



Teaching and learning for mathematical literacy

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Foreword

My interest in the teaching and learning of mathematics started in secondary school when I was helping a friend with her mathematics homework. I realised that to help my friend understand I had to challenge my own mathematical understanding. It was both fascinating and engaging. This fascination was the reason why I became a teacher and pursued a master's degree in mathematics education. It continued through my work as a mathematics teacher in lower secondary school.

Even though I enjoyed teaching, I missed the researching and learning that I experienced when doing my master. When Sogn og Fjordane University College (now Western Norway University of Applied Sciences) offered a 4-years full-time position as a PhD research fellow, I saw an opportunity to engage in research on mathematics teaching and learning and continue teaching, although at the teacher education level.

Many people deserve to be acknowledged for their support during these four years. First, I would like to thank the school leaders, teachers, and students who participated in the research. They generously welcomed me into their schools and classrooms and shared their stories and experiences with me.

A warm thank you to my colleagues at the Western Norway University of Applied Sciences and Volda University College for support and encouragement during the process. Also, I am grateful for the support from The Norwegian National Research School in Teacher Education (NAFOL) and its members.

Above all, I would like to thank my supervisors, Simon Goodchild and Frode Olav Haara, for their guidance and support. Your intelligent remarks, challenging questions, humour, and warmth have been of utmost importance for me in order to complete this work. I also want to give warm thanks to Professor Marilyn Goos and her colleagues in EPI*STEM, University of Limerick, for welcoming me as a visiting researcher for three months in the autumn of 2019.

Finally, and most importantly, to my mum, dad, sister, husband, family, and friends: Thank you so much for your patience and understanding, and for always believing in me and cheering me on.

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Abstract

This dissertation reports from research that investigates the nature of teaching and learning for mathematical literacy in three lower secondary schools in Norway. Mathematical literacy is a notion used to denote the competences required to meet the mathematical demands of life in modern society. The importance of education for mathematical literacy is emphasised by The Organisation for Economic Cooperation and Development (OECD), and mathematical literacy has become increasingly prominent in national curricula around the world. In the Norwegian curriculum, mathematical literacy is considered a basic skill that should be developed across school subjects.

This study of teaching and learning for mathematical literacy is framed within a cultural-historical perspective on teaching and learning. It draws on cultural-historical activity theory and the theory of objectification. Also, a multi-faceted model of mathematical literacy is used to analyse the data. The research uses a cross-sectional case study design involving six school leaders, three mathematics teachers, and their grade 9 students. A qualitative approach to data generation and data analysis was adopted, and the empirical material was generated through interviews and lesson observations.

The results of the study show that teaching and learning for mathematical literacy can be improved. Although the teachers recognise the importance of education for mathematical literacy and ways in which this can be done, they need a strategy for implementing it in their teaching. Also, there is an extensive focus on the contextual element of mathematical literacy. This emphasis may be overshadowing other important elements of mathematical literacy and, in this way, narrowing the meaning of mathematical literacy to only involve the use of mathematics in context. Consequently, opportunities for developing mathematical literacy through, for example, critically evaluating the use of mathematical knowledge and tools are not recognised and pursued.

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1 Introduction

On a mathematics test in lower secondary school, I gave my students the following task, derived from a national mathematics exam:

A farmer has 180 meters of fence. He wants to use the fence to make a grazing area. He wants the grazing area to have one of the shapes displayed below.



Square



Circle

The farmer wants the area of the grazing area to be as large as possible. Decide using calculations which shape the farmer should choose. (The Norwegian Directorate for Education and Training, 2015, p. 10, my translation)

One of my students answered; “I think he should choose the quadrilateral because that’s what I’ve seen that most farmers do.” This answer was not what I expected nor what I wished. The circle has the largest area. However, my student had a point. First, why would the farmer stand out and do something different from the other farmers? He might look stupid. Second, there might be good reasons for the other farmers to choose the square. Even though the circle has the largest area, the circle does not tessellate, which makes it difficult to make use of the rest of the field.

My point is that this task does not consider the issues which are essential in the context from which the task originates. In a mathematics context, my student’s answer might look silly. From a real-world perspective, it is, in fact, the task that is silly. An exploration of the task could have been the start of a fruitful discussion about mathematics and the real world. However, at the time, I did not engage in such exploration with my students, perhaps leaving them with the

wrongful impression that the “rules” of the real world do not apply in the world of mathematics.

Situations such as the one described above may be one reason for the constantly recurring question in mathematics classrooms; “Why do we have to learn this?” (Hernandez-Martinez & Vos, 2018; Wedege, 2009). Most mathematics teachers have been asked this question at some point in their careers, including myself. It may not always be easy to provide students with a satisfactory answer. “You will need it in the future” may silence them, but it does not always do the trick. Students want to know in which ways the mathematics curriculum content is or will be useful to them. They want what they learn in school to be relevant for their current and future lives, and sometimes they need assistance in seeing the actual use of it.

Niss (1996) analyses the justification and goals for mathematics education from historical and theoretical perspectives. He identifies three fundamental reasons for mathematics education:

- To contribute to the technological and socio-economic development of society at large.
- To contribute to society’s political, ideological, and cultural maintenance and developments.
- To provide individuals with the prerequisites which may help them to cope with different aspects of life.

These three reasons presuppose that mathematics education can contribute to such societal and individual development. However, teachers often complain that students are not able to use what they learn in school in different contexts (De Lange, 2003). They struggle to see the connections between different subject areas and situations. To be able to transfer their knowledge from one context to another, students need experience in solving problems in a range of different contexts (Steen, 2001).

The mathematics education research community has for a long time argued for the importance of involving students’ everyday lives in mathematics teaching (Blum, Galbraith, Henn, & Niss, 2007; De Lange, 2003; Freudenthal, 1973; Haara, 2011). Moreover, there is a political focus on promoting students’ motivation and learning through practical, varied, and relevant teaching by focusing on mathematical applications (De Lange, 1996; The Norwegian Ministry of Education and Research, 2010). Hence, making mathematics teaching realistic and relevant for life in the so-called “real” world (that is the

lived-in world outside the school) is emphasised by researchers, educators, students, and politicians. The question is “How can this be achieved”?

The research reported here investigates teaching in lower secondary school from the perspective of developing students’ competence to use mathematics in their everyday lives. In the present study, this competence is referred to as mathematical literacy. The study contributes to an understanding of the dynamics involved in teaching and learning mathematics across three levels in school. These levels involve school leaders, teachers, and students. The study also contributes to knowledge about the relationship between the rationales for teaching, the operationalised teaching, and the outcome of teaching with respect to mathematical literacy.

In this introductory chapter, the background for the study is presented in Section 1.1. In section 1.2, the Norwegian context is outlined, and the aims of the study are presented in Section 1.3. The final section, Section 1.4, contains an overview of the dissertation.

1.1 Background

There is a growing understanding that under-developed mathematical competences limit the individual’s prospects in terms of career aspirations, social well-being, financial security, and political participation (Geiger, Goos, & Forgasz, 2015). Rapidly developing technology, extensive use of numbers and quantitative measures in the media, and increasing use of quantitative thinking in personal life, the workplace, and society, in general, has led to a need for a set of competencies that involves more than pure mathematics (Steen, 2001). The competence to deal with the quantitative aspects of life is sometimes referred to as mathematical literacy. Mathematical literacy is defined by The Organisation for Economic Cooperation and Development (OECD) as

an individual’s capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to recognise the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens. (OECD, 2012, p. 25)

Mathematical literacy has gained increasing international attention, particularly through The Programme for International Student Assessment (PISA), carried out under the auspices of the OECD (Geiger, Forgasz, & Goos, 2015). PISA aims

to assess students' level of mathematical literacy. The PISA framework has had a great impact on the development of the participating countries' curricula and policy documents (Breakspear, 2012; Geiger, Goos, et al., 2015). Even though the notion of mathematical literacy may not be explicitly stated, an examination of curriculum documents shows that a wide range of aspects of mathematical literacy is implicit (Frejd & Geiger, 2017). One of the countries in which mathematical literacy has gained increased political and educational attention, and in which the current study is situated, is Norway.

1.2 The Norwegian context

Understanding of the natural sciences is important for the individual to understand the society we live in and to deal with everyday life. (...) We need to show that mathematics is important and provides possibilities – for society and for the individual. (The Norwegian Ministry of Education and Research, 2015, p. 6, my translation)

In the last 20 years in Norway, there has been great emphasis on students' competence in the STEM subjects (science, technology, engineering, and mathematics). This emphasis is in part due to the poor Norwegian results on international educational assessments such as PISA and the Trends in International Mathematics and Science Study (TIMSS), the latter run by the International Association for the Evaluation of Educational Achievement (IEA). When the first PISA test results were published in 2001, Norway experienced a "PISA shock". Norwegian students performed below the OECD average, which was lower than expected. Since then, several educational policy strategies have been initiated in order to deal with this "crisis" (Haugsbakk, 2013; Kongelf, 2019).

The curriculum reform implemented in 2006 (LK06) contained a set of basic skills, similar to the competences in the PISA framework (Kjærnsli & Olsen, 2013). This curriculum was operational at the time of the present study, and I will return to the basic skills in Chapter 2.

The Norwegian STEM strategy document entitled "Tett på realfag", which can be translated as "STEM in focus" (The Norwegian Ministry of Education and Research, 2015), was effective from 2015-2019. The document contains goals and strategies for developing children's and adolescents' understanding of the STEM subjects. One of the initiatives was to establish what were to be referred to as "STEM municipalities". These municipalities would receive government

funding to establish professional development networks for teachers in STEM subjects, and to develop local strategies to improve students' competence in these subjects.

Another measure to improve students' competence in the natural sciences was to renew the subject syllabuses. An expert group was appointed to evaluate the current subject syllabuses and make recommendations for revisions. The mathematics expert group reported that mathematics teaching involved little variety in learning activities:

Teaching is characterised by teacher demonstrations of theory and examples similar to textbook tasks. After that, students work mostly individually with tasks often connected to procedural knowledge. This form of teaching gives little room for cognitively challenging and complex problems. (The Norwegian Ministry of Education and Research, 2015, p. 17, my translation)

In 2020, a renewal of LK06, LK20, is being implemented in Norway. Reasons for this renewal are, among other things, that what students learn needs to be relevant in order to keep up with the rapid developments in society, work-life, and technology (The Norwegian Directorate for Education and Training, 2018). In this reform, the basic skills are extended, and a set of core elements are added. The core elements in mathematics describe methods of mathematical working and thinking, in addition to important mathematical subject areas. I will return to the core elements and LK20 later in the dissertation.

1.3 Aims and research question

As indicated above, the purpose of this study is to gain knowledge about teaching for mathematical literacy. According to Sfard (2014, p. 141) “the question of how to teach for mathematical literacy must be theoretically and empirically studied. When we consider the urgency of the issue, we should make sure that such research is given high priority.” It is believed that by investigating the current state of affairs, we can gain valuable insight into what we need to do in order to get where we want to be. The main research question guiding this study is:

What is the nature of teaching and learning for mathematical literacy in lower secondary schools in Norway?

1.4. Overview of the dissertation

The following chapter, Chapter 2, outlines the foundations of the notion mathematical literacy. Mathematical literacy is also discussed in relation to the Norwegian educational context. Models of mathematical competencies related to mathematical literacy are presented.

Chapter 3 outlines the theoretical background. A general description of cultural-historical activity theory and a more detailed outline of the theory of objectification is presented. The theory of objectification is a theory for conceptualising learning as processes of encounters with history and culture.

Chapter 4 gives an overview of the empirical background for the dissertation. A review of previous research in the area is provided. The chapter summarises research on mathematical literacy and research on teaching and learning mathematics through contexts and applications.

Chapter 5 elaborates the principal methodological issues in focus. The methodological approach, the research methods used, and data generation and analysis are presented. Ethical considerations are discussed at the end of the chapter.

The results from the three articles are presented in Chapter 6. The articles concern school leaders' and teachers' rationales for teaching for mathematical literacy, teachers' operationalisation of teaching for mathematical literacy, and students' encounters with mathematical literacy.

In Chapter 7, the results that were presented in the previous chapter are discussed with respect to the overarching aim of the project. The main research question is addressed, and the three articles are connected and discussed with relation to the theoretical framework and empirical background. Conclusions are drawn from the discussion. Finally, critical reflections and contributions of the study in terms of implications for practice and further research are offered.

2 Mathematical literacy

In modern society, the roles played by numbers are endless. Uses of quantitative thinking in the workplace, in education, and nearly every other field of human endeavour are rapidly increasing (Steen, 2001). Unfortunately, many educated adults lack the quantitative skills needed in today's world, and manifestations of such are prevalent (De Lange, 2003; Steen, 2001), for example, in terms of mathematical errors in newspapers. On August 5, 2019, a Norwegian newspaper reported from a party leader debate (Krekling, 2019). The topic was greenhouse emissions. One accused the other of not understanding the statistics he presented related to the decrease of greenhouse emissions in a particular area. She argued that emissions had increased by 22 per cent from 2011 to 2015. From 2015 emissions have decreased by 20 per cent. She concluded that this gives a total increase of 2 per cent. It appears as if they both have some challenges with the statistics.

Another example is displayed in Figure 1 below. The diagram is a screenshot from a Norwegian online newspaper article (Solgård, 2019) and shows the answers to the question "To what extent would you have a guilty conscience for the climate if you ordered a plane trip?" The darkest blue sector displays women, the lightest blue sector displays men, and the medium blue sector displays the total. In fairness, and fortunately, the newspaper later deleted the diagrams from the article.

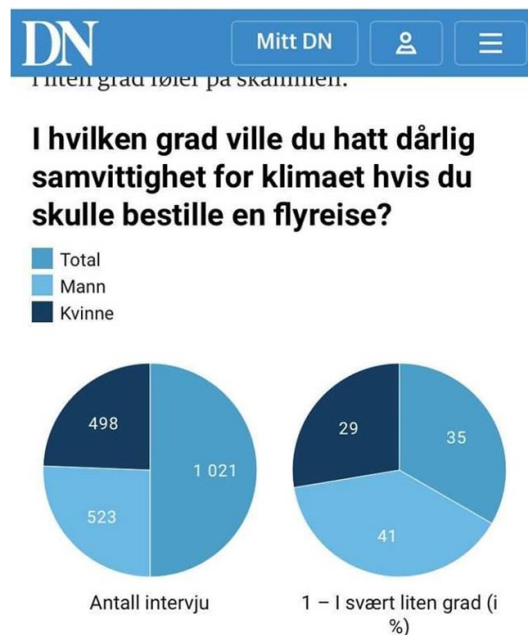


Figure 1. Screenshot from a Norwegian newspaper.

To discover and question the errors made in the examples above requires some level of mathematical knowledge and confidence. In this section, I describe mathematical literacy and related notions and skills considered important for coping with today's world. I give a brief historical outline and relate it to the Norwegian context. I also describe different elements involved in the different notions.

Mathematical literacy is a notion used to define the body of knowledge and competences required to meet the mathematical demands of personal and social life and to participate in society as informed, reflective, and contributing citizens (Geiger, Forgasz, et al., 2015). One of the first occurrences of the notion was in 1944 in the USA when a Commission of the National Council of Teachers of Mathematics (NCTM) required that the school should ensure mathematical literacy for all who can achieve it (Niss & Jablonka, 2014). However, no attempt to formulate an explicit definition was offered until the initial OECD framework for PISA 1999. The definition has been slightly revised for subsequent PISA studies but the version from PISA 2012, cited in the introduction of this dissertation, still stands.

Despite the international rooting of the definition in the OECD-PISA study, mathematical literacy has no universally accepted meaning. It is a difficult concept to translate as it lacks non-English equivalents (Jablonka, 2015). In some languages, the word literacy has such a narrow meaning that it can be impossible to convey the broad meaning intended by PISA (Stacey & Turner, 2015). For example, in Spanish, French, and Scandinavian languages, literacy is linked to very basic reading and writing competencies. As a result, concepts like *mathematical competence* and *mathematical culture* are used instead to avoid the narrow connotations of the term literacy in educational debates (Stacey & Turner, 2015).

Also, mathematics education literature contains several notions related to mathematical literacy. Some authors use concepts like mathematical literacy, *numeracy*, and *quantitative literacy* synonymously, while others distinguish between them (Niss & Jablonka, 2014). Other related concepts are *critical mathematical numeracy* (e.g. Frankenstein, 2010), *mathemacy* (e.g. Skovsmose, 2011), *matheracy* (e.g. D'Ambrosio, 2007), and *statistical literacy* (Watson, 2011). De Lange (2003) conceptualises mathematical literacy as the overarching concept comprising all others.

While the term mathematical literacy seems to be of American origin, the term numeracy has been principally used in countries influenced by the United Kingdom. It was coined as the mirror image to literacy in the Crowther Report of 1959 (Ministry of Education, 1959), meaning scientific literacy in the broad sense. A narrowing of the meaning was noted in the Cockcroft Report of 1982 (Department of Education and Science, 1982), describing numeracy as an “at-homeness” with numbers (Stacey & Turner, 2015). However, there are variations in the meaning of the term numeracy, ranging from the acquisition of basic arithmetic facts and procedures through to richer interpretations that involve problem-solving within authentic contexts and higher-order thinking (Geiger, Goos, et al., 2015; Steen, 2001). Still, the different interpretations of these concepts have in common that they stress awareness of the usefulness and competence to use mathematics in different areas (Niss & Jablonka, 2014).

PISA’s reports that compare students’ performance have been influential in shaping educational policies in several OECD countries, and curriculum developers/reviewers have tried to reflect PISA competences in their national curricula (Breakspear, 2012). Three approaches have been used internationally in efforts to promote mathematical literacy learning in schools (Bennison, 2015). One approach is to offer mathematical literacy subjects as an alternative to mathematics subjects. This approach is taken in South Africa. From 2000 to 2005, as much as 40 per cent of South African learners writing the grade 12 exam did not take mathematics as a subject (Pillay & Bansilal, 2019). As a consequence, a new subject called mathematical literacy was implemented in 2006 to help learners develop competence to understand and engage with mathematics in the real world. In South Africa, mathematical literacy is a compulsory subject for students who are not studying mathematics in grades 10-12 (Botha & van Putten, 2018). A second approach is to integrate mathematics and other subjects. For example, recent revisions to mathematics curricula in some European countries, have resulted in an increased emphasis on cross-curricular links. In these two approaches, the emphasis is on mathematics. However, a third approach sees mathematical literacy as part of all subjects across the curriculum. This third approach is taken, for example, in Australia (Bennison, 2015) and Norway (The Norwegian Directorate for Education and Training, 2012).

2.1 Mathematical literacy in Norwegian curriculum documents

The connection between mathematics and life outside school has been emphasised in several Norwegian curricula. In the curricular reform from 1997 (L97), it is stated that all subjects should promote inventive abilities, creativity, practical skills, and knowledge of nature, the environment, and technology (The Norwegian Ministry of Church Affairs Education and Research, 1996). The subject syllabuses emphasise practical activities and tasks and the connection between theory and practice. In primary school, teaching should be organised according to themes, and interdisciplinary work is valued. In the introduction to the mathematics syllabus, it is stated that “The syllabus emphasises making connections between school mathematics and the mathematics in the world outside school” (The Norwegian Ministry of Church Affairs Education and Research, 1996, p. 153, my translation). In addition to learning mathematical concepts and symbols, the syllabus emphasises the importance of mathematics to participate in society and to handle challenges in personal and work life. *Mathematics in daily life* is, in fact, a topic in the syllabus and is described as follows:

The students must get to know basic mathematical concepts which are in direct connection with experiences from their everyday. They must experience and become confident with the use of mathematics at home, in school, and their local community. They must learn to cooperate to describe and find solutions to situations and problems, discuss and explain their thinking, and develop confidence in their own possibilities. (The Norwegian Ministry of Church Affairs Education and Research, 1996, p. 158, my translation)

In 2006, a curricular reform (LK06) was implemented in Norway. LK06 included five basic skills that are “fundamental to learning in all subjects as well as a prerequisite for the student to show his/her competence and qualifications” (The Norwegian Directorate for Education and Training, 2012, p. 5). These skills should be integrated and developed in all subjects across the curriculum. The five basic skills are *reading, writing, oral skills, digital skills, and numeracy*.

Numeracy means applying mathematics in different situations. Being numerate means to be able to reason and use mathematical concepts, procedures, facts and tools to solve problems and to describe, explain and predict what will happen. It involves recognizing numeracy in different contexts, asking questions related to mathematics, choosing relevant methods to solve problems and interpreting validity and effect of the results. Furthermore, it involves being able to backtrack to make new choices.

Numeracy includes communicating and arguing for choices by interpreting context and working on a problem until it is solved.

Numeracy is necessary to arrive at an informed opinion about civic and social issues. Furthermore, it is equally important for personal development and the ability to make appropriate decisions in work and everyday life. (The Norwegian Directorate for Education and Training, 2012, p. 14)

The Framework for Basic Skills (The Norwegian Directorate for Education and Training, 2012) describes four sub-categories of numeracy: *recognise and describe*, *apply and process*, *communicate*, and *reflect and assess*. Recognise and describe involves being able to identify situations involving numbers, units, and geometric figures found in games, play, subject situations, work situations, and civic life. It also involves identifying, analysing, and formulating problems appropriately.

Apply and process involves being able to choose strategies for problem-solving. It also involves using appropriate measurement units, calculating, retrieving information from tables and diagrams, drawing and describing geometric figures, processing and comparing information from different sources. Communicate involves being able to express numerical processes and results in different ways, and to argue for and validate choices, explain work processes and present the results.

Reflect and assess involves interpreting results, evaluating the validity, and reflecting on the meaning of the results. It also involves using the results as the basis for a conclusion or an action.

Hence, numeracy involves having elementary, technical skills and factual knowledge and the competence to use these in practical and subject-related tasks and problems. It is something more than knowledge on an elementary level (Alseth, 2009). The students should be prepared to take a stand on societal issues and to make well-founded decisions in everyday life.

The Norwegian curriculum is translated to English by the Norwegian Directorate for Education and Training. The basic skill numeracy is a translation of the Norwegian notion *rekning som grunnleggende ferdighet*. There are several challenging issues with this Norwegian notion. In the following, I will outline some of these issues.

Numeracy is translated from the Norwegian word *rekning*. *Rekning* corresponds to *computation* or *arithmetic* in English and *rechnen* in German. The word *rekning* is challenging because it involves various interpretations in the

Norwegian language. There is no clear definition of *rekning*, and discussion about how to understand *rekning* in the mathematics subject happens frequently (Fauskanger & Mosvold, 2009; NOU 2015:8, 2015). One interpretation involves technical computations, another emphasises understanding and meaning involved in computations, and a third connects it to practical computations in everyday life. An acknowledged Norwegian encyclopaedia (Store norske leksikon) states that *rekning* “usually denotes the execution of the elementary arithmetic operations with numbers: addition, subtraction, multiplication, division, and partly also evolution” (Regning-matematikk, 2018, my translation).

The meaning of the word *basic* (*grunnleggande* in Norwegian) is also not clear. It creates associations to elementary, fundamental knowledge (Grønmo, 2014).

Skills also involve challenges. Historically, the word *skills* (*ferdigheiter* in Norwegian) has been understood as technical and routine symbol treatment. Brekke (2002) defines *skills* as well-established procedures in several steps which are automatised. *Skills* are used in this manner in Norwegian daily language (Grønmo, 2014), and this has led many to believe that numeracy as a basic skill should be understood in this manner.

Alseth (2009) problematises the use of the word *ferdigheiter* in LK06 because it may create misconceptions about what numeracy as a basic skill is. The discussion above indicates that the entire notion of *rekning som grunnleggande ferdighet* involves issues that can cause misconceptions. *Rekning* has always been a central part of mathematics. To add some more confusion to the notion, in Norway, both *rekning* and mathematics have been used to denote the subject, sometimes synonymously. The official name change from *rekning* to mathematics was in the 1960s (Botten & Sikko, 2009), but it can sometimes still be heard in use.

The Norwegian White Paper Nr. 30, is one of the founding documents for LK06 (Alseth, 2009). In this report, numeracy is explained as follows:

To compute and to be numerate is the competence to use addition, subtraction, multiplication, division, and ratios to solve a wide range of tasks and challenges in both everyday and subject situations. It also involves the competence to observe and interpret patterns and graphs. (The Norwegian Ministry of Education and Research, 2004, p. 34, my translation)

This definition is close to understanding *rekning* as meaning technical skills, even though it is connected to an everyday and subject context.

Starting from the fall of 2020, a revision of LK06, LK20, is being implemented. The basic skills are extended in the new curriculum. Also, six core elements are included in the common core subject of mathematics. These are; *inquiry and problem solving, modelling and applications, reasoning and argumentation, representation and communication, abstraction and generalisation*, and *mathematical knowledge areas* (The Norwegian Directorate for Education and Training, 2019). The five first core elements describe methods, procedures and ways of thinking in mathematics. The sixth describes central mathematical knowledge areas which the students should meet through the five first. LK20 emphasises mathematics as central in contributing to students' development of a language for reasoning, critical thinking, and communication through abstraction and generalisation. Critical thinking is described as involving a critical evaluation of reasonings and arguments.

In addition to the basic skills and the core elements, the general part of LK20 also introduces three cross-curricular topics, of which two are emphasised in the mathematics curriculum. These two topics are *public health and life management* and *democracy and citizenship*. The cross-curricular topics can also be related to mathematical literacy as they involve the competence to make responsible life choices, to explore and analyse real data and numbers and evaluate the validity of such, and to formulate arguments and contribute in the societal debate. It emphasised that the mathematics subject should contribute to students' competence to reason, think critically, understand patterns in nature and society, and make decisions in one's own life and society. In this sense, LK20 contains more about what to teach for mathematical literacy than LK06.

In Norway, as in several other countries, mathematical literacy is a cross-curricular commitment, and students' mathematical literacy is measured through international tests such as PISA and national mathematical literacy tests. However, this does not mean that the curriculum documents provide any guidance in operationalising the mathematical literacy demands and opportunities of the subjects they teach. Also, suggestions or advice about how to design tasks and learning sequences that embed mathematical literacy across the curriculum, or how to make decisions about pedagogies that support mathematical literacy learning are not provided (Liljedahl, 2015). Hence, teachers are expected to implement these ideas in their teaching, perhaps involving fundamental changes in their practices, only supported by a definition.

Changing teaching practice is a process that must take place within the teacher. Mosvold (2005) refers to Wilson and Cooney's (2002) findings that there are connections between teachers' beliefs about mathematics and their teaching. These beliefs are not necessarily rooted in the curriculum. Hence, a renewal of the curriculum does not alone lead to a change in teaching practice (Mosvold, 2005).

2.2 Elements of mathematical literacy

There seems to be agreement that the numerical demands of modern life require more and something else than pure mathematical knowledge. Several attempts have been made to identify the elements involved in mathematical literacy and related concepts.

Two competence models, both introduced around the year 2000, have been influential in the development of the mathematical literacy framework in PISA. In a comparison between the mathematical aims of LK06 and the PISA 2012 analytical framework, Nordtvedt (2013) concludes that the mathematical content covered by PISA is included in the Norwegian curriculum. It is therefore relevant to give the two models some attention in this dissertation. Also, the PISA modelling cycle is introduced, followed by a model for numeracy in the 21st century.

Kilpatrick, Swafford, and Findell (2001) have formulated five components of *mathematical proficiency*, which they believe comprise the mathematical knowledge, understanding, and skill people need today (see Figure 2). *Conceptual understanding* involves comprehension of mathematical concepts, operations, and relations. *Procedural fluency* involves skill in carrying out procedures with flexibility, accuracy, efficiency, and appropriateness. *Strategic competence* is the competence to formulate, represent, and solve mathematical problems. Adaptive reasoning is the capacity for logical thought, reflection, explanation, and justification. *Productive disposition* is the habitual inclination to see mathematics as sensible, useful, and worthwhile, and the belief in one's own efficacy. The five components are interwoven.

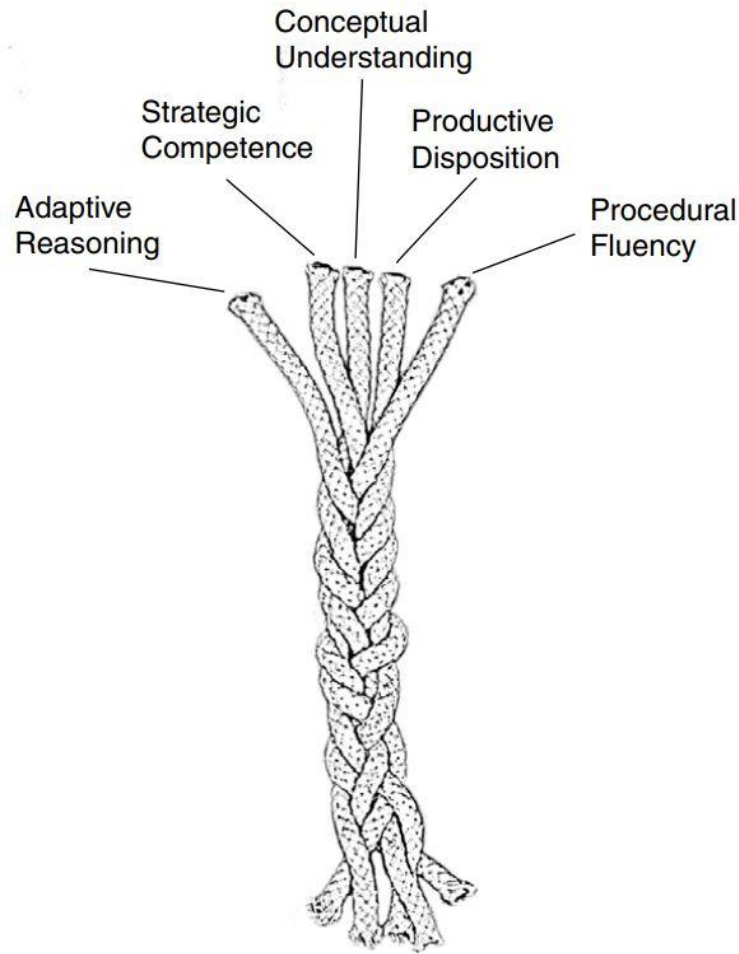


Figure 2. The intertwined strands of mathematical proficiency. Adopted from Kilpatrick et al. (2001, p. 117).

The strands of mathematical proficiency comprise the mathematical knowledge, skill and understanding people need. However, the competence to apply these in real-world contexts are not explicit in the model proposed by Kilpatrick and colleagues. Niss (2015) uses the term *mathematical competence* to denote the knowledge and insights needed to deal with mathematical challenges in a variety of situations successfully. In general,

mathematical competence comprises having knowledge of, understanding, doing, using and having an opinion about mathematics and mathematical activity in a variety of contexts where mathematics plays or can play a role» and «a mathematical competency is a well-informed readiness to act appropriately in situations involving a certain type of mathematical challenge. (Niss & Højgaard, 2011, p. 49)

Niss and Højgaard (2011) identify eight competencies, depicted as the petals of a flower, as illustrated in Figure 3. The first four comprise a group that forms the

composite competence to ask and answer questions. These are *mathematical thinking competency* (the nature and kinds of questions and answers that are typical of mathematics), *problem tackling competency* (identifying, posing, and solving mathematical problems), *modelling competency* (competence to deal with mathematics in extra-mathematical domains by way of explicit or implicit modelling), *reasoning competency* (construct, follow, and justify answers). The remainders form a group that concerns the competence to handle language and tools. This group comprises *aids and tools competency* (to handle physical instruments to assist in carrying out mathematical processes), *communicating competency* (to express oneself and to understand others' verbal, written or figural expressions), *symbols and formalism competency* (to deal with mathematical symbols rules, and formalisms), and *representing competency* (to interpret, employ, and translate between representations).

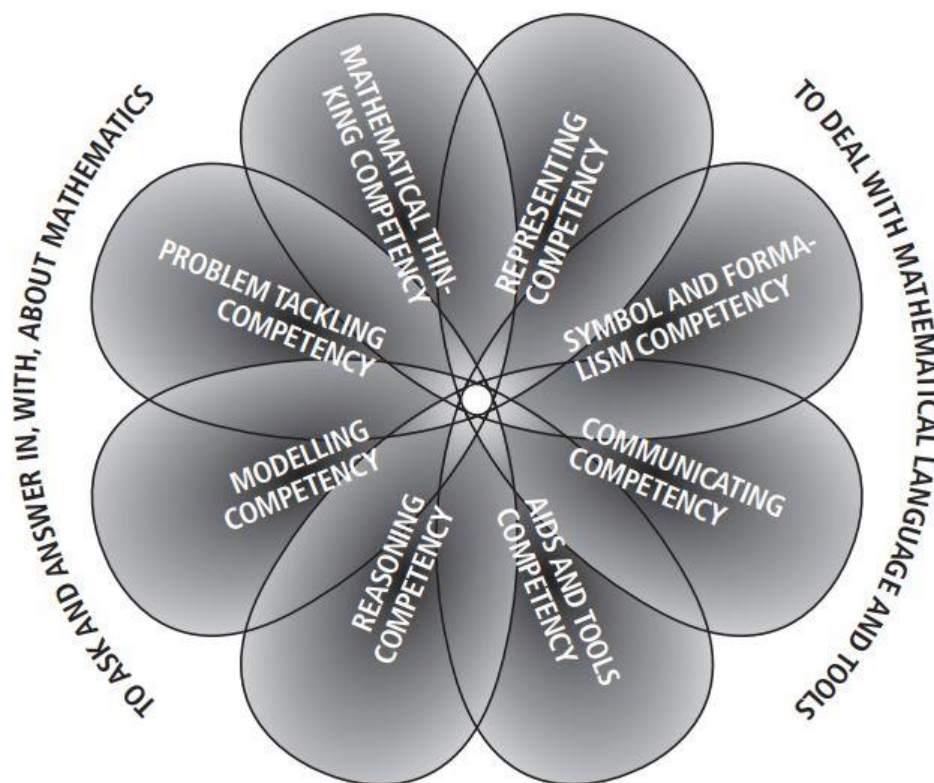


Figure 3. A visual representation of the eight mathematical competencies.

Adopted from Niss and Højgaard (2011, p. 51).

In PISA 2009, the KOM competencies were presented as key components of mathematical literacy (Niss, 2015). Niss (2015) perceives mathematical literacy

as a subset of mathematical competence. This perception implies that a mathematically competent person is also a mathematically literate person, but it does not hold the other way around. The reason it does not hold is the focus on the functional aspects of having learnt mathematics. According to Niss (2015), mathematical competence also involves working within purely mathematical structures that are never required in the physical world.

In the PISA framework, the notion of mathematical modelling is a cornerstone embedded within the definition of mathematical literacy (OECD, 2012). Modelling problems arise from the real world. To solve these problems, one has to draw upon different mathematical concepts, knowledge and skills. *The modelling cycle* in PISA involves the four processes; *formulate*, *employ*, *interpret*, and *evaluate*. A problem in context is transformed into a mathematical problem by identifying mathematical aspects in the context and formulating them mathematically. Mathematical concepts, procedures, facts, and tools are employed in order to obtain a mathematical result. The mathematical result is interpreted in the original problem in context, and the reasonableness of the whole process is evaluated. The modelling cycle is displayed in Figure 4.

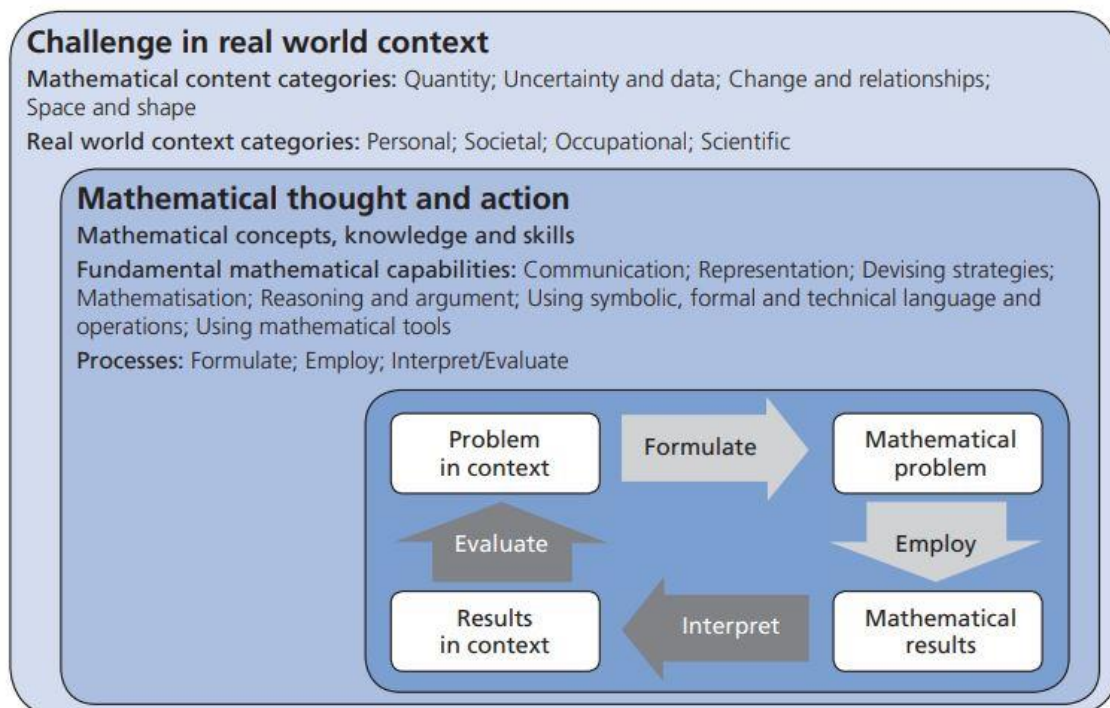


Figure 4. The PISA modelling cycle. Adopted from OECD (2012, p. 26).

The real-world problems may be set in a personal, occupational, societal, or scientific context. The mathematical processes involve several fundamental capabilities such as different forms of representations (i.e. formal symbols, language, graphs, diagrams), strategies, tool use, reasoning, and arguing. The competency flower comprises the competencies needed to deal with mathematics in various situations. However, as the proficiency strands, Niss and Højgaard’s (2011) competency flower does not explicitly emphasise real-world contexts. Besides, like written descriptions of mathematical literacy and related notions, it is not easily operationalised. Therefore, informed by relevant research, Merrilyn Goos has developed a model designed to capture the richness of current definitions of numeracy (Goos, Geiger, & Dole, 2010). The model, which she refers to as the *Numeracy model*, represents the multi-faceted nature of numeracy (see, i.e. Goos, Geiger, & Dole, 2014). The Numeracy model (see Figure 5) involves five elements: *mathematical knowledge, contexts, dispositions, tools*, and *critical orientation*. The elements in the model are interrelated and “represent the knowledge, skills, processes, and modes of reasoning necessary to use mathematics effectively within the lived world” (Geiger, Forgasz, et al., 2015, p. 614).

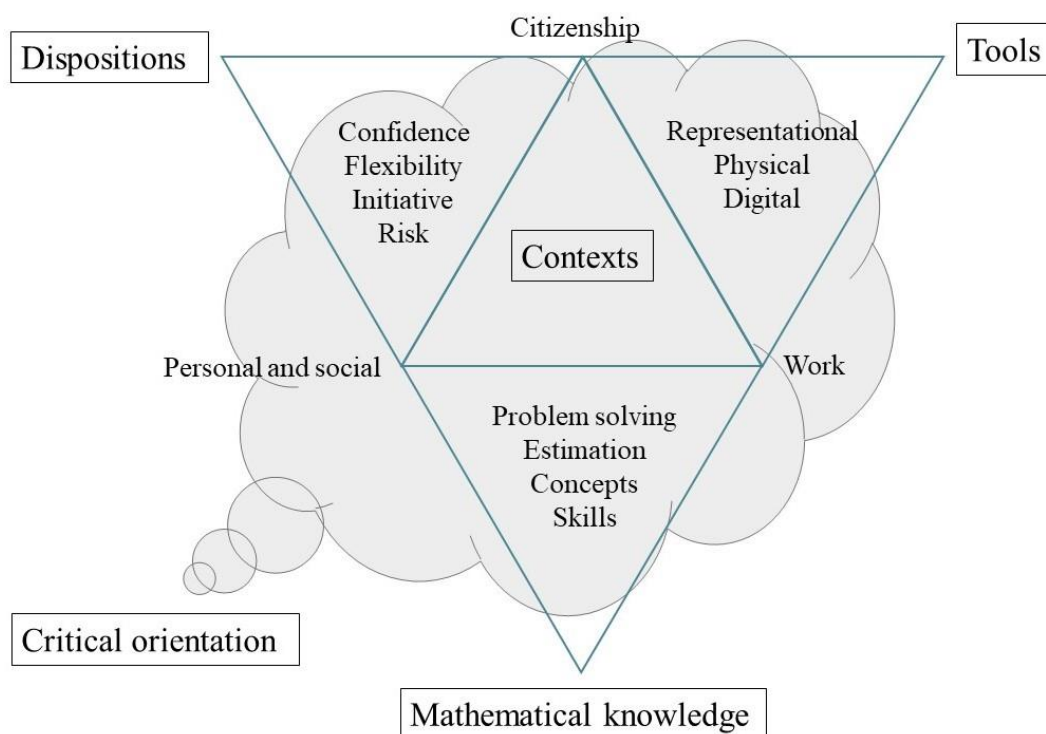


Figure 5. The numeracy model. Adapted from Goos et al. (2014, p. 84).

Goos and colleagues have used the model in a series of research and development projects related to teaching numeracy across the curriculum (Geiger, Goos, et al., 2015). I will return to their research findings in Chapter 4. The numeracy model is developed in the Australian context, but there are several reasons for its relevance in a Norwegian context. First, in Australia, numeracy has been interpreted in a broad sense similar to the OECD definition of mathematical literacy (Goos et al., 2010). Second, there are similarities between the Norwegian and Australian curriculums concerning the Norwegian basic skills and the Australian general capabilities. In both curricula, numeracy is considered a competence to be developed in all subjects, as well as in mathematics specifically. Both countries conduct national tests to assess students' numeracy level. Third, a cluster analysis of the cognitive items in mathematical literacy from PISA 2003, suggests that the Nordic countries' profiles strongly relate to the profiles of five of the six English-speaking countries participating in PISA (Olsen, 2006). Australia is one of these five countries. Hence, it is reasonable to use the model in the Norwegian context.

In Articles 2 and 3 (see Appendix C), Goos' model was adapted and interpreted in the context of mathematical literacy. The model was used to analyse teachers' operationalisation of and students' encounters with mathematical literacy. A short description of the elements involved in mathematical literacy is presented in Table 1 on the following page. See also Appendix C and Section 5.4 for a more detailed outline of the five elements.

Table 1

Descriptions and operationalisations of the elements involved in mathematical literacy

Element	Description
Mathematical knowledge	Mathematical concepts, skills, and problem-solving strategies
Contexts	The competence to use mathematical content in various situations in everyday life
Dispositions	Willingness and confidence to engage with mathematical tasks flexibly and adaptively
Tools	The use of physical, representational, and digital tools to mediate and shape thinking
Critical orientation	To use mathematical information to make decisions and judgements, add support to arguments, and challenge an argument or position

This chapter has provided an outline of the focus area of the research reported in this dissertation. As discussed above, mathematical literacy can be related to several notions and concepts. In the following chapter, I present the theoretical background in which this study of mathematical literacy is framed.

3 Theoretical background

Theory plays several roles in enabling research to address the problems of generating appropriate data and subjecting data to trustworthy and meaningful analysis (Silver & Herbst, 2007). By providing tools and language to describe, understand, and explain observed phenomena, theory can enable researchers to make predictions about relationships and structure the conduct of inquiry. Hence, theory can be understood as both guiding research practices and being the goal of research practices (Bikner-Ahsbabs & Prediger, 2014).

In my research, it is crucial that theories of teaching and learning account for the influence of history and culture and acknowledge that schooling, in addition to reproducing knowledge, also reproduces societal inequities. Therefore, I consider mathematical literacy as a social practice (Yasukawa, Jackson, Kane, & Coben, 2018). This perspective focuses on what people do with mathematical literacy through social interactions in particular contexts, not on people's performance of mathematical skills in isolation from context. A focus on practice entails viewing mathematical literacy activity as culturally, historically, and politically situated. The interest is in both visible and invisible mathematics.

The social practice perspective on mathematical literacy does not discount the importance of school-based learning or technical skills. However, it shows that mathematical knowledge and skills devoid of context do not enable people to be productive participants in a particular community. Cultural-historical activity theory enables the researcher to problematise the way that particular mathematical literacy practices have been shaped or disrupted by rules and traditions, the mediating tools and instruments available, and the community in which the mathematical literacy practices have meaning and value (Yasukawa et al., 2018). Hence, in the research reported here, I draw on the cultural-historical perspectives of teaching and learning.

In this chapter, I give a short historical overview of the development and foundations of cultural-historical activity theory. Next, I present the perspectives of a cultural-historical theory on mathematics teaching and learning, the theory of objectification. Finally, I connect cultural-historical perspectives to mathematical literacy.

3.1 Cultural-historical activity theory (CHAT)

Activity theory was developed from Russian cultural-historical psychology in the 1920s and 1930s (Kaptelinin & Nardi, 2009). Russian psychology was influenced by Marxist philosophy and the fundamental idea that the interaction between subjects and objects of activity is social. The Russian psychologist Lev Vygotsky is considered to have laid the foundations of activity theory. For Vygotsky, it was a fundamental issue that culture and society are directly involved in shaping the nature of the human mind. Human beings develop meanings and values by appropriating meanings and values already existing in the world. The ideas of cultural-historical psychology were carried further by a student of Vygotsky, Aleksey Leont'ev, who assimilated them into a system of concepts and principles known as activity theory (Kaptelinin & Nardi, 2009).

Activity should not be confused with activity as a series of actions and deeds (as *Aktivität* in German and *aktivnost* in Russian). Instead, Activity refers to the German *Tätigkeit* or Russian *deyatel'nost'*, which refers to a dynamic system geared to the satisfaction of collective needs. Therefore, I use a capital "A" when referring to Activity as a dynamic system and a lowercase "a" when referring to activity as a series of actions.

In CHAT, Activity is considered the central organising category. Activity is a structural moment of society that produces something for a generalised, common need as part of a division of labour (Roth & Radford, 2011). Hence, Activity produces the psychic aspects of everyday life where the inner and outer world are connected and irreducible to each other. In and through their participation, students reproduce schooling, society, and cultural practices.

Activity is a molar, not an additive unit of the life of the physical, material subject. In a narrower sense, that is, at the psychological level, it is a unit of life, mediated by psychic reflection, the real function of which is that it orients the subject in the objective world. In other words, activity is not a reaction and not a totality of reactions but a system that has structure, its own internal transitions and transformations, its own development. (Leont'ev, 1978, p. 50)

According to Leont'ev (1981), Activities are carried out in response to a subject's specific need. This need stands behind the activity *motive*. The main thing that distinguishes one Activity from another is the difference in their motives (Leont'ev, 1978). Students performing the same mathematical task, one with understanding the mathematics involved as the motive, the other with the motive

of passing the subject, engage in different Activities. The Activity depends on the subjects' possibility of enhanced life quality.

Needs can be represented in two different ways; *objectified* or *unobjectified* (Kaptelinin & Nardi, 2009). An unobjectified need is a need that is not associated with a specific object. It causes excitement which stimulates the search for an object that satisfies it. The subject may experience discomfort but cannot direct behaviour toward anything in particular that will satisfy the need. When a need is met, however, the need is transformed. It is coupled with an object; hence it is objectified. From that moment on, the object becomes a motive. The need stimulates and directs the subject, and an Activity emerges. Therefore, objectivity is a constituting characteristic of Activity which endows the Activity with a particular intent.

Students need to recognise their motives in their learning activities. The motive emerges through the teacher's and the student's joint action and is therefore also a product of the Activity. Students cannot recognise their motives on their own, and the teachers cannot tell them (Roth & Radford, 2011). Mathematical literacy is important for understanding and engaging in society. In this sense, teachers have an important role in facilitating students' engagement in Activities with developing mathematical literacy as a motive. The question is which Activity the students engage in, and, therefore, which motives they take up and pursue (Roth & Radford, 2011). I want to find out if this Activity is related to mathematical literacy.

To study Activity, one must study *actions*. Actions are what translate Activity into reality (Leont'ev, 1981). It is what we consciously do when we participate in Activity. According to Leont'ev (1978), human Activity does not exist except as action or a chain of actions. They are steps that eventually may result in attaining the motive. An action is subordinated to achieving a conscious goal. The goal is the immediate result to be attained if the subject engages in the Activity that will satisfy its motive. Hence, goals are related to the motive but are not equal to it. Several different goals and actions can relate to the same Activity and motive. For example, if students' motives are to perform at a satisfactory level, or better, in mathematics, one goal can be to pass all mathematics tests during a school year. One student's action related to that goal can be rote memorisation of mathematical rules and procedures. Another student's action may be directed toward developing an understanding of mathematical relations.

The actions are different but related to the same motive. Hence, the goals toward which actions are directed are framed by the individual (Roth & Lee, 2007).

Actions do not just exist; they have to be performed (Roth, 2012).

Methods for accomplishing actions are called *operations* (Leont'ev, 1978).

Operations are routine processes, automatised actions. For example, in learning mathematical procedures, holding the pen or writing the numbers are not attended to. The focus is on producing the algorithm and obtaining the correct answer. Operations are only performed because the goal-directed action requires them. Hence, operations do not possess their own goals but adjust actions to the *conditions* under which a goal is reached. Conditions can be both physical and psychological opportunities and constraints (i.e., accessible tools).

Initially, every operation emerges as an action, subordinated to a goal. The action is gradually internalised and included in another action (Huang & Lin, 2013). The action becomes a method for reaching the goal. For example, a student learning to solve equations initially solves different equations to practice the skill. Later, the student can use this method, for example, in problem-solving tasks. Mathematically literate students can use mathematics in various contexts. This means that the conditions change, and students must recognise which operations to perform. Operations can transform into actions, for instance when an operation fails to produce the desired outcome and the individual reflects on the reasons for the failure and how it can be solved (Kaptelinin & Nardi, 2009). Mathematically literate students engage in such reflection.

From the concepts outlined above, Activity can be represented as a hierarchical structure in three layers: the motive of Activity, the goals/actions, and the operations/conditions. The hierarchical structure of Activity can be visualised, as presented in Figure 6. As indicated in the discussion above, the three layers are interrelated and account for an inseparable relationship between the subject and the Activity (Roth & Radford, 2011).

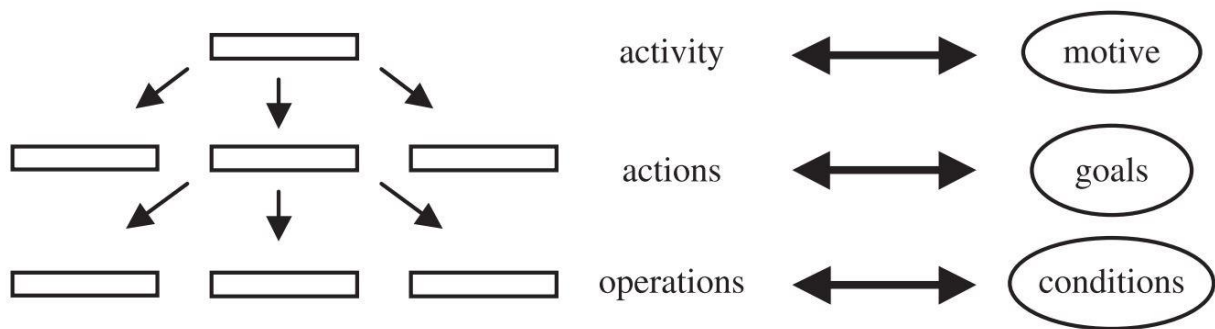


Figure 6. The hierarchical structure of Activity. Adopted from Kaptelinin and Nardi (2009, p. 64).

Vygotsky (1978) considered speech/language as important as action in attaining a goal. Speech serves as a tool in solving tasks and planning solutions before executing them. In this way, speech/language is what distinguishes the human use of tools from that of animals. Also, speech/language, in the sense of words and signs, is important as a means of social contact with others. Hence, speech/language takes on both an intrapersonal and an interpersonal function.

Following Vygotsky, Leont’ev considered tools as having a fundamental impact on the mind (Kaptelinin & Nardi, 2009). By learning how to use a tool, integrating it in activities, as well as the structure of the tool itself, we appropriate experience accumulated in culture. Tools are also important for understanding the role of signs and symbols, and for the development of concepts. Tools can serve as an embodiment of abstract concepts based on the generalisation of individual and collective processes. Using a tool for a specific purpose, for example, an axe to cut down a tree can lead to a generalisation of the experience of using the tool. One may compare the axe to the tree in terms of hardness and softness, and also compare cutting down the tree to other ways the axe can be used.

I see working to develop mathematical literacy as closely related to motives, goals and conditions. Students’ mathematical knowledge should “meet the needs” of their current and future lives (OECD, 2012). Activities, actions, and operations that students engage in should reflect how mathematics relates to “the real world”, and how the mathematics learned in school is useful in students’ daily lives. By analysing Activities, actions, and operations, and relating them to mathematical literacy, I can understand how teachers and

students work to develop mathematical literacy and whether this is a prioritised motive.

3.2 The theory of objectification (TO)

From the works of Vygotsky and Leont'ev, Luis Radford has developed a theory of knowledge objectification¹ (TO). TO focuses on how students and teachers produce knowledge against the backdrop of history and culture, and on how they co-produce themselves as subjects in general and subjects in education in particular.

The TO is inscribed within an understanding of mathematics education as a political, societal, historical, and cultural endeavor. Such an endeavor aims at the dialectic creation of reflexive and ethical subjects who critically position themselves in historically and culturally constituted mathematical practices, and ponder and deliberate on new possibilities of action and thinking. (Radford, 2016, p. 196)

The TO is a coherent theory of mathematics teaching and learning. Theories of teaching and learning differ from each other in their conceptions about the content to be learnt, the learner, and how learning occurs. Constructivist views perceive the learner as the constructor of his or her own knowledge.

According to the theory of knowledge objectification, learning does not consist in constructing or reconstructing a piece of knowledge. It is a matter of actively and imaginatively endowing the conceptual objects that the student finds in his/her culture with meaning. (Radford, 2008, p. 223)

According to Radford (2008), mathematics learning has often been reduced to merely obtaining a certain concept, and knowledge to a sort of commodity. However, knowledge is not something that can be “possessed” or “attained”. Knowledge is general and in flux. “Knowledge is an ensemble of culturally and historically constituted embodied processes of reflection and action” (Radford, 2013, p. 10).

In the TO, knowledge involves *possibility* and *actuality*. Objects of knowledge have a potentiality for doing something. This potentiality is abstract or general interpretations or actions resulting from cultural and historical ways of thinking and doing (Radford, 2015a). Also, objects of knowledge can be actualised through something concrete and noticeable. For example, knowledge

¹ In the theory of objectification, the notions *objectification* and *subjectification* have specific meanings. It is important to note that the same notions have different meanings when used in other discourses, such as Sfard (2008).

of arithmetic calculations is a possibility. It is a culturally constituted way of thinking about numbers. In doing a specific arithmetic calculation, arithmetic calculation knowledge is actualised in a singular instance. However, Radford (2015a) remarks, the singular is not the symbols themselves, but the embodied, symbolic, and discursive actions and thoughts required in solving the arithmetic calculation. In the singular, knowledge appears as both concrete and abstract simultaneously.

Activity is what makes the movement from potentiality to actuality possible. Knowledge needs determinations in the form of specific problem-posing and problem-solving activities to be an object of thought and interpretation. Hence, objects of knowledge are mediated by Activity. Instead of Activity, Radford, in his more recent works, uses the notion of *joint labour*. This is to avoid confusion about what is meant by Activity. Joint labour is a social form of joint endeavour where humans engage themselves actively in the world. They produce to fulfil their needs which occur in social processes, and at the same time, they produce themselves. The joint labour involves matter, body, movement, action, rhythm, passion, language, signs, and thinking (Radford, 2018). In classroom teaching and learning, the concept of joint labour involves conceiving the teacher and the students as engaged in the same Activity. Teacher and students labour together, for example towards the production of a specific way of thinking about numbers. In this dissertation, I will continue to use the term Activity.

In the TO, learning is conceptualised as the outcome of *processes of objectification*. It means that the cultural objective knowledge is transformed into an object of consciousness (Radford, 2013). Consciousness is considered as a subjective reflection of the external world and is a product of historical-cultural and emergent contingent relations and mediations. It is the subjective process through which each of us as individuals reflect. Consciousness continuously emerges and transforms through processes of objectification.

Processes of objectification are social and collective processes of becoming conscious of cultural and historical systems of thought and action. Such processes happen through Activity. Radford uses the metaphor of *encounter*. Objectification is our encounter with the knowledge that exists in our culture. Our encounters with cultural and historical systems of thought happen gradually and endlessly. Therefore, learning is perceived as something that never really ends (Radford, 2018).

For learning to occur, the realm of the possible and the virtual has to appear in a concrete manifestation in the students' consciousness. This in turn requires that the general be mediated by the particular –a specific activity that makes the general appear in the concrete world, to become endowed with a particular conceptual content. If the general is a form of thinking algebraically about sequences, the particular is the Activity that would require the teacher and the students to engage in some type of reflection and action that features the target algebraic conceptual content, so that the general finds itself embodied in the resulting singular –maybe even in novel ways. (Radford, 2013, p. 30)

Knowing is the instantiation or actualisation of knowledge. Hence, knowing is the interrelation of general knowledge, the actualisation of knowledge, and the result of knowledge actualisation. Knowing is, therefore, the concrete conceptual content through which knowledge is instantiated. It is modes of cognition and forms of knowability which frame the scope of concepts that can be produced at a specific time in a specific culture. Knowing is what is grasped of the singular, concepts which have become objects of consciousness in the course of the joint labour. The dynamic system of potentiality, Activity, actuality, and knowing is illustrated in Figure 7.

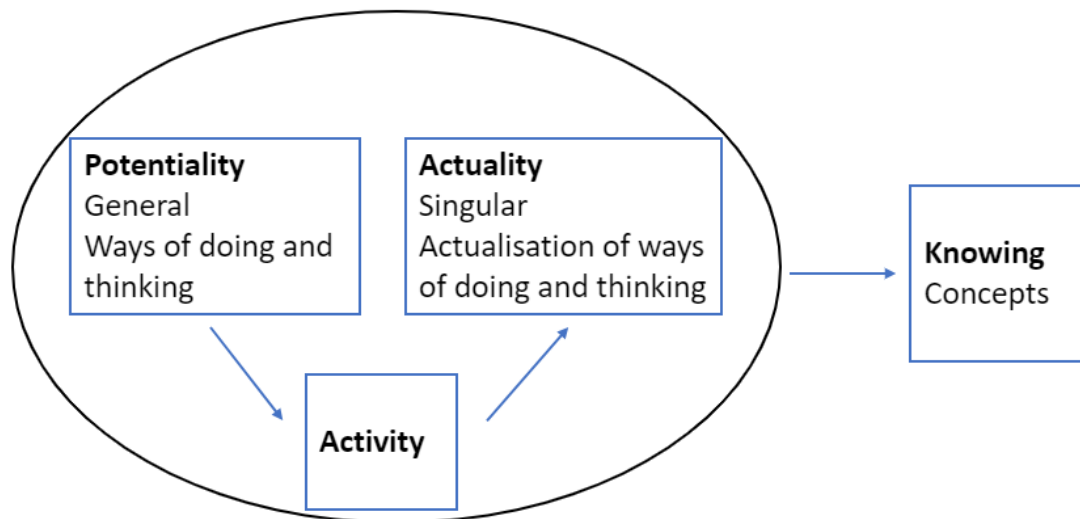


Figure 7. Knowing as what is grasped by the individuals in the realisation or actualisation of knowledge through Activity. Adapted from Radford (2015a, p. 140).

As mentioned earlier, Activities are characterised by their motive. The motive of a mathematics classroom activity may be the encounter with mathematical

literacy. As students may not be aware of the motive of Activity, the teacher may introduce related goals. Specific tasks are introduced to reach these goals. The tasks of Activity correspond to actions in Leont'ev's scheme (see Figure 6). In the TO, the motive-goal-task structure is a central part of the design of classroom Activity. It corresponds to the left arrow in Figure 7, from potentiality to Activity. The motive-goal-task structure is illustrated in Figure 8. According to Radford (2015b), the left arrow refers to the pedagogical intention of the classroom activity, in other words, the activity design. The design involves an epistemological analysis of the target mathematical content and a reflection of how things might occur in the classroom. However, as many educators may have experienced, one can never know how things might occur in the classroom. The middle arrow, from Activity to actuality, in Figure 7 indicates the specific actualisation of knowledge as produced by Activity. It refers to how things actually turn out, the implementation of the activity in the classroom. In my interpretation, this involves Leont'ev's operations/conditions level. The actualisation of knowledge is an emergent process, meaning that the classroom is conceived as an evolving system. The evolution of the system depends on how the teacher and students engage in the Activity and cannot be predetermined.

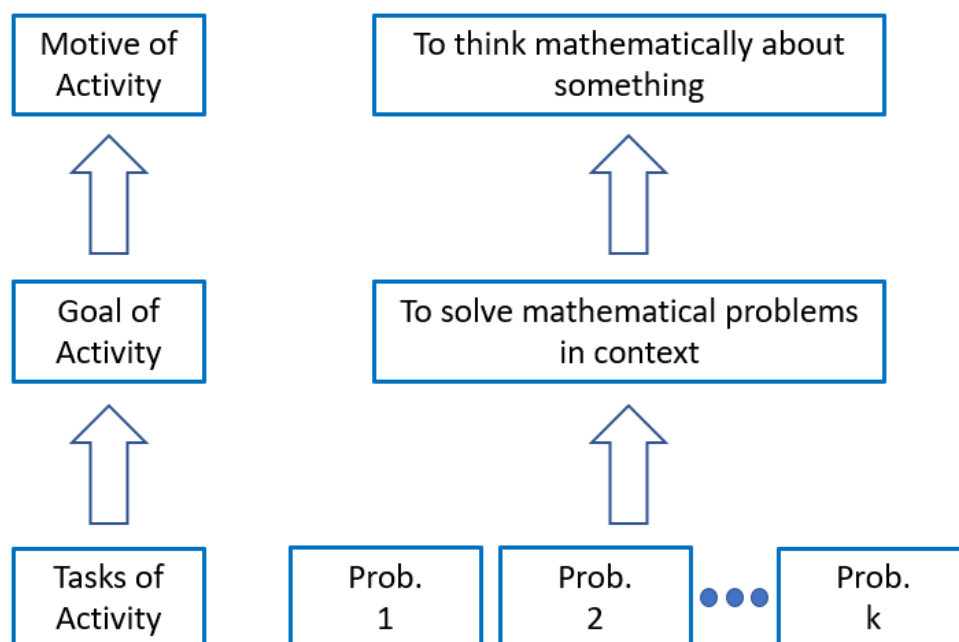


Figure 8. The motive-goal-task structure. Adapted from Radford (2015b, p. 555).

In terms of the TO, learning is defined as the outcome of processes of objectification. Classrooms do not only produce knowledge; they also produce

subjectivities. This is what Radford terms *processes of subjectification*. Processes of subjectification are “the processes where, co-producing themselves against the backdrop of culture and history, teachers and students come into presence” (Radford, 2018, p. 140). Hence, learning is also about becoming. A person undergoes subjectification in every Activity s/he participates in during the day, week, or month. School, personal life, work-life, and societal life belong to different forms of Activity. During these activities, the person continuously changes and becomes more knowledgeable. This is connected with how the TO considers emotions and thinking. In the TO, emotions and thinking are considered as part of human nature. Learning involves thinking and emotions that affect us as human beings. Therefore, as we learn, we also come into being. Radford (2015a) states that there is a dialectical relationship between knowing and becoming. We are becoming because we are knowing, and we are knowing because we are becoming.

3.3 Connecting CHAT, TO, and mathematical literacy

During a day, a person participates in several Activity systems and participates in realising multiple motives. The different motives that orient our Activities have different degrees of importance to the individual (Roth, 2014). Hence, the motives are hierarchically organised. This hierarchy is created in the system of social relations that the individual enters through Activity. The motives and their hierarchies form the individual’s personality. The individual weaves together his/her involvements in different activities and prioritises the different motives. It is how the hierarchy of motives are formed that constitutes personality. If motives of mathematical literacy have low priority, this becomes an integral part of the personality. Mathematical literacy as a social practice and in the light of CHAT and TO means that mathematical literacy is part of one’s personality. Personality and subjectification describe and theorises the experience of the person (Roth, 2014).

Mathematical literacy involves using mathematics in various contexts. This means to draw on mathematical experiences from one context to solve problems in another. Mathematical experiences come from the different Activities in which the individual engages. As the hierarchy of motives is formed within social relations, the individual’s encounters with mathematical literacy in different Activities influence to what extent mathematical literacy is prioritised as a motive.

I propose a conceptualisation of mathematical literacy within the concepts of TO. I relate the five elements of mathematical literacy to the system illustrated in Figure 7. Hence, I see mathematical literacy as a particular kind of knowledge needed to participate in society. This knowledge involves potentiality, Activity, and actuality. Potentiality is the general and abstract mathematical knowledge. The Activity is determined by particular contexts, participants' dispositions, and available tools. The specific task contexts and problems students engage in are the actualisations of the general in the singular. Critical orientation is conceptualised as knowing, the conscious, subjective process through which the individual reflects and orients her/himself in the world. In this sense, I conceptualise developing mathematical literacy as a process of objectification. As mathematical literacy involves recognising how mathematical information can be used for different purposes in society, developing mathematical literacy is closely related to processes of subjectification. It is about becoming an informed, reflected, and ethical citizen. A model of my conceptualisation of mathematical literacy within the concepts of TO is illustrated in Figure 9.

The left arrow in Figure 9 illustrates the teacher's pedagogical intentions and planning. The teacher's motive of Activity is related to a specific goal. Different tasks are planned in order to reach the goals that satisfy the motive. The mathematical literacy model serves as a tool in planning and organising the tasks of Activity, but also to understand mathematical literacy. The elements of mathematical literacy can be used as a tool for planning teaching but also a way to understand what mathematical literacy is. This is illustrated by the inclusion of the elements in the process of objectification. I, therefore, propose this model as a tool to understand, plan, and analyse holistic teaching and learning for mathematical literacy.

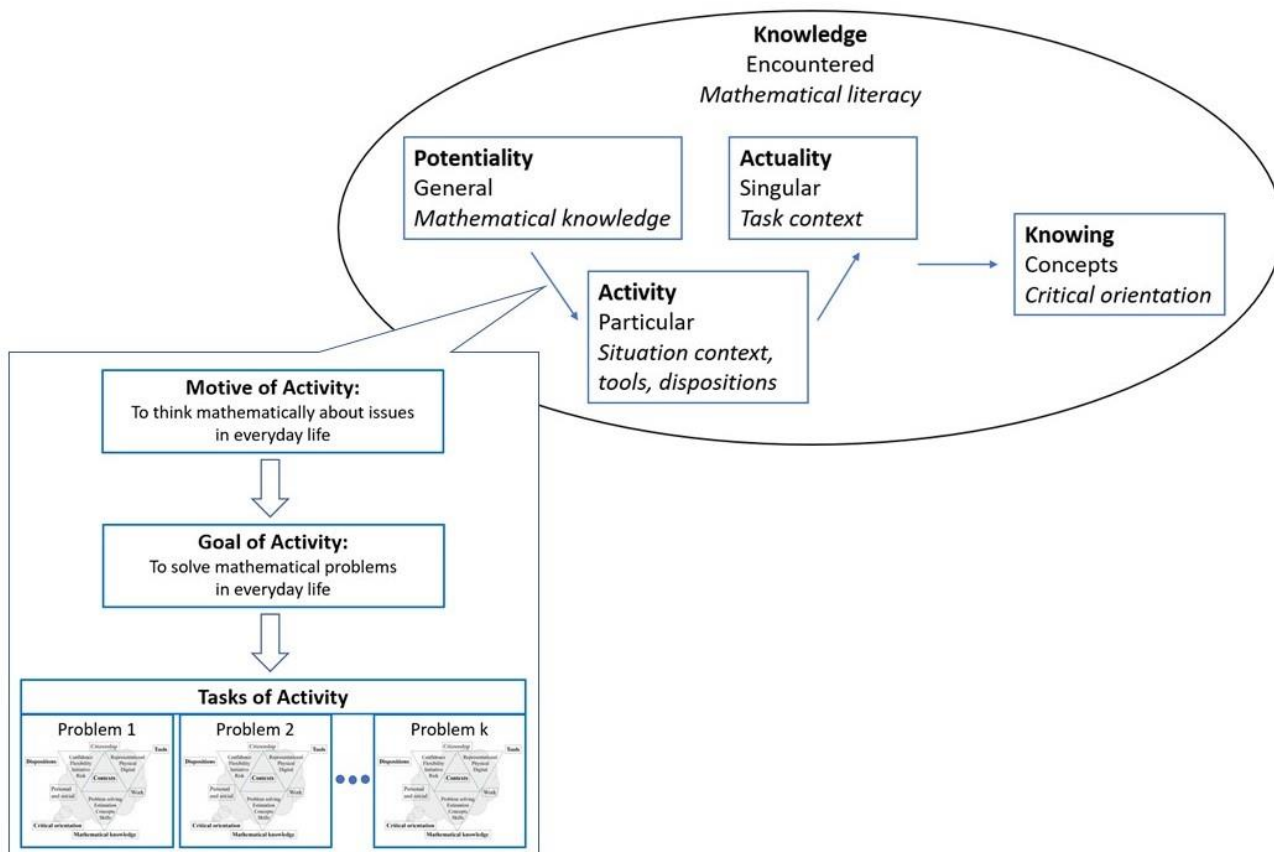


Figure 9. A model for the objectification of mathematical literacy.

In these three first chapters, I have outlined the background and foundations on which this research on teaching for mathematical literacy is built. In the following chapter, I present an overview of previous research on mathematical literacy.

4 Empirical background

The impact of the PISA studies and the increased curricular emphasis on mathematical literacy have made mathematical literacy an important research field. In South Africa, mathematical literacy is a compulsory subject alternative to the mathematics subject in secondary school. Hence, several studies focus on teaching the mathematical literacy subject. My research focuses on mathematical literacy as a competence to be developed as part of the regular mathematics subject and across curriculum subjects. Therefore, studies specifically addressing teaching the South African mathematical literacy subject is not included in this chapter.

Another issue that I will not go into here is the research on “street mathematics”. Much research has been done looking at the mathematical competencies of child street traders. For example, Nunes, Schliemann, and Carraher (1993) and Saxe (1991) investigated the mathematical practices of Brazilian child candy-sellers. The candy-sellers were performing quite complex calculations in terms of, for example, working out profit and loss, change to be given, and engagement with very large numbers. They were very efficient in using a range of informal methods that did not find any place in school, and the children’s competence in the street mathematics appeared unrelated to their measured performance in school mathematics. Hence, it could be argued that this research is relevant to my work. However, I argue that it is not because my interest is in the development of mathematical literacy in the formal context of regular schooling. Although I do not wish to deny the possibility of informal out of school approaches, these lie outside my main study focus and Norwegian context.

In Section 4.1, I briefly summarise some main results from the Norwegian PISA reports. In this section, I also report from a review of empirical articles which mostly were founded upon PISA data.

As described earlier, mathematical literacy involves using mathematics in contexts. This is illustrated in the mathematical literacy model (see Figure 5 in Chapter 2), where contexts are placed in the centre. However, working with mathematical literacy tasks is more than a simple matter of applying mathematical knowledge and rules to a new situation. It involves engagement with attaining the various attributes of the context, and success in these tasks depends on one’s competence to use the rules of the context, to understand the

language of the context, and to engage in reasoning about the context. In this chapter, I also look at previous research on teaching and learning mathematics in context. The chapter involves two sections with slightly different approaches to contexts. Section 4.2 concerns research related to using contexts from the real world outside the school in mathematics teaching and learning. Section 4.3 concerns using other curriculum subjects as contexts in which mathematical literacy can be developed. The sections are organised by their approach to teaching mathematics in contexts, and not by the contexts per se. Therefore, a context or task may be used in both approaches, depending on whether it is connected to curriculum goals in other subjects than mathematics or everyday life in general.

4.1 PISA studies and research on mathematical literacy

The Norwegian part of the PISA project is financed by the Norwegian Ministry of Education and Research and conducted by the Department of Teacher Education and School Research at the University of Oslo. The Norwegian PISA reports are publicly accessible at the University of Oslo website (<https://www.uv.uio.no/ils/forskning/prosjekter/pisa/publikasjoner/>).

The first PISA study was conducted in 2000, and students' performance in reading was the main focus. On the PISA 2000 mathematics test, Norwegian students scored one point below the OECD average (Lie, Kjærnsli, Roe, & Turmo, 2001). In 2003, when mathematics was the main focus of assessment, Norwegian students scored 495 points, which is five points below the OECD average of 500. It is also below the other Nordic countries, which all scored above average (Kjærnsli, Lie, Olsen, Roe, & Turmo, 2004). Despite that problem solving had been a focus area in Norwegian curriculum since 1987, Norwegian students also performed below the OECD average and the other Nordic countries in problem-solving.

Mathematics was not the main focus in the PISA studies in 2006 and 2009. The Norwegian mathematics results in PISA 2006 were disappointing, as this was the first time Norwegian students scored significantly below the OECD average (Kjærnsli, Lie, Olsen, & Roe, 2007). In 2009, however, Norwegian students performed two points above the OECD average, and there are fewer students at the lowest performance level (Kjærnsli & Roe, 2010)

In 2012, mathematics was again the main focus. Norwegian students performed slightly under the OECD average, and the share of low performing

students had increased since 2009 (Kjærnsli & Olsen, 2013). This decline in students' mean performance seemed to be related to an increase in the share of low performers since 2009, while the share of top performers remained stable. Less than 10% of the students performed at a high level and more than 20% performed at a low level.

Norwegian students performed significantly above the OECD average for the first time in 2015 (Kjærnsli & Jensen, 2016) and in PISA 2018, the Norwegian mathematics results remained unchanged (Jensen et al., 2019). In the 2018 study, 19% of the students were considered low performing and 12% were considered high performing. From 2000 till 2018, the Norwegian PISA results have remained stable. Several measures have been taken to contribute to enhancing mathematics teaching and learning in Norwegian schools (Kjærnsli & Jensen, 2016). The latest PISA studies may indicate a small increase in Norwegian students' performance (Jensen et al., 2019). However, the results are not significantly higher than in the PISA 2003 survey. Therefore, the results from PISA 2021 will be particularly interesting. In 2021, mathematics is, again, the main focus, and the results from this study may reveal whether the positive development is, in fact, a trend.

Research on mathematical literacy comprises both qualitative and quantitative methods and focuses on teachers' understanding, teaching practice, student learning, and curriculum assessment. In a literature review of 28 empirical articles on mathematical literacy, Haara, Bolstad, and Jenssen (2017) conclude that quantitative approaches dominate the field. Research focuses mainly on school outcomes, and not on what goes on in the classroom. One reason may be that PISA test results provide large-scale data samples from a range of countries and do not inform about practices within classrooms. Quantitative research studies on mathematical literacy focus on implications for national school and society matters. Aksu and Güzeller (2016) analyse quantitative data from PISA 2012, attempting to classify successful and unsuccessful students in terms of mathematical literacy according to interest, attitude, motivation, perception, self-efficacy, and anxiety. In studying factors related to adolescents' reading, mathematics and science literacy, Areepattamannil (2014) underlines the relationship between gender, metacognitive learning strategies, and students' positive perception of the classroom and school environment to students' academic performance. Comparing successful and unsuccessful countries in PISA 2009, Danju, Miralay,

and Baskan (2014) stress the importance of government investment in education. Other matters investigated include affective variables, social and cultural status, use of representations, and computer use, and how these relate to students' mathematical literacy (İş Güzel & Berberoğlu, 2010; Jürges, Schneider, Senkbeil, & Carstensen, 2012; Koğar, 2015; Lin & Tai, 2015; Matteson, 2006; Papanastasiou & Ferdig, 2006; Tai & Lin, 2015).

PISA results and quantitative research on factors affecting students' mathematical literacy are important. They provide a picture of mathematical literacy levels and illuminate factors related to mathematical literacy development. However, they do not provide real knowledge about what goes on in the classroom. In my opinion, these results are not properly exploited if they are not also used to design research studies which provide knowledge about aspects of teaching and learning where mathematical literate students are the desired outcome. However, it is important to note that in the study by Haara et al. (2017), mathematical literacy was the only search word. This may have excluded studies using related concepts, such as numeracy, and hence also studies taking a qualitative approach. Studies situated in South Africa were also not included in the review.

4.2 Research on mathematical literacy and real-world contexts

Several aspects of students' mathematical literacy and use of mathematics in real-world contexts have been investigated. Aligning with the PISA framework, several studies connect mathematical literacy with mathematical modelling. As discussed in Chapter 2, modelling problems arise from the real world and involves the competence to formulate, employ, interpret, and evaluate. Gatabi, Stacey, and Gooya (2012) use the modelling process as a theoretical framework when analysing seventh-grade textbooks. They compare one Iranian and two Australian textbooks in terms of mathematical literacy problems. The textbooks problems are limited in requiring students to formulate and interpret. It is suggested that in order to foster mathematical literacy, textbooks should include problems with various contexts, problems requiring formulation by the students, and problems closer to mathematical modelling (Gatabi et al., 2012).

Another approach is taken by Kaiser and Willander (2005), who adapt Bybee's levels of scientific literacy to mathematics and use this hierarchy of mathematical literacy levels as a framework. These levels refer to students' understanding and use of mathematical concepts. The study is part of an

evaluation of a larger project which investigates teaching and learning processes which emphasise real-world contexts and modelling in German schools. Data comprise 31 grade 7 and 8 students' responses to a pre- and post-test, each comprising four tasks. The tasks involve connections between mathematics and the real world. The study focuses on the students' competence to work out problems using mathematical methods, the competence to reason mathematically, and the competence to use mathematical concepts and methods in a flexible and reflected manner. The authors conclude that students find relations between mathematics and the real world problematic and suggest that students should work with open problems with real-world contexts to develop mathematical literacy. This work, they assert, should start at the primary school level.

Much of the research on mathematical literacy is concerned with higher grades. It may, therefore, be challenging to see what teaching for mathematical literacy might look like in the lower grades, especially when it comes to embedding a critical orientation. Sikko and Grimeland (2020) investigate what critical mathematical literacy might look like in a Norwegian second-grade classroom. The study is part of a larger project where the researchers and teachers use a Lesson Study approach to develop and redevelop lessons inspired by inquiry-based learning. Data comprise video and audio recordings of two student groups, composed of 14 and 16 students, working with adding numbers up to twenty in the contexts of Norwegian coins. Also, students' worksheets were collected, and observation schemes were filled out. The tasks involve problems with a single solution, several solutions, infinitely many solutions, and no solution. According to the authors, students are not used to problems with no solutions. By providing such tasks in the early grades, students may experience the need to challenge the given problem situations, and this contributes to critical thinking. To understand what a solution means, under which circumstances solutions can be found, and to see that a change in circumstances can lead to other solutions is an important part of learning mathematics, of developing a critical orientation, and of solving problems in society (Sikko & Grimeland, 2020).

Sikko and Grimeland (2020) conclude that it is possible to work with critical orientation in a real-world context in the lower school grades. This is an important finding, as results from a study by Hunter, Turner, Russell, Trew, and Curry (1993) imply that the perception of mathematics as divorced from reality

starts already in primary school, if not before. Hunter et al. (1993) interviewed 144 students in grades 2 to 5 in Northern Ireland about their everyday uses of mathematical operations. Their results show that students initially suggest pure rather than applied uses of mathematical operations. Mathematics is viewed as “a school activity, engaged in to gain academic ‘kudos’, and not as a way to make sense of the world, to communicate or to address practical tasks” (Hunter et al., 1993, p. 25). Hence, there appears to be a dichotomy between everyday mathematics and school mathematics in the sense that formal learning fails to benefit from the intuitive knowledge students bring to the classroom, and students are unable to generalise their mathematical knowledge to situations outside school spontaneously. Hunter et al. (1993) also note that the students’ parents were concerned in their children’s mathematical performance and were practising routine rules and number facts with them at home. The authors suggest that this may have strengthened the students’ perceptions of mathematics as a formal exercise. They propose that homework could be used to find the real-world use for each of the mathematical techniques they encounter in school. In this way, parents could be included in the child’s mathematical learning, and the child is alerted to the applications of mathematics in real-world contexts.

An argument for using real-world contexts in mathematics teaching is the belief that it enhances students’ interest and motivation for learning mathematics. According to Lee (2012), pre-reservice teachers are positive to the purpose and effectiveness of real-life-connected word problems. Seventy-one pre-service teachers were asked to collect, create, and evaluate various word problems according to the level of reality, clarity of wording, and grade appropriateness. The pre-service teachers believe that contexts relatable to students’ everyday life will provide richer conditions for students to engage in the learning process. However, they show a discrepancy on how reality is defined. Almost half of the pre-service teachers included in the study accepted imaginary contexts as possible real-life connections, whereas the others did not.

There are different views in the mathematics education community regarding what counts as real. For instance, in realistic mathematics education (RME), a fantasy world can be a suitable context as long as it is real in the student’s mind. Studies show that students can engage productively with mathematics when it is explored in imaginative settings. For example, Nicol and Crespo (2005) studied 36 pre-service teachers and 50 grade 6/7 students working with imaginative tasks. Data comprise field notes, copies of student work, and

video recordings. They found that both student groups posed their own questions and explored the mathematical ideas embedded in the contexts without asking for relevance. Therefore, the authors suggest that imaginative contexts can help students support and sustain their engagement with the mathematics in the task.

On the other hand, Rellensmann and Schukajlow (2017) investigated 100 German grade 9 students' interests in solving problems with and without connections to reality. The students were asked to solve 12 problems and, immediately after solving each task, indicate their interest in the problem on a questionnaire. The tasks involved four modelling problems, four "dressed up" word-problems, and four problems without real-life connections. It was expected that students would be more interested in the problems connected to reality. However, students reported more interest in solving problems without a real-life connection. The authors explain this as students' low interest in the particular contexts offered in the problems. Therefore, contexts should be individualised to suit the particular group of students (Rellensmann & Schukajlow, 2017). Julie and Mbekwa (2005) support this view when they argue that students' interests should be considered when developing the curriculum, learning resources, and tests.

However, predicting students' interests is not always straight forward. In their study on students' interest in solving word-problems, Rellensmann and Schukajlow (2017) also investigated whether pre-service teachers could accurately predict students' interest in solving problems with or without a real-world connection. One hundred and sixty-three pre-service teachers were provided with the same 12 tasks as the students. They were asked to judge the task difficulty and fictitious grade 9 students' interest in solving the problems. The findings show that pre-service teachers overestimated students' interest in solving real-world problems and underestimated students' interest in solving problems without a real-world connection.

Besides affecting students' engagement and motivation in solving tasks, it is also known that contexts may affect students' methods (Boaler, 1993). Six questions were given to 100 grade 8 students. Also, classroom observations, interviews with teachers, and review of classroom materials were made. Boaler (1993) reports variation in students' performance and procedures across contexts. Meaney (2007) uses the framework adapted from Kaiser and Willander (2005) when studying how different problem contexts affect students' judgments concerning mathematical literacy. Her study is based upon data from the

National Education Monitoring Programme in New Zealand. Seventy-two students in grades 4 and 8 were video recorded in one-to-one interviews working on a task of ordering four equally shaped and sized boxes according to weight. The task contained three sections, and the particular demands of each section caused students to provide different types of arguments. According to Meaney (2007), task contexts affect students' approaches to solve the tasks. It also affects the external perception of their level of mathematical literacy. Therefore, teachers need to be aware that students will likely not show higher-order thinking if they do not perceive that the task requires it.

Another issue concerns the relationship between students' experiences of different contexts and their decisions to engage with mathematics. In a study involving surveys, interviews, blogs, and logbooks from 38 Swedish upper secondary school students, Andersson, Valero, and Meaney (2015) found that the contexts in which mathematics is introduced affect students' engagement in learning mathematics. Students act in particular ways at specific times, and they make decisions to engage in learning of mathematics in some situations but not in others. These findings resonate with the findings by Boaler (1993). A context which may facilitate understanding and transferability for one student may inhibit understanding for another. Therefore, contexts should not be viewed simply as motivators. Students' predispositions to transfer mathematics learning to other contexts are complex and varied because contexts are part of an interaction between students' experiences, goals, and perception of the mathematical environment (Boaler, 1993). According to Andersson et al. (2015), making contextual changes to the way mathematics is introduced and allowing students to influence the classroom discourse can alter students' perceptions and decisions for learning. For example, introducing mathematics in relation to societal and critical issues and acknowledging students' discussions can contribute to engagement and experiences of meaningfulness.

When introducing mathematics in contexts, it is important to consider students' personal backgrounds and how these can affect teaching and learning. Sandström, Nilsson, and Lilja (2013) aim to exemplify students' mastering of mathematical literacy. Their study involves 75 grade 5 students from six different Swedish schools. The study is a comparative case study constituted by three groups of students: students with mathematical difficulties, students with a first language other than Swedish, and students without mathematical difficulties. Three activities were carried out in the classroom and observed by a researcher.

The first activity involved basic arithmetic with mixed operations. In the second activity, the pupils were asked to solve mathematical problems, and in the third, they were asked to construct their own mathematics word-problems.

The activities were followed by group interviews with the students. The interviews were aimed at finding out how the students related the activities to four aspects of mathematical literacy: 1) reasoning mathematically and using mathematical concepts, 2) recognising the role that mathematics plays in the world, 3) making well-founded judgements and decisions, and 4) solving problems set in the student's lifeworld context. The students with difficulties in mathematics related word-problems to understanding the role of mathematics. They also displayed bad self-confidence and attributed themselves the responsibility for failure. Students with another native language were affected by a lack of linguistic understanding. With help from classmates, they related problem-solving to mathematical literacy as a connection to their lifeworld. The students without mathematical difficulties talked about mathematical literacy in all three activities. The authors relate these results to students with difficulties being subject to a self-fulfilling prophecy in which students' assumed mathematical capacity is confirmed, and students with another first language being affected by cultural differences. Sandström et al. (2013) express concern that students in mathematical difficulties and students with another first language run the risk of being made invisible when working to develop mathematical literacy. It is, therefore, important to be aware of students' personal and cultural obstacles when working to develop mathematical literacy.

The studies referred to above suggest that developing mathematical literacy in terms of teaching and learning mathematics in various contexts and making real-world connections involves challenges when studied from the students' perspective. This impression is sustained when looking at research on teaching and learning mathematics through real-world contexts from the teachers' perspective.

A cross-case analysis of interviews with 16 Turkish upper secondary school teachers revealed seven emergent categories for teachers' conceptions of mathematical literacy (Genc & Erbas, 2019). The teachers hold various but interrelated conceptions about mathematical literacy as involving 1) formal mathematical knowledge and skills, 2) conceptual understanding, 3) problem-solving skills, 4) the ability to use mathematics in everyday activities, 5) mathematical thinking, reasoning, and argumentation, 6) motivation to learn

mathematics, and 7) innate mathematical ability. The various conceptions may, on the one hand, indicate an ambiguous and confusing conception of mathematical literacy, or it may, on the other hand, reflect richness in one's understanding of its various aspects. In general, teachers seem to recognise the contextual and applied aspect of mathematical literacy. However, according to Gainsburg (2008), teachers count a wide range of practices as real-world connections. In her study, 62 teachers responded to a written survey, and five of these were selected for classroom observation and subsequent interviews. The focus was on the nature of connections between mathematics and the real world. According to Gainsburg (2008), teachers make such connections frequently, but they are brief and do not require any thinking from the students. The study concludes that teachers' main goal is to impart mathematical concepts and skills, and the development of students' competence and disposition to recognise applications and solve real problems is of lower priority.

One reason that teachers' focus on mathematical concepts and skills may come from their lack of experience with connections to the real world. When investigating seven secondary mathematics teachers' recognition of mathematics in museum exhibits, Popovic and Lederman (2015) found that the teachers searched for explicitly represented concepts such as numbers, graphs, and shapes. Only after instruction from the researchers they started looking for exhibits that would make abstract mathematical concepts more concrete. The teachers realised that explicit representation by numbers, shapes, and figures is not vital in order to identify mathematics. It is therefore important to note that for teachers to incorporate real-life connections to their teaching, they must be able to make such connections themselves.

Taking an activity theory perspective, Venkat and Winter (2015) relate the challenge of incorporating contexts to boundary-crossing. From their study of one pre-service teacher's lesson concerning the reading of a map, they conclude that teachers of mathematical literacy need familiarity with artefacts at the boundary from the perspectives of both mathematical and contextual activities. As students bring awareness of traditional mathematical goals and conventions alongside mathematical literacy goals and conventions, teachers also need the competence to negotiate the different goals and conventions involved in both mathematics and mathematical literacy. "Thus, rather than being a member of one or other activity, the numeracy teaching role is centrally configured at the

boundary of both activities with the need for extensive comfort with boundary crossing around boundary artefacts” (Venkat & Winter, 2015, p. 584).

Crossing boundaries between mathematical goals and mathematical literacy goals may be particularly challenging in a diverse classroom. Students from other national backgrounds and cultures need adequate opportunities to develop mathematical literacy competence, as this can be a crucial gateway to participation in a new society. In a case study on migrant students’ opportunities to develop mathematical literacy, Nortvedt and Wiese (2020) interviewed four Norwegian grade 8 mathematics teachers about how they adapt their classroom practice and assessment situations to this student group. The study is part of the Erasmus + Study: Aiding Culturally Responsive Assessment in Schools (ACRAS). The four participating teachers focused on student-oriented practices that involve problem-solving, applied problems, and investigations, which are elements that support mathematical literacy development. Also, the teachers had positive attitudes toward inclusive education and diversity.

On the other hand, the teachers believed that mathematics is culture free and did not show awareness of and attention to students’ cultural backgrounds. For example, they were aware that migrant students might not be familiar with student-oriented classrooms and problem-solving situations but did not relate this to their culture. According to Nortvedt and Wiese (2020), the neglect of mathematics as a cultural practice involves a risk of diminishing migrant students’ opportunities to work with mathematical problem-solving in ways that promote mathematical literacy. It can, for example, cause challenges in finding appropriate contexts in which students can apply mathematical competence in real-world problems. Although this study focuses on migrant students, I believe that the research has implications for teaching for mathematical literacy in general. If mathematics is viewed as a neutral subject, the development of mathematical literacy and a critical orientation can be at risk for all students.

Hence, the process of working with context-based tasks is a complex process for both teachers and students. Wijaya, Van den Heuvel-Panhuizen, and Doorman (2015) developed a framework for coding teachers’ teaching practice related to context-based tasks. The framework comprises four stages which students pass through in solving context-based tasks and descriptions of teachers’ practice supportive or non-supportive of students’ opportunities to learn to solve such tasks. The authors used the framework to analyse 27 Indonesian teachers’ questionnaire responses and observations of 4 teachers’ mathematics lessons.

The participating teachers claim that they are offering students opportunities to learn to solve context-based tasks. However, they also reflect a mechanistic view of school mathematics as pure mathematics and context-based tasks as plain word-problems. The observed teachers mainly used a direct instructional approach. Instead of asking students to paraphrase the problem, they told the students what the problem was about. Instead of encouraging students to identify the relevant mathematical procedures, they told the students how to solve the problem. Instead of verifying the reasonableness of the solution, they told them whether it was correct or not without connecting it to the context (Wijaya et al., 2015). Although there are limitations to the study (which the authors are explicit about), it illustrates the importance for teachers to look at their practice critically when working with context-based tasks in the classroom.

Haara (2018) proposes pedagogical entrepreneurship as an approach to develop students' mathematical literacy. "Pedagogical entrepreneurship is action-oriented teaching and learning in a social context where the student is active in the learning process and where personal features, abilities, knowledge and skills provide the foundation and direction for the learning processes" (Haara, 2018, p. 254). According to Haara, problem-solving, local cultures and resources, authenticity and action competence are key elements both in the development of mathematical literacy and in pedagogical entrepreneurship in mathematics. His study takes an action research, self-study approach and involves literature studies and his own classroom experience when teaching to groups of about 25 Norwegian teacher education students. Data comprise personal notes from the lesson experiences made in two rounds; the first immediately after the lessons and the second months later. From working with the students on two problem-solving tasks, Haara (2018) concludes that pedagogical entrepreneurship supports students' mathematical literacy development in that it helps them develop self-regulation and competence in choosing and applying the right mathematics when relevant. However, this approach requires that the teacher is aware of his/her role as a tutor prioritising student participation and does not assume the role of the expert instructor.

4.3 Research on mathematical literacy across the curriculum

There is a large body of research focusing on embedding mathematical literacy across the curriculum. Mathematical literacy is considered a general capability in

the Australian curriculum, and this has led to several research projects on developing mathematical literacy across the curriculum.

Thornton and Hogan (2004) comment on the teacher's role in developing students' mathematical literacy. The authors report findings from The Middle Years Numeracy Across the Curriculum Project, involving nineteen Australian teachers in grades 5 to 10. Based on teacher group discussions, teachers' records of their own action research projects, and formal written results, the authors identified three idealised types of teacher orientations toward teaching mathematical literacy across the curriculum. *The separatist* recognises that mathematical skills are important, however, sees it as the mathematics teacher's job to teach such skills. Mathematical concepts may be encountered within other areas of the curriculum. If the students struggle to understand this concept, the mathematics teacher has not done his/her job well enough. *The theme-maker* recognises that mathematics is connected to other subjects and the real world. He/she develops tasks or projects that incorporate mathematics and other curriculum areas, often based around a theme. *The embedder* recognises that quantitative elements are embedded within the context of other learning areas. He/she believes that every teacher is a teacher of mathematical literacy, and hence, all share the responsibility to help students develop a mathematical view of the world. Thornton and Hogan (2004) conclude that developing students' mathematical literacy is everyone's responsibility. As learning is situated, students need to encounter mathematics as embedded in other curriculum areas, and not only through word-problems in the mathematics lessons. This requires that teachers develop the confidence and disposition to be embedders.

The Australian Numeracy Project conducted by Merrilyn Goos and her colleagues intended to assist teachers in becoming embedders. Goos' Numeracy Model (outlined in Chapter 2) is central in this work. Goos and her colleagues have used the model in several studies concerned with teachers' professional development related to teaching mathematical literacy across the curriculum.

The model has been used in curriculum studies to audit the mathematical literacy demands and opportunities of the curriculum (Goos, Dole, & Geiger, 2012; Goos et al., 2010). For example, Goos et al. (2012) evaluated the Australian history curriculum. Each member of the research team independently read and qualitatively evaluated the curriculum demands in terms of the five elements, before meeting to discuss each person's findings. The findings suggest that the history curriculum can provide engaging and meaningful contexts for

developing students' mathematical literacy, and that mathematics can provide tools to support historical inquiry. However, the authors distinguish between mathematical literacy demands and mathematical literacy learning opportunities. In the online version of the Australian curriculum, the mathematical literacy demands are explicitly identified by icons and online filters. In contrast, the mathematical literacy learning opportunities are not visible unless one knows what to look for. Such opportunities are, to some extent, exemplified by identifying contexts and relevant mathematical knowledge. However, elements such as tools, dispositions, and critical orientation are not included. Goos et al. (2012) therefore propose that the model, which involves these elements, can direct attention to the mathematical literacy demands and opportunities of the curriculum.

The model has also been used in studies of teachers' professional development as an instrument for planning and reflection (Geiger, Forgasz, et al., 2015; Goos, Dole, & Geiger, 2011; Goos et al., 2014). Goos et al. (2014) report from twenty teachers' involvement in a yearlong action research project aimed at developing strategies for planning and implementing mathematical literacy teaching. The professional development approach included three whole-day workshops, two rounds of school visits for lesson observations, teacher and student interviews, and collection of student work. Teachers also completed written surveys regarding their confidence in mathematical literacy teaching and their use of the model for planning. The teachers entered the project with concern for students' mathematical knowledge, dispositions, and competence to use mathematics in contexts. The results from the surveys showed that the project increased teachers' confidence in terms of recognising the mathematical literacy learning opportunities and demands in their own curriculum areas. Their confidence also increased in terms of determining students' mathematical literacy learning needs to inform planning, demonstrating effective mathematical literacy teaching strategies, and modelling ways of dealing with the mathematical literacy demands of their curriculum area. Several of the teachers involved in the project commented on the usefulness of the model in planning teaching. However, Goos et al. (2014) acknowledge that they cannot be sure that the changes achieved were sustainable after the project ended.

On the other hand, one of the teachers involved in this development project reported that both her practice and understanding of good teaching had changed (Goos et al., 2011). In the final interview, she commented on both

professional and personal development in terms of knowing about mathematical literacy from the activities in the workshop and doing mathematical literacy when she tried out activities in her own classroom. As she developed her knowing and doing, her approach to teaching mathematical literacy became part of her being, saying “This is just part of my teaching now... it’s part of who I am now” (Goos et al., 2011, p. 143). This statement suggests that there is a possibility that the professional development project will have some long-term effects.

Geiger, Goos, and Dole (2014) report from another case study coming from the previously mentioned development project. Here, grade 8 students’ perspectives regarding their experiences with the mathematical literacy Health and Physical Education lessons developed by their teacher during the professional development project are studied. The teacher was chosen because of her progress in the projects. The teacher nominated students for interviews based on their competence to articulate themselves. In the interviews, the students reported that they enjoyed the lessons because they were allowed to work in groups and to use digital tools. Also, they enjoyed participating in extended investigations, and many felt that they were learning mathematics without realising it. Also, they got the opportunity to be engaged in their own learning as the activities were relevant to students’ interests and had a genuine purpose. The students acquired new mathematical knowledge, used digital tools, demonstrated positive dispositions to using mathematics in a context, and engaged in critical review of the results.

However, developing activities that students’ find engaging and relevant can be challenging. Relating to Goos’ model, Geiger (2018) studies how teachers investigate ideas to be used for mathematical literacy activities. The research is a case study of two teachers, and part of a larger professional development project involving workshops and school visits. Data are drawn from classroom observations and interviews. Geiger (2018) identified that an in-depth knowledge of the curriculum across subjects helped generate ideas for teaching across subjects. In this approach, the teacher looked for ideas by making connections between curriculum goals of different learning areas. Hence, the curriculum was used as a lens to look for teaching ideas. Another approach was to connect teaching ideas to the curriculum and park them until a suitable time to use them. In this way, the curriculum was not used as a lens, but rather as a way of facilitating a broader educational purpose. In this approach, the already identified

ideas were fitted to serve the requirements of the curriculum. Hence, mathematical literacy tasks can be generated in different ways and yet comply with the elements in the model.

According to Liljedahl (2015), designing mathematical literacy tasks is crucial to understand what it means to be mathematically literate. From his role as a facilitator for a Numeracy Design Team consisting of 13 Canadian grade 5 and 8 teachers with the objective to develop two mathematical literacy tasks, he found that the task designing process led to a change in the teachers' teaching practice. As the team's facilitator, Liljedahl based his study on field notes taken during and after meetings, interviews with individual participants, and field notes from classroom visits. During the meetings, the design team discussed their understanding of what mathematical literacy is, what qualities a mathematically literate student possesses, and what a mathematical literacy task should look like. The team developed mathematical literacy tasks which they piloted with their own students, and subsequently discussed their experiences and refined the tasks. Because definitions of mathematical literacy are not pragmatic and clear, Liljedahl (2015) argues that the task design process makes the embodied qualities of mathematical literacy more clear and concrete. Therefore, designing mathematical literacy tasks is an important exercise both in terms of understanding mathematical literacy and in developing mathematical literacy competence.

One aspect of developing mathematical literacy tasks involves reflecting on mathematical literacy opportunities in tasks across the curriculum. Dole, Hilton, and Hilton (2015) use Goos' model to analyse proportional reasoning tasks and activities to theorise their capacity for supporting students' mathematical literacy capabilities. The data comprise surveys from 40 teachers where they are asked to reflect upon activities and tasks implemented in different subject areas. The number of tasks and activities were quantified and categorised according to the five elements of mathematical literacy. The results show that the development of proportional reasoning can occur in all learning areas and can promote all elements of mathematical literacy. In mathematics lessons, moments emphasising mathematical knowledge occurred 32 times, followed by 17 moments emphasising context, and four moments emphasising tools and critical orientation. In mathematics, no moments were classified as emphasising dispositions. In fact, in all learning areas, moments emphasising mathematical knowledge occurred more often than moments emphasising the other

mathematical literacy elements, and moments emphasising dispositions did not occur in any. However, it is conjectured that the reported moments of proportional reasoning could be categorised as relating to several of the mathematical literacy elements. Also, the teachers nominated 395 moments of proportional reasoning across all curriculum learning areas, which suggests that the teachers had a cross-curricular approach to promoting students' proportional reasoning.

On the other hand, it may be easier to identify moments that potentially contribute to developing students' mathematical literacy than to create such moments oneself. Therefore, teachers need support in developing their teaching for mathematical literacy. Forgasz, Leder, and Hall (2017) present findings from two studies; one involving 62 prospective teachers' experiences of a compulsory course entitled Numeracy for Learners and Teachers in which Goos' model was central, and the other involving 500 Australian, US, and Canadian practising teachers' views about mathematical literacy, its relationship to mathematics, and their mathematical literacy capabilities. The pre-service teachers responded to a pre- and a post-course survey about their views of mathematical literacy, and their confidence to recognise and take opportunities to develop students' mathematical literacy competencies across curriculum subjects. The practising teachers responded to the same survey, but with minor modifications to ensure suitability. Many respondents in each group struggled to articulate what mathematical literacy is and did not seem to appreciate contemporary understandings of the relationship between mathematics and mathematical literacy. A teacher education course on mathematical literacy seems to foster students' confidence in incorporating mathematical literacy in their teaching. Therefore, the authors argue for the implementation of mathematical literacy courses in teacher education. It is anticipated that this will support teacher education students in becoming practising teachers who consciously incorporate mathematical literacy in their teaching. This will, in turn, benefit students' mathematical literacy development. Also, practising teachers need to broaden their understanding of mathematical literacy and recognise its importance. Therefore, professional development programs for practising teachers on how to incorporate mathematical literacy in their teaching, whatever subject they teach, is needed.

In order to prepare students for the data-rich modern society, more holistic approaches to teaching and learning mathematical literacy are necessary (Geiger,

Dole, & Goos, 2011). Such an approach is possible if teachers have a model for teaching, which draws their focus to additional elements of mathematical literacy other than mathematical knowledge alone. The model of mathematical literacy developed by Goos can support teacher learning and development in terms of mathematical literacy task design in mathematics (Goos, Geiger, & Dole, 2013). Implications for further research include the question of how the model supports mathematical literacy planning and pedagogies at the whole school level.

To summarise this chapter, qualitative studies relate mathematical literacy to teaching mathematics in real-world contexts. Students perceive mathematics as a formal exercise detached from everyday life. This perception is already present in primary school. One reason for this perception is the challenge teachers face in finding meaningful contexts with everyday relevance. The studies show that teachers and students have various and varying conceptions of the relevance of different contexts. However, it seems as if the pedagogical approach is more important than the actual contexts. The research studies reported suggest that mathematical modelling tasks, pedagogical entrepreneurship, and a cross-curricular approach may all contribute to students' development of mathematical literacy. Hence, the organisation of classroom activities seems to play a more important role.

Mathematical literacy problems should be open-ended and require students to make investigations without direct instruction from the teacher. As the studies show, this kind of teaching approach is challenging for several reasons. The most important issue involves the teacher's understanding of what mathematical literacy is. A rich understanding of mathematical literacy does not come from reading definitions and descriptions but from discussions, practice, and task development. The studies show that with support and practice, teachers can develop their understanding of and teaching for mathematical literacy, and help students recognise the role mathematics plays in the world.

Most studies discussed in this chapter concern teachers' and prospective teachers' understanding and teaching related to aspects of mathematical literacy. Some studies provide frameworks for evaluating students' mathematical literacy and investigate students' mathematical work in contexts, but very little research concerns students' experiences of teaching for mathematical literacy. Hardly any research on mathematical literacy concern school leadership. Although there is an extensive body of literature related to mathematical literacy, few studies focus on all three different levels in school involving school leaders, teachers, and

students. In fact, Goos et al. (2011) state that research on mathematical literacy looking at the school as a whole is needed.

The research reported in this chapter and the notion of “rekning som grunnleggande ferdighet” in the Norwegian curriculum emphasises the cross-curricular aspect of mathematical literacy. The cross-curricular aspect of mathematical literacy also involves the mathematics subject. Therefore, an important part of teaching and learning for mathematical literacy concerns what goes on in the mathematics classrooms. However, there is little research that specifically concerns mathematics teachers’ teaching for mathematical literacy. In this research, teaching and learning for mathematical literacy in mathematics classrooms is investigated.

Mathematical literacy has been an explicit part of the Norwegian curriculum since 2006. However, there is a lack of research on teaching for mathematical literacy in Norway specifically, and in Scandinavia in general. There is a lack of knowledge about how this competence is interpreted in Norwegian schools, how it is implemented in the teaching, and how it is experienced by students. In the research reported in this dissertation, the aim is to understand the nature of teaching and learning for mathematical literacy in Norwegian schools as a whole. The research questions addressed in the three studies on which this dissertation is founded are:

- What are school leaders’ and teachers’ rationales for teaching for mathematical literacy?
- How do teachers operationalise students' learning for mathematical literacy in lower secondary school mathematics classes?
- What are the characteristics of students’ encounters with mathematical literacy?

In the following chapter, I outline the methods used in order to answer these questions.

5 Methodology

In this chapter, I describe the data generation and analysis of data for the three articles and discuss ethical considerations and trustworthiness. First, an elaboration of the research paradigm is provided in Section 5.1. Next, an outline of the research design is provided in Section 5.2, and a brief outline of the research participants in Section 5.3. Section 5.4 is devoted to an elaboration on the data analysis. The chapter ends with a discussion of trustworthiness in Section 5.5 and ethical considerations in Section 5.6.

5.1 Research paradigm

A paradigm is a set of beliefs that underpin how the researcher sees the world and acts in it. It encompasses four concepts: ontology, epistemology, methodology, and ethics (Denzin & Lincoln, 2000). Ontology concerns the nature of reality and the human being in the world. It involves the question of whether entities should be perceived as objective and independent of social actors or subjective and created from the perceptions and consequent actions of those social actors concerned with their existence. Epistemology concerns the knowledge of the world and the relationship between the inquirer and the known. It deals with the sources and limitations of knowledge and whether observable phenomena or subjective meanings provide acceptable knowledge. Methodology concerns the best means for gaining knowledge of the world. It involves the research design and methods for data generation and analysis. Ethics is about the way to be a moral person in the world. It involves considerations of the research aims, the treatment of research participants, and the handling and presentation of data (Denzin & Lincoln, 2000).

The research reported here is framed within the interpretative paradigm. In this view, the subject matter of social sciences is fundamentally different from that of natural sciences (Bryman, 2008). The social world cannot be studied according to the same principles as the natural sciences. In my research, data is generated from regular classrooms, which are complex social organisations that vary across time, location, and cultural context. Reality is a construct of the human mind. Individuals interpret their social world often in similar ways, but not necessarily the same. Social reality constantly emerges and shifts (Bassegy, 1999). Interpretative researchers see descriptions of human actions as based on social meanings. The researcher is part of the world s/he is observing and, by observing they change the situation they are studying. Social objects and

categories are socially constructed and not objective facts beyond reach and influence. Organisation and culture are products of negotiations between the parts involved, and are constantly being established and renewed (Bryman, 2008). In my research, I consider the classrooms and schools as social entities and constructions built up from the actions and perceptions of the social actors involved.

The research presented here is conducted within the cultural-historical theoretical framework and the theory of knowledge objectification, as outlined in Chapter 3. This framework has influenced the research questions and methods of the study. The historical epistemology of TO draws on Hegel's work (Radford 2015). Knowledge is not something one possesses or constructs. It is a social-cultural-historical entity resulting from and produced through doing, thinking, and relating to others and the world. These entities are experienced and known through Activity; a social and active engagement in the world.

The object of study is the teaching for mathematical literacy in lower secondary school. The aim is to see how teaching to develop mathematical literacy is understood, facilitated and experienced in schools. The aim of the study is not to generate a theory per se, but to understand how mathematical literacy is worked with in schools. This aim has resulted in testing theories, using them to understand what is going on, and also framing suggestions for adjustment of theories.

To the interpretative researcher the purpose of research is to advance knowledge by describing and interpreting the phenomena of the world in attempts to get shared meanings with others. Interpretation is a search for deep perspectives on particular events and for theoretical insights.

(Bassegy, 1999, p. 44)

According to Bryman (2008), qualitative research is typically associated with generating theory. In the inductive approach, theory emerges after the generation and analysis of data. However, research can also take a deductive approach. In the deductive approach, research is done in order to answer questions posed by theoretical considerations and theory guides the generation and analysis of data. The inductive and deductive approaches are general orientations to the link between theory and research, and research is not always one or the other. For instance, deduction entails an element of induction due to the possibility of revising theory based on the research findings. In this way, although I take a deductive approach, my study also entails inductive elements.

5.2 Research design

The research design is a multiple case study where I study the activity system of three schools. According to Stake (2005), several cases may be studied jointly in order to investigate a phenomenon or to examine an issue. I study three cases simultaneously, and each case study is a concentrated inquiry into a single case. The cases facilitate understanding of the issues investigated but are in themselves of secondary interest.

My research interest is in the school leaders', teachers' and students' understanding, teaching, and experience of mathematical literacy. Embedded in this lies their judgement on the focus of mathematics teaching, and the knowledge and competence one should aim to develop. The research sets out to study the exemplifying case where the aim is to capture the conditions and circumstances of a common situation. The cases are selected because they will provide a suitable context for answering the research questions. According to Cohen, Manion, Morrison, and Bell (2011), case studies give a unique picture of real people in real situations. It is a specific case designed to illustrate a more general principle, "the study of an instance in action" (Cohen et al., 2011, p. 253). Case studies are not a choice of methods, but a choice of the subject of study. Regardless of the research methods, one chooses to study a particular case (Stake, 2005). A model of the cases and methods used is displayed in Figure 10. Rationales, operationalisation, and encounters refer to the research questions for the three articles (see also Table 2).

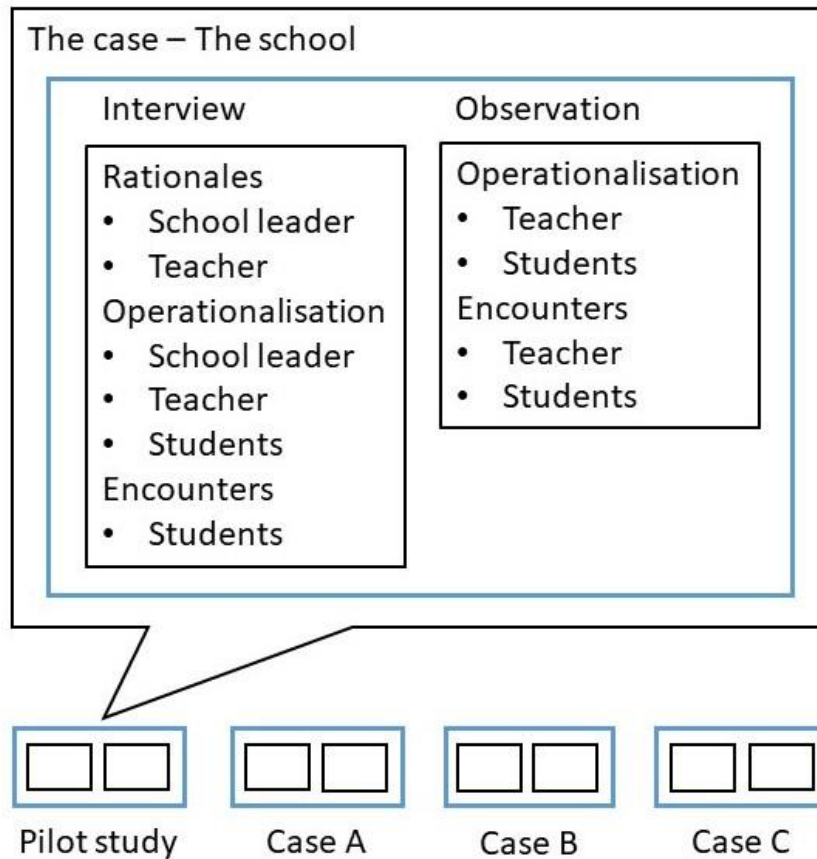


Figure 10. Cases and methods for data generation.

Figure 11 illustrates the design of the research project. First, a pilot study was conducted in one school. The aim was to test and practice the methods for data generation, such as the interview guides and the technological equipment. Also, it provided a sense of what to expect from the data generation. In the subsequent data generation phase, all the data were generated parallel. In a six-week-period, I interviewed school leaders, teachers, and students, and observed mathematics lessons in the three schools. This way of generating data differs from other research projects where data is generated at various stages during the project, and the type of data generated depends on results derived from previous data generation and analysis. Generating data at various stages of the research project makes it possible to pursue issues of interest arising from previously generated data to get deeper insight. On the other hand, generation of all the data in parallel ensures the project’s coherence because the focus and type of data for each study follow a predetermined and common plan. Also, the participants were informed about the whole data generation process in advance, and there was no need to get renewed consent. However, this way of generating data involves a risk of

missing the opportunity to explore unexpected issues and the disadvantage of not being able to process and analyse all the data immediately after generating it.

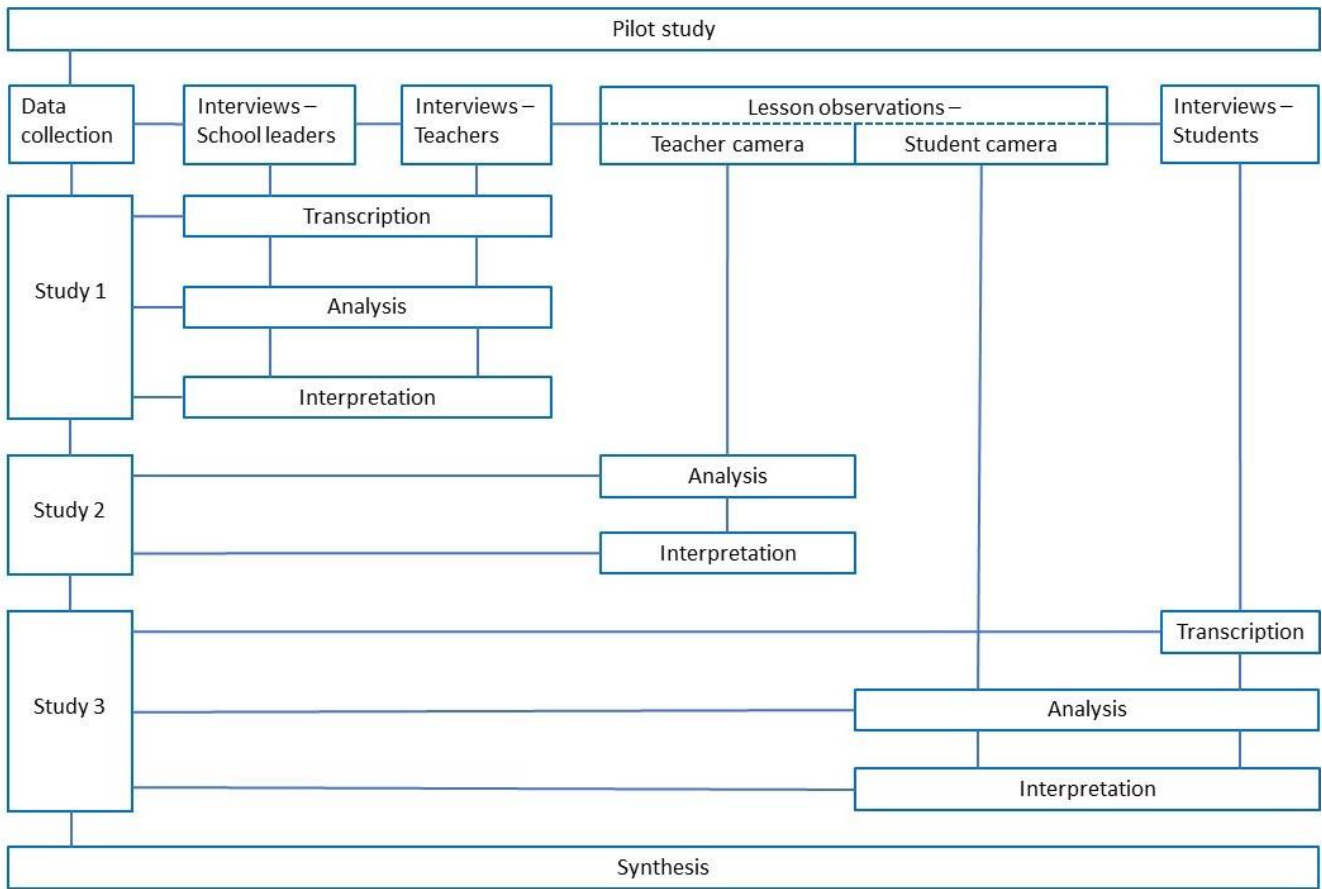


Figure 11. The design of the research project.

After generating the data, attention was directed toward one research question at a time. Table 2 displays the three research questions and corresponding methods for data generation. First, interviews with school leaders and teachers were transcribed, analysed, interpreted, and reported (Study 1). Second, lesson observations from the teachers’ cameras were analysed, interpreted, and reported (Study 2). Third, student interviews were transcribed, and the interviews and lesson observations from the students’ cameras were analysed, interpreted, and reported (Study 3). Although Figure 11 gives the impression that the three studies were worked on in isolation from each other, this is not the case. As all the data was generated by me, my knowledge of the data as a whole may have influenced the analysis performed in each study. Also, as the teachers’ and students’ cameras recorded the same mathematics lessons, Studies 2 and 3 are closely connected in terms of context and content. The three studies are reported in three

independent journal articles (see Chapter 6 and Appendix C). Therefore, finally, the studies were synthesised and interpreted as a whole and reported in this dissertation.

Table 2

Research questions and corresponding methods

Research question	Method/data
What are school leaders' and teachers' rationales for teaching for mathematical literacy?	Interviews with school leaders Interview with mathematics teachers
How do teachers operationalise mathematical literacy in their teaching?	Observations of classroom teaching focusing on the teacher
What are the characteristics of students' encounters with mathematical literacy?	Observations of classroom teaching focusing on the students Interviews with students

As mentioned, the methods for generating data are interviews and observations of classroom teaching. According to Corbin and Strauss (2015), there are three basic types of interviews, all used in qualitative research. These are unstructured, semi-structured and structured interviews. The unstructured interview was irrelevant for me because I had specific issues that I wanted the interviewees to address. The structured interview was also irrelevant because I wanted the participants to be able to introduce topics important to them, and I wanted to be able to ask follow-up questions. In semi-structured interviews, some topics are selected before beginning the research. The same topics are covered in each interview, and many researchers feel comfortable having a list to fall back on. When and how the topics are presented is not determined in advance or pursued by adhering to a predetermined schedule. The researcher can ask additional questions to clarify points, and the participants are given opportunities to add anything else that they feel is relevant. If the researcher has a reasonably clear focus regarding what the research is about, the semi-structured interview allows for more specific issues to be addressed (Bryman, 2008). I conducted semi-structured interviews with the school leaders, teachers, and students. The

interviews with the school leaders and teachers lasted for about an hour, while student interviews lasted for 15-20 minutes. I prepared interview guides with questions and topics I wanted to learn more about to make sure that I did not forget anything. The interview guides are included in Appendix A. However, the questions were rarely asked in the exact way they are formulated in the interview guides. The wording was adapted to suit the context and the individual participant.

There were several issues I had to consider regarding interviewing. For example, periods of silence can be a difficult aspect of interviewing (Corbin & Strauss, 2015). The participant may not have thought thoroughly about an issue and needs time to think before answering. It is important that the researcher does not jump in with questions, breaking the participant's thought process. For example, the students may not have reflected on or articulated answers to the kind of questions I posed. I, therefore, had to give them enough time to think. However, if I waited too long, they could feel uncomfortable. I also emphasised that there were no right or wrong answers, and my interest was in their reflections. This issue was particularly important in the student interviews to prevent them from saying what they thought I wanted to hear instead of their own reflections.

A skilled interviewer lets the participant guide the course of the interview. Bryman (2008) recommends recording interviews to ensure that answers are captured accurately and in the interviewees' own terms. I video recorded all the interviews. The video recordings allowed me to examine the answers more thoroughly, and during the interviews to be more responsive to the interviewees' answers and to follow them up because I was not preoccupied with taking good notes. Compared to audio recordings, the video recordings were useful when transcribing the interviews, because I could see the participants' faces and their lips if the audio was, for some reason, difficult to hear. Also, the recordings provided me with gestures which could support or contradict what they said. However, I was aware that video cameras might be off-putting for the interviewees. I, therefore, tried to place the camera discreetly and bit to the side of the table, so that they would not look straight at it while looking at me.

I started all interviews with general questions about participants' backgrounds and interests. For example, I asked the school leaders and teachers about their educational background and subject interests. I asked the students about their interests and hobbies before moving on to thought about what they

want to do when they get older in terms of further studies and work. These questions served two purposes. First, I wanted them to tell me something of personal relevance to make them feel more relaxed in the interview situation. Second, I wanted to use these responses as a way into the topics I was interested in. To emphasise that there were no right or wrong answers, I tried to start the questions with phrases such as “Can you think of...” and “What do you think...”. I also tried to acknowledge their responses and encourage them to continue talking by giving short statements such as “mmm” and “yes”. In Table 3, on the following page, is a short excerpt from one student interview to illustrate this. The first column shows the original Norwegian transcript and the second show an English translation.

Interviews are distinct from conversations in everyday situations. For instance, everyday situations are governed by social norms depending on trust and mutuality. In contrast, interviews are more hierarchical in the sense that the interviewer initiates topics, directs the flow of talk, decides when a response is adequate and does not have to disclose his/her views (Mishler, 1991). Trying to respond to these issues, I asked the participants whether they could think of something else or wanted to add something to let them decide when the response was adequate. Although I tried to create an informal and relaxed atmosphere, I acknowledge the hierarchical structure of the interview situation, especially in the student interviews. This concern particularly regards situations in which participants may have wanted me to respond to their views. When interviewing, however, it is important to try to be non-judgemental to prevent distortion of later answers (Bryman, 2016).

Nevertheless, in social interaction, we always interpret our surroundings. The same way I interpreted the participants’ responses, tone of voice, and body language in the context of the interviews, they interpreted mine. It is, therefore, possible that from their interpretations, they tried to respond in the way they believed I wanted them to. Hence, all our interpretations in the interview context affect our communication in the situation. These interpretations also affect my subsequent interview analysis.

Table 3

Excerpt from transcribed student interview

	Norwegian original	English translation
Oda:	Mmm. Ja. Eh... Kjenner du nokon som, nokon andre, som du veit brukar mykje matte i jobben sin, eller kjem, veit du om nokon yrke som, som du trur brukar, har bruk for mykje matematikk?	Mmm. Yes. Uhm... Do you know anyone that, anyone else that you know uses a lot of mathematics in their work, or do you know any occupations that, that you think uses, uses a lot of mathematics?
Student:	Lærarar.	Teachers.
Oda:	Lærarar? (Ler) Mattelærarar, ja? Mmm (bekreftande). Kan det vere andre?	Teachers? (Laughs) Mathematics teachers, yes? Mmm (affirmative). Can there be anyone else?
Student:	Eh... Folk som, eg veit ikkje, eg hugsar ikkje kva det heiter, men folk som bygger hus og sånt.	Uhm... People that, I don't know, I don't remember what it's called, but people that build houses and stuff.
Oda:	Ja. Snikkarar, for eksempel?	Yes. Carpenters, for example?
Student:	Ja.	Yes.
Oda:	Mmm (bekreftande). Kva trur du dei har bruk for å kunne matte til, då?	Mmm (affirmative). What do you think they need mathematics for?
Student:	For å vite kor lang ein vegg skal vere og sånt. Og i forhold til alle andre veggar og, liksom å setje opp.	To know how long a wall should be and stuff. And compared to all the other walls and, like, to place it.
Oda:	Kan du gi eit eksempel på... ja, kva i matematikken dei brukar då?	Can you give an example of... well, what kind of mathematics they use?
Student:	Meter og...	Meters and...
Oda:	Mmm (bekreftande).	Mmm (affirmative)
Student:	Meter og centimeter og alt såne ting.	Meters and centimetres and all that stuff.
Oda:	Mmm (bekreftande).	Mmm (affirmative)
Student:	Og kvadratmeter og... Ja.	And square metres and... Yes.

As mentioned above, the interviews were semi-structured, allowing the school leaders, teachers, and students to talk about their interests and concerns. The school leaders and teachers picked up on open questions and often carried the conversation in other directions than intended. In the student interviews, this was not the case. The students' responses were short and to the point, and this is illustrated in the interview excerpt above. As Mishler (1991, p. 7) remarks, "questioning and answering are ways of speaking that are grounded in and depend on culturally shared and often tacit assumptions about how to express and understand beliefs, experiences, feelings, and intentions". The school culture involves questioning and answering as a form of assessment. Although I emphasised in all the interviews that there are no right or wrong answers and that I was interested in the participants' thoughts and meanings, the culture of schooling may have affected the students' perception of the interview and their answer. Because of my background as a lower secondary school teacher and the three schools' cooperation with the University College in which I was employed, the school leaders and teachers may to a greater extent have interpreted the interviews as a conversation among peers.

I also observed classroom teaching in order to see what and how teachers teach and how students work. Sixteen mathematics lessons were observed, and video recorded. In each lesson, the teacher and three students wore head-mounted cameras (Go Pros). Head cameras enabled me to capture the participants' visual fields, get deeper insight onto the direction and timing of participant attention, and document participant actions. For example, when the teacher approached the students, it was important to capture what s/he might write or point at in their books. It was also important to capture the students' working process and interactions with each other in terms of written statements and gestures related to these. Therefore, audio recordings and collection of student work were not considered as sufficiently documenting the joint endeavour of the teacher and students in the classrooms.

The use of head cameras is widespread in sports and studies of wildlife, but less prevalent in education research. By wearing head cameras, participants have a more active role in the data generation, and this, in a way, blurs the lines between participants and researcher (Blikstad-Balas & Sørvik, 2015). The use of head cameras can provide a new dimension in education research in terms of capturing students' and teachers' perspectives. In this study, the head cameras provided valuable insight in students' conversations, the tasks and students'

written accounts, and their attention toward the blackboard (or elsewhere), all in one recording.

However, the head cameras caused some challenges. A couple of times, a memory card error occurred in the middle of the lesson, causing the recording to stop. As a result, some recordings were incomplete. Also, the camera makes a sound when switched on and off, and this caused some distraction. The battery lifespan is limited, and sometimes (in long lessons) they did not last the whole lesson. In this case, the recording was also incomplete. It was sometimes difficult to adjust the angle of the cameras to capture what the teachers were watching. They sometimes had to adjust the head strap where the camera was mounted, and this caused the camera to move. Head cameras are also limited in that they can only capture a subset of participants' visual fields, potentially leaving activities under-documented (Maltese, Danish, Bouldin, Harsh, & Bryan, 2015).

In addition to the head cameras, a stationary camera in the back of the classroom helped me capture an overview of the whole setting. The stationary camera served as a valuable backup. With help from the recording from this camera, I could still see most of what happened in the classroom when the head cameras did not work.

The observations are supplementary to the interviews and provide data that exemplify or contradict statements given in the interviews. I was present in the classroom during the lessons, taking notes and helping with the cameras. I did not intervene in the teaching. The teacher was instructed to plan and conduct the lessons in the manner of his/her regular practice. I did not provide any guidelines regarding lesson content or teaching methods.

Before starting the main data generation, I conducted a pilot study. The pilot study was conducted using the same approaches that I intended for the main study. Piloting plays an important role in ensuring that the research instrument functions well (Bryman, 2008). Piloting the interview guide gave me experience with interviewing and made me more confident as an interviewer. Also, it gave me the opportunity to evaluate and revise the questions. The pilot included a small set of respondents comparable to the sample of respondents intended for the main study. The respondents in the pilot were not included in the main study. A pilot study also enhanced the quality of the coding. It provided some preliminary results, and I had the chance to practice the coding process, to develop codes, and form some hypotheses on what I might find during the main study.

5.3 Research participants

The three participating schools and the pilot school are all located in the region that until January 1, 2020, was known as Sogn og Fjordane county (following a national reorganisation of local authorities in 2019, it is now part of Vestland county). Students in the Sogn og Fjordane region regularly perform, in general, higher than average on national tests (called “Nasjonal prøve”). These findings are consistent over five years (Langfeldt, 2015). The national tests aim to provide schools with knowledge of students’ basic skills in reading, English, and numeracy. The project “Learning regions” (originally “Lærende regioner”) seeks reasons for the disproportionately good results. Cooperation with the university college, a focus on well-educated teachers, and integrated ideas and beliefs about how students learn are some of the explanations proposed (Langfeldt, 2015). My study aims to find examples of good practice for organising teaching and learning, and the results from “Learning regions” suggest that schools in Sogn og Fjordane may provide me with the data I need to achieve this. In other words, my sample was intended to be representative of better than average practice.

The three schools participating in the study were not randomly selected. There were no specific criteria other than that I selected schools with which I was familiar, either through my teacher education studies or my work as a teacher. Schools are busy places with a lot going on all the time, and it is easier to agree to participate in a project when one knows the person who is asking. Hence, the schools were thus recruited based on convenience and prior acquaintance.

I refer to the schools as A, B, and C. The schools are situated in small communities where the population is homogenous in terms of cultural and social background. The Norwegian school system is based on principles of equal opportunity and individually adapted learning for each student within an inclusive environment. Therefore, students are taught in mixed attainment groups. The schools’ total number of students on roll range from 220 to 370 and all three schools teach grades 1 through 10. I contacted the school leaders, and they recruited teachers and their respective classes. Criteria for the selection of classes were that they were grade 9 (students aged 14-15 years) and that they agreed to participate. I needed consent from both the students and their parents. All parties involved received written information explaining my interest in studying teaching with respect to concepts in policy documents. To ensure informed consent, I attended meetings with the teachers, the students, and the

parents. Details concerning consent and other ethical issues are elaborated in Section 5.6.

Data comprises interviews and lesson observations. Six school leaders, three mathematics teachers, and 22 students were interviewed. Two school leaders and one teacher from each school were interviewed in addition to eight students from School A, seven students from School B, and seven students from School C. The only criteria for the selection of students, were that they had signed the consent form and agreed to be interviewed. Hence, the interviewed students are a random mix of gender and attainment. All school leaders have previous experience as teachers, and school leaders and teachers have between 10 and 40 years of experience from working in schools. The teachers teach several other subjects in addition to mathematics. I did not collect any background information about the students as my focus was on the teaching.

In total, there are 24 students in Class A, 14 students in Class B, and 28 students in Class C. To get information about classroom activities, 16 mathematics lessons were video recorded. I recorded six mathematics lessons in School A, and five mathematics lessons in Schools B and C. Class C was divided into two groups according to which students had consented to participate in the research. Therefore, 18 students were present in the observed lessons. There were originally 28 students in Class C. They had two mathematics teachers, and the class was usually divided into two groups for all mathematics lessons. Hence, this arrangement was not made because of the research project. Only the group composition was changed according to consent.

In Schools A and B, all the observed lessons concerned the topic equations. In School C, the two first observed lessons concerned equations and the remainder concerned percentages. The lessons varied in length from 45 to 90 minutes. I was a non-participant observer and did not intervene in the lessons, other than by being present. I asked the teachers to plan and conduct the teaching as they would normally as I was interested to observe, as far as possible, regular mathematics lessons. In retrospect, it is reasonable to wonder whether the outcome of the study would have been different if I had chosen to observe at a time when the mathematical topic was more likely to be related to mathematical literacy. Algebra and equations are commonly described as abstract and more challenging to relate to real-world contexts. Per cent, on the other hand, can easily be connected to several real-world contexts. Therefore, one could expect that the lessons about per cent would involve a stronger relation to mathematical

literacy than those on equations. As I will discuss later, the lessons on per cent did involve more contextual tasks than the lessons on equations. However, as discussed in Chapter 4, the contexts do not alone determine whether it is a good task for the education of mathematical literacy. The structure of the task and the organisation of classroom activity plays an important part in developing mathematical literacy. As shown in Study 2, the lessons and tasks on per cent and equations are structured in similar ways. This similarity may indicate that the results would have been the same for other topics as well.

On the other hand, the teachers may have had experiences and encounters with mathematical literacy in everyday life that are connected to other topics. For instance, statistics provides many opportunities for students to formulate a question, make inquiries, model their results, and interpret and evaluate them. Statistics is also commonly used in the media, and this can provide opportunities to critically interpret and evaluate the way statistical measures are presented, for examples in newspaper articles and advertisements.

Another point concerns the teachers' motives and goals. If the classroom Activities and their motives and goals involve developing mathematical literacy in all lessons, the specific mathematical topic will not matter. What matters is whether the students take up this motive and become aware of the culturally constituted mathematical meanings involved in mathematical literacy.

5.4 Analysis

In TO, the central theoretical category and methodological unit of analysis is Activity (Radford, 2016). Activity, as outlined in Chapter 3, is the social process where individuals engage themselves in the world to meet their needs. In the research reported here, I investigate Activity in terms of teaching for mathematical literacy. Mathematical literacy is conceived as knowledge about how to engage mathematically in the world. This knowledge is created and recreated through sensuous cultural-historical Activity by school leaders, teachers, and students working together. The way this Activity unfolds is, as mentioned above, qualitatively studied.

One of the challenges with qualitative research is that it often generates an extensive data corpus, and there are few well-established and widely accepted rules for data analysis. There are, however, some general approaches, such as analytic induction, grounded theory, thematic analysis, and narrative analysis (Bryman, 2016). In most of these approaches, coding is the key process and main

feature. It is important to keep in mind the risk of losing the context and narrative flow of what is said when plucking chunks of data out of the context in which they originally appear. This risk is part of the critique of this coding approach. However, measures, such as creating memos, can be taken to reduce this risk. Coding is central in the analysis of my research data. Taking notes about significant remarks and observations, generating theoretical ideas about the data, and connecting and interpreting emerging concepts and categories were important steps in the process (Bryman, 2016).

My data comprise video recordings of individual semi-structured interviews and observations of classroom teaching. Video recordings offer opportunities for analysing issues beyond the content of what is said, such as interpersonal interaction. This makes video analysis a time-consuming process (Kvale & Brinkmann, 2009). By recording interviews with school leaders, teachers and students, and recording mathematics lessons in three different schools, I generated a lot of data. I, therefore, had to make some preparations before starting with the analysis. The recordings were imported into the computer-assisted qualitative data analysis software NVivo. This software allowed me to cut recordings into smaller sections, to code them by use of the computer, and to retrieve the coded video sections easily. NVivo enabled me to collect and retrieve all video sections coded in a particular way. The software could not help me with decisions about coding and interpretation of findings, but it helped me with the manual labour involved, such as structuring and organising the data (Bryman, 2008).

I transcribed the interviews into the NVivo software. In NVivo, it is possible to watch the recordings while transcribing and connect time spans in the recordings to the different parts of the transcript. This made it easy to go back to the recordings later if needed while working on the analysis.

To transcribe means to transform and a transcript is a translation from an oral narrative to a written narrative. Oral speech and written texts entail different language games. Therefore, transcriptions may produce artificial constructs that are adequate to neither oral nor written language (Kvale & Brinkmann, 2009). Transcription is a challenging process and interpretative in the sense that the transcriber's own perceptions may influence the process (Kvale & Brinkmann, 2009). This challenge raises questions of transcriber reliability. Although an effort was made to be as accurate as possible, there might be discrepancies due to poor recording, mishearing, or difficult audible segments. It can also be difficult

to know where a sentence ends and where to insert periods and commas. The same written words can convey quite different meanings depending on punctuation.

Transcribing can also be a tiresome and time-consuming task. However, it is a valuable way to get to know the content of the data before starting the coding. I, therefore, did my own transcribing. In this way, I also ensured that relevant details for the analysis were not left out. I transcribed the interviews verbatim in order to capture as much as possible of what was said. The exact wordings of the participants have been used in the transcriptions, which means that incomplete sentences and bad grammar were not corrected, and “ums” and “mmms” were included (see excerpt earlier in this chapter). Inaudible words were marked in brackets: (inaudible). Non-verbal and emotional elements are included in brackets, for example: (points to the definition), (laughs). Short pauses were marked with three following dots, (...), and longer pauses were marked as; (long pause).

The categories presented in studies 1 and 3 are documented with excerpts from the interview transcripts. In studies 2 and 3, specific examples from classroom teaching considered important are transcribed and reported. The transcriptions are originally in Norwegian and have been translated by the researcher. The aim has been to provide as authentic a translation as possible. As a consequence, some excerpts had to be adjusted to fit the standards of written English. The analysis was carried out in Norwegian and, therefore, only the particular excerpts used in the three articles were translated.

The three articles focus on teaching for mathematical literacy at three different levels of practice. Together they form a whole, involving school leaders' and teachers' rationales, the operationalisation of teaching in the classroom, and the students' processes of objectification. In general, qualitative content analysis was used. From a hermeneutic phenomenological approach, consciousness is not separate from the world, but a formation of historically lived experience (Laverty, 2003). In this way, a person's history and cultural background present ways of understanding the world. The cultural-historical background is an important issue in the analysis from both the participants' and the researcher's perspective. In the same way as the participants' understanding of the world is influenced by their backgrounds, my understanding of the data is influenced by my historical and cultural background. As a researcher, it is

important to be aware of this influence and try to set aside personal assumptions and prejudice.

Meaning is contextually and culturally grounded (Mishler, 1991). In coding data, my task as a researcher is to determine the meaning of interview and classroom discourse. The participants may represent different subcultures outside the culture of the school, and hence, only partially share a common culture. The combinations of various cultures affect the participants' responses and meanings. Also, different culturally grounded rules and norms guide how we act in various situations. Such rules and norms apply in the interview and classroom situation to both the researcher and the participants. Therefore, the data and the analysis are influenced by the research participants' and my own (the researcher's) "joint production" (Mishler, 1991) or Activity (Radford, 2018) in the sense that we work together to develop our understanding of teaching for mathematical literacy.

Study 1 was exploratory, and data were analysed qualitatively using an inductive approach and "meaning coding". Meaning coding involves attaching keywords to text segments in order to permit later identification of a statement (Kvale, 2007). I engaged in multiple close readings and interpretations of the data. I tried to get an overall understanding of the data and to identify text segments. Sometimes whole sections of transcripts elaborated on the same issue, and sometimes only short sentences. In this way, the text segments differ in length. The initial coding resulted in numerous codes, with names close to the empirical data. At this stage, coding was conducted manually, with pencil and paper. This approach was necessary to get a full overview of all the text segments and codes.

During the initial coding, many codes were given names related to similar concepts. Next, the text segments were compared for similarities and differences. Through interpretations, the text segments were evaluated to belong to different categories. Text segments containing similar topics or issues were grouped together to make broader categories. Thus, the broader categories were developed with respect to key themes in the text segments. Additionally, this coding phase also reduced data, as some codes were interpreted as not relevant. After developing the categories, more general theoretical ideas were considered. Connections were outlined between the categories and concepts from existing literature.

Finally, category descriptions were made based on theoretical concepts. Figure 12 illustrates an example of the process of developing one of the categories in Study 1.

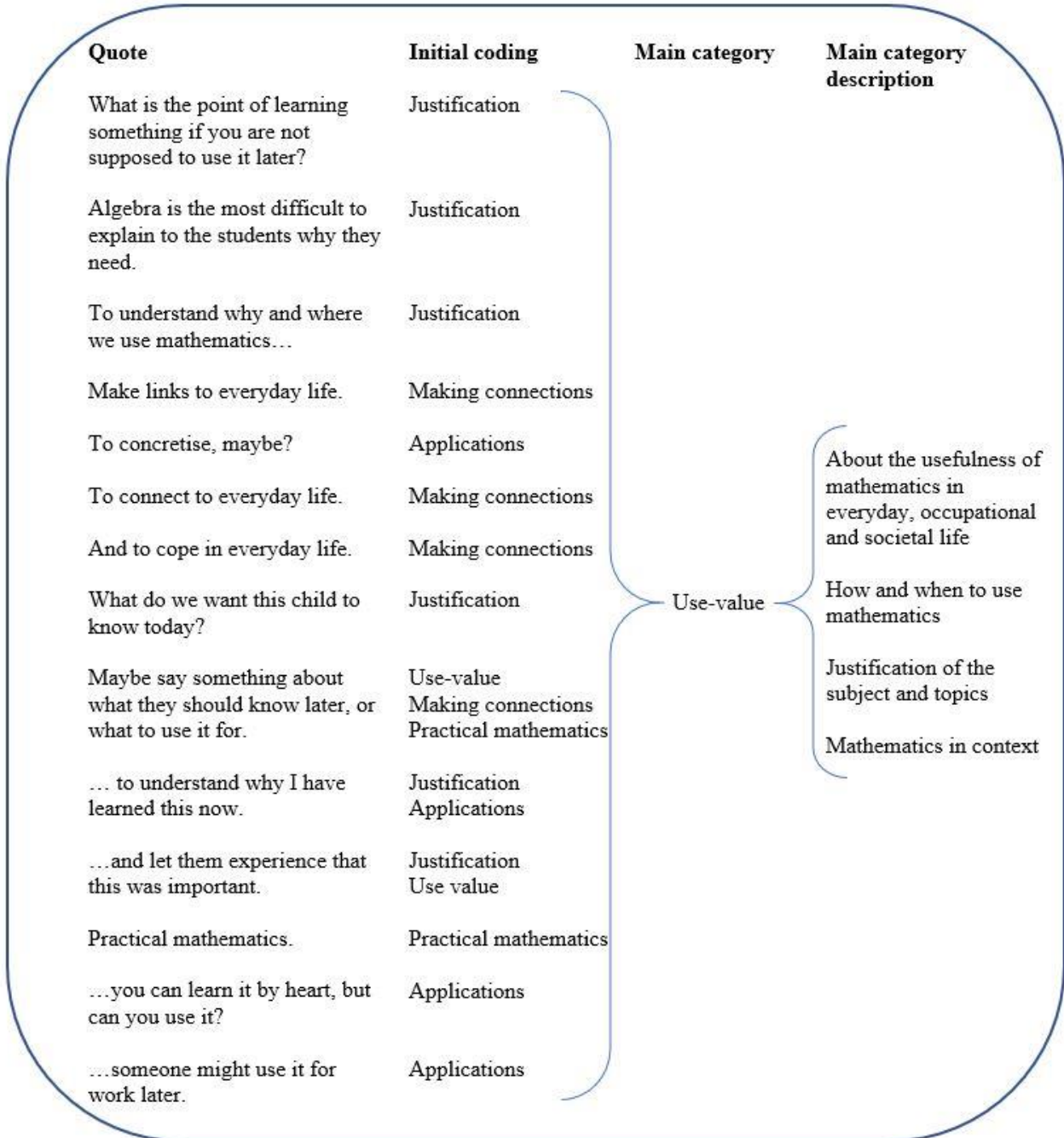


Figure 12. The process of category development.

The transcripts were then coded again, using NVivo, with respect to the category descriptions. A colleague, provided with the category descriptions and a sample

of the transcriptions, also coded the data. A comparison of our codes using the inter-rater reliability test in NVivo showed an agreement of 92.2%.

In Studies 2 and 3, the analysis processes were slightly different than in Study 1. In these studies, the model developed by Marilyn Goos, described in Chapter 2 served as an important tool for analysis. Mathematical literacy involves the competence to use mathematics in various contexts. Therefore, in the classroom, I look for tasks, examples, and discussions where mathematics is used to solve a problem or make sense of an issue in personal life, work-life, and citizenship. In the interviews, I look for encounters with mathematics in real-life situations. Such encounters may be the participants' own experiences of using mathematics to solve a problem, or it may be them referring to other's experiences with using mathematics. The contexts, in the classroom and interviews, involve an evaluation of authentic aspects of the situations and the mathematics used.

Developing mathematical knowledge is usually the main focus in the mathematics subject. In my analysis of the lessons, I look at the kind of mathematical knowledge worked with and how. For instance, I look at how the participants explain and discuss various solution methods and strategies and how they verbalise and connect concepts. I also look at the type of tasks worked with in the classroom, for instance, problem-solving tasks, drill tasks, word problems, open-ended tasks, or inquiries, and what knowledge they are aimed at developing. The type of tasks is important because they reflect the teacher's pedagogical intention and the goal and motive of the classroom Activity (see Chapter 3). In the interviews, mathematical knowledge is related to mathematical facts, concepts, procedures, methods, and strategies that the participants have encountered in situations in everyday life.

Dispositions is another important element of mathematical literacy. In the classrooms, dispositions may be noticeable in various ways. The participants may display their emotions verbally, or by facial expressions or body language. Dispositions may also be displayed through the way the participants work. For example, the teachers may encourage and praise the students. They may try to make connections to topics of students' interests. The students may show interest by engaging in discussions and asking questions. Also, dispositions may be visible in their reactions if stuck on a problem or getting the wrong answer to a task. In such situations, the students may give up, or they may revise their work and try a different strategy. In the interviews, dispositions can be connected to

the participants' view of mathematics. Their dispositions involve recognising encounters with mathematics in real-life situations, and perceiving mathematics as something worthwhile. However, the perception of mathematics as distinct from real life does not necessarily mean that the participants do not hold positive dispositions toward mathematics. They may still have the willingness and confidence to engage with mathematical tasks. Some also appreciate solving tasks using formal mathematics without any real-world contexts.

Tools involve the use of physical, representational, and digital tools to mediate and shape thinking. In the analysis, I looked for uses of physical tools such as manipulatives, models, and measuring instruments. Representational tools may include graphs, tables, maps, drawings, symbols, or language (written and spoken). Computers, calculators, software, and the internet are examples of digital tools. In the classroom, the actual use of such tools was investigated. In the interviews, I looked for references to tools used to reason and act in the world.

The elements of mathematical literacy are embedded within a critical orientation. Critical orientation involves holding dispositions to critically reflect on the contexts in which claims are made, to critically evaluate the mathematical knowledge drawn upon and the tools used to display or support the claims. It also involves being critical to one's own and other's judgements and arguments. In the classroom, this construct can be noticeable through classroom discussions where concepts, methods, solutions, and tools are questioned, justified, evaluated, and validated in relation to the problem context. In the interviews, a critical orientation can be noticeable through participants' experiences with mathematics in real-world contexts. For example, they may talk about situations where they or someone they know have used mathematical information to make decisions and judgements. Also, they may have encountered situations where they have had to critically evaluate the use of mathematical knowledge and tool use in real-world contexts.

The operationalisations of the mathematical literacy elements in the lessons and interviews are displayed in Table 4 below.

Table 4

Operationalisations of mathematical literacy elements

Elements	Lessons	Interviews
Context	<p>Tasks and examples that support the development of the competence to use mathematical content in various situations in everyday life.</p> <p>Discussion of applications of mathematics in different contexts.</p> <p>Discussions of authentic aspects and certifications.</p>	<p>Examples of contexts and situations where mathematics is or can be useful</p>
Mathematical knowledge	<p>Tasks and examples that support the development of mathematical concepts, methods, skills, and problem-solving strategies.</p>	<p>Examples that show how mathematical topics, concepts, methods, skills, and problem-solving strategies are or can be useful.</p>
Tools	<p>Tasks and examples that support the development of digital tools, representations, models, and communication, and the use of such tools.</p>	<p>Examples that show how digital tools, representations, models, and communication are or can be useful.</p>
Dispositions	<p>Tasks and examples that support the development of the willingness and confidence to engage with mathematical tasks flexibly and adaptively, and foster curiosity, and interest.</p>	<p>Examples that show a willingness and confidence to engage with mathematical tasks flexibly and adaptively and that shows curiosity and interest in mathematics.</p>
Critical orientation	<p>Tasks and examples that support developing the competence to use mathematical information to make decisions and judgements, add support to arguments, challenge an argument or position, and discuss, question, explain, evaluate, and validate methods and solutions.</p>	<p>Examples related to the use of mathematical information to make decisions and judgements, add support to arguments, challenge an argument or position, and discuss, question, explain, evaluate, and validate methods and solutions.</p>

Hence, for Studies 2 and 3, the categories were determined in advance. The data in Study 2 consisted of video recordings of classroom teaching, which were not transcribed. The previously outlined model developed by Goos (i.e. Goos et al., 2014) was used to analyse the teaching, and the categories used corresponded to the five elements of the mathematical literacy model. In preparing the analysis, the descriptions of the model elements were adjusted to fit the purposes of the study. The elements were connected to relevant existing literature. This literature was specifically focused on teachers and teaching.

The recordings were imported into NVivo, and sections of recordings were coded with respect to the elements and corresponding operationalisation of the mathematical literacy model. As the elements are closely connected, the teachers may attend to several elements simultaneously. Therefore, some sections of recordings were coded to several elements. In Study 2, NVivo was used as a tool to structure the recordings according to the elements and made it easier to retrieve the different coded sections. Some sections of the coded data were considered as representative for the general findings. These sections were transcribed and served as examples in the study report (Article 2).

In Study 3, the data comprised both interviews and video recordings. Interviews and lesson recordings were analysed using the mathematical literacy elements, as mentioned. The operationalisations of the elements developed for Study 2 were revised to suit the video data from the students' perspective. Also, the interview questions were connected to the elements of mathematical literacy and their operationalisations.

The process of interview analysis in Study 3 was similar to that of Study 1. However, in Study 3, the coding categories were predetermined. First, tables based on the students' answers were constructed in order to get an overview of the interview data. Next, several close readings were done in order to get to know the data and to interpret and code students' answers according to the developed element descriptions. Sections of the transcribed interviews were coded with respect to the elements of mathematical literacy. NVivo served as a structuring tool in the coding process.

The video recordings were analysed using the same procedure as in Study 2. Using NVivo, the lesson recordings were coded according to the descriptions of the elements of mathematical literacy. Particular sections of recordings were transcribed to serve as examples in the article reporting the study.

Finally, the elements of mathematical literacy were reinterpreted in the context of TO, outlined in Chapter 3. The analysed data was conceptualised within this framework.

5.5 Trustworthiness

Trustworthiness is an important aspect of the quality of research. Issues of reliability and validity are important in positivistic research but are associated with some challenges in assessing the quality of qualitative studies. As qualitative studies are based on the researcher's subjective interpretations, the quality of the data, methods, and interpretations must be assessed. Lincoln and Guba (1985) and Guba and Lincoln (1994) (referred to by Bryman, 2008), propose trustworthiness and authenticity as such assessment criteria.

Trustworthiness is made up of credibility, transferability, dependability, and confirmability. In order to support trustworthiness in my research, I have to reflect upon how believable the findings are, and if they apply to other contexts and at other times. I also have to be aware of how my values may have affected the analysis (Bryman, 2008).

Credibility involves the degree to which the findings corresponds with reality (Bryman, 2016). One way to help ensure credibility is respondent validation. During the interviews, I asked questions to confirm that I had understood them correctly (i.e. "Do you mean that...", "Do I understand you correctly if..."). The aim was to make sure that my findings are in line with the participants' views. Another way to help ensure credibility is to triangulate methods and sources of data. For instance, ethnographers often check their observations with interview questions to determine whether they have understood what they have seen (Bryman, 2016). Hence, triangulation means to investigate the same research issue from different perspectives. This study used data from different sources; school leaders, teachers, and students, to investigate teaching for mathematical literacy in school. Moreover, interviews and lesson observations served as two different methods of data generation (see Section 5.2). As mentioned above, in Study 1, a colleague also coded the interview data, and our coding was tested for inter-rater reliability.

Transferability is related to external reliability, which refers to the degree to which a study can be replicated. Replication is difficult in qualitative research because one cannot "freeze" the social setting, as social settings are "in flux" (Roth & Radford, 2011). Instead, the researcher must provide thick descriptions

of the case. Thick descriptions are detailed and rich accounts of the culture of study, in order to provide others with sufficient information to make judgements about the possible transferability of findings (Bryman, 2008). In this dissertation and the articles reporting the studies, I have tried to provide detailed descriptions of the methods and procedures.

Another issue of transferability and dependability concerns the researcher's role as the main instrument of data generation (Bryman, 2016). This issue is related to confirmability. Confirmability is concerned with ensuring that the researcher has not allowed personal values to influence the conduct of research and the findings. In social research, there are numerous points where biases can occur. The researcher's values can, for instance, affect the choice of the research area and research questions, choice of research design and methods, and analysis and interpretation of data (Bryman, 2008). It is important to recognise and acknowledge that research cannot be value-free, and the researcher has to reflect on and discuss the possibility of such value intrusions. This research project is influenced by my interests and experiences. In my opinion, this is only natural. I believe that a certain level of interest is necessary to find the motivation to conduct research.

As a mathematics teacher, the students often asked me "When will I ever need this in real life?" It is important for me to learn more about how teachers make the subject relevant for students' current and future lives and to prevent it from being just a school subject with no relevance to the world outside of school. During the analysis, it was important to keep in touch with the data and the theory. I needed to be attentive to the possibility of seeing what I wanted or hoped to see instead of what was there. I also had to carefully consider what and how information was presented to the participants. If participants were too well informed, it might cause them to say or do what they thought I wanted them to say or do, and in that way jeopardise the validity of data and the results. I argue that by providing transparency in terms of insights into the research process in this dissertation, issues of credibility are addressed. Hence, I can assure the reader that the research has been conducted in good faith.

As the research reported here is based on a small-scale study that relies on data generated from a convenience sample, it is difficult to argue for generalisability of the findings. It is impossible to argue that the cases reported here are typical of all schools in Norway, and this is not the intention. In

qualitative research, it is the quality of the theoretical inferences that is crucial to the assessment of generalisation (Bryman, 2016).

5.6 Ethical considerations

According to the Oxford English Dictionary (oed.com), *ethics* is “a system or set of moral principles, a set of social or personal values”. Ethical considerations are something everyone must deal with. In social research, ethical issues have been gaining increased awareness (Bryman, 2016). A qualitative study is characterised by a close relationship between the researcher and participants. For this reason, it is important to treat the participants with respect. The research starts already with the preparations for data generation and continues until the research report is finished. The researcher must, therefore, make ethical considerations before, during and after the data generation (Postholm, 2005).

Diener and Crandall (1978, in Bryman, 2008) have broken down four main areas regarding ethical issues in social research: whether there is harm to participants, whether there is a lack of informed consent, whether there is an invasion of privacy, and whether deception is involved.

Harm to participants can entail different aspects, such as physical harm, harm to participants’ development, loss of self-esteem, and stress. The researcher must consider the possibility of causing harm to the participants carefully. Data analysis may cause some challenges with respect to the participants. According to Stake (2005, p. 459) “qualitative researchers are guests in the private spaces of the world. Their manners should be good and their code of ethics strict.”

Researchers working with case studies have a great interest in personal viewpoints and conditions. This interest also carries a risk of embarrassment and lost self-esteem to those having their viewpoints and conditions exposed. In my study, it is possible that the school leader’s and the teacher’s goals for mathematics teaching and learning do not coincide, or that the students do not perceive the teacher’s goals for the subject. There is a risk that the results can cast some participants in a negative light. I have discussed this thoroughly with the school leaders and the teachers. I have also specified that my aim is not to evaluate their work but to describe the state of the art and look for examples of good practice

Interviews and observations may also involve information about a third party not directly involved in the research. In the interviews conducted in my study, the school leaders and teachers mention their colleagues and the students

talked about their family members. In the classroom observations, discussions concerning other persons than the ones involved in the research are audible on the video recordings. It is therefore very important to be attentive to situations where a third party is involved. A way to prevent harm to participants and third parties is to make sure that all records remain confidential and that participants remain anonymous. I had to ensure that individuals were not identified or identifiable in the published report.

To secure anonymity, I did not keep written records of the participants' names and names of the schools and places. In the published research, the participants are referred to as "School leader", "Teacher", and "Student", and the schools are labelled "A", "B", and "C". The files with the recorded interviews and lesson observations were also named in this way. The data material was saved on the university college's secure server for research data. A back up of the data was saved on a password-protected external hard drive which was kept in a locked cabinet to prevent unauthorised access to the data. The data will be permanently destroyed by August 1, 2021.

The second issue concerns informed consent. According to Cohen et al. (2011), informed consent contains four elements. First, it means that the participants are competent to decide whether to participate or not. The researcher is responsible for making sure of this. Second, participation in the study must be voluntary. Third, participants must fully understand the situation they are putting themselves in by participating in the study. Fourth, participants must have full information about the study in which they are participating.

Taking part in this research project was entirely voluntary for all participants. I provided information about the aims and procedures, both oral and in writing to the school leaders, teachers, students, and parents. The information letters (included in Appendix B) contain reasons for conducting the research, methods for generating data, time span, and desired outcomes. They make it clear that participation is voluntary, and that all participants will be anonymous and untraceable in the published report. Participants may withdraw from the study at any point, without giving any reason for doing so. The letters also informed about who will have access to the data, how the data will be stored, and when it will be deleted. I also gave a sense of the time and commitment that was expected.

Four schools agreed to participate in the study. One of them is the location for the pilot study, while data from the three other schools are used in this final

report. To establish a good dialogue with the schools, the school leaders and mathematics teachers at the four schools were invited to a meeting where I informed them about the different aspects of the study. These aspects involved clarification of my role as a researcher and the participants' roles and rights. It can be difficult to fully inform participants about the project because qualitative research is often open-ended, and new knowledge and insights can develop during the study (Bryman, 2008). Therefore, I have informed the participants that the project may turn in a different direction than what was originally planned, based on the findings and ensuring their right to withdraw consent at any time.

Although participants were informed about the possibilities of a change or narrowing of focus, I faced an ethical dilemma concerning the level of detail required in the information. My research interest is in mathematical literacy and how the concept is connected to perceptions about teaching and learning in Norwegian schools. Mathematical literacy is not a concept that exists explicitly in the Norwegian curriculum. I want to find out whether school leaders and teachers are familiar with the concept, how they understand it, and how teaching to develop mathematical literacy is conducted. By explicitly revealing my intentions, I would risk that they prepared for interviews and planned their teaching with respect to this, and my results would not be valid. I solved this by giving a more general description of my aims, as looking at concepts in policy documents in general, and how they work with these concepts in schools. I made it clear that I was not trying to assess them or their work, but rather to explore the connections between school leaders' and teachers' rationales, teachers' operationalisations, and students' experiences. Therefore, I do not believe that this caused any harm to the participants.

Another issue I have dealt with is whether participants are able to understand the information they are given about the project. In the group where I did the pilot study, there were two minority language speaking students. They had just arrived in Norway and did not know Norwegian nor English very well. Hence, it is likely that they would not understand what they were participating in. The teacher and I agreed that they would not be part of the participant group.

The participants were recruited by convenience, and I knew some of them on a professional level. However, during a research project, the close relationship between researcher and participant can evolve almost into a friendship. The participant may give confidential statements directed toward the researcher as a friend. At the same time, the researcher is dependent on a relationship of trust

with the participants, in order to ensure openness, honesty, and that they do not withhold important information (Postholm, 2005). Due to my background as a teacher, I found it likely that the teachers might reveal personal thoughts and opinions regarding different issues concerning the school. It was therefore important for me as a researcher to be aware of these issues and to distinguish between information given in confidence and information that helped me answer my research questions.

In addition to the Norwegian National Committees for Research Ethics' General guidelines for research ethics, most higher education organisations have ethics committees that issue guideline about ethical practice (Bryman, 2008). NSD is the Data Protection Official for research and educational institutions in Norway. They assist institutions in fulfilling their statutory duties relating to internal control and quality assurance of their own research. If a researcher will be processing participants' personal data, the research project is subject to notification to NSD. My project is subject to notification, and my plans for generation and processing of data have been approved.

The third issue, invasion of privacy, is concerned with people's right to privacy and that transgression of that right is not acceptable (Bryman, 2016). Although participants have given informed consent, they have not invalidated their right to privacy. For example, I had to be conscious that the participants in my study might perceive some questions as too personal and would not want to answer. This issue also touches upon the issue of confidentiality and harm to participants. The teachers might be concerned that the responses given in interviews could, in some way, be revealed to the school leaders. Also, the students wearing head cameras might worry that recordings of them not paying attention in class would be revealed to the teacher. If participants forgot the fact that their conversations and actions were being recorded, sensitive information, not intended for me, might be disclosed. It was, therefore, very important that the plans for generating and processing data were followed.

Deception is the fourth issue and occurs when researchers present their work as something else than what it is (Bryman, 2016). Deception may involve the deception of research participants, or it may relate to fabricating, falsifying, or withholding research data. In any case, it may cause harm to participants and those intended to benefit from the research in addition to endangering the reputation of social researchers. As discussed above, measures have been taken to avoid causing harm to the participants of this study. In this dissertation, I have

also tried to provide sufficient information about the research process and
conduction of the study to ensure transparency.

This methodology chapter is rather long and comprehensive in order to
inform the reader about the procedures carried out. In this chapter, I have
explained and justified the choice of methodology. I have presented the
participants and the data generated to address the research question. I have also
explained how data was analysed. In the following chapter, I present the three
articles on which this dissertation is based.

6 Presentation of articles

In this chapter, I present the results in the three articles. The three articles are closely connected, and together they aim to describe teaching for mathematical literacy from three different perspectives. More specifically, the chapter concerns school leaders' and teachers' rationales for teaching for mathematical literacy (Article 1), teachers' operationalisation of mathematical literacy (Article 2), and students' encounters with mathematical literacy (Article 3).

6.1 Article 1: Teaching for mathematical literacy: School leaders' and teachers' rationales

The study is rooted in an exploration of the meanings school leaders, and teachers hold about mathematical literacy. Teaching for mathematical literacy is connected to school leaders' and teachers' contradictory rationales for teaching. The rationales are identified as connected to five main categories. The categories are use-value, meaning, teaching practice, teacher competences and knowledge, and universality.

Use-value

The school leaders and teachers have contradictory rationales concerning use-value. They are concerned with the use-value of mathematics. That is, they argue that teaching for mathematical literacy should focus on how to use mathematics in societal, occupational, and personal life. However, it can be challenging to find suitable contexts for teaching use-value, as students have different rationales for learning and different conceptions of mathematics. Also, teaching for mathematical literacy should involve solving practical tasks. However, the school leaders and teachers comment that the curriculum lacks focus on mathematics in everyday life.

Meaning

There are also contradictions regarding meaning. Mathematics can be seen as a language, and mathematical literacy was connected to the competence to use and understand mathematical language and concepts. In other words, communication was seen as an important element of mathematical literacy and an important part of the learning process. However, the school leaders and teachers also

commented that there is not enough focus on reasoning and reflecting and that we think students understand concepts to a greater extent than they do.

Teaching practice

School leaders and teachers have contradictory rationales concerning teaching practice. On the one hand, teachers must teach according to the curriculum. The school leaders and teachers comment that there are strong connections between the textbook and the curriculum competence goals. However, this connection leads to a heavy reliance on textbooks in teaching, which they do not consider as the best way to teach for mathematical literacy. Also, the curriculum's basic skills are connected to mathematical literacy and interdisciplinary work. However, the school leaders and teachers report that it is difficult to implement the basic skills and interdisciplinary work in their teaching in a natural and meaningful way.

Teacher competences and knowledge

Teacher competences and knowledge was also an area of contradictions. In Norway, there is an increasing focus on teachers' subject knowledge. On the one hand, the school leaders were concerned that specialised subject teachers come at the expense of effective student-teacher relations. Also, the high mathematics admission requirements in teacher education may exclude potentially good teachers.

The teachers were concerned that specialised subject teachers would make interdisciplinary work more challenging. Here, the contradiction is between valued knowledge in teacher education and teaching practice. Also, there is a contradiction among teachers regarding their own competence and their goals for teaching.

Universality

Mathematical literacy is conceived as comprehensive and wide, just like the curriculum. Everything is part of a big whole.

To sum up, the school leaders and teachers comment on several contradictions regarding teaching for mathematical literacy. However, the universality category suggests that teaching for mathematical literacy is a goal for teaching. The findings indicate that mathematical literacy is both difficult to

understand and teach in a way that is consistent with curriculum goals, policy expectations, their own convictions, and students' requests.

6.2 Article 2: Secondary teachers' operationalisation of mathematical literacy

This article reports a qualitative study of teachers' operationalisation of mathematical literacy. A model representing the multifaceted nature of mathematical literacy (see also Chapter 2) is used to analyse video recordings of mathematics teaching in three grade 9 classes. The observed lessons concern the topics equations and percentages. The results are organised according to the five elements in the mathematical literacy model; *mathematical knowledge, contexts, dispositions, tools, and critical orientation*.

Mathematical knowledge

In terms of mathematical knowledge, teachers prioritise developing students' procedural fluency. Classroom activities mostly concern practising procedures and methods for solving equations. The students are rarely asked to explain or justify their methods and solutions. The focus is on how to solve the tasks and to get the correct answers. There are few discussions of alternative solution methods. The teachers do however try to draw on students' previous mathematical knowledge, for example, that adding number fractions and adding fractions with unknowns is in nature the same.

Contexts

Most of the activities and tasks in the observed lessons do not contain any contexts. They are used to practise procedures and skills. That is, they focus on mathematical knowledge. There are, however, some traditional word problems that involve contexts connected to personal and social life. There are few examples of contexts concerned with citizenship or political, scientific, technological or occupational issues. Some contexts are (potentially) concerned with issues related to citizenship, such as a decrease in the number of libraries and birth rates. The teachers focus on task authenticity and certification to a limited extent. Even though the task contexts may stem from real life, the questions posed in the tasks or methods used to solve the tasks are not questions one would pose or methods one would use when faced with the problem in a

context outside school. Also, contexts are sometimes used to help students understand how to perform calculations that are not originally set in a context.

Dispositions

To help students develop positive dispositions, the teachers rely heavily on communication. They talk to the students about how they are doing, praise them, and try to encourage and motivate them. At the end of a conversation with the students, they often say “good” or “well done”.

As mentioned earlier, the teachers spend much time demonstrating tasks on the chalkboard, and students spend much time practising methods. Procedural fluency may contribute to developing students’ confidence in mathematics. Hence, demonstrating tasks and practising methods to develop procedural fluency is also a way to develop positive dispositions. Also, by engaging students in demonstrating how to solve the tasks, the teachers expose the students to the risk of demonstrating an incorrect solution. Handling such risks can also be connected to having positive dispositions.

Tools

The teachers use different representations as mediating tools. For example, communication is an important tool in the teachers’ lessons. The teachers talk a lot, explaining concepts and demonstrating procedures. In this way, language serves as a tool to mediate mathematical knowledge. To model the solutions of geometry word problems, the teachers draw geometric figures on the chalkboard. The drawings serve as representational tools to mediate thinking in order to represent the situation with symbols and to solve the equation. A number line and mathematical symbols are other representational tools used. However, there is no observed use of digital or physical tools.

Critical orientation

The lessons do not involve tasks or activities where mathematical information is used to make decisions and judgements, add support to arguments, or challenge arguments. One teacher talks about how mathematics is used to make decisions. However, there are no observations where students are asked to use mathematical information to make decisions and judgements, add support to arguments, and challenge an argument or position themselves. Hence, critical orientation is less obvious in these lessons.

In the remaining part of the article, I discuss these results and make concluding remarks. Analysis indicates that operationalisation of mathematical literacy appears to be fragmented and teaching is focused towards developing procedural fluency. Even though all elements in the mathematical literacy model can be identified to some extent, they are not connected. Students may develop competencies connected to all five elements. However, they are left to make the connections between the elements on their own. As a result, they have to develop the competence to use mathematics in real-world situations on their own. It seems like the focus is on developing mathematical knowledge, and that the other elements are just instruments to achieve this. Mathematical literacy was introduced in the Norwegian curriculum in 2006 and is considered a basic skill which should be developed across subjects. However, it appears that teachers still struggle to implement teaching to develop this competence.

6.3 Article 3: Lower secondary students' encounters with mathematical literacy

In this article, students' encounters with mathematical literacy are investigated. The study is framed within the TO. The elements of mathematical literacy are used to analyse recordings of mathematics lessons and interviews with students.

Analysis of the lessons shows that although some tasks involve word problems in real-life contexts, students' encounters with mathematical literacy are limited. The word problems are word problems in the traditional sense, aimed at providing variation, motivation, and practising technical skills. There is an emphasis on conceptual understanding and procedural fluency in the sense that students spend most of the time practising the procedures. Also, the task contexts involve inauthentic elements and therefore, have limited possibilities of showing how mathematics is used in the real world. However, on a couple of occasions, one teacher provides certifications.

Tool use involves calculators to perform calculations. On a few occasions, students are encouraged to make drawings, and one teacher frequently emphasises that students should discuss methods and strategies. In this way, language serves as a tool for thinking. The tasks do not invite students to be creative or inquire, and the students display varying emotions and engagement in the tasks. There is no collective focus on critical orientation in terms of engaging critical discussions, justifications, and evaluations of methods, solutions, and concepts, and the contexts in which they are used. Although methods are the

topic of whole-class and peer-group talk, it is, to a large extent, up to the individual to make the critical judgements in his/her own mind.

Analysis of the interviews suggests that students have encountered mathematical literacy. They describe situations involving mathematics from personal and social life and work-life. For example, they see mathematics as relevant in terms of shopping, cooking, and redecorating the house. In this sense, they see mathematics as useful and relevant in life outside school. However, their encounters are connected to specific mathematical topics in specific contexts from personal and work-life in terms of short-term utility and performing basic procedures aimed at producing a specific number. The contexts do not involve citizenship and societal issues. The lack of contexts from citizenship suggests that the students have not encountered mathematical literacy in such contexts.

The students believe that mathematics is useful and, as such, hold positive dispositions. However, they do not comment on how mathematics is used to form an argument or justify a position. Students have a narrow view of mathematics as numbers, calculations (the four arithmetic operations), and a way to find solutions. A few students relate these solutions to problems in everyday life, such as shopping and cooking. Mathematics is related to practising procedures and performing calculations, and not as a way to make sense of the world.

In this study, encounters of mathematical literacy are characterised by an emphasis on developing mathematical knowledge. In our culture, mathematics is valued as important for the individual and society, but how it is important is not evident through the classroom activity. I argue that TO can provide a useful perspective in terms of understanding and developing mathematical literacy. This perspective should be further explored.

7 General discussion and conclusions

This research set out to investigate the nature of teaching and learning for mathematical literacy in Norwegian lower secondary schools. In Chapter 2, I presented a variety of notions related to the overarching concept of mathematical literacy. Some of these notions have similar meanings, while some appear to have contrasting meanings. However, these notions share a common ground in their emphasis on an awareness of and competence to use mathematics to deal with issues in personal, occupational, and societal life. The background for this study of mathematical literacy is an increasing role that number, quantity, measures and numerical comparison plays in society. Another reason for the study is the important function of school in equipping students with the competence necessary to deal with mathematical issues in the variety of situations encountered by adults in everyday life. The research was framed within cultural-historical activity theory (CHAT) and the theory of objectification (TO). Qualitative methods were used within a naturalistic research design, and the main sources of data were interviews and video observations. In the previous chapter, the results of the three studies were presented. The results indicate that school leaders and teachers aim at developing students' mathematical literacy. However, they experience challenges in the implementation of actions that will achieve their goals. These challenges are evident in the teaching, and also influence how students experience mathematical literacy and their perceptions of how mathematics is used in the everyday lives of people outside school.

In this concluding chapter, I discuss the findings reported in the three articles and the project as a whole. The discussion is structured around the three studies. In the following section, Section 7.1, I focus on the school leaders' and teachers' rationales for teaching for mathematical literacy. Section 7.2 considers the teachers' operationalisation of mathematical literacy. In section 7.3, I discuss the students' encounters with mathematical literacy. The main research question is addressed in Section 7.4 in terms of teaching and learning for mathematical literacy, and conclusions are drawn. In Section 7.5, I reflect critically on my approach and consider the implications of the research findings for practice and further research. The dissertation closes with a brief conclusion in Section 7.6.

7.1 Rationales for teaching for mathematical literacy

Study 1 focuses on school leaders' and teachers' rationales for teaching for mathematical literacy. It was based upon interviews with school leaders and

mathematics teachers in three schools where they discuss the definition of mathematical literacy and their interpretation of it, how it relates to the Norwegian curriculum, and how to teach for mathematics literacy. The study showed that the school leaders and teachers have contradictory rationales for teaching for mathematical literacy. They recognise the value placed upon mathematical literacy by the Norwegian curriculum, and they see mathematical literacy as a desired outcome and a motive of schooling. However, the specific content, actions and goals of the teaching and learning Activity are sources of challenges. The findings concern issues related to what they perceive teaching for mathematical literacy should involve, and the reasons why teaching does not involve these issues. Hence, the main findings are about the challenges in terms of teaching for mathematical literacy.

The first challenge is related to contexts. The importance of students' encounters with mathematics in real-world contexts is emphasised in respect of its use-value. The contextual aspect of mathematical literacy is also emphasised by the teachers in Genc and Erbas (2019) in terms of possessing mathematical knowledge and skills. Depending on its usage, such knowledge can be basic level knowledge necessary to meet the general demands of everyday life, or more advanced level knowledge needed for scientific or technical developments. In this way, their findings are also related to use-value. However, in Study 1, the school leaders and teachers report that providing students with encounters that show the use-value of mathematics can be challenging. For example, as suggested by the school leaders and teachers as well, it is difficult to predict what contexts the students will engage in and consider meaningful (Nicol & Crespo, 2005; Rellensmann & Schukajlow, 2017). In Rellensmann and Schukajlow (2017), it was expected that the students would be more interested in the tasks with real-life contexts. However, this was not the case. The same study also showed that the pre-service teachers that participated in the research were not able to predict the students' interest in such problems.

Unlike in the previously mentioned study, the school leaders and teachers in my research are experienced teachers who know their students well. Nevertheless, they report that students' various interests and backgrounds make it challenging (if not impossible) to use contexts that are deemed meaningful by all the students all the time. This challenge is exemplified by one of the school leaders, stating that the students may learn how to calculate the area of something, as length times breadth, but may not have had real-life experiences

that support the understanding of the area concept (see Article 1 in Appendix C). Another school leader commented that students should experience the usefulness of mathematics themselves and that it is not enough just to tell them that it is or will be useful. Hence, the question is; how can students be provided with experiences of the usefulness of mathematics in real-life when it is not possible to be sure which contexts they engage in or how their lives will look like in ten or twenty years. The context alone is, it seems, insufficient to demonstrate the use-value of mathematics and to develop mathematical literacy. This leads to another reported challenge; one related to teaching practice.

Again, from the results in Study 1, a teaching practice supporting mathematical literacy development should not rely too much on the textbook. This view can be supported by similar results from the study by Gatabi et al. (2012). They found that the analysed textbooks were limited in providing problems where students have to formulate and interpret. Formulating and interpreting are parts of the modelling process, which is a key process involved in mathematical literacy (Stacey & Turner, 2015). However, the strong connection between the textbooks and the curriculum makes it challenging for teachers to put the textbook aside. The Norwegian curriculum, as several other national curricula, does not provide any specific guidance about how to design tasks and learning sequences or how to make decisions about pedagogies that support mathematical literacy learning (Liljedahl, 2015). Therefore, the textbook is the closest thing teachers get to such guidance. They do not recognise following the textbook to be the best way to teach for mathematical literacy, but they comment that they still struggle to put it away. The reason for this may be the lack of a better alternative.

On the other hand, the teachers and school leaders mention that cross-curricular work is a way to teach for mathematical literacy. The cross-curricular aspect of mathematical literacy in the Norwegian curriculum has been elaborated in Chapter 2. Also, several research studies, reported in Chapter 4, emphasise the benefits of teaching mathematical literacy across the curriculum. Steen (2001), for example, suggests that a cross-curricular approach to mathematical literacy has greater potential to empower students to meet the mathematical demands of modern life than approaches that seek to develop mathematical literacy solely through mathematics subjects. Connecting mathematics to other curriculum subjects may be a way of providing meaningful contexts for mathematics

learning because of the possibility for students to learn about both mathematics and the context.

The school leaders and teachers interviewed for this study report that teaching across the curriculum needs to be addressed in teacher education. As Norwegian teacher education is becoming more specialised, newly qualified teachers are qualified to teach fewer subjects than their more experienced colleagues. This specialisation makes a cross-curricular approach more challenging, as one has fewer subjects to draw on. To be able to teach mathematics in contexts which highlight the use-value of mathematics, teachers need to see connections between mathematics and other subjects (Popovic & Lederman, 2015). With only a few subjects to draw upon, the teacher must, to a greater extent, rely on his/her informal knowledge and personal experiences in order to connect mathematics with different contexts. This supports the argument for including mathematical literacy courses and a cross-curricular approach to teaching in Norwegian teacher education, as is done, for example, in Australia (Forgasz et al., 2017).

The school leaders and teachers have encountered mathematical literacy at the level of potentiality. They articulate that mathematical literacy is a motive of the Activity of schooling. They are concerned with the goals of this Activity in terms of use-value of mathematics. When it comes to the tasks of Activity, the school leaders and teachers have a general pedagogical opinion on the type of problems that should be posed. They mention problem-solving tasks and practical activities. However, the school leaders and teachers do not provide specific examples of problems that could be used in teaching for mathematical literacy or more detailed characteristics of such tasks.

In this section, I have discussed challenges perceived by the school leaders and teachers related to teaching for mathematical literacy, as identified in Study 1. In the following section, I discuss the findings from Study 2, which concern the teachers' operationalisation of mathematical literacy in the classroom.

7.2 Operationalisation of mathematical literacy

Study 2 is concerned to explore teachers' operationalisation of mathematical literacy. In terms of TO, the study focuses on the actualisation of teaching for mathematical literacy. That is the concrete and noticeable embodied, symbolic, and discursive actions involved in teaching. It is based on lesson observations of teachers in their classrooms. The model of mathematical literacy (see Figure 5)

was used to investigate how teaching was organised around the five elements (contexts, mathematical knowledge, tools, dispositions, and critical orientation).

In the observed lessons, few of the tasks assigned to students involved contexts. This is particularly the case with the tasks focusing on equations. The tasks focusing on per cent do, for the most part, involve contexts. However, regardless of the mathematical topic, the tasks and contexts have similar characteristics. The tasks are traditional textbook problems aiming at providing students with exercises to practise specific procedures. Although the students are intended to translate a word problem into mathematical symbols, the problems are already mathematised. They are limited in the extent to which they require students to formulate and evaluate. In this way, the tasks in the observed lessons are similar to the textbook problems described by Gatabi et al. (2012). The task contexts are related to personal life and work-life, but they are not discussed in terms of authenticity and the real-world relevance of the methods used to solve the contextualised problems. Hence, real-world aspects of the contexts are not discussed.

As the observed lessons are mathematics lessons, developing mathematical knowledge is the main focus. Much time is spent on developing the students' procedural fluency in terms of solving equations and performing calculations involving per cent in terms of practising routine procedures. The teacher and students demonstrate the procedures and solutions on the chalkboard. However, they do not discuss these procedures and solutions; they only accept or reject the answers. In this way, the teaching in the observed lessons is consistent with the finding by Gainsburg (2008), who suggests that teachers' main focus is to impart mathematical concepts and skills.

The model by Goos et al. (2014) highlights tool use in terms of physical tools, digital tools, and representations. The observed lessons do not involve much use of such tools. There are a few examples where the teachers emphasise how making drawings can help organise the information in the word problems, and one teacher draws a number line to illustrate what the solution to an inequality means. Also, the students can use calculators when solving the tasks. The mathematical literacy model does not explicitly emphasise the importance of communication as a tool. However, from a CHAT perspective, speech is regarded as an important tool in planning solutions and solving tasks before actually executing them (Vygotsky, 1978). Communication is evident in the observed lessons. The teachers and students spend much time talking. Therefore,

as the teachers demonstrate task solutions and explain how they think, they create encounters with ways of thinking and doing. Teacher B, to a greater extent than the other two teachers, explicitly emphasises to the students the importance of oral communication. He constantly tells them to discuss and talk to each other about their strategies and solutions. They are also required to talk to themselves. In this way, he emphasises both the intrapersonal and interpersonal function of speech.

The teachers try to foster students' positive dispositions through the use of praise and positive feedback. It is difficult to make claims about whether the lessons foster students' curiosity and interest. However, the focus on practising methods and developing procedural fluency may contribute to students' confidence and feeling of mastery. Also, contexts are often considered as affecting students' dispositions in terms of engagement and motivation (Boaler, 1993; Gainsburg, 2008; Lee, 2012). In Gainsburg (2008), motivating students and helping them understand mathematical concepts were more often mentioned as reasons for making real-life connections than helping students see how mathematics is used in the world or their lives. In this sense, contexts can serve the goal of developing students' positive dispositions. The observed lessons do not provide explicit evidence of this. However, there are examples of the teachers trying to help students see how mathematics is used in the real world. Teacher C's talk about Black Friday sales is one example where she tries to relate to students' interests. It is, therefore, possible that the teachers also see real-world contexts as a way to develop positive dispositions.

The critical orientation element is an analytical and evaluative demand embedded in all the other elements. It involves critically evaluating and discussing the contexts, the mathematical knowledge, and the tools. There are very few examples of this in the lessons. Hence, it seems that critical orientation is the most challenging element to address. This finding is consistent with Geiger, Forgasz, et al. (2015), who report that teachers struggled with this, even after two years of engagement with the idea. Sikko and Grimeland (2020) argue that a classroom culture that values questions, inquiry, and where errors are seen as a prerequisite for learning, is important for developing a critical orientation. The observed lessons did not involve inquiry and the questions asked were posed by the teacher and usually had one correct answer, either a number or a method.

On the other hand, the focus on the students' participation in demonstrating the task solutions may suggest that the teachers aim at developing

such a culture. Sikko and Grimeland (2020) claim that to understand what a solution means, under which circumstances it can be found, and to see that a change in the circumstances can lead to other solutions are important for developing a critical orientation. Such issues were not emphasised in the lessons observed.

Study 2 indicates that teachers' operationalisation of mathematical literacy, at least in the lessons observed, is fragmented. Although the five elements of mathematical literacy can be found in the lessons, the challenge seems to be to integrate them coherently in the lessons. They are there, but they are not working together to provide students with a coherent experience of mathematical literacy. Hence, the overriding impression I have formed from the observations and analysis of the data, is that in the observed lessons there is little evidence of coherence between the elements of the mathematical literacy model that is proposed by Goos. It is difficult to find explicit evidence for either arguing a strong case for coherence or lack of coherence because the analysis has led me to focus on the elements rather than on their coherence. However, the analysis has not drawn attention to anything that may contribute to the coherence of the elements. Therefore, I make no stronger claim than that I have not been able to expose evidence of the coherence between the elements in the observed lessons.

7.3 Encounters with mathematical literacy

Study 3 is concerned to explore and expose students' encounters with mathematical literacy. Encounters refer to the TO, where encounters are social and collective processes of becoming conscious of cultural and historical systems of thought and action (Radford, 2013). The study is based on observations of the same lessons as Study 2 but analyses these data from the students' perspective. Also, the study is based on interviews with students in the three classes. The interviews involve questions about the students' hobbies and interests and whether they need mathematics in everyday life. Further, the questions seek to explore students' knowledge of their parents' need for mathematics in their personal life or work-life and whether there are occupations where there are no mathematical requirements (see also Appendix A). The mathematical literacy model was central in this study as well.

The lesson observations are the same as for Study 2. However, it is possible to get some more insight into students' dispositions. The students' head-mounted cameras show varying degrees of interest. Some students engage in the

tasks throughout the lessons, and others spend most of the lesson talking to their classmates about other things. Also, related to critical orientation, the students do not discuss methods or contexts in terms of justifying, evaluating, or validating. They are just describing what they have done and the answers they got.

In the interviews, the students give several examples of situations and occupations where mathematics is or can be used. The students acknowledge different contexts in which mathematics can be used in the real world. In this way, one can argue that they might also hold positive dispositions in that they see that knowing mathematics can be useful and worthwhile.

As in Nosrati and Andrews (2017), a utilitarian connection between mathematics and the real world was seen, but the students did not provide any examples concerning citizenship. The students may have interpreted everyday life as personal life, and hence answered accordingly. Some students stated that mathematics is something you need in life. The notion of “life” might involve citizenship, but when I asked them to give examples, the contexts concerned personal and social life. Hence, the students may have greater difficulty to relate mathematics to societal contexts. One reason may be that the observed lessons lacked activities that focus on mathematics in the context of citizenship. Hence, issues of citizenship may not be something that the students have encountered through school mathematics or at home.

The contexts and the mathematical knowledge that the students mention are very specific and closely connected. The contexts are basic everyday activities that they themselves have experienced, such as shopping or cooking. The mathematical topics involve calculations, per cent, and units of measurements. The situations students connect to the use of mathematics in the real world and involve typical everyday contexts, similar to contexts commonly used in traditional word problems. They involve only the type of mathematics that is explicit and visible, and not, for instance, the use of mathematics in newspapers or advertisements. They describe that shop assistants need mathematics to calculate the sum that the customer has to pay and to calculate the amount of change to give back. However, the mathematical knowledge needed by shop assistants is generally low because machines calculate the sum of the cost of goods, how much change the customer should get back (if the customer pays in cash), and also delivers the correct amount. Hence, the shop assistant does not even need to know how to count. As computers perform most of the mathematics done in society, students may be left with the impression that

only a few people do mathematics. This impression creates a tension between what goes on in society and what goes on in schools (Gravemeijer, Stephan, Julie, Lin, & Ohtani, 2017).

The students' encounters with mathematical literacy are confined to such word-problem contexts. They do believe that mathematics is important and that they need it somehow, but they are not sure what they need it for. It is not clear whether the students' perception of the usefulness of mathematics comes from their own experiences and encounters or the value placed upon mathematics by society.

Several students answer that they see mathematics as a school subject and that they do mathematics in their everyday life when they are doing homework. They also say that they may use it without being aware of it, unconsciously, for instance, by estimating when to leave their house in order to get where they need to be in time. This unconscious use of mathematical Activity can be related to the operations level in CHAT. Everyday Activity involves cultural and social norms of participation, relatively routine sets of activity, and material tools. The norms, routines, and tool use change over time in relation to the change in the individuals who participate in the Activity and their responses to new challenges. Contrary to mathematics reasoning in out-of-school contexts, learning mathematics in school is scheduled for specific times in the school day (Nicol, 2002). In this way, there appears to be a boundary between school and everyday life in terms of mathematics that goes beyond elementary topics and operations (Venkat & Winter, 2015). Hence, the students in this research seem to view mathematics predominantly as a school activity (Hunter et al., 1993).

7.4 Teaching and learning for mathematical literacy

The research reported in this dissertation set out to investigate teaching and learning for mathematical literacy. The main research question guiding the research was

What is the nature of teaching and learning for mathematical literacy in lower secondary schools in Norway?

As mentioned in Section 5.3, the schools participating in this research are located in a county in which students regularly perform well above average on the nationally conducted mathematical literacy test. Therefore, best-practice

examples of teaching for mathematical literacy was anticipated. Also, it was believed that by investigating the current state of affairs, valuable insight into teaching and learning for mathematical literacy could be gained. The research reveals that teaching and learning for mathematical literacy in the three schools are affected by challenges at the different levels of the schools' structures. These challenges are related to the three curriculum levels described by van den Akker (2010); the intended, the implemented, and the attained.

The intended curriculum is the formal, written curriculum document. In terms of TO, the intended curriculum involves potentiality. The description of mathematical literacy in the curriculum is an idealisation of what mathematical literacy can potentially be and achieve. Challenges involved with mathematical literacy in the intended curriculum in Norway have already been discussed in Chapter 2. These involve conflicting historically and culturally embedded meanings and interpretations of the concept "mathematical literacy", and a lack of guidelines regarding how to work to develop students' mathematical literacy in terms of tasks or methods. In this way, the teacher is on his/her own when it comes to identifying mathematical literacy opportunities in the intended curriculum.

Goos et al. (2010) and Goos et al. (2012) evaluated Australian curriculum documents in terms of identifying mathematical literacy opportunities. In a similar way as with the Australian curriculum, the Norwegian online version of the new curriculum (LK20) contain filters that serve to exemplify subject-specific competence goals that are linked to the basic skills and the core elements. These may be helpful to the teachers when planning their lessons, presupposing that they have strategies for planning and implementing mathematical literacy. If they do not have such strategies, the filters and connections are of little use, and mathematical literacy is at risk of remaining no more than an ideal. Hence, it is up to the teacher to identify relevant contexts and tools and to plan to develop positive dispositions and critical orientation. How the teacher deals with these challenges affects the level of the implemented and attained curriculum.

The implemented curriculum relates to the world of schools and teachers (van den Akker, 2010). This level involves two sublevels. One is the perceived curriculum, which concerns the curriculum as interpreted by its users, for example, school leaders and teachers. The second level is the operational level. This level involves the actual teaching and learning. The two levels of the

implemented curriculum are closely related. In terms of TO, the implemented curriculum concerns the move from potentiality to actuality through the mediating Activity and the pedagogical intent of the classroom Activity in terms of goals, actions, and tasks. This level influences the outcome of teaching in terms of the attained curriculum. Hence, the attained curriculum involves the experiential level, how the learners perceive the teaching and the learning. In terms of TO, this level relates to knowing. It concerns the concrete conceptual content through which knowledge is instantiated.

The emphasis in policy documents and curriculum does not necessarily equip teachers with adequate conceptions of mathematical literacy (Bennison, 2015). Understanding teachers' conceptions of mathematical literacy provide insights into why teachers make particular instructional decisions regarding mathematical literacy (Goos et al., 2014). These conceptions also concern challenges involved in teaching for mathematical literacy.

As Study 1 has shown, the teachers perceive and acknowledge the importance of developing students' mathematical literacy and they have an understanding of how this can be done. For instance, they mention that teaching should emphasise use-value, meaning and reasoning, and involve practical tasks rather than a reliance on the textbook. However, their operationalisation in the observed lessons does not match this understanding. Teaching does not correspond with the way the teachers perceive that teaching for mathematical literacy should be. Teachers rely heavily on the textbook, and the tasks therein are procedural, they involve few contexts, do not involve inquiry and mathematising, and they are not practical. In Study 1, the teachers emphasise the importance of meaning and reasoning, and they try to include this in the lessons. However, the questions they ask are closed, requiring only numerical or procedural answers. As a result, students' attainment of mathematical literacy seems to be limited. The situations they describe are similar to the tasks in the lessons. The students seem to have an understanding that mathematics is something they do in school, and the tasks in the lessons seem to support or be the reason for this view.

Hence, the research reported here shows that the challenges experienced by the teachers in terms of transforming the intended curriculum into the operational teaching for mathematical literacy have an impact upon the students' attainment. These results suggest that the teachers do not have a clear understanding of what mathematical literacy is and how it is developed, which is

related to the perceived and operational curriculum and is a result of the challenges of the intended curriculum. Although they have a notion of what mathematical literacy is, the teachers do not seem to have developed strategies for implementing it. At least, they did not reveal such strategies, and the contradiction between their understanding of teaching for mathematical literacy and their actual teaching supports this claim. According to Liljedahl (2015), designing mathematical literacy tasks is crucial to understanding what mathematical literacy is. By designing such tasks, the teachers in his study changed their teaching practice.

It is not clear whether the challenges reported by the school leaders and teachers are reasons for not engaging in designing mathematical literacy tasks. Difficulty in relating mathematics to meaningful contexts and lack of knowledge of how to teach across the curriculum may cause the teachers to settle for the textbook. The close connection between the curriculum and the textbooks helps to ensure that they are teaching the students what is intended to be learned.

Another issue worth mentioning is related to professional development, such as the studies discussed in Chapter 4. Such professional development programmes seem to have a positive influence on the teachers' understanding of mathematical literacy and teaching (Goos et al., 2011). Through these professional development programmes, teachers are engaged in the Activity with the motive of developing their competence to teach for mathematical literacy. They work together in order to become conscious of the cultural and historical ways of thinking and doing. In the study by Liljedahl (2015), the participants were also engaged in discussions, sharing of experiences, and revising the tasks. Changing practice is challenging, and the social interaction involved in the studies mentioned above, both in terms of working with other teachers and working with the students, seems paramount. It may be difficult to find the time and energy to engage in such collegial discussions, and, therefore, the challenges involved in developing mathematical literacy tasks may seem too big to handle on top of all other issues the teachers have to deal with.

Therefore, the students' encounters with mathematical literacy are confined to the contexts in textbook problems and the teachers' descriptions of their encounters with mathematical literacy. Their experiences of mathematics in everyday life involve specific topics and situations where the mathematical concepts and procedures are explicit and visible. As the teaching does not emphasise the invisible uses of mathematics in the world, the role mathematics

plays in education for citizenship is not emphasised in terms of positive dispositions, tool use, and critical orientation.

The framing of this research within CHAT and TO has allowed me to gain insight into the cultural and historical ways of thinking about mathematical literacy in the three schools. Teaching and learning for mathematical literacy require that one is aware of cultural norms that are part of mathematical activities. From a cultural-historical perspective, mathematical literacy must be learnt. The teachers and students have to become aware of the cultural and historical ways of acting and thinking in terms of mathematical literacy. In terms of TO, previous research is often focused on the students' learning and processes of objectification and subjectification. However, the teacher also engages in processes of objectification in the sense that s/he engages with different student groups and different curriculum documents, in addition to colleagues and parents. Both teachers and students bring with them their encounters with culture, history, and society into the classrooms in their joint Activity. Hence, the teacher is also in a constant process of learning. TO sees the classroom as producing subjectivities. These subjectivities involve both teachers and students. In this way, TO has broadened my perspective of teaching and the teachers' learning.

Also, TO challenges the idea that best teaching practices are only about the mathematical content. Pedagogical understanding has to move beyond the traditional interpretation of learning as the reproduction of known procedures to solve familiar problems, and beyond the constructivist view that it is the student who constructs her or his own knowledge (Radford, Miranda, & Lacroix, 2018). According to Radford et al. (2018), teaching practices have to include the dimension of the student as a social being in the making. The study of Geiger et al. (2014), where students were working in groups and making their own investigations, supports this claim. The teachers' role in TO is to engage with the students and try to challenge them to move their strategies further or to suggest new paths. In this sense, the teacher is not merely assisting the students. Through students' and teachers' joint labour, knowledge is produced in the sense that it is brought forward. I believe that this conceptualisation brings a new dimension to what the nature of teaching and learning for mathematical literacy can be.

As Yasukawa et al. (2018) point out, a researcher interested in mathematical literacy 'sees' mathematical literacy as the motive of the Activity system. However, for the members of the Activity, the motive is rarely, if ever, that. For them, mathematical literacy is not visible, but useable in producing the

outputs of the task at hand. Even though mathematical literacy is a motive of the school Activity system, it may not be the motive with the highest priority. Besides, several actions and goals can be directed toward the same motive (Leont'ev, 1978). If developing students' mathematical literacy is a motive, there is more than one goal that relates to that motive. There may be several goals and actions related to the different mathematical literacy elements that satisfy the motive of developing mathematical literacy. This contributes to the complexity of the nature of teaching for mathematical literacy.

From the above discussion, some assertions about the nature of teaching and learning for mathematical literacy in the three schools can be made. The teaching and learning for mathematical literacy in the participating schools could be improved. As also noted by Yasukawa et al. (2018), there has been progress in terms of how mathematical literacy is conceptualised with respect to historical interpretations of the concept. These interpretations are outlined in Chapter 2. However, how mathematical literacy teaching and learning is enacted reflects and reinforces narrow conceptions of what constitutes mathematical literacy.

The teachers need a strategy for how to implement teaching for mathematical literacy. They have a notion of what such teaching should look like, but they need something to help them organise and operationalise their teaching. It is in this operationalisation that the mathematical literacy model has proven useful in previous studies. However, an important part of these studies involves social interaction with others in terms of discussing, testing, and revising mathematical literacy tasks. Hence, teachers need to encounter how teaching for mathematical literacy can be operationalised.

The lack of guidance in the curriculum regarding how to teach for mathematical literacy contributes to the dominant use of the textbook. Extensive use of textbooks confines the understanding of what mathematical literacy is and how it can be developed because the development of such understanding is supported by designing, implementing, and revising mathematical literacy tasks. By relying on the textbook, the teachers are transmitting someone else's understanding of mathematical literacy.

From the emphasis on real-life situations in the mathematical literacy definition and the placing of contexts at the centre of the mathematical literacy model, it is easy to think that mathematical literacy is just about mathematics in context. However, the mathematical literacy model tells us that mathematical literacy involves much more than having appropriate contexts for mathematics.

Even in the observed lessons involving abstract equation solving, there were opportunities to engage in the mathematical tasks in ways that are aligned to an education focused on mathematical literacy. Such opportunities could be taken, for example, by focusing on developing positive dispositions, the use of different tools, and developing a critical orientation toward the answers. There is much work to do in mathematics classrooms that is abstract and difficult to relate to contexts. However, that does not mean that one can ignore the need to educate for mathematical literacy because it involves much more than contexts. Also, because it involves more than just contexts, developing mathematical literacy is not confined to working with contexts. It can be supported by working with abstract mathematics as well. For instance, a critical orientation should be embedded in all the mathematical literacy elements. Emphasising critical discussions of mathematical procedures and tools involved in more abstract mathematical topics also supports the development of mathematical literacy in terms of developing certain dispositions and ways to engage with and think about numbers, whether in contexts or not.

7.5 Limitations and implications for practice and further research

Qualitative research can be subject to various criticisms, such as being too subjective, difficult to replicate, and lacking generalisability and transparency (Bryman, 2016). In this section, I will engage in some critical reflection of the limitations and implications of the research reported in this dissertation.

First, I claim that the research reported here has its strength in that it addresses school leaders, teachers, and students. It provides an encompassing hierarchical insight into an important issue of the curriculum. I do not just address what is happening in the classroom but look beyond single classroom scenes or isolated student's thinking, reasoning, and solving mathematics problems, and seeing those as removed from what the teachers' goals are and what the classroom culture is. Rather, I investigate education for mathematical literacy in a cross-section, which addresses Goos et al.'s (2011) request for research on mathematical literacy at the whole school level.

Second, I see the teaching and learning for mathematical literacy from a naturalistic perspective. I have asked the participants to wear head cameras and to participate in interviews. However, I have not, in any way, interfered with the implementation of the curriculum. Hence, I am seeing the school leaders, teachers, and students, as far as possible in their regular, routine practice.

Third, although the schools are a convenience sample, they are public schools using a textbook series that is commonly used throughout Norway. They are situated within a county, with its particular culture, and the schools' results are good in terms of the national mathematical literacy tests. I am not claiming the representativeness of the results. However, the schools do have a special characteristic in terms of the above-average results on the mathematical literacy tests. Therefore, I expected to see evidence of better practice in these schools. Also, I have engaged in the schools in a prolonged fashion. I have spent time in the schools, not only for the specific times of data generation, but also in meetings and written correspondences with the school leaders, teachers, students, and parents. Hence, the research is based on more than just a short visit. This gives me confidence and support in saying that the fact that mathematical literacy is not emphasised to any great extent in these schools is something that should be taken notice of.

Fourth, I am using and applying a mathematical literacy framework that has been developed and used by other researchers in other countries in significant projects investigating mathematical literacy. A potential weakness of the research concerns the influence of my subjectivity as the sole researcher involved in the project. However, I believe that this is mitigated by the use of the mathematical literacy model. The research is not based only on my subjectivities regarding what mathematical literacy is and how to teach for it. Furthermore, I have spent substantial time in the research community of the person who developed the mathematical literacy model. Therefore, I have had an important personal experience of this model that I use as a lens to look at the data I generated. Also, I have used TO, which provides me with a theoretical lens to observe with greater objectivity because it has been developed and used by researchers other than me.

Although there are potential weaknesses, the above assertions point to the overall strengths of the research and contribute to the trustworthiness of the messages that I am bringing to the community through my research. Other issues regarding trustworthiness are previously discussed in Chapter 5.

As also discussed in Chapter 5, educational research should be ethical research. As described there, ethical considerations have been made throughout the research to ensure informed consent and anonymity and to prevent harm to the participants. However, Hostetler (2005) argues that good education research is not only a methodological question or a matter of sound procedures. Ethical

research also means that the research is done so that good may come from the research in terms of beneficial aims and results. In this sense, this research is ethical because I am drawing attention to an important issue within the Norwegian mathematics curriculum and an important issue within the education of Norwegian children (and children worldwide). This research shows that there are things in that education which could be improved to the benefit of the students.

An important issue to investigate further is how teacher education programmes prepare prospective teachers for teaching for mathematical literacy. As this study has shown that the implemented and attained curriculum do not match the intention of education for citizenship in the curriculum, it is relevant to investigate how teacher educators perceive and implement the curriculum and how they support prospective teachers in implementing it. With a new curriculum being implemented in August 2020, this is particularly relevant in the Norwegian context. Some teacher education programmes involve compulsory mathematical literacy courses. In Forgasz et al. (2017, p. 6), the Numeracy for Learners and Teachers course described aims for students “to develop understanding of what numeracy is and how it relates to mathematics; to learn to recognise numeracy opportunities across all learning areas of the curriculum; and to identify ways to engage their future students in relevant, critically challenging, curriculum-based activities that would build numeracy skills”. To my knowledge, Norwegian teacher education does currently not involve corresponding courses.

In relation to this, a further investigation of how to develop teachers’ teaching for mathematical literacy should be investigated. The model developed by Goos has been used and proven valuable in this respect in several research projects. It should, therefore, be investigated whether the model can support teachers’ understanding and teaching of, and students experience with, mathematical literacy in Norway.

Another educational issue I have not given much attention to in this research concerns students’ learning in terms of the assessed curriculum (Porter & Smithson, 2001). The assessed curriculum involves high-stakes tests, such as exams. In my research, the focus has not been on formal assessment. It would, however, be relevant to investigate how (or whether) national exams assess students’ mathematical literacy. Exams are set to test the knowledge and competences in the curriculum, and teaching is oriented at developing such knowledge and competences. Therefore, investigating exams in terms of

mathematical literacy could provide a new perspective and, hence, deeper insight into teaching for mathematical literacy. Similarly, a study of the tasks in the Norwegian national mathematical literacy test would be interesting. Such a study can add an even broader perspective to this research project and would be a possible continuation of the research. To my knowledge, such research has not been conducted in the Norwegian context.

Taking a critical perspective on Goos' model in terms of teaching mathematical literacy and teacher education, Venkat and Winter (2015) suggest a possibility for extending the model. Tools at the boundary of mathematics and contexts can be viewed and used differently depending on the situation's vantage point. In teaching for mathematical literacy, teachers need to be aware of this. My research shows that awareness of contexts is the main emphasis of teaching for mathematical literacy. The contextual focus, although important, may be overshadowing the importance of the other elements of mathematical literacy. From my view, language is the tool with the potential of mediating between the different mathematical literacy elements. Therefore, adding and relating to Venkat and Winter (2015)'s suggestion, I propose that from a CHAT and TO perspective, language should be given more attention.

7.6 Closure

I will close this dissertation by returning to the story about my student and his response to the grazing area task. By engaging the students in a discussion about why most farmers choose a quadrilateral instead of a circle, I could have assisted them in becoming conscious about cultural and historical ways of thinking about and doing mathematics in the Activity of farming. I could have assisted them in recognising a context in which mathematical knowledge can be used to make a well-founded decision. Also, I could have assisted them in critically evaluating the results obtained. In a way, this is what my student did.

Moreover, by engaging with the students in such discussions, I might have assisted them in developing positive dispositions toward mathematics in terms of recognising how mathematics is used in the world, in a meaningful way. I might have supported an understanding of mathematics in the real world as being more than just using a predetermined procedure to find one correct numerical answer.

Through my work with this research, I have become conscious of cultural and historical ways of thinking about teaching and learning for mathematical literacy. I hope and believe that if faced with a similar situation today, I would

have provided the students with such encounters as well. Hopefully, others can benefit from this research in the way I have.

8 References

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9 Appendices

Appendix A: Interview guides

Norwegian original	English translation
Skuleleiar:	School leader:
Kor lenge har du jobba som rektor?	For how long have you worked as a school leader?
Kva bakgrunn har du? <ul style="list-style-type: none"> • Tidlegare yrke • Fagkombinasjon (som lærar) • Fagleg interesse/motivasjon 	What is your background? <ul style="list-style-type: none"> • Former profession • Teaching subjects • Subject interest/motivation
Kva er måla for matematikkopplæringa på skulen? <ul style="list-style-type: none"> • Personlege mål • Basert på læreplan eller andre politiske dokument • Basert på læreboka 	What are the goals for the mathematics education in this school? <ul style="list-style-type: none"> • Personal goals • Goals based on the curriculum or other policy documents • Based on the textbook
I kor stor grad påverkar du/ynsker du å påverke matematikklærarane i høve deira undervisning?	To what extent do you (which to) influence the mathematics teachers in their teaching?
Kva vil du seie karakteriserer undervisninga på skulen?	How would you characterise the teaching in this school?
Kva er det viktigaste elevane bør lære i matematikkfaget?	What is the most important thing students should learn in the mathematics subject?
Kva vil det seie å vere god i matematikk?	What does it mean to be good at mathematics?
Kjenner du omgrepet «mathematical literacy»? <ul style="list-style-type: none"> • Viss ja, korleis forstår du omgrepet? • Viss nei, kva trur du det kan handle om? 	Do you know the term “mathematical literacy”? <ul style="list-style-type: none"> • If yes, how do you understand the term? • If no, what do you think it is about?
OECD har formulert denne definisjonen på «mathematical literacy», og eg har omsett den til norsk. Korleis forstår du denne definisjonen?	The OECD has formulated this definition of “mathematical literacy”, and I have translated it into Norwegian. How do you understand this definition?

På kva måte er dette i tråd med LK06 (eller ikkje)?	In what way is this in accordance with LK06?
Korleis kan du som rektor leggje til rette for at elevane skal utvikle denne kompetansen?	How can you, as the school leader, accommodate students' development of this competence?

Lærer:	Teacher:
Kor lenge har du jobba som rektor?	For how long have you worked as a teacher?
Kva bakgrunn har du? <ul style="list-style-type: none"> • Tidlegare yrke • Fagkombinasjon (som lærar) • Fagleg interesse/motivasjon 	What is your background? <ul style="list-style-type: none"> • Former profession • Teaching subjects • Subject interest/motivation
Kva er måla for matematikkopplæringa i klassen? <ul style="list-style-type: none"> • Personlege mål • Basert på læreplan eller andre politiske dokument • Basert på læreboka 	What are the goals for the mathematics education in your class? <ul style="list-style-type: none"> • Personal goals • Goals based on the curriculum or other policy documents • Based on the textbook
I kor stor grad påverkar rektor deg i høve di undervisning?	To what extent do the school leaders influence your mathematics teaching?
Kva vil du seie karakteriserer di matematikkundervisning og matematikkundervisninga på skulen generelt?	How would you characterise your mathematics teaching and the mathematics teaching in this school in general?
Kva er det viktigaste elevane bør lære i matematikkfaget og korleis legg du opp undervisninga i høve til dette?	What are the most important things students should learn in the mathematics subject, and how do you organise your teaching according to this?
Kva er matematikk?	What is mathematics?
Kva vil det seie å vere god i matematikk?	What does it mean to be good at mathematics?

Korleis veit ein når elevane forstår? Kva skal til for at dei forstår?	How do you know that the students understand? What does it take for the students to understand?
Korleis vil elevane svare på dette spørsmålet: Brukar de matematikken de lærer på skulen utanfor skulen? Brukar læraren noko av det de gjer utanfor skulen i matematikktimane?	How do you think the students will respond to this question: Do you use the mathematics you learn in school outside of school? Does the teacher use some of the things you do outside of school in the mathematics lessons?
Kjenner du omgrepet «mathematical literacy»? <ul style="list-style-type: none"> • Viss ja, korleis forstår du omgrepet? • Viss nei, kva trur du det kan handle om? 	Do you know the term “mathematical literacy”? <ul style="list-style-type: none"> • If yes, how do you understand the term? • If no, what do you think it is about?
OECD har formulert denne definisjonen på «mathematical literacy», og eg har omsett den til norsk. Korleis forstår du denne definisjonen?	The OECD has formulated this definition of “mathematical literacy”, and I have translated it into Norwegian. How do you understand this definition?
På kva måte er dette i tråd med LK06 (eller ikkje)?	In what way is this in accordance with LK06?
Korleis kan du som lærar leggje til rette for at elevane skal utvikle denne kompetansen?	How can you, as the teacher, accommodate students’ development of this competence?

Elev:	Student:
Fortel om dine interesser <ul style="list-style-type: none"> • Fritidsaktivitetar • Framtidig yrke • Fagleg interesse/motivasjon 	Tell me about your interests <ul style="list-style-type: none"> • Hobbies • Future occupation • Subject interest/motivation
Kva synest du kunne vore kjekt eller nyttig å lære om i matematikk? <ul style="list-style-type: none"> • Personlege interesser/mål • Basert på læreplan eller andre politiske dokument • Basert på læreboka 	What do you think would be fun or useful to learn in mathematics? <ul style="list-style-type: none"> • Based on personal interests/goals • Based on the curriculum or other policy documents • Based on the textbook
Er det noko du har lært om i matematikk på skulen som du har fått bruk for utanom skulen? I så fall kva?	Is there anything you have learned in mathematics in school that has been useful outside of school? If yes, what?

Trur du at du brukar noko matematikk i kvardagen? I så fall, kor tid og kor ofte?	Do you think that you use any mathematics in your everyday life? If yes, when and how often?
Kjenner du nokon som brukar matte i jobben sin?	Do you know anyone who uses mathematics in their work?
Brukar læraren eksempel frå kvardagen i matematikktimane? Kan du gi døme?	Does the teacher use examples from everyday life in the mathematics teaching? Can you give an example?
Kva er matematikk?	What is mathematics?
Tenk på ein person som er flink i matematikk. Kva er det som gjer denne personen flink i matematikk? Kva er det denne personen kan?	Think of a person who is good at mathematics. What makes this person good in mathematics? What does this person know?
Tenk på talet 50. Er det eit stort eller eit lite tal? Kvifor?	Think of the number 50. Is it a small or a large number? Why?
Viss læraren ber deg lage reknestykke i staden for å løyse reknestykke, er det matematikk?	If the teacher asks you to make a mathematical problem, is that mathematics?
Kor mange måtar er det å komme fram til svaret på i matematikk? Er det ulike måtar å løyse eit reknestykke på?	How many ways are there to get the answer in mathematics? Are there different ways to solve a problem?
Kven spør du om hjelp med matematikkleksene? <ul style="list-style-type: none"> • Nokon andre? • Brukar dei matte i jobben sin? • Har du snakka med dei om korleis dei brukar matte? 	Whom do you ask for help with your mathematics homework? <ul style="list-style-type: none"> • Anyone else? • Do they use any mathematics in their work? • Have you talked with them about how they use mathematics?

Appendix B: Information letters with letters of consent

Information letter and letter of consent to the students and parents

Oda Heidi Bolstad
Høgskulen på Vestlandet
odabo@hvl.no
Røyrgata 6
6856 Sogndal

Telefon: 57676334
E-post:

Sogndal, 23.08.2017

Til foreldre/føresette for elevar på 9. trinn ved ... skule

DOKTORGRADSPROSJEKT OM SKULEN SITT ARBEID MED FOKUSOMRÅDE FOR MATEMATIKKUNDERVISNINGA

I august 2016 begynte eg som doktorgradsstipendiat i matematikdidaktikk ved Høgskulen på Vestlandet (tidlegare Høgskulen i Sogn og Fjordane). Eg er i gang med å planlegge datainnsamlinga til forskingsprosjektet mitt, og ynskjer med dette å rette ein førespurnad til dykk om løyve til at dykkar barn kan delta i prosjektet. Føremålet med dette brevet er å informere om prosjektet, og å be om løyve til å gjere video- og lydopptak, å gjere intervju med elevane, og eventuelt samle inn relevant skriftleg materiale.

Prosjektet har som mål å få ei dynamisk forståing av læring og undervisning i matematikk. Eg vil ha fokus på alle tre nivåa på ein skule, med rektor, lærar og elevar. I tillegg til personlege oppfatningar kring læring og undervisning i matematikk, er det også ulike styringsdokument som spelar inn (t.d. læreplanen, stortingsmeldingar, strategidokument og lærebøker). Med dette prosjektet ynskjer eg å sjå på korleis omgrep og fokusområde i ulike dokument blir oppfatta, arbeidd med/for og erfart på dei ulike nivåa i skulen. Prosjektet er meldt inn til Personvernombudet for forskning, NSD - Norsk senter for forskningsdata AS. Dette skal sikre at deltakarane blir ivaretekne både med omsyn til personvernopplysingar og etiske retningslinjer.

Video- og lydopptak vil bli gjennomført i vanleg klasseromsundervisning og under intervju med einskildelevar. Det blir vektlagt at prosjektet i minst mogleg grad skal gripe inn i elevane sin skulekvardag, og at undervisninga skal gå som normalt. Eg er interessert i å sjå korleis den ordinære undervisninga i klasserommet går føre seg. Videoopptaka blir gjort ved hjelp av hovudkamera (nytta av både elevar og lærar) og eit

stasjonært kamera bak i klasserommet. Eg vil besøke klassen i forkant av undersøkinga, for at elevane skal bli kjende med meg, og for at dei skal få prøve utstyret. Spørsmåla i intervjuet vil handle om korleis eleven opplever matematikkfaget og matematikkundervisninga på skulen. Foreldre/føresette har høve til å sjå intervjuguide i forkant av intervjuet, om det er ynskjeleg.

Innsamlinga av datamateriale vil skje over 4-8 undervisningsøker i perioden mellom 1. september og 31. desember 2017. Nærare fastsetjing av dato skjer i samråd med lærar og rektor. Videoopptaka vil berre bli sett av meg og mine rettleiarar, Frode Olav Haara (Høgskulen på Vestlandet, frode.olav.haara@hvl.no) og Simon Goodchild (Universitetet i Agder, simon.goodchild@uia.no), og eventuelt andre kollegaer om det skulle vere naudsynt i samband med analysearbeidet. Opptaka vil bli lagra på ekstern harddisk, og vil ikkje bli nytta i andre samanhengar enn i arbeid med dette prosjektet. Resultata blir publiserte i mi doktorgradsavhandling, planlagt avslutta vår/sommar 2020. Skulen og alle involverte personar vil bli anonymisert, og det vil ikkje vere mogeleg å spore attende til einskildindivid. Datamaterialet vil bli sletta så snart avhandlinga er levert og vurdert godkjend.

Eg vil presisere at deltaking i prosjektet er heilt frivillig. Det blir gitt eit likeverdig pedagogisk tilbod i matematikk til dei som ikkje ynskjer å delta i prosjektet. Sjølv om ein har gitt samtykkje kan ein til ei kvar tid trekke seg att frå deltaking, utan å måtte oppgi nokon grunn til dette. Det vil ikkje påverke forholdet til skulen dersom ein ikkje ynskjer å delta, eller vel å trekke seg frå deltaking.

Eg håpar de synest dette er interessant og viktig, og håpar de vil late dykkar barn delta. Eg ber om at de gir skriftleg løyve til å gjere video- eller lydopptak og samle inn skriftleg materiale til prosjektet. Dette gjer de ved å fylle ut den vedlagde svarslippen og levere han til læraren i klassen. Føresetnaden for løyvet er at alt innsamla materiale blir handsama med respekt og blir anonymisert, og at prosjektet elles føl gjeldande retningslinjer for personvern.

Eg vil vere til stades på foreldremøtet den ... for å svare på eventuelle spørsmål og komme med utfyllande opplysingar om prosjektet. De kan også kontakte meg på e-post eller telefon dersom de ynskjer det (sjå øvst for detaljar). Mine rettleiarar kan også kontaktast (sjå e-postadresser i teksten).

Beste helsing
Oda Heidi Bolstad

SVARSLIPP

Eleven sitt førenamn og

etternamn:.....

(Set eitt eller to kryss)

Eg/me har motteke informasjon om prosjektet. Eg/me gir løyve til at Oda Heidi Bolstad kan nytte video- og lydopptak der mitt/vårt barn er med, og skriftleg materiale frå mitt/vårt barn, i sitt doktorgradsprosjekt. Eg/me har snakka med jenta/guten vår om dette, og han/ho har også gjeve sitt samtykkje.

Eg/me gir også løyve til at Oda Heidi Bolstad kan intervjuje mitt/vårt barn i samband med sitt doktorgradsprosjekt.

Dato:

Underskrift av føresette

Ver vennleg å returnere svarslippen til læraren i klassen innan ...

Information letter and letter of consent to the school leaders and teachers

Oda Heidi Bolstad
Høgskulen på Vestlandet
odabo@hvl.no
Røyrgata 6
6856 Sogndal

Tel.: 57676334
E-post:

Sogndal, 23.08.2017

Til rektor og matematikklærer på 9. trinn ved ... skule

DOKTORGRADSPROSJEKT OM SKULEN SITT ARBEID MED FOKUSOMRÅDE FOR MATEMATIKKUNDERVISNINGA

I august 2016 begynte eg som doktorgradsstipendiat i matematikdidaktikk ved Høgskulen på Vestlandet (tidlegare Høgskulen i Sogn og Fjordane). Eg er i gang med å planlegge datainnsamlinga til forskingsprosjektet mitt, og ynskjer med dette å rette ein førespurnad til dykk om å delta i prosjektet. Føremålet med dette brevet er å informere om prosjektet, og å be om løyve til å gjere video- og lydopptak, å gjere intervju med dykk, og eventuelt samle inn relevant skriftleg materiale.

Prosjektet har som mål å få ei dynamisk forståing av læring og undervisning i matematikk. Eg vil ha fokus på alle tre nivåa på ein skule, med rektor, lærar og elevar. I tillegg til personlege oppfatningar kring læring og undervisning i matematikk, er det også ulike styringsdokument som spelar inn (t.d. læreplanen, stortingsmeldingar, strategidokument og lærebøker). Med dette prosjektet ynskjer eg å sjå på korleis omgrep og fokusområde i ulike dokument blir oppfatta, arbeidd med/for og erfart på dei ulike nivåa i skulen. Prosjektet skal meldast inn til Personvernombudet for forskning, NSD - Norsk senter for forskningsdata AS. Dette skal sikre at deltakarane blir ivaretekne både med omsyn til personvernopplysingar og etiske retningslinjer.

Video- og lydopptak vil bli gjennomført i vanleg klasseromsundervisning og under intervju med rektor, matematikklærer og einskildelevar. Det blir vektlagt at prosjektet i minst mogleg grad skal gripe inn i elevane og dei tilsette sin skulekvardag. Videoopptaka blir gjort ved hjelp av hovudkamera (nytta av både elevar og lærar) og eit stasjonært kamera bak i klasserommet. Eg vil besøke klassen i forkant av undersøkinga, for at elevane og matematikklæraren skal bli kjende med meg, og for at dei skal få prøve utstyret. Spørsmåla i intervjuet vil handle om korleis deltakarane opplever

matematikkfaget og matematikkundervisninga på skulen. Deltakarane har høve til å sjå intervjuguide i forkant av intervjuet, om det er ynskjeleg.

Innsamlinga av datamateriale vil skje over 4-6 undervisningsøker i perioden mellom 1. august og 1. desember 2017. Videoopptaka vil berre bli sett av meg og mine rettleiarar, Frode Olav Haara (Høgskulen på Vestlandet, frode.olav.haara@hvl.no) og Simon Goodchild (Universitetet i Agder, simon.goodchild@uia.no), og eventuelt andre kollegaer om det skulle vere naudsynt i samband med analysearbeidet. Videoopptaka vil bli lagra på ekstern harddisk, og vil ikkje bli nytta i andre samanhengar enn i arbeid med dette prosjektet. Materialet vil bli sletta så snart avhandlinga er levert og vurdert godkjend. Resultata vil bli publiserte i mi doktogradsavhandling, planlagt ferdig vår/sommar 2020. Skulen og alle involverte personar bli anonymisert, og det vil ikkje vere mogeleg å spore attende til einskildindivid.

Eg vil presisere at deltaking i prosjektet er heilt frivillig. Sjølv om ein har gitt samtykkje kan ein til ei kvar tid trekke seg att frå deltaking, utan å måtte oppgi nokon grunn til dette.

Eg håpar de synest dette er interessant og viktig, og håpar de ynskjer delta. Eg ber om at de gir skriftleg løyve til å gjere video- eller lydopptak og samle inn skriftleg materiale til prosjektet. Dette gjer de ved å fylle ut den vedlagde svarslippen. Føresetnaden for løyvet er at alt innsamla materiale blir handsama med respekt og blir anonymisert, og at prosjektet elles føl gjeldande retningslinjer for personvern.

De kan gjerne kontakte meg på e-post eller telefon dersom de har spørsmål eller ynskjer meir informasjon (sjå øvst for detaljar). Mine rettleiarar kan også kontaktast (sjå e-postadresser i teksten).

Beste helsing
Oda Heidi Bolstad

SVARSLIPP

(Set kryss)

Eg har motteke informasjon om prosjektet. Eg gir løyve til at Oda Heidi Bolstad kan nytte video- og lydopptak av intervju og undervisning og skriftleg materiale, i sitt doktorgradsprosjekt.

Dato:

Underskrift:

Appendix C: Articles 1-3

Article 1: Teaching for mathematical literacy: School leaders' and teachers' rationales

This article was published in the European Journal of Science and Mathematics Education, Vol. 7, No. 3, 2019, 93-108.

Article 2: Secondary teachers' operationalisation of mathematical literacy

This article was published in the European Journal of Science and Mathematics Education, Vol. 8, No. 3, 2020, 115-135.

Article 3: Lower secondary students' encounters with mathematical literacy

This article was submitted on 23.06.2020 to Nordic Studies in Mathematics Education. The article is under review.



Teaching for mathematical literacy: School leaders' and teachers' rationales

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Abstract

This article reports a qualitative inquiry into school leaders' and teachers' rationales for teaching to develop students' *mathematical literacy*. The study is rooted in an exploration of the meanings that the school leaders and teachers hold about the term mathematical literacy. Six leaders and three grade 9 mathematics teachers from three schools were interviewed. Analysis framed within cultural-historical activity theory indicates that mathematical literacy is perceived as a desired outcome of schooling, and that teaching for mathematical literacy is connected to school leaders' and teachers' contradictory rationales for teaching. The rationales are connected to use value, meaning, teaching practice, teacher competences and knowledge, and universality.

Keywords: Cultural-historical activity theory, mathematical literacy, mathematics education, teaching

Introduction

The Programme of International Student Assessment (PISA) has provoked much interest and discussion about the notion *mathematical literacy*(ML)(Fried & Dreyfus, 2014). The Organisation for Economic Co-operation and Development (OECD)(2012, p. 25) defines ML as

an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to recognise the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens.

In spite of the international rooting of the definition in the OECD-PISA study, ML has no universally accepted meaning and attempts have been made to clarify the expression (Fried & Dreyfus, 2014). In its broadest sense, ML concerns the ability to perform, understand, and use mathematics in daily life(Colwell & Enderson, 2016). It concerns the ability to use mathematical knowledge and competence to formulate and solve mathematical problems in a range of different situations. Sford (2014) explains ML as the ability to participate in mathematical communication whenever this is necessary for our understanding and manipulation of the world. It is therefore important that students learn how to speak mathematically, when to speak mathematically, and what to speak mathematically about.

Mathematics education literature contains several notions related to ML. Some authors use concepts like ML, *numeracy*, and *quantitative literacy* synonymously, while others distinguish between them (Niss & Jablonka, 2014). Other related concepts are *critical mathematical numeracy*(e.g. Frankenstein, 2010), *mathemacy* (e.g. Skovsmose, 2011), *matheracy* (e.g. D'Ambrosio, 2007), and *statistical*

literacy (Watson, 2011). Still, the different interpretations of these concepts have in common that they stress awareness of the usefulness and ability to use mathematics in different areas (Niss & Jablonka, 2014).

Another problematic issue with the term ML is that although it is widely used internationally, it lacks non-English equivalents (Jablonka, 2015). This means that it is difficult to translate the concept into other languages and retain the meaning. In some languages, the word literacy has such a narrow meaning that it can be impossible to convey the broad meaning intended by PISA (Stacey & Turner, 2015). For example in Spanish, French, and the Scandinavian languages, literacy is linked to very basic reading and writing abilities. As a result, concepts like *mathematical competence* and *mathematical culture* are used instead to avoid the narrow connotations of the term literacy in educational debates (Stacey & Turner, 2015).

PISA's reports comparing students' performance have been influential in shaping educational policies in several OECD countries, and curriculum developers/reviewers have tried to reflect PISA competences in their national curricula. In the latest curricular reform in Norway (LK06), there is an explicit attempt to align with PISA frameworks by including basic skills in all subject syllabuses (Breakspear, 2012; Det kongelige utdannings- og forskningsdepartement, 2004). The basic skills correspond to the English notion *literacy* (Det kongelige utdannings- og forskningsdepartement, 2004). The five basic skills are *reading, writing, oral skills, digital skills, and numeracy*.

Numeracy means applying mathematics in different situations. Being numerate means to be able to reason and use mathematical concepts, procedures, facts and tools to solve problems and to describe, explain and predict what will happen. It involves recognizing numeracy in different contexts, asking questions related to mathematics, choosing relevant methods to solve problems and interpreting validity and effect of the results. Furthermore, it involves being able to backtrack to make new choices. Numeracy includes communicating and arguing for choices by interpreting context and working on a problem until it is solved.

Numeracy is necessary to arrive at an informed opinion about civic and social issues. Furthermore, it is equally important for personal development and the ability to make appropriate decisions in work and everyday life. (Utdanningsdirektoratet, 2012, p. 14)

The lack of a universal understanding of ML and the range of similar notions affect teaching to promote ML. Colwell and Enderson (2016) studied pre-service teachers' perceptions of ML to inform program changes in teacher education in the USA. The pre-service teachers emphasised writing, communication, and application skills as important factors of ML. However, the pre-service teachers were uncertain of how to integrate such practices into their teaching. In the Norwegian context, an evaluation of LK06 shows that the basic skills have been understood in a more confined way than intended (NOU 2015:8, 2015), and Grønmo (2014) calls for discussion and measures on how to implement the basic skills in a satisfactory way. This is consistent with the findings in Haara, Bolstad, and Jenssen's (2017) review of empirical studies of ML. If students are to become problem solvers that use concepts, procedures, facts, and tools to reason, describe, explain, and predict in various contexts, teachers need to implement instructional techniques to promote this development. To support implementing such techniques, it is valuable to investigate and build on teachers' own understanding of practice in relation to the notion ML.

Teachers experience tension between wanting students to have time to "understand" the mathematics while making sure they "cover the syllabus" for the upcoming test. This tension between performance on tests and depth of understanding is one that reaches out beyond the classroom, implicating policy, texts, tests, and assessments (Williams, 2011). Hence, the immediate needs of performance conflicts

with future use demands. Studying how to teach for ML is therefore of interest both in a Norwegian and an international context.

Teaching for ML requires a notion of what it means to be mathematically literate. As noted above, literacy has no equivalent word in Norwegian, and the notion of literacy is not made explicit in the Norwegian curriculum. However, the development of students' ability to use mathematical procedures and tools to solve problems in different contexts is an explicit goal. Thus, it is relevant to study the influence of the OECD competence framework on Norwegian school education. In addition, knowledge about the Norwegian education system may contribute to insights in international educational contexts. According to Sfard (2014, p. 141) "the question of how to teach for ML must be theoretically and empirically studied. When we consider the urgency of the issue, we should make sure that such research is given high priority."

In the educational context of the school and classroom, school leaders and teachers are the decision-makers, planners, and organisers of activity. They focus their plans and actions towards a desired outcome. Hence, school leaders and teachers have rationales for teaching mathematics. To learn more about teaching for ML, this study seeks to investigate the rationales for teaching in the context of their understanding of ML.

School leaders' and mathematics teachers' understanding of ML involves some prior knowledge of the concept, different interpretations of the notion, and the aspects of mathematical knowledge and skills it encompasses. Their understanding includes their ideas about how ML is related to or manifested within the curriculum. It also includes their rationales and goals for teaching so that students develop ML. The study reported here focuses on how the concept ML is understood in Norwegian lower secondary schools, and the research question addressed in this article is:

What are school leaders' and teachers' rationales for teaching with respect to their understanding of ML?

Theoretical Perspectives

In this article, students' ML development is conceived as a goal of a sub-set of teachers' actions. In this respect, "education" and schooling are taken as a culturally and historically situated activity. In other words, the study is framed within cultural-historical activity theory (CHAT).

In CHAT, *activity* is the unit of life that is mediated by mental reflection, which functions to orient the subject in the world of objects (Roth & Radford, 2011). Activities answer to a subject's specific need. This need stands behind the activity *object* or *motive*. Object/motive consist of material reality and its ideal reflection in consciousness and between current and future material/ideal states. In other words, the object/motive drives activity from the experienced "here and now" to a desired future state envisaged in the imagination. Hence, it consist of both the object-sensory practical activity and the ideal object reflected in consciousness during activity.

The main thing that distinguishes one activity from another is the difference in their objects/motives (Leont'ev, 1978). In the activity of education, students pursue different objects/motives. Students performing the same mathematical task, one with the ability to use mathematics in everyday life as motive, the other with the motive of achieving examination success in the subject, engage in different activities. The object/motive that drives learning activity is accessible to students only as an outcome of the activity. Students cannot know what they are supposed to learn before they have learnt it. They cannot recognise the objects/motives on their own, and the teachers cannot tell them. The object/motive emerges through the teacher's and the student's joint action. In learning activity, the

teachers have to take on the regulative function that in other productive human activities exist in the known object/motive (Roth & Radford, 2011). Teachers have to facilitate students' engagement in activities concerned with developing ML. The question is which activity the students engage in, and, therefore, which motives they take up and pursue.

Actions are initiated by the object/motive and translate activity into reality (Leont'ev, 1981). An action is directed toward a conscious *goal*. Several different goals and actions can relate to the same object/motive, but are not equal to it. ML contains goals connected to mastery of mathematical procedures, understanding of mathematical concepts, and the ability to use all of this in different contexts. Several actions can contribute to reaching these goals. For students in the process of learning, there is an absence of a concretisation of the object/motive. Learning goals are stated by the curriculum (and the teacher), and not by the students themselves. Therefore, their actions do not and cannot make sense. "Students may realize a task without taking up the object/motive, in which case they do not expand their action possibilities in the intended way, and do not learn what they are invited to learn" (Roth & Radford, 2011, p. 97). An example is students using rote-learned procedures to solve mathematical tasks without understanding the underlying processes and structures. Teachers have to launch an objectifying process where room is created for joint work for the object/motive to emerge for the student. The teacher's actions have to open up new possibilities for student action.

The intention behind ML is to enhance students' possibilities for action in their everyday life. It emphasises the use value of mathematics. Williams (2011) argues that within CHAT values must have a crucial role in shaping subjective needs, and that values are bound up in ideal outcomes and subjectively perceived needs mediated by cultural norms. If the object is a mathematical task, the goal of the acting subject may be to understand mathematics or to get the correct answer. In his study, the *use value* of mathematics was discussed in relation to mathematics in daily life (such as shopping) and connections between mathematics and vocational activity. Williams also found that students talk of a kind of currency of mathematics qualifications and grades. This currency is required for entry to universities and courses, and will eventually result in a respectable career. Williams sees this currency of mathematics as akin to *exchange value*. Exchange value is a quantitative relation between two values in use. "As use values, commodities are, above all, of different qualities, but as exchange values they are merely different quantities, and consequently do not contain an atom of use value" (p. 28). In relation to Ernest (2004), this can be denoted as utility, meaning narrowly conceived usefulness that can be demonstrated in the short term. He contrasts utility with relevance. Relevance is relative according to the person is using it. It is a relation between an activity or object, a subject, and a goal. Varying goals give rise to different conceptions of relevance in mathematics education (Ernest, 2004). Hence, teaching with respect to use value is complex.

The term ML concerns the use value of mathematics in everyday life and includes using mathematical *tools*. Tools are used for some purpose, in order to achieve something. They are embedded within a cultural-historical form of thinking (Roth & Radford, 2011). Tools can be external items (e.g. a calculator or an abacus), thinking tools (e.g. different forms of representations such as graphs and algebraic expressions), and communicative tools (e.g. language, text, and speech). They assist us to see something through something or someone else, in other words the tools *mediate*. Tools can also mediate mathematical understanding. Mathematical tools help us describe, explain, and predict phenomenon, and to understand the world. Mathematics, written language, speech, gestures, and every sign system are communicative systems developed for different purposes. They are tools to activities. To make clear the historical intelligence embedded in tools requires that other people who know this intelligence helps us acquire it (Radford, 2008).

Activity is social, and *communication* is an indissoluble part of the activity process. In ML the ability to formulate, interpret, reason, describe, and explain refer to different forms of communication. As

stated in the introduction, ML involves communicating mathematically in order to understand and manipulate the world. Communication is a system of goal-directed and motivated processes, which ensure the interpersonal components of activity. It is through communication that ideas are shared, strategies developed, and projects carried out (Mellin-Olsen, 1987).

Language and concepts are important for communication and learning. Language *mediates* significations, or meanings, which constitutes a practical consciousness for others and constitutes one of the main contents of collective consciousness. “As meaning exists in the form of language, language is shared socially as an objective reality. The meaning which language conveys, however, is interpreted subjectively by the individual (Mellin-Olsen, 1987, p. 44). Concepts are the result of the objectification of historically achieved significations. A word reflects the social, political, and theoretical position of the person uttering it. Knowing, as an outcome of learning, refers to the possibilities that become available to the participants for thinking, reflecting, arguing, and acting in a certain historically contingent cultural practice (Roth & Radford, 2011).

Radford (2008) defines learning as the social process of *objectification* of those external patterns of action fixed in the culture. Learning is not merely acquiring, possessing or mastering something, but seeking to find “something” in culture. It is a subjective awareness of cultural objects. Related to ML, it involves recognising the role that mathematics plays in the world. Objectification entails the process of *subjectification*—i.e., the becoming of the self. In this process, the learner objectifies cultural knowledge and finds himself objectified in a reflective move. This is the making of the subject, and it is the outcome of the act of learning. In knowing mathematics, the student enters into a historically mediated relationship with mathematics and other people. This historically mediated relationship not only makes mathematics noticeable to the student, but also the student to himself through the available forms of subjectivity and agency of the culture. Hence, it enables the student to make well-founded judgements and decisions in everyday life. Objectification and subjectification should be seen as two mutually constitutive processes leading to students’ engagement with cultural forms of thinking and a sensibility to issues of interpersonal respect, plurality, and inclusiveness (Radford, 2008).

Hence, CHAT embodies both the individual and the society as a unity. The individual acts on society at the same time as s/he becomes socialised into it (Mellin-Olsen, 1987). ML concerns the individual’s ability to act in and be a part of society by knowing mathematics.

Students learn about what is important knowledge, expectations, future prospects etc. for their local community and they develop rationales for learning. School may or may not be part of these rationales (Mellin-Olsen, 1981). The *S-rationale* is the rationale for school learning. “It is the rationale for learning evoked in the pupil by a synthesis of his self-concept, his cognition of school and schooling, and his concept of what is significant knowledge and a valuable future, as developed in his social setting” (Mellin-Olsen, 1981, p. 357). The conception of what is significant knowledge will differ between geographical regions and between social classes. Students can face contradictory rationales for learning according to their relation to different social groups. Additionally, there is a rationale for learning related to school as an instrument for a good future or qualifying the students so that they obtain a good price for their commodity of labour. Mellin-Olsen (1981) calls it the *I-rationale*. The *I-rationale* creates learning that shows no interests in the content itself, rather the purpose is to demonstrate knowledge to obtain good marks or a degree. This could mean rote learning of mathematics procedures and facts. The optimal situation is when the *S-* and the *I-rationales* coincide. This is when the curriculum that leads to good marks (the procedures and facts) is the same as that which the students experience as significant knowledge (knowledge useful in everyday life). However, the most common situation in the classroom is when the *S-* and the *I-rationales* overlap. The teachers’ task is to make this overlap as large as possible.

The task of making the overlap is challenging due to students' differing S- and I-rationales. In addition, teachers' conflicts regarding teaching for understanding and teaching to cover the syllabus suggest that teachers also have rationales for teaching. This study seeks to investigate school leaders' and teachers' rationales for teaching for ML.

Research Design

According to Roth and Radford (2011), language is the vehicle of consciousness and words constitute aspects of consciousness. By conducting interviews with school leaders and mathematics teachers, I wanted to learn about their rationales for teaching, and their object/motive, actions, and goals in relation their understanding of ML.

However, words are addressed to an interlocutor, and will depend on the interlocutor's social role/status. Therefore, words are not solely a property of the person uttering them. In my research, the school leaders' and teachers' responses will be affected by my role as a researcher and teacher and the social relationship between them and me. Another researcher might get different answers or interpret the answers differently.

Subjects

This study aims to investigate school leaders' and teachers' rationales for teaching with respect to their understanding of ML. I conducted interviews with six school leaders (three male and three female) and three grade 9 mathematics teachers (one male and two female) in three schools in Western Norway. The schools' number of students on roll range from 220 to 370 and all 3 schools teach grades 1 through 10. The school leaders have previous experience as teachers. All the participants have more than ten years of experience from working in school.

The three schools cooperate with the author's university teacher education programme. They were therefore recruited as an outcome of acquaintance. I first contacted the school leaders and they recruited the teachers. The criteria for selection of teachers were that they were teaching grade 9 (students aged 14-15 years) mathematics the current school year, and that they agreed to participate. All participants received an information letter explaining my interest in studying their understanding of concepts in policy documents regarding mathematics teaching and learning. Because of the lack of a Norwegian equivalent and a universal understanding of ML, I did not include this notion in the letter. In addition to the information letter and e-mail correspondence, I also attended meetings with them to ensure informed consent.

Mathematics teachers plan and conduct teaching in order to enable students to obtain the goals stated in the syllabus. In Norway, mathematics is a discrete subject within the curriculum. The syllabus contains sets of competence goals, which the students are to obtain. The goals are connected to the different mathematical topics. Through grades 1 to 10, students are expected to study mathematics on average about 11 hours each week (given that one school year consists of 38 weeks). As noted above, numeracy is expected to permeate the whole curriculum. Teaching is likely influenced by how the teacher interprets, understands, and conceptualises the ideas and concepts in the syllabus and the textbook. In Norway, the school leaders have pedagogical, administrative, and staff responsibilities at the school. School leaders are responsible for students' learning environments and outcomes, and expected to make professional decisions rooted in subject knowledge. They are also responsible for school development. In this way, the school leaders have to prioritise the issues worked with and the extent to which they are emphasised. It is expected that the school leaders' priorities influence the teaching.

Design and Procedures

The school leaders have the pedagogical responsibility at the schools, but the teachers conduct the teaching. I was therefore interested to explore both groups' understanding of ML. I conducted individual semi-structured interviews with all nine participants. I developed an interview guide with questions and topics I wanted them to consider/reflect upon but without a predetermined sequence. I used the same interview guide for all interviews to get perspectives from the different groups' standpoint. I asked four questions about ML. First, I asked whether they had heard about ML. I wanted them to give their own explanation of the concept. Second, I presented the OECD definition of ML and asked them to comment on it. Third, I asked whether this definition corresponds with the Norwegian curriculum. Fourth, I asked how they would conduct teaching in order to develop students' ML. Each interview lasted for about one hour. The interviews were video recorded and transcribed.

Process of Analysis

This study was exploratory and data was analysed qualitatively using an inductive approach. I analysed the data using the computer-assisted qualitative data analysis software NVivo. First, the transcribed data were categorised according to the questions I asked in the interviews. In this way, I had some general structuring principles, and I focused the analysis on the answers to the questions concerning ML.

Second, I engaged in multiple close readings and interpretations of the data. I tried to get an overall understanding of the data and to identify text segments. Text segments are statements connected to a specific topic or issue. They contain school leaders' and teachers' meanings and concerns related to the four questions about ML. Sometimes whole sections of transcripts elaborated on the same issue, and sometimes only short sentences. In this way, the text segments differ in length.

Third, I grouped together text segments containing similar topics or issues to make broader categories. The broader categories were developed with respect to key themes in the text segments. The categories are *use value, meaning, teaching practice, teacher competences and knowledge, and universality*. The categories are closely connected, and not mutually exclusive. Table 1 shows category descriptions and the categories' relation to the theory outlined in section 2.

Table 1. Category descriptions and their relations to theory

Category	Description	Theory
Use value	Examples of how and in what areas mathematics is used. Examples of teaching activities that show mathematics in context. Related to the justification and relevance of mathematics as a school subject, and highlighting contextualisation of mathematics.	Subjective needs and goals for teaching and learning mathematics. Connected to I-rationales and the exchange value of mathematics, or S-rationales and use value of mathematics.
Meaning	Concerns meaning making and the ability to understand and communicate. Students should develop this ability. It relates to use value in the sense that ability to communicate has use value, but in a more abstract way than i.e. to use mathematics in an occupation.	Communication and language as mediator of cultural-historical aspects of mathematics. Mathematical language can be perceived as both I- and S-knowledge.
Teaching practice	Concerns what teachers do, or should (not) do, generally, in teaching mathematics. It includes	Actions related to goals and rationales.

class organisation, teaching materials and factors influencing practice (like curriculum, textbooks, etc.), and the way teaching is conducted. It relates to use value and meaning, but is more general.

Teacher competences and knowledge	Concerns knowledge and skills needed by the teacher. It involves subject knowledge, professional knowledge, and personal qualities. It relates to teaching practice, but focuses on the teacher rather than teaching.	Goals and rationales for teaching.
Universality	Everything is linked together. Teaching, learning, mathematics, and the world are all part of a whole.	Seeing mathematics as part of the cultural-historical activity. Both I- and S-knowledge are valued.

After developing the categories, I coded the transcriptions again using these categories. To test for reliability, I provided a colleague with a sample of the transcriptions and my preliminary category descriptions. We discussed the data sample and the category descriptions to get a mutual understanding of them. She then engaged in the data sample with respect to the coding and categories. We then compared and discussed our coding, and agreed on some category revisions. Finally, we both coded the data using the revised categories. The inter-rater reliability test in NVivo showed a 92, 2% agreement (Kappa value 0, 47).

Categories of Rationales for ML

In this section, I will describe the results of analysis as five categories based on the responses given to questions about the OECD ML definition, teaching for ML, and correspondence between ML and the curriculum. I present the results with respect to the two groups as a whole. However, not every individual commented on all the specific issues.

Use Value

The responses concerned with use value focused on utility. Both school leaders and teachers referred to students' request for justification of mathematical topics. The students want to know why they have to learn mathematics and how it will (or can be) useful to them, hence they ask for the use value of mathematics. The school leaders and teachers argued that teaching for ML should focus on how to use mathematics in societal, occupational and personal life. That is, try to relate to students' S-rationales. However, it is difficult to know what will or will not come of use in the students' future life. The school leaders and teachers thus highlighted the way of thinking about a problem and the ability to use mathematics as a tool as a goal for teaching.

Students have different S-rationales for learning, and this makes teaching challenging. The school leaders pointed to challenges with finding suitable contexts for teaching related to the use value of mathematics. "You can learn the area of... a building, and calculate it, but still the student doesn't see it, how it is in reality, and do not understand the concepts. (...) Because when you, as an adult, have made a judgement on... Right, you have an image of a concept in your mind, based on several experiences, right? Perhaps you have measured the size of a floor, and you know how many square meters you have to paint, and then, right... You have seen it practically, but an eight year old has not" (School leader A2).

The textbook is a much-used teaching tool. According to the school leaders, teaching should involve practical tasks, and not rely too heavily on solving routine tasks in the textbook. Teaching should aim to get the students to experience the use value of mathematics themselves: "To let them experience things, I think. We telling them does not always do the trick, but to let them experience that this was important. (...) For them to see that 'Wow! You know, now it was really helpful that I knew this'" (School leader B2).

The teachers were concerned about developing students' ability to see mathematics in everyday life: "I wish I was able to do what that sentence says; to assist individuals to recognise the role that mathematics plays in the world" (Teacher A). They gave examples from lessons where they had tried to do this. Regarding mathematics in everyday life, the school leaders commented that the curriculum competence goals do not focus too much on the use value of mathematics. "I think you need to think very creatively about the competency goals before you're able to, to assist individuals to recognise the role that mathematics plays in the world" (School leader B2).

Meaning

The school leaders commented that mathematics can be seen as a language. They connected ML to the ability to use mathematical language and understand mathematical concepts. They gave several reasons for this, for instance that mathematics gives you a foundation in communication with others. "It's important for everyone to understand what we talk about, when we speak the language" (School leader A1). In this sense, mathematical language can be seen as S-knowledge and important in order to function socially. An important part of learning to speak the language is to learn the concepts. According to the school leaders, we often think that students understand concepts to a greater extent than they actually do. Hence, language is socially shared but meaning is interpreted by the individual.

As stated earlier, communication is important for developing and sharing ideas. The school leaders and teachers highlighted language as important in the learning process. They said that it is important to talk about mathematics. Students should put into words what they know and discuss and justify their methods and interpretations.

The school leaders connected ML to reading as a basic skill. They highlighted that reasoning, reflection and interpretation is important. "The capacity to formulate, interpret, for example, in various contexts. Reasoning. It was these things we just worked with... in our development project on reading in all subjects. (...) As I told you before, we've had a great focus on reflecting on a text. What lies between the lines? What...? You know? What's the message, really, instead of just finding the answer directly in the text, because that's often what happens in school, right? (...) There's too little focus on reasoning and reflecting and such" (School leader A2). Reasoning and reflecting concern the significations and meanings that language mediates.

Teaching Practice

The premises for teaching lies in the curriculum. The curriculum contains the object/motive for education, and different goals to satisfy the object/motive. The school leaders pointed to different aspects of the curriculum that affect teaching practice and the actions carried out to reach the goals. The Norwegian curriculum is structured in competence goals for each school subject. The textbooks often refer to these goals. The school leaders pointed to this as one reason for the heavy reliance on textbooks. "I don't think that we managed to crack the code with the curriculum. If we had cracked the code with the curriculum, I think maybe we had managed to put the textbook aside. But we were so focused on the learning goals, and the textbook contained learning goals, so it was an easy way out for us" (School leader B1). The textbook as a teaching tool's close connection to the teaching goals may have a limiting effect on the teachers' teaching actions.

Both school leaders and teachers commented that heavy reliance on textbooks does not support the development of ML. "You know, theory got too much attention in a way, and we put aside a lot, or some, of the practical. (...) Now it's the desk, and to sit and work with the textbook. I can see that the textbook steers too much, and we have to dare to put it away if we're to achieve that students get different skills, understanding, and competence" (School leader B1). According to both groups, interdisciplinary work and connecting different subjects is a way to develop ML. The teachers gave examples from their own teaching practice where they tried to connect mathematics with other school subjects. "When I teach arts and crafts I draw on mathematical knowledge, like with measuring, and to be accurate and... It's going to be a certain length in centimetres and we have to measure... If we're to find the middle... All the time" (Teacher C).

The school leaders connected the curriculum to ML through the basic skills framework. They also connected the basic skills to interdisciplinary work. The challenge is to implement it in the teaching practice in a natural way. "They have tried, with the curriculum, to include every subject in every subject. Or the basic subjects [Norwegian, English, and mathematics]. You know, to... That you should have basic skills in all areas, right. Reading, numeracy, in all subjects. However, I'm not sure that they have quite succeeded. You know... that it has worked. Because it's... I think that it's sometimes a bit artificial. Like, 'Oh, by the way, we have to remember to put in something about numeracy in this project about reading', right? Because we have to make sure that that goal is also reached. Instead of doing it in a more natural way" (School leader B2).

The teachers have different views on whether the curriculum supports teaching for ML. On one hand, the teachers connected ML to the basic skills, and in this way also found support for interdisciplinary work. "We work with numeracy in all subjects, like we work with reading and writing in all subjects. And we try to be more conscious that, well, in social subjects for example, with tables, that's mathematics. We work with it in Norwegian, with study techniques, different ways to work with a text. And a table is a text too" (Teacher A). On the other hand, the teachers conceived the curriculum as focused on subject knowledge, specifying what students should learn in the different subjects and not on connections between subjects.

Teacher Competences and Knowledge

Both school leaders and teachers expressed concern about the increasing subject knowledge requirements for teachers. However, they had differing concerns. School leaders were worried that increasing requirements might be at the expense of good student-teacher relations. "What I fear a bit in the future, if it turns out as suggested, is that we'll get, like in the old days, more specialised subject teachers. I'm not sure that will benefit the student, because for me it's important to attend to the whole student, to see the whole picture, and to see the responsibility the school has for the student as a whole, and not just the one subject. I can go in and teach mathematics, and out again, barely knowing who the student is. Right? And just go on to the next class" (School leader C2). In addition, the school leaders were worried that school will miss out on good teachers because admission to teacher education requires higher mathematics grades than Norwegian.

The teachers were concerned that as teacher education becomes more specialised it will be challenging for the teacher to work interdisciplinary. The inclusion of mathematical topics in other school subjects can be challenging if mathematics is not part of the teacher's curriculum. "If we're going to accomplish that (ML)... Well, then we need a minimum in our education where we have the opportunity to work interdisciplinary. (...) We get more and more specialisation within subjects. And that can be a good thing. But you don't get that interdisciplinary... if you just specialise within a subject. So I'm thinking that there has to be a connection between what we're supposed to do and the

education. (...) It's difficult to implement if you don't have the competence to work interdisciplinary" (Teacher C).

According to the school leaders, teachers can have different intentions with their teaching and one needs to be conscious about what one wants to accomplish. "First and foremost there needs to be an awakening with the mathematics teachers, right? What do I want with this subject, and should I focus on this (points to the ML definition)? Should I... So... It depends on the person. Some teachers teach... as long as they get through the syllabus. Then they're in the clear, they have done it, and the students' results are insignificant. We have this kind of teachers. And it's difficult to shift the focus and to give them another approach, right? What do I really want to accomplish, is it that you've done your job or that the students understand?" (School leader A2).

Teachers' competences and knowledge also concerned textbook use. The school leaders commented that a reason for using the textbook might be low confidence with one's own competences. "We are strange, us teachers. We don't trust that we know our subject. We think that the textbook decides what the students should know" (School leader B1).

Universality

Universality consist of text segments suggesting that everything is connected. Teaching, learning, mathematics, and the world are all part of a whole. The school leaders commented that the curriculum is wide and the ML definition is wide. It has to be this way, because they are supposed to cover all students. The school leaders see ML as comprehensive, something that concerns all subjects, not just mathematics.

The teachers commented that mathematics is everywhere all the time. It is important in several contexts. Hence, mathematics is part of a cultural-historical activity.

Both the school leaders and the teachers expressed that ML is a desired outcome of mathematics education and something that they work with. "It is what teaching, or a subject is all about. That's what I think" (School leader B2). "Isn't this in fact what we're doing?" (Teacher B). This suggests that ML is a goal for their teaching actions.

Discussion

In the previous section, I described five categories of school leaders' and teachers' rationales for ML. The remaining part of this article will focus on interpreting the rationales in the categories in light of the previous outlined theoretical perspectives.

From the perspective of this article, the object/motive is mathematics education and a goal and desired outcome of teachers' actions is students' ML. By objectifying cultural knowledge embedded in mathematical concepts, procedures, facts and tools, the students find themselves objectified. They find agency and individual capacity to make well-founded judgements, and they recognise the role mathematics plays in the world. In this way, ML contributes to subjectification. Learning is both a process of knowing and becoming. The teaching process consists in offering the students rich activities where they encounter cultural objects supported by meaning in tools and social interaction.

Teachers plan different learning actions related to learning goals in order to satisfy the object/motive. Their planning relates to the goals in the curriculum, but also to their own goals for teaching. In the teachers' competences and knowledge category, the school leaders expressed that teachers need to be conscious about their goals for teaching. The teachers' S- and I-rationales influence teaching.

I see ML as the optimal situation where the S- and the I-rationales coincide. ML contains aspects of mathematics theory (concepts, procedures, tools), and the ability to use the theory in various contexts. Local, national, and international tests and examinations assess students' level of ML. ML mastery will lead to good marks. This connects ML to the I-rationale in the sense that knowledge can be perceived as instrumental. At the same time, ML is supposed to be valuable and useful for the future and life outside of the classroom. Knowledge should meet personal and societal needs. Hence, ML is also connected to the S-rationale. Therefore, teaching for ML means that the mathematical instruments taught in school are the ones perceived as useful in social life.

The school leaders and teachers highlighted that students often question the use value of mathematical topics and tasks. This may indicate students' search for an S-rationale. The challenge for the teacher is to relate the topic to the student's conception of use value. Hence, the use value category is related to the S-rationale for teaching. The school leaders and teachers focused on the utility aspect, although relevance and appreciation of mathematical ideas might also answer to students' S-rationales. The school leaders and teachers expressed that students must recognise that mathematics is important for their total life situation, not just life in school. The meaning category is closely connected to use value and the S-rationale. To learn to understand mathematical language and to interpret and reflect upon mathematical results is also important in society outside of school. As stated earlier, communication is important for sharing ideas, developing strategies, and carrying out projects, and hence is an important part of activity.

The students have S-rationales for learning. Therefore, they seek justification and use value of mathematics. However, it is difficult for the teacher to identify students' activities outside the classroom. Teachers have to observe the students' actions to learn something about their activity. The teachers can provide students with situations intended to initiate constructive activities, but the individual decides whether they will engage in them. By focusing on the use value of mathematics and real world contexts, the school leaders and teachers try to create educational situations that relate to the students activity. They try to help students endow conceptual objects of mathematics and culture with meaning. The school leaders and teachers commented that mathematics is a tool for solving problems and for our understanding of the world. By focusing on use value, teachers' try to help students acquire the cultural-historical intelligence embedded in mathematics and relate it to students' S-rationales.

The I- and S- rationales are connected to the object/motive of the activity. If ML is a desired outcome and goal for teaching, the teacher has to help students discover the object/motive of their actions. Interdisciplinary work and real life contexts are regarded as approaches to teaching students the use value of mathematics. They provide a means of materialising the object/motive for the students. The use of specific situations and contexts are designed to help students understand the use of mathematics in general. They are particular instances of the general objects/motives. However, the design and selection of teaching materials and tasks may involve consideration of their attractiveness to the student. Attractiveness and attention may lead to the false assumption that the presence of the curriculum materials in students' consciousness will lead to the intended learning. In fact, the elaborations that such materials include may actually detract learners from engaging in the real activity, that is, in discovering the real object of their activity. The inner actions that are to be structured by the students require the abstraction from the materially objective content of the presentations, and this abstraction is more difficult the richer the content is (Roth & Radford, 2011). For example, students may perceive practical tasks intended to highlight specific mathematical content as a fun break from the regular teaching activities, without reflections concerning the mathematics involved. Hence, the students realise the task without taking up the object/motive. In that case, they do not expand their action possibilities in the intended way and do not learn what was intended. That is, the teachers have an S-rationale for teaching, and they try to relate it to the

students' S-rationales, but if the students do not recognise the social significance of the mathematical content (related to their S-rationales), the task may be pursued with an I-rationale or be reduced to a welcome break. This is an important issue when it comes to teaching practice, and teachers need to consider this when they plan their teaching.

The teachers do not agree on whether the curriculum supports teaching for ML. This may relate to their rationales for teaching. On the one hand, the teachers said that the curriculum does not support ML and that the curriculum focuses on the individual subjects. I suspect that the rooting of this statement comes from the competence goals, which are presented subject by subject. In this sense, the curriculum represents the I-rationale for teaching. The curriculum goals state the knowledge students should attain. Teachers base the students' grades and examination results on their level of goal attainment. Therefore, the teachers have to make sure that they work on all the goals. If the teachers do not teach according to the specific topics in the curriculum, the students will miss out on opportunities for education and employment later in life. Hence, the I-rationale also relates to the exchange value of mathematics in that the student has to learn because it will pay out in terms of grades and exams.

On the other hand, the teachers said that the curriculum supports ML through the basic skills framework. The basic skills are fundamental to learning in school, work and social life. The basic skills represents the S-rationale where teaching means teaching something more than just school knowledge. Teaching should prepare students for life in the real world, to help them reason, interpret, reflect, and solve problems. Hence, this relates to use value and meaning.

The school leaders' statement about not managing to crack the code with the curriculum may relate to teacher rationales and their perception of the curriculum rationales. The competence goals represent I-rationales for teaching and the basic skills represent the S-rationales. The challenge is to get the two to coincide, or at least to make the overlap as great as possible.

The school leaders pointed to challenges in planning actions and goals that satisfy the object/motive of mathematics education. The curriculum goals guide the teaching and therefore the textbook becomes an important mediating tool in mathematics teaching. The textbooks can make teachers' lives easier when it comes to what to teach and what tasks to use. However, there was agreement among the participants that extensive textbook use does not satisfy the object/motive of mathematics teaching. To reach the goals involved in mathematics education requires more than solving routine textbook tasks. A heavy reliance on textbooks does not seem to relate to students activities and does not meet their S-rationale for learning. Textbook use was related to teachers' competences and knowledge. The school leaders suggested that one reason for textbook use might be that teachers do not trust their own professional competences. This supports the claim about challenges regarding teachers' S- and I-rationales and their challenges to make them overlap. If the teachers perceive the curriculum as focusing on I-knowledge, it will require a great deal of effort from the teachers to be able to present it for the students as S-knowledge. If this is the case, they may struggle to plan actions that answer to the object/motive of mathematics education. This could suggest that they do not have agency, and have not subjectified the teacher role.

Another suggested reason for textbook use was that teachers might have different perceptions about the object/motive of mathematics education. This relates to the school leaders' comment regarding curriculum interpretation and the heavy focus on the competence goals. The teachers do not agree on whether ML is a goal for education supported by the curriculum. The curriculum has to be interpreted, and the teacher has to recognise this as an object/motive when working with the curriculum. S/he will also be affected by other objects/motives in their hierarchy of objects/motives, and the related actions and goals.

The teachers must be conscious of their own objects/motives and how they relate to teaching actions and goals.

Activities represent how a particular individual decides to act in her world, according to the make-up of this world. Individuals do not always agree on which Activities are the important ones to carry through, or how to carry out any particular Activity for which the goal is agreed" (Mellin-Olsen, 1987, p. 37).

Even if the teachers and school leaders agree on ML as an object/motive, and on the curriculum goals, they may not agree on how to act in order to pursue these. One teacher's I-rationale can be another's S-rationale. Teachers with different rationales for teaching will plan their teaching actions differently. Another difficulty for the teacher is that it will vary within the same class what will pass as I-knowledge and what will pass as S-knowledge. This means that the teachers sometimes have to choose which group of students to favour.

To plan for the actions to reach the goals that satisfy the object/motive, teachers need relevant education. The teacher requirements must work together with the object/motive of mathematics education. The teachers' concern about more subject specialisation can also be connected to their rationales. The subject requirements for teachers do not correspond with the teachers' rationales for teaching. The increasing focus on subject knowledge may lead to less competence in interdisciplinary work. Teachers want knowledge about learning, pedagogy, students, about all aspects of teaching and learning that are important for their school community. Thus, they have an S-rationale for teaching. They perceive requirements as an I-rationale, something they have to do to qualify for teaching. The school leaders and teachers worry that in the next step this may lead to a focus on teaching I-knowledge rather than S-knowledge.

Usually the I- and S-rationales work together (Mellin-Olsen, 1987). The rationales for teaching is a combination of the two. The mathematics content is important both in terms of examinations and in itself. Teachers have to cope with curricula designed to gather all the students under one umbrella of knowledge and to provide a coherent education. "This is the major contradiction of the comprehensive school and the most severe problem the didactician faces when he attempts to design a curriculum which applies to all pupils" (Mellin-Olsen, 1981, p. 357). Teachers' and students' sometimes differing rationales make teaching challenging. "There is no lack of exercises in which the pupils experience what the numbers and their relations stand for. But it is often a coincidence whether or not the use of mathematics proves to be of any significance to the pupil" (p. 361). This depends on whether the student takes up the object/motive of teaching, and if the teacher's rationales matches the students'.

The school leaders and teachers want to teach mathematics that has use value for the students. They want the students to learn and experience mathematical meaning and the universality of mathematics. However, it can be challenging to teach the use value, meaning, and universality of mathematics. The mathematics subject content is stated in curriculum goals, which school leaders and teachers do not always feel match the mathematics they want to teach. This influences their teaching practice. The close connection between the curriculum goals and the textbook also affect teaching practice. They feel they need to finish the textbook to make sure they cover the syllabus. Teaching practice also connects to school leaders' and teachers' competences and knowledge. They experience increasing subject requirements, but these do not always correspond with the school leaders' and teachers' own rationales and perceptions about competences and knowledge important for teaching. Hence, school leaders and teachers experience contradictory rationales for teaching with respect to

their understanding of ML. Their contradictory rationales arise according to their relation to the curriculum, the students, the textbook, and the policy makers.

Concluding Remarks

In this article, I reported school leaders' and teacher rationales for teaching with respect to their understanding of ML. Although ML does not have an equivalent notion in Norwegian language, the school leaders and teachers seem to recognise the ideas connected to ML. They see ML as a desirable outcome, and an object/motive, of schooling. However, the specific content, actions and goals of the teaching and learning activity cause some challenges. Hence, the school leaders' and teachers' experience contradictory rationales for teaching for ML when it comes to use value, meaning, teaching practice, teacher competences and knowledge, and universality.

School leaders and teachers connect ML to their S-rationales for teaching mathematics. The S-rationales concern the use value of mathematics. School leaders and teachers want the students to recognise the role mathematics plays in the world, and to use mathematics as a tool for solving problems. Meaning is also part of their S-rationales for teaching mathematics. They see mathematics language, reflection, and conceptual knowledge as important to understand the world.

I-rationales are also connected to teaching for ML. These are connected to teaching practice. Curriculum goals do not always support school leaders' and teachers' goals to teach use value and meaning. Additionally, the school leaders and teachers experience increasing requirements for subject specialisation. These are connected to teacher competences and knowledge. They perceive these requirements as I-rationales because they are not directed to helping them become better teachers and improve their teaching pedagogy, but rather as qualification.

The school leaders' and teachers' rationales are concerned with all areas of teaching, not just subject issues. There seems to be challenges related to the overlap between school leaders' and teachers' S- and I-rationales for teaching for ML. This may suggest that ML is difficult to both understand and teach in a way that is consistent with curriculum goals, policy expectations, their own convictions, and students' requests.

This study focuses on rationales at the school leader and teacher level based on interviews. It is a multiple case study based on a convenience sample and the results are not generalizable. However, this study of teachers' and school leaders' rationales may contribute to knowledge about the complexity of teaching and learning mathematics in general, and teaching for ML in particular.

To teach for ML, school leaders and teachers need to be conscious about their rationales for teaching and to see the totality of mathematics education in addition to the particular lesson. There is a need for further research on this issue.

Further research should also focus on teachers' rationales related to their teaching practice. This may provide fruitful insight on how to teach for ML, and possible challenges. It will also be of interest to study students' rationales for learning with respect to ML. Student learning is what teaching is all about. By gaining more insight in teachers' rationales related to their teaching practice and students' rationales for learning mathematics, we might get closer to answering the question of how to teach for ML.

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Secondary teachers' operationalisation of mathematical literacy

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Abstract:

This article reports a qualitative study of teachers' operationalisation of mathematical literacy. A model representing the multifaceted nature of mathematical literacy is used to analyse video recordings of mathematics teaching in three grade 9 classes. Analysis indicates that teachers' operationalisation of mathematical literacy appears to be fragmented and that teaching is focused on developing procedural fluency. Mathematical literacy was introduced in the Norwegian curriculum in 2006 and is considered a basic skill which should be developed across subjects. However, it appears that teachers still struggle to implement teaching to develop this competence.

Keywords: mathematical literacy, numeracy, mathematics education, teaching

Introduction

One goal of schooling is for students to acquire knowledge and competences that meet the needs of modern society. Mathematical literacy (ML) is a notion used to define the body of knowledge and competences required to meet the mathematical demands of personal and social life and to participate in society as informed, reflective, and contributing citizens (Geiger, Forgasz, & Goos, 2015). There are several notions related to ML, for example, numeracy and quantitative literacy. While the term numeracy is more common in English-speaking countries, such as the UK, Australia, and New Zealand, quantitative literacy and ML are used in the USA (Geiger, Forgasz, et al., 2015). Some use these notions synonymously while others distinguish between them. The meaning of numeracy varies from the acquisition of basic arithmetic skills through to richer interpretations related to problem-solving in real-life contexts (Geiger, Goos, & Forgasz, 2015). Quantitative literacy is associated with the requirements connected to the increasing influence of digital technology in society and the forms of thinking and reasoning related to problem-solving in the real world (Steen, 2001). Other perspectives, such as critical mathematical numeracy (e.g. Frankenstein, 2010), mathemacy (e.g. Skovsmose, 2011), matheracy (e.g. D'Ambrosio, 2007), are concerned with competences for challenging social injustices and for working to promote a more equitable and democratic society. Although these notions do not share the same meaning, their definitions share many common features (Goos, Geiger, & Dole, 2014; Niss & Jablonka, 2014).

ML is one of the competences assessed in the Programme for International Student Assessment (PISA), carried out under the auspices of the Organisation for Economic Cooperation and Development (OECD, 2012). PISA's reports that compare students' performance have been influential in shaping educational policies in several OECD countries, and curriculum developers have tried to reflect PISA competences in their national curricula (Breakspear, 2012). The first PISA results were a wake-up call for several of the participating countries. Norway, which considered itself having one of the world's best educational systems, performed (and has continued to perform) around the OECD average.

The Norwegian “PISA shock” led to a focus on the skills required to deal with life in school, work, and society. Pupils presently at school follow the curriculum introduced in 2006 (LK06). LK06 describes ML as a basic skill “fundamental to learning in all subjects as well as a prerequisite for the student to show his/her competence and qualifications” (The Norwegian Directorate for Education and Training, 2012, p. 5). ML should be integrated and developed in all subjects across the curriculum. More specifically, LK06 states that

Numeracy means applying mathematics in different situations. Being numerate means to be able to reason and use mathematical concepts, procedures, facts and tools to solve problems and to describe, explain and predict what will happen. It involves recognizing numeracy in different contexts, asking questions related to mathematics, choosing relevant methods to solve problems and interpreting validity and effect of the results. Furthermore, it involves being able to backtrack to make new choices. Numeracy includes communicating and arguing for choices by interpreting context and working on a problem until it is solved.

Numeracy is necessary to arrive at an informed opinion about civic and social issues. Furthermore, it is equally important for personal development and the ability to make appropriate decisions in work and everyday life. (The Norwegian Directorate for Education and Training, 2012, p. 14)

This broad definition of the Norwegian basic skill is similar to the ML definition in the PISA framework (see OECD, 2012, p. 25 for comparison). Hence, the development of students’ ML is a goal for mathematics teaching. Bolstad (2019) reported that Norwegian school leaders and teachers relate ML to the use-value of mathematics and the ability to use mathematics in contexts in personal, occupational and societal life. In their understanding, teaching for ML should involve practical and cross-curricular tasks and not rely too much on solving traditional textbook problems. However, the school leaders and teachers participating in the study experienced challenges in terms of finding suitable contexts in which students will experience the use-value of mathematics. Also, they do not feel competent enough to take a cross-curricular teaching approach, and the close connection between textbooks and curriculum makes it difficult to put the textbook aside.

In a similar study from Turkey, Genc and Erbas (2019) elicited seven categories related to teachers’ conceptions of ML. The teachers hold various but interrelated conceptions about ML as involving 1) formal mathematical knowledge and skills, 2) conceptual understanding, 3) problem-solving skills, 4) the ability to use mathematics in everyday activities, 5) mathematical thinking, reasoning, and argumentation, 6) motivation to learn mathematics, and 7) innate mathematical ability. The various conceptions may, on the one hand, indicate an ambiguous and confusing conception of ML, or it may, on the other hand, reflect richness in one’s understanding of its various aspects.

Teachers seem to recognise the contextual and applied aspect of ML. However, according to Gainsburg (2008), teachers count a wide range of practices as real-world connections. They make such connections frequently, but they are brief and does not require any thinking from the students. The study concludes that teachers’ main goal is to impart mathematical concepts and skills, and the development of students’ ability and disposition to recognize applications and solve real problems is of lower priority. To support ML, teachers should devise a teaching style that includes conventional and applied knowledge and create situations where formal knowledge and mathematical activities can be combined in understanding the subject matter (Höfer & Beckmann, 2009). Steen (2001) suggests that a cross-curricular approach to ML has greater potential to empower students to meet the mathematical demands of modern life than approaches that seek to develop ML solely through mathematics subjects. A cross-curricular approach means finding other curriculum areas in which mathematics can play an important part.

Another approach may be to draw on contexts arising from life outside school. Kaiser and Willander (2005) suggest that students should work with open problems with real-world contexts such as mathematical modelling problems to develop mathematical literacy. Modelling problems are open tasks in which students have to formulate a problem, develop a mathematical model, solve the problem, and interpret the solution in terms of mathematics and the problem context (Blum, Niss, & Galbraith, 2007). Modelling problems have gained increasing importance in mathematics education, and mathematical modelling is considered a key process in ML in the PISA framework and the Norwegian framework for basic skills (Nordtvedt, 2013). However, everyday mathematics teaching involves few modelling activities (Blum & Ferri, 2009). One reason may be that it makes lessons less predictable for the teacher. Teachers find it difficult to think on their feet if students give unexpected responses. Also, teachers report difficulty in anticipating students' potential responses in advance and identifying productive teaching strategies to overcome these (Jones & Tanner, 2008). Therefore, open problems and mathematical modelling require a high level of pedagogical knowledge and skill and a willingness to explore and respond to pupils' thinking. For many teachers, this represents a challenge to current practices, especially if they have a model of teaching which is based on knowledge transmission and practicing skills (Tanner & Jones, 2013).

A second reason for the challenge of mathematical modelling problems, which teachers experience, is that such problems require teachers' real-world knowledge. In Gainsburg's (2008) study, the teachers reported that the ideas for real-world connections mostly came from their minds and experiences. Therefore, teachers' understanding of how to apply mathematics in out-of-school contexts is an important factor for providing students with the learning experiences necessary to adapt the knowledge they learn in school to the outside world (Popovic & Lederman, 2015).

Theoretical framework

The lack of consensus about a definition for ML and related notions makes it difficult to ensure that the same constructs are being considered. In this research, a model developed by Marilyn Goos (see figure 1) is used because it is helpful in defining the complexity and scope of the domain under consideration. Goos' model is research-informed and designed to capture the richness of current definitions of ML (Goos, Geiger, & Dole, 2010). The model represents the multifaceted nature of ML and involves five elements: *mathematical knowledge, contexts, dispositions, tools, and critical orientation* (i.e. Goos et al., 2014). The elements in the model are interrelated and "represent the knowledge, skills, processes, and modes of reasoning necessary to use mathematics effectively within the lived world" (Geiger, Forgasz, et al., 2015, p. 614).

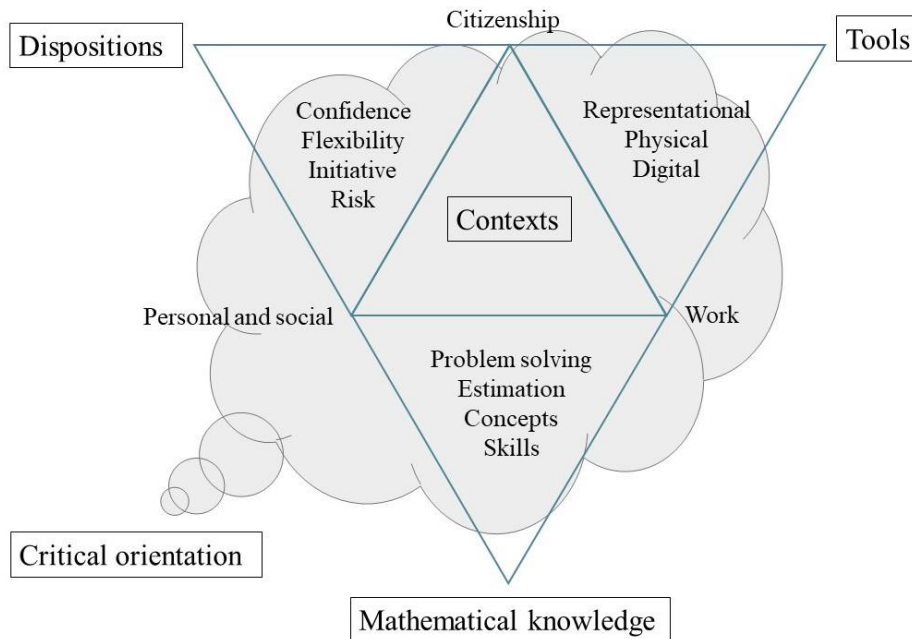


Figure 1. The multifaceted nature of ML, derived from (Goos et al., 2014)

In the following, the elements in the above model are interpreted in relation to relevant mathematics education research and in the context of teaching for ML.

Mathematical knowledge

ML requires mathematical knowledge. Researchers distinguish different kinds of mathematical knowledge. Hiebert and Lefevre (1986) discuss notions of *conceptual* and *procedural knowledge* in mathematics. Conceptual knowledge is characterised by as knowledge that is rich in relationships and connections between pieces of information. Procedural knowledge is made up of the formal language of mathematics and the algorithms and rules for completing mathematical tasks. The two notions are related to what Skemp (1976) denotes *instrumental understanding*, which he explains as “rules without reason”, and *relational understanding*; knowing what to do and why.

Kilpatrick, Swafford, and Findell (2001) formulated an illustration composed of five interwoven elements, or strands, to provide a framework for discussing the mathematical knowledge, skills, abilities, and beliefs that enable students to cope with the challenges of daily life. These elements are *conceptual understanding*, *procedural fluency*, *strategic competence*, *adaptive reasoning*, and *positive dispositions*. I relate four of them to mathematical knowledge, although they are also involved in the other elements of ML. Conceptual understanding and procedural fluency relate to the previously mentioned concepts. Strategic competence is connected to problem-solving and refers to the ability to formulate, represent, and solve mathematical problems. Adaptive reasoning concerns thinking logically about relationships among concepts and situations. It involves knowledge of justification and validation (Kilpatrick et al., 2001).

Contexts

Numbers and data play a significant role in modern society (Steen, 2001). ML is the competence to use mathematical content in real and various contexts. The ML model highlights three contexts, *personal and social*, *work*, and *citizenship*. Personal and social contexts arise from daily life with the perspective of the individual being central. Such contexts may involve personal finance, making decisions about personal health, and participation in different leisure activities. Work contexts arise from professional life. According to Noss, Hoyles, and Pozzi (2000) practitioners use mathematics in their work, but what they do and how they do it may not be predictable from considerations of general mathematical

methods. Particular occupations have specific requirements and tasks related to different kinds of mathematical knowledge, like financial transactions or drug administration. Citizenship concerns societal contexts arising from being a citizen, local, national, or global. Every major public issue depends on different types of data, for example understanding a voting system, social security funding, or international economics.

Wedegé (1999) distinguishes between two kinds of contexts in mathematics activity, *task context* and *situation context*. Situation context has to do with, for example historical, social, psychological matters and relations. It is a context for learning, using and knowing mathematics (i.e. in school, everyday life, workplace), or context of mathematics education (i.e. educational system, educational policies). Task context is about representing reality in tasks, word problems, examples, textbooks, and teaching materials. In this sense, context is often normatively employed, e.g. in curriculum documents as a requirement that teaching and teaching materials shall contain “real-life context” or “meaningful and authentic contexts”.

A typical way of connecting mathematics to real life is through task contexts like word problems. A word problem is a narrative that describes an artificial, pseudo-realistic situation that ends with a question requiring a number for the answer (Vos, 2018). According to Frankenstein (2010), word problems use real numerical data as “window dressing” to practice mathematical skills, and Vos (2018) argues that word problems are inauthentic and prevent students from experiencing the usefulness of mathematics. Vos proposes a model for analysing tasks concerning different aspects of authenticity; *authentic methods and tools* for solving the problem, *authentic problem context*, and *authentic questions*. However, authentic contexts from real life do not necessarily mean authentic questions that real people in the context would pose. Therefore, Vos highlights the importance of *certification*. Authenticity should be made explicit to the students. For an aspect in education to be considered as authentic, it requires an *out-of-school origin* that ensures that it does not originally have an educational purpose, and certification of provenance either physically or by an expert (Vos, 2018). These are important issues in analysing contexts involved in teaching for ML.

Dispositions

Dispositions are related to Kilpatrick et al.’s (2001) fifth strand, *productive dispositions*. To develop ML to the full requires positive dispositions towards using mathematics and an appreciation of mathematics and its benefits (Jablonka, 2003). Mathematically literate individuals possess willingness and confidence to engage with mathematics. Confidence is the opposite of “math anxiety”. Empirical studies of ML show that affective factors like high anxiety and low confidence affect students’ ML development (i.e. İş Güzel & Berberoğlu, 2010; Tzohar-Rozen & Kramarski, 2013). Also, affective factors such as self-efficacy, interest, and classroom environment influence students’ ML development (Aksu & Güzeller, 2016; Areepattamannil, 2014). People need the disposition to look at the world through mathematical eyes (Steen, 2001).

Problems occurring in everyday life usually do not come with an already existing solution. To figure out how to solve these problems requires one to think flexibly about mathematics and adapt the methods and procedures to the current context (Schoenfeld, 2001). Therefore, the competence to think creatively is an integral part of life and ML. Creativity involves taking initiative and risks.

Tools

Tools are essential in every aspect of life, for example, in communication, in education, in work life, and technology. ML concerns using mathematics as a tool to understand and uncover social and political issues. Tool use involves understanding how the use of, for example, statistical data can both deepen our understanding and change our perception of these issues (Jablonka, 2003).

Tools are important to enable, mediate, and shape mathematical thinking and are, therefore, an important part of ML. Tools are used for some purpose to achieve something (Roth & Radford, 2011). They can be physical items (e.g. measuring instruments or concretes), thinking tools (e.g. different forms of representations such as graphs and algebraic expressions), communicative tools (e.g. language, text, and speech), and digital tools (a calculator or computer software). They assist one to see something through something or someone else; in other words, the tools *mediate*. Tools can also mediate mathematical meaning. Mathematical tools help us describe, explain, and predict phenomenon, and to understand the world. Mathematics, written language, speech, gestures, and every sign system are communicative systems developed for different purposes.

Critical orientation

The model is grounded in a critical orientation to ML. ML is about recognising the powers and dangers of numbers. Mathematically literate people not only know and use efficient methods (formulate and employ), but also evaluate the results obtained (Goos et al., 2014). They evaluate mathematical solutions and reason about the context of the problem and determine whether the results are reasonable and make sense in the situation (OECD, 2012, p. 25).

Mathematically literate individuals can recognise the role mathematics plays in the world, for example, how mathematical information and practices can be used to persuade, manipulate, disadvantage, or shape opinions about social or political issues (Jablonka, 2003). In this way, ML involves the competence to use mathematics to make well-founded judgements and decisions in our personal, occupational, and societal life. Hence, mathematical reasoning is an important part of ML. To participate successfully in modern society, people need competence in ML to think through issues expressed in modern forms of communication. They also need to express themselves in these forms of communication to function as a well-educated citizen (Steen, 2001).

Frankenstein (2010) highlights the importance of understanding the meaning of numbers in real life. By using mathematics, one can illuminate how the world is structured. One can describe the world, reveal more accurate descriptions, understand the meaning of numbers used to describe, understand the implications hidden by numbers, and understand the meanings that numbers cannot convey.

The elements of the ML model can be related to the definition in the Norwegian curriculum cited in the introduction. The use of symbolic language and mathematical concepts, methods, and strategies can be related to the mathematical knowledge element. Tools for calculations, communication, and modelling relates to tools in the ML model. Contexts are described as situations in work, civic, and everyday life. Critical orientation concerns communication, validation, and evaluation of methods and solutions, and the ability to describe situations where mathematics is used. To describe and explore situations mathematically and deal with problems using mathematics also involve positive dispositions. Hence, the model can serve as a framework to analyse teaching in terms of ML in the Norwegian context. The importance of developing students' ML is recognised and prioritised internationally. Hence, the study is also of international interest.

Goos and colleagues have used the model in a series of research and development projects related to teaching ML across the curriculum (Geiger, Goos, et al., 2015). Still, few studies have used the model to analyse teaching in mathematics classrooms. If the model is suited for planning and evaluating teaching in other learning areas, it may well be suited for planning and evaluating ML in mathematics.

Even though developing students ML is deemed important, few studies have investigated teaching in this area. The purpose of this study is to investigate mathematics teaching for ML. It is believed that understanding teachers' operationalisation of ML will facilitate better support not only for students' ML development but also for teachers in terms of ongoing professional development.

Due to the vast body of quantitative data provided by the PISA studies, research on ML is predominated by quantitative studies (Haara, Bolstad, & Jenssen, 2017). There is, however, a lack of qualitative studies on teaching for ML, and it is argued that such research should be given priority (Haara et al., 2017; Sfard, 2014). In this article, I investigate teaching for ML in mathematics lessons. I address the following research question: How do teachers operationalise students' learning for mathematical literacy in lower secondary school mathematics classes?

In the next section, I outline the methods for data collection and my use of the model in the analysis.

Method

The research reported here is conducted within the interpretive paradigm. Social objects and categories are socially constructed and not objective facts beyond our reach and influence. Organisation and culture are products of negotiations between the parts involved, and are continually being established and renewed (Bryman, 2008). In this research, the classrooms are considered as social entities and constructions built up from the actions and perceptions of the social actors involved.

Sampling and subjects

Mathematics teachers plan and conduct teaching to enable students to obtain the goals stated in the syllabus. Teaching is influenced by the teacher's interpretation, meaning, and conceptualisation of the ideas and concepts in the curriculum and the textbook. As this study aims to investigate what teachers do in the classroom in terms of teaching for ML, data were collected through classroom observations.

Data were generated in three rural public schools in a county in Western Norway. The schools are situated in small communities where the population is homogenous in terms of cultural and social background.

The Norwegian school system is based on principles of equality of opportunity and individually adapted learning for everyone within an inclusive environment. Therefore, students are taught in mixed ability/attainment groups. The schools' total number of students on roll range from 220 to 370 and all three schools teach grades 1 through 10. All three schools cooperate with the author's university teacher education programme. They were therefore recruited for convenience and as an outcome of acquaintance. I contacted the school leaders, and they recruited the teachers. Criteria for selection of teachers were that they were teaching grade 9 mathematics and that they agreed to participate. In Norway, grade 9 students are aged 14-15 years. As PISA measures 15-year-olds' ML, it is reasonable to study teaching for ML to students within this age group.

To make video recordings in the classroom, I needed consent from the students and their parents. All parties involved received written information explaining my interest in studying teaching concerning concepts in policy documents. To ensure informed consent, I attended meetings with the teachers, the students and the parents. In case some students were reluctant to participate, an equivalent teaching alternative was arranged for them.

Data are composed of video recordings of classroom teaching. I observed, and video recorded three grade 9 mathematics teachers, one male and two females. I refer to the teachers as A, B, and C. Teacher A has 37 years of teaching experience and teaches mathematics, natural sciences, social studies, and Norwegian. There are 24 students in her class. Teacher B has 11 years of teaching experience. He teaches mathematics, natural sciences, and physical education. His class has 14 students. Teacher C has 15 years of teaching experience. In addition to mathematics, she also teaches natural sciences, social studies, religion, food and health, and arts and crafts. There are 28 students in her class. In mathematics,

Norwegian, and English lessons, class C is divided into two groups. During the fieldwork, the class was divided according to which students had consented to participate in the research. Therefore, teacher C had 18 students in her group during my visits.

Design and procedures

During my visits, I video recorded six mathematics lessons for teacher A, and five mathematics lessons for teachers B and C. The lessons varied in length from 45 to 90 minutes. I was a non-participant observer and did not intervene in the lessons, other than by being present. I instructed the teachers to plan and conduct the teaching as they would normally. I wanted to observe the teachers in their regular mathematics lessons.

I placed one static camera in the back of the classroom. This camera was focused toward the chalkboard but was intended to capture as much of the classroom as possible. The teacher wore a head camera. In this way, I could capture everything the teacher did and said, both to the whole group and to individual students.

Head cameras provide a unique opportunity to capture the teachers' perspective. They enable one to capture the participants' visual fields, get more in-depth insight onto the direction and timing of participant attention, and document participant actions. However, head cameras are also limited in that they can only capture a subset of participants' visual fields, potentially leaving activities under-documented (Maltese, Danish, Bouldin, Harsh, & Bryan, 2015). The use of head cameras is widespread in sports and studies of wildlife, but less prevalent in education research. By wearing head cameras, participants have a more active role in the data collection, and this, in a way, blurs the lines between participants and researcher (Blikstad-Balas & Sørvik, 2015).

My presence and the cameras, the head camera, in particular, may have affected both the teachers' and the students' behaviour. However, the teachers commented on several occasions that they forgot about the cameras, even the head camera. They also said that they could not notice any changes in students' behaviour. Although there were no evident indicators, I cannot be sure that the cameras and my presence did not have any effect on the teachers' and students' behaviours. According to Blikstad-Balas (2017), the issue of reactivity is somewhat overrated when it comes to the use of video research. She claims that there is no such thing as completely "natural data" and expecting participants in a video study to pretend that nobody is recording or hiding their awareness of the camera is unnatural.

Process of analysis

I used the previously outlined elements of ML to analyse the observations to investigate teachers' operationalisation of ML. Sections of recordings were analysed and categorised to the five elements of mathematical knowledge, dispositions, context, tools, and critical orientation. To be able to identify the different elements in the classroom, I developed descriptions of what the teacher might do to address the various elements in his/her teaching.

In developing students' mathematical knowledge, the teacher can ask students to explain and discuss various solution methods, verbalise connections among representations and concepts, to represent mathematical situations in different ways, and to invent their own procedures. For computational procedures to be efficient, accurate, and correct, it is important that the teacher focuses on students' understanding, and that students get time to practice. To develop flexibility in mathematics, teachers can expose students to non-routine problems for which they do not immediately recognise a suitable solution method. Students may also benefit from a focus on several approaches to these non-routine problems. It involves urging students to explain, justify, and prove solution methods, problem solutions, and mathematical results.

When working with tasks in context, the teacher can offer certification of authentic aspects of the task. Students could also be involved in discussing the authenticity of different aspects of the contexts. The contexts used may originate from life outside school and not originally have an educational purpose in terms of practising mathematical skills. However, the tasks can offer new insights and knowledge about the contexts in which they are situated, either real-world contexts or cross-curricular contexts.

“Developing a productive disposition requires frequent opportunities to make sense of mathematics, to recognize the benefits of perseverance, and to experience the rewards of sense making in mathematics” (Kilpatrick et al., 2001, p. 131). Teachers can encourage students to maintain positive attitudes toward mathematics. Expectations guide teachers’ interactions with students. Therefore, the interaction can focus on students’ capability of learning and the expectation of success. In this respect, it is important to note that success comes with hard work and learning orientation, rather than resulting from fixed abilities (Kilpatrick et al., 2001). To maintain motivation and appreciation of the value of what they are learning, teachers can help students to think about how they can apply what they are learning in different contexts. Other ways of motivating students can be to emphasise topics of student interest, communicate enthusiasm for the content, stimulate curiosity, provide opportunities to interact with peers, and introduce game-like activities.

When working with tools, it is important that the teacher helps students to see the relevant mathematical aspects involved in different tools, and makes links to concepts, symbols, and procedures. The teacher can model how tools can be used and encourage students’ tool use in solving problems and tasks.

All elements are embedded in critical orientation. The teacher can engage students in activities and discussions concerning real problems. Such activities may focus on verifying, following the logic of an argument, understanding how numerical descriptions originate, using calculations to restate information, using calculations to explain information, and using calculations to reveal unstated information. To develop critical orientation, teachers can pose open-ended questions and encourage students to pose their own questions. Teachers can bring up social, political, cultural, historical, environmental, and scientific issues and help students analyse and reflect on these.

A summary of the element descriptions and operationalisations used in the analysis is displayed in table 1 below:

Table 1. Descriptions and operationalisations of the elements involved in ML

Element	Description	Operationalisation
Mathematical knowledge	Mathematical concepts, skills, and problem-solving strategies	Explain and discuss methods, connect concepts and representations, practice skills, solve non-routine problems, justify
Contexts	The competence to use mathematical content in various situations in everyday life	Certification, authentic question, authentic methods, authentic context
Dispositions	Willingness and confidence to engage with mathematical tasks flexibly and adaptively	Encouragement, expectations, enthusiasm, stimulate curiosity and interest
Tools	The use of physical, representational, and digital tools to mediate and shape thinking	Model and encourage the use of digital tools, representations, and models

Critical orientation	To use mathematical information to make decisions and judgements, add support to arguments, and challenge an argument or position	Discuss, question, explain, reveal, evaluate, and validate issues in everyday life
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The elements are closely connected, and teaching may focus on several elements simultaneously. Therefore, some sections were categorised to more than one element. In the following section, I present an analysis of the observations of teaching concerning the five elements.

Observations

In all of teacher A's and B's lessons, the mathematical topic is equations. Teacher C teaches equations for two lessons, and the remaining lessons' topic is percentages. The three teachers do not use the same textbook.

Mathematical knowledge

In the observed lessons, classroom activity usually starts with simple examples or tasks, or repetition from the previous lesson. The task difficulty increases throughout the lesson. In their lessons, the teachers prioritise developing students' procedural fluency. The following excerpt is from teacher A's lesson 2. Writes the equation $x + 7 = 12$ on the chalkboard.

Teacher: What do we do here? (Student 1 raises his hand) Student 1, do you want to come to the board, or do you want to dictate?

Student 1 comes to the board and solves the equation without saying anything, and then returns to his seat.

Teacher: Good stuff. Only Student 1 solved the equation in this way on the test. The same way I do it because Student 1 and I are a bit lazy. We do not write more than we have to. What has he really done? Student 2?

Student 2: He has subtracted on both sides.

Teacher: Yes. He has... (writes $x + 7 - 7 = 12 - 7$ on the chalkboard) He has done this line mentally. When we have these kinds of tasks, and we are lazy, like me and Student 1, then we think, we want the x alone on one side, and move the numbers to the other side. We say, change the side, change the sign. And as I told some of you yesterday, you are going to solve the equations. Then you must show how you find the answer. You cannot do it just mentally and write only the answer.

The teacher praises Student 1 for solving the equation correctly, without asking for any explanation or justification of the procedure. Student 1 does not explain what he is doing or why while solving the equation, suggesting that this usually is not requested. Hence, the focus is on how to do it and to get the correct answer. The focus on procedures is also emphasised in the last line, where the teacher urges the students to write the whole solution on paper, even though it can easily be solved mentally, which also involves less work and is less time-consuming.

The rule "change the side, change the sign" is used by all teachers. The rule is not explained or discussed in any of the observed lessons. Teacher A consequently offers both the "lazy" method and the full solution of the equations, which might suggest that she wants the students to understand the procedure. On the other hand, the two solutions are presented as two different methods, when one of them is only a "shortcut". Also, there is no emphasis on why one subtracts the same number on both sides of the equals sign. The teachers do not demonstrate or discuss different or alternative solution

methods, like for example, guess-and-check, and the students are not encouraged to use or look for such methods.

The lessons contain a lot of questioning and answering. The teachers ask students questions about procedures to solve the tasks, answers to calculations during task solving, and facts about concepts and procedures. Questions typically asked when starting a new task are: "What do I have to do here?", "What is the first thing I must do?", and then "What do I do next?". Why-questions are related to procedures, like "Why do I write this?". The teachers' questions are also concerned with the answers to specific calculations, such as "x plus four x; how much is that?" (Teacher A, lesson 1). The questions are closed and, therefore, limited when it comes to discussions about solution methods, connections, and justifications.

In lesson 1, teacher B focuses on developing students' strategic competence by providing a strategy for attacking word problems. It is a kind of problem-solving procedure consisting of a list of points intended to help the students structure the information given in the tasks:

1. Read the task carefully
2. Find out what they are asking for
3. Find the best point of departure (who/what do we have least knowledge of?)
4. Form the equation
5. Check if the equation makes sense
6. Solve the equation to find the unknown
7. See if you have the answer to the task

The teacher refers to this list when they solve word problems and encourages the students to use it as well. Students are set to practice solving word problems individually or in pairs to practice their strategic competence. However, it seems that the preferred solution method is equations.

Another example of strategic competence is the rule, "change the side and change the sign". Demonstrating this rule is showing that one can replace or simplify initial procedures with more effective procedures, and this is part of having strategic competence. However, if the rule is not understood, and demonstrated without justification, it is pure instrumental knowledge.

The teachers work to develop conceptual understanding by helping students to draw on previous knowledge. For example, when working with equations with fractions, teacher B's students struggle solving them. The teacher writes the following on the chalkboard: $\frac{x+2}{x} + \frac{4}{x+2} =$

Teacher: What is the rule? I am going to add them; how do I do that?

Student: You need a common denominator

The students realise that they must find a common denominator, but they do not know how to find it. The teacher then writes the $\frac{1}{2} + \frac{1}{3} =$ on the chalkboard and asks, "What do I have to do here?" The students know straight away how to find the common denominator in this task. He then returns to the first two fractions.

Teacher: Here, then?

Student: The same.

The teacher tries to draw on the students' previous knowledge about adding number fractions to help them add fractions with unknowns. He tries to show them that the procedure is the same, even though there are x-es involved. The students see the connection, but they are still not sure how to do it. The

teacher refers to “the rule” and does not ask for justification, which points toward procedural knowledge. On the other hand, this is something the teacher expects the students to know already, and he is trying to relate it to what they are currently working on to help students see connections between mathematical topics.

The focus on isolated procedural and factual knowledge exemplified above does not indicate a focus on developing ML.

Contexts

Most of the activities and tasks in the observed lessons do not contain any contexts. They are used to practise procedures and skills. That is, they focus on mathematical knowledge and therefore are less concerned with developing ML.

However, in the last part of teacher A’s lesson 5 and the first part of lesson 6, and teacher B’s last parts of lessons 1 and 2, they focus specifically on solving word problems using equations. The lesson topics are “Equations and problem solving” (A) and “From text to equation” (B). The topics indicate that the solution method (equations) is the real focus and not the problem situation.

The word problems contain contexts connected to personal and social life. Teacher A used tasks from the textbook, such as:

Hanna buys 5 pizzas and 10 soft drinks for a class party. The total cost is 650 NOK. How much does one pizza cost if one soft drink bottle costs 18 NOK? Solve the task by equations. (Hjardar & Pedersen, 2014a, p. 56)

The task context is authentic in the sense that it is likely that someone buys pizzas and soft drinks for a class party. However, the question and method are not authentic. If someone were arranging a class party, they would likely know the price of both the soft drink and the pizza before buying it. If not, they would look at the price list or ask the cashier.

Teacher B displays the tasks on a PowerPoint slide. They are not collected from the textbook, but the structure is similar. For example:

The ages of two brothers and a sister add up to 35 years. The oldest brother is twice the age of the sister. The youngest brother is three years older than the sister is. Altogether, they have 12 arms and legs. How old are each of them? (Teacher B, lesson 1.)

Here, context and question are authentic. It is common wanting to know someone’s age. However, usually when asking, one will get the answer straight away. Hence, the method is not authentic. Other examples of contexts concern finding an amount of money, or the number of fish caught. In general, the contexts are (at least to some extent) authentic, but questions and methods are not. There are no certifications.

On some occasions, the teachers use contexts to help students understand how to perform calculations. For example, in teacher A’s lesson 4, a student is unsure how to calculate $-3x + 5x$. Teacher A says: “X is chocolate bars. You owe me three chocolate bars. You get five from your mother. How many do you have left?” Nothing about this context is authentic. Indicating that x is a subject may damage students’ conceptual understanding and cause misconceptions related to students’ mathematical knowledge.

In lesson 1, teacher C uses the context of debt when explaining subtraction of negative numbers to a student who is unsure how to perform the calculation: “You lack five kroner, and then you lack one

more. How much do you lack then?" She constructs a narrative to fit the symbolic expression. The lack of money is not unusual. In this case, the lack is not connected to a specific situation, and it is difficult to evaluate authenticity. The teacher uses the context to support the student's understanding of negative numbers and mathematical knowledge.

Teacher C's three lessons on percentages contain everyday life contexts. The tasks concern situations from personal and social life. They concern sale and discount, salary increase, and comparison of prices, for example, in lesson 3:

Anne's annual salary increased from 276000 NOK to 285400 NOK. How many per cent was the salary increase? (Bakke & Bakke, 2006, p. 114)

In this task, context, question, and method are authentic. However, it is sometimes more interesting to know the amount rather than the per cent, at least from a personal point of view.

In the observed lessons, issues of authenticity concerning the contexts are only commented on two occasions, both in teacher B's classroom. In a task about a pasture (which will be cited in the Tools section), the teacher comments that "This is a problem that many horse owners have", in a humorous tone, followed by laughter, suggesting that this is just a joke. Also, in lesson 1, after obtaining the answer to a task about the price of a pack of chewing gum, a student comments to the teacher that "Chewing gum is not that cheap". These two comments are not subject to further discussion, suggesting that real-world aspects are not of real concern.

In lesson 4, teacher C refers to a discussion they had earlier about Black Friday sales. She talks about how to use percentages to evaluate if it is a good buy. She also talks about a web page that compares prices on commodities in different stores. This web page contains graphs that show how the prices have changed, and the teacher explains how this helps evaluate a buy. In lesson 5, teacher C talks about two newspaper articles, one that compared prices in general stores and another that compared the municipal taxes in neighbouring municipalities. She uses these newspaper articles as examples of how comparisons of percentages are used in personal and social contexts. In both these examples, the teacher provides certification of how percentages are used in daily life. However, the teacher is doing most of the talking, and there are few opportunities for the students to explore the contexts themselves.

Dispositions

To help students develop positive dispositions, the teachers rely heavily on communication. They talk to the students about how they are doing, praise them, and try to encourage and motivate them. For example, at the end of a conversation with the students, they very often say "good" or "well done". In lesson 3, teacher B comments to the class that

I think that the way you work now, that you discuss, you compare, you stop when you feel that "I cannot get any further, this cannot be right, something's wrong here". It is excellent, the way you work now. No-one is sitting there and just "I don't know anything about this".

The teacher is commenting on their strategic competence, which complies with what he expects of them. He wants them to be confident, to reflect on what they do, and not give up if the first strategy does not work.

The teachers try to encourage students if they are frustrated, like teacher B in lesson 1: A student asks for help, thinking that she is unable to solve the task. She erases what she has written and solves the task again with the teacher standing beside her. She discovers that what she had erased was correct and says, "That is in fact, what I had written." The teacher replies, "Yes, it is exactly what you had

written. I have said it before; you have to trust yourself!" Similarly, teacher A comments: "You know, the point is that if you sit down and think that 'This is too difficult, I will never make it, this will never work', then you get negative thoughts, and then it gets difficult."

Teacher A encourages the students to come to the chalkboard and show their solutions. By agreeing to share their solutions in front of the whole class, the students show confidence in the work they have done. Also, focusing on the students' solutions may serve as an inspiration for the rest of the class. It may also be a factor in developing an inclusive class environment. The students take risks by showing their solutions to the rest of the class, but the students' methods may open up for further discussions on the topic, for example, if the solution on the chalkboard contains an error, like in teacher A's lesson 4: A student instructs the teacher how to solve the equation $64 = 4x^2$, and gets the answer $x = \pm 4.24$. When they test the answer, the left side is not equal to the right, and they conclude that the solution must be wrong.

Teacher: What do we do? If these two are not equal? If I gave you a test and you got this answer, what would you do? What would you do?

Student: Try again.

Teacher: Try again. How many of you would think that 'Oh, I cannot do it', and moved on to the next task, without trying again? How many of you would do that?

In this situation, the teacher tries to focus on flexibility. If the solution is wrong, the students need the confidence to try again and flexibility to adapt the method to get it right. The students also need to see that there is nothing wrong with not getting it right the first time as long as they do not give up. Not giving up is also connected to strategic competence.

Teacher C's comments regarding Black Friday sales and the local taxes referred in the previous section are also ways to foster positive dispositions toward ML. Trying to relate to students' interests and give examples of how mathematics is used in everyday life may help them to see mathematical knowledge as something useful and worthwhile and motivate them to engage in the subject.

As mentioned earlier, the teachers spend much time demonstrating tasks on the chalkboard. Procedural fluency may contribute to developing students' confidence in mathematics because it provides them with a strategy for obtaining the correct answer. Hence, demonstrating tasks to develop procedural fluency is also a way to develop positive dispositions.

Tools

Communication is an important tool in the teachers' lessons. The teachers talk a lot, explaining concepts and demonstrating procedures. In this way, language serves as a tool to mediate mathematical knowledge. Particularly teacher B stresses that students should discuss with each other. He encourages the students to talk about what they are doing. When they are working individually, he approaches them and tells them to talk. However, the talk mainly concerns procedural steps and what to do next to solve the tasks. The students are not asked to justify or explain procedures to each other. Therefore, it seems as if the talk is oriented towards developing students' mathematical knowledge. Also, talk may support students' positive dispositions. Students may become more motivated by being allowed to work together. It can also be easier to ask questions or demonstrate the solution to the whole class if the problem has been discussed with a peer first.

There are examples from all three teachers where they use drawings in their modelling of a task solution. The drawings serve as representational tools to mediate thinking to represent the situation with symbols and to solve the equation. The example below is from teacher B's lesson 2.

In a pasture, the length is three times the breadth. The perimeter is 240 meters. What is the area of the pasture?

A group of students start discussing how to solve the task. The teacher says, "Here we have to start with discussing the geometrical figure. It may be smart to make a drawing." He walks around the classroom listening to the students' discussions. He sits down with two students

Teacher: We have a pasture, and it has length and breadth. What figure is that?"

Student: Rectangle.

Teacher: (Nods) Draw a rectangle. (The students draw a rectangle.) What is the length and what is the breadth? (The student points on the drawing). If the breadth is x , what is the length?

Student: Three x .

The teacher has assisted the students in representing the task information with a drawing of a geometrical figure. He directs the students' attention to the length of the perimeter and suggests that they start discussing the formula for finding the perimeter. Then he moves on to another group of students. The concepts breadth, length and perimeter refer to a shape, and the students must draw on their conceptual knowledge to connect the concepts with the figure. The drawing of the figure makes it easier to formulate the equation to solve the task.

The word problems are in themselves examples of representations. Here, a situation or problem is stated with written language. The students are supposed to represent the situation or problem using mathematical symbols. As mentioned, drawings can mediate students' thinking. However, a gradual process from written language to mathematical symbols is also possible, such as the following example in teacher A's lesson 6. The task is:

Three buckets have different colours and volumes. Five blue buckets have the same volume as three red buckets. Two yellow buckets and one blue bucket have the same volume as one red bucket. How many yellow buckets have the same volume as one blue bucket? Solve using equations. (Hjardar & Pedersen, 2014b, p. 40)

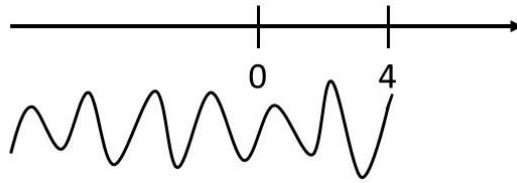
The teacher says, "Five blue equals three red, and two yellow plus one blue equal one red." At the same time, she writes:

$$5b = 3r$$

$$2y + 1b = 1r$$

"Five blue buckets" in the task is "five blue" in the teacher's oral representation and "5b" in her written mathematical symbols. "Have the same volume as" in the task is "equal" in the teacher's oral representation and is represented written with the equal sign. The symbols are expressed in natural oral language to structure the information given in the tasks. Gradually, they move towards formal mathematical notations to solve the task. Natural language serves as a tool to help the student formulate the task using mathematical symbols to solve it.

In lesson 6, teacher A demonstrates how to solve inequalities. She solves $x + 4 < 8$ on the chalkboard. To mediate students' understanding of what the solution $x < 4$ means, she draws a number line on the chalkboard:



The number line is a representation of which the students are familiar. The intention is to help students understand the meaning of the solution, that is, to show that there are several values for x , which makes the inequality true. The teacher is using the representation as a tool to develop their conceptual understanding and mathematical knowledge.

Critical orientation

The activities in the lessons mostly concern tasks without contexts. The tasks' focus is to develop procedural fluency. Therefore, the teachers do not emphasise the role mathematics plays in the world.

However, a few situations are worth mentioning (see also the Context section above). In lesson 4, on percentages, teacher C refers to an earlier discussion about Black Friday. She talks about the importance of knowing percentages to avoid being tricked by the stores. She explains how the stores often raise prices before the sales so they can advertise big discounts, and that it is smart to compare prices in different stores before buying.

In lesson 5, teacher C refers to a newspaper article they had discussed earlier in social studies. The article compared municipal taxes in neighbouring municipalities. There were great differences between the municipalities, and they had discussed different reasons for this. The teacher did most of the talking, but with these two examples, she tries to show students how mathematical information is used to make decisions and judgements. In this way, critical orientation is, to some extent, involved in these two lessons. However, in general, the teachers do not pose open-ended questions concerning social, political, cultural, historical, environmental, or scientific issues for the students to analyse and reflect upon.

A summary of the results is presented in Table 2. In general, it seems that the main objective is to develop mathematical knowledge, and the other elements serve as a means to this goal. In the remaining part of the article, I discuss these results and make concluding remarks.

Table 2. Summary of results

Element	Results
Mathematical knowledge	The teachers explain and practice methods to develop procedural fluency. They try to connect concepts and representations. Teacher B introduces a strategy for problem-solving, but students are not encouraged to develop their own strategies. There are no observations where teachers include non-routine problems or discuss or encourage the use of alternative methods. They are also not observed requesting students' justifications and explanations.
Contexts	Few tasks involve contexts. Task contexts from personal life are used, mainly through traditional word problems. The task contexts are authentic, but certifications are not observed. Some certified contexts are provided by the teacher as examples and do not involve any observed student activity. The task questions and methods are not certified and are rarely authentic.

Dispositions	There is extensive use of praise and encouraging comments directed at how students work and their task solutions, aimed to develop confidence. Comments aimed at relating mathematics to students' interests are made. Teacher enthusiasm and stimulation of students' curiosity are not evident.
Tools	The teachers use language to mediate knowledge. The teachers model and encourage the use of visual representations. There is no observed use of digital or physical tools.
Critical orientation	There is no evidence of numbers presented in and derived from word problems being discussed, questioned, evaluated, or validated. Teacher C talks about how mathematics is used to make decisions. However, there are no observations where students are asked to use mathematical information to make decisions and judgements, add support to arguments, and challenge an argument or position themselves.

Discussion and conclusions

In this article, I study how teachers operationalise students' learning for ML in Grade 9 secondary school mathematics classes in terms of five elements. As noted in the outline of the ML model, the elements are closely connected. That means that one can be addressing several elements at the same time. This is also demonstrated in the previous section, where several examples involve more than one element. What seems to be recurring is the emphasis on mathematical knowledge. Contexts, dispositions, tools, and critical orientation appear to be mainly oriented towards developing aspects of mathematical knowledge, and not so much for the development of knowledge of contexts, dispositions, tools and critical orientation. The findings support those by Gainsburg (2008), suggesting that teachers' main focus is to impart mathematical concepts and skills.

Previous research has demonstrated that teachers relate teaching for ML to teaching the use-value of mathematics through problem-solving tasks and practical tasks with contexts related to other curriculum subjects or everyday life (Bolstad, 2019; Genc & Erbas, 2019). Teaching for ML involves the challenge of promoting students' mathematical knowledge at the same time as providing the conditions under which they learn to use mathematics in context. As summarised in Table 2, there is little emphasis on creating opportunities for students to learn to use mathematics in context. The few tasks that involve contexts use them merely as "window dressing" to practice a mathematical skill. Word problems in mathematics can, therefore, sometimes appear nonsensical, leading to jokes like "Maths, the only place people can buy 64 watermelons, and no one wonders why." Such issues affect students' dispositions. According to Vos (2018), students are more motivated by authentic questions than by authentic contexts. The contexts used in classrooms should, therefore, be selected with care to help students appreciate how understanding numbers and calculations can illuminate meaning in real life.

Teaching ML across the curriculum is emphasised by researchers (Geiger, Goos, & Dole, 2014; Steen, 2001), teachers (Bolstad, 2019), and policymakers (The Norwegian Directorate for Education and Training, 2013). In the observed lessons, there is little evidence of such cross-curricular work. This may be surprising as the three teachers are all experienced professionals and they teach several other subjects where they could find suitable contexts. When investigating teachers' recognition of mathematics in museum exhibits, Popovic and Lederman (2015) found that the teachers searched for explicitly represented concepts such as numbers, graphs, and shapes. Only after instruction from the researchers they started looking for exhibits that would make abstract mathematical concepts more concrete. In the same way as for students, we cannot expect teachers to make real-world connections out of the blue. Hence, the lack of meaningful contexts in the observed lessons may be explained by teachers' lack of experience with how to teach mathematical knowledge in meaningful contexts for

example through a cross-curricular approach (Steen, 2001) or mathematical modelling activities (Blum & Ferri, 2009).

Another related point involves the use of textbooks. Practical non-textbook tasks, problem-solving, mathematical modelling, and open-ended problems are related to ML development (Blum et al., 2007; Bolstad, 2019; Genc & Erbas, 2019; Kaiser & Willander, 2005; The Norwegian Directorate for Education and Training, 2013). The teachers in the study rely heavily on the textbook, and the students spend most of the time practising textbook tasks. These tasks do not fall under the categories of problem-solving or modelling. Although textbook tasks may provide opportunities for engagement in meaningful contexts, it requires that the teacher takes these opportunities. For example, the task about salary increase referred in the Observations section does not show meaningful use of mathematics. It only serves to practice mathematical skills. Discussing the reasons for posing such a question could provide meaningful reflections on issues from real life. For example, a person could find it interesting to calculate the percentage salary increase to compare it with the national average and retail price index. A comparison of salary increases may lead to investigations concerning who gets the larger increase, and what is fair. These kinds of investigations and discussions involve social, political, and environmental issues. By focusing on such issues, students get the opportunity to understand how numbers can both conceal and reveal descriptions of the world. Such understanding is connected to critical orientation.

In the ML model, all the elements are grounded in critical orientation, making critical orientation an overarching construct. Critical orientation is hence an important part of using mathematics in contexts. Critical orientation and contexts appear to be the most challenging elements to implement (see Table 2). Geiger, Forgasz, et al. (2015) also report that implementing activities that integrate a critical orientation is challenging. Their research shows that teachers struggled with this, even after two years of engagement with the idea, despite an indication from the teachers that developing a critical orientation is an important goal for schooling and one worth pursuing. So, even when there is a desire to embed a critical orientation within ML tasks, time, opportunity, and experience are still necessary to develop rich tasks that best support the implementation of this aspect. It is therefore not surprising that the teachers in my study did not integrate critical orientation in their lessons. Critical orientation involves complex and demanding issues, both for the teacher and the students because mathematics in real life are not as black and white as in traditional word problems in textbooks.

The heavy reliance on textbooks may also be connected to the lack of experience with cross-curricular and modelling tasks discussed earlier. In addition to the real-life aspect of such tasks, they also involve unpredictability and uncertainty. One never knows what issues or strategies students take up. The topics may be far from the teacher's knowledge area, and the mathematical content may not comply with the curriculum. Therefore, the textbook provides a structured and predictable plan that ensures that the curriculum content is covered.

The observed lessons involve a textbook guided and rather traditional teaching approach, which is different than what is recommended in the research literature on developing ML. Tanner and Jones (2013) confirm the difficulty teachers have in moving on from traditional teaching practices. Changing expectations of teaching and learning is complex, and it takes time. On the other hand, some teachers may not have the will or see the need to change practice. However, teaching for ML calls for something else than the traditional teaching of mathematics (Haara et al., 2017; Steen, Turner, & Burkhardt, 2007). The inclusion of real-world problems and a cross-curricular approach requires a different way of thinking about teaching. However, teachers feel that they do not have sufficient knowledge of how to work interdisciplinary (Bolstad, 2019) and how to teach modelling (Steen et al., 2007). Therefore, in-service teachers and pre-service teachers must get the necessary support in their professional

development, for example, from professional development programmes and teacher education courses. In that respect, the ML model can serve as a useful tool for teachers in their planning.

Based on the analysis and observations reported in the previous section (see Table 2), the conclusion is that in the observed lessons, teachers' operationalisation of ML appears to be fragmented rather than integrated. Even though the ML elements are involved in the lessons, the connections between them are not apparent. In other words, the teaching is concerned with the elements in isolation and not holistically to develop ML. Students may develop competences connected to all five elements, but they are left to make the connections between the elements on their own, and as a result, to develop the ability to use mathematics in real-world situations on their own. In that sense, one can question whether the teachers are operationalising students' learning for ML. Several suggestions have been made in this article regarding possible reasons for teachers' challenges.

Teaching for ML requires an integrated approach, connecting the elements in mathematical activities. This does not necessarily require drastic changes from the teacher, but rather a slight move of emphasis and awareness.

Closing remarks

It is important to note that the research in this article is based on a few teachers' teaching in a limited number of lessons. The topics are also limited. Visiting these teachers at another time could have given a different result. The teachers' reflections regarding the lessons could also have provided a more nuanced picture of their teaching. Therefore, the analysis is not a characteristic of the teachers, but of the specific teaching in the specific classrooms at specific times as observed by me.

It is also important to emphasise that it is not expected that teachers teach across the curriculum or modelling all the time. Knowledge of procedures and facts are essential elements in mathematics education. The challenge is to connect these procedures and facts to the other elements to make teaching for ML more coherent and connected.

Therefore, there is a need for further research on how to assist teachers in implementing teaching for ML in mathematics classrooms. Research on teachers' understanding of ML and reflections on how to teach for it can be a starting point. A study of students' experiences of teaching for ML may also prove useful in this respect.

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Lower secondary students' encounters with mathematical literacy

Abstract:

World-wide, there has been increased emphasis on enabling students to recognise the real-world significance of mathematics. Mathematical literacy is a notion used to define the competencies required to meet the demands of life in modern society. In this article, students' encounters with mathematical literacy are investigated. The data comprises interviews with 22 students and observations of 16 mathematics lessons in three grade 9 classes in Norway. The analysis shows that students' encounters with mathematical literacy concern specific mathematical topics and contexts from personal and work life. Students' encounters with ML in school is characterised by an emphasis on developing mathematical knowledge within the school context.

Keywords: mathematical literacy, numeracy, the theory of objectification, mathematics education

Introduction

One goal of schooling is for students to acquire knowledge and competences that meet the needs of modern society. Mathematical literacy (ML) is a notion used to define the body of knowledge and competences required to meet the mathematical demands of personal and social life and to participate in society as informed, reflective and contributing citizens (Geiger, Forgasz, & Goos, 2015). ML has many related concepts, such as *numeracy* and *quantitative literacy*. While the term numeracy is more common in the UK, Australia, and New Zealand, quantitative literacy and ML are used in the USA (Geiger, Forgasz, et al., 2015). Some use these notions synonymously while others distinguish between them. The meaning of numeracy varies from the acquisition of basic arithmetic skills through to richer interpretations related to problem-solving in real-life contexts (Geiger, Goos, & Forgasz, 2015). Quantitative literacy is associated with the requirements connected to the increasing influence of digital technology in society and the forms of thinking and reasoning related to problem-solving in the real world (Steen, 2001). Other perspectives, such as *critical mathematical numeracy* (e.g. Frankenstein, 2010), *mathemacy* (e.g. Skovsmose, 2011), and *matheracy* (e.g. D'Ambrosio, 2007), are concerned with competences for challenging social injustices and for working to promote a more equitable and democratic society. Although these notions do not share the same meaning, their definitions share common features in that they stress awareness of the usefulness of, and ability to use, mathematics in different contexts (Niss & Jablonka, 2014). Typically, they do not discriminate between contexts from employment and everyday life, but the main orientation appears to be toward everyday life

and citizenship (Gravemeijer, Stephan, Julie, Lin, & Ohtani, 2017). In this article, ML is conceptualised in a broad way, composed of the others.

ML is one of the educational competencies emphasised by the Organisation for Economic Cooperation and Development (OECD). Curriculum documents around the world have been restructured to include this competence (Stacey & Turner, 2015). For instance, in Norway, ML is one of five basic competences to be developed across school subjects. The Norwegian curriculum describes ML as

applying mathematics in different situations. Being numerate¹ means to be able to reason and use mathematical concepts, procedures, facts and tools to solve problems and to describe, explain and predict what will happen. It involves recognizing numeracy in different contexts, asking questions related to mathematics, choosing relevant methods to solve problems and interpreting validity and effect of the results. Furthermore, it involves being able to backtrack to make new choices. Numeracy includes communicating and arguing for choices by interpreting context and working on a problem until it is solved.

Numeracy is necessary to arrive at an informed opinion about civic and social issues. Furthermore, it is equally important for personal development and the ability to make appropriate decisions in work and everyday life. (The Norwegian Directorate for Education and Training, 2012, p. 14)

The current worldwide emphasis on ML is based on the recognition that students are completing compulsory education without the mathematical skills required in life and work. Formal mathematics alone is not helping them meet these demands (Liljedahl, 2015). In several places, (i.e. Popovic & Lederman, 2015; Vos, 2018) students' view of mathematics is described as detached from reality, and the most frequently asked question in mathematics classrooms is "When will we ever use this?"

Some researchers discuss the purposes of mathematics education, but few research studies are concerned with the purpose of mathematics from the students' perspective (Nosrati & Andrews, 2017). Students can contribute with valuable insider perspectives on mathematics education and there is a need for more research concerning the issue. Such research must also consider the environments in which students learn (Mellin-Olsen, 1981). Situations may occur where students are unable to place the learning situation in any other context than that of school. In such cases, years of mathematics studies may seem to have unclear purposes. Therefore, research needs to consider the nature of students' learning processes, i.e. in terms of teaching, tasks, culture and society.

¹ In the English translation of the Norwegian curriculum, the word numeracy is used. However, the PISA framework (OECD, 2012) has influenced the description of this competence and resemblances can be found between the two descriptions. Therefore, in the Norwegian context, and for the purpose of this article, the two notions are taken to mean the same.

This article reports from a study that investigates students' learning regarding the purpose of mathematics in terms of connections between mathematics and real life. The aim is to investigate how classroom activities are connected to students' perception of the contexts in which they need mathematics and their learning of ML. The research question addressed is

What are the characteristics of students' encounters with mathematical literacy?

Students' learning processes are viewed as situated within social, historical, and cultural forms of thinking and doing. Therefore, the study is framed within a cultural-historical theory of mathematics teaching and learning. The theoretical perspectives are presented in the following section.

Theoretical perspectives

The theory of objectification

From the works of Vygotsky and Leont'ev, Luis Radford has developed the theory of objectification (TO). TO focuses on how students and teachers produce knowledge against the backdrop of history and culture, and on how they co-produce themselves as subjects in general and subjects in education in particular.

The TO is inscribed within an understanding of mathematics education as a political, societal, historical, and cultural endeavor. Such an endeavor aims at the dialectic creation of reflexive and ethical subjects who critically position themselves in historically and culturally constituted mathematical practices, and ponder and deliberate on new possibilities of action and thinking. (Radford, 2016, p. 196)

In TO, knowledge involves potentiality and actuality (Radford, 2015). Potentiality means general and abstract interpretations or actions resulting from cultural and historical ways of thinking and doing, for example, general knowledge about doing calculations. Actuality means that these general interpretations and actions are actualised through something concrete and noticeable, for example, doing a specific calculation. Therefore, in TO, knowledge is not something one possesses but rather something one *encounters*.

Learning happens when the general interpretations are actualised and, in this way, becomes part of the individual's consciousness. That is, when, through doing the specific calculation, the individual encounters and becomes aware of the general knowledge about doing calculations. The process of recognising such encounters with knowledge is what Radford terms processes of objectification (Radford, 2015).

The process of subjectification is closely connected to processes of objectification (Radford, 2016). As the individual becomes more knowledgeable, s/he also changes and develops as a person. Therefore, we are learning because we are becoming, and we are becoming because

we are learning. In the research reported here, developing ML is considered as both learning and becoming.

Mathematical literacy

Merrilyn Goos has developed a model designed to capture the richness of current definitions of ML and related concepts (Goos, Geiger, & Dole, 2014). The model has been used in professional development programmes as a tool to plan ML teaching. The model represents the multifaceted nature of ML and involves five interrelated elements: *contexts*, *mathematical knowledge*, *tools*, *dispositions*, and *critical orientation*. The model is presented in Figure 1 below.

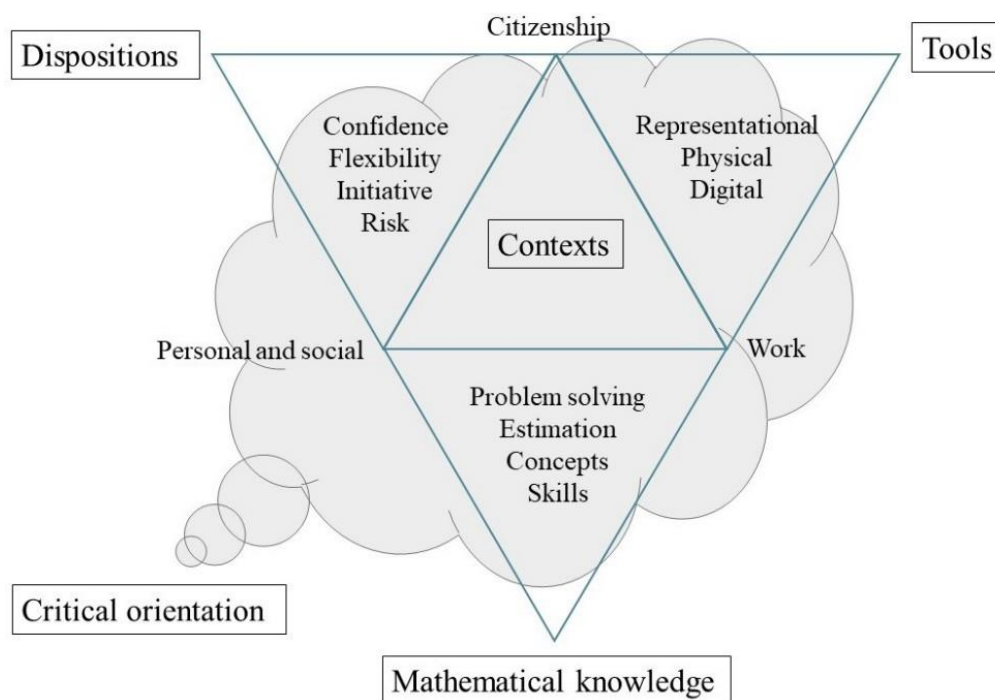


Figure 1. A model of ML, derived from Goos et al. (2014, p. 84)

Contexts are placed at the centre of the model because ML concerns the ability to use mathematics in contexts. Goos et al. (2014) highlight three contexts in which ML is important; personal and social life, work-life, and citizenship. Personal and social contexts arise from daily life with the perspective of the individual being central, involving, for instance, personal finance and participation in different leisure activities. Work contexts arise from professional life. People use mathematics in their work, but what they do and how they do it may not be predictable from considerations of general mathematical methods (Noss, Hoyles, & Pozzi, 2000). Occupations have specific requirements and tasks related to different kinds of mathematical knowledge, such as financial transactions or drug administration. Citizenship concerns societal contexts arising from being a citizen, local, national, or global. Every major public issue depends on different types of data, for instance, in understanding a voting system or international economics.

Mathematical knowledge is composed of knowledge of mathematical concepts, procedures, and facts, and using these in problem-solving strategies and estimations to describe, explain and predict. Hence, a part of being mathematically literate means being able to perform calculations and use procedures and algorithms successfully (Steen, Turner, & Burkhardt, 2007).

Tools can be physical items (e.g. measuring instruments or concretes), thinking tools (e.g. different forms of representations such as graphs and algebraic expressions), communicative tools (e.g. language, text, and speech), and digital tools (a calculator or computer software). Tools can enable and shape mathematical thinking. They are used for some purpose, in order to achieve something, (Roth & Radford, 2011).

Developing ML requires positive dispositions toward using mathematics and an appreciation of mathematics and its benefits (Jablonka, 2003). This involves willingness and confidence to engage with mathematics. Figuring out how to solve problems occurring in everyday life requires one to think flexibly about mathematics and adapt the methods and procedures to the current context (De Lange, 2003; Schoenfeld, 2001). Therefore, the competence to think creatively is an important part of life and ML. It involves both taking the risk of not succeeding and the initiative to try different approaches.

All the elements are grounded in a critical orientation. ML is about recognising the power and risk when issues are expressed numerically and to critically consider the contexts, mathematical knowledge, and tools involved. Mathematically literate individuals recognise the role mathematics plays in culture and society, for example, how mathematical information and practices can be used to persuade, manipulate, disadvantage, or shape opinions about social or political issues (Jablonka, 2003). Hence, they know and can use efficient methods and evaluate the results obtained (Goos et al., 2014).

Teaching and learning mathematics in contexts

Although teachers recognise the contextual and applied aspect of ML (Genc & Erbas, 2019), they count a wide range of practices as real-world connections (Gainsburg, 2008). Teachers make such connections frequently, but the connections are brief and do not require any thinking from the students. Therefore, Gainsburg claims that teachers' main goal is to impart mathematical concepts and skills, and the development of students' competence and disposition to recognise applications and solve real problems is of lower priority. Wijaya, Van den Heuvel-Panhuizen, and Doorman (2015) argue that to create opportunities for students to learn to solve contextualised tasks, teachers can ask the students to paraphrase the problem, encourage them to identify the relevant mathematical procedures, and verify the reasonableness of the solution.

It is usually expected that students are more interested in contextualised problems. Andersson, Valero, and Meaney (2015) report that students experience meaningfulness and engagement when mathematics is related to societal issues, and that their engagement in mathematics learning is influenced by experiences related to task, situation, school organisation, and the

socio-political. However, if the particular context is of low interest, students are more interested in solving problems without real-life connections (Rellensmann & Schukajlow, 2017). Therefore, various aspects of the context need to be considered. Authentic contexts do not necessarily involve authentic questions that people in the real context would pose or authentic methods that people in the real context would use. For an aspect in education to be considered as authentic, it requires an *out-of-school origin* and certification of provenance either physically or by an expert (Vos, 2018). Also, there are different views in the mathematics education community regarding what counts as real. For instance, in realistic mathematics education (RME), a fantasy world can be a suitable context as long as it is real in the student's mind and students can engage productively with mathematics when it is explored in imaginative settings (Nicol & Crespo, 2005).

Hence, students' predispositions to transfer mathematics learning in school to real-life situations are complex and varied because contexts are part of an interaction between students' experiences, goals, and perceptions of the mathematical environment (Boaler, 1993). Students' view of mathematics as a school activity and not as a way to make sense of the world, creates a dichotomy between everyday mathematics and school mathematics in the sense that formal learning fails to benefit from the intuitive knowledge students bring to the classroom, and students are unable to generalise their mathematical knowledge to situations outside school (Hunter, Turner, Russell, Trew, & Curry, 1993). As teachers' ideas for making real-world connections come from their own experiences (Gainsburg, 2008), teachers' understanding of how to apply mathematics in real-world contexts is important for providing students with the learning experiences necessary to adapt the knowledge they learn in school to the outside world (Popovic & Lederman, 2015).

Method

Subjects and procedures

Data were collected in three schools in Western Norway. I refer to the schools as A, B, and C. The schools' total number of students on roll range from 220 to 370 and all three schools teach grades 1 through 10. The three schools cooperate with the author's university teacher education programme and were therefore recruited for convenience.

I contacted the school leaders, and they recruited teachers and their respective classes. Criteria for selection of classes were that they were grade 9 (students aged 14-15 years) and that they agreed to participate. I needed consent from both the students and their parents. All parties involved received written information explaining my interest in studying teaching concerning concepts in policy documents. To ensure informed consent, I attended meetings with the teachers, the students, and the parents.

Methods for data collection are interviews and lesson observations. The number of participants involved from each school is displayed in Table 1 below.

Table 1. Overview of collected data and the number of participants involved

	Number of observed lessons	Number of students interviewed	Number of students in the class
School A	6	8	14
School B	5	7	28
School C	5	7	18 ^a
Total	16	22	60

^aIn school C, the class was divided into two groups according to which students had consented to participate in the research. There were 28 students in total in the class.

I instructed the teachers to plan and conduct the teaching as they would normally as I was interested to observe, as far as possible, regular mathematics lessons. Therefore, I was not involved in decisions regarding the mathematical topics taught, or the activities worked with in the lessons. In schools A and B, all lessons concerned the topic equations. In school C, the first two lessons concerned equations and the rest concerned percentages. The lessons varied in length from 45 to 90 minutes. I was a non-participant observer and did not intervene in the lessons, other than by being present.

Individual semi-structured interviews were conducted with 22 students. To investigate students' encounters with ML, I asked questions about what mathematical knowledge they need and in which contexts they need it. Also, I asked questions about what their parents or other people they know use mathematics for. The belief was that by thinking of someone they know, students would have a starting point for further reflection about the use of mathematics in the real world. I developed an interview guide with questions and topics I wanted them to reflect upon but without a predetermined sequence. Each interview lasted about fifteen to twenty minutes. The interviews were recorded and transcribed.

To capture the students' perspective of mathematics teaching, I video recorded 16 mathematics lessons using head-mounted cameras. For each lesson, three different students wore head cameras, recording the classroom activity. Head cameras enabled me to capture the participants' visual fields, get more in-depth insight onto the direction and timing of participant attention, and document participant actions. They also provided me with valuable insight in students' conversations, the tasks and students' written accounts, and their attention toward the blackboard (or elsewhere), all in one recording.

Process of analysis

The interviews and lesson recordings were loaded into the computer-assisted qualitative data analysis software NVivo. Interviews were transcribed verbatim. To get an overview of the interview data, I constructed tables based on the students' replies to the questions. The frequency of students' examples of different occupations, everyday situations, and mathematical topics was recorded. As students' encounters with ML was the topic of study, the interviews and the lesson observations were closely studied and analysed according to the

elements of ML. The operationalisations of the elements of ML in the lesson observations and interviews are presented in Table 2 below.

Table 2. Operationalisations of ML elements

ML elements	Lessons	Interviews
Context	Do tasks involve contexts from personal and social life, work-life, or citizenship? Are applications of mathematics in different contexts discussed? Are authentic aspects of the tasks and certifications discussed?	Do students describe different contexts where mathematics is or can be useful? What characterises these contexts?
Mathematical knowledge	Which concepts and procedures are worked with, and how? Are various methods explained and discussed? Are concepts and representations connected? Are solutions justified?	Do students describe mathematical topics, concepts, procedures, and methods that they or others use or might use?
Tools	Are digital tools, representations and models used to solve or model problems? How?	Do students connect using tools to doing mathematics in context? Do students view tools as a mediator of thought?
Dispositions	Are students engaged in tasks and discussions? Do they show curiosity, interest and confidence by engaging in investigations and discussions? How does the teacher motivate and encourage?	What are students' views of mathematics? Do students see the benefits of mathematical knowledge? Do students see themselves as mathematics learners and users?
Critical orientation	Are the students involved in discussing, questioning, explaining, evaluating, and validating methods and solutions? Is mathematical information used to make decisions and judgements, add support to arguments, and challenge an argument or position?	Do students recognise the role mathematics plays in society in general, as a tool to understand, inform, and make judgements? Do students provide examples of situations where they have used, or might use, mathematics to make informed decisions and judgements, or critically evaluate others'?

Findings

In the following, I present the findings from the observed lessons and the findings from the interviews.

Lessons

Most of the tasks in the observed lessons on equations are strictly mathematical and do not involve contexts. However, the students work on a few word problems with contexts from personal and social life. These tasks contain inauthentic questions and solutions methods. An example is the following task from school B, lesson 2:

In a pasture, the length is three times the breadth. The perimeter is 240 meters. What is the area of the pasture?

First, a farmer is unlikely to express the length and breadth of a pasture in terms of an unknown. Second, to find the length and breadth, s/he would go out and measure it. Authentic aspects are not critically discussed in the lessons. Therefore, the tasks do not demonstrate the role mathematics plays in the world. Also, the teachers do not give any certification of contexts where equations are used, even though the students have requested it. The following excerpt suggests that the students do not see when they would use equations in life outside school, and the teacher cannot provide them with one.

Teacher: I remember what you said to me then (in the previous chapter on algebra).

“When will we use this?”, you said when we worked with all those expressions.

Student: To use it in the next chapter, that was not what we meant. We meant in life.

(School B, lesson 2)

The lessons on percentages in school C all contain task contexts from personal and social life, aimed at showing the use of percentages in the real world. However, these tasks are also traditional word problems and contain inauthentic aspects. Still, in the lessons, the teacher provides certifications by referring to contexts in real life where knowing percentages are useful. For example, she talks about how some stores advertise discount in terms of money while others use per cent. It is, therefore, useful to calculate percentage in order to evaluate which is the better buy. She also talks about her own experiences when shopping at sales and states:

Teacher: There are many things that you learn in mathematics where you ask me “What do we need this for?” But I know from experience that this will be very useful for you later.

The observed lessons involve great emphasis on developing mathematical knowledge. Conceptual understanding and procedural fluency are emphasised in the sense that students spend most of the time practising the procedures demonstrated on the chalkboard. The procedures concern how to solve linear and quadratic equations, equations with fractions, how to test their solutions, inequalities, and word problems. All the observed lessons are organised in similar ways with the teacher demonstrating or explaining a concept or technique on the chalkboard, followed by students working with textbook tasks. Some tasks are solved either by students or the teacher on the chalkboard. The questions and answers concern carrying out the correct procedure and finding the correct number, and do not involve critical discussions

about concepts, relationships or alternative solution methods. However, it can be argued that testing a solution is a way of critically evaluating the answer.

The students frequently use calculators to perform calculations. On a few occasions, the teachers use and encourage students to use representational tools. For example, teacher A draws a number line to represent the solution of an inequality, and students are encouraged to make drawings to represent the problems and to mediate their thinking. Also, teacher B emphasises language as an important part of thinking and often tells the students to discuss the methods and strategies with each other or oneself.

Both peer-work and comments about the real-world significance of mathematical knowledge are ways to motivate and engage the students. Also, the teachers try to foster students' positive dispositions and engagement in the tasks through praise and supportive feedback on their work. The tasks worked with do not invite students to be creative and inquire. The students display great varieties in terms of emotions and engagement. Some work concentrated on the tasks throughout the lessons, while others are distracted and unfocused. Some express feelings of enjoyment while others express dislike.

In terms of critical orientation, there is a lack of critical discussion, justification, and evaluation of methods, solutions, concepts, and contexts in which they are used. Although methods are the topic of whole-class and peer-group talk, it is to a large extent up to the individual to make the critical judgements in his own mind. There is no collective focus on engaging in critical discussions. The goal is to find the correct number and the contexts (and numbers) are not given any further attention. However, three episodes from the classroom may, to some degree, be related to critical orientation. Two episodes come from the lessons on percentages in school C. One concerns Black Friday sales and evaluating a purchase. The teacher talks about a webpage comparing prices and displaying the price history of different commodities. The second comes from a previous lesson in social sciences where they compared local taxes in neighbouring municipalities and discussed reasons for the large differences. The third comes from school B and concerns equations. The teacher provides the students with a list of points to help them structure word problems and instructs them to read the task carefully, and to look for information not relevant for solving the task:

1. Read the task carefully
2. Find out what they are asking for
3. Find the best point of departure (who/what do know the least about?)
4. Form the equation
5. Check if the equation makes sense
6. Solve the equation to find the unknown
7. See if you have the answer to the task

This list is easily transformed into a general strategy for solving problems and for addressing issues connected to critical orientation such as using mathematics to support an argument. However, the focus is on the equations and the list's potential for developing critical

orientation is not fulfilled. A common feature in all three examples is that they are all led by the teachers and do not involve any student action.

Interviews

In the interviews, the students mentioned 11 different examples of situations from daily life involving mathematical knowledge. In total, there were 45 examples, as some students mentioned the same situations. There were 32 different examples of occupations involving mathematics, and a total of 84. The students also connected different mathematical topics and knowledge to everyday and occupational situations. Table 3 shows the number of times different mathematical topics were connected to contexts in everyday life or occupations.

Table 3. Mathematical topics in everyday life

Mathematical topic	Everyday	Occupations
Geometry (area, length)	14	23
Money and finances	3	8
Calculations and counting (mental arithmetic, the four arithmetic operations)	23	21
Measurements (time, weight, volume)	27	12
Percentages	7	3
Equations and algebra	0	5
Fractions	1	1

In the interviews, students gave examples of the mathematical knowledge they, their parents or others need in everyday life. Their responses concerned the topics geometry, money and finances, calculations and counting (arithmetic), percentages, measurements, fractions, and equations and algebra. The students were unsure about the need for equations. Some students commented that some occupations might require equation solving, but they could not provide an example of what they need equations for. Two students also mentioned digital tools (spreadsheets) as relevant in some occupations. In the interviews, students only commented on specific mathematical topics and did not talk about problem-solving strategies or mathematical skills, except doing mental calculations.

The students connected specific mathematical knowledge and tools to specific contexts. They reflected on situations in which they, their parents, or someone they know need to formulate, represent, and solve a mathematical problem. The contexts in the students' examples concern personal and social life and work life. The students commented that mathematics is necessary to manage personal finances, i.e. to pay bills, plan what to spend money on, and "At the store, if I am buying several things, to calculate how much it costs" (Student in school B).

Mathematics is also required when cooking, planning a journey, or redecorating the house. For example, a student in school C commented: "Not long ago, I wanted to buy a new desk, and then I had to measure my room to find out if it would fit or not". The students relate

mathematical knowledge to performing basic procedures aimed at producing a specific number. Some students relate mathematics to school. For example, a student in school A said: “I use it for homework and stuff, of course. And here.” This could indicate that students see mathematics as relevant for their further education in terms of admission to schools and further studies. Also, they spend a big part of their day in school and therefore connected their use of mathematics in everyday life to school. On the other hand, it may display a view of mathematics as something detached from life outside school.

From contexts in work life, students referred to different occupations and examples of mathematics needed by professionals in their work. Carpenters need knowledge about mathematics in order to build houses correctly, for example, find the area of the rooms or “to measure how long that plank has to be” (Student in school B). Shop assistants need to do mental arithmetic and percentages to calculate prices and sums of commodities. The students also commented that doctors and nurses use mathematics for calculations so that the patients get the right medicine dosage. Leaders and economists need mathematics to deal with budgets, salaries, and purchases. The students believed that mathematics is needed in most occupations. No one was able to give examples of occupations where mathematics is not needed. However, they believed that some occupations require more mathematics than others, or as a student in school C stated, “It is smart to know maths either way”.

The students did not give any examples of mathematics used in contexts concerning citizenship or societal issues, which suggests that they have not had sufficient encounters with ML in such contexts. Societal issues are important in the development of ethical and reflective subjects. The contexts students mention are contexts that are certified, either by parents or relatives, or by their own experiences.

The fact that all students were able to give examples of how mathematics is used in the real world suggests that they, at least to some extent, appreciate the role mathematics plays in the world and as such hold positive dispositions toward mathematics. Some students express that mathematics is difficult and that they do not think they use it often. Still, they acknowledge that there may be situations where they are involved in mathematical activity without reflecting upon it. One can argue that in such situations, they use mathematics that they have encountered several times and has become part of them. On the other hand, it might be that the mathematics involved has not yet become part of their consciousness.

The interviews contain little evidence of a critical orientation. Although students can recognise some of the role mathematics plays in specific contexts, they do not comment on how mathematics is used to form an argument or justify a position. Students have a narrow view of mathematics as numbers, calculations (the four arithmetic operations), and a way to find solutions. A few students relate these solutions to problems in everyday life, such as shopping and cooking. Mathematics is related to practising procedures and performing calculations, and not as a way to make sense of the world.

Discussion

In ML, context is the central element, but from the observed lessons and interviews, formal mathematical knowledge seems to be central. Although teachers believe that they are making mathematics relevant to the students by offering contextualised tasks, they may be reinforcing students' narrow view of the subject by only considering the importance of the mathematical topic and not the significance or authenticity of the contexts and tasks and their potential to teach about the context (Gainsburg, 2008). There is a lack of certifications and critical discussions about context, mathematical knowledge, and tools in the lessons. This may contribute to the narrow view of mathematics displayed in the interviews.

However, some points in the list provided by teacher B can be related to Wijaya et al. (2015)'s framework for teaching practice supportive for students opportunities to solve contextualised tasks. However, the list involves specific references to using equations, which do not encourage students to explore various procedures to solve the problem. It may support a view of mathematical problems as having only one approach and one solution (Vos, 2018). Also, the list is used for solving traditional word problems where there is, in fact, a preferred procedure and a fixed solution. If presented in a general way, the list might help students develop strategies for solving all kinds of problems in which they initially do not know how to solve, and in that way might contribute to developing students' ML.

Nosrati and Andrews (2017) express disappointment in that the students in their study did not see mathematics as a cultural artefact or as an education for citizenship. From the observed lessons reported here, such views of mathematics could not be expected. Research has shown that teachers struggle to implement authentic and meaningful contexts and activities involving citizenship (Goos et al., 2014). This seems to be the case in the observed lessons as well. Therefore, if the students have not encountered citizenship and cultural issues in the mathematics classroom, how can we expect them to be part of their consciousness? The interviews show that although students are conscious of the use of mathematics in several contexts, this consciousness is confined to very basic mathematical operations performed in word-problem-like contexts. This resonates with the findings of Nosrati and Andrews (2017). If these findings are prevailing in other classrooms as well, we are currently not preparing students for the demands of the twenty-first-century workplace and world (Gravemeijer et al., 2017).

Manifestations of mathematical illiteracy are prevalent in society, for example, in terms of mathematical errors in newspapers (De Lange, 2003). Either the content of mathematics learned in school is not making citizens mathematically literate, or the structural design of teaching practices are not helping students make connections to real-life situations. From the results reported here, I argue that the problem lies with the teaching practices. Although ML has been considered a basic competence in Norway since 2006, and problem-solving and real-world connections even longer, it appears that teaching is still following the findings of Wijaya et al. (2015). If teaching practice fails to involve students in posing and answering questions, making inquiries and solving open-ended problems, students will continue to view

mathematics only as a school activity, and the contexts to which students relate the use of mathematics will continue to be limited to basic everyday activities. The social justification of mathematics depends on its potential use in real-life situations. For individuals to develop their ML learning and becoming, they need to encounter the use of mathematics in real-life situations a sufficient number of times, and the situations need to be significant to the students. According to Mellin-Olsen (1981, p. 362), “the determination of this ‘sufficient number’ and of the significant situations is, of course, the difficult crux of our problem, where we have to focus our energies when preparing practice”. As students still hold the view of mathematics as detached from the reality outside school, and teaching still supports this view, it seems like this crux is just as challenging almost 40 years later.

Not every mathematical topic that students learn in school has an apparent application in their daily lives. The application of equations and algebra seems to be particularly challenging to demonstrate. Equations may, therefore, not be the best-suited topic of study when investigating students’ encounters with ML. However, this was the topic at the time of my visits. Besides, the issues arising from the analysis have a didactical dimension that goes beyond the specific mathematical topic. Further research should focus on how teaching can provide students with encounters of mathematics in real-life to support their objectification of ML, for example through tasks involving learning about both context and mathematical topic, such as mathematical modelling tasks (Steen et al., 2007; Vos, 2018). Research on how a critical orientation can be implemented in teaching in all school levels is of great importance.

In this study, ML is framed within the perspective of TO. The tasks and examples in the observed lessons and interviews are actualisations of the potential knowledge of ML. The teachers’ and students’ thoughts and actions are a result of cultural and historical ways of thinking and doing. Such cultural and historical ways of thinking and doing characterise students’ encounters with ML. These encounters concern developing mathematical knowledge for personal advancement (Nosrati & Andrews, 2017) instead of becoming ethical and reflexive subjects in the world (Radford, 2016), and they are also results of our history and culture. I believe that interpreting ML in terms of TO can provide a new perspective on how ML can be understood and developed. This perspective should be further explored.

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