Has the Introduction of Bitcoin Futures on Regulated Exchanges Decreased Price Volatility?

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Preface

This thesis marks the end of our five-year Business Administration Master´s programme with specialization in Financial Economics. It has been an educational journey and we would like to express our gratitude towards those who have helped us along the way. Despite hours of hard work, trouble with software and numerous other challenges, we are honoured to present this thesis.

We would like to express our deepest appreciation to our supervisor, Professor Jochen Jungeilges. He provided the proposition for this thesis and also a theoretical foundation through his courses in econometrics. We took on the challenge of solving many of the problems ourselves. However, this would not have been possible without the econometric framework, toolbox and support provided by our supervisor.

Finally, we would like to extend gratitude to our family and friends for the guidance and support we have received.

Abstract

Bitcoin is a tremendously debated phenomenon in the world of finance and in recent the scientific literature on the topic has expanded. In this thesis, the bitcoin to US dollar exchange rate is examined through various conditional variance models to describe its highly volatile nature. We examine whether the introduction of bitcoin futures contracts in late 2017 has had a decreasing impact on price volatility by estimating the unconditional variance.

The log-return of the bitcoin exchange rate is analysed, and there is evidence of volatility clustering and time-varying volatility. Consequently, the variance is modelled through the GARCH(1,1), EGARCH(1,1) and GJR-GARCH(1,1) models with innovations following three distributions. The in-sample selection method selected the EGARCH(1,1) model where innovation terms follow a generalized error distribution as the most parsimonious model.

The findings show that volatility has not decreased after the introduction of bitcoin futures on regulated exchanges.

Keywords: Bitcoin, conditional variance modelling, bitcoin futures, price volatility exchange rate, statistical analysis

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1. Introduction

The introduction of bitcoin futures contracts on the Chicago Mercantile Options Exchange and Chicago Mercantile Exchange in December 2017 marked an important step in bitcoin history. For the first time, it was possible to trade bitcoin futures contracts on regulated exchanges. The highly volatile nature of bitcoin is one argument against it being adopted as a currency. This is because high price volatility negatively affects bitcoin´s ability to function as a medium of exchange and a store of value. Due to this, some argue that bitcoin rather should be classified as a speculative asset.

There exists financial research stating that the introduction of futures contracts has had a volatility-decreasing effect on underlying spot markets for stocks and currencies. Consequently, some bitcoin advocates hoped that the introduction of bitcoin futures could help stabilize the price volatility, and in turn open up for widespread acceptance of bitcoin as a currency. The extreme price fluctuations have possibly scared potential users and overshadowed the ideas underlying bitcoin. A decrease in price volatility might help uncover some of the intentional functions of the Bitcoin network such as lower transaction fees, peer-to-peer transfers and independence from central entities.

Bitcoin is a revolutionary concept in the world of finance, but there is limited research regarding the impact of bitcoin futures. The ability to trade bitcoin futures contracts allows participants to hedge against risk and to speculate on price fluctuations. Furthermore, it brings liquidity to the market and opens up the investment space to new participants. The purpose of this thesis is to by examine the impact of bitcoin futures on price volatility. The ability to trade bitcoin futures occurred fairly recently. We believe that incorporating a longer time period after the introduction, compared to similar studies, can better capture the possible effects of bitcoin futures. In order to examine this relationship, we will model the volatility through different conditional variance models. These conditional variance models are popular in finance as they are helpful in capturing some of the characteristics financial returns often display. In the process of examining the impact of bitcoin futures, we select the model that best explain the volatility. The variable studied is the log-returns of the bitcoin to U.S. Dollar exchange rate.

The conditional variance models used to model volatility in this thesis are the GARCH model, EGARCH model and the GJR-GARCH model. In order to estimate these models it is necessary to determine the distribution of innovation terms. This is a difficult challenge in statistical modelling, and especially considering the behaviour of the exchange rate of bitcoin. The fitting of the optimal distribution is not the focus of this thesis. However, we acknowledge that the heavy-tailed distribution of the log-returns might require a different distribution of innovation terms than the Gaussian distribution. Consequently, we model the different conditional variance models under the assumption that the innovation terms follow a student's t distribution and generalized error distribution in addition to the gaussian distribution. These are the three different distributions that STATA allows us to choose.

These nonlinear conditional variance models mentioned above are rooted in the autoregressive conditional heteroskedasticity model (ARCH). The ARCH model was developed to capture volatility clustering and time-varying volatility characteristics in financial returns. To test that the bitcoin log-returns fulfil these prerequisites, the LM-test for ARCH effects and the BDS-test for nonlinearity is applied. The results from these tests deem the conditional variance models as appropriate in modelling the variance. These procedures lay the theoretical basis for this thesis.

We provide a brief insight to the technology behind bitcoin and its revolutionary aspects to give the reader a better understanding of its behaviour and the controversy surrounding Bitcoin. The first part of the thesis consists of an introduction to currencies and highlights the differences between Bitcoin and traditional fiat currencies. In addition, we also provide information about the Bitcoin payment system and distributed ledger. These are two of the revolutionary features praised by advocates and condemned by critics. From here a short summary of bitcoin history and an overview of bitcoin futures contracts is given.

The goal of the thesis is to select the best conditional variance model and estimate the unconditional variance as a measure of realized volatility in the period before the introduction of futures and after. In order to do this, we will use in-sample selection techniques. We will also address the adequacy of the preferred models by applying different model diagnostic tools.

The thesis consists of seven chapters. In the second chapter an introduction to the bitcoin phenomenon is given as a starting point. In the third chapter, we examine literature surrounding the introduction of futures contracts for stock indexes, single-stocks and commodities. We will also discuss one research effort that has examined the relationship between bitcoin futures and volatility. In the following chapter, information about the data used is given. The justification of using a bitcoin price index consisting of the average price from multiple exchanges above a single-exchange price is also provided. Plots of the exchange rate and log-returns are given alongside a table of statistical properties of the log-returns. In chapter 5, the preliminary tests for ARCH effects and nonlinearity, the conditional variance models and model diagnostic tools are explained.

As the software used in this thesis, STATA, does not provide a point estimate for unconditional variance when modelling the conditional variance, the procedure of retrieving this estimate is given in chapter 6. We introduce a dummy variable that alters the form of the different conditional variance equations for the models. In chapter 7, we present the results from the preliminary tests regarding ARCH effects, nonlinearity and stationarity properties. Then the estimated results from the nine different conditional variance models and model diagnostic tools are presented and commented on. The in-sample information criteria results are included in these results. Finally the summary is given alongside challenges we encountered and proposals to future research within this field.

2. The Bitcoin Phenomenon

2.1 Introduction to Currencies

The role of a currency is to serve as a store of value, a medium of exchange and a unit of account (Ali, Barrdear, Clews, Southgate, 2014). Until the second half of the 20th century, most of the world's currencies were tied to physical commodities such as gold, silver and deer skin as guarantees for the holder. These tangible currencies had intrinsic value derived from the scarcity and non-monetary utility of the physical commodities. However, most currencies in the 21st century are fiat currencies. Fiat currencies are not tied to physical commodities and have no intrinsic value. The value of a fiat currency is determined by supply and demand and relies on users´ trust to the system. Fiat currencies are issued and backed by governments and this paves the way for governmental control regarding economic factors such as interest rates, credit supply and liquidity (Chen,2019).

Bitcoin is a type of digital currency called cryptocurrency due to its use of cryptography. Cryptography is a collection of methods consisting of technological and mathematical techniques used to hide information in communication. Digital currencies differ from fiat currencies in that they are only available in digital form and not physical form. Satoshi Nakamoto introduced the concept of Bitcoin in 2008 with the purpose of addressing some of the weaknesses of traditional fiat currencies (Rothstein,2017, p.3).

2.2 Bitcoin Payment System and Distributed Ledger

One aspect that sets Bitcoin apart from fiat currencies is that it is neither controlled nor issued by any government or bank. In contrast, bitcoin is created by a decentralized network. At this stage it is worth clarifying the difference between the uses of capital “B” and lower case “b”. “Bitcoin” with a capital “B” refers to the whole payment network or protocol. On the other side, “bitcoin” with a lower case “b” refers to the currency (Blockchain Team, 2014).

In order for an asset to operate as a medium of exchange, there needs to be a functioning payment system that is able to transfer values (Ali, Barrdear, Clews, Southgate, 2014). In traditional money markets, intermediaries such as banks, PayPal etc. are central for the transfer of value. Most of the money in the world is recorded electronically in bank deposits. A transfer between two accounts in the same bank can be done internally. A transfer of money between two accounts in different banks is made by transferring claims on a central entity, usually a central bank (Ali, et al. 2014). The payment system related to Bitcoin is fundamentally different. It allows users to transfer value directly without the use of an intermediary.

For a payment system to be trusted there needs to be a record of stored value - this is called a ledger. The role of a ledger is to avoid the double spending problem. In the physical transfer of money, the double spend problem is absent. If an individual only has $1, it is not possible to give that $1 to two different individuals. On the other hand, electronic payment systems need to deal with the double spend problem as digital records can be edited or copied. To address this, banks keep control of customers´ accounts on their own ledgers. Furthermore they also keep accounts in the central entity that again has its own ledger. The ledgers allow the controllers to keep a record of individual´s money and block transactions they deem non-valid. For such a system to work, the users need to trust that the banks don't tamper with the ledgers. Bitcoin uses a distributed ledger where users agree on transactions and all transactions are available for everyone to see. This concept requires no intermediary to ensure that that the ledger is reliable, as each and every user of the Bitcoin network has access to the ledger. Bitcoin removes the need to trust a single entity; rather trust is put in the entire network.

2.3 The Process of a Successful bitcoin Transaction

The creator of Bitcoin, who is only known under the pseudonym Satoshi Nakamoto, defines bitcoin as a chain of digital signatures (Nakamoto,2008, p.1). During a bitcoin transaction information consisting of the sender's digital signature, the receiver's digital signature and the amount of bitcoin to be transferred is sent to the blockchain. The blockchain checks that the digital signatures are correct and that the sender has a sufficient amount of bitcoin. If these aspects are fulfilled, the blockchain verifies the transaction and updates the record.

The fundamental technology underlying bitcoin is the aforementioned blockchain. The blockchain enforces the security of bitcoin as it removes the possibility of counterfeiting. It does so by keeping a record (ledger) of all bitcoins in existence and the owners of these. The blockchain updates the ledger when transactions have been completed. Thus one cannot replicate bitcoin as they would need to be accounted for in the ledger. If one tried to spend one of these replicated coins, checking the blockchain would quickly expose the perpetrator.

The blockchain is constantly being updated through a verification process called mining. The objective of the verification process is twofold. Firstly, miners validate that transactions are correct by matching digital signatures and secondly they try to achieve consensus by presenting a proof of work. The validation step is completed rather quickly whilst the proof of work is a more complex process that involves the use of cryptography to show that it has taken computing time to accomplish. The reason for why establishing consensus is difficult is to motivate a trustworthy behaviour. If rent-seeking individuals wished to verify fraudulent transactions they would need powerful computing resources. To be specific, those individuals or groups would need more than 51% of the total processing power in the network. However, such actions would lead to distrust in the system and to the decline of bitcoin value. Instead they are given incentives to verify only correct transactions and given bitcoins as reward. Transactions are gathered by miners in blocks and once they have achieved consensus from miners, the blocks are added to the blockchain. After the blockchain has been updated, information is stored and available for all participants to check. Consequently, this process ensures that there is no need for a central authority to verify transactions and keep and updated ledger.

A version of the updated blockchain is sent to all computers connected to the bitcoin network called nodes. Each block in the blockchain contains information about transactions that are connected to transactions in previous blocks. Consequently, there is an information dependence that makes it very difficult to alter, change or remove transactions once updated.

In order to store bitcoin, one needs a bitcoin wallet that in fact is a software program that stores the user's private keys. A private key is a complex sequence of numbers and letters constructed by cryptography that allows the user to create a unique digital signature and consequently sign transactions. Due to the uniqueness of every signature, it is impossible to alter or change a transaction once signed and broadcasted to the bitcoin network. The private key needs to be kept secure because a loss of it will block the ability to spend, transfer and withdraw bitcoins from the wallet (Frankenfield, 2018).

Based on the private key, a complicated algorithm is used to develop the public key. When a bitcoin user wishes to make a transaction, it signs the transaction with the help of the private key. This private key makes sure that the signature is unique. However, the information regarding the private key is not known to any other than the user itself. In order to prove that digital signature in fact came from the user’s private key, the public key is applied. This can be done as the public key is derived from the user's private key. The loss of a public key is not as severe compared to the loss of a private key. This is because a new public key can be derived from the private key and allow the user to proceed as earlier (Frankenfield, 2018) Once the transaction is deemed valid by the network, the bitcoin is sent to the receiver's public address. The public address is a compressed form of the public key and can be viewed as the receiver's bank account. The receiver is then able to use the private key to transfer, spend or withdraw the bitcoin stored at the public address.

2.4 Bitcoin History

In October 2008, Satoshi Nakamoto outlined publicly for the first time the intentions and ideas behind Bitcoin in the white paper “Bitcoin: A Peer-to-Peer Electronic Cash System” (Nakamoto,2008, p.1). Shortly, the very first bitcoin software was released in February 2009. A peer-to-peer system in its simplest form is the sharing of resources or files between two computers without the need of a separate server. Nakamoto computed the initial coding for the network, but also collaborated with developers by responding to questions and fixing problems. In December 2010, a period of inactivity from the creator began as the maintenance of the network had been taken over by others. It is still unknown whether Satoshi Nakamoto is one individual or a collection of individuals, as the creator(s) has remained anonymous.

One part of the motivation behind the concept of bitcoin is to allow for irreversible transactions. Traditional payment systems involving financial institutions cannot allow irreversible transactions as they might be involved in disputes between two parts. As a result of this, Nakamoto argued that transaction costs are higher than necessary and that this limits the possibility of small transactions. Furthermore, reversible transactions increase the need of trust between parties involved and consequently the need of personal information. Nakamoto proposed a solution to this by implementing a peer-to-peer payment system allowing irreversible transactions without the need of a third party.

After the initial release of the software one could only acquire bitcoins through mining. In July 2010 the very first Bitcoin exchange was launched, Mt. Gox. The exchange was based in Tokyo, Japan and quickly grew to become the leading Bitcoin exchange in the world. At its peak Mt. Gox was handling over 70% of the world’s bitcoin transactions (Ogun, 2015, p.47).

The Tokyo based exchange was shut down in 2014. A hole in the security of Mt. Gox was discovered and exploited by hackers, and it is estimated that 744,000 bitcoins with a value of $350 million were stolen during the breach. At the time this was 6% of the 12,4 million bitcoins in circulation. This is one of many examples of criminal activity connected to bitcoin. The extreme price fluctuations in the bitcoin exchange rate is powered by different factors. Bad news stories such as governmental inference and bankruptcy of exchanges like Mt. Gox and Yapian caused panic and drove down the value of bitcoin. Numerous hacker attacks on large exchanges has also instilled fear in the bitcoin environment.

The perceived value of bitcoin differs, and some argue that it holds the same hedging opportunities as gold. This means that the demand of bitcoin can increase when there is uncertainty in the market, causing the price to rise. These are some of the factors driving the high price volatility of bitcoin.

The blockchain and cryptography technology surrounding cryptocurrencies and Bitcoin is continually growing, and it is now becoming part of the medium-term strategy for global financial institutions (Castillo, 2018). Bitcoin has managed to catch the attention of the mainstream media, and one of the hot topics has been the Bitcoin price. In 2017, an exponential growth in the bitcoin price was seen when it reached its peak on December 17th, with a price of $19,783.21 per bitcoin.

Due to the popularity of bitcoin and the interest amongst investors, bitcoin derivatives have been launched. The Commodity Futures Trading Commission ruled in favor of accepting Bitcoin options trading in October 2017 (Hankin, 2017). In December 2017 both the Chicago Board of Options Exchange and the Chicago Mercantile Exchange launched bitcoin futures (CME Group, 2017). Although contrary to common belief, the possibility to buy and sell bitcoin derivatives was already present at unregulated exchanges such as BitMex and Bittrex. However, the introduction of bitcoin futures contracts on CBOE and CME marked a significant breakthrough, as these were the first regulated exchanges to offer such products. Regulatory exchanges have to adhere to rules and guidelines set by the Financial Conduct Authority (FCA), which is a regulatory body in the United Kingdom. This is an attractive feature that makes them popular amongst institutional investors.

2.5 Futures Contract

"A futures contract is a legal agreement to buy or sell a particular commodity or asset at a predetermined price at a specified time in the future" (Chen, 2019).
There is a long position and short position in every futures contract. The long side is obligated to buy the asset at the expiration date and the short side is obligated to provide the asset at that same date. Futures contracts are standardized in terms of quantity and quality. Companies might use futures contracts to hedge against risk and be able to plan ahead.
A gold mining company could be interested in selling futures contracts. If the company knows it will have a certain amount of gold in the future, futures contracts allow them to

know for certain at what price they can sell the gold. Speculators can use futures contracts to bet on future price movement and possibly make a profit.
In the bitcoin marketplace, the hedgers can be bitcoin mining companies or other users that know they will have a certain amount of bitcoin in the future. Likewise, there will also be speculators in the bitcoin universe trying to forecast price movements and profit from this.

Unlike the bitcoin futures offered on unregulated cryptocurrency exchanges, CBOE and CME offer hedgers and speculators more security and protection. Moreover, the bitcoin futures contracts offered at both exchanges are settled in cash. At the expiration date, the seller transfers the associated cash position (Chen, 2019). The result is that the buyers and sellers actually do not need to own bitcoin at any time in the process. This enables participants in countries where bitcoin trading is banned, to take part.

Table 1. Bitcoin futures overview

Variable	**CBOE Futures**	**CME Futures**
Product Code	XBT	BTC
First Traded	10th of December 2017	18th of December 2017
Contract Unit	1 bitcoin	5 bitcoins
Position Limits	5000 Net long or short contracts	1000 Contracts
Price Limits	Not subject to price limits	7% above and below settlement price, +/-13% previous settlement, +/- 20% for prior settlement
Minimum Price Fluctuation	10$ Per contract	25$ Per contract
Settlement	The Final Settlement Value of an expiring XBT futures contract shall be the official auction price for Bitcoin in U.S. dollars determined at 4:00 p.m. Eastern Time on the Final Settlement Date by the Gemini Exchange Auction.	Cash settlement with reference to the Final Settlement Price, equal to the CME CF Bitcoin Reference Rate on the last day of trading.

(CME,2017), (CBOE,2017).

The monthly trading volume of bitcoin futures has declined on the Chicago Board Options Exchange (CBOE) from the introduction in late December of 2017 as presented in figure 1. On the other side, the volume of bitcoin futures has increased on the Chicago Mercantile Exchange (CME). It is also worth mentioning that each future contract unit on the CME is five bitcoins compared to the CBOE futures contract unit of one bitcoin. Consequently, the volume traded in dollars is significantly higher on CME especially in the second half of the time period.

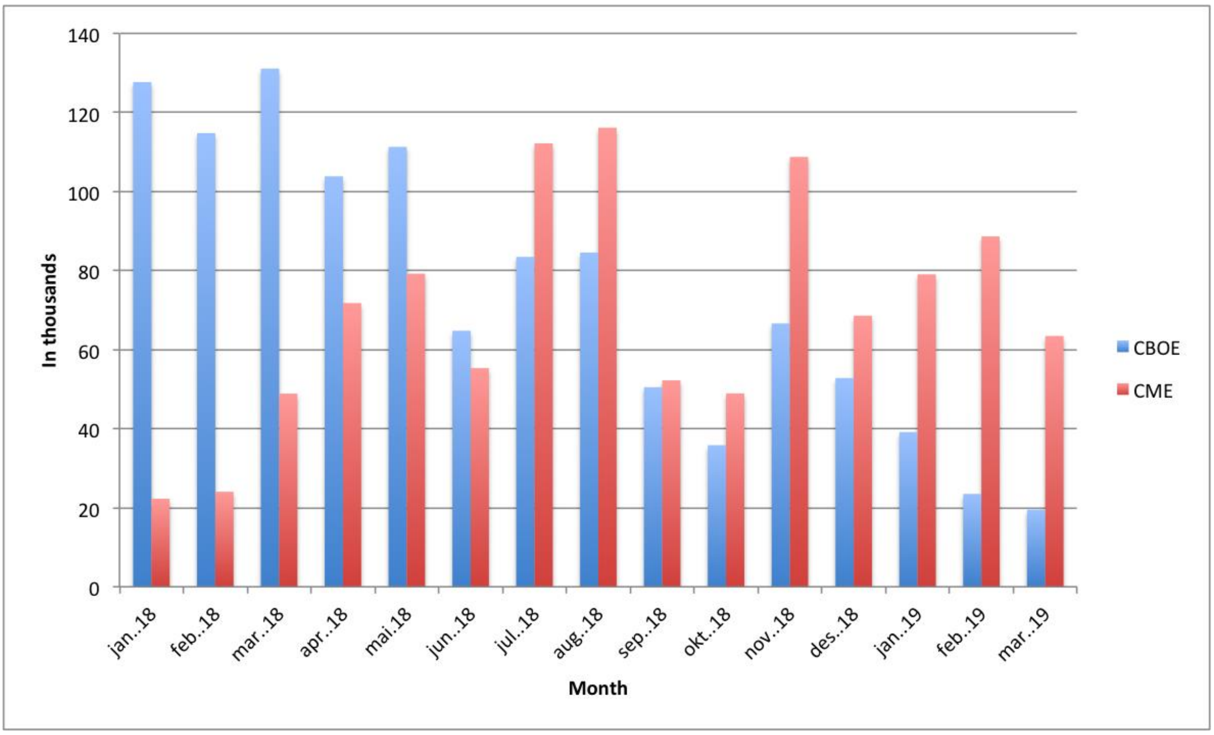

Figure 1. Monthly traded volume of bitcoin futures on CBOE and CME from January 2018 to March 2018

https://www.investing.com/crypto/bitcoin/cboe-bitcoin-futures-historical-data

https://www.investing.com/crypto/bitcoin/bitcoin-futures-historical-data

3. Literature Review

Research addressing the introduction of futures contracts in the bitcoin environment has been sparse. However, we will present research that has contributed to this specific part of the literature. To the best of our knowledge, the following two research papers are the only studies addressing the impact of bitcoin futures on price volatility.

In the research article “The Impact of Futures Trading on Intraday Spot Volatility and Liquidity: Evidence from Bitcoin Market” (Shi, 2017), Shi claims to conduct the first empirical analysis surrounding the impact of bitcoin futures on the cash market. The researcher uses high-frequency log-returns data with 5-min intervals and estimates an EGARCH (1,1) model where innovation terms follow a generalized error distribution. The result presented is that there is a statistically significant decrease in realised spot volatility after the introduction of bitcoin futures. The researcher applies the Ljung-Box and McLeod-Li test to assess the adequacy of the model. There is one possible drawback with the time period studied in the research article. The time period spans from the 3rd of December 2017 to the 17th of December 2017. This implies that only seven days before and after the introduction of bitcoin futures is included in the time period. According to CNBC, the traded volume of futures was very low in the days after the introduction at the 10th of December 2017 (Cheng, 2017). Consequently, it is difficult to say if the full effect of the futures was present in the post-period.

The research article of Corbet, Lucey, Peat and Vigne examines the impact of bitcoin futures on price volatility after the introduction on Chicago Board of Options Exchange and Chicago Mercantile Exchange. The time period studied spans from 26th of September 2017 to 22nd of February 2018 and includes two subsamples (pre- and post-introduction). The researchers apply an ARMA (1,1) - GARCH (1,1) model to examine change in volatility and find that spot volatility significantly increased during the announcement of bitcoin futures and in the time that followed. The method of change point detection is used to assess a change in volatility. Specifically, the Mood statistic for changes in volatility (scale) is estimated from the raw returns Mood statistic (Corbet, Lucey, Peat, Vigne, 2018).

The existing literature of the impact of futures contracts in relation to the underlying spot markets of stocks and currencies is substantially larger compared to that of bitcoin futures. As bitcoin has been classified as both an asset and a currency, we will discuss two studies addressing the impact of futures on these markets.

According to Yermack, bitcoin fails to function as a medium of exchange, a store of value and a unit of account. The researcher states that bitcoin behaves more like an investment (Yermack, 2015). Stocks are equity investments and the study conducted by Danielsen, Van

Ness and Warr examined the impact of single-stock futures on short selling and trading activity of US securities. The futures were listed on the OneChicago LLC exchange which is a joint venture between CBOE and CME, and NQLX exchange which is joint venture between NASDAQ and LIFFE. The time period spans from the introduction of futures in November 2002 to the end of 2003. The introduction of single-stock futures allows investors to take leveraged short and long positions at a lower cost than in the spot market. Interestingly, the researchers found that single-stock futures markets significantly reduced intra-daily volatility measured by the standard deviation of quote midpoint. (Danielsen, Van Ness & Warr, 2008). In addition, the researchers also claim that trading activity, measured in transaction volume and size, was reduced.

Although some classify bitcoin as a speculative asset, it is technically defined as a cryptocurrency. The study by Kumar, “Impact of Currency Futures on Volatility in Exchange Rate: A Study of Indian Currency Market”, examined the impact of the introduction of currency futures on the euro to rupee exchange rate volatility. The variable studied is the daily exchange rate values from the 1st of January 2006 to the 30th of September 2014. The log-returns of the exchange rate is computed and tested for stationarity with the Augmented Dickey-Fuller test. In addition, the log-returns are also tested for ARCH effects by applying the ARCH-LM test. These are of the preliminary tests that will be applied to the log-returns of the bitcoin exchange in our thesis. From here, the volatility is modelled by a GARCH (1,1) model and a GJR-GARCH (1,1) and there is evidence of news impacting the volatility asymmetrically. The t-test, Welch t-test, f-test and Welch F-test provides evidence that the volatility has significantly decreased in the period after the introduction of futures for EUR/INR exchange rate (Kumar, 2015).

We believe that our thesis will contribute to the literature surrounding the impact of bitcoin futures on price volatility. One reason is that we include a longer time period after the introductions of the futures. This will arguably allow the impact of the futures more time to come into effect compared to previous studies of bitcoin futures (Shi, 2017) (Corbet. et al., 2018). As shown by figure 1, traded volume of bitcoin futures was low in the start but experienced an increase a couple of months in. We will also apply a different framework in modelling the volatility by estimating additional conditional variance models and allow the innovation terms to follow different distributions compared to previous studies. As we will see in the next section, the behaviour of the log-returns of the exchange rate might require a different distribution than Gaussian distribution.

The literature discussed above has given us valuable insight in the theory of volatility estimation. Although the change in volatility is assessed differently in the studies mentioned, most use conditional variance models in the process of volatility estimation. These models will also be included in the framework of our thesis and we hope that our results can expand the literature regarding bitcoin futures.

4. The Data

4.1 Bitcoin Price Index (XBP)

The data used in this thesis is the Bitcoin Price Index (ticker: XBP), which is the bitcoin exchange rate to US Dollars, gathered from www.coindesk.com. The dataset was transformed and downloaded as an excel file before it was imported into Stata 15.0. It is the American company Digital Currency Group (DCG) that owns and operates the website www.coindesk.com after acquiring it from its original founder Shakil Kahn in 2016. The Bitcoin Price Index was launched in 2013 and its price consists of average prices from different worldwide leading bitcoin exchanges. To be included in the index, the exchanges have to meet certain criteria:

- USD exchanges must serve an international customer base.
- Exchange must provide a bid-offer spread for an immediate sale (offer) and immediate purchase (bid)
- Minimum trade size must be less than 1,500 USD (9,000 CNY) or equivalent.
- Daily trading volume must meet minimum acceptable levels as determined by CoinDesk.
- Exchanges must represent at least 5% of the total 30-day cumulative volume for all the exchanges included in XBP.
- The stated and/or actual time for a majority of fiat currency and bitcoin transfers (whether deposits or transfers) must not exceed two business days.

The Bitcoin Price Index is an average of four leading bitcoin to US Dollar (USD) exchanges and one bitcoin to Chinese Yuan exchange (CNY). The exchanges included are: Bitstamp (USD), Coinbase (USD), itBit (USD), Bitfinex (USD) and OKCoin (CNY). Its value is the midpoint of the bid/ask spread. The index is updated every 60 seconds and operated 24 hours a day. A simple average of the exchanges is applied instead of volume weighting. Not all international exchanges are accessible for different national trading participants. Consequently, the simple average property ensures that the index is meaningful for global participants and does not favour a particular regional exchange with high trading volume.

Liquidity and infrastructure issues related to different exchanges make arbitrage opportunities difficult to exploit. The result is that the price of one bitcoin can differ from one exchange to another. This is an argument for why the Bitcoin Price Index is applied in this study as it incorporates an average of several leading bitcoin exchanges. Moreover, the index has been quoted in several prestigious newspapers such as The New York Times, Wall Street Journal, CNBC and Fortune. It also covers the period before and after the introduction of bitcoin futures well.

The dataset consists of daily bitcoin closing prices denominated in US dollars from 1st of October 2013 to 25th of March 2019. There is in total 1,993 observations but 10 missing observations: 13th July 2018, 20th September 2018, 6th of June – 10th of June, 6th of July 2018, 29th December 2018 and 30th December 2018. The reason for why these observations are missing is unclear. In addition, two days are listed twice: 31st of May 2018 and 1st of June 2018. The price listed on these dates at 00:00 has been included instead of the price listed at 04:00.

4.2 The Exchange Rate and Log-returns

The studied variable is the daily closing value of the BTC/USD exchange rate.

Figure 2 BTC/USD daily closing price

Figure 5 shows the daily exchange rate from the start of the Bitcoin Price Index at the 1st of October 2013 to the 25th of March 2019 that marks the end of the time period studied. Due to the exponential growth in late 2017 it is difficult to use this graph to comment on price movements. Consequently, the time period is divided into four parts:

Figure 4 XBP/USD 01.10.2013-09.02.2015

Figure 3 XBP/USD 10.02.2015-21.06.2016

Figure 5 XBP/USD 22.06.2016-01.11.2017

Figure 6 XBP/USD 02.11.2017-25.03.2019

The Bitcoin Price Index was established in October 2013 and the first recorded closing price of one Bitcoin was $123. The price continued its increase until 5th of December 2013 when it peaked at around $1,140. Fast forward a year and the price of one Bitcoin had declined to $193 (15th of January 2015). It was not until February 2017 the daily closing price would surpass the $1000 level last seen in 2013. During the year of 2017 the price increased to a daily closing price of $19,166 (17th of December 2017) that is the highest recorded closing value in the Bitcoin Price Index. Following this peak, a sharp price decline can be observed and at the 7th of March the closing price was $10,735. The remainder of the year the price moved in the $6,000-$9,000 range. From the 13th of November 2018 to 16th of December 2018 the price almost halved from $6,282 to $3,200. In the last period in the dataset, the price of one Bitcoin was fairly steady around the $3,000 mark.

The continuously compounded returns (log-returns) are calculated using the following formula:

$$r_t = \ln(\frac{p_t}{p_{t-1}}) \qquad (1)$$

where p_t is the Bitcoin price at time *t* and *ln* is the natural logarithm.

It is preferable to work with returns rather than direct prices, as returns are unit-free. Furthermore, log-returns are continuously compounded returns and thus the frequency of compounding is not important. This makes it easier to compare returns across assets. The price series of an asset is often non-stationary which implies that mean and variance change over time. It is found that most returns series from financial data exhibit preferable stationary properties that make them easier to work with (Fan & Yao, 2017, p.12). The concept of stationarity is elaborated on in chapter 5.

The figure under graphically shows the log-returns of the Bitcoin Price Index in the time period studied. One can clearly observe volatility clustering which is the tendency of volatility to appear in bursts. As mentioned earlier, this is a result of large returns following large returns and small returns following small returns – a phenomenon often observed in financial markets. From the graph there are two periods of volatility bursts that stick out, the last half of 2013 as well as the last half of 2017 and early 2018.

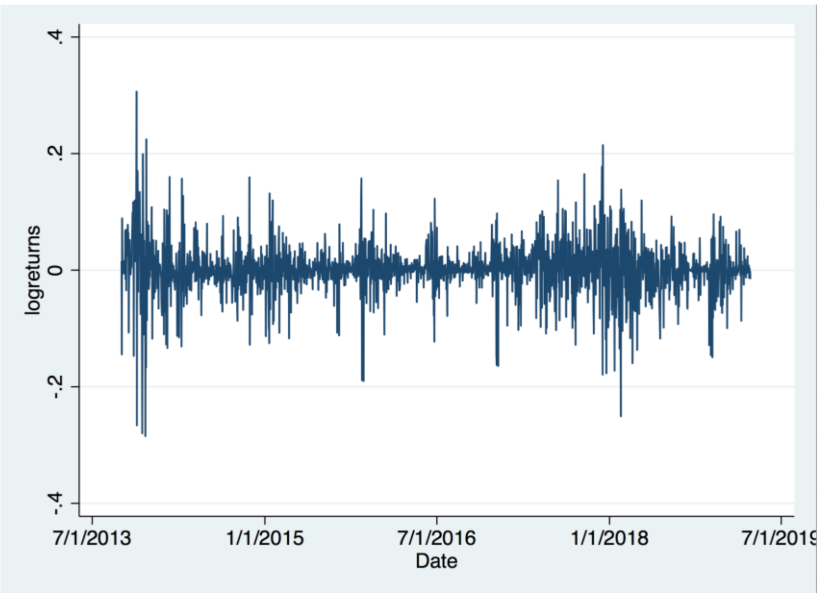

Figure 7 Log-returns XBP/USD

4.3 Statistical Properties

We perform the descriptive statistics command *summarize returns, detail* in STATA to exhibit some of the basic statistical properties of the marginal distribution of log-returns.

Table 2. Descriptive statistics log-return XBP/USD

	Percentiles	Smallest		
1%	-0.134976	-0.2844803		
5%	-0.0700623	-0.2797923		
10%	-0.0427742	-0.2661844	**Obs**	1992
25%	-0.0130687	-0.250672	**Sum of Wgt.**	1992
50%	0.0015095	**Largest**	**Mean**	0.0017343
75%	0.0187142	0.19919734	**Std. Dev.**	0.0443648
90%	0.04668	0.214508	**Variance**	0.0019682
95%	0.0694027	0.2245069	**Skewness**	-0.3691399
99%	0.1235613	0.3063761	**Kurtosis**	9.616139

The figure above displays the different percentiles and from this one can see that the median of returns (50^{th} percentile) is 0.0015095. There are in total 1992 observations as one observation is removed when transforming the closing prices into returns. The four smallest and largest observations are substantial values when transformed into percentages. They fluctuate between $+/-$ 20-30%.

The skewness is -0.3691399 and thus the marginal distribution is negatively skewed. The skewness of a distribution measures the lack of symmetry around the mean and a normal distribution has zero skewness. In this case, a negative skewness results in a larger left tail compared to the right tail.

The kurtosis of 9.616139 is very large. It refers to the peakedness of the distribution and for a normal distribution the value of the kurtosis is 3. If the kurtosis exceeds a value of 3 it is characterized as a leptokurtic distribution.

A leptokurtic distribution has more values around the mean, a higher peak and fatter tails compared to a normal distribution. Moreover, leptokurtic distributions are in fact very likely to be observed in financial time series (Brooks p.162). Fat tails imply that the probability of extreme events are more likely to occur than under normal distributions and in finance they are often regarded as additional risk. The use of nonlinear models such as the GARCH model is advantageous to explain the effects of this phenomenon better than linear models. The histogram in figure 11 displays the frequency distribution of data values and a normal distribution is overlaid to highlight the differences.

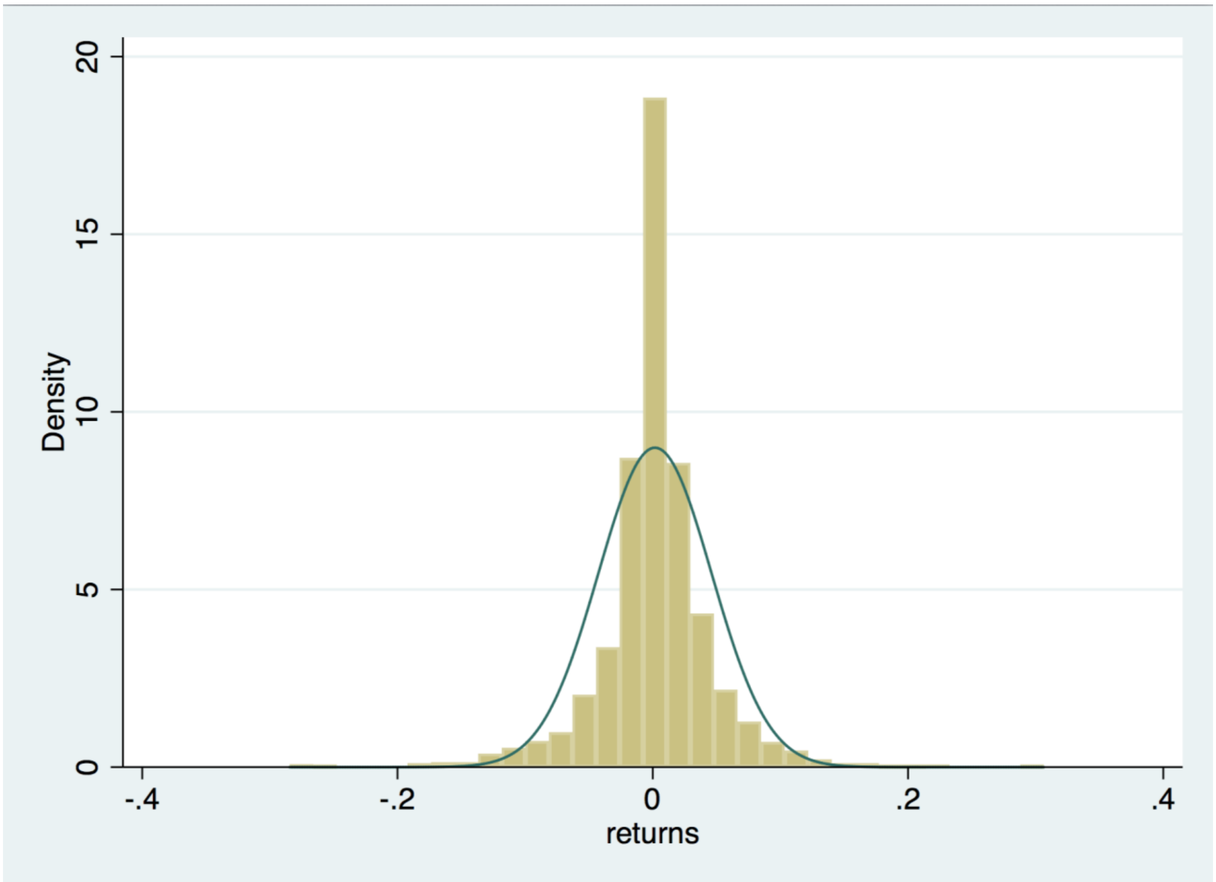

Figure 8 Histogram of bitcoin log-returns. The green line shows perfect normality and the yellow bars are the sample.

4.4 Distributions of Innovations

Bitcoin is a new phenomenon and only recently did scientific research on the subject start. Due to this, we attempt to fit a model that describes the data set optimally. In our research we fit a GARCH(1,1), GJR-GARCH(1,1) and EGARCH(1,1) model under different types of distributions. According to the thesis of Filipovic and Nilgård, the log-return of the bitcoin exchange rate should be described by flexible distributions (Filipovic & Nilgård, 2018, p. 40). In their thesis, eleven distributions were fitted to the log-return of the exchange rate and the skewed generalised t distribution was found to be the optimal fit. However, the STATA software used in this thesis only allows for three different distributions of innovations when modelling conditional variance. The distributions used are a Gaussian distribution, student's t-distribution and a generalized error distribution (GED).

The student's t distribution is leptokurtic, meaning that the kurtosis of a student's t distribution is greater than that of a normal distribution. The t-distribution is characterized by having heavier tails compared to the normal distribution. In turn this means that values far from the mean are more likely to be produced. The generalized error distribution is a distribution that includes all normal and Laplace distributions, this distribution is to be used when the focus of the study are the values around the mean and tails.

The normal distribution also known as the Gaussian distribution (Gauss, 1809):

$$f(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma}} e^{\frac{(x-\mu)^2}{2\sigma^2}} \tag{2}$$

for $x \in \mathbb{R}, \mu \in \mathbb{R}$ and $\sigma > 0$

The distribution has the following moments:

Mean = μ

Variance = σ^2

Skewness = 0

Kurtosis = 3

Student’s t distribution (Gosset,1908):

$$f(x;\mu,\sigma,\nu) = \frac{K(\nu)}{\sigma}\left[1 + \frac{(x-\mu)^2}{\sigma^2\nu}\right]^{\frac{-(1+\nu)}{2}} \tag{3}$$

for $x \in \mathbb{R}, \mu \in \mathbb{R}$ and $\sigma > 0$ and $\nu > 0$,

$where\ K(\nu) = \sqrt{\nu}\mathrm{B}(\frac{\nu}{2},\frac{1}{2}), and\ \mathrm{B}\left(\frac{\nu}{2},\frac{1}{2}\right) = \int_0^1 t^{\frac{\nu}{2}-1}(1-t)^{-\frac{1}{2}}\,dt$

The distribution has the following moments:

Mean = μ

Variance = σ^2

Skewness = 0 for df >3, otherwise not defined.

Kurtosis = $\frac{6}{(df-4)}$, The kurtosis is infinite when df. is <4

The generalized error distribution (Giller, 2005):

$$f(x|\mu,\sigma,k) = \frac{e^{-\frac{1}{2}|\frac{x-\mu}{\sigma}|^{\frac{1}{k}}}}{2^{k+1}\sigma\Gamma(k+1)} \tag{4}$$

The distribution has the following moments:

Mean = μ

Variance = $2^{2k}\sigma^2\frac{\Gamma(3k)}{\Gamma(k)}$,

Skewness = 0

Kurtosis = $\frac{\Gamma(5k)\Gamma(k)}{\Gamma^2(3k)}$

The innovation terms under all distributions are assumed to be independent and identically distributed. This is a strong assumption about the data that must be fulfilled in order to perform maximum likelihood estimation.

5. Conditional Variance: Models and Related Methods

The assumption that the variance of error terms is constant at all given points is called homoscedasticity $var(u_t) = \sigma^2$. To ensure the BLUE properties of the estimators in the least squared method, the assumption of homoscedasticity must be fulfilled. However, in many financial time series, the variance of error terms might be larger at specific points in time and this change in variance is called heteroscedasticity. More specifically, heteroscedasticity implies that the variance of the error terms is not constant. In the presence of heteroscedasticity, the regression coefficients estimated are still deemed unbiased, but the estimated confidence intervals and standard errors are too narrow and imprecise.

Financial time series are often characterized by some time periods being riskier than others. In other words this implies that the magnitude of errors terms has a greater expected value in some time periods. Furthermore, the risky time periods are not randomly distributed. This is another characteristic surrounding financial time series that is called volatility clustering: “large changes seem to be followed by large changes (of either sign) and small changes seem to be followed by small changes (of either sign)” (Brooks, p.387).

5.1 ARCH

The autoregressive conditional heteroscedasticity (ARCH) model addresses the problem of volatility clustering and time-varying volatility in the modelling of variance. To understand the use of the ARCH model, there is a need to elaborate on conditional and unconditional variance. The conditional variance is the variance of a random variable given other variables and the unconditional variance is the standard measure of variance. Consequently, the conditional variance can be written as:

$$\sigma_t^2 = var(u_t | u_{t-1}, u_{t-2}, \dots) \tag{5}$$

In an ARCH model, the autocorrelation in volatility is captured by allowing the conditional variance to be depended on previous lagged values.

An example of a partial ARCH (1) model where the conditional variance is only depended on the most recently lagged value of the squared error is presented below:

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 \tag{6}$$

To show a full ARCH (1) model, a conditional mean equation has to be included. In this case the conditional mean consists of only a constant:

$$y_t = \beta_1 + u_t \qquad u_t \sim N(0, \sigma^2) \tag{7}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 \tag{8}$$

And in the generalized case, a partial ARCH (q) model where" q" is the number of lags of squared errors to be included, could be expressed as follows:

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2 + \cdots + \alpha_q u_{t-q}^2 \tag{9}$$

The conditional variance σ_t^2 must have a strictly positive value, as it does not make sense to have negative variance. To make sure that the estimate of conditional variance is valid, the coefficients are required to be non-negative. A larger negative coefficient compared to squared lagged error terms could result in an invalid negative estimate. This is known as the non-negative constraints and is one of the limitations connected to the ARCH model. As more parameters are included in the conditional variance equation, there is an increase of the chance that non-negative estimates will occur. Another problem connected to the ARCH model is how to determine the amount of lags (q) to be included. In fact, there could be a need to include a large amount of lags to capture all the dependence in the conditional variance. These limitations have made the ARCH model an unpopular choice when modelling conditional variance.

5.2 Testing for Autoregressive Heteroscedasticity Effects

The presence of ARCH effects in returns time series occur when values seems to be changing rapidly from period to period in an unpredictable manner and large changes are followed by

large changes and small changes are followed by small changes. As mentioned earlier, these concepts are referred to as time-varying volatility and volatility clustering.
In addition, returns time series often display “leptokurtic” properties. This distribution involves more observations around the mean and in the tails and consequently its associated histograms display a peaked mean and fat tails. As mentioned in the section about the ARCH model, an ARCH (q) model can be written as:

$$y_t = \beta_1 + \mu_t \qquad \mu_t \sim N(0, h_t) \tag{10}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \mu_{t-1}^2 + \alpha_2 \mu_{t-2}^2 + \cdots + +\alpha_q \mu_{t-q}^2 \tag{11}$$

The ARCH model explains the variance as a function of errors and it is intuitively appealing as it captures two aspects: large shocks or news will result in large volatility and large changes are followed by large changes. This is due to large changes in μ_t being fed into σ_t^2 by the lagged terms μ_{t-q}^2.

To test for the presence of ARCH effects a Lagrange multiplier (LM) test is applied. The first step of this test is to estimate the mean equation by for instance regressing the variable on a constant. Secondly, the estimated residuals $\hat{\mu}_t$ are saved and the squared estimated residuals computed $\hat{\mu}_t^2$. To test for first order ARCH effects, the squared estimated residuals $\hat{\mu}_t^2$ are regressed upon the lagged squared estimated residuals $\hat{\mu}_{t-1}^2$:

$$\hat{\mu}_t^2 = \gamma_0 + \gamma_1 \hat{\mu}_{t-1}^2 + v_t \tag{12}$$

where v_t is a random term.

The null-hypothesis and alternative hypothesis are:

$$H_0: \gamma_1 = 0 \qquad\qquad H_1: \gamma_1 \neq 0 \tag{13}$$

If there are no ARCH effects present, $\gamma_1 = 0$ and the associated R^2 will be low. On the other hand, if there are ARCH effects present, then $\gamma_1 \neq 0$ and the associated R^2 will be relatively high. This will be the case when the magnitude of $\hat{\mu}_t^2$ is expected to depend on lagged values.

The Lagrange-Multiplier test statistic is as follows:

$$LM = (T - q)R^2 \tag{14}$$

T = sample size
q = the number of $\hat{\mu}_{t-j}^2$ terms
R^2= coefficient of determination

If the null-hypothesis is true, the test-statistic is chi-squared distributed $\mathrm{X}_{(q)}^2$ where q is the order of lag and T-q is the number of complete observations. If $(T - q)R^2 \geq \mathrm{X}_{(1-\alpha,q)}^2$, the null hypothesis is rejected, and there is evidence of ARCH effects being present.

5.3 GARCH

The general autoregressive conditional heteroskedasticity model was developed by Bollerslev (Bollerslev, 1986). It is an extension of the ARCH model that allows the conditional variance to be dependent upon its own lags, making the simplest model a GARCH (1,1):

$$y_t = \beta_1 + u_t \qquad u_t \sim N(0, \sigma^2) \tag{15}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{16}$$

The GARCH model is a parsimonious model compared to the ARCH model, and in the form above it is a weighted function of a long-term average value α_0, information regarding previous volatility $\alpha_1 u_{t-1}^2$ and fitted variance from the previous period $\beta\sigma_{t-1}^2$. The GARCH model is a restricted infinite form of the ARCH model making it less likely to breach the non-negative constraints. It allows an infinite number of squared errors to impact conditional variance with only three parameters in the equation. The model can be extended to a GARCH (p,q) capturing more lags of the squared errors and fitted conditional variance:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \tag{17}$$

However, in the Introduction to Econometrics for Finance by Brooks, the author states that in many cases a GARCH (1,1) model is sufficient to capture volatility clustering (Brooks p.394).

Estimation of the many different types of GARCH models is achieved by invoking the maximum likelihood principle. The maximum likelihood method chooses parameter values such that the probability for the occurrence of the observed sample is maximised. To be specific, a log-likelihood function is created and the values of the parameters that maximise this function are found. Consequently for models involving conditional heteroscedasticity such as an AR (1)-GARCH (1,1):

$$y_t = \mu + \phi y_{t-1} + u_t \quad u_t \sim N(0, \sigma_t^2) \tag{18}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{19}$$

The error variance is no longer constant, rather it has been modified to be time-varying. For the AR (1)-GARCH (1,1) model above, the log likelihood function (LLF) under normality assumption is given by (Brooks p.395):

$$L = -\frac{T}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\log(\sigma_t^2) - \frac{1}{2}\sum_{t=1}^{T}\frac{(y_t - \mu - \phi y_{t-1})^2}{\sigma_t^2} \tag{20}$$

The estimation process of maximizing the LLF involves minimizing $\sum_{t=1}^{T}\frac{(y_t-\mu-\phi y_{t-1})^2}{\sigma_t^2}$ and $\sum_{t=1}^{T}\log(\sigma_t^2)$. Not all software programs employ the same methods in this procedure, although the objective of searching for the values that maximize the LLF is the same. It is worth mentioning that this divergence in approach can lead to dissimilar coefficient and standard error estimates. The default optimization technique in Stata employs the algorithm by Berndt, Hall, Hall and Hausman (BHHH). In this technique the first derivatives are calculated analytically as well as approximations to the second derivative. This ensures computational speed but at the cost of inferior approximation if the maximum value is far away. Stata also employs an optimization technique algorithm developed by Broyden, Fletcher, Goldfarb and Shanno (BFGS). However, this technique will not be discussed in detail.

5.4 EGARCH

The EGARCH and GJR-GARCH are two extensions of the GARCH model that includes the "leverage effect" when modelling variance. The leverage effect is a result of the asymmetry effect where negative shocks are believed to have a larger impact on volatility than equal positive shocks. For equities, a negative shock will lower the value of a firm and consequently increase the debt to equity ratio. This can lead to shareholders viewing the stock as more risky as they discount their future cash flows. Although bitcoin is not an equity that provides cash flow, the asymmetry effects could possibly have an impact. In behavioural economics, the term "loss aversion" has been widely accepted. This phenomenon states that the pain of losing is twice as powerful than the joy of winning (Kahneman & Tversky, 1979). In this thesis, the Nelson´s exponential GARCH (EGARCH) model will applied to model the conditional variance:

$$\ln(\sigma_t^2) = \omega + \beta \ln(\sigma_{t-1}^2) + \gamma \frac{u_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \alpha \left[\frac{|u_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}} \right] \tag{21}$$

One of the advantages of the EGARCH model is that it allows for the asymmetry effect to be taken into account. From the conditional variance equation above, γ will be negative in the case of negative relationship between volatility and returns. Furthermore, there is no need for the non-negative constraints. As mentioned earlier in the ARCH section, the conditional variance cannot be negative. However, if the parameters in the equation are negative the $\ln(\sigma_t^2)$ will still be positive. Nelson initially assumed a generalized error distribution for the errors terms (Nelson,1991).

5.5 GJR-GARCH

The GJR-GARCH model is a version of the GARCH model that incorporates an extra term to allow for asymmetry effects (Glosten, Jagannathan, Runkle, 1993). The conditional variance equation is given by (Brooks, p.405):

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta\sigma_{t-1}^2 + \gamma u_{t-1}^2 I_{t-1} \tag{22}$$

$I_{t-1} = 1\ for\ u_{t-1} < 0$ (bad news)

$I_{t-1} = 0\ for\ other\ cases$ (good news)

When positive news (shocks) occur, the GJR-GARCH model converges to a standard GARCH model and the impact on volatility is α_1. However, when negative news (shocks) occur, the impact on volatility is $\alpha_1 + \gamma$. This model incorporates the fact that negative shocks have a stronger impact on volatility σ_t^2 compared to positive shocks. A GJR-GARCH(1,1) model with a constant-only mean equation can be expressed as follows:

$$y_t = \beta_1 + u_t \qquad u_t \sim N(0, \sigma^2) \tag{23}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta\sigma_{t-1}^2 + \gamma u_{t-1}^2 I_{t-1} \tag{24}$$

5.6 Weak Stationarity and the Augmented Dickey-Fuller Test

In order to make any valid inference regarding time series data, it has to exhibit stationarity properties. In a data series that exhibits strict stationarity, the distribution of its values remain unchanged as time passes. In other words, the probability measure for the sequence y_t is the same as for the sequence y_{t+k}. More formally, a strictly stationary process can be expressed as (Brooks, p.207):

$$Fy_{t1}, \dots, y_{t2}, \dots, y_{tr}(y_1, \dots, y_T) = Fy_{t1+k}, \dots, y_{t+k2}, \dots, y_{tr+k}(y_1, \dots, y_T) \tag{25}$$

$$F = joint\ distribution\ function\ of\ the\ set\ of\ random\ variables$$

However, when examining the data for stationarity properties, the literature is usually referring to weak stationarity properties. A weak stationary time series y_t has the following three properties: constant mean, constant variance and constant autocovariance structure.

$$\text{Constant mean: } E(y_t) = \mu \tag{26}$$

$$\text{Constant variance: } E(y_t - \mu)(y_t - \mu) = \sigma^2 \tag{27}$$

$$\text{Constant autocovariance structure: } E(y_{t1} - \mu)(y_{t2} - \mu) = \gamma_{t1-t2} \tag{28}$$

To test if the Bitcoin returns data used are stationary, a Dickey-Fuller test is applied. This test examines whether there exists a unit root in the time series. An AR(1) process $y_t = py_{t-1} + u_t$ is not stationary when p=1. Moreover, if p=1 the process is said to contain a unit root. In cases where p<1, the process fulfils the stationarity properties. The Dickey-Fuller test is done through a regression of the lagged variable:

$$\Delta y_t = \theta y_{t-1} + u_t \tag{29}$$

With the accompanying null-hypothesis and alternative-hypothesis:

H0: $\theta = 0$
H1: $\theta < 0$

The test statistic $\frac{\hat{\theta}}{SE(\hat{\theta})}$ follows a non-standard distribution where critical values are based on simulations. A rejection of the null-hypothesis provides evidence that the series is stationary.

The aforementioned test is only valid if u_t is regarded as white noise. A white noise process has a constant mean and variance like a stationary process. However, there is no autocorrelation present except at lag 0. This implies that each observation is not correlated with any other values.

To account for a situation where u_t is not white noise, an Augmented Dickey Fuller test can be applied:

$$\Delta y_t = \theta y_{t-1} + \sum_{i=1}^{p} \alpha_i \, \Delta y_{t-1} + u_t \tag{30}$$

The difference between the Dickey-Fuller test and the Augmented Dickey-Fuller test is that lags of the dependent variable are added to capture dynamic structure and account for autocorrelation in u_t. The same critical values based on simulation are valid.

5.7 The Ljung-Box Test

To determine whether the specified and estimated model is adequate, we apply the Ljung-box test. This implies checking the estimated residuals for evidence of linear dependency. In the case of such evidence, the model will be deemed inadequate to capture the data features. The Ljung-Box approach tests whether there is any autocorrelation in the squared standardized residuals and such evidence will lead to an under-parameterized model. One of the weaknesses of the Ljung-box test is that the model will only reveal a model that is under parameterized and not overparameterized.

The Ljung-Box Q-statistic is given by (Verbeek, p.285):

$$Q^2 = T(T+2) \sum_{k=1}^{K} \frac{1}{T-k} r_k^2 \mathrm{I} \sim \chi_m^2 \tag{31}$$

$T = sample\ size$
$r_k = estimated\ autocorrelation\ coefficients\ of\ residuals$
$K = number\ chosen\ by\ researcher\ but\ should\ fulfill\ K > p + q + 1$

Q^2 is approximately chi-squared distributed with K-p-q-1 degrees of freedom under the null-hypothesis.

When using the Ljung-Box test as a model diagnostic tool, the null-hypothesis and alternative hypothesis can be expressed in the following way:

H0: The model is not exhibiting a lack of fit
H1: The models is exhibiting a lack of fit

5.8 Kolmogorov-Smirnov Test

The Kolmogorov-Smirnov test is a goodness-of-fit test that tests if a sample comes from a population with a specific distribution (Chakratavi, Laha, Roy, 1967, p.392). One appealing feature of this test is that the underlying distribution of the test-statistic does not depend on the cumulative distribution function tested. However, one severe limitation is that the distribution must be fully specified, as the test is not valid if the location, scale and shape parameter are estimated from the data. Another limitation is that it only applies to continuous distributions.

The test is defined by:

H0: The data follows a specific distribution
H1: The data does not follow a specific distribution

And the Kolmogorov-Smirnov test statistic D can be expressed as:

$$D = \ max_{1 \leq i \leq N} \left(F(Y_i) - \frac{i-1}{N}, \frac{i}{N} - F(Y_i) \right) \tag{32}$$

$F(Y_I) = theoritcal\ cumulative\ distribution\ of\ distribution\ examined$

5.9 In-sample Evaluation: Model Selection Criteria

The objective in model specification is to select a parsimonious model. This implies selecting a model that describes all data features with as few variables as possible. There are two reasons for why a parsimonious model is desirable (Brooks, p.231):

1. A model that contains irrelevant lags of a variable will usually lead to increased coefficients for standard errors. The result will be an increased difficulty in finding significant relationships in the data.
2. An extravagant model might fit to specific data features that would not be replicated out-of-sample. This implies that the model might seem to fit the data very well but will give imprecise forecasts. The objective is to fit a model that encapsulates the signal (data-specific features) and not the noise (random component).

One model selection technique, which removes the subjective assessment of autocorrelation plots and partial autocorrelation plots, is information criteria. Information criteria consist of two parts, one part that penalizes the adding of parameters due to the loss of degrees of freedom and one part that is a function of the residual sum of squares. The information criteria are therefore composed in such a way that adding new parameters will increase parameter penalty but decrease the residual sum of squares. The objective is to select the model with parameters that minimizes the value of information criteria. Moreover, adding a parameter in the model will only decrease information criteria if the parameter penalty is outweighed by the fall in the residual sum of squares. In this thesis the different GARCH models will be assessed through the Akaike´s information criteria (AIC) and Schwarz Bayesian information criteria (SBIC). The difference between the two aforementioned information criteria is the stiffness of the penalty term. The SBIC imposes a much stiffer penalty when adding parameters compared to the AIC.

$$AIC = \ln(\hat{\sigma}^2) + \frac{2k}{T} \tag{33}$$

$$BIC = \ln(\hat{\sigma}^2) + \frac{k}{T} lnT \tag{34}$$

$\hat{\sigma}^2 = residual\ variance = \frac{RSS}{T}$

$k = total\ number\ of\ parameters = p + q + 1$

$T = number\ of\ observations$

5.10 Testing for Nonlinearity

In order to use nonlinear models such as the GARCH model there is a need to test for the relationship between variables. To clarify, nonlinear models should only be applied on data where the relationship between the variables is of the nature deeming them appropriate. The use of autocorrelation functions and partial autocorrelation functions is of little use in this case. The reason for this is that even if these functions do not find a linear structure between variables, this does not automatically imply that the observations are independent of another (Brooks, p.381).

In order to determine if nonlinear models are appropriate in modelling variance we apply the Brock, Dechert and Scheinkman test also known as the BDS test. The details of this test are complicated and will be discussed briefly:

The starting point is a time series $\{x_t, t = 1, \ldots, T\}$ and a point in the m-dimensional Euclidean space $x_t^m = (x_t, x_{t-1}, \ldots, x_{t-m+1})$. The correlation integral $C_{m,\delta}$ estimates the probability that any two pair of points x_t^m and x_s^m are in a distance δ of each other.

$$C_{m,\delta} = \frac{2}{T_m(T_m - 1)} \sum_{m \le s} \sum_{t \le T} I(\, x_t^m, x_s^m : \delta) \tag{35}$$

where $T_m = T - m + 1$ and I is an indicator function.

If the time series x_t is generated from an independently identical distributed (IID) process, it is expected that $C_{m,\delta} = C_{1,\delta}^m$.

$$C_{1,\delta}^m = PR(|x_t - x_s| < \delta)^m \tag{36}$$

The researchers constructed a test statistic to test the null hypothesis that in fact $C_{m,\delta} = C_{1,\delta}^m$. In other words, the null-hypothesis is that the data is independent and identically distributed. The test-statistic follows normal distribution N (0,1)

The BDS test statistic can be expressed as:

$$V_{m,\delta} = \sqrt{T}\frac{C_{m,\delta} - C_{1,\delta}^{m}}{s_{m,\delta}} \tag{37}$$

where $s_{m,\delta}$ is the standard deviation.
(Zivot & Wang, 2006, p. 651)

5.11 The Sign bias Test for Asymmetries in Volatility

The sign bias test proposed by Engle and Ng is applied to the residuals of a fitted GARCH model. Its objective is to determine whether asymmetries in volatility are present and thus also the need for an asymmetric GARCH model. The test is not complicated and involves regressing residuals on a constant and dummy variable (Engle, Ng, 1993):

$$\hat{u}_t^2 = \theta_0 + \theta_1 S_{t-1}^{-} + v_t \tag{38}$$

v_t= iid error term
S_{t-1}^{-} = dummy variable
= 1 if $\hat{u}_t^2$ < 0 and 0 otherwise

A significant θ_1 will indicate that positive and negative shocks to $\hat{u}_t^2$ effects the conditional variance dissimilarly.

The positive size-biased test and negative size-biased test are applied to test if the size of the shock to $\hat{u}_t^2$ effects the shock in volatility asymmetrically or not. In the negative size-bias test the dummy variable S_{t-1}^{-} becomes a slope dummy variable:

$$\hat{u}_t^2 = \theta_0 + \theta_1 S_{t-1}^{-} u_{t-1} + v_t \tag{39}$$

As S_{t-1}^{+} = 1 - S_{t-1}^{-}, the positive size-bias test can be expressed as:

$$\hat{u}_t^2 = \theta_0 + \theta_1 (1 - S_{t-1}^{-}) u_{t-1} + v_t \tag{40}$$

The significance of θ_1 in the size-bias test will reveal if the volatility reacts asymmetrically to the magnitude of the shocks to error term. All in all, a significant θ_1 estimate in any of the tests on GARCH residuals will indicate a miss-specified model and that the use of asymmetric models such as the EGARCH or GJR-GARCH might be more appropriate. However, in this thesis we have applied the sign bias test and size-bias test as model diagnostic tools to evaluate the adequacy of the asymmetric models. This implies that the when testing the residuals from the EGARCH and GJR-GARCH, the θ_1 estimate should not be statistically significant if the models are correctly specified.

6. Estimating Change in Volatility

In order to research whether the level of volatility has decreased after the introduction of bitcoin futures, we need to estimate the level of unconditional variance. From the ARCH section, the conditional variance equation in an ARCH (1,1) process can be expressed as:

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 \tag{41}$$

A full ARCH model must consist of both a mean equation and a conditional variance equation as the one in equation 6.1. The mean equation describes how the dependent variable behaves and can be constructed in many different ways. In this thesis, the choice of a mean equation with only a constant and no explanatory variables, was made for all the estimated models. The form of the conditional variance equation will depend on what GARCH model is estimated, but the mean equation will be in the form of equation 6.2 for all GARCH models .A full ARCH (1,1) model can be expressed as:

$$y_t = \beta_1 + u_t \qquad u_t \sim N(0, \sigma^2) \tag{42}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 \tag{43}$$

To capture the possible change in unconditional variance before and after the introduction of bitcoin futures in the dataset, we introduce the use of a dummy variable in the conditional variance equation. The dummy variable x_t takes the value 1 in the period before bitcoin futures were introduced and the value 0 at and after Bitcoin futures entered the marketplace.

The conditional variance equation can now be expressed as:

$$\sigma_t^2 = \exp(\lambda_0 + \lambda_1 x_t) + \alpha_1 \beta \sigma_{t-1}^2 \tag{44}$$

The statistical significance of the dummy variable will demonstrate if the change in unconditional variance has been statistically significant.
From our interpretation of the STATA manual, this is the method to follow as there is no point estimate for unconditional variance estimated when modelling conditional variance. However, one of the drawbacks related to this approach is the additional assumptions that the GARCH coefficients remain unchanged throughout the sample.

Similarly the conditional variance equation in a GARCH (1,1) model is given by:

$$\sigma_t^2 = \omega + \alpha \mu_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{44}$$

And with a dummy variable x_t it can be transformed to:

$$\sigma_t^2 = \exp(\lambda_0 + \lambda_1 x_t) + \alpha \mu_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{45}$$

There are many ways to express the conditional variance equation for the EGARCH (1,1) model and one expression is the following (Levendis, 2018)

$$ln(\sigma_t^2) = \alpha_0 + a_{11} z_{t-1} + \alpha_{12}\left(|z_{t-1}| - \sqrt{\frac{2}{\pi}}\right) + \gamma_1 \ln(\sigma_{t-1}^2) \tag{46}$$

And with a dummy variable it can be transformed to:

$$\ln(\sigma_t^2) = \lambda_0 + \lambda_1 x_t + a_{11} z_{t-1} + \alpha_{12}\left(|z_{t-1}| - \sqrt{\frac{2}{\pi}}\right) + \gamma_1 \ln(\sigma_{t-1}^2) \tag{47}$$

where $z_{t-1} \sim N(0,1)$ is a standard normal variable. This conditional variance equation differs from the one under a GARCH(1,1) as the variance is already log-specified (StataCorp, 2017)

For the GJR-GARCH (1,1) model, the conditional variance equation can be expressed as:

$$\sigma_t^2 = \omega + \alpha\mu_{t-1}^2 + \beta\sigma_{t-1}^2 + \gamma\mu_{t-1}^2 I_{t-1} \quad (48)$$

And with a dummy variable it is transformed to:

$$\sigma_t^2 = \exp(\lambda_0 + \lambda_1 x_t) + \alpha\mu_{t-1}^2 + \beta\sigma_{t-1}^2 + \gamma\mu_{t-1}^2 I_{t-1} \quad (49)$$

7. Volatility Estimation and Model Selection

7.1 Augmented Dickey-Fuller Test

The choice of estimating the log-returns of the bitcoin exchange rate was made to achieve weak stationarity properties. Such properties are desirable in order to make valid statistical inferences in finance. The Augmented Dickey-Fuller test statistic is of such size that the null-hypothesis is rejected at a 1% critical level. This indicates that there is evidence of weak stationarity properties present and we can proceed with the analysis.

Table 3. Augmented Dickey-Fuller test log-returns

Augmented Dickey-Fuller test					
	Test-statistic	**Critical values**			
	$\frac{\hat{\theta}}{SE(\hat{\theta})}$	**1%**	**5%**	**10%**	**P>|t|**
Z(t)	-45.500	-3.430	-2.860	-2.570	0.0000

7.2 The BDS Test

To test if the log-returns data exhibited nonlinear characteristics, we applied the BDS test. This was done using the software Eviews, as we were not able to compute it in STATA. Hsieh points out that there are four types of non-IID behavior: linear dependence, non-stationarity, chaos and stochastic nonlinearity. To apply the BDS test as a test for nonlinearity he suggests that one must remove the linear dependence. This is done by using an autoregressive model AR(p) where the lag-length is selected by the AIC or SBIC information criteria (Hsieh, 1990). The table below presents the information criteria AIC and SBIC for an AR(p) lag length 1-20. As mentioned before, the best models are those with the lowest information criteria value. In this case that would be the AR(1) and AR(12) models as highlighted in the table below.

Table 4. Model selection of AR(p) model based on information criteria

AR(p)	AIC	SBIC
1	-6754.228	**-6737.437**
2	-6754.737	-6732.349
3	-6752.883	-6724.898
4	-6753.003	-6719.422
5	-6758.574	-6719.395
6	-6767.371	-6722.596
7	-6765.482	-6715.11
8	-6763.494	-6707.525
9	-6765.085	-6703.519
10	-6776.545	-6709.383
11	-6776.971	-6704.211
12	**-6777.305**	-6698.949
13	-6775.41	-6691.457
14	-6774.108	-6684.558
15	-6772.195	-6677.048
16	-6770.876	-6670.132
17	-6770.11	-6663.769
18	-6768.371	-6656.433
19	-6767.843	-6650.308
20	-6766.258	-6643.126

We estimated an AR (1) and AR (12) model on the log-returns times series, saved the residuals and applied the BDS-test on the residuals. Eviews allows four different methods to select the epsilon value and in our approach the epsilon value was set as a fraction of standard deviation. The standard choice of six dimensions was used. From the tables below we reject the null-hypothesis stating that the time series is generated from an independent and identically distributed process (IID). Due to the preliminary removal of linear dependence, this implies that there is nonlinearity present in the data and that the use of nonlinear conditional variance models is appropriate going forward.

Table 5. Results of the BDS-test applied on residuals from an AR(1) model in Eviews.

Dimension	BDS Statistic	Std. Error	z-Statitic	Prob.
2	0.035347	0.002539	13.92184	0.0000
3	0.067720	0.004041	16.75816	0.0000
4	0.090847	0.004822	18.84174	0.0000
5	0.105044	0.005036	20.85671	0.0000
6	0.111587	0.004868	22.92046	0.0000

Table 6. Results of the BDS-test applied on residuals from an AR(12) model in Eviews

Dimension	BDS Statistic	Std. Error	z-Statistic	Prob
2	0.034273	0.002531	13.54286	0.0000
3	0.066545	0.004025	16.53388	0.0000
4	0.090579	0.004798	18.89800	0.0000
5	0.104966	0.005008	20.95857	0.0000
6	0.111040	0.004837	22.95481	0.0000

7.2 The ARCH-LM Test

The next step is to apply the ARCH-LM test on the log-returns series to check for the presence ARCH effects. Time-varying volatility and volatility clustering are two examples of such effects. As mentioned earlier, this is characterized by big changes in volatility being followed by big changes and small changes being followed by small changes. From the table below it can be shown that the null-hypothesis is rejected at a 1% critical level and there is strong evidence of ARCH-effects present in the log-returns series.

Table 7. Testing for ARCH-effects in squared residuals

<table>
<tr><th colspan="6">ARCH-LM test</th></tr>
<tr><td></td><td>Test-statistic</td><td colspan="3">Critical values</td><td></td></tr>
<tr><td></td><td>$LM = (T - q)R^2$</td><td>1%</td><td>5%</td><td>10%</td><td>P>|t|</td></tr>
<tr><td></td><td>350.035</td><td>23.209251</td><td>18.307038</td><td>15.987179</td><td>0.000</td></tr>
</table>

7.2.1 GARCH(1,1) – Gaussian Distribution

The tables below show the estimated parameter coefficients of a GARCH (1,1) model under the assumption that innovations follow gaussian distribution. The four parameter estimates are statistically significant at a 1% critical level. A negative and statistically significant λ_1 estimate indicates that the volatility in the period after the introduction of the bitcoin futures has been significantly reduced. The values of $\alpha + \beta$ are very close to 1 which indicates a high persistence of volatility (Verbeek, 2012, p.299). The Ljung-Box test on squared standardized residuals is 7.0838 with a p-value of 0.7175. This indicates that the null hypothesis cannot be rejected and the model is deemed adequate.

Table 8. GARCH(1,1) – Gaussian distribution

GARCH - Gaussian	Estimate	P-Value
λ_0	-9.629327	0.000
λ_1	-0.7365804	0.000
α	0.1761087	0.000
β	0.8196084	0.000
AIC	-7635.711	
SBIC	-7607.726	
Q^2 (10)	7.0838	0.7175

7.2.2 GARCH(1,1) – Student´s t Distribution

This table shows the parameter estimates of a GARCH (1,1) model under the assumption that innovations follow student´s t distribution. The λ_1 estimate connected to the dummy variable is in this case not statistically significant even at a 10% critical level. Therefore no conclusion can be made regarding the change in volatility after the introduction of futures. The α and β estimates are statistically significant at a 1% critical level. Moreover, $\alpha + \beta > 1$, this indicates that the volatility is highly persistent and growing. The result is a non-stationary process which might indicate that this model is not suited to explain the data well. In the presence of non-stationarity in variance, the unconditional variance is not defined (Brooks, 2003, p.194). In the limitations part of this thesis we go further into probable causes of this issue. The Ljung-Box test-statistic has a p-value of 0.6651 which implies that the null-hypothesis cannot be rejected. The result according to this test is that we have an adequate model.

Table 9. GARCH(1,1) - Student´s t distribution

GARCH - Student's t	Estimate	P-Value
λ_0	-11.38997	0.000
λ_1	0.5801025	0.552
α	0.3343851	0.000
β	0.811822	0.000
AIC	-8027.353	
SBIC	-7993.772	
Q^2 (10)	7.6279	0.6651

7.2.3 GARCH(1,1) – Generalized Error Distribution

This table shows the parameter estimates of a GARCH (1,1) model under the assumption that the innovations follow a generalized error distribution. As for the GARCH (1,1) model under the student´s t distribution, the λ_1 estimate is not statistically significant. Therefore no conclusion regarding the volatility change after the introduction of bitcoin futures can be made. Again $\alpha + \beta$>1 and both parameter estimates are statistically significant at a 1% critical level. The result is non-stationarity in variance. This indicates a persistent and growing volatility. The Ljung-Box test deems this model adequate based on the same arguments as the two preceding examples.

Table 10. GARCH(1,1) – Generalized error distribution

GARCH-GED	Estimate	P-Value
λ_0	-10.55447	0.000
λ_1	-0.2653826	0.455
α	0.2135245	0.000
β	0.8119971	0.000
AIC	-8047.895	
SBIC	-8014.313	
Q^2 (10)	8.8539	0.5460

7.3.1 GJR-GARCH (1,1) – Gaussian Distribution

This table shows the parameter estimates of an asymmetric GJR-GARCH (1,1) under the assumption that the innovations follow a Gaussian distribution. The estimate of the λ_1 parameter connected to the dummy variable is statistically significant at a critical level of 1% and is negative. This indicates that the volatility has decreased significantly after the introduction of bitcoin futures. The parameter estimate of the dummy variable connected to the leverage effect Υ is positive and statistically significant at a 1% critical level. This indicates that the leverage effect is present and positive, which means that positive shocks impact volatility less than negative shocks. The estimates of α and β are statistically significant at a 1% critical level and the sum of the coefficients is close to 1. This is evidence of high volatility persistence. Again the Ljung-Box test deems this model adequate, as we do not reject the null-hypothesis.

Table 11. GJR-GARCH (1,1) – Gaussian distribution

GJR-GARCH - Gaussian	Estimate	P-Value
λ_0	-9.623769	0.000
λ_1	-0.7800734	0.000
α	0.15218	0.000
β	0.8209827	0.000
Υ	0.0477626	0.007
AIC	-7637.952	
SBIC	-7604.371	
Q^2 (10)	8.6367	0.5669

7.3.2 GJR-GARCH (1,1) – Student´s t Distribution

In the following table, the parameter estimates for a GJR-GARCH (1,1) model are presented. The underlying assumption is that the innovations follow student´s t distribution. The estimate of the parameter connected to the dummy variable is not statistically significant. Consequently, there is no evidence regarding change in volatility after the introduction of bitcoin futures. The value of $\alpha + \beta > 1$ and both parameter estimates are statistically significant at a 1% critical level. Which indicates non-stationarity in variance. In turn this indicates growing and persistent volatility. The dummy variable related to the leverage effect is statistically significant but only at a 10% critical level. The positive and statistically significant estimate indicates the presence of a positive leverage effect. The model is also deemed adequate based on the Ljung-Box test result on squared standardized residuals.

Table 12. GJR-GARCH (1,1) – Student´s t distribution

GJR-GARCH - Student's t	Estimate	P-Value
λ_0	-11.4135	0.000
λ_1	0.5417651	0.572
α	0.2717419	0.000
β	0.8132719	0.000
Υ	0.1248552	0.068
AIC	-8029.961	
SBIC	-7990.783	
Q^2 (10)	9.6030	0.4760

7.3.3 GJR-GARCH (1,1) – Generalized Error Distribution

This table provide the result from a GJR-GARCH (1,1) model estimation. It is assumed that the innovations follow a generalized error distribution. The parameter estimate connected to the dummy variable measuring volatility in the two periods before and after bitcoin futures were introduced is negative. Even though the p-value is substantially lower compared to the previous model, it is still not statistically significant at a 10% critical level. This implies that there is no evidence that the volatility has decreased in the second period. Both the α and β estimates are statistically significant at a 1% critical level and their sum indicates high volatility persistence. The Υ parameter estimate that measures leverage effect is not statistically significant at a critical level of 10% indicating its absence. As for the previous models, the Ljung-Box test result deems this model adequate.

Table 13. GJR-GARCH(1,1) – Generalized error distribution

GJR-GARCH - GED	Estimate	P-Value
λ_0	-10.55544	0.000
λ_1	-0.3109162	0.381
α	0.1810695	0.000
β	0.8134084	0.000
Υ	0.0640857	0.141
AIC	-8048.742	
SBIC	-8009.564	
Q^2 (10)	10.8986	0.3655

7.4.1 EGARCH (1,1) – Gaussian Distribution

The following table presents the results from fitting an EGARCH (1,1) model where innovations follow a gaussian distribution. The λ_1 estimate is negative and statistically significant. This implies that there is evidence that volatility has decreased in the period after the bitcoin futures have been introduced. In this case, the leverage effect is connected to the parameter a_{11}. The estimate of this parameter is positive and statistically significant at a 1% critical level. The parameter γ is related to the volatility persistence in the EGARCH model. In this case, its estimate is negative and statistically significant at a 1% critical level which indicates a lack of persistence in volatility. The α_{12} represents the magnitude effect of the model and its estimate is positive and statistically significant at a 1% critical level. The Ljung-Box test also deems this model adequate although the p-value is substantially lower compared to the other models estimated this far.

Table 14. EGARCH(1,1) – Gaussian distribution

EGARCH - Gaussian	Estimate	P-Value
λ_0	- 11.1132	0.000
λ_1	- 1.09969	0.000
γ	-0.9120776	0.000
a_{11}	0.0133693	0.000
α_{12}	0.11661	0.000
AIC	-6807.009	

SBIC	-6773.427	
Q^2 (10)	771.3520	0.000

7.4.2 EGARCH (1,1) – Student´s t Distribution

The following table presents the result from an EGARCH (1,1) model estimation where innovations follow a student's t distribution. The λ_1 estimate is not statistically significant making it invalid to draw a conclusion regarding change of volatility. Furthermore, the leverage effect is also not statistically significant at a 10% critical level. The volatility persistence parameter estimate of γ is close to 1 and statistically significant at a 1% critical level. This indicates a persistence in volatility. The magnitude effect measured by α_{12} is also statistically significant at a 1% critical level. This model is adequate according to the Ljung-Box test on squared standardized residuals.

Table 15. EGARCH(1,1) – Student´s t distribution

EGARCH Student's t	Estimate	P-Value
λ_0	-0.245376	0.680
λ_1	0.0031307	0.837
γ	0.9816151	0.000
a_{11}	0.040268	0.100
α_{12}	0.4389442	0.000
AIC	-8040.378	
SBIC	-8001.200	
Q^2 (10)	9.2391	0.5096

7.4.3 EGARCH (1,1) – Generalized Error Distribution

The last model estimated is an EGARCH (1,1) model where the innovations follow a generalized error distribution. As in the previous model, the λ_1 estimate is not statistically significant and there cannot be made any statistical inference regarding change in volatility. The leverage effect measured by the a_{11} parameter estimate is not statistically significant. The estimates of γ and α_{12} are both statistically significant at a 1% critical level indicating high volatility persistence and magnitude effect presence. The Ljung-Box test deems this model adequate and it is also the model with the lowest scores regarding the information criteria AIC and SBIC. This implies that it is the most parsimonious model of the 9 models estimated. In the original formulation, the innovation terms actually followed a generalized error distribution (Nelson, 1991).

Table 16. EGARCH (1,1) – Generalized error distribution

EGARCH - GED	Estimate	P-Value
λ_0	-0.159485	0.002
λ_1	**-0.0061056**	**0.686**
γ	0.9700836	0.000
a_{11}	0.0237147	0.268
α_{12}	0.359347	0.000
AIC	**-8059.753**	
SBIC	**-8020.574**	
Q^2 (10)	11.8532	0.2950

7.5 Kolmogorov-Smirnov Test

The Kolmogorov-Smirnov (Chakravati, Laha, Roy, 1967, p.392) test for the model chosen by the in-sample evaluation, indicate that the underlying distribution assumption of generalized error distribution for the EGARCH (1,1) model is appropriate. The p-value in is of such magnitude that we cannot reject the null-hypothesis at a 5% critical level for the EGARCH (1,1) model. In other words we cannot reject the notion that the innovation terms do not follow the specific distributions.

Table 17 Kolmogorov-Smirnov test

Kolmogorov-Smirnov test			
GARCH(1,1)			
Distribution	Gaussian	Student´s t	GED
KS-statistic (p-value)	0.095 (0.000)	0.027 (0.099)	0.027 (0.112)

EGARCH (1,1)			
Distribution	Gaussian	Student´s t	GED
KS-statistic (p-value)	0.122 (0.000)	0.026 (0.149)	**0.028** **(0.087)**
GJR-GARCH			
Distribution	Gaussian	Student´s t	GED
KS-statistic (p-value)	0.093 (0.000)	0.025 (0.180)	0.026 (0.134)

7.6 Sign bias Test

The sign bias test and size-bias test based on the residuals of the asymmetric models indicate that they are correctly specified. The asymmetry parameter estimate is not statistically significant at even a 15% critical level as highlighted by p-values in parenthesis. This is the second model diagnostic tool applied for the models in addition to the Ljung-Box test.

Table 18 Sign bias test and size-bias test

	Sign bias test	Negative size biased test	Positive size biased test
	S_{t-1}^{-}	S_{t-1}^{-}	S_{t-1}^{+}
EGARCH			
Gaussian	0.0065 (0.953)	-1.6921 (0.376)	0.0720 (0.972)
Student´s t	0.0796 (0.460)	0.1042 (0.955)	-2.526 (0.205)
GED	**0.0925** **(0.499)**	**-0.4866** **(0.837)**	**-2.319** **(0.358)**

GJR-GARCH			
Gaussian	0.0392 (0.721)	-1.6136 (0.395)	-0.9785 (0.630)
Student´s t	0.0994 (0.387)	0.5363 (0.786)	-3.0700 (0.149)
GED	0.0966 (0.458)	-0.665 (0.976)	-3.016 (0.208)

8. Conclusions

8.1 Discussion

The volatility of the bitcoin to US Dollar exchange rate has been studied by estimating nine different conditional variance models. This event study was conducted to examine if the introduction of bitcoin futures on regulated exchanges has had a decreasing effect on price volatility. Based on the in-sample selection technique, we selected the most parsimonious model to estimate the volatility. The EGARCH (1,1) model where innovation terms follow a generalized error distribution, was the preferred model based on the AIC and SBIC information criteria. The model was deemed adequate and correctly specified by its favourable p-values in the Ljung-Box test, Kolmogorov-Smirnov test, Sign bias test and Size bias test. Specifically, the model was used to produce an estimate of the unconditional variance as a measure for volatility in the period before and after the introduction of bitcoin futures. There was no evidence that the introduction of bitcoin futures has had a volatility-decreasing impact.

This result differs from the two studies previously conducted on the impact of bitcoin futures on price volatility. We will comment on possible causes by highlighting differences in the research methods used. The study from Shi (2017) found evidence that intraday spot volatility decreased in the seven days after the introduction of bitcoin futures. Although the researcher estimates an EGARCH (1,1) model similar to us, the change in volatility is addressed by using a linear regression method. The post-introduction sample covers a substantially smaller time-frame and the high-frequency five-min data interval also differs from our use of the daily closing price data. In the study by Corbet, Lucey, Peat and Vigne, there is evidence that price volatility actually increased after the introduction of bitcoin futures (Corbet et al., 2018). The researchers also estimate a conditional variance model but apply change point detection to address change in volatility. In our study, we use the statistical significance of a dummy variable in the conditional variance equation to address the change. The study also covers a smaller post-introduction period.

There is a debate whether bitcoin should be classified as a speculative asset or a currency mainly due to its high volatility. The studies that examined the impact of futures contracts for US stocks and the EURO/INR exchange rate, both presented evidence of lower price volatility after the introduction of futures contracts. This effect was not found for the bitcoin exchange rate and a possible explanation could be that the bitcoin market is yet to mature. In fact, on the 21st of May 2019 the SEC postponed the approval of bitcoin exchange traded funds due to the lack of market maturity (Morris, 2019). It is also important to remember that the ability to trade bitcoin derivatives on unregulated exchanges existed before 2017 and this could have an impact on the results as the true effect of introducing futures might not be present. There are other factors than bitcoin futures that can impact the bitcoin price volatility. As mentioned earlier, the bitcoin price is sensitive to security hacking attacks on exchanges, trade manipulation as well as regulatory news regarding the subject. According to a report from 2019 provided by Bitwise Asset Management, up to 95% of reported traded volume is artificial (Yates, 2019).

Bitcoin is still a controversial topic and there seems to be a clear distinction between its advocates and critics. One of the arguments against the use of bitcoin as a currency, is its highly volatile and unpredictable nature. There are many factors driving the bitcoin price volatility and it will be interesting to follow the development. A price stabilization could possibly help uncover some of the intended functions of Bitcoin and implement it as a global currency.

8.2 Limitations

A full ARCH, GARCH, EGARCH or GJR-GARCH model consists of a mean equation and conditional variance equation. In this thesis, the choice of a mean equation with a constant and no explanatory variable was selected. However, it is recommended to include explanatory variables in the mean equation to capture the behavior of returns if there is serial correlation present (Tsay, 2005, p.104). This is because the presence of serial correlation can lead to the conclusion that estimates of parameters are more precise than in reality.

Consequently, this can lead to the null-hypothesis being rejected in cases where it should not be rejected. The problem of serial correlation can be adressed by estimating GARCH models with mean equations containing an autoregressive model AR(p) of order p. The Ljung-Box test on the standardized residuals is applied to check for serial correlation.

If serial correlation is still present, its recommended to add more lags to the autoregressive model in the mean equation (Tsay, 2005, p. 104)

To address this we estimated fifteen GARCH (1,1) models under gaussian distribution with a mean equation process described by an AR (p) model with $p = [1,15]$, estimated the standardized residuals and tested for serial correlation by applying the Ljung-Box test. The results are in 9.1 Appendix A and it shows that there needs to be included 15 autoregressive lags in the mean equation for the null-hypothesis of no serial correlation to not be rejected at a 5% critical level. This is a peculiar result and unlike anything we can find in the books of Brooks, Verbeek or Tsay. When plotting the autocorrelation function and partial autocorrelation function of the log-returns in STATA by using the command *ac* and *pac,* the significant autocorrelation points seem random as there is no clear pattern. One possible explanation is that bitcoin is a highly volatile and unpredictable phenomenon that is unlike most concepts in the financial world. It may also be that these plots are not suited to explain serial correlation in bitcoin log-returns.

Although the data in this thesis covers a different time period and is from another source, the structure of the autocorrelation plots are similar to those presented by Filipovic and Nilgård in their master thesis. There is no clear structure and there seems to be some randomness involved in the distribution of the autocorrelation points. Filipovic and Nilgård address this randomness of the autocorrelation plots and in effect treat the data as non-correlated (Filipovic & Nilgård, 2018, p.19).

In this thesis no concrete conclusion will be made regarding the presence of serial correlation, but the point is that choosing a mean equation without explanatory variables could be less problematic than originally assumed.

Estimating GARCH models with a dummy variable in the mean equation or variance equation can present some problems. Especially in event studies, the use of dummy variables is popular. Event studies are widely used in finance to measure the consequences of an event such as introduction of futures. These event studies are built on the framework of Fama (Fama, Fisher, Jensen, Roll, 1969) and uses abnormal observations around the event occurrence to determine statistical significance through hypothesis testing.

Chen and Lu (2011) showed that the use of dummy variables in the variance equation of a GARCH model could produce an overstated variance deflation resulting in unreliable statistical inference for event studies. This implies that the t-statistic distribution of the dummy variable coefficient in the variance equation is more negatively skewed compared to normal distribution under the null-hypothesis. The result is that the use of dummy-variables in financial event studies can lead to limited hypothesis testing power. This is especially relevant for short event windows defined by Chen and Lu as less than 100 data points (Chen & Lu, 2011). As we have 642 data points for the closing price of bitcoin from the first introduction of bitcoin futures at the 10^{th} of December 2017, this might not be a severe problem for us. However, it is important to be aware of when drawing conclusions.

Another concept related to time series analysis is structural breaks. A structural break implies that the coefficients of a model differ before and after a policy change such as the introduction of bitcoin futures. If GARCH models that suffer from parameter changes in the conditional variance (structural breaks) are estimated and these are not taken into account, the result is error in the estimation. Specifically this implies that even if the parameters $\alpha + \beta$ from the GARCH (1,1) conditional variance equation will be close to 1, indicating a high persistence in volatility, might be wrong. In other words, there will be spurious high persistence in volatility results (Hillebrand, 2005).

Moreover, Pesaran and Timmerman found that structural breaks in both mean and variance of forecasting models can be costly to ignore. They found that selecting the correct window to estimate forecasting models based on identifying the structural breaks is important (Pesaran & Timmerman, 2004).

Some of the model estimations were computed under restrictions in STATA set manually by the researchers. To be able to produce results for two of the models we had to disable the gradient criterion by using the maximization option *gtolerance(999)*. The drawback of using this command is that even though results are produced, it is not certain that the global maximum likelihood has been discovered. The command *iterate()* was also applied in the estimation of one of the models due to problems of flat log likelihood.
In this case, the maximum number of iterations were set so results were produced. However, the consequence of this is that the convergence was not reached. More details are provided in the do-file.

8.3 Proposal for Future Research

Future research could extend the modelling of bitcoin price volatility by applying higher-order conditional variance models or use other extensions of the GARCH model. It could also be interesting to use different mean equations to capture some of the serial correlation. The mean equations could take the form of an AR model or ARMA model. Furthermore, the estimation of conditional and unconditional variance might be improved if the framework of structural breaks is applied. Due to the problem of including dummy variables in conditional variance equations, it would be interesting to use software that does provide point estimates of unconditional variance when modelling conditional variance. It is in the belief of the researchers that the software package R provides this estimate.

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9. Appendices

9.1 Appendix A – Autocorrelation Plots

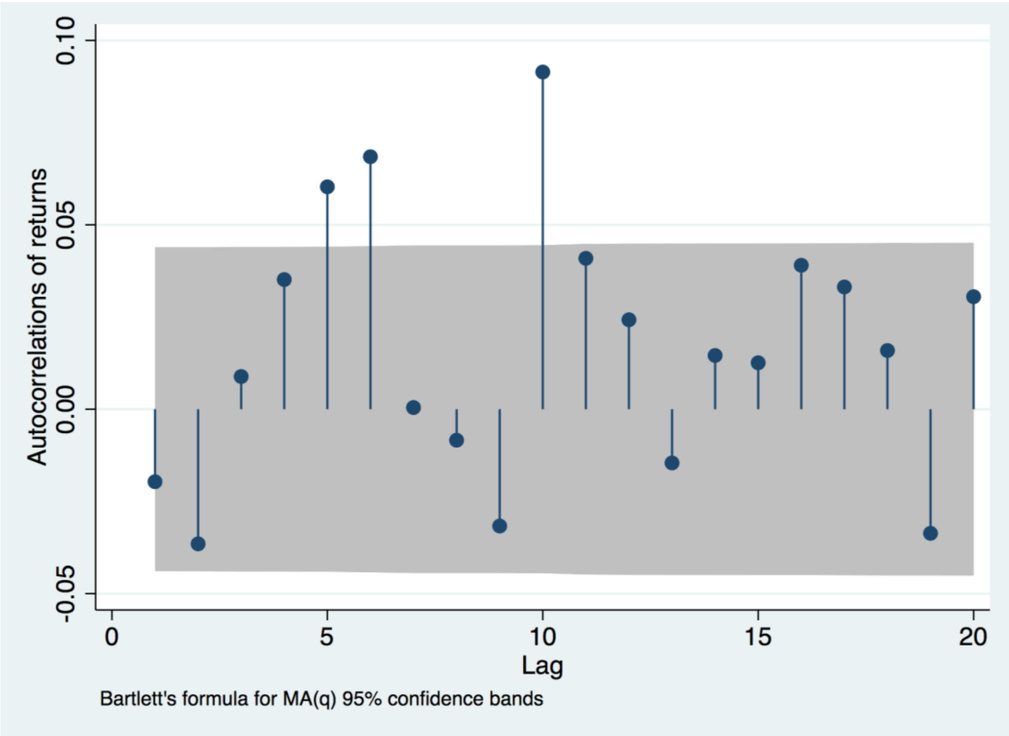

Figure A1. The autocorrelation function for log-returns

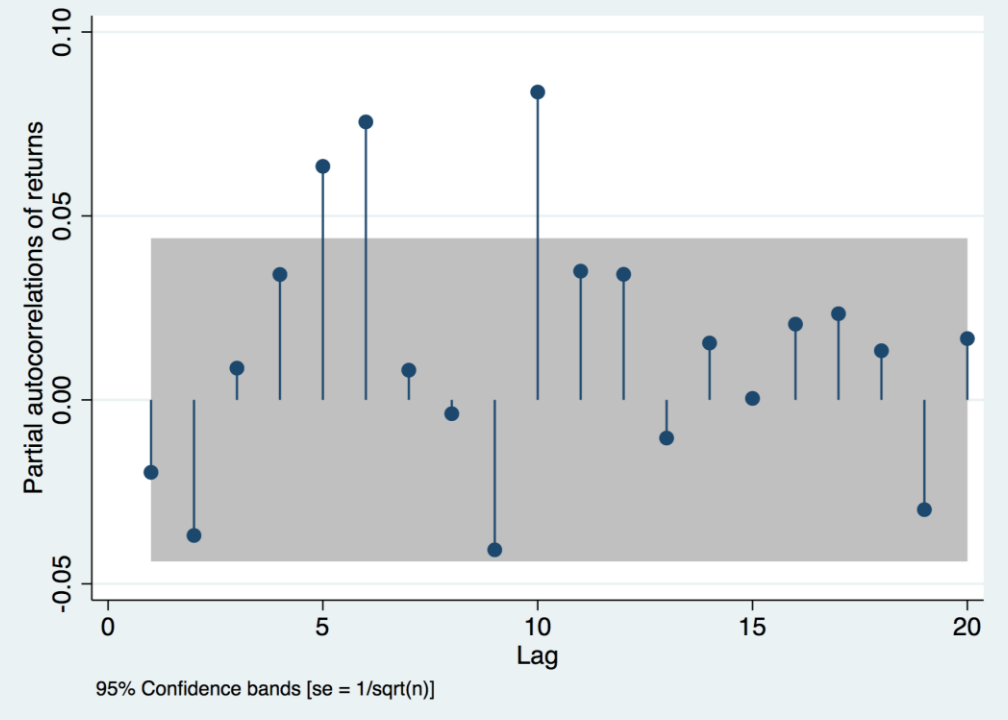

Figure A2. The partial autocorrelation plot for log-returns

Table A3 – Test for serial correlation in log-returns

AR(p)	Q-test standardized residuals
1	85.6517 (0.000)
2	85.0728 (0.000)
3	84.8919 (0.000)
4	88.1064 (0.000)
5	89.5785 (0.000)
6	67.6793 (0.004)
7	67.2115 (0.0045)
8	67.7133 (0.004)
9	69.0372 (0.0029)
10	56.8111 (0.0410)
11	56.3649 (0.0447)
12	57.2843

	(0.0375)
13	56.0574 (0.0473)
14	56.1412 (0.0466)
15	54.4529 (0.0634)

9.2 Appendix B – STATA do-file

```
// Conditional variance modelling bitcoin log-returns
import excel "/Users/thomaszernichow/Desktop/BTCXBP.xls", sheet("BTCXBP.csv") firstrow
tsset Date
gen returns = ln(ClosingPriceUSD[_n]) - ln(ClosingPriceUSD [_n-1])
browse returns
//Generate dummy variable
gen time = _n
gen dummy = 0
replace dummy = 1 if time<1532
// run STATA's Lagrange multiplier test for ARCH effects
regress returns       // fit constant only model (OLS)
archlm, lags(10)
//Augmented Dickey-Fuller test log-returns
dfuller returns

//GARCH(1,1) gaussian distribution
arch returns, arch(1) garch(1) het(dummy)
//Kolmogorov-Smirnov test via "archqq" command
archqq
// Ljung-Box test squared standardized residuals
predict e, resid
predict v, variance
gen eSqrtV = e/sqrt(v)
gen e2v = (e/sqrt(v))^2
wntestq e2v, lags(10)
//Predict conditional variance to use in statistical loss functions
predict variance
display variance

//GARCH(1,1) student´s t distribution
arch returns, arch(1) garch(1) het(dummy) distribution(t)
//Kolmogorov-Smirnov test via "archqq" command
archqq
// Ljung-Box test squared standardized residuals
predict e, resid
predict v, variance
```

```
gen eSqrtV = e/sqrt(v)
gen e2v = (e/sqrt(v))^2
wntestq e2v, lags(10)
//Predict conditional variance to use in statistical loss functions
predict variance
display variance

//GARCH(1,1) ged distribution
arch returns, arch(1) garch(1) het(dummy) distribution(ged) iterate(168)
//Kolmogorov-Smirnov test via "archqq" command
archqq
// Ljung-Box test squared standardized residuals
predict e, resid
predict v, variance
gen eSqrtV = e/sqrt(v)
gen e2v = (e/sqrt(v))^2
wntestq e2v, lags(10)
//Predict conditional variance to use in statistical loss functions
predict variance
display variance

//EGARCH (1,1) gaussian distribution
arch returns, earch(1) egarch(1) het(dummy) gtolerance(999)
//Kolmogorov-Smirnov test via "archqq" command
archqq
// Ljung-Box test squared standardized residuals
predict e, resid
predict v, variance
gen eSqrtV = e/sqrt(v)
gen e2v = (e/sqrt(v))^2
wntestq e2v, lags(10)
//Predict conditional variance to use in statistical loss functions
predict variance
display variance

//EGARCH(1,1) student´s t distribution
arch returns, earch(1) egarch(1) het(dummy) distribution(t)
//Kolmogorov-Smirnov test via "archqq" command
archqq
// Ljung-Box test squared standardized residuals
predict e, resid
predict v, variance
gen eSqrtV = e/sqrt(v)
gen e2v = (e/sqrt(v))^2
wntestq e2v, lags(10)
//Predict conditional variance to use in statistical loss functions
predict variance
display variance

//EGARCH(1,1) ged distribution
arch returns, earch(1) egarch(1) het(dummy) distribution(ged) gtolerance(999)
//Kolmogorov-Smirnov test via "archqq" command
```

```
archqq
// Ljung-Box test squared standardized residuals
predict e, resid
predict v, variance
gen eSqrtV = e/sqrt(v)
gen e2v = (e/sqrt(v))^2
wntestq e2v, lags(10)
//Predict conditional variance to use in statistical loss functions
predict variance
display variance

//GJR-GARCH(1,1) gaussian distribution
arch returns, arch(1) tarch(1) garch(1) het(dummy)
//Kolmogorov-Smirnov test via "archqq" command
archqq
// Ljung-Box test squared standardized residuals
predict e, resid
predict v, variance
gen eSqrtV = e/sqrt(v)
gen e2v = (e/sqrt(v))^2
wntestq e2v, lags(10)
//Predict conditional variance to use in statistical loss functions
predict variance
display variance

//GJR-GARCH(1,1) student´s t distribution
arch returns, arch(1) tarch(1) garch(1) het(dummy) distribution(t)
//Kolmogorov-Smirnov test via "archqq" command
archqq
// Ljung-Box test squared standardized residuals
predict e, resid
predict v, variance
gen eSqrtV = e/sqrt(v)
gen e2v = (e/sqrt(v))^2
wntestq e2v, lags(10)
//Predict conditional variance to use in statistical loss functions
predict variance
display variance

//GJR-GARCH(1,1) ged distribution
arch returns, arch(1) tarch(1) garch(1) het(dummy) distribution(ged)
//Kolmogorov-Smirnov test via "archqq" command
archqq
// Ljung-Box test squared standardized residuals
predict e, resid
predict v, variance
gen eSqrtV = e/sqrt(v)
gen e2v = (e/sqrt(v))^2
wntestq e2v, lags(10)
//Predict conditional variance to use in statistical loss functions
predict variance
display variance
```

```
//Sign-bias test and size-biased test
import excel "/Users/thomaszernichow/Desktop/BTCXBP.xls", sheet("BTCXBP.csv") firstrow
gen time=_n
tsset time
gen returns = ln(ClosingPriceUSD[_n]) - ln(ClosingPriceUSD [_n-1])
//Generate dummy variable
gen dummy = 0
replace dummy = 1 if time<1532

//For EGARCH (1,1) with gaussian distribution
arch returns, earch(1) egarch(1) het(dummy) gtolerance(999)
predict e, resid
predict var, variance
gen s_var = sqrt(var)
gen se_var = e/s_var
gen se2_var = se_var^2
//Sign-bias test
gen d_var = 0
replace d_var = 1 if L.e<0
replace d_var = . if missing(L.e)
regress se2_var d_var
//Negative size bias test
gen dneg_var = d_var*L.e
replace dneg_var = . if missing(L.e)
regress se2_var dneg_var
//Positive size-bias test
gen d1pos_var = (1-d_var)*L.e
replace d1pos_var = . if missing(L.e)
regress se2_var d1pos_var

//For EGARCH (1,1) with student´s t distribution
arch returns, earch(1) egarch(1) distribution(t) het(dummy)
predict e_, resid
predict var, variance
gen s_var = sqrt(var)
gen se_var = e/s_var
gen se2_var = se_var^2
//Sign bias test
gen d_var = 0
replace d_var = 1 if L.e<0
replace d_var = . if missing(L.e)
regress se2_var d_var
//Negative size bias test
gen dneg_var = d_var*L.e
replace dneg_var = . if missing(L.e)
regress se2_var dneg_var
//Positive size-bias test
gen d1pos_var = (1-d_var)*L.e
replace d1pos_var = . if missing(L.e)
regress se2_var d1pos_var

//For EGARCH (1,1) with ged distribution
```

```
arch returns, earch(1) egarch(1) distribution(ged) gtolerance(999) het(dummy)
predict e_, resid
predict var, variance
gen s_var = sqrt(var)
gen se_var = e/s_var
gen se2_var = se_var^2
//Sign-bias test
gen d_var = 0
replace d_var = 1 if L.e<0
replace d_var = . if missing(L.e)
regress se2_var d_var
//Negative size bias test
gen dneg_var = d_var*L.e
replace dneg_var = . if missing(L.e)
regress se2_var dneg_var
//Positive size-bias test
gen d1pos_var = (1-d_var)*L.e
replace d1pos_var = . if missing(L.e)
regress se2_var d1pos_var

//For GJR-GARCH with gaussian distribution
arch returns, arch(1) tarch(1) garch(1) het(dummy)
predict e, resid
predict var, variance
gen s_var = sqrt(var)
gen se_var = e/s_var
gen se2_var = se_var^2
//Sign bias test
gen d_var = 0
replace d_var = 1 if L.e<0
replace d_var = . if missing(L.e)
regress se2_var d_var
//Negative size bias test
gen dneg_var = d_var*L.e
replace dneg_var = . if missing(L.e)
regress se2_var dneg_var
//Positive size-bias test
gen d1pos_var = (1-d_var)*L.e
replace d1pos_var = . if missing(L.e)
regress se2_var d1pos_var

//For GJR-GARCH with student´s t distribution
arch returns, arch(1) tarch(1) garch(1) distribution(t) het(dummy)
predict e_, resid
predict var, variance
gen s_var = sqrt(var)
gen se_var = e/s_var
gen se2_var = se_var^2
//Sign bias test
gen d_var = 0
replace d_var = 1 if L.e<0
replace d_var = . if missing(L.e)
```

```
regress se2_var d_var
//Negative size bias test
gen dneg_var = d_var*L.e
replace dneg_var = . if missing(L.e)
regress se2_var dneg_var
//Positive size-bias test
gen d1pos_var = (1-d_var)*L.e
replace d1pos_var = . if missing(L.e)
regress se2_var d1pos_var

//For GJR-GARCH with ged distribution
arch returns, arch(1) tarch(1) garch(1) distribution(ged) gtolerance(999) het(dummy)
predict e_, resid
predict var, variance
gen s_var = sqrt(var)
gen se_var = e/s_var
gen se2_var = se_var^2
//Sign bias test
gen d_var = 0
replace d_var = 1 if L.e<0
replace d_var = . if missing(L.e)
regress se2_var d_var
//Negative size bias test
gen dneg_var = d_var*L.e
replace dneg_var = . if missing(L.e)
regress se2_var dneg_var
//Positive size-bias test
gen d1pos_var = (1-d_var)*L.e
replace d1pos_var = . if missing(L.e)
regress se2_var d1pos_var

// AR(p)-GARCH(1,1) modelling of log-returns and Ljung-Box test on standardized residuals
import excel "/Users/thomaszernichow/Desktop/BTCXBP.xls", sheet("BTCXBP.csv") firstrow
tsset Date
gen returns = ln(ClosingPriceUSD[_n]) - ln(ClosingPriceUSD [_n-1])
ac returns
pac returns
//AR(1)-GARCH(1,1)
import excel "/Users/thomaszernichow/Desktop/BTCXBP.xls", sheet("BTCXBP.csv") firstrow
tsset Date
gen returns = ln(ClosingPriceUSD[_n]) - ln(ClosingPriceUSD [_n-1])
arch returns, arch(1/1) garch(1/1) arima(1,0,0)
predict e, resid
predict var, variance
gen s_var = sqrt(var)
gen se_var = e/s_var
wntestq se_var
//AR(2)-GARCH(1,1)
import excel "/Users/thomaszernichow/Desktop/BTCXBP.xls", sheet("BTCXBP.csv") firstrow
tsset Date
gen returns = ln(ClosingPriceUSD[_n]) - ln(ClosingPriceUSD [_n-1])
arch returns, arch(1/1) garch(1/1) arima(2,0,0)
```

```
predict e, resid
predict var, variance
gen s_var = sqrt(var)
gen se_var = e/s_var
wntestq se_var
//AR(3)-GARCH(1,1)
import excel "/Users/thomaszernichow/Desktop/BTCXBP.xls", sheet("BTCXBP.csv") firstrow
tsset Date
gen returns = ln(ClosingPriceUSD[_n]) - ln(ClosingPriceUSD [_n-1])
arch returns, arch(1/1) garch(1/1) arima(3,0,0)
predict e, resid
predict var, variance
gen s_var = sqrt(var)
gen se_var = e/s_var
wntestq se_var
//AR(4)-GARCH(1,1)
import excel "/Users/thomaszernichow/Desktop/BTCXBP.xls", sheet("BTCXBP.csv") firstrow
tsset Date
gen returns = ln(ClosingPriceUSD[_n]) - ln(ClosingPriceUSD [_n-1])
arch returns, arch(1/1) garch(1/1) arima(4,0,0)
predict e, resid
predict var, variance
gen s_var = sqrt(var)
gen se_var = e/s_var
wntestq se_var
//AR(5)-GARCH(1,1)
import excel "/Users/thomaszernichow/Desktop/BTCXBP.xls", sheet("BTCXBP.csv") firstrow
tsset Date
gen returns = ln(ClosingPriceUSD[_n]) - ln(ClosingPriceUSD [_n-1])
arch returns, arch(1/1) garch(1/1) arima(5,0,0)
predict e, resid
predict var, variance
gen s_var = sqrt(var)
gen se_var = e/s_var
wntestq se_var
//AR(6)-GARCH(1,1)
import excel "/Users/thomaszernichow/Desktop/BTCXBP.xls", sheet("BTCXBP.csv") firstrow
tsset Date
gen returns = ln(ClosingPriceUSD[_n]) - ln(ClosingPriceUSD [_n-1])
arch returns, arch(1/1) garch(1/1) arima(6,0,0)
predict e, resid
predict var, variance
gen s_var = sqrt(var)
gen se_var = e/s_var
wntestq se_var
//AR(7)-GARCH(1,1)
import excel "/Users/thomaszernichow/Desktop/BTCXBP.xls", sheet("BTCXBP.csv") firstrow
tsset Date
gen returns = ln(ClosingPriceUSD[_n]) - ln(ClosingPriceUSD [_n-1])
arch returns, arch(1/1) garch(1/1) arima(7,0,0)
predict e, resid
predict var, variance
```

```
gen s_var = sqrt(var)
gen se_var = e/s_var
wntestq se_var
//AR(8)-GARCH(1,1)
import excel "/Users/thomaszernichow/Desktop/BTCXBP.xls", sheet("BTCXBP.csv") firstrow
tsset Date
gen returns = ln(ClosingPriceUSD[_n]) - ln(ClosingPriceUSD [_n-1])
arch returns, arch(1/1) garch(1/1) arima(8,0,0)
predict e, resid
predict var, variance
gen s_var = sqrt(var)
gen se_var = e/s_var
wntestq se_var
//AR(9)-GARCH(1,1)
import excel "/Users/thomaszernichow/Desktop/BTCXBP.xls", sheet("BTCXBP.csv") firstrow
tsset Date
gen returns = ln(ClosingPriceUSD[_n]) - ln(ClosingPriceUSD [_n-1])
arch returns, arch(1/1) garch(1/1) arima(9,0,0)
predict e, resid
predict var, variance
gen s_var = sqrt(var)
gen se_var = e/s_var
wntestq se_var
//AR(10)-GARCH(1,1)
import excel "/Users/thomaszernichow/Desktop/BTCXBP.xls", sheet("BTCXBP.csv") firstrow
tsset Date
gen returns = ln(ClosingPriceUSD[_n]) - ln(ClosingPriceUSD [_n-1])
arch returns, arch(1/1) garch(1/1) arima(10,0,0)
predict e, resid
predict var, variance
gen s_var = sqrt(var)
gen se_var = e/s_var
wntestq se_var
//AR(11)-GARCH(1,1)
import excel "/Users/thomaszernichow/Desktop/BTCXBP.xls", sheet("BTCXBP.csv") firstrow
tsset Date
gen returns = ln(ClosingPriceUSD[_n]) - ln(ClosingPriceUSD [_n-1])
arch returns, arch(1/1) garch(1/1) arima(11,0,0)
predict e, resid
predict var, variance
gen s_var = sqrt(var)
gen se_var = e/s_var
wntestq se_var
//AR(12)-GARCH(1,1)
import excel "/Users/thomaszernichow/Desktop/BTCXBP.xls", sheet("BTCXBP.csv") firstrow
tsset Date
gen returns = ln(ClosingPriceUSD[_n]) - ln(ClosingPriceUSD [_n-1])
arch returns, arch(1/1) garch(1/1) arima(12,0,0)
predict e, resid
predict var, variance
gen s_var = sqrt(var)
gen se_var = e/s_var
```

```
wntestq se_var
//AR(13)-GARCH(1,1)
import excel "/Users/thomaszernichow/Desktop/BTCXBP.xls", sheet("BTCXBP.csv") firstrow
tsset Date
gen returns = ln(ClosingPriceUSD[_n]) - ln(ClosingPriceUSD [_n-1])
arch returns, arch(1/1) garch(1/1) arima(13,0,0)
predict e, resid
predict var, variance
gen s_var = sqrt(var)
gen se_var = e/s_var
wntestq se_var
//AR(14)-GARCH(1,1)
import excel "/Users/thomaszernichow/Desktop/BTCXBP.xls", sheet("BTCXBP.csv") firstrow
tsset Date
gen returns = ln(ClosingPriceUSD[_n]) - ln(ClosingPriceUSD [_n-1])
arch returns, arch(1/1) garch(1/1) arima(14,0,0)
predict e, resid
predict var, variance
gen s_var = sqrt(var)
gen se_var = e/s_var
wntestq se_var
//AR(15)-GARCH(1,1)
import excel "/Users/thomaszernichow/Desktop/BTCXBP.xls", sheet("BTCXBP.csv") firstrow
tsset Date
gen returns = ln(ClosingPriceUSD[_n]) - ln(ClosingPriceUSD [_n-1])
arch returns, arch(1/1) garch(1/1) arima(15,0,0)
predict e, resid
predict var, variance
gen s_var = sqrt(var)
gen se_var = e/s_var
wntestq se_var
//AR(16)-GARCH(1,1)
import excel "/Users/thomaszernichow/Desktop/BTCXBP.xls", sheet("BTCXBP.csv") firstrow
tsset Date
gen returns = ln(ClosingPriceUSD[_n]) - ln(ClosingPriceUSD [_n-1])
arch returns, arch(1/1) garch(1/1) arima(16,0,0)
predict e, resid
predict var, variance
gen s_var = sqrt(var)
gen se_var = e/s_var
wntestq se_var
```

9.3 Appendix C – Eviews

We fit an AR(1) by the following method:

Quick-> Estimate Equation (Method is LS – Least Squares (NLS and ARMA)

In the Equation Estimation window we use the following command:

closing_price c closing_price (-1)
BDS is computed as follows:
closing_price.bdstest (m=s, e=1.5, d=6, 0=bdsvec)

We fit an AR(12) as well by the following method:
Quick-> Estimate Equation (Method is LS – Least Squares (NLS and ARMA)
In the Equation Estimation window we use the following command:
closing_price c closing_price (-1 to -12)
BDS is computed as follows:
closing_price.bdstest (m=s, e=1.5, d=6, 0=bdsvec)

9.4 Appendix D – Reflection note by Thomas Zernichow

This reflection note is related to the master thesis written by Edhem Misic and myself in the last semester of our study programme. It has been a demanding but educational process where we have gained much experience in problem solving. The thesis examines the impact of bitcoin futures contracts on bitcoin price volatility. The log-returns of the bitcoin to dollar exchange rate was examined for volatility clustering, time varying volatility and nonlinearity to justify the use of conditional variance models. We wanted to model volatility through the use of these conditional variance models, as we believe they capture the behaviours of the log-returns better than the alternatives. From here, the unconditional variance was estimated as a measure for realized volatility. We found that the GJR-GARCH (1,1) with student´s t distribution and EGARCH(1,1) with generalized error distribution were the best models according to different model selection techniques. The result from the estimation was that the unconditional variance has not decreased in the period after futures contracts were introduced in the bitcoin environment.

Although bitcoin has become a topic in mainstream media, it is in my opinion that a more people could benefit from studying its intentions, the underlying technology and philosophy. The extreme price fluctuations have possibly painted a picture of bitcoin that covers its potential. The themes of internationalization, innovation and accountability are three aspects central in every bitcoin debate taking place. Bitcoin is revolutionary concept that has the potential to turn the financial world up side down. It can reduce the cost of money transfers across national borders, decentralize the payment system process by removing the need of a central entity, enforce the security and anonymity of its users and connect the world´s population through the use of a global currency. At the same time, there are also problems related to these potential benefits that need to be addressed.

Bitcoin has the potential to become a global currency transcending national borders. One of the problems addressed in the bitcoin white paper is related to the transferring of money across borders by private persons. In fact, I have personal experience that transferring money from my own bank account to a relative in a third world country is both costly in money and time. In the bitcoin network, transaction fees and transfer time is significantly lower. However, it is important to point out that these benefits are only present if bitcoin can be used as a medium of exchange in the same way as fiat currencies.
For now, bitcoin transfers are more complicated than intended. If I were to send bitcoin to a relative in another country, I would need to convert Norwegian Krone to bitcoin. Furthermore, the relative receiving bitcoin must convert the bitcoin to the fiat currency used in his country. This is one of the reasons why bitcoin advocates believe that mainstream acceptance of bitcoin, as a payment method, is an important next step. Reducing the time and cost of transferring money across border is two of the benefits of bitcoin that can promote internationalization and unite the worlds population through a global currency.

The cryptography and blockchain technology are not innovative aspects of bitcoin, as these existed several years before the concept of bitcoin was presented. Actually, Satoshi Nakamoto refers to this in the white paper. It is the combination of the two technologies in addition to the proof of work and incentive layer that represents the true innovation of bitcoin. The combination of this represents a solution to the double spend problem often related to digital currency and incorporates game theory to promote desired behaviour. The miners of bitcoin have more to gain by confirming transactions and establishing proof of work than to trick the system. Introducing cryptography to link blocks of transactions results in a distributed ledger that protects users´ privacy. Consequently, the need of a central entity is removed in payment system. Clearly, this is a disturbing development in the financial world where banks make a lot of money on transaction fees.

Interestingly, critics condemn the privacy policy of bitcoin advocated by supporters. There has been a lot of suspicious behaviour related to bitcoin and cryptocurrencies in general. The dark-web site Silk Road was an illegal marketplace were buyers and users bought and sold narcotics with bitcoin. It had a similar structure as Amazon or eBay. This website was active between 2011-2013 and has often been used as an argument against the anonymity of bitcoin. Contrary to popular belief however, it is more difficult to stay anonymous using bitcoin because it requires numerous precautionary efforts, Security experts refers to bitcoin as pseudonymous, similar to writing a book under a pseudonym. If the pseudonym is linked to you, all your bitcoin transactions can be uncovered. This is where personal accountability and responsibility must be advocated.

The concept of bitcoin is still a hot debate topic in the mainstream world. Although most news is regarding price volatility or suspicious activity, it will be interesting to see if some of the benefits of bitcoin will be highlighted in the near future. This could lead to aacceptance of bitcoin as a payment method and begin its mainstream use. In the process of writing this thesis, some of the potential of bitcoin has become clearer for us. It will be interesting to follow its development.

9.5 Appendix E - Reflection note by Edhem Misic

The following reflection note will describe the main theme and findings of the master's thesis that has been completed in the final semester of the Business and Administration Master's programme at the University of Agder. Additionally I will relate the main themes of the School of Business and Law and explain how they are connected to our master's thesis. The themes are as follows: international, innovation and responsibility.

The thesis is about Bitcoin, specifically the price exchange rate between bitcoin and U.S. dollar. We study if the introduction of futures has had a statistically significant effect on the price volatility. In order to do this we use conditional variance models, several tests were conducted to assure the validity of the methodology in our thesis. We use the GARCH (1,1), EGARCH (1,1) and GJR-GARCH (1,1) model. Each of the models is also fitted under three different distributions, namely: gaussian, student's t and the generalized error distribution. Several model selection techniques were used to determine the best models. The GJR-GARCH (1,1) with student's t distribution and the EGARCH (1,1) with GED distribution were deemed to be the best models according to the selection techniques.
However no evidence of a decrease in volatility after the introduction of futures was found.

Bitcoin has gained much attention over the last few years by the media, mainly because of its exponential price growth. Bitcoin is still a new phenomenon and it will be very interesting to watch the development of Bitcoin. Furthermore Bitcoin offers a new way to send money across borders, trade without the need of a third party, and reduced fees when transferring money. The ledger stores all information and trade history and is constantly updated. Bitcoin also offers its users anonymity. However Bitcoin has been mis-used as well as, it has been used for embezzlement, investment scams and money laundering.
Digital money has existed for quite some time, however what makes Bitcoin special is that it has been built using an innovation, namely the blockchain. Bitcoin is still in an early stage of its development, how it will develop further only time will tell. Supporters of Bitcoin believe it will develop into a currency that is accepted worldwide, now that would be quite innovative. Perhaps one of the biggest innovative aspects of Bitcoin is the fact that it does not need a third party in order to transfer money from one person to another.

This thesis however does not explore the development of Bitcoin itself rather our focus has been the use of statistical techniques in order to determine if the introduction of bitcoin futures has had an impact on the price volatility of bitcoin to U.S. dollar exchange rate.

The second theme that the School of Business and Law values is internationalisation. This applies to Bitcoin as Bitcoin aims to transcend borders and be used across the world. Furthermore the development has led the world to become much more connected. Bitcoin could play a major role if it were to develop into a currency that can be used internationally. As mentioned previously Bitcoin offers faster transactions and lower fees, as well as removing the need for a third party.

We also see that more and more companies are accepting bitcoin as payment. The potential for Bitcoin is there but I cannot predict whether it will become a global currency or not in the future.

The final theme that I will comment on is responsibility. Bitcoin has been used on the dark web, it was used on a site called Silk Road. Users could buy and sell illegal substances, guns and much more. We also read about a case where an assassin was hired and paid in bitcoin. However even though Bitcoin can be abused in many ways the same can be said about fiat currencies. In my opinion every individual has his or her own responsibility when it comes to how they wish to use bitcoins, the same can be said about fiat currencies. The phenomenon is still new, and it is not clear how it will develop in the future, nevertheless it will be very interesting to watch.