PAPER E

Robust Transmit Beamforming for Underlay D2D Communications on Multiple Channels

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Abstract

Underlay device-to-device (D2D) communications lead to improvement in spectral efficiency by simultaneously allowing direct communication between the users and the existing cellular transmission. However, most works in resource allocation for D2D communication have considered single antenna transmission and with a focus on perfect channel state information (CSI). This work formulates a robust transmit beamforming design problem for maximizing the aggregate rate of all D2D pairs and cellular users (CUs). Assuming complex Gaussian distributed CSI error, our formulation guarantees probabilistically a signal to interference plus noise ratio (SINR) above a specified threshold. In addition, we also ensure fairness in allocation of resources to D2D pairs. We accommodate the probabilistic SINR constraint by exploiting a Bernstein-type inequality.The resulting problem is a mixed integer non-convex problem, and we propose to approximately solve it by exploiting a semi-definite relaxation (SDR) and a quadratic transformation, which leads us to an alternating optimization method. Simulation results corroborate the merits of the proposed approach by illustrating higher network throughput and more reliable communication.

Index Terms

D2D communications, resource allocation, robust beamforming, semi-definite relaxation.

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I. Introduction

D2D communications in the underlay framework, improve the spectral efficiency by simultaneously allowing transmissions of D2D pairs and traditional cellular network in the same spectrum [1]. However, simultaneous transmissions in the same spectrum bands increase interference, which must be deliberately handled by devising judicious algorithms for the assignment of channels to D2D pairs and the control of transmission powers. The allocation of resources must also be fair while guaranteeing the desired Quality of service (QoS) and also robust to errors in CSI.

Most works on resource allocation problems for underlay D2D communication have assumed single antenna transmission [2], [3]. These schemes also restrict D2D pairs from accessing more than one channel and are also assumed to have prefect CSI. Scenarios of imperfect CSI for single antenna transmission are considered in [4], [5].

Under the multi antenna transmission framework, [6] provides a comprehensive analysis for joint beamforming in D2D underlay cellular networks. However, this work is restricted to a single D2D pair with the further assumption of perfect CSI. In [7] multiple D2D pairs are considered; however, once again knowledge of perfect CSI is assumed at the base station (BS). Quantization error in CSI due to limited feedback is assumed in [8] to study the

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conventional maximum ratio transmission and interference cancellation based beamforming techniques. Recently, a joint beamforming and power control strategy is presented in [9] under the assumption of both perfect and imperfect CSI. Here the objective is to minimize the total transmit power of BS and D2D pairs while ensuring QoS (SINR) requirements.

Design of robust beamformers for general multiuser communication has also been investigated in past research works [10]–[12]. Under the assumption of Gaussian CSI uncertainties, analytical methods based on Bernstein-type inequality and decomposition based large deviation inequality are proposed in [10] to approximate the probabilistic rate outage constraints. Similarly, under the assumption of Gaussian channel distribution, the probabilistic rate outage constraint is handled by SDR relaxation and sequential convex approximation in [11]. Further, authors in [12] have proposed a decentralized approach to design the robust beamformers considering elliptically bounded CSI errors. In all these works the objective is either minimization of transmit energy, or sum rate maximization; however, in underlay D2D communication jointly optimizing the power allocation and channel assignment poses additional analytical and computation challenges.

Despite the above research efforts, none of the existing approaches provide a robust beamforming design while performing joint channel assignment and power allocation to the D2D pairs and CUs. The main contributions of this work are:

- We formulate a robust beamforming design problem under the assumption of complex Gaussian distributed CSI error in the channel gain vector. Our objective is to maximize the aggregate rate of all D2D pairs and CUs with a penalty on unfair channel assignment, under a constraint on the minimum SINR requirement to guarantee a specified outage probability. The probabilistic constraints are handled by exploiting the Bernstein type inequality [10], [13] for a quadratic form of Gaussian random variables.
- Since the resulting problem is a mixed integer non-convex problem, with aid of auxiliary variables and with no loss in optimality, we decompose the problem into multiple power allocation subproblems and a channel assignment subproblem. The power allocation subproblems are solved by alternating optimization obtained after applying semi-definite relaxation [14] and fractional programming via a quadratic transformation [15]. The channel assignment subproblem is solved by integer relaxation.
- Finally, numerical experiments are performed to corroborate the merits of the proposed approach by illustrating a higher throughput and more robust communication.

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II. SYSTEM MODEL

Consider the cellular communication setup shown in Fig. 1 where a BS with K_B transmit antennas communicates with N_C single antenna CUs through N_C downlink channels[1]. The set of CUs (equivalently, channels) are indexed by $\mathcal{C} = \{1, ..., N_C\}$. In an underlay configuration, N_D D2D pairs, indexed by $\mathcal{D} = \{1, ..., N_D\}$, wish to communicate using the aforementioned N_C downlink channels. The D2D transmitters are assumed to have K_D antennas communicating with respective single antenna D2D receivers. Furthermore, we denote the channel gain between the BS and the i-th CU by $\boldsymbol{g}_{B_i} \in \mathbb{C}^{K_B \times 1}$; and between the j-th D2D pair by $\boldsymbol{g}_{D_j} \in \mathbb{C}^{K_D \times 1}$. Similarly, the interference channel gain[2] between the BS and the receiver of the j-th D2D by $\boldsymbol{h}_{B_j} \in \mathbb{C}^{K_B \times 1}$; and between the transmitter of the j-th D2D pair and the i-th CU by $\tilde{\boldsymbol{h}}_{D_{j,i}} \in \mathbb{C}^{K_D \times 1}$. We assume minimum cooperation from CUs in estimating the gain of the interference channel; thus, $\tilde{\boldsymbol{h}}_{D_{j,i}}$ is modeled as a random vector with complex circular Gaussian distribution, i.e., $\tilde{\boldsymbol{h}}_{D_{ji}} \sim \mathcal{CN}(\boldsymbol{h}_{D_{ji}}, \boldsymbol{M}_{ji})$ where $\boldsymbol{h}_{D_{ji}}$ and $\boldsymbol{M}_{ji}$ are assumed to be known or learned in advance. The additive white noise power is denoted by N_0.

[1]Even though the formulation is done for downlink, the same formulation can be directly extended to uplink

[2]In principle, $\boldsymbol{g}_{D_j}$ and $\boldsymbol{h}_{B_j}$ should also depend on the i-th channel; however, this subscript is dropped as the proposed scheme carries over immediately to accommodate such dependence.

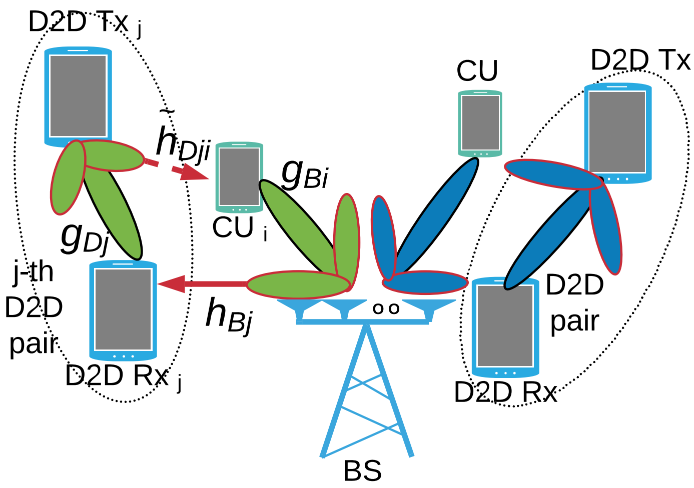

Fig. 1: Illustration of the overall system model.

The BS assignment of channels to D2D pairs is denoted by the indicator parameters $\{\beta_{i,j}\}_{i\in\mathcal{C},j\in\mathcal{D}}$, where $\beta_{i,j}=1$ indicates assignment of the i-th channel to the j-th D2D pair and $\beta_{i,j}=0$ otherwise. For higher throughput, we allow a D2D pair to simultaneous access multiple channels. However, to restrict interference, no more than one D2D pair can access each channel, i.e., $\sum_{j=1}^{N_D}\beta_{i,j}\leq 1,\ \forall i$. We denote the transmit precoder vector for the BS to communicate with the i-th CU as $\boldsymbol{p}_{B_i}\in\mathbb{C}^{K_B\times 1}$ and as $\boldsymbol{p}_{D_{j,i}}\in\mathbb{C}^{K_D\times 1}$ for the j-th D2D transmitter on the i-th channel. The precoders are constrained as $||\boldsymbol{p}_{B_i}||_2^2\leq p_{B,\max}$ and $||\boldsymbol{p}_{D_{j,i}}||_2^2\leq p_{D,\max}$. To ensure successful communication, the SINR should also be enforced to be greater than a certain threshold $\eta_{D,\min}$ for the D2D pairs and $\eta_{C,\min}$ for the CUs with a maximum allowed outage ratio of ϵ.

III. Problem Formulation

Due to limited cooperation of CUs in estimating the interference channel $\tilde{\boldsymbol{h}}_{D_{ji}}$, the objective of this work is to guarantee a maximum outage probability ϵ to the CUs, i.e., we maximize the minimum total network rate that is achieved at least a $(1-\epsilon)$ portion of the time. This can be realized by defining a lower bound on the total rate that can be achieved over every channel. To this end, let $\Gamma(z):=\mathrm{BW}\log_2(1+z)$, where BW is the channel bandwidth. For the i-th channel, this rate can be expressed as $R_i^{LB}:=(1-\sum_{j\in\mathcal{D}}\beta_{i,j})R_{C_{i,0}}+\sum_{j\in\mathcal{D}}\beta_{i,j}[R_{D_{j,i}}+R_{C_{i,j}}^{LB}]$, where:

- $R_{C_{i,0}}:=\Gamma(p_{B_{max}}||\boldsymbol{g}_{B_i}||_2^2/N_0)$, rate of the i-th CU without assignment of D2D pairs, i.e., $\beta_{ij}=0\ \forall j$.
- $R_{D_{j,i}}:=\Gamma(|\boldsymbol{p}_{D_{ji}}^H\boldsymbol{g}_{D_j}|^2\ /(N_0+|\boldsymbol{p}_{B_i}^H\boldsymbol{h}_{B_j}|^2))$, rate of the j-th D2D pair when assigned with the i-th CU, i.e., $\beta_{ij}=1$.
- $R_{C_{i,j}}^{LB}:=\Gamma(z_{C_{i,j}}^{LB})$, where $z_{C_{i,j}}^{LB}$ is such that $\Pr\{z_{C_{i,j}}^{LB}\leq|\boldsymbol{p}_{B_i}^H\boldsymbol{g}_{B_i}|^2/(N_0+|\boldsymbol{p}_{D_{ji}}^H\tilde{\boldsymbol{h}}_{D_{j,i}}|^2)\}=1-\epsilon$, rate that must be exceeded a $(1-\epsilon)$ portion of the time by the i-th CU when assigned with the j-th D2D pair, i.e., $\beta_{ij}=1$.

Finally, the minimum total network rate is defined as $R(\mathbb{B},\mathbb{P}_B,\mathbb{P}_D):=\sum_{i\in\mathcal{C}}R_i^{LB}$, where, $\mathbb{B}:=[\beta_{i,j}]$, $\mathbb{P}_B:=[\boldsymbol{p}_{B_i}]$, $\mathbb{P}_D:=[\boldsymbol{p}_{D_{j,i}}]\ \forall i\in\mathcal{C}$ and $j\in\mathcal{D}$.

In order to have fairness in the channel assignment, we introduce a secondary objective that penalizes greedy channel assignments to the D2D pairs. We consider an unfairness measure $\delta(\mathbb{B})=1/(N_Dc^2)\sum_{j=1}^{N_D}(x_j-x_0)^2$ along the lines in [16], [17], where $x_j:=\sum_{i=1}^{N_C}\beta_{ij}$ is the number of channels assigned to the j-th D2D pair, and $x_0 := N_C/N_D$ is the fairest assignment. Summing up, the overall problem can then be formulated as:

$$\underset{\mathbb{B},\mathbb{P}_B,\mathbb{P}_D}{\text{maximize}} \;\; R(\mathbb{B},\mathbb{P}_B,\mathbb{P}_D) - \gamma\delta(\mathbb{B}) \tag{1a}$$

$$\text{subject to } \beta_{i,j} \in \{0,1\}, \;\; \sum_{j=1}^{N_D} \beta_{i,j} \leq 1 \,\forall\, i; \tag{1b}$$

$$||\boldsymbol{p}_{B_i}||_2^2 \leq p_{B,\max} \;\; \forall\, i, \quad ||\boldsymbol{p}_{D_{j,i}}||_2^2 \leq p_{D,\max} \;\; \forall\, j,i; \tag{1c}$$

$$\Pr\left\{ \frac{|\boldsymbol{p}_{B_i}^H \boldsymbol{g}_{B_i}|^2}{N_0 + |\boldsymbol{p}_{D_{ji}}^H \tilde{\boldsymbol{h}}_{D_{j,i}}|^2} \geq \eta_{C,\min} \right\} \geq 1 - \epsilon, \tag{1d}$$

$$\frac{|\boldsymbol{p}_{D_{ji}}^H \boldsymbol{g}_{D_j}|^2}{N_0 + |\boldsymbol{p}_{B_i}^H \boldsymbol{h}_{B_j}|^2} \geq \eta_{D,\min} \quad \text{if } \beta_{ij} = 1, \;\forall\, i,j. \tag{1e}$$

The regularization parameter $\gamma > 0$ is selected to balance the trade-off between the minimum total rate and the fairness in channel assignment. Problem (1) is a non-convex mixed-integer stochastic program, which involves exponential complexity. In the next section, we propose efficiently solving problem (1) by exploiting semi-definite relaxation and a quadratic transformation.

IV. Proposed Optimization Algorithm

The complexity to obtain the solution of (1) can be reduced by decomposing the problem into multiple sub-problems of lower complexity. Thus, we first rewrite the sum rate as:

$$R(\mathbb{B},\mathbb{P}_B,\mathbb{P}_D) = \sum_{i\in\mathcal{C}} \left[\sum_{j\in\mathcal{D}} \beta_{i,j} v_{i,j}(\boldsymbol{p}_{B_i}, \boldsymbol{p}_{D_{ji}}) + R_{C_{i,0}} \right], \tag{2}$$

where $v_{i,j}(P_{B_i}, P_{D_{ji}}) := R_{C_{i,j}}^{LB} + R_{D_{j,i}} - R_{C_{i,0}}$ represents the minimum rate increment due to the assignment of channel i to the D2D pair j, relative to the case where the channel i is only used by the CU. Next, notice that the objective of (1), with substitution of (2), can be equivalently expressed by replicating $\{\boldsymbol{p}_{B_i}\}$ with multiple auxiliary variables $\{\boldsymbol{p}_{B_{ij}}\}$ and removing the constant terms from the objective function. The resulting equivalent problem can be stated as:

$$\begin{aligned} \underset{\mathbb{B},\mathbb{P}_B,\mathbb{P}_D}{\text{maximize}} \quad & \sum_{i\in\mathcal{C}} \sum_{j\in\mathcal{D}} \left[\beta_{i,j} v_{i,j}(\boldsymbol{p}_{B_{ij}}, \boldsymbol{p}_{D_{ji}})\right] - \gamma\delta(\mathbb{B}) \\ \text{subject to} \quad & \text{(1b), (1c), (1d), (1e).} \end{aligned} \tag{3}$$

To recover the optimal $\{\boldsymbol{p}_{B_i}^*\}$ of (1) from the optimal $\{\boldsymbol{p}_{B_{i,j}}^*\}$ of (3), one just needs to find, for each i, the value of j such that $\beta_{i,j} = 1$ and set $\boldsymbol{p}_{B_i}^* = \boldsymbol{p}_{B_{i,j}}^*$. If no such j exists, i.e. $\beta_{i,j} = 0 \;\forall j$, then channel i is not assigned to any D2D pair and the BS can transmit with maximum power.

In addition, similar to [17], it can be shown that (3) decouples across i and j into $N_C \times N_D$ power allocation sub-problems and a final channel assignment problem. The power allocation sub-problem can be stated as:

$$\begin{aligned} \underset{\boldsymbol{p}_{B_{ij}},\, \boldsymbol{p}_{D_{ji}}}{\text{maximize}} \quad & R_{C_{i,j}}^{LB} + R_{D_{j,i}} \\ \text{subject to} \quad & \text{(1c), (1d) and (1e),} \end{aligned} \tag{4}$$

which should be solved $\forall i \in \mathcal{C}, \forall j \in \mathcal{D}$. The subsequent channel assignment problem is discussed in subsection IV-D. We can notice that problem (4) is still a non-convex stochastic problem. Hence, we derive next closed-form expressions for the stochastic terms.

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A. Closed-form stochastic constraints

In order to bring the stochastic terms from the objective (4) to the constraints, we introduce slack variables $\boldsymbol{z} \triangleq [z_C,\ z_D]^T$ as follows:

$$\underset{\boldsymbol{p}_{B_i},\boldsymbol{p}_{D_{ji}},\boldsymbol{z}}{\text{maximize}} \quad \log_2(1+z_C)+\log_2(1+z_D) \tag{5a}$$

$$\text{subject to } \Pr\left(z_C \leq \frac{|\boldsymbol{g}_{B_i}^H \boldsymbol{p}_{B_i}|^2}{N_0+|\tilde{\boldsymbol{h}}_{D_{ji}}^H \boldsymbol{p}_{D_{ji}}|^2}\right) \geq 1-\epsilon \tag{5b}$$

$$z_D \leq \frac{|\boldsymbol{g}_{D_j}^H \boldsymbol{p}_{D_{ji}}|^2}{N_0+|\boldsymbol{h}_{B_j}^H \boldsymbol{p}_{B_i}|^2}, \quad \text{(1c), (1d) and (1e)} \tag{5c}$$

Representing the random interference channel vector $\tilde{\boldsymbol{h}}_{D_{ji}} = \boldsymbol{h}_{D_{ji}} + \boldsymbol{e}_{ji}$, where $\boldsymbol{e}_{ji} \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{M}_{ji})$, the stochastic inequality, $z_C \leq \frac{|\boldsymbol{g}_{B_i}^H \boldsymbol{p}_{B_i}|^2}{N_0+|\tilde{\boldsymbol{h}}_{D_{ji}}^H \boldsymbol{p}_{D_{ji}}|^2}$ can be equivalently expressed as $N_0+|\boldsymbol{h}_{D_{ji}}^H \boldsymbol{p}_{D_{ji}}|^2 + \boldsymbol{e}_{ji}^H \boldsymbol{p}_{D_{ji}} \boldsymbol{p}_{D_{ji}}^H \boldsymbol{e}_{ji} + 2\,\mathrm{Re}\left\{\boldsymbol{e}_{ji}^H \boldsymbol{p}_{D_{ji}} \boldsymbol{p}_{D_{ji}}^H \boldsymbol{h}_{D_{ji}}\right\} \leq \frac{1}{z_C}|\boldsymbol{g}_{B_i}^H \boldsymbol{p}_{B_i}|^2$. Further, letting $\boldsymbol{e}_{ji} := \boldsymbol{M}_{ji}^{1/2} \boldsymbol{v}_{ji}$ where, $\boldsymbol{v}_{ji} \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{I})$, this stochastic inequality can equivalently stated as:

$$N_0+|\boldsymbol{h}_{D_{ji}}^H \boldsymbol{p}_{D_{ji}}|^2 + \boldsymbol{v}_{ji}^H \boldsymbol{Q}_{ji} \boldsymbol{v}_{ji} + 2\,\mathrm{Re}\left\{\boldsymbol{v}_{ji}^H \boldsymbol{u}_{ji}\right\} \leq \frac{1}{z_C}|\boldsymbol{g}_{B_i}^H \boldsymbol{p}_{B_i}|^2 \tag{6}$$

where, $\boldsymbol{Q}_{ji} := \boldsymbol{M}_{ji}^{1/2} \boldsymbol{p}_{D_{ji}} \boldsymbol{p}_{D_{ji}}^H \boldsymbol{M}_{ji}^{1/2}$ and $\boldsymbol{u}_{ji} := \boldsymbol{M}_{ji}^{1/2} \boldsymbol{p}_{D_{ji}} \boldsymbol{p}_{D_{ji}}^H \boldsymbol{h}_{D_{ji}}$. Thus, the stochastic constraint (5b) can be re-stated as:

$$\Pr(\boldsymbol{v}_{ji}^H \boldsymbol{Q}_{ji} \boldsymbol{v}_{ji} + 2\,\mathrm{Re}\left\{\boldsymbol{v}_{ji}^H \boldsymbol{u}_{ji}\right\} \leq c_{ji}) \geq 1-\epsilon, \tag{7}$$

where $c_{ji} = \frac{1}{z_C}|\boldsymbol{g}_{B_i}^H \boldsymbol{p}_{B_i}|^2 - N_0 - |\boldsymbol{h}_{D_{ji}}^H \boldsymbol{p}_{D_{ji}}|^2$. Next, in order to obtain a closed from expression for (7), we use a Bernstein-type inequality for the quadratic form of Gaussian vectors [13].

Lemma 1. *Let $G = \boldsymbol{v}^H \boldsymbol{Q} \boldsymbol{v} + 2\,\mathrm{Re}\left\{\boldsymbol{v}^H \boldsymbol{u}\right\}$ where $\boldsymbol{Q} \in \mathbb{H}^{K_D}$ is a complex Hermitian matrix, $\boldsymbol{u} \in \mathbb{C}^{K_B}$ and $\boldsymbol{v} \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{I})$. Then for any $\delta > 0$, we have:*
$Pr\{G \leq Tr(\boldsymbol{Q}) + \sqrt{\delta}\sqrt{||\boldsymbol{Q}||_F^2 + 2||\boldsymbol{u}||_2^2} + 2\delta s^+(\boldsymbol{Q})\} \geq 1-e^{-\delta}$,
where $s^+(\boldsymbol{Q}) = \max\{\lambda_{\max}(\boldsymbol{Q}), 0\}$, $\lambda_{\max}(\boldsymbol{Q})$ denotes the maximum eigenvalue of $\boldsymbol{Q}$, and $||\cdot||_F$ denotes the matrix Frobenius norm.

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Considering Lemma 1 and setting $\delta = -\ln(\epsilon)$, equation (7) holds if the following inequality is satisfied:

$$\mathrm{Tr}(\boldsymbol{Q}_{ji}) + \sqrt{\delta}\sqrt{||\boldsymbol{Q}_{ji}||_F^2 + 2||\boldsymbol{u}_{ji}||_2^2} + 2\delta s^+(\boldsymbol{Q}_{ji}) \leq c_{ji}. \tag{8}$$

Rearranging the terms in (8) and exploiting Lemma 1 for constraint (1d), problem (5) can be stated as,

$$\underset{\boldsymbol{p}_{B_i},\boldsymbol{p}_{D_{ji}},\boldsymbol{z}}{\text{maximize}} \quad \log_2(1+z_C)+\log_2(1+z_D) \tag{9a}$$

$$\text{subject to } z_C \leq \frac{|\boldsymbol{g}_{B_i}^H \boldsymbol{p}_{B_i}|^2}{f(\boldsymbol{p}_{D_{ji}}, \boldsymbol{Q}_{ji}, \boldsymbol{u}_{ji})}, \quad z_D \leq \frac{|\boldsymbol{g}_{D_j}^H \boldsymbol{p}_{D_{ji}}|^2}{N_0+|\boldsymbol{h}_{B_j}^H \boldsymbol{p}_{B_i}|^2} \tag{9b}$$

$$\frac{|\boldsymbol{g}_{B_i}^H \boldsymbol{p}_{B_i}|^2}{f(\boldsymbol{p}_{D_{ji}}, \boldsymbol{Q}_{ji}, \boldsymbol{u}_{ji})} \geq \eta_{C,\min}, \quad \text{(1c) and (1e)} \tag{9c}$$

where, $f(\boldsymbol{p}_{D_{ji}}, \boldsymbol{Q}_{ji}, \boldsymbol{u}_{ji}) := N_0 + |\boldsymbol{h}_{D_{ji}}^H \boldsymbol{p}_{D_{ji}}|^2 + \mathrm{Tr}(\boldsymbol{Q}_{ji}) + \sqrt{\delta}\sqrt{||\boldsymbol{Q}_{ji}||_F^2 + 2||\boldsymbol{u}_{ji}||_2^2} + 2\delta s^+(\boldsymbol{Q}_{ji})$. Notice that the constraints (9b) involve (i) a ratio of a convex and a non-convex function, and (ii) a ratio between two convex functions, which are non-convex. In the next subsection, we use fractional programming to relax the non-convexity due to these ratios.

B. Fractional Programming by Quadratic Transformation

Taking a partial Lagrangian by considering only the constraints related to the slack variables z_C and z_D (9b), we have:

$$L(\boldsymbol{p}, \boldsymbol{z}, \boldsymbol{\lambda}) = \log_2(1 + z_C) + \log_2(1 + z_D) -$$
$$\lambda_C \left(z_C - \frac{|\boldsymbol{g}_{B_i}^H \boldsymbol{p}_{B_i}|^2}{f(\boldsymbol{p}_{D_{ji}}, \boldsymbol{Q}_{ji}, \boldsymbol{u}_{ji})} \right) - \lambda_D \left(z_D - \frac{|\boldsymbol{g}_{D_j}^H \boldsymbol{p}_{D_{ji}}|^2}{N_0 + |\boldsymbol{h}_{B_j}^H \boldsymbol{p}_{B_i}|^2} \right)$$

At a stationary point, $\frac{\partial L}{\partial \boldsymbol{z}} = 0$; thus, the optimal values of the Lagrange variables can be computed as $\lambda_C = \frac{1}{1+z_C}$ and $\lambda_D = \frac{1}{1+z_C}$. Furthermore, the optimal values of the slack variables are achieved when the inequality constraints (9b) are satisfied with equality. Thus, by calculating λ_C^* and λ_D^* and substituting them in problem (9a) we obtain:

$$\begin{aligned} \underset{\boldsymbol{p}_{B_i}, \boldsymbol{p}_{D_{ji}}, \boldsymbol{z}}{\text{maximize}} \quad & \log_2(1 + z_C) + \log_2(1 + z_D) \\ & - z_C + \frac{(1 + z_C)|\boldsymbol{g}_{B_i}^H \boldsymbol{p}_{B_i}|^2}{|\boldsymbol{g}_{B_i}^H \boldsymbol{p}_{B_i}|^2 + f(\boldsymbol{p}_{D_{ji}}, \boldsymbol{Q}_{ji}, \boldsymbol{u}_{ji})} \\ & - z_D + \frac{(1 + z_D)|\boldsymbol{g}_{D_j}^H \boldsymbol{p}_{D_{ji}}|^2}{N_0 + |\boldsymbol{g}_{D_j}^H \boldsymbol{p}_{D_{ji}}|^2 + |\boldsymbol{h}_{B_j}^H \boldsymbol{p}_{B_i}|^2} \\ \text{subject to (9c)} & \end{aligned} \tag{10}$$

Next, we absorb the fractions in the objective by introducing two auxiliary variables y_C and y_D through a quadratic transformation [15], obtaining:

$$\begin{aligned} \underset{\boldsymbol{p}_{B_i}, \boldsymbol{p}_{D_{ji}}, \boldsymbol{z}, \boldsymbol{y}}{\text{maximize}} \quad & \log_2(1 + z_C) + \log_2(1 + z_D) - z_C - z_D \\ & + 2y_C\sqrt{(1 + z_C)|\boldsymbol{g}_{B_i}^H \boldsymbol{p}_{B_i}|^2} + 2y_D\sqrt{(1 + z_D)|\boldsymbol{g}_{D_j}^H \boldsymbol{p}_{D_{ji}}|^2} \\ & - y_C^2 \left(|\boldsymbol{g}_{B_i}^H \boldsymbol{p}_{B_i}|^2 + f(\boldsymbol{p}_{D_{ji}}, \boldsymbol{Q}_{ji}, \boldsymbol{u}_{ji})\right) \\ & - y_D^2 \left(N_0 + |\boldsymbol{g}_{D_j}^H \boldsymbol{p}_{D_{ji}}|^2 + |\boldsymbol{h}_{B_j}^H \boldsymbol{p}_{B_i}|^2\right) \\ \text{subject to} \quad & \text{(9c)} \end{aligned} \tag{11}$$

The optimal values of y_C and y_D can be readily obtained as:

$$\begin{aligned} y_C^* &= \sqrt{(1 + z_C)|\boldsymbol{g}_{B_i}^H \boldsymbol{p}_{B_i}|^2}/(|\boldsymbol{g}_{B_i}^H \boldsymbol{p}_{B_i}|^2 + f(\boldsymbol{p}_{D_{ji}}, \boldsymbol{Q}_{ji}, \boldsymbol{u}_{ji})), \\ y_D^* &= \sqrt{(1 + z_D)|\boldsymbol{g}_{D_j}^H \boldsymbol{p}_{D_{ji}}|^2}/(N_0 + |\boldsymbol{g}_{D_j}^H \boldsymbol{p}_{D_{ji}}|^2 + |\boldsymbol{h}_{B_j}^H \boldsymbol{p}_{B_i}|^2) \end{aligned} \tag{12}$$

Notice that for the updated values of the auxiliary variables z_C, z_D, y_C and y_D, optimization problem (11) is still jointly non-convex in $\boldsymbol{p}_{B_i}$ and $\boldsymbol{p}_{D_{ji}}$. We propose performing a semi-definite relaxation in (11) on the variables $\boldsymbol{p}_{B_i}$ and $\boldsymbol{p}_{D_{ji}}$ as shown in the following subsection.

C. Semi-definite Relaxation

In order to obtain convex sub-problems from (11), let us denote $\boldsymbol{P}_{B_i} := \boldsymbol{p}_{B_i}\boldsymbol{p}_{B_i}^H$ and $\boldsymbol{P}_{D_{ji}} := \boldsymbol{p}_{D_{ji}}\boldsymbol{p}_{D_{ji}}^H$. We consider the SDR sub-problem of (11) to optimize $\boldsymbol{P}_{B_i}$ and $\boldsymbol{P}_{D_{ji}}$

for given values of z_C, z_D, y_C and y_D, which can be stated as follows:

$$\begin{aligned}
\underset{\boldsymbol{P}_{B_i},\boldsymbol{P}_{D_{ji}},w_1,w2}{\text{maximize}} \quad & 2y_C\sqrt{(1+z_C)\boldsymbol{g}_{B_i}^H\boldsymbol{P}_{B_i}\boldsymbol{g}_{B_i}} - y_C^2\boldsymbol{g}_{B_i}^H\boldsymbol{P}_{B_i}\boldsymbol{g}_{B_i} \\
& - y_D^2\boldsymbol{h}_{B_j}^H\boldsymbol{P}_{B_i}\boldsymbol{h}_{B_j} + 2y_D\sqrt{(1+z_D)\boldsymbol{g}_{D_j}^H\boldsymbol{P}_{D_{ji}}\boldsymbol{g}_{D_j}} \\
& - y_D^2\boldsymbol{g}_{D_j}^H\boldsymbol{P}_{D_{ji}}\boldsymbol{g}_{D_j} - y_C^2\left(\boldsymbol{h}_{D_{ji}}^H\boldsymbol{P}_{D_{ji}}\boldsymbol{h}_{D_{ji}} + w_1 + 2\delta w_2\right) \\
\text{subject to} \quad & 0 \le \mathrm{Tr}(\boldsymbol{P}_{B_i}) \le p_{C,\max}, \quad \boldsymbol{P}_{B_i} \succeq 0, \qquad (13) \\
& 0 \le \mathrm{Tr}(\boldsymbol{P}_{D_{ji}}) \le p_{D,\max}, \quad \boldsymbol{P}_{D_{ji}} \succeq 0, \\
& \eta_{C,\min}\left(N_0 + \boldsymbol{h}_{D_{ji}}^H\boldsymbol{P}_{D_{ji}}\boldsymbol{h}_{D_{ji}} + w_1 + 2\delta w_2\right) - \boldsymbol{g}_{B_i}^H\boldsymbol{P}_{B_i}\boldsymbol{g}_{B_i} \le 0, \\
& \eta_{D,\min}\left(N_0 + \boldsymbol{h}_{B_j}^H\boldsymbol{P}_{B_i}\boldsymbol{h}_{B_j}\right) - \boldsymbol{g}_{D_j}^H\boldsymbol{P}_{D_{ji}}\boldsymbol{g}_{D_j} \le 0, \\
& \sqrt{\delta}\left\|\begin{bmatrix}\mathrm{vec}(\boldsymbol{M}_{ji}^{1/2}\boldsymbol{P}_{D_{ji}}\boldsymbol{M}_{ji}^{1/2}) \\ \sqrt{2}(\boldsymbol{M}_{ji}^{1/2}\boldsymbol{P}_{D_{ji}}\bar{\boldsymbol{h}}_{D_{ji}})\end{bmatrix}\right\| \le w_1 - \mathrm{Tr}(\boldsymbol{M}_{ji}^{1/2}\boldsymbol{P}_{D_{ji}}\boldsymbol{M}_{ji}^{1/2}), \\
& \boldsymbol{M}_{ji}^{1/2}\boldsymbol{P}_{D_{ji}}\boldsymbol{M}_{ji}^{1/2} - w_2\boldsymbol{I} \preceq 0,
\end{aligned}$$

where, $w_1, w_2 \in \mathbb{R}$ are slack variables. Note that the obtained optimal solution for the relaxed problem in (13) may not be rank one; thus, additional rank one approximation procedures may be needed to obtain the $\boldsymbol{p}_{B_i}, \boldsymbol{p}_{D_{ji}}$ vectors from the respective $\boldsymbol{P}_{B_i}^*, \boldsymbol{P}_{D_{ji}}^*$ matrices. We propose solving this by scaling the eigen-vector $\boldsymbol{v}_{\max}$ corresponding to the highest eigenvalue $\lambda_{\max}$ with the square root of this eigenvalue. In case that the obtained vectors are not feasible with respect to the original constraints in (9c), we propose using the following equation to obtain a feasible solution:

$$\begin{aligned}
\boldsymbol{p}_{B_i} &= \alpha\sqrt{\lambda_{\max}(\boldsymbol{P}_{B_i})}\boldsymbol{v}_{\max}(\boldsymbol{P}_{B_i}) + (1-\alpha)\sqrt{p_{B,\max}}\frac{\boldsymbol{g}_{B_i}}{\|\boldsymbol{g}_{B_i}\|}, \\
\boldsymbol{p}_{D_{ji}} &= \alpha\sqrt{\lambda_{\max}(\boldsymbol{P}_{D_{ji}})}\boldsymbol{v}_{\max}(\boldsymbol{P}_{D_{ji}}),
\end{aligned}$$

where α is the highest number $\in [0,1]$ such that $\boldsymbol{p}_{B_i}$ and $\boldsymbol{p}_{D_{ji}}$ are feasible, and $\alpha = 0$ will lead to a solution where the D2D pair is not transmitting while other constraints are satisfied. To sum up, our power optimization problem in (4) is solved by iteratively updating the auxiliary variables z_C, z_D, y_C, y_D followed by solving the relaxed convex sub-problem (13) for updating the values of $\boldsymbol{p}_{B_i}$ and $\boldsymbol{p}_{D_{ji}}$ until convergence. Once (4) is solved $\forall i \in \mathcal{C}, \forall j \in \mathcal{D}$, the next step is to perform channel assignment to D2D pairs, as explained next.

D. Channel Assignment via Integer Relaxation

For the channel assignment to D2D pairs, sub-optimal values $\tilde{v}_{i,j}$ (obtained after solving (4) $\forall i \in \mathcal{C}, \forall j \in \mathcal{D}$) are substituted into (3) and then we need to maximize with respect to $\mathbb{B}$.The resulting channel assignment sub-problem can be stated as:

$$\begin{aligned}
\underset{\mathbb{B}}{\text{maximize}} \quad & \sum_{i\in\mathcal{C}}\sum_{j\in\mathcal{D}}\beta_{i,j}v_{i,j}^* - \gamma\delta(\mathbb{B}), \qquad (14) \\
\text{subject to} \quad & \beta_{i,j} \in \{0,1\}\ \forall\ i,j, \quad \sum_{j\in\mathcal{D}}\beta_{i,j} \le 1\ \forall\ i.
\end{aligned}$$

Due to the integer constraints, solving (14) involves prohibitive computational complexity even for reasonable values of $N_C,\ N_D$. Thus, similar to [17], we relax the integer constraints to $\beta_{i,j} \in [0,1]\ \forall i,j$ to obtain a differentiable strongly convex objective function with linear constraints which can be efficiently solved using the Projected Gradient Descent algorithm. The obtained solution is finally discretized back to satisfy the original constraints $\beta_{i,j} \in \{0,1\}\ \forall i,j$. This is done by setting the highest positive value in every row of $\mathbb{B}$ to 1 while setting other values in the same row to 0.

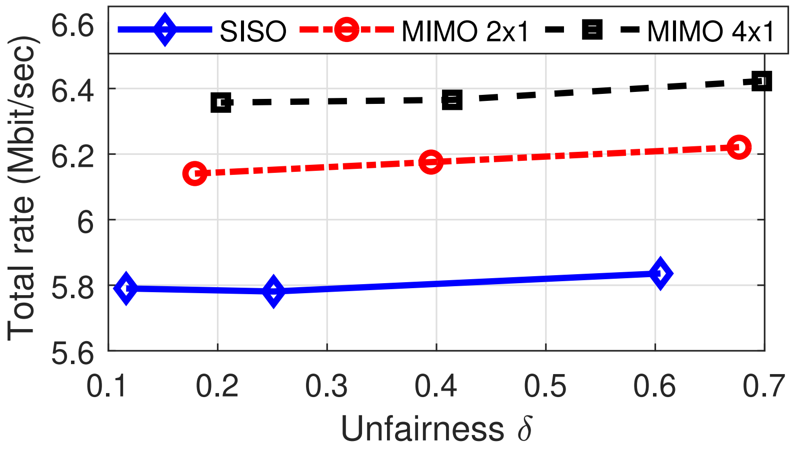

Fig. 2: Total average rate R vs. Unfairness δ

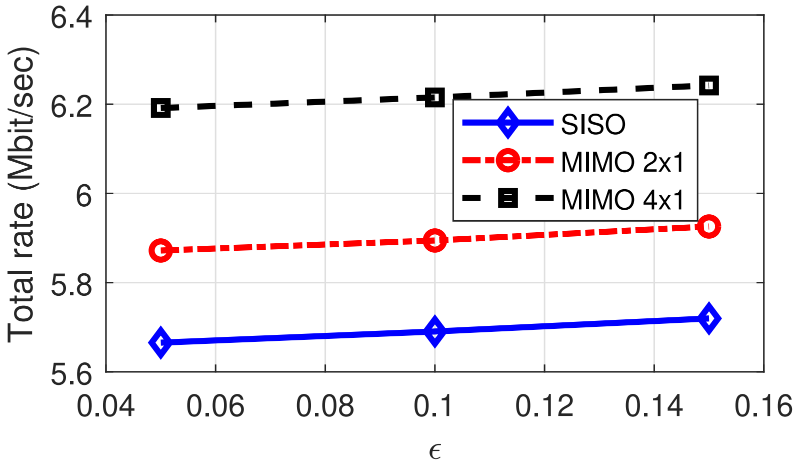

Fig. 3: Total average rate R vs. ϵ

V. Simulations

The simulation setup comprises a circular cell of 500 m radius in which the CUs and D2D transmitters are placed uniformly at random. Each D2D receiver is placed uniformly at random inside a circle of radius $5m$ centered at the corresponding transmitter. The channel gains are calculated using a path loss model with exponent 2 and gain -5 dB at a reference distance of $1m$. Averages over 1,000 independent realizations with parameters BW $=$ 15 kHz, $\gamma = 50 \times$ BW, $N_D = 5$, $N_C = 5$, $N_0 = -70$ dBW. Two values of the number of transmit antennas were tested $K_B = K_D = 2$ and $K_B = K_D = 4$. The proposed method is compared with the method by Elnourani et al. [18], which to the best of our knowledge is the best existing method for the SISO case.

Fig. 2 shows that the proposed methods achieve higher rate than the method by [18] for $\epsilon = 0.1$ and for different values of γ between 10 and 30. The increment in the total rate is around 3% for all values of γ for the 2×1 MIMO case and around 9% for the 4×1 MIMO case. In general, all rates decrease when γ increases.

Fig. 3 shows that the proposed methods achieve similar increment in rate for different values of ϵ with $\gamma = 100$. In general, all rates increase when ϵ increases. The unfairness values for all the tested cases are between 0 and 0.035, which are very small and very close. This indicates that the selected methods were able to achieve a good rate-fairness trade-off for the specified values of γ and ϵ.

Fig. 4 and Fig. 5 show a comparison between the proposed method and an unreliable beamforming method in [15]. The unreliable method achieves better rate compared to the

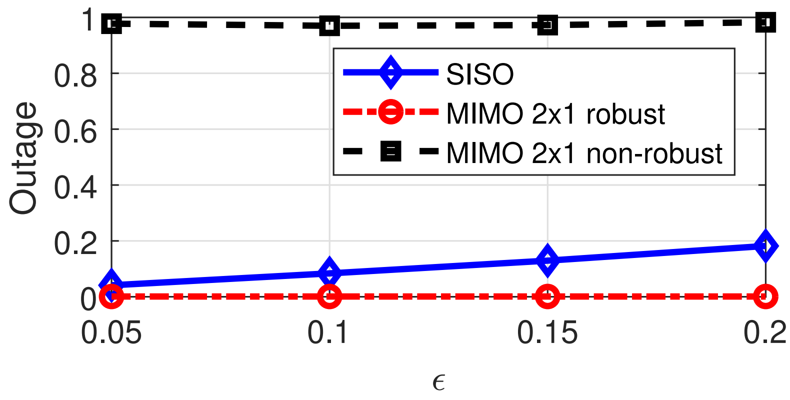

Fig. 4: Outage probability vs. ϵ.

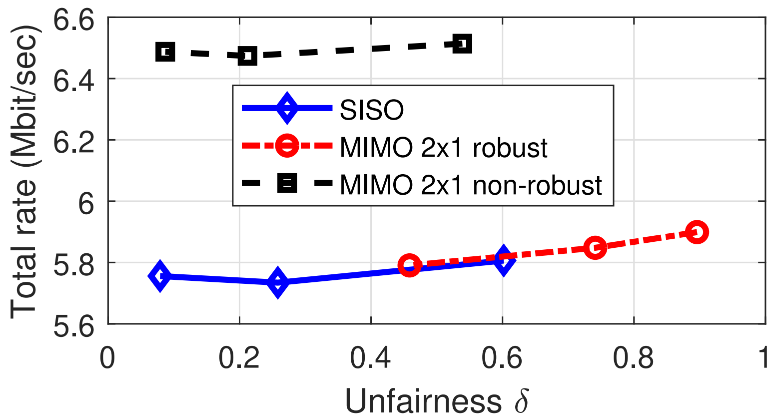

Fig. 5: Total average rate R vs. Unfairness δ ($\epsilon = 0.1$ and γ from 10 to 30)

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proposed method, however the outage probability is close to 100%. On the other hand, our proposed method achieves almost 0 outage probability, since the Bernstein-type inequality is a very conservative approximation for the reliability constraint.

In general, MIMO beamforming can be considered as adding more degrees of freedom to the system. In the cases where a D2D pair is considered inadmissible by [18], due to infeasiblility in the power allocation, beamforming might render the power allocation problems feasible, and thus, resulting in better fairness. Moreover, it will also results in a higher total rate, since it usually generates lower interference.

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