Adaptive Backstepping Control of a 2-DOF Helicopter

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***Abstract*—This paper proposes an adaptive nonlinear controller for a 2-Degree of Freedom (DOF) helicopter. The proposed controller is designed using backstepping control technique and is used to track the pitch and yaw position references independently. A MIMO nonlinear mathematical model is derived for the 2DOF helicopter based on Euler-Lagrange equations, where the system parameters and the control coefficients are uncertain. Unlike some existing control schemes for the helicopter control, the developed controller does not require the knowledge on the system uncertain parameters. Updating laws are used to estimate the unknown parameters. It is shown that not only the global stability is guaranteed by the proposed controller, but also asymptotic tracking and transient performances are quantified as explicit functions of the design parameters. Simulations and experiments are carried out on the Quanser helicopter to validate the effectiveness, robustness and control capability of the proposed scheme.**

***Index Terms*—adaptive control, backstepping, 2-Degree of Freedom helicopter, position control**

I. Introduction

Air vehicles such as helicopters and unmanned aerial vehicles have received an increased attention in the past years. As with regular helicopters, these provide great accessibility and are utilized in many areas of application, such as transport, search and rescue, police or military operations, photography and filming, and for hobby use. These applications require the air craft capable of executing complicated maneuvers in adverse flight conditions, while being stable, reliable, and safe. Increasing demands in these areas increases the necessity of advanced control systems. This has also resulted in lots of research later years for control of such systems, because it is still a challenge in controller design due to its nonlinear behavior, its coupling, parameter perturbation, model uncertainties, and external disturbances. Therefore it is desired to continue the development and research of advanced control systems.

Control of helicopter system has been investigated using many control techniques. This includes conventional PID control in [1], linear quadratic regulator (LQR) control in [2], and some advanced controllers including model predictive control (MPC) in [3], [4], robust sliding mode control in [5], nonlinear $\mathcal{H}_\infty$ control in [6]. In practice, it is often required to consider the case where the plant to be controlled is uncertain. As well known, adaptive control is a useful and important approach to deal with system uncertainties due to its ability of providing online estimations of unknown system parameters with measurements. Adaptive control of helicopter systems has been investigated in [7] and [8]. In [7], model reference adaptive control (MRAC) was designed for a linear helicopter system. The simple adaptive control was designed in [8] for the quadrotor helicopter with loss of control effectiveness. However, [7] and [8] considered a linearized system which made the controller design simple.

Since backstepping technique was proposed, it has been widely used to design adaptive controllers for uncertain systems [9]–[12]. This technique has a number of advantages over the conventional approaches such as providing a promising way to improve the transient performance of adaptive systems by tuning design parameters. Because of such advantages, research on adaptive control of helicopter using backstepping technique has also received great attention, see for examples [13], [14]. In [13], an adaptive backstepping control for a model helicopter in presence of external disturbances was presented and only the simulation was done which showed that the control design was robust to disturbances. In [14], an adaptive integral backstepping control scheme for tracking control for a 2DOF helicopter was proposed, where only the uncertainty in the mass was considered.

In this paper, a mathematical model of a 2-DOF helicopter is obtained from its Euler-Lagrange equations with pitch and yaw axes, which includes the parameter perturbation and uncertainties. A nonlinear adaptive backstepping controller is developed to track the pitch and yaw position references independently. A theoretical proof of stability with the proposed adaptive control is given with the use of constructed Lyapunov functions, where asymptotic tracking and the global boundedness of all signals in the closed loop system are achieved. Also, the transient performance for the tracking errors in terms of $\mathcal{L}_2$ norm is derived, where the tracking error performance can be improved by adjusting the design parameters. Simulations and experiments are conducted on a Quanser helicopter called Aero. To examine the robustness of the proposed controller, disturbances have been added, including fault in one propeller and added external torques. The performance of the controller is compared with the performance of an LQR, and the results shows a lower total error for the adaptive backstepping controller. Simulation and experimental results validate the effectiveness, robustness and control capability of the proposed scheme.

II. Plant Description and Model

The Quanser Aero shown in Figure 1 is a two-rotor laboratory equipment for flight control-based experiments. The setup is a horizontal position of the main thruster and a vertical position of the tail thruster, which resembles a helicopter with two propellers driven by two DC motors. The forces on the

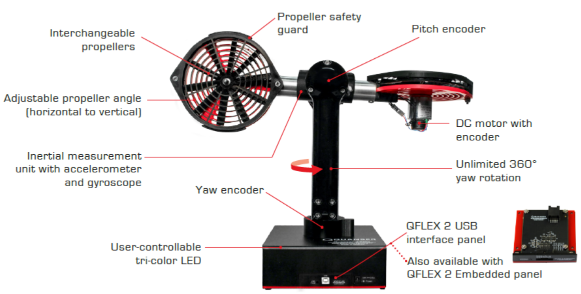

Fig. 1. Quanser Aero

Aero body is visualized in Figure 2 showing both a free body diagram (FBD) and a kinetic diagram (KD). The main motor is producing two forces, one main force, F_{Mz}, in the z_b-direction that will give a positive pitch angle, and also a force, F_{My}, in the y_b-direction, meaning this will give a yaw angle. This last force is due to the aerodynamic forces. The tail motor is also producing two forces, F_{Tz} and F_{Ty}. This motor is basically here to counteract the yaw from the main motor and thus control the yaw while the main motor is controlling the pitch. These forces are functions of the two system inputs V_p and V_y. Viscous damping, proportional to the velocity of the Aero body, is also present.

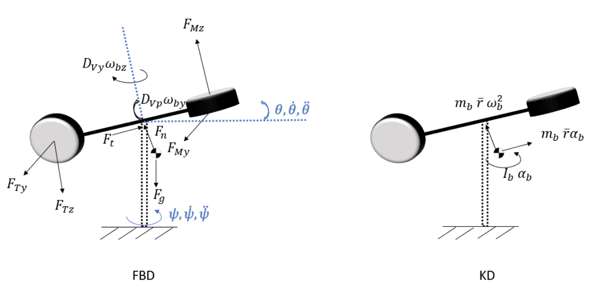

Fig. 2. Free body diagram and kinetic diagram of the Aero body

The Aero is considered as a rigid body and the equations of motion are derived using Euler-Lagrange equations given by

$$(I_p + ml_{cm}^2)\ddot{\theta} = K_{pp}V_p + K_{py}V_y - D_{Vp}\dot{\theta} - F_g l_{cm}\sin\theta + ml_{cm}^2\dot{\psi}^2\cos\theta\sin\theta, \tag{1}$$

$$I_y\ddot{\psi} = K_{yy}V_y + K_{yp}V_p - D_{Vy}\dot{\psi} - 2ml_{cm}^2\dot{\theta}\dot{\psi}\cos\theta\sin\theta, \tag{2}$$

where θ and ψ are pitch and yaw angles, $\dot{\theta}$ and $\dot{\psi}$ are angular velocities of pitch and yaw angles, inputs V_p and V_y are the voltages applied to the main and tail motors, K_{pp} and K_{yy} are torque thrust gains from main and tail motors, K_{py} is cross-torque thrust gain acting on pitch from tail motor, K_{yp} is cross-torque thrust gain acting on yaw from main motor, l_{cm} is the distance between the center of mass and the origin of the body-fixed frame, I_p and I_y are the moments of inertia of the pitch and yaw respectively, $F_g = mg$ is the gravity force, m is the total mass of the Aero body and D_{V_y} and D_{V_p} are the damping constants for the rotation along the yaw axis and pitch axis separately. Note that the equivalent moment of inertia of the yaw will change with changing pitch angle, but this is assumed small and is not included.

A. System transformation

The objective is to control the attitude of the Aero with control of pitch and yaw angles. The state variables are defined as

$$\mathbf{x}^T = [\theta, \psi, \dot{\theta}, \dot{\psi}], \tag{3}$$

and the control variables are defined as

$$\mathbf{u}^T = [u_p, u_y]. \tag{4}$$

where the variables u_p and u_y are the control inputs for pitch and yaw defined as

$$u_p = K_{pp}V_p + K_{py}V_y, \tag{5}$$

$$u_y = K_{yy}V_y + K_{yp}V_p. \tag{6}$$

Equations (1) and (2) are rewritten into the state space given by

$$\dot{\mathbf{x}} = \begin{bmatrix} x_3 \\ x_4 \\ b_p u_p + \Phi_1^T(x)\Theta_1 \\ b_y u_y + \Phi_2^T(x)\Theta_2 \end{bmatrix}, \tag{7}$$

where Φ_1 and Φ_2 are known nonlinear functions defined as

$$\Phi_1 = \begin{bmatrix} -x_3 \\ -\sin x_1 \\ x_4^2\cos x_1\sin x_1 \end{bmatrix}, \quad \Phi_2 = \begin{bmatrix} -x_4 \\ -x_2x_4\cos x_1\sin x_1 \end{bmatrix} \tag{8}$$

Θ_1 and Θ_2 are unknown constant vectors defined as

$$\Theta_1 = \frac{1}{I_p + ml_{cm}^2}\begin{bmatrix} D_{Vp} \\ mgl_{cm} \\ ml_{cm}^2 \end{bmatrix}, \quad \Theta_2 = \frac{1}{I_y}\begin{bmatrix} D_{Vy} \\ 2ml_{cm}^2 \end{bmatrix}. \tag{9}$$

and b_p and b_y are unknown constants defined as

$$b_p = \frac{1}{I_p + ml_{cm}^2}, \quad b_y = \frac{1}{I_y}. \tag{10}$$

The control objective is to design a control law for $u_p(t)$ and $u_y(t)$ to force the outputs $x_1(t)$ and $x_2(t)$ to asymptotically track the reference signals $x_{r1}(t)$ and $x_{r2}(t)$ for pitch and yaw respectively. To achieve the objective, the following assumptions are imposed.

Assumption 1: The reference signals $x_{r1}(t)$ and $x_{r2}(t)$ and first and second order derivatives are known, piecewise continuous and bounded.

Assumption 2: All unknown parameters Θ_1, Θ_2, b_p and b_y are positive constants.

III. Adaptive Control Design

We begin by introducing the change of coordinates

$$z_1 = x_1 - x_{r1}, \tag{11}$$
$$z_2 = x_2 - x_{r2}, \tag{12}$$
$$z_3 = x_3 - \alpha_1 - \dot{x}_{r1}, \tag{13}$$
$$z_4 = x_4 - \alpha_2 - \dot{x}_{r2}. \tag{14}$$

where α_1 and α_2 are the virtual controllers and will be determined later.

• *Step* 1: The virtual controls are chosen as

$$\alpha_1 = -c_1 z_1, \tag{15}$$
$$\alpha_2 = -c_2 z_2, \tag{16}$$

where c_1 and c_2 are positive constants. A control Lyapunov function is chosen as

$$V_1 = \frac{1}{2} z_1^2 + \frac{1}{2} z_2^2. \tag{17}$$

The derivative of V_1 is

$$\begin{aligned}\dot{V}_1 &= z_1 \dot{z}_1 + z_2 \dot{z}_2 \\ &= z_1(z_3 + \alpha_1) + z_2(z_4 + \alpha_2) \\ &= -c_1 z_1^2 + z_1 z_3 - c_2 z_2^2 + z_2 z_4. \end{aligned} \tag{18}$$

If z_3 and z_4 are zero, then $\dot{V}_1$ is negative and z_1 and z_2 will converge towards zero.

• *Step* 2: The derivative of z_3 and z_4 are expressed as

$$\dot{z_3} = b_p u_p + \Phi_1^T(x)\Theta_1 + c_1(x_3 - \dot{x}_{r1}) - \ddot{x}_{r1}, \tag{19}$$
$$\dot{z_4} = b_y u_y + \Phi_2^T(x)\Theta_2 + c_2(x_4 - \dot{x}_{r2}) - \ddot{x}_{r2}. \tag{20}$$

The control inputs u_p and u_y will now be designed so that z_1, z_2, z_3 and z_4 all converge towards zero. Then, the adaptive control law is designed as follows:

$$u_p = \hat{\rho}_1 \bar{u}_p, \tag{21}$$
$$u_y = \hat{\rho}_2 \bar{u}_y, \tag{22}$$
$$\bar{u}_p = - z_1 - \Phi_1^\top \hat{\Theta}_1 - c_3 z_3 - c_1(x_3 - \dot{x}_{r1}) + \ddot{x}_{r1}, \tag{23}$$
$$\bar{u}_y = - z_2 - \Phi_2^\top \hat{\Theta}_2 - c_4 z_4 - c_2 (x_4 - \dot{x}_{r2}) + \ddot{x}_{r2}, \tag{24}$$

and the parameter updating laws are chosen as

$$\dot{\hat{\Theta}}_1 = \Gamma_1 \Phi_1 z_3, \tag{25}$$
$$\dot{\hat{\Theta}}_2 = \Gamma_2 \Phi_2 z_4, \tag{26}$$
$$\dot{\hat{\rho}}_1 = -\gamma_1 \bar{u}_p z_3, \tag{27}$$
$$\dot{\hat{\rho}}_2 = -\gamma_2 \bar{u}_y z_4, \tag{28}$$

where c_3 and c_4 are positive constants, Γ_1 and Γ_2 are the adaption gain matrices and positive definite, γ_1 and γ_2 are positive constants, $\hat{\Theta}_1$, $\hat{\Theta}_2$, $\hat{\rho}_1$ and $\hat{\rho}_2$ are the estimates of Θ_1, Θ_2, $\rho_1 = \frac{1}{b_p}$ and $\rho_2 = \frac{1}{b_y}$. Let $\tilde{\Theta}_1 = \Theta_1 - \hat{\Theta}_1$, $\tilde{\Theta}_2 = \Theta_2 - \hat{\Theta}_2$, $\tilde{\rho}_1 = \rho_1 - \hat{\rho}_1$ and $\tilde{\rho}_2 = \rho_2 - \hat{\rho}_2$ be the parameter estimation errors. Note that using (21) and (22), $b_p u_p$ and $b_y u_y$ in (19) and (20) can be expressed as

$$b_p u_p = b_p \hat{\rho}_1 \bar{u}_p = \bar{u}_p - b_p \tilde{\rho}_1 \bar{u}_p \tag{29}$$
$$b_y u_y = b_y \hat{\rho}_2 \bar{u}_y = \bar{u}_y - b_y \tilde{\rho}_2 \bar{u}_y \tag{30}$$

We define the final Lyapunov function as

$$\begin{aligned} V_2 = & V_1 + \frac{1}{2} z_3^2 + \frac{1}{2} z_4^2 + \frac{b_p}{2\gamma_1} \rho_1^2 + \frac{b_y}{2\gamma_2} \rho_2^2 \\ & + \frac{1}{2} \tilde{\Theta}_1^\top \Gamma_1^{-1} \tilde{\Theta}_1 + \frac{1}{2} \tilde{\Theta}_2^\top \Gamma_2^{-1} \tilde{\Theta}_2 \end{aligned} \tag{31}$$

The derivative of (31) along with (19) to (30)

$$\begin{aligned} \dot{V}_2 = & - c_1 z_1^2 - c_2 z_2^2 - c_3 z_3^2 - c_4 z_4^2 \\ & + \Phi_1^T \tilde{\Theta}_1 z_3 + \Phi_2^T \tilde{\Theta}_2 z_4 - \tilde{\Theta}_1^T \Gamma_1^{-1} \dot{\hat{\Theta}}_1 - \tilde{\Theta}_2^T \Gamma_2^{-1} \dot{\hat{\Theta}}_2, \\ & - b_p \tilde{\rho}_1 \bar{u}_p z_3 - \tilde{\rho}_1 \frac{b_p}{\gamma_1} \dot{\hat{\rho}}_1 - b_y \tilde{\rho}_2 \bar{u}_y z_4 - \tilde{\rho}_2 \frac{b_y}{\gamma_2} \dot{\hat{\rho}}_2 \\ = & - c_1 z_1^2 - c_2 z_2^2 - c_3 z_3^2 - c_4 z_4^2 \\ & - \tilde{\Theta}_1^T \Gamma_1^{-1} (\dot{\hat{\Theta}}_1 - \Gamma_1 \Phi_1 z_3) - \tilde{\rho}_1 \frac{b_p}{\gamma_1} (\dot{\hat{\rho}}_1 + \gamma_1 \bar{u}_p z_3) \\ & - \tilde{\Theta}_2^T \Gamma_2^{-1} (\dot{\hat{\Theta}}_2 - \Gamma_2 \Phi_2 z_4) - \tilde{\rho}_2 \frac{b_y}{\gamma_2} (\dot{\hat{\rho}}_2 + \gamma_2 \bar{u}_y z_4), \end{aligned} \tag{32}$$

where the update laws (25)-(28) eliminate the last four terms in equation (32). Then

$$\dot{V}_2 = -c_1 z_1^2 - c_2 z_2^2 - c_3 z_3^2 - c_4 z_4^2. \tag{33}$$

We then have the following stability and performance results based on the control scheme.

Theorem 1: Considering the closed-loop adaptive system consisting of the plant (7), the adaptive controllers (21) and (22), the virtual control laws (15) and (16), the parameter updating laws (25)-(28) and Assumptions 1-2. All signals in the closed loop system are ensured to be globally bounded. Furthermore, the asymptotic tracking is achieved, i.e.

$$\lim_{t \to \infty} = [x_i(t) - x_{ri}(t)] = 0 \quad \text{for} \;\; i = 1, 2. \tag{34}$$

Proof: The stability properties of the equilibrium follow from Equations (31) and (33). By applying the LaSalle-Yoshizawa theorem, $V_2(t)$ is globally bounded. This implies that z_1, z_2, z_3, z_4 are bounded and are asymptotically stable and z_1, z_2, z_3, $z_4 \to 0$ as $t \to \infty$ and also $\hat{\Theta}_1$ and $\hat{\Theta}_2$ are bounded. Since $z_1 = x_1 - x_{r1}$ and $z_2 = x_2 - x_{r2}$, tracking of the reference signals is also achieved, and x_1 and x_2 are also bounded since z_1 and z_2 are bounded and since x_{r1} and x_{r2} are bounded by definition, cf. Assumption 1. The virtual controls α_1 and α_2 are also bounded from Equation (15) and (16) and then x_3 and x_4 are also bounded. From Equations (21) and (22) it follows that the control inputs also are bounded. ■

Now, considering the error state $\mathbf{z}$ including the tracking errors. Bounds for transient performance can be derived, and bound on the $\mathcal{L}_2$ norm will now be proven.

Theorem 2: The transient tracking errors performance are given by

$$||x_1(t) - x_{r1}(t)||_2 \leq \frac{1}{\sqrt{c_1}}\sqrt{V_2(0)}, \tag{35}$$

$$||x_2(t) - x_{r2}(t)||_2 \leq \frac{1}{\sqrt{c_2}}\sqrt{V_2(0)} \tag{36}$$

where initial values for $\mathbf{z}$ are set to $z_i(0) = 0,\ i = 1, 2, 3, 4$ and

$$\begin{aligned} V_2(0) =& \frac{1}{2}\tilde{\Theta}_1(0)^T\Gamma_1^{-1}\tilde{\Theta}_1(0) + \frac{1}{2}\tilde{\Theta}_2(0)^T\Gamma_2^{-1}\tilde{\Theta}_2(0) \\ &+ \frac{b_p}{2\gamma_1}\tilde{\rho}_1(0)^2 + \frac{b_u}{2\gamma_2}\tilde{\rho}_2(0)^2 \end{aligned} \tag{37}$$

and the transient velocity tracking errors are given by

$$||\dot{x}_1(t) - \dot{x}_{r1}(t)||_2 \leq \left(\frac{1}{\sqrt{c_3}} + \sqrt{c_1}\right)\sqrt{V_2(0)}, \tag{38}$$

$$||\dot{x}_2(t) - \dot{x}_{r2}(t)||_2 \leq \left(\frac{1}{\sqrt{c_4}} + \sqrt{c_2}\right)\sqrt{V_2(0)}. \tag{39}$$

Proof: The Lyapunov function V_2 is non increasing from (33) and bounded from below by zero, and then

$$||z_1||_2^2 = \int_0^\infty |z_1(\tau)|^2 d\tau \leq \frac{1}{c_1}V_2(0), \tag{40}$$

$$||z_2||_2^2 = \int_0^\infty |z_2(\tau)|^2 d\tau \leq \frac{1}{c_2}V_2(0), \tag{41}$$

$$||z_3||_2^2 = \int_0^\infty |z_3(\tau)|^2 d\tau \leq \frac{1}{c_3}V_2(0), \tag{42}$$

$$||z_4||_2^2 = \int_0^\infty |z_4(\tau)|^2 d\tau \leq \frac{1}{c_4}V_2(0). \tag{43}$$

Thus the inequalities (35) and (36) are achieved, where the bounds can be reduced by increasing c_1 and c_2 or by increasing the adaptation gains Γ_1, Γ_2, γ_1 and γ_2. For the velocity tracking errors we have

$$\begin{aligned} ||\dot{x}_1 - \dot{x}_{r1}||_2 &= ||\dot{z}_1||_2 = ||z_3 - c_1 z_1||_2 \leq ||z_3||_2 + c_1||z_1||_2 \\ &= \left(\frac{1}{\sqrt{c_3}} + \sqrt{c_1}\right)\sqrt{V_2(0)}, \end{aligned} \tag{44}$$

$$\begin{aligned} ||\dot{x}_2 - \dot{x}_{r2}||_2 &= ||\dot{z}_2||_2 = ||z_4 - c_2 z_2||_2 \leq ||z_4||_2 + c_2||z_2||_2 \\ &= \left(\frac{1}{\sqrt{c_4}} + \sqrt{c_2}\right)\sqrt{V_2(0)}, \end{aligned} \tag{45}$$

where (38) and (39) are achieved and the bounds depend on c_i, Γ_i and γ_i. The $\mathcal{L}_2$ transient performance of the $\mathbf{z}$ system can be improved by increasing the control parameters c_1, c_2 or by increasing the adaptation gains Γ_1, Γ_2, γ_1 and γ_2. We can see that increasing c_1 or c_2 also increase the velocity error of pitch or yaw. This suggests fixing the gain c_1 or c_2 to some acceptable value and adjust the other gains such as Γ_i and γ_i.

Remark 1: From Theorems 1 and 2 the following conclusions can be obtained:

- The $x_i(t) - x_{ri}(t)$ $(i = 1, 2)$ can be made smaller by increasing the design parameters c_i, Γ_i and γ_i .
- The transient performance depends on the initial estimate errors $\tilde{\Theta}_i(0)$, $\tilde{\rho}_i(0)$ and the explicit design parameters. The closer the initial estimates $\hat{\Theta}_i(0), \hat{\rho}_i(0)$ to the true values Θ_i, ρ_i, the better the transient performance.
- The bound of the angular tracking error $\parallel x_i(t) - x_{ri}(t) \parallel_2$ is an explicit function of design parameters and thus computable. We can decrease the effects of the initial error estimates on the transient performance by increasing the adaptation gains Γ_i and γ_i.
- To improve the angular tracking error performance we can also increase the gain c_1 or c_2. However, increasing c_1 or c_2 will influence the velocity tracking performance such as $\parallel \dot{x}_i - \dot{x}_{ri} \parallel_2$.

IV. Experimental Results

The proposed controller was simulated and tested on Quanser Aero using MATLAB/Simulink. A fixed sample time of $0.002\ s$ was used, and the initial condition for angles and angular velocities were set to zero. The values for the constants c_1, c_2, c_3 and c_4 and for the adaption gain Γ_i, γ_i were found by trial and error.

A. Test 1- Track a Sinusoid Signal

The first test was a sine wave with amplitude of 40 degrees and frequency of 0.05 Hz applied to pitch, while there should be no rotation about yaw.
The results from simulation and testing on the Aero with the adaptive controller is shown in Figures 3 and 4, where blue is the real time experiment result and red is the simulation result. Figure 3 shows that the absolute error was ≤ 2 deg for pitch angle and ≤ 0.9 deg for yaw angle. The maximum error occurred when the input voltages changed the sign, seen in Figure 4, meaning that the rotors changed direction. Both simulation and testing on the Aero show that the desired trajectory for a sine wave in pitch can be followed using the proposed adaptive controller. The input voltages were similar for both simulation and testing as shown in Figure 4.

B. Test 2- Disturbance of Added Mass

1) Negative Torque: A mass of 5.8 grams in form of a washer was added to the main thruster, a distance of 23.8 cm from the pivot point. This gave a negative torque to the Aero and a new equilibrium point, moved from zero degrees to a negative pitch angle of 24.5 deg. The results from this test are shown in Figures 5 to 7 for both the LQR and for the adaptive backstepping controller. Both controllers had a maximal error that was less than 1 degree for yaw. Both followed the sine wave, and both had a maximal error of approximately 4 deg. The LQR had the biggest error when the pitch trajectory reached max and min values, while the adaptive controller had the biggest error around where the Aero had its new equilibrium point, a pitch angle of -24.5 deg. This was also when the rotors changed from a positive to a negative voltage.

2) Positive Torque: Now the mass was added to the tail thruster at the same distance as in the previous test, giving a positive torque to the Aero. The results are shown in Figures 5 and 7. The results for this disturbance was similar to the one

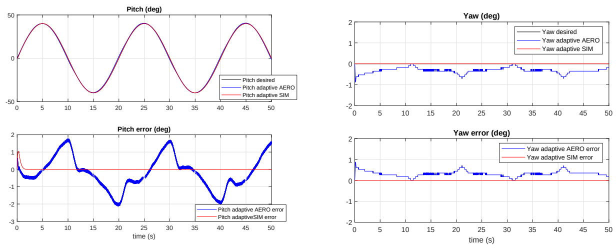

Fig. 3. Left: Pitch angle and error for test 1; right: Yaw angle and error for test 1

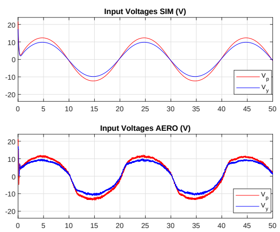

Fig. 4. Voltage for test 1 from simulation and testing on the Aero

with a negative torque, with the main difference of changed sign for errors. From the test, the LQR had same behavior as before, with largest error for max and min points on the pitch trajectory, while the adaptive controller had largest error when the Aero had a pitch angle of 24.5 deg, which was at the new equilibrium point of the Aero in pitch.

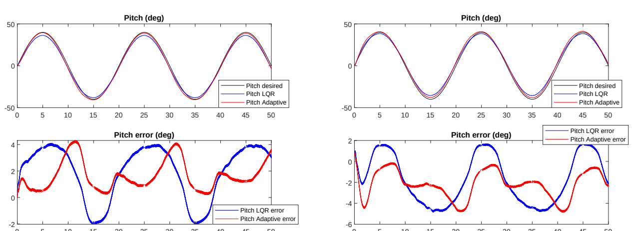

Fig. 5. Pitch angle and error for test 2 with disturbance. Left: negative torque; right: positive torque

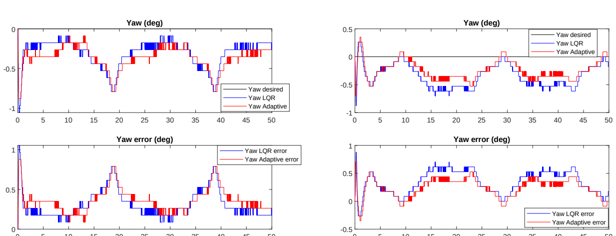

Fig. 6. Yaw angle and error for test 2 with disturbance. Left: negative torque; right: positive torque

C. Test 3- Disturbance of Changing Propeller

The tail propeller was changed from a low- to high efficiency propeller shown in Figure 8, meaning that the main

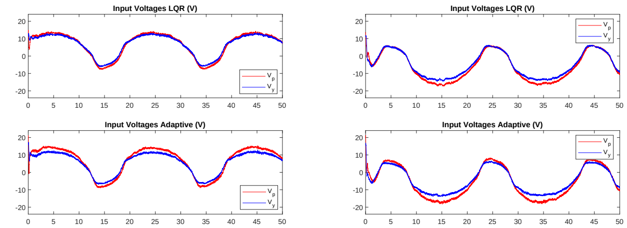

Fig. 7. Voltage for test 2 with disturbance. Left: negative torque; right: positive torque

and cross-torque gains produced by the input V_y were changed. Because this new propeller had a lower weight, a mass was also included to the tail thruster so that the Aero retained a horizontal position when at rest. This disturbance illustrate actuator damage where the dynamics are changed. Results from testing are shown in Figures 9 and 10. Once again the LQR had biggest error when the pitch trajectory reached maximum and minimum values, with a maximal error of 4 deg, a little higher than without added disturbance. The adaptive controller had biggest error at the equilibrium point with maximal error of 2 deg, just as without the disturbance. From Figure 10 one can see that a higher voltage was needed to reach the highest points on the curve, where both input voltages had increased for both controllers, but input V_y had changed most.

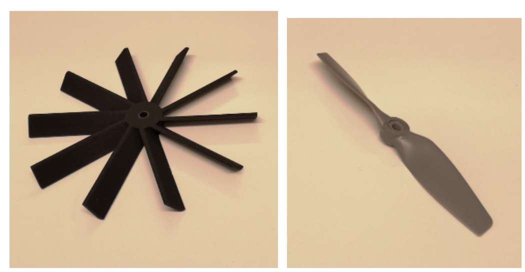

Fig. 8. Propellers: Left-low efficiency, Right-high efficiency

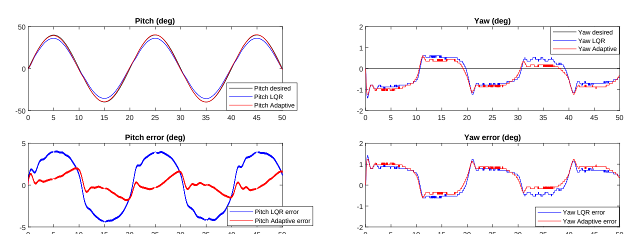

Fig. 9. Left: Pitch angle and error for test 3; right: Yaw angle and error for test 3

D. Results

The LQR and the adaptive backstepping controller were compared with a measurement of tracking error and of the total voltage used. The more accurate the controller is, meaning the error is smaller, the more voltage is needed to hold the

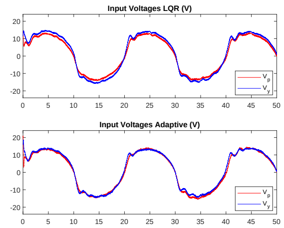

Fig. 10. Voltage for test 3 with disturbance of changing tail propeller

trajectory closer to the reference and so there is a trade-off between these two. The measurement of the total error is

$$\|\mathbf{z}_\lambda\|^2 = \int_0^t |\mathbf{z}_\lambda(\tau)|^2 d\tau, \tag{46}$$

where $\lambda = [\theta, \psi]$, and the measurement of the total voltage, i.e. input to the system, is

$$\|\mathbf{v}\|^2 = \int_0^t |\mathbf{v}(\tau)|^2 d\tau, \tag{47}$$

where $\mathbf{v} = [V_p, V_y]$. Table I shows the total tracking errors from equation (46) and table II shows the measurement of the total voltage from equation (47). The results show that the proposed adaptive backstepping controller had a lower tracking error than the LQR for all tests, but used a little more voltage than the LQR. Note that the LQR needs a linearized model with fully knowledge of system parameters, while the proposed nonlinear adaptive controller can be used for uncertain nonlinear system without knowledge of the system parameters. Therefore the results validate the effectiveness, robustness and control capability of the proposed nonlinear adaptive scheme.

V. Conclusion

In this paper, an adaptive backstepping controller is developed for a 2-DOF helicopter to track the pitch and yaw angles independently. A mathematical model of the 2-DOF helicopter is obtained from its Euler-Lagrange equations. The system parameters are not required to be fully known for the controller design. A theoretical proof of stability with adaptive backstepping is given with the use of constructed Lyapunov functions, where asymptotic tracking and the boundedness of all signals in the closed loop system are achieved. Also, the transient performance for the tracking errors in terms of $\mathcal{L}_2$ norm is derived, where the tracking error performance can be improved by adjusting the design parameters. Simulations and experiments are conducted on a Quanser helicopter called Aero. The robustness of the proposed controller is evaluated by adding disturbances to the system, including fault in one propeller and added external torques. Simulation and experimental results validate the effectiveness, robustness and control capability of the proposed scheme.

TABLE I
Comparison of controllers via total tracking error

Total tracking error					
Test	Controller	No dist	Dist, added mass -	Dist, added mass +	Dist, changed tail propeller
Sine	LQR	0.0938	0.1228	0.1245	0.1625
Sine	Adaptive	0.0153	0.0616	0.1001	0.0195

TABLE II
Comparion of controllers via total voltage

Total voltage					
Test	Controller	No dist	Dist, added mass -	Dist, added mass +	Dist, changed tail propeller
Sine	LQR	6572	8953	8623	11666
Sine	Adaptive	7244	9107	9028	11848

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