Optimized Portfolios vs. the Naive-Diversification Strategy

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Preface

This master thesis is written as the final part of my master degree in Economics and Business Administration, with specialization in financial economic at School of Business and Law at the University of Agder. The masters program has given me a deeper understanding of financial theory. My interest for financial economics inspired me to explore these topics in a master thesis. I want to thank my supervisor Valeriy Zakamulin for always being available and gave me clear and constructive guidance throughout the process of writing this thesis. His guidance helped me a lot and without the supervision it would not have been possible. I will also thank my sister Lina, and also my nearest classmates for the working atmosphere and the writing process with inputs at The University of Agder.

Willy Ly

Kristiansand, June 2019

Abstract

The novelty of my thesis is to add to the academic debate introduced by DeMiguel, Garlappi, and Uppal (2009) an attempt to answer the question whether optimized portfolio strategies consistently outperform the naïve-diversification strategy. Earlier academic studies that have tried to defend the optimized portfolios strategies against the naïve-diversification strategy are Kritzman, Page & Turkington (2010) and C.Kirby and B. Ostdiek (2010). But there are also some weaknesses by these studies that the datasets provided by Kenneth French and the performance is measured by means of the Sharpe Ratio. The study by Zakamulin (2017) aims to provide a cautionary note regarding the use of Kenneth French datasets in portfolio optimization, without controlling whether the superior performance appears due to better mean-variance efficiency or due to exposures to established factor premiums. Also, almost all datasets in the Kenneth French online data library contains the low volatility anomaly. In this thesis I want to do a research and find out the answer of the thesis by using 16 datasets provided by Kenneth French (2018), to find out if the optimized portfolio strategies can consistently outperform the naïve-diversification strategy. Optimized strategies are simulated over a period from January 1963 to December 2018. The performance is measured by means of Sharpe ratio and Alpha, the Capital Asset Pricing Model and the Fama-French 3-factor model. I will focus especially on the Fama-French 3-factor. Additionally to the naïve-benchmark strategy, this thesis covers a study of 4 optimized strategies. The results show that the optimized portfolio strategies cannot outperform the naïve-diversification, either can the naïve-diversification beat the optimized portfolio strategies. By the 95% significance we cannot reject the hypothesis and the hypothesis will be equal like the paper of DeMiguel et al (2009). But with that choosing a specific optimization portfolio strategy does not guarantee a poorly performance than the Naïve-diversification strategy when the set of portfolios are chosen arbitrarily.

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1 Introduction

The Modern Portfolio Theory (MPT) was proposed by Harry Markowitz during the decade of 1960´s, the MPT was further extended to the Capital Asset Pricing Model by William Sharpe and others. Subsequently, even though the early empirical tests of the CAPM cast doubt on the validity of this theory, both the MPT and CAPM were accepted by academics during 1970s. Only by the end of 1990s some academics raised the concerns that the popular investment advice contradicts the MPT because in the MPT the composition of the optimal portfolio does not depend on the invertors risk aversion. Also in MPT the composition of the optimal portfolio does not depend on the investment horizon length (Time Diversification Puzzle).

Later, DeMiguel et al. (2009) and Duchin and Levy (2009), raised the concerns that there is no scientific evidence that optimized (in accordance with the MPT) portfolios outperform the naïve diversification strategy. There are lots of academic studies related to this topic. I will mention these in literature review like Kritzman, Page and Turkington (2010), Tu and Zhou (2011), Kirby and Ostdiek (2012), and many others. Kritzman et al (2010) argue that optimized portfolios (mean-variance and minimum-variance) are better than the naïve portfolio. However, Kritzman et al (2010) do not present any scientific evidence. They compute the Sharpe ratios, but do not test the hypothesis of equal Sharpe ratios. When it comes to deficiencies of studies that defend the optimization strategies, it is that all of them have some common. All the datasets from Kenneth French contains low-volatility.

The problem for the thesis is optimized portfolios can consistently beat the naïve diversification strategy. The problem is related to the academic debate initiated by the study by DeMiguel, Garlappi, and Uppal (2009). Kenneth French provides the data for the thesis. All datasets represent value-weighted portfolios formed using different criteria. In this thesis I want to do a research and find out the answer of the thesis by using 16 datasets provided by Kenneth French to find out if the optimized portfolio strategies can consistently outperform the naïve-diversification strategy, even when low-volatility are in all datasets.

Almost all of the datasets have return on 10 stock portfolios, and only one with returns on 25 stock portfolios. The data for all datasets is at a monthly frequency. As the start date use January 1963 and end at December 2018. In addition to the naïve diversification strategy, the following of optimized portfolios are minimum-variance, maximum-diversification and risk parity. Figures that are explained in the thesis have been placed in the appendix out of respect to the text space and flow.

The methodology of out-of-Sample testing is where I am using 5-year rolling window to estimate the variance-covariance matrix and the vector of expected return. Then the composition of the optimized portfolios each month is rebalanced. There are no transaction costs.

The outcome of the empirical study contains the out-of-sample Sharpe Ratios, alpha CAPM and 3-factor Fama-French of each portfolio as well as the p-values of the tests of Sharpe Ratio, alpha CAPM and 3-factor Fama-French. This problem is of interest because I wanted to find out if any datasets could reach to beat the naïve diversification with the optimized portfolios, by picking more and different datasets from earlier research, especially with the Fama-French 3-factor model. Also, to test with the portfolios strategy that I just mentioned.

In Modern Portfolio Theory (MTP) managing when trying to calculate the correct expected return on a portfolio, it is essential that the optimal volatility return will be calculated correctly (Markowitz [1952]). The Capital Asset Pricing Model (CAPM) by Sharpe (1964), and others, contributed to the modern portfolio theory and built further on the assumption that the individual investor´s desire and preference for a portfolio can be explained by a utility function of the two parameters, expected return and risk.

By using 16 datasets all provided by Kenneth French and three different optimized strategies, I did research into this interesting topic by testing for the equality of performance of the optimized strategies and naïve rule using the Sharpe Ratio and also CAPM alpha and Fama-French 3 factor-model. My goal is to find if these 16 datasets can beat the naïve diversification strategy by the optimized strategies, and if there is any results and conclusion instead of the papers from earlier research. The biggest goal is to find if there are some different result with Fama-French 3 factor model.

The novelty of my thesis is structured as follows. I will start with the literature review where I will tell about the Modern Portfolio Theory (MTP) and some others research on the topic. Then I will provide the descriptive statistics for the data that are used, the methodology following the strategies and then the empirical results and finally discussion of the assumed results and conclusion. The literature review is to give a background to the research and to defend the paper as contribute to the debate. The section of data describes and presents the datasets that I have chosen to download and used in the research of the paper. The methodology has the different optimized portfolio strategies, that are briefly derived and referenced to relevant literature. The empirical results there are tables of the tests that are presented and analyzed. Some of the figures that come from the empirical results have been placed in the appendix, because of the space in the paper. Then, the last section I will give a discussion of the findings and the research, with a conclusion.

Further, I can conclude that the optimized strategies either outperform naïve diversification frequently or consistently for the data at hand in the period that is chosen, and the other way around.

The returns of the strategies are constructed using the datasets and the methodology. As follow, the estimated measures of performance are presented and analyzed. The findings of the empirical result and the approach of the research are later discussed.
The cumulative returns of the strategies of the dataset are represented in time series in the figures.

2 Literature review

2.1 Modern portfolio theory and capital asset pricing theory

MTP. In this chapter of the thesis I will describe some research studies that previously has been completed on related topics that I cover in this thesis. Many studies claim defend the value of portfolio optimization since the publication of the paper by DeMiguel et al. (2009). But first we can take a look back to the theoretical framework that past into the history of the Modern Portfolio Theory (MTP), to get it from a historical perspective. It starts with Markowitz (1952), where he wrote "Portfolio Selection", introduced as fundamental concepts of MTP.

CAPM. The Sharpe Ratio was derived in 1966 by William Sharpe, where he introduced the performance in "Mutual Fund Performance" (Sharpe, 1966, p.123) and has been one of the most referenced risk/return measures used in finance, and this popularity can be attributed to its simplicity. Some years earlier Sharpe (1964) formed on MTP with "Capital Asset Prices: A theory of market equilibrium under conditions of risk" where he introduced an important concept for that later, and with others (Treynor (1961), Lintner (1965) and Mossin (1966)), that creates what is known as the Capital Asset Pricing Model (CAPM). The CAPM have a relationship with Jensen´s Alpha, which was a performance introduced by M. Jensen. The CAPM relationship uses to estimate abnormal returns (Jensen, 1968, p.394). W. Sharpe also mentioned this in his model a few years earlier (Sharpe, 1963, p.283).

2.2 Anomalies within CAPM

The low-volatility portfolios and the minimum-variance portfolio are often essential in these studies, and therefor interesting for this thesis. Low-volatility strategies are portfolios of less risky assets, with the function of lowering the portfolios volatility. Based on traditional assumptions about the risk-reward connection, such strategies would expect to deliver lower risk-adjusted returns. This is based when taking more risk, one would expect to be compensated by earning higher return. There is expected to hold for another important factor in low-volatility investing, which is low-beta. This is according to CAPM, portfolios with high beta that have higher expected returns than portfolios with low beta. By testing the low-beta strategies out-of-sample, they find such portfolios frequently deliver equal or better risk-adjusted returns than high-beta strategies.

The prediction of the risk-award relationship and the debate that concerning the CAPM is not new. In the studies of Black, Jensen and Scholes (1972) they criticized the CAPM, where they discovered that low-beta stocks deliver better risk-adjusted returns than high-beta stocks. In more recent times, the topic of low-volatility strategies has become more regularly mentioned and several papers got the same conclusions.

The literature proposes some different explanations as for why the minimum-variance portfolio outperforms the market. One has to implement factor models in an attempt to explain the returns by their exposure to different sources of risk. Blitz and van Vliet (2007) found that it was still significant alpha present in their low-volatility portfolios after controlling for size, value and momentum effects. They also concluded that regression analysis with classical risk factors could not explain the volatility effect in full and that low-volatility stocks had low betas.

A recently popularized low-volatility asset-allocation that mentioning is the risk parity strategy. There are several approaches to constructing a risk-parity portfolio, the general objective is to weight each asset in proportion to their risk so that every asset will have an equal risk contribution to the total risk of the portfolio. The portfolio overweighs less volatile assets and underweights assets with higher volatility. An advantage of the risk-parity portfolio, similar to that of the minimum-variance portfolio, is that it only requires the covariance matrix in its construction.

Chow, Hsu, Kuo and Li (2013) provide a comprehensive survey of low-volatility strategies. The paper points out that since global financial crisis, low volatility portfolios based on U.S assets have outperformed the market by delivering higher returns and Sharpe Ratios, with only two/thirds the volatility risk. They also found that low-volatility portfolios generally deliver superior returns in the long term across several countries. The different low-volatility strategies, they do not find that one construction method is better than another from a return perspective.

2.3 Research on the performance of optimized portfolios

The topic of naïve diversification being able to consistently outperform the optimized strategies has been a discussion in modern time, where the paper considered by V. DeMiguel, L. Garlappi and R. Uppal have found that none of the optimized portfolio strategies consistently perform out-of-sample in standings of Sharpe Ratio related to naïve-diversification. Further, a paper by R. Duchin and H. Levy (2009) concluded that naïve diversification, while having similar risk as measured by standard deviation with the mean-variance optimized strategy, had a greater average expected return when the assets in the portfolio was small.

Some academic studies backing the idea of optimized portfolio strategies being able to consistently outperform the naïve diversification as well. In a study by Turkington, Page and Kritzman the confidence that naïve diversification may be superior to optimization, where DeMiguel et. al. did not claim this, was rejecting from improbable assumptions and presenting the opinion that optimized portfolio strategies outperform the naïve rule. (Kritzman, Page & Turkingston, 2010, p. 37). Further, J.Tu and G. Zhou (2011) recommended a combination of the naïve diversification rule and one of the four sophisticated strategies. Most of the cases they outperform the naïve diversification, but they also found a combination with the sophisticated strategies. Then, C.Kirby and B. Ostdiek (2012) introduce two optimization strategies: volatility timing and reward-to-risk timing. And both the strategies outperform the naïve diversification, even when there were high transaction costs.

Mostly of these academic studies we can see, they tried to defend the optimized portfolio that can beat the naïve-diversification strategy. The weakness to these studies is that different portfolio optimization methods are implemented, using the datasets generously provided by Kenneth French and the performance is measured by means of the Sharpe Ratio. Zakamulin (2017) aims to provide a cautionary note regarding to the use of Kenneth French datasets in portfolio optimization without controlling whether the superior performance appears, due to better mean-variance efficiency or due to exposures to established factor premiums. And the low-volatility effect is present in virtually all datasets in the Kenneth French online data library. Further, when there are used a few simple portfolio optimization models that are said to outperform the naïve-diversification, it shows that these portfolios are tilted towards assets with lowest volatilities, and when the low-volatility effect have been controlled, there is absolutely no evidence of superior performance.

3 DATA

The data used in the research are 16 arbitrary chosen datasets provided by Kenneth French (2018) and are listed in Table 1. They have constructed the first dataset by get each stock from NYSE, AMEX and NASDAQ into book-to-market portfolios. The datasets represent value-weighted portfolios formed using deciles of the variables in the table.

The datasets originally beginning in July 1927 with exception of some dataset like E/P and CFP that starts in July 1951. But the starting period in this thesis is January 1963 excluding a 5-year in sample. This is called training period prior to this. This is to show the understanding of the out-of-sample or testing period. The frequency in the datasets is monthly rate of returns from July 1963 to December 2018, for a total of 672 monthly observations. In addition, the Fama-French factors provide the risk-free rate of return, the market premium rate of return and two other market factors that will be presented in the methodology. In the robustness tests, we also consider the set of portfolios from the Fama-French 3-factor model.

Table 1 shows the name and number of portfolios for all the datasets. For all these datasets there are one more dataset containing the Fama-French factors that was used. There is also a table of the summary statistics for the risk-free rate in the testing period that are handed in appendix. This was to extract the risk-free rate to compute the excess return, as a restriction in one of the strategies and to perform the equality of alphas test.

3.1 Data

Table 1: Kenneth French datasets

#	**Dataset**	**No.**
1	Book-to-market	10
2	Size	10
3	EP	10
4	Industry	10
5	Size and BM	25
6	Investment	10
7	Momentum	10
8	Profit	10
9	Accruals	10
10	Cashflow Price	10
11	Dividend Yield	10
12	Short Term Reversal	10
13	Long Term Reversal	10
14	Market Beta	10
15	Net Share Issues	10
16	Residual Variance	10

# denotes the dataset number, whereas **No.** denotes the number of portfolios in each dataset.

4 METHODOLOGY

In this section the techniques that are implemented to relevant portfolios will be presented. The CAPM and Fama/French 3 factor-model will be presented, to estimate alpha. The statistical software R is the source that is used for all estimation and computational resolutions. The resolutions for R code will be in the appendix.

4.1 Strategies

This thesis applies five portfolio optimization strategies. All portfolio optimization strategies have the advantage of utilizing information of the parameters of portfolio returns.

Table 2: List of strategies used in this thesis.

	Strategy	Abbreviation
1	Naïve-diversification	Naïve.div.
2	Minimum-variance	Min.-var
3	Maximum-diversification	Max.-div.
4	Risk-parity	R.port

4.1.1 Naïve-diversification

The naive-diversification strategy is independent of the statistical properties of historical returns and is completely dependent on the number of assets. The Naïve-diversification strategy is often credited to the Babylonian Talmud, where it was stated that one´s money should always be equally divided into three; land, merchandise and for keeping ready at hand, which can be interpreted as a Naïve-diversification of three assets. The weights of assets in the Naïve-diversification strategy are given by the

$$w_{i=} \frac{1}{N} > 0 \tag{1}$$

Where N is the total number assets.

4.1.2 Minimum-variance portfolio

Traditional portfolio construction depends on estimates of future returns. The estimations are not very precise and exceptionally to estimation error, which in turn leads to suboptimal performance. When we hypothesis a minimum variance portfolio we only need the covariance between historical returns. This reduces the risk as the covariance matrix can be estimated more precisely (Kempf and Memmel (2002)). The concept of minimum variance dates back to the effort of Markowitz (1952), which shaped modern portfolio theory, as we know it.

By the fact that the variance-covariance matrix, Σ, of stock returns can be estimated much more precisely than the mean returns, Clarke, de Silva, and Thorley (2006) proposed to implement the minimum-variance portfolio (the goal is to minimize the risk of the diversification). It turns out that this strategy not only has low risk, but also a quite high Sharpe ratio.

The return on the on the investors portfolio is given as

$$X_p = \Sigma w_i x_i \text{ , } subject\ to\ \Sigma\ w_i = 1 \tag{2}$$

Where x_p is the return on the portfolio, w_i is the weight invested in asset *i,* and x_i is the return from asset *i*. Now the relevant weights of the minimum variance portfolio are computed so that the resulting portfolio has the lowest return variance, σ_p^2, for a given covariance matrix Σ. The covariance matrix will consist of variances and covariances of the returns. While the variance of the portfolio returns can be expressed as

$$\sigma_p^2 = w'\Sigma \mathrm{w} \tag{3}$$

The minimum-variance portfolio can further solve the weights for the following minimization problem, according to

$$\min_{w} \frac{1}{2}\ w'\Sigma \mathrm{w}, \text{subject to } \sum_{\mathrm{i}=1}^{\mathrm{N}} w_i = 1 \tag{4}$$

Where 1 is a column vector of one and w is a column vector of the portfolio weights. Solving the minimization problem we get relevant weights for constructing the minimum variance portfolio, and assume that all mean returns are alike.

In theory, the MVP can be constructed with weights taking both long and short positions. However, in practice one often imposes long positions constraint. The reasoning for this is based on many factors. In reality short sales are not possible for every asset and can also be a very risky strategy, so a long/short portfolio might be unwanted for many investors in real life.

4.1.3 Maximum-diversification

The weights of Maximum-diversification strategy is to find the solution of the maximum diversification portfolio and then we use

$$\max_w \frac{w'\sigma}{\sqrt{w'\Sigma w}}, subject\ to\ \sum_{i=1}^{n} w_i = 1 \qquad (6)$$

where $\boldsymbol{\sigma} = [\sigma_1, \sigma_2, \dots, \sigma_n]$ is the vector of standard deviations.

The ratio $\frac{w'\sigma}{\sqrt{w'\Sigma w}}$ is called the diversification ratio.

Choueifaty and Coignard (2008) defined the diversification ratio as the ratio of the weighted average of standard deviations devided by the standard of the strategy. The Maximum-diversification strategy provides similar results to other strategies under certain conditions. The maximization problem in the formula, is to constraint stating that the sum of weights must equal 100% if the expected excess return of portfolios is proportional to their standard deviation. The maximum-strategy is similar to the Minimum-variance strategy, only if all portfolios have the same standard deviation. If the correlation matrix is invertible and some other conditions are fulfilled, then the Maximum-diversification strategy provides similar results to the Risk-Parity strategy (Choueifaty & Coignard, 2008, p. 41-43)

The portfolio that has the minimum variance is heavily concentrated in the asset with the lowest standard deviations. The maximization of the diversification ratio helps to mitigate this problem and invest more in the other assets.

4.1.4 Risk parity

The Risk-Parity strategy allocates risk equally between all portfolios. The weights are computed by the formula and you can also see the normalization factor calculated by the formula. Relative to the Tangency strategy, the Risk-parity overweighs safer portfolios (Asness, Frazzini & Pedersen, 2012)

The weights of the asset *i* is inversely proportional to its standard deviation and is determined by

$$w_{i,t} = \frac{1}{\sigma_{i,t}} k_t, \tag{7}$$

where k_t is the normalization factor (that makes the sum of all weights to equal to one)

$$k_t = \frac{1}{\sum_{i=1}^{N} \frac{1}{\hat{\sigma}_{i,t}}} \tag{8}$$

4.2 Performance measures

4.2.1 Sharpe Ratio

The Sharpe Ratio was developed in 1966 by William Sharpe. The Sharpe-Ratio is a single measure of risk and returns and is always defined against the risk-free rate of return. It is given an indication to evaluate the performance of the portfolios, also procedures the relationship between risk and return. This is one of the most famous and well-used ratios. The ratio can explain a lot suitable to the performance and it is simple to implement, because it only consists of three components. It is given by

$$\mathrm{SR_p} = \frac{E\,(\mu_p - r_{f)}}{\sigma_p} \tag{9}$$

Where μ_p = Return from portfolio p

r_f = risk free rate

σ_p = standard deviation of μ_p

E $(\mu_p - r_{f)}$= expected excess return

The higher Sharpe Ratio, the more excess return portfolios can expect to deliver for extra volatility that they are exposed to by holding a riskier asset, that mean the best performing portfolio would be the one with the highest Sharpe Ratio.

The Sharpe Ratio is derived as the slope of the CAL. The Sharpe Ratio measures a portfolios performance by its expected excess return divided by expected risk.
It is expected that different portfolios would have different Sharpe Ratios. Test given by Jobson and Korkie (1981) could enquire whether the Sharpe Ratios are significantly different or not. The null hypotheses are as follows:

$$H_0: SR_{1/N} = SR_{opt.} \quad \wedge \quad H_1: SR_{1/N} \neq SR_{opt}$$

To measure the performance of strategies, the Sharpe ratio by W. Sharpe (1994) is estimated. The Sharpe Ratio test have the null-hypothesis $H_0 : SR_1 - SR_2 = 0$. The Sharpe Ratio are the test statistic divided by its asymptotic standard deviation, estimated by the formula where SR_1 and SR_2 is the Sharpe Ratio of strategy 1 and 2 and ρ is the correlation coefficient between the excess return of the two strategies

$$Z = \frac{SR1-SR2}{\sqrt{\frac{1}{T}}[2(1-\rho)+\frac{1}{2}(SR_1^2+SR_2^2-2SR_1SR_2\rho^2)]} \tag{10}$$

Where T = number of observations

p = correlation coefficient between the levels of SR_1 and SR_2

Which is asymptotically distributed as a standard normal. When the p-value of the test is below than the predetermined significance level (usually 0.05), then one rejects the null hypothesis.

4.2.2 Alpha CAPM

Usually the single factor model is motivated by the CAPM. In the CAPM, the Capital Allocation Line (CAL) is derived as possible combinations between a portfolio and a risk-free asset. The Capital Market Line (CML) is explained as the specific CAL, which is tangent to the efficient frontier, at the point of which the Sharpe Ratio is the highest.

Alpha estimates are also a performance. The Alphas are estimated by Ordinary Least Squares (OLS) of factor models, which also called intercepts of linear regressions. The single-factor model is estimated by the OLS to abtain the Alpha estimate

$$R_t = \alpha + \beta R_{mkt,t} + \epsilon_t \tag{11}$$

where the factor model R_t explain the excess return of a strategy, by factors of the market. $R_{mkt,t}$ are the single-factor model that are explained by CAPM where the factor of the model is the excess market return.

If the market return fully explains the return on the risky asset, then the model predicts and the model is estimated by running OLS and then testing the hypothesis

$$H_0: \alpha_{1/N}^{CAPM} = \alpha_{opt.}^{CAPM} \wedge H_1: \alpha_{1/N}^{CAPM} \neq \alpha_{opt.}^{CAPM}$$

4.2.3 Fama-French 3-factor

The Fama-French 3-factor model by Fama and French is a multiple factor model (1993). Also to the excess market return, the Fama-French 3-factor model includes factors related to firm size and book-to-market equity (BE/ME). The factors that are used is High-Minus-Low (HML) and Small-Minus-Big (SMB). The HML-factor is meant to capture the risk in returns related to BE/ME and is measured by monthly difference between the average of returns on two high-BE/ME portfolios and the average of returns on two low-BE/ME portfolios. The SMB-factor are meant to capture the risk in returns related to size and is measured by the monthly difference between the average of return on three small-stock portfolios. The multiple factor model that the formula show is estimated by the OLS to find an Alpha estimate (Fama & French, 1993, p. 9)

$$R_t = \alpha + \beta_1 R_{mkt,t} + \beta_2 SMB_t + \beta_3 HML_t + \epsilon_t \tag{12}$$

Where SMB (Small-minus-Big) is the risk factor related to size and HML (High-minus-Low) is the risk factor related to value.

The alphas are tested with tested with the hypothesis

$$\text{Test } H_0: \alpha_{1/N}^{FF3} = \alpha_{opt.}^{FF3} \wedge H_1: \alpha_{1/N}^{FF3} \neq \alpha_{opt.}^{FF3}$$

It is interest to test the equity of Alphas between strategies. The Alpha is presented with a t-test to the test statistic, where σ is the standard error of the Alpha estimate, and it is calculated by the standard deviation of the residuals, where the ρ is the correlation between the residuals

$$Z = \frac{\alpha 1 - \alpha 2}{\sqrt{\frac{1}{T}(\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2)}} \tag{13}$$

4.3 Back-testing

Back-testing, also called out-of-sample simulation, is a method for empirically testing strategies based on historical data, and to also measure the performance of strategies in practice. Back-testing is useful because there are many possible strategies, and a portfolio manager wants to find out which strategy performs best. The back-testing requires the look-back period, the testing period and the training period to be defined. The training period is the sample that is the data that is known and utilized at the first investment. The testing period is the out-of-sample, where the returns of the strategies are simulated. The look-back period is the rolling window, which is a constant time length in the past that the investment is based on. The training period has the same length as the look-back period. In back-testing the portfolios are functional, which means that the weights in the strategies are re-balanced each month. The weights are re-computed by the optimization of the strategies with the data in the look-back period.

Back-testing must be considered in any market frictions such as transaction costs. Therefore the re-estimation may be done in some other frequency than monthly.

The length of the rolling window is 5 years that is the same length as the training period. For all other estimations, the illustrative statistics that are shown for the period is from July 1968 to the end of available data, December 2018. This is to show the understanding of the out-of-sample or testing period.

In the back-testing, the historical mean and covariance are estimated by Shrinkage. The Shrinkage intention is to decrease estimation error. The Shrinkage estimation has the historical mean that applies the method of James and Stein (1961) and the Shrinkage estimation of the covariance matrix applies the method of Ledoit and Wolf (2003). The Shrinkage estimator of covariance and mean are estimated by the formulas.

$$\hat{\mu}_t^{Shrink} = (1 - \delta_t)\hat{\mu}_t^{Hist} + \delta_t \hat{\mu}^{Target} \tag{14}$$

$$\hat{\sum}_t^{Shrink} = (1 - \delta_t)\hat{\sum}_t^{Hist} + \delta_t \hat{\sum}^{Target} \tag{15}$$

where $\hat{\sum}_t^{Shrink}$ is the shrink estimated covariance matrix, the δ_{t} is called the Shrinkage constant. The shrinkage constant takes on a value between 0 and 1 and determines the combining ratio between the rolling historical estimate and the Shrinkage target in the Shrinkage estimate. The optimal value δ_t is the one that minimizes the expected value of the quadratic loss function between the shrinkage estimator and the true covariance matrix. This constant is an estimate that is why it is denoted with a hat on Equation. The optimal Shrinkage constant minimizes the expected distance between the Shrinkage estimator and the true covariance matrix (Ledoit & Wolf, 2003, p.7)

The estimation is a weighted convex linear combination of the rolling historical estimate, and the sample estimate based on the look-back period, and the Shrinkage target. The rolling historical mean and covariance are estimated by the formulas. Both are calculated on the data in the rolling window. (Ledoit & Wolf, 2003, p. 5-6)

$$\hat{\mu}_t^{Hist} = \frac{1}{L}\sum_{i=t-L}^{t-1} r_i$$

$$\hat{\Sigma}_t^{Hist} = \frac{1}{L}\sum_{i=t-L}^{t-1} \epsilon_i \epsilon'_i$$

Here is L the length of the look-back period

The Shrinkage target to the rolling historical mean is the expected return to the Naïve-diversification strategy in the look-back period. The Shrinkage target to the rolling historical covariance is the covariance matrix of the Constant Correlation Model (CCM). In the model the correlations are assumed to be identical for all portfolios return (Ledoit & Wolf, 2003, p. 6-11)

5 Empirical results

Annualized Sharpe Ratios with the p-values of the Sharp Ratio test in parentheses. Sharpe Ratios that are significantly different from the Sharpe Ratio of the naïve-diversification strategy at a 5%-significance level are bolded.

Table 3 shows Sharpe Ratio with p-values of the Sharpe ratio test and the p-values of the significance test are in parentheses of each optimized strategies. Further, Table 4 shows Alpha estimates from the OLS estimation of the CAPM motivated Single-factor model. P-values of the Alpha test are in parentheses of each optimized strategies. After that, Table 5 show that we have Alpha estimates from the OLS estimation of the Fama-French 3-factor model. The p-values of the significance test are in parentheses of each optimized strategies.

Table 3: SR: Sharpe Ratio

		Strategy			
#	**Dataset**	**Naive**	**Min.var**	**Max.div**	**R.P.**
1	BEME	0.53	0.51 (0.75)	0.48 (0.07)	0.53 (0.24)
2	Size	0.47	0.42 (0.37)	0.46 (0.66)	0.47 (0.81)
3	EP	0.56	0.50 (0.30)	0.53 (0.76)	0.54 (0.75)
4	Ind	0.48	0.51 (0.69)	0.49 (0.88)	0.50 **(0.02)**
5	BEME25	0.50	0.49 (0.92)	0.44 (0.12)	0.52 (0.06)
6	Inv	0.54	0.58 (0.28)	0.54 **(0.00)**	0.56 **(0.00)**
7	Mom	0.38	0.44 (0.11)	0.42 (0.23)	0.40 **(0.006)**
8	Profit	0.47	0.59 **(0.01)**	0.44 **(0.00)**	0.49 **(0.00)**
9	Acc	0.50	0.61 **(0.01)**	0.51 **(0.00)**	0.51 **(0.00)**
10	CFP	0.54	0.51(0.42)	0.52 (0.49)	0.54 (0.58)
11	DY	0.48	0.52 (0.43)	0.47 (0.80)	0.49 **(0.02)**
12	STR	0.44	0.44 (0.91)	0.38 (0.14)	0.44 (0.18)
13	LTR	0.48	0.53 (0.18)	0.46 (0.35)	0.49 (0.15)
14	MaBe	0.49	0.65 **(0.05)**	0.55 (0.13)	0.53 **(0.00)**
15	NSI	0.44	0.48 (0.30)	0.49 **(0.01)**	0.45 **(0.00)**
16	ResVar	0.43	0.62 **(0.01)**	0.36 **(0.02)**	0.47 **(0.00)**

P-value of SR-test. Annualized Sharpe Ratios with the p-values of the Sharp Ratio test in parentheses. Sharpe Ratios that are significantly different from the Sharpe Ratio of the naïve-diversification strategy at a 5%-significance level are bolded.

Table 4: Alpha CAPM

		Strategy			
#	**Dataset**	**Naive**	**Min.var**	**Max.div**	**R.P.**
1	BEME	0.13	0.16 (0.95)	0.14 (0.08)	0.15 (0.41)
2	Size	0.08	-0.00 (0.20)	0.09 (0.36)	0.10 (0.43)
3	EP	0.11	0.12 (0.47)	0.12 (0.95)	0.13 (0.57)
4	Ind	0.09	0.17 (0.20)	0.12 (0.52)	0.11 **(0.01)**
5	BEME25	0.16	0.12 (0.65)	0.44 (1.34)	0.18 (0.07)
6	Inv	0.00	0.01 (0.25)	0.10 **(0.01)**	0.10 **(0.00)**
7	Mom	0.01	0.01 (0.46)	0.01 (1.13)	0.01 (0.35)
8	Profit	0.02	0.13 **(0.05)**	0.08 **(0.00)**	0.06 **(0.05)**
9	Acc	0.01	0.10 **(0.02)**	0.01 **(0.00)**	0.05 **(0.03)**
10	CFP	0.05	0.06 (0.20)	0.10 (0.32)	0.08 (0.42)
11	DY	0.15	0.20 **(0.05)**	0.17 **(0.01)**	0.16 **(0.04)**
12	STR	0.00	0.02 (0.68)	-0.07 (0.02)	0.01 (0.22)
13	LTR	0.01	0.01 (0.48)	0.01 (5.88)	0.01 (0.90)
14	MaBe	0.02	0.15 **(0.04)**	0.03 (0.40)	0.05 **(0.02)**
15	NSI	0.01	0.01 (0.09)	0.03 **(0.02)**	0.00 **(0.01)**
16	ResVar	0.01	0.10 **(0.02)**	0.01 (0.27)	-0.01 **(0.00)**

P-value for testing equality of two alphas. Alpha estimates from the OLS estimation of the CAPM motivated by single-factor model. The p-values of the Alpha test are in the parentheses. Alpha estimates that are significantly different from the Alpha estimate of the Naïve-diversification strategy at a 5% significance level are bolded.

Table 5: FF3

		Strategy			
#	**Dataset**	**Naive**	**Min.var**	**Max.div**	**R.P.**
1	BEME	-0.00	0.03 (0.31)	0.00 **(0.03)**	0.01 (0.03)
2	Size	-0.02	0.04 (0.22)	0.00 **(0.05)**	-0.02 (0.08)
3	EP	0.05	0.01 (0.13)	0.05 (0.44)	0.05 (0.37)
4	Ind	0.04	0.11 (0.29)	0.46 (0.95)	0.06 **(0.03)**
5	BEME25	0.00	0.00 (0.31)	0.03 (0.43)	0.08 (0.17)
6	Inv	0.01	0.03 (0.62)	0.10 **(0.01)**	0.07 (0.21)
7	Mom	0.01	0.01 (0.66)	0.01 (2.28)	0.01 (0.68)
8	Profit	0.15	0.07 **(0.05)**	0.01 **(0.02)**	0.06 **(0.05)**
9	Acc	0.03	0.10 **(0.01)**	0.07 **(0.01)**	0.06 **(0.03)**
10	CFP	0.08	0.05 (0.36)	0.01 (0.59)	0.08 (0.53)
11	DY	0.10	0.15 (0.74)	0.16 (0.14)	0.05 (0.24)
12	STR	0.01	0.01 (0.62)	0.01 (0.08)	0.01 (0.25)
13	LTR	0.01	0.01 (0.40)	0.01 (0.39)	0.01 (0.97)
14	MaBe	-0.02	0.10 **(0.04)**	0.01 **(0.00)**	0.03 **(0.01)**
15	NSI	-0.01	-0.00 (0.20)	-0.01 (0.08)	0.15 (0.08)
16	ResVar	0.02	0.11 **(0.01)**	0.04 **(0.05)**	0.05 **(0.03)**

P-value for testing equality of two alphas. Alpha estimates from the OLS estimation of the Fama-French 3-factor model. The p-values of the Alpha test are in parentheses. Alpha estimates that are significant at a 5%-significance level are bolded and Alpha.

If the P-values of the alphas are below 0.05, then we can say that the portfolio is significant. We are 95% sure that the optimized strategy has higher optimized portfolios than that of the naïve-diversification strategy. That means that if the null hypothesis were rejected at the 5% level, the result of the test would be statistically significant. If the null hypothesis were not rejected, it would be said that the results of the test is insignificant.

5.1 Sharpe ratio

The null hypothesis of the Sharpe Ratios being insignificantly different from zero was rejected of the test. Sixteen datasets and three optimization strategies were used in addition to naïve diversification. This grants us a total of 48 p-values, were 18 which below the chosen significant level of 5%. Some of the SR of the optimized portfolio was actually smaller than the naïve strategy. This result uses the Sharpe Ratio as the performance measure and is only statistically valid given the chosen period. These results cast some doubt on the ability of optimized portfolios to outperform naïve diversification.

From the output of Sharpe Ratios represented in Table 3, it is of interest to infer if optimized portfolio strategies outperform the Naïve-diversification strategy. A significantly different and higher Sharpe Ratio would imply that the strategy outperforms the Naïve-diversification strategy. However, the frequency of significantly different Sharpe ratios for different datasets provides an indication as to if the strategy can consistently outperform the Naïve-diversification strategy. In Table 3, there are significant and higher Sharpe ratios for optimized portfolio strategies in 9 of 16 datasets. This implies that in 9 out of 16 cases, depending on the variable on which the portfolios are formed on, the Naïve-diversification strategy is outperformed by an optimized portfolio strategy. Some variables have more strategies that outperform the Naïve-diversification strategy than other variables. Further, portfolios formed on Operating Profitability and Accruals have the most optimized portfolio strategies that outperform the Naïve-diversification strategy, which will say in all three strategies. There is also Market Beta that has the highest significant Sharpe Ratio with 0.65. As for the strategies, the Risk-Parity strategy outperforms the Naïve-diversification strategy in 9 of 16 cases. Even though the Sharpe ratios of the Risk-parity are significantly different and higher than the Sharpe Ratios of the Naïve-diversification strategy, the differences are quite small. The Max-diversification strategy performs poorly with no higher significantly Sharpe Ratios than the naïve-diversification.

5.2 Alpha CAPM

From the estimates of Alphas CAPM that is represented in Table 4, it is possible to conclude whether there is significant expected excess return, unexplained by the market, of a specific strategy and whether the expected excess return is significant higher than the Alpha estimate of the Naïve-diversification strategy. If an Alpha estimate were significantly different and higher than the Alpha estimate of the Naïve-diversification strategy, then it would imply that the strategy of the Alpha estimate outperforms the Naïve-diversification strategy.

From the output of Alpha represented in Table 4, it is possible to infer whether there is significant expected excess return, unexplained by the market, of a specific strategy and whether the expected excess return is significantly higher than that of the Naïve-diversification strategy. If an Alpha estimate is significantly different and higher than the Alpha estimate of the Naïve-diversification strategy, then it would imply that the strategy of the Alpha estimate outperforms the naïve-diversification strategy. In Table 4, there is significant and higher Alpha for optimized portfolio strategies in 8 of 16 datasets. This implies that in 8 out of 16 cases, depending on the variable on which the portfolios are formed on, the Naïve-diversification strategy is equally weighted with the optimized portfolio strategy. Some variables have more strategies that outperform the Naïve-diversification strategy than other variables. Further, datasets formed on Operating Profitability, Accruals and Dividend Yield have the most optimized portfolio strategies that outperform the Naïve-diversification strategy, which will say in all three strategies. Dividend Yield has also the highest significant alpha with 0.20. As for the strategies, the Risk-Parity strategy outperforms the Naïve-diversification strategy in 8 of 16 cases. Even though the Alpha of the Risk-parity are significantly different and higher than the Alpha of the Naïve-diversification strategy, the differences are quite small. None of the strategies performs poorly with lower significantly Alpha than the naïve-diversification. There were several significant estimates of alpha in the results, yet few of the estimated alphas for the optimized strategies were concluded to be significantly larger than the estimate alphas for the naïve strategy.

5.3 Alpha FF3

In the analytical interpretation it is applied to the Alpha estimates estimated in the Fama-French 3-factor model. From the output of Alpha represented in Table 5, it is possible to infer whether there is significant expected excess return, unexplained by the market, of a specific strategy and whether the expected excess return is significantly higher than that of the Naïve-diversification strategy. If an Alpha FF3 estimate is significantly different and higher than the Alpha FF3 estimate of the Naïve-diversification strategy, then it would imply that the strategy of the Alpha FF3 estimate outperforms the naïve-diversification strategy. In Table 5, there is significant and higher Alpha for optimized portfolio strategies in 8 of 16 datasets. This implies that in 8 out of 16 cases, depending on the variable on which the portfolios are formed on, the Naïve-diversification strategy is equally-weighted with the optimized portfolio strategy. Some variables have more strategies that outperform the Naïve-diversification strategy than other variables. Further, datasets formed on Operating Profitability, Accruals and Market Beta and Residual Variance have the most optimized portfolio strategies that outperform the Naïve-diversification strategy, which will say in all three strategies. Residual Variance has also the highest significant alpha with 0.11. As for the strategies, the Maximum-diversification strategy outperforms the Naïve-diversification strategy in 8 of 16 cases. Even though the Alpha FF3 of the maximum-diversification are significantly different and higher than the Alpha FF3 of the Naïve-diversification strategy, the differences are quite small. None of the strategies performs poorly with lower significantly Alpha FF3 than the naïve-diversification.

Table 6: Growth period

	Naïve	**Risk Parity**	**MaxDiv**	**Minvar**	**Average**
Book-to-market	550.19	**551.90**	*405.17*	431.10	484.59
Size	**549.96**	524.98	350.51	*198.30*	405.94
EP	458.35	451.98	**510.51**	*301.22*	430.52
Industry	333.60	**356.64**	281.81	*266.76*	309.70
Size and BM	684.11	**741.17**	*320.26*	352.03	524.39
Investment	*180.23*	**194.31**	190.39	181.07	186.5
Momentum	*191.85*	218.86	**310.83**	275.92	128.47
Profit	107.57	117.14	*83.81*	**205.34**	128.47
Accruals	*130.78*	139.78	144.95	**206.04**	155.39
Cashflow Price	**427.58**	425.65	413.34	*316.29*	395.72
Dividend Yield	328.36	346.21	*291.14*	**354.79**	330.13
Short Term Reversal	250.74	**252.71**	*153.82*	217.65	218.73
Long Term Reversal	*452.24*	457.60	462.64	**558.35**	482.71
Market Beta	*145.33*	163.34	152.95	**172.94**	158.64
Net Share Issues	*87.30*	95.29	**122.52**	102.74	101.96
Residual Variance	101.63	129.42	*40.60*	**157.83**	107.37
Average	311.24	**322.93**	*264.70*	268.65	-

1$ growth for the period (1963,1) – (2018,12)

Table 6 shows the performance of datasets and strategies in terms of financial value, ignoring any market resistances like transaction costs. The table shows the answer of how much one would have in the end of the year 2018 if they had invested $1 in the beginning of the testing period. The largest amount is highlighted in **bold** and the smallest inn *cursive* for each dataset. Risk Parity has the largest average, and an end return for five out of 16 datasets. Minimum-variance ends up with quite impressive return that had the largest datasets in six out of 16, but has also three of the smallest. Maximum-diversification ends up with the smallest end amount in six out of the 16 datasets, but has also three of the biggest return. Naïve-diversification has the largest return for two out of the 16 datasets, but has like minimum-variance six of the smallest returns. As for the strategy averages, the Risk-parity has the highest average out of all the strategies and the maximum-diversification the lowest. The naïve-diversification is not far away to have the highest average, and so do the minimum-variance to the smallest average. Still, the transactions costs and other market frictions have not been accounted for. It should be before taking any conclusions based on the observation from the table.

The Sharpe Ratio favors the Risk-parity strategy. The Risk-parity strategy has mostly higher Sharpe Ratios than the Naive-diversification strategy that are significantly different. Minimum-variance has a lot of Sharpe Ratios that are higher than the Naïve-diversification strategy. The output of OLS estimation of the Single-factor model CAPM favors the maximum-diversification strategy. The maximum-diversification has the highest and most significant Alpha estimations that are also significantly different to the Naïve-diversification strategy. And in the Alpha estimates estimated by OLS of the Fama-Franch 3-factor model, there are the maximum-diversification the highest and most significant estimations compared to the naïve-diversification.
Some datasets perform a lot greater for many of the strategies, while others perform weakly for the strategies.

By the empirical results implementing the out-of-sample test period, it shows that the minimum-variance has fewest significant Sharpe Ratios and Alphas that implement worst of the strategies portfolios. These results accord with what DeMiguel et al. (2009) found, that it is the equally weighted is the best performing strategies. By my research of statistical significance of the difference between the Sharpe Ratios, I reached the same conclusions as DeMiguel et al. (2009) that nobody of the optimized portfolios statistically outperforms the naïve-diversification strategy. However, this also holds the other way around, in that naïve-diversification strategy do not statistically outperform the optimized portfolios. That means it is equally-weighted. In DeMiguel et al. (2009), the Sharpe Ratios estimated are higher for the naïve-diversification than the Mean-variance strategy. In Kritzman et al (2010), the Sharpe Ratios of the minimum-variance strategy are higher than the Mean-variance. Even though data from my thesis is different to what they implemented in the academic studies, I still find similar results.

Despite of that how well the Sharpe Ratios and Alphas reflect on the performance of the strategies are often debated. The Sharpe Ratio as a measure of performance in incomplete in the sense of explaining the source of performance gains. By the interest it would be to consider different performance measures, and to measure their quality in explaining the performance of strategies.

Based on the Sharpe alone, the minimum-variance strategy is the worse strategy. This would be the strategy that delivers lowest risk-adjusted return of all the portfolio models considered in the academic study.
Kritzman et al. (2010) show that the minimum-variance strategy is superior to the naïve-diversification.

From the results by the Sharpe Ratio I got that if the P-values of the Sharpe Ratio are below 0.05, then we can say that the portfolio is significant. We are 95% sure that the optimized strategy has higher optimized portfolios than that of the naïve-diversification strategy. The P-values of Sharpe Ratio were 18 significant portfolios with a total of 48 p-values. That means the optimized does not beat the naïve-diversification. But this means also that the naïve-diversification do not statistically outperform optimized portfolios. With the hypothesis testing we will reject the H_1 and hold the H_0. Further, the p-values of Alpha CAPM single-factor were 18 significant portfolios of 48 p-values. That means the optimized portfolios foes not beat the naïve-diversification. In the hypothesis testing we will also reject the H_1 and hold H_0. Then we have the Alpha Fama-French 3-factor model that had 16 significant portfolios of those 48 p-values. And there is no evidence to prove that the optimized portfolios do not outperform the naïve-diversification strategy in the Fama-French 3-factor model by these 16 datasets that are chosen.

In the minimum-variance, maximum-diversification strategy and the Risk-parity strategy have often the same expected return and risk. This could be because of the Shrinkage estimation, as the covariance matrix becomes more structured in a way that makes the risk allocation and diversification similar.

Then we have the back-testing that usually is to fulfill market frictions and the avoidance of the look-ahead bias. The look-ahead bias has been avoided to the best for the capabilities and the accounting for market frictions, such as transaction costs has not been included which could have affect the results in this thesis. In practice this is not true that there are no transaction costs.

6 CONCLUSION

When looking at the literature at empirical performance of different strategies, there is little question on whether or not optimization leads to performance. Can optimized portfolio strategies consistently outperform the naïve-diversification strategy? Of the research I have pointed out two academic papers with different conclusions. DeMiguel et al. (2009) raised the concerns that there is no scientific evidence that optimized portfolios outperform the naïve-diversification strategy. The other paper with Kritzman et al. (2010) claimed that optimized portfolios are better than the naïve-diversification strategy.

The goal of this thesis has been to compare the out-of-sample performance of optimized portfolios and naively diversified portfolios. By studying the empirical performance of the minimum-variance, max-diversification and risk-parity in order to control if some of the portfolio strategies deliver better performance. To measure the performance it was used the out-of-sample Sharpe Ratio, and also the Alphas CAPM single-factor and Fama-French 3-factor model. Although a strategy could outperform one of the strategies of the Sharpe Ratio, Alpha CAPM and FF3, the results of data shows that the ratios are equal to each other. I conclude that the performance of the optimized portfolios and the naïve-diversification do not outperform each other. Further, I find that the difference between the Sharpe Ratios of the optimized portfolios were not consistently statistically significant. That means the optimized portfolio do not outperform the naïve-diversification strategy, but also the other way around. The results also suggest that the choice of datasets does not affect the performance of portfolios when compared to each other.

The research problem is which is whether optimized portfolio strategy consistently outperform the naïve-diversification strategy cannot be answered by a yes or no. The estimates to answer the research problem have been estimated. The empirical results presented in the thesis, it can be specified that optimized portfolio strategy does not outperform the naïve-diversification strategy, either the naïve-diversification outperform the optimized strategy. There is no evidence to prove that the optimized portfolio does outperform the naïve-diversification or the other way around where we do not reject the significance at least 95%. A finish to this thesis is inspire the development and testing of optimized portfolio strategies and to not reject the Naïve-diversification as a well-meaning performance benchmark.

7 Referances

Asness, C. S., Frazzini, A., & Pedersen, L. H. (2012). Leverage aversion and risk parity. "*Financial Analyses Journal*", 68 (1), 47-59.

Asness, C. S., Frazzini, A., & Pedersen, L. H. (2014). "Low-Risk Investing without Industry Bets", *Financial Analyses Journal*, 68 (1), 47-59.

Choueifaty, Y., & Coignard, Y. (2008), Toward maximum diversification. "*The Journal of portfolio management*", 35 (1), 40-51.

Danthine, J. B., Jean-Pierre; Donaldson. (2014). "*Intermediate financial theory*".

DeMiguel, V., Garlappi, L., & Uppal, R. (2009). Optimal versus naive diversification: How inefficient is the 1/n portfolio strategy? "*The Review of Financial Studies"*, 22 (5), 1915-1953.

Duchin, R., & Levy H. (2009). Markowitz versus the talmudic portfolio diversification strategies. "The Journal of Portfolio Management", 35 (2), 71-74.

Fama, E. F., & French, K. R. (1993). Common risk factors in the returns on stocks and bonds. "Journal of Financial Economics", 33(1), 3-56.

French, K. R. (2018). Data library: U.S. research return data.

Garlappi, L., Uppeal, R., & Wang T. (2007) Portforolio section with parameter and model uncertainty; A multi-prior approach. Review of Financial Studies, 20(1), 41-81.

Jensen, M. C. (1968) The performance of mutual funds in the period 1945-1964. "The Journal of Finance", 23(2), 389-416.

Kirby, C., & Ostdiek, B. (2012). It´s all in the timing: Simple active portfolio strategies that outperform naïve diversification. “Journal of Financial and Quantitative Analysis”, 47 (2), p- 437-467.

Kritzman, M., Page, S., & Turkington, D. (2010). In defense of optimization: The fallacy 1/N. “Financial Analysts Journal”, 66 (2), p. 31-39.

Ledoit, O., & Wolf, M. (2008). Robust performance hypothesis testing with the Sharpe Ratio. “Journal of Emporical Finance”, 15 (5), p. 850-859.

Lintner, J. (1965). The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. The Review of Economics and Statistics, 47 (1), p. 13-37.

Markowitz, H. (1952). Portfolio selection. “The Journal of Finance”, 7(1), 77-91.

Scherer, B. (2011). “A Note on the Returns from Minimum Variance Investing”, *Journal of Empirical Finance*, 18 (4), 652-660.

Sharpe, W. F. (1963) A simplified model for portfolio analysis. Management Science, 9 (2), 277-293.

Sharpe, W. F. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk. “The Journal of Finance”, 19(3), 425-442.

Tu, J., & Zhou, G. (2011). Markowitz meets Talmud: A combination of sophisticated and naïve diversification strategies. “Journal of Financial Economics”, 99 (1), p. 204-215.

Zakamulin, V. (2019) “Superiority of optimized portfolios to naïve diversification: Fact or fiction?”

APPENDIX

The 16 datasets have all each graph with four portfolios with four different strategies that are starting in year 1963 and ends in year 2018. All graphs show a cumulative growth through time/years. All the portfolios in each graph have a minor downfall about year 1975. After that we can see that the portfolios have a trend upwards, before it is a new downfall in year 2008. Mostly of the portfolios we can clearly see that all portfolios in each graph are following the same path and behaves the same way against time.

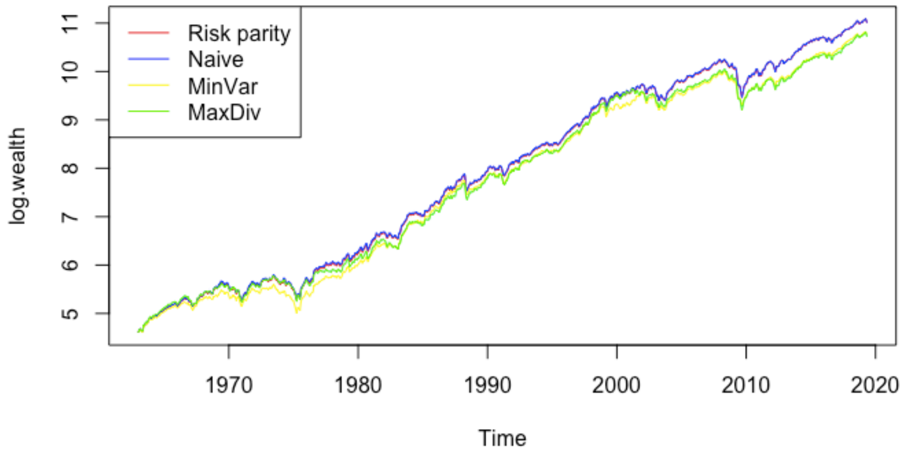

Figure 1: Book-to-market

The naïve-diversification strategy and risk parity are always near each other. They are also the highest portfolios, especially after the downward fall around 1975. Minimum variance are lowest of the portfolios in the beginning, but from 1983 and to 2018 it is maximum diversification.

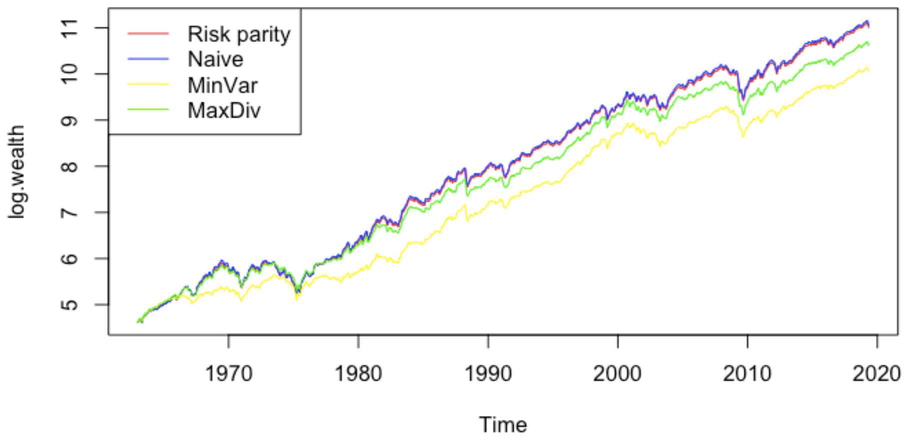

Figure 2: Size

Also here it is naïve-diversification strategy that is highest portfolios, and risk parity is very near. Minimum variance is the lowest all the time.

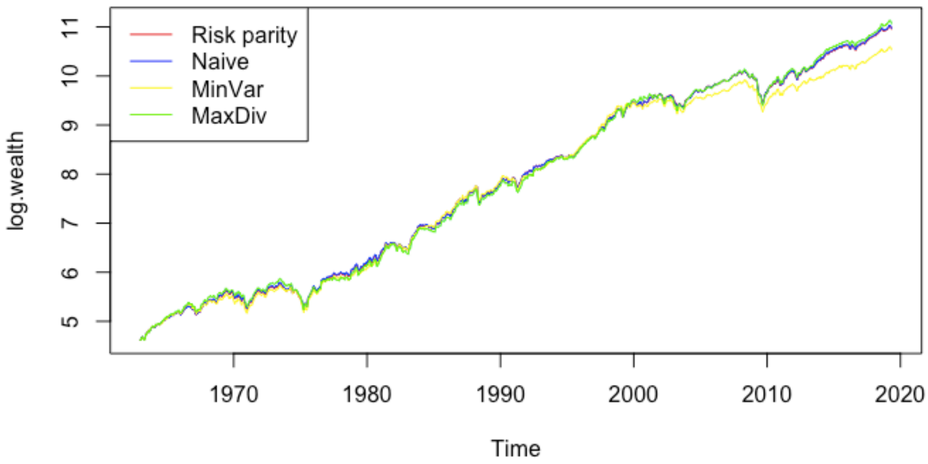

Figure 3: Earning Price

Very close between MaxDiv, Naïve and RP. MaxDiv highest, Minvar lowest.

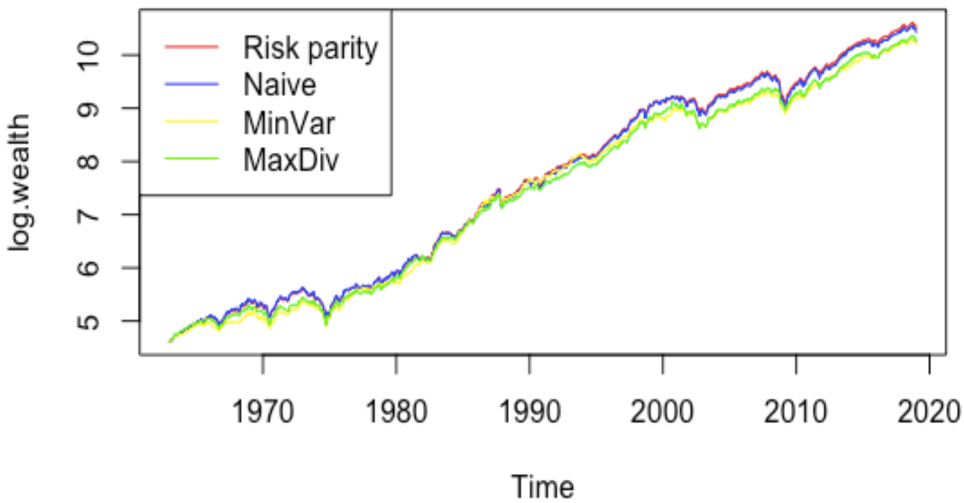

Figure 4: Industry

All four very close 1995, then Naive and Risk parity highest and maxdiv and minvar lowest.

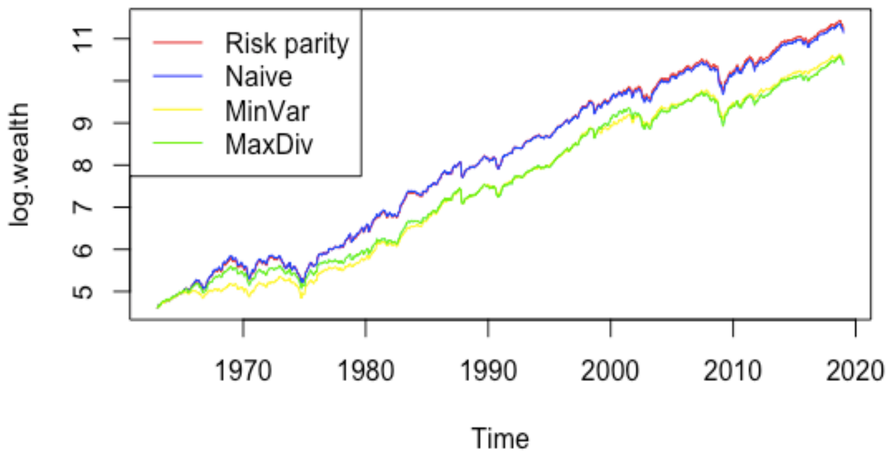

Figure 5: Size and BM

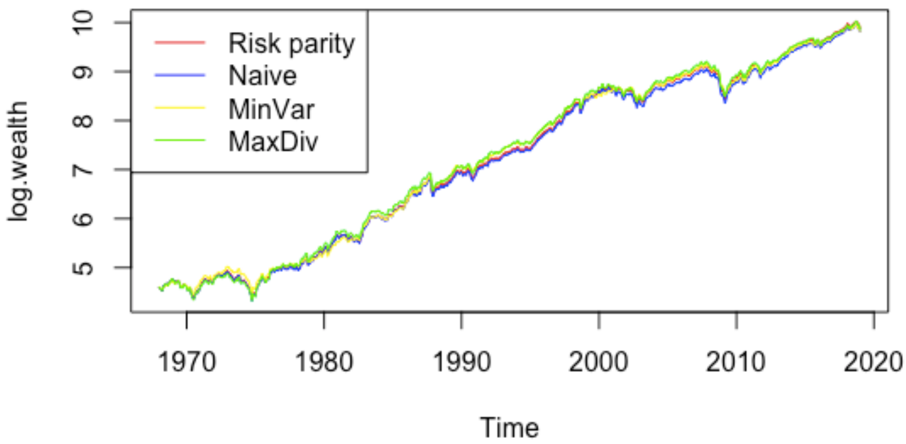

Figure 6: Investment

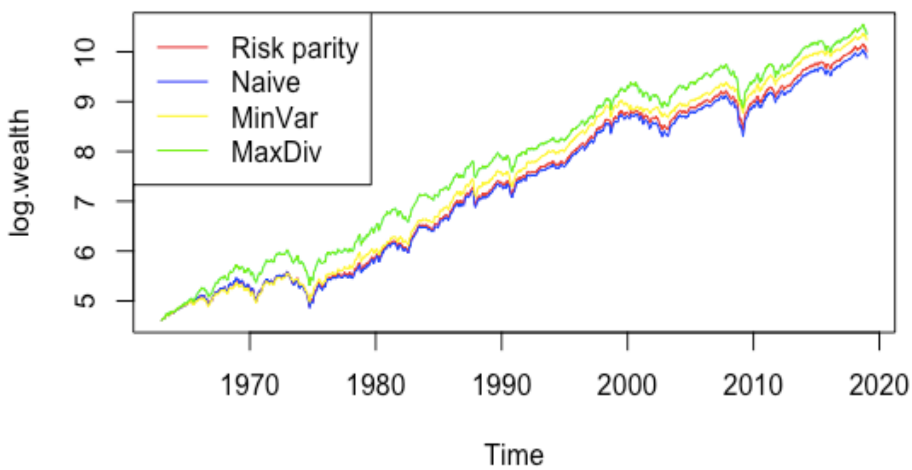

Figure 7: Momentum

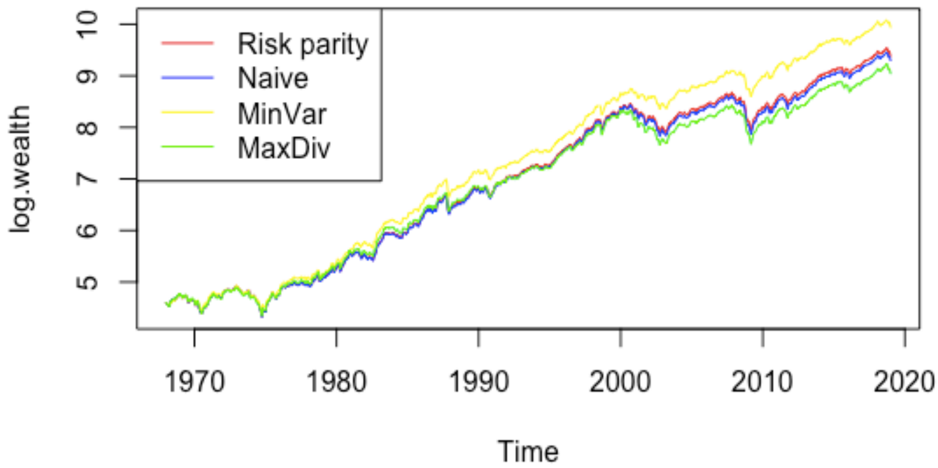

Figure 8: Profit

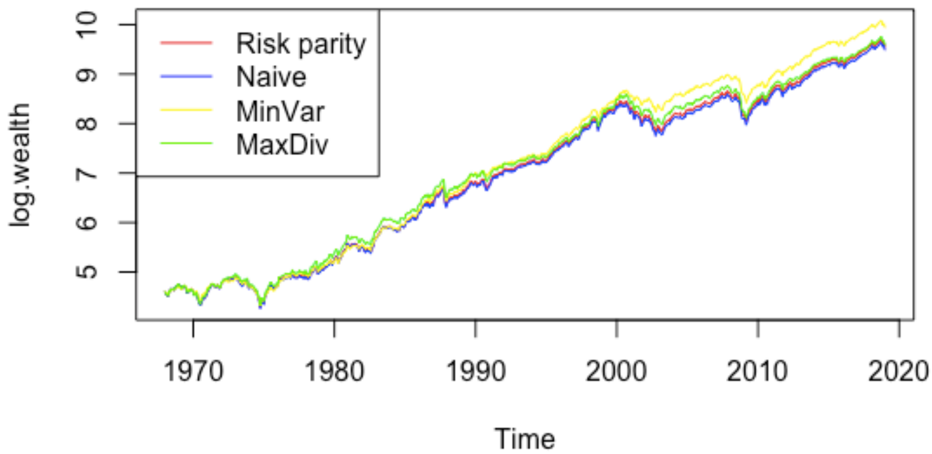

Figure 9: Accruals

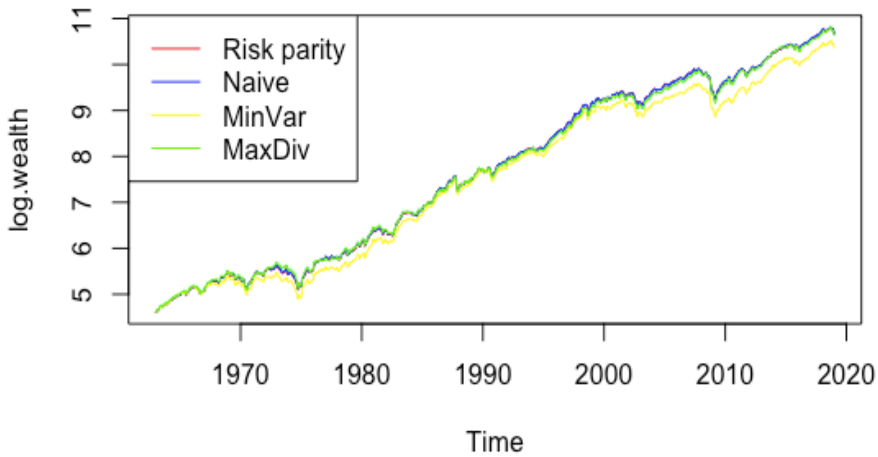

Figure 10: Cashflow Price

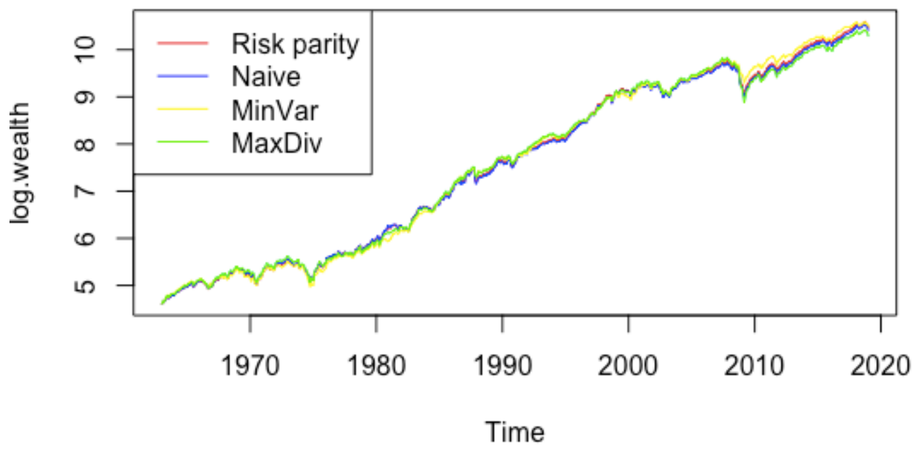

Figure 11: Dividend Yield

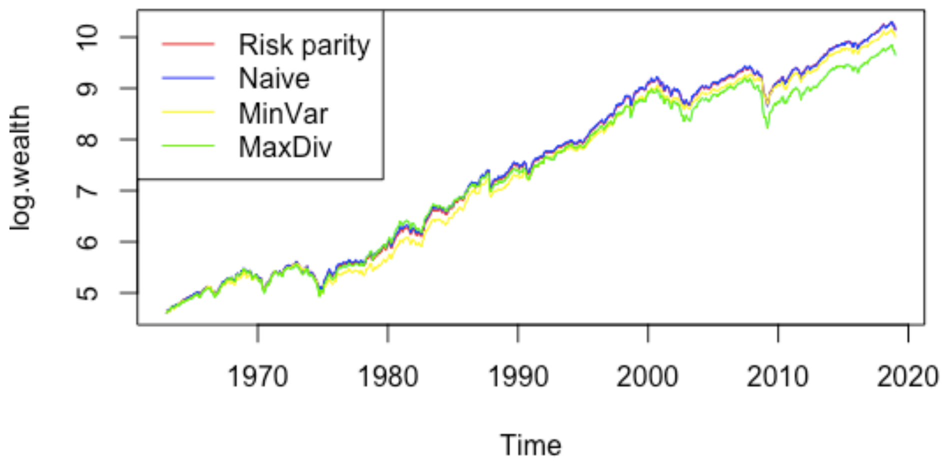

Figure 12: Short Term Reversal

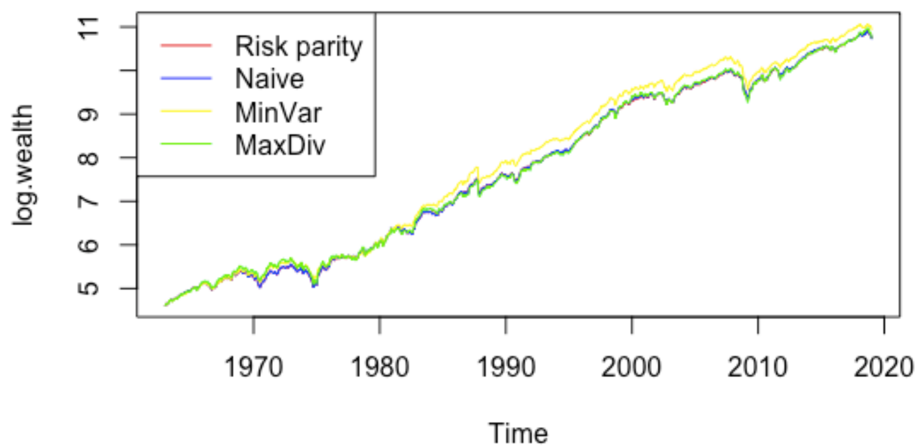

Figure 13: Long Term Reversal

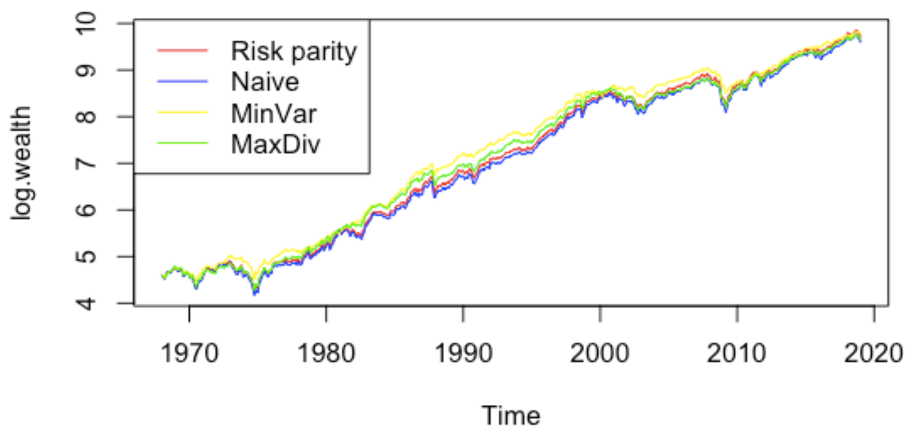

Figure 14: Market Beta

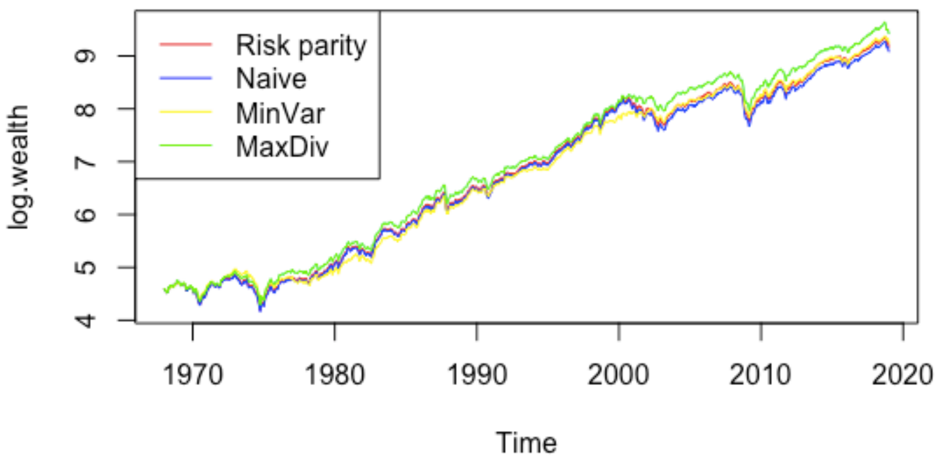

Figure 15: Net Share Issues

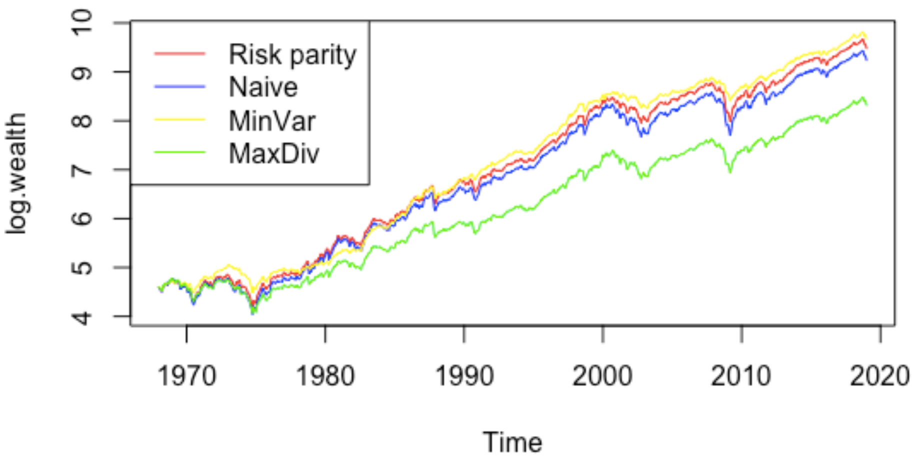

Figure 16: Residual Variance

Each portfolio has 10 different decile portfolios and consists a return by market equity, where the first column starts at the lowest and upwards to the highest. The means, median, standard deviations, minimum and maximum values and the range of the decile portfolios sorted. (1963,1 – 2018,12)

Table 7: Book-to-market decile portfolios

BE-ME	Lo10	Dec2	Dec3	Dec4	Dec5	Dec6	Dec7	Dec8	Dec9	Hi10
Mean	0.87	0.97	1.00	0.94	0.957	1.067	0.997	1.115	1.25	1.30
Median	0.96	1.17	1.17	1.10	1.21	1.25	1.17	1.285	1.665	1.64
Std.dev	5.02	4.57	4.51	4.58	4.37	4.30	4.53	4.585	4.88	6.06
Min	-22.72	-24.8	-25.72	-23.59	-23.50	-23.08	-24.32	-24.88	-19.35	-26.36
Max	23.03	19.55	17.08	18.51	17.57	18.44	22.19	22.70	22.28	34.84
Range	45.75	44.35	42.80	42.10	41.07	41.52	46.51	47.58	41.63	61.20

Table 8: Size decile portfolios

ME	Lo10	Dec2	Dec3	Dec4	Dec5	Dec6	Dec7	Dec8	Dec9	Hi10
Mean	1.16	1.12	1.20	1.13	1.15	1.09	1.10	1.08	1.00	0.88
Median	1.3	1.365	1.60	1.45	1.63	1.32	1.28	1.3	1.32	1.10
Std.dev	6.22	6.27	5.96	5.73	5.51	5.18	5.08	4.94	4.52	4.16
Min	-28.92	-30.48	-28.93	-29.45	-28.12	-26.15	-26.23	-24.32	-22.27	-19.72
Max	29.54	28.40	25.74	24.33	24.80	20.90	22.41	19.11	18.14	18.12
Range	58.46	58.88	54.67	53.78	52.92	47.05	48.64	43.43	40.41	37.84

Table 9: Earnings Price decile portfolios

E/P	Lo10	Dec2	Dec3	Dec4	Dec5	Dec6	Dec7	Dec8	Dec9	Hi10
Mean	0.91	0.82	0.93	0.88	0.91	1.02	1.10	1.10	1.18	1.23
Median	1.10	0.97	1.13	0.93	1.09	1.25	1.16	1.34	1.38	1.6
Std.dev	5.61	4.67	4.47	4.34	4.39	4.24	4.34	4.48	4.74	5.19
Min	-25.89	-23.85	-22.52	-23.35	-22.53	-23.86	-19.83	-19.07	-18.95	-22.26
Max	22.69	21.52	17.53	15.31	18.61	21.3	19.37	23.64	26.25	26.07
Range	48.58	45.37	40.05	38.66	41.14	45.16	39.2	42.71	45.2	48.33

Table 10: Industry decile portfolios

Ind	Lo10	Dec2	Dec3	Dec4	Dec5	Dec6	Dec7	Dec8	Dec9	Hi10
Mean	1.04	0.83	0.96	0.97	1.01	0.85	1.04	1.07	0.83	0.94
Median	1.04	0.75	1.24	0.91	1.03	1.02	1.00	1.15	0.91	1.36
Std.dev	4.21	6.20	4.85	5.43	6.34	4.58	5.09	4.81	3.96	5.21
Min	-21.03	-32.63	-27.33	-18.41	-25.96	-16.36	-28.23	-20.46	-12.65	-23.58
Max	18.88	42.63	17.5	24.56	20.76	21.36	25.86	29.52	18.84	20.22
Range	39.91	75.26	44.83	42.97	46.72	37.72	54.09	49.98	31.49	43.8

Table 11: Size and BM decile portfolios

BM&ME	Lo10	Dec2	Dec3	Dec4	Dec5	Dec6	Dec7	Dec8	Dec9	Hi10
Mean	0.70	1.17	1.18	1.39	1.50	0.90	1.16	1.25	1.31	1.38
Median	1.09	1.41	1.23	1.44	1.51	1.32	1.45	1.44	1.52	1.77
Std.dev	7.84	6.86	5.92	5.64	5.93	7.08	5.92	5.37	5.20	5.97
Min	-34.22	-30.95	-28.77	-28.89	-28.87	-32.72	-31.66	-28.40	-25.03	-28.83
Max	38.90	41.05	28.16	27.84	33.88	28.18	26.12	26.33	27.58	29.71
Range	73.12	72.00	56.93	56.73	62.75	60.90	57.77	54.73	52.62	58.55

Table 12: Investment decile portfolios

INV	Lo10	Dec2	Dec3	Dec4	Dec5	Dec6	Dec7	Dec8	Dec9	Hi10
Mean	1.11	1.14	1.04	0.99	0.96	0.92	0.95	0.89	0.90	0.63
Median	1.23	1.23	1.19	1.12	1.20	1.11	1.12	0.96	1.16	0.86
Std.dev	5.41	4.78	4.36	4.12	4.21	4.37	4.44	4.78	5.46	6.17
Min	-26.96	-21.40	-21.00	-18.04	-18.49	-21.47	-23.79	-23.06	-24.31	-28.56
Max	20.51	17.65	16.63	17.46	14.72	15.83	15.16	22.50	20.30	19.73
Range	47.47	39.05	37.63	35.50	33.21	37.30	38.95	45.56	44.61	48.29

Table 13: Momentum decile portfolios

MOM	Lo10	Dec2	Dec3	Dec4	Dec5	Dec6	Dec7	Dec8	Dec9	Hi10
Mean	0.20	0.69	0.85	0.89	0.83	0.90	0.90	1.07	1.12	1.48
Median	0.20	0.67	0.83	0.91	1.18	1.16	1.13	1.27	1.54	1.71

Std.dev	7.92	6.10	5.22	4.70	4.38	4.43	4.27	4.38	4.73	6.09
Min	-26.12	-24.85	-23.31	-18.65	-21.48	-23.77	-24.28	-20.53	-26.27	-26.74
Max	45.46	35.50	33.78	21.66	20.81	16.68	17.49	19.75	21.83	23.07
Range	71.58	60.35	57.09	40.31	42.29	40.45	41.77	40.28	48.10	49.81

Table 14: Profit decile portfolios

Profit	Lo10	Dec2	Dec3	Dec4	Dec5	Dec6	Dec7	Dec8	Dec9	Hi10
Mean	0.25	0.39	0.53	0.60	0.75	0.48	0.57	0.74	0.74	0.63
Median	0.92	1.12	0.91	0.86	0.95	1.29	0.96	1.15	0.98	0.89
Std.dev	7.19	5.73	5.02	4.70	5.11	4.46	4.19	4.13	4.08	3.90
Min	-25.57	-24.51	-16.73	-23.68	-19.79	-14.56	-16.51	-13.90	-16.29	-15.47
Max	15.98	15.21	15.86	12.45	15.68	11.34	11.25	11.73	11.71	10.23
Range	41.55	39.72	32.59	36.13	35.47	25.90	27.76	25.63	28.00	25.70

Table 15: Accruals decile portfolios

ACC	Lo10	Dec2	Dec3	Dec4	Dec5	Dec6	Dec7	Dec8	Dec9	Hi10
Mean	1.07	0.99	0.96	0.93	0.95	0.86	0.89	0.96	0.81	0.62
Median	1.25	1.26	1.10	1.14	1.12	1.09	1.09	0.99	1.15	0.98
Std.dev	5.70	5.21	4.60	4.32	4.06	4.43	4.55	4.96	5.32	6.03
Min	-25.08	-22.57	-25.04	-18.80	-19.07	-20.78	-19.25	-23.98	-29.28	-28.90
Max	20.40	19.83	16.39	16.67	14.34	21.68	16.72	16.73	24.61	19.20
Range	45.48	42-40	41.43	35.47	33.41	42.46	35.97	40.71	53.89	48.10

Table 16: Cashflow Price decile portfolios

CFP	Lo10	Dec2	Dec3	Dec4	Dec5	Dec6	Dec7	Dec8	Dec9	Hi10
Mean	0.86	0.91	0.93	0.92	0.98	0.89	1.03	1.08	1.14	1.23
Median	1.14	0.94	1.04	1.20	1.17	1.14	1.09	1.37	1.38	1.52
Std.dev	5.47	4.63	4.47	4.46	4.47	4.43	4.31	4.37	4.42	5.17
Min	-26.02	-23.44	-22.11	-22.97	-25.85	-23.01	-23.68	-20.02	-20.27	-22.80
Max	23.09	22.65	15.75	15.77	20.03	14.79	18.47	22.70	24.98	26.14
Range	49.11	46.09	37.86	38.74	45.88	37.80	42.15	42.72	45.25	48.94

Table 17: Dividend Yield decile portfolios

DY	Lo10	Dec2	Dec3	Dec4	Dec5	Dec6	Dec7	Dec8	Dec9	Hi10
Mean	0.88	0.87	0.92	0.96	0.86	0.98	0.99	1.05	1.00	0.96
Median	1.15	1.12	1.07	1.21	1.01	1.09	1.17	1.68	1.10	0.98
Std.dev	5.60	4.98	4.88	4.60	4.67	4.33	4.29	4.10	4.09	4.41
Min	-26.88	-26.01	-23.36	-24.01	-25.13	-22.61	-21.97	-20.06	-18.38	-29.14
Max	22.97	21.09	20.25	26.59	21.46	18.69	13.96	19.38	18.48	28.33
Range	49.85	47.10	43.61	40.60	46.59	41.30	35.93	39.44	36.86	57.47

Table 18: Short Term Reversal decile portfolios

STR	Lo10	Dec2	Dec3	Dec4	Dec5	Dec6	Dec7	Dec8	Dec9	Hi10
Mean	0.98	1.11	1.14	1.02	0.98	0.88	0.89	0.87	0.73	0.64
Median	1.09	1.18	1.37	1.33	1.16	1.23	1.13	1.18	0.84	0.79
Std.dev	7.14	5.65	5.03	4.66	4.41	4.27	4.23	4.36	4.66	5.39
Min	-29.14	-25.56	-24.07	-21.09	-21.39	-18.47	-20.70	-20.47	-26.84	-27.10
Max	34.93	27.27	22.21	20.69	19.07	13.96	15.34	16.45	20.11	24.41
Range	64.39	52.83	46.28	41.78	40.46	32.63	36.04	36.92	46.95	51.51

Table 19: Long Term Reversal decile portfolios

LTR	Lo10	Dec2	Dec3	Dec4	Dec5	Dec6	Dec7	Dec8	Dec9	Hi10
Mean	1.22	1.07	1.08	1.01	1.02	1.00	0.99	0.98	0.88	0.87
Median	1.04	1.13	1.23	1.21	1.37	1.22	1.21	1.34	1.04	1.14
Std.dev	6.65	5.20	4.73	4.43	4.33	4.21	4.28	4.32	4.68	5.80
Min	-29.92	-28.59	-25.75	-25.35	-20.86	-22.49	-20.40	-17.16	-23.11	-24.41
Max	39.08	30.86	23.66	22.96	21.33	17.66	21.73	15.69	16.75	25.58
Range	69.00	59.45	49.41	48.31	42.19	40.15	42.13	32.85	39.86	49.99

Table 20: Market Beta decile portfolios

MaBe	Lo10	Dec2	Dec3	Dec4	Dec5	Dec6	Dec7	Dec8	Dec9	Hi10
Mean	0.93	0.91	0.95	1.06	0.90	0.99	0.84	0.97	0.92	0.85
Median	1.10	0.94	1.01	1.12	1.15	1.29	1.21	1.11	1.23	0.92
Std.dev	3.49	3.85	4.09	4.65	4.82	5.51	5.50	6.11	6.74	7.95

Min	-13.05	-15.12	-20.32	-23.99	-24.36	-24.78	-27.16	-26.26	-29.69	-33.14
Max	18.66	18.71	15.46	18.76	18.25	20.08	18.24	26.66	31.92	33.5
Range	31.71	33.83	35.78	42.75	42.61	44.86	45.50	52.92	61.61	66.66

Table 21: Net Share Issues decile portfolios

NSI	Lo10	Dec2	Dec3	Dec4	Dec5	Dec6	Dec7	Dec8	Dec9	Hi10
Mean	0.95	0.85	0.87	1.00	1.02	1.02	0.92	0.75	0.65	0.40
Median	1.02	1.04	1.04	1.10	1.15	1.48	1.24	1.06	1.05	0.71
Std.dev	4.45	4.50	4.71	4.89	4.88	5.45	5.58	5.57	5.40	5.70
Min	-20.58	-20.50	-23.68	-25.59	-26.17	-25.93	-21.03	-23.31	-24.93	-25.99
Max	16.83	18.87	18.26	21.26	16.53	18.72	23.46	18.93	18.74	21.63
Range	37.41	39.37	41.94	46.85	42.70	44.65	44.49	42.24	43.57	47.62

Table 22: Residual Variance decile portfolios

ResVar	Lo10	Dec2	Dec3	Dec4	Dec5	Dec6	Dec7	Dec8	Dec9	Hi10
Mean	0.94	0.97	0.90	0.99	0.98	1.13	0.90	1.09	0.83	0.78
Median	1.04	1.20	1.09	1.28	1.16	1.35	1.07	1.33	1.04	0.66
Std.dev	3.60	4.61	4.51	4.85	5.12	5.56	6.04	6.66	7.46	8.55
Min	-14.32	-20.93	-20.36	-24.51	-24.71	-26.90	-29.73	-30.27	-31.42	-32.66
Max	13.91	17.81	17.88	20.70	16.40	21.98	21.19	30.86	31.23	33.03
Range	28.23	38.74	38.24	45.21	41.10	48.88	50.92	61.13	62.65	65.59

Table 23: Risk-free rate

	Mean	Sd	Min	Max	Range
RF	0.38	0.26	0.00	1.35	1.35
MKTMRF	0.67	0.586	-29.13	38.85	67.98
HML	0.49	0.391	-13.28	35.46	48.74
SMB	0.40	0.333	-9.88	36.7	46.58

The mean, standard deviation, minimum and maximum value and the range of the risk-free rate for (1963,1 – 2018, 12)

Implemented R code for achieving the empirical results

```
rm(list=ls(all=TRUE))

library(tseries)
library(quadprog)
library(Rsolnp)
library(xtable)
library(psych)
library(zoo)
source("markowitz.r")
source("performance.r")
source("maxdivport.R")
source("ols.r")

#********************************************#
# Read data
data = read.table("BE-ME.txt", header=TRUE)
data <- ts(data[,11:ncol(data)], start=c(1927,1), frequency = 12)

# Read the factor data
factors = read.table("FF-data.txt", header=TRUE)
factors = ts(factors[,2:ncol(factors)], start=c(1926,7), frequency = 12)
factors <- as.data.frame(factors)/100

# find the time index of portfolio start
year.start <- 1963
ind.start <- which(time(data) == year.start)
assets <- as.matrix(data)/100 # transform ts to a matrix
nobs <- nrow(assets)
nAssets <- ncol(assets)
r.tbill=factors$RF

#********************************************
# Portfolio management part of the program
#********************************************

lookback.period = 5*12    # length of lookback period in a number of months
n <- nobs - ind.start + 1 # number of monthly portfolio returns
r.port <- rep(0,n)       # reserve the space for portfolio returns
r.minvar <- rep(0,n)
r.maxdiv <- rep(0,n)
r.naive <- rep(0,n)

ws=rep(0,n)
w.naive <- rep(1/nAssets, nAssets) # weights of assets in 1/N portfolio

for (i in 1:n) {
  # find the indices of the lookback period
  period.end <- ind.start + i - 2
```

```
 period.start <- period.end - lookback.period + 1
 if (period.start < 1) period.start <- 1 # index cannot be less than 1
 # estimate the standard deviations in the lookback period
 covmat <- cov(assets[period.start:period.end,])
 std <- sqrt(diag(covmat)) # vector of standard deviations
 #Risk parity
 w <- 1/std    # initial vector of weights
 w <- w/sum(w) # normalized vector of weights
 ws[i] <- w[1] # keep the weight of stocks
 r.port[i] <- sum(w*assets[period.end+1,])
 #gmv (minvar)
 w=gmvportfolio(covmat, shorts = FALSE)
 ws[i] <- w[1]
 r.minvar[i] <- sum(w*assets[period.end+1,])
 #Naive
 r.naive[i] <- sum(w.naive*assets[period.end+1,])
 #Maxdiv port
 w=maxdivport(covmat)
 r.maxdiv[i] <- sum(w*assets[period.end+1,])
}

factors <- factors[ind.start:nobs,]
mktmrf <- factors$MKTMRF # market factor
r.tbill=factors$RF
smb <- factors$SMB
hml <- factors$HML

X <- cbind(mktmrf, smb, hml)

# Computes the Sharpe ratio
Sharpe.port = SR(r.port-r.tbill)
Sharpe.port
Sharpe.naive = SR(r.naive-r.tbill)
Sharpe.naive

Sharpe.minvar = SR(r.minvar-r.tbill)
Sharpe.minvar
Sharpe.maxdiv = SR(r.maxdiv-r.tbill)
Sharpe.maxdiv

# perform the test of equality two Sharpe ratios
pvalport = SharpeTest(r.port-r.tbill, r.naive-r.tbill)
pvalminvar = SharpeTest(r.minvar-r.tbill, r.naive-r.tbill)
pvalmaxdiv = SharpeTest(r.maxdiv-r.tbill, r.naive-r.tbill)

# illustrate the growth of wealth
wealth.port = cumprod(c(100,1+r.port)) # from portfolio of stocks and bonds
wealth.naive = cumprod(c(100,1+r.naive)) # from stocks
wealth.minvar = cumprod(c(100,1+r.minvar)) #from mean variance
```

```
wealth.maxdiv = cumprod(c(100,1+r.maxdiv)) #from maximum div

# the final wealth
Wealth.Final.port = round(wealth.port[n+1], digits=2)
Wealth.Final.naive = round(wealth.naive[n+1], digits=2)
Wealth.Final.minvar = round(wealth.minvar[n+1], digits=2)
Wealth.Final.maxdiv = round(wealth.maxdiv[n+1], digits=2)

# construct a ts object and plot the growth of wealth
wealth = ts(cbind(wealth.port,wealth.naive,wealth.minvar,wealth.maxdiv),
start=c(year.start,1),
        frequency = 12)
log.wealth=log(wealth)

plot(log.wealth, plot.type = "single", col=c("red","blue","yellow","green"))

legend(x="topleft", legend=c("Risk parity","Naive","MinVar","MaxDiv"),
    col=c("red","blue","yellow","green"), lty=1)

my.summary <- function(r) {
  cat("\n**************************************** \n")
  cat(" SUMMARY STATISTICS \n")
  cat("**************************************** \n")
  cat("Number of observations = ", length(r), "\n")
  cat("Mean = ", mean(r), "\n")
  cat("Median = ", median(r), "\n")
  cat("Variance = ", var(r), "\n")
  cat("Standard deviation = ", sd(r), "\n")
  cat("Min = ", min(r), "\n")
  cat("Max = ", max(r), "\n")
  cat("Range = ", max(r)-min(r), "\n")
}

my.summary(assets[(1110-677):1110,10])

my.summary(r.naive)
my.summary(r.port)
my.summary(r.minvar)
my.summary(r.maxdiv)

#**************************************************
# Compute the performance measures
#**************************************************

# *********** ALPHA ESTIMATION AND TESTING ***********

# first portfolio
r.naivemrf <- r.naive - r.tbill
r.portmrf <- r.port - r.tbill
```

```
r.minvarmrf <- r.minvar - r.tbill
r.maxdivmrf <- r.maxdiv - r.tbill

# estimate the single-factor model for asset 1
res1 <- regstats(r.naivemrf, mktmrf)
alpha.naive <- res1$beta[1]
resid.naive <- res1$resid

# estimate the single-factor model for asset 2
res2 <- regstats(r.portmrf, mktmrf)
alpha.rport <- res2$beta[1]
resid.rport <- res2$resid

# estimate the single-factor model for asset 3
res3 <- regstats(r.minvarmrf, mktmrf)
alpha.rminvar <- res3$beta[1]
resid.rminvar <- res3$resid

# estimate the single-factor model for asset 4
res4 <- regstats(r.maxdiv, mktmrf)
alpha.rmaxdiv <- res4$beta[1]
resid.rmaxdiv <- res4$resid

# conduct the test about equality of alphas
a1 <- alpha.naive + resid.naive
a2 <- alpha.rport + resid.rport
a3 <- alpha.rminvar + resid.rminvar
a4 <- alpha.rmaxdiv + resid.rmaxdiv

res <- t.test(a1, a2, paired = TRUE)
pval.rport <- res$p.value # access the p-value of the test
pval.rport

res <- t.test(a1, a3, paired = TRUE)
pval.rminvar <- res$p.value # access the p-value of the test
pval.rminvar

res <- t.test(a1, a4, paired = TRUE)
pval.rmaxdiv <- res$p.value # access the p-value of the test
pval.rmaxdiv

a1eq0 = res1$pval[1]
a2eq0 = res2$pval[1]
a3eq0 = res3$pval[1]
a4eq0 = res4$pval[1]

alpha.naive = alpha.naive*12*100
alpha.rport = alpha.rport*12*100
alpha.rminvar = alpha.rminvar*12*100
```

```
alpha.rmaxdiv = alpha.rmaxdiv*12*100

# estimate the CAPM (single-factor model)
result <- lm(r.naivemrf ~ mktmrf)
summary(result)


# estimate the FF3 model for asset 1
res1 <- regstats(r.naivemrf, X)
alphaFF3.naive <- res1$beta[1]
resid.naive <- res1$resid

# estimate the FF3 model for asset 2
res2 <- regstats(r.portmrf, X)
alphaFF3.rport <- res2$beta[1]
resid.rport <- res2$resid

# estimate the FF3 model for asset 3
res3 <- regstats(r.minvarmrf, X)
alphaFF3.rminvar <- res3$beta[1]
resid.rminvar <- res3$resid

# estimate the FF3 model for asset 4
res4 <- regstats(r.maxdiv, X)
alphaFF3.rmaxdiv <- res4$beta[1]
resid.rmaxdiv <- res4$resid

# conduct the test about equality of FF3
a1FF3 <- alphaFF3.naive + resid.naive
a2FF3 <- alphaFF3.rport + resid.rport
a3FF3 <- alphaFF3.rminvar + resid.rminvar
a4FF3 <- alphaFF3.rmaxdiv + resid.rmaxdiv

res <- t.test(a1FF3, a2FF3, paired = TRUE)
pval.rportFF3 <- res$p.value # access the p-value of the test
pval.rportFF3

res <- t.test(a1FF3, a3FF3, paired = TRUE)
pval.rminvarFF3 <- res$p.value # access the p-value of the test
pval.rminvarFF3

res <- t.test(a1FF3, a4FF3, paired = TRUE)
pval.rmaxdivFF3 <- res$p.value # access the p-value of the test
pval.rmaxdivFF3

a1FF3eq0 = res1$pval[1]
a2FF3eq0 = res2$pval[1]
a3FF3eq0 = res3$pval[1]
a4FF3eq0 = res4$pval[1]
```

```
alphaFF3.naive = alpha.naive*12*100
alphaFF3.rport = alpha.rport*12*100
alphaFF3.rminvar = alpha.rminvar*12*100
alphaFF3.rmaxdiv = alpha.rmaxdiv*12*100

# estimate the FF3 (3-factor model)
result <- lm(assetmrf ~ mktmrf + smb + hml)
summary(result)


-----
# now the backward conversion to data frames
port <- as.data.frame(port)
factors <- as.data.frame(factors)

# select the portfolio (next to smallest and most valued)
asset <- port$S2BM5
# compute the excess return
assetmrf <- asset-rf

# estimate the CAPM (single-factor model)
result <- lm(r.naivemrf ~ mktmrf)
summary(result)
```

Reflection notes

In my master thesis my main topic is portfolio management in finance theory.
I wanted to write about this thesis because the empirical study was interesting were I could learn about programming and statistical tools used in the real world analysis and modelling of fincancial data in modern finance.

I used the software program R to find the results by historical financial data from Kenneth French data. The strategies I used in the program was minimum-variance portfolio (MVP), maximum-diversification portfolio (MDP) and risk-parity portfolio (RPP) to test against the naive-diversification strategy. Further the optimized portfolios I used was Sharpe Ratio, Alpha CAPM and Fama-French 3-Factor model. The main goal was to find if the optimized portfolio strategy could outperform the naive-diversification strategy. By my main findings were i used the software program R, I tested the hypothesis if the significance could hold or be rejected. First when i picked out the datasets, i thought the optimized portfolios would beat the naive-diversification strategy. In the end after findings the results, I found that the optimized portfolio strategy could not outperform the naive-diversification strategy. Either could the naive-diversification not outperform the optimized strategy.

To start with what the thesis should have to get better results, it is unfortunately that it should be added more portfolio optimization strategies. Mean-variance (Utility) and Minimum-variance (S) are the portfolio optimization strategies.
Further, the thesis could have better or another results if it was picked other dataset instead of those 16 dataset from Kenneth French that was picked. That would have given another conclusion. Also, the datasets only have one with 25 portfolios, and the rest are with 10 portfolios. That could have given another conclusion.

This reflection note is written as a part of a master´s in Business Administration at the University of Agder. The purpose of the reflection note is to draw on the knowledge generated from across the whole master program and discuss how the thesis topic relates to three broad terms: International trends, innovation and responsibility.

International

There are a lot of academic paper about these findings and many debates where the conclusion is not the same. With these datasets in the website to Kenneth French, everybody in the world can download them. All they need is to learn to use the software program R, or maybe there are other programs that can be used, to test and find out the results of the portfolios in the datasets. The weaknesses with the research is that all the datasets have in common that each of them have low-volatility. The portfolio management is very interesting, but doing with that you should know a bit about the economic and finance.

Innovation

The fundamental innovation capability to portfolio management is dynamically plan and optimize investments. It serves the providing corporate and business unit leaders with insight and analytics. It also optimizes projects to maximum value creation and gives alternative scenarios to investment within different areas. Today in the 21.century the technology do it easier to analyze portfolios by different programs, and with that it can be easier to do investments. But what I think that none of the portfolios will in the long-run beat the market portfolio.

Responsibility

Investment is essential individual for everyone. Therefore it plays a role in deciding the best investment for an individual as per his income, age and ability to take risks. If a person are interest in portfolio management it is essential to do a analyze of different portfolios and know the risks that are involved. Most people in the world wants to earn fast money with minimum risks involved and maximum returns, but they also have to know the capacity to invest.

There are some questions that are introduced and unexplained that are out of scope in this thesis. It can be interesting topics for future research to follow up this project. One could pick another datasets that are applied and try with other datasets to test for similar results. The same if one could try to change the start of the testing period and length of training period. One could also try to account the market friction, and see what the difference is. But the transactions costs are in practice, and maybe it will be different to in the thesis. One could also present other optimized strategies and other performance measures to add more deepness in the analysis. There are some possibilities here that have not been completed by this project.