

On Switching between Motion and Force Control

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Abstract—In motion control technologies, an automatic switching between trajectory following and set reference force, upon the impact, is a frequently encountered requirement. Despite both, motion and force controls, are something of well-understood and elaborated in the control theory and engineering practice, a reliable switching between them is not always self-evident. It can lead to undesired deadlocks, limit cycles, chattering around switching point and, as consequence, to wearing or damages in the controlled plant and its environment. This paper contributes to analysis and understanding of the autonomous switching from the motion to force control and vice versa. Simple output and state feedback controllers are assumed, and the conditions to be held in vicinity to the switching state are explored. A simple yet robust hysteresis-relay-based switching strategy is shown to be suitable for such type of motion control applications. The hybrid automaton, as most general tool, is used for exploring and analyzing the transients. A multiple Lyapunov function approach is applied for stability analysis of the switched control system. A second-order system, with uncertain nonlinear dynamics, is demonstrated as an illustrative numerical example.

I. INTRODUCTION

Motion and force controls are something of well-understood and elaborated in the theory and practice, and (since decades) have attracted attention of both, academic research communities and practicing engineers in the various application fields. The common paradigm of motion and force controls, operating on the similar or even same system plants, has been formerly established for robotics [1] and mechatronics [2], in general, as two of the most appealing application fields. Since there, multiple works have been dedicated to a combination of both, while taking different focus on the robustness, parameters adaptivity, impedance variation, and possibility to have a common architecture of the control system. Just to mention few of them here, in [3] a hybrid position-force control of manipulators with a common matched control action has been proposed and investigated experimentally. Another simulation study of hybrid force-motion control has been shown in [4] for robotic manipulators, yet without explicitly defining and analyzing a suitable switching strategy. An analysis and experimental evaluation of the force control, applied to a robotic manipulator and incorporating the controlled motion before an impact with environment, has also been provided in [5].

In spite of numerous strategies and techniques have been elaborated and applied in the motion and force control applications, especially in robotics, a reliable switching between both is not self-evident and remains further on demanding for theoretical research and experimental evidence. At that

point, it can be mentioned that a hybrid motion-force control clearly falls into the focus of switching (also hybrid) dynamic systems, for which the theory has already been established in the last two decades, see e.g. [6], [7], [8], [9], [10].

The aim of this paper is to address an autonomous switching between the feedback motion and force controls. To this end, we demonstrate suitability of a simple hysteresis-relay-based strategy, which enables stable and robust changes between both control actions and elaborates on a boundary layer in the associated three-dimensional state-space. We note that this switching strategy has been empirically tried and attested in [11], while the potential applications and not limited to hydraulic actuators and spread out from the tactile instruments in medical technologies to the large robotic manipulators and handling- or construction-machines.

The paper is organized as follows. The problem statement of the second-order motion system with both, trajectory following and set force, controls is given in Section II. In Section III, we provide the required notations and analyze the dynamic behavior of both feedback control systems. Also a hysteresis-relay-based switching between both is introduced. An associated hybrid automaton suitable for formalizing and investigating the switched closed-loop system dynamics is described in Section IV. The multiple Lyapunov function approach is demonstrated in Section V for analyzing stability of the switched control system. A numerical example of the second-order motion control system with nonlinear dynamics and uncertainties is provided in Section VI, and that for trajectory following and force controllers, including the proposed autonomous switching. The paper is briefly summarized and concluded in Section VII.

II. PROBLEM STATEMENT

We consider a relative motion system of type

$$\begin{aligned}\dot{x}_1 &= x_2, \\ \dot{x}_2 &= -f(x_1, x_2) - sK(x_1 - X_s) + u.\end{aligned}\quad (1)$$

The exogenous control signal, to be addressed later, is u . The generalized motion coordinates are denoted by x_1 , here and in the following as scalar value (meaning a 1-DOF system) for the sake of simplicity. The nonlinear map of the system dynamics is $f(\cdot)$, this allowing for uncertainties such that for $F = \hat{f} - f$ it holds $\|F, \dot{F}\| < \eta$. Here the nominal system dynamics is \hat{f} , and η is some positive constant of uncertainties norm. The threshold of a constrained displacement is denoted by X_s and the binary switching variable is

$$s = \begin{cases} 1, & \text{if } (x_1 - X_s) \geq 0, \\ 0, & \text{otherwise.} \end{cases}\quad (2)$$

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The environmental stiffness of the impact, i.e. after unconstrained relative displacement, is $K \gg |f|$. Note that, for the sake of simplicity, neither restitution coefficient nor structural damping of environmental contact are taken into account. This simplifying assumption can be, however, lifted on the costs of extending the system dynamics (1). Note that such extensions might potentially improve accuracy in describing the transients between the motion and force control. However, they are less significant as for principles and, therefore, omitted in the recent work. It is worth noting that X_s constitutes boundary of a load-free displacement under control, i.e. motion without counteracting environmental forces. Beyond that value, cf. (2), a relative displacement will be inherently restricted by a high stiffness K . Thus, a penetration $0 < x_1 - X_s < \delta$ into environment is assumed as always limited by some small positive constant δ .

Now we are in position to formulate the problem statement that will be addressed and discussed in the rest of the paper. For the hybrid dynamic system (1), two control laws are assumed in the following:

$$u = \begin{cases} \alpha(X_r, x_1, t) & \text{while no impact,} \\ \beta(P_r, K(x_1 - X_s), t) & \text{otherwise.} \end{cases} \quad (3)$$

The first control is for tracking the reference trajectory $X_r(t)$, assumed as at least once differentiable. The second control is for set the reference force P_r . At this stage, the case difference (3) constitutes some non-strict switching conditions that should be then formalized, hereupon resulting in an autonomous switching control law. Note that both controllers in (3) are defined as explicit functions of time, since the derivative and integral terms can be equally involved in addition to the system output value. Note that the controllers $\alpha(\cdot)$ and $\beta(\cdot)$ are understood to be each asymptotically stable and capable of $|X_r(t) - x_1(t)| < e_\alpha$ and $|P_r(t) - K(x_1(t) - X_s)| < e_\beta$ for all $t > t_r$. Here t_r is the transient response time, and e_α, e_β are the residual control errors correspondingly.

The question that arises now is which autonomous switching, simple as possible, can be defined for ensuring both controllers (3) keep the above defined performance, once the system (1) changes from the controlled steady-state motion to the controlled force upon an impact with environment. The reversal switching, i.e. back from the force to motion control, can be captured when surpassing some lower assigned P_r set-value, below which the α -control takes over back, i.e. to follow again a reference trajectory $X_r(t) < X_s$.

III. FEEDBACK CONTROLLERS WITH HYSTERESIS SWITCHING

A. Motion control

First, consider the motion control of reference following, while the ramp $X_r(t) = Rt$ with a constant reference velocity R is assumed, that for the sake of simplicity yet without loss of generality. Obviously, a standard linear feedback control should equally include an integral error term so as to compensate for $f(\cdot)$ and its uncertainties. Introducing the

corresponding feedback gains k_1, k_2 for the dynamic system states, and k_0 for the integral error state, one can write the closed-loop control system (for the range $x_1 < X_s$) as

$$\dot{x}_2 + f(x_1, x_2) + k_2 x_2 + k_1 x_1 + k_0 \int x_1 dt = Rt(k_1 + k_0 \frac{1}{2}t). \quad (4)$$

Taking time derivative of the left- and right-hand side of (4), and assuming zero initial conditions, one obtains

$$\ddot{x}_2 + \dot{f}(x_1, x_2) + k_2 \dot{x}_2 + k_1 x_2 + k_0 x_1 = k_1 R + k_0 Rt. \quad (5)$$

It can be seen that from steady-state, including time-derivative of nonlinear dynamics $\dot{f}(\cdot) = 0$, a coefficients' comparison of the left- and right-hand side of (5) yields

$$\begin{aligned} \bar{x}_2 &= R, \\ \bar{x}_1 &= Rt, \end{aligned} \quad (6)$$

while the bar (i.e. \bar{x}) emphasizes the steady-state of corresponding dynamic variable. The stationary behavior (6) implies the α -control in (4) can exactly track the reference trajectory, provided the eigen-dynamics i.e. left-hand side of (5) remains asymptotically stable despite uncertainties. The latter can be assessed considering the full derivative

$$\frac{d}{dt}f(x_1, x_2) = \frac{\partial f}{\partial x_1}x_2 + \frac{\partial f}{\partial x_2}x_1. \quad (7)$$

Substituting (7) and $x_2 = \dot{x}_1$, and rewriting the left-hand-side of (5) one obtain the characteristic (polynomial) equation of the closed-loop control system as

$$\ddot{x}_1 + k_2 \dot{x}_1 + \left(\frac{\partial f}{\partial x_1} + k_1\right)\dot{x}_1 + \left(\frac{\partial f}{\partial x_2} + k_0\right)x_1 = 0. \quad (8)$$

Obviously, the stability of characteristic equation (8) can be guaranteed by an appropriate assignment of k_0, k_1, k_2 gains, while the partial derivatives of the motion dynamics should be taken into account. For systems with uncertainties, i.e. $\eta \neq 0$ cf. with Section II, a robust design of the feedback gains should be ensured, for instance, an interval polynomials test of stability can be performed based on the Kharitonov's theorem, see [12], [13] for details. A more detailed analysis of the robust gains in (8) will be not further addressed, as being out of main scope of the recent work.

B. Force control

Due to the fact that the motion dynamics becomes constrained upon an impact, meaning the motion system loses a free integrator, the position feedback in control becomes not longer required since $-K(x_1 - X_s)$ is already acting in feedback. Assuming $X_s = 0$, for the sake of simplicity, one can write the β -controlled motion system (1)-(3) as

$$\dot{x}_2 + f(x_1, x_2) + k_2 x_2 + Kx_1 = P_r, \quad (9)$$

that understands the set point command of the desired force P_r . Since the environmental stiffness is assumed to be relatively high, a sufficient control damping k_2 should be assigned to suppress oscillatory behavior at the impact excitation. This is also with respect to uncertain dynamic nonlinearities $f(\cdot)$ which might additionally contribute to

the transient oscillations around $X_s \leq x_1 X_s + \delta$. Obviously, ensuring a robust damping of the system at impact, a steady-state $K(x_1(t) - X_s) = P_r$ for $t > t_r$ can be guaranteed, which means setting of the reference force value.

Further we note that in case of some specific nonlinear system dynamics, an additional state feedback $-k_1 x_1$, equally as an auxiliary gain i.e. $G P_r$, can be desirable for shaping the closed-loop behavior, cf. e.g. [2]. However, this case-specific design measures will be omitted here, so that a generalized, to say rigidly matched, force-controlled motion system as in (9) will be considered in the following.

C. Autonomous switching

From viewpoint of the system dynamics, cf. (1), the threshold X_s at impact constitutes a boundary layer within the (x_1, x_2, x_3) state-space, starting from which the controlled motion (4) will come to an equilibrium state in case either $k_0 = 0$ or the actuator boundaries $U_{\min} \leq u(t) \leq U_{\max}$ apply. A corresponding trajectory of the controlled motion is exemplarily drawn in Fig. 1. One can see that a forced

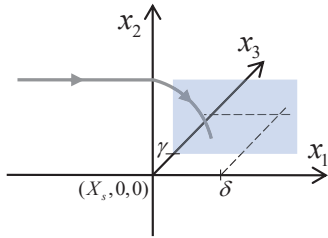


Fig. 1. Phase portrait of (x_1, x_2, x_3) -trajectory of the controlled motion system at impact, with $k_0 = 0 \vee U_{\min} \leq u(t) \leq U_{\max}$ control setting.

motion stops within a bounded force-displacement region, while δ limits the penetration into environment after an impact, cf. Section II. It is apparent that an autonomous switching between the motion and force controls should appear within this bounded region, while a force threshold value γ is to be assigned as a design parameter. From Fig. 1 one can recognize that γ defines a vertical plane that should always be traversed by a motion trajectory before the system comes into an idle state. We recall that an ideal positioning control possesses infinite stiffness, cf. with [2], so that the γ -assignment is rather application-specific.

It is evident that at $x_3 = \gamma$ the plane becomes essential for switching between the motion and force control, and its boundary layer becomes relevant for a transient system dynamics. We recall that a robust switching in one way should be ensured without appearance of any deadlocks, limit cycles, or sliding modes around the switching plane. Whichever controller $\alpha \vee \beta$ is in place, the transient overshoots and transient oscillations might occur within some boundary range, so that a direct switching at $x_3 = \gamma$ appears to be less suitable. Apart from the transient effects, the process- and above all measurement-noise can furthermore limit the performance of an autonomous switching. Another issue to be taken into account, when designing the switching

law, is that the motion control (4) includes integrator of the position control error. That one will keep an accumulated control value even after switching to (9) force control. In order to avoid an undesired, to say spurious, control coaction when switching back, i.e. from β to α action, the integrator in (4) should be reset. This will be taken into account when specifying the hybrid automaton in Section IV.

An obvious strategy for avoiding the above mentioned problems is using a non-ideal relay, i.e. with hysteresis, which will create a force tolerance layer around $x_3 = \gamma$ plane. Note that a hysteresis switching, especially scale-independent hysteresis switching logic, has already been used for hybrid dynamical systems, see e.g. [14]. A symmetric hysteresis-relay with input $p_\gamma = x_3(t) + \gamma$, which is the force biased by the switching threshold γ , is given by

$$z^+(t) = \min \left[\text{sign}(p_\gamma(t) + P_s), \max[z(t), \text{sign}(p_\gamma(t) - P_s)] \right], \quad (10)$$

while the relay's initial state at t_0 is given by

$$z_0 = \begin{cases} \text{sign}(p_\gamma(t_0)), & \text{if } p_\gamma(t_0) \in (-\infty, -P_s] \vee [P_s, \infty), \\ \{-1, 1\}, & \text{otherwise,} \end{cases} \quad (11)$$

see [15] for details. The hysteresis relay has memory of the recent state, and keeps its value as long as $p_\gamma \in (-P_s, P_s)$, where $0 < P_s < \gamma$ is the switching threshold parameter to be assigned. Note that the hysteresis relay (10) contains discontinuities, and switches immediately upon the threshold values. Therefore, each change between the α and β controllers takes place without transient delays in the phase.

Apparently, the codomain of the relay function is $z \in \{-1, 1\}$, so that the not-completed switching control law (3) can now be rewritten to

$$u = \begin{cases} \alpha(X_r, x_1, t) & \text{if } z = -1, \\ \beta(P_r, x_3, t) & \text{if } z = 1. \end{cases} \quad (12)$$

Apart from the linear feedback controllers α and β , the γ and P_s values constitute additional design parameters.

IV. HYBRID AUTOMATON OF SWITCHED DYNAMICS

For analyzing the switched feedback control system, with plant dynamics (1) and controllers (12), we use an autonomous-switching hybrid system notation, cf. with [6], and write the compact state-space notation as

$$\dot{\mathbf{x}}(t) = A_{m_i} \mathbf{x}(t), \quad (13)$$

$$m(t^+) = \varphi(\mathbf{x}(t), m(t)). \quad (14)$$

Here m_i with $i \in \{1, 2, 3\}$ are the switching modes of both, the motion controlled system with varying-structure plant (1), captured by m_1, m_2 , and the forced controlled system captured by m_3 . Note that for the motion control, an autonomous system with zero reference is assumed, while for the force control the $P_r = \text{const}$ is the single exogenous quantity incorporated into (13). The state vector is $\mathbf{x} = [x_0, x_1, x_2, x_3]^T$, while x_0 captures the (integral) control error and x_3 constitutes a counteracting force at impact with environment. The discrete transition (or switching) function

is φ , while $m(t^+)$ denotes the discrete system state (here and further on also denoted as ‘mode’) which is piecewise-continuous from the left. That means $m_i(t^+)$ is a so-called ‘successor’ of the recent mode $m_j(t)$ while $i \neq j$. The above notations allow for representing the switched dynamical system (13), (14) as a hybrid automaton, see e.g. [10] for details, which is well-comprehensible for analyzing individual subsystems and switched transitions between those. Beforehand, the single modes system matrices are specified below, for the sake of completeness.

Assuming a linearized system nonlinearity (around an operation point) to be $f(x_1, x_2) \approx L_1 x_1 + L_2 x_2$ and extracting the states dynamics from (1), (4) yield

$$A_{m_{1,2}} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -k_0 & -(L_1 + k_1) & -(L_2 + k_2) & -s \\ 0 & 0 & Ks & 0 \end{pmatrix}, \quad (15)$$

with $s = 0$ for m_1 -mode, cf. (1), (2). For the m_3 -mode, with the same states notation as above, one obtains

$$A_{m_3} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -L_1 & -(L_2 + k_2) & -1 \\ 0 & 0 & K & 0 \end{pmatrix}, \quad (16)$$

based on the force control dynamics (9).

The hybrid automaton of the switched force-motion control system (13), (14) is depicted in Fig. 2. One can recognize that the constrained motion control mode m_2 at the impact occurs always between the unconstrained motion control m_1 and force control m_3 modes. Note that a supervised switching-back to the motion control, omitted here due to an autonomous system assumption without external reference input, will require setting $P_r < \gamma - P_s$. Thereupon, the force control would drive the system to $x_3 < \gamma - P_s$ that implies an activation of the corresponding switching function and transition back from m_3 to m_2 .

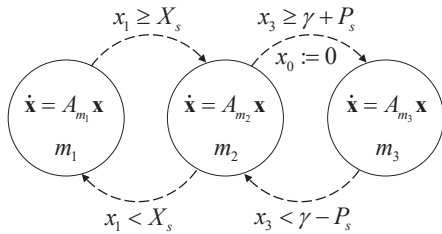


Fig. 2. Hybrid automaton of switched motion/force feedback control.

V. MULTIPLE LYAPUNOV FUNCTIONS

For the defined hybrid automaton, we are interested to show that the relay-based switching keeps the overall control system asymptotically stable. Recall that the individual subsystems, i.e. governed by the feedback controllers α and β , are asymptotically stable as has been assumed and discussed in Section III. Despite this fact, an autonomous switching between the asymptotically stable subsystems may lead to

destabilizing the trajectory solution as a whole, examples of which are well-known in the literature, see e.g. [7].

Traditionally, the fact that a common Lyapunov function guarantees stability for arbitrary switching has led researchers to search for conditions under which a common Lyapunov function exists [8]. Although the necessary and sufficient conditions for the existence of a common quadratic Lyapunov function for switching between two stable linear systems have been shown in [16], the finding of such suitable Lyapunov function is nontrivial and rather case-specific. In view of the $i = 3$ subsystems and varying structure between the $m_{1,2}$ - and m_3 -switched systems, cf. Section IV, we abstain from temptation of finding a common Lyapunov function. Instead, we will make use of the so-called multiple Lyapunov functions, see [7] for details, that can be assumed for a slow switching between the single stable modes. Note that a slow switching solely ensures that the intervals between two consecutive switching times are large enough, while the switching itself appears instantaneously, that is without additional time delays or phase lags.

For m_1 and m_2 system modes, consider the Lyapunov function candidate

$$V_{1,2} = \frac{s}{2K} x_3^2 + \frac{1}{2} x_2^2 + \frac{L_1 + k_1}{2} x_1^2 + k_0 x_0 x_1, \quad (17)$$

which is positive definite for all $\mathbf{x} \in \mathbb{R}^4$ and radially unbounded. Note that (17) is equally valid for A_{m_1} and A_{m_2} , driven by the same α -controller, with a single difference of $s = 0$ for the m_1 mode. Taking the time derivative of (17) and substituting the corresponding dynamics (15) results in

$$\frac{d}{dt} V_{1,2} = -(L_2 + k_2) x_2^2 + k_0 x_1^2, \quad (18)$$

which is negative definite if and only if

$$|x_1| < \frac{L_2 + k_2}{k_0} |x_2|. \quad (19)$$

Since the α -control includes the integral error feedback, it comes as not surprising that the time derivative of Lyapunov function equally contains the square of x_1 , which is contributing with positive sign. Recall that for zero reference and non-zero initial conditions – assumption taken already in Section IV for comprehensibility of analysis – the x_1 state constitutes the position control error. Only when either $k_0 = 0$, meaning no integral control action, or $x_1 = 0$, meaning the motion system is in the set position, (18) becomes negative definite for all x_2 values including zero velocity. Otherwise, when stopping outside of $x_1 = 0$, the closed-loop control system increases its energy level, i.e. $d/dt V_{1,2} > 0$, which is in meaning of an integral control action. The above consideration reveals (19) as a limiting relationship between the relative displacement and velocity for which a stable switching can take place. If (19) holds, the energy increase rate, due to the integral control error, is balanced by the energy dissipation rate due to the system and control damping. This case a switched control system will not exhibit an energy storage which, otherwise, might lead to transient instabilities when switching from m_2 to m_3 .

Further we note that once the velocity starts decreasing after impact (cf. Fig. 1), the condition (19) can be used for optimally designing the threshold force $\gamma = K(x_1 - X_s)$ of autonomous switching. Then, the instantaneous (decreasing) velocity is not longer required, and some reference velocity, e.g. $|x_2| \in [0.5R, R]$, can be assigned in (19) for determining the x_1 -value of an optimal switching.

In the same manner of developments as above, we introduce the Lyapunov function candidate

$$V_3 = \frac{1}{2K}x_3^2 + \frac{1}{2}x_2^2 + \frac{L_1}{2}x_1^2 \quad (20)$$

for the m_3 mode. Note that since the β -controller does not have an integral control action, the x_0 state is not contributing to (20). The V_3 function is positive definite and radially unbounded, while

$$\frac{d}{dt}V_3 = -(L_2 + k_2)x_2^2 \quad (21)$$

is only negative semi-definite since $d/dt V_3 = 0$ for $x_2 = 0$. That is the control system in m_3 mode has an invariant set

$$\Omega = \{x \in \mathbb{R}^3 \mid x_2 = 0, x_1 > X_s\}.$$

This comes as not surprising since an idle state with zero velocity, behind the impact boundary layer, corresponds to the force setting point requested by an external reference.

Now we can employ multiple Lyapunov functions, i.e. (17), (20), for which one assumes the switching times $t_0 < t_1 < \dots < t_i$ so that a switching signal (z in our case) is continuous from the right everywhere: $z(t_i) = \lim_{t \rightarrow t_i^+} z(t)$ for each i . For such switching sequence, following is sufficient to be shown for a family of Lyapunov functions $\{V_i : i \in m_i\}$ while V_i is decreasing on each interval where m_i mode is active. If for every m_i the V_i -value at the

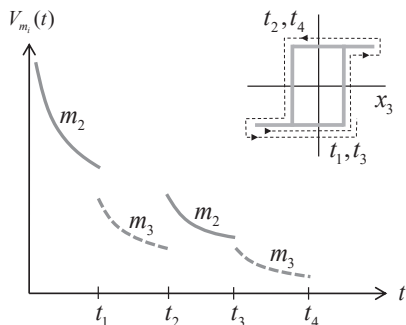


Fig. 3. Two Lyapunov functions for m_2 and m_3 modes.

end of each interval (immediately before switching) exceeds its value at the end of next interval in which m_i is active, then the entire switched system is asymptotically stable; for formal notation and proof we refer to cf. [7]. An alternating V_i is schematically shown in Fig. 3 for the m_2 and m_3 modes. Here t_1, t_3 are the time instants where the hysteresis relay is switching up, thus changing to the force control β . Respectively, t_2, t_4 are those times where the hysteresis relay is switching down, thus changing to the motion control α .

For (17) one can show that $V_2(t_1) > V_2(t_3)$ due to reduction of velocity $x_2(t_1) > x_2(t_3)$ and apparently integral control error state $x_0(t_1) > x_0(t_3)$. Note that state values of relative displacement and counteracting force remain the same $x_1(t_1) = x_1(t_3)$, $x_3(t_1) = x_3(t_3)$ for both instants. With the same argumentation line, one can show for (20) that $V_3(t_2) > V_3(t_4)$ and that due to the velocity reduction i.e. $x_2(t_2) > x_2(t_4)$. The above conditions allow concluding an asymptotic stability of the switched system (13), (14), provided the hysteresis amplitude $2P_s$ is sufficient to ensure the required slow switching. That is the switching intervals $t_{i+1} - t_i$ are long enough for all i , so that the transient effects disappear, upon which one can guarantee the reduction of relative velocity and integral error of the motion control. We notice that a qualitative measure of such intervals of slow switching, and an associated selection of the hysteresis relay parameters, are outside the scope of the recent work and rather subject to our further investigations in the field.

VI. NUMERICAL EXAMPLE

Following numerical example is taken for evaluating the autonomous switching motion-force control system designed as above. The nonlinearity of dynamics is modeled as

$$f(x_1, x_2) = 0.2 \cos(x_1) + 0.5 \tanh(10x_2).$$

Note that the assumed $f(\cdot)$ approaches typical situation of a mechanical manipulator, where the trigonometric function of position state is due to the gravity, and the hyperbolic tangent of the velocity state approximates the Coulomb friction while smoothing discontinuity around zero crossing. The environmental stiffness is assumed to be $K = 10000$.

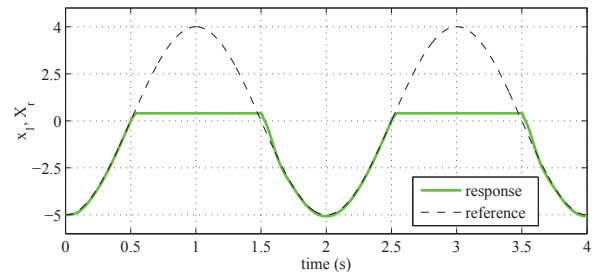


Fig. 4. Reference motion trajectory X_r versus the controlled relative displacement x_1 constrained by $X_s = 0$ with switching to $P_r = 4000$.

The assigned feedback gains are $k_0 = 100000$, $k_1 = 4000$, $k_2 = 150$, and the hysteresis relay is parameterized with $\gamma = 1500$ and $P_s = 500$. The impact state is assumed at $X_s = 0$, that for the ease of interpretation. Furthermore, the initial conditions are assumed to be $\mathbf{x}(t_0) = [0, -5, 0, 0]^T$.

For evaluating the autonomous switching between the motion and force controls, and that in both directions, a periodic reference trajectory $X_r(t) = 5 \sin(0.5t - \pi/2)$ is applied. Simultaneously, a square-pulse sequence (of the same period) with $P_r = 4000$ is assigned for the force set point. This case, the reference force (for the β -controller) will drop to zero each time the reference trajectory becomes $X_r < T_s$. Thereupon, the switched back α -controller should

repetitively track the motion trajectory before coming again to the impact with environment.

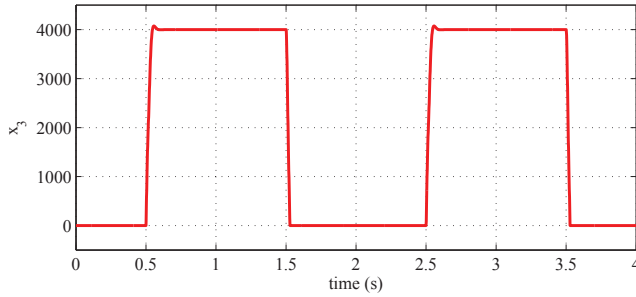


Fig. 5. Controlled force x_3 with $P_r = 4000$ setting point at impact.

The α -controlled relative displacement is shown in Fig. 4 versus the reference trajectory X_r . It can be seen that after impact (at $x_1 = 0$) certain penetration into environment appears due to $P_r = 4000$. When passing the threshold

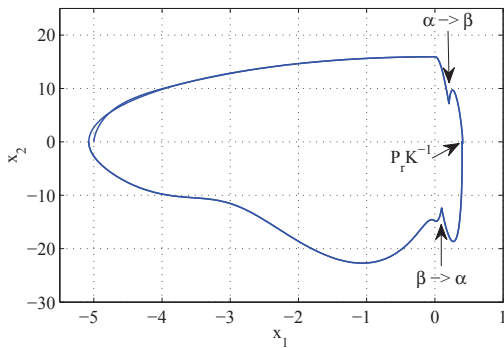


Fig. 6. Phase portrait of periodic motion with indicated switching points from α - to β -controller, from β - to α -controller, and force setting at P_r .

force value, the control loop is switching to β -controller while neither transient oscillations nor signs of any temporal instabilities can be observed. The corresponding controlled force is shown in Fig. 5. The counteracting (environmental) force x_3 continuously increases, starting from the time instants where x_1 exceeds the X_s value. Note that no transient changes in the x_3 trajectory can be observed when switching from α - to β -controller and vice versa. To complete the picture about the system dynamics, the phase portrait in the (x_1, x_2) coordinates is shown in Fig. 6. Here the half-plane $x_1 > 0$ corresponds to the constrained system motion upon the impact with environment. Both switching points between the α - and β -controllers are indicated. One can recognize that the associated transient changes in relative velocity are minor in both directions. The equilibrium point $(P_r K^{-1}, 0)$ corresponds to the set force value of the β -controller.

VII. CONCLUSIONS

The problem of autonomous switching between the motion and force controls has been addressed. For both asymptotically stable control subsystems it has been shown how a simple yet robust hysteresis-relay-based switching strategy can be efficiently incorporated. Autonomous-switching

hybrid system notation, with an associated hybrid automaton, have been derived as suitable for an uniform system description and related analysis. For analyzing stability of the autonomous-switching motion-force control system, the multiple Lyapunov functions approach has been used, relying on the so-called slow switching conditions. The proposed developments and analysis allow for further investigations towards optimal parametrization of the switching hysteresis relay, correspondingly time scales of the switching delay and associated dynamic transients. Numerical example of a second-order system with nonlinear dynamics has been shown for motion and force control and autonomous switching between those. Application of the proposed approach to more complex control and system dynamics are thinkable.

Acknowledgments

This work has received funding from the European Union Horizon 2020 research and innovation programme H2020-MSCA-RISE-2016 under the grant agreement No 734832.

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