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DOES OPTIMIZED PORTFOLIOS ADD ANY VALUE?

A empirical study conducted to examine the performance of various optimized portfolios using the naive diversification rule as benchmark.

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Abstract

DeMiguel, Garlappi, and Uppal (2009) conducted a study where they demonstrated that none of several optimized portfolios consistently outperform the naive allocation rule, initiating a heated debate in the academic community. Subsequently, several researchers conducted studies with the aim to defend the optimized strategies. The common problem with these studies is that they do not control if superior performance appear due to mean-variance efficiency or due to exposure to established factor premiums. We conducted an empirical study, examining the performance of six optimized strategies, across 13 portfolios. We test if the performance of the optimized strategies is superior to the performance of the naive allocation rule. We account for some known financial anomalies, and control for these using various performance measures. Compared to other researchers, the novelty of the conducted study is that we examine a extended time period for USA market, as well as introducing Norwegian data. Based on the findings from the conducted study our main conclusion is that optimized strategies do not consistently beat the naive allocation rule.

1. Introduction

Around the fourth century Rabbi Issac bar Aha proposed a rule for asset allocation, stating: "*One should always divide wealth into three parts: a third in land, a third in merchandise, and a third ready to hand*". The discussion on how wealth should be allocated across assets has existed for a long time, and is still ongoing. The naive allocating rule is one of the oldest allocation strategies, stating that wealth should be distributed equally among all assets. The naive strategy does not rely on any estimation or optimization, it simply divides wealth equally across all assets. In 1952 Markowitz introduced the mean-variance strategy with the aim to optimize tradeoff between risk and return. The mean-variance optimization theory is considered a cornerstone in modern portfolio management, and it is an important part of every financial textbook. Virtually all portfolio optimization methods are based on the mean-variance portfolio optimization theory, the Capital Asset Pricing Model (CAPM) and the Sharpe ratio are also based on this theory. Modern portfolio management is based on using quantitative methods, where each portfolio is an active dynamic portfolio that is rebalanced periodically. How a manger should allocate wealth across assets, balancing risk against performance, is key to effective portfolio management.

Several researchers have identified some unexpected patterns in average returns often referred to as average-return anomalies, tying average return to various factors. A part of these anomalies is the size-anomaly, which refers to when small market cap stocks historically outperforms big market cap stocks. The value-anomaly, refers to when high book-to-market stocks outperforms their low book-to-market counterparts. These researchers have found that the unexpected patterns in average return cannot be explained by the CAPM or the Sharpe ratio. Fama and French (1993) introduced the 3-factor model which is used to correct for the size- and value- anomaly. This model is an extension of the CAPM, tying expected return on a portfolio to be explained by its sensitivity to three factors.

Low-volatility anomaly was first documented by Blitz and Van Vliet (2007) in the USA, European and Japanese equity market, and in a follow-up paper by Blitz, Pang, and Van Vliet (2013) they documented the low-volatility anomaly in emerging equity markets. Low-volatility anomaly can be explained as when stocks with low volatility earns higher risk-adjusted returns, their results indicates that equity investors overpays for risky stocks. The low-volatility anomaly is a part of a more general "low-risk anomaly" in the financial market. Black, Jensen, and Scholes (1972) observed that the relationship between beta and expected excess return is flatter than predicted by the CAPM. The Fama-French 2-factor model is used by several researchers, such as Scherer (2011), Goldberg, Leshem, and Geddes (2013) and Zakamulin (2017), to correct for the low-volatility anomaly. This model is an extension of the CAPM model, and the expected return on a portfolio can be explained by its sensitivity to two factors.

DeMiguel et al. (2009) conducted a highly influential study, where they compared the performance of optimized strategies to the naive allocation rule. They define the naive rule as: "*One in which a fraction 1/N of wealth is allocated to each of the N assets available for investment at each rebalancing time*". The paper by DeMiguel et al. (2009) concluded that none of the considered optimized strategies consistently outperforms the naive diversification rule, in term of Sharpe ratio. This study initiated a heated debate in the financial academic community on whether portfolio optimization adds any value.

Kritzman, Page, and Turkington (2010) as well as Kirby and Ostdiek (2012) are among researchers defending the optimized strategies, arguing that the research presented by DeMiguel et al. (2009) cast the mean-variance strategy in an unfavourable light. Kritzman et al. (2010) argues that when one has at least some information on expected returns, riskiness and diversification properties of an asset, the optimized strategies should outperform the naive allocation rule. Kirby and Ostdiek (2012) argues that the results presented by DeMiguel et al. (2009) are obtained due to their research design, as they use portfolios that are subject to high estimation risk and extreme turnover.

Zakamulin (2017) points out that researchers such as Kritzman et al. (2010) and Kirby and Ostdiek (2012) defends the value of optimized portfolios, using the data provided by Kenneth French and the Sharpe ratio as a performance measure. First, he demonstrates the presence of low-volatility effect in virtually all datasets provided by Kenneth French. Second, evaluating a few simple optimized portfolios methods that are said to outperform the naive allocation rule, he shows that after controlling for the low-volatility effect, there are no evidence of superior performance. He argues that the Sharpe ratio is not sufficient as a performance measure, as it does not control for whether superior performance appears due to mean-variance efficiency or due to the presence of some known anomalies.

When conducting an empirical study we evaluate ten datasets based on the USA market and three datasets based on the Norwegian market, across six optimized strategies. The limited (same as the specific time period used by other researcher) time period for the USA market is set from July 1973 to 2008, extended time period is set from January 1694 to December 2018, and the data is collected from the Kenneth French data library. For the Norwegian market we set the time period from July 1981 to December 2016, and the data is collected from Bernt Arne Ødegaard data library. To examine the performance of several optimized portfolios we formulate a hypothesis, where the null hypothesis states that: *performance by optimized portfolio is equal to the performance by the naive allocation strategy*.

The goal of the conducted study in this thesis is to examine the performance of several optimized portfolios using the naive strategy as benchmark. First, we examine if any evidence indicates that the optimized strategies are superior to the naive allocation strategy, using the Sharpe ratio and the alpha in the CAPM. These results are compared to findings of other researchers. Second, we determine if superior performance of optimized portfolios can be attributed to mean-variance efficiency or due to exposure to financial anomalies using the 3-factor and 2-factor model. Third, we compare the results for the limited and the extended time period for the USA market, and check the robustness of the results. Fourth, we compare the results for the limited time period for the USA market, against the results obtained for the Norwegian market. The results from the conducted study shows that using Sharpe ratio and alpha from the CAPM as performance measure, controlling for financial anomalies, comparing two time period for the USA market and comparing the Norwegian and the USA market all indicates that no optimized strategy consistently beat the naive allocation rule.

The rest of the thesis is organized as following. Section 2 reviews literature and Section 3 presents the data used in the empirical study. Section 4 describes the methodology used in the empirical study. Section 5 presents and discusses the results and Section 6 concludes the thesis.

2. Literature Review

The mean-variance methodology introduced by Markowitz (1952) relies on estimates of the portfolio mean and variance, and is considered a cornerstone in modern portfolio theory and financial theory. Modern portfolio theory (MPT) aims to optimize or maximize the expected return for a given level of risk, one using quantitative methods, where each portfolio is an active dynamic portfolio that is rebalanced periodically. According to the MPT one can construct a “efficiency frontier”, representing the optimal combination of risk and return.

The CAPM is a part of statistical measurement used in modern portfolio theory, the model takes into account the portfolios sensitivity to systematic risk, expected return and the expected return on a risk-free asset. Using the CAPM as statistical measurement, we use the Alpha (α) as a measurement of performance. If the alpha value is statistically different compared to the benchmark value, this indicates that the strategy has managed to beat the market. The Sharpe ratio was introduced by Sharpe (1966) and it is also a part of statistical measurement used in modern portfolio theory. The Sharpe ratio takes into account portfolios expected returns, risk-free rate and the standard deviation of the portfolios excess returns. This model is used to understand the return of an investment compared to its risk. We notice a problem using these performance measures, as they do not control for whether superior performance is due to mean-variance efficiency or due to the presence of some anomalies

Several researchers over time has discovered some unexpected patterns in average return, challenging the assumption of efficient market. These unexpected patterns in average returns are not captured by the performance measures α in the CAPM or the Sharpe ratio, and are typically referred to as average-return anomalies. A part of these average-return anomalies are: (i) size-anomaly that can be explained as when small market cap stocks historically outperforms big market cap stocks; (ii) value-anomaly can be explained as when high book-to-market stocks have outperformed their low book-to-market counterparts. Fama and French (1993) introduced the 3-factor model tying expected return on a portfolio to be explained by its sensitivity to three factors: (i) the excess return on a broad market portfolio (MKT-RF); (ii) difference between return on a portfolio of small stocks and return from a portfolio of large stocks (SMB); (iii) difference between return on a portfolio with high-book-to-market stocks and returns on a portfolio with low-book-to-market stocks (HML). Using these three factors they were able to capture much of the cross-sectional variation in average return.

Blitz and Van Vliet (2007) documented the existence of low-volatility anomaly in USA, European and Japanese equity market, and in a follow-up paper Blitz et al. (2013) reported similar anomaly in emerging equity markets. The low-volatility anomaly can be explained as when portfolios consisting of stocks with low volatilities significantly outperforms portfolios of stocks with high volatilities. Research that supported their findings was already introduced by Black et al. (1972), they presented evidence that indicates that the expected excess return on an asset is not strictly proportional to its beta. Also Fama and MacBeth (1973) test the relationship between average return and risk for New York Stock Exchange. They found that for the overall period there is no clear relationship between risk and return. Multiple optimization portfolios invest heavily in assets with low-volatility stocks, when controlling for the low-volatility effect, the superior performance shown by these optimized strategies disappear. Scherer (2011), Leote, Lu, and Moulin (2012) and Goldberg et al. (2013) finds that the High Minus Low (HML) Fama-French factor can be used to control for the low-volatility anomaly. Blitz (2016) constructs a distinct low-volatility risk factor and find that this factor is highly positively correlated with the HML factor. We control for the low-volatility anomaly by augmenting the CAPM with the HML Fama-French factor, and using α as a performance measure.

The volatile and turbulent financial markets of the 2000s decade, boosted the interest in investment optimization strategies with focus on optimal portfolio risk control across assets and time. Here one explicitly acknowledges that it is impossible to predict the mean returns, but it is possible to predict the variance-covariance matrix. In the conducted study we evaluate several optimized portfolios, the motivation behind each optimized portfolio is different. The mean-variance portfolio aims to optimized the tradeoff between risk and return, selecting a specific mean return value and minimizing the risk associated with the return value. This strategy relies on estimates of the portfolio mean return and the portfolio variance. Clarke, de Silva, and Thorley (2006) proposed the minimum variance portfolio where the aim is to minimize the risk, motivated by the fact that the variance-covariance matrix can be estimated much more precisely than the mean returns. A problem with the minimum-variance model is that it is poorly diversified, and the investment is heavily concentrated in assets with the lowest standard deviation. Choueifaty and Coignard (2008) introduces the maximum diversification ratio, as a way to invest more in assets with higher standard deviation. Asness, Frazzini, and Pedersen (2012) proposed the risk parity portfolio where the aim is to construct portfolio with equal risk contribution from all assets. Kirby and Ostdiek (2012) introduced the volatility timing portfolio, where the portfolio weights are rebalanced monthly based solely on changes in the variance of asset returns. Kirby and Ostdiek (2012) also introduced the reward-to-risk timing portfolios, where the portfolio weights are rebalanced monthly based on changes in the expected return.

The objective of DeMiguel et al. (2009) is to understand under which conditions mean-variance optimal portfolio models can be expected to outperform the naive strategy, even in the presence of estimation risk. They evaluate the out-of-sample performance of 14 models across seven empirical datasets of monthly return, using the naive strategy as benchmark and the Sharpe ratio as performance measure. DeMiguel et al. (2009) demonstrated that none of the optimized allocation strategies they considered consistently outperformed the naive allocation strategy, initiating a debate in the economic academic community. They also argue that the estimation window needed to be much longer before one would be able to realize any gain promised by the optimal portfolio choice. Many researchers have challenged their result in favour of optimized strategies.

Duchin and Levy (2009) conducted a study with the aim to answer the question: if it is better to employ the naive strategy and avoid choosing the wrong portfolio mix due to sampling errors of the various parameters. Or if it is better to employ the optimal mean-variance strategy which takes into account various parameters, albeit with some possible sampling errors. When using a mean-variance strategy with the same standard deviation as the naive strategy, they compare the average return of each strategy. Their result shows that the naive strategy, in some cases outperform the optimal strategy. However, a critic of their study is that they consider only three portfolios. When considering only tree portfolios, there is a chance that the results they present are not robust and does not justify the conclusion drawn.

Kritzman et al. (2010) argues that when one has at least some information about expected return, riskiness and diversification properties of the asset, the optimization portfolio should outperform the naive portfolio. They evaluate 13 datasets, with which they construct 50,000 optimized portfolios and evaluate the out of sample performance. They computed the Sharpe ratio and obtained higher values for the optimized portfolio. However, one criticism of the result they present, is that they do not test if the different Sharpe ratios are significantly different. Another criticism of their study is that they use only one performance measure, the Sharpe ratio. This performance measure does not adjust for any anomaly, that may be present in the dataset.

Kirby and Ostdiek (2012) argues that the results by DeMiguel et al. (2009) cast mean-variance optimization in a unfavorable light, mostly because their research design targets conditional expected return that exceeds the conditional expected return of the naive strategy. They argue that if the mean-variance model is implemented by targeting the conditional expected return of the naive strategy, the mean-variance generally performance better than the naive portfolio. They use four datasets with monthly returns, implementing four strategies, using the Sharpe ratio as performance measure. They introduced two new methods of mean-variance portfolio selection that exploits sample information in a manner that mitigates estimation risk impact. Volatility timing strategy and reward-to-risk timing strategy, these portfolios are characterized by four notable features: (i) require no optimization, (ii) require no covariance matrix inversion, (iii) does not generate negative weights, (iiii) allow the sensitivity of the weights to volatility to change and be adjusted via a tuning parameter. By the use of these two strategies they obtain results that outperform the naive portfolio performance.

Zakamulin (2017) aim to provide a cautionary note regarding the use of datasets provided by Kenneth French, without controlling for if superior performance is due to mean-variance efficiency, or due to established factor premiums. He demonstrates the presence of low-volatility anomaly in virtually all datasets provided by Kenneth French. Evaluating a few optimized strategies that are earlier proven to outperform the naive allocation strategy, such as claimed by Kritzman et al. (2010) and Kirby and Ostdiek (2012), he shows that after controlling for the low-volatility effect there is no evidence of superior performance. The main conclusion Zakamulin (2017) reaches, is as following. A convincing demonstration of portfolio optimization value cannot be made without controlling for some known anomalies before evaluating superior performance.

Serval researchers conducted studies, either defending superior performance by optimized strategies or claiming that optimized strategies do not add any value. Researchers use USA based data for a specific time period, making it difficult to determine if the results are robust across other markets and time periods. As well as very few of them control for whether superior performance is due to mean-variance efficiency or due to exposure to financial anomalies.

3. Data

In order to evaluate the performance of the different allocation strategies, an analysis of several empirical dataset is conducted. The data used was obtained from two separate data libraries maintained by Kenneth R. French[1] and Bernt Arne Ødegaard[2], all datasets are value-weighted portfolios sorted using different criterias. A list over the datasets considered is given in Table 1, with a brief description. The purpose of using several datasets is to evaluate if the result obtained from one dataset, is consistent with the result from the other datasets.

We use several factors when computing the performance measures, these are also collected from Kenneth French and Bernt Arne Ødegaard´s web sites. Factors SMB short for Small Minus Big, is the average return on the three smallest portfolios minus the average return on the three largest portfolios. HML stands for High Minus Low, and is the average return on the two value portfolios minus the average return on two growth portfolios. RF is the risk-free rate, and MKT-RF denotes excess return on the market. All factors collected from Kenneth R. French database are factors based on US market, while the factors for the Norwegian marked are collected from Bernt Arne Ødegaard´s web site.

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[1] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html
[2] http://finance.bi.no/~bernt/financial_data/ose_asset_pricing_data/index.html

Table 1, Datasets

#	DATA AND SOURCE	N	TIME PERIOD
1.	Portfolios based on firm size	10	07/1926–12/2018
2.	Portfolios based on book-to-market value	10	07/1926–12/2018
3.	Portfolios based on operating profitability	10	07/1926–12/2018
4.	Portfolios based on investment	10	07/1926–12/2018
5.	Portfolios based on size and book-to-market	6	07/1926–12/2018
6.	Portfolios based on size and book-to-market	20	07/1926–12/2018
7.	Portfolios based on earnings/price ratio	10	07/1951–12/2018
8.	Portfolios based on cashflow/price ratio	10	07/1951-12/2018
9.	Portfolios based on dividend yield	10	07/1927–12/2018
10.	Portfolios based on accruals	10	07/1963–12/2018
11.	Portfolios based on firm size	10	01/1980-12/2017
12.	Portfolios based on book-to-market,	10	01/1980 12/2017
13.	Portfolios based on spread,	10	01/1980-12/2017

Table 1 is a list over various dataset that are analyzed in the conducted study. # denotes the dataset number, N denotes number of portfolios in a dataset. Portfolio 1-10 are USA based data and are obtained from Kenneth French data library. Portfolio 11-13 are Norwegian based data and collected from Bernt Arne Ødegaard data library.

Data collected from Kenneth R. French´s website are based on the New York Stock Exchange (NYSE), American Stock Exchange (AMEX) and National Association of Securities Dealers Automated Quotations (NASDAQ). The ten first portfolios are constructed at the end of June using the June market equity and NYSE breakpoint. In the empirical study the datasets used are the monthly return from January 1964 to December 2018, and from July 1973 to December 2008.

In Table 1 the last three portfolios are collected from Bernt Arne Ødegaard´s website. The data are portfolio returns based on the Norwegian market collected from Oslo Stock Exchange (OSE), and interest rate are collected from Norges Bank[3]. Equity size used are the observation from the previous period, t-1. The data is portfolio returns from July 1981 to December 2016, with a monthly interval.

[3] https://www.norges-bank.no/en/

As previous researchers such as DeMiguel et al. (2009) and Zakamulin (2017), we select a limited time period from July 1973 to December 2008. This is done to make the results comparable with their finding. We use this subperiod only on the USA market, as this time period is not available for the Norwegian based data, and earlier research is done using USA market data. Compared to earlier research we have a longer time period available for the USA market, and we conduct an analysis of the extended time period from January 1694 to December 2018, we also use the extended time period to check the robustness. For the Norwegian market we select a time period with equal length as the sub-period used for the USA market.

4. Methodology

This section provides a description of the methodology used in the conducted empirical testing providing theoretical framework for the concept of backtesting, numerical solutions, performance measures as well as portfolio strategies listed in Table 2.

Table 2, Allocation strategies

#	MODEL	ABBREVIATION
1.	Naive strategy	1/N
2.	Mean-variance strategy	Mean-var.
3.	Minimum variance	Min-var.
4.	Maximum-diversification	Max.div.
5.	Risk-parity	Risk-par.
6.	Volatility timing $\eta \in [2, 4]$	VT
7.	Reward-to-risk timing $\eta \in [2, 4]$	RRT

This table lists the various asset-allocation models considered, and # denotes the number of the strategy. Volatility timing and reward-to-risk timing strategies are used twice, with $\eta \in [2, 4]$. The last column of the table gives the abbreviation used to later refer to the strategy.

We set a budget constrain on various strategies, indicated by $\sum_{i=1}^{N} w_i = 1$. This sets an upper limit of 1, as the total wealth that can be allocated across all assets. To make the results comparable with those of DeMiguel et al. (2009), Kritzman et al. (2010) and Zakamulin (2017) we imposed the short sale constraint on the mean-variance, minimum variance and the maximum diversification strategy. With the short sale restriction, we consider long-only portfolios, short sales strategies are also considered to be very risky strategies. The asset weights in those models are restricted to be non-negative, this can be written as $w_i \geq 0$.

4.1 Backtesting

Modern portfolio management is based on using quantitative models, where each portfolio is an active dynamic portfolio that is rebalanced periodically. One usually starts with the estimations of parameters, such as variance-covariance matrix and mean returns, then solving a specific optimization problem, such as minimizing risk or maximising diversification. Each step is repeated periodically. At each rebalancing time, a portfolio manager must re-estimate model parameters and re-solve the optimization problem. This is done to ensure that the asset weights are according to the chosen strategy aim.

Backtesting method relies on using historical data to compare performance of portfolio strategies. One stimulates a portfolio strategy using a specific time period, determining how a strategy would perform. Stimulating the performance of various allocation strategies, this allows us to compare these strategies against each other. This method ensures estimation of the "real-life performance" of a portfolio, therefore a portfolio manager must stimulate a real-life trading. As a way to stimulate real-life trading the dataset is divided into "in-sample" and "out-of-sample" segments, setting time t as the split point between the segments. At time t a portfolio manager needs to construct a portfolio according to an allocation strategy. The manager is only allowed to use historical data up to time t, the period from time 1 to time t is used to estimate the model parameters.

The "rolling data window" approach is used to estimate parameters, such as vector of mean returns and the variance-covariance matrix. This approach is also used to estimate asset weights, when considering portfolios consisting of only risky assets. These weights are later used to compute return in month t + 1. This process is continued throughout the sample, by adding return for the next period, dropping earliest returns. The outcome of this approach is a series of out-of-sample returns, generated by all of the portfolio strategies listed in Table 2 for all datasets listed in Table 1. In the conducted study we evaluate datasets of various lengths, as specified in

Section 3, for all datasets the length of the rolling data window is the same. The main problem one encounters while using this method, is choosing how long the look-back period should be. DeMiguel et al. (2009) argue that using 60 months compared to 120 months, produce not very different results, hence we choose to use a rolling data window of 60 months

4.2 Numerical methods

When we impose the short sale constrain on various allocation strategies, there is no closed-form solution possible. One has to rely on using numerical methods to find the asset weights. Numerical method can be explained as using algorithms to construct an approximate solution. In the empirical study we use the statistical program R[4], to conduct the statistical analysis. R is a statistical programming language, that provides a complete environment in which one can import data, perform computations, run statistical analysis and plot graphical models. RStudio[5] environment is preferred compared to the environment used in R, as RStudio has a more user-friendly environment. Analysis conducted in this paper is done using the RStudio environment. Numerical solution for the mean-variance and minimum-variance strategy with the short sale constrain are found using the R package "quadprog" function solve.QP. The numerical solution for the maximum-diversification strategy with the short sale constrain are found using the R package "Rsolnp" and function solvnp.

4.3 Portfolio strategies

4.3.1 Naive portfolio strategy

The idea of the naive portfolio is inspired by Rabbi Issac bar Aha, who proposed around the fourth century, a rule for asset allocation. "*One should always divide wealth into three parts: a third in land, a third in merchandise, and a third ready to hand*". This simple thought was the idea behind the asset allocation strategy known as the naive portfolio strategy. One would assume that such a simple strategy would have been outperformed by optimized strategies, but that is not the case. A definition provided by DeMiguel et al. (2009) defines the naive strategy

[4] https://www.r-project.org/
[5] https://www.rstudio.com/

as, "*we define the naive rule to be one in which a fraction 1/N of wealth is allocated to each of the N assets available for investment at each rebalancing date*". The intention is not to propose that one should use the naive strategy, but it is rather used as a benchmark to evaluate the performance of other strategies. The weight of asset i is found using the following equation

$$w_i = \frac{1}{N},$$

where w_i denotes the weight of an investor's wealth invested in asset i, 1 is the total wealth and N denotes number portfolios in the dataset considered. An explanation provided by DeMiguel et al. (2009) describes a well performed naive portfolio as following: "*Diversified portfolios have lower idiosyncratic volatility than individual assets, the loss from naive opposed to optimal diversification is much smaller when allocating wealth across portfolios*".

4.3.2 Optimized strategies

The optimized strategies rely on estimations of the portfolio mean and the variance. Variance can be explained as a number that tell how varied or spread out the numbers in a set are, we associate variance with risk. We can find the portfolio mean return and variance using the following equation

$$\mu_p = \boldsymbol{w}'\boldsymbol{\mu} \quad \text{and} \quad \sigma_p^2 = \boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w} \quad \text{subject to} \quad \boldsymbol{w}'\mathbf{1} = 1,$$

where μ_p denotes the portfolio mean return and σ_p^2 denotes the portfolio variance. We denote $\boldsymbol{w}$ as the weight matrix, $\boldsymbol{w}'$ denotes the transpose of the weight matrix, $\boldsymbol{\mu}$ denotes the mean return matrix, $\boldsymbol{\Sigma}$ denotes the variance-covariance matrix and $\mathbf{1}$ is a vector of ones. Subjecting the equation to $\boldsymbol{w}'\mathbf{1} = 1$ refers to the budget constrain implemented on the equation. This constrain is setting an upper limit of one, this limits how much can be totally allocated across all risky assets. In the conducted study these parameters are found with the rolling data window method, the parameters are later used when computing the optimized portfolios.

4.3.3 Mean-Variance strategy

The mean-variance model introduced by Markowitz (1952), allows the investor to optimize the tradeoff between the mean and variance of portfolio return. This model relies on the assumption that the investor will make a rational decision, if they have complete information. If one is able to construct two portfolios with different variance, for a particular mean value, the investor would prefer the portfolio with the lowest variance. The weights of the mean-variance strategy are the solution to the following quadratic program

$$\min_{w} \boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w} \quad \text{subject to} \quad \boldsymbol{w}'\boldsymbol{\mu} = \mu^*, \; \boldsymbol{w}'\boldsymbol{1} = 1 \text{ and } w_i \geq 0$$

where μ^* denotes the particular value of the portfolio mean we want to construct the portfolio with the lowest variance. When setting the equation subject to $\boldsymbol{w}'\boldsymbol{\mu} = \mu^*$, this is a condition that one finds the weights given the particular mean value set to μ^*. $\boldsymbol{w}'\boldsymbol{1} = 1$ sets a upper limit of one as the total wealth that can be allocated across all assets. The $w_i \geq 0$ denotes the short sale constrain. There is no explicit solution to the quadratic equation when we impose the short sale constrain, one has to rely on numerical method to find the asset weights. We use the R package "quadprog", function solve.QP to find the numerical solution.

4.3.4 Minimum variance model

Clarke et al. (2006) introduced this model where one focus only on estimates of the variance-covariance matrix, rather than estimations of expected returns. One way to consider this model is as a reduced form of the mean-variance model, where the aim is to minimize the risk. The variance-covariance matrix provides estimates that are more precise than the estimated expected returns, thus providing more reliable results. The asset weights for the minimum variance strategy is the solution to the following quadratic program

$$\min_{w} \frac{1}{2}\sum_{i=1}^{N} \boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w} \quad \text{subject to} \quad \boldsymbol{w}'\boldsymbol{1} = 1 \quad \text{and} \quad w_i \geq 0\,,$$

where $\boldsymbol{\Sigma}^{-1}$ denotes the transpose of the variance-covariance matrix. In this thesis the model is implemented with short sale constrain, and therefore it is not possible to use closed-form solution to find the model weights. The solution can be obtained using a numerical method, we use the R package "quadprog" function solve.QP.

4.3.5 Maximum diversification

The mean-variance strategy invest heavily in assets with low standard deviation, resulting in poorly diversified portfolio. Choueifaty and Coignard (2008) introduced the diversification ratio defined as $\frac{w'\sigma}{\sqrt{w'\Sigma w}}$, this is used to increase investment in assets with higher standard deviation. The asset weights is the solution to the following program

$$\min_{w} \frac{\boldsymbol{w'\sigma}}{\sqrt{\boldsymbol{w'\Sigma w}}} \quad \text{subject to} \quad \sum_{i=1}^{N} w_i = 1 \quad \text{and} \quad w_i \geq 0.$$

The diversification ratio helps to reduce the problem of investing heavy in few assets and the distribution of wealth is more versatile. This model is implemented with the short sale restriction, indicated by $w_i \geq 0$. As earlier pointed out, models with short sale restriction have no closed-form solution, and one relies on numerical method to find the asset weights. We find the asset weights by using R package "Rsolnp" and function solvnp.

4.3.6 Risk-parity model

The idea with the risk parity model is to construct a portfolio with equal risk contribution from all assets. This strategy is motivated by the observation that in traditional asset management with 60/40 portfolio of stock and bonds, 80-90% of the risk comes from stocks. Asness et al. (2012) proposed a method where one does not consider the variance-covariance matrix, but rather use estimates of standard deviation. To constructs the portfolio of N assets, using a rolling window, one estimates the standard deviation $\hat{\sigma}_{i,t}$ $i = 1,2, \dots. N$. The asset weights are found using the following equation

$$w_{i,t} = \frac{1}{\hat{\sigma}_{i,t}} k_t, \qquad k_t = \frac{1}{\sum_{i=1}^{N} \frac{1}{\hat{\sigma}_{i,t}}},$$

where k_t is the normalization factor that makes the sum of all weights equal to one. Note that the risk-parity portfolio overweights the less volatile assets, and underweights more volatile assets.

4.3.7 Volatility timing

Fleming, Kirby, and Ostdiek (2003) study a class of portfolio strategies where portfolio weights are rebalanced monthly, based on changes in the estimated covariance matrix of returns. They find that rebalancing according to change in the estimated mean-variance matrix, the volatility timing strategies (VT) outperform mean-variance optimized portfolios significant. The asset weights are found using the flowing equation

$$w_i(\eta) = \frac{\left(\frac{1}{\sigma_i^2}\right)^{\eta}}{\sum_{i=1}^{N}\left(\frac{1}{\sigma_i^2}\right)^{\eta}},$$

where σ_i^2 denotes the variance of asset i and η is a tuning parameter. η parameter is introduced as a *"measure that determines how aggressively the investor adjust portfolio weights in response to volatility changes"* (Kirby & Ostdiek, 2012), page 448). As $\eta \rightarrow 0$ one recovers the naive strategy and as $\eta \rightarrow \infty$ the weight distribution is heaviest in the asset with the lowest volatility. In this paper this strategy is used twice, with $\eta \in [2, 4]$.

4.3.8 Reward-to-risk timing

While volatility timing strategy ignores information about expected return, it is natural to ask if one can improve performance by incorporating information on expected return. Kirby and Ostdiek (2012) suggest the reward-to-risk timing (RRT) strategy as a method to incorporating conditional expected return. The asset weights are calculated using the following equation

$$w_i(\eta) = \frac{\left(\frac{(\mu_i - r_t)^+}{\sigma_i^2}\right)^{\eta}}{\sum_{i=1}^{N}\left(\frac{(\mu_i - r_f)^+}{\sigma_i^2}\right)^{\eta}},$$

where μ_i denotes mean return of asset i, and r_f is the risk free rate of return. $(\mu_i - r_f)^+$ is equivalent to max $(\mu_i - r_f, 0)$, which guarantees the non-negativeness of asset i. As $\eta \to 0$ one recovers the naive strategy and as $\eta \to \infty$ the weight distribution is heaviest in the asset with the maximum estimated reward-to-risk ratio. In the conducted study we use this strategy twice, with the tuning parameter $\eta \in [2, 4]$.

4.4 Performance measures

4.4.1 Sharpe ratio

Sharpe (1966) introduced the Sharpe ratio, used to evaluate the performance of different portfolio strategies. The Sharpe ratio is given by

$$SR = \frac{r_i - r_f}{\sigma_i},$$

where r_i is the return on asset i, r_f is the risk free rate of return and σ_i is the standard deviation of asset i. Then quantities are computed as

$$r_i = \frac{1}{T} \sum_{i,t=1}^{T} x_i \quad \text{and} \quad \sigma_i = \sqrt{\frac{1}{T-1} \sum_{i=1}^{T} (x_i - \mathrm{E}[x])^2}\,,$$

where T is the number of months used in the rolling window, and x_i denotes return in month i. We compute the Sharpe ratio of each optimized strategy SR_j, and the naive benchmark strategy $SR_{1/N}$. The Sharpe ratios are compared against each other, testing whether SR_j and $SR_{1/N}$ are statistically distinguishable. Formulating this as a hypothesis, we test the null hypothesis

$$H_0 : SR_{1/N} = SR_j\,,$$

with a test statistic

$$Z = \frac{SR_1 - SR_2}{\sqrt{\frac{1}{T}\left[2(1-\rho) + \frac{1}{2}(SR_1^2 + SR_2^2 - 2SR_1 SR_2 \rho^2)\right]}},$$

where ρ denotes the estimated correlation coefficient over the sample of size T. To test the null hypothesis, we compute the p-value of the difference between optimized portfolio and naive allocation rule. Comparing the values against each other, for this purpose such as DeMiguel et al. (2009), Zakamulin (2017) and other researchers, we employ the Jobson and Korkie (1981) test with the Memmel (2003) correction. The decision rule is as following: if the p-value is below α reject the null hypothesis, else fail to reject.

4.4.2 Alpha in the CAPM

The CAPM was developed by Sharpe (1964), Lintner (1965) and Jensen (1969), with the purpose of estimating the expected return and risk composition of an asset. The advice regard asset allocating provided by the CAPM is as following: investors should hold the market portfolio, levered according to investors risk preference. The single-factor model is usually motivated by the CAPM, and can be considered to be a implementation of the CAPM. In this model the single factor is represented as the market return at time t, and it is the single source of risk. Some assumptions regarding this method are, i) expected value of the disturbance term is zero, ii) firm-specific risk is uncorrelated, and (iii) firm specific risk is uncorrelated with market risk. The alpha can be found using following equation

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_M \left(r_{M,t} - r_{f,t} \right) + \varepsilon_t,$$

rewriting this model, the excess return is given by

$$R_{i,t} = \alpha + \beta R_{M,t} + \varepsilon_t,$$

where r_t denotes the return on risky asset at time t of asset i and $r_{f,t}$ is the risk-free rate of return at time t. The α_i denotes the expected return of asset i if the market is neutral, $r_{M,t}$ denotes the market return at time t. The β_i denotes asset i´s responsiveness to market movements, while ε_t is the disturbance term. If one can fully explain the return of a risky asset with the use of this model, estimated value of α_i is zero. Calculating the alpha for the optimized strategy α_j^{CAPM}, and the naive allocating strategy $\alpha_{1/N}^{CAPM}$, we compare the two values against each other, testing whether α_j^{CAPM} and $\alpha_{1/N}^{CAPM}$ are statistically distinguishable. When comparing the alphas we calculate p-values that are later compared against the critical value α, set to 5%. The decision rule is as following: if the p-value is below α reject the null hypothesis, else fail to reject. The test is formulated as a hypothesis

$$H_0 : \alpha_{1/N}^{CAPM} = \alpha_j^{CAPM}.$$

4.4.3 Alpha in the Fama-French 3-factor model

As earlier established by researchers in the presence of size- and value-anomalies, these unexpected patterns cannot be captured by the Sharpe ratio or CAPM. Fama and French (1993) introduced the 3-factor model as a method to correct for the size- and value-anomalies. They identified three risk factors, tying average return to various firm characteristics. This model augment the CAPM equation with two factors: i) firm size factor (SMB_t) and ii) book to market equity factor (HML_t). SMB factor account for publicly traded companies with small market caps that generate higher return. While HML factor account for value stocks with high book-to-market ratios, that generate higher returns in comparison to the markets. The model can be represented as following

$$R_{i,t} = \alpha + \beta_M R_{M,t} + \beta_j SMB_t + \beta_k HML_t + \varepsilon_{i,t},$$

where the intent of the model is to examine if the alphas for the optimized strategy and the naive strategy are statistically distinguishable, when taking into account some unexpected pattern in average return. Calculating the alpha for the optimized strategy α_j^{FF3},and the naive allocating strategy $\alpha_{1/N}^{FF3}$, we compare the alphas calculating the p-values that are later compared against the critical value α, set to 5%. The decision rule is as following: if the p-value is below α reject the null hypothesis, else fail to reject. The test is formulated as a hypothesis

$$H_0 : \alpha_{1/N}^{FF3} = \alpha_j^{FF3}.$$

4.4.4 Alpha in the Fama-French 2-factor model

This model is used by Scherer (2011), Goldberg et al. (2013) and Zakamulin (2017) to determine if superior performance by optimized models can be attributed to low-volatility anomaly. The model augment the CAPM with the book to market equity factor (HML_t) we calculate the alphas, using following equation

$$R_{i,t} = \alpha + \beta_M R_{M,t} + \beta_j HML_t + \varepsilon_{i,t}.$$

The intent of the model is to examine if the alphas for the optimized strategy and the naive strategy are statistically distinguishable, when taking into account the low-volatility anomaly. We start by calculating the alpha for the optimized strategy α_j^{FF2} and the naive allocating strategy $\alpha_{1/N}^{FF2}$, when comparing the alphas we calculate p-values that are later compared against the critical value α, set to 5%. The decision rule is as following: if the p-value is below α reject the null hypothesis, else fail to reject. The test is formulated as a hypothesis

$$H_0 : \alpha_{1/N}^{FF2} = \alpha_j^{FF2}.$$

5. Empirical Results

In this section we have an interest to infer results of the performance for optimized strategies compared to the naive strategy. We compare the performance for all datasets listed in Table 1, across all strategies listed in Table 2. In each table the columns refer to the different strategies, while the rows refer to various datasets. When testing the null hypothesis for the various performance measure, if the values are significantly different, we indicate this by highlighting the values in the tables. When the values are statistically different, we reject the null hypothesis of equal performance. The results are described using a significantly different ratio, as an indication of numbers of significantly different values observed in each performance measure. We compute the ratio of significantly different values in percentage for each performance measures. We count all significantly different values that are higher and none-negative compared to the benchmark strategy, valued against numbers of p-values. The empirical study is conducted using the methodology presented in Section 4.

First, we examine the limited time period using the Sharpe ratio as performance measure, the results are presented in Table 3. When comparing the optimized strategies against the naive allocation strategy, we observe that the majority of Sharpe ratios of optimized strategies are higher compared to the naive allocation strategy, even if the values are not considered to be statistically different. These results are consistent with the findings of DeMiguel et al. (2009), who reported higher Sharpe ratio for optimized strategies. In Table 3 we observe the ratio of significantly different values to be 25%, where the optimized strategies outperform the naive allocation rule.

Using the alpha in the CAPM as performance measure for the limited time period the results are presented in Table 4. When comparing the significantly different values for the optimized strategies compared to the naive allocation rule, we observe a 22.5% ratio as where optimized strategies outperform the naive allocation rule. All of the significantly different values are higher compared to the naive strategy, however one value is negative and not included in the significantly different ratio. The majority of significantly different values are observed using the risk-parity model. Similar with the Sharpe ratio of higher values for optimized strategies, the majority of α^{CAPM} are higher compared to the benchmark strategy, even if they are not statistically different.

Second, Fama and French (1996) argues that some unexpected patterns in average return cannot be explained by the Sharpe ratio or CAPM. They proposed using the 3-factor model to control for unexpected patterns in average return. Fama and French (1993) shows that using the 3-factor model is a good description of returns on portfolios formed on size and book-to-market. Fama and French (1996) shows that this applies also on portfolios formed on earning/price and cashflow/price. Considering that portfolios # 1, 2, 5,6,7 and 8 are formed on those specifics, we are interested to see if numbers of significantly different values are changed when using the 3-factor model compared to the CAPM. In the significantly different ratio we just consider the significantly different values observed in listen portfolios. If one is able to observe a reduction in significantly different values, this implies that the superior performance of optimized portfolios can be attributed to the average-return anomaly. For the limited time period considering just the listed portfolio numbers using the alpha in the CAPM we observe a 8,33% ratio of significantly different values, compared to one significantly different alpha when using the 3-factor model. However the significantly different value using 3-factor is negative. These results suggest that when controlling for average-return anomalies there is no evidence of superior performance of optimized strategies.

Consistent with other researchers, we use the 2-factor model as a method to control for the low-volatility anomaly. For the limited time period for the USA market we observe a ratio of 25% of significantly different ratio using the Sharpe ratio, a 22.5% ratio when using α in the CAPM, compared to a 2.5% ratio of significantly different values when using the 2-factor model. The reduction of significantly different values in the 2-factor model compared to the Sharpe ratio and α in the CAPM, shows that the majority of superior performance by optimized strategies can be attributed to low-volatility anomaly and not mean-variance efficiency. When controlling for the low-volatility, anomaly superior performance by optimized strategies disappear, these results are consistent with the findings of Zakamulin (2017).

Third, comparing the results for the limited and extended time period for the USA market, we want to determine if our results are robust. The ratio of significantly different Sharpe values in the extended time period is 25%, compared to 25% in the limited time period. For both time periods we observe an identical ratio of significantly different values, but there are some changes in how the significantly different values are distributed across the optimized strategies. In the limited time period the values are more spread out across strategies risk-parity, minimum-variance, volatility timing $\eta = 2 \,\&\, 4$ and reward-to-risk $\eta = 2 \,\&\, 4$. Compared against the extended time period where the values are more concentrated in strategies risk-parity, minimum-variance and volatility timing $\eta = 2 \,\&\, 4$. In both time periods the majority of significantly different values are observed when using the risk-parity strategy. When using the α^{CAPM} for the limited time period we observe 22.5% ratio of significantly different values, and in the extended time period we observe a ratio of 22.5%, the ratios are identical. Across both time periods among the optimized strategies, the majority of significantly different values are observed when using the risk-parity strategy. It is interesting to observe that for both time periods when using the risk-parity strategy in portfolios based on operating profitability, we observe a significantly different value that is negative.

We compare the performance of specified portfolios when evaluating the alpha from the 3-factor model, as a model used to correct for the average-return anomaly in earlier listed portfolios. In the limited time period we observe only one significantly different value, however the value is negative. In the extended time period we observe an 20.8% ratio of significantly different values. That is after correcting for a negative value and one value lower than the benchmark. There is no obvious reason we can point out to explain the changes of significantly different values. The last performance measure we evaluate is the 2-factor model that is used to correct for the low-volatility anomaly. In the limited time period observed a significantly different ratio of 2.5% compared to 11.25% observed in the extended time period. Blitz (2016) shows that the value effect cannot be explained for the USA data pre-1963, nor post-1984. Given that for the extended time period a large portion of the data is post-1984, there is possible that there exist a distinct low-volatility effect that cannot be explained by the value effect. However, compared against the results for the Sharpe ratio and α^{CAPM}, the α^{FF2} reports the lowest number of significantly different value.

Fourth, we compare the results for the Norwegian market listed in Table 11 against the results for the limited USA market, as the time horizon for the limited USA market and the Norwegian market is equal. For the Norwegian market we observe a significantly different ratio of: 8.3% when using the Sharpe ratio, 33% using the α^{CAPM}, 18.75% using the α^{FF3}, 37.5% using the α^{FF2}. In the USA market the Sharpe ratio is the performance measure with the highest ratio of significantly different values, while in the Norwegian market the ratio is the smallest across all performance measures. When correcting for the average-return anomaly in the Norwegian market we observe a reduction in the significantly different ratio using the α^{FF3} compared to α^{CAPM} , consistent with the findings for the USA market. The results for the Norwegian market using the 2-factor model are the most different results compared to the USA market, the ratio of significantly different values is the highest among all performance measure when using α^{FF2}. Showing that when controlling for the low-volatility anomaly, there is an increase in optimized portfolio outperforming the naive allocation rule. One possible explanation for this result can be that the SMB factor is much higher in Norway compared to the USA, therefor the α^{FF3} is the most relevant performance measure when evaluating the Norwegian market.

Throughout the conducted study, portfolios based on operating profitability have the most optimized portfolios that outperform the naive allocation rule. The only exception is observed for the extended time period based on the USA data, when using the 3-factor model. This makes us question, if there are any fluctuations in the data that are not captured by our study. Compared against the results presented by Zakamulin (2017), he observed no odd behavior by such portfolios. In this paper the parameters are estimated using a rolling data window of 60 months, compared to 120 months used by other researchers. It is possible when using a shorter time period while estimating the parameter, that some fluctuations in the data are not captured.

The motivation behind each optimized strategy is different, but we observe that some optimized portfolios perform better than others. Risk-parity portfolios had the highest amount of significantly different values consistent over the extended and limited time period. In the limited time period we observe that 9 out of 10 portfolios have a higher Sharpe ratio and last value is equal to the value of naive allocation strategy, and six out of 10 values are statistically different. For the extended time period all ten risk-parity portfolios have a higher Sharpe ratio than the naive allocation rule, and six out of ten values are statistically different. When using the maximum-diversification strategy we observe that none of the Sharpe ratios where statistically distinguishable for the limited and extended time period. In our results we observe that for the limited time period 7 out of 10 portfolios have a lower Sharpe ratio and for the extended time period in 8 out of 10 portfolios have a lower Sharpe ratio, compared to the naive allocation strategy. These results indicates strongly that one does not obtain higher Sharpe ratio when using the maximum diversification strategy compared to the naive allocation rule, as proposed by Choueifaty and Coignard (2008).

In Section 2 of this thesis we reviewed contribution of other researchers, and we criticized some of them for not using enough datasets when conducting the empirical study. For the USA market we include 10 datasets over two different time periods, we have a large number of observation as a foundation for the drawn conclusion. For the Norwegian market we only consider three datasets for one time period, and we obtain some unexpected results. Considering the Norwegian market we evaluate only three datasets, we cannot with certainty claim that there are no fluctuation in the data that may be responsible for the unexpected results.

Table 3, Sharpe ratio (1973 - 2008)

Strategy

#	1/N	Risk-par.	Min-var.	Mean-var.	Max-div.	VT $\eta = 2$	VT $\eta = 4$	RRT $\eta = 2$	RRT $\eta = 4$
1	1.40	1.40 (0.95)	0.39 (0.97)	1.51 (0.69)	1.31 (0.55)	1.40 (0.99)	1.39 (0.94)	1.47 (0.52)	1.47 (0.69)
2	1.65	1.68 (0.06)	1.69 (0.82)	1.42 (0.33)	1.49 (0.18)	1.73 (0.08)	1.78 (0.12)	1.70 (0.59)	1.64 (0.91)
3	1.20	**1.26** (0.00)	**1.66** (0.00)	1.22 (0.90)	1.04 (0.10)	**1.37** (0.00)	**1.44** (0.00)	1.25 (0.56)	1.24 (0.74)
4	1.52	**1.60** (0.00)	1.77 (0.20)	1.50 (0.89)	1.52 (0.99)	**1.73** (0.00)	**1.77** (0.02)	1.65 (0.21)	1.63 (0.37)
5	1.55	**1.62** (0.03)	1.76 (0.33)	1.84 (0.17)	1.47 (0.32)	1.70 (0.14)	1.69 (0.37)	**1.81** (0.04)	1.81 (0.11)
6	1.59	**1.68** (0.01)	1.78 (0.47)	1.84 (0.29)	1.44 (0.40)	**1.81** (0.05)	1.85 (0.14)	**1.89** (0.01)	**1.92** (0.04)
7	1.64	1.66 (0.11)	1.62 (0.94)	1.63 (0.96)	1.65 (0.87)	1.71 (0.19)	1.73 (0.30)	1.80 (0.09)	1.79 (0.26)
8	1.63	1.65 (0.22)	1.59 (0.83)	1.50 (0.58)	1.56 (0.53)	1.67 (0.39)	1.67 (0.64)	1.68 (0.55)	1.63 (0.97)
9	1.61	**1.69** (0.01)	2.14 (0.11)	1.29 (0.28)	1.69 (0.73)	**1.91** (0.01)	**2.13** (0.02)	1.49 (0.43)	1.48 (0.58)
10	1.26	**1.32** (0.00)	**1.75** (0.01)	1.33 (0.76)	1.28 (0.83)	**1.46** (0.00)	**1.59** (0.00)	1.24 (0.80)	1.26 (0.99)

This table present the Sharpe ratio over the subperiod from July 1973 to December 2008 for USA, # denotes the portfolio used , this matches the portfolio number listed in Table 1. The highlighted values is the Sharpe ratios that is statistically significant at a 5% α level, and each value is annualized.

Table 4, alpha in the CAPM (1973 - 2008)

Strategy

#	1/N	Risk-par.	Min-var.	Mean-var.	Max-div.	VT $\eta = 2$	VT $\eta = 4$	RRT $\eta = 2$	RRT $\eta = 4$
1	0.93	0.87 (0.58)	0.50 (0.72)	1.62 (0.62)	1.31 (0.42)	0.69 (0.57)	0.53 (0.59)	1.18 (0.67)	1.16 (0.80)
2	1.73	**1.83** (0.05)	2.14 (0.57)	1.16 (0.61)	1.05 (0.20)	2.09 (0.06)	2.33 (0.07)	1.99 (0.48)	1.81 (0.88)
3	-0.44	**-0.14** (0.00)	**1.71** (0.00)	-0.02 (0.68)	-1.12 (0.12)	**0.37** (0.00)	**0.69** (0.00)	-0.11 (0.47)	-0.15 (0.61)
4	1.05	**1.37** (0.00)	2.20 (0.12)	1.33 (0.76)	1.14 (0.71)	**1.91** (0.00)	**2.11** (0.02)	1.70 (0.13)	1.67 (0.25)
5	1.45	1.72 (0.08)	2.32 (0.35)	3.34 (0.07)	1.03 (0.26)	1.98 (0.25)	1.94 (0.49)	2.76 (0.32)	2.86 (0.06)
6	1.79	**2.17** (0.04)	2.46 (0.57)	3.42 (0.17)	0.96 (0.31)	2.64 (0.12)	2.78 (0.24)	**3.22** (0.01)	**3.38** (0.04)
7	1.66	1.76 (0.09)	1.92 (0.73)	2.04 (0.70)	1.66 (0.99)	1.96 (0.16)	2.07 (0.24)	2.36 (0.06)	2.37 (0.18)
8	1.61	1.69 (0.15)	1.67 (0.93)	1.45 (0.88)	1.34 (0.60)	1.84 (0.28)	1.87 (0.47)	1.89 (0.45)	1.76 (0.79)
9	1.59	**1.89** (0.00)	**3.91** (0.04)	0.74 (0.47)	2.08 (0.56)	**2.79** (0.00)	**3.72** (0.01)	1.21 (0.55)	1.39 (0.82)
10	-0.10	**0.14** (0.00)	**2.08** (0.01)	0.64 (0.49)	0.01 (0.77)	**0.76** (0.00)	**1.32** (0.00)	-0.14 (0.94)	0.08 (0.81)

Table 5 presents the CAPM over the subperiod from July 1973 to December 2008 for USA, # denotes the portfolio used, this matches the portfolio number listed in Table 1. The highlighted values is the CAPM values that is statistically significant at a 5% α level, and each value is annualized.

Table 5, alpha in the 3-factor model (1973 - 2008)

Strategy

#	1/N	Risk-par.	Min-var.	Mean-var.	Max-div.	VT $\eta = 2$	VT $\eta = 4$	RRT $\eta = 2$	RRT $\eta = 4$
1	-0.33	-0.33 (0.93)	0.52 (0.11)	0.84 (0.33)	-0.28 (0.94)	-0.28 (0.76)	-0.16 (0.49)	-0.02 (0.51)	0.08 (0.56)
2	-0.30	-0.25 (0.29)	-0.62 (0.64)	-1.87 (0.14)	-0.36 (0.91)	-0.16 (0.44)	-0.15 (0.62)	-0.67 (0.26)	-1.07 (0.14)
3	-0.88	**-0.63** (0.00)	**0.85** (0.00)	-0.59 (0.77)	-1.16 (0.49)	**-0.18** (0.00)	**0.09** (0.00)	-0.70 (0.68)	-0.73 (0.78)
4	0.45	**0.59** (0.05)	0.40 (0.93)	0.13 (0.73)	0.15 (0.37)	0.76 (0.15)	0.72 (0.42)	0.44 (0.97)	0.26 (0.69)
5	-0.50	**-0.39** (0.05)	-0.13 (0.54)	0.02 (0.59)	-0.80 (0.35)	-0.32 (0.35)	-0.41 (0.80)	-0.23 (0.56)	-0.37 (0.84)
6	-0.45	-0.31 (0.13)	0.27 (0.37)	0.81 (0.28)	-1.12 (0.30)	-0.15 (0.24)	-0.01 (0.32)	0.13 (0.16)	0.10 (0.35)
7	0.45	0.46 (0.85)	0.07 (0.60)	-0.32 (0.43)	0.59 (0.77)	0.46 (0.95)	0.43 (0.96)	0.44 (0.98)	0.24 (0.66)
8	0.36	0.32 (0.36)	-0.55 (0.18)	-0.44 (0.41)	0.07 (0.56)	0.15 (0.22)	-0.10 (0.14)	0.03 (0.35)	-0.31 (0.19)
9	0.22	0.30 (0.32)	0.89 (0.51)	-1.32 (0.19)	-0.33 (0.49)	0.58 (0.32)	0.86 (0.34)	-0.47 (0.28)	-0.71 (0.31)
10	0.35	**0.51** (0.01)	**2.00** (0.02)	0.97 (0.56)	0.48 (0.69)	**0.93** (0.02)	**1.33** (0.02)	0.13 (0.68)	0.28 (0.93)

This table present the results from the 3-factor model over the subperiod from July 1973 to December 2008 for USA, # denotes the portfolio used, this matches the portfolio number listed in Table 1. The highlighted values is the 3-factor value that is statistically significant at a 5% α level, and each value is annualized.

Table 6, alpha in the 2-factor model (1973 - 2008)

Strategy

#	1/N	Risk-par.	Min-var.	Mean-var.	Max-div.	VT $\eta = 2$	VT $\eta = 4$	RRT $\eta = 2$	RRT $\eta = 4$
1	1.20	1.06 (0.20)	0.45 (0.54)	1.41 (0.88)	0.05 (0.44)	0.69 (0.21)	0.35 (0.25)	1.05 (0.79)	0.79 (0.65)
2	-0.23	-0.18 (0.34)	-0.46 (0.74)	-1.61 (0.20)	-0.02 (0.97)	-0.10 (0.48)	-0.08 (0.63)	-0.51 (0.41)	-0.90 (0.21)
3	-0.79	**-0.59** (0.01)	**0.70** (0.01)	-0.71 (0.94)	-0.08 (0.76)	**-0.22** (0.01)	**0.01** (0.01)	-0.77 (0.96)	-0.86 (0.91)
4	0.49	0.59 (0.15)	0.24 (0.69)	0.26 (0.81)	0.02 (0.51)	0.68 (0.38)	0.57 (0.80)	0.49 (0.98)	0.30 (0.69)
5	0.53	0.47 (0.64)	0.01 (0.55)	1.06 (0.58)	-0.00 (0.12)	0.19 (0.39)	-0.18 (0.26)	0.64 (0.81)	0.48 (0.94)
6	0.90	0.87 (0.83)	0.42 (0.68)	1.90 (0.40)	-0.04 (0.09)	0.65 (0.59)	0.47 (0.56)	1.21 (0.49)	1.07 (0.79)
7	0.24	0.23 (0.88)	-0.25 (0.50)	-0.03 (0.52)	0.05 (0.54)	0.19 (0.79)	0.13 (0.73)	0.26 (0.98)	0.05 (0.70)
8	0.17	0.13 (0.32)	-0.75 (0.18)	-0.49 (0.50)	0.00 (0.80)	-0.05 (0.19)	-0.29 (0.14)	-0.11 (0.41)	-0.45 (0.22)
9	-0.22	-0.13 (0.31)	0.64 (0.40)	-1.64 (0.23)	-0.05 (0.67)	0.16 (0.28)	0.49 (0.28)	-0.95 (0.25)	-1.13 (0.32)
10	0.48	0.59 (0.13)	1.82 (0.08)	1.07 (0.57)	0.05 (0.77)	0.89 (0.13)	1.21 (0.11)	0.15 (0.53)	0.27 (0.78)

This table present the results from the 2-factor model over the subperiod from July 1973 to December 2008 for USA, # denotes the portfolio used, this matches the portfolio number listed in Table 1. The highlighted values is the 2-factor value that is statistically significant at a 5% α level, and each value is annualized.

Table 7, Sharpe ratio (1964 - 2018)

Strategy

#	1/N	Risk-par.	Min-var.	Mean-var.	Max-div.	VT $\eta = 2$	VT $\eta = 4$	RRT $\eta = 2$	RRT $\eta = 4$
1	1.34	1.36 (0.21)	1.34 (0.98)	1.37 (0.92)	1.30 (0.69)	1.41 (0.30)	1.42 (0.53)	1.37 (0.88)	1.39 (0.82)
2	1.59	1.61 (0.08)	1.61 (0.87)	1.51 (0.70)	1.45 (0.12)	1.66 (0.10)	1.70 (0.13)	1.71 (0.46)	1.67 (0.63)
3	1.25	**1.30** (0.00)	**1.60** (0.00)	1.26 (0.96)	1.14 (0.11)	**1.39** (0.00)	**1.45** (0.00)	1.30 (0.79)	1.31 (0.75)
4	1.49	**1.54** (0.00)	1.61 (0.38)	1.47 (0.92)	1.49 (0.99)	**1.62** (0.01)	**1.64** (0.05)	1.62 (0.47)	1.63 (0.49)
5	1.48	**1.55** (0.00)	1.69 (0.24)	1.91 (0.08)	1.45 (0.61)	**1.65** (0.04)	1.66 (0.17)	1.76 (0.17)	1.79 (0.15)
6	1.45	**1.54** (0.00)	**1.79** (0.04)	1.83 (0.08)	1.40 (0.56)	**1.70** (0.01)	**1.75** (0.05)	1.74 (0.11)	1.76 (0.12)
7	1.61	1.62 (0.37)	1.51 (0.43)	1.55 (0.77)	1.59 (0.78)	1.63 (0.60)	1.63 (0.81)	1.75 (0.36)	1.71 (0.56)
8	1.59	1.60 (0.34)	1.52 (0.63)	1.70 (0.62)	1.53 (0.55)	1.61 (0.59)	1.60 (0.86)	1.74 (0.34)	1.72 (0.47)
9	1.54	**1.59** (0.01)	1.73 (0.41)	1.27 (0.29)	1.50 (0.76)	**1.73** (0.02)	**1.84** (0.04)	1.48 (0.73)	1.45 (0.67)
10	1.34	**1.38** (0.00)	**1.69** (0.02)	1.48 (0.51)	1.39 (0.38)	**1.49** (0.00)	**1.59** (0.00)	1.52 (0.25)	1.54 (0.27)

This table present the results of the Sharpe ratio over the time period from January 1964 to December 2018 for USA, # denotes the portfolio used, this matches the portfolio number listed in Table 1. The highlighted value is the Sharpe ratio that is statistically significant at a 5% α level, each value is annualized.

Table 8, alpha in the CAPM, (1964 - 2018)

Strategy

#	1/N	Risk-par.	Min-var.	Mean-var.	Max-div.	VT $\eta = 2$	VT $\eta = 4$	RRT $\eta = 2$	RRT $\eta = 4$
1	0.52	0.58 (0.69)	0.24 (0.78)	1.27 (0.58)	0.15 (0.50)	0.65 (0.77)	0.58 (0.96)	0.96 (0.66)	1.07 (0.63)
2	1.59	1.61 (0.08)	1.61 (0.87)	1.51 (0.70)	1.45 (0.13)	1.71 (0.13)	0.16 (0.12)	2.16 (0.27)	2.08 (0.37)
3	-0.23	**-0.01** (0.00)	**1.41** (0.00)	0.33 (0.56)	- 0.72 (0.14)	**0.38** (0.00)	**0.66** (0.00)	0.26 (0.48)	0.35 (0.43)
4	0.86	**1.07** (0.00)	1.49 (0.24)	1.39 (0.59)	0.94 (0.78)	**1.41** (0.01)	**1.50** (0.04)	1.79 (0.22)	1.83 (0.22)
5	1.12	**1.38** (0.03)	1.87 (0.34)	**3.73** (0.02)	0.86 (0.41)	1.70 (0.14)	1.69 (0.35)	2.70 (0.08)	2.90 (0.07)
6	1.35	**1.80** (0.00)	2.92 (0.07)	**3.73** (0.03)	1.07 (0.49)	**2.11** (0.05)	2.22 (0.17)	2.63 (0.07)	2.74 (0.08)
7	1.47	1.51 (0.34)	1.19 (0.62)	1.77 (0.74)	1.43 (0.92)	1.56 (0.53)	1.57 (0.71)	2.25 (0.21)	2.13 (0.32)
8	1.35	1.40 (0.27)	1.29 (0.91)	2.46 (0.24)	1.19 (0.70)	1.46 (0.45)	1.46 (0.66)	2.30 (0.16)	2.29 (0.21)
9	1.19	**1.40** (0.01)	2.33 (0.16)	0.66 (0.59)	1.24 (0.94)	**2.01** (0.01)	**2.58** (0.01)	1.21 (0.98)	1.25 (0.95)
10	0.18	**0.37** (0.00)	**1.78** (0.01)	1.41 (0.21)	0.45 (0.33)	**0.84** (0.00)	**1.26** (0.00)	1.29 (0.12)	1.43 (0.11)

Table 9 presents the results of the CAPM model over the time period from January 1964 to December 2018 for USA, # denotes the portfolio used, this matches the portfolio number listed in Table 1. The highlighted value is the CAPM value that is statistically significant at a 5% α level, each value is annualized.

Table 9, alpha in the 3-factor model (1964 - 2018)

Strategy

#	1/N	Risk-par.	Min-var.	Mean-var.	Max-div.	VT $\eta = 2$	VT $\eta = 4$	RRT $\eta = 2$	RRT $\eta = 4$
1	-0.20	**- 0.11** (0.03)	0.45 (0.13)	1.11 (0.29)	- 0.25 (0.92)	**0.16** (0.02)	**0.35** (0.03)	0.70 (0.32)	0.84 (0.29)
2	1.43	**1.52** (0.01)	0.32 (0.48)	1.64 (0.94)	- 0.19 (0.78)	**0.32** (0.03)	0.45 (0.07)	0.52 (0.36)	0.37 (0.53)
3	-0.54	**- 0.34** (0.00)	**0.98** (0.00)	0.19 (0.45)	- 0.74 (0.52)	**0.05** (0.00)	**0.33** (0.00)	0.13 (0.34)	0.23 (0.30)
4	0.51	**0.61** (0.04)	0.42 (0.84)	0.89 (0.70)	0.37 (0.61)	0.53 (0.18)	0.65 (0.56)	1.25 (0.33)	1.20 (0.38)
5	-0.20	**0.01** (0.00)	**0.87** (0.04)	**2.34** (0.02)	-0.18 (0.95)	**0.36** (0.00)	**0.49** (0.03)	1.34 (0.08)	1.55 (0.06)
6	-0.13	**0.11** (0.00)	0.83 (0.11)	1.82 (0.06)	-0.61 (0.18)	**0.43** (0.00)	**0.67** (0.02)	0.91 (0.13)	0.97 (0.13)
7	0.58	0.57 (0.92)	0.05 (0.34)	0.19 (0.67)	0.79 (0.59)	0.53 (0.73)	0.45 (0.62)	0.98 (0.51)	0.74 (0.81)
8	0.44	0.43 (0.76)	0.03 (0.44)	0.90 (0.62)	0.34 (0.79)	0.35 (0.53)	0.23 (0.40)	0.86 (0.52)	0.69 (0.73)
9	0.16	0.25 (0.18)	0.31 (0.84)	- 0.32 (0.63)	- 0.48 (0.25)	0.51 (0.19)	0.70 (0.27)	0.30 (0.85)	0.16 (0.99)
10	0.57	**0.71** (0.00)	**1.82** (0.01)	1.80 (0.20)	0.81 (0.37)	**1.05** (0.01)	**1.36** (0.01)	1.60 (0.15)	1.68 (0.15)

This table present the result of the 3-factor model over the time period from January 1964 to December 2018 for USA, # denotes the portfolio used, this matches the portfolio number listed in Table 1. The highlighted value are the 3-factor value that is statistically significant at a 5% α level, each value is annualized.

Table 10, alpha in the 2-factor model (1964 - 2018)

Strategy

#	1/N	Risk-par.	Min-var.	Mean-var.	Max-div.	VT $\eta = 2$	VT $\eta = 4$	RRT $\eta = 2$	RRT $\eta = 4$
1	0.27	0.32 (0.66)	0.37 (0.29)	1.32 (0.43)	0.02 (0.66)	0.43 (0.68)	0.45 (0.79)	0.99 (0.45)	1.05 (0.48)
2	-0.01	**0.09** (0.02)	0.33 (0.51)	-0.11 (0.95)	-0.15 (0.80)	**0.34** (0.04)	0.47 (0.08)	0.55 (0.37)	0.39 (0.54)
3	-0.52	**- 0.33** (0.00)	**0.93** (0.00)	0.20 (0.46)	- 0.67 (0.62)	**0.04** (0.00)	**0.29** (0.00)	0.13 (0.36)	0.22 (0.33)
4	0.53	0.62 (0.09)	0.35 (0.71)	0.93 (0.68)	0.41 (0.68)	0.69 (0.33)	0.60 (0.77)	1.27 (0.33)	1.21 (0.39)
5	0.13	0.28 (0.21)	0.85 (0.37)	**2.57** (0.03)	0.03 (0.75)	0.50 (0.35)	0.52 (0.523)	1.55 (0.11)	1.74 (0.10)
6	0.23	0.40 (0.18)	0.81 (0.56)	1.23 (0.37)	- 0.65 (0.17)	0.66 (0.29)	0.77 (0.43)	1.20 (0.21)	1.21 (0.25)
7	0.55	0.54 (0.82)	- 0.02 (0.31)	0.18 (0.69)	0.78 (0.55)	0.48 (0.66)	0.40 (0.56)	0.94 (0.52)	0.69 (0.83)
8	0.41	0.40 (0.70)	- 0.03 (0.42)	0.89 (0.61)	0.34 (0.85)	0.32 (0.49)	0.19 (0.38)	0.83 (0.52)	0.66 (0.73)
9	0.05	0.14 (0.18)	0.25 (0.79)	- 0.41 (0.64)	- 0.53 (0.30)	0.41 (0.19)	0.61 (0.26)	0.18 (0.86)	0.04 (0.99)
10	0.62	**0.74** (0.03)	1.75 (0.06)	1.84 (0.21)	0.85 (0.40)	**1.04** (0.04)	**1.32** (0.05)	1.61 (0.16)	1.69 (0.17)

This table present the result of the 2-factor model over the time period from January 1964 to December 2018 for USA, # denotes the portfolio used, this matches the portfolio number listed in Table 1. The highlighted value are the 2-factor value that is statistically significant at a 5% α level, each value is annualized.

Table 11, Norwegian data (1981 - 2016)

Strategy

	1/N	**RISK-PAR.**	**MIN-VAR.**	**MEAN-VAR.**	**MAX-DIV.**	**VT $\eta = 2$**	**VT $\eta = 4$**	**RRT $\eta = 2$**	**RRT $\eta = 4$**
(1) $H_0: SR_{1/N} = SR_j$									
# 11	4.64	4.59 (0.15)	4.43 (0.50)	5.27 (0.10)	**5.10** (0.05)	4.45 (0.22)	4.19 (0.10)	4.88 (0.26)	4.86 (0.48)
# 12	2.71	2.71 (0.79)	2.66 (0.86)	2.18 (0.10)	2.66 (0.75)	2.74 (0.79)	2.78 (0.74)	2.47 (0.16)	2.27 (0.10)
# 13	3.80	3.83 (0.28)	3.97 (0.52)	4.53 (0.07)	**4.38** (0.01)	3.84 (0.69)	3.77 (0.89)	4.19 (0.08)	4.26 (0.19)
(2) $H_0: \alpha_{1/N}^{CAPM} = \alpha_j^{CAPM}$									
# 11	0.15	**0.15** (0.03)	0.15 (0.71)	0.23 (1.48)	**0.18** (0.01)	0.14 (0.13)	0.13 (0.21)	**0.18** (0.01)	**0.19** (0.01)
# 12	0.04	0.043 (0.92)	0.05 (0.68)	0.03 (0.52)	0.04 (0.93)	0.05 (0.83)	0.05 (0.47)	0.03 (0.35)	0.03 (0.37)
# 13	0.10	0.10 (0.43)	0.12 (0.14)	**0.17** (0.00)	**0.14** (0.00)	0.10 (0.65)	0.11 (0.71)	**0.13** (0.02)	**0.14** (0.01)
(3) $H_0: \alpha_{1/N}^{three} = \alpha_j^{three}$									
# 11	0.09	**0.09** (0.03)	0.09 (0.64)	**0.16** (0.00)	**0.11** (0.04)	0.08 (0.14)	0.08 (0.22)	0.11 (0.18)	0.12 (0.12)
# 12	0.04	0.04 (0.32)	0.06 (0.10)	0.03 (0.73)	0.05 (0.23)	0.05 (0.20)	0.06 (0.09)	0.03 (0.52)	0.03 (0.58)
# 13	0.06	0.06 (0.92)	0.07 (0.54)	**0.11** (0.01)	**0.08** (0.01)	0.06 (0.79)	0.05 (0.75)	0.07 (0.10)	0.09 (0.07)
(4) $H_0: \alpha_j^{two} = \alpha_j^{two}$									
# 11	0.15	**0.15** (0.03)	0.15 (0.85)	**0.23** (0.00)	**0.18** (0.01)	0.14 (0.14)	0.13 (0.21)	**0.18** (0.01)	**0.19** (0.01)
# 12	0.04	0.04 (0.98)	0.05 (0.59)	0.03 (0.60)	0.04 (0.79)	0.05 (0.75)	0.05 (0.42)	0.04 (0.40)	0.03 (0.43)
# 13	0.10	0.10 (0.43)	0.12 (0.15)	**0.16** (0.00)	**0.14** (0.00)	0.10 (0.66)	0.11 (0.72)	**0.13** (0.02)	**0.14** (0.01)

This table present the Sharpe ratio, alpha CAPM, alpha 3-factor model and alpha 2-factor model over the period from July 1981 to December 2016 for Norway based data, (1), (2), (3) and (4) denotes the hypothesis that is tested for each of the portfolios. # denotes the portfolio number evaluated, matches the portfolio number listed in Table 1. The highlighted values indicates the statistically different value compared to the naive strategy, compared against a critical value α, set to 5%. Each value is annualized.

6. Conclusion

DeMiguel et al. (2009) conducted a highly influential study initiating the debate about the value of optimized strategies compared to the naive allocation strategy. Several studies claim to demonstrate the superiority of optimized portfolios. Motivated by the ongoing debate whether the value of optimization portfolios, the goal of this thesis is to provide answer to following question: *can optimized strategies beat the naive allocation rule?*.

First, in this paper we duplicate earlier research, examining the performance of several optimized portfolios over the limited time period using the Sharpe ratio and alpha in the CAPM as performance measure. The results when using the Sharpe ratio are consistent with other researchers, such as DeMiguel et al. (2009), the results shows that no single optimized portfolio consistently outperforms the naive allocation rule. The alpha in the CAPM shows similar to the Sharpe ratio, that no single optimized strategy consistently outperforms the naive allocation rule. Some of the optimized portfolios performs better than other, such as the risk-parity portfolios, but it is still not good enough to determine that the portfolios are consistently outperforming the naive allocation rule. Second, using the 3-factor model and 2-factor model we control for some known anomalies, examining if superior performance of optimized portfolios can be attributed to mean-variance efficiency or due to exposure to financial anomalies. Consistent with the conclusion drawn by Zakamulin (2017), our results shows that after controlling for anomalies there is little or no evidence of superior performance by optimized strategies.

Third, we compare the results for the limited time period against result for an extended time period using USA based data, examining the robustness of our results. The results for the Sharpe ratio and alpha from the CAPM are surprisingly consistent. There are no variation in number of significantly different values and small variations in how these values are distributed across strategies and datasets. The results using the alpha from the 3-factor model in the extended time period are somewhat surprising relative to the results for the limited time period due to the large increase of significantly different values. Using 2-factor model there is also an increase in significantly different values for the extended time period compared to the limited time period, but the results somewhat indicate the same conclusion. Fourth, we compare the performance over the limited time period for the USA market against results obtained for the Norwegian market. The results for the Norwegian market are surprising in relation to the results for the USA market, in term of the results using the Sharpe ratio and 2-factor model. However, none of the optimized portfolios are observed to consistently beat the naive allocation rule.

Our results do not advocate that one should use the naive allocation rule, it is merely used as a benchmark to assess the value performance of various performance measures. The results clearly indicates the danger of concluding with superior performance of optimized portfolios without controlling for financial anomalies. When controlling for anomalies we observe a large reduction of significantly different values of optimized portfolios outperforming the naive allocation rule. The findings from the conducted study shows that optimized portfolios do not consistently outperform the naive allocation rule. For further studies we encourage researchers to include data from other countries, additional to USA. The results from the Norwegian market compared to USA based data, indicate that the existing theory is not sufficient to explain the contrary results.

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Appendix:

A. Reflection note

The theme in my thesis has been asset allocation, examining the performance of optimized portfolio using the naive allocation strategy as benchmark. The optimized strategies are more complex as they require estimation and optimization, the main purpose is to optimize the tradeoff between risk and return. Optimized portfolio are a part of modern portfolio theory, where one uses quantitative methods and where each portfolio is an active dynamic portfolio that is rebalanced periodically. The naive strategy is a very simple allocation strategy, where one divides wealth equally across all assets. How a manger should allocate wealth across assets, balancing risk against performance, is key to effective portfolio management.

Performing an empirical study with the aim of answering the question, *can optimized strategies beat the naive allocation rule?*. First, I examine the performance using Sharpe ratio and alpha in the CAPM over an limited time period across the USA market. Second, controlling for some known financial anomalies I use the alpha in the 3-factor and alpha in the 2-factor as performance measure. Third, I compare the performance over the limited time period for the USA market against an extended time period for the Norwegian market. Fourth, the limited time period for the USA market is compared against the Norwegian market. From the conducted study my main conclusion is that no optimized portfolios consistently outperform the naive allocation rule.

In my thesis all theory and methodology on asset allocation is driven by research at an international level. The data used in this thesis uses both national and international data, as I use the Norwegian and USA based markets. When including the USA based data I consider also the international perspective of our results, and we can compare the performance at an national and international level. Most of the research covering this topic uses USA based data over an specific time period, one can argue that when one just consider data from one market the results are not robust, as there are large variations between different countries. In my paper the performance measures using the Norwegian data shows the importance of including the small minus big factor, and indicates that the 3-factor model is the most important performance measure when considering the Norwegian market. Contrary to the USA marker where the impact of small minus big factor is not as relevant as in the Norwegian market, and one prefer the 2-factor model. The movements in one countries financial market are much tied up to movements in the global financial market, and therefore the research and methodology covering this subject are heavily impacted by the movements in the global financial market.

If we consider the topic of asset allocation from an innovative perspective, we consider the new allocation strategies developed over time. The idea of asset allocation has been around for centuries, in the 1950 Markowitz introduced the men-variance theory and it is considered to be a cornerstone in modern portfolio management. The volatile and turbulent financial markets of the 2000s decade, boosted the interest in investment optimization strategies with focus on optimal portfolio risk control across assets and time. A result of this interest in investment optimization strategies resulted in the development of several optimized strategies such as the minimum-variance, maximum-diversification and risk-parity. Several researchers has come out in the defence of optimized portfolio while other has argued that optimized strategies does not add any value. This ongoing discussing is highly relevant in today's market and several researcher point of different perspective of this topic that can impact the research conclusion. As over time we also observe that researcher discover financial anomalies that are may be an explanation for the obtained results, and after controlling for these new anomalies the conclusion can be very different from the initial observation. The discussion is optimized portfolios adds any value was initiated by DeMiguel, Garlappi, and Uppal in 2009 and is still ongoing, several researcher has come out in defence of the optimized strategies while other claim that optimized strategies adds no value. There is an gap in existing literature as an large amount of the research covering this topic uses USA based data for an specific time period, and researcher does not consider if the one obtain the same results for other markets and time periods. The novelty of my conducted study is that I include the Norwegian market and one extended time period for the USA market to determine how the results varies over different times and across different markets.

If we consider the topic from an accountability perspective, one consider the responsibility of portfolio managements to their customers. All of the optimized strategies are based on estimations, an there is a level of uncertainty associated with using estimations even when one can estimate with some level of certainty. While the naive allocation rule that is simple and wealth is equally divided across all assets. Given that optimized portfolios are more expensive to manage as they require estimating and rebalancing portfolio weights, one can argue that using the optimized portfolio does not add any value and therefore should not be used. There can be ethical challenging for portfolio manager to recommend using optimized portfolio, if they are more expensive and no evidence support the claim that they performs better than the simple naive allocation rule.