 6.2 and Section 7.1. The other two lessons I observed in this class was ordinary lessons planned by the teacher. I wanted to get a first experience on how students in a vocational class acted in the mathematics classroom and found it valuable to see the same students in their Common Programme Subjects. In this class I tested what to record, with video cameras, sound recorder and field notes. I also tested out how to work together with the teachers in order to design vocationally connected tasks. I began to analyse the data from this class before I started the data gathering in my other two cases, in order to reflect on what kind of data I needed to collect to answer my research questions, and what data might be missing from this first data collection. In the research reported in this thesis I have used the data from this class together with data from the two other classes.

5.3.2 The Technical and Industrial Production Class

In Osp secondary school I followed a Technical and Industrial Productions[18] class comprising 16 male students. In Norway, about 89 % of students on this education programme is male (The Norwegian Directorate for Education and Training, 2014b). The mathematics teacher in this class was female and is called Ingeborg in this thesis. Students who study on the technical and industrial production education programme can become engine vehicle mechanics, drill operators, welders, laboratory technicians or vehicle sprayers, depending on their later specialisations (The counties’ information service for applicants to upper secondary education and training, 2013a).

The students in this class were not so comfortable with being video recorded, but ten of them agreed to participate in the research. One of the students

[18] Norwegian: Teknikk og industriell produksjon.

withdrew from the school during the school year, so the data in this class are gathered from nine students. The students who declined to be part of the research were not recorded but agreed to me being present in the classroom, and I made sure that students could participate in the mathematics class without being recorded. This meant that I pointed cameras and recorders away from those students, and that I stopped recording parts of the classroom activities if necessary.

I visited the class eight days in their mathematics lessons (12 lessons of 45 minutes), and twice in their practical lessons in the Common Programme Subject. In this period three vocationally connected tasks were implemented, and the rest of the lessons was ordinary mathematics teaching. The theoretical elements of the Common Programme Subject were taught in a classroom setting, while practical work took place in a workshop[19] at the school. Here, students had opportunities to learn how to spray paint, weld, use turning lathes and practice other vocational skills with instruction and help from a teacher. In my visits to the workshop, I talked to the students while they were working on different projects and observed how this learning environment was organised. In addition, I discussed the connection between mathematics and the technical and industrial production education programme with the Common Programme Subject teacher.

5.3.3 The Media and Communication Class

In Gran secondary school, I followed a Media and Communication[20] class in the vocational education programme. The class comprised seven male students and nine female students, who all agreed to be participants in the research. The mathematics teacher in this class was female and is called Ragnhild in this thesis. In the Media and Communications education programme, students can continue to a third year in secondary school and obtain the general education certificate, or continue with an apprenticeship period and enter professions such as photography, media design, and media graphics (The counties' information service for applicants to upper secondary education and training, 2013c). In the class that I followed, several of the students told me that they had chosen Media and Communications as a planned alternative route to a general education certification. I observed and recorded 21 lessons (of 45 minutes over 14 days) in their mathematics course 1P-Y. In this period two vocationally connected tasks

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[19] Norwegian: Verkstedhall.
[20] Norwegian: Media og kommunikasjon.

were implemented, and the rest of the lessons was ordinary mathematics teaching. I also visited the students once in the Common Programme Subject in the beginning of the research period.

5.4 Data Collected

I chose to concentrate on classroom activity as the main empirical data on students' work with vocationally connected tasks. I therefore observed the students as they worked on the vocationally connected tasks and when they worked on tasks from their mathematics textbooks or other sources. During my observations, the students worked individually and in small groups.

All the mathematics lessons I observed were video recorded, and I also conducted video interviews with students and audio only interviews with the three teachers. I made field notes to record my observations of the Common Programme Subject lessons. In addition, I had short conversations with both teachers and students during my time in the schools, which were recorded as field notes. I also took photographs of some student work. Table 5.1 summarises the different kinds of data collected.

Table 5.1: Table of recorded data.

Types of data	Timespan	Number of visits
Visits to the class Design, Arts, and Crafts; video recordings, field notes, pictures of student work	April to May 2012	3 days: A total of 5 lessons of 45 minutes
Visits to the class Technical and Industrial Production; video recordings, field notes, pictures of student work	October 2012 to April 2013	8 days: A total of 12 lessons of 45 minutes
Visits to the class Media and Communication; video recordings, field notes, pictures of student work	October 2012 to May 2013	14 days: A total of 21 lessons of 45 minutes
Interviews with Media and Communication students	November 2012	2 students
Interviews with Technical and Industrial Production students	October 2012	5 students
Informal discussions with the teachers	During the data gathering period	10 documented with sound recorder
Observations in the Common Programme Subjects	In the beginning of the data gathering periods	1 or 2 visits (about a half day) to each of the programmes.

Classroom visits listed in the “Number of visits” column include observing the students working with the vocationally connected tasks and their work with tasks from their textbooks, see also Table 5.3. The following sections provide further detail on the different kinds of data that I collected.

5.4.1 The Tasks

To gather data about students’ interactions with vocationally connected tasks I needed such tasks for implementation in the classrooms. Ideas for tasks were found through study of the students’ mathematics textbooks, reading of the common programme subject’s curriculum, and the teachers or my knowledge of the students’ future workplaces. Some suggestions for tasks came from the common study programme teachers as well. The tasks should fit in the curriculum goals of the course and be connected to the planned progression in the different classes. The tasks were intended to be used in mathematics classrooms without need for too much special adaptation. The tasks also needed to fit into the time available.

Therefore, tasks that depended on photographs and did not need tools from the workshops or collaboration with participants from the vocational practice were made. It was possible to implement the tasks in a regular classroom without the need for special equipment, but this did constrain the possibilities for authentic aspects in the tasks. I do not regard the tasks as data sources as such, but they are an important part of the setting when I present the various episodes. I therefore discuss the selected tasks in detail in Chapter 6.

The vocationally connected tasks used in this study were designed in collaboration between the three mathematics teachers and me. Not all tasks were produced with the same amount of collaboration, but all were approved by the teachers before they were implemented in the classroom. The amount of collaboration depended on how much time we had to plan the lesson and the tasks before the lessons. After the tasks were designed, they were implemented by the teachers in the media and communication and technical and industrial classes; in the design, arts and crafts class I introduced the task myself. In the design, arts and crafts class one vocationally connected mathematics task was implemented. Two tasks were implemented in the media and communication class and in the technical and industrial class three tasks were implemented. Below are a table with an overview over the implemented tasks and the amount of time I spent in regular lessons.

Table 5.2: Tasks implemented in the classrooms

Case	Tasks	Time used on tasks in classroom
Design, Arts, and Crafts. Total of 5 lessons of 45 minutes.	Hair Salon Budget Task	3 lessons of 45 minutes over two days
	Regular classroom work	2 consecutive lessons of 45 minutes in one day
Technical and Industrial Production. Total of 12 lessons of 45 minutes.	Gear Task	2 consecutive lessons of 45 minutes in one day
	Jack Stand Task	1 lesson of 45 minutes
	Engine Cylinder Task	2 consecutive lessons of 45 minutes in one day
	Regular classroom work	7 lessons in total, over five days
Media and Communication. Total of 21 lessons of 45 minutes.	Golden ratio	3 lessons of 45 minutes, over two days
	Frifond Project Task	2 consecutive lessons of 45 minutes in one day
	Regular classroom work	16 lessons in total, over 11 days

As shown in Table 5.2, I observed the classes both while they worked on regular tasks in lessons planned by the teacher, and when they worked on vocationally connected tasks. In Section 5.5.2 I explain why the three tasks, the hair salon budget task, the engine cylinder task, and the Frifond project task, were chosen for deeper analysis. I found it valuable to also be present while the classes did not work with vocationally connected tasks, to get a feel of usual interaction patterns in the classroom. In addition, I got to know the students better.

The main goal of the tasks was that they should be connected to the vocational practice and at the same time be relevant to the mathematics curriculum. Possible methods that were important for vocational practices could be, among others, trial-and-error methods and estimation. The vocationally connected tasks often offered the possibility for more than one answer and some were intentionally designed with missing information so that the students themselves needed to figure out estimates. The tasks were intended to be connected to students' future vocational practice, and one way we did this was by referring to objects from the vocational practice. This was done by including photographs, websites or other references to boundary objects representation in the tasks. The boundary object representations were intended to facilitate communication in the classroom between the teacher and the students.

5.4.2 Classroom Observations

The classroom observations are my main data source in the three different cases. It was important to have prolonged exposure in the classrooms (Bassey, 1999) to familiarise me with the students, their work practices and the classroom interactions. The most important data source in this study is classroom interactions. Before I started my classroom observations, I explained to the students that I wanted to study how students worked on vocational tasks, and that such knowledge may help future students in mathematics. See the Information letter (Appendix 11.1) for the written information the students and teachers were given. When and if students were curious about my research in later visits, I explained more about what they wanted to know. I took a role which Gold (1958) characterised as "participant as observer" when conducting classroom observations: the students and teachers knew that I was there for research purposes, but over time, I became less of an oddity and more a familiar participant in the classroom practice. I was present in the classroom because of my research, but the students often regarded me as an assistant teacher and asked me questions related to the mathematics they worked on. I found such conversations to be a good way of observing the students' actions and reasoning.

While I interacted with the students, I tried to encourage them to articulate their thinking to clarify where they had questions and problems. I tried to maintain a good tone with the students, and Ingeborg, the Technical and Industrial production mathematics teacher, said in one of our conversations that "you can just, sort of, be a friend with them, yes, I think they like you a bit[21]".

I documented my classroom observations in field notes and video recordings. I used two cameras to ensure multiple views of the classroom with a few exceptions (where I had forgotten to take two tripods or to make sure that both cameras were charged). When I collected data, it was important to consider bias (Miles & Huberman, 1994; Schoenfeld, 2008; Walker, 1983). When I decided what and whom the cameras were to be aimed at, I made choices that shape what is included as part of my analysis. Blikstad-Balas (2017) argues that even though video data is often rich, it is still only parts of what happens in the classroom that is captured, and this is the case in my study. I chose to have one camera which recorded the teacher when she or he was talking to the whole class. I had one camera which recorded one group of students, or two students sitting next to each

[21] Norwegian: Du kan bare være venn med de på en måte. Ja, jeg tror nok at de liker deg og litt.

other when the students worked individually or in groups, ideally for the whole lesson. I tried to choose groups or students which I had previously observed as fairly active verbally in the classes, and I also tried to observe different groups and students over time. I moved the other camera around in the classroom in order to document my conversations with the students, and I also used this camera to take photographs of the students' notes and calculations. In this way I obtained a more dynamic selection of conversations during the lessons, because I would talk to students that wanted to ask questions and could try to capture situations which were interesting. However, I still only have video recordings of parts of the dynamics in the classrooms.

The teacher and I asked the students to work as they normally would and encouraged them to try not to be disturbed by the presence of the camera. It seemed that the students got used to the cameras quickly, and most of the time, they ignored them. However, there are instances during my observations where the students sing for the camera or ask me about the angle of the camera and whom I had recorded. I would move the camera if the students seemed uncomfortable. Some students stated that I would not get much good data from them or similar concerns. In such instances I explained that it was important to capture what really happened in the classroom. When I asked the teachers, they mostly stated that the students worked as usual when the camera was present.

I had originally planned to photocopy the students' notes, but I decided after my pilot study to take photographs instead of some of their written work. I did this because the students tended to write few or no notes on their worksheets and the worksheets were mostly used to record their answers. An example of this from the pilot study is presented in Figure 5.1. It would have been possible to ask the students to note more, but I deemed that this would interfere too much with their ordinary actions in the classroom.

Inntekter	Kroner	Forklaring/regnestykke
Dameklipp :	63000	
salg hårprodukter	98000	
herreklipp :	42000	
Farging hår :	126000	
Sum inntekter		

Figure 5.1: Student’s notes on her calculation of a budget, where there are no notes in the ‘explanation/calculation’ cells.

I therefore found it more informative to record the students’ gestures and discussions with each other, the teacher and myself. I tried to ask the students after the fact what they had done and thought as they solved the tasks, but they often struggled with retelling their actions. It thus became important to capture their actions in real time. I explain in Section 5.5.1 how I transcribed the classroom episodes and noted gestures and so on.

5.4.3 Interviews and Conversations

I planned to supplement the classroom observations by conducting what Bryman (2008) calls semi-structured interviews with the students. This approach turned out to be less than successful, especially in the Technical and Industrial Productions class. Four students did not want to be interviewed, and the other five answered with short answers, and seemed uncomfortable with the setting. I also tried to interview two students in the Media and Communication class, and they were clearly more comfortable in the interview setting, but still it seemed advantageous to change my plan from interviews toward recorded informal, unplanned conversations in the classroom while they were working with the tasks. I decided against conducting focus group interviews, given the possibility of intruding too much on the students’ time and a possibility for increased negativity toward the research project.

Because of my problems with conducting interviews, I aimed to have more informal conversations with the students during the mathematics classes. It was important not to distract the students too much from their work, and therefore I took the opportunity to talk when the students themselves asked for help with the mathematics tasks. I tried to answer their questions and engage in conversations about what their thoughts of mathematics were and what they thought of the tasks. An additional advantage of talking with the students in the classroom was that other students sitting close by, could and would, chime in when they had opinions. When several students talked about issues, I argue that it minimised the issue of them giving different answers to a researcher than they would have given to their friends and classmates. I also tried to minimise problems as much as possible by trying to keep a level of trust when working with students, and by trying to maintain a close relationship between the students and myself while I talked with them.

After most lessons that I observed, I talked with the teachers about the tasks and the whole lesson. These conversations were sometimes quite short, and limited in length, since teachers might need to go to other lessons. The discussions were documented with an audio recorder or field notes. The few interviews with students and the discussions contribute to triangulation of the research (see Section 5.6), giving me the possibility of confirming whether my observations of the students' actions in the classroom were concurrent with what they expressed in the interviews and discussions.

5.5 Data Analysis

Qualitative research relies on the gathering and analysis of mostly unstructured data, and consistent and systematic analysis is not necessarily straightforward (Bryman, 2008). This study involved documentation of various activities through video recordings, audio recordings, photographs and field notes. My overarching research aim is to understand how students interact with vocationally connected mathematics tasks in school. The ontological issue of what mathematics is in the mathematics classroom and in a vocational practice, and how students, workers and teachers recognise it, leads to complex outcomes for attempts to analyse the students' interactions with the vocationally connected tasks.

Common to the analysis as a whole is that I take Radford's (2009) stance on thinking, which is that it "does not occur solely in the head but also *in* and *through* a sophisticated semiotic coordination of speech, body, gestures, symbols and tools" (p. 111, italics in original). I take this to mean that when we observe gestures, we are not just observing clues for interpreting what thinking is taking place, but that we are observing part of the thinking itself. Therefore, my analysis focuses on both spoken words and gestures when possible. The methods of data collection that I used in this study enabled me to observe much (although not all) of the students' gestures, expressions, discussions and written notes.

5.5.1 The Transcription Process

The lessons selected for analysis (see Section 5.5.2) were transcribed in Norwegian, as close to the spoken words as possible, but I replaced dialect words in some cases to avoid possible identification of participants. However, I chose to keep most of the dialect in the Norwegian transcripts in order to keep as close to the spoken words as possible. Since language is important for learning (Mercer, 2000; Säljö, 2001; Vygotsky, 1978), it was important to capture students' and teachers' use of language as correctly as possible. Transcription involves a degree of interpretation; I made choices of which words to emphasise and what pauses, overlaps, etc. to note. I tried to listen closely and keep to what I believed was the intention in the statements, which is highlighted by Ochs (2006) as important.

When possible, I noted gestures, movements, and explanations in the transcripts to support the process. I recorded gestures and movement when they were visible on the recordings. When I translated the Norwegian transcriptions to English for the purposes of inclusion in this thesis, I concentrated on keeping the meanings of the text, while staying close to the Norwegian syntax. Such translations add a degree of interpretation, and therefore the transcripts were analysed in the original Norwegian. The transcription key is presented in Appendix 11.2. Transcripts are presented with turn number, the English translation, and the original Norwegian statements. If a quotation is comprised of several turns, the turn numbers indicate this. I present a short example of a transcription below.

Table 5.3: Transcript example from the jack stand task (Section 1.1).

Turn	English translation	Norwegian	Comments
1	Ingeborg: 3.14 times 43 that is 145 (faint voice: no), 135.	Ingeborg: 3,14 gange 43 det blir 145 (svakt: nei) 135.	Ingeborg writes 135.02 on the blackboard while she talks.
2	Martin: It isn't, damn what is it called, it's millimetres. I think when you are supposed	Martin: Det er jo ikke, faen hva heter det for noe, det er jo millimeter der. Tenker jo når du	Martin explains his calculation of 3.14·43=135.02, and

	to measure, you get millimetres, and it isn't as though we can measure down to zero point two, so then I just rounded off to just.	skal måle lissom, så har du jo millimeter. Det er ikke som vi kan måle heilt ned til null komma to så da runda æ av til bare.	that he rounds off the answer. Martin shows his knowledge of the vocational practice (… see more in Section 1.1)
3	Ingeborg: So, we round it off?	Ingeborg: Så vi runder av?	Ingeborg asks a clarifying question about Martin's statement.

The transcriptions and gesture descriptions carried out within the qualitative data analysis software NVivo[22], and linked with the video recordings, so I could consult the original data source during further analysis.

5.5.2 Selection of Tasks for Analysis

In order to select tasks for analysis I created data summaries (Miles & Huberman, 1994) and coded selected lessons. It is important to note (as discussed in Section 5.4.2) that video recordings only cover selected parts of classrooms activities. I recorded a total of 38 classroom lessons of 45 minutes, which were organised by vocational education programme and dates (see Table 5.1 and Table 5.2). Most of the lessons were ordinary lessons in mathematics, but they also included the implementation of six vocationally connected tasks. I first undertook a data summary process in order to obtain an overview of the recordings. This consisted of summarising the recording of one lesson to a maximum of one or two pages per camera, depending on whether the given lesson took one or two 45-minute periods. In the summaries I divided each lesson into sections, for example "introduction by teacher", "group starts to work", "group is talking about non-subject or school-related issues", and "student is working on task about…" and so on. Within each section, I noted who took part and how, and possible links to the research aim.

I used the data summaries in combination with my field notes to decide which of the classroom observations to transcribe. Since my aim is to study students' interactions with vocationally connected tasks, the lessons where these

[22] QSR NVivo 10 for Windows.

took place were transcribed in full. I also transcribed parts of some of the ordinary lessons where students talked about what they thought about mathematics or its relevance for their future or for out-of-school practices, or if they mentioned their educational programme. In addition, I transcribed the interviews with the teachers and the students.

I also carried out a basic coding of the video recording of lessons with vocationally connected tasks, with the aim of identifying events related to authority, authenticity, norms and connections to vocational practice. *Potential normative activity* meant that there was potential to inform about enacted norms. *Potential authority* was an issue I identified during the coding process; this described situations where the participants showed their knowledge about vocational or mathematical practices, and who they regarded as knowers. I also coded *sociomathematical norms, social norms,* and *suspension of sense-making*. *References to vocational practice* meant that the participants referred to vocational routines or practice, while *references to mathematical classroom practice* concerned how one acts in a mathematics classroom. Such references could be explicit or implicit.

With *references to potential authentic aspects,* I coded episodes that could indicate whether the tasks were regarded as having authentic aspects, based on the presence of talk about the task and what could happen in out-of-school settings, on the use of special language and the kind of questions that were asked. *References to potential boundaries or boundary objects* were episodes where potential boundaries, boundary objects or boundary object representations were referred to, or discussed. *Solution strategies and working methods,* either from mathematics practices or from vocational practices, were coded when students or teachers referred to how they had solved the tasks, or if they talked about what they could have done. An example of the application of codes is shown in Figure 5.2.

In this example, the students in the technical and industrial production education programme had worked on a vocationally connected task concerning sizes of cylinders in engines (discussed in Section 6.3). The excerpt in Figure 5.2 is taken from our talk after the lesson. I told the teacher (Ingeborg) about a student’s reaction to my (mistaken) explanation of how cylinders were placed in a car engine. In the coding I used one or several codes for one turn, and in this example, both turns 1 and 2 have been marked with four different codes, namely *authentic, potential normative activity, potential authority, reference to vocational practice.*

authentic,
normative,
authority,
vocational practice

Turn	English	Norwegian	Comments
1	Trude: It is stuff like that I said that the engine had eight cylinders [in a row], and he just ''no, they have not. They stand like that.''	Trude: Det er jo sånne ting som at jeg sier at den motoren hadde åtte sylindere [etter hverandre], og han bare ''nei, det har de ikke. De står sånn''.	Trude repeats the student's reaction to her wrong statement about cylinders in an engine.
2	Ingeborg: It was like that with Fredrik also. [I say] today we calculate on car engines. And he sort of looked at the sheet and ''uh, that's not a car engine, that's a moped engine''.	Ingeborg: Det var jo sånn på Fredrik au. [Jeg sier] i dag skal vi regne på bilmotorer. Og han liksom så på arket og sånn, ''øh, det der er jo ikke en bilmotor, det er en mopedmotor''.	Ingeborg continues with giving another example of a student reaction on her statement that they would calculate on car engines, while she showed a photograph of a moped engine.

Figure 5.2: Example of codes in the transcripts.

In the statements above, Ingeborg and I referred to two incidents that we had noticed when the students worked on the vocationally connected task. In Turn 1, the student had corrected my explanation of the engine, and referred to his knowledge of what an engine looks like. Similarly, in Turn 2, Ingeborg referred to Fredrik's first reaction to the photograph where he identified it as a moped engine, not a car engine.

Turn 1 is coded with *authentic* because in the turn a student's reaction to the V8 car engine is described, and I was interested in whether he regarded the context as authentic or not. Likewise, Turn 2 is also coded with *authentic* because of the possibility of confirmation or denial of the authentic aspects of the task. The turns were also coded with *potential normative activity* (shortened to *normative*) because the statements could indicate something about the norms in the classroom. *Potential authority* (shortened to *authority*) was also applied here because the statements indicate something about who the knowers were. In addition, there was *reference to vocational practice* (shortened to *vocational practice*), indicating that the students referred to knowledge from out of the mathematics classroom.

I had originally planned to use a coding system in order to analyse the data, adding and revising the codes to a point where it would be possible to compare across the tasks. However, because my theoretical framework suggested that a deeper and more holistic analysis of individual tasks and students' interactions with them was appropriate, I used the codes in the first part of the analysis process only, in order to identify which three tasks to look at more closely. The first criterion for selection of which vocationally connected tasks to study was that the data should be rich, and the students should be verbally active, in order to have something to work on. I also wanted one task from each programme for variety across the educational programmes. After this first narrowing down, I looked for notes in my data summaries and at the initial coding for indications that students or teachers referred to vocational practices. I also looked more closely at the tasks that I had noted had surprising elements in the implementation, either because they seemed to be more engaging than I would have anticipated, or because things did not go as planned. This selection process resulted in identifying three tasks for further analysis:

- The task about production of a budget for a hair salon in the design, arts, and crafts class. The main rationale is that it was the only vocationally connected task tried out in the class, but it was also important because students discussed what would be done in a real hair salon.
- The task about engine volume in the Technical and Industrial production class. The main rationale is that the students clearly referred to the vocational practice several times during the lesson.
- The task about producing a budget for a chosen project in the media and communication class. When I planned the task with the teacher, it had possibility for authentic aspects and had a real-world connection, but during implementation it seemed as though the students did not engage with this. Therefore, this task implementation seemed worth further analysis.

This selection meant that I did not analyse the 'jack stand task', the 'gear task' or the 'golden ratio task' in great detail. However, these lessons, as well as the rest of the lessons where I was present, are part of the background material of the thesis. I drew on these data in order to confirm the analysis, especially with regards to the classroom practices and normative activity.

The jack stand task was ruled out because it was too short and had relatively few verbal exchanges. This was a task suggested by the vocational teacher as something the students needed to know, so it was interesting for that reason; however, the engine cylinder task seemed much more engaging for the students.

Likewise, the gear task was connected to issues in the related vocation, since gear trains are important in mechanical engineering. However, the implementation of the task did not seem to engage the students, and they made few references to out-of-school practices.

In the case of the media and communication class the choice was between the golden ratio task and the Frifond project task. The golden ratio task was about ratios in photographs, and what makes ratios attractive. Both tasks had relevance to out-of-school practices, or vocational practices. However, when the students worked on the golden ratio task they barely spoke or reflected out loud on what they did, and therefore the task implementation is difficult to analyse. In addition, the Frifond project task seemed to be quite authentic, but the students seemed like they lost track of this in the implementation, and luckily worked in groups so that they discussed with each other.

Following this selection of the three tasks for further analysis I reread the transcripts closely and reflected on different parts of the lessons and how these related to the different research questions. This led to a closer analysis of students' interactions in the classroom, with triangulation where possible from the teachers' reflections and the student interviews. This process and the operationalisation of concepts from the theory is discussed in Section 5.5.3.

5.5.3 Operationalisation of the Theoretical Framework

In order to address the research questions, I needed to operationalise the theoretical concepts introduced in Chapter 4. To study enacted norms, I focused on students' working practices in mathematics, how they solved the task questions, and how they referred to mathematical practices and vocational practices.

As discussed earlier, norms emerge from interactions between teacher and students, and I therefore used the local classroom community as the reference point, as suggested by Cobb et al. (2001). I identified norms by looking for regularities and patterns in social interactions, and incidents where there were possibilities for developing or negotiating shared meanings. I also looked for evidence that the students and the teachers had a joint repertoire when they worked with the tasks. I noted how participants used words and actions to express what constituted expected and unexpected actions, and the distribution of knowledge of these practices. In addition, I looked for confirmation of the proposed norms, by analysing other parts of the data, where participants violated or challenged what were identified as norms in the classroom. I would then note whether their actions were regarded as legitimate or illegitimate, because such interactions indicate what the norms in the classrooms are.

To identify social norms, I looked for references to what were expected and unexpected actions by students and teachers. Such norms are likely to be most visible when something unexpected happens, and participants comment or react to what others do. Examples of this could be that a student or a teacher could say that 'remember that you always should explain your solution'. An important issue here was to distinguish between social and sociomathematical norms. Recognising normative activity with regard to mathematics as opposed to purely social norms is dependent on the ontological issue of what mathematics is in different settings, and it was important to consider if a norm could be considered a sociomathematical norm in different practices.

To identify sociomathematical norms, I looked for references to norms of mathematics practices such what is a properly expressed solution, what is a different solution, what is treated as appropriate sense-making action in a task (and what is not). These norms could be indicated by teacher comments such as 'great that you found a different mathematical way to solve this task'. Situations where teachers and students discussed solutions and how to solve the tasks were of particular interest, as were situations which included justifications from the vocational practice or mathematical practice. For example, this could be as in the vignette (Section 1.1) where the student mentioned that millimetres are rounded off, because it is not possible to measure less than that in the workshop. In the same episode, the teacher says that it is better to wait until later before rounding off, referring implicitly to what is usually done in the mathematics classroom. How such differences of opinion were resolved indicated possible breakdown moments and reestablishment of norms. I also looked closely at participants' roles and the perceived purpose of the tasks, noting any shifts in roles in terms of who the 'knowers' were, and issues regarding why and how one should work with tasks in mathematics classrooms.

To study connections students make with workplace practices and out-of-school knowledge the identification of references to workplace practices and out-of-school knowledge was required. Noticing use of language, including specialist terminology, and traces of brokers, boundary objects and boundaries was also important here. I wanted to assess whether the participants seemed to recognise the mathematics as connected to a school setting or to vocational practice. The analysis drew on what I identified as normative activity, but also on the students' demonstration of knowledge of the vocational practices.

I noted references to and use of strategies and working methods from out-of-school practices or vocational contexts, and discussions of the purpose of the vocationally connected tasks. I also looked for indications of whether the task was used to practice or develop formal school mathematics, or if it was treated as an opportunity to use different kinds of competences, for instance both formal and informal practices. I also noted explicit and implicit references to the different communities of practice related to school, workplace settings or experiences from out-of-school. Examples of this could be that a student or a teacher could say that 'in the workplace they need to open before the customers arrive' or '(in the workplace) they just estimate the size they need'. I noted whether the students asked the teacher or other students or asserted knowledge of the vocational context themselves. I looked for students' use of the task context in terms of explaining or discussing relevant vocational experiences. I also studied the episodes for indications that teachers or students were engaged in sharing of unfamiliar practices. For example, I looked for evidence of appropriation of terminology or actions from the vocational practice.

I also analysed the task implementations with regards to the concept of authenticity. To do this I first identified situations in the lessons where aspects of authenticity were directly or indirectly discussed, and thereafter I analysed whether the participants referred to an out of school origin, and how or if the aspect got a certification of originality. I would analyse whether the vocational context was disregarded or used, after it was recognised by the participants.

I also looked for evidence of discontinuities between different communities of practice and whether the intended boundary object representation was acknowledged and used by the participants. To do this I noted situations where vocational practices were discussed explicitly.

5.6 Trustworthiness of the Research

Validity and reliability of the research findings is important. As mentioned in Section 5.2, some commonly cited problems with case study research are cross-checking, validity, and subjectivity (Bassey, 1999; Cohen et al., 2013; Stake, 1995; Yin, 2014). In case studies and other qualitative research, it is usual to talk about trustworthiness of the research instead of validity (Bassey, 1999; Lincoln

& Guba, 1985). Some important issues for research to be trustworthy are prolonged engagement, triangulation, sufficient details, and an adequate audit trail (Bassey, 1999). To ensure a prolonged engagement, I was present in the classes several times over about a half year. I visited the students in their vocational classes to get a connection with the students out of the mathematics lessons. I also had the possibility to understand some of the issues that the students meet as participants in the practice of vocational education. I argue that I present the cases and the episodes with enough detail to ensure that the issues are recognisable, and I have tried in Section 5.5 to present an audit trail of how I analysed my data.

Triangulation of data is also mentioned in Schoenfeld's (2008) list of criteria which can be used to evaluate empirical and theoretical work in mathematics education. He argues that theories and empirical work can be judged by their "descriptive power, explanatory power, scope, predictive power, rigor and specificity, falsifiability, replicability, generality, and trustworthiness and multiple sources of evidence (triangulation)" (p. 487).

Descriptive, explanatory and predictive power, and scope all deal with the theoretical underpinnings which are used to explain and analyse the data. Does the theory capture and have the possibility to explain all relevant episodes? Rigor and specificity concerns how well-defined the objects and concepts in the framework are. In my research, this concerns among other issues how to identify norms and whether it is possible to identify authentic aspects of tasks.

I have used triangulation as a source of trustworthiness. Schoenfeld (2008) writes that "the more independent sources of confirmation there are, the more robust a finding is likely to be" (p. 494). I used the video recordings of the observed lessons as my main data and triangulated the observations with the discussions and interviews with the teachers and students. Another way of triangulation is by using multiple observers (Schoenfeld, 2008). I have discussed my analysis with colleagues at UiA and my supervisors and discussed my results when I presented parts of my data in conferences and seminars.

Bassey (1999) points to the importance of checking the data with informants. I have presented my CERME-paper (Sundtjønn, 2013) to the participating teacher, but I had no opportunity to get the students to comment. This "member check" of the data would have increased the trustworthiness of my analysis.

Unfortunately, the students had finished at their school when I had completed my analysis.

Another threat to the trustworthiness of my research is the difficulty of reproduction. Even if I describe all circumstances and how the research was conducted, it would be impossible to recreate the same environment. Schoenfeld (2008) argues that not all studies of education should be replicable, but that non-replicable studies still deepen our understanding. In my research, it may be possible to have similar findings in other vocational education mathematics classrooms.

5.7 Ethical Considerations

Tangen (2014) divides ethical considerations of research into three domains: ethics within the research community, protection of participants, and the value and role of the research in society (see Figure 5.3).

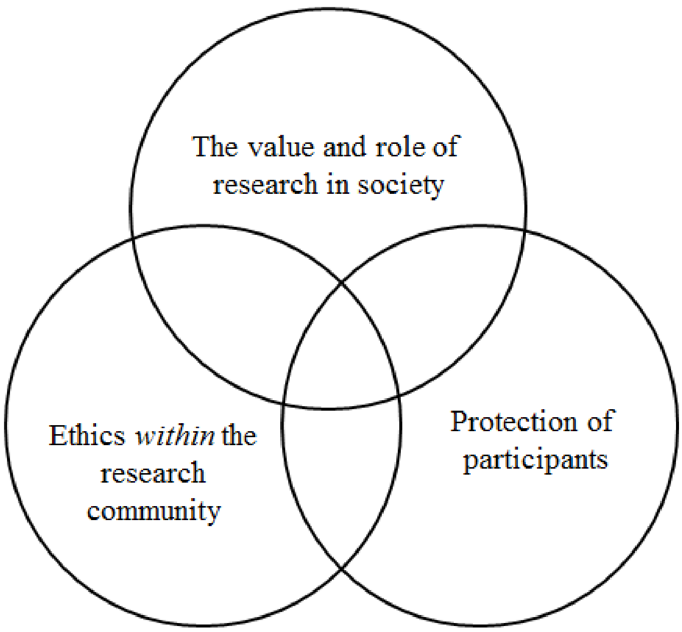


Figure 5.3: The three domains of research ethics (Tangen, 2014, p. 460).

Issues in the domain of ethics within the research community are professional integrity, good practice and independent research. These issues are discussed in the previous section on the trustworthiness of my research. The issues I will discuss here are protection of the participants and the value and role of the research. One important concern in case study research is informed consent from the participants. Participants should know what they are agreeing to be a part of, and the possible consequences of their participation. I tried to ensure this by giving oral and written information of the project. After my first informal contact, I gave the students more oral and written information about the research. The students and the teachers then decided if they would participate in the research. The approval/information letter to the students and teachers is presented in Appendix 11.1. Participation was voluntary, and it was possible to withdraw from the research at any time. In addition, I always listened to the students if they said that they did not want to be recorded at the specific moment.

I analysed three teachers and in total about 30-45 students from their classes. Such small numbers of participants mean it is possible to identify specific students and teachers. I have used pseudonyms for the teachers and students and changed the names of schools. This anonymisation makes identification of participants difficult for people outside the region, but since I was present in classes over a longer period, staff, administration, and other students in the schools knew which teachers and students I was in contact with.

Another issue about the protection of the participants is how to act as a researcher in the classroom. I did not want to interrupt “usual” classroom activity or undermine the teacher’s authority, but I wanted the students to discuss and share their opinions. Therefore, if the students asked me questions in mathematics, while the teacher was otherwise engaged, I choose to answer and offer them help if I deemed it non-interruptive of the teacher’s classroom management. I chose to ignore most of the disciplinary issues that a teacher would react to, to minimise my presence in the classroom. This could be when students played games on their cell phones or discussed issues related to their spare time.

This is ethically questionable, for what are the consequences when an adult figure in the classroom ignores such issues? Does this change the students’ patterns of interactions in future mathematics class, and should I have taken a stronger role as a complete observer? Or should I have reacted as I would as a participant from the teacher community? I will argue that all options are viable and can have positive or negative consequences. I chose to interact in a friendly and non-authoritative way, but I think that I still was regarded as a representative from the “adult world”.

In the data analysis phase, the responsibility for choosing which episodes and incidences to analyse was mine. My intention in the research was to choose explanatory episodes in connection with the vocationally connected tasks, and I hope that the participants feel that they recognise the descriptions of the situations and that they have been represented in a fair way. As mentioned earlier, it is not possible to escape bias when selecting and representing episodes (Bassey, 1999; Walker, 1983).

Regarding the value and role of the research in and for society, I discuss both its instrumental value for applicability, and the research's potential for systemic critique (Tangen, 2014). I argue that the research is applicable for teachers and policymakers in vocational education, and that it is important to research on vocational mathematics courses.

Hostetler (2005) writes that the aim of the research should be considered when doing research and defining what good research is. He expresses it as follows:

> Good research is a matter not only of sound procedures but also of beneficial aims and results. Our ultimate aim as researchers and practitioners is to serve people's well-being — the well-being of pupils, teachers, communities, and others. Education research can have a profound impact on people's well-being (p. 17).

In my research, I have tried to understand how students interact with vocationally connected mathematics tasks in school. I propose that if students get opportunities to understand mathematics both in school and in their daily practices and work life, they will be better equipped to be a part of our knowledge-based society and take part in democratic decisions. Therefore, I argue that my research could be considered valuable, relevant, and good research.

5.8 Reflections on Methodological Choices

All methods and methodologies have strengths and weaknesses. Although I have endeavoured to ensure that my research is trustworthy and that the methods used provided an insight into the research questions, there are of course weaknesses. My intention was to summarise and transcribe as much as possible of the empirical material parallel to the data collection, but that was too time consuming. Therefore, much of this work was done after the data collection was finished. If I had managed to summarise the data during the data collection phase, it would have been possible to strengthen the data collection, for instance such insight could have been used in the design of later tasks.

During and after the data gathering period I reflected further upon my role in the classroom. I think that it would have been useful to have taken a more passive role than I did in order to better see how the students and teacher interacted in the classroom without my presence. However, this would have lessened my opportunities to talk with the students, which I have found quite valuable. Turning to technical issues, I could have been more attentive to collecting good video recordings. Some of the recordings are difficult to transcribe and analyse, because of poor sound and images out of focus. I also wish for more pictures of students' notebooks. It would also been useful implement more vocationally connected tasks, or implementing the same tasks in several classrooms.

To get a stronger grip on the norms in the classrooms, and the establishment of the norms, I think it would have been quite useful to follow the mathematics classes and their teacher from the first day of school. Situations where there were different expectations between the participants would be more obvious in the start of the school year. While getting to know each other students and teachers work together with establishing norms and routines about what mathematics look like in this new community. When I met the classes, they were well into their school year, and already had routines for what to do, and when.

The research would have benefited from more successful student interviews. When working with students and other interviewees, there is a possibility that people do not say everything consistently (Mellin-Olsen, 1996); for example, a student could be hesitant to present negative aspects of his school experience with an adult in an interview setting, but be eager to discuss such experiences with his friends. The physical setting can likewise be important. Is it better to interview students in the classroom, in the workshop or in out-of-school settings? The interviews I conducted with the students were done outside the mathematics classroom, but within the school. This may have contributed to problems with the interviews. I could have interviewed the students out of the school setting, and maybe they would have been open to revealing more from their experiences with mathematics. Group interviews could be helpful for making students more talkative about mathematics. Having studied the data, I have noticed places where I really would have wished for the participants' own voices explaining why some tasks seemed have more connections to practices out of school.

The task design could have been strengthened by further observations of the students in their Common Programme Subject and a broader knowledge of what mathematics may look like in different communities of practice. Working together with full participants from different communities to design the tasks would have been valuable. The following chapter describes the selected tasks in detail, before I in Chapter 7 and 8 discuss the analysis of the implementations in the classrooms with regards to the research questions.

6 Design and Implementation of Three Vocationally Connected Tasks

In this chapter, I present the three vocationally connected tasks which are an important part of the setting in the different cases. I present the background framework of how the tasks were designed in Section 6.1. The design and intended learning outcomes of each of the tasks are presented in Sections 6.2, 6.3 and 6.4. I discuss the relevance of the tasks in terms of their fit with the curriculum for 1P-Y, their connection with the common programme subjects, and possible connections to students' vocational practice. In addition, I explain authentic aspects of the tasks (see Section 4.3) and how I regard the boundary object representations (see Section 4.2) in the tasks. In Chapter 7 I present an analysis of what happened during the implementation in the classrooms. The exact wordings and layout of the tasks in Norwegian are given in Appendix 11.3.

6.1 Background for Design of the Tasks

The design of the vocationally connected tasks was strongly influenced by Skovsmose's framework, which distinguishes between two different milieus of mathematics teaching and potential learning experiences in a school setting (Alrø & Skovsmose, 2002; Skovsmose, 2001, 2011). The different milieus are *the tradition of exercises*[23] and *landscapes of investigations*.

Many will recognise the tradition of exercises as the normal pattern of participation in mathematics classrooms. The teacher starts with a presentation of some mathematics, and then the students practice with exercises. The tasks given in the tradition of exercises are usually small variations of the examples the teacher presented on the blackboard, and will often have one, and only one, correct answer (Skovsmose, 2001; Grønmo et al., 2010). This is what can be thought of as some of the routines in a community of practice in a mathematics classroom. For instance, it is usual that the information in the task is neither more nor less than what is needed, and the task can be solved by combining the numbers given in the task with appropriate mathematical operations. As discussed in the literature review, and the theory chapter, school mathematics is different from mathematics in out-of-school practices.

[23] Some different names are used in the literature: tradition of exercises (Skovsmose, 2001), paradigm of exercise (Alrø & Skovsmose, 2002), and sequence of exercises (Skovsmose, 2011).

In contrast to the tradition of exercises, Skovsmose (2011) argues that one can have mathematics lessons in landscapes of investigation. In landscapes of investigation, the tradition of exercises is replaced with an open scene setting (Alrø & Skovsmose, 2002) which invites “students to formulate questions and to look for explanations” (Skovsmose, 2001, p. 125). Students are invited to ask and solve questions that they are interested in; these can be issues from their daily lives or investigations into mathematics.

In both education milieus, there is the possibility for division into mathematics, semi-reality, and real-life references (see Figure 6.1).

	Tradition of exercises	Landscapes of investigation
References to pure mathematics	(1)	(2)
References to a semi-reality	(3)	(4)
Real-life references	(5)	(6)

Figure 6.1: Milieus of learning.

Skovmose (2011) argues that this matrix is meant as a simplification and possible discussion point of complex classroom structures. It is possible to move between different parts of the figure over time (Skovsmose, 2011). The divisions between the different cells are not unambiguous, and there are cases where “some exercises can provoke problem solving activities, which might turn into genuine mathematical investigations” (Skovsmose, 2001, p. 128).

In addition to the separation between landscapes of investigation and tradition of exercises, the horizontal separation in Figure 6.1 is important. In a task, the context can refer to pure mathematics, semi-reality or real-life. Tasks with references to pure mathematics can be found in both education milieus. Tasks within the tradition of exercises with references to pure mathematics can, for example, be tasks such as, “calculate 23 + 45” and “calculate the area of a square with sides of 5 cm”. It is also possible to refer to pure mathematics in a milieu dominated by landscapes of investigation; for example, one can explore grids of the multiplication table or investigate different variables in formulas and so on.

But tasks can also refer to events and contexts outside the realm of mathematics. In Skovsmose's framework, this is known as either semi-reality or reality. Tasks with references to semi-reality refer to a situation which has similarities with events in the real world, but the situation in the task is artificial and would not be discussed in that way out of school education. For example, this could be word problems in which someone wants to buy 15 kg of apples, or a question of finding the time used to get a boat across a river by using vector calculations. The situation which the task refers to is familiar or known to the students, but the problems that are to be solved are in some ways artificial. In this way students are socialised into a school mathematic practice.

In the framework it is likewise possible to have mathematical activities or tasks with real-world references. It is important that "the references are real, and they provide the activities (and not only the concepts) with meaning" (Skovsmose, 2001, p. 128). This means that for a task to have real-world references, it should be based in reality, and be solved in ways which make the solutions relevant and appropriate for the real context.

I drew on Skovsmose's framework when designing the tasks and wanted to make tasks which were in the landscape of investigations and had connections to the real-world. I discovered the theoretical concept of authenticity and boundary objects after the tasks had been designed and implemented and was also not conscious of the concept of norms in the design phase. Still, these theoretical concepts have been helpful in analysing the tasks implementations, although they were not in use in the task design phase. To summarise I thought in the terms of Skovsmose's framework when designing the tasks and I wanted to make tasks which had connections to the students' future vocations, in line with what Skovsmose calls real-life references.

6.2 The Hair Salon Budget Task

The Design, Arts, and Crafts class worked with the competence aims in economics in mathematics. Among these objectives are that students should be able to "compose budgets and accounts using various tools" and "calculate taxes" (The Norwegian Directorate for Education and Training, 2010b, my translation). In the students' two Common Programme subjects, Production and Quality, and Documentation, they are supposed to "carry out basic marketing surveys, and evaluate the need for crafts and services in the market" and "calculate the price of materials, products and services" (The Norwegian Directorate for Education and

Training, 2006a, my translation). If the students continue toward a crafts certificate in hairdressing, the competence aims in the third and fourth year specify that they should “carry out basic marketing surveys, and evaluate the need for hairdressing services”, “give an account of the relationship between cost and profitability in hairdresser”, and “set up a basic budget and calculate price of products, raw materials and services” (The Norwegian Directorate for Education and Training, 2008, my translation). Therefore, the teacher and I expected that the students would need to estimate customer turnover in their future vocation, and from this estimation, calculate income and expenses of a small business. We expected that the students, regardless of their plans for their future vocation, would have the experience of being customers at a hair salon. Thus, the goal of the task was to make a budget for an imaginary hair salon. The intention was that the students would work together in small groups and plan their future hair salon.

The task had an accompanying slideshow presentation which I used to introduce the questions during the lessons. The students also received a written task handed out on paper. The first slide in the slideshow presentation is presented in Figure 6.2 and displays a copy of a real receipt for a hair cut from a salon in Kristiansand, and a price list from a hair salon in Oslo.

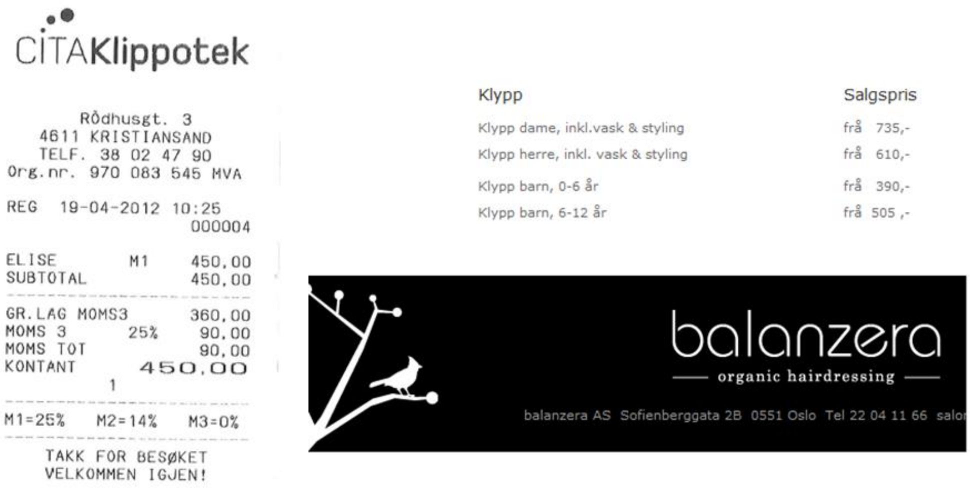


Figure 6.2: Receipt for lady’s haircut in Kristiansand and price list from a hair salon in Oslo presented in the PowerPoint presentation.

The receipts are boundary object representations of objects from the vocational practice. The receipts and the price list represent items the students have met as customers and will meet in their potential profession as hairdressers. Even if the students are not aiming to be hairdressers, they would have seen similar objects as customers, so the receipts had the potential to connect different practices. The receipt and the pricelist are items which are in daily use in businesses, but in this task, it was used to promote collaboration between the students about different prices on haircuts.

We intended the photographs presented in Figure 6.2 to be a discussion point for the first questions in the task which were, “What is an ordinary price for a lady’s haircut?”, and “What is the highest price you could have paid for a lady’s haircut?”. The receipt presented from Kristiansand showed that the cost of the haircut was 450 Norwegian Kroner, but the example from Oslo gives a price from 735 Norwegian Kroner, almost twice as much.

The start of the task is presented in Figure 6.3, and the slideshow presentation and the whole written task is presented in Appendix 11.3.1. The layout of the task included spaces for students to write their answers.

Hair Salon Budget

Question 1:

What is an ordinary price for a lady’s haircut?
What is the highest price you (in the role of the customer) would have paid for a lady’s haircut?
What is the lowest price you (in the role of the hairdresser) could have asked for a lady’s haircut? (Remember that you will not get the whole amount yourself.)

	Price	Reason
Lady’s haircut ordinary price		
Highest price you could have paid		
Lowest possible price you could have asked for		

What do you need to know to decide the prices?

____

____

____

____

Figure 6.3: First sheet of the hair budget task.

As presented in Figure 6.3 the hair salon budget task is divided into a sequence of several questions and starts with questions about prices in hair salons, intended to trigger thinking about why haircut prices could vary. The intention of these questions was that students should discuss haircut prices in their group and then fill out price suggestions and reasons for the suggested prices. They were asked to reflect upon what information is needed to decide prices. We wanted the students to draw on their experiences as customers at a hair salon, but with a view towards being the owners of the hair salon. The next set of questions, see Appendix 11.3.1, asked the students what factors the prices depended on, and to note what they thought would be the important monthly expenses and income sources of a hair salon.

After this introduction, we provided students with a budget template on paper, and asked them to make a budget for their hair salon. In order to help the students develop some of the items in their budget, they were given information on rental fees (per square metre), sample prices of haircuts, and information on electricity, phone prices and so on (see Figure 6.4 and full list in Appendix 11.3.1)

Husleie: 1200 kr pr kvadratmeter pr måned

Telefon:

	ISDN Pluss
Pris per mnd	265,-
Etablering/eierskifte	fra 899,-
Minuttpris	Ring gratis til fasttelefoner i Norge
Pris til mobil hos Telenor og Netcom	0,68
Pris til mobil i andre nett	0,76
Startpris	0,59

Renovasjon: 3625 kr i året + 171 kr pr tømming av 1000 liters dunk

Figure 6.4: Some of the information given as part of the task: rent of location, telephone prices and garbage collection costs.

The information we gave the students was not exhaustive, and the students were encouraged to include other budget items from their own experience. We intended that the students would estimate or find the numbers they needed by gathering appropriate information and discussing it as a group. We asked them to calculate loss or profit for their salon and included a reminder to consider value-added tax[24] when budgeting, because this is an important factor when pricing a service for customers. This information could have been left out of the task, and the students could have gotten the responsibility for finding all the information needed to make the budget. The items could be regarded as boundary object representations, and especially the information about phone prices, shown in Figure 6.4, is a snippet taken from a real phone company's information brochure.

The students started out making budgets on paper, and later in the process they were encouraged to set up the budget in a spreadsheet. After the students had made a draft budget, the task asked "What does the hair salon earn in a month?" and "What do you earn in a month… [and] … an hour?" These reflections were intended to be used for revising the budget. We wanted the groups to reflect on, and possibly revise, their budget so that they could be satisfied with the profit of their imaginary hair salon. They could also figure out what their intended profit margin would mean for the hair salon prices. After the students had finished the paper versions of the budget, they were asked to use a software spreadsheet (Excel) to further refine their budget and prices. With a

[24] Norwegian: Merverdiavgift (moms). In Norway, value-added tax is 25 % added on the cost.

spreadsheet it is possible to try out different scenarios and simulate how changes impact profit or loss.

To work on the budget the students need to discuss, estimate and decide prices for the different services they would offer, how many and what kinds of haircuts they could manage in a day, figure out which kind of expenses the salon would have, and so on[25]. There were opportunities for students to work on budgeting, estimation of prices, modelling (how many customers to expect per day and need for various services), calculate value added tax and determine rental fees by estimating the required salon area. Much of this information connects to rate of change, such as haircuts per day, area required per worker and rent per month. These rates of change are also connected to each other; for example, if one has many hair colourings per day, the salon needs to have more employees or do fewer regular haircuts. It would be possible to make a workflow diagram or an appointment calendar to systematise information on number of haircuts of different kinds a hair salon can do in a day or a month.

To start on the budget, the students mostly needed addition and multiplication for their calculations, however the students needed to decide which budget items are necessary and estimate important information in order to produce a realistic budget. The task could have provided a budget model on a spreadsheet for the students to try out with different scenarios, but we chose to ask the students to generate the spreadsheet themselves. This gave them the opportunity to learn or practice how to build formulae in spreadsheets.

We wanted the task to provide opportunities for students to collaboratively inquire into the mathematics of budgeting and decide what an effective and realistic budget is. The task refers to a possible future practice where the students are the owners of a hair salon. The issue of making budgets for a future business has clear connections to workplace practices. But the students are given sequential questions, which are closely related to traditional practices in a school mathematics classroom. At the same time, when making a budget for a hair salon, there are no definitive answers on how to estimate budget items and

[25] In retrospect the students could also have been asked to find real information by asking in a local hair salon or having an oldtimer from the hairdressing practice being part in the process. How does a real hair salon ensure that they do not overbook customers, and that their prices are right for the market? Such connections could have enabled the students to decide if their calculations were realistic. A similar idea would have to use a real budget for an existing hair salon and share this with the students. However, by getting the students to start from scratch they needed to plan and figure out which items to include by themselves.

therefore the corresponding budget. There were several possible authentic aspects in the task. I will discuss the certification in Chapters 7 and 8, since this occurred within the implementation.

There were a number of potential authentic aspects with regards to what Palm (2008) calls event, question, purpose, language and information. The event that the students will, in the future, need to make a budget for a small business is probable. The purpose of the task is to figure out a budget for a hair salon, and such a budget has an out-of-school origin. But at the same time: the propose of the task in the mathematics classroom is to learn about budgeting, not starting a hair salon, so that it this sense the task do not have an out-of-school purpose.

My language when presenting the task was connected to the practice of the community of hairdressers. The task referred to objects from hair salon practices, and the images presented in the PowerPoint presentation are from real hair salons (see Figure 6.2). The images of the receipt and the price list have an out-of-school origin, and therefore fulfil the first part of being authentic objects. The information provided in the task is similar to what could be known when setting up a preliminary budget for a small business. However, the information given to the students about telephone bills, rent and so on is not specific to hair salon rentals, although it was designed to be specific to the area. This information is also not presented in the same way as it would be if the students would set up their own hair salon. Then they would need to gather such information themselves.

Other aspects of the task that were not authentic were that the task is set up with many sub-questions, and this made it look more like a traditional mathematics task. Additionally, in real life, when a budget is made, there would probably be possibilities to learn from other hair salons' budgets and estimates. The students would then have a better understanding of what would be realistic numbers of customers, prices, and expenses.

The hair salon budget the students developed can also be regarded as a possible boundary object. The budget could be shared with people working as hairdressers, and the budget could have been used to communicate across the boundary between school and workplace practice. Thus, in total, I regarded the task as having some possible authentic aspects, with references to a future possible workplace practice. In Section 7.1 I present and analyse the implementation of the task.

6.3 The Engine Cylinder Task

During the Spring of 2013 the industrial and technical production class worked on geometry. The competence objectives in mathematics included that the students should be able to ‘‘solve practical problems involving length, angle, area and volume’’ and ‘‘calculate using different measurement units, use different measuring tools and evaluate measurement accuracy’’ (The Norwegian Directorate for Education and Training, 2010b, my translation).

The intention was to develop a task which would address these competence aims and be relevant for the vocational programme and the target students. The teacher and I knew that many of the students in the class drove and owned mopeds. In Norway one has to be 18 years old to obtain a car driver’s licence, but it is legal to drive a moped if one is 16 years old or above with the appropriate driver’s licence, so mopeds are a common mode of transportation for young people. There are certain rules about cylinder volume of the engine: a moped is street legal if the cylinder volume is maximum 50 cm^3, and has a maximum speed of about 45 km/h on a flat road. Although there are mopeds with engines with cylinder volumes bigger than 50 cm^3 and with higher maximum speeds, these are only legal to use in dirt track racing and similar activities. We knew that ‘tuning’[26] of mopeds was a topic of conversation among young people and occurred in the area[27]. Such tuning can be done in several different ways: for example, by removing seals, changing electronic chips or swapping the engine cylinder with a bigger cylinder. We also knew that the students had worked with different kinds of engines and worked on engine cylinders as part of their common programme subject.

With regard to the relevance for the vocational education programme the curriculum for the common programme subject in Technical and Industrial Productions specifies that students are supposed to ‘‘plan and conduct preventive maintenance on machinery and equipment’’ (The Norwegian Directorate for Education and Training, 2006c, my translation). The teacher from the Common Programme Subject had suggested to the mathematics teacher Ingeborg earlier that the students could be interested in mathematics with connections to engines and engine cylinders. I had previously observed the students when they worked

[26] Norwegian: Trimming. People have been known to illegally modify the engines of street mopeds to be able to drive faster.
[27] 5 of 20 mopeds were illegally tuned in a control in the centre of Kristiansand autumn 2014 http://www.fvn.no/lokalt/kristiansand/article2676821.ece

on a lawnmower engine in the workshop and had learned that some of the students had been on a short internship in a car repair shop. With regards to the future workplace practices of the students, one of the possible craft certificates after finishing the Technical and Industrial Production Education Programme is Motor Vehicle Mechanic.

With of Ingeborg's support, I designed a task which had references to moped and car engine cylinders and was meant to be relevant for the competence aims in geometry. The task is presented in Figure 6.5 (translated from the original task in Norwegian).

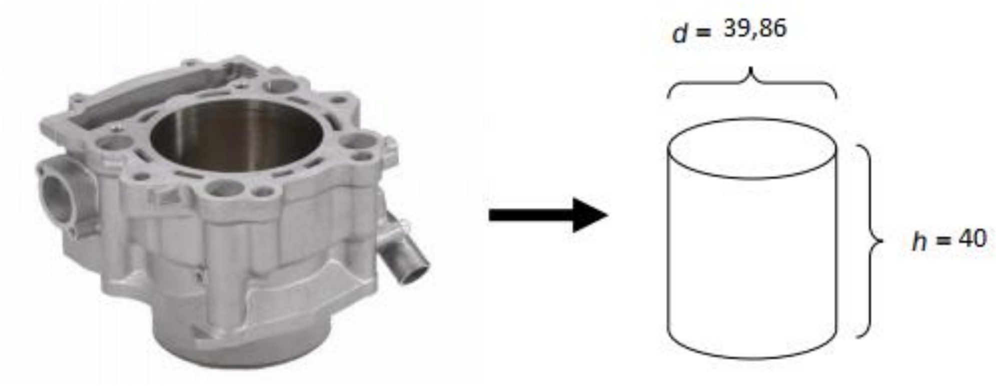
Moped Engine Task

The maximum legal stroke volume in a moped in Norway is 50 cubic centimetre (50 cc). In the table, there is one example of an engine which fulfils this[28].

Technical specifications

	Senda R DRD
Engine	Single cylinder 2 T EURO 1
Bore x Stroke	39.86 x 40
Cylinder capacity	49.9 cc
Carburettor	Dell'Orto PHVA-14
Cooling system	Liquid
Starting system	Kick starter
Compression relation	13.0: 1
Fuel	Unleaded petrol

[28] Sources:
Brochure ''2003 Range –Derbi Senda – Making a Difference'' and
https://en.wikipedia.org/wiki/Cylinder_(engine)#/media/File:Malossi_70cc_Morini_cylinder.jpg



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Bore: bore/diameter of the cylinder.

Stroke: stroke volume/height of the cylinder.

a) Check that the engine in the table above has a cylinder volume less than 50 cm^3.
b) What is the new volume if you bore out the whole cylinder internally such that the diameter is 2 mm bigger?
c) How large can the stroke/height maximal be to have a legal moped if the bore/diameter is
   1) 50 mm?
   2) 30 mm?

Car Engine Task

A car engine often has 4, 6 or 8 cylinders.

a) An engine has 4 cylinders with bore/diameter 81 mm and stroke/height 77.6 mm. What is the total stroke volume of the engine?
b) A 3.5 litre V8 engine (8 cylinders) has a stroke/height of 65.6 mm. How large must the bore/diameter be?
c) Suggest what the stroke/height and bore/diameter can be in a 3.5 litre engine with 4 cylinders.

Figure 6.5: The engine cylinder task.

In this task, the students can work with the volumes, stroke/ height[30] and bore/diameter[31] of different cylinders. The teacher and I hoped that they would

[29] Sources:
http://www.atvriders.com/images/yamaha/2006raptor/RaptorParts800/cyl.jpg and Utdanningsdirektoratet: Eksamen MAT0010 Matematikk 10. årstrinn 20.05.2011.

[30] Norwegian: Slaglengde/høyde.

[31] Norwegian: Borring/diameter.

draw on their knowledge from previous experiences with engines as they worked on the questions, especially in order to estimate, suggest and evaluate cylinder sizes. In this task, the photograph of the moped cylinder and the brochure extract with specifications of a moped engine are boundary object representations. They are representations of objects from the vocational practice so their origins are from an out-of-school practice. They are not the real objects but representations of them. The intended role of these boundary object representations was to connect the mathematics practice in the classroom with the students' future vocational practice.

As can be seen in Figure 6.6, taken from a moped brochure (see footnote 28), the units of the numbers for stroke and bore (39.86 × 40) are not specified, but the cylinder capacity is given as 49.9 cc (i.e. 49.9 cm^3).

Technical specifications

	Senda R DRD
Engine	Single cylinder 2 T EURO 1
Bore x Stroke	39.86 x 40
Cylinder capacity	49.9 cc
Carburettor	Dell'Orto PHVA-14
Cooling system	Liquid
Starting system	Kick starter
Compression relation	13.0: 1
Fuel	Unleaded petrol

Figure 6.6: Enlargement of table given in the engine cylinder task.

In Figure 6.6, stroke and bore are specified in millimetres, but the measurement unit is not written in the table, being a part of the tacit knowledge. The cylinder capacity is specified in cc (cm^3), so to calculate the volume of the cylinder a conversion needs to take place, either from millimetres to centimetres, or vice versa. Such tacit knowledge regarding which units are used, but not written, is a part of the routine practice of those who work with engines.

The students had previously seen and worked with the formula for cylinder volume, both earlier this year and in their prior schooling. The formula could also be found by looking in their mathematics textbook, or other sources. Question 1a can be answered by inserting the relevant information from the given table into the formula for cylinder volume: $V = \pi \cdot r^2 \cdot h$ and using the formula appropriately for a mathematics classroom practice. As noted, it is necessary to convert from diameter to radius at some point in the calculations (or modify the formula for cylinder volume with regards to diameter), and to use appropriate units. The same formula and method can be used in 1b, with the revised diameter. If one has experience with similar engines, the answers could be compared with known engines, and conclusions based on this knowledge. The difficulties in these questions are threefold – being able to use a correct formula, correctly applying the formula, and correctly using appropriate units. The first two problems are clearly worked on in mathematics lessons, but the third is closely connected to knowing in an out of school practice.

In question 1c, the height is the unknown variable in $V = \pi \cdot r^2 \cdot h$. This again can be done by applying the appropriate formula. The height is possible to figure out by solving the formula as an equation, or by choosing different heights and using by trial and error to work out which would give the wanted volume. If solving this problem by trialling different heights it would be an advantage to have a feeling for what numbers with appropriate units could work. However, these ways of solving the questions are clearly based in practices from school mathematics.

In the next set of questions, the context is car engine cylinders. Car engines have different numbers of cylinders depending on the engine size (for example 4, 6, or 8). Similar to question 1a, the first question (2a) is solvable with calculations using the $V = \pi \cdot r^2 \cdot h$ formula and the given information about height/stroke and diameter/bore. Here the total volume of the engine is found by multiplying the cylinder volume by four, since the car engine is a four-cylinder engine.

Question 2b is more difficult if solved as an equation, because of the need to calculate with square roots. One possible solution method for this question is presented below.

$$V = 8 \cdot \pi \cdot r^2 \cdot h$$

$$3.5 \text{ l} = 8 \cdot \pi \cdot r^2 \cdot 65.6 \text{ mm}$$

$$3500 \text{ cm}^3 = 8 \cdot \pi \cdot r^2 \cdot 6.56 \text{ cm}$$

$$r^2 = \frac{3500 \text{ cm}^3}{8 \cdot \pi \cdot 6.56 \text{ cm}}$$

$$r = \sqrt{\frac{3500\ \text{cm}^3}{8 \cdot \pi \cdot 6.56\ \text{cm}}}$$
$$r \approx 4.6074\ \text{cm}$$
$$d \approx 2 \cdot 4.6074\ \text{cm} = 9.2148\ \text{cm}$$
$$\underline{\underline{\text{The bore/diameter is then 92,1 mm}}}$$

This is written as what would be judged as a good mathematical solution in a mathematics classroom. Another way of solving this question is to work out the bore/diameter by estimating a solution and checking it in the formula until the correct volume is found.

In question 2c several (actually infinite) solutions will fit the criteria; the intention of this question was to give students opportunities to explore the solution space and make their own decisions regarding appropriate solutions. One solution that would be mathematically correct is height equal to 44.57 m (4457 cm) and diameter equal 0.5 cm; see calculation below.

$$V = 4 \cdot \pi \cdot \left(\frac{0.5\ \text{cm}}{2}\right)^2 \cdot 4457\ \text{cm}$$
$$V \approx 3500\ \text{cm}^3 \approx 3.5\ \text{dm}^3 \approx 3.5\ \text{l}$$

However, this would make an unstable and useless cylinder for an engine. Such a cylinder would be prone to breakage because of its dimensions. However, if students decide to base their answer on their vocational knowledge, they might suggest a more appropriate answer to this question. Most cylinders in engines have approximately the same size of stroke and bore, which in this case could for example be stroke equal to 10.3 cm and bore equal to 10.4 cm as follows.

$$V = 4 \cdot \pi \cdot \left(\frac{10.4\ \text{cm}}{2}\right)^2 \cdot 10.3\ \text{cm} \approx 3500\ \text{cm}^3 \approx 3.5\ \text{l}$$

As shown above, the tasks are possible to solve within the routines of mathematics classrooms. The question of how similar issues would be resolved in a car repair shop is not known, but it is likely that both solution processes, and demands for calculations would differ. The task is in many ways quite reminiscent of tasks done in school mathematical practices. Much of the task is possible to answer by utilising knowledge of the formula of cylinder volume and applying it to the relevant numbers. Students need to figure out appropriate units to use, and then convert the units necessary. I argue that the task has closed answers on most of the questions, and the layout is quite traditional with questions arranged by increasing difficulty. However, in question 2c ‘‘Suggest stroke/height and bore/diameter in a 3.5 liter engine with four cylinders’’, students can investigate different answers. There are several (infinitely many) correct *mathematical* answers to this question. To find appropriate answers in a workplace practice one would need to draw on knowledge of how engines are usually designed to be stable and durable[32]. As mentioned, one solution that is mathematically correct is height equal to 44.57 m and a diameter of 0.5 cm.

There were several authentic aspects planned in the task. As discussed in Section 4.3, an authentic aspect needs to have an out-of-school origin and a certification of originality (Vos, 2015). The certification of authenticity could have been confirmed, for example, by experts on motor engines or the teacher in the vocational context, but neither of these persons were available in the mathematics class. The mathematics teacher and the researcher were therefore the only adults who could have given such certifications. However, as I will discuss in Chapter 7, the certification was confirmed several times by students themselves in the class. In the remainder of this section, I explain which aspects of the task had out-of-school origins, while the certification will be discussed in Chapters 7 and 8, since this occurs within the task implementation.

The event that a cylinder would be replaced by another sized cylinder has an out-of-school origin. As explained earlier, in Norway street legal mopeds must have engines with maximum 50 cm^3 cylinder volume, but one common way of making such engines more powerful is to exchange the original cylinder with a cylinder with a bigger volume. When one orders a new cylinder, the information about cylinder volume is ordinarily supplied in addition to the bore and stroke. However, there may be cases where this information is unavailable. Thus it is arguable that identifying the volume, the stroke or the bore can be related to an out-of-school need.

The words used in the task were informed by the language I had seen used on Internet forums about tuning of moped engines. In the task, one could have used only stroke and bore, but we chose to write “stroke/height” and “bore/diameter” in order connect words from the vocational practice with the mathematical terms,

[32] With reflection after the task was implemented, it would have been very useful to ask students to elaborate different dimensions and the advantages and disadvantages of them with regards to the engine context.

with the additional advantage that students without experience of the vocational words could understand what they meant. I will again argue that the vocational terms bore and stroke words have an out-of-school origin. However, in an out-of-school practice these words would not be juxtaposed with height and diameter as they were in the task. There is also a clear structure in the task similar to mathematics tasks from the school practice.

Some inauthentic aspects of the task are that it is presented in a mathematics classroom, by a mathematics teacher, and with no defined production goals for the activity. A production goal in a vocational practice could be that a car repair shop needed to figure out which cylinders would fit and still be legal. The task is more structured and provides information that would not be given in a vocational setting, such as the outlined plain cylinder, and use of both words “bore/diameter” and “stroke/height”. The information is also modified by giving the students an excerpt of small part of the brochure, instead of the brochure itself.

As presented in Figure 6.5, illustrations of a moped engine cylinder were included. This included two illustrations of moped engine cylinders, and one outlined stylised cylinder. In this task these illustrations are boundary objects representation, intended to support communication between the mathematics and the vocational practice. The photographs are of objects from an out-of-school context, but they do not depict the moped engine itself[33].

To summarise, the task was seen before the implementation as connected to out-of-school practice, and as providing an opportunity for students to engage with calculations of volume. The implementation of the task is described and discussed in Section 7.2.

6.4 The Frifond Project Task

In the Media and Communication education programme, the students were given the task of making a project proposal including a budget for a real small-scale grant scheme called Frifond[34]. This grant is aimed at the students’ age group, and funds support for small scale youth activities. Activities that have received

[33] What could have been different would be to have real physical cylinder from an engine. The task could have been presented in the workshop, with real engines cylinders, where the students could have measured stroke and bore themselves, and then worked out the volume. The volume could have been checked by both calculations and by measuring with filling it with a liquid.
[34] http://www.frifond.no/

support are for instance building a skate ramp, arranging a street party or making a music video. The task was intended to be relevant for the mathematics competence aim that students should be able to ‘‘compose budgets and accounts using various tools’’ (The Norwegian Directorate for Education and Training, 2010b, my translation). Additionally, the task was intended to be relevant for the students’ vocational education since we knew that in the Common Programme Subject, they were supposed to learn to ‘‘calculate cost in a media production’’ (The Norwegian Directorate for Education and Training, 2006b, my translation).

We thought that they might choose to plan something connected to their educational programme, for instance that they could make a short movie. The ability to make a cost estimation of a production is one of the basic skills of numeracy in media and communication (The Norwegian Directorate for Education and Training, 2006b). The students worked with a genuine grant scheme and had responsibility for finding the relevant information they needed. The translated task is presented in Figure 6.7. For the Norwegian version, see Appendix 11.3.3

PROJECT TASK - BUDGET

Frifond was established by the Stortinget in 2000 to improve economic conditions for voluntary and democratic activities for and by young people in their local community. You can apply to grants of up to NOK 25 000 to do what you would like, where you live! Read more here (frifond.no)! Please also read the grant application guidelines you can find on the same page.

1. Pick a project and make a short description of the idea that you want to implement. The project should be suitable for an application for a grant from Frifond. This should be presented to the whole class in 15 minutes. (The project could be to build a skate ramp, make a film, create a magazine, arrange a LAN party, arrange a photo exhibition, and arrange a festival, courses or seminars.)
2. Write a short project proposal for what you need in terms of equipment, premises etc.
3. Set up a budget for the project.

   NB! You must show your working and explain the numbers you get, i.e. write down where you find prices and your calculations. This can be done in an appendix to the budget.

Figure 6.7: The Frifond project task.

In the Frifond project task, the students needed to estimate costs and make a budget. The task was introduced by showing the webpage for the grant and mentioning previous proposals that had been funded. The students were then given the task above, and we supplied them with some places online which could be useful for items needed for making a movie. Our original idea was that the students would be interested in something like making a movie or a music video because of their chosen educational programme. However, the task was completely open with regards to what kind of project the students would like to plan, if it was suitable for the grant scheme. We intended that the task could make the students define an interesting (for them) project. They were asked to read about the grant scheme online on the webpages, and propose projects that they were interested in.

The task clearly has connections to the out of school setting since the students were to make a project proposal and budget for a genuine, existing grant scheme. If the students wanted and found a project that they wanted to work on they could send in an actual application to the grant scheme. The webpage of the grant scheme is a boundary object which facilitates communication between the grant scheme administration and the students in the mathematics class. Here the students could interact with the real webpage, and not a facsimile of it. The task is quite open, and the students needed to define themselves the parameters of their project.

The task and the website do not give instructions on how present their budget, and so the students had the opportunity to decide how to communicate the budget in writing to the teacher, the rest of the class and possibly the grant administrators. This task was designed after the hair salon budget task, which also asked the students to make a budget. Therefore, the experiences with that task implementation was part of the experiences I had when this task was designed. In the Frifond project task we did not specify how this budget should be set up, so that the information the students were given would reflect the grant webpage. The task started with the students brainstorming to propose a project they were interested in working out a budget for, then the student groups would present their ideas to the whole class and hopefully get feedback on their idea. Then they would refine and plan their project and figure out what would be needed for a budget for the project. The students had approximately 3 back to back lessons of 45 minutes to work with the task altogether, and the goal was that during this time should make a short project description, and a budget for their project.

Depending on the students' own choice of project, they would work with estimation, budgeting, modelling, proportional reasoning and problem-solving skills. One possible solution to the task can be a simple budget such as the one presented in Table 6.1, this example budget was not shown to the students.

Table 6.1: Example of simple budget for a short movie.

Item	Income	Cost
Rent of location		1000 per day
Participation fee	100 per participants	
Food to participants per day		500 per day
Other items	…	…
Total sum	5000	30000
Grant needed	Cost-income: 30 000 - 5 000 = 25 000	

If the students wanted to make a short movie or some other complex project, the budget could be more advanced.

As in the other tasks, I can identify several possible authentic aspects that were planned in the task. I will discuss the certification in Chapters 7 and 8. The purpose of the task is to write a proposal for a grant application to Frifond. The purpose clearly has an out-of-school origin however it is set in a school setting. The students had to make a project, regardless of their interest for this. They could choose what to focus on, but it is probable that the students were not driven by a real need for the money for a project. The purpose of the task was to work with budgets in a mathematics class, although the students could have used the budget for a real application.

The language used to present the task is simple, and not specific to mathematics classrooms. The students were asked to read and navigate the real Frifond-website, and they needed to interpret what is written there to make a good proposal and budget. This meant that they had exactly the same information as young people would have when starting to make a proposal for a grant to Frifond. I regard the Frifond website as a boundary object, since it facilitates communication between the students and the grant administrators. With regard to authenticity, the website has an out-of-school origin. This Frifond-website is not constructed for the school task but is the actual website for such a proposal.

Even though the task has several aspects with out-of-school origins, there are still elements that show that this is a school task. The student groups are put together by the teacher, and they work under time constraints. Even though time constraints are common in out-of-school settings, a three-hour time limit seems unrealistic for planning without any previous ideas. It is important to remember that the students did not have a real need to apply for grant money, however it is possible that some of the students had previously thought about ideas that could be fitting for Frifond. The group composition is also artificial, and some of the participants will have less knowledge and interest in the group's chosen proposal. However; it seemed like the planned Frifond project task has the potential to provide students with opportunities to engage with mathematics with clear out-of-school connections.

6.5 Similarities between the Tasks

In this chapter, I have presented three different vocationally connected tasks. All three tasks were potentially relevant for competence aims in 1P-Y, and indirectly, some competence aims in the students respective Common Programme Subjects. I would argue that all tasks have relevance for a vocational practice that the students can meet in their possible future vocation. However, some of the students were not familiar with engines, did not want to become hairdressers, do not need to make budgets for projects or will not set up their own business in the future.

All three tasks had some authentic aspects, at least aspects that fulfil the first part of being authentic, namely having an out-of-school origin. For example, the photographs of the engine cylinders, the hair cut receipts and the webpage of the Frifond have out-of-school origins. Additionally, the questions in the tasks could be asked in a vocational practice or outside of a mathematics classroom, however not necessarily in the same manner. Therefore, I argue that the tasks gave opportunities for interaction with out-of-school practices. I cannot argue that one or two of the tasks are more or less authentic than the others, but they have different authentic aspects. I would also argue that the tasks did not contain any boundary objects in the traditional sense, apart from the Frifond website. There are several objects in the tasks that can be seen as boundary object representations, such as the photographs of the engines, but the “real” objects are not present in the mathematics classrooms.

The tasks have several inauthentic aspects, with an important aspect being that the tasks are made for, and worked with in mathematics classes in school. In this practice the students will be in their roles as mathematics learners, and they know that they are participants in the subject mathematics. They met their mathematics teachers and are presented with tasks that are reminiscent of tasks they have seen before in the mathematics class practices. In the next chapters I will discuss the implementation of the tasks.

7 Findings: Norms and Practices in the Task Implementation

In this chapter, I report on the implementations of the tasks presented in Chapter 6. For each task, I first present a short narrative of the lessons together with reflections recorded by the teacher and myself as a preliminary overview of the task implementation, in order to set the stage for a more detailed description and analysis of episodes in the lessons. I then address issues related to research question 1, what characterises the enacted norms in the classroom when students work with vocationally connected mathematics tasks and research question 2, what connections do students make with workplace practices and out-of-school knowledge when engaged with vocationally connected tasks. I illustrate the analysis with example episodes, excerpts and explanations, chosen from each task implementation. In Chapter 8, I present an analysis of the commonalities and differences between the cases.

7.1 The Hair Salon Budget Task

The students worked on the task during three 45-minute lessons over two days (one lesson on Day One and two lessons on Day Two). The teacher and I decided together that I should present the task, since I had had the main responsibility for preparing it, and I knew the planned implementation. The lesson began by discussing hair cut prices in hair salons with the students. I presented the two different price examples (see Figure 6.2) in a PowerPoint presentation and asked the students to try to explain how prices were different and possible reasons for the price difference. The students discussed this in groups of three or four, followed by a short review on the blackboard of possible income and expenses suggested by the students. I displayed the budget template on the blackboard and presented a few examples of possible calculations (see Figure 7.1).

Income	**Kroner**	**Explanation/calculation**
Lady's haircut	$450 \cdot 15 \cdot 20 =$	450 kroner for haircut, 15 haircuts one day, and 20 working days.

Sum income		
Expenses	**Kroner**	**Explanation/calculation**
Rent		

Figure 7.1: Part of the budget template presented on the blackboard with calculation for ladies' haircuts. The example noted is 450 kroner times 15 cuts/day times 20 days/month.

The students started to fill out and calculate the budget for their hair salon for the remaining time of the first lesson. They were given sample prices on rental fees, telephone fees, renovation fees and other relevant information (see Appendix 11.3.1 for a list of the information given to the students) and worked on calculating the numbers for their specific circumstances. On the second day (lessons two and three), the students continued to work in the same groups. The lesson started with a short reminder about the task, and then the students continued to figure out their budget. Some of the groups, depending on their overall progress, started to work on their budget in Excel. During the lessons the students' patterns of interaction varied between the students from eager discussions, through off-task behaviour such as singing and dancing, to reluctant participation in whole class discourse.

I mainly observed the group consisting of Emma, Linn, and Sara. During the lessons this group interacted with their neighbouring group made up of Bente, Gunn, and Mari. These two groups were chosen because of their placement at the back of the classroom. Both groups asked the teacher Alexander, and me for help at various points during the lessons.

7.1.1 Enacted norms in the Classroom

Analysis of the data highlighted particular social and sociomathematical norms regarding who was perceived as having relevant knowledge in relation to the task, and what the purpose of the task was. Overall, the students approached the task as a mathematics classroom task in which the teacher was the authority, although they recognised their own authority over certain aspects of knowledge related to the practice of hair dressing. For example, the students asked the teacher and/or me for confirmation or views on what would be the right calculations but consulted with the other students to check answers. Another issue to notice is that the students did not check *how* other groups solved their questions, but only what their final answers were. When the students discussed the issues closely connected to the hair salon, they did not ask the teacher, but knowledgeable other students about hair salon practices.

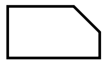
At the start of the lesson, the group consisting of Emma, Linn, and Sara started to discuss the different expenses involved in running a hair salon. They realized that they needed to rent a location but were unsure of possible rental fees. The group figured out that they could estimate the rental fee by using the classroom as a way of measuring what would be an appropriate size for their hair salon. They asked the teacher, Alexander, for help in figuring out the size of the classroom, agreeing with his suggested values at once, without discussing other possible estimates. The girls agreed that the length of the walls would be approximately 7 metres and 10 metres (Turn 1, Table 7.1), and they started calculating after some confusion about the difference of circumference and area.

Here one can notice that when the teacher gave a suggestion (Turn 13, Table 7.1), Emma and the rest of the group listened to that suggestion and did not discuss further. However, when Emma asked how to calculate the area, both Sara and Linn suggested the right number and formula (Turns 4, 5 and 6, Table 7.1), but their ideas were not taken up by Emma. This pattern suggests that the teacher alone was regarded as the authority in the mathematics domain.

Table 7.1: The teacher's mathematical suggestions are the valued suggestions.

Turn	English	Norwegian	Comments
1	Emma: 10 10 7 7 *(pointing around the room on the walls).*	Emma: 10 10 7 7 *(peker rundt i rommet på veggene).*	Emma uses the formula that would calculate the perimeter of a room seven times ten metres. Alexander corrects her and asked her to
2	Alexander: That would be the perimeter; you should have area.	Alexander: Det hadde vært omkretsen, areal skal du ha.	

			calculate the area instead.
3	Emma: We should find square metres.	Emma: Vi skal finne kvadratmeter.	Emma realises that area needs to be measured in square metres.
4	Sara: It is 70, 70 *(gesturing with her hands in the air).*	Sara: Det blir 70, 70 *(virrer med hånda i lufta).*	Sara states the correct answer for the first time.
5	Linn: Area is side times side.	Linn: Areal er jo side gange side.	Linn states the correct formula for finding the area of a rectangular area.
6	Sara: That is area.	Sara: Som er arealting.	Linn’s statement in Turn 5 is confirmed by Sara.
7	Alexander: Mm.	Alexander: Mm.	
8-11	(Sara and Linn continue to talk about area and try to illustrate it by gesturing about a surface area.)		
12	Emma: But we should concentrate; we must find square metres. How do we find square metres?	Emma: Men så skal vi, kons, vi må finne kvadratmeter. Hvordan finner vi kvadratmeter?	Emma does not acknowledge Linn’s correct formula in Turn 5 and Sara’s correct solution in Turn 4, instead asking for help from the teacher, who repeats the formula.
13	Alexander: If you should find square metres of a square, you have to take side times length.	Alexander: Viss du skal finne kvadratmeter av et kvadrat så skal du gange side med lengde.	
14	Emma: Okay, so you have ten times seven, so that will be seventy.	Emma: Okei, viss du har ti gange syv, så blir det søtti.	Emma then calculates the area from the formula.
15	Alexander: Yes.	Alexander: Ja.	
16	Emma: Is it seventy square metres in this room? *(Waving her hand in a circle and looking at Alexander).*	Emma: Er det søtti kvadratmeter dette rommet? *(Vifter med hånda i en sirkel og ser på Alexander).*	Emma asks for confirmation of the calculations from the teacher, and

17	Alexander: I think it's less than that, maybe fifty since it is not a [rectangle].	Alexander: Jeg tror det er mindre enn det, kanskje femti siden det ikke er en [rektangel].	Alexander gives a new estimation, which includes the fact that the room was not completely rectangular, but more pentagon shaped, shown here: 
18	Emma: We will note 50 then *(leaning down toward the paper),* 50 times 1200.	Emma: Da skriver vi 50 da*, (lener seg ned mot arket),* 50 gange 1200.	Emma (and the rest of the group) immediately accepts Alexander's statement as the proper number to use in their calculations.

As seen in Turns 4 and 5 above, Sara and Linn state the right formula and answer immediately. But Emma seemed to listen only to Alexander's suggestions (Turns 13, 15 and 17), even though his suggestions were identical with Linn and Sara's ideas. Likewise, when the teacher made a new suggestion about the room size in Turn 17, this number was immediately taken to be a fact, and used unquestioningly in their further calculations. I interpret the episode as indicative of a sociomathematical norm in this classroom that the teacher is the authority in the mathematics class.

Another example of the teacher being perceived as the authority in the mathematics domain was when the students wanted to calculate the value added tax, and Emma asked me for help. Even though it seemed that Emma, Linn, and Sara did not understand my explanation, the group accepted the method I suggested and the answer without question[35]. When some minutes later the neighbouring group (Bente, Julie, and Mari) asked Emma how they should find out value added tax they were immediately referred to me with the statement,

[35] The students had a price of haircuts and wanted to figure out how much value added tax would be. If the price for the customer is for instance 400 kr, their suggestion was that one should calculate 25 % of 400 kr, namely 100 kr in tax. However; the correct calculation of value added tax would have been price without tax multiplied by 1.25 equals 400 kr, and therefore a value added tax of 400 divided by 1.25 equals 320 kroner, which gives 80 kr in tax.

“You don’t do it by multiplying [the number of transactions by 25 %]; she can explain,” and pointed at me. In another episode, the students referred to the teacher to give the correct answer when they wondered if the correct unit term was square metres.

In contrast, when the students discussed items that were closely connected to their real-world knowledge, or the hair salon context, they positioned other students as knowers of the workplace practice. This was evident when the students discussed how many days a week one should work, and which salary they should use in their calculations. In the following example, the group consisting of Emma, Linn, and Sara started discussing hairdressers’ salaries. The girls did not know what a likely salary would be and discussed how it might be calculated by the hour or according to commission (Turns 1, 2, 3, Table 7.2). The following example shows that the students draw on the collective knowledge of the group, and valued the different input and considerations brought up by the group members.

Table 7.2: Discussion of how hairdressers’ salaries are calculated.

Turn	English	Norwegian	Comments
1	Linn: Do you get paid by the hour, or?	Linn: Får man sånn timebetalt, eller sånn?	The students discuss how salaries are calculated.
2	Emma: Hourly based, everyone gets paid by the hour.	Emma: Timebetalt, alle får jo timebetalt.	Emma first argues that all salaries are based on number of working hours, and Linn refers to commission-based work.
3	Linn: But someone can get paid by how many clients you have in a day.	Linn: Men noen kan også få sånn hvor mange kunder du tar i løpet av dagen.	
4	Sara: Yes, that had (unclear).	Sara: Ja, det hadde (utydelig).	Sara agrees with Linn’s suggestion.
5	Emma: Yes, for one that takes ten customers one day, and then the rest get just as much.	Emma: Ja, for det er en som tar ti kunder en dag, og så får de like mye.	Emma refers to the injustice in paying people equally if they do not bring in equal revenue.
6	Linn: Mm.	Linn: Mm.	
7	Emma: But then they don’t work by the hour?	Emma: Men da jobber de ikke med timer	Emma seems to accept what the

	(Unsure of what she says here).	heller? (Litt usikker på hva hun sier her).	others say, however she wants to find what the alternative way of calculating salaries is.
8	Sara: But I've heard that the bosses, get many as customers, such that *(pointing with pencil in the sheet).*	Sara: Men jeg har hørt atte, de som er sjefen ikke sant, de får så mye så som kunder, sånn som *(peker med blyanten i arket).*	Sara draws on information she has gathered from the hair salon practice. Sara's meaning is unclear here.

Here, Emma changed her original idea of an hourly-based salary when Linn and Sara told her about the option of commission-based work. Emma seemed to accept the others' contribution but wanted to check what the alternative way of calculating salaries was. The students discussed what they had heard and knew (Turns 2, 3 and 8, Table 7.2), and pieced together a common understanding on how salaries of hairdressers could work.

The students drew on their experiences of hair salons to reflect on their numbers and consequently revise their considerations. In the following episode, the group have started working on the budget in Excel and calculated the number of hair colourings they would do in a month.

Table 7.3: The group revises their previous estimates.

Turn	English	Norwegian	Comments
1	Emma: Then we colour 240 people's hair each month. We can't colour 240 people's hair a month.	Emma: Da er det 240 vi farger, i måneden. Kan faen meg ikke farge 240 stykk i måneden.	Emma strongly expresses her disagreement with the group's earlier assumption.
2	(Linn shakes her head).		The whole group seems convinced that the numbers they had previously used are not realistic.
3	Sara: No, I think we're going to colour like 100 persons, no.	Sara: Nei, æ tror vi kommer til å farge sånn 100 stykk, nei.	
4	Emma: No, not even that, maybe like 50.	Emma: Nei, ikke det engang, kanskje sånn 50.	The group start to discuss new numbers of hair colouring

5	Sara: 50, yes.	Sara: 50, ja.	customers, and drastically change the number from 240 persons to 50 or 70 persons.
6	Emma: 70, something Then it's wrong, right *(referring to their previously used numbers).* (…)	Emma: 70, ett eller annet. Da blir jo det feil, ikke sant *(henviser til tidligere brukt tall).* (…)	
7	Emma: Yes, okay, how many should we say we colour then? 70 or 50?	Emma: Ja, okei, hvor mange skal vi si vi farger da? 70 eller 50?	Emma tries to get the group to reach an agreement.

Emma strongly expressed her disagreement with the group's earlier assumption (Turn 1, Table 7.3), and the whole group seemed convinced that the numbers they have previously used were not realistic. The group started to discuss new numbers of hair colouring customers, and Emma tried to get the group to reach an agreement. The group changed their estimated number of hair colouring customers drastically, but the changes seemed to be made on the basis of tacit knowledge, rather than on the basis of any arguments for deciding on the new estimated number of customers. It can seem like the students are adjusting the numbers by feel of the numbers, not by doing a mathematically driven estimate or nor drawing on a practical estimate with basis in hair salon routines.

Another related issue was that the students wanted immediate answers. Like in the previous episode, the students did not use mathematical knowledge, or knowledge from out of the school setting for figuring out how much to pay for electricity. For instance, Mari was impatient to figure out which numbers they should use in the budget, and did not seek explanations, only answers. In the following episode, she wants to know how much one pays for electricity (Turns 1 and 4, Table 7.4).

Table 7.4: Mari wants immediate answers to fill in the budget.

Turn	English	Norwegian	Comments
1	Mari: Electricity, how much does it cost per month?	Mari: Strøm, hvor mye koster det i måneden?	Mari wants to note how much one pays for electricity.
2	Gunn: What has she written?	Gunn: Hva har hun skrevet?	The students look to a neighbouring group to find what
3	Student: 6000 or something.	Elev: 6000 eller noe.	

			that group had estimated.
4	Mari: Hello, does anyone know how much it costs for electricity per month? Electricity per month?	Mari: Hallo, er det noen som vet hvor mye det koster for strøm i måneden? Strøm i måneden?	Mari here seems impatient to get an answer. She does not ask for how or why, and only asks for the final figure.
5	Linn: 5000.	Linn: 5000.	

Mari reacted similarly when she started on working with value added tax, and impatiently demanded to know how to calculate it. She did not ask for explanations, but only wanted the instrumental method or a direct answer. These episodes are examples of students wanting uncomplicated answers to their questions and being willing to ignore their real-world experience in order to simplify the group's work. However, some students, like Bente (Turn 1, Table 7.6), argued for more realistic budgets. It seems that the introduction of a vocational context into the practice of mathematics in school settings leads to, for some of the students, shifts where they need to reposition themselves and figure out which norms to value.

For instance, the issue of the potential realism of numbers and fees and how to decide what were "proper" numbers, were one situation where the students needed to position themselves with respect to which practices to value. The students were especially in doubt about the rental fees, which were given as 1200 Norwegian kroner per square metre36. They had no experience with rental prices and therefore were unable to build on familiarity with the context in order to decide on the amount. They seemed surprised by the numbers that they then arrived at (Turns 1 and 3, Table 7.5), and I interpret this as an indication that they were thinking about the numbers with regards to their real value rather than as imaginary costs in a mathematics task. To the students, 60 000 kroner seemed excessive because it is equivalent to approximately three months' salary at the start of their apprenticeship.

[36] This rental fee was given to the students as information in the task. This is around double the proper fee, which has been identified as about 600 kroner per square metre in high pressure areas. For example, a shop with 87 square metres on a main street would cost about 50 000 in rent per month. http://www.fvn.no/okonomi/Kafkonkurs-i-Markens-gate-2757749.html

Table 7.5: Students' concern about rental prices.

Turn	English	Norwegian	Comments
1	Mari: You cannot pay 42 000 in month in rent.	Mari: Man kan ikke betale 42 000 i måned, i husleie.	The group had calculated 35 square metres times 1200 kroner per square metre to get 42 000 kroner.
2	Trude: You can.	Trude: Du kan jo det.	Here I think I want to persuade the students to accept the rental fee.
3	Emma: We have 60 000 a month.	Emma: Vi har 60 000 i måneden.	Emma is in the neighbouring group to Mari and points out that their group's suggested hair salon needed to pay even more in rent.
4-9	(In these interactions I tried to convince the students that the rental fee was plausible.)		
10	Mari: Not 42 000 kroner.	Mari: Faen meg ikke 42 000 kroner.	Mari was not convinced.

Even though Mari was quite certain that the calculated price for the rent of the salon was wrong (Turn 10, Table 7.5), the group did not change the numbers, instead continuing to the next expense item. Their willingness to ignore the problematic rental fee might have been encouraged by my efforts to convince them. Here the students acted regarding to norms and routines of a mathematics classroom, where it is normal to ignore numbers that do not make sense, to be able to solve the tasks.

Later, Mari was again willing to ignore her real-world experience in order to solve the questions and simplify the group's work, despite a long discussion about hours and realism. The group discussed how many hours they would need to work a day, in order to figure out their hourly rate of earnings. Mari informed the other students that they needed to start working one hour before the hair salon opened, to get the salon ready for the day. After a short discussion about whether this hour should be paid, Bente said that a usual working day is eight hours. Mari then referred to her experience from her after-school job and said that the salon she worked for had opening hours from 10:00 to 21:00. Bente acknowledged this but argued that someone would not work the whole time that the salon was open. Mari suggested working nine hours a day, while Emma from the neighbouring group said that they would work seven hours; finally the group decided on using eight hour working days. Bente then said “Okay, just use eight then, I think it’s horrible to work eight hours”, suggesting that she actually imagined herself working in the hair salon.

Emma then asked if they worked six days a week, Mari replying that they worked seven, and Emma that they worked only five days. Bente (who was in Mari’s group) then reacted to the seven-day working schedule (Turn 1, Table 7.6). In the following excerpt, it seems that there is a difference between how Bente and Mari looked at the task and the need for realistic considerations. Mari realised that using 30 working days a month in a budget was not correct (Turn 1, Table 7.6). But she argued that since the hair salon was imaginary, and therefore that the need for realism was less important, so then they could avoid the additional work required to recalculate their budget (Turn 9, Table 7.6).

Table 7.6: Students’ different expectations of the importance of the realism of the budget.

Turn	English	Norwegian	Comments
1	Bente: We cannot be open on Sundays.	Bente: Vi kan jo ikke ha åpent på søndag.	Bente points to realistic constraints of a hair salon.
2	Mari: Yes, but we have calculated all this for 30 days a month. *(Pointing at the sheet).* So then we need to be open on [Sundays].	Mari: Jo, men vi har jo regnt alt det her på tretti dager i måneden. *(Peker på arket).* Da må vi jo ha åpent på [søndager].	Mari argues because of their previous decision to calculate with 30 days a month, they would need to be open on Sundays.
3	Emma: Then you get more than us anyway, because you work more.	Emma: Da får jo dere mer enn oss uansett, for dere jobber jo mer.	Emma and Mari take the opportunity to explain the discrepancy between the groups’ budgets on the basis of their different assumptions.
4	Mari: Therefore, we earn a million a month *(points with the pen to convince Bente).*	Mari: Derfor tjener vi en million i måneden *(dunker med pennen for å overbevise Bente).*	

5	Mari: Oh (indistinct). Now we pretend like this, and so we leave it (...)	Mari: Åh (utydelig). Nå later vi som dette, og så bruker vi han (…)	Mari points out that they would need to do the calculations again if they were to use more realistic considerations of either 20 or 25 working days a month. She points out to her fellow group members that the hair salon was just imaginary.
6-8	(Some indistinct comments. The group read numbers out loud.)		
9	Mari: And then times 30 this month, Bente, now are we finally finished, then we have to do it all again [if you want to change]. We will not have this job for real, so it will be fine!	Mari: Og så gange tretti denne måneden, Bente, nå har vi endelig ferdig, må gjøre alt på nytt [om du vil forandre]. Vi skal ikke ha denne jobben på ekte, så det går fint!	

Here, Bente seemed to regard the pressure of working on Sundays as real (Turn 1, Table 7.6), and connected to her future work situation. But Mari argued that the group would not really do this job, and that to make changes in assumptions that were already in the budget was not worthwhile (Turn 9, Table 7.6). It would mean that the group would need to do all the work again, but the hair salon was not real (Turn 9). So even though Mari had good knowledge of the working of a hair salon, and earlier had used it to argue for realistic assumptions, she disregarded these considerations in order to finish the budget task. Here, there is a difference in opinion between Bente's and Mari's treatment of the vocational context presented in the task, where Mari invokes a standard mathematics classroom norm of simplifying the task as much as possible even though it bears little resemblance to real-world events.

The prioritisation of traditional mathematics classroom norms was again apparent when I tried to have a short review of the work the students had done on the blackboard at the start of the second lesson, where the students barely contributed, offering single word answers only. The discourse fell into a classical IRE pattern as presented in the episode below (Table 7.7). After asking for suggestions on possible income sources, I asked for possible expenses. In the excerpt presented below, one can notice that the students barely contributed, and that I completely controlled the discourse (Turns 1-2, 4-5, 9 and 11, Table 7.7).

Table 7.7: An example of a classroom conversation with mostly teacher-led questions and few student contributions.

Turn	English	Norwegian	Comments
1-2	Trude: And then we have to think about what are expenses? Okay, we have expenses, it is rent *(pointing to the blackboard)*, and how much is it (...) Do we have any more expenses we think we need?	Trude: Og så må vi tenke på, hva i alle verden er utgifter? Okei vi har utgifter, det er husleie *(peker på tavla)*, og hvor mye koster det (…) Har vi flere utgifter vi tror vi trenger?	I engage in a long monologue where I repeated the question of which expenses the hair salon would have.
3	Student: Electricity.	Elev: Strøm.	Two suggestions from students are noted on the blackboard. I seem in charge of the dialogue.
4-5	Trude: Electricity. More expenses? *(Silence).*	Trude: Strøm. Flere utgifter? *(Stillhet).*	
6	Student: Water.	Elev: Vann.	
7	Trude: Water.	Trude: Vann.	
8	Student: And if you have like a coffee machine.	Elev: Og viss du har sånn derre kaffemaskin.	A new expense is suggested by a student.
9	Trude: I heard expenses of products here, but I think that no one else heard that. What were you saying?	Trude: Jeg hørte innkjøp av produkter her, men det tror jeg ingen andre hørte. Hva var det du sa?	I repeat another students' suggestion.
10	Student: If you have like a coffee machine.	Elev: Viss du har sånn derre kaffemaskin.	The students suggest a coffee machine again.
11	Trude: Coffee, so it's an expense, of course *(note onto the blackboard).* Yes, so we fill the income and the expenses we think we have.	Trude: Kaffemaskin, så det er jo en utgift, selvfølgelig *(noterer inn på tavla).* Yes, så vi fyller nedover sånn, de inntektene og de utgiftene vi tror vi har.	I then end the dialogue with a suggestion on how to proceed with making the budget.

As mentioned, the students barely contributed, and I took control over the discourse. I also chose which contributions to acknowledge in the classroom. This is quite different from when the students compared their own assumptions and estimates in their group discussion. The patterns of interactions are closely connected to traditional mathematics classrooms routines. In this setting, with a teacher (me), at the front of the classroom, and a teacher-lead discussion of the task the students did not show their vocational knowledge, like they did in the small group discussions of the vocational practice.

7.1.2 Connections with Out-of-School Practices

As seen in the previous section the students used knowledge of hair salons in the discussions of hair salon practices but abandoned such considerations when confronted with the mathematics task. In the work with the mathematical parts of the task, the students acted according to standard classroom norms. In this section I will look closer at connections students made with the hair salon practice and their out-of-school knowledge.

The following episodes illustrate how the students connected the hair salon budget task with experiences they had from outside of the mathematics classroom, using their knowledge of the hair salon context to reason in their work with the questions. The episodes highlight contributions by Emma, Linn, and Sara when they discussed the hair salon budget. In the following, the students discussed the start questions in the task: “What is an ordinary price for a lady’s haircut” and “what is the highest price you could have paid for a lady’s haircut”. During their discussions, they referred to the hair salon practice and issues they need to figure out.

Table 7.8: What to consider when thinking about reasonable haircut prices.

<table>
<tr><th>Turn</th><th>English</th><th>Norwegian</th><th>Comments</th></tr>
<tr><td>1</td><td>Emma: Then we need to think about long hair and short and stuff.</td><td>Emma: Da må vi tenke på langt hår, og kort og sånt.</td><td rowspan="2">Emma refers to important issues to consider while deciding prices, namely that different kinds of haircuts need different amounts of time to be completed.</td></tr>
<tr><td>2-3</td><td colspan="2">(Irrelevant comments from the neighbouring group).</td></tr>
</table>

4	Linn: That will be more expensive.	Linn: Det blir dyrere.	Linn and Emma discuss how it would be more expensive to cut long hair (because you use more time on it).
5	Emma: Yes, but I think if we take a regular price, and then suddenly go up to 500? [Indistinct].	Emma: Ja, men jeg tenker hvis vi tar det er vanlig pris, og så plutselig gå opp til 500? [Utydelig].	
6	Linn: The price you could have paid. Imagine if it had been Jan Thomas[37].	Linn: Pris du kunne betalt. Tenk om det hadde vært Jan Thomas.	Linn refers to a famous hairdresser, suggesting that not all haircuts will cost the same, depending on the importance of the hairdresser.
7-14	(The girls discuss if one would be willing to pay higher prices for the 'same' haircut).		
15	Emma: More. He is better. Maybe not 1500, but at least over 1000.	Emma: Mer. Han er jo flinkere da. Kanskje ikke 1500, men iallfall over 1000.	Emma agrees that haircuts could have various values. Customers can pay more to get a perceived better haircut.

Emma pointed out that deciding on an ordinary price of a lady's haircut is not trivial (Turn 1, Table 7.8). In the hair salon practice, one needs to consider the cases of short and long hair, and time used on different haircuts. Linn seemed to agree with Emma in Turn 4, when she argued that some cuts would be more expensive. In addition, they discussed possible price differences with reference to the assumption that there are quality differences between hairdressers (Turns 6 and 15, Table 7.8).

Later, when the group discussed what they needed to know to decide their prices, they referred to specific brands of hair products, which are familiar from hair salon practice. The students also mimicked how hairdressers work to sell products. In the next episode (Table 7.9), the students' use of brand names and their mimicking of how hairdressers talk about such products indicates that they

[37] Jan Thomas is a famous Norwegian hairdresser.

connected the task to their experiences as customers, and for some, to their experience of part-time work in a hair salon.

Table 7.9: Students' use of language and interaction patterns from the practice.

Turn	English	Norwegian	Comments
1	Emma: We need to know income, no, yes, we need to buy shampoos *(gesturing in the air)* and stuff.	Emma: Vi må vite inntekt, nei, jo, vi må jo kjøpe inn sånn derre shampoer og *(gestikulerer i lufta)* og sånt.	The students are discussing how they need to stock products which will be used in the hair salon.
2-5	(Students talk about unrelated issues.)		
6	Emma: Like Redken products. You get lots of stuff; you have to know prices when you're deciding the main prices. You have to bring in money to earn it back.	Emma: Redken-produkter, liksom. Du får jo alt mulig, du må jo tenke på prisane på det når du skal bestemme prisane. Du må jo få inn penger for å tjene det igjen.	Emma refers to a specific brand of shampoo which is only sold in hair salons. She implies that one needs to know prices of products before deciding on their own prices.
7	Linn: But it's extra that we need to sell. Like if you say, oh, you have dry hair, you need [a hair cure or another product].	Linn: Men det er jo sånn ekstra vi må selge. Sånn åh du har tørt hår, du trenger [en hårkur eller annet produkt].	Linn gesticulates with her hands to illustrate touching the customer's hair. Linn illustrates her perception of what hairdressers do to (increase) sales.

Both Emma's use of the brand name "Redken" and Linn's imitation of a hairdresser's actions (Turn 7, Table 7.9) indicate that the group seemed to interpret the task through their experiences of hair salons. Also, in the next lesson, the number of sales of hair products was again discussed by the same group. At the start of the discussion Emma (Turn 1, Table 7.10) used the same argument as Linn in Turn 7 (Table 7.9). They both implied that hairdressers make sales pitches toward their customers. Now Emma suggested that they estimate the number of sales in relation to the number of customers (Turns 2 and 3, Table 7.10), suggesting that half of the customers would buy two products. The episode below presents how the students used their knowledge of the hair salon practice to argue for their assumptions, which are then used in the budget.

Table 7.10: Students' use of knowledge from practice to argue for their assumptions.

Turn	**English**	**Norwegian**	**Comments**
1	Emma: Yes, but think of a lady who colours her hair. And then you say, that yes, you have brittle hair; you should buy a [product] for newly dyed hair, and then it will be two products. It's not often you buy one product.	Emma: Ja, men, en dame, da, som skal farge håret. Og så sier du da at ja, du har litt slitt, du bør kjøpe en for nyfarga hår, så to produkter. Det er ikke ofte du kjøper ett produkt.	Emma emulates a hairdresser and how they talk when selling hair products to customers. She argues that one does not buy only one hair product at a time.
2-3	Emma: We have to, somehow, think about it in relation to [the number of customers]. Here there are ten (pause). Linn: Yes.	Emma: Vi må liksom, vi må tenke på, i forhold til det [antall kunder]. For der er det ti (tenkepause). Linn: Ja.	Emma then points out that they need to use their customer estimation to figure out a reasonable number.
4-5	Emma: If each of them buys two products each (two seconds of silence). Or if we take one, there are some who buy two, and some who do not buy anything, then it becomes….	Emma: Viss hver av de kjøper to produkter hver, (to sekunders stillhet) ikke sant. Eller viss vi tar en, det noen som kjøper to, og noen som ikke kjøper noe liksom, da er det blir det.	Emma reasons with the knowledge that not all would buy hair products and uses this to establish the estimation.
6	Linn: Each day?	Linn: Om dagen?	
7	Emma: Yes.	Emma: Ja.	
8	Linn: There are not many men who buy such things	Linn: Det er ikke så mange menn som kjøper sånne ting.	Linn uses knowledge about men's buying habits to suggest

			adjustments to the group’s estimate.
9-10	Emma: No, there are five more (unsure what they say). Ooh, yes, it will be five a day.	Emma: Nei, da er det fem til (usikker på hva de sier). Ææh, ja, fem om dagen, det blir.	When the group agree on a number that seemed reasonable, they used this mathematically.
11	Emma: Five a day each? Five times five? Hmm, that is fifteen.	Emma: Fem om dagen hver? Fem gange fem? Hmm, det blir femten.	Emma calculates 3 hairdressers times the half of 10 customers and gets 15 sales a day. Why she says five times five is not clear.

Here, the students drew on their real-world experiences in order to estimate the numbers needed to work with the budget. Emma figured out that there needed to be a reasonable connection between the number of customers, and the possible sale of hair products. In addition, Linn used knowledge about men’s buying habits to suggest adjustments to the group’s estimate (Turn 8, Table 7.10). There were also other instances during the work when the students referred to a particular salon in the city they all seemed to be familiar with, and they talked about why this hair salon was more expensive. Linn emphasised that this hair salon was expensive for all treatments, however she justified this by bringing up the perceived quality of the hair salon. The students repeatedly brought in their real-world experiences from hair salons or daily life. They seemed to make realistic decisions based on their experiences, although this realism didn’t extend as far as it might have done – for example, they could have discussed the manpower needs of the number of different services they wanted to offer.

The group situated next to Emma, Linn, and Sara in the classroom also drew on their out-of-school experiences. During the lesson, I learned that Mari worked after school as a helper in a hair salon. In the episode presented below (Table 7.11), she used her experience to convince the group on an appropriate figure when the group discussed how many hair products (shampoo, conditioner, and so on) they would sell each day (Turn 2, Table 7.11).

Table 7.11: Mari argues with basis in her own experience as a helper in a hair salon.

<table>
<tr><th>Turn</th><th>English</th><th>Norwegian</th><th>Comments</th></tr>
<tr><td>1</td><td>Julie: Because it’s not everyone who buys something.</td><td>Julie: For det er jo ikke alle som kjøper.</td><td>Julie points out that not all customers buy hair products.</td></tr>
<tr><td>2</td><td>Mari: When I work at the hair salon, I stand at the till all the time. People [come by] all the time who buy products themselves, and they tend to buy 2 - 3 each.</td><td>Mari: Sånn som jeg jobber jo på frisørsalongen, jeg står kun i kassa hele tida. [Det kommer] folk hele tida som kjøper produkter sjøl, og de pleier å kjøpe 2-3 hver, liksom.</td><td>Mari counters this argument with a fact from her workplace experience, namely that hair salons have walk in customers that only buy products.</td></tr>
<tr><td>3</td><td>Gunn: Seven products each, every day?</td><td>Gunn: Syv produkter hver, hver dag?</td><td rowspan="2">This information leads to a decision about appropriate numbers of sales. Why they use seven is not clear from their statements.</td></tr>
<tr><td></td><td colspan="2">(Mari then used her calculator to find the answers, and the girls note their answers.)</td></tr>
</table>

The students built on Mari’s experience from her after-school job at the hair salon to estimate a realistic number of sales of hair products. Mari, with her experience from a hair salon, had opportunities to certify the authenticity of aspects of the task. Here she indirectly certificated that the issue of sales towards customers is authentic, and she told the group about her experiences.

Even though all the students had a passing familiarity with hair salons, they had many questions about them. They wondered about the salary of a hairdresser, whether it was fixed, or varied according to the number of customers. In addition, the students were surprised about some of the suggested expenses, like fees for garbage collecting, electricity, and water. They voluntarily compared and contrasted their budget estimations with each other and were especially interested in comparing the profits (Turns 1 and 5, Table 7.12). In the following episode Emma listened to the neighbouring group discussing their answers, and then joined the conversation and offered her group’s results in Turn 5.

Table 7.12: The groups compare their results.

<table>
<tr><th>Turn</th><th>English</th><th>Norwegian</th><th>Comments</th></tr>
<tr><td>1</td><td>Mari: We are only four hairdressers, and we earn nearly one million a month.</td><td>Mari: Vi er bare fire frisører, og så tjener vi nesten en million i måneden.</td><td>Mari tells how much money they earn.</td></tr>
<tr><td>2-4</td><td colspan="2">(The group asked me if it would be unlikely to earn this much in a month).</td><td rowspan="2">Emma compares Mari’s estimate with the estimate of her group.</td></tr>
<tr><td>5</td><td>Emma: We have 114 000.</td><td>Emma: Vi har 114 000.</td></tr>
<tr><td>6-7</td><td colspan="3">(I asked the students to evaluate why the difference was so marked).</td></tr>
<tr><td>8</td><td>Mari: Oh my god. We have a million a month.</td><td>Mari: Å herregud. Vi har en million i måneden.</td><td>Mari stretches to look at Emma’s notes.</td></tr>
<tr><td>9</td><td>Emma: Huh? (Sharp voice).</td><td>Emma: Hæ? (Skarp stemme).</td><td rowspan="2">The girls sound surprised and disbelieving.</td></tr>
<tr><td>10</td><td>Mari: Yes. (Laughter in her voice).</td><td>Mari: Ja. (Latter i stemmen).</td></tr>
<tr><td>11</td><td>Emma: But, no, this is one week.</td><td>Emma: Men, nei, dette er en uke.</td><td rowspan="3">The argument that it was the difference between a week and a month was accepted without further reflection. The group then continued arguing that employees’ salaries will reduce the profits.</td></tr>
<tr><td>12-14</td><td colspan="2">(The group made confirming comments about the budget being for one week).</td></tr>
<tr><td>15</td><td>Julie: I think actually it is quite (pointing at the sheet) right when you think we should pay the workers and expenses. Then probably that sum is quite right.</td><td>Julie: Æ tror faktisk det er ganske (peker på arket) riktig når du tenker skal ha lønn til antall ansatte og utgiftene. Da er nok den summen ganske riktig.</td></tr>
</table>

Emma compared Mari’s estimate with the estimate of her group (Turn 5, Table 7.12). The girls sounded surprised and disbelieving of Mari’s budget (Turns 9 and 10, Table 7.12), which had an income of over one million krone a month. The girls then discovered that one group had calculated weekly income, and the other monthly income (Turns 8 and 11, Table 7.12). The argument that the difference in the amount of income was because of the difference between a week and a month was accepted without further reflection. The group then continued arguing that salary for employees will reduce the profits.

Similar comparisons between groups are evident in other episodes, and it seemed like the students valued designing a hair salon that had realistic amounts of income. This can be seen in the way the students certified the question aspect of the task, and they seemed to regard the budget as something which would really be done in the vocational practice. In the next excerpt, the two groups have started to reflect over their calculated monthly profit. Mari argued that the neighbouring group had too much profit (Turn 1, Table 7.13), but the girls went on to discuss enthusiastically about implications and which hair salon would be best (Turns 2 and 3, Table 7.13).

Table 7.13: Another comparison between the groups.

<table>
<tr><th>Turn</th><th>English</th><th>Norwegian</th><th>Comments</th></tr>
<tr><td>1</td><td>Mari: It’s way too much.</td><td>Mari: Det er alt for mye.</td><td>Mari comments on the realism.</td></tr>
<tr><td>2</td><td>Emma: I get more money than you, much bigger house than you.</td><td>Emma:Jeg får mer penger enn deg, mye større hus enn deg.</td><td>Emma ignores Mari’s critical comment and focuses on the implications of their budget.</td></tr>
<tr><td>3</td><td>Mari: You get a much harder job than me, you need to own it all by yourself.</td><td>Mari: Du får mye vanskeligere jobb enn meg, du må eie alt aleine.</td><td>Mari comments with regards to real concerns (work pressure).</td></tr>
<tr><td>4-8</td><td colspan="3">(The students were asked to work with their budget in Excel but continue to discuss their earnings.)</td></tr>
<tr><td>9</td><td>Mari: We earn a million a month. (Eeeh! Pointing to Emma, seeming to say, “We beat you”.)</td><td>Mari: Vi tjener en million i måneden. (Eeeh! Peker på Emma, som i “vi slår dere”).</td><td rowspan="3">Here, the difference between week and month is again evident, but the students did not compare their actual results adjusted for time span.</td></tr>
<tr><td>10</td><td>Emma: But that is weekly.</td><td>Emma: Men det er i uka.</td></tr>
<tr><td>11-12</td><td colspan="2">(Mari said that they had not calculated weekly.)</td></tr>
<tr><td>13</td><td>Mari: They do not even earn half of what we do.</td><td>Mari: De tjener ikke halvparten av oss engang.</td><td>Here, the difference between income and profit are mentioned,</td></tr>
</table>

14	Emma: But still, we have more profit.	Emma: Men fremdeles, vi har mer overskudd.	but the students do not explore this further.

During the students’ discussion, Emma ignored Mari’s critical comment, and focused on the implications of their budget (Turn 2). Mari then commented regarding real concerns such as work pressure (Turn 3), and thereby certified that this was regarded as authentic in the moment. When the students compared the budget with each other, they seemed very eager, and expressed perplexity with the numbers. As noted before, the students themselves certified that making a budget is something which is done in the vocation. They discussed their profits and compared them and discussed their own future working conditions and what salaries are based on. However, they also dismissed this when Mari (see Table 7.6) pointed out that they would not really work in this hair salon.

The objects referred to in the task, the receipt and the hair salon pricelist, was not discussed by the students, however, they referred themselves to a famous hairdresser and different brands. This is an indication of certification of the language used in the task, and by the students themselves. The students needed to find some information themselves and this could have been an authentic aspect in the task. But the students did not use out-of-school resources to find the information they needed but kept their discussions to asking each other and the teacher.

It seems that the hair salon budget was interesting for the students and they drew heavily on real world knowledge. As presented earlier, the students discussed their calculated numbers, and seemed eager to compare their budgets. The students discussed questions connected to the real world context of the hair salon, such as what is an ordinary salary, reasons for different salary levels, what the interior the hair salon needed, the cost of advertising, and the importance of a coffee machine. Such discussions showed that the students considered that they themselves could be knowers of the practices of a hair salon. That they started discussion of such issues shows that the students certified themselves that the event of working in and managing a hair salon was possible. However, when the mathematics was visible the students reverted to traditional mathematics classroom norms, where the teacher is the authority and it is more important to finish the task, than to make and argue for realistic assumptions. This can be seen in the fact that the students voluntarily argued and discussed between the groups when they worked on the questions but did not participate and discuss when the questions were reviewed on the blackboard.

7.2 The Engine Cylinder Task

The engine cylinder task was implemented in a double lesson of 2 x 45 minutes. In their previous lesson, the students had worked with formulas for cuboids and cylinders and how to convert between different cubic units. Ingeborg, the teacher, started the lesson by reviewing on the blackboard how to convert between units and the formula for the volume of a cylinder (see Figure 7.2). She told the students that they would work with calculations on engine cylinders. The students were instructed to work individually and ask for the next worksheet (car engine cylinder) when they had finished the first worksheet (moped engine cylinder).

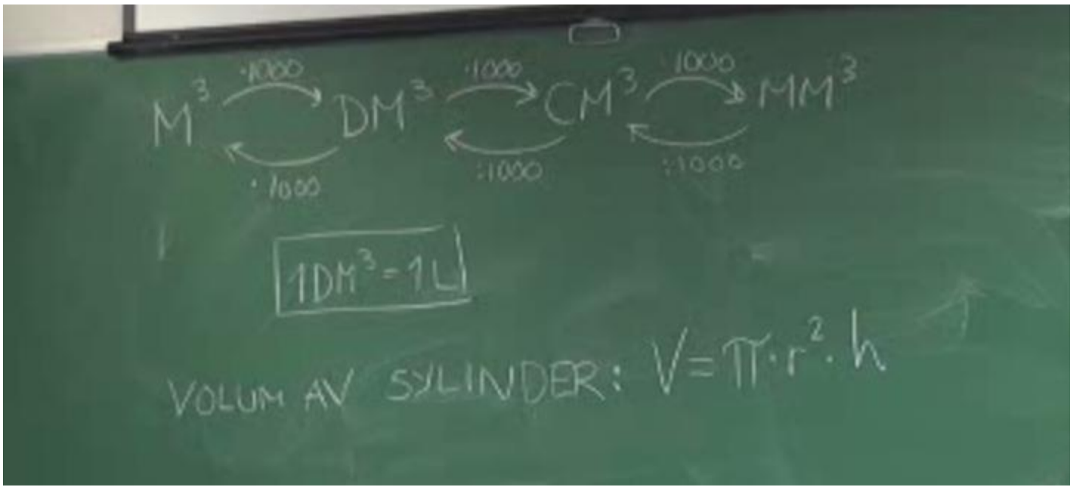


Figure 7.2: The blackboard inscriptions at the start of the lesson.

The students worked mostly individually, but while they worked, they would talk to each other, both about the task and about non-mathematical issues. I had a camera which captured whole-class interactions, but I mainly observed the students Erik and Fredrik in this lesson. This was largely a choice of convenience with regard to Erik, who was seated at the back of the classroom which is easy for camera placement, and I knew from earlier visits that he was quite talkative. Fredrik arrived approximately five minutes late to the lesson and worked mostly alone. I had noticed that Fredrik sometimes seemed uninterested in mathematics during earlier mathematics classes, but in this lesson I soon noticed that he was working on the task and seemed more interested. Therefore, I thought that he could be interesting to observe in this lesson. Erik worked individually but talked to the students sitting close to him, Jens and Martin. During the lesson both Erik and Fredrik asked Ingeborg and me for help with the tasks.

The class worked on the task for approximately 60 minutes. Ingeborg used the last 30 minutes to review students’ solutions at the blackboard, asking the students how they had worked out their solutions and what the answers were. She noted the different solution methods the students explained on the blackboard. Ingeborg tried to encourage the students to present their solutions and write on the blackboard but did not get any volunteers.

In the analysis below, I address RQs 1 and 2 in turn. This means that the illustrative episodes are not necessarily in chronological order. Where relevant for the narrative, I describe whether the episodes are from the start of the lesson, the middle or the teacher-led review. Some episodes were captured on the main camera, while others are from the camera close to Erik and Fredrik or from the hand-held camera.

7.2.1 Enacted Norms in the Classroom

In this section, I analyse and discuss both social and sociomathematical norms as the students worked on this particular task. One noticeable pattern of interaction in this classroom was that discourse often followed the social norms of a typical IRE pattern. This can be seen when Ingeborg reviewed questions and answers at the end of the lesson at the blackboard. An example of this is shown in the excerpt below about finding the volume of the moped engine cylinder, where Ingeborg asked a series of questions, requiring only short answers (Turns 1, 3-4, 6 and 14, Table 7.14).

Table 7.14: Ingeborg leads the class discussion on how to find the volume of a cylinder.

Turn	English	Norwegian	Comments
1	Ingeborg: Then we need to find the volume, and for that we need the radius, and what is it?	Ingeborg: Så skal vi finne volumet, og da trenger vi radiusen, og hva blir den?	Ingeborg repeats the question aloud, and only a short answer is needed. The answer can be found by identifying that the diameter was given in the task, and knowing the relationship between radius and diameter
2	Student: 20.8.	Elev: 20,8.	

			(calculate the division 41.86 : 2).
3-4	Ingeborg: Then we divide 41.86 by two, and then we got 20.93 so that we are ready to calculate the volume. And then we wrote, what is it?	Ingeborg: Da deler vi 41,86 delt på to, og så fikk vi 20,93 sånn at vi er klare til å regne på volumet. Og da skrev vi, hva blir det da?	Ingeborg talks through what should be done and notes the calculations on the blackboard. She again asks a question where the students can only give a short answer.
5	Student: I think it is 3.14	Elev: Jeg tror det er 3,14	
6	Ingeborg: 3.14 times	Ingeborg: 3,14 gange	Ingeborg notes the calculations on the blackboard.
7-13	(Ingeborg continued to get short answers when going through the calculation process and ended with $3.14 \cdot 20.93 \cdot 20.93 \cdot 40 = 55020.9$).		
14	Ingeborg: Point nine, so we round it off then, and then I heard that you had converted it to cubic centimetres instead, so what did you get?	Ingeborg: Komma ni, så vi runder den av da, og så hørte æ at du hadde gjort det om te kubikkcentimeter i stedet for, så da fikk du?	Ingeborg asks the students question to again.
15	Student: 55 point	Elev: 55 komma	The answer is 55 cm^3 is written on the blackboard
16	Ingeborg: Approximately 55 cubic centimetres.	Ingeborg: Ca 55 kubikkcentimeter.	

In this and similar episodes, the students are left to mostly fill in one or two words in the teaching sequence (Turns 2, 5 and 15, Table 7.14). However, at other points in the lesson, there was a departure from the standard IRE pattern and what may seem as a shift in sociomathematical norms: the students did not hesitate to correct the teacher or myself in relation to references to the vocational practice. One example of such a correction is presented in the excerpt below, where I mixed up bore and stroke, and used bore as height, not diameter (Turns 4 and 6 Table 7.15). Here, it is noticeable that Erik did not hesitate to correct me; he was clearly familiar with the bore and stroke and appeared sure and confident in his statements (Turns 5 and 7, Table 7.15).

Table 7.15: Erik corrects me when I mix up bore and stroke.

Turn	English	Norwegian	Comments
1	Erik: I remember the bore of it.	Erik: Jeg husker borringa på han.	I ask how big Erik's own cylinder was. He says that he remembered the bore.
2	Trude: Yes, what was the bore then?	Trude: Ja, hva var borringa, da?	
3	Erik: 48.	Erik: 48.	

4	Trude: 48. Was it the same, do you remember, do you have a clue on what the diameter might have been then?	Trude: 48. Var han sa[mme], husker du, har du peiling på diameter den kunne vært, da?	I mix up the words, and use “bore” as height, not diameter. I am corrected by Erik, who seems familiar with bore and stroke.
5	Erik: That was the diameter.	Erik: Det var det som var diameteren.	
6	Trude: Oh, was the bore not downwards? No, it is not.	Trude: Åh, var ikke borringa nedover? Nei, det er det ikke.	
7	Erik: No.	Erik: Nei.	

This exchange contrasted to several of the dialogues about mathematical solutions, such as in Table 7.14, where the students did not voluntarily contribute to the conversations. Overall, I found that the social and sociomathematical norms regarding the contributions and roles taken up by the students, the teacher or myself differed according to whether the discussion focused on mathematics or on vocational practices. In addition to correcting myself or the teacher, the students also corrected aspects of the task several times, as presented in Table 7.29 and Table 7.30 (see Section 7.2.2). For example, they pointed out that a V8 would not be the size given in the task. This contrasts with my general observation that the students did not correct the teacher’s mathematical explanations, solutions or presumptions.

Another place where sociomathematical normative issues was open to debate was accuracy, and where and when to round off in calculations and answers. In this lesson, it appeared that the students and teacher negotiated how to treat the issue of accuracy: should this be according to rules from within school mathematics discourse or from the students’ experiences with workshop practice? For example, Frederik queried what was accurate enough as he worked on the first question, regarding whether the cylinder given in the table (see Figure 6.5) was street legal. To figure this out, one needed to calculate the radius by dividing the given diameter (39.86 mm) in half. Fredrik asked how accurate he had to be, and then asked more specifically if he could use 20 as radius. He received the reply that he could try with 20 and be pretty close.

In the episode presented below, Fredrik had used this cell phone calculator to calculate the size of the engine cylinder to be approximately 50.26 cubic centimetres (Turn 1, Table 7.16). He had taken 20 mm as the (rounded up) radius and got an answer that indicated that the moped engine was illegal, whereas if he had not rounded up, it would be legal. I commented on this and pointed out that another student had calculated an answer of less than 50 cubic centimetres from the same information (Turns 2 and 3, Table 7.16).

Table 7.16: The possible consequences of accuracy in the task with real world connections.

Turn	English	Norwegian	Comments
1	Fredrik: 50.	Fredrik: 50.	Fredrik has used his calculator to calculate the cylinder volume to multiply $3.14 \cdot 20 \cdot 20 \cdot 40 = 50240$. Fredrik rounded his answer 50240 to 50000 from the calculator[38].
2-3	Trude: 50 point a tiny bit. Your neighbour who had taken the exact number he got 49 point something.	Trude: 50 komma bittelitt. Han naboen som hadde tatt akkurat tallet han fikk 49 komma bittelitt.	I read Fredrik's calculator, and then point out that another student, who used a more accurate radius, has gotten a smaller answer.
4	Fredrik: Yes.	Fredrik: Ja.	
5	Trude: So if you had been a little bit more accurate, it would have been legal. When you are a little bit inaccurate it is not completely legal, but it is as close as you get	Trude: Så viss du hadde littegrann mer nøyaktig, så hadde det vært lovlig. Når du er bittelitt unøyaktig så er det ikke heilt lovlig, men det er så nærme du kommer.	I point to reasons from the real world to use 19.93 instead of 20 (the need for the engine to be legal).
6	Fredrik: Mm.	Fredrik: Mm.	

Here, I pointed in Turns 3 and 5 to the fact that Fredrik's choice of when to round off affected the answer he got. His rounding off meant that the answer, interpreted into real-world context, would suddenly make the engine illegal, instead of legal. Even though abandoning standard mathematical practice might go against the real-world need to be legal, Frederik's action challenges standard classroom practice to follow a rounding norm from the real-world context.

[38] The calculator showed the answer as 50.240, using the . as a separation sign for thousandths. This meant that it is easy to ignore that what Fredrik had calculated is 50240 mm^3, and just read it as 50.240 cc.

Another normative issue relates to how to communicate solutions appropriately in the classroom. When Fredrik solved the first questions about moped engines, he was asked to note down what he had done. Until then, he had calculated on his cell phone calculator without any written notes. I pointed out that he needed to write down what he had done and justified this by saying that it was needed so Ingeborg could understand. He then started to note down his answers; Figure 7.3 shows his inscriptions on question 1c1.

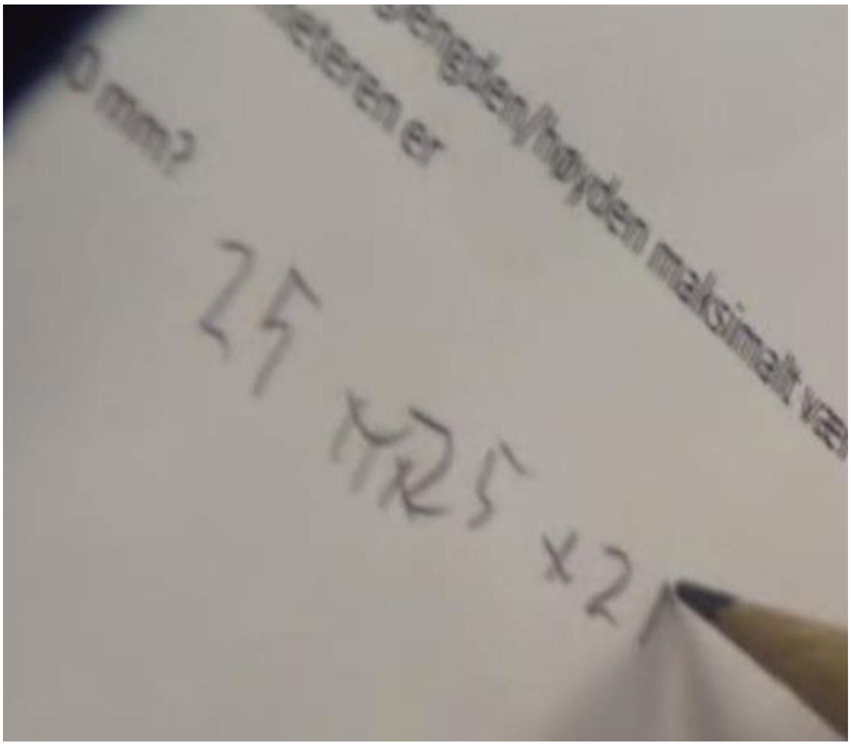


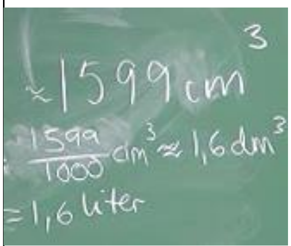
Figure 7.3: Fredrik's notes on question 1c1.

Erik made the excuse to Ingeborg that he "hadn't noted so much on the sheet[39]" when she asked what he had done. This is an example of the enactment of a social norm in the classroom, namely, that one should not just solve problems, but write down what one has done. This extends to a sociomathematical norm when it is agreed that a solution must be mathematically clear, for instance that units must be made clear. The students and the teacher seemed to have a common understanding of the norm that one should write down answers and procedures, although this was a rule which some of the students disregarded unless prompted. However, they did demand that the teacher adhere to the rule of mathematical clarity: later in her review of the task, Ingeborg skipped a step when converting

[39] Norwegian: Har ikke skrevet så mye på arket.

from cubic centimetres to cubic decimetres, and was rebuked by the students for not noting enough on the blackboard (Turns 2, 4 and 5, Table 7.17). Martin told Ingeborg that one needed to note that they had divided by 1000.

Table 7.17: What to note on the blackboard (or in a notebook).

<table>
<tr><th>Turn</th><th>English</th><th>Norwegian</th><th>Comments</th></tr>
<tr><td>1</td><td>Ingeborg: Divide by a thousand. Then I’ll take about 1.6 cubic decimetres, in other words 1.6 litres.</td><td>Ingeborg: Dele på tusen. Da tar jeg ca 1,6 kubikkdesimeter, altså 1,6 liter.</td><td rowspan="2">As she talks, Ingeborg writes 1599cm^3 ≈1.6 dm^3 = 1.6 l on the blackboard.</td></tr>
<tr><td>2</td><td>Student1: Did you divide with a thousand?</td><td>Elev: Delte du på tusen?</td></tr>
<tr><td>3</td><td>Ingeborg: Yes.</td><td>Ingeborg: Ja.</td><td rowspan="4">The students ask what she has calculated and want her to specify the division with 1000 on the blackboard. Ingeborg does as the students ask and adds $\frac{1599}{1000}$ dm^3, as seen in the picture.

</td></tr>
<tr><td>4</td><td>Student1: Must write it down, don’t you have to?</td><td>Elev1: Må skrive det, må du ikke det?</td></tr>
<tr><td>5</td><td>Student2: Yes.</td><td>Elev2: Ja.</td></tr>
<tr><td>6</td><td>Ingeborg: Must perhaps do that. If I write 1599 divided by 1000, cubic decimetres, 1.6 cubic metres, like that, are you happy now?</td><td>Ingeborg: Må vel kanskje det. Viss æ skriver 1599 delt på 1000, kubikkdesimeter, også 1,6 kubikkmeter, sånn, er dere fornøyd nå?</td></tr>
<tr><td>7</td><td>Student: Okay then.</td><td>Elev: Okei da.</td><td></td></tr>
</table>

In this episode, the students invoke the rule that one should explain in writing with the proper amount of reasoning. However, the class’s emphasis on written conversion in this example was not repeated later when Martin explained how to convert units (Turn 2, Table 7.18). When he converted from cubic millimetres to cubic decimetres, the class did not react to his short explanations, as seen in the example below, suggesting that the norms of how to communicate and explain were not completely established.

Table 7.18: Here the students did not ask for Ingeborg to note down the conversion of the units.

Turn	English	Norwegian	Comments
1	Ingeborg: Must convert; we had litres, and here we have it measured in millimetres and this is then 3.5 cubic decimetres. And if we get cubic millimetres when we should calculate, how is it?	Ingeborg: Må gjøre om, vi hadde liter, og her har vi jo målt i millimeter og dette her er jo da 3,5 kubikkdesimeter. Og vi får kubikkmillimeter viss vi skal regne, hvordan blir det?	Ingeborg asks how the students want to convert between units, since some of the dimensions are in millimetres and some in litres.
2	Martin: First to cm as it becomes 3500 and then it becomes - add three zeros more.	Martin: Først til cm så blir det 3500 og så blir det, ææh, legge på tre nuller te.	See Figure 7.4 for the written notations. Here, the students do not ask Ingeborg to write down as much as in the previous example.
3	Ingeborg: Put on 3 zeroes more, yes. *(Smiling voice).* Like that. Then we can put it in here. Yes, and so?	Ingeborg: Hive på 3 nuller te, ja. *(Smilende stemme).* Sånn. Da kan vi putte det inn her. Ja, og så?	

Martin explained the conversion as just adding three more zeroes (see Figure 7.4), and neither the teacher nor the other students reacted to this explanation.

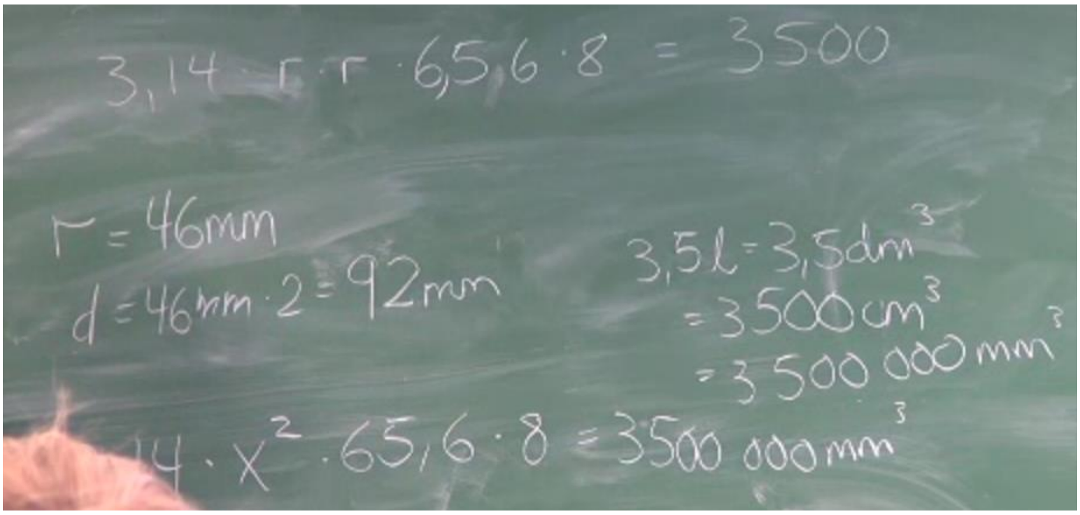


Figure 7.4: Blackboard inscriptions regarding the change from litre to cubic millimetres.

In the oral explanations by Martin, there was no mention of multiplication by one thousand, but conversion was done by the addition of three zeroes, and the change of unit. However, earlier, the students had explicitly asked for the teacher to note the division by one thousand, and the teacher and the class then seemed to agree that such notation was useful. In these episodes, the students are inconsistent in their application of the rules of explaining mathematically, apparently applying these to the teacher's action but not to their own. Working on the engine cylinder task appeared to invoke norms both of the mathematics classroom and of the vocational context. It seems that the students and the teacher had diverging expectations about how the task should be solved, and the sociomathematical norms concerning appropriate and effective solution methods were an issue in several episodes.

The examples below are mostly taken from the review part of the lesson, since during the review the issue of the possible acceptance of different methods by the teacher and the students arose several times. The analysis shows that most of the students had solved the questions with trial-and-error methods, but the teacher concentrated on moving from this solution strategy to solving the questions algebraically or with equations. Several times during the review of the lesson on the blackboard, Ingeborg acknowledged trial-and-error as a valid solution method, and then asked for other ways of solving the question. The following excerpt (Table 7.19) is taken from an exchange between Fredrik and Ingeborg about Question 1c1, how big can the stroke of a legal moped engine be if the diameter is 50 millimetres? Here, one can see that Ingeborg acknowledged Fredrik's trial-and-error method and regarded this as one possible solution method (Turn 4, Table 7.19). She also indirectly pointed to the norm that answers should be explained (Turns 1 and 10, Table 7.19).

Table 7.19: Fredrik solves the question by trial-and-error.

Turn	English	Norwegian	Comments
1	Fredrik: 25.	Fredrik: 25.	Fredrik answers Ingeborg's question about the maximum stroke size of the particular cylinder. Ingeborg points indirectly to the norm that one should explain answers.
2	Ingeborg: Then you have gotten 25, and how did you get that? (Silence 10 seconds).	Ingeborg: Da har du kommet frem til 25, og hvordan kom du frem til det? (Stillhet 10 sekunder).	
3	Trude: We tried, but not with so many numbers.	Trude: Vi prøvde, men det var ikke så mange tall.	Fredrik does not seem to have the words to explain his actions, but I had seen him work, so I try to help.
4	Ingeborg: You guessed your way to it; yes, that is quite possible.	Ingeborg: Dere prøvde dere frem, ja, det går fint ann.	Ingeborg confirms that trial-and-error is a possible solution

			method. She has a positive intonation in her voice.
5-9	(Students discuss if 25.4 would be better than 25. Both give an answer in which the cylinder volume is less than 40 cubic centimetres.)		
10	Ingeborg: How did you do it, Fredrik?	Ingeborg: Åssen gjorde du det, Fredrik?	Ingeborg asks again. She probably wants to get more information about how he picked numbers and entered them into the formula.
11	Fredrik: I tried until I got [a number that fit].	Fredrik: Æ prøvde meg frem heilt te [jeg fikk et tall som stemte].	Fredrik repeats my explanation.
12-21	(Ingeborg calculated with the numbers Fredrik suggested in the cylinder volume formula on the blackboard.)		

In Turn 4, Ingeborg acknowledged this solution method by saying that that is a possible way to solve the question. She asked him again to explain what he has done (Turn 10, Table 7.15). Ingeborg then calculated on the blackboard and showed that it gives the correct answer. After she has calculated with the suggested numbers in the cylinder volume formula, Ingeborg asked if anyone has tried to solve it with methods other than trial-and-error (for example, with equations). This suggests that solving the question by equation is a preferred goal for Ingeborg (and the mathematical education community she represents) in comparison with trial-and-error methods. Alternatively, she may want to present a range of solution methods, without any preference for any of them.

As shown in Table 7.20, Ingeborg continued the review on the blackboard by asking for other solution methods (Turn 1, Table 7.20), and quickly concentrates on solving the question by equations (Turn 2, Table 7.20). None of the students volunteered this method, but when Martin is prompted, he explained his attempt to find a solution.

Table 7.20: Ingeborg asks for other solution methods for finding the unknown height of the cylinder.

Turn	English	Norwegian	Comments
1	Ingeborg: Is there anyone who did not try trial-and-error, but who has tried to calculate as an equation? (Silence).	Ingeborg: Er det noen som ikke har prøvd å feile, men som har prøvd regne med likning? (Stillhet).	Ingeborg asks for other solution methods.
2	Ingeborg: But should we try, if it is possible, to	Ingeborg: Men skal vi prøve om det går an å	Ingeborg had talked to Martin earlier, and

	also calculate what it is, not just use trial-and-error? Martin, didn't you try that?	regne seg frem til det au, ikke bare prøve og feile? Prøvde ikke du på det, Martin?	seen that he had tried using equations.
3	Martin: Huh?	Martin: Hæ?	Ingeborg asks Martin what he has done to solve the question with an equation.
4	Ingeborg: To calculate it?	Ingeborg: Å regne ut?	
5	Martin: Yes.	Martin: Jo.	
6	Ingeborg: (Indistinct) How did you do it?	Ingeborg: (Utydelig) Hvordan gjorde du da?	
7-31	(Ingeborg led Martin and the class through how to solve the question as an equation. She started with 3.14 times 2.5 times 2.5 times unknown height equals 50 cubic cm, and then solved the equation for the unknown height. Martin explained during this interaction how he had solved the equation).		Martin explains, and Ingeborg asks several leading questions during their interactions.
32	Ingeborg: Yes, so then we found approximately this, point 425, just as you, who had tried your way to it.	Ingeborg: Ja, så da traff vi ca på dette her, komma 425, som dere snakka om dere som hadde prøvd dere frem.	Ingeborg connects the two different solution methods, and notes that they both give same answer.

Ingeborg used considerable time to obtain and present this solution, even though Fredrik had answered the question correctly. She pointed out that the two different solution methods give same answer (Turn 32, Table 7.20). When reviewing how to solve the question as an equation on the blackboard, the dialogue is mostly controlled by the teacher, with short answers from the students. It seems from Turns 1 and 2 that Ingeborg wanted the students to do or know about the algebraic solution method.

An additional example of this can be seen on a later occasion when Question 2b was reviewed. Fredrik offered his solution found by trial-and-error (Turns 1, 5 and 7, Table 7.21). Ingeborg again asked for explanations and acknowledged trial-and-error as one way of solving the question (Turns 6, Table 7.21). She seemed accepting of Fredrik and his solution (Turns 6 and 55, Table 7.21), but then asked if someone had tried to solve it as an equation (Turn 20, Table 7.21).

Table 7.21: Ingeborg asks for other solution methods when reviewing question 2b, finding the unknown radius in a cylinder volume.

Turn	English	Norwegian	Comments
1	Fredrik: 46.	Fredrik: 46.	Fredrik continues with his short answers without explanations.
2-3	(Ingeborg asks Fredrik if that was the radius, which he confirms).		

4	Ingeborg: So how did you find it [the radius 46 mm]?	Ingeborg: Så hvordan kom du frem til det da [radiusen på 46 mm]?	Ingeborg asks again for explanations.
5	Fredrik: I tried my way to it.	Fredrik: Prøvde mæ fram.	Fredrik gives only a short answer.
6	Ingeborg: You tried your way again. You're good at trying; it seems like you mostly get the answer, so that is good. Yes and how do you do that when you try your way forward then?	Ingeborg: Du prøvde deg frem igjen. Du er god til å prøve deg frem virker det som, du treffer på det meste, så bra. Yes, og hvordan gjør du det når du prøver deg frem da?	Asks here for further explanations of what "tried" meant. Seems positive to Fredrik and his solution.
7	Fredrik: Take different numbers.	Fredrik: Tar forskjellige tall.	Again, a short answer from Fredrik.
8-19	(Ingeborg goes through the formula on the blackboard: "3.14 times some radius times some radius times 65.6 times 8". Fredrik confirms that this is how he has done it.)		
20	Ingeborg: (...) Is there anyone who has calculated differently or tried their way forward? (...) Are there more who have tried and got the same? (Silence 4 seconds). No, but Fredrik has at least figured it out. Okay, there are some who have tried as equation then?	Ingeborg: (…) Er det noen som har regnet annerledes, eller prøvd seg frem? (…) Er det fler som har prøvd seg frem og fått det samme? (Stillhet 4 sek). Nei, men Fredrik har iallfall funnet ut av det. Okei, er det noen som har prøvd med likning da?	Ingeborg tries again to find other solution methods that have been used by students, but has problems getting the students to answer.
21	Martin: I tried, but it did not go well.	Martin: Æ prøvde, men det gikk ikke så bra.	Martin seems unsure of his solution method. Ingeborg then tries to get him to explain what he has tried.
22	Ingeborg: You tried, but it did not go well. How did you do it then?	Ingeborg: Du prøvde, men det gikk ikke så bra. Hvordan gikk du frem, da?	
23-54	(Ingeborg leads Martin and the class through how to solve the question as an equation. She starts with calculating 3.14 times 65.6 times 8, then isolates the unknown variable and finally uses square root to find the bore/diameter. She gets step-by-step suggestions from Martin at the start of the calculations.)		
55	Ingeborg: 46, well, then we have a very good trial-and-error solver here.	Ingeborg: 46, det vil si at vi har en som er veldig god på og prøving og feiling her.	Ingeborg's answer is 46 when solving the equation, the same answer as Fredrik has found by trial-and-error. Ingeborg connects the two

			solution methods and points out that they give the same answer.

In this discussion in Turn 6, Ingeborg first complimented Fredrik for his work with the trial-and-error method, before she asked in Turn 20 for other solutions and again encouraged Martin to admit that he had tried with equations. Martin seemed dissatisfied with his own work in Turn 21, but with help from Ingeborg, he explained what he has tried to do. In Turn 55, Ingeborg pointed out that the trial-and-error method gives the same result as the algebraic method.

When I discussed the lesson afterwards with Ingeborg, she confirmed that her intention was that the students could move from trial-and-error methods to more algebraic methods, thus attempting to shift the sociomathematical norms of the classroom. However, Erik had expressed his opinion about solving the question algebraically to me earlier the lesson. In the following excerpt, it can be seen that Erik is averse to solving mathematical questions algebraically; he even said that working with such methods is something he hates to do (Turn 6, Table 7.22).

Table 7.22: Erik's reaction to solving the task algebraically when I talk with him in class, before the review on the blackboard.

Turn	English	Norwegian	Comments
1	Trude: Can you manage to find out what it should be if we want the diameter to be 50. How high must it then be to be legal?	Trude: Kan du klare å finne ut hva det skal være viss vi skal ha diameteren til å være 50. Hvor høy må den da være for å være lovlig?	I try to direct Erik's attention to the main points in the question.
2	Erik: How high? How can we find out that?	Erik: Hvor høy? Hvordan kan vi finne ut av det?	Erik confirms that he understands what the question asks for, but he indicates that he does not understand how to solve it.
3-5	Trude: No, I wonder about that too. How can you figure it out? You can try to write down the numbers. (...) If it is times with 40 then, so. No, times, 19.8 times 25, what must the number be to get just	Trude: Nei, det lurer jeg au på. Åssen kan du finne ut av det? Du kan prøve å skrive opp de tallane. (…) Viss den gange med 40 da, så. Nei, gange med 19,8 gange med 25, hva må da det tallet være om du	I help Erik by repeating the numbers into the formula he has used in the previous question and point out which number would be the unknown.

	under 50? Do you get it?	skal komme rett under 50? Skjønner du?	
6	Erik: Yes, I realize that, but I hate things like that.	Erik: Ja, jeg skjønner det, men sånt hater jeg.	Erik confirms that he understands how to set up the solution of the question however he shows a negative emotional response to my suggested solution method. I point out that he could solve it by trial-and-error.
7	Trude: We can try; we do not have to make a formula for it. We can try our way a little.	Trude: Vi kan jo prøve, vi trenger ikke lage noe formel for det for å si det sånn. Vi kan bare prøve oss litt frem.	

Here, I tell Erik in Turn 7 that it was not necessary to solve it algebraically. One interpretation of his reaction in Turn 6 is that he thinks that the correct solution method for such questions in mathematics is to solve the question algebraically. Earlier, in an interview with Erik about his experiences with mathematics, he had stressed that he never understood algebra and always got confused when working with algebra. It appears that he has an assumption of what “proper” mathematics is.

Fredrik also seemed to be frustrated with Ingeborg’s insistence on explanations. He had solved the questions by trial-and-error methods, and when Ingeborg asked for explanations, he pointed out several times that he had just calculated it or just “saw it” (Tables 7.15 and 7.17). Another example of this comes later in the lesson, when Ingeborg tried to get him to explain how he had worked out his solution. Fredrik replied briefly to Ingeborg that he “only calculated it[40]” and seemed frustrated with the pressure to elaborate. It appears that Fredrik does not understand why Ingeborg asked for further explanations when he has found a answer.

These episodes suggest that the students and the teacher diverged in their expectations about how to answer the questions. I interpret this in terms of different expectations of the sociomathematical norms regarding appropriate solution methods in mathematical class itself, and a clash with the norms which arise from the students’ experience in the vocational practice or the common programme subject. Most of the students solved the questions with trial-and-error methods, but the teacher concentrated on moving from this solution strategy to

[40] Norwegian: Æ bare regnte ut, æ.

solving the questions algebraically or as equations. It can seem as though the students do not see the point that Ingeborg was trying to make, that algebraic methods can have an advantage over trial-and-error methods. This is a reasonable argument from the students, because trial and error worked, and working on their phones, without written notes, worked for them to solve the tasks. An additional sociomathematical norm that was important in the classroom discourse was accuracy when calculating. Here, students and the teacher negotiated whether accuracy should be treated with rules from within mathematics discourse or from the students' experiences within the common programme subject or out of school practices.

In addition, there were diverging opinions of the norms which regulated how answers should be explained. If the students conferred with each other about the mathematics tasks, they did not ask for explanations, only the answer. However, the teacher wanted the students to give explanations when they presented their solutions to her or the rest of the class, and this norm was clearly established in the classroom. This suggests that the students knew the normative activity was to explain their answers, however figured that it was a rule which could be disregarded unless the teacher demanded that they pay attention to it.

7.2.2 Connections with Out-of-School Practices

As we have already seen with respect to norms, there is evidence that the students connected the task with their experiences from outside of the mathematics classroom. They used words from their future vocation when they discussed the task in the classroom, in addition to, or instead of, words from mathematics. They demonstrated their knowledge of the practice and used this to inform or correct the teacher or researcher. However, even though the students seemed to certify authentic aspects in the task, they disregarded their knowledge of engines when they solved the questions. The authentic aspects of the tasks were not always recognised by the students, and there are instances in the empirical material when students questioned planned authentic aspects. In this section I present episodes in which we see examples of the students using their experiences and knowledge about engines, mopeds and cars. The episodes highlight contributions from Fredrik and Erik where there are indications of connections to their previous experiences from outside/workplace practice.

In the following episode, Fredrik had solved the first question, in which he was supposed to check if the given engine had a cylinder capacity of less than 50 cm^3. He used his cell phone calculator and the formula for volume of cylinders to solve the problem. Even though he had found the correct answer, he continued to work intensely on his calculator. When I asked what he was calculating, he explained that he was trying to figure out the cylinder capacity of his own cylinder (Turn 1, Table 7.23). This calculation was done completely on his own initiative, and he found that his cylinder was 78 cm^3 (see Figure 7.5).

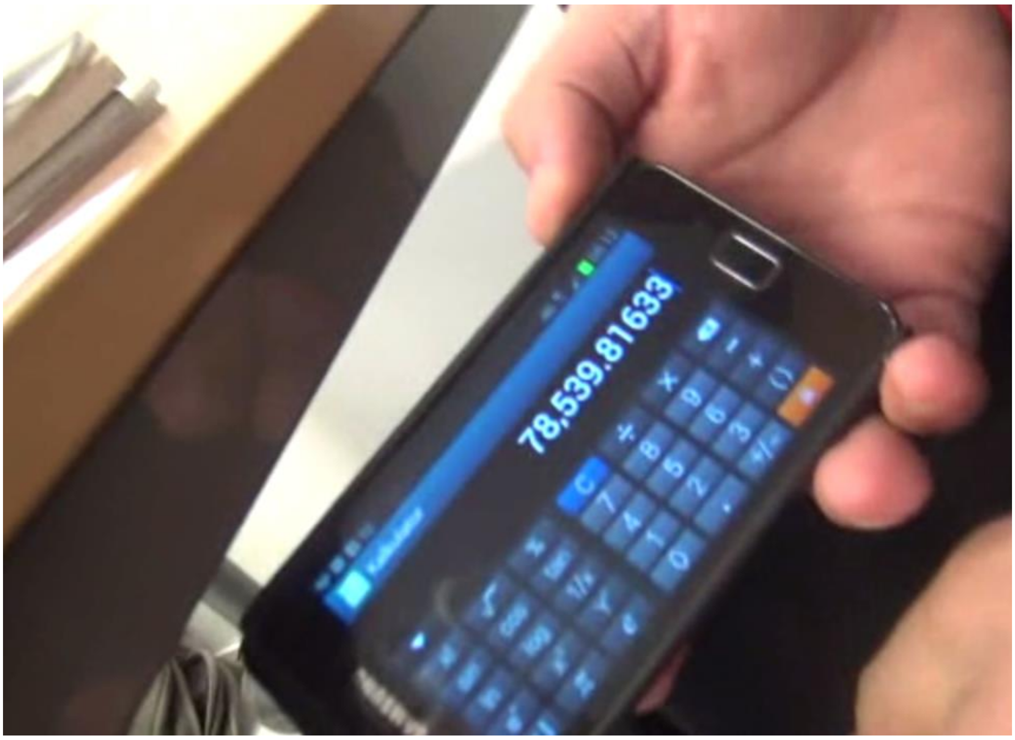


Figure 7.5: Fredrik's calculations of the volume of his own cylinder.

Fredrik then explained that the difference between his original and new cylinder was only a difference in diameter (Turn 3, Table 7.23). He could answer quickly without hesitation and showed that he had knowledge connected to the vocational practice.

Table 7.23: Fredrik explains the differences between the cylinders.

Turn	English	Norwegian	Comments
1	Fredrik: I'm trying to figure out my cylinder.	Fredrik: Jeg prøver å finne ut av sylinderen min.	Fredrik replies to a question about what he is working on.
2	Trude: Your cylinder, yes. Yes, but this is interesting. Do you know what it looks like?	Trude: Av sylinderen din, ja. Ja, men dette er interessant. Vet du åssen den ser ut?	I ask Fredrik if he is familiar with the dimensions of his moped cylinder.

3	Fredrik: It's larger only in diameter.	Fredrik: Han er bare større i diameter.	Fredrik answers quickly without hesitation and shows that he is knowledgeable about the moped cylinder.

Fredrik worked on solving a question with relevance to his own out of school experiences, namely what the volume of his own moped engine cylinder was (Turn 1, Table 7.23), connecting the issues raised in the task to his own experiences and real world practice. In this sense, Fredrik himself confirmed that the question of calculating the volume of an engine cylinder is an authentic aspect in the task. He did not need the confirmation of someone from the practice to do so and certified the authentic aspect himself by asking his own similar question. His actions in the classroom confirm that the work of figuring out volume of a cylinder is an authentic aspect of the task.

After this first episode, he continued to work on his calculator and told me that he was calculating what would happen if the cylinder was taller. Thus, Fredrik solved not just one question of his own making, but two. Here, I observed that Fredrik worked without the teacher prompting him, and he demonstrated an engagement far different from the engagement shown in the dialogue (Table 7.24) that I had with him few minutes earlier. The following exchange is between Fredrik and me at the beginning of the lesson (Turns 1, 3 and 8, Table 7.24), four minutes before Fredrik started to work on his own questions about his own engine. Fredrik followed my verbal clues, and asked one question (Turn 7, Table 7.24), but the dialogue was driven by me, and there are no signs of Fredrik taking initiatives.

Table 7.24: Fredrik follows my clues.

Turn	English	Norwegian	Comments
1	Trude: Should we look at it together, Fredrik?	Trude: Skal vi se på det sammen, Fredrik?	I take the initiative, and seem to drive the conversation forwards.
2	Fredrik: Yes.	Fredrik: Jaha.	Fredrik answers with a disinterested tone.
3	Trude: We should find out how much the volume is.	Trude: Vi skal finne ut hvor stor volum det er.	The question is repeated without any reference to an engine.

4	Fredrik: Yes.	Fredrik: Ja.	Fredrik answers with a disinterested tone.
5-6	(Trude talks about the task.)		
7	Fredrik: How do we find the radius?	Fredrik: Åssen finner vi radiusen?	Fredrik asks a short question regarding how to find the radius and get a leading answer
8	Trude: Yes, we have the diameter. Do you know the relationship between diameter and radius?	Trude: Ja, vi har diameteren. Vet du sammenhengen mellom diameter og radius?	
9	Fredrik: Divide by two.	Fredrik: Dele på to.	Fredrik's contribution is reduced to a very short procedural answer to a factual question.
10	Trude: Sounds pretty sensible.	Trude: Hørtes nokså fornuftig ut.	Confirm his reply.

In these interactions, the student and teacher seem to have quite firm roles. I framed the conversation with leading questions (Turns 1, 3 and 8, Table 7.24) and the student filled in with short sentences or facts (Turns 2, 4, 7 and 9, Table 7.24). Here the roles of the teacher and student seems like typical student-teacher roles in stereotypical mathematics classrooms, and the real world connections are not mentioned at all. This is in sharp contrast to the episode mentioned above where Fredrik started to work on his own questions, just few minutes later.

Later in the lesson Fredrik's own cylinder seemed to be interesting to the rest of the class when Ingeborg reviewed the tasks on the blackboard. Ingeborg knew that Fredrik had checked his own engine cylinder from her observations while the students worked on the task. She led the class through calculating with Fredrik's dimensions. Even though Fredrik supplied quite brief responses to Ingeborg's questions (see Table 7.25), the other students asked questions about and showed interest in his moped engine cylinders.

Table 7.25: Ingeborg leads Fredrik through a review of his calculations.

Turn	English	Norwegian	Comments
1	Ingeborg: When we talk about calculating illegal mopeds, then we have Fredrik, he has calculated his. How did it go? The volume of Fredrik's engine? 3.14	Ingeborg: Når først vi er inne på å regne på ulovlige mopeder, så har vi Fredrik, han har regnet på sin, han. Åssen gikk det? Volum på Fredrik sin? 3,14	Ingeborg knows that Fredrik has checked his own engine cylinder from her observations while the students worked on the task. She

	times, what was your numbers, Fredrik?	gange, hva hadde du Fredrik?	leads the class through how to calculate Fredrik’s dimensions.
2	Fredrik: 25.	Fredrik: 25.	
3-9	(Ingeborg led Fredrik through his calculations on the blackboard).		
10	Ingeborg: And what did you get?	Ingeborg: Og da fikk du?	The final answer for Fredrik’s illegal moped engine cylinder is 88 cm^3.
11	Fredrik: 88[41].	Fredrik: 88.	
12	Ingeborg: 88 cubic centimetres.	Ingeborg: 88 kubikkcm.	

Fredrik responded with short answers throughout the dialogue with Ingeborg, and the class was quiet. But when Ingeborg and Fredrik arrived at 88 cubic centimetres in volume some students suddenly took the initiative and asked Fredrik questions (Turns 1, 2 and 6, Table 7.26). The classroom dialogue pattern changed, and Ingeborg was no longer the driving force in the classroom discourse. I present this in Table 7.26, which is a contrast to the situation in Table 7.25, where Fredrik answered Ingeborg’s questions passively.

Table 7.26: The students started to ask Fredrik questions.

Turn	English	Norwegian	Comments
1	Student: Look at that.	Elev: Se det ja.	The student sounds like he admires the size. Another student asks a question about how the engine runs with such a big cylinder volume.
2	Student: Which pistons? How do the pistons go around?	Elev: Åssen stempler, åssen går stemplane rundt?	
3-5	(Some irrelevant exchanges).		
6	Student: Which cylinder do you have?	Elev: Åssen sylinder har du?	A student wants to know which brand of cylinder Fredrik has. Fredrik answers with the make and brand of his cylinder.
7	Fredrik: Hmm? A [brand name].	Fredrik: Hmm? En [merkenavnet]	

In this example, the students seemed to admire the size of the engine cylinders, and clearly treated them in terms of their connection to out of school practice. During the lessons, I observed that the students had lively discussions about engines when the questions were realistically posed, drawing on their own experiences while tuning in out of school practice. One noticeable characteristic

[41] Fredrik calculated two different engine cylinders known to him; one had a total volume of 78 cm^3 (Figure 7.2) and one had a total volume of 88 cm^3 (Table 7.3).

of the interaction patterns of the students was that they asked questions or commented if I or the teacher made a mistake regarding the vocational context. This can, for example, be seen when I mixed up the bore and the stroke, as presented in Table 7.15 (see further discussion of this in Section 7.2.1). Erik said that he remembered the bore of his cylinder, and he then corrected me when I mixed up bore and stroke. Erik's ease in using these terms confirm that these words are an authentic aspect of the task, and he and the other students certified themselves that such words are in use in the vocation.

Another aspect of references to out-of-school practice is related to which words were used during the work with the task. When Fredrik explained his bigger moped cylinder to me, he used the word "diameter" rather than the word "bore" (Turn 3, Table 7.23). As mentioned earlier, both words are given in the task. But later, when he was asked to clarify how the tuning of cylinders was done in his experience, he used a word which is specific to the engine context: "crank[42]" (Turn 4, Table 7.27). This connection to the use of words from the vocational practice is an indication that the task contained out-of-school references for the students.

Table 7.27: Fredrik's use of words from vocational practice.

Turn	English	Norwegian	Comments
1	Trude: Have you changed the cylinder?	Trude: Har du bytta ut sylinderen?	I ask Fredrik if he has changed his original cylinder.
2	Fredrik: Yes (…)	Fredrik: Ja (…)	Fredrik confirms this.
3	Trude: Then you switched the piston that goes up and down? Replaced whole thing?	Trude: Da må du bytte den derre stempelet som går opp og ned au? Du bytter heile greia?	I am not using the correct words in a natural way, and seem unsure.
4	Fredrik: The crank?	Fredrik: Veiva?	Fredrik uses the work crank from the engine context.
5	Trude: Yes, sure?	Trude: Ja, sikkert?	I am unsure of the word Fredrik used.
6	Fredrik: Yes.	Fredrik: Ja.	

I suggest that, here, Fredrik certifies that the event of changing a cylinder is authentic, and he also positions himself as a participant in the vocational practice through his word use. I am not familiar with the same words (Turns 3 and 5,

[42] Norwegian: Veiva.

Table 7.27), and therefore clearly not a member of the same practice. Further examples of this occurred when Fredrik commented that he recognised approximately how big the numbers should be, when he worked with Question 1c, which asked for the stroke/height if the bore/diameter was 50 mm in a legal moped engine. Fredrik confirmed that he knew and recognised the size of the numbers since and that he had knowledge about moped engine cylinders. Later, when he worked on the questions about car engines, he said that he did not know common dimensions of car engine cylinders.

Another example in which students drew on knowledge from the outside world can be observed in the following episode. Kristian solved Question 2a, about the size of the stroke volume of a V4 engine, with given dimensions for stroke/height and bore/diameter. He asked if the correct answer was 1.8. To this, I replied that I thought it was supposed to be 1.6, and that 1.8 sounded like a small calculation mistake. Kristian then corrected my assumption that this was something he had calculated and said that he had figured out the answer mentally. When I asked if he envisaged a familiar car when he read the sizes, he answered in the affirmative, and said that he just “thought of another [known] engine and compared them[43]”. Here he indirectly certifies that the given dimensions of this engine. It seems like Kristian connected the task with his knowledge of the vocational practice since he tried to solve the problem by comparing the engine described in the task with an engine he knew. He seems like a competent knower of vocational knowledge, especially since his mental calculation was not far off.

Several other examples of the students’ familiarity with engine cylinders were found in the data material. Erik and Jens showed knowledge of moped cylinders and confirmed that they knew the size of their own cylinders when asked if they knew how big a cylinder they had in their moped. Likewise, when Erik, Jens and Fredrik talked about their moped engines, they used words such as “nozzle”, “carburettor”, and “filings[44]”, and had obviously common language with regards to moped engines and possible tuning of such engines. Altogether, the task seemed to be treated as connected to a vocational practice by the students, and they themselves used words connected to the engine context and

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[43] Norwegian: Ser bare på en annen motor og sammenlikner dem.
[44] Norwegian: Dysa, forgasser og plombene.

treated the information given as worth discussing in regard to their knowledge of the out of school practice.

Another finding was that the students demonstrated that they had relevant knowledge about the engine context and used this knowledge to inform or correct the teacher or myself. One example is when Fredrik corrected the task question 1b (file to create a bigger cylinder) and said that this did not concur with his experiences (Turns 6 and 10, Table 7.28). Here he did not certificate a planned authentic aspect in the task. Such filing was the background of task 1b, where the question was to find the volume if the cylinder was filed internally such that the diameter increased by 2 millimetres.

Table 7.28: Fredrik explains that the whole cylinder is swapped and not filed larger.

Turn	English	Norwegian	Comments
1	Trude: Is it, you know, what I see people doing is replacing the whole thing.	Trude: Er det, vet du, det jeg ser at folk gjør er å bytte ut heile den greia der.	I ask about Fredrik's experience on how the tuning of moped engines is conducted.
2	Fredrik: Yes.	Fredrik: Ja.	
3,5	Trude: But is it possible to just polish it little larger; it there a point, or is it [just wasted?]	Trude: Men går det an å bare pusse han litt større au på en måte, (…) Men er det noe vits? Eller er det [bare bortkastet?]	
6	Fredrik: It does not help that much.	Fredrik: Hjelper ikke så mye.	Fredrik sees sure of himself, and does not confirm my idea of filing.
7-8	(Repeated that it does not help to file).		
9	Trude: You need to replace the whole thing?	Trude: Du må bytte heile greia?	I ask for confirmation of my understanding.
10	Fredrik: Yes.	Fredrik: Ja.	Fredrik again seems sure in his knowledge.

In Turns 1,3 and 5, I asked Fredrik to explain what he knew from his previous experiences with moped engines. In both Turns 6 and 10, he drew on his knowledge of the practice to inform me about the way this is done in his experience. He seemed sure of his explanations and took the role of the knower in a vocational practice. An additional example of the students rejecting an aspect of authenticity is in a later episode with Kristian, when he explained the size of a four-cylinder engine. Earlier in the lesson, he had reacted to the given information in the questions, and in the excerpt shown in (Table 7.29), he explained the size of a V4 engine.

Table 7.29: Kristian explains the size of a V4 engine.

Turn	English	Norwegian	Comments
1	Trude: Could you have like an eeeh, 3, 4, or a 3.5 litre four cylinder?	Trude: Viss du skulle hatt en eeeh, 3, 4, går det an å ha en 3,5 liter i med fire sylindere?	I ask what he thinks would be plausible engine sizes.
2	Kristian: Yes, one could, but it is not common.	Kristian: Ja, det gjør det, men det er ikke normalt.	Kristian can explain to me what a usual cylinder volume in a V4-engine would be.
3-4	(I then ask Kristian what the size of a common V4 engine in his experience would be, and Kristian enlists the help of another student).		
5	Student: 2.3, 2.4 for example. You can get 2.5 also; there are two-and-a-halves.	Elev: 2,3, 2,4 for eksempel. Du kan få 2,5 au, det finnes to og halvlitere.	The students point out that the size (3.5 l) given in the question is too high, the common size; would be a maximum of 2.5 litres.
6	Trude: But it does not go up to 3.5? That is crazy. *(Meaning impossible).*	Trude: Men det går ikke opp til 3,5? Det er crazy. *(Crazy som 'helt umulig').*	
7	Kristian: Ehh, yes I think it is too large. 2.8 in my [known engine].	Kristian: Ehh, ja jeg tror det er for stort det. 2,8 i min sin	Kristian confirms again that the size is too big.
8	Trude: Okay, thanks. Then I learned something new.	Trude: Okei, takk. Da lærte jeg noe nytt	The student points out that it could be possible, but that such sizes are unlikely.
9	Student: It possible to get more, you know, but.	Elev: Det går sikkert an å få mere, vet du, men.	
11	Trude: No, no, then we learned new things today too.	Trude: Nei, neinei, da lærer vi nye ting i dag også.	

Kristian were able to explain to me what the usual cylinder volume in a V4-engine would be. He and the other student pointed out that the size (3.5 l) given in the question is too high; the common size would be a maximum of 2.5 liters. Here, Kristian and the other student took on roles as the knowers in Turns 2, 5 and 7. In their explanation, they shared their expertise with me, and seemed comfortable with their role as knowers of an out-of-school practice. The students showed a shared repertoire and competence in the vocational practice.

The students’ competence with regards to the vocational practice was likewise obvious when Erik corrected my explanation of a V8 car engine. Erik asked for help on Question 2b about finding the bore/diameter for a 3.5-liter V8 engine with stroke/height 65.6 mm. He commented that the question was “really hard”. I then tried to point out some important information in the question with regards to mathematics, that there were 8 cylinders and it was quite similar to a question he already had solved. But I made a mistake when talking about the physical composition of the cylinder placement (Turn 1, Table 7.30 and Figure 7.6). I said that the cylinders were placed in a row, but was corrected instantly by Erik (Turn 2, Table 7.9 and Figure 7.7). In the following example, one can see that he was both comfortable and sure of himself in his correction of my explanation.

Table 7.30 Erik corrects my explanation of the placement of cylinders in a V8 engine.

Turn	English	Norwegian	Comments
1	Trude: Okay. We know that there will be eight cylinders like that across.	Trude: Okei. Vi vet at det skal være åtte sånne sylindere bortover.	I read parts of the task out loud and refer to important information.
2	Erik: Not across.	Erik: Ikke bortover.	Erik corrects me to say that the eight cylinders are in a row, in quite a firm voice.
3	Trude: Ah.	Trude: Aahh.	I seem embarrassed or annoyed. Erik laughs, but in a good-natured way.
4	Erik: (laughs)	Erik: (ler)	
5	Trude: Sorry. Don’t they stand four on one side, and four on the other side?	Trude: Sorry. Står de ikke med fire på den sida, og fire på den sida?	I show four and four fingers straight up next to each other. (See Figure 7.6)
6	Erik: Yes, **like that.**	Erik: Ja, **sånn.**	Erik shows four and four fingers with about 90 degrees of separation (See Figure 7.7). He uses a firm voice and finger movements when he shows the correct placement.

Figure 7.6: I show my impression of how the cylinders in a V8 are placed.

Figure 7.7: Erik shows the correct placement of cylinders in a V8 engine.

In Turn 2, Erik corrected my explanation of car engines. He seemed sure of his knowledge in Turns 2 and 6 and illustrated this with the placement of his fingers as shown in Figure 7.7. I agreed with his illustration and expressed knowledge with both words and gestures.

However, his knowledge of the engine was soon ignored by Erik himself. Even though in Turns 2-6 in Table 7.30 Erik seemed to treat the engine as something he was familiar with from his experiences, he was willing to disregard his experiences to solve the question. In the ensuing dialogue, Question 2b was repeated, and I stated that there are supposed be eight cylinders, and all together they are 3.5 litres. Erik confirmed this, and I continued with remarking that the height is given. However, Erik now reacted to the question which stated that a

V8 engine is 3.5 litres, challenging the planned authentic aspect of the information in the question in Turns 1 and 3a in Table 7.31. But continuing in Turn 3b he stated that the problematic part of the question was not important for solving the question.

Table 7.31: Erik challenges the question's vocational connection, but then does not use his vocational experience.

Turn	English	Norwegian	Comments
1	Erik: It's too small.	Erik: Det er jo lite.	Erik connects the question to his previous knowledge of engines.
2	Trude: I thought 3.5 litres was pretty much in an engine.	Trude: Jeg trodde 3,5 liter var nokså mye i en motor jeg.	I seem a bit unsure, and in want of an explanation.
3a	Erik: Not in a V8. (Pause 3 seconds)	Erik: Ikke på en V8. (Pause 3 sekunder).	Erik confirms his previous statement, but then seemed to move on to ask how to solve the question.
3b	Erik: Whatever.	Erik: Samma det, samma det.	Erik seems impatient, and he accepts that the incorrect size of the engine volume does not matter for the mathematical solution process.
4	Trude: Yes, okay. You're probably right about that. (...)	Trude: Ja, okei. Du har nok rett i det. (…)	I accept that the engine size of the V8 probably was wrong.
5-7	(Erik explained that a V8 would be about 4-5 litres).		
8	Trude: Yes. But, but anyway. If we pretend that we call the (indistinct) for almost reasonable.	Trude: Ja. Men, men anyway. Viss vi later som vi kaller den (utydelig) for nesten fornuftige.	I agree with Erik's Turn 19b and ask Erik to think of it as almost reasonable.
9	Erik: Yes.	Erik: Ja.	Erik agrees to continue with the given sizes.

Turns 3b-4 are interpreted as instances in which Erik chose to not use what he knew of the engine context. He continued to concentrate on the procedural part of solving the question instead of working with the task with his knowledge from the vocational practice. Such disregard of knowledge from the practice seemed to be treated as unproblematic by both Erik and myself in Turns 8-9, before we continued to work on solving the question with the given 3.5 litre engine.

After the lesson, Ingeborg told me that the students had commented on the task in various ways which referenced the outside world; for example, they had asked if one was supposed to "sandpaper the cylinders bigger". To this, she paraphrased her own reply as "I only eeeh…", and referred to that she did not know the answer to the question, but she seemed pleased with the student interaction. Ingeborg explained that when she told Fredrik that he was supposed to calculate about car engines and presented Worksheet 1 to him, he instantly replied that that wasn't a car engine; it was a moped engine. She argued that he probably had recognised a moped engine from the photographs. Ingeborg said that she was happy that words from the workplace were used in the questions, and pointed out that, without this use of words, she would not have known what the students talked about when they said "bore" or "stroke volume". In Fredrik's recognition of a moped engine in the task boundary object representation, the issue of authenticity is prevalent. Fredrik certificates that the boundary object representation is real and shows his competence of engines.

The students certified themselves that it is quite common that a cylinder is replaced by one of a different size. Fredrik also calculated his own cylinder size, showing that replacing the cylinder do not mean that you know the size of the new stroke volume. The students themselves used language that seemed appropriate in the practice, and by their use of the words certified that bore and stroke was used. Fredrik's calculations of his own cylinder volumes, and his fellow students' admiration of the size of his engine cylinder, is an indication that for him, the goal of the lesson, to calculate volumes, became something useful and worthwhile out of the mathematics classroom. However, the answers to calculation of the original questions had no exchange value out of the mathematics classroom, and the students pointed out that some of the sizes given in the task was unrealistic.

To summarize, I found evidence that the students connected the task with experiences that they had with moped and car engines. The students used words with relevance to the engine context in addition to, or instead of words, from mathematics. The students referred to their own engine knowledge, but there is an inconsistency in which routines and working methods they would use. The students used their knowledge of the practice to inform and correct the questions and the teacher or researcher, but they were still inclined to disregard their own knowledge in order to solve the mathematical problems within routines from mathematics classrooms. This will be further discussed in Chapter 8.

7.3 The Frifond Project Task

The Frifond project budget task was implemented in three 45-minute lessons, back to back. The students were divided into groups, two groups of three students and two groups of four students. The task was to find and develop an idea for a project application, including the project budget (the task is further discussed in Section 6.3). The teacher started the lesson by explaining about the Frifond grant scheme, informing the students that Frifond is a scheme for funding local activities for young people. She told the class that they should develop projects that they could apply for.

The groups then worked for about 20 minutes, looking at the Frifond webpages, and thinking up and agreeing on a project idea. After the 20 minutes, the groups presented their ideas to the rest of the class. The four groups had different ideas: a computer event (LAN-party) in the local sports hall, building a climbing forest, organising a volleyball tournament, and organising a photography exhibition.

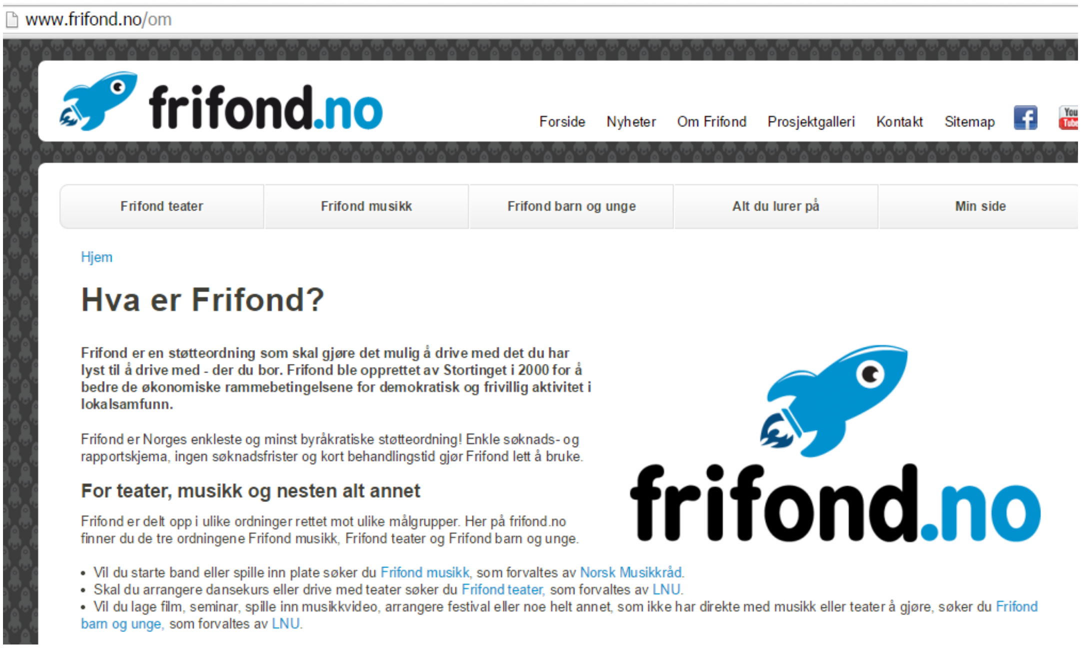

Figure 7.8: The Frifond webpage, which the students were asked to read.

The teacher then asked the students to specify what equipment and other items would be necessary for their projects, and to write a project proposal. For example, the group that wanted to arrange a volleyball tournament needed to decide how many teams could play and what the rent for the sports equipment and volleyball fields would be. The groups then worked on specifying the project proposal, including equipment rental prices and other expenses, using internet searches and estimates based on their own experience. The groups continued to work with their project proposal and budget for the rest of the allotted time.

In the following sections, I present and analyse episodes illustrating enacted norms and the students' interactions with the task in relation to out-of-school knowledge. The video recordings from these three lessons mainly focus on the group that made a proposal for a photography exhibit. This was also the only group that worked on a project directly related to a possible future vocation, namely being a photographer. The other three groups' projects - the volleyball tournament, the climbing forest and the LAN-party - are related to the students' daily life, but not directly connected to their vocational education programme. For both of these reasons most of the episodes are taken from the group that wanted to make a photography exhibit.

7.3.1 Enacted Norms in the Classroom

My observations of the students' work with the Frifond project task provided insights into the enacted norms in the classroom regarding what tasks should look like: I found that the students favoured project proposals that fitted into their notion of what kind of task is appropriate in a mathematics lesson. The students operating much of the time in accordance with norms which are familiar in traditional mathematics classrooms, including minimizing effort.

In the students' work with the Frifond project task there are clear indications that they were working in accordance with norms relating to how (conventional) mathematics tasks are supposed to look and how one should work on them. This is evident both in the students' selection of project ideas to develop, and in their work with the task. I will first discuss the students' choices in the pre-planning phase, before I discuss episodes from the rest of the implementation.

The issue of which project to plan had implications for both the mathematics classroom, and the grant scheme. The students needed to negotiate between their expectations of how to act in a mathematics class, and how to respond to the grant scheme information. In the pre-planning phase, when the students discussed which idea to develop, the group that ended up with choosing a climbing forest had first discussed the idea of a youth party. However, this idea was dismissed as being too complicated, as shown in Table 7.32.

Table 7.32: The students argue that they need something easier to plan.

Turn	English	Norwegian	Comments
1	Aud: Party at school. Watch movies and stuff, like a Halloween party.	Aud: Fest på skole. Se film og sånn, for eksempel Halloween fest.	The students discuss an idea of arranging a youth party.
2	Nora: (indistinctly) where they have that kind of foam	Nora: (utydelig) når de he sånn derre skum	One idea was to rent foam machines.
3	Aud: Google it	Aud: Google det	
4	Nora: No. Need to figure out something easier. (A little laugh in her voice)	Nora: Nei. Må finne på noe lettere noe. (Litt latter i stemmen)	Nora wants the group to pick something that would be a bit easier to work on.
5	Aud: But I mean, we can, for example, have something like that [unclear] something.	Aud: Men æ meiner, vi kan jo for eksempel ha noe sånn [utydelig] noe.	The girls continue to discuss other ideas.
6	Nora: Or like a festival, summer festival	Nora: Eller sånn festival, sommerfestival.	

Nora stated in Turn 4, Table 7.32, that they need to find something ‘easier’ to make a proposal about, than figuring out what was needed for a youth party. The students continue to exchange ideas, suggesting a festival, but it is unclear at this point whether they are looking for something that fits their ideas of what to do in a mathematics class, or whether they think this would be unfit for the grant schema.

A bit later in the discussion, see Table 7.33, Nora suggested that one thing she missed was a proper fitness centre, so she noted this as an idea. Aud, who was a part of her group, protested that this would not be feasible for the amount of money they could apply for (Turn 1, Table 7.33). After this interaction they agreed to record it as an idea, and seemed to regard the suggestion as interesting, but not realistic for this grant (Turn 4 and 5, Table 7.34). Here it is clear that the students rejected the idea on the basis of their considerations of the grant, rather than of what would be suitable for a mathematics class. The students’ discussion of the possibility of implementation of the idea of a fitness centre is presented in the example below.

Table 7.33: The students point out that the suggested projects need to be suitable for the grant size.

Turn	English	Norwegian	Comments
1	Aud: Do you want to start a fitness centre for 20 000? (Pointing at the sheet).	Aud: Skal du starte treningssenter for 20 000? (Peker på arket).	Aud seems very sceptical of the idea of using the grant money for a fitness centre.
2	Nora: But it’s just to get some support.	Nora: Ja, men det er liksom for å få litt støtte, her da.	Both Aud and Nora giggle, and seem to regard the idea as a bit far-fetched.
3	Ragnhild: You can create a budget of more than 25 000 of course, but then you must find a way to bring in the rest of the money.	Ragnhild: Dere kan jo lage et budsjett på mer enn 25 000 selvfølgelig, men da må dere finne en måte å få inn resten av pengene på.	The teacher tries to offer a solution: that the group could make a budget for large-scale ideas if they figure out how to raise additional money.
4	Nora: So lame, I’ve just thought of something, Aud.	Nora: Så teit, jeg har jo bare kommet på noe, Aud.	Nora defends her statement by saying that it was just a suggestion.
5	Aud: It would have been smart to have.	Aud: Det hadde vært smart og hatt det.	Aud agrees that it would be nice to have a fitness centre.

Both Aud and Nora seemed to agree that a fitness centre is something that they would like to have in their community, but they quickly realised that it was not appropriate for Frifond. Ragnhild, the teacher, pointed out in Turn 3 that it was possible to make project proposals which would need additional founding to implement, but the students did not pick up on this point, and continued to discuss other ideas. In the continuation the students in the group discussed what they missed in their neighbourhood, and suggestions of a shopping mall and a Starbucks came up but were dismissed as not relevant.

Because of the camera placement, there are no data about the decision-making processes of the group that decided to arrange a photo exhibit. However, one of the students in that group, Marte, wanted to be a photographer as a career, and it is reasonable to guess that the idea is connected to suggestions from her.

In the process of figuring out what projects to work on, the students dismissed some ideas because they were considered irrelevant for the grant scheme, and at least one idea was dismissed as too difficult. Why the student groups decided on their final projects is difficult to trace in the data. When the students presented their ideas to the rest of the class after the 20 minutes, one student in the group would tell a bit about their idea, and the teacher would have a short interaction with them about it, before the next group presented their idea. The teacher asked the other students what they thought about the projects and if they would have been interested. From the following presentation it seems that projects that were interesting to the other students were valued.

Table 7.34: The students share their project ideas.

Turn	English	Norwegian	Comments
1	Lukas: We have a photography exhibition	Lukas: Vi har fotoutstilling.	The group tell that they wanted to arrange a photography exhibition, and the rest of the class seemed like they thought this was an interesting idea. Ragnhild told them that they had a good start on the idea.
2-5	(Some exchanges around this idea.)		
6	Ragnhild: Fine, what do you others think?	Ragnhild: Fint, hva synes dere andre?	
7	Student: Cool.	Elev: Kult.	
8	Student: Pretty cool that they dare to do [this].	Elev: Ganske kult at de tør å gjøre [det].	
9	Ragnhild: You have a good basis to continue working with.	Ragnhild: Dere har et godt grunnlag å jobbe videre med.	
10	(Irrelevant exchange)		
11	Student: Climbing forest here. *(Points out the window)*	Elev: Klatreskog her. *(Peker ut vinduet)*	The students present their idea of a climbing forest and are asked by the teacher if they have anything specific that they know they need.
12	Ragnhild: Nothing more concrete than that yet?	Ragnhild: Ikke noe mer konkret enn det enda?	
13	Student: Like it is at Evje. We have not started yet [on the planning].	Elev: Sånn som er på Evje. Vi har ikke begynt på enda [med planlegginga].	Here the students refer to a climbing forest in a small town close by.
14	Ragnhild: What do you others think about it, would it be fun?	Ragnhild: Hva synes dere andre om det, hadde det vært gøy?	The teacher then asks if it would be fun to do this, and if anyone has tried that climbing forest. The students start to discuss if it is just
15	Students: Yes, yes, fun	Elever: Ja, jo, dritgøy	
16	Ragnhild: Is there any who have been to Evje and tried it?	Ragnhild: Er det mange som har vært i Evje og prøvd det?	

17-21	(Some more exchanges on this idea, and if it is for just kids or adults as well).		for kids or for adults as well.
22	Ragnhild: Yes, that sounds good.	Ragnhild: Yes, men det høres greit ut det.	The teacher confirms the idea and move on to the next group.
24	Student: Volleyball tournament on the city beach, maximum 16 teams, hire referees.	Elev: Volleyballturnering på bystranda, maks 16 lag, leie inn dommere.	The next group tell that they are thinking of a volleyball tournament at the local beach, and the teacher asks the rest for the class if that would be of interest.
25	Ragnhild: Yes, is that something you would like to be a part of?	Ja, er det noe dere ville dratt på?	
26	Student: No, it sucks. (irony)	Elev: Nei, det suger. (ironi)	
27-45	(Discussions of numbers of teams and contestants.)		
46	Ragnhild: Concretize a bit more when you work with the project next.	Ragnhild: Konkretiser det litt mer etterpå når dere lager prosjektet.	The students is asked to be more specific when they continue to plan.
47	Student: Data party in the hall, it lasts for 3 days, kiosk, internet and electricity and everything.	Elev: Data-party i hallen, det varer i 3 dager, kiosk, internett og strøm og alt som er.	The last group give brief keywords for their plan for a data party, and what they need, and the teacher acknowledges this.
48	Ragnhild: Yes, that sounds good.	Ragnhild: Ja, det høres bra ut.	

The teacher asked if the other students would like to take part in the suggested projects and validated the students' ideas as worthwhile to explore further and make project proposals about. There was no lively whole class discussion about the projects, and they were all accepted as something to continue working with. In this part of the planning the teacher did not ask the students for specific mathematical issues, and the students did not ask. The groups were then asked to continue working. As the groups continued on their budgets, indications of the norms underpinning this work with the task began to be visible, as the students questioned how the task should be solved, and how precise they needed to be.

After the students had worked in their groups for a while, the group that wanted to arrange a photography exhibition made a suggestion for a budget. Here the issue of how much money to apply for came up, showcasing a possible expectation that in a mathematics class, a budget would be tailored to fit the suggested amount. In Table 7.35 the students have made a budget of around 7000 NOK, under a third of the maximal amount of 25 000. When I ask in Turn 3 if the group needed more money, one student, Ingrid, responds by asking in Turn 4 if it is necessary to get close to the maximal amount.

Table 7.35: Discussion on if the projects has to be close to the maximal amount possible.

Turn	English	Norwegian	Comments
1	Trude: Location, picture frames, camera, ads, printing of programs, food. How much does it cost in total? Oh, yes, this was not expensive.	Trude: Lokale, bilderammer, kamera, annonser, printa program, mat. Hvor mye koster det her til sammen? Åh, ja, det var ikke dyrt det her.	Trude is reading the students' budget suggestion out loud. Then I comment that this is not expensive, before the student tells how much the total budget is.
2	Ingrid: 6900	Ingrid: 6900	
3	Trude: 7000 yes. Can't you get closer to 25? What else can you spend money on? Do you need more money?	Trude: 7000 ja. Kan dere ikke klare å komme dere nærmere 25? Hva mer kan dere bruke penger på? Trenger dere mer penger?	Here I ask if they want to get closer to 25 000, and if they are sure that they do not need more money.
4	Ingrid: Do we need to get close as possible to 25?	Ingrid: Må vi være nærmest mulig 25?	Ingrid asks a relevant question – do they need to be close to the maximal amount? Which is indicative for that she is worried that the budget may not be appropriate.
5	Trude: If you are going to apply for money, is it wise to [apply for] much money? No, you do not need to do it.	Trude: Om dere skal søke om penger, så er det lurt med mye penger? Neida, dere trenger ikke det.	

Picking up my suggestion that they could apply for more money, Ingrid asked whether the budget needs to be close to 25 000. My reply in Turn 5 indicates two different things: that in real life when a budget and application is made it can be a good idea to apply for more than the bare minimum to take into account unforeseen expenses, but also that they do not need to make up expenses just to get to the maximal amount possible. This suggests that I am responding to the students' unfamiliarity with the task, and their uncertainty about whether they had done what they were supposed to do.

The same group also questioned how carefully they needed to plan after I tried to ask then to explain what kind of printed programme for the exhibition they planned to make. Here the group needed to decide what the cost for making of a programme would be, and when I thought out loud about a possible design

(Turn 3), Ingrid asked if they needed to plan as carefully as my questions suggested.

Table 7.36: How do detailed do one need to plan for the photography exhibition?

Turn	English	Norwegian	Comments
1	Trude: Should you print the program, or ?	Trude: Skal dere trykke program, eller ?	The students wanted a programme for the exhibit. Marte explains that they would make it in a design programme.
2	Marte: Use InDesign.	Marte: Bruke InDesign.	
3	Trude: Use InDesign, so you only need a printer, regular printer?	Trude: Bruke InDesign, så trenger bare printer, vanlig printer?	Trude then want to figure out if the students could print it themselves.
4-10	(Short discussion about this)		
11	Trude: Should it be like that 1-2-3-4. (mimics a folded A4 sheet)	Trude: Skal det være sånn 1-2-3-4. (mimer et bretta A4 ark)	I try to figure out if the programme could have been made of a A4 sheet.
12	Ingrid: Do we have to think that carefully?	Ingrid: Må vi tenke så nøye?	Ingrid queries what kind of detail is expected for creating a budget.
13	Trude: No, I do not know, but if it's just a sheet, is it easy to make it yourself?	Trude: Nei, jeg vet ikke, men om det bare er et ark er det lett å lage selv?	

In this exchange, Ingrid asked in Turn 12 if they needed to think that carefully about how the programme should be designed. Here there is a difference between my intention that they reflect on what kind of printer that was needed, and Ingrid's reaction which seems to reflect a concern about not understanding how to plan the project, which is perhaps mixed with a worry that they will have to do more work than they thought. This is indicative of a worry of not understanding how to act in the solution of the task. After this interaction, the students quickly decided that they could print the programme on an ordinary printer, and then they add a cost for printing the programme in their budget.

At the end of the lesson when the teacher talked to this photography exhibit group, she pointed out that "this is a nice project, which could have been implemented[45]". Also, in the teacher's closing remarks of the lesson she asked

[45] Norwegian: Dette er et fint prosjekt, som kunne blitt gjennomført.

the students if they believed they could have implemented the projects and got a chorus of yes.

Table 7.37: The teacher's comment at the end of the lesson.

Turn	English	Norwegian	Comments
1	Ragnhild: Everyone has worked relatively well, at least most, and made fun projects, eh, hush, does anyone think that you could have gone through with this?	Ragnhild: Alle har jobba forholdsvis bra, i alle fall mange, og lagt gøye prosjekter, eh, hysj, er det noen som tror at dere kunne gjennomført dette her?	
2	Students: Yes	Elever: Ja	Several voices of agreement from the students.

These examples are indications that the students were questioning when they worked on finding projects. The students seemed a bit unsure about what would be proper solutions and how carefully they would need to plan. In her discussions with the students, the teacher showed interest in whether the students liked the idea of participating in the different projects and praised those that were possible to implement. It is possible that the students were conforming with the norms of working in a mathematics class rather than feeling that they had autonomy to choose to investigate more complex projects that they were interested in.

As in the moped engine cylinder task, uncertainty about how and what to write down in order to perform the task was also evident in the data. The group working on the photography exhibition decided to use a predefined budget spreadsheet, while the group planning the volleyball tournament decided not to note down some of their assumptions. The students appeared to trust in mathematical tools without reflection on when and why the tools would be appropriate. In Table 7.36, discussed above, the students questioned how carefully one needed to plan an item in the budget, and Table 7.38 below records an episode that highlights the students' reasoning about what to write down. The episode is from the group that planned a volleyball tournament, and their discussion about how much soda they needed for the kiosk.

Table 7.38: What is necessary to note down?

Turn	English	Norwegian	Comments
1	Trude: Should you write on the side what have you done?	Trude: Om du skriver på sida hva det er du har gjort?	Trude asks if they want to note down their calculations for figuring out the amount of soda needed.
2	Student: We don't bother	Elev: Det gidder vi ikke	The student replies that that is not necessary, because they remember how much soda they had used in their calculations
3	Trude: How much soda is it then?	Trude: Hvor mye brus er det da?	
4	Student: We know how much it is.	Elev: Me vet hvor mye det er	

In Turn 2 and 4 the students told me that noting down the calculation is not necessary, because they knew what they have done. This is an indication of what the students find worthwhile to do in this budget, and it may seem like writing down a number is more important than explaining for a reader of the budget how that number is arrived at.

A particular problem arose with the students in the photography exhibition group, who chose to utilise a predefined budget template which they had used in earlier lessons. This template was designed to work with particular textbook questions, where the students used given data to add accounting information to complete a budget. The given task, and the Frifond website, did not specify a budget format, which may account for why the students looked for what they had done earlier in their mathematics class, and decided to use this budget template.

However, this predefined budget template did not fit the Frifond project task, since the spreadsheet was designed to be appropriate for making monthly budgets of a specific type. The spreadsheet had cells that asked for "inventory at the start of the month[46]", "inventory at the end of the month[47]" and "profit/loss[48]". In addition, there were columns entitled "budget" and "accounting". These headings meant that the template did not fit the students' budget requirements. For instance, "inventory at the start of the month", has no meaning in the context of a arranging photography exhibition. Also, the notions of profit/loss, and

[46] Norwegian: Pengebeholdning i beg. av md
[47] Norwegian: pengebeholdning i slutten av md
[48] Norwegian: Overskudd/underskudd

accounting were not necessary for a budget of this type. Nevertheless, the students attempted to use it. A translated version of the students' draft budget for the photography exhibit is shown in Figure 7.9.

	Budget	Accounting
Expenses		
Location	2000	2000
Frames	1000	1000
Camera	500	500
Newspaper advertisements	150	150
Printer	2700	2700
Programme	50	50
Food and drink	500	500
Total	6900	6900
Profit/loss	0	
Inventory at the start of the month		
Inventory at the end of the month		

Figure 7.9: Translated example of the students' input into the predefined budget template

As shown in Figure 7.9, the students listed expenses such as location, frames, camera, newspaper advertisements, printer, programme and food and drink. When the group started to fill in the budget template they got stuck on the issue of "inventory at the start of the month". They asked if it would be how much they had borrowed (Turn 1, Table 7.39), and the teacher, Ragnhild, replied that the grant money would have to be in the income column.

Table 7.39: A discussion of how to fill in the budget template.

Turn	English	Norwegian	Comments
1	Ingrid: But you, inventory at the beginning of the month, there is not that much money there. That is probably what we have borrowed.	Ingrid: Men du, pengebeholdning i begynnelsen av måneden, det er jo ikke så mye penger der. Det er vel det vi har lånt.	Ingrid seems confused by what should appear where in the budget and talks about borrowed money when she means the grant money.

2	Trude: Inventory at the beginning of the month, it's in a way how much, inventory at the beginning of the month, it does not make sense in such a budget.	Trude: Pengebeholdning i begynnelsen av måneden, det er jo på en måte hvor mye, pengebeholdning i begynnelsen av måneden, det gir jo ikke noe mening på et sånn et budsjett.	Trude tries to argue that the cell does not make sense for a budget such as the one the students were making.
3	Ingrid: But it is the money?	Ingrid: Men det er jo sånn penger?	The teacher helps the students place the grant money in the part of the spreadsheet template for income (not shown in Figure 7.9)
4	Ragnhild: No, put that on income. Put the loan on income. Or, not a loan, but the grant money.	Ragnhild: Nei, det setter dere på inntekter. Sett lånet på inntekter. Eller det er ikke lånet, men støtten.	
5	Trude: How much does the income have to be for this to be okay?	Trude: Hvor mye må inntekta være for at dette skal gå greit?	
6-9	(Some irrelevant exchanges)		
10	Ingrid: Those numbers [under accounting]?	Ingrid: De tallene [under regnskap]?	Ingrid asks about the numbers in the accounting cells, and what should be done there. Trude and Ragnhild explain that these cells do not need to be considered.
11	Trude: It's because you do not have the real numbers.	Trude: Det er fordi du ikke har de reelle tallene.	
12	Ragnhild: So really you can only fill in the budget now, you do not have to fill in both.	Ragnhild: Så egentlig kan dere bare fylle ut budsjettet nå, dere trenger ikke fylle ut begge.	
13	Ingrid: Oh, okay.	Ingrid: Åh, ja.	

In this interaction the teacher had to explain that the students did not need to fill in all the cells. As shown in Figure 7.9 the students had filled in the same numbers in both the budget and accounting column, and in Turn 10 Ingrid questioned the numbers under accounting. Ragnhild argued in Turn 12 that the students do not need to fill in both columns.

It appears that the norms that regulate what to do in a mathematics classroom exert a strong influence here. When the students work with the task, they are in a mathematics classroom, with their mathematics teacher present, and are supposed to make a budget. They have worked with budgets in their mathematics lessons before, using premade budget templates, and the fact that they looked for a familiar template to use for the Frifond project task is maybe not surprising. According to the standard norms of the mathematics classroom, there is no need to do anything other than figuring out which procedure they should use. The teacher needed to tell them that the budget template did not fit, and they did not reflect on the appropriateness of the tool.

However, this incident contrasts with other episodes where students' knowledge of relevant practice was valued. There are snippets where the students connect what they are planning with what would happen in the real world, and they use the Internet and the teacher to figure out what information they need, and to confirm their estimates. One example of this valuing of information from fellow students can be seen in the excerpt below, where the photo exhibition group estimated how many photographs they could sell. The students first discuss and agree that they could expect to sell approximately half of the 25 photographs they wanted in the exhibit and finally agree on 10 sales in the budget (this is shown in Table 7.49, Section 7.3.2). The students then had to decide the sale prices of the photographs. In the following example, one can see that the students did not randomly decide these prices. Instead they had a discussion based on the purchase cost of necessary items like frames and printing, combined with their experiences of a common price of such photographs. This is an indication that the students saw this aspect of the task (figuring out prices) as authentic in this moment. Here the students discussed the price point with Trude, and Marte started by telling the rest of the group that the prices vary.

Table 7.40: Students' use of knowledge of the practice to estimate prices.

<table>
<tr><th>Turn</th><th>English</th><th>Norwegian</th><th>Comments</th></tr>
<tr><td>1</td><td>Ingrid: Well, how much do we take for a photograph, then?</td><td>Ingrid: Ja, hvor mye tar vi for et bilde, da?</td><td rowspan="2">Marte points to the fact that photographs and art do not have a fixed price point.</td></tr>
<tr><td>2</td><td>Marte: It differs.</td><td>Marte: Det er jo forskjellig.</td></tr>
<tr><td>3</td><td>Trude: Let's see, you have to think that the photo frame costs something, and the print of the photos cost something, then you need to at least sell it for more than that.</td><td>Trude: Skal vi se, dere må tenke at bilderamma koster et eller annet, og selve printinga av bildene koster et eller annet, så må iallefall selge det for mer enn det.</td><td>I try to point out that it is important in sales to at least break even, so that one can pay for the cost of printing and framing the photographs.</td></tr>
<tr><td>4</td><td>Ingrid: I believe we noted 3700 for print [in our budget].</td><td>Ingrid: Printe det blir 3700, tror jeg vi tok [i budsjettet].</td><td>The students use their previously found prices to</td></tr>
</table>

6	Marte: Yes, it was 25 photographs.	Marte: Ja, 25 bilder var det.	identify a minimum price point. (In between turns 4, 6, 9, 11, 13 there are confirmations, like 'mm' from the rest of the group.)
9	Marte: We have to divide all that by 25, because that is the total!	Marte: Vi må jo dele det der på 25, for det der er jo totalt!	
11	Ingrid: And then we need to divide the photo frame price with 25.	Ingrid: Og så må vi dele bilderamma på 25.	
13	Ingrid: 108.	Ingrid: 108.	The minimum price is found to be at least 108 kroner per printed photograph to break even.
14	Trude: 108 kroner, and then you need to sell it for more than 150 kroner.	Trude: 108 kroner, og så dere må jo selge det for mer enn 150 kroner.	
15	Group: Yes, yes.	Gruppen: Ja, ja.	The group agrees.

Having identified a minimum price point (Turn 13, Table 7.40), they started to consider the knowledge they have of art exhibitions, as seen in Table 7.41. Marte pointed out that the price of photographs she has seen has been much higher (Turn 1, Table 7.41), but that the group as amateurs cannot ask for such prices. Both in the excerpts presented above and below, Marte is clearly treated as the expert, unlike the usual practice in this mathematics class. In the excerpt below, one can see that she first talked about her previous experiences, before she gave a suggestion which is taken up by the group.

Table 7.41: A discussion of what would be proper pricing of the photographs.

Turn	English	Norwegian	Comments
1	Marte: There are a lot of the paintings that, eh, are expensive. Since they are probably like a square metre. So they were up to one thousand and two thousand and three four five thousand, prices like that, but I do not know if we should sell the pictures for so much.	Marte: Det er jo mange av de maleriene som, eh, men de er dyre da. Siden de er sånn, de er sikkert sånn kvadratmeter. Så de var jo opp mot tusen og totusen og tre fire fem tusen og sånne ting, men jeg vet ikke om vi skal selge bildene for så mye.	Marte shares her previous experiences with the rest of the groups but seems unsure if that would be a fitting price range for their project.
2-4	Trude: It is perhaps a bit much again, I think we need to be below a thousand.	Trude: Det er kanskje litt mye igjen, æ tror under tusenlappen, det må vi [nok].	Ragnhild and I agree with Marte's hesitation and suggest a lower price range. Marte suggests a price
5	Ragnhild: Maybe I would have tried that.	Ragnhild: Kanskje jeg ville prøvd det.	

6	Marte: Shall we take 500, 500 for all the photos then?	Marte: Skal vi ta 500, 500 for alle bildene da?	below her previous experiences, but above the minimum price they had calculated in Table 7.40.

The episode ended with Marte and Ingrid calculating that they would budget for an income of 500 kroner times ten photographs sold, so 5000 kroner was noted in the income column in their spreadsheet. Marte participated in this decision with suggestions which were valued in the conversation. She listened to the teacher's and my input, but she was the one that suggested the number that the students finally ended up with in the budget. In this episode a student's knowledge of the practice is valued and her suggestions are used, suggesting that Marte was regarded as the knower in the practice, and was treated here as the authority.

Generally, the students drew on other students and the teacher to figure out issues they were unsure of. When the volleyball tournament group started on their budget for the kiosk, they needed to calculate how many sodas, hotdogs and hamburgers were needed. Here they needed to estimate the amount needed and find the relevant prices for the inventory. In the following dialog Hanne asks if they could just write down a number (Turn 5, Table 7.42), apparently instead of calculating a reasonable price or taking the time to identify a plausible price.

Table 7.42: How to find numbers for the budget for the kiosk in the volleyball tournament.

Turn	English	Norwegian	Comments
1	Trude: How much does a case of soda cost?	Trude: Hvor mye koster ei kasse brus?	The students need to calculate the cost of sodas, and the student first tries to figure out a price, before suggesting that they can just write something.
2	Hanne: A small [soda], how much are they? 0.5?	Hanne: En liten [brus], hvor mye er de? 0,5?	
3	Trude: 0.5, you can probably find [the price].	Trude: 0,5, finnes sikkert [pris] på.	
4	Hanne: Can't we just write something?	Hanne: Kan vi ikke bare skrive noe?	
5	Trude: How much does a soda cost then?	Trude: Hvor mye koster ei brus da?	Trude tries to engage the student in an estimate, but the student claims not to know the price of sodas.
6	Hanne: Do not have a clue.	Hanne: Har ikke peiling.	

7	Trude: Then it's just, how many holes are there in a case?	Trude: Da er det jo bare, hvor mange hull er det i ei kasse?	Trude continues with asking how many sodas they would have in a case, and Oscar knows this.
8	Oscar: 24 [in a case].	Oscar: 24 [i ei kasse].	
9	Hanne: But you, you do not pay as much for a case, when you buy a whole case.	Hanne: Men du, du betaler ikke like mye for ei kasse, når du kjøper ei kasse.	While she says this Hanne calculates on her cell phone and tries with a soda price of 13 kroner.
10	Hanne: Oh, wow.	Hanne: Å herremin	Shows her cell phone with 312, from the calculation 13·24
11	Hanne: 300?	Hanne: 300?	
12	Oskar: Yes.	Oskar: Ja.	
13	Hanne: It is a bit much.	Hanne: Det ble litt mye.	Hanne uses her fingers to recount, then goes on to work on the calculator.
14	Trude: No, but take a little less then?	Trude: Nei, men ta litt mindre da?	
15	Student: Let's say 250.	Elev: Vi sier 250.	

Here there are several things happening. At the start, Hanne seemed like she would prefer to just write down a number (Turn 4). However, when the other students started to calculate the price of a case of sodas, Hanne pointed out that, in real life, a case of sodas is less expensive than the same amount of sodas sold separately (Turn 9). Then she calculated the price of 24 sodas and seemed surprised by her answer (Turn 11). In the end the students decided to take my advice to record a lower price, but in fact opt for a number that is a lot less than the number Hanne has calculated. The students do not discuss the numbers, and their exchanges of ideas are generally rather short before they decided they are satisfied and could note down a number.

The students used the teacher more as a sounding board for the price-ranges, and there are few interactions connected to the mathematics issues of making budgets. I found that the students were insecure at the beginning of the task about how much they could choose and define, but quickly settled into a pattern of interactions where they found some information, and tried to verify the information they were unsure about with the teacher. This suggests that the teacher was seen as the authority, and the person that would define if they had done a good enough job. Having decided on their project ideas, the students did not refer to Frifond as the recipient of the project application and budget. In this respect the task became embedded in the mathematics classroom routines, although the students did try to find prices online and drew on the knowledge of students with relevant experience.

However, in this classroom implementation there is one more issue that is noticeable from the video recordings. There was quite a lot of off task talk as the students were working, with the students apparently starting to work only when the teacher is close. For instance, in the last of the three lessons, the group that worked on planning the volleyball tournament used the first 18 out of 45 minutes to talk about applying for school, rules for scholarships and were playing on their cell phones. When the teacher came close to the group, they started to talk about the task. Likewise, in the group that planned the climbing forest there was one student that did not engage in the discussions in the group at all. Overall, it appeared that the students were inclined to minimize their effort on the task. Thus, despite evidence that the students were at times acting in accordance with recognition of the out-of-school experience demonstrated by students such as Marte, for much of the time they were operating more in accordance with norms which are familiar in traditional mathematics classrooms, including minimizing effort.

7.3.2 Connections with Out-of-School Practices

As seen in the previous section, the students drew on their own experiences as they worked on this task. In this section I explore how the students interacted with the task in relation to their experience of out-of-school practice in more detail. The task refers to a real website, and a real grant scheme, and therefore there are several potential authentic aspects to it: developing a project proposal and a related budget is clearly possible also outside a mathematics classroom, and the students were asked to gather and use information from the real world. The students referred to places and experiences from the local community as they discussed the projects and their expenses, providing evidence that they connected the task with experiences from outside the mathematics classroom, and that some aspects of the task were regarded as authentic. However, despite this, they sometimes reduced the complexity of the proposed projects in order to make working on their proposals easier.

In the following episodes, I present examples of students referring to, and using information from, outside the mathematics classroom. As shown in Section 7.3.1, both the students and their teacher appeared to look for proposals that would be realistic and possible to implement. For example, one group talked about how they missed a fitness centre, a shopping mall and a Starbucks (see Table 7.33), but these ideas were dismissed by the students themselves as not relevant for this grant. When another student group discussed their idea of a climbing forest, they referred to a familiar example of a climbing forest in a nearby small town. The teacher used the same local example to validate this project idea (see Table 7.34).

After the groups had decided which project idea to explore further, they needed to figure out what items they would have to include in their budget, and then find reasonable estimates or real prices for them. In this process, the students drew on their own previous knowledge of prices, but also tried to find prices on the internet. In the exchanges shown in Table 7.43, the teacher initially provides information that copy shops print photographs in big sizes (Turn 2, 4 and 6), but then Lukas, having looked for prices on the internet, commented on his experiences with online shops, that most of them do not reveal their prices online (Turn 15).

Table 7.43: A discussion on where to print photographs.

Turn	English	Norwegian	Comments
1	Marte: That's the kind of printer that will print A3 paper.	Marte: Det er jo sånn printer som skal skrive ut A3-ark.	Marte explains that they are looking to buy a printer to print the photographs, before Ragnhild argues that this can be done at copy shops.
2	Ragnhild: Can’t you do it copy shops, Allkopi, for example?	Ragnhild: Gjør det ikke det på sånn kopi, Allkopi, for eksempel?	
3	Marte: Like that big, I thought they just printed smaller sizes?	Marte: Sånn derre svære, jeg trodde de bare skrev ut sånne små.	
4	Ragnhild: No, Allkopi is that kind of copying place, they print photo wallpapers and posters and.	Ragnhild: Nei, Allkopi er sånn kopieringsgreier, de skriver ut fototapeter og plakater og.	Ragnhild argues that this can be done at a printing shop, and Marte says that she had been thinking of the photography stores, where she knew that A3 printing wasn’t available.
5	Marte: I was thinking of places like Japanphoto and Elitefoto and stuff.	Marte: Jeg tenke på sånn Japanphoto og Elitefoto og sånt.	
6	Ragnhild: Oh, but those kinds of copy companies print anything.	Ragnhild: Åh, ja, sånne kopieringsfirmaer skriver ut hva som helst.	
7-14	(The students look for prices on the website and talk about the where to click on the website.)		
15	Lukas: It's very difficult to find prices for these things, because they do not bother to give you	Lukas: Det er veldig vanskelig å finne priser på sånne ting, for de gidder ikke gi deg	Here Lukas argues that finding prices online is difficult.

	prices until you get there. They change them all the time.	priser før du kommer der. De endrer på de hele tida.	

At the start of this exchange, Marte and the teacher referred to real shops, and Marte drew on her experience with photoshop chain stores, that they do not print photographs in size the students want for their project. However, Marte did not seem to know that printing shops, which are geared towards posters, signs and similar items, can usually print photographs in large sizes as well. Then the students tried to find prices on the webpage for the printing service that the teacher talked about. Here Lukas drew of his experience and argued that finding prices online is difficult. He also implied a reason for this, which is that businesses can then tailor their offers if they get customers to get in direct contact.

This point was also made clear during the group's attempt to find rental prices for the exhibition location. Here the students looked at the webpages of their two suggested locations, the local art museum or a youth club. First, they do not find prices for renting the art museum, and then they move on to try the youth club webpage. There they found for rent information, but found that the price information is unavailable, and that it was necessary to contact the youth club by mail to get an offer for rental prices. Then, in Turn 2, Lukas repeated his earlier point that prices are never revealed online.

Table 7.44: Lukas argues that businesses do not reveal their prices online

Turn	English	Norwegian	Comments
1	Trude: Oh, no I've seen, to rent. Maybe that's right. Oh - for booking, mail.	Trude: Oh, nei jeg har sett, utleie utlån. Tror kanskje det er riktig. Hø – for booking mail.	The correct place on the webpage is found, but after opening it says to mail for bookings.
2	Lukas: They never give prices online.	Lukas: Kommer aldri med noe pris på nett.	Lukas repeats his earlier argument that prices are not online.
3	Trude: What do we reckon the youth club costs to rent then?	Trude: Hva kan vi tro ungdomshuset koster å leie da?	I then suggest that we think about what a youth club could be rented for, and Lukas refers to a price he has found previously for another location.
4	Lukas: I searched for a price list earlier, and then I found a rental price.	Lukas: Æ søkte på prisliste i sta, og da fant jeg leie på.	
5	Marte: 1800 per day.	Marte: 1800 pr dag.	

6	Marte: Shall we just guess that it costs 1800, then.	Marte: Skal vi bare gjette at det koster 1800, da.	Marte suggests that they just continue with 1800 kroner and argues that it cannot be expensive.
7	Marte: It cannot be very expensive.	Marte: Det kan jo ikke være veldig dyrt.	

After Lukas's argument that prices never are revealed online, the students needed to find a way of figuring out a realistic cost. Lukas argued for his suggestion on the basis an earlier pricelist he found for another place available for rent. The group took up this suggestion without much argument and added that they do not believe that the youth club would be very expensive. They did not consider contacting the rental places or using more time on this part of the task, having exhausted their options of looking online at their two suggested locations. However, working on finding their own prices and using real sources is an authentic element in the task.

The students found other prices online. For instance, they managed to find a suggested price for printing the photographs. Here they discussed the price briefly before moving on.

Table 7.45: The students used the internet to find the price for printing photographs.

Turn	English	Norwegian	Comments
1	Trude: But it's poster, I do not know if [that's right] It's not that kind of photo paper. You can try that custom [print button].	Trude: Men det er jo plakat, vet ikke om atte. Det er jo ikke sånn fotopapir. Du kan prøve sånn egendefinert [knapp].	The students first try to use the "poster" option; however, this would not be printed on photography paper.
2-6	(Students talk about where to navigate on the website.)		
7	Lukas: For now I added 25 pictures.	Lukas: For nå tok jeg 25 bilder.	Lukas identifies a suggested price for printing 25 photographs.
8	Trude: Oh, yes, you did, yes, then it became a lot less expensive per picture.	Trude: Åh, ja, det gjorde du ja, da ble det en del mindre dyrere pr bilde.	
9	Lukas: 2700	Lukas: 2700	After Lukas said this price out loud, Marte wonders if it was for one or for all. Here she evaluates the price as to expensive if it was for only one photograph.
10	Marte: For all or for one?	Marte: For alle eller for ett?	
11	Trude: For all.	Trude: For alle.	
12	Marte: Yes, because I started to wonder about if it was for one.	Marte: Ja, for jeg begynte å lure på for på ett.	

The students wanted to know if the price was for one photograph, or for 25. Had the price been for one photograph, it seems that they would have objected, but having noted that the price was for all the photographs, they did not reflect any further on it. However, there were other occasions where the students questioned the information they found online. For instance, the students thought about renting a camera to take photographs at the exhibition. They found a potential camera for rent online, but the rental asked for a deposit, and a rent per day price. Here the students questioned the price of the deposit, and also seemed to realise that if they actually were to rent the camera, they would have a problem with paying for the deposit.

Table 7.46: Discussion of the deposit amount for a camera.

Turn	English	Norwegian	Comments
1	Lukas: Rent 5000, 40 000 deposit.	Lukas: Leie 5000, 40 000 i depositum.	Lukas has found a camera for rent online and reads up the deposit and the rent per day.
2	Trude: Oh, that was lots. 40 000 deposit?	Trude: Oi, det var mye. 40 000 i depositum?	
3	Ingrid: Hey, what, if you rent, then it's cheaper to buy one.	Ingrid: Hæ, hva da, viss du leier, da er det billigere å kjøpe et.	Ingrid points out that they could rather buy a camera.
4	Trude: But you get back the 40,000, but still it's a bit of an overkill with 40 deposit.	Trude: Men du får tilbake de 40 000, men likevel litt overkill med 40 i depositum.	The students echo my questioning of the deposit amount.
5	Marte: For a day? That's really weird.	Marte: For en dag? Det er jo helt rart.	
6-8	(Oskar rereads the information and Marte asks what kind of camera it is.)		
9	Ragnhild: The deposit tends to be like three times the rental price.	Ragnhild: Depositum pleier å være sånn tre ganger leiepris.	Ragnhild contributes a suggestion about how deposits are normally priced.
10-14	(A student rereads the information and there is talk about what deposits are.)		
15	Trude: 500 kroner a day was not expensive.	Trude: 500 kroner dagen var ikke dyrt.	Trude comments that the rent per day seems reasonable, and Lukas asks if they should note that down. However, Marte is worried about the payment of the deposit.
16	Lukas: Shall we write it down?	Lukas: Skal vi skrive det?	
17	Marte: But then we get problems with the payment.	Marte: Men da får vi jo problemer med betalinga.	
18-24	(Students work on finding another camera to rent. After about two minutes Lukas asks the group the following question.)		

25	Lukas: Is it normal to rent at all?	Lukas: Er det normalt å leie i det hele tatt?	Lukas starts to wonder if it is usual to rent cameras, and Marte says no to this.
26	Marte: No, it's not. It is normal to bring your own.	Marte: Nei, det er jo ikke det. Det vanlige er at en tar med sitt eget.	

In this episode the students questioned the information found online, and the teacher confirmed in Turn 9 that the amount seems unreasonably high, since deposits are normally three times the rental price. In Turn 15 I suggested that the price for one day's rent is not unreasonable, and Lukas were about to write down that suggestion before Marte rightly pointed out that in real-life they would need to pay the deposit (Turn 17). This led to further discussion in the group, and they started to question if renting a camera is normal. Marte argued that the normal practice is to bring your own camera. However, in their budget (see Figure 7.9), the students have added a line 'Camera 500', indicating that they entered this cost into the budget without solving the real-life issue that they would not have enough money for the deposit.

I had talked to Marte previously and knew that she wanted to be a photographer. During the group discussions she referred several times to her experiences as a visitor at art exhibitions. One example of this was when the students considered whether the visitors to their exhibition would need food. Marte referred to her experience to argues that it is usual to serve apple juice. This was considered by the group, and they used her estimates to add this to their budget.

Table 7.47: Marte knows what to serve at exhibitions.

Turn	English	Norwegian	Comments
1	Marte: We used to have apple juice like that, in sort of champagne glasses.	Marte: Vi har pleid å ha sånn eplesider, i sånne champagneglass.	Marte refers to her own experiences at art exhibitions, and Ingrid wonders how the juice is served.
2	Ingrid: Do you walk around and give it to those who are at the exhibition?	Ingrid: Pleid å gå rundt og gi til de som er på utstillinga?	
3	Marte: It's like, on a table, so that people serve themselves.	Marte: Det står liksom, på et bord, som de kan ta selv.	
4	Trude: You can put that in the budget. Food and drink for [guests].	Trude: Det kan dere jo legge inn i budsjettet.	Trude points out that this should be a part of their budget.

		Mat og drikke til [gjestene].	
5-14	(The students talk about how to add a row in Excel.)		
15	Trude: How much does a guest drink, a glass?	Trude: Hvor mye drikker en gjest, et glass?	Here Marte use her experience to estimate how much apple juice to buy.
16	Marte: No, I think they take two. If it comes in champagne glasses.	Marte: Nei, jeg tror de tar to, jeg. Om det kommer i champagneglass.	

Marte argued first that they could serve apple juice (Turn 1) and then she argued how much per guest (Turn 16). The group took up her suggestions; Marte clearly argued on the basis of her previous experiences at similar events.

When Marte and the rest of the group discussed which camera they should rent, they used the brand names “Canon” and “Nikon” and later, they specified that the leaflet for the exhibition would be made using the InDesign program (Table 7.36). The use of terms and brand names and references to practice in the planning of the exhibition indicated that the students certified by their own use of language that they regarded finding these prices authentic. Such certification was also evident in their planning of where to stage the exhibition - they referred to two potential real locations in the neighbourhood, the local art museum or the youth club.

Table 7.48: A conversation about where to rent, the local art museum or the youth club.

Turn	English	Norwegian	Comments
1	Marte: Where is the location [in the budget]?	Marte: Hvor er det lokale henne [i budsjettet]?	In the budget the students had noted location and needed to figure out the rental price.
2	Trude: Where were you going to be then?	Trude: Hvor var dere hadde tenkte å være da?	
3	Marte: The art museum in the city.	Marte: Det kunstmuseet i byen.	Marte refers to a real art museum in the closest city.
4	Trude: Try to search and see if it is possible to rent.	Trude: Prøv å søk og se om det går ann å leie.	
5-11	(Talk about what to search for online, and then they find the website for the art museum.)		
12	Marte: I know that it is possible to [have an exhibition] at the art museum, but I do not know if photo	Marte: Vet at det går ann å ha på kunstmuseet, men vet ikke om fotoutstilling [er mulig]	Marte seems sure that it is possible to have art exhibitions at the museum but is unsure if photography

	exhibitions [are possible]		exhibitions are possible.
13	Lukas: We'll take it to the youth club then.	Lukas: Vi tar det på ungdomshuset da.	Lukas suggests they go for the youth club.

Here Marte argued that she knew that holding exhibitions is possible at the local art museum, but she was unsure if photography exhibitions are possible (Turn 12). Instead of figuring this out, Lukas who had been looking at the website for the art museum without finding information about rentals, suggested moving on to their second suggestion. Here the students referred to real places, visited their websites and engaged with real locations in their planning. However, as seen in both Turn 13, Table 7.48 and Turn 5, Table 7.49, they were quite eager to move on if the information is difficult to find or figure out.

Marte's knowledge of how things usually were at exhibitions was also evident when the students planned the number of photographs to show. Marte referred to what she thought was normal practice, arguing that the usual number of photographs was 20 to 25.

Table 7.49: The students figure out number of photographs they want to show.

Turn	English	Norwegian	Comments
1	Trude: How many photos are you going to have?	Trude: Hvor mange bilder skal dere ha?	Trude asks about how many photographs they think of showing, and Marte argues that that depends on the location, but then refers to what she knew was normal.
2	Marte: It depends on how big the room is	Marte: Kommer ann på hvor stort rommet blir	
3	Trude: Either choose the room first, or number first?	Trude: Enten velge rommet først, eller antall først?	
4	Marte: It is normal to have around 20-25 photos	Marte: Det er normalt å ha rundt 20-25 bilder	
5	Ingrid: Just type 25 (quickly)	Ingrid: Bare skriv 25 (raskt)	The group agree quickly on this number

Here the students used Marte's suggestion, and it appears that Ingrid was happy as long as she got a quick decision on what number to work from. Marte also referred to her own experiences when the group needed to figure out the opening hours of the exhibition and argued that this would usually be about three hours, which meant that they only needed to rent an exhibition location for one day.

Many of the discussions about the group's project plan were decided with references to her experiences of exhibition routines and her knowledge of photography.

Likewise, the cost of the photographs was decided mostly on the basis of Marte's knowledge of what would be needed. The students needed to decide sizes of the photographs before they could do the cost calculation on the website for a local printing firm (see also discussion around Table 7.43). They discussed sizes A1, A2 and A3, but the issue was decided when Marte stated "It needs to be A2, because A3 is too small to hang [up on the wall]" (Turn 11, Table 7.50).

Table 7.50: A discussion of what size the photographs need to be.

Turn	English	Norwegian	Comments
1	Marte: But will A3 be too small, or will it okay?	Marte: Men blir A3 for lite, eller blir det nok?	The students wonder what size they should have the photographs printed in and have earlier noted A3.
2	Ragnhild: What room are you going to be in then?	Ragnhild: Hvilket rom skal dere være i da?	
3	Ragnhild: A3 is well…	Ragnhild: A3 er jo …	
4	Marte: Is it A2 or A1 or something like that?	Marte: Er det A2 eller A1 eller noe sånn ting?	Marte mentions other larger paper formats.
5	Ragnhild: I'm not that good at paper formats.	Ragnhild: Jeg er ikke så god på papirformater, jeg.	
6	Trude: Shall we see.	Trude: Skal vi se.	*Trude is pointing at A3 sheets in the classroom.*
7	Marte: They are too small.	Marte: De blir jo for små.	
8	Ragnhild: Can be twice as big.	Ragnhild: Kan ha dobbelt så stor.	Trude and Ragnhild talk about paper formats.
9	Trude: A1 is a square meter.	Trude: A1 er jo en kvadratmeter.	
10	Ragnhild: A2 is like 2 A3.	Ragnhild: A2 er sånn 2 A3.	
11	Marte: I think A2, then we cannot have A3, it will be too small. It will be A2, because A3 is far too small to hang [on the wall].	Marte: Jeg tenker A2, da kan vi ikke ha A3 det blir for lite. Det blir jo A2, for A3 er alt for lite til å henge [på veggen].	Marte argues for a larger size than A3 and suggests that A2 is best for hanging on the wall.
12	Trude: A1 is large, then you are up in.	Trude: A1 er jo svært, da er du oppe i.	Marte points out that there are some that would use A1 but it seems like she thinks it is not fitting here.
13	Marte: There are some who have A1, but ….	Marte: Det er jo noen som har A1 og, men ...	

The size of the photographs was decided with respect to Marte's arguments about what sizes that would be too small to hang. It seems that she referred to photographs she had seen previously, and the rest of the group let her decide without coming with suggestions.

The purpose of the task is to write a proposal for a grant application to Frifond. The teacher certified this by referring to the real webpage, and several times asking the students if they would want to participate in the suggested projects. The students could have applied with their project to the grant schema; however, they needed an idea that they really were interested in. The information aspect in the task was the same as they would have elsewhere, since the students were directed to the webpage. However, in a project planned out-of-school the students would probably have greater access to the needed information.

These data show how the students frequently drew on real out of school experiences and would become quite involved in discussions about what information was available and what was likely. In this sense, they treated the Frifond project task as connected to issues out of the mathematics classroom. However, despite the extent of these discussions, their final decisions in relation to their budgets tended to fall back on those options which were easiest to note down, and least complicated. They drew the line at extensive investigations in order to find out important details (eg what the options were for locations to rent) and omitted important details for a budget task such as the deposit for camera rental. In Chapter 8 I will look across the cases for commonalities and differences both with regards to normative activity, and the connections with out-of-school practices found in implementation of the tasks.

8 Cross-case Analysis: Opportunities and Challenges

In the previous chapter, I reported on the implementations of the hair salon budget task (see Sections 6.2 and 7.1), the engine cylinder task (see Sections 6.3 and 7.2), and the Frifond project task (see Sections 6.4 and 7.3), with respect to RQ1 and RQ2. Various issues arose in these individual analyses that highlighted how the tasks disrupted the norms of the classrooms, raising questions over where expertise lay, and the purpose of tasks in relation to work and school practices. In this chapter, I explore these disruptions further.

First, in section 8.1, I look at how the students juggled the routines and practices from the two different communities of practice, the mathematics classroom and the vocational community. Here the tasks lead to fluctuations in the routines of the mathematics classroom. I found that the students had two ways of working with the vocationally connected tasks. They would either continue to act according to the sociomathematical norms of the mathematics classroom, or sometimes they would draw on norms from the vocational practice.

In section 8.2 I will explore how the students' novice membership of the communities of vocational practice connected to the perceived authenticity in the tasks. I discuss how the students showed through language and routines that the tasks were perceived as connected to the vocational practice. In section 8.3 I discuss opportunities and challenges when working with the tasks.

8.1 Changes in Normative Activity: The Impact of the Tasks

The three tasks differ with regards to vocational education programmes. However, the students and the teachers interacted with each other and with the mathematics and renegotiated the classroom norms in similar ways. In this section, I focus on how the tasks disturbed the routines of the communities of practice. The students would either continue to assume the sociomathematical norms of the mathematics classroom when they worked on the tasks, or sometimes would draw on norms from the vocational practice. There were changes in roles in the classrooms, as the students sometimes took on the role as knowers of the vocational practice.

8.1.1 Sociomathematical Norms

In a community of practice the participants are engaged in a joint enterprise and relations of mutual accountability (Wenger, 1998). These relations include ‘‘what matters and what does not, what is important and why it is important, what to do and not to do, (…) what to talk about and what to leave unsaid, what to justify and what to take for granted'' (Wenger, 1998, p. 81). Taking the three classrooms in this study as communities of practice, I noted similarities in relations or norms in terms of how the students' interactions with each other and with the teacher were regulated. Likewise, there were similarities with regards to expectations about how students should work with mathematics tasks in terms of which issues were taken up for discussion and what were considered good or appropriate solution strategies. As discussed earlier, assumptions about how to act in a mathematics classroom can be analysed in terms of the concept of sociomathematical norms (Yackel & Cobb, 1996).

The students and teachers face similar conditions across the schools; they share artefacts such as the curriculum, mathematics textbook series and resources, and they have Norwegian schooling culture as a common background. Most of the students come straight from 10 years of school and have been socialised during this period of time into how to act in mathematics classrooms. Some common normative issues across the cases are connected to interaction patterns in the classroom. The students and the teacher show that they know how they are expected to act in the classroom, and what roles to take. The teacher is treated as the authority on what to do in mathematics. Students know how to act while working with mathematics tasks, and 'know' that a task should be solved quickly (or not at all), and that problematic real-world considerations can be disregarded if necessary. These shared histories of mathematics education could be seen to influence patterns of participation in the classroom in terms of where expertise was seen to lie, and how the tasks were treated in the classroom.

The students and the teachers interacted with each other with the tasks as the focal point. The tasks drew on contexts and issues from the relevant vocational community of practice and were intended to make connections between the two communities of practices. The first issue that arose from the analysis was that the students kept drawing on sociomathematical norms from previous schooling when they worked on the tasks. It seemed like the vocationally connected tasks were accepted by the students as "proper" tasks if they appeared to be an appropriate size and seemed to be solvable without too much fuss. In addition in all three cases, there was a continuity between standard classroom sociomathematical norms and those which applied when students were working with the tasks, namely that the students accepted that any related vocational knowledge that they might have could be disregarded if such disregard made the solution process in the mathematics tasks easier.

We can understand these findings if we look at the mathematics classrooms as communities of practices, where students are old-timers at being students in a mathematics classroom, and therefore know how one is supposed to act in the mathematics classroom, including how to work with mathematics tasks. As seen in Alrø and Skovsmose’s (2002) work on the exercise paradigm, students know that it is expected that all the information needed to solve a mathematics exercise is given, and that a mathematics question will have one, and only one, answer. Likewise, Verschaffel et al. (2000) identified that stereotypical word problems would often be solved uncritically by arithmetic procedures with the given numbers in the problems.

My analysis shows that the students treated mathematics tasks as something that should not be too complex and should be possible to solve in a short amount of time. It is evident that the students I observed would often look for easy and immediately found answers, instead of taking the invitation to engage with the vocational context. The need for the task to be of appropriate size was evident when the students in the media and communication class planned their Frifond application. The students wanted to know if their proposed Frifond project would be of an appropriate size for the work they were supposed to do in the mathematics lesson, or if they needed to plan a bigger project. Another group in the same class dismissed an idea because they said that it would be too complex to figure out the project proposal and budget, and that they wanted an easier and more feasible idea.

The students in the Design, Arts, and Crafts class showed the same tendency to prioritise finishing the budget instead of discussing the figures and assumptions which might affect the correctness of their budget proposal. This can be seen when Mari argued for disregarding the fact that they had calculated with 30 working days in a month (Table 7.6). Such issues were likewise evident with the students in the Technical and Industrial Production class. At the end of the engine cylinder task, the students were asked to find several solutions for possible cylinder dimensions but stopped when they had identified one solution. The student Erik also pointed out that the size of the engines given in the task was unrealistic, however chose to continue to work on the task anyway.

The expectation that tasks should be possible to solve quickly contrasted with the expectations of many of the projects the students worked on in their Programme Subjects; often these would be worked on over several days with the students responsible for progress and working methods. Jensen (2017) describes how students' competence is constructed differently in the mathematics classroom and the vocational education subjects. She argued that in the mathematics classroom the students had little room for making choices and sharing insights, but in the vocational education the students were accountable for finding solutions and being critical to their own and others' ideas. Likewise, I found that it seemed that there was little space in the mathematics classroom routines for the students to take responsibility for exploring and using time on tasks without contradicting assumptions about how to act in a mathematics class.

Another sociomathematical norm observed in the classrooms was that students would disregard practice knowledge if it complicated the task solution. This norm appeared to be connected to their earlier experiences of interaction patterns in mathematics classrooms. A task in mathematics is something which should be solved quickly and with the available information, and where the context is not important. This appears to be an example of attitudes to mathematics which appear in a number of research studies, for example disregard of sense-making (Schoenfeld, 1991), the concept of exercise paradigm (Skovsmose, 2001), studies of students' work on word problems (Verschaffel et al., 2000), and norms regarding contextual reasoning (Busse, 2005).

It is evident from the data that the students considered which information and knowledge was relevant to use or if the knowledge of the practice should be disregarded when solving tasks. I observed that they often disregarded their knowledge from the vocational practice in the setting of the mathematics classroom, finding it acceptable and advantageous to ignore information that could lead to difficulties with a solution.

When working with the engine cylinder task, students discussed issues relevant to the vocational practice: which brands of cylinders to use when tuning engines, and the need to exchange other parts as well to get good results from tuning. But here the students were willing to disregard their knowledge of the vocational practice to move on with the mathematics task. An example of this is when Erik worked on the question about a V8 engine and corrected my explanation of the placement of the cylinders (Table 7.30). He also pointed out that the engine size was incorrect since a V8 engine would have a bigger cylinder volume than 3.5 litres (Table 7.29). But immediately after he had corrected me on these two issues, he said that this mistake in engine size did not matter and indicated that he was impatient to find the correct calculation procedure to figure out the question. Here, the problematic aspect of the question was disregarded, and both he and I continued to solve the question without discussing or problematizing the given information. Also Fredrik solved the question even though he informed me that one replaced the cylinders instead of filing them bigger (Table 7.28).

The sociomathematical norm that it is possible to disregard real-world knowledge when solving mathematics tasks came to the forefront in the hair salon budget task when a group discovered that they had calculated with 30 working days in a month instead of 20 (Table 7.6). The students in the group agreed that working 30 days a month was not realistic. Nevertheless, their solution to the lack of realism was to ignore the problem with the 30 days and continue to use their previous calculations. In this situation, one can see a clear difference between the students Bente and Mari. Bente talked about herself working in the salon and what she would feel about working that much, while Mari argued that they could ignore their mistaken assumption since the hair salon was only imaginary. She pointed out that if they changed their assumptions from 30 days a month to 20 days a month, they would need to do the calculations again, and that was not worth their trouble. I interpret this episode as an instance where the group accepted a sociomathematical norm which states that it should be possible to finish mathematics tasks quickly. This norm again leads to the sociomathematical norm which allowed the students to ignore (their own) real-world considerations. In a real budget, ignoring realistic assumptions in this way would make the budget proposal unusable.

Such a sociomathematical norm (that mathematics tasks can be made less realistic) is similar to the statement of a teacher cited in Gellert and Jablonka (2009) who said, “I’m not talking about reality” when his students queried his introduction to a mathematics task. Norms which permit learners to disregard knowledge from practice were also observed in Busse’s (2005) study of norms that regulated to what degree contextual reasoning was appropriate for solving problems. These issues are also evident in Skovsmose’s concept of semi-reality and word problems where the context has little or no role in the task solutions (Verschaffel et al., 2000).

Students are used to mathematics problems which do not need to make sense in the world outside school. In stereotypical word problems, all given numerical information should be used, but the context is often not necessary to use to find the solution (Verschaffel et al., 2000). Therefore, students that have worked with such word problems are used to ignoring contextual information and disregarding any practice knowledge which complicates the task. Students seem to make sense of word problems in the school context, and know that the usual way of solving mathematics tasks is to ignore knowledge from the outside world which would make the mathematical reasoning in the word problem problematic or impossible (Schoenfeld, 1991). In contrast, in a workplace setting, a question or answer that does not make sense with regards to a workers' knowledge would be scrutinised and discussed to make sure that no one had made a mistake or a wrong assumption (Magajna & Monaghan, 2003).

An important interaction pattern that was evident in the cases was that the students treated the teachers as the authority on mathematics. They were regarded as able to answer questions that arose and as authorities in mathematical practices. This is not surprising due to the students' experiences with earlier school mathematics practices – as I noted in Chapter 3, mathematics in school settings differs from mathematics in workplaces. Grønmo et al. (2010) found that Norwegian students report that they often solve problems that are similar to problems in their textbooks. Often a standard mathematics lesson often involves the teacher demonstrating a procedure in mathematics, followed by students practicing it with exercises from the textbook or worksheets; the teacher alone is responsible for checking if exercises are solved in the correct way, and knowing the correct answer.

The tasks in this project provided the potential for making connections and using routines from both the mathematics classroom and the vocational practice. One obvious disturbance in the routines of the mathematics classroom as the students were doing the tasks was that they challenged the norm of the teacher as expert, and there were instances where the students corrected or helped the teacher or researcher. However, such challenges were more likely to be related to the vocational practice, not the mathematical practices. When mathematical practices were the focus, the students reverted to treating the teacher as the authority.

For example, in the hair salon budget task, the group first asked what rent prices would usually be and I gave them information on rental prices per square metres. They then struggled with how to calculate the area of their hair salon (Table 7.1), and it was clear that when the teacher came up with suggestions for the floor area, these suggestions were taken to be correct without further consideration from the students. Earlier in the conversation, two other group members had tried to explain the same formula and calculations as the teacher, but their explanations were not immediately taken into account in the same way. The teacher in this episode clearly held the position of mathematical authority, and the interaction patterns in the classroom seemed grounded in traditional mathematics classroom norms.

In the technical and industrial production class, the interaction patterns revealed that the teacher had the role as the mathematical authority when she helped the students with calculating engine cylinder volumes. Yackel and Cobb (1996) point out that sociomathematical norms include what is deemed as an acceptable mathematical explanation. In this episode the teacher judged if the students' calculations of cylinder volumes were correct, and she tried to progress their solution strategies from trial-and-error methods to algebraic methods (Table 7.19 and Table 7.20). Here the teacher showcased a solution method that most of the students had not tried. This movement from solution methods that are based on trial-and-error to algebraic methods is an indication of what the teacher regards as mathematical efficiency in this classroom, an approach which was clearly connected to the mathematics classroom, not the vocational practice.

Likewise, the teacher in the Frifond project task was the one that helped the students understand that the budget template did not need to be filled out in all the cells, when their project budget did not fit a premade budget template (Table 7.39).

In all the cases I found that the teachers clearly steered the mathematical discourse when the mathematics were in forefront, but this interaction pattern changes when the vocational practices are dragged into the mathematics discussions. There was a potential for students to adapt or change the sociomathematical norms that regulate traditional mathematics classes or adapt norms that regulate interaction patterns in the vocational practice. I will discuss such fluctuations in the norms in the next section.

8.1.2 Fluctuations in Enacted Norms in the Classroom

Even though students were clearly holding on to interaction patterns which had a basis in the routines and norms of traditional mathematics classrooms, the tasks created potential for fluctuations in routines between the two communities of practice. As we have seen, the students had two ways of working with the tasks, often continuing to draw on regular sociomathematical norms, but sometimes seeming to draw on, or wanting to draw on, the sociomathematical norms of the relevant vocational practice. Thus the students' shifted between different ways of working with the tasks, and there were associated shifts in the roles in the classroom, where sometimes the students emerged as authorities on what to do in the vocational setting, in contrast with standard mathematics classroom practices where the teacher is regarded as the authority in mathematics.

The tasks contributed to fluctuations in norms regarding roles in the classroom. I found that sociomathematical norms regarding who possessed knowledge and who occupied positions as knowers tended to fluctuate as the students worked. There were break down moments where the students showed that they had experience of the vocational context, but as shown in Section 8.1.1, they often ignored such knowledge if this enabled them to solve the mathematics tasks more easily.

However, I did find that the students used their vocational knowledge to discuss authentic aspects in the tasks, and they also corrected the teacher's descriptions of vocational practices, but at the same time the teacher often seemed to be in control of the mathematical discourse. Overall, I found that the prevalent sociomathematical norm was that the teacher was regarded as the knower of mathematical practices, and the students were regarded as knowers of vocational practices. As they worked on the tasks, the students themselves took up the role of knowers of vocational practices and it was evident that some were even regarded as experts in issues relating to these practices. The other students would ask them for help or clarifications of how things should be done in the vocational practice. In this sense, these students were beginning to be regarded by the others as legitimate participants in the vocational communities of practice.

For example, as the students discussed the sizes of different engines in the engine cylinder task, they would inform me and teacher that some of the information was wrong. Kristian explained to me that a V4-engine could be 3.5 litre (as given in the task), but the common size would be about 2.3 or 2.4 litres (Table 7.29). In this episode the student was clearly the person with relevant knowledge, compared to my own lack of this vocational knowledge. Erik took ownership over the vocational practice aspects of the task when he corrected my explanation of the setup of cylinders in a V8 and what an appropriate size of a V8 engine is (Table 7.29 and Table 7.30), as in Martin and LaCroix's (2008) research, where a student could identify if the solution looked correct.

However, in contrast to Martin and LaCroix's findings, Erik did not use this knowledge in the mathematics process. When we continued the discussion about how to solve the mathematics question, he relied completely on my explanations. Here Erik showed that he knew vocational practices, but he then relied on the teacher to explain how to work within the practice of the mathematics class. Another example of a student in an expert role was Fredrik's explanation of how tuning is done in practice (Table 7.28). Here he was in his element and could explain that it was not usual to file the cylinder so that it was bigger, but to replace it with another bigger cylinder model. These episodes show that when they worked on the moped engine cylinder task, the students were able to take up expert roles as knowers of vocational practices in the classroom.

The students who worked on the Frifond project task with an application for support for a photography exhibition used Marte's and other students' knowledge of exhibitions to make decisions about what they needed for their proposal (Table 7.33). Marte told the other students what number of photographs would commonly be needed for an exhibition. When the group asked the teacher for help in choosing which sizes to print the photographs, the teacher joined the discussion, but the issue was decided based on Marte's knowledge. However, the teacher helped the group figure out issues connected to the budget template they used, for example explaining that they did not need to fill in all the cells since the budget for a proposal is different from a monthly budget. Here, the teacher was regarded as the knower of how to use the mathematical tool (the spreadsheet), and she helped the students to use the spreadsheet in order to make a budget proposal. Even though the students had real world knowledge of the context, they did not have experience with budgets in the context. The vocational task depended both on knowing how to set up a budget in a mathematics classroom, and on using vocational knowledge to utilise realistic expenses.

In the Design, Arts, and Crafts class the students made use of Mari's experiences and knowledge from her after school job to estimate the number of sales of hair products and how many hours hairdressers worked (Table 7.2). In another group, the students clearly mimicked hairdressers as they set up the budget, drawing on their knowledge of how they sell products and the need to calculate different prices for different haircut types (Table 7.8, Table 7.9 and Table 7.11). Although the different student groups used each other to compare their budgets (Table 7.12 and Table 7.13), the students used the teacher as the expert when it came to mathematical issues, such as how to calculate area (Table 7.1).

That the students themselves were regarded as the knowers of vocational practices is in contrast to Hahn's (2000) finding that students rejected the realism of word problems from a jewel shop context. In another configuration of students and experts, Rangnes' (2012) study had pupils construct drawings of a fisherman's cabin, where they had the opportunity to ask a professional carpenter for advice. In that project, the carpenter occupied the role of the knower of norms and routines from the workplace when he worked with the pupils. While Rangnes' study involved an experienced vocational practice representative, clearly this was not the case in the tasks in this research, and as we have seen, some of the students emerged as 'experts' while discussing the tasks.

However, it was noticeable in my study that most of the students had limited experience of the vocational practices of hair salons, engine cylinders, jack stands, exhibitions, or tournaments, so many questions emerged as they worked with the tasks. In this sense they were similar to the students in LaCroix's (2014) study, who had difficulties solving mathematics problems, not because they lacked the necessary mathematical knowledge, but because they had limited experience of the objects in vocation.

I observed that the students wondered about the salary of hairdressers and how many hours a hairdresser would work (Table 7.2). They expressed surprise that they would need to pay for running expenses such as garbage removal, telephone, and water bills. The students probably had little or no experience with rental fees, and what size of location they would need. But they had a lively discussion about what was important to have in a salon so that its design would appeal to customers. The students' knowledge or lack of knowledge shaped how they could budget for the hair salon. For example, they discussed how long a normal working day would be, and Mari pointed out that hairdressers needed to start work one hour before the customers arrive so as to be ready, meaning that they had one hour of paid work that did not generate any income. Other groups did not calculate this extra cost. As seen here the students had different knowledge of the vocational practices.

These changes in the classroom routines suggest that the tasks created disruptions in participants' roles. Teachers were regarded as knowers of mathematical practices, while students could be regarded as experts in vocational practices, leading to a different distribution of who occupied the role of the expert in comparison to many standard classroom practices, where the teacher always is the authority. Even when students contribute their opinions and knowledge, ultimately it is the teacher who judges and examines solutions and defines what the correct knowledge is. The role of the students as knowers in the vocational practices may give a new classroom dynamic which can lead to productive and interesting negotiations of meanings in the classroom community.

I found that the students had two ways of working with the tasks. They often continued to use regular sociomathematical norms when they worked on the tasks, but sometimes the students wavered and would draw on sociomathematical norms from the vocational practice. Research have shown that mathematics and mathematical processes in workplaces are used for the purpose of producing or checking a product (Hudson, 2008; Magajna & Monaghan, 2003), and V. Lindberg (2003) found that tasks in vocational education in the Nordic countries were often part of a practice which aimed to produce a product. In contrast to this aim of producing a product, tasks in mathematics classes have the aim of helping students to learning mathematics.

As in these studies, I found that there were contradictions between the norms and routines of mathematics classes and vocational practices: the vocational aspects of the task had implications for the sociomathematical norms in a mathematics class. In the vocational practice it would seem like an answer regardless of procedure was enough, however in the mathematics class the solution process is just as important as (or even more than) the answers. Depending on the goals of a lesson, a teacher may want the students to understand mathematical principles or practice procedures. Mathematics tasks can give students opportunities for engaging with and understanding mathematical practices. The students solve tasks, such as setting up a budget or calculate cylinder volumes, because the teacher wants the students to learn mathematics. But in a workplace setting the goal of mathematical activity is to produce a product (Magajna & Monaghan, 2003). There were several episodes where the students and the teacher negotiated implicitly which norms to conform to when solving the vocationally connected tasks.

For instance when students wrote down answers without any notes on how they found the answer (Figure 7.3), and in Fredrik's annoyance with having to explain what and how he had calculated (Table 7.17 and Table 7.19) there were fluctuations in the norms of how to solve a task. This was also seen in the Frifond project task were the students did not want to note down what they had calculated (Table 7.38). If the important part of solving the mathematics questions was to reach correct and usable answers, then finding the answers should have been enough. In contrast, the group that worked on the budget for their hair salon and disregarded the problem of their assumed 30 working days a month (Table 7.6), were more interested in getting through the process of making a budget for the task to be done, than that the finished product should be usable. The students who worked on the proposal for a photography exhibit were praised by the teacher for producing an implementable proposal, indicating that the teacher valued the usability of the students' finished product. There were fluctuations between the norms and routines of the vocational practice, where an answer regardless of procedure to find the answer was enough, and the norms of the mathematics class, where the solution process is just as important as (or more than) the answers.

It is possible to see these episodes as instances where norms were being negotiated. Norms are established over time and the vocationally connected tasks only show a snapshot of practice. The students and teachers needed to negotiate how to work on these tasks: should they be solved according to routines and norms of the vocational practice, or should they be solved according to the established sociomathematical norms of the mathematics classroom? Since the tasks were implemented and solved in the mathematics class it is not surprising that the norms established in the mathematical classroom are prominent in the data. It is interesting, though, to note that the vocational practices leave their mark on the students' discussions and some of the suggested solutions.

8.2 Task Authenticity and Membership across Practices

In this section, I discuss my findings of what connections students make with workplace practices and out-of-school knowledge when engaged with vocationally connected tasks. I discuss perceived authenticity of the tasks in connection to the students' membership of different practices. The students showed through language and routines that they sometimes connected the tasks to vocational practice. I will then explore issues related to why the students use or not use the context. Do the students act as though aspects in the tasks are authentic for them, and do they have relevant experiences to work meaningfully within the vocational context?

The work on the tasks seemed to bring together routines from different communities of practice, and the students (and the teachers) fluctuated between routines from each. I found that the students treated the connection to vocational practices in various ways during their work: the context was confirmed, used, disregarded, or debated. The students also utilised their experiences of the vocational practice to determine estimates and assumptions needed for the tasks. I found that the students would shift between discussing the questions in relation to the vocational practice or to school mathematics.

8.2.1 Students' use of Language Connected to the Different Communities

I found that the vocationally connected tasks led to changes in language use in the classroom. Students went back and forth between using terms from mathematics and from the vocational practice as they worked on the tasks. Wenger (1998) argues that one of the characteristics of a community of practice is that the participants share a repertoire, including words, gestures and genres. I found no evidence that the students were conscious of their alternating use of words, but there are episodes where it is obvious that the language and meaning of words from the vocational practice is more familiar to some of the students than others.

Here it is important to remember that the students are still newcomers into that community, having studied on the vocational education programme for less than one year. Some students had probably been interested in relevant hobbies such as working on their mopeds, or had part-time employment at a hair salon, for several years. These enterprises and hobbies were related to issues and practices from their chosen vocational programme, and therefore those students could have more substantial knowledge of the vocational practice than students that did not have such hobbies. In addition, some students had experience of a vocational practice, for example through part-time jobs or family businesses.

An example of practice-related repertoire occurred when the students worked on the task about engine cylinders. The written text of the task used the terms “stroke”, “bore”, “height” and “diameter” (Section 6.3). The students used all these terms when they discussed and worked on the questions. The students I interacted with knew the terms “stroke” and “bore”, and when I used “bore” wrong I was immediately corrected by Erik (see Table 7.15). For me, these terms were not familiar, and I needed to translate them mentally to the more (for me) familiar words of “diameter” and “height”. Generally, the mathematics teachers were less familiar than students with the repertoire of different vocational practices. Several of the students also used terms from the vocational practice that were not introduced in the task such as “crank”, “nozzle”, “carburettor”, and “fillings” (Table 7.26 and Table 7.27). I take such use of terms as evidence that the students were participants in a community of practice connected to the introduced vocational setting of the task. It appeared that the use of the terms “stroke” and “bore” in the task instructions lowered the threshold for using other vocationally relevant terms.

The students who worked on the hair salon budget task likewise used practice-relevant words and gestures and showed knowledge of the routines of the practice which neither I nor the teacher had introduced. They talked about specific brands of hair products (e.g. Redken) and mimicked how hairdressers pitch sales to customers in their discussions of sale of hair products (Table 7.9). Similarly, when the students from Media and Communication worked on their budget for a photography exhibition, they discussed the camera brands “Nikon” and “Canon” as options for camera rentals, and noted that they could not print the photographs in the desired size at certain chain photography stores. They referred to real locations for rent for their photography exhibition, and also referred to the design programme InDesign when they wanted to print programmes for the exhibition.

In all these instances, the students seemed to be more or less familiar with the repertoire of the related vocational practice, as demonstrated through their use of words, gestures, stories and ways of doing things. I interpret this as indicating that the tasks gave the students opportunities to identify and use relevant experiences from outside of the mathematics classroom as in Lowrie’s (2011) study of students who worked with authentic artefacts, Bonotto's (2005) study of students' work with supermarket receipts, and Martin and LaCroix's (2008) study of ironworker apprentices. However, not all students showed a familiarity with vocational repertoires, and they would keep to the vocabulary and routines from the mathematics classroom. As in LaCroix's (2014) study, some students in my research had only novice levels of knowledge of the objects from the vocational practices and therefore could not draw on their experiences in their solution processes.

It may be the case that the boundary object representations used in the tasks (eg, photograph of an engine and the receipt from a hair salon) may have been a contributing factor in the students' use of terms and routines from the vocational practices. I noticed that the students would often direct their attention towards discussing the vocational practice rather than the mathematical assumptions or models. Often vocational practice was discussed more than the mathematics that was intended to be emphasised.

8.2.2 Authenticity and Students' Discussion of Boundary Object Representations

The nature of the students' talk about the boundary object representations, routines and practices highlighted issues of authenticity in the introduction of the vocational context in the tasks. Vos (2015) claims that authenticity is agreed upon through a social process, and she defines an authentic aspect as something that has an out-of-school origin and a certification of originality. In her work, one way that certification of originality can be vouched for is by "experts" from practice or other stakeholders. I found that the students in my study confirmed or denied authentic aspects given in the task, and thereby certified authenticity themselves. In addition, my findings indicate that that the students were keen to discuss the vocational practice, asking each other questions and talking about their experiences. However, when they discussed mathematics, there were fewer interactions between them.

The finding that students pointed out authentic or inauthentic aspects in the tasks is similar to other studies where vocational practices or contexts have been used in a mathematics classroom. For example, in Hahn's (2000) study of jewel shop apprentices, the students rejected the realism of word problems from the workplace given in mathematics lessons. She found that the physical setting affected the students' solution methods, and that they solved questions with procedures learned in their professional practice if they were asked to solve word problems from the workplace outside of the physical mathematics classroom.

In my study, I found that the students often discussed what they saw as problematic aspects of the tasks. For example, they rejected the idea that a V4 engine would be 3.5 litres, as suggested in the task, and Fredrik disagreed with the idea that cylinders would be filed to a bigger size (Table 7.28 and Table 7.29). The students connected their experiences from the vocational practice to the boundary object representations referred to in the tasks and decided what would be appropriate sizes in the real world.

I interpret episodes where the students chose to continue to investigate similar questions for their own interest, as confirmation of an authentic aspect of the tasks, namely that the question seemed authentic for the students. For example, when the students worked with the engine cylinder task, Fredrik calculated on his own initiative the size of his own moped engine cylinders (Table 7.23 and Table 7.25), indirectly confirming that calculating such volumes had an authentic purpose. Also, when Kristian estimated the volume of the given engine to be 1.8 l instead of 1.6 l, he indirectly confirmed that the given cylinder dimensions seemed to be authentic (Section 7.2.2). He indicated that he knew normal engine sizes, and that the given dimensions of the cylinder in the task could be compared to cylinders he had seen in his workplace experiences.

When working on the hair salon budget, the students treated their proposed hair salon as real in some episodes. They treated the task as real when they mimicked hairdressers and used product names that are connected to hair salon practice (Table 7.9, Table 7.10 and Table 7.11), connecting and using routines and repertoire from a hair salon. Likewise, the students were interested in discussing Mari's experiences as a helper in the hair salon, and her opinion of what were the important aspects of managing a hair salon.

In my study, the tasks sometimes inspired close connections between the representation boundary objects in the task, the students' interest in the vocational practice and the mathematics at stake. As already noted, Fredrik started to figure out the cylinder capacity of the cylinder of his own moped and did this completely on his own initiative (Table 7.23 and Table 7.25). He had his own experiences of engine tuning and knew enough to hold a position as an expert in the classroom in this regard. He calculated the volume of two different cylinders he was familiar with (78 cm^3 and 88 cm^3), and when the teacher asked about the dimensions of these cylinders Fredrik answered quickly, without any hesitation. Fredrik indicated that he recognised the boundary object representations (the picture of the moped engine cylinder) in the task (Section 7.2.2), and he clearly connected these objects to his own moped cylinders. In the hair salon budget tasks, the students were eager to compare and discuss differences in their profit with the other groups (Table 7.12 and Table 7.13).

The students used their experiences of the vocational practices to determine assumptions and estimates, as in several other studies which show that students can and will use their previous knowledge in mathematics tasks (Bonotto, 2005; Lowrie, 2011; Martin & LaCroix, 2008). A common finding of these studies is that students use their knowledge to make sense of the problem setting. Bonotto (2005) found that they drew on previous knowledge to support a movement between understanding the problem and reviewing the results. Lowrie (2011) found that students used their personal knowledge of theme parks, although they found it difficult to develop shared solutions in their groups. Martin and LaCroix (2008) showed that students are capable of treating and reflecting on mathematical vocational context problems as realistic and using their contextual knowledge to identify mistakes in their calculations.

In my study the students used Marte's knowledge to estimate what number of photographs and what sizes they should be, and likewise the students used Mari's experiences as a helper to estimate sales and the number of customers. However, the estimates would be quickly decided, and as noted earlier, the students were willing to disregard their knowledge if necessary.

The tasks seemed to inspire the students to discuss issues about the vocational practice that were not directly needed for the mathematics tasks at hand, for example, discussion of the different brands of moped engines and their potential for fast acceleration, or their experiences the last time they were at the hairdresser. However, the students did not always link the vocational and mathematical practices in this way. When they discussed the vocational practice, they asked each other questions and talked about their experiences, but there were fewer interactions between the groups when they discussed mathematics, and less references to the vocational context. When the students discovered that they had calculated with 30 working days a month, Mari quite firmly pointed out that they would not really work there (Table 7.6), and therefore, they could ignore this mistake in assumptions of number of working days. Similarly, when one group discovered that they had calculated with too many hair colouring customers, they were unhappy about the need to recalculate (Table 7.3). Although the context was interesting to the students, often this did not affect their routines for working in a mathematics classroom, where realism often can be disregarded.

Thus, students shifted between discussing the questions with regards to the vocational practice and with regards to school mathematics, and in the discussions, they used words and routines both from vocational practices and mathematics. Although they were, up to a point, members of two communities, they appeared often be simply focused on ‘getting by’ and seemed to lack intellectual autonomy in the mathematics classroom. To summarise, the students in my study used their insight and knowledge from the vocational practice to work with and solve the tasks. They utilised experiences from the vocational practice to make decisions about assumptions and estimates, and to figure out which units they should work with. However, I found that overall, they were willing to disregard their knowledge of the vocational practice in order to solve tasks as quickly and easily as possible, prioritising the norms of the standard mathematics classrooms which they were used to.

8.3 Opportunities and Challenges in Vocationally Connected Tasks

In this section, I discuss my findings in relation to what opportunities and challenges arise in employing vocationally connected tasks in school-based mathematics classes, building on my responses to research questions 1 and 2. First, in Section 8.3.1, I revisit the design considerations which led to the three tasks and their potential for bridging between the different communities of practice, before turning in Section 8.3.2 to an analysis of how this potential interacts with issues of classroom norms and authenticity, presenting opportunities and challenges in task design.

8.3.1 Opportunities: The Potential of the Tasks for Bridging between Communities of Practice

The vocationally connected tasks were planned with the intention that they should be connected to the vocational practice and be relevant for the mathematics curriculum. The tasks were intended to be connected to students’ future vocational practice. The tasks were planned with the intention of engaging students in a ‘landscape of investigations’ (Skovmose, 2001) with connections to vocational practices, in what in Skovsmose’s framework is known as real world connections. The intention was for the tasks to create a bridge between two communities of practice, the school mathematics classroom and vocational practice. To use Wenger's concepts, I wanted to create opportunities for the students and the teacher to have a mutual engagement, be involved in a joint activity and to establish a shared repertoire across the two practices.

The tasks were created to contain boundary objects representations, and as seen in Section 8.2.1 and 8.2.2 the students to some degree recognised, discussed and used their knowledge of the vocational context. With regards to task authenticity, some aspects of the tasks were treated by the students as authentic, for instance the aspect event, while they did not treat other aspects as authentic. It was most noticeable that the students often discussed issues from the vocational context. However, they disregarded their out-of-school knowledge and solved the tasks in accordance with traditional mathematics classrooms norms for solving problems.

In the engine cylinder task, the students were expected to explore calculations of cylinder volume in engines, and some, but not all, of the students had earlier experiences with engines, and the cylinders in them. The boundary object representation was the picture of an engine cylinder, and the mathematical idea at stake was volume of cylinders. This task was engaging for many of the students, and the students talked about their own moped engines, or what they knew about engines, and thereby drew connections to their experiences in the real world. They recognised the task as something that had authentic aspects and connections to the real world, and they themselves incorporated their knowledge from a context other than the mathematics classroom. There were clearly students in the class who were on their way to becoming full members of the community of practice in the vocation, and they used their fledgling expertise to discuss the context. The students confirmed or denied authentic aspects in the task and had or took a role as knowers of the vocational practices (Section 8.1 and Section 8.2).

These findings resonate with the literature on authenticity. As in Verschaffel et al.'s. (2000) discussion of authenticity with regards to different aspects of a problem, we saw how, in the engine cylinder task, one student changed a question posed by the teacher into one which he had wondered himself: ''what about my own moped cylinder?'' (see Table 7.23), where the goal of answering his own question was not to succeed in the classroom, but to have a purpose in the real world; he also came up with the relevant data himself. In Vos' (2011, 2015) terms, calculating the volume of engine cylinders has an out-of-school origin, and the students themselves partly certified the originality of aspects in the task. In Palm's (2008) framework, the evaluation of authenticity is concerned with the event, question, purpose, language and information or data in the task. The event of calculating the volume of an engine cylinder is realistic and some of the language in the task is connected to the vocational practice.

The task of making a budget for a hair salon had vocational relevance and is something that would be done in the vocational practice if one were to start a business. This is in line with Vos' (2015) concept of the task having an out of school origin, and Palm's (2008) concept of event; the task could have been something that might happen in the real world. The task contained representation boundary objects like receipts and prices of different expenses. It gave opportunities for discussing the context of working in, and running, a hair salon. The students had experiences being customers at hair salons and could discuss what made some hair salons more attractive than others for the customer. In this task, the students seemed to notice the relevance of the context, referring to their own experiences as customers, and one student's part time job in a hair salon, thereby giving the task a certification of originality with regards to the situation of setting up a hair salon. In the hair salon budget task, the questions were pre-formulated, and some information was given, however the students could in addition work together and find more necessary sources themselves. The task invited the students to engage in exploring different issues of budgeting: what different choices for numbers of haircuts, hair colourings, and the overall size of the salon of would mean for their overhead.

The Frifond project task, where the students were supposed to make a budget for a youth project, was in many ways the task with most authentic aspects of the three tasks before the implementation. The students were directed to a real website, with real possibility for applying for money for a given project. The event aspect is clearly fulfilled because making a project and associated budget could really be used for applying for a grant. I will also argue that the question aspect had an out-of-school origin, and in line with Verschaffel et al.'s (2000) point of possible consequences of the task, the result of the task (project description and accompanying budget) could, if sent in, get a genuine reaction from the grant managers. Here the problem was quite open, and the students were supposed to find necessary sources themselves, thereby being close to authentic in the aspect of data. The language had out-of-school origins in regard to the students being directed to a real website where they were supposed to read the grant guidelines and so on. With regard to a community of practice out of school, the students could use routines and practices of the real world to apply for the grant, and they could use their own experiences in their chosen project to judge what would be necessary and possible expenses. They had the freedom to choose their own project to work on.

The mathematics in the Frifond project task concerns budgeting, and this is relevant both for the mathematics community, and for the students' future vocational practice. The representation boundary object was the Frifond grant website, and the task provided an opportunity to bridge between the students' roles as mathematics learners in the mathematics classroom, and as participants producing a usable budget for a real-world project.

To summarise, the tasks all had boundary object representations, and were connected to an out-of-school community of practice. Different elements of the tasks had the potential for being regarded as authentic; with regards to Vos' (2015) criteria all three tasks can be said to have aspects that have an out-of-school origin and some of the aspects in the tasks got a certification of authenticity by the students. The students were engaged in the contexts of the tasks, and used their knowledge of routines, language and practices in the vocational practices, indicating that they sometimes took practice knowledge into serious consideration as they solved the tasks. Some students got or took roles as knowers of the vocational practice. This change in roles provided opportunities to engage through their involvement in joint activities of the mathematics task in the classroom. They were able participate and communicate with each other and start to master the shared cultural meanings of mathematical and vocational procedures connected to mathematics. Thus, the tasks had potential for bridging between the two communities of practice, the mathematics classroom, and vocational practice.

8.3.2 Challenges: Difficulties when Working on the Boundary

In all cases, the students recognised the context, and referred to it in their conversations as they worked on the tasks. They used the representation boundary objects and drew on words and references to workplace practices. However, they were also in the physical mathematics classroom, which may have contributed to their willingness to stick to its norms. Studies (Hahn, 2000 and Numes et al., 1993) shows that students work differently on the same tasks if they are carried out in the mathematics classroom, or out of the mathematics classroom. Also, Frejd and Muhrman' (2020) study showcase the importance of the physical placement and how it affects the students' working routines.

The engine cylinder task was in many ways successful, because many of the students recognised and used the context. They were interested, and worked on the mathematics, and as argued in Section 8.3.1, aspects in the task has an out-of-school origin and is certified by the students themselves. However, the original task was proposed by the teacher, even though some of the students started exploring their own questions. The information required for solving the task was already present, and solutions were to be judged according to mathematical criteria; solving the tasks led to no other consequences than succeeding in the mathematics classroom. Although on several occasions, there were glimpses of a change in roles in the classroom towards the students being knowers of the vocational practice, leading to fluctuations in the normative activity, the students tended to revert to the norms of the mathematics classroom, perhaps because the task had these ultimate consequences which were rooted in the mathematics classroom.

In retrospect, it is possible that the engine cylinder task was too closed, and built up with many sub-questions, but it did have potential for exploring different cylinder parameters by changing stroke volume/height and bore/diameter. There was no explicit instruction that students should connect the different choices with their vocational knowledge. For instance, it would have been possible to ask what would happen if one were to choose stroke volume/height and bore/diameter in ways that would be good or bad for real life use. Nevertheless, the task still resulted in some of the students exploring and connecting the work they did to understandings that they needed in the vocational practice, as when Fredrik checked his own engine cylinder.

Likewise, the hair salon task drew on a context which made the students discuss their own experiences at hair salons. However, like the other two tasks, the problem was initially suggested by the teacher, and the goal of the task was to solve a mathematics problem in the classroom. This was particularly apparent when the students discussed the context of hair salons but appeared to have an ambivalent view of what to do in the task with regard to the mathematics. It seemed that there was ambivalence in whether the students should act as though the proposed hair salon was something that could become real, or whether they should act as usual in a mathematics classroom, where context and unrealistic answers in the tasks can be overlooked.

This ambivalence appeared to have its roots in the fact that, in the hair salon task, the students did not see an immediate use for the proposed budget outside the mathematics classroom, as clearly stated by one student (Table 7.6). Thus, the solution of the task – a proposed budget for a hair salon – was seen as something to be judged by solely mathematical criteria, not in terms of its feasibility for planning a hair salon. In a real hair salon those setting up the budget would be more experienced than the students were at the time and would also work in a different way than they did in the mathematics classroom. In such a process, one would talk to realtors, check competitors' prices and so on. But the students did not take the opportunity to explore their own questions in this sense, instead focusing on getting a result that would be good enough for the mathematics classroom. The task could have been improved by for instance a cooperation with a hairdresser teacher, or an old-timer in the hair salon business, who could have talked about how they would work for making such a budget. In Frejd and Muhrman (2020) study, there was a collaboration between the mathematics teacher and the vocational teacher, but they argue that the students got the impression from the vocational teacher that the mathematics they were doing were not a real part of the vocational practice. Therefore, it would be important to make a task which really encapsulate how budgets are made in the vocation. The task could also have been adapted to encourage the students' own questions and given the students the task of making a budget to be judged by criteria from the practice.

The Frifond project task had clearly authentic aspects in the sense that it had an out-of-school origin - the students were supposed to relate to the real website of the grant. As noted above, if the students had sent in a proposal for a project with a budget, they could have received a genuine reaction from the grant administrators. The task was also open in the sense that the students themselves should choose their own project, and there was no pre-planned way of setting up the budget. The students should also identify their own expenses and find the necessary sources themselves. Despite these positives, and although the students did discuss possible projects, they did not engage much in finding real estimates and prices. As we have seen, they were more concerned with what would be the ‘right’ size of project and budget, and they would quickly accept suggested numbers. This may have been because they did not have any strong motivation for developing a project which required funding, so that, however authentic the event and question aspects of the task, it remained a task imposed in a mathematics lesson - the students did not themselves see it as something that would they would do outside the mathematics classroom.

All in all, the students did not use the references to workplaces as intended, in terms of supporting their mathematical engagement and conceptual development; rather, they tended in all three tasks to disregard their knowledge of the vocational practice. Often this appeared to be in order to simplify their workload, or to be driven by indirect reference to the norms of the mathematics classroom - mathematics tasks are not supposed to make sense in the real world.

One important issue to remember is that the students had varying degrees of experience and competence in the vocational practice the tasks were supposed to be connected to and were at different stages on their path to becoming members of the community of practice in question. Their experiences were limited to being a customer in a hair salon, changing their own engine cylinder, and using leisure activities - they knew *about* the context. But they did not have experience as someone working within the context, setting up budgets or calculating the volume of engine cylinders. In this sense, they did not have first-hand experience of *using* the specific mathematics of the vocational practice. Even though they were often interested in the context, and even though they probably realised that budgeting or calculating would sometimes be involved, that is not the same as feeling the need for doing what is perceived as a mathematics task when it is given in a mathematics classroom context.

Whatever the number of authentic aspects in a task, and however experienced the students may be in the vocational context which a task refers to, we might still expect fluctuations in norms when the students and teacher are positioned between two communities of practice. One such fluctuation concerns the norms which regulate participants’ roles in the classrooms. I found that often the teachers were regarded as experts in mathematical practices and the students were regarded as experts in vocational practices. Jensen (2017) argues that students construct their competence in differently in the mathematics classroom, and in the vocational education, where the students in the vocational education had more accountability for their own solutions. This accountability differs from many mathematics classrooms where the teacher is always regarded as the expert. The students may contribute their opinions and knowledge, but in the end, it is the mathematics teachers that judge and examine solutions and define what the correct knowledge is.

This research shows that designing vocationally connected tasks is no magic fix for students and teachers bridging the gap between practices. However, enabling students to take the role of experts of vocational practices may provide opportunities for a new classroom dynamic which can lead to productive and interesting negotiations of meanings in the classroom community if these are made an explicit focus of the activity. The students' participation can give opportunity for what Yackel and Cobb (1996) call intellectual autonomy, where the students themselves use their own mathematical knowledge when making mathematical decisions, instead of relying on the expertise of the teachers. If the norms of the classroom over time change towards students' autonomy, in not just in vocational discussions, but also when discussing the mathematics at stake, there would be opportunities for students to engage more in mathematics. Likewise, Wenger (1998) argue that learning is about becoming an active participant in a community. So, for Wenger, learning is about gradually being able to negotiate meanings and actions within the practice. Such a new classroom dynamic can enable students to construct an identity as a competent participant, hopefully both in mathematics and in vocational practices. In the next chapter I will discuss both theoretical and pedagogical implications of my study.

9 Conclusions and Implications

In this concluding chapter I will present and discuss opportunities and challenges that arises when students in vocational education programmes work with mathematics tasks designed to be connected to vocational contexts. As we know, much time is spent on tasks in mathematics classrooms, and therefore tasks are an important aspect of both learning mathematics and learning about what counts as mathematics and how to act in mathematics classrooms. I start this chapter with the conclusions of the study, before turning to its contribution to the field of mathematics education research in terms of theoretical contributions and the research design. Lastly, I discuss implications for pedagogical practice, and future research needed in the field.

9.1 Conclusions

Recalling the aim of the study which was to understand how students interact with classroom mathematics tasks designed to draw on their future vocational contexts, I present some conclusions to my research questions:

- RQ1: What characterises the enacted norms in the classroom when students work with vocationally connected mathematics tasks?
- RQ: What connections do students make with workplace practices and out-of-school knowledge when engaged with vocationally connected tasks?
- RQ3: What opportunities and challenges arise in employing vocationally connected tasks in school-based mathematics classes?

Vocationally connected tasks made it possible for the students to use routines and knowledge they had about the vocational practice, and I found that there were changes in the roles of who was regarded as experts in the classroom. Since the tasks were implemented and solved in the mathematics class, it is not surprising that previously established norms of mathematical classrooms were evident in the data, and these were closely connected to the routines and practices of traditional mathematics classrooms. The students acted in accordance with traditional norms regarding the nature and purpose of mathematics tasks. They also regarded the teacher as the authority on mathematics. It is interesting, though, to note that the vocational practices did leave some marks on the students' discussions and suggested solutions, in the cases where there were diverging opinions on what could be considered good solutions, and where it was implicit whether solutions should be judged according to the norms of vocational practices or the mathematics class.

In my data I find that the students treated the vocationally connected tasks in the same way as the norms of a normal mathematics classroom dictate. They assumed that a task should be of appropriate size, and possible to solve in a limited amount of time. I found that the students were quite willing to ignore the vocational context of the tasks, and I took this as evidence of a prevailing sociomathematical norm in their classrooms: vocational knowledge can be disregarded if this makes the solution process in the mathematics tasks easier. It seemed that it was both acceptable and advantageous to disregard problematic issues from the outside world when these caused conflicts with the solution process of the task. I found that the students treated the tasks in different ways during their work: the context was confirmed, discussed or disregarded. Although the students were on the verge of becoming members of a vocational community, they were still newcomers in this practice, and lacked the knowledge of the practice as old-timers have.

I found that the students could be the certifiers of authentic aspects of the tasks. This is a finding, which I believe, can be generalised to other cases where students work with mathematics tasks with authentic aspects. Even though the tasks had several inauthentic aspects, the students still took the opportunity to discuss and consider vocational connections. However, it seemed that the students mostly discussed the vocational practices, not the mathematics connected to these practices.

One issue, evident in my analysis, concerned whether vocational considerations should be valued or disregarded. In several episodes, students disregarded problematic aspects of the tasks in order to finish their mathematical work. Such issues can be connected to what Skovsmose (2001) refers to as semi-reality and the exercise paradigm, and Gellert and Jablonka's (2009) observations of a teacher who argued that he was not talking about reality. The students seemed to be used to mathematics that was separate from their everyday or vocational experiences. However, the ontological issue of what mathematics is in different practices, and how students and teachers recognise it, leads to both opportunities and challenges in the classroom implementation. An important question to ask after studying students' interactions with vocationally connected tasks: are vocationally connected tasks really about the vocational practice, and what the students know, or are they just disguised school mathematics tasks? This is supported by the fact that the tasks are implemented in the physical context of a mathematics classroom, and the analysis shows that the students understand, and act, as though their solutions are judged by previously established norms of the mathematics classroom.

There were opportunities for discussion about issues connected to vocational practices the students knew, wondered and cared about, but it was a challenge to get their discussions to be about mathematics as well. This was for instance observed in the hair salon budget task, where the students eagerly discussed how salaries were calculated (Table 7.2) and compared the size of their budgets (Table 7.12 and 7.13), but wanted quick answers to finish their budgets (Table 7.4 and Table 7.6). How to work with these challenges is something that needs to be considered when planning and implementing vocationally connected mathematics tasks.

I argue that vocationally connected tasks generated fluctuations in classroom norms, which can create space for change and growth, and opportunities to loosen up or change interaction patterns in mathematics classes. Quite a few of the students I met seem bored or disillusioned with their own learning opportunities in mathematics classes. Although not all students had insight into the vocational subjects, vocationally connected mathematics tasks meant that they could show what they knew of vocational practices, instead of continuing earlier experiences of school mathematics. Opportunities for changes in student and teacher roles can give possibilities for discussion that mathematics does not need to be rule based and consist of a fixed set of procedures that need to be followed. To understand and use mathematics in appropriate ways for the given situation is an important aspect of having intellectual autonomy in mathematics.

In addition, I argue that tasks with vocational connections have the possibility to stimulate and provide opportunities for discussion about why mathematics tasks are solved as they are, and why mathematics is relevant. However, the teachers' demands with regards to how to act and argue in a mathematics class did not seem to change with the introduction of the vocationally connected mathematics tasks. This situation is made more complex by the fact that, in the first year of vocational education, there exists a double problem with respect to mathematics, namely that students are novices in the vocational practices, and old-timers in school mathematics classrooms. Their expectations on how they are supposed to act are influenced by the expectations from the different communities of practice. The differences in what mathematics is in these practices needs to be made explicit when working with vocationally connected mathematics tasks. Overall, there were fleeting situations of change towards students' autonomy, but these were not sustained throughout the lessons.

9.2 Contribution to Mathematics Education Research

In my study, I wanted to provide a research-based insight into what happens when students' future vocational contexts are used as a basis for the design of classroom mathematics tasks. As the literature review showed, it is not easy to integrate vocational mathematics into educational practices. The ontological issue of what mathematics actually is in different settings, and how students, workers and teachers recognise it, leads to complex outcomes for attempts to support both school and vocational mathematics learning.

My contribution builds on and extend the work of Yackel and Cobb (1996). While Yackel and Cobb (1996) studied sociomathematical norms in regular mathematics classrooms, my study addresses enacted norms in mathematics in vocational education programmes. In my study, there are explicit links to various vocations, and students are likely to see themselves as on a trajectory into those vocations. Although there are studies that investigate the nature of mathematics in workplaces (see Section 3.2), and how learners learn mathematics, there is little research on school-based mathematics learning for vocations. Therefore, my study can be considered a bridge between these lines of research and the opportunities and challenges when vocationally connected tasks are implemented in school-based mathematics classes.

My study has shown that there is a connection between tasks and norms in classrooms, and that it is not possible to use tasks alone to change classroom norms. In a way, I would say that tasks and norms are mutually constitutive, because working with tasks together means that students and teachers negotiate norms on how to do mathematics, and what is perceived as mathematically valid arguments and what should be valued in mathematics. Conversely, what is perceived as mathematics and how mathematics should be worked on, influences the norms concerning how students should work with mathematics tasks. In my research, I found that the prevailing norms of the mathematics classrooms are difficult to shift even when students and teacher work with tasks with vocational connections.

This connection means that there is a need to consider the role of task design in attempts to change normative activities in the classroom. Cobb, Zhao, and Visnovska (2008) argue that it is important for teachers to understand the underlying rationale of instructional sequences. This implies that getting a better understanding of how teachers present vocationally connected tasks, how teachers might shift norms and exploit the potential for a shift in norms is an important part of implementing vocational connections in mathematics. I have also seen in my study that authenticity is difficult to manipulate, and that exploiting authenticity needs to go hand in hand with recognising the force of norms, and how norms can be changed. In my cases, the task which on paper seemed to have most authentic aspects and connections to the real world, the Frifond project task, was treated by the students as a normal mathematics task, while the task of calculating engine volumes had students engaging with the context and figuring out their own questions.

9.2.1 Theoretical Framework

I struggled to decide which theoretical concepts to use in this study, finally bringing together a range of sociocultural theoretical standpoints which enabled some insights into the effectiveness of my task design. The theoretical concepts of boundary objects, sociomathematical norms, communities of practice and authenticity were central to my analysis.

When I designed the tasks, I did so with Skovsmose's (2001) concepts of 'tradition of exercises' and 'landscapes of investigations', and 'real-life references' in mind. The concepts enabled me to design tasks which had connections to out-of-school practices. This did, in hindsight, yield surprising results; the Frifond project task, which had both real-life references and could be in a landscape of investigations had students worrying if they had chosen projects fitting for the mathematics class. In comparison, the engine cylinder task, which is built up as in the tradition of exercises sparked some students' explorations of their own engine cylinders. Skovsmose (2001) points out that the invitation into the landscape of investigations is just that – an invitation. Whether or not the students choose to take the invitation, will depend on the classroom community of teacher and students, and what they perceive as an invitation to explore. This seems to be in line with my research, where I need to use the concept of communities to capture the point of students' membership in different practices for figuring out in which moments the students took such invitations.

Several researchers have used the concepts of 'boundary objects', 'boundary crossings' and 'brokers' to study mathematics in and between different practices, including studies of workplace mathematics. However, I found the original concept of 'boundary object' problematic to use. A commonly used definition of boundary objects is that they are "object[s] that cross boundaries" (Akkerman & Bakker, 2011b, p. 2), and the criterion for an object to be a boundary object is that it needs to bring about and help to coordinate collaborations between different practices (Wenger, 1998). I did not have objects in my study which facilitated communications between participants from different practices in a classical sense, with the exception of the Frifond webpage where there potentially could have been communication between the students and the grant administrators.

Instead, I had tasks which contained items intended to facilitate communication in the mathematics classroom. The tasks were made for use in the mathematics classroom and were not objects which carried different meanings in the different practices. If the tasks had meaning in the vocational practice, were worked on in the mathematics classroom, and the results were then made available to participants from both the vocational practice and the mathematics classroom, it would have been more fitting to use the concept of boundary object. Since this were not the case, the idea of a boundary object representation opens up a new avenue of investigation relevant to vocational education, particularly as it is currently constituted in Norway.

Recognising the particular properties of a boundary object representation entailed recognising in turn the relevance of the students' membership of different practices - what Wenger (1998) defines as multimembership. Employing Wenger's theory of communities of practice enabled further insights into the students' dual role as old-timers in the community of practice of the mathematics classroom, and novices in the vocational community of practice, and the implications of this duality for vocational education.

Wenger's emphasis on communities of practice and their shared repertoire and norms also connected with Yackel and Cobb's work on sociomathematical norms. As noted above, the connections between tasks and norms is important, and the use of Yackel and Cobb's concept enabled an analysis which went beyond a consideration of whether or not tasks 'worked' to a wider understanding of how they were understood in the classroom in terms of expectations about the tasks' role in that particular community of practice.

A duality is also seen in the concept of 'authenticity'. In my study, I found that authentic aspects can be certified by students with experiences from vocational practices. And the other way around means that the students demonstrate their intellectual autonomy by certifying authenticity. That students themselves can certify opens possibilities for investigating when and how such student authentication takes place.

9.3 Reflections on the Research Design

After considering the contributions that this study has made to the field of mathematics education, I find that there are some potential improvements on the research design. The first issue I will discuss is how to use out-of-school contexts for task design, before I turn to discuss how the task design process could have been improved.

9.3.1 Implementing Tasks with Out-of-school Connections in Classroom Settings

As shown in Chapter 3, implementing out-of-school contexts in a school like setting is difficult. In my task design, I used Skovsmose's (2001) concepts of 'tradition of exercises' and 'landscapes of investigations', and 'real-life references' to design tasks which had connections to out-of-school practices. After I had gathered my data, I among other concepts used the concept of authenticity to analyse the implementation of the tasks. In hindsight, designing the tasks with emphasis on authentic aspects could have been another way to implement real-life references.

The task design could have been strengthened by further observations of the students in their Common Programme Subject and a broader knowledge of what the relevant future workplace mathematics could look like for the students. One way to increase the number of authentic aspects in the tasks could have been with a closer collaboration with vocational teachers or insiders from workplaces. My research is only a snapshot of a longer history of the students in their mathematics classes, and further cooperation with the common programme subjects and workplaces would have increased the authentic aspects in the tasks. Not only would the tasks contain more precise information, but also the certification of originality of different aspects would be more plausible.

Vos (2018) argues that several studies show that it is the authentic questions which is most important for students' motivation. This seems in line with what happened in the engine cylinder task, Fredrik started exploring his own cylinder volume, and for him that was an authentic question. However, the students' various degrees of knowledge (and in many cases lack of vocational practices knowledge) made it challenging to design and implement vocationally connected tasks with authentic aspects, especially with the current curriculum in Norway where mathematics is placed in the students' first year of vocational education.

This placement means that the students barely have started to become participants in a vocational community of practice and may not know or recognise vocational practices. An issue to be aware of is that aspects that would be recognised by experienced practitioners as authentic may not be recognised by the students as authentic.

9.3.2 Researching Educational Design

This study was designed to trial materials just once and collect data from the implementation in one classroom for each task. This is similar to what Burkhardt and Pead (2020) calls engineering methods, where fast prototyping, small scale test, and rich feedback are important design principles. This gave me the possibility to study the task implementation in a detailed manner, however it would have been valuable to study how the tasks could have been redesigned. The tasks could have been tried out with students, refined or redesigned, before new implementation could have been done with students.

An interesting change in my research design would have been to keep the accompanying pedagogical design of the tasks in mind and make a clear plan for how the tasks should be introduced, worked with and evaluated in the classroom setting. This could have been done in collaboration with mathematics teachers, and I could have asked them to reflect upon how they were teaching the tasks, before planning further implementations. In such a collaboration challenging issues with the implementation could have been addressed, in combination with reflection on mathematics role in school and the vocation. Important discussions on the normative issues could then have taken place.

A collaboration between mathematics teachers, vocational teachers, maybe members of the vocation, and researchers on design and implementation of tasks could also been a fruitful method to discuss what vocational mathematics should be in the Norwegian context. In such a collaboration one could discuss in which way the students work with the tasks could be of importance in the vocational practice.

It would also have been worthwhile to gather systematic data on the teachers' talk and their treatment of the materials. In which ways do the teachers change their practice in the vocational mathematics classrooms, compared to their practice in a mathematics classroom for general education students? In my data, the norms of the classrooms did not change noticeable from what can be thought of as traditional mathematics classroom norms. In connection to this, another way of implementing the tasks is to have them introduced and supported by a professional from the vocational practice. This could have given credibility that the issue discussed in the tasks is done in the vocation. Even though the students had some background and sometimes authenticated parts of the tasks, they are newcomers into the vocational practices, and an old timer in the practice would have more tacit knowledge of the vocational practices gained through experience.

However, as long as the work with vocational mathematics takes place in a mathematics classroom, it may not matter who authenticate aspects of the tasks and how many authentic aspects the tasks has, if the sociomathematical norms do not change from traditional sociomathematical norms. This leads into the pedagogical implications of my study.

9.4 Pedagogical Implications

Two important questions have arisen from my study: ‘‘What kind of mathematics should be taught in vocational education?” and ‘‘how can vocationally connected mathematics tasks best be used in a school setting?” The first question addresses aspects of the mathematics curriculum while the second question addresses aspects of pedagogic practice. In the following, I start with discussing implications for mathematics curriculum and follow up with discussing implications for pedagogical practice.

9.4.1 The Mathematics Curriculum

It is important to reflect on which mathematical practices we want students to have the opportunity to experience. When designing the curriculum for a mathematics course it is possible to design it with the intention to further the students' mathematics competence with regards to their future vocation, or more pure mathematical competence, or a mixture of both. This is a political issue, connected to mathematics place as a gatekeeper to higher education and school mathematics exchange value. In Norway students have the option to take one additional year after their vocational education to get a certification of admission to higher education, and for both this, and for obtaining their trade certificate, they need to pass the mathematics course. Mathematics courses that are closely connected to the students' vocational education can lessen the mathematics exchange value for the students in the future. This was one of the major changes in the Reform-94. Vocational educations in Norway went from quite narrow paths to a specific vocation, where mathematics was taught to support the specific vocation, to the situation today, where the mathematics course is more general.

The placement of a mathematics course in the students' first year of vocational education means that the students have barely begun to become participants in a vocational community of practice at the time they are taught mathematics and may not know or recognise many vocational practices. I found that some students had limited experience from vocational practices, and the students' various levels of knowledge (and in many cases lack of vocational knowledge) can make it challenging to design and implement vocationally connected tasks.

The placement of the mathematics course is also an opportunity for introducing new routines and practices, since the students have for the first time in their educational history, taken an active choice in which education programme they want to study and started on a path towards their future vocation. The students and teachers in my study tried a few implementations of vocationally connected tasks. For such an approach to vocational mathematics to be successful, work with connecting mathematics with the practices in the vocation should be a sustained effort through the whole school year. And in this effort the curriculum influence what kind of tasks the students work with. The findings of this study suggest that in vocational education it is important for the curriculum to be conscious to the differences of what mathematics actually are in different communities.

When I observed the students, they would sometimes disregard realistic considerations, even though they notice problematic areas with regard to their experience from the vocational context. To disregard realistic considerations is a clever strategy for finishing tasks fast, but I believe that this is not the intention of vocationally connected tasks. There is no need for further examples in mathematics classes where the students experience that one can disregard knowledge and experiences from out-of-school settings. Therefore, curriculum design of these courses should carefully consider how to give students opportunities in mathematics where consideration of realistic aspects should be a part.

In 2020 a new curriculum in mathematics is implemented in Norway. One of the changes in this curriculum is that the vocational mathematics courses (1P-Y and 1T-Y) has some common curriculum aims, and some curriculum aims which are specific to the different vocational educational programmes. For instance, for students wanting to be hairdressers there are now, amongst others, the following two aims specific for this educational programme:

> The aims of the course are that the pupils should be able to
>
> - obtain data from the practice field, do estimates and calculations and make purposeful representations of the results and present these
> - read, use and make spreadsheets in work with budgets, tenders and cost calculation related to hairdresser, flowers, interior and display design, and assess how different factors affect the result
>
> (The Norwegian Directorate for Education and Training, 2020b, my translation)

For students who wants to be mechanics, these are the aims specific for that educational programme:

> The aims of the course are that the pupils should be able to
>
> - do calculations and assessments related to measurement uncertainty and tolerance
> - explore and use the properties of geometric figures; and calculate lengths, angles, area, volume, ratio and scale in problem solving within technology and industry subjects
>
> (The Norwegian Directorate for Education and Training, 2020c, my translation)

With this shift toward adapted curriculums for the different vocational education programmes teacher knowledge of mathematics in vocations will be important to manage to teach mathematics in a meaningful way. I found that the students had somewhat limited experiences of the workplace practices. This means that even though contexts that are relevant to their chosen educational programme there is no guarantee that the students have relevant experiences to apply when they work with such tasks. However, this change gives opportunities for discussions of workplace mathematics with students. It is possible to discuss in mathematics classes why mathematics in vocational practices is difficult to observe and the difference between school mathematics and mathematics used in out-of-school settings. Vocationally connected tasks can give opportunities for discussing if there exist only one answer to questions in mathematics and what different answers may mean.

An important question that will need careful discussion is if vocationally connected mathematics tasks should be solved with regards to norms and routines from either vocational practices or the mathematics classroom, or perhaps the answer is something in between. If the students were to work within norms from the vocational practices, the mathematics could seem more relevant and useful for their future vocation. The students could work within their vocational contexts and therefore establish more connections from within school mathematics to vocational practices. However, it is also useful for the students to know mathematics in a general form which may not have a direct application to their limited knowledge of the vocational practices. General mathematics knowledge is also relevant for their future participation in our society and democratic processes. Therefore, curriculum design for these courses is a tricky business – combining mathematics learning for general education, for different vocations, and for general knowledge may not be compatible.

9.4.2 Pedagogic Practice

Teaching vocational mathematics can be quite different from teaching ‘regular’ mathematics courses. One possible fruitful way of working on vocational mathematics could be through collaboration between mathematics teachers and vocational teachers. The teachers of the vocational subjects have experiences from the vocational practice and know what the students learn in their common programme subjects. They will probably have experienced problematic mathematical issues in the common programme subjects for the students and can suggest and help with authentic questions and other authentic aspects. In my study, the common programme teacher suggested working with cylinder engines and the jack stand diameter. Such collaborations are time consuming and need support from the administration to be successful over time (L. Lindberg & Grevholm, 2011). Teachers I have spoken with also mention that it would be useful to teach students from the same vocational education programmes over several years to get to know the vocational practices better, but since 1P-Y is such a small course for the teachers' workload it is often shifted around for scheduling reasons.

My study shows that teachers who teach mathematics in vocational education programmes need more and different knowledge of mathematics than what is required in an upper secondary mathematics course in the general studies programme. While research has been conducted on what kind of mathematics knowledge teachers need for teaching mathematics (Ball, Thames, & Phelps, 2008), my study suggests that teachers in mathematics in vocational education programmes would benefit from knowledge of the mathematics in vocational practices. The teachers need to know workplace mathematics as it is seen and utilised in the vocation, because there are differences in what mathematics is in mathematics classes and in vocational settings. If the teachers have such knowledge, they can broker across the practices, and can facilitate explicit discussions about the differences between mathematics in the different practices.

I argue that when teachers implement vocationally connected tasks, there is an added complexity in their teacher role. One can discuss if some solution procedures and answers are more correct than others, and how such correctness can be judged. The teachers need to be able judge if solutions are acceptable and help students reflect on the relevance of the answer for the vocational practice. Teachers will for example need to consider what good approximations and estimates are both in school mathematics practice and in the vocational practice. Judging such issues are different and maybe more difficult than 'just' checking answers in a regular mathematics lesson. One can discuss if some solution procedures and answers are more appropriate than others, and how such appropriateness can be judged. There are opportunities to reflect upon the difference between why and how to solve problems in mathematics class and in vocational practices. Here one can discuss if the product (answer) or the process of getting to the answer is the important matter. This demands that teachers that are reflective on their own teaching practice in connection to the student group and curriculum.

In the implementation of the tasks in this study the students were resources for certification of authentic aspects, and they sometimes knew routines and common solution methods from the vocational practice. The students had clear ideas of what authentic aspects the tasks had, and that it was advantageous that the students had met the context themselves before it was discussed in mathematics classes.

I found that having boundary object representations and some authentic aspects in tasks is no guarantee for the students to apply vocational practice knowledge. One way of increasing students' engagement would be emphasising authentic questions and working with tasks to which the answers have a value outside of the mathematics classroom. Solutions which are judged by old timers in a vocational practice, and in the best case; actually put into use, seems like a way to introduce productive social and sociomathematical norms.

Most mathematics teachers in Norway have an academic background, and do not have experiences from vocational practices that the students are beginning to become participants in. Therefore, that the students themselves can authenticate vocational aspects of tasks, can be one solution which lessens the need for participants from vocational practices. Another possibility would be to involve the vocational programme teachers in a certification process of authentic aspects.

9.5 Further Research

Both Bakker (2014) and FitzSimons (2014) argue that vocational mathematics in education is an under-researched, but important area of mathematics education. Although my study provides an insight into just some aspects of school mathematics learning in combination with context, it is nevertheless a start on a difficult, but fascinating and exciting, field of mathematics education.

Several suggestions for further research arise from the discussion in this chapter, and different research designs would be valuable and generate different data sets. Further research on possible models of collaboration between mathematics and vocational teachers for implementing vocational mathematics is necessary. Both collaborations for task design, and collaborations for understanding how teachers implement designed tasks are important.

It would be worthwhile researching teachers of mathematics in the vocational education. In which ways do the teachers change their practice in the vocational mathematics classrooms, compared to their practice in a mathematics classroom for the general education students? In my research, I found that the norms of the classrooms did not change noticeable from what can be thought of as a traditional mathematics classroom.

Cobb, Zhao and Visnovska (2008) argue that teachers have a central role in turning instructional sequences into learning situations in the classroom. Therefore, teachers should access, and understand, the underlying rationale of instructional sequences so that they can productively adapt tasks in their own classrooms. In this situation, a collaboration between both mathematics and vocational teachers is necessary to promote discussion and reflection on the rationale of vocationally connected tasks. Students can then be supported to participate in mathematical practices, both in school mathematics and in mathematical practices in the workplace. I argue that further research on how teachers use and adapt vocationally connected mathematics tasks would be important. This could be done by working together with the teachers as co-researchers, where they could reflect on the students' participation in mathematics.

Another approach could be close analysis of teacher responses during the implementation of vocationally connected tasks. Such analysis could give information on how norms in the classrooms are negotiated, and how teachers give students access to important mathematical and vocational issues. This could be done by analysing which ideas are taken up for discussion, what view of mathematics the teachers convey, which routines and practices are valued and how they encourage intellectual autonomy of the students. This provides the information needed for designing future successful vocational mathematics classes.

Building theoretical understanding through further exploration of the idea of boundary object representations and how the concept can be used for task design seems essential, given the issues of authenticity that have arisen in this thesis. For instance, co-designing tasks with authentic aspects with representatives from the vocation, before implementing, analysing the implementation and adjusting the tasks could help create tasks that draw on both boundary object representations and have authentic aspects.

Related work would be further study into students' boundary crossing as they move between the mathematics classroom and the vocational training. Vocational pathway students are in transition towards becoming legitimate participants in a vocational practice, and it would be fruitful to find out more about the role mathematics has and how it may be transformed in this transition. This could be done by following students in their mathematics class and in their programme subject, and then into their vocational training as apprentices in an enterprise. Studying the students' interactions with mathematics in all three settings and combining this with interview data on students' perceptions of mathematics and how it is used in the vocational context, could shed light on students' views at different stages of the transitions.

This study has shown that the norms of the classrooms were closely aligned with norms of traditional mathematics classrooms. This issue could be pursued further by extending Yackel and Cobb's work on social and sociomathematical norms to a detailed analysis of the norm structure in vocational mathematics classrooms. It is possible that a comparative analysis of the enacted norms in these different practices may shed further light on the nature of sociomathematical norms. An important related research, into the social and sociomathematical norms of workplaces, would help to inform educational design of mathematics courses in vocational education.

Another potential area of research would be comparative study of vocational education across the world, since there are many different models and educational pathways. The Norwegian context differs from that of many other countries in terms where mathematics is places in the vocational education. In some countries, mathematics is its own subject, in other countries it is integrated into vocational training at school or integrated into vocational training at workplaces. Research on sociomathematical norms in the different educational settings would be worthwhile for understanding more about how students perceive and have the opportunity to work with and learn mathematics. In my study the students were active when they discussed vocational practices, but less so when discussing mathematics. How can students be persuaded to discuss mathematics as well as the vocational context of the tasks? One way to explore this could be to involve students as co-researchers in order to understand their responses to vocationally connected tasks.

All these issues can help us further improve vocational mathematics education for students. Hopefully in the future we will have good practice and research-based replies to the response: "We are not doing this job for real, so that's no problem!"

10 References

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11 Appendices

11.1 Information Letter

UNIVERSITETET I AGDER

Dato: Februar 2012

Besøksadresse: Gimlemoen 25
Direkte: 38 14 15 38
Faks: 38 14 10 71

Til DEG
som er elev ved videregående skole
Kopi til foreldre/foresatte

Prosjektet: Matematikkmestring i videregående skole

Universitetet i Agder (UiA) har et forskningsprosjekt med tittelen, *Matematikkmestring i videregående skole*. Vi samarbeider med flere videregående skoler i Kristiansand og nabokommunene og ønske at dette skal hjelpe deg som elev til å få mer utbytte av matematikken i skolen.

Prosjektet drives av en forskningsgruppe i matematikkdidaktikk ved UiA og ledes av professor Anne Berit Fuglestad. To PhD-stipendiater vil arbeide i prosjektet sammen med didaktikere (forskere/lærere) ved UiA. Målgruppe er videregående skole, spesielt i yrkesfaglig studieretning og prosjektets mål er gjennom forskning og utviklingsarbeid å bidra til bedre undervisning og læring for elever i videregående skole og få dypere innsikt i utvikling av undervisning. Lærere og forskere vil samarbeide om design av oppgaver som er egnet til å vise om elevene forstår matematikken og analysere resultatene med tanke på bruk av resultatene i videre utvikling. Videre planlegges design av utfordrende oppgaver og bruk av disse i undervisning for å stimulere hvordan oppgavene kan bidra til at elevene blir aktive og utvikler kunnskaper og ferdigheter.

Karakteristik for prosjektet er å arbeide sammen og diskutere løsninger. Å stille spørsmål, undre seg og utforske matematikken er gode arbeidsmåter og vi ønsker at dere elever skal bli aktive idette arbeidet. Og vi vil svært gjerne ha innspill til hva som er lett eller vanskelig, hva er meningsfylt og hva oppleves relevant for deg og den studieretningen du har valgt. Det vil komme anledninger til å gi kommentarer til dette underveis. Læreren din vil delta i utvikling av oppgaver og undervisning sammen med andre læree og didaktikere og vil prøve ut oppgaver sammen med dere i klassen.

For forskningen ved UiA ønsker vi å dokumentere prosessen ved å samle inn ulike typer av informasjon. Det vil bli gjort observasjoner av undervisningen og arbeid i klassen, foretatt intervju med lærere og elever og gjennomført spørreskjemaundersøkelser. I forbindelse med dette vil det bli tatt lydopptak og videoopptak. Opplysningene vi får fra dette vil være av samme type som en lærer normalt får gjennom sitt arbeid, og det er ikke aktuelt å samle inn spesielt følsomme data. Spørsmålene vil dreie seg om matematikkoppgaver, løsninger av disse og hva elevene mener om forskjellige sider ved faget og arbeidet i matematikk. Observasjonene vil ikke ha noen innflytelse på elevenes eventuelle karakterer i faget. Vi som forskere og lærere er underlagt taushetsplikt, all informasjon vil bli behandlet konfidensielt og alle navn vil bli erstattet med et pseudonym eller bli gitt et nummer når forskningsresultater offentliggjøres eller lagres. Vi ønsker å presentere resultater fra prosjektet på konferanser for lærere og forskere og i artikler og andre vitenskapelige publikasjoner. I den sammenheng kan det bli aktuelt å vise korte utdrag av video-opptak eller bilder som illustrerer arbeidet. Prosjektet er meldt til Personvernombudet for forskning, Norsk samfunnsvitenskapelig datatjeneste som har godkjent opplegget.

Det foreliggende prosjektet i skolene avsluttes i 2015, mens forskningsarbeidet fortsetter og avsluttes i løpet av høsten 2023. Lyd- og video-opptak vil bli bli oppbevart på ubestemt tid og kan være tilgjengelig for forskning innenfor samme område på et senere tidspunkt. Innen utgangen av 2023 vil øvrige data bli anonymisert ved at eventuelle navnelister bli slettet. Dersom data blir brukt i nye forskningsprosjekter, vil nødvendige tillatelser bli innhentet fra Personvernombudet for forskning.

Vi ber om tillatelse fra deg som elev og fra foreldre/foresatte for de som er yngre enn 16 år. Deltakelse i prosjektet er frivillig og det er selvsagt mulig å reservere seg. Det er mulig å trekke tillatelsen tilbake senere

UNIVERSITY OF AGDER
SERVICE BOX 422 NO-4604 KRISTIANSAND NORWAY
PHONE +47 38 14 10 00 FAX +47 38 14 10 01
ORG.NO. 970 546 200 MVA postmottak@uia.no www.uia.no

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UNIVERSITETET I AGDER

uten å gi noen grunn for det. Vær vennlig å returnere svarslippen nedenfor eller melde fra til matematikklæreren eller kontaktlæreren. Det kan gjøres i vanlig post, e-post eller i elev/foreldresamtale.

Vi ønsker gjennom prosjektet å lære mer om hvordan vi best kan legge til rette for gode oppgaver i matematikk, slike som stimulerer forståelse og gjør at elevene lykke i matematikk. Det er viktig for oss at flest mulig deltar så vi håper på velvilje fra elever og foresatte. Vi er takknemlige for hjelpen vi får ved at **du** deltar, og dermed bidrar til å tilrettelegge for bedre matematikklæring i skolen.

Med hilsen

Anne Berit Fuglestad

Anne Berit Fuglestad
Prosjektleder/Professor   Rektor

✂ ---

Svarslipp:

Angående prosjektet: **Matematikkmestring i videregående skole**

Jeg har gjort meg kjent med informasjonen om prosjektet og tillater deltakelse.

Elevens navn/ underskrift: ______________________

Skole: _____________________

Foresattes underskrift: ______________
(for elever yngre enn 16 år)

Svar kan leveres til elevens lærer eller sendes i posten. Eventuelle spørsmål kan også sendes via e-post til: Anne.B.Fuglestad@uia.no

11.2 Transcription Key

,	Comma
.	Full stop
?	Question mark
!	Exclamation mark
(*x* pause)	Pause of *x* seconds
(Italic)	Description of things that are done/gestures/actions or other comments from the transcribers
Bold	Emphatic speech
(...)	Shorted in transcript when presented in thesis
(Indistinct)	Not possible to hear what is said or when in doubt of what is said – best guess
[ ]	Continued sentence to clarify statement

Overlapping or interrupted speech is noted in comments.

Transcript Example:

Turn	English	Norwegian	Comments
2	Ingeborg: Then you have gotten 25, and how did you get that? (Silence 10 seconds).	Ingeborg: Da har du kommet frem til 25, og hvordan kom du frem til det? (Stillhet 10 sekunder).	
3	Trude: We tried, but not with so many numbers.	Trude: Vi prøvde, men det var ikke så mange tall.	Fredrik does not seem to have the words to explain his actions, but I had seen him work, so I try to help.
4	Ingeborg: You guessed your way to it; yes, that is quite possible.	Ingeborg: Dere prøvde dere frem, ja, det går fint ann.	Ingeborg confirms that trial-and-error is a possible solution method. She has a positive intonation in her voice.

11.3 Tasks

11.3.1 Hair Salon Budget Task - Design, Arts, and Crafts

The PowerPoint Presentation:

Frisørsalong

- Gruppa driver en frisørsalong i byen
- Dere skal bestemme prisene, og lage et budsjett for en måned.

Hva skal en dameklipp koste?

- Hva er en vanlig pris for en dameklipp?
- Hva er det meste DU er villig til å betale?
- Hva er laveste pris DU kunne tatt for en dameklipp?
- Hva må dere vite for å bestemme prisene?

Hva påvirker prisene?

- Hvilke utgifter har frisørsalongen?
- Hvilke inntekter har frisørsalongen?

Lag et budsjett for frisørsalongen

- Bruk opplysningene dere får utdelt
- Om noen viktige kostnader eller inntekter mangler må dere diskutere dere frem til realistiske tall.
- Husk at noen inntekter og utgifter har en sammenheng – f.eks salgsinntekter og mva

---

- Hvor mye tjener dere i løpet av en måned?
- Hvor mye tjener DU i timen?
- Hvordan kan dere tjene mer?
  - Juster budsjettet deres slik at du blir fornøyd med månedslønna
  - Hva betyr denne justeringen for prislista?
    Husk å tenke på mva…

Lage budsjett i Excel

I Excel kan vi lage et budsjett hvor vi kan lett variere antall hårklipp og priser. Da kan man justere på disse til man er fornøyd med budsjettet.

Lag et budsjett i Excel, og bruk mest mulig formler.

	A	B	C
1	Budsjett Trudes Frisørsalong		
2			
3	Arbeidsdager	20	
4			
5		Pris	Antall
6	Dameklipp	450	8
7	Herreklipp		
8	Farging		
9			
10	Husleie	1200	40
11			
12			
13			
14			
15	**Inntekter**	**Kroner**	
16	Dameklipp	=B3*B6*C6	
17	Herreklipp		
18	Farging		
19			
20			
21			
22	**Sum inntekter**		
23			
24			
25	**Utgifter**	**Kroner**	
26	Husleie	=B10*C10	
27			
28			
29			
30			
31			
32	**Sum utgifter**		
33			

Budsjettarbeid frisørsalong

Oppgave 1:

Hva er en vanlig pris for en dameklipp?

Hva er den høyeste prisen du kunne betalt for en vanlig dameklipp?

Hva er laveste pris DU kunne tatt for en dameklipp? (Husk at du ikke får utbetalt hele prisen selv)

	Pris	Begrunnelse
Dameklipp vanlig pris		
Høyest pris du kunne betalt		
Lavest mulige pris		

Hva må dere vite for å kunne bestemme prisene?

____

____

____

____

____

____

____

____

Oppgave 2: Hva påvirker prisene?

Skriv ned de viktigste utgiftstypene frisørsalongen har i løpet av en måned.

____

____

____

____

____

____

____

____

____

____

____

Skriv ned de viktigste inntektstypene frisørsalongen har i løpet av en måned.

____

____

____

____

____

____

____

____

____

____

____

Oppgave 3. Lag et budsjett for frisørsalongen

Bruk opplysningene dere får utdelt. Om noen viktige kostnader eller inntekter mangler må dere diskutere dere frem til realistiske tall.

FORKLAR TALLENE DERES…

HUSK AT STATEN TAR MVA (moms) PÅ ALT DERE SELGER…

Inntekter	Kroner	Forklaring/regnestykke
Dameklipp		
Sum inntekter		
Utgifter	Kroner	Begrunnelse/regnestykke
Husleie		

Sum utgifter		
Overskudd/underskudd		

Oppgave 4:

Hva tjener frisørsalongen i løpet av en måned? ____

Hva tjener du i løpet av en måned? ____

Hva tjener du i løpet av en time?____

Hvordan kan dere tjene mer?

____

____

____

____

Hvordan vil dette påvirke budsjettet deres?

____

____

____

____

Lag nytt budsjettutkast slik at dere er fornøyd med inntekten deres. (Husk at en del av utgiftene forandrer seg når dere øker eller minsker inntekten.)

Hva betyr disse forandringene for prislista? Hvor mye vil en dameklipp koste nå?

____

____

____

____

____

INNTEKTER OG UTGIFTER – printes og deles i lapper

Husleie: 1200 kr pr kvadratmeter pr måned

Telefon:

	ISDN Pluss
Pris per mnd	265,-
Etablering/eierskifte	fra 899,-
Minuttpris	Ring gratis til fasttelefoner i Norge
Pris til mobil hos Telenor og Netcom	0,68
Pris til mobil i andre nett	0,76
Startpris	0,59

Renovasjon: 3625 kr i året + 171 kr pr tømming av 1000 liters dunk

Reparasjon og nyanskaffelser av utstyr: 3000 kr i måneden

Forbruk hårprodukter: 15 kr pr kunde

Forbruk hårfarge: 100 kr pr fargebehandling

Reklame:

Kaffe/te: 5 kr pr kunde

Mva (moms): 25 % av prisen på klipp og annet varesalg

Forsikring: 20000 kr i året

Hårklipp dame:

Hårklipp mann:

Farging hår:

Fortjeneste salg hårprodukter:

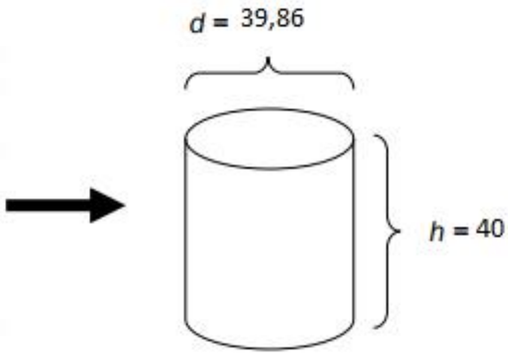
11.3.2 Engine Cylinder Task - Technical and Industrial Production

Oppgave mopedmotor

Maksimalt lovlig slagvolum på moped i Norge er 50 kubikkcentimeter (50 cc).
I tabellen er det ett eksempel på en motor som oppfyller det.

Technical specifications

	Senda R DRD
Engine	Single cylinder 2 T EURO 1
Bore x Stroke	39.86 x 40
Cylinder capacity	49.9 cc
Carburettor	Dell'Orto PHVA-14
Cooling system	Liquid
Starting system	Kick starter
Compression relation	13.0: 1
Fuel	Unleaded petrol



Bore: boringa/diameteren til sylinderen.
Stroke: slaglengde/høyde i sylinderen.

a) Kontrollregn at motoren i tabellen over har sylindervolum under 50 cm^3.

b) Hva blir nytt volum om du pusser hele sylinderen innvendig slik at diameteren blir 2mm større?

c) Hvor stor kan slaglengden/høyden maksimalt være for å ha en lovlig moped om boringa/diameteren er

3) 50 mm?

4) 30 mm?

Oppgave bilmotor

En bilmotor har ofte 4, 6 eller 8 sylindere.

a) En motor består av 4 sylindere med boring/diameter 81 mm og slaglengde/høyde 77,6 mm. Hvor stort totalt slagvolum har motoren?

b) En 3,5 liter V8 motor (8 sylindere) har en slaglengde/høyde på 65,6 mm. Hvor stor må boringa/diameteren være?

c) Foreslå hva slaglengde/høyde og boring/diameter kan være i en 3,5 liter motor med 4 sylindere.

11.3.3 Frifond Project Task - Media and Communication

PROSJEKTOPPGAVE – BUDSJETT

Innledning

Frifond ble opprettet av Stortinget i 2000 for å bedre de økonomiske rammebetingelsene for demokratisk og frivillig aktivitet i lokalsamfunn. Her kan dere søke om støtte på inntil kr 25 000,- for å gjøre det du har lyst til der du bor!

Les mer her! Les også retningslinjene for å søke som du finner link til på samme side.

1. Velg et prosjekt som dere vil søke om støtte for hos Frifond, og lag en ideskisse. Denne skal dere presentere for klassen om 15 min.
   (prosjektet kan være f.eks. bygge skateramper, lage film, lage magasin, arrangere dataparty, lage fotoutstilling, arrangere festivaler, kurs eller seminarer…)

2. Skriv en kort prosjektbeskrivelse med hva dere trenger av utstyr, lokaler osv.

3. Sett opp et budsjett for prosjektet.
   NB! Dere må dokumentere tallene, dvs. skriv hvor dere henter priser fra og evt. utregninger. Dette kan dere sette opp i et vedlegg til budsjettet.

Lenker

Greie ressurser:

http://filmklubb.no/starte-en-filmklubb/filmkonomi

http://filmforbundet.wordpress.com/dine-rettigheter/honorarsatser/

http://blogg.norgeskreativefagskole.no/2012/02/slik-skriver-du-et-budsjett/

http://www.lovdata.no/for/sf/ku/xu-20090907-1168.html

Leie kamerating:

http://tv-nor.com/leie-videokamera-oslo/

http://robinlund.no/utleie/

http://puzzlefilm.no/index.php?option=com_content&view=article&id=57&Itemid=75

http://www.doppler.as/Dokumenter/Priser_2.htm (prisliste med fotograf + kamera)

http://www.eurotek.no/ (mye rart, såpeboblemaskin og flammekaster…)

Leie kostymer:

http://www.festmagasinet.no/default.aspx?gid=11

http://kostyme-spesialisten.no/default.php?cPath=4&osCsid=8pq7rcojp7r9bguuuk0eeir9c7

http://verdal-teaterlag.no/kostymeutleie/ (tipper rosegården har ca samme priser)

Leie utstyr:

http://injection.no/tv-media/tjenester/76-utleie-av-videoutstyr

http://injection.no/tv-media/prisliste

11.3.4 Jack Stand Task - Technical and Industrial Production

Bruk av omkretsen av en sirkel

Oppgave 1

a) Tenk deg at du skal lage en støttebukk med tre bein. Det utvendige røret har en diameter på 48mm. Du skal sveise på tre bein. Hvor langt fra hverandre må du sette merkene langs omkretsen av røret?

b) Hvor langt fra hverandre må du sette merkene dersom røret har en diameter på 43mm?

c) Hva ville svarene i a og b blitt dersom støttebukken skulle ha fire bein?

Oppgave 2

Du skal bore tvers gjennom et rør med diameter 8 cm. For å vite hvor boret skal komme ut, vil du sette et merke på motsatt side av røret. Hvordan vil du finne ut hvor du skal sette merket?