Distributed Adaptive Asymptotically Consensus Tracking for Uncertain Nonlinear Systems with Intermittent Actuator Faults and Directed Communication Topology

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***Abstract*—In this paper, we investigate the output consensus tracking problem for a class of high-order nonlinear systems with unknown parameters, uncertain external disturbances and intermittent actuator faults. Under directed topology condition, a novel distributed adaptive controller is proposed. The common time-varying trajectory is allowed to be totally unknown by part of subsystems. Therefore, the assumption on linearly parameterized trajectory signal in most literatures is no longer needed. To achieve the relaxation, extra distributed parameter estimators are introduced in all subsystems. Besides, to handle the actuator faults occurring possibly infinite times, a new adaptive compensation technique is adopted. It is shown that with the proposed scheme, all closed-loop signals are globally uniformly bounded and asymptotically output consensus tracking can be achieved.**

***Index Terms*—distributed adaptive control, multi-agent systems, asymptotically consensus tracking, directed topology, intermittent actuator faults.**

I. Introduction

Due to its various applications in modern engineering, cooperative control of multi-agent systems (MAS) has attracted huge attention over the past decades [1]–[7]. Leader-following consensus is one of the most typical and hot research issues in this area. It aims to achieve an agreement for the states/output of each subsystem, by designing distributed controllers for all subsystems [8]–[13]. In most available results, the desired reference trajectory is specified by a known agent (called the leader), which has zero or known input and shares similar system structure with the followers. For some more general cases, the common reference trajectory is represented by a time-varying function. Thus such a research topic is sometimes referred to as distributed consensus tracking control [11]–[14].

It is worth mentioning that in contrast to classical tracking control of single system, the main challenge in distributed consensus tracking problem lies in the limitation that only a small portion of subsystems can directly access full information of the desired trajectories. So far, some early effective control protocols have been presented for solving this issue; see for examples in [9], [15]–[19].

Note that all the results mentioned above are established based on relatively simple systems with linear or precisely known system dynamics. However, almost all systems in practical engineering are inherently nonlinear and unavoidably involve model uncertainties. Therefore, developing distributed control strategies for nonlinear subsystems with unknown parameters and uncertain disturbances are of significance in both theory and practice. As we know, adaptive control is an effective approach to handle the uncertainties involved in systems as it can offer on-line estimation for the uncertain parameters [20]. However, valid distributed control protocols employing adaptive techniques are still limited, especially for the cases with directed communication graphs. This is because directed topology is associated with asymmetric Laplacian matrix, which will bring about new challenges to design distributed adaptive laws based on Lyapunov stability theory. In [21], a distributed coordination control scheme is proposed for uncertain first-order nonlinear systems, by incorporating adaptive neural network with robust control techniques. Bounded synchronization error can be shown if the control gains are selected to be sufficiently large. The results are generalized to second-order and high-order uncertain nonlinear systems in [14] and [10], respectively. Based on the assumption that the common reference trajectory is linearly parameterized with basis functions known by all subsystems, a distributed tracking control scheme for uncertain first-order nonlinear systems is presented in [22]. The result is extended to solve finite-time consensus problem of uncertain high-order systems in [23]. In [24], by utilizing the similar assumption, backstepping based distributed adaptive controllers for parametric strict-feedback nonlinear multi-agent systems are designed under directed graph condition. The Lyapunov function is carefully chosen for the overall system with only local estimation errors involved. Thus the coupling terms in the derivative of Lyapunov function as in [21], [14] and [10], which are induced by directed graph condition and related to local consensus errors as well as parameter estimation errors in the neighbors, can be avoided. Hence, asymptotically consensus tracking is finally achieved. Based on the same assumption on the reference trajectory, an

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adaptive iterative learning control scheme is proposed in [25] for a class of high-order nonlinear multi-agent systems with directed communication topology. In [26], the derivative of the reference trajectory is assumed to be linearly parameterized. A distributed adaptive tracking control scheme is presented for a class of uncertain first/second multi-agent systems such that asymptotically cooperative tracking can be achieved under undirected communication topology codition.

In [13], [27] and [12], new distributed adaptive consensus control schemes are presented to relax the aforementioned linearly parameterized assumption on the common desired trajectory signals. In [13], the desired trajectory $y_r(t)$ is allowed to be totally known by part of subsystems. By introducing a differentiable function of consensus errors and certain positive integrable functions in each step of virtual control design, asymptotically output consensus tracking can also be achieved for high-order nonlinear multi-agent systems. However, the proposed distributed protocol is only valid when interaction topology is undirected. The results are then extended to second-order Euler Lagrange systems with balanced and weakly connected digraph in [27]. In [12], by introducing an nth-order filter and a group of n estimators for counteracting the effects due to totally unknown trajectory information in each agent, a new backstepping based smooth distributed adaptive tracking control protocol is proposed. With the proposed scheme, each agent needs collect n dimensional parameter estimates from its neighbors to complete the design of distributed controllers. The control performance will be influenced when the bandwidth of communication network and computation capacity are limited.

Motivated by the above observations, we shall focus on the distributed adaptive control problem for a class of high-order nonlinear subsystems in the presence of unknown parameters and uncertain external disturbances under the directed graph condition. The main contributions of this paper can be summarised as follows.

- This paper achieves the relaxation of the assumptions required in [9], [17], [22], [23] and [24] that the desired trajectory $y_r(t)$ is linearly parameterized and the basis functions are known by all subsystems. By introducing the differentiable function of consensus errors and positive integrable functions of similar forms with those in [13], $y_r(t)$ is allowed to be totally unknown by the subsystems without direct access to $y_r(t)$.
- The results in [13], [27] are extended to the case with high-order nonlinear systems under the directed communication topology condition. To handle the aforementioned coupled terms in the derivative of Lyapunov function which are induced by directed graph issue, extra distributed estimators are introduced in each subsystem. Besides, by incorporating the tuning function technique, the computational complexity and communication burden among connected subsystems can be effectively reduced with comparison to [12].
- Moreover, uncertain intermittent partial-loss-of-effectiveness (PLOE) type of actuator faults possibly occurring in each subsystem are considered. Different from [28] where only globally uniform boundedness of all closed-loop signals is ensured, asymptotically output consensus tracking can be achieved in this paper.

The rest of this paper is organized as follows. In Section II, the considered system model, communication graph condition and intermittent actuator fault model are introduced. Then the distributed adaptive fault tolerant controllers are developed in Section III followed by the closed-loop system stability analysis in Section IV. In Section V, simulation results are provided to illustrate the effectiveness of our proposed scheme. Finally, a conclusion is drawn in Section VI.

II. Problem formulation

A. System model

In this paper, we consider a group of N nonlinear subsystems modeled as follows.

$$y_i^{(n)}(t)-\sum_{l=1}^{p_i}\theta_{il}\varphi_{il}(y_i,\dot{y}_i,\ldots,y_i^{(n-1)}) = b_i u_i(t)+d_i(t) \quad (1)$$

where $y_i(t) \in R$ is the output of subsystem i for $i = 1,\ldots,N$. $u_i(t) \in R$ is the input acting on subsystem i, which is also the output of the actuator. $b_i \in R$, $\theta_{il} \in R$ are unknown constant parameters and b_i is nonzero. $\varphi_{il} : R^n \to R$ is a known smooth nonlinear function. $d_i(t) \in R$ represents uncertain external disturbances. The same class of nonlinear systems are widely considered in the literatures including [29], [30] and [31].

By defining the state variables as $x_{i,q} = y_i^{(q-1)}$, $q = 1,\ldots,n$, system (1) can be rewritten as

$$\begin{aligned}\dot{x}_{i,q} =& x_{i,q+1}, \quad q = 1,2,\ldots,n-1;\\ \dot{x}_{i,n} =& b_i u_i(t) + \varphi_i^{\mathrm{T}}\theta_i + d_i(t) \quad (2)\\ y_i =& x_{i,1} \quad (3)\end{aligned}$$

where $\varphi_i = [\varphi_{i1},\varphi_{i2},\ldots,\varphi_{ip_i}]^T$ and $\theta_i = [\theta_{i1},\theta_{i2},\ldots,\theta_{ip_i}]^T$.

B. Communication condition among the N subsystems

Suppose that the interaction topology can be represented by a fixed directed graph $\mathcal{G} \triangleq (\mathcal{A},\mathcal{V},\varepsilon)$, where $\mathcal{A} = [a_{i,j}] \in R^{N\times N}$ with nonnegative elements is the adjacency matrix associated with $\mathcal{G}$. $\mathcal{V} = \{1,...,N\}$ denotes the set of indexes corresponding to each subsystem. $\varepsilon \subseteq \mathcal{V}\times\mathcal{V}$ is the edge set of ordered pairs of nodes. An edge $(i,j) \in \varepsilon$ indicates that subsystem j can obtain information from subsystem i, but not necessarily vice versa [32]. In this case, subsystem i is called a neighbor of subsystem j. We adopt $\mathcal{N}_j$ to represent the collection of neighbors of subsystem j, i.e. $\mathcal{N}_j \triangleq \{i \in \mathcal{V} : (i,j) \in \varepsilon\}$. The elements in the adjacency matrix $a_{i,j} = 1$ if $(j,i) \in \varepsilon$, $a_{i,j} = 0$ otherwise. Note that self-edges (i,i) are not allowed unless otherwise stated. Thus $(i,i) \notin \varepsilon$ and $i \notin \mathcal{N}_i$. Hence, the diagonal elements are all zeros, i.e. $a_{i,i} = 0$. An in-degree matrix is defined as Δ, which is a diagonal matrix with diagonal elements being $\Delta_i = \sum_{j\in\mathcal{N}_i} a_{i,j}$. Then, the Laplacian matrix of N subsystems is defined as $L = \Delta - \mathcal{A}$.

A directed path exists, which is originated from node i and terminated at j if there are a sequence of successive edges $\{(i,k),(k,m),\dots,(n,l),(l,j)\} \subseteq \varepsilon$ with $i,k,m,...,n,l,j \in \mathcal{V}$. A directed graph contains a directed spanning tree if there exists at least a root node i such that all the remaining nodes in the graph can be reached from i through a directed path. In this paper, we use $\mu_i = 1$ to denote the case that the subsystem i can directly access the full information of the common reference trajectory $y_r(t)$, otherwise $\mu_i = 0$.

C. Intermittent actuator fault model

Suppose that the internal dynamics of the actuator in the subsystems is negligible. $u_{ci} \in R$ represents the control signal (i.e. the input of the actuator) in subsystem i, which will be generated through controller design. An actuator is fault-free if it can execute the control command with 100% effectiveness, i.e. $u_i = u_{ci}$. In this paper, an intermittent partial loss of effectiveness (PLOE) type of actuator faults as modeled below are considered.

$$u_i(t) = \rho_i(t)u_{ci}(t) \tag{4}$$

with

$$\rho_i(t) = \rho_{ih}, t \in [t_{ih,s}, t_{ih,e}), h \in Z^+ \tag{5}$$

where $0 < \underline{\rho}_i \leq \rho_{ih} \leq 1$ with ρ_{ih} and $\underline{\rho}_i$ being unknown constants. h is a positive integer, which represents the number of actuator faults occurred in subsystem i. $t_{ih,s}$ and $t_{ih,e}$ denote the time instants when the hth actuator fault occurs and ends, respectively. There is $0 \leq t_{i1,s} < t_{i1,e} \leq t_{i2,s} < t_{i2,e} \leq \cdots \leq t_{ih,s} < t_{ih,e} \leq \infty$. Both $t_{ih,s}$ and $t_{ih,e}$ for the hth actuator fault in subsystem i are allowed unknown by designers. Equations (4) and (5) imply that the actuator in subsystem i will lose $(1-\rho_{ih}) \times 100\%$ of its effectiveness to execute the control command $u_{ci}(t)$ from time $t_{ih,s}$ to $t_{ih,e}$.

Remark 1. *As explained in [28], (4)-(5) indicate that a faulty actuator may be recovered to a normally working mode or changes from one faulty mode to another faulty mode intermittently without offline repairment. In contrast to this, the fault models with single occurrence time are normally considered in the existing results on adaptive fault tolerant control; see [33]–[35] for instance. Since the latter actuator fault model can also be represented by (4)-(5) with $h = 1$ and $t_{ih,e} = \infty$, the fault model in this paper is more general than those in [33], [34] and [35].*

The control objective of this paper is to design distributed adaptive controllers for all the N subsystems (1) with directed communication topology such that:

- all closed-loop signals are globally uniformly bounded despite possible occurrence of intermittent PLOE actuator faults (4)-(5);
- all subsystem outputs can reach a consensus by tracking a common desired trajectory $y_r(t)$ asymptotically though only a small fraction of subsystems know $y_r(t)$ directly.

To achieve the control objective, some necessary assumptions are imposed.

Assumption 1. *The directed graph $\mathcal{G}$ contains a spanning tree and the root nodes n_l can directly access the full information of $y_r(t)$, i.e. $\mu_{n_l} = 1$.*

Assumption 2. *The first nth-order derivatives of $y_r(t)$ are bounded, piecewise continuous and are directly available to the subsystem i with $\mu_i = 1$.*

Assumption 3. *The sign of b_i is known by each subsystem i.*

Assumption 4. *The disturbance $d_i(t)$ is bounded such that $|d_i(t)| \leq D_i$, where D_i is an unknown positive constant.*

Remark 2. *Note that $y_r(t)$ is allowed to be totally unknown by the subsystem i with $\mu_i = 0$. In contrast to currently available results in [9], [17], [22] and [24], the assumption that $y_r(t)$ is linearly parameterized with basis functions known by all subsystems is no longer needed. Besides, although a novel distributed adaptive consensus tracking scheme is proposed in [13] for nth-order nonlinear multi-agent systems with a relaxed assumption on $y_r(t)$ as similar to Assumption 2, the results are only applicable to the case with undirected communication topology.*

III. Design of distributed adaptive controllers

Before we proceed to present the detailed procedure of distributed adaptive controller design, the following lemmas are introduced, which will play important roles in control design and stability analysis.

Lemma 1. *[32] Based on Assumption 1, the matrix $(L+B)$ is nonsingular where $B = diag\{\mu_1, \dots, \mu_N\}$. Define*

$$\begin{aligned}
\bar{q} =& [\bar{q}_1, \dots, \bar{q}_N]^T = (L+B)^{-1}[1, \dots, 1]^T \\
P =& diag\{P_1, \dots, P_N\} = diag\{\frac{1}{\bar{q}_1}, \dots, \frac{1}{\bar{q}_N}\} \\
Q =& P(L+B) + (L+B)^T P,
\end{aligned}$$

then $\bar{q}_i > 0$ for $i = 1, \dots, N$ and Q is positive definite.

Lemma 2. *[36] The following inequality holds*

$$0 \leq |z| - z \cdot sg(z) \leq \eta$$

for any scalars $z \in R$, $\eta > 0$ and $sg(z) = \frac{z}{\sqrt{z^2+\eta^2}}$.

Lemma 3. *[32] Based on Assumption 1, the system*

$$\dot{X}(t) = -(L+B)X(t) \tag{6}$$

where $X(t) \in R^N$ is globally exponentially stable.

Backstepping technique [20] is adopted to generate the adaptive control law for each subsystem. The change of coordinations is firstly introduced as below.

$$z_{i,1} = \sum_{j=1}^{N} a_{ij}(x_{i,1} - x_{j,1}) + \mu_i(x_{i,1} - y_r) \tag{7}$$

$$z_{i,k} = x_{i,k} - \alpha_{i,k-1}, k = 2, \dots, n. \tag{8}$$

$z_{i,1}$ is often known as the local neighborhood consensus errors of each subsystem i [24]. $\alpha_{i,k}$ is the virtual control signal to be designed in each recursive step k for subsystem i.

• Step 1. From (7), we have

$$z_1 = H\delta = (L+B)\delta \tag{9}$$

where $z_1 = [z_{1,1}, ..., z_{1,N}]^T$, $\delta = [\delta_1, ..., \delta_N]^T$. $\delta_i = x_{i,1} - y_r$ is the actual tracking error between the output of subsystem i and the common desired trajectory y_r.

From (2), (8) and (9), the derivative of z_1 is computed as

$$\begin{aligned}\dot{z}_1 &= H\begin{bmatrix} x_{1,2} - \dot{y}_r \\ \vdots \\ x_{N,2} - \dot{y}_r \end{bmatrix} = H\begin{bmatrix} z_{1,2} + \alpha_{1,1} - \dot{y}_r \\ \vdots \\ z_{N,2} + \alpha_{N,1} - \dot{y}_r \end{bmatrix} \\ &= H\left[z_2 + \alpha_1 - 1_N \otimes \dot{y}_r\right] \end{aligned}\tag{10}$$

where $z_2 = [z_{1,2}, \ldots, z_{N,2}]^T$, $\alpha_1 = [\alpha_{1,1}, \ldots, \alpha_{N,1}]^T$ and $1_N = [1, \ldots, 1]^T$.

The virtual control law $\alpha_{i,1}$ is chosen as

$$\alpha_{i,1} = -c_1 z_{i,1} - sg(z_{i,1})\hat{F}_i \tag{11}$$

where $\hat{F}_i$ for $i = 1, \ldots, N$ is the estimate of F, which is the upper bound of $|\dot{y}_r(t)|$. c_1 is a positive constant. $sg(z_{i,1})$ is a smooth function defined in Lemma 2. Equation (11) can be compactly reperented as

$$\alpha_1 = -c_1 z_1 - diag\left\{sg(z_{i,1})\right\}\hat{F}_t \tag{12}$$

with $\hat{F}_t = [\hat{F}_1, \hat{F}_2, \ldots, \hat{F}_N]^T$ and

$$diag\left\{sg(z_{i,1})\right\} = \begin{pmatrix} sg(z_{1,1}) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & sg(z_{N,1}) \end{pmatrix}.$$

The local parameter update law for $\hat{F}_i$ is designed as

$$\dot{\hat{F}}_i = -\sum_{j=1}^{N} a_{ij}(\hat{F}_i - \hat{F}_j) - \mu_i(\hat{F}_i - F) \tag{13}$$

Clearly, the design of $\dot{\hat{F}}_i$ also exhibits a distributed manner based on additional parameter estimates $\hat{F}_j$ collected from the neighboring subsystems j of subsystem i with $a_{ij} = 1$. Equation (13) can be compactly rewritten as

$$\dot{\hat{F}}_t(t) = -(L+B)\hat{F}_t(t) = -H\hat{F}_t(t) \tag{14}$$

From Lemma 3, this distributed parameter estimate system is stable. Thus $\hat{F}_i(t)$ is bounded, and so is $\dot{\hat{F}}_i(t)$ for all $t \in [0, \infty)$. The property of this distributed parameter estimate system will be employed in designing the virtual control law in Step 2.

The Lyapunov function candidate in this step is defined as

$$V_1 = \frac{1}{2}{z_1}^T P z_1 + \frac{1}{2\gamma_{F_t}}\tilde{F}_t^T P \tilde{F}_t \tag{15}$$

where γ_{F_t} is a positive constant and $\tilde{F}_t$ represents the column vector of parameter estimation errors, i.e. $\tilde{F}_t = 1_N \otimes F - \hat{F}_t$. From (10), (12) and (13), the derivative of V_1 is computed as

$$\begin{aligned}\dot{V}_1 =& {z_1}^T P \dot{z}_1 + \frac{1}{\gamma_{F_t}}\tilde{F}_t^T P(-\dot{\hat{F}}_t) \\ =& {z_1}^T PH z_2 - {z_1}^T P(\Delta - A)diag\{sg(z_{i,1})\} \cdot 1_N \otimes F \\ &- {z_1}^T PBdiag\{sg(z_{i,1})\} \cdot 1_N \otimes F - c_1 {z_1}^T PH z_1 \\ &- {z_1}^T PH \cdot 1_N \otimes \dot{y}_r + {z_1}^T PHdiag\{sg(z_{i,1})\}\tilde{F}_t \\ &- \frac{1}{\gamma_{F_t}}\tilde{F}_t^T PH\tilde{F}_t \end{aligned}\tag{16}$$

Based on Lemma 2 and $|sg(\cdot)| \le 1$, the following two terms can be computed as
i)

$$-{z_1}^T P(\Delta - A)diag\{sg(z_{i,1})\} \cdot 1_N \otimes F \le \sum_{i=1}^{N} F P_i \Delta_i \eta_i \tag{17}$$

ii)

$$\begin{aligned}&-{z_1}^T PBdiag\{sg(z_{i,1})\} \cdot 1_N \otimes F - {z_1}^T PH \cdot 1_N \otimes \dot{y}_r \\ &\le \sum_{i=1}^{N} P_i \mu_i F \eta_i \end{aligned}\tag{18}$$

Substituting (17) and (18) into (16) yields that

$$\begin{aligned}\dot{V}_1 \le& {z_1}^T PH z_2 - \frac{1}{2}c_1 {z_1}^T Q z_1 + \sum_{i=1}^{N}(\Delta_i + \mu_i)P_i F \eta_i \\ &+ {z_1}^T PHdiag\{sg(z_{i,1})\}\tilde{F}_t - \frac{1}{2\gamma_{F_t}}\tilde{F}_t^T Q \tilde{F}_t \\ \le& \|z_1\|\|PH\|\|z_2\| - \frac{1}{3}c_1\lambda_{\min}(Q)\|z_1\|^2 \\ &+ \sum_{i=1}^{N}(\Delta_i + \mu_i)P_i F \eta_i - \frac{\lambda_{\min}(Q)}{4\gamma_{F_t}}\left\|\tilde{F}_t\right\|^2 \\ &+ \Bigg(-\frac{1}{6}c_1\lambda_{\min}(Q)\|z_1\|^2 + \|z_1\|\|PH\|\left\|\tilde{F}_t\right\| \\ &- \frac{3\|PH\|^2}{2c_1\lambda_{\min}(Q)}\left\|\tilde{F}_t\right\|^2\Bigg) \\ &+ \left(\frac{3\|PH\|^2}{2c_1\lambda_{\min}(Q)} - \frac{\lambda_{\min}(Q)}{4\gamma_{F_t}}\right)\left\|\tilde{F}_t\right\|^2 \\ \le& \|z_1\|\|PH\|\|z_2\| - \frac{1}{3}c_1\lambda_{\min}(Q)\|z_1\|^2 \\ &+ \sum_{i=1}^{N}(\Delta_i + \mu_i)P_i F \eta_i - \frac{\lambda_{\min}(Q)}{4\gamma_{F_t}}\left\|\tilde{F}_t\right\|^2 \\ &+ \left(\frac{3\|PH\|^2}{2c_1\lambda_{\min}(Q)} - \frac{\lambda_{\min}(Q)}{4\gamma_{F_t}}\right)\left\|\tilde{F}_t\right\|^2 \end{aligned}\tag{19}$$

Note that γ_{F_t} can be chosen to satisfy

$$0 < \gamma_{F_t} < \frac{c_1\lambda_{\min}(Q)}{6\|PH\|^2} \tag{20}$$

such that the last term with respect to $\|\tilde{F}_t\|^2$ in (19) can be rendered negative. Then $\dot{V}_1$ is further derived as

$$\begin{aligned}\dot{V}_1 \le& \|z_1\|\|PH\|\|z_2\| - \frac{1}{3}c_1\lambda_{\min}(Q)\|z_1\|^2 \\ &+ \sum_{i=1}^{N}(\Delta_i + \mu_i)P_i F \eta_i - \frac{\lambda_{\min}(Q)}{4\gamma_{F_t}}\left\|\tilde{F}_t\right\|^2 \end{aligned}\tag{21}$$

Remark 3. *It is worth noting that the positive constant γ_{F_t} only appears in the Lyapunov function candidate, which is chosen in this step to analyze the stability of $(\dot{z}_1, \tilde{F}_t)-subsystem$. Condition (20) indicates that for an arbitrary choice of control gain $c_1 > 0$ and directed graph $\mathcal{G}$ satisfying Assumption 1, there always exists a sufficiently small positive constant γ_{F_t} such that by choosing the Lyapunov function V_1 as in (15), inequality (21) holds.*

• Step 2. From (7) and (11), it implies that $\alpha_{i,1}$ is the function of η_i, $x_{i,1}$, $\hat{F}_i$, $a_{ij}x_{j,1}$ and $\mu_i y_r$.

Differentiating $z_{i,2}$ with respect to t yields that

$$\begin{aligned}\dot{z}_{i,2} =& z_{i,3} + \alpha_{i,2} - \dot{\alpha}_{i,1}\\ =& z_{i,3} + \alpha_{i,2} - \frac{\partial \alpha_{i,1}}{\partial x_{i,1}} x_{i,2} - \frac{\partial \alpha_{i,1}}{\partial \eta_i}\dot{\eta}_i - \frac{\partial \alpha_{i,1}}{\partial \hat{F}_i}\dot{\hat{F}}_i\\ & - \sum_{j=1}^{N} a_{ij}\frac{\partial \alpha_{i,1}}{\partial x_{j,1}} x_{j,2} - \mu_i \frac{\partial \alpha_{i,1}}{\partial y_r}\dot{y}_r \end{aligned} \tag{22}$$

The virtual control law $\alpha_{i,2}$ is chosen as

$$\begin{aligned}\alpha_{i,2} =& - \hat{c}_{i,2} z_{i,2} + \frac{\partial \alpha_{i,1}}{\partial x_{i,1}} x_{i,2} + \frac{\partial \alpha_{i,1}}{\partial \eta_i}\dot{\eta}_i\\ & - \frac{\partial \alpha_{i,1}}{\partial \hat{F}_i} sg\left(z_{i,2}\frac{\partial \alpha_{i,1}}{\partial \hat{F}_i}\right)\hat{B}_i + \sum_{j=1}^{N} a_{ij}\frac{\partial \alpha_{i,1}}{\partial x_{j,1}} x_{j,2}\\ & + \mu_i \frac{\partial \alpha_{i,1}}{\partial y_r}\dot{y}_r \end{aligned} \tag{23}$$

where $\hat{c}_{i,2}$ is the local estimate of a positive constant c_2 to be defined later. $\hat{B}_i$ is the estimate of B_i, which is a constant upper bound of $|\dot{\hat{F}}_i|$.

The parameter update law for $\hat{c}_{i,2}$ is designed as

$$\dot{\hat{c}}_{i,2} = \gamma_{c_{i,2}} {z_{i,2}}^2 \tag{24}$$

where $\gamma_{c_{i,2}}$ is a positive design parameter. To avoid the over-parameterization problem [20] about B_i in the next step, the tuning function technique is adopted. The first tuning function term $\tau_{i,1}$ is defined as

$$\tau_{i,1} = \gamma_{B_i} z_{i,2}\frac{\partial \alpha_{i,1}}{\partial \hat{F}_i} sg\left(z_{i,2}\frac{\partial \alpha_{i,1}}{\partial \hat{F}_i}\right). \tag{25}$$

where γ_{B_i} is a positive design parameter.

Define the Lyapunov function candidate as

$$V_2 = V_1 + \frac{1}{2}\sum_{i=1}^{N} {z_{i,2}}^2 + \sum_{i=1}^{N}\frac{1}{2\gamma_{c_{i,2}}}\tilde{c}_{i,2}^2 + \sum_{i=1}^{N}\frac{1}{2\gamma_{B_i}}\tilde{B}_i^2 \tag{26}$$

where $\tilde{c}_{i,2} = c_2 - \hat{c}_{i,2}$ and $\tilde{B}_i = B_i - \hat{B}_i$. Define $z_3 = [z_{1,3}, z_{2,3}, \ldots, z_{N,3}]^T$. From (21), (22), (23), (24) and (25), the derivative of V_2 is calculated as

$$\begin{aligned}\dot{V}_2 =& \dot{V}_1 + \sum_{i=1}^{N} {z_{i,2}}^T \dot{z}_{i,2} + \sum_{i=1}^{N}\frac{1}{\gamma_{c_{i,2}}}\tilde{c}_{i,2}(-\dot{\hat{c}}_{i,2})\\ & + \sum_{i=1}^{N}\frac{1}{\gamma_{B_i}}\tilde{B}_i(-\dot{\hat{B}}_i)\\ \leq & \|z_1\|\,\|PH\|\,\|z_2\| - \frac{1}{3}c_1\lambda_{\min}(Q)\|z_1\|^2\\ & + \sum_{i=1}^{N}(\Delta_i+\mu_i)P_i F\eta_i - \frac{\lambda_{\min}(Q)}{4\gamma_{F_t}}\left\|\tilde{F}_t\right\|^2\\ & + {z_2}^T z_3 - c_2\|z_2\|^2 + \sum_{i=1}^{N}\tilde{c}_{i,2}\left({z_{i,2}}^2 - \frac{1}{\gamma_{c_{i,2}}}\dot{\hat{c}}_{i,2}\right)\\ & + \sum_{i=1}^{N}\left[\left|z_{i,2}\frac{\partial \alpha_{i,1}}{\partial \hat{F}_i}\right| - z_{i,2}\frac{\partial \alpha_{i,1}}{\partial \hat{F}_i} sg\left(z_{i,2}\frac{\partial \alpha_{i,1}}{\partial \hat{F}_i}\right)\right]B_i\\ & + \sum_{i=1}^{N}\frac{1}{\gamma_{B_i}}\tilde{B}_i\left[\gamma_{B_i} z_{i,2}\frac{\partial \alpha_{i,1}}{\partial \hat{F}_i} sg\left(z_{i,2}\frac{\partial \alpha_{i,1}}{\partial \hat{F}_i}\right) - \dot{\hat{B}}_i\right]\\ \leq & -\left(\frac{c_1}{6}\lambda_{\min}(Q)\|z_1\|^2 - \|z_1\|\,\|PH\|\,\|z_2\|\right.\\ & \left. + \frac{3\|PH\|^2}{2c_1\lambda_{\min}(Q)}\|z_2\|^2\right) - \frac{c_1}{6}\lambda_{\min}(Q)\|z_1\|^2\\ & + {z_2}^T z_3 + \left(\frac{3\|PH\|^2}{2c_1\lambda_{\min}(Q)} - c_2\right)\|z_2\|^2 + \sum_{i=1}^{N}\eta_i B_i\\ & + \sum_{i=1}^{N}(\Delta_i+\mu_i)P_i\eta_i F + \sum_{i=1}^{N}\frac{1}{\gamma_{B_i}}\tilde{B}_i(\tau_{i,1} - \dot{\hat{B}}_i)\\ & - \frac{\lambda_{\min}(Q)}{4\gamma_{F_t}}\left\|\tilde{F}_t\right\|^2\\ \leq & -\frac{c_1}{6}\lambda_{\min}(Q)\|z_1\|^2 + \sum_{i=1}^{N}(\Delta_i+\mu_i)P_i\eta_i F + {z_2}^T z_3\\ & + \left(\frac{3\|PH\|^2}{2c_1\lambda_{\min}(Q)} - \frac{c_2}{2}\right)\|z_2\|^2 - \frac{c_2}{2}\|z_2\|^2 + \sum_{i=1}^{N}\eta_i B_i\\ & + \sum_{i=1}^{N}\frac{1}{\gamma_{B_i}}\tilde{B}_i(\tau_{i,1} - \dot{\hat{B}}_i) - \frac{\lambda_{\min}(Q)}{4\gamma_{F_t}}\left\|\tilde{F}_t\right\|^2 \end{aligned} \tag{27}$$

where the Lemma 2 has been applied in handling the term containing unknown parameter B_i. Clearly, by defining c_2 as a positive constant satisfying that

$$c_2 > \frac{3\|PH\|^2}{c_1\lambda_{\min}(Q)} \tag{28}$$

$\dot{V}_2$ is further derived as

$$\begin{aligned}\dot{V}_2 \leq & -\frac{c_1}{6}\lambda_{\min}(Q)\|z_1\|^2 + \sum_{i=1}^{N}(\Delta_i+\mu_i)P_i\eta_i F + {z_2}^T z_3\\ & + \sum_{i=1}^{N}\frac{1}{\gamma_{B_i}}\tilde{B}_i(\tau_{i,1} - \dot{\hat{B}}_i) - \frac{\lambda_{\min}(Q)}{4\gamma_{F_t}}\left\|\tilde{F}_t\right\|^2\\ & - \frac{c_2}{2}\|z_2\|^2 + \sum_{i=1}^{N}\eta_i B_i \end{aligned} \tag{29}$$

Remark 4. *It can be observed from (27)-(29) that c_2 needs be sufficiently large to obtain (29). Since parameters P, H and Q in (28) are matrices related to global graph information, they are normally unknown to local subsystems. Thus for each subsystem i, a parameter estimator of c_2 is introduced and the generated estimate $\hat{c}_{i,2}$ is adopted as an adaptive feedback gain of $z_{i,2}$ in the design of $\alpha_{i,2}$ in (23).*

Remark 5. *It is observed from (23) that the term* $-\frac{\partial\alpha_{i,1}}{\partial\hat{F}_i}sg\left(z_{i,2}\frac{\partial\alpha_{i,1}}{\partial\hat{F}_i}\right)\hat{B}_i$ *is introduced to compensate* $\frac{\partial\alpha_{i,1}}{\partial\hat{F}_i}\dot{\hat{F}}_i$ *instead of substituting* $\frac{\partial\alpha_{i,1}}{\partial\hat{F}_i}\dot{\hat{F}}_i$ *directly, even though the information of* $\frac{\partial\alpha_{i,1}}{\partial\hat{F}_i}\dot{\hat{F}}_i$ *is fully available for each subsystem* i *in Step 2. The main reason of doing this is that* $\dot{\hat{F}}_i$ *in (13) involves its neighbors' instantaneous parameter estimates* $a_{ij}\hat{F}_j$ *for* $j\in\mathcal{N}_i$*. Then in the subsequent step of the backstepping technique, the time derivative of* $\alpha_{i,2}$ *needs to be computed while the derivative* $a_{ij}\dot{\hat{F}}_j$ *may involve the information beyond subsystem* i*'s neighborhood area.*

- Step 3. From (8) and (23), it implies that $\alpha_{i,2}$ is the function of $\hat{c}_{i,2}$, $x_{i,1}$, $x_{i,2}$, η_i, $\dot{\eta}_i$, $\hat{F}_i$, $\hat{B}_i$, $a_{ij}x_{j,1}$, $a_{ij}x_{j,2}$, $\mu_i y_r$ and $\mu_i\dot{y}_r$. Differentiating $z_{i,3}$ with respect to t, we have

$$\begin{aligned}\dot{z}_{i,3}=&x_{i,4}-\dot{\alpha}_{i,2}\\=&z_{i,4}+\alpha_{i,3}-\dot{\alpha}_{i,2}\end{aligned}\tag{30}$$

where

$$\begin{aligned}\dot{\alpha}_{i,2}=&\frac{\partial\alpha_{i,2}}{\partial\hat{c}_{i,2}}\dot{\hat{c}}_{i,2}+\sum_{l=1}^{2}\frac{\partial\alpha_{i,2}}{\partial x_{i,l}}x_{i,l+1}+\sum_{l=1}^{2}\frac{\partial\alpha_{i,2}}{\partial\eta_i{}^{(l-1)}}\eta_i{}^{(l)}\\&+\frac{\partial\alpha_{i,2}}{\partial\hat{F}_i}\dot{\hat{F}}_i+\frac{\partial\alpha_{i,2}}{\partial\hat{B}_i}\dot{\hat{B}}_i+\sum_{j=1}^{N}a_{ij}\sum_{l=1}^{2}\frac{\partial\alpha_{i,2}}{\partial x_{j,l}}x_{j,l+1}\\&+\mu_i\sum_{l=1}^{2}\frac{\partial\alpha_{i,2}}{\partial y_r{}^{(l-1)}}y_r{}^{(l)}\end{aligned}\tag{31}$$

The virtual control law $\alpha_{i,3}$ is designed as

$$\begin{aligned}\alpha_{i,3}=&-z_{i,2}-c_3z_{i,3}+\frac{\partial\alpha_{i,2}}{\partial\hat{c}_{i,2}}\dot{\hat{c}}_{i,2}+\sum_{l=1}^{2}\frac{\partial\alpha_{i,2}}{\partial x_{i,l}}x_{i,l+1}\\&+\sum_{l=1}^{2}\frac{\partial\alpha_{i,2}}{\partial\eta_i{}^{(l-1)}}\eta_i{}^{(l)}+\sum_{j=1}^{N}a_{ij}\sum_{l=1}^{2}\frac{\partial\alpha_{i,2}}{\partial x_{j,l}}x_{j,l+1}\\&+\mu_i\sum_{l=1}^{2}\frac{\partial\alpha_{i,2}}{\partial y_r{}^{(l-1)}}y_r{}^{(l)}-\frac{\partial\alpha_{i,2}}{\partial\hat{F}_i}sg\left(z_{i,3}\frac{\partial\alpha_{i,2}}{\partial\hat{F}_i}\right)\hat{B}_i\\&+\frac{\partial\alpha_{i,2}}{\partial\hat{B}_i}\tau_{i,2}\end{aligned}\tag{32}$$

where c_3 is a known positive constant and $\tau_{i,2}=\tau_{i,1}+\gamma_{B_i}z_{i,3}\frac{\partial\alpha_{i,2}}{\partial\hat{F}_i}sg\left(z_{i,3}\frac{\partial\alpha_{i,2}}{\partial\hat{F}_i}\right)$.

We define a Lyapunov function candidate as

$$V_3=V_2+\frac{1}{2}\sum_{i=1}^{N}z_{i,3}^2\tag{33}$$

Let $z_4=[z_{1,4},z_{2,4},\ldots,z_{N,4}]^T$. From (29), (30), (31) and (32), the derivative of V_3 can be calculated as

$$\begin{aligned}\dot{V}_3=&\dot{V}_2+\sum_{i=1}^{N}z_{i,3}\dot{z}_{i,3}\\\le&-\frac{c_1}{6}\lambda_{\min}(Q)\|z_1\|^2+\sum_{i=1}^{N}(\Delta_i+\mu_i)P_i\eta_iF+z_3{}^Tz_4\\&-\frac{\lambda_{\min}(Q)}{4\gamma_{F_t}}\left\|\tilde{F}_t\right\|^2-\frac{c_2}{2}\|z_2\|^2-c_3\|z_3\|^2+2\sum_{i=1}^{N}\eta_iB_i\\&+\sum_{i=1}^{N}z_{i,3}\frac{\partial\alpha_{i,2}}{\partial\hat{B}_i}(\tau_{i,2}-\dot{\hat{B}}_i)+\sum_{i=1}^{N}\frac{1}{\gamma_{B_i}}\tilde{B}_i(\tau_{i,2}-\dot{\hat{B}}_i)\end{aligned}\tag{34}$$

- Step k ($k=3,\ldots,n-1$). From Step 1, Step 2 and Step 3, we can conclude that $\alpha_{i,k}$ is the function of $\hat{c}_{i,2}$, $\hat{F}_i$, $\hat{B}_i$, $x_{i,1}$,…,$x_{i,k}$, η_i,…,$\eta_i{}^{(k-1)}$, $a_{ij}x_{j,1}$,…,$a_{ij}x_{j,k}$, μ_iy_r,…,$\mu_iy_r{}^{(k-1)}$. From (8), the derivative of $z_{i,k}$ is computed as

$$\dot{z}_{i,k}=z_{i,k+1}+\alpha_{i,k}-\dot{\alpha}_{i,k-1}\tag{35}$$

with

$$\begin{aligned}\dot{\alpha}_{i,k-1}=&\frac{\partial\alpha_{i,k-1}}{\partial\hat{c}_{i,2}}\dot{\hat{c}}_{i,2}+\sum_{l=1}^{k-1}\frac{\partial\alpha_{i,k-1}}{\partial x_{i,l}}x_{i,l+1}+\sum_{l=1}^{k}\frac{\partial\alpha_{i,k-1}}{\partial\eta_i{}^{(l-1)}}\eta_i{}^{(l)}\\&+\frac{\partial\alpha_{i,k-1}}{\partial\hat{F}_i}\dot{\hat{F}}_i+\frac{\partial\alpha_{i,k-1}}{\partial\hat{B}_i}\dot{\hat{B}}_i+\mu_i\sum_{l=1}^{k-1}\frac{\partial\alpha_{i,k-1}}{\partial y_r{}^{(l-1)}}y_r{}^{(l)}\\&+\sum_{j=1}^{N}a_{ij}\sum_{l=1}^{k-1}\frac{\partial\alpha_{i,k-1}}{\partial x_{j,l}}x_{j,l+1}\end{aligned}\tag{36}$$

The virtual control law $\alpha_{i,k}$ is designed as

$$\begin{aligned}\alpha_{i,k}=&-z_{i,k-1}-c_kz_{i,k}+\frac{\partial\alpha_{i,k-1}}{\partial\hat{c}_{i,2}}\dot{\hat{c}}_{i,2}+\sum_{l=1}^{k-1}\frac{\partial\alpha_{i,k-1}}{\partial x_{i,l}}x_{i,k+1}\\&+\sum_{l=1}^{k-1}\frac{\partial\alpha_{i,k-1}}{\partial\eta_i{}^{(l-1)}}\eta_i{}^{(l)}+\frac{\partial\alpha_{i,k-1}}{\partial\hat{B}_i}\tau_{i,k-1}\\&-\frac{\partial\alpha_{i,k-1}}{\partial\hat{F}_i}sg\left(z_{i,k}\frac{\partial\alpha_{i,k-1}}{\partial\hat{F}_i}\right)\hat{B}_i\\&+\sum_{j=1}^{N}a_{ij}\sum_{l=1}^{k-1}\frac{\partial\alpha_{i,k-1}}{\partial x_{j,l}}x_{j,l+1}+\mu_i\sum_{l=1}^{k-1}\frac{\partial\alpha_{i,k-1}}{\partial y_r{}^{(l-1)}}y_r{}^{(l)}\\&+\sum_{l=3}^{k-1}\frac{\partial\alpha_{i,l-1}}{\partial\hat{B}_i}z_{i,l}\gamma_{B_i}\frac{\partial\alpha_{i,k-1}}{\partial\hat{F}_i}sg\left(z_{i,k}\frac{\partial\alpha_{i,k-1}}{\partial\hat{F}_i}\right)\end{aligned}\tag{37}$$

with

$$\tau_{i,k-1}=\tau_{i,k-2}+\gamma_{B_i}z_{i,k}\frac{\partial\alpha_{i,k-1}}{\partial\hat{F}_i}sg\left(z_{i,k}\frac{\partial\alpha_{i,k-1}}{\partial\hat{F}_i}\right)\tag{38}$$

where c_k is a positive constant. Let $z_k=[z_{1,k},z_{2,k},\ldots,z_{N,k}]^T$. Define the Lyapunov function candidate as

$$V_k=V_2+\frac{1}{2}\sum_{l=3}^{k}\sum_{i=1}^{N}z_{i,l}{}^2\tag{39}$$

whose derivative is calculated as

$$\begin{aligned}\dot{V}_k\le&-\frac{c_1}{6}\lambda_{\min}(Q)\|z_1\|^2+\sum_{i=1}^{N}(\Delta_i+\mu_i)P_i\eta_iF\\&+z_{i,k}{}^Tz_{i,k+1}-\frac{\lambda_{\min}(Q)}{4\gamma_{F_t}}\left\|\tilde{F}_t\right\|^2\\&-\frac{c_2}{2}\|z_2\|^2-\sum_{l=3}^{k}c_l\|z_l\|^2+(k-1)\sum_{i=1}^{N}\eta_iB_i\\&+\sum_{i=1}^{N}\sum_{l=3}^{k}z_{i,l}\frac{\partial\alpha_{i,l-1}}{\partial\hat{B}_i}(\tau_{i,k-1}-\dot{\hat{B}}_i)\end{aligned}$$

$$+\sum_{i=1}^{N}\frac{1}{\gamma_{B_i}}\tilde{B}_i(\tau_{i,k-1}-\dot{\hat{B}}_i) \quad (40)$$

- Step n. From (8) for $k=n$, then

$$\begin{aligned}\dot{z}_{i,n}=&\dot{x}_{i,n}-\dot{\alpha}_{i,n-1}\\=&b_iu_i(t)+{\varphi_i}^T\theta_i+d_i(t)-\dot{\alpha}_{i,n-1}\\=&b_i\rho_i(t)u_{ci}(t)+{\varphi_i}^T\theta_i+d_i(t)-\dot{\alpha}_{i,n-1}\\=&b_i\rho_i(t)u_{ci}(t)+{\varphi_i}^T\tilde{\theta}_i+d_i(t)-\dot{\alpha}_{i,n-1}+{\varphi_i}^T\hat{\theta}_i\end{aligned} \quad (41)$$

where $\tilde{\theta}_i=\theta_i-\hat{\theta}_i$ with $\hat{\theta}_i$ being the estimate of θ_i.

Define $\overline{\alpha}_{i,n}={\varphi_i}^T\hat{\theta}_i+sg(z_{i,n})\hat{D}_i-\alpha_{i,n}$. The actual control law for each subsystem is designed as

$$u_{ci}(t)=-sgn(b_i)\frac{z_{i,n}\hat{\varrho}_i^2{\overline{\alpha}_{i,n}}^2}{\sqrt{{z_{i,n}}^2\hat{\varrho}_i^2{\overline{\alpha}_{i,n}}^2+{\eta_i}^2}} \quad (42)$$

where $\hat{\varrho}_i$ and $\hat{D}_i$ are the estimates of $\frac{1}{b_i\underline{\rho}_i}$ and D_i with D_i being the upper bound of d_i, respectively. $sgn(x)$ is the sign function with the property that $sgn(x)=\frac{x}{|x|}$ if $x\neq 0$, otherwise $sgn(x)=0$. $\alpha_{i,n}$ is determined by equation (37) for $k=n$. With the designed control law (42) and Lemma 2, then

$$\begin{aligned}z_{i,n}b_i\rho_i(t)u_{ci}(t)\leq&-|b_i|\,\underline{\rho}_i\frac{{z_{i,n}}^2\hat{\varrho}_i^2{\overline{\alpha}_{i,n}}^2}{\sqrt{{z_{i,n}}^2\hat{\varrho}_i^2{\overline{\alpha}_{i,n}}^2+{\eta_i}^2}}\\\leq&|b_i|\,\underline{\rho}_i\eta_i-|b_i|\,\underline{\rho}_i\hat{\varrho}_i\,|z_{i,n}\overline{\alpha}_{i,n}|\end{aligned} \quad (43)$$

which will be applied in the stability and consensus analysis for the entire closed-loop system given later.

The local parameter update laws are designed as follows:

$$\dot{\hat{B}}_i=\tau_{i,n-1} \quad (44)$$

$$\dot{\hat{D}}_i=\gamma_{D_i}z_{i,n}\mathrm{sg}(z_{i,n}) \quad (45)$$

$$\dot{\hat{\theta}}_i=z_{i,n}\Gamma_i\varphi_i \quad (46)$$

$$\dot{\hat{\varrho}}_i=\gamma_{\varrho_i}\,|z_{i,n}\overline{\alpha}_{i,n}| \quad (47)$$

where γ_{D_i}, Γ_i and γ_{ϱ_i} are all positive design parameters with appropriate dimension.

IV. System stability and consensus analysis

The main results of this paper are formally stated in the following theorem.

Theorem 1. *Consider the closed-loop system consisting of N uncertain nonlinear subsystems modeled in (1) with possibly intermittent actuator faults as modeled in (4), (5) under Assumptions 1-4. With the distributed adaptive fault tolerant controllers (42) and the parameter update laws (13), (24), (44)-(46) and (47), all the closed-loop signals are globally uniformly bounded. Asymptotically consensus tracking of all the subsystems' outputs to $y_r(t)$ is achieved, i.e. $\lim_{t\to\infty}\delta_i\to 0$ for $i=1,\ldots,N$. Furthermore, the states of N subsystems in lth order can also track the $(l-1)$th order derivative of $y_r(t)$, i.e. $\lim_{t\to\infty}x_{i,l}\to y_r(t)^{(l-1)}$ for $l=2,\ldots,n$.*

Proof. We define a Lyapunov function candidate for the entire closed-loop system as

$$\begin{aligned}V_n=&V_{n-1}+\frac{1}{2}\sum_{i=1}^{N}{z_{i,n}}^2+\sum_{i=1}^{N}\frac{|b_i|\,\underline{\rho}_i}{2\gamma_{\varrho_i}}\tilde{\varrho}_i^2+\frac{1}{2}\sum_{i=1}^{N}\tilde{\theta}_i^T{\Gamma_i}^{-1}\tilde{\theta}_i\\&+\sum_{i=1}^{N}\frac{1}{2\gamma_{D_i}}\tilde{D}_i^2\end{aligned} \quad (48)$$

From (41), the derivative of V_n can be calculated as

$$\begin{aligned}\dot{V}_n=&\dot{V}_{n-1}+\sum_{i=1}^{N}\frac{|b_i|\,\underline{\rho}_i}{\gamma_{\varrho_i}}\tilde{\varrho}_i(-\dot{\hat{\varrho}}_i)+\sum_{i=1}^{N}\tilde{\theta}_i^T{\Gamma_i}^{-1}(-\dot{\hat{\theta}}_i)\\&+\sum_{i=1}^{N}\frac{1}{\gamma_{D_i}}\tilde{D}_i(-\dot{\hat{D}}_i)+\sum_{i=1}^{N}z_{i,n}\mathrm{sg}(z_{i,n})\tilde{D}_i+\sum_{i=1}^{N}\eta_iD_i\\&+\sum_{i=1}^{N}z_{i,n}b_i\rho_i(t)u_{ci}(t)+\sum_{i=1}^{N}z_{i,n}{\varphi_i}^T\tilde{\theta}_i\\&+\sum_{i=1}^{N}z_{i,n}[{\varphi_i}^T\hat{\theta}_i+\mathrm{sg}(z_{i,n})\hat{D}_i-\alpha_{i,n}]\\&+\sum_{i=1}^{N}z_{i,n}(\alpha_{i,n}-\dot{\alpha}_{i,n-1})\end{aligned} \quad (49)$$

Substitute the inequality (43) into equation (49), then

$$\begin{aligned}\dot{V}_n\leq&\dot{V}_{n-1}+\sum_{i=1}^{N}z_{i,n}(\alpha_{i,n}-\dot{\alpha}_{i,n-1})+\sum_{i=1}^{N}|b_i|\,\rho_i\eta_i\\&+\sum_{i=1}^{N}\tilde{\theta}_i^T{\Gamma_i}^{-1}(z_{i,n}\varphi_i\Gamma_i-\dot{\hat{\theta}}_i)+\sum_{i=1}^{N}\eta_iD_i\\&+\sum_{i=1}^{N}\frac{|b_i|\,\underline{\rho}_i}{\gamma_{\varrho_i}}\tilde{\varrho}_i\left\{\gamma_{\varrho_i}\left|z_{i,n}[{\varphi_i}^T\hat{\theta}_i+\mathrm{sg}(z_{i,n})\hat{D}_i-\alpha_{i,n}]\right|\right.\\&\left.-\dot{\hat{\varrho}}_i\right\}+\sum_{i=1}^{N}\frac{1}{\gamma_{D_i}}\tilde{D}_i[\gamma_{D_i}z_{i,n}\mathrm{sg}(z_{i,n})-\dot{\hat{D}}_i]\end{aligned} \quad (50)$$

From (36) (37) and (40), the solution of the term $\dot{V}_{n-1}+\sum_{i=1}^{N}z_{i,n}(\alpha_{i,n}-\dot{\alpha}_{i,n-1})$ can be straightforwardly computed as

$$\begin{aligned}&\dot{V}_{n-1}+\sum_{i=1}^{N}z_{i,n}(\alpha_{i,n}-\dot{\alpha}_{i,n-1})\\&=-\frac{c_1}{6}\lambda_{\min}(Q)\|z_1\|^2+\sum_{i=1}^{N}(\Delta_i+\mu_i)P_i\eta_iF\\&-\frac{\lambda_{\min}(Q)}{4\gamma_{F_t}}\left\|\tilde{F}_t\right\|^2-\frac{c_2}{2}\|z_2\|^2-\sum_{l=3}^{n}c_l\|z_l\|^2\\&+(n-1)\sum_{i=1}^{N}\eta_iB_i+\sum_{i=1}^{N}\sum_{l=3}^{n}z_{i,l}\frac{\partial\alpha_{i,l-1}}{\partial\hat{B}_i}(\tau_{i,n-1}-\dot{\hat{B}}_i)\\&+\sum_{i=1}^{N}\frac{1}{\gamma_{B_i}}\tilde{B}_i(\tau_{i,n-1}-\dot{\hat{B}}_i)\end{aligned} \quad (51)$$

Finally, takeing (44), (45), (46), (47) and (51) into inequality (50), we have

$$\begin{aligned}\dot{V}_n \leq &-\frac{c_1}{6}\lambda_{\min}(Q)\|z_1\|^2 + \sum_{i=1}^{N}(\Delta_i+\mu_i)P_i\eta_i F \\ &-\frac{\lambda_{\min}(Q)}{4\gamma}\left\|\tilde{F}_t\right\|^2 - \frac{c_2}{2}\|z_2\|^2 - \sum_{l=3}^{n}c_l\|z_l\|^2 \\ &+(n-1)\sum_{i=1}^{N}\eta_i B_i + \sum_{i=1}^{N}|b_i|\,\underline{\varrho}_i\eta_i + \sum_{i=1}^{N}\eta_i D_i \quad (52)\end{aligned}$$

Integrating both sides of (52) yields that

$$\begin{aligned}&V_n(t)+\frac{c_1}{6}\lambda_{\min}(Q)\int_0^t\|z_1(\tau)\|^2 d\tau + \frac{c_2}{2}\int_0^t\|z_2(\tau)\|^2 d\tau \\ &\quad+\sum_{l=3}^{n}c_l\int_0^t\|z_l(\tau)\|^2 d\tau + \frac{\lambda_{\min}(Q)}{4\gamma_{F_t}}\int_0^t\left\|\tilde{F}_t(\tau)\right\|^2 d\tau \\ &\quad\leq V_n(0)+\sum_{i=1}^{N}[(\Delta_i+\mu_i)P_iF+(n-1)B_i \\ &\quad+|b_i|\,\underline{\varrho}_i + D_i]\bar{\eta}_i \quad (53)\end{aligned}$$

where $\bar{\eta}_i = \int_0^t \eta_i(\tau)d\tau$ with $\bar{\eta}_i$ being a constant. From (53) and the definition of V_n in (48) along with (26), we conclude that all the signals $z_{i,l}$ for $l=1,\ldots,n$, $\hat{F}_i$, $\hat{c}_{i,2}$, $\hat{B}_i$, $\hat{\varrho}_i$, $\hat{\theta}_i$, $\hat{D}_i$ for $i=1,\ldots,N$ are bounded. Besides, $z_{i,l}\in\mathcal{L}_2$.

From (7) and Assumption 2, it implies $x_{i,1}$ is bounded. From (11), (13) and Lemma 3, $\alpha_{i,1}$ is bounded. From (8), it follows that $x_{i,2}$ is bounded. From (23) and the boundedness of $y_r(t)$ and $\dot{y}_r(t)$ given in Assumption 2, $\alpha_{i,2}$ is bounded. By following similar analysis, the boundedness of $x_{i,l}$ and $\alpha_{i,l}$ for $l=1,\ldots,n$ can be shown. From (42), the control u_i is bounded. Therefore, the boundedness of all the signals in the closed-loop system can be guaranteed. Thus, from (10), $\dot{z}_{i,1}$ is bounded. By applying Barbalat's lemma, it can be concluded that $\lim_{t\to\infty} z_{i,1}\to 0$. Since H is nonsingular in (9), it further follows that asymptotic consensus tracking of all the N subsystems' outputs to a common desired trajectory $y_r(t)$ can be achieved, i.e. $\lim_{t\to\infty}\delta_i(t)\to 0$ for $i=1,\ldots,N$.

We now continue to analyze the consensus of the states for the subsystems. Differentiating $\dot{z}_1$ in (10) with respect to time t, there is $z_1^{(2)} = H[x_3 - 1_N\otimes y_r(t)^{(2)}]$. Since $x_3=[x_{1,3},x_{2,3},\ldots,x_{N,3}]^T$ and $y_r(t)^{(2)}$ are bounded, $z_1^{(2)}$ is bounded. Based on Barbalat's lemma, it implies that $\dot{z}_1$ will approach to zero when t goes to infinity, thus $\lim_{t\to\infty}x_{i,2}\to\dot{y}_r(t)$. Similarly, with the boundedness of $x_{i,l}$ and Assumption 2, the states of N subsystems in lth order can also track $y_r(t)^{(l-1)}$ asymptotically, i.e. $\lim_{t\to\infty}x_{i,l}\to y_r(t)^{(l-1)}$ for $l=3,\ldots,n$. □

Remark 6. *From the proof of Theorem 1, we notice that the two sufficient conditions in (20) and (28) are essential to achieve our main results. Although the two conditions include global topology parameters P, Q and H, they only indicate the existence of γ_F and c_2 which are needed in stability analysis. In other words, global topology information is not needed in the design of adaptive control law. Observing (42), (13), (24), (44)-(46) and (47), it can be summarized that the required knowledge to generate u_{ci} in each subsystem i include: (i) the common reference trajectory $\mu_i y_r^{(l)}$ if $\mu_i=1$ for $l=0,1,\ldots,n-1$; (ii) the local states $x_{i,q}$ for $q=1,\ldots,n$ and local parameter estimates $\hat{F}_i,\hat{c}_{i,2},\hat{B}_i,\hat{\varrho}_i,\hat{\theta}_i,\hat{D}_i$; (iii) its neighbors' states and parameter estimates $a_{ij}x_{j,q}, a_{ij}\hat{F}_j$ with $a_{ij}=1$ for $q=1,\ldots,n$. Therefore, the proposed adaptive control scheme is totally distributed.*

Remark 7. *To handle the issue that reference trajectory information $y_r(t)$ is totally unknown by a fraction of high-order subsystems with $\mu_i=0$, an alternative approach is proposed in [12]. In [12], for each subsystem i of order n, an auxiliary $n-$th order pure integrator type of filter is introduced. Then n distributed parameter estimators are designed to handle the unknown $n-$th order derivative of $y_r(t)$. Moreover, each subsystem i needs collect $n-$ dimensional parameter estimates from its neighbors to complete the design of its local controller. Different from [12], the auxiliary filter can be avoided and only one dimensional parameter estimates need be transmitted among connected agents. Therefore, though both results achieve asymptotically consensus tracking, the computational and communication cost can be effectively reduced in this paper with compared to [12].*

Remark 8. *In this paper, the results in Wang et al. [13], [27] are successfully extended to uncertain high-order nonlinear systems with uncertain intermittent actuator faults under the directed communication condition. Besides, unlike the representative results in this area including Das et al, [21] Zhang et al, [10] and Yoo, [11] where only semi-global uniform ultimate boundedness of tracking errors are guaranteed, perfect output consensus tracking can be achieved in this paper.*

V. Simulation Results

A. An Application Example

In the simulation, we firstly consider a group of 4 second-order continuous torsional pendulum systems [37] modeled as follows

$$\begin{aligned}\dot{\vartheta}_i &= \omega_i; \\ J_i\dot{\omega}_i &= u_i - M_i g l_i \sin(\vartheta_i) - f_{di}\omega_i;\ i=1,\ldots 4. \quad (54)\end{aligned}$$

where ϑ_i and ω_i are the angle and angular velocity, respectively. $M_i=1/3kg$ and $l_i=2/3m$ are the mass and length of each torsional pendulum, respectively. $g=9.8m/s^2$ denotes the acceleration of gravity. $J_i=4/3M_i l_i^2$ represents the rotary inertia. $f_{di}=0.2$ is the frictional factor. M_i, l_i, J_i, g and f_{di} are all unknown system parameters. Define $b_i=J_i^{-1}$, $\theta_i=[J_i^{-1}M_i g l_i, J_i^{-1}f_{di}]^T$ and $\varphi_i=[-\sin(\vartheta_i),-\omega_i]^T$. The desired reference trajectory is $y_r(t)=\sin(0.1t)$. The communication topology among the 4 subsystems and the reference trajectory $y_r(t)$ is represented by a directed graph as shown in Fig. 1. The faulty model in (4) is modeled as $u_i(t)=\rho_i(t)u_{ci}(t)$, $t\in[hT^*,(h+1)T^*)$, $h=1,3,\ldots$, where $\rho_i(t)=0.3$ and $T^*=10$ seconds, which are both unknown parameters in the design of controllers.

In simulation, the initial states including $\omega_1(0)$, $\omega_2(0)$, $\omega_3(0)$, $\omega_4(0)$, $\hat{F}_i(0)$, $\hat{c}_{i,2}(0)$, $\hat{B}_i(0)$, $\hat{\theta}_i(0)$, $\hat{\varrho}_i(0)$ for $i=$

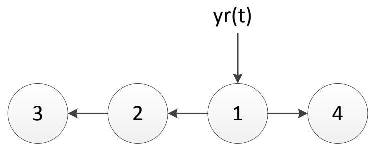

Fig. 1. Communication topology for a group of 4 subsystems.

$1, \ldots, 4$ are set as zeros. $\vartheta_1(0) = 1$, $\vartheta_2(0) = 0.8$, $\vartheta_3(0) = 1.2$, $\vartheta_4(0) = 0.5$. The design parameters are chosen as $c_1 = 1$ and $F = \gamma_{c_2} = \gamma_{B_i} = \gamma_{\varrho_i} = 1$, $\Gamma_i = diag\{1, 1\}$, $\eta_i(t) = 0.2e^{-0.03t}$. The tracking performance, the tracking error and the control inputs are respectively shown in Fig. 2-5, which validate the effectiveness of the proposed distributed adaptive consensus tracking control scheme. Besides, the method presented in this paper is compared to that in [12] in terms of both computational complexity and communication burden as shown in TABLE I and II, respectively. Note that both methods can achieve asymptotically consensus tracking in this example. In [12], to compensate the effect of unknown trajectory, an auxiliary second-order filter and two distributed parameter estimators, as given in TABLE I, need be introduced in each subsystem i. And four signals (i.e. the states ϑ_j, ω_j, $\hat{F}_{j1}$, $\hat{F}_{j2}$) need be collected from its neighbors through the communication network. However, in this paper, only one distributed parameter estimator is introduced and the number of communication signals is reduced to 3, i.e. ϑ_j, ω_j and $\hat{F}_j$. Hence, the communication channel bandwidth and computation resource can be saved with the method in this paper.

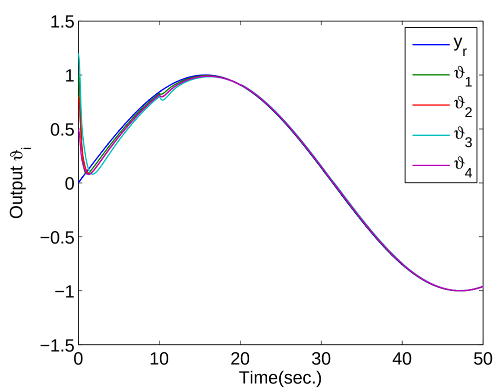

Fig. 2. The outputs ϑ_i, $i = 1, \ldots, 4$.

B. A Numerical Example

A group of 4 high-order subsystems modeled as follows are considered.

$$\begin{aligned} \dot{x}_{i,1} =& x_{i,2}; \\ \dot{x}_{i,2} =& x_{i,3}; \\ \dot{x}_{i,3} =& b_i u_i(t) + {\varphi_i}^{\mathrm{T}} \theta_i + d_i(t), i = 1, \ldots, 4. \end{aligned} \tag{55}$$

where $b_1 = 1$, $b_2 = -2$, $b_3 = 0.5$, $b_4 = 3$, $\theta_1 = 1$, $\theta_2 = 0.5$, $\theta_3 = 2$ and $\theta_4 = 3$ are all unknown system parameters.

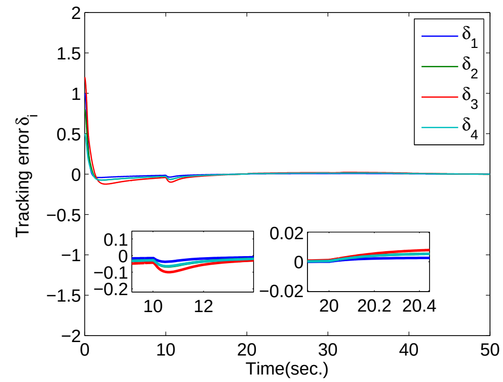

Fig. 3. Tracking errors $\delta_i = \vartheta_i - y_r$, $i = 1, \ldots, 4$.

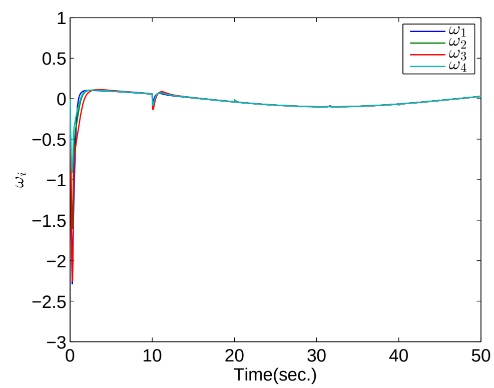

Fig. 4. The states ω_i, $i = 1, \ldots, 4$.

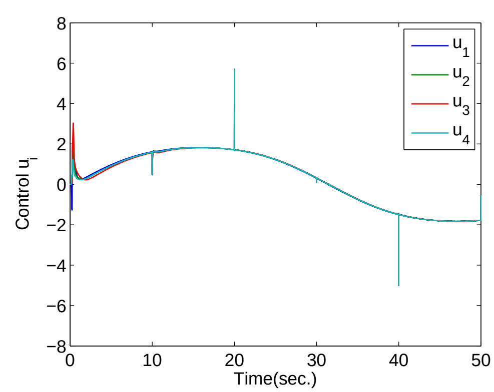

Fig. 5. Control inputs u_i without actuator faults, $i = 1, \ldots, 4$.

$\varphi_1 = x_{1,3}^3$, $\varphi_2 = x_{2,3}^2$, $\varphi_3 = x_{3,3}$ and $\varphi_4 = x_{4,2}x_{4,3}$ are the subsystem structure. $d_1(t) = 0.1\sin(t)$, $d_2(t) = 0.2\sin(t)^2$, $d_3(t) = 0.05\sin(t)$ and $d_4(t) = 0.15\sin(t)$ denote uncertain external disturbances. Neither the detailed function of $d_i(t)$ nor its upper bound will be used in the design of distributed adaptive controllers. The communication topology among the 4 subsystems is the same as that in the former application example.

In simulation, the state initials including $x_{i,1}(0)$, $x_{i,2}(0)$, $x_{i,3}(0)$, $\hat{F}_i(0)$, $\hat{c}_{i,2}(0)$, $\hat{B}_i(0)$, $\hat{\theta}_i(0)$ for $i = 1, \ldots, 4$ in the closed-loop system are set as zero and $\hat{\varrho}_i(0) = 2$,

TABLE I
THE COMPARISON OF THE COMPUTATIONAL COMPLEXITY FOR THE METHODS PRESENTED IN THIS PAPER AND [12].

	To handle the issue of unknown trajectory information, the additional filter and distributed parameter estimators are introduced in each subsystem i.
Reference [12]	$\dot{q}_{i,1} = q_{i,2}$; $\dot{q}_{i,2} = v_i$. $\dot{\hat{F}}_{ik} = \sum_{j=1}^{4} a_{ij}(\hat{F}_{jk} - \hat{F}_{ik}) + \mu_i(F_k - \hat{F}_{ik})$; $k = 1, 2$.
The present paper	$\dot{\hat{F}}_i = \sum_{j=1}^{4} a_{ij}(\hat{F}_j - \hat{F}_i) + \mu_i(F - \hat{F}_i)$

TABLE II
THE COMPARISON OF THE COMMUNICATION BURDEN FOR THE METHODS PRESENTED IN THIS PAPER AND [12].

	For each subsystem i, the signals need be collected from its neighboring subsystem j through communication network, if $a_{ij} = 1$.
Reference [12]	ϑ_j, ω_j, $\hat{F}_{j1}$, $\hat{F}_{j2}$
The present paper	ϑ_j, ω_j, $\hat{F}_j$

TABLE III
THE COMPARISON OF THE COMPUTATIONAL COMPLEXITY FOR THE METHODS PRESENTED IN THIS PAPER AND [12].

	To handle the issue of unknown trajectory information, the additional filter and distributed parameter estimators are introduced in each subsystem i.
Reference [12]	$\dot{q}_{i,1} = q_{i,2}$; $\dot{q}_{i,2} = q_{i,3}$; $\dot{q}_{i,3} = v_i$. $\dot{\hat{F}}_{ik} = \sum_{j=1}^{4} a_{ij}(\hat{F}_{jk} - \hat{F}_{ik}) + \mu_i(F_k - \hat{F}_{ik})$; $k = 1, 2, 3$.
The present paper	$\dot{\hat{F}}_i = \sum_{j=1}^{4} a_{ij}(\hat{F}_j - \hat{F}_i) + \mu_i(F - \hat{F}_i)$

$\hat{D}_i(0) = 1$. The reference trajectory is generated by a time-varying function $y_r(t) = \cos(0.1t)$, whose information is directly available only for subsystem 1 as shown in Fig. 1. The faulty model in (4) is modeled as $u_i(t) = \rho_i(t)u_{ci}(t)$, $t \in [hT^*, (h+1)T^*), h = 1, 3, \ldots$, where $\rho_i(t) = 0.3$ and $T^* = 15$ seconds, which are both unknown parameters in practical application. The design parameters are chosen as $c_1 = c_3 = 0.05$ and $F = \gamma_{c_2} = \gamma_{B_i} = \gamma_{D_i} = \gamma_{\varrho_i} = \Gamma_i = 1, \eta_i(t) = 0.2e^{-0.03t}$.

The tracking performance of all the agents' outputs $y_i(t)$ and tracking errors are shown in Fig. 6 and Fig. 7. It can be seen that the satisfactory asymptotic output consensus tracking for all the subsystems can be achieved with our proposed distributed adaptive control scheme. Furthermore, the consensus of states $x_{i,2}$ and $x_{i,3}$ can also be ensured as shown in Fig. 8 and 9. Control inputs as designed in (42) is shown in Fig.10. From Fig. 6-10, it can be observed that all the closed-loop signals are bounded, even though there exist unknown intermittent actuator fault happening in the actuators with time progresses. Similar to previous example, TABLE III-IV are given to compare the computational complexity and communication burden for the methods in [12] and this paper. It can be seen that the improvement achieved with the method in this paper becomes more significant with the increase of the subsystem order.

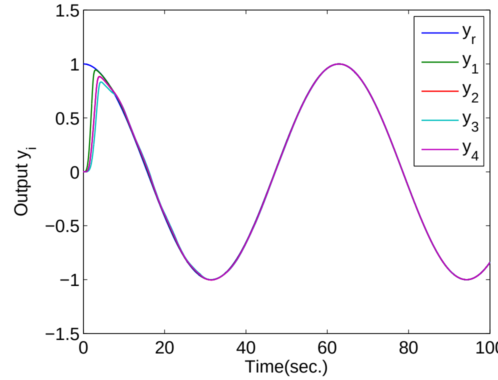

Fig. 6. The outputs y_i, $i = 1, \ldots, 4$.

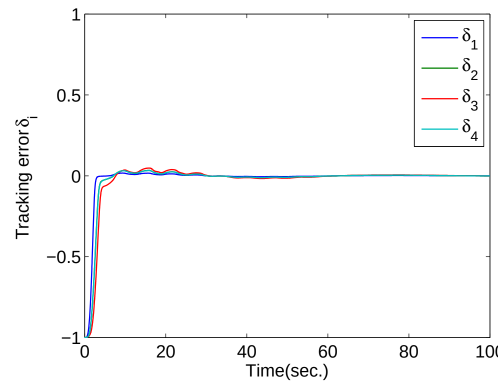

Fig. 7. Tracking errors $\delta_i = x_{i,1} - y_r$, $i = 1, \ldots, 4$.

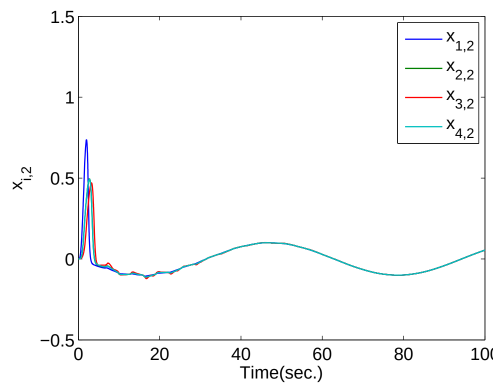

Fig. 8. The states $x_{i,2}$, $i = 1, \ldots, 4$.

TABLE IV
THE COMPARISON OF THE COMMUNICATION BURDEN FOR THE METHODS PRESENTED IN THIS PAPER AND [12].

	For each subsystem i, the signals need be collected from its neighboring subsystem j through communication network, if $a_{ij} = 1$.
Reference [12]	$x_{j,1}$, $x_{j,2}$, $x_{j,3}$, $\hat{F}_{j1}$, $\hat{F}_{j2}$, $\hat{F}_{j3}$
The present paper	$x_{j,1}$, $x_{j,2}$, $x_{j,3}$, $\hat{F}_j$

VI. CONCLUSION

In this paper, the output consensus tracking problem for uncertain high-order nonlinear systems with intermittent ac-

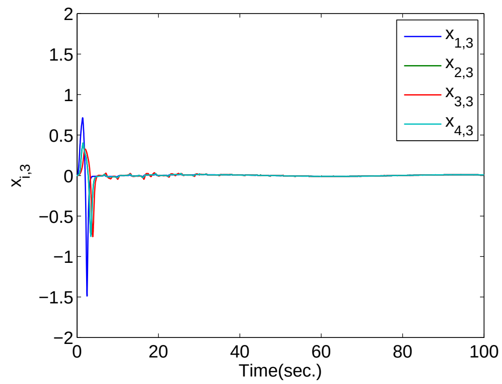

Fig. 9. The states $x_{i,3}$, $i = 1, \ldots, 4$.

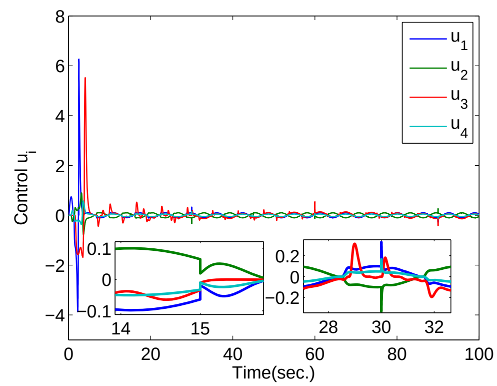

Fig. 10. Control inputs u_i with actuator faults, $i = 1, \ldots, 4$.

tuator faults is studied under directed topology condition. A novel backstepping based distributed adaptive consensus controller is proposed. By introducing extra distributed parameter estimators in all subsystems to account for the unknown but constant bound regarding the first order time derivative of the desired trajectory function, the assumption on linearly parameterized trajectory in most relevant results can be successfully relaxed. Besides, to handle the intermittent actuator faults, a new adaptive compensation technique is adopted. It is shown that with the proposed distributed adaptive control protocol, asymptotically consensus output tracking can be achieved while all closed-loop signals are ensured globally uniformly bounded.

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References

[1] R. Olfati-Saber and R. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *IEEE Transactions on Automatic Control*, vol. 49, no. 9, pp. 1520–1533, Sep. 2004.
[2] W. Ren and R. Beard, "Consensus seeking in multiagent systems under dynamically changing interaction topologies," *IEEE Transactions on Automatic Control*, vol. 50, no. 5, pp. 655–661, May 2005.
[3] L. Moreau, "Stability of multiagent systems with time-dependent communication links," *IEEE Transactions on Automatic Control*, vol. 50, no. 2, pp. 169–182, Feb. 2005.
[4] R. Olfati-Saber, "Flocking for multi-agent dynamic systems: Algorithms and theory," *IEEE Transactions on Automatic Control*, vol. 51, no. 3, pp. 401–420, Mar. 2006.
[5] W. Ren and E. Atkins, "Distributed multi-vehicle coordinated control via local information exchange," *International Journal of Robust and Nonlinear Control*, vol. 17, no. 10-11, pp. 1002–1033, Jul. 2007.
[6] S.-L. Du, W. Xia, X.-M. Sun, and W. Wang, "Sampled-data-based consensus and L-2-gain analysis for heterogeneous multiagent systems," *IEEE Transactions on Automatic Control*, vol. 47, no. 6, pp. 1523–1531, Jun. 2017.
[7] S.-L. Du, X.-M. Sun, M. Cao, and W. Wang, "Pursuing an evader through cooperative relaying in multi-agent surveillance networks," *Automatica*, vol. 83, pp. 155 – 161, Sep. 2017.
[8] M. Arcak, "Passivity as a design tool for group coordination," *IEEE Transactions on Automatic Control*, vol. 52, no. 8, pp. 1380–1390, Aug. 2007.
[9] H. Bai, M. Arcak, and J. Wen, "Adaptive design for reference velocity recovery in motion coordination," *Systems & Control Letters*, vol. 57, no. 8, pp. 602–610, Aug. 2008.
[10] H. Zhang and F. Lewis, "Adaptive cooperative tracking control of higher-order nonlinear systems with unknown dynamics," *Automatica*, vol. 48, no. 7, pp. 1432 – 1439, Jul. 2012.
[11] S. Yoo, "Distributed consensus tracking for multiple uncertain nonlinear strict-feedback systems under a directed graph," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 24, no. 4, pp. 666 – 672, Apr. 2013.
[12] J. Huang, Y. Song, W. Wang, C. Wen, and G. Li, "Smooth control design for adaptive leader-following consensus control of a class of high-order nonlinear systems with time-varying reference," *Automatica*, vol. 83, pp. 361–367, Sep. 2017.
[13] W. Wang, C. Wen, and J. Huang, "Distributed adaptive asymptotically consensus tracking control of nonlinear multi-agent systems with unknown parameters and uncertain disturbances," *Automatica*, vol. 77, pp. 133–142, Mar. 2017.
[14] A. Das and F. L. Lewis, "Cooperative adaptive control for synchronization of second-order systems with unknown nonlinearities," *International Journal of Robust and Nonlinear Control*, vol. 21, no. 13, pp. 1509–1524, Sep. 2011.
[15] Y. Hong, J. Hu, and L. Gao, "Tracking control for multi-agent consensus with an active leader and variable topology," *Automatica*, vol. 42, no. 7, pp. 1177–1182, Jul. 2006.
[16] W. Ren, "Multi-vehicle consensus with a time-varying reference state," *Systems & Control Letters*, vol. 56, no. 7, pp. 474 – 483, Jul. 2007.
[17] H. Bai, M. Arcak, and J. Wen, "Adaptive motion coordination: Using relative velocity feedback to track a reference velocity," *Automatica*, vol. 45, no. 4, pp. 1020–1025, Apr. 2009.
[18] W. Dong, "Adaptive consensus seeking of multiple nonlinear systems," *International Journal of Adaptive Control and Signal Processing*, vol. 26, no. 5, pp. 419–434, May 2012.
[19] Z. Li, X. Liu, W. Ren, and L. Xie, "Distributed tracking control for linear multiagent systems with a leader of bounded unknown input," *IEEE Transactions on Automatic Control*, vol. 58, no. 2, pp. 518–523, Feb. 2013.
[20] M. Krstic, I. Kanellakopoulos, and P. Kokotovic, *Nonlinear and adaptive control design*. New York, NY: Wiley, 1995.
[21] A. Das and F. Lewis, "Distributed adaptive control for synchronization of unknown nonlinear networked systems," *Automatica*, vol. 46, no. 12, pp. 2014–2021, Dec. 2010.
[22] H. Yu and X. Xia, "Adaptive consensus of multi-agents in networks with jointly connected topologies," *Automatica*, vol. 48, no. 8, pp. 1783 – 1790, Aug. 2012.
[23] H. Yu, Y. Shen, and X. Xia, "Adaptive finite-time consensus in multi-agent networks," *Systems & Control Letters*, vol. 62, no. 10, pp. 880–889, Oct. 2013.
[24] W. Wang, J. Huang, C. Wen, and H. Fan, "Distributed adaptive control for consensus tracking with application to formation control of nonholonomic mobile robots," *Automatica*, vol. 50, no. 4, pp. 1254 – 1263, Apr. 2014.
[25] X. Jin, "Adaptive iterative learning control for high-order nonlinear multi-agent systems consensus tracking," *Systems & Control Letters*, vol. 89, pp. 16–23, Mar. 2016.
[26] J. Wang, "Distributed coordinated tracking control for a class of uncertain multiagent systems," *IEEE Transactions on Automatic Control*, vol. 62, no. 7, pp. 3423–3429, Jul. 2017.

[27] W. Wang, C. Wen, J. Huang, and H. Fan, "Distributed adaptive asymptotically consensus tracking control of uncertain Euler-Lagrange systems under directed graph condition," *ISA Transactions*, vol. 71, no. 1, pp. 121–129, Nov. 2017.
[28] W. Wang and C. Wen, "Adaptive compensation for infinite number of actuator failures or faults," *Automatica*, vol. 47, no. 10, pp. 2197–2210, Oct. 2011.
[29] C. Su, Y. Stepanenko, J. Svoboda, and T. Leung, "Robust adaptive control of a class of nonlinear systems with unknown backlash-like hysteresis," *IEEE Transactions on Automatic Control*, vol. 45, no. 12, pp. 2427–2432, Dec. 2000.
[30] J. Zhou, C. Wen, and Y. Zhang, "Adaptive backstepping control of a class of uncertain nonlinear systems with unknown backlash-like hysteresis," *IEEE Transactions on Automatic Control*, vol. 49, no. 10, pp. 1751–1759, Oct. 2004.
[31] J. Zhou and C. Wen, "Adaptive backstepping control of uncertain nonlinear systems with input quantization," in *52nd IEEE Conference on Decision and Control*, Dec. 2013, pp. 5571–5576.
[32] W. Ren and Y. Cao, *Distributed Coordination of Multi-agent Networks: Emergent Problems, Models, and Issues*, ser. Communications and Control Engineering. Springer London, 2010.
[33] J. Boskovic, J. Jackson, R. Mehra, and N. Nguyen, "Multiple-Model Adaptive Fault-Tolerant Control of a Planetary Lander," *Journal of Guidance Control and Dynamics*, vol. 32, no. 6, pp. 1812–1826, Nov.-Dec. 2009, AIAA Guidance, Navigation and Control Conference and Exhibit, Honolulu, HI, Aug. 18-21, 2008.
[34] Z. Zhang and W. Chen, "Adaptive output feedback control of nonlinear systems with actuator failures," *Information Sciences*, vol. 179, no. 24, pp. 4249–4260, Dec. 2009.
[35] X. Tang and G. Tao, "An adaptive nonlinear output feedback controller using dynamic bounding with an aircraft control application," *International Journal of Adaptive Control and Signal Processing*, vol. 23, no. 7, pp. 609–639, Jul. 2009.
[36] Z. Zuo and C. Wang, "Adaptive trajectory tracking control of output constrained multi-rotors systems," *Control Theory & Applications Iet*, vol. 8, no. 13, pp. 1163–1174, Sep. 2014.
[37] Y.-X. Li and G.-H. Yang, "Model-Based Adaptive Event-Triggered Control of Strict-Feedback Nonlinear Systems," *IEEE Transactions on Nueral Networks and Learning Systems*, vol. 29, no. 4, pp. 1033–1045, Apr. 2018.