# Reliable Underlay Device-to-Device Communications on Multiple Channels

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#### Abstract

Device-to-device (D2D) communications provide a substantial increase in spectrum usage and efficiency by allowing nearby users to communicate directly without passing their packets through the base station (BS). In previous works, proper channel assignment and power allocation algorithms for sharing of channels between cellular users and D2D pairs, usually require exact knowledge of the channel-state-information (CSI). However, due to the non-stationary wireless environment and the need to limit the communication and computation overheads, obtaining perfect CSI in the D2D communication scenario is generally not possible. In this work, we propose a joint channel assignment and power allocation strategy for D2D pairs and cellular users to maximize the overall aggregate throughput, under imperfect knowledge of CSI, while guaranteeing the outage probability for all users and encouraging fairness among D2D pairs. The proposed solution does not restrict the D2D transmitters to operate on a single channel, allowing each D2D pair to simultaneously access multiple channels and increase the overall throughput. We propose both a centralized and a decentralized method to solve our problem, where the computation load of the BS is alleviated by decomposing our problem into several subproblems, each of them being solved iteratively at the individual D2D pairs. Numerical experiments corroborate the merits of the proposed schemes when compared with state-of-the-art alternatives.

#### **Index Terms**

D2D communications, power allocation, channel assignment, reliability, convex relaxation.

#### I. INTRODUCTION

The exponentially increasing throughput demand in cellular communication networks [1] can no longer be met by increasing the *spectral* efficiency of point-to-point links, since existing systems are already approaching the channel capacity [2]. D2D communications constitute a prominent example in improving *spatial* efficiency, where mobile users are allowed to communicate directly with each other without passing their messages through the BS [3], [4]. Thus, users operating in D2D mode need half of resources of those operating in the traditional cellular mode. Moreover, channels (e.g. frequency bands or time slots) used by D2D users can be simultaneously used by a traditional cellular user under restricted interference configuration, a framework termed *underlay*. It is necessary to devise algorithms that judiciously assign cellular channels to D2D users and prudently control the power to limit interference to cellular users and guarantee quality of service (QoS) (e.g. SINR, reliability) to all users. In addition, algorithms must be computationally inexpensive and reliable in imperfect CSI cases.

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Early works on D2D communications typically rely on simplistic channel assignment schemes, where each D2D pair communicate through a cellular sub-channel (hereafter referred to as *channel*) selected uniformly at random by the BS [5]. However, these approaches control interference in a simplistic way. To overcome these limitations, [6] proposes a scheme where each D2D pair simultaneously transmits in all cellular channels. However, every pair control its own power independently. In [7], channels are judiciously assigned by the BS. However, this work does not incorporate power control. In [8]–[10], joint channel assignment and power allocation was proposed. These schemes restrict D2D users to access at most one channel. In addition, all the aforementioned works have adopted a model with perfect CSI. There are few works that consider reliability by guaranteeing the desired outage probabilities for cellular users (CUs) under imperfect CSI. In [11]-[13], powers and channels are assigned while restricting D2D users to access at most one cellular channel. To sum up, non of the existing approaches provide a reliable joint channel assignment and power allocation for the scenario where D2D users can simultaneously operate on multiple cellular channels, which is very relevant for maximizing throughput among D2D pairs. In addition, considering energy and infrastructure cost at BS, investigation on reducing the computational load at BS also needs appropriate attention.

This paper considers the above challenges by proposing two *reliable* joint channel assignment and power allocation solutions (centralized and decentralized) that allows each D2D pair to use more than one cellular channel while guaranteeing certain SINR and outage probability under *imperfect CSI* scenario. Here, we consider the downlink scenario, however, with minor modifications, the proposed algorithms can easily be adapted to the uplink scenario. The key research contributions of this work can be summarized as:

- We formulate an optimization problem to jointly assign channels and allocate power to D2D pairs in a downlink cellular environment. We also consider guaranteeing a certain outage probability to address imperfect CSI. In addition, we include an unfairness measure which penalizes assigning most channels to a small fraction of D2D pairs. The resulting optimisation problem is a mixed integer non-convex problem.
- We show that our overall problem can be decomposed into several power allocation subproblems and a channel assignment problem without loss of optimality, and propose an efficient centralized algorithm performed at the BS in order to solve our problem.
- We also propose a decentralized algorithm that reduces the computation load at the BS by performing alternating maximization over each of the power allocation subproblems associated to each of the D2D pairs. Moreover, some of the computations for the channel assignment problem are performed by the D2D pairs.

Our simulations show good performance relative to the state-of-the-art alternatives. The rest of this paper is structured as follows. Sec. II describes the system model. Sec. III introduces the joint channel assignment and resource allocation problem and proposes two efficient algorithms to solve it. Finally, Sec. IV provides the simulations.

## II. SYSTEM MODEL

Consider a cell (or sector) where a BS communicates with  $N_C$  CUs through  $N_C$  downlink channels<sup>12</sup>. For convenience, the set of CUs (or, equivalently, channels) will be indexed by  $C = \{1, ..., N_C\}$ . In this cell,  $N_D$  D2D pairs, indexed by  $\mathcal{D} = \{1, ..., N_D\}$  (typically  $N_D < N_C$ ), wish to communicate using the aforementioned downlink channels at the same time as the BS (underlay communications). The assignment of channels to D2D pairs will be represented

<sup>&</sup>lt;sup>1</sup>Recall that a channel here may stand for resource blocks, or time slots.

 $<sup>^{2}</sup>$ In general, a CU can use multiple channels simultaneously. The same model can be used by putting similar CUs in every channel that was assigned to the same user.



Fig. 1: Illustration of the system model.

by the indicators  $\{\beta_{i,j}\}_{i\in\mathcal{C},j\in\mathcal{D}}$ , where  $\beta_{i,j} = 1$  when the D2D pair j uses channel i and  $\beta_{i,j} = 0$  otherwise. It is assumed that each D2D pair can access multiple channels at the same time, but no channel can be used by multiple D2D pairs simultaneously, which implies that  $\sum_{j=1}^{N_D} \beta_{i,j} \leq 1$ ,  $\forall i$ . The transmission power used by the BS to communicate with the i-th CU is represented by  $P_{B_i}$  and is constrained to lie in the interval  $0 \leq P_{B_i} \leq P_{B_{\max}}$ . Similarly,  $P_{D_{ji}}$  is the transmission power used by the j-th D2D pair when utilizing the i-th channel and is constrained to  $0 \leq P_{D_{ji}} \leq P_{D_{\max}}$ . Successful communications require the signal-to-interference-plus-noise ratio (SINR) to be greater than  $\eta_{\min}^C$  for CUs and  $\eta_{\min}^D$  for D2D receivers, and cellular users have a maximum allowed outage ratio of  $\epsilon$ .

Fig. 1 illustrates the system model. Specifically,  $g_{B_i}$  denotes the gain between the BS and the *i*-th CU;  $g_{D_j}$  denotes the gain<sup>3</sup> of the *j*-th D2D link;  $\tilde{h}_{C_{j,i}}$  denotes the gain of the interference link from the transmitter of the *j*-th D2D pair to the *i*-th CU, which is modeled as a random variable since it is usually estimated at the receiver with a minimum cooperation with the CU (as opposed to the model in [14]);  $h_{B_j}$  denotes the gain of the interference link between the BS and the receiver of the *j*-th D2D pair; and  $N_0$  the noise power.

between the BS and the receiver of the *j*-th D2D pair; and  $N_0$  the noise power. Given  $g_{B_i}$ ,  $g_{D_j}$ ,  $h_{B_j}$ , the distribution of  $\tilde{h}_{C_{ji}} \forall i, j$ , as well as  $N_0$ ,  $\eta^C_{\min}$ ,  $\eta^D_{\min}$ ,  $\epsilon$ ,  $P_{C_{\max}}$ , and  $P_{D_{\max}}$ , the goal is to choose  $\beta_{i,j}$ ,  $P_{B_i}$ ,  $P_{D_{ji}} \forall i, j$  to maximize the aggregate throughput of the D2D pairs and CUs while ensuring fairness among D2D pairs, by discouraging assigning channels to D2D pairs unequally, and preventing detrimental interference to CUs by guaranteeing the desired outage probability.

#### **III. JOINT CHANNEL ASSIGNMENT AND POWER ALLOCATION**

We next formulate the optimization problem. Then in Sec. III-B, we propose two efficient algorithms. To simplify the notation, let us collect the requested variables in vector-matrix form as  $\boldsymbol{B} = [\beta_{i,j}] \in \mathbb{R}^{N_C \times N_D}$ ,  $\boldsymbol{P}_{\boldsymbol{D}} = [P_{D_{j,i}}] \in \mathbb{R}^{N_D \times N_C}$ , and  $\boldsymbol{p}_{\boldsymbol{B}} = [P_{B_i}] \in \mathbb{R}^{N_C}$ .

#### A. Problem formulation

The first step is to select a criterion that quantifies the desirability of a given channel assignment and power allocation  $(B, p_B, P_D)$ . To guarantee a desired outage probability  $\epsilon$ ,

<sup>&</sup>lt;sup>3</sup>Note that  $g_{D_j}$  and  $h_{B_j}$  should in principle depend also on *i* since the associated gains generally depend on the channel selected by the *j*-th pair; however, this subscript is dropped for simplicity since the proposed scheme carries over immediately to accommodate such dependence.

we adopt a criterion to maximize the minimum network rate, which must be at least achieved for a  $(1 - \epsilon)$  portion of the overall time. Additionally, an unfairness term that penalizes unfair channel assignments is included in the objective function. Let us define the rate  $\Gamma(z) := BW \times \log_2(1 + z)$ , where BW is the channel bandwidth and z is the SINR. The minimum network rate can be considered by analyzing the lower bound of the total rate at channel *i*, which is defined as:  $R_i^{LB} := (1 - \sum_{j \in D} \beta_{i,j}) R_{C_{i,0}} + \sum_{j \in D} \beta_{i,j} [R_{D_{j,i}} + R_{C_{i,j}}^{LB}]$ , where:

- $R_{C_{i,0}} = \Gamma(P_{B_{max}}g_{B_i}/N_0)$  is the rate of the *i*-th CU when it does not share its channel with D2D pairs *i.e.*  $\beta_{ij} = 0 \forall j$ ,
- $R_{D_{j,i}} = \Gamma(P_{D_{ji}}g_{D_j} / (N_0 + P_{B_i}h_{B_j}))$  is the rate of the *j*-th D2D pair when sharing the channel with the *i*-th CU *i.e.*  $\beta_{ij} = 1$ ,
- $R_{C_{i,j}}^{\text{LB}}$  denotes the lower bound (which must be at least achieved  $(1 \epsilon)$  portion of the time) of the rate of the *i*-th CU when sharing the channel with the *j*-th D2D pair  $(\beta_{ij} = 1)$ . Since  $\tilde{h}_{C_{j,i}}$  is random, we can compute  $R_{C_{i,j}}^{\text{LB}} = \Gamma(z_{C_{i,j}}^{\text{LB}})$  where  $z_{C_{i,j}}^{\text{LB}} : \Pr\{z_{C_{i,j}}^{\text{LB}} \leq P_{B_i}g_{B_i}/(N_0 + P_{D_{ii}}\tilde{h}_{C_{i,i}})\} = 1 \epsilon$ .
- The minimum network rate is therefore  $R(\boldsymbol{B}, \boldsymbol{p}_{\boldsymbol{B}}, \boldsymbol{P}_{\boldsymbol{D}}) := \sum_{i \in \mathcal{C}} R_i^{LB}$ .

We consider a second term of the objective that penalizes channel assignments where a small fraction of D2D pairs use a large part of the channels. To this end, the *unfairness* measure  $\delta(B)$  from [7], [14] is considered in this work. It is given by  $\delta^2(B) = 1/(N_D x_0^2) \sum_{j=1}^{N_D} (x_j(B) - x_0)^2$ , where  $x_j := \sum_{i=1}^{N_C} \beta_{i,j}$  is the number of channels assigned to the *j*-th D2D pair and  $x_0 := N_C/N_D$ . If  $N_C$  is an integer multiple of  $N_D$ , then  $x_j = x_0 \forall j$  would be fairest channel assignment possible.  $\delta(B)$  can be interpreted as the root mean deviation of  $\{x_j\}_{j=1}^{N_D}$  from their fairest value  $x_0$  and thus the more unevenly channels are assigned among D2D pairs, the larger it is.

The overall problem can then be formulated as:

$$\underset{\boldsymbol{B},\boldsymbol{p}_{\boldsymbol{B}},\boldsymbol{P}_{\boldsymbol{D}}}{\text{maximize}} \quad R(\boldsymbol{B},\boldsymbol{p}_{\boldsymbol{B}},\boldsymbol{P}_{\boldsymbol{D}}) - \gamma \delta^2(\boldsymbol{B})$$
(1a)

subject to 
$$\beta_{i,j} \in \{0,1\}, \quad \sum_{j=1}^{N_D} \beta_{i,j} \le 1 \,\forall i$$
 (1b)

$$0 \le P_{B_i} \le P_{B_{max}} \,\forall i, \ 0 \le P_{D_{ji}} \le P_{D_{max}} \,\forall j, i \tag{1c}$$

$$\Pr\left\{\frac{P_{B_i}g_{B_i}}{N_0 + P_{D_{ji}}\tilde{h}_{C_{j,i}}} \ge \eta_{min}^C\right\} \ge (1 - \epsilon) \quad \text{if } \beta_{ij} = 1, \ \forall i, j, \tag{1d}$$

$$\frac{P_{D_{ji}}g_{D_j}}{N_0 + P_{B_i}h_{B_j}} \ge \eta_{min}^D \text{ if } \beta_{ij} = 1, \ \forall i, j.$$

$$(1e)$$

The parameter  $\gamma > 0$  is a regularization parameter that balances the rate-fairness trade-off, which is selected in the scale of BW to ensure that the rate and the fairness are of comparable values. Problem (1) is a non-convex mixed-integer program, which involves combinatorial complexity. The next subsection provides two efficient methods to find a solution of (1), namely centralized and decentralized methods.

#### **B.** Proposed Optimization Algorithms

Several approaches can be applied to decompose (1) into multiple sub-problems of lower complexity without loss of optimality.

Given the statistical distribution of  $h_{C_{j,i}}$ , the probabilistic constraint in (1d) can be rewritten as follows:

$$\Pr\left\{\tilde{h}_{Cj,i} \leq \frac{P_{B_{ij}}g_{B_i} - \eta_{min}^{C}N_0}{P_{D_{ji}}\eta_{min}^{C}}\right\} \geq 1 - \epsilon$$
$$\frac{P_{B_{ij}}g_{Bi}}{N_0 + P_{D_{ji}}F_{\tilde{h}_{C_{j,i}}}^{-1}}(1-\epsilon)} \geq \eta_{min}^{C},$$

where  $F_{\tilde{h}_{C_{j,i}}}^{-1}$  is the inverse CDF of  $\tilde{h}_{C_{j,i}}$ . Similarly,  $R_{C_{i,j}}^{LB}$  can be written as:  $R_{C_{i,j}}^{LB} = \Gamma(P_{B_i}g_{B_i}/(N_0 + P_{D_{ji}}F_{\tilde{h}_{C_{j,i}}}^{-1}(1-\epsilon))).$ 

Next, we rewrite R in a simpler form to facilitate the decomposition of (1) into subproblems of lower complexity. From the definitions of R and  $R_i$  in Sec. III-A,

$$R(\boldsymbol{B}, \boldsymbol{p}_{\boldsymbol{B}}, \boldsymbol{P}_{\boldsymbol{D}}) = \sum_{i \in \mathcal{C}} \left[ \sum_{j \in \mathcal{D}} \beta_{i,j} v_{i,j} (P_{B_i}, P_{D_{j_i}}) + R_{C_{i,0}} \right],$$
(2)

where  $v_{i,j}(P_{B_i}, P_{D_{ji}}) := R_{C_{i,j}}^{LB} + R_{D_{j,i}} - R_{C_{i,0}}$  represents the minimum *rate increment* due to the assignment of channel *i* to D2D pair *j* relative to the case where the channel *i* is only used by the CU.

Next, we show that problem (1) can be decomposed without loss of optimality into several power allocation problems and a channel assignment problem. Notice that (1) can be equivalently expressed by replicating  $\{P_{B_i}\}$  to multiple  $\{P_{B_{ij}}\}$  and removing the constant terms from the objective function. The resulting problem can be stated as:

$$\begin{array}{ll}
\begin{array}{l} \underset{B,P_{B},P_{D}}{\text{maximize}} & \sum_{i \in \mathcal{C}} \sum_{j \in \mathcal{D}} \left[ \beta_{i,j} v_{i,j} (P_{B_{ij}}, P_{D_{ji}}) \right] - \gamma \delta^{2}(B) \\
\text{subject to} & (1b), (1c), (1e) \\
\frac{P_{B_{ij}} g_{B_{i}}}{N_{0} + P_{D_{ji}} F_{\tilde{h}_{C_{j,i}}}^{-1} (1 - \epsilon)} \geq \eta_{min}^{C}, \ \forall i, j, \\
\end{array}$$
(3)

where  $P_B := [P_{B_{ij}}]_{i,j}$ . To recover the optimal  $\{P_{B_i}^*\}$  of (1) from the optimal  $\{P_{B_{i,j}}^*\}$  of (3), one just needs to find, for each *i*, the value of *j* such that  $\beta_{i,j} = 1$  and set  $P_{B_i}^* = P_{B_{i,j}}^*$ . If no such a *j* exists, i.e.  $\beta_{i,j} = 0 \forall j$ , then channel *i* is not assigned to any D2D pair and the BS can transmit with maximum power  $P_{B_i}^* = P_{B_{\max}}$ .

In addition, it can be be shown that optimizing (3) with respect to  $P_B$  and  $P_D$  decouples across *i* and *j* into the following  $N_C \times N_D$  subproblems:

$$\begin{array}{ll} \underset{P_{B_{ij}}, P_{D_{ji}}}{\text{maximize}} & v_{i,j}(P_{B_{ij}}, P_{D_{ji}}) & (4) \\ \text{subject to} & 0 \le P_{B_{ij}} \le P_{B_{max}}, \ 0 \le P_{D_{ji}} \le P_{D_{max}} \\ \\ \frac{P_{B_{ij}}g_{B_i}}{N_0 + P_{D_{ji}}F_{\tilde{h}_{C_{j,i}}}^{-1}}(1-\epsilon) \ge \eta_{min}^C, \ \frac{P_{D_{ji}}g_{D_j}}{N_0 + P_{B_{ij}}h_{B_j}} \ge \eta_{min}^D, \end{array}$$

which should be solved  $\forall i \in C, \forall j \in D$ . We propose two methods for solving each of the power allocation subproblems in parallel.

### a) Centralized Resource Allocation Algorithm

In this case, each power allocation subporblem is solved at the BS by parallelly executing closed-form solutions. The closed-form solutions are obtained based on the fact that optimal power assignment lie on the border of the feasibility region and the objective is convex on those regions [14].

Once (4) has been solved  $\forall i \in C, \forall j \in D$ , it remains to substitute the optimal values  $v_{i,j}^*$  into (3) and minimize with respect to **B**. If (4) is infeasible for a given (i, j), then we can set its optimal value to  $v_{i,j}^* = -\infty$ . Thus, the resulting channel assignment subproblem can be stated as:

$$\begin{array}{ll} \underset{B}{\text{maximize}} & \sum_{i \in \mathcal{C}} \sum_{j \in \mathcal{D}} \beta_{i,j} v_{i,j}^* - \gamma \delta^2(B), \\ \text{subject to} & \beta_{i,j} \in \{0,1\} \ \forall i,j, \quad \sum_{j \in \mathcal{D}} \beta_{i,j} \leq 1 \ \forall i. \end{array}$$

$$(5)$$

Problem (5) is an integer program of combinatorial nature and exhibits prohibitive computational complexity even for reasonable values of  $N_C$ ,  $N_D$ . However, by relaxing the integer constraints to  $\beta_{i,j} \in [0,1] \quad \forall i, j$ , we have a differentiable strongly convex objective function with linear constraints. The resulting problem can be solved efficiently using the projected gradient descent algorithm. The obtained solution should be finally discretized back to satisfy the original constraints  $\beta_{i,j} \in \{0,1\} \quad \forall i, j$ . This is done by setting the highest positive value in every row of **B** to 1 while setting other values in the same row to 0. The resulting algorithm is summarized in Algorithm 1.

Algorithm 1 Centralized Resource Allocation

Initialize:  $B^0$ ,  $\alpha^0$ , k = 0for all  $j \in D$ ,  $i \in C$  do BS calculates:  $P_{B_{i,j}}$ ,  $P_{D_{i,j}}$ ,  $v_{i,j}$ end for repeat k = k + 1BS uses projected gradient descent algorithm to find  $B^k$ until B converges BS discretize B.

## b) Decentralized Resource Allocation Algorithm

Next, we describe a decentralized method to solve problem (4). First, to convert each of the  $N_C N_D$  subproblems to a form suitable for fractional programming, we introduce two auxiliary variables  $z_1, z_2$ , which represents lower bounds of the SINR of the *i*-th CU and the *j*-th D2D pair, as follows:

$$z_1 \le \frac{P_{B_{ij}}g_{Bi}}{N_0 + P_{D_{ji}}F_{\tilde{h}_{Ci}}^{-1}(1-\epsilon)} \text{ and } z_2 \le \frac{P_{D_{ji}}g_{Dj}}{N_0 + P_{B_{ij}}h_{B_j}}$$

The resulting objective function culminates to  $\log_2(1+z_1) + \log_2(1+z_2)$ , with two additional constraints for the SINR bounds defined by  $z_1$  and  $z_2$  above. Next, we find the Lagrangian of this objective function with respect to those two constraints and solve it with respect to the dual variable. We obtain the following objective function by substituting the optimal values of the dual variables in the Lagrangian:

$$\log_2(1+z_1) + \log_2(1+z_2) + \frac{(1+z_2)P_{D_{ji}}g_{D_j}}{P_{D_{ji}}g_{D_j} + N_0 + P_{B_i}h_{B_j}} + \frac{(1+z_1)P_{B_i}g_{B_i}}{P_{B_i}g_{B_i} + N_0 + P_{D_{ji}}F_{\tilde{h}_{C_{j,i}}}^{-1}(1-\epsilon)} - z_2 - z_1.$$

This objective function includes a sum of concave over convex fractions. The quadratic transformation proposed in [15] is identified as suitable for such extensions of fractional programming. However, this transformation will lead to a convex problem with an optima corresponding to a local optima of the original problem<sup>4</sup>. Moreover, the resulting function have a closed-form solution for every group of variables when fixing the values of other groups of variables. This makes it a suitable candidate for alternating optimization. The following iterative closed-form solution can be obtained:

$$\begin{split} z_1^{(k)} &= \frac{P_{B_{ij}}^{(k-1)} g_{B_i}}{N_0 + P_{D_{ji}}^{(k-1)} F_{\tilde{h}_{C_{j,i}}}^{-1} (1-\epsilon)}, \ z_2^{(k)} = \frac{P_{D_{ji}}^{(k-1)} g_{D_j}}{N_0 + P_{B_{ij}}^{(k-1)} h_{B_j}} \\ y_1^{(k)} &= \frac{\sqrt{(1+z_1^{(k)}) P_{B_{ij}}^{(k-1)} g_{B_i}}}{P_{B_{ij}}^{(k-1)} g_{B_i} + N_0 + P_{D_{ji}}^{(k-1)} F_{\tilde{h}_{C_{j,i}}}^{-1} (1-\epsilon)}, \\ y_2^{(k)} &= \frac{\sqrt{(1+z_2^{(k)}) P_{D_{ji}}^{(k-1)} g_{D_j}}}{P_{D_{ji}}^{(k-1)} g_{D_j} + N_0 + P_{B_{ij}}^{(k-1)} h_{B_j}}, \end{split}$$

<sup>4</sup>We omit the full details of the proof here due to lack of space.

$$\begin{split} P_{B_{ij}}^{(k)} &= \operatorname{proj}_{S} \left( \frac{(y_{1}^{(k)})^{2}(1+z_{1}^{(k)})g_{B_{i}}}{((y_{1}^{(k)})^{2}g_{B_{i}}+(y_{2}^{(k)})^{2}h_{B_{j}})^{2}} \right), \\ P_{D_{ji}}^{(k)} &= \operatorname{proj}_{S} \left( \frac{(y_{2}^{(k)})^{2}(1+z_{2}^{(k)})g_{D_{j}}}{((y_{2}^{(k)})^{2}g_{D_{j}}+(y_{1}^{(k)})^{2}F_{\tilde{h}_{C_{j,i}}}^{-1}(1-\epsilon))^{2}} \right). \end{split}$$

where  $y_1, y_2$  are generated from the quadratic transformation, and  $\text{proj}_S(.)$  denotes the projection on the set S defined by feasible set of (4) which has linear constraints.

Once (4) is solved  $\forall i \in C, \forall j \in D$ , substituting the optimal values  $v_{i,j}^*$  into (3), leads to (5). Furthermore, the same relaxation, algorithm, and discretization can also be used. Due to the iterative nature of the proposed power allocation algorithm and the projected gradient descent, the problem can be solved in a partially decentralized fashion. However, the fairness part of the object function contains a quadratic term that can not be easily decoupled across D2D pairs. Thus, classical decomposition methods can not be directly used. Nevertheless, the gradient of the object function can be decoupled across D2D pairs. Thus, by having each D2D pair perform a gradient descent on the corresponding column of B, part of the computations can be done by the pairs. The projection and the discretizations must be done centrally at the BS. Each iteration will lead to a feasible solution that can be used instantaneously by the BS and the D2D pairs for communications even before converging to the final solution as shown in Algorithm 2.

#### Algorithm 2 Decentralized Resource Allocation

Initialize:  $B^{(0)}, P_B^{(0)}, P_D^{(0)}, k = 0$ for all  $j \in \mathcal{D}$  do BS send to D2D pair j:  $B_j^0, P_{B_j}^0, P_{D_j}^0$ end for repeat k = k + 1for all  $j \in \mathcal{D}$  do D2D pair j calculates  $z_2^{(k)}, y_2^{(k)}, P_{D_j}^{(k)}$  and send them to the BS D2D pair uses the gradient decent algorithm and sends to BS:  $B_j^{(k)}$ BS calculates  $z_1^{(k)}, y_1^{(k)}, P_{B_j}^{(k)}$  and send them to pair j. end for BS projects B and sends each column to the corresponding D2D pair. until  $B, P_B, P_D$  converges

#### **IV. SIMULATIONS**

The simulation setup comprises a circular cell of 500 m radius in which the CUs and D2D transmitters are placed uniformly at random. Each D2D receiver is placed uniformly at random inside a circle of radius 5 m centered at the corresponding transmitter. The channel gains are calculated using a path loss model with exponent 2 and gain -5 dB at a reference distance of 1 m. We assume  $\tilde{h}_C$  to be exponentially distributed with the mean value obtained from the mentioned path-loss model. Averages over 100,000 independent realizations of the user locations with parameters BW = 15 kHz,  $\gamma = 50 \times BW$ ,  $N_D = 10$ ,  $N_C = 10$ ,  $N_0 = -70$  dBW ( $\gamma$  is scaled with BW to ensure that the unfairness and the achieved rate are of comparable values). The two proposed algorithms are compared with; (i) the method by Feng et al. [12], which to the best of our knowledge is the best existing alternative; and (ii)





the closed-form method in [14], where the average of  $\tilde{h}_C$  is an estimate of the instantaneous channel gain values.

Fig. 2 shows that the proposed methods achieve high average rates, which are also very close to the case where the probabilistic outage constraint is ignored. Notice that the gap decreases with increasing the desired outage probability  $\epsilon$ . This is in contrast to the method in [12] whose achieved rates are considerably lower than the proposed methods.

Fig. 3 shows that the method in [12] achieves the best fairness as expected, since a D2D pair can not use more than one channel at a time. The proposed methods achieve similar *fairness* level where the decentralized method achieves slightly better fairness. Moreover, our



Fig. 5: The convergence rate of a single realization.

proposed methods provides a rate vs fairness trade-off flexibility by changing the scaling parameter  $\gamma$ .

Fig. 4 shows that the proposed centralized method and the method in [12] achieve similar outage probabilities which are exactly equivalent to the desired outage values. On the other hand, the decentralized method achieves slightly lower outage probability, since (4) converges to a local optima where the rate and risk are lower than the optima.

Finally, Fig. 5 shows that the decentralized method converges to a rate close to the one achieved by the centralized method in relatively few iterations.

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