Mathematical interventions based around local bridges in Agder County

Design research on principles of accessible, realistic, and open mathematics tasks starting from authentic infrastructure objects

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Foreword

When beginning on my master thesis, I had no idea about the scope of the paper and how long time it would take to complete all of it. Never before had I written so many words. Probably because at heart, I am a mathematician — and we usually prefer working with formulas and short descriptions. However, it is not to ignore that certain negative changes to my personal life did not help me staying focused on the task.

Because of this, I want to thank all of the people that helped me through this process. First of these, I want to thank my supervisor for being extraordinarily patient with me. Especially when I would put myself in less than ideal situations, forcing myself to work constantly for days at a time. Secondly, I want to thank my 'work partner' Martin Sletten for motivating me to get started early on my own work, and for pushing me in the right direction. Furthermore, I want to thank my good friend Fredrik for always helping me out, and for offering to proofread the draft of this thesis. Additionally I want to thank my mother and father who allowed me to stay at their place when I required peace and quiet to work on the task.

Finally, I want to thank all the pupils, students and teachers who I have worked with and that were available to participate in my study. I would not have been able to do this without you.

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Abstract

Many pupils find school mathematics arcane, and not very useful in their daily lives. Seeing, as the educational system exists to prepare its pupils for society — this seems counterintuitive. The Ministry of education and research is already addressing this in their new curriculum for 2020. However, they do not yet provide any specific information on how to implement these changes into educational practice. These are the reasons why I am addressing situated learning through the creation of certain tasks in this design research study. The tasks of this study exists as a part of a bigger project to create a product similar to the Mathbridges calendars made by the University of Münster, together with other students who perform their own research. Both these reasons have an influence on how the tasks will look and perform — and makes the base for a set of characteristics promoting a particular type of task.

The theoretical framework of this thesis focuses on defining and characterizing traits of particular tasks with the intention to teach pupils about the use of application of mathematics to real situations. This bases itself in the theories of Boaler (2001), Ärlebäck (2009), Borromeo Ferri (2018), Skovsmose (2003), and Vos (2018).

The method of the study is design research as presented by (van den Akker, Bannan, Kelly, Nieveen, & Plomp, 2013). This method of research closely resembles action research, but focuses on the creation of *design principles* in addition to interventions of educational practice [in this study this means 'mathematical tasks']. This method is cyclic. Hence, the tasks goes through several stages of testing and development in order to consider how the tasks correlate with the principles that relate to the task creation process. In this study, there are three stages: 1) a try-out of the initial prototype, 2) an expert appraisal of the second prototype, and 3) a field test and interview of the final revision.

The initial design principles are as following:

- ***Accessible.*** All representation used are understandable for all pupils, and the task can be solved at different levels of complexity.
- ***Realistic.*** The task has a clear real-world connection, and contains at least one authentic aspect.
- ***Open.*** The task lacks strategical information and certain necessary information — in order to promote reasoning and argumentation among the pupils.

Initially, seven tasks were designed for this study. This was primarily a measure made so that I could test several different procedures of task design at once. Additionally this was a strategy so that I could rather discard some tasks rather than having to make modifications on all the tasks between each stage of the cycle. In the end, only two tasks [with alterations] had the traits required to promote use of 'everyday mathematics' in a format suitable to the calendar.

In addition, another design principle arose from the results of the study:

- ***Immersive.*** The relationship between the context and question makes the task believable, and promotes discussion.

The study concludes that both these two tasks and the design principles may provide useful tools for addressing situated learning in a classroom context, and the oncoming change that is to arrive with the new curriculum in 2020.

Sammendrag

Mange elever finner skolematematikken mystisk, og ikke veldig nyttig i sitt daglige liv. Med tanke på at utdanningssystemet eksisterer for å forberede sine elever for samfunnet - virker dette kontraintuitivt. Utdannings- og forskningsdepartementet tar dette allerede opp i sin nye læreplan for 2020. De gir imidlertid ikke noen spesifikk informasjon om hvordan man implementerer disse endringene i pedagogisk praksis. Det er av disse grunnene at jeg henvender meg mot situert læring gjennom å lage bestemte oppgaver i denne designforskningsstudien. Oppgavene i denne studien eksisterer som en del av et større prosjekt for å lage et produkt som ligner på 'Mathbridges' kalenderne laget av Münster Universitet, sammen med andre studenter som utfører egen forskning. Begge disse grunnene har innflytelse på hvordan oppgavene kommer til å se ut og utføres - og legger grunnlaget for et sett av egenskaper som fremmer en bestemt type oppgave.

Det teoretiske rammene for denne avhandlingen fokuserer på å definere og karakterisere egenskaper til bestemte oppgaver med formål om å lære elevene bruk av matematikk i virkelige situasjoner. Dette baserer seg i teoriene til Boaler (2001), Ärlebäck (2009), Borromeo Ferri (2018), Skovsmose (2003), og Vos (2018).

Metoden for studien er designforskning som presentert av van den Akker, Bannan, Kelly, Nieveen, & Plomp (2013). Denne forskningsmetoden ligner i stor grad aksjonsforskning, men fokuserer på opprettelse av prinsipper i tillegg til inngrep i pedagogisk praksis [i denne studien gjelder dette 'matematiske oppgaver']. Denne metoden er syklisk. Derfor går oppgavene gjennom flere stadier av testing og utvikling for å vurdere hvordan oppgavene samsvarer med prinsippene som relateres til oppgavelageprosessen [:prosessen å lage oppgaver]. I denne studien er det tre trinn: 1) en utprøving av den opprinnelige prototypen, 2) en ekspertvurdering av den andre prototypen, og 3) en felttest og intervju av den endelige revisjonen.

De opprinnelige designprinsippene er som følger:

- ***Tilgjengelig.*** Alle brukte representasjoner er forståelige for alle elever, og oppgaven kan løses på ulike kompleksitetsnivåer.
- ***Realistisk.*** Oppgaven har en klar forbindelse med den virkelige verden, og inneholder minst ett autentisk aspekt.
- ***Åpen.*** Oppgaven mangler strategisk informasjon og visse nødvendige opplysninger — for å fremme elevers resonnement og argumentasjon.

I utgangspunktet ble syv oppgaver designet for denne studien. Dette var først og fremst et tiltak gjennomført slik at jeg kunne teste flere forskjellige prosedyrer av oppgavedesign samtidig. I tillegg var dette en strategi slik at jeg heller kunne forkaste noen oppgaver i stedet for å gjøre endringer i alle oppgavene mellom hvert trinn i syklusen. Til slutt var det bare to oppgaver [med endringer] som hadde de nødvendige egenskapene som trengs for å fremme bruken av "hverdagsmatematikk" i et format som passer til kalenderen.

I tillegg oppsto et annet designprinsipp fra resultatene av studien:

- ***Oppslukende.*** Forholdet mellom konteksten og spørsmålet gjør oppgaven troverdig, og fremmer diskusjon.

Studien konkluderer med at begge disse to oppgavene og designprinsippene kan være nyttige verktøy for å henvise seg situert læring i et klasseromskontekst, og den imøtekommende forandringen som ankommer med den nye læreplanen i 2020.

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1 Introduction

1.1 Background

While growing up, I was doing quite well within the subject of mathematics. I enjoyed [and still do] learning about new methods and ways of solving different problems. However, I can rarely recall thinking 'this is something I can make use of when I leave school'. Mathematics were always some sort of game to me — with its own clear-cut rules and strategies for 'winning'. Sort of like football. It was all about scoring points. The more correct answers you discovered, the better your grade. Similar, you did not really use it when you were not at the 'playing field'. Only after playing the game of mathematics for many years did I finally acquire an understanding of its relevance. However, I am not the only one to have had such a problem — and many do not stay in the game for as long as I have. "For many students, already in lower secondary schools, the relevance of mathematics is not apparent, and the question 'why do we have to learn this?' is frequently asked" (Vos, 2018, p.1). Many students experience school mathematics as distant and irrelevant for their everyday lives. They experience the subject as a series of rituals and rules where form and presentation is more important than the content (Botten, 2016).

Through a collaboration project supervised by prof. Pauline Vos, a couple of other students and myself were presented with the opportunity to make tasks for a mathematical product —a calendar about bridges in Agder County and mathematical tasks. Seeing this as an opportunity to create, and learn about the process behind creating, rich and realistic tasks that pupils could find interesting and relate to their daily lives — of course I was interested!

1.2 Political influences

The Norwegian directorate for education and training (udtanningsdirektoratet; udir), is the executive branch of the ministry of education and research. This directorate provide guidelines, curriculums, and laws in order to secure an equally high level of educational practice within all schools. Although these regulations are predominately based in scientific discoveries, some are culturally bound or hold certain political liability.

At the current time, the Norwegian directorate for education and training is working on a new curriculum for primary- and secondary-schools[1] that is estimated to be finished by 2020 (Ministry of education and research, 2018). They had primarily three reasons for making these changes. The first reason is to keep the learning outcomes relevant and up to date — as new techniques and technologies appear, and society changes accordingly. The second reason is to reduce the width of each subject so that it is possible to learn certain topics in depth, rather than to have a shallow understanding of more topics. The third reason is so that they could revise and strengthen the connection between the different subjects, and between the topics within the subject itself.

While they have retained some elements of the old curriculum - such as the attention to 'basic skills'; reading, talking, calculating, speaking, and ICT [information and communications technology] - they also emphasize the importance of reflection, evaluation, critical thinking, and democratic participation. Because of this, the new curriculum is going to focus more strongly towards practical and explorative procedures. These changes will apply to all subjects through the introduction of 'core elements' - specific didactical themes.

[1] The school system in Norway are made up of a primary school (Barneskole, ages 6 - 13), a lower secondary school (Ungdomsskole, ages 13 - 16), and an upper secondary school (Videregående skole, ages 16 - 19)

Core element	Description
Exploring and problem solving	The search for and finding, mathematical connections through problem solving. Strategies and methods are more important than the answers.
Models and application	The making of mathematical models out of real problems - in order to gain insight into how to use mathematics in everyday life, science, and technology.
Reasoning and argumentation	The understanding and assessment of mathematical reasoning - in order to argue and follow the argumentation of others.
Representation and communication	The use of mathematical language in order to explain and justify their solutions.
Abstraction and generalization	The development of abstract knowledge through working with gradually more generalized tasks.
Mathematical knowledge*	Central aspects of mathematical competence. *This is to be learned through the other core elements.

Table 1.1: The six core elements in mathematics education according to the new curriculum (Ministry of education and research, 2018, p.15).

While this study is going to address several of these core elements to some extent, it is primarily going to focus on the use of mathematics in everyday life. A trait that connects to the 'models and application' core element.

1.3 Goals of the study

One overarching objective of this study is to address situated learning in schools and to present a way to bridge the gap between traditional school mathematics and everyday mathematics. Addressing this, I am primarily going to focus on the use of mathematical tasks and specific practices within the subject itself. By further separating this objective into smaller parts, we get two goals of this study:

1. Testing and validating principles given by previous research on this subject.
2. Creating additional principles through the testing of mathematical interventions.

This means that this study both has the goal of creating certain mathematical interventions [according to some principles] for use in school practice, and generate explicit principles that are useful when creating additional mathematical interventions.

Another goal of this study is to create specific tasks that, when compiled with the tasks of related studies, makes up a mathematical resource similar to the 'mathbridges international calendar' by the University of Münster (AFO Arbeitsstelle Forschungstransfer, 2018). The primary difference is going to be that this mathematical resource is going to refer to local bridges in Agder County rather than international ones. In that regard, the interventions designed in this study has the goal of assuming a similar format to the ones presented there.

Image 1.1: Example of an intervention, where the problem description reads as following: "*Lutosławski Bridge in Lublin is unofficially called the 'Bridge of Culture', because inhabitants of Lublin can take part in many cultural events on this bridge. For example concerts, parties, and exhibitions have been arranged here. How many people can safely fit on the bridge during a concert?"* (AFO Arbeitsstelle Forschungstransfer, 2018, p.16).

The tasks of the Mathbridges calendar each consist of an image [of the bridge], a short description, and a problem. I am going to include two of these three traits in my interventions. Namely the image in conjunction with the problem. I am initially excluding the description of the bridge both because it does not provide additional information required for solving the task, and it can make the tasks look more intimidating to the pupils.

1.4 Limits of the study

The goal of this study assumes that realistic aspects are generally lacking in modern [mathematical] educational practices. However, there exists little to no data on whether such practices is sufficiently embedded into Norwegian educational practice or not: 'how common they occur' or 'how the teachers plan around such aspects'. Because of this, it becomes difficult to assume the actual relevance of this study according to modern mathematics education. I am neither going to address this matter in my study - as the focus of this study lies purely with the mathematical interventions and the pupils interactions with them, not with the teachers. This is a limit of the study.

1.5 The structure of this paper

Each consecutive chapter of this paper revolves around different and unique parts of this study. The next chapter [chapter 2] presents previous research and explains the theoretical framework that constitutes the basis for this study. Chapter 3 describes and gives an overview of the methods conducted - presenting the design principles and tasks while focusing on a design research approach in an interpretivist paradigm. Chapter 4 presents the result, a brief discussion, and semi-summative conclusions of the formative assessment phases. Chapter 5 presents all of the results of the summative assessment phase according to noticeable 'trends'. Chapter 6 discusses these findings in accordance with the goal of this study. The final chapter [chapter 7] comes with concluding remarks and discusses possibilities for future research.

1.6 Temporary research question

The goals of this study were to create mathematical interventions similar to the Mathbridges tasks, and to produce principles for designing similar tasks. When addressing this, it becomes apparent that there not a sufficient amount of proper theoretical framework to address the goals directly. Because of this, it is difficult to define a formal research question — instead I will define a temporary research question.

> **Temporary research question:**
> 'What does it take to make tasks similar to the Mathbridges tasks?'

The formal research question will be rephrased later.

2 Theory

2.1 Mathematics as a social construct

There are several ways to view the nature and learning of mathematics. Most of these fall under either one of the two opposing ontological paradigms - objectivism and constructivism. The first of these views knowledge as something that exists on its own, and learning as the process of exploring something true and objective. However, if all education of mathematics essentially teaches the same principles, how can we explain why some pupils feel alienated by the subject? Why is it that some pupils disregards the subject as less than relevant for their own daily lives? For the sake of this study, I will therefore apply a constructivist view; mathematical knowledge is not something that exists on its own, but rather something created by people - undeniably shaped by our experiences, comprehension, and abilities of understanding.

Individuals do not construct this knowledge isolated from the world. As stated by situated theories, knowledge is to emerge from a series of interactions between the subject and the world. Because of this, “it becomes important to engage students in opportunities to use and apply knowledge, not only because such opportunities may afford the development of deeper knowledge, but because students engage in practices they will need to use elsewhere” (Boaler, 2001, p.1). People are not isolated entities, but they live and act within a given setting. Each setting is the holistic sum of all the factors and rules that dictates what actions they do and why they do it. The most important of these being, according to sociocultural theory, is the social conditions. This states that no learning happens without the person being part of a social environment (Imsen, Elevens verden: innføring i pedagogisk psykologi, 2015). Development goes from a state of doing things together, to doing things alone. In other words - we have to consider mathematics primarily not as a subjective construct, but as a social construct: knowledge that has been ‘agreed upon’ and imbedded into our culture.

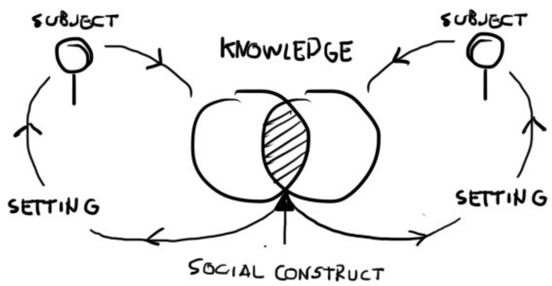


Figure 2.1: Mathematical knowledge is in continuous developed by its practitioners - which again change the development within the practitioner itself.

2.2 Critical competence

Certain educational practices holds a heavy emphasize on the pupils using predetermined methods in order to solve tasks and reach a particular goal or answer. Just as in any other learning situation, this practice will construct a certain view upon the practitioners. Because of this, it is then crucial that we - as educators - asks ourselves ‘what particular view is this going to be?’.

Skovsmose (2003) expresses a general concern that mathematics education fails to prepare pupils for a democratic society. Due to a trait of the common practice, he argues that, mathematical competences often develop detached from critical competences. Such competences include a democratic attitude in conflictual situations; communication in the classroom and consideration for pupils’ interests; and mathematics as a tool for critical thinking (i.a.). This traditional mathematical education is characterised by how it revolves around demonstrations by the teacher, and the procedure of training these techniques through short, closed questions. He refers to this type of education as the ‘task paradigm’ [org. oppgaveparadigmet] - and puts it in contrast other, more inquiry-based, methods of education.

A ‘landscape of discovery’ [org. undersøkelseslandskab] is a setting where the pupils are invited to, and cannot refrain from asking themselves questions such as ‘what if?’ or ‘why?’ (Skovsmose, 2003). A state of working where the pupils’ curiosity becomes the driving force of direction for their exploration. Furthermore it is important to emphasize the fact that this may only occur if the pupils ‘accept the call of adventure’; it is not to be simply understood as a trait of the intervention itself, but rather as a product of the pupils’ interaction with the intervention. Because of this, the characteristics of a landscape of discovery is relative - and it is up to the teacher to assess what sort of interventions would fit according to the traits and preferences of their pupils.

Boaler (2001) studied the impact of inquiry-based educational practice - such as modelling tasks and project work - as opposed to a ‘traditional’ task-based education. Her results concluded that these different practices did indeed shape the forms of knowledge produced. The pupils who had been learning mathematics through practicing traditional textbook-tasks did well at similar problems to the ones they were familiar - procedural replication. However, they were generally less likely to apply their mathematical knowledge in practical situations or use it in discussion. These pupils had likely constructed boundaries around their knowledge and only affiliated it with the school setting and nothing else. “When I asked the students whether the mathematics they used outside school was similar or different to that which they used inside school, the students at the two schools gave very different responses. All of the Amber Hill students said that it was completely different, and that they would never make use of any of the methods they used in school” (Boaler, 2001, p.124). On the other hand, the pupils who had been learning mathematics through open-ended tasks and group projects developed knowledge that was flexible and useful in a variety of situations - including authentic assessments and conceptual understanding. “The students at Phoenix Park responded very differently and three-quarters of the students said that there were no differences between mathematics of school and the real world, and that in their jobs and lives they thought back to their school mathematics and made use of it.” (Boaler, 2001, p.124). The final aforementioned form of knowledge seem to correlate with what Skovsmose describes as a critical competence.

Because it is the goal of this study to create mathematical tasks to promote the use of everyday mathematics, I will adopt the principles of an inquiry-based practice. It will be a part of the educational practice Skovsmose refers to as ‘critical mathematics education’, but it will not define the only method of such practice. Just as different pupils will accept different problems as landscapes of discovery, different pupils will need different methods of practice. Critical mathematical education should not base itself in one singular method of practice, but rather employ a set of methods (Skovsmose, 2003, p.144). It is not impossible that certain pupils may flourish within the safe confines of the ‘task paradigm’.

2.3 Mathematical modelling

The inquiry-based practice I will use in this study is the process and application of mathematical models on problems based in the real world. According to the core elements of the new curriculum (Ministry of education and research, 2018), this means to take a problem from reality, restate it to a mathematical model, and evaluate the model in light of the initial situation. While this is a general description of mathematical modelling, it is not sufficient in providing guidelines for creating such interventions or how to analyse them. Hence, I will apply the perspective of Ärlebäck (2009) on mathematical modelling as a “…complex (iterative and/or cyclic) problem solving process” (p.336) that can be further divided into sub-processes, such as those described by Borromeo Ferri (2018). Through making these choices, it is possible to construct a cognitive view on the modelling process -

which is a good instrument for the diagnostic purpose of creating tasks and analysing them once tested.

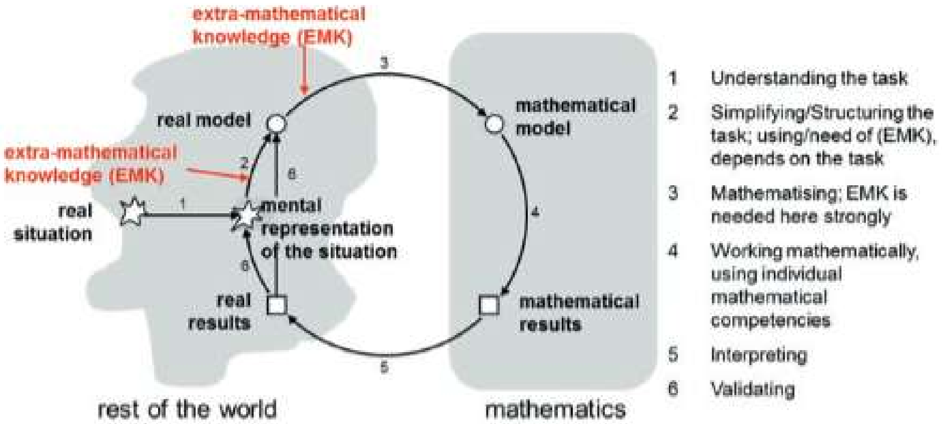


Figure 2.2: The modelling cycle from a cognitive perspective (Borromeo Ferri, 2018)

The activity of transitioning back and forth between reality and mathematics is an essential feature of mathematical modelling. This activity always starts with a problem presented in a 'real situation' that the participants has to interpret. Primarily, two parts affect this interpretation process. Namely, 'how the participant understand the problem' and 'what associations the participant has to the real situation'. Because both these elements will affect the participant's understanding of the task, it is important to separate the real situation [in itself] from the mental representation of the situation constructed by the individual.

After understanding the task, the situation will need further structuring in order for mathematical procedures to be applicable. This part of the modelling process refers to the actual creating of both mathematical and real models. Before one is able to work with a mathematical model, the participant needs to make a 'real model'. Features of this process involves making certain specifications and simplification in order to make structure of the situation through the appliance of extra-mathematical knowledge (EMK). EMK is all the data not given in the task, but that is required to solve it. This invokes participants' experiences or their need to make reasonable estimations, hence connecting mathematics and reality in a way that makes the usefulness of mathematics apparent. The 'mathematical model' is the appliance of mathematical knowledge and understanding on the real model. It is what defines which mathematical procedures the participants will use in order to get a result or solution.

The results are separated into the 'mathematical results' and the 'real results'. The mathematical results are simply the results of the mathematical procedures determined by the mathematical model and the mathematical competencies of the participant. It has value, but is without the context of the real situation. The real result on the other hand is the interpretation and validation of the results within the context of the situation. This is what separates mathematical modelling from other mathematical tasks. The return of the mathematical results into the context of reality; either a conclusion or continuation of the modelling cycle. If the pupils' does not question the reality of their results, mathematical modelling makes no sense.

In order to persist under this definition of modelling, the tasks of this study has to follow a set of principles. The first of these is that the task provides a real situation, which is understandable and associable by the participating pupils. The second is that it is open to several interpretations according to what EMK is being applied to the situation. This will allow the use of different models and strategies while still yielding effective results.

2.4 Realistic Fermi problems

Because the current curriculum do not emphasize the use of modelling problems in mathematics education, I am not going to assume that the pupils have previous knowledge of modelling procedures. For that reason, certain complications could arise. These complications are in no way reduced by the fact that I only have time to introduce the pupils to the procedures of mathematical modelling due to the short span of this study. It is with this in mind that I adopt the method of modelling presented by Ärlebäck (2009), where he concludes that pupils working in small groups with 'realistic Fermi problems' do provide a fruitful opportunity to introduce mathematical modelling at upper secondary school level (p.355).

Criteria	Brief description
Accessible	The task do not require any specific pre-mathematical knowledge. All pupils can approach the task. It is solvable at different levels of complexity.
Realistic	The task has a clear real-world connection.
Open	The task should not be immediately associated with a certain procedure, but rather urge the pupils to invoke strategical knowledge.
Lack of numerical data	The task have an absence of numerical data, so that the pupils need to make reasonable estimates.
Momentum for discussion	The task should promote discussion on matters such as 'what is relevant for the problem' and 'how to estimate the physical quantities'.

Table 2.1: Criteria for *realistic Fermi problems* (Ärlebäck, 2009, p.339-340)

A *Fermi problem* is a type of problem that requires a series of 'well-informed' estimates in order to reach a solution. Some examples of such problems are 'how many railroad cars are there in the US?' and 'how thick can the dirt on a window pane get?' (p.332-333). The origin of the term comes from the physicist Enrico Fermi (1901 - 1954), who was known to ask similar questions about everyday situations and phenomena he noticed or experienced. Different terms for this kind of problem can also be *back-of-envelope calculation problems* or *order of magnitude problems*.

Even though Fermi problems have a history both within mathematics and physics education, there are certain differences to the practices of the disciplines. In physics education, Fermi problems relate closely to physical phenomena in the real world and makes use of the same set of skills that professional physicists use in their everyday work. The same view is not predominantly apparent with the use of Fermi problems in mathematics education. Here it holds a certain niche to promote a more nuanced picture of mathematics (pp.334) and it does not necessarily relate to a real situation. However, certain arguments exists that - while mathematical Fermi problems can be purely intellectual in nature - it is better and more useful to situate them in an everyday context (pp.334). In that way, it opens the possibility to make a bridge between mathematics and the real world, or otherwise provide connection between mathematics and other school subjects. Differences aside, both disciplines do describe [to some extent] the same procedures on how to approach and solve such problems by *making simplified assumptions, estimating* and *doing rounded calculations*. Recent references from both field also argue for the inherent potential in the problems for festering pupils' critical thinking (p.334).

In order for a Fermi problem to be realistic according to Ärlebäck (2009), it requires a set of five criteria (p.339). The first of these criteria is *accessibility*, which means that the task is approachable and understandable by all pupils. It includes that the task does not require any specific extra-mathematical knowledge (EMK) and that it has a self-differentiating in nature - meaning that it is solvable at different levels of complexity by pupils at different educational levels. The second criteria is that the task is *realistic* by having a real-world connection. Consequently, the task provides more than just an intellectual exercise but also grants knowledge of real contexts rather than situated knowledge. The third criteria is that the task has an *open* formulation, and that it is not immediately associated with a certain strategy or procedure. This urges the pupils to specify and structure the relevant information to tackle the problem, and to invoke prior skills in approaching it. The fourth criteria is that the task *lack numerical data*. The absence of numerical data promotes the need to make reasonable estimates of relevant quantities. By extension, this criterion also implies that the context of the problem has to be familiar to the pupils if they are going to be able to make such estimations. The fifth, and final, criteria requires the task to hold an inner momentum to *promote discussion*. This is something that should occur in conjunction with the two previous criteria.

2.4 Authenticity as a social construct

Certain principles of the task creation focuses on the aspect of *realism* when creating interventions. Ärlebäck (2009) describes his Fermi problems to be 'realistic'; Borromeo Ferri (2018) bases modelling tasks in 'real situations'; and Skovsmose (2003) describes a certain field of tasks to have 'real references'. In all these cases, there seem to be a certain unity in understanding realism as 'something that has a real-world connection'. However, then two questions arise: 'what does *a clear real-world connection* mean?' and 'how can we assert that tasks have this specific trait?'.

In her text, Vos (2018) writes that the term *authenticity* plays an important role in criticising mathematics that seemingly relates to real life (p.2). However, she advises against the common use of authenticity as a holistic attribute that occurs when a series of elements work together. This is because it opens up the possibility of designing 'authentic' tasks that may contain clearly inauthentic aspects. Furthermore, such an understanding may make us ask questions like "how much correspondence with reality is sufficient to qualify for authenticity?.. …Or in other words: what essential aspects can, and what cannot be cut for the sake of education?" (p.5). Responding to this, she suggests another way to define the term authenticity according to two requirements. The first of these requirements ensures that the artefact did not originally have an educational purpose, while the second requirement certify the task as authentic in accordance with the community where it is from. This certification can come from either an acknowledged expert (professional workers, stakeholders, or the pupils themselves), or through interaction with physical media (video, image, newspaper clippings etc.) [or both].

In order to consider an aspect of education as authentic, it requires (p.8):

1. an out-of-school origins
2. a certification of provenance

Authenticity is a binary qualification; an intervention is either authentic or not. However, it is possible that an intervention contains some authentic aspects. Such aspects can be [but is not limited to] its context and its question. It is important to notice that authentic contexts do not imply authentic questions; these can occur simultaneous, but are not required to. A question is authentic to a context if, and only if, people within that context would ask it. Such questions are arguably a more suitable way to show the usefulness of mathematics in accordance with the real world than just adding authentic contexts to interventions. An intervention with an authentic context, but not an authentic question, may just help to prove the uselessness of mathematics in the real world because they are calculating things they never will need (p.3). Authentic contexts can, but do not necessarily improve pupils' motivations or performance in mathematics education. In response, one might ask 'is it then better to have interventions where all its aspects are authentic [aka. authentic interventions]?'. While some uses of authentic aspects seems to undermine its purpose, I am not under the impression that educational practices should idealize the use of truly authentic interventions [or the lack of any authentic aspects, for that matter]. An authentic intervention contains all aspects of real responsibility and consequences. It is in the nature of education, as separated from professional work, to allow for failure. In that way, the pupils may learn from their mistakes and improve accordingly.

2.5 Defining the research question

When addressing the temporary research question from the introduction — 'What does it take to make tasks similar to the Mathbridges tasks?' —using the theoretical framework we have defined, it becomes clear that we have to assess *criteria* or *characteristics* of these tasks. Boaler (2001) and Skovsmose (2003) further addresses an inquiry-based practice to further critical learning. The practice I have adapted is realistic Fermi problems, which is a type of modelling tasks. This type of problem are determined by a set of criteria . Furthermore, it becomes important to define both realistic and authentic — as addressed by Vos (2018). This brings us back to the formal research question:

'What are some characteristics of a mathematical intervention based in realistic Fermi tasks starting from an authentic infrastructure object in the pupil's region?'

3 Method

3.1 Methodology

I will conduct most of my research on people. In contrast to natural objects, both people and their institutions hold meaning and intention. It is important to understand this distinctiveness of humans as opposed to the natural order. Because of this, it is beneficial to apply an interpretivistic paradigm to this study.

"…It is founded upon the view that a strategy is required that respects the difference between people and the objects of the natural sciences and therefore requires the social scientist to grasp the subjective meaning of social action..."

(Bryman, 2016, p.26)

According to Bryman (2016) it is not sufficient to simply explain human behaviour, but also to interpret those actions and their social world from their point of view. Human actions hold meaning for those involved in the action. To understand and interpret the meaning behind these actions is what it means to take an interpretivistic approach.

3.2 Design research

Design research is an approach that closely resembles action research. "Action research can broadly be defined as an approach in which the action researcher and members of a social setting collaborate in the diagnosis of a problem and in the development of a solution based on the diagnosis" (Bryman, 2016 p.387). Both research methods appear cyclic in nature, and is conducted in a real-world setting with the involvement of practitioners of the field. While the goal of both research methods is to improve practice, design research - as opposed to action research - is primarily aimed at generating design principles (van den Akker, Bannan, Kelly, Nieveen, & Plomp, 2013, p.44). This essential difference between the *generation of interventions* and the *generation of design principles* is what differentiates design research from action research.

> "The practical contribution of design research lies in developing empirically grounded prototypical learning trajectories that may be adopted by others" (van den Akker, Bannan, Kelly, Nieveen, & Plomp, 2013, p.26).

There are primarily two different approaches to design research: *development studies* and *validation studies*. Although this distinction is conceptually important, design researchers often combine the two approaches in order to strengthen the value of their research.

Development studies aim to develop interventions, and [through the process of doing so] construct design principles. This process starts with the identification of a problem with no or few validated principles available to support the design and development of activities in that setting (van den Akker, Bannan, Kelly, Nieveen, & Plomp, 2013, p.23). From there, the researchers - in collaboration with practitioners of the field - develop effective interventions by carefully examining their prototypes. These reflections on the development process has the eventual purpose of yielding design principles for developing innovative interventions: 1) procedural design principles (characteristics of the design approach), and 2) substantive design principles (characteristics of the design itself). However, it is important to recognize that these design principles will be heuristic statements and provide no guarantee for success in other contexts.

The format of design principles in design research:

> "If you want to design <intervention X> for the <purpose/function Y> in <context Z>, then you are best advised to give <that intervention> the <characteristics A, B, and C> [substantive

emphasis], and to do that via <procedures K, L and M> [procedural emphasis], because of <arguments P, Q, R>."

(van den Akker, Bannan, Kelly, Nieveen, & Plomp, 2013, p.24)

Validation studies has the purpose of developing and/or validating existing theory (such as design principles). In validation studies - since the researcher work in the real world: a complex environment of interacting systems and elements - experiments are conducted by designing appropriate elements and anticipating how they will function together in the given setting. This may either be studies on micro-theories (such as instructional activities), local instruction theories (e.g. instructional sequences), or domain-specific instruction theories.

The process of design research is cyclic. Akker et. al. (2013) emphasizes the following distinguished phases (p.19):

- *Preliminary research.*
  Context analysis, review of literature, and development of a conceptual or theoretical framework for the study.
- *Development or prototyping phase.*
  Design phase consisting of iterations, with formative evaluation aimed at improving and refining the intervention.
- *Assessment phase.*
  Summative (or semi-summative) evaluation to conclude whether the solution or intervention meets the pre-determined specifications. This phase often results in recommendation for improvements of the intervention.

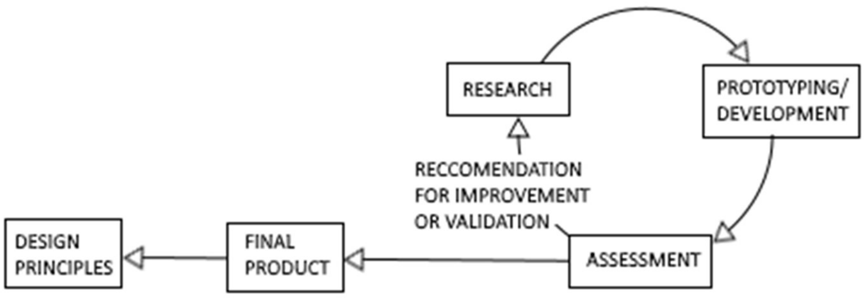


Figure 3.1: The cyclic process of design research

3.3 Ethical complications and quality criteria

"Knowledge resulting from a design research project will strongly increase in value when it is justified by theoretical arguments, well-articulated in providing directions, and convincingly backed-up with empirical evidence about the impact of those principles."

(van den Akker, Bannan, Kelly, Nieveen, & Plomp, 2013, p.24)

Just as it is in action research, there are certain implications when performing design research. While some of these are universal to all qualitative research studies of the social world - the aforementioned types of research are unique in that they primarily struggle with the problem of having the researchers both work with and assess their own creation. Zeni (1998) depicts this as the ambiguous relationship between the researcher and the social situation. As opposed to classic ethnographers, who observe change, but do not usually try to cause it, the action researcher actively tries to change and improve his or her own practice. Since this aspect holds true in design research as well - it is important that I overtly discuss what measures I can take in order to protect myself from 'seeing what I hope to see' and in such a way convolute and devaluate my findings.

Several methods and criteria for assessing qualitative studies exist. In this study, I will apply the *trustworthiness* and *authenticity* criteria as presented by Bryman (2016). The first of these, trustworthiness, refers to the validity and reliability of the study. This is something I hope to achieve through: 1) specifying my position in a specific theoretical doctrine, 2) having transparency of my methods and assumptions, and 3) applying several stages of semi-summative and summative assessment, in a hope of achieving some degree of triangulation. The second criterion, authenticity - which refers to the broader political impact of the study - was already addressed in the introduction of this study. Additionally, through the process of developing my mathematical intervention, two experts is evaluating my tasks in addition to myself. While this evaluation primarily assesses the practicality and effectiveness of the tasks, they will also [however subliminal] address the tasks' validity.

3.4 Principles of task design

Following the format of design principles given through the design research, we get the following design principles from the theory:

If you want to design a realistic Fermi task starting from an authentic infrastructure object in the pupil's region, then you are best advised to give that task the following characteristics:

- *Accessible.* The task is understandable. This means that all representation used - such as language, imagery, or physical objects - is understandable as a whole and in no need of further clarification. Furthermore, it means that the pupils are familiar with the representations used in the task, and can solve it either individually or in groups at different levels of complexity. It is important that the tasks are accessible because if the pupils cannot understand the problem of the task they will most probably not answer it.
- *Realistic.* The task has a clear connection between the task and the daily life of people. This means that the task contains certain authentic aspects or is completely authentic. However, it is not a goal in itself to make a completely authentic task. This is because of the limits posed upon an educational situation is often opposed to that of a real life situation. Furthermore, pupils should retain the option to fail without facing harsh consequences.
- *Open.* The task lacks numerical data and strategic formulations. The pupils need to make reasonable estimates, and ask themselves "what" and "why" they use the numbers they use, and think critically about how you are going to solve the task. This way of working is similar to how professionals use mathematics to solve problems in their work, and is a method of introducing the concept of modelling to pupils at upper secondary school.

3.5 Task development process

According to the cyclic nature of design research, procedural principles are going to sprout from the tempering of interventions through several stages of testing and evaluation. There are three such stages of development occurring in this study. The first of these is the development of the initial prototype, based on the initial research conducted. The second of these is the development of the first revision [prototype II] based on the results of the try-out of the initial prototype. Thirdly, and finally, is the development of the final revision based on the results of the expert appraisal of the second prototype. No further development occurs in this study.

		Development phase		Assessment phase	
Stages in prototype development →		**Initial prototype**	**First revision (prototype II)**	**Final revision**	
		Users (n=2)	Users (n=2)	Users (n=22)	Users (n=3)
Validity			ea		
Practicality	Expected	to	ea		
	Actual			ft	iv
Effectiveness	Expected	to	ea		
	Actual			ft	iv
to: try-out ea: expert appraisal ft: field-test iv: interview					

Table 3.1: The different stages of development and their corresponding method of evaluation, adapted from (van den Akker, Bannan, Kelly, Nieveen, & Plomp, 2013)

3.6 Overview of the tasks

As a design choice, the interventions created in this study are going to be tasks about bridges in Agder County. This has the purpose of compiling these, together with those of other studies, into a product similar to the 'mathbridges calendar' made by the University of Münster (AFO Arbeitsstelle Forschungstransfer, 2018). It was decided early on that my contribution to this product should consist of about three tasks. However, I did create a significantly higher number of tasks during the development of the initial prototype. This was because it is arguably easier to remove certain tasks that did not work as intended - rather than to change aspects within each task. Furthermore, it served as a method of gathering more data to develop the tasks that I kept. In total, I made seven tasks where some went through revision. The reason that I initially chose these particular seven bridges, was primarily because they had satisfying visual aesthetics [according to my own personal opinion].

Task #	Bridge	Topic
1	Bakke Bridge	Weight (relations)
2	Bankebroa	Money
3	Dorga Bridge	Volume & weight
4	The Fedafjord Bridge	Length
5	Rosnes Bridge	Time (relations)
6	Skjærnøysund Bridge	Height
7	The old timber slide in Vennesla	Area & volume

Table 3.2: Tasks, bridges and topics.

All the tasks designed in this study are fermi problems. By extension, they are trying to be both inquiry-based and open-ended. The primary trait of this type of problems is that they relate to finding certain magnitude of some measurable quantity. According to the design of these tasks, each relates to unique types of measurable quantities. Tasks 4 and 6 relates to spatial dimensions - respectively length and height. Tasks 1 relates to matter, while task 3 relates to correlation between volume and matter. Task 7 relates to a flat area and a proportional volume size. Task 2 is the only one that does not relate to a measurable physical attribute. Instead, it relates to a more abstract quantity - money.

During the design process, there became a divide between two 'subgroups' of tasks because of a trait of the Norwegian language. This trait translates relatively well into the English language, hence I will attempt to explain it - and how it affects the tasks. The divide stemmed from the types of question asked, and the use of either asking 'how much' or 'how many'.

The word 'much' were used when asking 'how much' of something there were. Tasks that applied these kinds of questions were tasks 2, 5, and 7. Some examples of these questions are 'how much time do you save taking the bridge instead of walking around?' and 'how much money was lost?' Neither question asks about a countable quantity, but we do expect the pupil to understand implicitly that we want an answer that is a countable quantity such as 'Norwegian kroner' or 'hours'. This may seem counterintuitive for use in Fermi problems, but it has a certain benefit. While these questions could have been changed to 'how many Norwegian kroners were lost?' or 'how many hours do you save by taking the bridge instead of walking around?', I kept these as they were in order to keep the tasks as open as possible. One could argue that this is certainly not necessary in the prototypes of task 5. However, it became an important aspect as I changed its wording from 'how much time do you save by taking the bridge instead of walking around?' to 'If the bridge would collapse, how much more time would it take for the inhabitants to get to the other side?' in the final revision. When I changed the method in the problem from simply 'walking' to opening up for more choices of transportation, the question benefited from using 'much' rather than 'many'. For example, a person who solves the task relating to 'walking around' would perhaps use the measurement of hours while someone who solves it relating to 'swimming' would use seconds or minutes.

3.7 Participants

3.7.1 Participants of the development phase with formative evaluation

Participants in the try-out of the initial prototype

The first group of participants consisted of a couple students who major in mathematics. For the sake of simplicity, I recruited suitable participants from the body of students that are attending the same university as myself through my personal network. In addition to having sufficient mathematical abilities, I also required all participants of this group to have some pedagogical background as well. This is because, they are supposed to have the necessary theoretical and practical expertise to work with, and evaluate the work around, interventions in mathematics education.

Participants in the expert appraisal of the second prototype

The second group of participants consisted of a couple students who both studies mathematics education, respectively at their fourth and fifth year. One of these additionally works as a teacher of physics at upper secondary school level. These are not the same participants as in the try-out of the initial prototype, as I wanted new eyes to assess my tasks. I recruited these participants through an email invitation that I sent to several possible respondents with sufficient competences including students, teachers and professors — trying to accommodate for expected low response ratings by reaching out through my own personal network.

3.7.2 Participants of the assessment phase with summative evaluation

The final group of participants consisted of 21 pupils attending Norwegian upper secondary school with mixed mathematical backgrounds. About 80 % of these pupils followed the practical mathematical course (1P), while the remaining 20 % followed the theoretical mathematical course (1T). All of the pupils had little to no previous experiences with modelling problems. This class were recruited to my study through a teacher I had previously worked with. Although I knew the teacher beforehand, I did not know any of the pupils.

3.8 Methods of data collection

3.8.1 Participating observation

Both during the try-out of the initial prototype and the field test of the final revision, I am going to observe the participants as they explore the tasks. These observations are overtly conducted and the participants is notified explicitly beforehand (see appendix B). A set of accompanying field notes of interesting events and behaviour, and my initial reflections of them, will be written down. They will initially be as short as possible in order not to draw my attention away for too long, or to upset the group dynamic by making the participants overly self-conscious (see appendix C). Only after — and not long after — exiting the setting, will I completely write up the final field notes. It is important to acknowledge that I cannot completely remove my presence as a foreign element in both groups, but through assuming a teacher role and keeping any excess writing to a minimum during the session - I am hoping to suppress it to an insignificant level.

3.8.2 Self-administered questionnaire

As a part of my development process, I want to evaluate the expected practicality and effectiveness of the tasks according to the principles I have deducted. It is undeniable that I am partial to my own creation. Because of this, it is crucial that someone else does this evaluation. However, as the principles I have in question are already determined, it is not beneficial for this study to allow a completely open evaluation of the tasks [as this allows for 'hit-or-miss' scenarios]. Hence, it is why I see it necessary to ask predetermined questions during the expert appraisal.

I chose the format of a self-administered questionnaire (see Appendix D), because it would allow myself to gain written answers - which are arguably quicker and easier to transcribe than oral answers. The questions of the questionnaire were open-ended because I wanted to tap into the respondents' knowledge and allow for unusual responses. Nonetheless, there are certain drawbacks to such an approach. One major disadvantage to open-ended questions is how time-consuming — and by extension intimidating — it would be for the respondents. Seeing how it is difficult to remove this aspect, I instead focused on keeping the number of questions reduced and the general layout of the questionnaire to be easy on the eye. This is especially visible in the first task, which easily could have been separated into nine questions rather than three. In an attempt to motivate the respondents further, I added a few sentences that appraise their knowledge of the subject matter at hand. Yet, when presenting the questionnaire to possible candidates, I did both mention the scope of the evaluation and the approximated time it would take to complete it.

3.8.3 Semi-structured interview

As a part of the assessment of the final revision, I conducted interviews with some of the pupils to gain further insights into how they worked with - and solved - their task. This allowed the pupils to explain further their actions and strategies, in addition to their feelings when working with the tasks. Because of this, I chose to perform the interviews in person directly after the pupils had concluded their work with the task of the field test. This way they would still have their work freshly in mind. The interviews were semi-structured: primarily having a predetermined set of questions, but also open to pursuit interesting answers for further explanation. There are three main questions in the interview guide (see appendix E), where each of these has a number of suggested follow-up questions. The main questions are open-ended short questions devoid of technical terms. These choices was made to accommodate for the fact that the pupils have little to no previous experience with mathematical modelling, in addition to keeping the questions open for unexpected answers and not leading the pupils to certain answers. However, there are certain obvious flaws of these questions as well. Because of the abstinence of technical terms and the focus on keeping the questions short, there are certain questions that become very general. Furthermore, some words such as 'similar', 'encountered', and 'earlier' holds a certain ambiguity to them, which makes it more difficult to understand the answer of a pupil unless they also elaborates with a detailed explanation. In total there were three pupils volunteering for these interviews, all of which were aware of the scope and goal of the study beforehand.

3.9 Tools of analysis

There is no way to monitor the cognitive processes of the pupils directly. Because of this, I need some tool in order to make sense of their cognitive processes without being able to perceive it. First, I will assume that a pupil's interactions with others in a social context is determined by its individual choices; that human interaction holds meaning, and that one can interpret the intent of an action by understanding the action according to the social context. Because of this, I will examine the pupils wording and actions during the field tests and the interview accordingly. When the pupil addresses aspects of extra-mathematical knowledge (EMK), either when interpreting, simplifying or mathematizing the problem in the task, I will interpret that as a sign that the pupil acknowledge the realism of the task. When the pupil assesses one or several methods for solving the task, either according to its estimations or to strategic procedures, I will interpret that as the pupil acknowledges the openness of the task. Finally, I will argue that it is difficult to interpret that the pupils find a task accessible. This is because usually, when things goes without any problems, the pupils probably will not commend this. On the other hand, if anything goes wrong or if something is difficult they will probably express concerns about that. Because of this, I am going to look for signs of confusion among the pupils, and interpret that as them not finding the task accessible. If there are no signs of confusion, I will interpret that as the task being accessible.

What I am interpreting it as	**The pupil acknowledges the realism of the task**	**The pupil acknowledges the openness of the task**	**The pupil does not find the task accessible***
What am I looking for	The pupil invoke real world knowledge / extra-mathematical knowledge (EMK)	The pupil assesses methods of solving the task.	The pupil expresses confusion about the task.
Examples of things the pupil might say	- The size / length / weight / speed of a… - If it was I, then…	- We can write this as an equation, because… - If we find the area, then…	- What do this mean? - Which part is the task asking about?
*I will argue that it is easier to recognize when the pupils does not find the task accessible, rather than when they do.			

Table 3.2: Matrix describing how I will interpret the actions of the subjects participating in this study.

While investigating the results, I will consider how the EMK invoked by the participants relates to their methods of solution. For this, I apply the stages in the modelling cycle (Borromeo Ferri, 2018) as a reference - addressing how the EMK supports (or do not support) the modelling process.

4 Development results

4.1 Initial prototype

4.1.1 Results

The participants of the initial try-out had some previous experience with mathematical modelling and fermi problems. This is not something I expect to be the case for most pupils attending upper secondary school, as mathematical modelling has not been a distinguished part of the current curriculum [although it is about to change (Ministry of education and research, 2018)]. Hence, the results of the initial try-out will only yield an estimate of the actual practicality and effectiveness of the tasks (see appendix F). Nonetheless, it goes without saying that if elements of the tasks are difficult or confusing to these 'ideal' participants they will undoubtedly be challenging to other participants with significantly less experience in handling such tasks. Because of this, I am using the initial try-out to improve certain aspects of the intervention as a whole — and beginning to assess the design principles of this study. This is the first of three evaluation stages.

The first element I am going to assess in my results is whether the participants acknowledge the realistic aspects of the task. According to my tool of analysis, I examined the participants' interactions for actions and dialogue where the participants would invoke extra-mathematical knowledge (EMK). In total, there were five such interactions spread over four of the seven tasks. Three of the tasks did not invoke any perceivable EMK. For the sake of simplicity, I am not going to consider the possibility of cognitive processes that does not connect to expressive actions. The five perceivable interactions did in general invoke three categories of knowledge. The first of these relates to a category of EMK that I am going to define as 'common knowledge', the second as 'specific knowledge', and the third category as 'authentic certification'.

Instances of the first category of EMK, invocation of common knowledge, occurred explicitly during the participants' work with task 1 and 7. In task 1, the participants were comparing the weight of a bus in relation to the weight of people. Here, they used certain information when simplifying the situation and constructing their real model. Noticeably, this was 'the weight of a bus', and 'the average weight of a person'. The first of these was not something they had knowledge of beforehand - the required information were too specific. Hence, I am not considering it either 'common knowledge' or EMK that they had (before working with the task). On the other hand, the average weight of a person was not something they had to look up. When addressing the modelling task, they already had certain preconceptions about what an average person weights. This is a prime example of invoked EMK that falls under the category 'common knowledge'. Another instance of this occurred during their work with task 7, as they had to assume the height of a person on the image in order to assume the height of the bridge's 'railings' [in the lack of a better word]. This, they were also able to do without the use of any external aid.

It is fair to say that the second category of EMK, invocation of specific knowledge, was the rarest during the initial try-out. Only one instance occurred, and this was during the participants' work with task 7. In this task, they took into consideration that the wood of the bridge would likely need at least two strokes of wood stain because of how exposed it was to moisture and extreme weather conditions. In contrast to the common knowledge, this type of information is not something most people would have.

The third and final category of EMK, authentic certification, occurred twice. Authentic certification manifest itself through the participants' invocation of knowledge not to provide information that specifically work to structure and simplify a model of the task, but rather to authenticate aspects of the intervention or its results as real [opposed to realistic]. This occurred once while working with task 2, and once with task 7. Working with task 2, one of the participants expressed familiarity to the events described in the task, and of the bridge itself. This was something that the participant could vouch for was a real event — even referring to have read about it in the local newspaper. While the yield of vouching for the authenticity of the setting in this task did not directly determine the course of their modelling process, it did however change their outlook on the task as a whole. A similar incident occurred during task 7, as one of the participant made comparisons to staining a terrace — which was something the participant had experienced first-hand.

Task #	The student acknowledges the realism of the task	The student acknowledges the openness of the task	The student does not find the task accessible*
1	Information: weight of a person	Comparisons (weight)	Problematic wording in the task description
2	Vouch for authenticity: familiarity to the bridge. Have read about it in the local newspaper. Information: comparison to another local bridge**	1) Looking up old news articles 2) Assessing the price of the whole bridge, relating to the parts that were lost	
3		Volume proportions	The use of 'how many' - and how this relates to their answer
4	Vouch for authenticity: search for construction documents**	Geometric principles (length)	
5		Interactive satellite maps	Image: lacking overview of the area
6			Image: parts of the bridge is missing Problematic wording in the task description
7	Information: weight of a person Information: staining wood exposed to moisture Vouch for authenticity: comparisons to staining terrace	Geometric principles (area)	

*All tasks had (to some degree) the need to make assumptions not based in their own common-sense making.
**These relates to the method of approaching an answer.

Table 4.1: Summary of the results concluding the try-out of the initial prototype

In addition to these categories, there were certain instances that could indicate additional appliances of EMK to the tasks. Such instances did relate more closely with how the participants chose to address the task, rather than explicitly applying preconceived knowledge. This occurred while the participants were working on task 2 and 4. In task 2, the participants tried to look up the article of the local newspaper [that one of them had read about the events of the task] to see if it held the solution to the task. It did not. Instead, they had to make comparisons to another local bridge in order to solve the task. In task 4, the participants tried to look up the building documents of the bridge. This seems to indicate that they, to some extent, accepted some of the realistic aspects of this task as well - although never mentioning it explicitly.

During the try out, I did not notice the participants to assess many different ways of solving each task. Usually, either one of the participants suggested a method that they preferred - while the other complied. Why either one of them chose a certain method, I do not know. I did not notice any mathematical reasoning for applying one method rather than another. This could be a consequence of the scope of the try out and the fact that there were many different tasks to solve in a very limited timeframe. However, what is interesting is that their strategy always seemed to be twofold. Their first attempt at finding a solution always relied on the use of EMK to see if they already knew - or could easily find - the answer without the need to make a mathematical model (as seen in the example above, with the newspaper). When this failed, they turned to modelling. However, making models just from their EMK was impossible to the participants. This was because a common feature of all the tasks was that they lacked some specific information [that the participants needed to solve the task]. Hence, the strategy of the participants were shaped by this; nearly all the strategies involved the use of digital aid to gather additional information.

Several elements of the tasks appeared confusing to the participants. The first of these, were the wording of certain problems. This includes primarily task 1, 3 and 6. In task one; this confusion arose due to the wording seemingly presenting the problem as logically strange. While they, as students of mathematical didactics, understood how I wanted them to solve the task - they still argued that the task description could present a problem to participants less experienced participants because of the wording. They continued by asking 'why it was so that this restriction only applied to buses and not all heavy transportation' and 'if the problem were asking about how many people the bridge could hold in addition to the bus or without it'. I provided further clarification that the intention of the task was to calculate how many people the bridge could hold on its own [without any buses on it]. This was not the end of the confusion. While working with the task, another interpretation of the wording arose - as it could seem as if 'suddenly after 1950 the bridge couldn't hold 10 tons anymore'. In task 3, there were a slight confusion with the wording of the task - as it seemed to specify wanting a number due to it asking 'how many stone blocks are needed?'. However, as the participants pointed out, due to the nature of modelling tasks and the estimations needed to be done, 'how many' doesn't necessarily relates to a number. It can also relates to quantitative operations such as 'double' or 'triple' the already existing amount - and in this way presenting an answer in the form of an expression relative to the unknown previous amount. In task 6, there were several confusing elements. The first of these being the wording of the problem. While the question is relatively short, asking 'how tall are the pillars of the bridge?', instead of providing clarity - it became a source of confusion in conjunction with the image. As there are several different pillars of different heights in the image, the problem given in the task is more than twofold and several answers are required. Furthermore, as the wording of the text specifies '…of the bridge?' and the image does not show the complete bridge, it lacks specific information about the real situation in order to be solvable.

In addition to task 6, also task 5 had a lacking imagery. Because the task required the participants to evaluate the complete environment around the bridge, but the image did not show any of this. It could seem unsolvable if it had not been a real object, which the participants could interact with through other media. Such media included images and maps made and published by other people online.

As a final question to the participants, I asked them if they could rate the tasks from most interesting to least interesting. This was not to assume that it would be representative for how others would feel about the tasks, but rather to see the general likeability of my tasks in comparison to each other. This additionally allowed me to consider how the confusion of the participants could correlate with their interest in the different tasks. Although it is not a goal of the study, I would prefer to make interesting tasks to disinteresting ones.

Task 7 (most interesting)	Task 5	Task 2	Task 3	Task 1	Task 4	Task 6 (least interesting)

Table 4.2: The participants' rating of how much they enjoyed each tasks in relation to the other

4.1.2 Discussion

From analysing the results, in regards of how the invoked extra-mathematical knowledge (EMK) connects to the solution, we can notice two trends. In the first of these, there appears to be a clear lack of connection between the use of EMK and the models created. In the second trend, there is a connection between these stages in the modelling cycle. Two of the tasks were impossible to assess according to either of these trends. Both because they lacked explicit incidents where the participants invoked EMK, and because their method of solution differed significantly from the other tasks. However, it is noticeable that tasks do require the participants to make reasonable estimates about very specific information. Such information includes knowledge about 'a particular area in the real world' or 'the dimensions of a bridge in the real world'. Both of these examples have very few reasoning options that allows someone to assume this information and get it relatively correct. One either know this sort of information, or do not. Hence, the participants struggles to make assumptions according to the missing information. However obvious the lack of numerical data in the tasks may appear, it does not correspond with the 'lack of numerical data' criterion of Fermi problems.

The first trend is apparent in task 4. Here, the participants applied their EMK to understand the task as a whole - including all associations they had to the context. The context of this task [just like all the others] was that of a particular bridge from the real world. Similar to all kinds of infrastructure in the real world, they do not merely exist on their own. Both their maintenance and development starts in a collective effort that includes many people - some designers and builders among others. Together, all of these associations constructs what is a mental representation of the task. It is fair to say that the participants' mental representation of the task evaluated the bridge as authentic, due to the explicit association to 'building documents'. However, this was the only incident of expressed EMK as the participants solved the task. 'Why is this?' may we ask ourselves. Certainly, there should have been more instances of invoked EMK? One reason for why this is not the case, could be the relationship between the question and the context. The question is inauthentic, and simply asks '*how many meters of steel wire was used to build the bridge*?'. The task, on the other hand, is open to several strategic formulations. While a number of strategies can solve this task, none of these methods actually requires the participant to invoke EMK. This is the biggest flaw of this task, and goes to show that even if this had not been an authentic bridge, the question could still have been answered to much of the same degree as it was. In this regard, the participants were not wrong in seeking out the 'building documents' as this was probably the most realistic method of acquiring the answer that the task were seeking. However, if the method had worked, then there would have been no need for mathematical modelling. In this sense, this task is a great example of what modelling tasks should not do; showing the uselessness of real experiences and EMK in performing mathematical tasks. Arguably, much of the same argumentation applies to task 6 as well — although I cannot be sure since no instances of EMK occurred as the participants where working with this task. The gravity of its confusing aspects might well have overshadowed other aspects of that task. However, if we assume both these tasks to be similar — it is also not so strange that they both rank similarly on the participants' interest scale.

The second trend is apparent in task 1, 2 and 7. This trend represents the opposite of the previous one. Here, the participants started from a similar starting point as in the aforementioned tasks with a mental representation of the context as authentic. We can notice this by looking at what information they utilize in their strategic development of a model. In task 1, they used the 'weight of a person' to solve the problem; in task 2, the 'comparisons to another local bridge'; in task 7, the 'height of a person'. In addition, there was also the incidents where the participants expressed familiarity to aspects in both task 2 and 7. Together, these instances points toward the fact that the participants viewed the context as authentic. Furthermore, what is interesting to notice is that the aforementioned bits of information - except for the familiarity - were explicitly used when solving the tasks. Let us look at task 7. Here, they do not know the height of the bridge. Just like in task 4, this information is difficult to acquire through reasonable assumptions. However, something is different with this task and the participants are able to assume a height after all. Looking at their method of solving, they were able to do this because of the person in the image acted as a sort of catalyst. In task 1, the 'weight of a human' does not work in a similar catalytic manner. However, it does serve a purpose required to solve the task; it gives meaningful input to their strategic procedure. The EMK invoked in task 2 serves a similar purpose as the one in task 1. In all the tasks, EMK was a requirement for their solution. What is curious to notice is that these tasks rank very differently on the participants' interest scale.

As for task 3 and 5, they did not invoke any explicit instances of EMK. Because of this, it is difficult to judge the relation between the authentic aspects of the task and the different processes in the modelling cycle. However, in task 5, the participants did use an interactive computer map to solve the task. Even though this was probably one of the few ways to solve the task, it does insinuate [to some degree] that the participants considered the task authentic and a part of the out-of-school world. None the less, the participants did not create mathematical models to solve the task. Instead, they used their computer competences to solve the task. One could argue that this is much similar to their strategic approach in task 4, but this time it actually yielded results. In task 3, the participants did not seem to invoke any EMK, and it was particularly difficult to assess their strategic arguments - and whether or not they made any models at all. It is also difficult to assess if they were aware of their simplification and structuring during the mathematizing step as they never displayed an overt strategy.

There were several difficult and/or confusing aspects of the tasks. In order to improve the tasks, I have to address these aspects and consider 'why' they are detrimental. Once I have that knowledge, I can also address ways of improving the tasks themselves. Three of the tasks had no particular confusing aspects. Namely, these tasks were 2, 4, and 7. I am not going to address these further. Starting with task 1, this task had a confusing description where it was unclear whether or not buses could drive over the bridge at all, or just without its passengers. The initial description was '*Bakke Bridge in Flekkefjord is Norway's oldest suspension bridge. It was opened in 1844. From the 1950s were bus passengers denied to be driven across the bridge because it could not handle the collective weight. How many people do the bridge handle?'* (translated from Norwegian). In this description, there is a lot of information about the bridge itself, but very little information about the apparent sudden change in policy regarding buses. Changing the first and the second sentence of the task description could accommodate for this. Additionally, the ending of the third sentence, which sets the premise for the modelling task, is very cryptic. To make sure that the participants would understand that it refers to both the bus and the passengers together, the ending of the third sentence could be changed to '*…could not handle the collective weight of both the bus and its passengers (together).'* Furthermore, it is also to notice that the question takes very little consideration to other authentic aspects of the context. '*How many people the bridge can handle*' is probably a sufficiently high enough number so that it will never be relevant (due to space and safety measures). A question that takes consideration to more aspects of the context, would probably ask something along the lines of '*how many people can be on the bridge simultaneously'*. In task 3, the wording of the problem was not strictly detrimental in the sense that the task became unsolvable. As previously discussed, the participants did get an answer. However, as it has been addressed, this solution did not include any visible incidents of the modelling process. Rather than specifying, 'how many stone blocks', this could have been changed to 'how many kilos of stone'. Furthermore, the task gives an inauthentic representation of the context of the bridge - as the width given in the task does not represent the actual width of the bridge in real life. It is an assumption made by myself. This is not ideal, but because of I do not own a car - I have not been able to drive out and measure the bridge myself.

4.1.3 Semi-summative remarks

I already knew before the initial try-out that I was not going to keep all the tasks. The only thing I needed to do was figure out which task was not going to make it, and why. Both task 1 and 6 are very similar to other tasks, but generally more confusing. Because of this, I have decided to stop developing these tasks. This is to allow myself to test the principles of my designs according to different performing tasks rather than towards similar ones. For the same reason, but also because the specific knowledge required to solve it is too difficult to assume, task 2 is discontinued. It is also noticeable that this task described a timely event, which might not have been relevant if the task were going to be included in a calendar some years down the line.

Tasks 3, 4, 5 and 7 are going to be continued. However, there are certain changes applying to them. The first change comes from adding aspects of specific information into the task descriptions of all the tasks. Some of this contradicts the absence of numerical data, as a part of the 'lack of numerical data' criterion of Fermi problems. However, all of the tasks still have lacking numerical data - so the use of assumptions and estimates are still essential to the problem solving process. There is just no longer an absence of numerical data. The reason for this change was so that pupils solving the tasks could rely on making reasonable estimates. Additional changes occurred to all the tasks except for 4 and 7 - who otherwise did not invoke incidents of confusion. In task 3, there are primarily two things changing. The description of task 3 changes into a more accurate description of the width of the bridge, and its question changes into a more specific question about 'kilos' in order to see if this will trigger explicit modelling procedures. In task 5, in addition to adding specific information in its task description, I also needed to add a map displaying the area surrounding the bridge. This is the same one that the participants used when solving this task [except it is not interactive]. Together, it should now be able to solve this task without the need of further specific information - only reasonable estimates.

In addition to the changes within each tasks, there are certain implications for my design principles as well. The first of these are the *openness* principle; 'The task lacks numerical data and strategic formulations'. Previously understood as an absence of numerical data, it is more fruitful to understand this as a lack of key data - and not numerical data in general. This is because we live in a world were much information and data are available at all times, either through the knowledge of people in our society or through established institutions. Disallowing such information in our task context does not serve to keep it authentic. Key data are the numerical information required to solve the task. It is by keeping this open, that we encourage our pupils to make reasonable estimates and ask themselves 'what and 'why' they use the numbers they use. We are still to understand 'openness to strategic formulations' as 'a lack of strategic formulations'. Furthermore, according to the *accessible* principle, it is important that the pupils are able to estimate this using their extra-mathematical knowledge (EMK). Because of this, the design of the task has to be aware of the two categories of required key data. The first of these are 'specific knowledge' and the second is 'common knowledge'. The later of these refers to knowledge that all subjects of a social context have. If the key data falls under the category of specific knowledge, it is not fair to assume that the pupils can estimate this through reasoning and EMK. For the task to be *accessible,* it requires either common knowledge to be the key data [of the task] or the possibility to 'figure out' the key data using common knowledge (such as in task 7). As for the *realism* principle, it becomes clear that an authentic context alone does not make the participants enjoy the tasks more or perform better. This seems to agree with Vos (2018); '…authentic contexts can, but don't necessarily improve students' motivation or performance in mathematics education.' (p.3).

4.2 Second prototype

4.2.1 Results

The respondents of the expert appraisal had some previous experience with mathematical modelling tasks, but they had more experience planning and executing interventions of educational practice. Because of this, while the results of this stage will give some knowledge of the expected practicality and effectiveness of the task, it will mostly allow the tasks validation by experts that were not a part of the initial or second task development process. This is the second of three assessments stages.

Task #	The expert acknowledges the realism of the task*	The expert acknowledges the openness of the task	The expert does not find the task accessible**
3		Estimation and calculation of volume	Difficult word: 'trunk road' Lack of specific information (length of bridge)
4	Making comparisons to the work of a construction engineer		Image: missing a part of the bridge Lack of specific information (dimensions of the bridge)
5	Information: real terrain features	Possible use of interactive map programs (Google maps)	Image: lack of terrain features on the map
7	Information: 'standard plank sizes'	1) Estimation of the width according to: a) the person on the image, b) the surroundings, c) standard plank sizes 2) Geometry (calculating area in order to find the required volume)	Lack of specific information (width of bridge)***
*There are no clear realism in any of the tasks [according to respondent B] **All the tasks lacks certain information [according to respondent B] ***This does not hinder the accessibility of the task			

Table 4.3: Summary of the results concluding the expert appraisal of the second prototype.

Addressing the realistic aspects of the tasks, the experts seem to be disagreeing. The first respondent [A] is the only one that connects the tasks to the real world through examples based on real situations and in an authentic context. This is noticeable in task 4 explicitly, and in task 5 and 7 implicitly. In task 4, this respondent announces that 'one could easily imagine this question raised by a construction engineer who is going to estimate an amount of steel that would [be expended] on the wires'. This is the only example where the respondents explicitly affirm the realism of the tasks. Otherwise, the appliance of extra-mathematical knowledge (EMK) occurs implicitly as the respondents are either taking consideration to the accessibility or the openness of the tasks. In task 5, the respondent [A] is addressing features of the environment that did not occur in the task but rather features one might expect in a real terrain. Such features were the 'paths in the area' and 'the steepness of the terrain'. In task 7, the respondent [A] expressed EMK while considering possible methods of solving the tasks: applying the existence of 'a standard size for wooden planks'. The second respondent [B] did not address the tasks singularly according to their realistic aspects, but expressed a more general impression that '…there is no clear link to the daily lives of people - even though the problems posed in the task might appear realistic'. However, both respondents did acknowledge the real aspect of the bridges and the effects it would have on the pupils. The second respondent [B] did acknowledge that 'the addition of actual bridges in the tasks might make them more realistic to the pupils'. The first respondent [A] found it '…exciting that [I] chose to use real bridges in the local area…', further expressing that '…such tasks could have an impact on the participants - in the sense that the next time they notice a similar phenomenon in real life they will consider how to calculate the various measurements'.

A few things affected the accessibility of the tasks according to the respondents. The first of these was the format of modelling problems itself, and the lack of information apparent in such tasks. According to the first respondent [A] this could lead to '…some participants [struggling] to start, seeing as [these are types] of task with *lacking information* in comparison to much of the other school mathematics they have worked with'. The second respondent [B] held a similar argument, and '…would not use such tasks in education at upper high school. This is because the pupils themselves has to gather the information, and that can lead to certain pupils not knowing what to do or how to solve the task'. In addition to these general remarks, there were also some specific comments about aspects that could cause a task to be less accessible. In task 3, respondent [B] advices to be '…careful with the use of the word trunk road as it can be confusing for many'. This was the only instance of a confusing word - as all the other tasks had understandable language. In task 4, the first respondent [A] finds it '…a bit suspicious that one cannot see the whole bridge in the image, [although] it is supposedly not that many meters missing'. In task 5, the same respondent [A] criticises the lack of a more detailed map in the task: 'with just the information given in the task, this is a tad too difficult - when one gets a map without elevations in the terrain, small paths etc'. Furthermore, expressing a concern for the participants of this task because '…the lack of terrain would be frustrating for participants.' and make this task 'uncomfortable to answer'. Task 7 was the only task that did not get specific comments in regards to confusing elements, but rather certain praise of its accessibility by the second respondent [B] who commented that '…the task seems easy to solve, although we do not get to know the width of the bridge'.

In total, the respondents made suggestions of procedures for solving three out of four tasks. Task 4 was the only one not addressed among the possible solutions. In task 3, the first respondent [A] addresses both the openness of the task and a possible method for solving. Commenting that '[t]he fact that one only got to know the width of the bridge adds up to the participant needing to make nice *mental leaps* in order to assume the length of the arc, which is required to calculate right volume needed to get the right estimate of rocks'. In task 5, the second respondent [B] is actually addressing the accessibility of the task [and its lack of certain information], when suggesting that '[i]t is possible to use Google Maps as an interactive tool instead of making mathematical calculations'. This is an example of a method of solving that does not rely on the use of mathematical competencies. In task 7, both respondents give similar suggestions on how to solve the problem. The first respondent [A] suggests that '…it is possible to estimate a width and from there calculate a surface area per meter of length (with uncertainties) and hence get a relatively clear image of how much area is going to be covered in wood stain'. Furthermore, this respondent [A] asks a question suggesting the possibility of several methods of estimating the required width; '[w]hat [do] they base their dimensions of; the person, the environment, or do they assume standard sizes of the planks?'. The second respondent [B] only addresses one course of solution, suggesting that '[a] possibility could be to assume the width of the bridge according to the person of the image'.

When asked if they [the respondents] had any final comments about the tasks, both respondents expressed concluding remarks that seemed to focus on the possibility of such tasks developing both creativity and confidence. The second respondent [B] wrote that 'I am certain that such tasks would not fit all the pupils… …However, the tasks do seem more exciting and more fun than *normal* mathematical tasks. Many pupils would probably find this interesting. Both because it demands more of their own efforts, and because it requires more creativity. Such tasks could probably fit better to pupils that wanted more challenge than the normal tasks'. Extending beyond the scope of this study, the first respondent [A] expresses an interest in possible future research. 'As an extension to this study, it would additionally be interesting to examine how participants of such tasks have constructed a more developed tool for analysing and studying real sizes, and not the least if they have gotten a confidence that says *this is something I can do!*'.

4.2.2 Discussion

The methods of solution that appears in these results are more or less the same as in the first stage of evaluation. This could point towards the tasks not being as *open* as intended despite the lack of strategic formulations and required key data. What is noticeable is that the tasks were never open ended, as they required a single quantitative value as their solution. However, the tasks had an open beginning allowing reasonable estimates and individual strategic procedures to play a part in the path to the solution. This is something the first respondent [A] exemplifies through arguments that there are several ways to estimate the width of the bridge in task 7. Yet, this does not explain why different people end up using the same methods on a seemingly open task. One implication could be that the solution occurring in both results are the simplest and most direct way of solving the task. That it is 'the path of least resistance', or in other words the most reasonable option. Another influence is the fact that the newly added information bases itself in the 'missing information' from the last test of the tasks. It could be that, by inserting this information, I have unintentionally removed the openness of the tasks.

The confusing aspects of these tasks were not as prominent as in the first stage of assessment. However, a few incidents occurred. The first of these were in regards to the use of a difficult word in task 3. It should be possible to circumvent the use of the difficult word, '*trunk road',* by using a slightly different word - such as '*main road'* or just *'road'*. The original wording had an underlying intention of teaching the pupils a particular type of specific knowledge, and in such a way 'prove' the usefulness of competencies learned through mathematics to the pupils. However, this did not work as intended on the respondents. Because of this, it is fair to assume that the pupils would not gain anything from it either. Secondly, the second respondent [B] expresses a desire for more information in all tasks. In some of the tasks, the first respondent [A] backs up these claims. For example, the first respondent [A] expresses the urgency of a more detailed map describing the area of task 5. Both respondents seems to be in unison that filling out more details of the context is crucial to the tasks. For the first respondent [A] this seems to relate primarily to the *realism* of the task, while for the second respondent [B] this seem to relate to the *accessibility* of the task.

Both respondents [A and B], as educators, expresses further interest in the wider context of how pupils respond to and are affected by this type of tasks. As the second respondent [B] suggest, 'these types of tasks would be a better fit for pupils that wanted more of a challenge; not all the pupils'.

From this suggestion alone, a question could arise about how different pupils with different mathematical skills and background responds to this type of tasks. Conversely, one could additionally question the impact of such tasks in relation to the pupils confidence within the subject and its relation to how the pupils perceive mathematics used outside of school - as suggested by the first respondent [A]. It is possible to assume some of the possible effects such tasks might have on pupils according to the study of Boaler (2001); where the pupils who had been working with modelling tasks [among other projects] saw the usefulness of mathematics outside of school (while those who had not, did not). However, both of the respondents' suggestions - and the implications that follows from them - are aspects beyond the scope of this study.

4.2.3 Semi-summative remarks

According to the results of the questionnaire, there were certain changes required. This was apparent both in task 3 and 5. In the first of these tasks, this change relates to a certain word. However, it was not the only problem of this task. While not addressed by the respondents, it was obvious to me that all of the tasks in this version had longer descriptions than they had during the initial prototype [because of the added specific information]. Longer tasks descriptions could be intimidating to the pupils, and it was essential to address this problem. In order to reduce such influences, I could replace the description of the tasks by adding additional imagery that would give the same information as the initial description. Even though this changes the format of the tasks [being one image and a task description], I had already passed that barrier by adding the map in task 5. Task 3 was the task with the longest description of them all. However, it was not possible to change the description using the aforementioned method without changing the fundamental dynamics of the task itself. It is also probable that the current version of the task were similar to task 4 in how pupils would have approached and solved it. Because of these factors, I discontinued task 3. As for the change in task 5, this was primarily concerning the lack of contextual information in the map. Fixing this, I only needed to change the old map into a new image that displayed more of the terrain and general environment around the bridge. However, there were another apparent 'problem' I wanted to attend. Namely, the curious case that the tasks still lead to the participants and respondents performing the same strategic choices in order to solve the tasks. In order to attend this, I changed the question of task 5 in an attempt to make it open ended - and to see if this would change the outcome of this trend.

The results further seem to imply some changes to the design principles, primarily the principle of *realism*. As the first respondent [A] seemed to be arguing, this relates to the amount of information given in certain tasks. In order for the context to be believable, a sufficient amount of information is required. This is similar to the 'transferability' trait of academic studies, and focuses on the pupil being able to both replicate EMK into the modelling process of real tasks, but also to be able to replicate the competences learned into the work with authentic problems in their everyday or professional life. However, one might ask 'what is a sufficient amount of information?'. Although, it is not certain at this point, it seem to being 'all the information you would have' when attempting to answer a particular question in that particular authentic context. To some extent, this varies with the correlation of the question and the context. For instance in task 4, the first respondent [A] proclaims that it is believable a construction engineer could have asked such a question. Considering this, it is not so strange that the first reaction to the participants of the initial try-out was to survey for construction documents - as this is something a construction engineer would have at disposal. The example represents the general idea of what could qualify as a 'sufficient amount of information'. In accordance with this, it is beneficial to apply a different method for approaching the task making process than the one previously applied [when making the prototypes of the tasks]. This method takes focus around the question to fit the context, rather than to put constraints on the context in order for the question to seem relevant. When considering 'what information is required to solve the task', it is perhaps better to consider 'what information would the person asking this question have at their disposal’. There is no reason to address the pupils’ common knowledge, but it is crucial to address the ‘additional’ specific knowledge of the context. Adapting the tasks [or creating additional ones] with this method makes it clear that it is affected by the principles of *openness* and *accessibility* as well. When taking consideration of the principle of *openness*, there should be a lack of key data and strategic formulations in the task. Because of this, it becomes important to consider the relationship between the question asked in the task and its context. If the answer to a question would already be at the disposal of the people in that context, then that question is not realistic.

5 Results

We conducted the field test during the last hours of lecture on a Monday. These hours were going to be in a different subject, but it was changed (to mathematics) a few days in advance because we noticed that the lecture I was originally assigned would fall away due to a mock exam. Some pupils seemed aware of this change, but clearly not everyone. Especially one of the pupils expressed contempt that we were going to do mathematics - and was clearly not interested in doing that. At some point later, I realized that the pupil who had previously expressed contempt had left the class. In general, it seemed fair to say that spirits were not high.

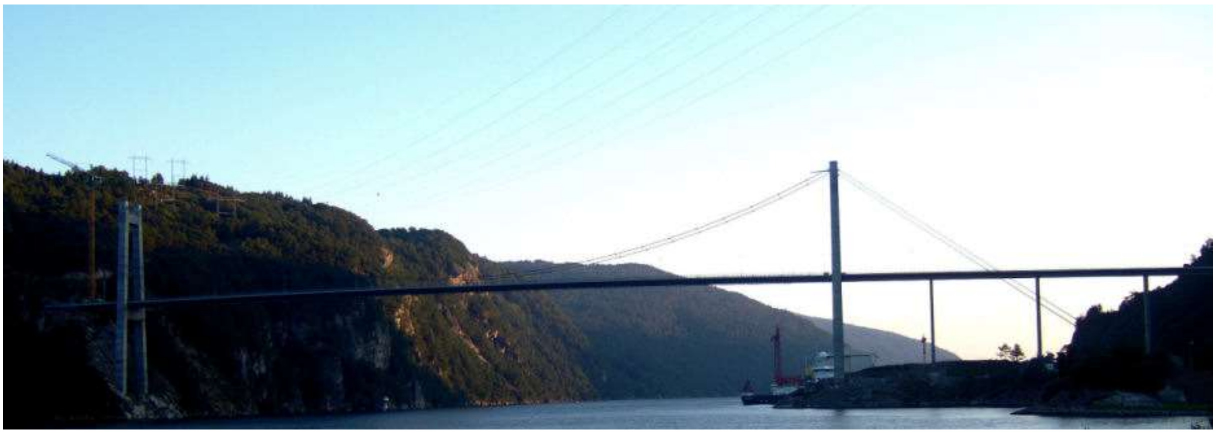
I determined the groups at random by attributing each pupil a number between one and five. This gave us four groups of four and one group of five. No group worked with more than one task because of the limited time available. We distributed the tasks among the groups as evenly as possible. Both the teacher and I talked with the pupils as they were working and, if necessary, offered help and advice. This was primarily to gain insights into their method of solution, but also to keep them active throughout the lecture. The teacher had not seen the final revision of the tasks before this lecture.

5.1 Task 4 - the Fedafjord Bridge

5.1.1 Details about the task

Picture 5.1: The final revision of task 4.

The final revision of the task about the Fedafjord Bridge introduces additional information about the 'free height' of the bridge and adds a second image while removing certain text. The additional image is supposed to present an alternative way of gathering the information required rather than to get it explicit - as were the case with the previous text. This is a measure to assert the task as 'different' in comparison with traditional word problems, and to reduce the amount of information in the task description. The description now asks 'how many meters of wire was required to build this bridge?' All the numerical details of the bridge is in an 'information box', which is separate from the task description.



Picture 5.2: The first image used in task 4 (Jarvin, 2006; retrieved from: https://commons.wikimedia.org/wiki/File:Fedafjordbridge2(Jarvin).jpg [15. February 2019]).

Picture 5.3: The second image used in task 4 (2010; retrived from: https://avisenagder.no/glatt-pa-fedafjorden-bru/19.9462 [8. April 2019]).

5.1.2 Results of the field test and interview

Two groups were working with this task and one pupil were available for an interview afterwards. Both groups were rather unmotivated to start working with the task, and required more encouragement than the other groups in order to initiate. An overall impression was that they used most of their time discussing other things than the tasks.

The pupil acknowledges the realism of the task	The pupil acknowledges the openness of the task	The pupil does not find the task accessible*
	1) Pythagoras' theorem 2) Qualified 'guessing'	Lack of information and strategic guidelines Image: lacking quality of the print Difficult word: 'free height'

Table 5.1: Summary of the final assessment of task 4.

During the field test of this task, none of the pupils invoked any extra-mathematical knowledge (EMK) when attempting to solve it. During the interview, as the pupil were explaining their work process in a bit more in detail, no additional references to EMK occurred either. When asked if the task was similar to previously experienced problems, the pupil initially denied without explaining. After further prying as whether the first response was in relation to real situations or mathematical tasks, the pupil explained that the response was in relation to previously experienced mathematical tasks. Further explaining that the task did not look like a mathematical task; '[w]hen you are doing mathematics, you most often have all the information written in the task that is required to solve it' [00:44]. I did not pursue why there was a lack of connection to the realistic aspects any further.

There were several confusing elements in this task. The first of these were the use of the word 'free height', which the pupils was not accustomed to using. The second confusing element was the lacking quality of the images in the print produced by my personal printer. Because of the lack of quality, both groups expressed that it was difficult to see the whole wire [in the first image] and to count the amount of parallel wires [in the second image]. There was a lot of confusion about the lack of information and clear strategic formulations in this task. Possibly even more so than in the other tasks, as it was mentioned four times during the interview. An example of the pupil [B] expressing a desire for more strategic formulations is in the excerpt from the transcription [below].

(translated from Norwegian)

[01:10] I: Could you have done this [referring to the task] again?

[01:11] B: Well, I could have done this again, but it is a bit difficult to… and to solve it like such when we do not know how to do it.

[01:20] I: Yes

[01:21] B: Yes, and such as showing formulas and stuff or something, then it would have been easier.

Two strategic approaches arose from the field test. The first of these were to use the Pythagoras's theorem. Although both groups considered this method, none of them actually ended up using it. Instead, they used the second method, which was a more implicit and arcane 'mathematical' method that appeared similar to guessing. What is interesting about this observation is that one of the groups did manage to get a much similar answer to the task as the one I myself got [when attempting to find possible solutions a day in advance]. They did not use Pythagoras's theorem, but I did. The interview gave some further insights when the pupil [B] responded to the second question of my interview guide (see Appendix).

(*translated from Norwegian*)

[01:42] I: Well, no. Could you describe how your group decided to solve the task? Or, you and your group - together - how you decided to solve the task?

[01:49]	B: Yeah. So we started to try and find some Pythagoras-options and such - for where the wires went downwards, but then it became a bit difficult and to determine the length where they touch down so… there was a lot of ca. and so. We took and estimated that it was so long that… went and calculated - from that information and then [unhearable].
[02:12]	I: Yes. There is going to be some assumptions… but did you end up going away from using Pythagoras's [theorem], or?
[02:19]	B: Yes.
[02:20]	I: Yeah [affirmative]. Was this because it got too difficult or because it took to much time?
[02:25]	B: No, I just do not think we were very motivated to struggle/bother with it.

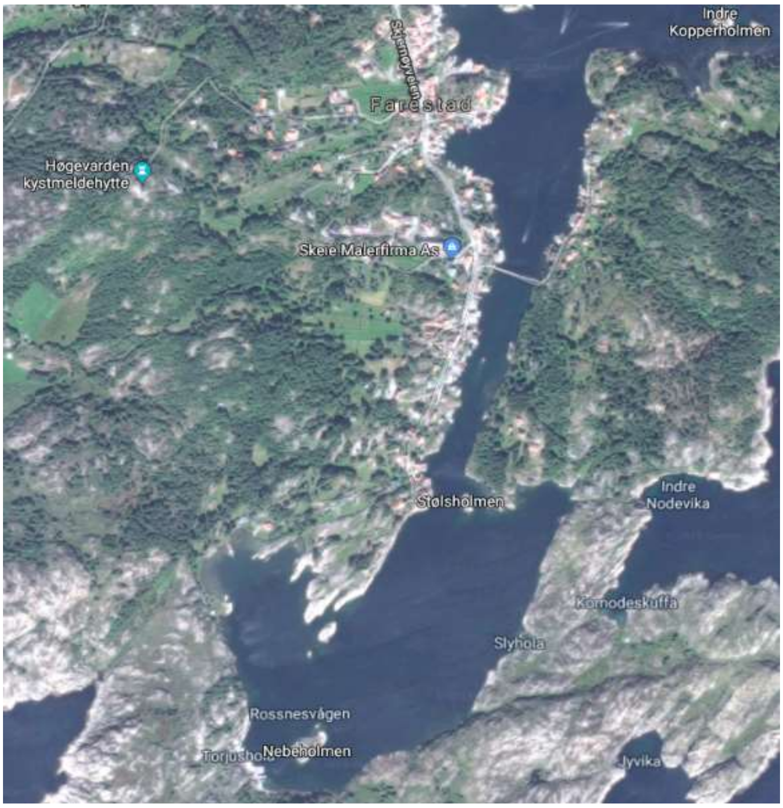
5.2 Task 5 - the Rossnes Bridge

5.2.1 Details about the task

Picture 5.4: The final revision of task 5.

Instead of the previous map, there is now a satellite image added to the task in order to give more information about the environment and terrain that surrounds the bridge. While the essence of the question, being about 'what if the bridge collapses', has not changed - the question now does not specify 'walking' as an option. Instead, the pupils may pick a preferred method of crossing according to their personal preferences. This makes the task more focused around the aspect of how the collapse would affect the inhabitants of this peninsula, rather than just being about calculating a time difference. Furthermore, by moving the numeric information in this task - the length of the bridge - from the text into a separate 'information box' it will presumably take less focus away from the nature of the task itself. It is also noticeable that the numeric information has been changed [from 110 meters to 76 meters], as it became apparent to me that it previously were wrong. Furthermore, to keep the task as open as possible, I had the information box block the view of the boat in the picture of the bridge. In this way, I tried to make sure that they would not immediately assume 'taking a boat' to be the superior option without arguing about the pros and cons of several other possible options as well. The final revision of the description reads: '*If the bridge would have collapsed, how much more time would it take the inhabitants to get to the other side?*'.

Picture 5.5: The first image used in task 5; the Rossnes Bridge during summer (Abrahamsen, 2007; retrieved from: https://no.wikipedia.org/wiki/Rossnesbroa#/media/File:Rossnes_bridge.jpg [15. February 2019]).



Picture 5.6: The second image used in task 5; a satellite image of the bridge and the area surrounding it (retrieved from Google Maps © [8. April 2019]).

5.2.2 Results of the field test and interviews

Two groups were working with this task during the field test, and two pupils were available for interviews afterwards. Both groups were highly active during the field test of the tasks, and discussed a lot. However, only a few pupils in each group took it upon themselves to design mathematical models to the task. This led to one of the pupils expressing a desire to change group during the session. The teacher denied this because the pupil otherwise got along seemingly well with the group.

The pupil acknowledges the realism of the task	The pupil acknowledges the openness of the task	The pupil does not find the task accessible*
Discussing elements of the authentic context around their preferred method Denying elements of fantasy Addressing 'real' people Association to 'similar' events Information: time it takes to swim 25 meters (experiences)	Discussing efficiency (time related to other factors) Applying time, stretch and speed formulas	

Table 5.2: Summary of the final assessment of task 5.

There were several instances where the pupils invoked extra-mathematical knowledge (EMK) both during the field test and during the interviews. The first of these instances occurred to both groups [separately] when they were discussing associations and knowledge about the context in order to decide on a model for the problem. They used much time to consider different aspects of each options; 'what if they are going swimming? How much time is it going to take? Are they going to take their clothes of before swimming? What about extra equipment or transporting items?' etc. One pupil [A] exemplified this during the interview and even mentioned there being certain difficulties with 'having too many options':

(*translated from Norwegian*)

[05:15] I: Mhm. I understand. Eh, well yeah! Finally - was there anything that was difficult with the task or - yeah, or things there?

[05:23] A: The thing I thought was most difficult with the task was in any case that… it is so much you can include to make the tasks even more difficult. Like here with the boarding and disembarking we had to consider if they were going to wear lifejackets or not, and to take the lifejackets on and off again - [and] if they were going to swim, if they were going to have time to take their clothes off and on again when they get back on the shore and such. Because there is many such [things] that can be included, as input, all the time - it is actually just your own choice about how difficult you want the task to be.

The second instance occurred later during the field test, as one of the groups had already decided on what model they wanted to use. While still discussing influencing aspects around this model, the discussion seemed to stray into the borders of fantasy as one pupil in particular began to argue about the concerns of sharks — urging the other group members into considering the risk of it attacking or overturning the boat. In response to this, the other group members argued by invoking EMK — saying that 'this is irrelevant; there are no big sharks in Norway'.

During the interview, when answering the first question, the pupils gave very different answers. The first pupil [A] responded with associating the task to familiar situations, while the second pupil [C] did not express any such associations. However, when later addressing the solution of the task, the second pupil [C] did noticeable refer to the people in the model as *we* on several occasions: '…we came with many suggestions to what we could do. It was both that we could… that *they* could swim across.' [01:28] and '…actually, we agreed to take a… sufficiently big enough boat. Then we could just have taken [a number of] people across at once' [02:03]. The associations of the pupil [A] does not seem connected to the context of the task, but rather the question of the task.

	(translated from Norwegian)
[00:56]	I: Uhm… so, uhm… the task - was it similar to problems or tasks that you have previously experienced?
[01:09]	A: Uh, are you thinking within mathematics classes then or?
[01:13]	I: Uh, both in mathematics and everyday life - actually.
[01:17]	A: Uhm… it actually does simply remind me of when I am on my way home - I take the bus often, because I am from [place] - so I take the bus back and forth all the time. So if I am at some point - if the bus gets an accident or something like that, then I have to calculate which… uh… what is fastest for me to… take taxi instead of how it would cost according to the rest of what I would have paid for a bus ticket.

Later in the interview, when addressing the first group's method of solution, the pupil [A] described previous experiences with swimming. Especially how they had, as a part of Physical Education, measured how fast they themselves could swim 25 meters. This knowledge was something that this group used when calculating how long time it would take to swim across the strait. Another option both groups considered was 'by taking a boat'. According this option, both pupils [A and C] expressed certainty that each boat could only hold a set number of people. While the second pupil [C] assumed this number to be somewhere around 20, 'as there probably was not that many people living on the peninsula', the first pupil [A] used the satellite image in the task and counted the number of houses [however difficult this might have been due to its bad resolution]. Further assuming that each house had four inhabitants (one father, one mother, and two children).

The pupils discussed several procedures and models during the field test. While the question in itself only asks about 'how much more time will it take for the inhabitants to get to the other side of the strait?', the predominant discussion among the pupils seemed to be laying elsewhere. Instead of choosing an option and calculating a solution, many of the pupils were arguing about what methods would be either quickest, cheapest, and/or easiest to conduct. The focus of the problem naturally digressed into a discussion about efficiency. Because of this, they quickly disregarded *walking* as a viable option; 'No one would walk around. It takes too much time'. In regards to this, both groups ended up with *boat travel* as one of their models of crossing the strait. In the second group, they assumed to have only a single boat because this would take the shortest amount of time [as exemplified in the excerpt below]. The first group did not specify if they considered the inhabitants to take one or several boats, but in the interview with the first pupil [A] it became apparent that they used previous mathematical knowledge of speed, distance, and time to solve the task. It is however not clear how the second group calculated their answer. The interview with the second pupil [C] provides little clarification.

	(translated from Norwegian)
[02:03]	C: Yeah - actually, we agreed to take a… sufficiently big boat. Then we could just have taken an X amount of people across at once. And it was quicker or - took shorter time than going back and forth with smaller boats.

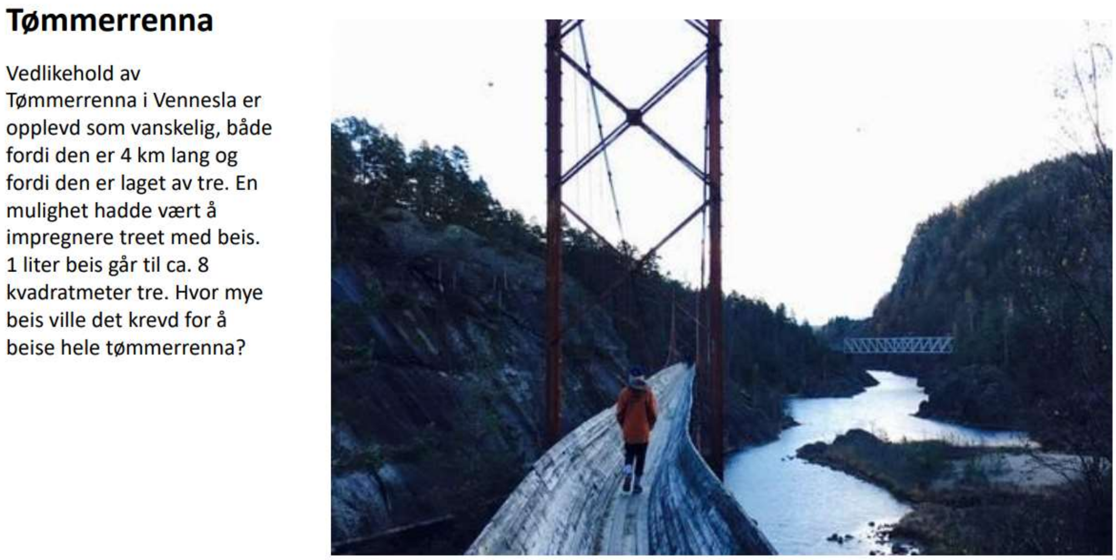
[02:24] I: Yes - did you use uh… or - what type of mathematics did you use when you solved - arrived at your answer?

[02:27] C: (Blowing air out of mouth) Yeah… what should I say to that? (Longer pause) Yeah… it was - we just calculated that - after looking at all the alternatives then uh, it was… easy to see that it was [the option] that took the shortest time. So we… maybe pretty quickly managed to calculate that it was easiest with both. So uh… I am not quite sure what to say to that - what we used, but…

None of the pupils [A or C] expressed any confusion about the task in relation to either the question or the pictures. I did not observe any confusion occurring during the field test beyond the initial confusion about the apparent lack of information in all tasks.

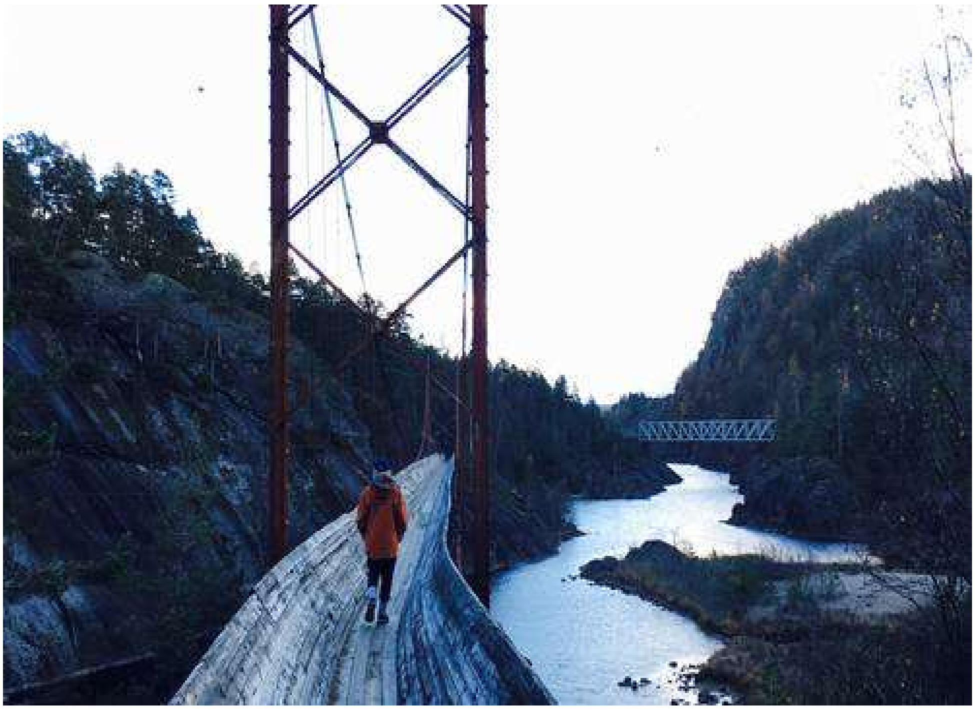
5.3 Task 7 - the old timber slide in Vennesla

5.3.1 Details about the final revision of the task



Picture 5.7: The final revision of task 7.

Even though there has been slight changes to the layout, this task is primarily the same as it was during its second prototype. This is to see if the pupils respond differently than the respondents of the expert appraisal. The current description reads: '*Maintenance of the old timber slide in Vennesla is experienced as difficult, both because it is 4 km long and because it is made of wood. An option could be to impregnate the wood with wood stain. 1 litre of wood stain goes to approximately 8 square meters of wood. How much wood stain would have been required to stain the whole timber slide?*



Picture 5.5: A person walking down the old timber slide (unkown; retrieved from https://prod.ut.no/tur/2.17290/ [15. February 2019]).

5.3.2 Results of the field test

Only one group worked with this task, and none of these pupils was available for an interview. This group expressed insecurities about their mathematical skills early on, referring to themselves as low achievers. Confusion later rose, as they did not remember how to produce square meters - which was an element in the task description. At this point, I helped them by handing them the formula [length x width] for calculating an area.

The pupil acknowledges the realism of the task	The pupil acknowledges the openness of the task	The pupil does not find the task accessible*
Information: comparison to own height	Geometry (calculating area in order to find the required volume)	Problematic term: square meters

Table 5.3: Summary of the final assessments of task 7.

During the field test, there were only one noticeable instance of extra-mathematical knowledge (EMK) being invoked. This was when the group was assuming the height and width of the bridge. They initially assumed the height of the bridge to be about 1.5 meters tall. However, at some point later, one of the pupils in the group said something along the lines of 'That can't be right. That is almost as tall as I am. It has to be shorter, look at the person in the image'. In regards to this remark, the group changed their assumption to a shorter length.

Using this estimated height, the group multiplied the collective width of the bridge [the sum of its sides and bottom] with its length. Through this operation, they found an area value they could use when finding an appropriate number of litres of wood stain. This estimate did assume the bridge both one-sided and flat, as they had neglected any influences by vertical surfaces of the objects. Towards the end of the field test, when I inquired if they had taken into account such influences when making their estimate — they responded that they had not.

6 Discussion

The development of the final revision of task 4 had two focus points. The first was about reducing the length of the task descriptions using imagery. The original picture displayed how the wires run the entire length of the bridge in a particular pattern. However, it did not display all of the wires as the the ones in front obscured the wires in the back. This was the reason why the extra information were in the task description to begin with. The second image shows the bridge from another angle and displays the eight wires of running along the edges of the bridge [four on each side]. Combining the information in both images, both groups working with this task were able to make good use of this - understanding that there were several similar wires running the entirety of the bridge. In their model, they accommodated for this by multiplying the length of one wire with the total number of wires (see Equation 6.1).

$$\frac{n\ [\text{meters}]}{1\ [\text{wire}]} \cdot k\ [\text{wires}] = nk\ [\text{meters}]$$

Equation 6.1

However, the two groups did not use the same number of wires: one group used eight and the other used ten. When I became aware of this, I inquired the group that used ten wires in their calculations about how they decided on this number. To answer my question, a pupil showed me the picture and explained that they had counted the number of wires. Knowing beforehand that there was only eight wires, I asked the pupil to count once more. The pupil did, and to my surprise managed to count to ten once more. This is where I became aware of a problem with the second image of the task. First, the quality of the print was less than ideal. What did originally look like a great image on the computer screen had now both low contrast and resolution once up-scaled to fit the A4 size of the task sheet. Secondly, depending on whether or not you looked at the left side of the bridge you can actually notice a number smaller wires running along the four heavy steel wires. These are difficult to see on the right side of the image, but on the left side - where there is a different angle and better contrast - they were far easier to notice. This is why, if you counted one side and multiplied it by two [to accommodate for both sides] you would get different numbers of wires. Eventually, this was not important; the purpose of the task was to have the pupils make estimations, argue about their strategic reasoning, and find give an accordingly reasonable answer - not to get the exact same answers. The second image might not have been ideal, but it did serve its purpose. The second focus point of the final revision of task 4 was to pose itself as different according to traditional mathematical tasks. The task managed this, according to the interview of the second pupil [B] who said that 'No [responding to whether or not the task seems similar to other math tasks], because when you take math then you often have that all the information is written in the task - that you need to solve that - in the task…' [00:44]. However, whether or not this was a good thing were disputable as the pupil [B] additionally gave an ambivalent response when I further inquired if this was an enjoyable trait of the task. It was different, for better or for worse, in comparison to the tasks that the pupil [B] were accustomed.

The final revision of task 5 also had some focus points. The first of these being the inclusion of a satellite image instead of a map in order to give a more complete picture of the area and environment surrounding the bridge. This was according to the updated principle of realism - and the idea that a rich context description makes the realistic aspects of the task more believable. The task did this quite well, looking at all the extra-mathematical knowledge (EMK) that the pupils invoked. Specifically the pupils 1) disregarded options because they were surrealistic [shark attacks or walking around], 2) imagined themselves in the events of the task, and 3) considered the effects of their strategies in respect of human lives (expression of empathy). Together, these incidents seem to distinguish the pupils' recognition of the task as believable and realistic. However, there were some disagreement between the two interviewed pupils [A and C] about whether the task were similar to problems they had met in their everyday life. Because of this, one might wonder if the task were actually as believable as it seemed. The first pupil [A] gave a positive response, comparing the problem to familiar everyday problems - ultimately vouching for the authenticity. On the other hand, the third pupil [C] disregarded my question quickly. Looking into this, there are some uncertainty in the meaning behind the third interviewed pupil's [C] answer due to the use of the word 'similar' and a lack of clarification by the pupil [C]. Neither less, because all pupils are unique and different from each other and there might be some differences in what they experiences as 'realistic', having a rich context is not counter intuitive. The second focus point of the final revision of task 5 was to assess how an open-ended task would affect the answers that the pupils would produce. While there were certainly differences in both groups' approach to the task, they ended up with choosing the same models: comparing the time it usually would take to cross the strait with the time it would take to either *swim* or *take a boat*. One of the pupils [A] argued that this was because these methods of crossing were the most intuitive ones: 'We solved the task by thinking out different methods to get across an ocean. When - as you can see on the map, you would have to travel all the way around just to get [over] in another way. Then the closest thing is to swim, [take a] boat, or… just - yeah, it is probably not many other options…' [02:41]. However, there is also the consideration of the influences of the teacher either directly or by working as a medium for suggestions and ideas to pass from one group to the other. The groups also were not inclined to work individually, and it could be that information passed between the groups directly. I am unsure to what degree the teacher or the groups affected each other during the modelling process, but one thing is certain: both groups ended up with choosing the same strategic methods. It would be interesting to see if another class would end up at the same decision, or if they would pick another solution. Furthermore, what is interesting with the final answer of both groups was that it involved the groups addressing an aspect that was not apparent in the question itself. While the question in itself only asked about 'how much more time would it take to get to the other side?', many pupils made the task more challenging for themselves by focusing on a second criterion that they had made up: 'which model is the most effective?'. This criterion seems based in the pupils' immersion, as they are displaying a consideration to problems beyond the scope of the question that would be crucial to the people of the context if the problem in the task were real.

The final revision of task 7 were the same as the second prototype of the task. This was because the experts did not assess there to be any confusing aspects of the task, and because it would be easier to make comparisons to the previous stages of formative assessment. When comparing the work of the different participants there were certain similarities of differences occurring. In the try-out of the initial prototype, the participants made use of the person in the image to determine an estimate of the height of the bridge. Making assumptions about that persons' height, and using her as a 'ruler' in order to measure the sizes of other objects in the picture. During the second stage of assessment, the respondents of the expert appraisal addressed this option as well - while also discussing additional strategies. Because of this, it was not so strange that in the final revision of the task the pupils applied the aforementioned method as well. This method has the format of applying common knowledge in order to estimate another specific type [of knowledge] that is required to solve the task. It is, perhaps, the most intuitive way of solving this task. Ultimately indicating that the *accessible* aspect works as intended. Furthermore, while their method was the same as in previous assessment stages, their argumentation was new. During the initial try-out of the tasks, the participants invoked the data without giving it much reason except that they 'knew' this information. In contrast, the pupils of the field test did not know such information beforehand. Instead, they made comparisons to themselves and argued accordingly. This gives to show that achieving and applying similar solutions does not necessarily equate a lack of openness in the task, as different argumentation may still lead to the same applied solutions. Furthermore, as previously discussed, the second prototype of the tasks were arguably more intimidating because of their longer task descriptions and increased use of numerical data in the text. This was the reason behind the changes to the final revision of task 4. Even though I did not get to interview any of the pupils that worked with this task [7], and hence cannot be completely sure, it seemed as if the length of the task description did not intimidate the pupils. Although there could be arguments that further the opposite agenda, considering the fact that they referred to themselves as ‘low-achievers’. They expressed this dismissive remark before being given the task - hence it is impossible to argue for a correlation. The members of the group did not complain once to me while working on the task. In the end, none of the tasks was particularly intimidating and the language used [with few exceptions] were understandable.

7 Conclusion

7.1 Principles of task design

Regarding the results of the summative assessment phase, as well as the previous formative ones, there have been several changes apparent in the design principles. In addition to the changes made to the previously addressed design principles, I will also suggest a 'new' design principle: *immersive*. This new principle adopts some of the attributes previously connected and discussed along with the realistic principle, and combines it with the *momentum of discussion* principle of realistic Fermi tasks by Ärlebäck (2009). There are certain reasons for this split, even though one could also argue for the aspects of this principle to continue as a part of the already existing ones. This these four characteristics of mathematical interventions based in a realistic Fermi task starting from an authentic infrastructure context.

Design principle	Description
Accessible	All representation used are understandable for all pupils, and the task can be solved at different levels of complexity.
Realistic	The task includes authentic aspects, and in such a way has a clear real world connection.
Open	The task lacks strategical information and certain necessary information in order to promote reasoning.
Immersive	The relationship between the context and question makes the task believable, and promotes discussion.

Table 7.1: A brief summary of the final revision of the principles of task design

The first principle is the *accessible* principle. This means that a task is approachable and understandable for all pupils. This includes that all representation used - such as language, imagery, or physical objects - is understandable as a whole, and that the task can be solved at different levels of complexity. In other words, the task has a low threshold and a high ceiling. It is not too intimidating either, by having a fine balance between too little and too much details in both its description and illustration. Furthermore, in accordance with the *open* principle, this means that the pupil needs to be able to solve the problem by making reasonable estimates without the need for additional extra-mathematical knowledge (EMK). Particularly if a task require specific information - that only a few pupils have - in order to address the main problem of the task, there has to be at least one method to for the pupils to acquire the specific information through another strategic operation using accessible [common] information. Put more simply: the task needs to be solvable by applying common information to it. An example of this can be to find the height of a bridge [which is difficult to assume on its own] by using the height of a person [easier to assume on its own].

The second principle is the *realistic* principle. For something to be realistic, it needs to have a clear connection to the real world. This means that the task contains one or more authentic aspects. An authentic aspect is any part of the tasks that have the following two criteria: 1) *it originally have an out-of-school context*, and 2) *it has a certification of authenticity*. A certification of authenticity may be anything from a physical proof [text, images, video etc.], an expert statement [professionals, scientists, etc.], or even the reactions of the participants themselves. Starting from an authentic infrastructure object of the real world, the context of the task will always be authentic as long as it is certifiable. In this study, because of the format of the tasks, the primary certification elements are the picture [of the bridges] and familiarity among the participants (due to the bridges presence in the local environment). If one had the time and resources available, it would also be possible to take the pupils physically to the bridges in order to work with tasks. The same can be said about other infrastructure objects in the local area. However, it is also important to recognize that an authentic context [alone] does not necessarily promote additional motivation or performance among the pupils.

The *open* principle is the third principle for creating mathematical interventions of this type. It means that the task lacks strategical information and certain required [key] data in order to solve the task. This principle relates to how the pupils are able to make meaningful choices for the task solving process and use their skills of evaluation, while additionally asserting what it means for a task to be a Fermi problem. The pupils needs to make reasonable estimates and critically assess different strategies in order to solve the task. In other words, the pupils need to ask themselves 'what' and 'why' they are using the numbers and models they use. Furthermore, this also aid the use of different solutions at different levels of complexity. We can notice that there are some overlap between the different principles - they are not mutually exclusive.

The final principle is the *immersive* principle. This means that a task is believable by the pupils, and that it promotes discussion. The purpose of this principle is to for the learning outcomes of the task to feel relevant for the pupils. This principle works together with the *realistic* principle, and directly addresses the possible lacking motivation among pupils [even though the task is realistic]. According to the results, what seems to motivate the pupils is how an intervention combines its problem to its context — and how rich the context feel. Allowing a task to make itself believable within the confines of reality. This lies in the correlation between the question and the description of the task. Does the question makes sense on its own? Does the task provide enough information that the question makes sense? Instead of putting strange and inauthentic confines on the premise of a task, it is better provide a sufficiently rich description that simulate the specific knowledge [as opposed to common knowledge] of a person asking the question [in that particular context] — allowing the pupils to immerse themselves. Immersion, by this definition, does not occur without the occurrence of the *realistic* principle. However, a task can be realistic and not immersive. Because of this, I have decided that it is best to separate these two principles [no matter how intertwined they are].

Together, these principles define some characteristics of a mathematical intervention based in a realistic Fermi task starting from an authentic infrastructure object in the pupil's region.

7.2 Tasks

Both task 5 and 7 performed well in the final assessment phase. While the pupils working with task 7 had fewer discussions than those of task 5, they still made some nice argumentation for making their assumptions — and they did produce a solution quicker. On the other hand, some of the pupils working with task 5 surpassed my expectations for both them and the task. Given that the previous versions of the task focuses on walking around, I was certain that someone would consider that as an option. I was wrong. Another interesting observation of the field test was how the format of the tasks [being different from traditional tasks] mixed the established patterns of 'high achieving' and 'low achieving' pupils. There was one pupil in particular, who had been giving detailed and reflected arguments all the way through the modelling process. When talking to this pupil, I was surprised to learn that this person was taking the practical mathematical course (P1) rather than the theoretical one. This is because the theoretical mathematics course (T1) is associated with 'higher achieving pupils' due to the topics that it teaches. These topics are similar to the scientific mathematics course (R1 & R2) that makes you eligible to study mathematics at higher education level. This is why most pupils with mathematical expertise prefer that subject. However, this pupil was not one of them — yet this pupil was the one making the most reasonable responses. Not a pupil from the theoretical mathematics course. This seem to correlate with the concerns of Skovsmose (2003), which says that new 'losers' and new 'winners' will emerge when you change the way we work with mathematics. Because of these reasons, I would think these tasks are almost ready for a possible compilation into the ‘mathbridges’ calendar. However, task 5 needs to merge the contextual information of both its images into one — as this were a premise of the calendar format. It would be possible to change this into one picture, by showing the bridge and its surroundings in a bird-eye perspective [looking away from the ocean].

Furthermore, another question arise — how do the final revision of the tasks perform according to the updated design principles? Are there eventually some things that needs to change in the tasks? When assessing both the tasks according to the principles, it becomes apparent that neither of the tasks are a fermi problems because of the added numerical data to the tasks. Changing this in task 5, the pupils would have to assess the width of the strait according to other objects in the area, not the bridge. However, this would arguably also make the task less accessible, but eventually demand more reasoning. There are both benefits and detrimental effects to this. On the other hand, if task 7 were to lose its numerical information it would be significantly more difficult to solve unless the required specific information can be acquired in another manner [pictures, physical objects, etc.]. Otherwise, both these tasks mostly satisfy the principles. Task 4, on the other hand, has proven that it is not immersive, and does no further the explicit application of EMK. Because of this, I would not include it into the compilation of authentic modelling problems started in local bridges of Agder County. Instead, it would probably be better to continue task 1 for this project, seeing as it had similar elements to that of task 7 during the initial development phase.

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Appendices

Appendix A: NSD Approval

NORSK SENTER FOR FORSKNINGSDATA

NSD sin vurdering

Prosjekttittel

Ferdigheter for den virkelige verden og noen kjennetegn for å promotere dem - et design studie

Referansenummer

453290

Registrert

21.03.2019 av Christoffer Haugland Paulshus - chrihp14@student.uia.no

Behandlingsansvarlig institusjon

Universitetet i Agder / Fakultet for teknologi og realfag / Institutt for matematiske fag

Prosjektansvarlig (vitenskapelig ansatt/veileder eller stipendiat)

Pauline Vos, pauline.vos@uia.no, tlf: 38142332

Type prosjekt

Studentprosjekt, masterstudium

Kontaktinformasjon, student

Christoffer Haugland Paulshus, chris.paulshus@live.no, tlf: 93866056

Prosjektperiode

01.04.2019 - 31.05.2019

Status

21.03.2019 - Vurdert

Vurdering (1)

21.03.2019 - Vurdert

Det er vår vurdering at behandlingen av personopplysninger i prosjektet vil være i samsvar med personvernlovgivningen så fremt den gjennomføres i tråd med det som er dokumentert i meldeskjemaet med vedlegg den 21.03.2019. Behandlingen kan starte.

MELD VESENTLIGE ENDRINGER
Dersom det skjer vesentlige endringer i behandlingen av personopplysninger, kan det være nødvendig å melde dette til NSD ved å oppdatere meldeskjemaet. Før du melder inn en endring, oppfordrer vi deg til å lese om hvilke type endringer det er nødvendig å melde:

https://nsd.no/personvernombud/meld_prosjekt/meld_endringer.html

Du må vente på svar fra NSD før endringen gjennomføres.

TYPE OPPLYSNINGER OG VARIGHET
Prosjektet vil behandle alminnelige kategorier av personopplysninger frem til 31.05.2019

LOVLIG GRUNNLAG
Prosjektet vil innhente samtykke fra de registrerte til behandlingen av personopplysninger. Vår vurdering er at prosjektet legger opp til et samtykke i samsvar med kravene i art. 4 og 7, ved at det er en frivillig, spesifikk, informert og utvetydig bekreftelse som kan dokumenteres, og som den registrerte kan trekke tilbake. Lovlig grunnlag for behandlingen vil dermed være den registrertes samtykke, jf. personvernforordningen art. 6 nr. 1 bokstav a.

PERSONVERNPRINSIPPER
NSD vurderer at den planlagte behandlingen av personopplysninger vil følge prinsippene i personvernforordningen om:
- lovlighet, rettferdighet og åpenhet (art. 5.1 a), ved at de registrerte får tilfredsstillende informasjon om og samtykker til behandlingen

- formålsbegrensning (art. 5.1 b), ved at personopplysninger samles inn for spesifikke, uttrykkelig angitte og berettigede formål, og ikke behandles til nye, uforenlige formål
- dataminimering (art. 5.1 c), ved at det kun behandles opplysninger som er adekvate, relevante og nødvendige for formålet med prosjektet
- lagringsbegrensning (art. 5.1 e), ved at personopplysningene ikke lagres lengre enn nødvendig for å oppfylle formålet

DE REGISTRERTES RETTIGHETER
Så lenge de registrerte kan identifiseres i datamaterialet vil de ha følgende rettigheter: åpenhet (art. 12), informasjon (art. 13), innsyn (art. 15), retting (art. 16), sletting (art. 17), begrensning (art. 18), underretning (art. 19), dataportabilitet (art. 20).
NSD vurderer at informasjonen om behandlingen som de registrerte vil motta oppfyller lovens krav til form og innhold, jf. art. 12.1 og art. 13.
Vi minner om at hvis en registrert tar kontakt om sine rettigheter, har behandlingsansvarlig institusjon plikt til å svare innen en måned.

FØLG DIN INSTITUSJONS RETNINGSLINJER
NSD legger til grunn at behandlingen oppfyller kravene i personvernforordningen om riktighet (art. 5.1 d), integritet og konfidensialitet (art. 5.1. f) og sikkerhet (art. 32).
Dersom du benytter en databehandler i prosjektet må behandlingen oppfylle kravene til bruk av databehandler, jf. art 28 og 29.

For å forsikre dere om at kravene oppfylles, må dere følge interne retningslinjer og/eller rådføre dere med behandlingsansvarlig institusjon.

OPPFØLGING AV PROSJEKTET
NSD vil følge opp ved planlagt avslutning for å avklare om behandlingen av personopplysningene er avsluttet.

Lykke til med prosjektet!

NSD - Personverntjenester
Tlf.: 55 58 21 17 (tast 1)

Appendix B: Project information and approval forms

Initial try-out

Vil du delta i forskningsprosjektet

"Ferdigheter for den virkelige verden og noen kjennetegn for å promotere dem - et design studie"?

Dette er et spørsmål til deg om å delta i et forskningsprosjekt hvor formålet er å finne kjennetegn ved mer virkelighetsnære og relevante matematikkoppgaver. I dette skrivet gir vi deg informasjon om målene for prosjektet og hva deltakelse vil innebære for deg.

Formål
Noen elever synes at matematikkundervisningen kan være uforståelig og irrelevant for matematikken som blir brukt i det daglige liv. Ettersom skolen har som jobb å forberede elever for samfunnet så er dette direkte motvirkende mot institusjonens mål. Det finnes allerede nok forskning på kjennetegn ved matematikkundervisning som skal fremme læringen av «ren» matematiske kunnskap og ferdigheter. Derimot finnes det overraskende lite forskning på kjennetegn ved matematikkundervisning som skal lære elever andre ferdigheter - som bl.a. kritisk tenkning og kreativitet.

Denne masteroppgaven skal prøve å definere noen slike kjennetegn:

> «Hva er noen kjennetegn ved en matematisk intervensjon for å promotere ferdigheter fra den virkelige verden hos norske elever ved videregående skole?»

Hvem er ansvarlig for forskningsprosjektet?
Masteroppgaven skrives av Christoffer H. Paulshus, og er veiledet av professor Pauline Vos ved Universitetet i Agder (UiA), institutt for matematiske fag.

Hvorfor får du spørsmål om å delta?
Som en masterstudent i matematikkfaget med pedagogisk bakgrunn er du antatt å ha nødvendig teoretisk og praktisk ekspertise til å arbeide med, og evaluere arbeidet rundt, intervensjoner i matematikkundervisning. Jeg ønsker å få innspill fra studenter med slik kompetanse og derfor blir du spurt.

Hva innebærer det for deg å delta?
Hvis du velger å delta i prosjektet, innebærer det at du arbeider med et sett matteoppgaver i en gruppe sammen med 1 - 2 andre personer. Disse oppgavene skal ta ca. 60 minutter å fullføre. Arbeidet deres med oppgavene vil bli observert og et sett feltnotater vil bli nedskrevet.

Det er frivillig å delta
Det er frivillig å delta i prosjektet. Hvis du velger å delta, kan du når som helst trekke samtykke tilbake uten å oppgi noen grunn. Alle opplysninger om deg vil da bli anonymisert. Det vil ikke ha noen negative konsekvenser for deg hvis du ikke vil delta eller senere velger å trekke deg.

Ditt personvern – hvordan vi oppbevarer og bruker dine opplysninger
Vi vil bare bruke opplysningene om deg til formålene vi har fortalt om i dette skrivet. Vi behandler opplysningene konfidensielt og i samsvar med personvernregelverket.

- Ingen navn eller personidentifiserende opplysninger vil bli nedskrevet i feltnotatet, og vil heller ikke fremkomme i den ferdigstilte masteroppgaven.

- Kun prosjektansvarlig - C. H. Paulshus (student) og P. Vos (veileder) - vil ha tilgang til feltnotatet.

Hva skjer med opplysningene dine når vi avslutter forskningsprosjektet?
Prosjektet skal etter planen avsluttes 31. mai 2019. Ved prosjektslutt skal det opprinnelige feltnotatet destrueres, men en transkribering blir vedlagt masteroppgaven for validering og eventuell videre forskning.

Dine rettigheter
Så lenge du kan identifiseres i datamaterialet, har du rett til:
- innsyn i hvilke personopplysninger som er registrert om deg,
- å få rettet personopplysninger om deg,
- få slettet personopplysninger om deg,
- få utlevert en kopi av dine personopplysninger (dataportabilitet), og
- å sende klage til personvernombudet eller Datatilsynet om behandlingen av dine personopplysninger.

Hva gir oss rett til å behandle personopplysninger om deg?
Vi behandler opplysninger om deg basert på ditt samtykke.

På oppdrag fra Universitetet i Agder har NSD – Norsk senter for forskningsdata AS vurdert at behandlingen av personopplysninger i dette prosjektet er i samsvar med personvernregelverket.

Hvor kan jeg finne ut mer?
Hvis du har spørsmål til studien, eller ønsker å benytte deg av dine rettigheter, ta kontakt med:
- UiA - Universitetet i Agder, institutt for matematiske fag, på epost (pauline.vos@uia.no) eller telefon: 38 14 23 32. Eventuelt ta kontakt med student Christoffer H. Paulshus, på epost (chris.paulshus@live.no) eller telefon: 93 86 60 56.
- Vårt personvernombud: Ina Danielsen, på epost (ina.danielsen@uia.no) eller telefon: 45 25 44 01
- NSD – Norsk senter for forskningsdata AS, på epost (personverntjenester@nsd.no) eller telefon: 55 58 21 17.

Med vennlig hilsen

Pauline Vos (Forsker/veileder) Christoffer Haugland Paulshus

Samtykkeerklæring

Jeg har mottatt og forstått informasjon om prosjektet «ferdigheter for den virkelige verden og noen kjennetegn for å promotere dem - en design studie», og har fått anledning til å stille spørsmål. Jeg samtykker til:

- ☐ å delta i utprøving av oppgavene, og da å bli observert mens et sett anonymiserte notater blir nedskrevet om arbeidet.

Jeg samtykker til at mine opplysninger behandles frem til prosjektet er avsluttet, ca. 31. mai 2019

(Signert av prosjektdeltaker, dato)

Expert appraisal

Vil du delta i forskningsprosjektet

"Ferdigheter for den virkelige verden og noen kjennetegn for å promotere dem - et design studie"?

Dette er et spørsmål til deg om å delta i et forskningsprosjekt hvor formålet er å finne kjennetegn ved mer virkelighetsnære og relevante matematikkoppgaver. I dette skrivet gir vi deg informasjon om målene for prosjektet og hva deltakelse vil innebære for deg.

Formål

Noen elever synes at matematikkundervisningen kan være uforståelig og irrelevant for matematikken som blir brukt i det daglige liv. Ettersom skolen har som jobb å forberede elever for samfunnet så er dette direkte motvirkende mot institusjonens mål. Det finnes allerede nok forskning på kjennetegn ved matematikkundervisning som skal fremme læringen av «ren» matematiske kunnskap og ferdigheter. Derimot finnes det overraskende lite forskning på kjennetegn ved matematikkundervisning som skal lære elever andre ferdigheter - som bl.a. kritisk tenkning og kreativitet.

Denne masteroppgaven skal prøve å definere noen slike kjennetegn:

> «Hva er noen kjennetegn ved en matematisk intervensjon for å promotere ferdigheter fra den virkelige verden hos norske elever ved videregående skole?»

Hvem er ansvarlig for forskningsprosjektet?

Masteroppgaven skrives av Christoffer H. Paulshus, og er veiledet av professor Pauline Vos ved Universitetet i Agder (UiA), institutt for matematiske fag.

Hvorfor får du spørsmål om å delta?

Du får spørsmål om å delta på grunn av din teoretiske og praktiske ekspertise til å arbeide med, og evaluere arbeidet rundt, intervensjoner i matematikkundervisning.

Hva innebærer det for deg å delta?

Hvis du velger å delta i prosjektet, innebærer det at du vurderer et sett matteoppgaver i henhold til et spørreskjema. Det er antatt at det vil ta deg ca. 45 minutter å besvare spørsmålene. Både spørreskjemaet og eventuelle tilhørende notater vil bli samlet inn.

Det er frivillig å delta

Det er frivillig å delta i prosjektet. Hvis du velger å delta, kan du når som helst trekke samtykke tilbake uten å oppgi noen grunn. Alle opplysninger om deg vil da bli anonymisert. Det vil ikke ha noen negative konsekvenser for deg hvis du ikke vil delta eller senere velger å trekke deg.

Ditt personvern – hvordan vi oppbevarer og bruker dine opplysninger

Vi vil bare bruke opplysningene om deg til formålene vi har fortalt om i dette skrivet. Vi behandler opplysningene konfidensielt og i samsvar med personvernregelverket.

- Ingen navn eller personidentifiserende opplysninger vil fremkomme i den ferdigstilte masteroppgaven.
- Kun prosjektansvarlig - C. H. Paulshus (student) og P. Vos (veileder) - vil ha tilgang til feltnotatet.

Hva skjer med opplysningene dine når vi avslutter forskningsprosjektet?
Prosjektet skal etter planen avsluttes 31. mai 2019. Ved prosjektslutt skal det opprinnelige spørreskjemaet (og tilhørende notater) destrueres, men en transkribering blir vedlagt masteroppgaven for validering og eventuell videre forskning.

Dine rettigheter
Så lenge du kan identifiseres i datamaterialet, har du rett til:

- innsyn i hvilke personopplysninger som er registrert om deg,
- å få rettet personopplysninger om deg,
- få slettet personopplysninger om deg,
- få utlevert en kopi av dine personopplysninger (dataportabilitet), og
- å sende klage til personvernombudet eller Datatilsynet om behandlingen av dine personopplysninger.

Hva gir oss rett til å behandle personopplysninger om deg?
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På oppdrag fra Universitetet i Agder har NSD – Norsk senter for forskningsdata AS vurdert at behandlingen av personopplysninger i dette prosjektet er i samsvar med personvernregelverket.

Hvor kan jeg finne ut mer?
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- Vårt personvernombud: Ina Danielsen, på epost (ina.danielsen@uia.no) eller telefon: 45 25 44 01
- NSD – Norsk senter for forskningsdata AS, på epost (personverntjenester@nsd.no) eller telefon: 55 58 21 17.

Med vennlig hilsen

Pauline Vos (Forsker/veileder)	Christoffer Haugland Paulshus

Samtykkeerklæring

Jeg har mottatt og forstått informasjon om prosjektet «ferdigheter for den virkelige verden og noen kjennetegn for å promotere dem - en design studie», og har fått anledning til å stille spørsmål. Jeg samtykker til:

- ☐ å besvare spørreundersøkelsen, og at min besvarelse blir samlet inn når jeg har svart.

Jeg samtykker til at mine opplysninger behandles frem til prosjektet er avsluttet, ca. 31. mai 2019

(Signert av prosjektdeltaker, dato)

Final assessment phase

Vil du delta i forskningsprosjektet

"Ferdigheter for den virkelige verden og noen kjennetegn for å promotere dem - et design studie"?

Dette er et spørsmål til deg om å delta i et forskningsprosjekt hvor formålet er å finne kjennetegn ved mer virkelighetsnære og relevante matematikkoppgaver. I dette skrivet gir vi deg informasjon om målene for prosjektet og hva deltakelse vil innebære for deg.

Formål

Noen elever synes at matematikkundervisningen kan være uforståelig og irrelevant for matematikken som blir brukt i det daglige liv. Ettersom skolen har som jobb å forberede elever for samfunnet så er dette direkte motvirkende mot institusjonens mål. Det finnes allerede nok forskning på kjennetegn ved matematikkundervisning som skal fremme læringen av «ren» matematiske kunnskap og ferdigheter. Derimot finnes det overraskende lite forskning på kjennetegn ved matematikkundervisning som skal lære elever andre ferdigheter - som bl.a. kritisk tenkning og kreativitet.

Denne masteroppgaven skal prøve å definere noen slike kjennetegn:

> «Hva er noen kjennetegn ved en matematisk intervensjon for å promotere ferdigheter fra den virkelige verden hos norske elever ved videregående skole?»

Hvem er ansvarlig for forskningsprosjektet?

Masteroppgaven skrives av Christoffer H. Paulshus, og er veiledet av professor Pauline Vos ved Universitetet i Agder (UiA), institutt for matematiske fag.

Hvorfor får du spørsmål om å delta?

Du får spørsmål om å delta siden du er en elev ved videregående skole, som er målgruppen til prosjektet. Dine innspill er viktige for å undersøke hvordan oppgavene fungerer i praksis.

Hva innebærer det for deg å delta?

Hvis du velger å delta i prosjektet, innebærer det at du arbeider med et sett matteoppgaver i en gruppe sammen med 2 - 4 andre elever og stiller deg disponibel til et intervju omhandlende hvordan din gruppe løste oppgavene. Oppgavene skal ta ca. 60 minutter å fullføre. Arbeidet deres med oppgavene vil bli observert og et sett feltnotater vil bli nedskrevet.

Kun 3 elever vil bli valgt ut til et delvis strukturert intervju. Intervjuet skal vare i ca. 15 minutter og det vil det bli tatt lydopptak. Det er mulig å bare ta del i oppgavejobbingen, men å ikke stille disponibel til intervju, dersom dette er ønskelig.

Det er frivillig å delta

Det er frivillig å delta i prosjektet. Hvis du velger å delta, kan du når som helst trekke samtykke tilbake uten å oppgi noen grunn. Alle opplysninger om deg vil da bli anonymisert. Det vil ikke ha noen negative konsekvenser for deg hvis du ikke vil delta eller senere velger å trekke deg.

Ditt personvern – hvordan vi oppbevarer og bruker dine opplysninger

Vi vil bare bruke opplysningene om deg til formålene vi har fortalt om i dette skrivet. Vi behandler opplysningene konfidensielt og i samsvar med personvernregelverket.

- Ingen navn eller personidentifiserende opplysninger vil fremkomme i den ferdigstilte masteroppgaven.
- Kun prosjektansvarlig - C. H. Paulshus (student) og P. Vos (veileder) - vil ha tilgang til feltnotatet.

Hva skjer med opplysningene dine når vi avslutter forskningsprosjektet?
Prosjektet skal etter planen avsluttes 31. mai 2019. Ved prosjektslutt skal det opprinnelige feltnotatet og lydopptaket destrueres, men anonymiserte transkriberinger blir vedlagt masteroppgaven for validering og eventuell videre forskning.

Dine rettigheter
Så lenge du kan identifiseres i datamaterialet, har du rett til:
- innsyn i hvilke personopplysninger som er registrert om deg,
- å få rettet personopplysninger om deg,
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- få utlevert en kopi av dine personopplysninger (dataportabilitet), og
- å sende klage til personvernombudet eller Datatilsynet om behandlingen av dine personopplysninger.

Hva gir oss rett til å behandle personopplysninger om deg?
Vi behandler opplysninger om deg basert på ditt samtykke.

På oppdrag fra Universitetet i Agder har NSD – Norsk senter for forskningsdata AS vurdert at behandlingen av personopplysninger i dette prosjektet er i samsvar med personvernregelverket.

Hvor kan jeg finne ut mer?
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- Vårt personvernombud: Ina Danielsen, på epost (ina.danielsen@uia.no) eller telefon: 45 25 44 01
- NSD – Norsk senter for forskningsdata AS, på epost (personverntjenester@nsd.no) eller telefon: 55 58 21 17.

Med vennlig hilsen

Pauline Vos
(Forsker/veileder)

Christoffer Haugland Paulshus

Samtykkeerklæring

Jeg har mottatt og forstått informasjon om prosjektet «ferdigheter for den virkelige verden og noen kjennetegn for å promotere dem - en design studie», og har fått anledning til å stille spørsmål. Jeg samtykker til:

- ☐ å delta i utprøving av oppgavene, og da å bli observert mens et sett anonymiserte notater blir nedskrevet om arbeidet.
- ☐ å stille disponibel til intervju hvor det blir tatt lydopptak, og diskutere oppgavene.

Jeg samtykker til at mine opplysninger behandles frem til prosjektet er avsluttet, ca. 31. mai 2019

(Signert av prosjektdeltaker, dato)

Appendix C: Template for taking field notes

Field Note: Participants: Date:

	The students invoke **real world knowledge** / extra-mathematical knowledge	The students assesses **methods of solving** the task	The students expresses **confusion** about the task	Personal feelings and analytic thoughts about the observations
Task 1				
Task 2				
Task 3				

Appendix D: Self-administered questionnaire

English

Expert Appraisal
The following tasks (see the handout) are being designed for use in Norwegian upper high school (vigeregående skole). As a part of this design process, I need to evaluate its expected practicality and relevance. For this reason, I appreciate constructive feedback from experts with mathematical and pedagogical knowledge - such as yourself. Thank you for taking part in this project.

1. Examine the tasks. Evaluate how each of them relate to the following criteria:

   a) ***Accessibility***
   The task is accessible for the students. This primarily means the task is understandable as a whole (language used, imagery, etc.) and in no need of further clarification. Additionally, this includes that the task does not need additional specific extra-mathematical knowledge beyond what the students should have - or what they should be able to estimate within reason (common sense).

   b) ***Realism***
   The task has a clear connection to the daily lives of people. This means that either the context of the task - or the question posed [by the task] - is perceivable as authentic/real by the students.

   c) ***Openness***
   The task is open. This means that several different strategies may be applied to solve the task, and the task does neither have a definite answer.

2. Evaluate if you would use the tasks on high school students. Explain why or why not:

3. Other comments about the tasks (if any):

Ekspertvurdering
Følgende oppgaver (se oppgavearket) er utformet for bruk i videregående skole. Som en del av den videre designprosessen trengs det informasjon om oppgavenes forventede funksjonalitet og relevans. Av denne grunn søker jeg [konstruktiv] tilbakemelding fra eksperter med matematisk og pedagogisk kunnskap. Takk for at du tar deg tid til å delta i dette prosjektet.

1. Undersøk oppgavene. Vurder hvordan hver av dem relateres til følgende kriterier:

   a) ***Tilgjengelighet***
   Oppgaven er tilgjengelig for elevene. Dette betyr i hovedsak at oppgaven er forståelig som en helhet (språk brukt, bilder, osv.) og at den er uten behov for ytterligere avklaring. I tillegg innebærer dette også at oppgaven gir tilstrekkelig [spesifikk] ekstramatematisk kunnskap utover hva elevene allerede har - eller hva de skal kunne tilnærme seg innenfor rimelighetens grenser (sunn fornuft).

   b) ***Realisme***
   Oppgaven har en tydelig forbindelse til dagliglivet. Dette betyr at enten konteksten til oppgaven eller spørsmålet som er oppgitt [av oppgaven] kan oppfattes som autentisk av studentene.

   c) Åpenhet
   Oppgaven er åpen. Dette betyr at flere forskjellige strategier kan brukes for å løse oppgaven, og oppgaven har heller ikke et bestemt svar.

2. Vurdere om du ville ha brukt oppgavene til undervisning på videregående skole. Forklar hvorfor [eller hvorfor ikke]:

3. Andre kommentarer om oppgavene (hvis du har noen):

Appendix E: Interview guide

Intervjuguide

Spørsmålene (se spørsmål 1 - 3) gis i henhold til hver oppgave så langt det lar seg gjøres; hvert intervju skal holdes innenfor 10 - 15 minutter. Intervjuet er delvis strukturert, og jeg kommer til å oppfølge besvarelser dersom de virker interessante for prosjektet eller om det virker nødvendig med ytterligere forklaring (se forslag til utdypningsspørsmål a, b, …).

1. Var oppgaven lignende på problemer du har møtt tidligere?
    a. I matematikkundervisningen?
    b. I ditt daglige liv?
2. Kan du beskrive hvordan gruppen din løste oppgaven?
    a. Hvordan begynte dere på oppgaven?
    b. Hvorfor valgte dere å løse oppgaven på denne måten?
3. Var det ting som var vanskelig med oppgaven?
    a. Var det noe du ikke forstod?

Appendix F: Tasks

Initial prototype

Bakke bru i Flekkjord er Norges eldste hengebro. Den ble åpnet i 1844. Fra 1950-årene ble busspasasjerer nektet å bli kjørt over broen, pga. at bruen ikke tålte denne samlede tyngden. Hvor mange mennesker tåler broen?

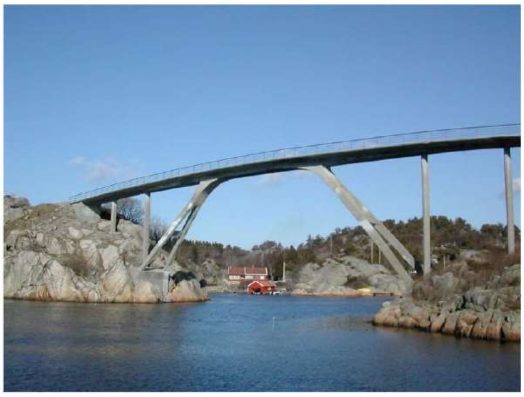
Under oppsettingen av ny sykkelvei, raste deler av Bankebroa i Mandal sammen februar 2018. Hvor mye penger gikk tapt?

Dorga bro i Sirdal kommune er en hvelvbro som krysser Dorgefossen. I 1972 ble den utvidet til 4 meters bredde, noe de fleste moderne bilister fremdeles synes er for trangt. Istedenfor å kjøre to i bredden, velger de fleste å stoppe før broen og slippe motgående trafikk forbi. Dersom man skulle utbygd broen til standard veibredde, hvor mange steinblokker hadde man trengt?

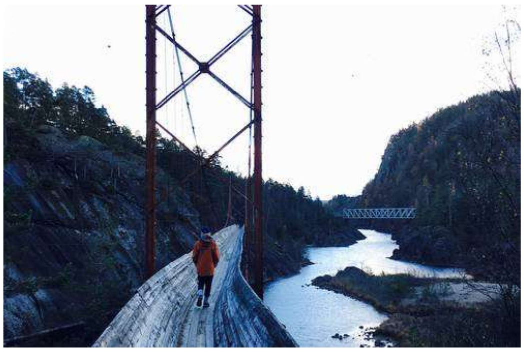
Fedafjorden bru er en hengebro over Fedafjorden. Den ble ferdigstilt i 2006. Hvor mange meter vaier ble brukt til broen?

Rossnesbrua er en del av rosnesveien på Skjernøya i Mandal. Vei går mellom Rossnes og Ytre Farestad. Hvor mye tid sparer man på å ta denne broen istedenfor å gå rundt?

Skjernøysund bru binder sammen Skjernøy med fastlandet i Mandal. Hvor høye er pillarene til denne broa?



Vedlikehold av Tømmerrenna i Vennesla ble opplevd som ekstra vanskelig når det var tørke sommeren 2018. En vedlikeholdsmulighet hadde vært å impregnere treet med beis. Hvor mye beis kreves for å beise hele tømmerrenna?



II prototype

Oppgave 1

Dorga bro i Sirdal kommune ble i 1972 utvidet til 7 meters bredde, noe de fleste moderne bilister fremdeles synes er for trangt. Istedenfor å kjøre to i bredden, velger de fleste å stoppe før broen og slippe motgående trafikk forbi. Dersom man skulle utbygd broen til 8,5 meter, vanlig minstebredde på stamvei, hvor mye stein hadde man trengt? Steintypene brukt i konstruksjonen veier ca. 3 tonn per kubikkmeter.

Oppgave 2

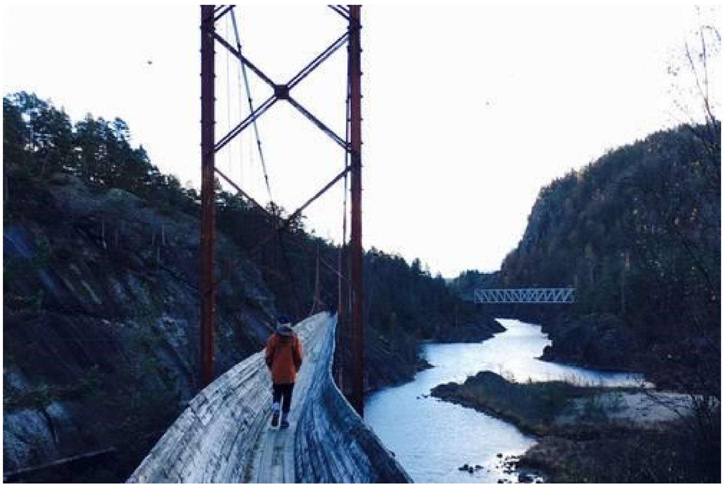
Fedafjorden bru er en 566 meter lang hengebro over Fedafjorden. 8 tykke vaiere (4 på hver side) går langs hele broen og stabiliserer den. Hvor mange meter vaier ble brukt til å bygge broen?

Oppgave 3

Rosnesbrua er en 110 meter lang gangbro. Den er en del av rosnesveien på Skjernøya i Mandal. Dersom broen skulle falt sammen, hvor mye ekstra tid ville det tatt å gå rundt?

Oppgave 4

Vedlikehold av Tømmerrenna i Vennesla er opplevd som vanskelig, både fordi den er 4 km lang og fordi den er laget av tre. En mulighet hadde vært å impregnere treet med beis. 1 liter beis går til ca. 8 kvadratmeter tre. Hvor mye beis ville det krevd for å beise hele tømmerrenna?



Rossnesbrua

Dersom broen hadde falt sammen, hvor mye lengre tid ville det tatt for beboerne å komme seg til den andre siden?

Broens lengde: 110 meter

Fedafjorden bru

Hvor mange meter vaier måtte til for å bygge denne broen?

Lengde: 566 meter
Fri høyde: 50 meter

Tømmerrenna

Vedlikehold av Tømmerrenna i Vennesla er opplevd som vanskelig, både fordi den er 4 km lang og fordi den er laget av tre. En mulighet hadde vært å impregnere treet med beis. 1 liter beis går til ca. 8 kvadratmeter tre. Hvor mye beis ville det krevd for å beise hele tømmerrenna?

Appendix G: Field notes

Field Note #1
Date: 15.2.2019
Stage of development process: first assessment - development phase (try out)
Participants: 2 students

General

Enter a large room at our university. The sun was shining outside, and natural lighting lit up the room. We had just participated in a similar study, and everyone were eager to work with the tasks.

Before beginning, I stated the importance of working and talking together - so that I could get insight into their thoughts. Additionally, I explicitly stated that I would be taking field notes - while sitting straight above them, at the same table they are working at. When necessary, I was open for questions and other interactions. This did not seem to make them overly self-conscious or otherwise affect them negatively.

They had computers at hand during all tasks. Before beginning, I suggested that they might need these - as I emphasized the importance of using the internet (to collect data) or otherwise make reasonable assumptions.

Task 1 - Bakke Bru

The participants expressed an initial confusion about the wording of the problem - asking 'why it was so that this restriction only applied to buses and not all heavy transportation' and 'if the problem were asking about how many people the bridge could hold in addition to the bus or without it'. To which I provided further clarification. However, as they were working with the task, another interpretation of the wording arose - as it could seem as if 'suddenly after 1950 the bridge couldn't hold 10 tons anymore'.

After the clarification of the intention behind the problem in the task, work picked up and progressed at a normal pace. Although they had a clear strategic approach to solve the problem, it was also clear that the lacked certain data. Using their computers, that found information about both buses and the bridge itself. One of the things they discovered was that the bridge could hold a bus and its driver, but without any passengers. They combined this knowledge with the knowledge they had about the average person in order to solve the problem.

When estimating the weight of an average person, one of them mentioned 75 kg. This was not something they looked up, but rather some knowledge or preconception they had. From there, they used that weight to calculate how many people would equate a bus (approximately 10 tons).

After finishing, one of the participants mentioned that the historical aspect of this task was enjoyable - as you rarely see such elements in mathematical tasks.

Task 2 - Bankebrua

The immediate reaction to the participants were positive. One of the participants expressed familiarity to the bridge and the event described in the task. This was something the participant could vouch for was a real event - even referring to have read about it in the local newspaper. From here, they went as far as to look up that particular article in order to see if it held the answer to the task. Unfortunately, it did not.

In contrast to the positive response in the beginning, there were some obvious problems with finding numbers to work with. The needed information was not available neither online or in the task description. In order to solve this task, they needed to make assumptions not based in their own common-sense making.

In the end, they had to make some comparisons to another bridge in the local area in order to solve the task.

Task 3 - Dorga bru

Initially, the participants had some problems with this task. They did not know what the width of a standard road was - or if there even was a 'standard road width' defined by the government. After looking up documents online, it appeared that there were not just one standard width - but rather different widths used for different terrains. This invoked the need for some extra-mathematical knowledge to evaluate the terrain surrounding the bridge accordingly.

After looking up facts, and gathering information, they decided that you would need twice as many stone blocks than what was already there. They asked if this answer were sufficient or if I wanted a number of rocks - as the description of the task was a bit confusing at this point. I agreed with their answer although I originally had intended for them to calculate a specific number of rocks. However, after observing their approach to the problem, I accepted the ambiguity of the task description. Additionally, I did not want them to spend an unnecessary amount of time attempting a much more difficult problem.

Task 4 - Fedafjorden bru

At this point, the participants were getting quite confident in their need and skill to find additional information online. So much so, that their first attempt at solving this problem was to search for the building documents of the bridge. They had no luck on that behalf.

Proceeding onwards, they did get to practice their proficiency with computers. They did acquire some additional information about the bridge itself. This was both descriptive information - like the length and height of the bridge, and additional imagery from different angles.

Using some general assumptions and geometric principles, they came up with a reasonable estimate. I got the impression that they used more mathematics here than in the other tasks so far - or it could be that the math here just seemed more explicit to me.

Task 5 - Rosnesbrua

Because of the lack of an overview of the area, the task would have been more or less impossible to solve. Because of this, the participants turned to use interactive satellite images and maps on their computers to solve the task.

They managed to get an answer using their computer proficiencies only - applying a built in 'waypoint' feature that would calculate the time necessary to move between two points of their choice.

Task 6 - Skjernøysund brua

There were in general a lot of confusion about this task. Both the description and the picture of the task failed to provide clarity. As all the pillars of this bridge were all of different heights (and not all pillars could be seen in the picture), the question - as pointed out by one of the participants - was ambiguous at best.

After I clarified the intention of this task, and that the problem was to estimate the height of each pillar, they eventually did that. However, both participants expressed their dissatisfaction with this task.

Task 7 - Tømmerrenna

Using internet, they found information about the length of the bridge in addition to different types of wood stain. At this point, they were both using digital devices to acquire different crucial pieces of information simultaneously. After looking at forums and recommendations, they ended up with the assumption that one litre of wood stain would go to eight square meters of wooden surface. They also took into consideration that the wood would likely need (at least) two strokes of wood stain because of how neglected it currently was - and because of how extreme weather conditions it was going to overcome. One participant made some comparisons to staining his terrace.

In order to assume the area of the bridge, they needed to assume the height of the person in the image. This, they were able to do without the use of digital aid. From that point, they used simple geometry to present an estimate.

After finding out how much stain they needed, I urged them to evaluate if they would consider this a good solution to the problem regarding this bridge. This led them down a rabbit hole of looking up prices and calculating estimates of how long it would take to apply all of it. Although they did not conclude on this matter, they seemed to have a lot of fun looking up all the possibilities.

Concluding Remarks

As a final question to the participants - I asked them if they could rate the tasks from most interesting to least interesting. This was the order they decided upon together:

Task 7 (most interesting)	Task 5	Task 2	Task 3	Task 1	Task 4	Task 6 (least interesting)

Field Note #2
Date: 8.4.2019
Stage of development process: final assessment (field test of final revision)
Participants: 22 (21 pupils and 1 teacher)

General

The class consisted of about 80 % of the pupils following the practical mathematical course (1P), and the remaining 20 % following the theoretical mathematical course (1T). Their teacher were excited to try out the tasks, but were otherwise unsure about how the pupils would respond since they had never worked with anything similar.

We conducted the field test during the last hours of lecture on this Monday. These hours were going to be in a different subject, but it was changed (to mathematics) a few days in advance because we noticed that the lecture I was originally assigned would fall away due to a mock exam. Some pupils seemed aware of this change, but clearly not everyone. Especially one of the pupils expressed contempt that we were going to do mathematics - and was clearly not interested in doing that. At some point later, I realized that the pupil who had previously expressed contempt had left the class. In general, it seemed fair to say that spirits were not high.

We chose the groups at random by attributing each pupil a number between one and five. This gave us four groups of four and one group of five. No group worked with more than one task because of the limited time available. We distributed the tasks among the groups as evenly as possible.

Both the teacher and I talked with the pupils as they were working and, if necessary, offered help and advice. This was primarily to gain insights into their method of solution, but also to keep them active throughout the lecture.

The teacher had not seen the final revision of the tasks before this lecture.

Task 1 - Tømmerrenna

Only one group worked with this task. This group expressed insecurities about their mathematical skills early on, referring to themselves as low achievers. Confusion later rose, as they did not remember how to produce square meters. At this point, I helped them by handing them the formula (length multiplied by width) for calculating an area.

When assuming the height of the bridge (or the length of one of the sides to be precise), they initially assumed it to be around 1.5 meters tall. However, at some point later, one of the pupils in the group said something along the lines of “That can’t be right. That is almost as tall as I am. It has to be shorter, look at the person in the image”. In regards to this remark, they changed their assumption to a smaller number.

The group did eventually estimate a reasonable answer. This estimate did assume the bridge to be a flat object in two-dimensions, and they neglected any influence by the vertical surfaces of the object. Because of this inexplicit simplification was done, I later inquired the pupils if they had considered this. They had not. As a response, they slightly changed their estimate to accommodate for my concerns.

Task 2 - Rosnesbrua

Two different groups were working separately with this task. In both groups there were a lot of discussion about hypothetical influences (Ex: “What if they’re going swimming? How much time is it going to take? Are they going to take of their clothes before swimming? What about extra equipment or transporting items... and so forth). Only a couple of the pupils in each group took it upon themselves to design mathematical models to the discussed situations. One of these pupils expressed a desire to change groups. The teacher denied this, as it otherwise appeared as if the pupil got along well with the other group members.

At some points, the scenarios almost went into the realm of fantasy. One pupil in particular was arguing about the concerns of a shark attack. However, as her other group members argued, this was irrelevant - "There are no big sharks in Norway".

In both groups, the task seemed to digress naturally into a discussion about efficiency. They quickly disregarded *walking* as a viable option - because "No one would walk around. It takes too much time". Several methods (of passing the water) did occur. However, in the end both groups ended up arguing that *boat travel* would be the preferred way of passing. The teacher was an active consultant to both groups during their decision-making process, but I am unsure to what degree they were affected.

After one of the groups had decided on *boat travel* as their preferred mode of passing, they seemed quite satisfied with their answer. In order to fuel the fire a bit more - I asked them if they had considered how the winter season would affect this decision. This led the students to consider many new factors such as bad weather and ice - "Perhaps we would even be able to pass by foot if the ice got thick enough".

Task 3 - Fedafjorden bru

Two different groups were working separately with this task. Both groups were rather unwilling to start and required more encouragement than the other groups (doing the other tasks) in order to initiate. The pupils expressed confusion primarily about two things: first the quality of the printed images, and second the term "free height" [org. fri høyde]. The image clearly showed that there were more than one wire on each side. However, because of the quality, the pupils expressed difficulties to see if it was either eight or ten wires. As for the term "free height", this was probably new to them.

When attempting to solve the task, they grazed upon the idea of using Pythagoras' theorem. Before making any real progress on that end, they quickly deemed that method as being too tedious to bother. Rather than applying Pythagoras' theorem, they would apply estimates made on other inexplicit mathematical principles - which I will regard as "qualified" guessing. One interesting thing to notice is that one of the groups' estimate came very close to my own, which I had scribbled down when trying to solve my tasks a day in advance. I had used Pythagoras' theorem. They had not.

Overall, I was under the impression that both these two groups used most of their time discussing other things than their task.

Concluding Remarks

In general, all groups expressed an initial confusion about the tasks - mainly towards the lack of certain data. They were clearly not used to this way of working, but most of them adapted quickly the new "rules of play".

Both the teacher and I could agree that much of the pupils' work exceeded our expectations.

Appendix H: Questionnaire responses

Questionnaire feedback
Stage of development process: second assessment - development phase (expert appraisal)
Respondents: 2

General

Respondent A were a bit unsure about what elements I was looking for feedback on, and did as such not give comments on all the aspects of the tasks.
A: It was a nice and professional information document where you additionally boost the participant's morale by addressing his or her mathematical expertise. Beautiful.
It is exciting that you have chosen to use real bridges in the local area, considering that you wish to examine mathematical tasks that is supposedly more realistic. Furthermore, such tasks could have an impact on the participants - in the sense that the next time they notice a similar phenomenon in real life they will consider how to calculate the various measurements. As an extension to this study, it would additionally be interesting to examine how participants of such tasks have constructed a more developed tool for analysing and studying real sizes, and not the least if they have gotten a confidence that says 'this is something I can solve!'.

Respondent B did generally think there was a lack of information in all of the tasks.
B: A common trait among the tasks is that the pupils' themselves has to find information in order to solve the tasks. According to this, it might seem like the tasks has several different answers - although the actual measurements of the bridges exists and is constant. The addition of actual bridges in the tasks might them appear more realistic to the pupils, but there is no clear link to the daily lives of people - even though the problems posed in the tasks might appear realistic.
Personally, I would not use such tasks in education at upper high school [org. videregående skole]. This is because the pupils themselves has to gather the information, and that can lead to certain pupils not knowing what to do or how to solve the task. From my experience, I am certain that such tasks would not fit all the pupils. Additionally, if it is required that they use internet to solve the tasks, it could become a problem that they do something else online. Something that they are not supposed to do. However, the tasks do seem more exciting and more fun than 'normal' mathematical tasks. Many pupils would probably find this interesting. Both because it demands more of their own efforts, and because it requires more creativity. Such tasks could probably fit better to pupils that wanted more challenge than the normal tasks.

Task 3 - Dorga bru

Accessible:

B: The language used is okay. However, be careful with the use of the word trunk road [org. stamvei] as it can be confusing for many. The image provide great as an illustration, but in relation to the problem of the task it would be fine having additional sketches according to what the tasks wants you to solve [like the length of the bridge and the radius of the arc]. If it is the purpose of the task that the pupils should find such information themselves, the measurements of the bridge can be found on Wikipedia. However, beware that they have two different measurements listed without explaining which relates to what.

Open:

A: The fact that one only got to know the width of the bridge adds up to the participant needing to make nice 'mental leaps' in order to assume the length and the arc, which is required to calculate right volume needed to get the right estimate of rocks.

Expected practicality:

A: I can imagine some participants will struggle to start, seeing as this is a type of task with 'lacking information' in comparison to much of the other school mathematics they have worked with.

Other notes:

A: It is a very fun geometrical task.

Task 4 - Fedafjorden bru

Accessible:

A: It is a bit suspicious that one cannot see the whole bridge in the image, but it is supposedly not that many meters missing.

B: Language is fine. Great illustration. We don't get to know the dimensions of the bridge.

Realistic:

A: I could easily imagine this question raised by a construction engineer who is going to estimate an amount of steel that would go on the wires.

Other notes:

A: Again, a quite amusing task that requires something extra rather than being a classical type of task where all information is given and a simple mathematical procedure performed.

Task 5 - Rosnesbrua

Accessible.

A: With just the information given in the task, this is a tad too difficult - when one gets a map without elevations in the terrain, small paths etc. In other words, there will be many assumptions about how the terrain around the water will be; are there a path? Is it steep?

B: Nice language. Great illustrations. We do not get to know how long time it usually takes to cross the bridge, so it will be impossible to calculate how long it takes to walk around it. It is possible to use Google Maps as an interactive tool instead of making mathematical calculations.

Expected practicality:

A: I think the lack of terrain would be frustrating for participants. Furthermore, after my first reading, I also assume this task as more challenging and uncomfortable to answer than the other tasks.

Other notes:

A: However frustrating, it could be interesting to see how the participants reflects over the factors of the terrain.

Task 7 - Tømmerrenna

Accessible:

B: Nice language and illustration. The task seems easy to solve, although we do not get to know the width of the bridge. A possibility could be to assume the width of the bridge according to the person in the image.

Open:

A: Here it is possible to estimate a width and from there calculate a surface area per meter of length (with uncertainties) and hence get a relatively clear image of how much area is going to be covered in wood stain.

What they base their dimensions of; the person, the environment, or do they assume standard sizes of the planks?

Other notes:

A: This task is more okay and straightforward. What is fun in this tasks I will claim to be how (or if) the participants relates to the uncertainty of the task. Does it increase with each meter of length, or does it stay constant? This task strikes me as smoother than task number three, in light of school mathematics.

Appendix I: Interview transcription

Transkrib 1
Interviewer (I), and subjects (A, B, C)

Intervju 1

[00:01]	I: Skal vi se… testing, testing. Det lyser. (flytter mikrofonen)
[00:05]	I: Da legger jeg den her… Okay, kan du si noe? Så skal vi sjekke at det…
[00:11]	A: Ja, skal jeg bare si at jeg heter [navn].
[00:13]	I: Neida. Det går fint. Sånn. Okay. Ehm…
[00:20]	I: Så… jeg kan jo egentlig starte å spørre - hva synes du oppgaven, sånn helhetsmessig? Hva var inntrykket ditt?
[00:28]	A: Sånn helhetsmessig så var det jo en ganske bra oppgave fordi at jeg får jo sett selv hvor god jeg er til å gå inn i forskjellige situasjoner og kunne se konsekvenser og hvordan konsekvensene kan utvikle seg. For så å være forberedt på sånne ting senere. Sånn som nå, drøfta jo vi ut ifra ett utgangspunkt og endte opp med tre forskjellige utgangspunkt.
[00:50]	I: Ikke sant (bekreftende), og det er jo - det er fantastisk. At dere har valgt å gjøre det på tre forskjellige måter der.
[00:56]	I: Ehm… så, ehm… oppgaven - var den lignende på problemer eller oppgaver du har møtt tidligere?
[01:09]	A: Eh, tenker du innenfor mattetimer da eller?
[01:13]	I: Eh, både matematikk og i hverdagen - egentlig.
[01:17]	A: Ehm… det minner meg jo egentlig bare om det at hvis jeg er på vei hjem - jeg tar jo ofte buss, for jeg er ifra [sted] - så jeg tar buss fram og tilbake hele tiden. Så hvis jeg en gang havner - hvis bussen får en ulykke eller noe sånt og vi må bytte buss, så må jeg kunne regne om hvilken… eh… hva som er raskest for meg å gjøre for å kunne rekke timene tidsnok og sånt. Om det er raskest for meg å… ta taxi i forhold til hvordan det vil koste i forhold til resten av hva jeg ville betalt for en bussbilett.
[01:48]	I: Jaja.
[01:49]	A: Så det er jo en oppgave jeg selv kunne kjent meg kjent i på flere forskjellige måter.
[01:55]	I: Mhm. Det er - det er jo interessant å høre. For det var jo ikke sånn at jeg visste noe om din situasjon før du fikk den oppgaven.
[02:03]	A: Nei (bekreftende).
[02:03]	I: Men, eh… flott å høre. Eh, men ja det er jo da i hverdagen og sånt - men i matematikkundervisningen eller lignende. Synes du den ligner på matteoppgaver da?
[02:14]	A: Ja, det gjør jo det fordi at det er masse beregninger på avstand og tid.
[02:21]	I: Mhm.
[02:23]	A: Og det blir jo t- det blir jo fort at du må jo kunne matte for å komme fram til et svar.
[02:29]	I: Ja.
[02:30]	A: Ja (puster lettet ut).
[02:31]	I: Eh… det er klart. Ehm, men vi kan jo gå litt videre. Kan du beskrive hvordan gruppen din - eller du da - hvordan dere løste oppgaven.
[02:41]	A: Eh, vi løste oppgaven bare ved å tenke ut forskjellige måter man kan komme seg over et hav. Når - som du ser på kartet måtte du jo reist helt rundt bare for å komme inn på en annen måte, hvis du ikke kunne krysse havet. Så det nærmeste er å svømme, eller båt, eller… bare - ja, det er vel ikke så veldig mye annet å velge mellom. Ehm… et [utydelig ord] fartøy på vann i alle fall. Så begynte vi bare å drøfte mellom det og hvor fort en båt ville kjørt den distansen og hvor lang tid selv bruker på å svømme 25-meteren som vi alle har hatt på ungdomsskolen også bare ta det ut ifra hvor fort vi selv svømte. (intervjuer avbryter neste setning)

[03:29]	I: Ja, dere brukte, eh… (subjekt fortsetter)
[03:30]	A: Også… pluss at vi kom til den avstanden vi hadde som mål.
[03:34]	I: Mhm. Også matematikken dere brukte her - hva… lyst til å forklare litt der også?
[03:42]	A: Ehm… vi hadde jo omregning i fra sekund til minutt, også hadde vi jo… ja… hvordan skal man si det? Ehm… vi hadde jo bare omregning sånn generelt med tid.
[04:04]	I: Tid, fart og strekning - type?
[04:06]	A: Ja.
[04:07]	I: Kanskje. Ja. Ehm… jeg vet ikke. (Jeg) Tror du på en måte har sagt - hvorfor dere valgte å løse det på denne måten?
[04:19]	A: Ja, vi valgte jo det på grunn av at det er ikke så veldig mange alternativer på hvordan man - du kunne krysse et hav.
[04:24]	I: Nei (bekreftende). Den «gåen» der (referer til gruppas oppgaveark), er det «gå broa»?
[04:27]	A: Ja, det var hvis du hadde tenkt på i forhold til hvor lang tid vi hadde brukt når brua ikke var til stede - i forhold til hvor fort det gikk når brua faktisk var der.
[04:36]	I: Mhm.
[04:40]	A: Hvis brua ikke hadde vært der hadde de jo ikk- (omformulerer seg) automatisk brukt mye lengre tid bare på å krysse - på å krysse… eh… elva, for å kunne komme seg hjem.
[04:50]	I: Ja. Vurderte dere også eh… å prøve… ehm, å regne ut hvor lang tid det ville gått å gå rundt?
[05:02]	A: Eh, nei - det tenkte vi ikke på sånn i det store. Vi tenkte bare at det - det er for langt til å kunne satse på det. Det var vel egentlig bare det vi tenkte, også droppa vi den muligheten.
[05:15]	I: Mhm. Jeg ser den. Ehm, men ja! Helt til slutt - var det noen ting som var vanskelig med oppgaven eller - ja, eller ting der?
[05:23]	A: Det jeg syntes var vanskeligst med oppgaven var i alle fall det at det… det er så masse du kan få inn av ting som kan gjøre oppgaven enda vanskeligere. Sånn som her på påstigning og avstigning så måtte jo vi tenke på om de skulle ha vest eller ikke ha vest, og ta av og på vest og - hvis de skulle ha svømt om de skulle ha tid til å ta av seg klær og på seg klær når de kommer på land og sånt. For det er jo masse sånn som kan komme inn, som innspill, hele tiden - og da blir det jo egentlig bare et valg selv om hvor komplisert du vil at oppgaven skal være.
[05:55]	I: Akkurat!
[05:56]	A: (sukker lettet) Følte vi i alle fall.
[05:58]	I: Jo, nei. Godt følt.
[06:01]	I: Ehm, men sånn selve teksten og bildene og sånt - var det et eller annet du, dere ikke forstod der? Var oppgaven...?
[06:09]	A: Nei, vi forstod jo det vi skulle forstå. Det vi brukte lengst tid på med bildene var mer å tenke ca. hvor mange er det som bor her? Det var det vi brukte lengst tid på, men eh… hvis du bare - vi tenkte jo at de reiste fra den største til den minste øya for å kunne komme hjem etter en arbeidsdag eller noe lignende. Ehm… så har vi bare tatt ut ifra hvor mange hus vi selv klarer å telle langs denne veien som går innover her (henviser til oppgavearket) og regne ut at det er ca. fire stykker i en familie - mor far og to barn - også gang antall hus med fire.
[06:51]	I: Åja (overrasket). Kult.
[06:54]	A: Ja (lettet).
[06:54]	I: Ja.
[06:55]	I: Ehm… tusen takk for at du tok del. (opptak avsluttet)

Intervju 2

[00:01] I: Så, nå lyser det grønt.

[00:04] (Noen roper i bakgrunnen)

[00:06] I: Eh, dere hadde den oppgaven som var… med den broa. Eh - skal se om jeg fant den, men du husker den kanskje?

[00:12] B: Mhm

[00:13] I: Ehm, var… nei hva synes du? Det er jo det første jeg kan spørre om. Hva synes du om oppsettet, oppgaven, timen?

[00:23] B: Nei, ehm… det var jo litt lite opplysninger da. Så du måtte liksom anta… ting og så deg cirka.

[00:30] I: Mhm.

[00:31] B: Så må du jobbe ut ifra det.

[00:34] I: Ja. Synes du dette lignet på problemer du har møtt tidligere?

[00:40] B: Nei.

[00:41] I: Innenfor matte? I hverdagen - jeg vet ikke?

[00:44] B: Nei, for at når du tar matte så har du så ofte at alle opplysningene står stort sett i… som du treng da for å løse det - i oppgava. Her var det litt vanskeligere… for dem som ikke har løst slike oppgaver før.

[01:01] I: Nei (bekreftende). Var det… var det en dårlig ting? Eh

[01:04] B: Hæ?

[01:05] I: Synes du… synes du det var en dårlig ting at det var på denne måten, eller…

[01:09] B: Nja…

[01:10] I: Kunne du gjort dette igjen?

[01:11] B: Kunne jo gjort det på nytt, men det er jo litt vanskelig å… også løse det slik når os ikke vet ossen du skal gjøre det.

[01:20] I: Ja.

[01:21] B: Ja, og slik som vist formler og slik eller et eller annet, så hadde det vært enklere.

[01:26] I: Mhm. Det var egentlig noe jeg hadde tenkt at jeg kunne gjøre hvis vi hadde hatt tid, og hvis vi trengte det, så kunne jeg vise hvordan jeg har valgt å løse det i forhold til det som dere har hatt om tidligere i [mattekurset]. Eh… [uhørlig ord]

[01:42] I: Men, nei. Kan du beskrive hvordan gruppen din løste oppgaven? Eller, du og gruppen din - sammen - hvordan dere valgte å løse den oppgaven?

[01:49] B: Ja. Så vi begynte å prøve også finne noen Pytagoras-muligheter og slik - for der som vaierne gikk nedover, men da vart det litt vanskelig og så fastslå lengda der dem treffer ned så… det vart jo mye lik circa og sånn. Vi tok og antok at det var så langt så… tok og så regna ut - ut ifra de opplysningene og så [uhørlig ord]

[02:12] I: Ja. Det blir jo litt antagelser… men endte dere opp med å gå vekk ifra Pytagoras eller?

[02:19] B: Ja.

[02:20] I: Ja. Var det fordi det ble for vanskelig eller at det tok for lang tid?

[02:25] B: Nei, tror ikke oss alle var så veldig motiverte for å begynne å krongle noe særlig med det der.

[02:33] I: Nei, det er kanskje sent på dagen også… jeg vet ikke. Ehm…

[02:42] I: Nei, ehm, nei. Jeg skulle hatt - jeg skulle virkelig hatt oppgaven foran meg. Men, eh - var det - var det ting som var vanskelig med oppgaven? Sånn - du har sagt at det var disse antagelsene, men sånn teksten, bildet. Var det noe - eller elementene der som gjorde… som kunne være vanskelig?

[03:02] B: Nja, det var jo det at du skulle liksom finne ut hvor lang den vaieren var - også går den litt oppover også nedover att så… det er jo litt vanskelig å se nøyaktig hvor lang den er da. Du må jo prøve å gjette deg fram litt.

[03:17] I: Og selve oppgaveteksten - hva synes du om den?

[03:20]	B: Nei, oppgaveteksten [uhørlig mumling - så fekk me kanskje veta?] at selve brua var 566 meter lang og at fri høyden var 50 meter.
[03:27]	I: Mhm
[03:29]	B: Så-eh… Vi syntes ikke det var nok opplysninger da til å… klare å regne ut det, egentlig.
[03:37]	I: Hvilke eh, andre opplysninger er det du kunne ønsket å hatt med?
[03:42]	B: Hmm… Hvor langt det var den der den mellomdistansen mellom de to bjelkene der.
[03:49]	I: Åja, sånn spennet på brua? Ja, det er jo faktisk noe som man kunne funnet ut lett.
[03:56]	I: Det, eh… det er jo litt i utkanten, men vurderte dere å bruke kalkulator eller sjekke internett for svar?
[04:06]	B: Ja. Jeg spurte om de skulle bruke kalkulator, men det ble nedstemt i gruppa så… så jeg antok også at det ikke var lov å bruke internett så… vi gjorde ikke det.
[04:20]	I: Ja. Nei - jeg sa ikke noe på det, men egentlig så er jo den meningen at dere skulle få gjøre hva dere ville - om det så hadde vært å bruke wikipdia og andre nettsider.
[04:39]	I: Eh, har du… har du noen andre kommentarer til eh… til oppgaven eller… ellers så er jeg egentlig ferdig.
[04:45]	B: Nei, heg har ikke noen kommentarer.
[04:47]	I: Nei, eh, tusen takk for at du tok deg del. (opptak avsluttes)

Intervju 3

[00:00]	I: Da starter jeg opptaket. Skal sjekke at det lyser grønt når jeg prater, ja? Ja. Okay. Så jeg har noen spørsmål angående oppgaven.
[00:10]	C: Okay.
[00:11]	I: Også etterpå så… jeg tar bare opptak fordi at det skal være lettere for meg å huske hva vi prater om. Hvis det kommer fram noen personopplysninger så bare fjerner jeg det når jeg skriver det om til eh… til å bruke i oppgaven.
[00:21]	C: (samtidig som intervjuer prater) Jaja, ok. Det går fint.
[00:29]	I: Yes, eh… så nei, hva synes du om oppgaven? Hva synes du om…
[00:35]	C: Nei. Var greit. Var litt… gøy - å få se hva de forskjellige på gruppa kom fram til da, men det var jo… ja. Gøy.
[00:50]	I: Gøy?
[00:50]	C: Ja. Grei oppgave.
[00:52]	I: Ja. Hva - var oppgaven lignende på problemer du har møtt tidligere?
[00:59]	C: Nei, det vil jeg ikke si.
[01:01]	I: Nei, du har ikke… ikke i matematikken?
[01:03]	C: Ikke ennå.
[01:04]	I: Ikke i det daglige hverdagslivet?
[01:09]	C: Nei.
[01:10]	I: Nei?
[01:11]	C: Det vil jeg ikke si. Ikke ennå.
[01:12]	I: Ikke ennå? (begge fniser smått)
[01:13]	C: Nei, ikke ennå. Man vet aldri.
[01:15]	I: (spøkfult) Ja. Før eller senere så går det en bro. (litt fnising igjen)
[01:19]	I: Ehm… men jo, har du lyst til å beskrive hvordan eh… hvordan gruppen din løste - løste oppgaven?

[01:28] C: Eh… kom f- vi kom med en del f-forslag til hva vi kunne gjøre da. Det var da både at vi kunne - atte de kunne bare svømme over. Det var - eller de kunne ta båt. Eller de kunne gå rundt - heile da. Og… så tenkte vi om det var på vinteren da - så kunne de enten stå på skøyter eller ski eller bare gått rett over isen. Kommer ann på hvor tykk den var da. Men så bestemte vi oss for atte… ja, vi ble da enige om at det ble båt da. (intervjuer holder på å si noe) Så, ja…

[02:01] I: Ja, nei - bare… fortsett.

[02:03] C: Ja - altså, da ble vi enige om at vi kunne ta en… nok så stor båt da. Så kunne vi bare tatt et X antall folk over på en gang da. Og det tok fortere tid da eller - kortere tid enn å måtte gå frem og tilbake med mindre båter.

[02:24] I: Ja - brukte dere eh… eller - hvordan type matematikk brukte der når dere løste - når dere kom fram til eh, svaret deres.

[02:27] C: (blåser luft ut av munnen) Ja… hva skal jeg si på den? (lengre pause) Ja… det var jo - vi regnte jo bare ut at - etter vi hadde sett på alle alternativene så eh, var det jo… enkelt å se at det var det a- det som tok kortest tid da. Så vi… kanskje ganske fort klarte å regne ut at det var lettest med båt da. Såeh… vet ikke helt hva jeg skal si på den - hva vi brukte, men…

[03:13] I: Nei, det var… det var en s- kanskje mer evaluering [uhørlig] i helhetsevaluering? Kanskje både tid og penger og jeg vet ikke?

[03:24] C: Ja.

[03:25] I: Ja. Ehm… men ser dere at dere kunne løst oppgaven på en annen måte? Hadde dere funn- fantes andre, eh… måter å løse oppgaven på?

[03:37] C: Det svaret vi kom fram til?

[03:39] I: Ja. Eller annen matematikk man kunne brukt - eller… enn det som dere kom fram til, ja?

[03:48] C: Det er jo sikkert noe her - skal vi se…

[03:54] I: [uforståelig]

[03:57] C: Om jeg kanskje hadde hatt bedre tid enn det vi hadde, så hadde jeg sikkert klart å komme på noen løsning da… om jeg var alene i tillegg.

[04:06] I: Åja. Følte du at det var tidspress og… eh, lite - lite hjelp fra gruppa?

[04:15] C: Ja. Så det var den, ja. For jeg ga meg egentlig på den… [uhørlig] egentlig bare lot gruppa bestemme.

[04:22] I: Ja. Nei, sånn er det jo i noen gruppe… gruppearbeid. Eh, videre - var det ting som var vanskelig med oppgaven? Eller hva / hvilke ting eventuelt?

[04:42] C: Det var ikke akkurat vanskelig noe av det. Det var bare - vi måtte - måtte bare komme frem til noen eksempler, og så bli enige om det - men det var ikke sånn direkte vanskelig.

[04:55] I: Nei. Så hva med oppgaveteksten, bildet, kartet - var det noe der som var uforståelig?

[05:02] C: Nei, det var egentlig - det er grei oppgavetekst. Bildene var veldig greie og [uforståelig] sånn generelt så er det veldig fint [uhørlig]. Var ikke for vanskelig - var ikke… noe problem med det.

[05:18] I: Nei, riktig. Ja men, skal vi se da…
(opptaket avsluttes)