Mathematical Competencies and E-Learning: A Case Study of Engineering Students' Use of Digital Resources

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This paper explores how an e-learning environment affords the execution of mathematical competencies in an undergraduate engineering context. Considering the students' mathematical practice as action mediated by the digital resources in a sociocultural sense, we employ the competence framework by (Niss & Højgaard, 2011) to make sense of students' learning. Case-study research design has been implemented to thoroughly observe the mathematical practices of a small group of participants. Observing students' group work and following their mathematical discussions elucidated the way this environment afforded the execution of competencies. Closer analysis revealed that the availability of online tools in this environment has the twofold effects on mathematical thinking, mathematical reasoning and problem-tackling competencies.

Keywords: Calculus, Engineering mathematics, E-learning, Mathematical competencies.

Introduction

The use of digital resources in mathematics education has started since the development of such tools and is still being researched to study its impact on mathematical learning. Increased dependence on digital tools for practicing mathematics is transforming the mathematics education, and to learn mathematics is not the same as it was before the introduction of digital technology. The use of digital resources is of particular relevance in engineering mathematics in the sense that modern-day engineers during their professional activities rely on technology for mathematical tasks (van der Wal, Bakker, & Drijvers, 2017). The framework for mathematics curricula in engineering (Alpers et al., 2013) also recommends how technology should contribute towards fostering the engineering students' mathematical competencies (Alpers et al., 2013). The notion of mathematical competence from the Danish KOM project (Niss, 2003; Niss & Højgaard, 2011) has been adopted to make sense of the engineering students' mathematical learning.

Previous research studies have also employed this competence framework, either to make sense of students' learning in mathematics or to analyse how these competencies are developed in particular situations or through certain activities. For instance, Jaworski (2012) used Niss's idea of mathematical competencies to design and analyse the tasks and to recognise the engineering students' mathematical learning. Jaworski pointed out that a potential use of the competence framework may be to create opportunities for students to achieve certain competencies (Jaworski, 2013). Furthermore, Albano and Pierri (2014) used a role play activity and identified the first-year engineering students' mathematical competencies through the questions students asked. Albano and Pierri concluded that students seemed to possess all the competencies by Niss (2003) which were evident through the words they used in their questions. García, García, Del Rey, Rodríguez, and De La Villa (2014) presented a model for the integrated use of CAS which they implemented and analysed in engineering classrooms. They suggested that the use of CAS in all learning and assessment activities has the potential to positively influence the development of mathematical competencies. Recently, Queiruga-Dios et al. (2016) analysed the development of mathematical competencies among industrial engineering students through their teamwork which included the use of CAS for solving mathematical problems as an integral part. While their main aim was to integrate these mathematical competencies with the required engineering competencies in Spain, they claimed that the students acquired all the mathematical competencies during this task.

Our study focuses particularly on nature of mathematical competence afforded by an e-learning environment. Realising the contemporary and the future state of mathematics education, we attempt to add to the research literature within the context of engineering mathematics education. In this paper, we analyse engineering students' engagement within a calculus course to report on how their mathematical competencies are supported within an e-learning situation. We attempt to answer the following research questions: What traces of mathematical competencies are observed in students' work when they practice mathematics digitally? How does this environment afford the execution of these mathematical competencies?

Theoretical perspective

We consider students' mathematical practice in the present situation as mediated action in sociocultural terms (Vygotsky, 1978). The provided resources which support the learning of mathematics serve as mediating artefacts between students and the mathematical concepts. The mediating artefacts used in the present situation are MyMathLab, tutorial videos, textbook, Maxima for programming, and other internet-based resources. The students' homework and eventually the students' assessments are done digitally. There were no regular face-to face lectures thus the situation is considered as e-learning in which students remotely work with the resources. A brief introduction of these resources follows.

MyMathLab is an online interactive learning environment for practicing mathematics digitally. While the main aim of this resource is to provide a platform for digital homework and assessments, it also facilitates in solving the tasks by providing illustrated worked examples and personalised feedback. The tutorial videos replace traditional university lectures and are linked topic-wise with the textbook sections. The videos are recorded by the mathematics teacher using a document camera, and they consist introduction to each mathematical topic along with worked examples. The tutorial videos and the homework in MyMathLab were clearly linked with the chapters in the textbook.

We employ the competence framework by Niss and Højgaard (2011) to make sense of engineering students' mathematical learning (Jaworski, 2012, 2013). The framework is complemented by sociocultural notion of resource mediation. The Danish KOM project (Niss & Højgaard, 2011) enlisted eight mathematical competencies, divided into two groups as follows (Figure 1):

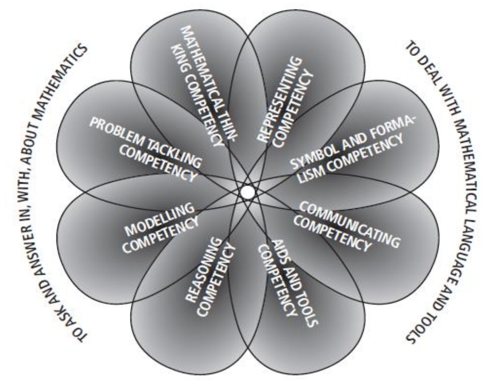

Figure 1: A visual representation of eight mathematical competencies (Niss & Højgaard, 2011, p. 51).

The Ability to Ask and Answer Questions in and with Mathematics

The first group comprises the competencies of mathematical thinking, mathematical reasoning, problem tackling, and mathematical modelling. Mathematical thinking competency involves "awareness of the types of questions which characterise mathematics" (Niss & Højgaard, 2011, p. 52) and "being able to recognise, understand and deal with scope of given mathematical concepts" (Niss & Højgaard, 2011, p. 53). Mathematical reasoning includes following and assessing chains of arguments, comprehending a mathematical proof, and devising formal and informal mathematical arguments (Niss, 2003). In the present study, the proofs were not a part of the mathematics curriculum. Thus, the reasoning competency is only observed within the context of problem solving. Mathematical modelling is neither a part of the curriculum in the present situation.

The Ability to Deal with Mathematical Language and Tools

The second group includes the competencies of representing mathematical entities, handling mathematical symbols and formalism, communicating in, with and about mathematics, and making use of aids and tools.

Research Design and Methods

This study is carried out following a case study design (Yin, 2013) and the data has been collected in a Norwegian public university. A small group of three male students, enrolled in the first year of an electronics engineering program, has been observed over the whole semester. The methods used to generate data include group observations, group interviews, individual weekly journals and field notes by the researcher.

For the participant observations, video recordings of their group work, and screen recordings to follow the activity on computer screens have been collected. Additionally, participants provided screen recordings of their individual work, and weekly journals containing self-reports about the use of resources for practicing mathematics. In this paper, we analyse three episodes of the students' group work in order to look for how these competencies are supported in an e-learning environment.

Analysis

The two sets of competencies are not mutually disjoint, in general, and are intertwined which is evident from the so-called competency flower. Although each competency has a well-defined identity in theory, execution of each competency in practical will draw on some other competencies. This makes it empirically challenging to disentangle one competency from the others (Niss, Bruder, Planas, Turner, & Villa-Ochoa, 2016). We adhere to these considerations and our purpose here is to rather we look for possibilities in which e-learning influences each sets of competencies.

In the quest for finding correct answers to the given tasks in present situation, participants needed to go through certain procedures where they could demonstrate these competencies. Geogebra (https://www.geogebra.org/) and WolframAlpha (https://www.wolframalpha.com/) were main tools used by the students to make sense of various mathematical functions, checking for the functions' behaviour and to look at the solutions of the tasks. Textbook served as a main written help material in terms of consulting for mathematical formulas, explanations or illustrations, and for checking whether their solutions were correct by comparing these with the answers to tasks provided in the end of textbook. At several occasions, the textbook served as an aid to get acquainted with the mathematical topics, as the students read the textbook to understand the mathematics. The introduction of Maxima was done in a project in this course, and the purpose was to make engineering students capable of using this programming language to solve mathematical problems thus it also served as a resource.

The exposure to Google and different online calculators, in this case, for finding solutions of the given tasks, has shared the role for computing and calculating the solutions. We noticed that in participants' arguments, the element of tool dependence was evident.

In this regard, WolframAlpha and GeoGebra have a central role, since it in the present situation supported students in making sense of the functions, expressions and mathematical concepts in different ways. For example, when the students were not able to solve an integral $\int_0^1 \frac{\sin(x)}{x}\,dx$ by programming with Maxima, they started wondering whether it was solvable at all, and they used WolframAlpha to make sense of the scope of the task or to know the answer:

Per: (…) Maybe it… (we) can't solve it? Have you tried Wolfram? [Per is addressing Jan and visits WolframAlpha website himself. Per has looked up $\int_0^1 \frac{\sin(x)}{x}\,dx$ on WolframAlpha (Figure 2)]

Per: No, you're supposed to get an answer.

In this example, when asked by Jan, Per was trying to handle the scope of this integral. He used WolframAlpha to see what this integral is all about, and based on the output, he decided that it could be solved. This example illuminates how the mathematical thinking and problem-tackling competencies are being executed along with the obviously observed aids and tools competency.

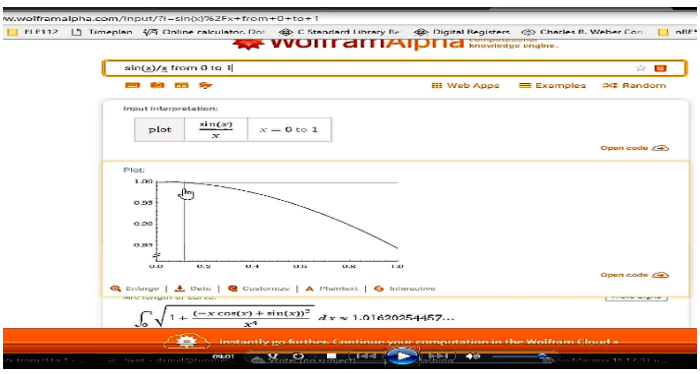

Figure 2: Screenshot of a participant's work on WolframAlpha.

The online tools mediated in the students' abilities to think and reason mathematically either by providing the complete calculations or the opportunities to explore the tasks at hand. By using paper and pencil techniques, both of these functions require a different kind of knowledge and skills as it says in the competence framework.

The following excerpt indicates how this environment is supporting the competencies of dealing with mathematical language and tools. While trying to solve a definite integral $\int_{-1}^{1} e^{-\sqrt{-1}wt}\,dt$ using Maxima, they got apparently a different outcome than what it said in the book.

Per: It is the same? It is the same thing, just written in a different way.
Jan: Yeah
Per: Simplify [Per tries to use the "simplify" command on the expression in Maxima]
Jan: Yeah, it just looks that much nicer when you do it in…
Per: In Wolfram.
Jan: Yeah. Yeah, or at that. Did you get…You got the same in Wolfram?

Per: Nnn… I haven't checked it. I assume I get what it says in the book.

[Per looks up $\int_{-1}^{1} e^{-\sqrt{-1}wt}\, dt$ in WolframAlpha.]

Per: Then I get sine w to…2 sine w divided by w, and that's exactly the same as it says in the book.

Jan: There, it… If you go back. Wolfram has moved -1 outside.

[Jan is trying to make Per aware how WolframAlpha has changed the representation.]

Per: Where?

Jan: Put the square root outside the parentheses.

Per: Yeah, but that's just if… I don't think it matters if...

[Meanwhile Per writes the original expression slightly differently in Maxima and gets the same output]

Per: It is exactly the same. I think it is correct.

Here, Per and Jan were trying to make sense of the different representations of the expression when both resources offered the result in a slightly different manner. The second set of competencies concerning representing mathematical entities, handling mathematical symbols and formalism, communicating in, with and about mathematics, and making use of aids and tools are *in action*.

An interplay of different resources had also been helping to approach a given task from different perspectives and to gain more information about the task in hand. Also, the use of Maxima apparently seemed as a short cut for getting ready-made answers. However, it has been observed that it required some effort from the students to decode the mathematical language into programming language.

Discussion

We intended to look for the execution of mathematical competencies in an e-learning environment in our case, and the findings of this study differ from the previous findings by (García et al., 2014). We found that while this environment supports some competencies, it does not ensure enhancing all of these in all learning environments. The way in which this online learning environment provides possibilities for practicing mathematics makes it different from the traditional way of doing mathematics in a paper and pencil environment.

For instance, from the first set, when the competencies of thinking and reasoning mathematically have to be executed in an online environment. We conjecture that the effects are twofold. On one hand, the resources are facilitating in computing, calculating and providing answers requiring less effort from the students thus limiting the possibilities for exploration. However, on the other hand, when used for comprehension of the tasks at hand they have potential to enhance the possibilities of exploration. We further observed that e-learning is certainly not on the same lines as it means to think and reason mathematically in a traditional way. In a traditional paper and pencil environment, students use their own knowledge and skills for performing the tasks at the hand.

The second set of competencies has more scope in the present context owing to the use of different tools and aids for practicing mathematics. When students used different tools for practicing mathematics, and each one of those tools uses different symbolism which provides some opportunities for the students to experience and handle varied mathematical formalism in a way.

Question for discussion: How to devise a better systematic scheme for analysing mathematical competencies in this environment?

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