Performance Analysis of Underlay Two-way Relay Cooperation in Cognitive Radio Network with Energy Harvesting

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Abstract

Cognitive radio and energy harvesting are two important approaches to solve the problem of spectrum scarcity and energy constraint in wireless communications. In this work, we study a two-way relay cooperation scheme in underlay cognitive radio networks (CRNs) with energy harvesting in which two secondary users exchange information via an energy harvesting relay node. Since the relay node collects energy from the received signal and utilizes it to forward the received signal, the secondary transmission power can be markedly reduced. Hence the interference of the secondary networks to the primary networks can be substantially reduced. We derive the outage probability of the secondary networks and analyze the ergodic sum-rate of the secondary networks. For the relay selection scheme, we find the optimal relay by interior point method based on penalty function. Numerical results show that the proposed scheme gives higher throughput for the secondary networks than other strategies.

Keywords: underlay cognitive radio networks, two-way relay cooperation, energy harvesting, outage probability, relay selection.

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1. Introduction

With the rapid development of the wireless communications, we have witnessed an exponential growth of demand for current wireless communications with convenient quality-of-service (QoS) over the last 20 years. Spectrum and energy are one of the most indispensable resources which need to be allocated reasonably and controlled properly in wireless networks. However, the measurements by Federal Communications Commission have shown that 70% of the allocated spectrum in US is underutilized [1]. Therefore, the fixed spectrum assignment policy becomes a bottleneck for more efficient spectrum utilization, under which a great portion of the licensed spectrum is in idle state most of the time [2]. In this situation, some regulatory agencies have suggested opening up the licensed band which is exclusively occupied by primary users to the secondary users subject to the guarantee of non-interference with primary users. The critical technology behind spectrum reuse is cognitive radio (CR) which has the ability to adapt the operating parameters via sensing the surrounding radio environment [3, 4].

CR has been proposed as one of the promising techniques for improving the utilization of the available spectrum bands in wireless systems. CR users, known as secondary users (SUs), sense the spectrum belonging to the licensed users, also known as primary users (PUs), and opportunistically utilize unused spectrum bands.

According to the aforementioned method in which PUs' spectrum is utilized by the SUs, there are three main CRNs paradigms: underlay, overlay, and interweave. In the overlay mode, the SUs use sophisticated signal processing and coding techniques to maintain or improve the communications of licensed radios while also obtain some additional bandwidth for their own communications. In the interweave mode, the SUs opportunistically exploit spectral holes to communicate without disrupting other transmissions [5]. The underlay mode allows cognitive users to operate if the interference to the licensed users is below a given threshold. While doing so, the SUs must keep the amount of interference power introduced into active PUs' sub-carriers at or below a previously configured threshold, commonly referred as the "interference temperature" limit [6]. In this paper, we focus on the performance investigation in the underlay CRNs. Due to restrictions imposed on the SUs' transmission in the underlay CRNs, the transmission rates are bound to be reduced. Therefore, the relay cooperation technology becomes an effective way to solve low speed transmission in the underlay mode.

Relay cooperation, known as a powerful technology that mitigates signal fading through multipath propagation in a radio environment [7], utilizes the spatial diversity offered by cooperative nodes. Therefore, the achievable data rate and reliability can be improved significantly. The relaying protocols for CRNs include amplify-and-forward (AF), decode-and-forward (DF), coded cooperation and so on [8]. In CRNs, a primary destination node receives multiple copies of signals from the primary transmitter and SUs. In this way, the transmission data rate and accuracy can be improved even if the wireless channels suffer serious path loss.

As one of the key enabling technologies in the next-generation wireless networks, cooperative communications have been extensively studied for the purpose of either spatial reusability enhancement or coverage range expansion; and recently there have been some research articles on energy harvesting in two-hop relay or cooperative communications. Though this technology significantly improves the spectrum efficiency, it obviously leads to much higher energy cost. Hence, it could be a double-edged sword in terms of transmission performance and energy consumption. On the one hand, cooperative capacity could be significantly improved under excellent relay channel condition with one or more relay nodes creating additional data links between source nodes and destination nodes, while on the other hand this could also bring an additional "load" for energy expenditure of relay nodes.

The promising solutions to prolonging the lifetime of power-constrained wireless nodes consist of power control design, energy harvesting and wireless power transfer (WPT) technique. To date, there are some literatures about the energy harvesting technique powering the wireless networks in the underlay spectrum sharing area [9-12]. The energy harvesting technology for underlay CRNs was considered [9] and the optimal transmission power and energy transfer policy was obtained in a single slot for maximizing the number of bits transmitted by SU under the primary sum-rate constraint in an offline setting. In [10], radio frequency (RF) energy harvesting underlay CR system which operated in a slotted fashion was considered. Firstly, that paper formulated the problem of maximizing the achievable secondary sum rate under primary receiver's protection criteria as a convex optimization problem, and secondly, obtained the optimal time sharing and secondary transmission power under offline setting. An online solution for the optimal time allocation between the energy harvesting phase and the information transfer phase in an underlay CRNs was proposed [11], which harvested the RF energy originating from the primary system. Performance of CRNs under imperfect channel state information where a SU transmitted data using DF relay was studied [12].

The energy saving and the spectrum efficiency are two very important aspects in wireless communications. However, existing papers either consider the spectrum efficiency or energy saving under the assumption of the mutual interference between the primary networks and the secondary networks in the underlay CRNs. To the best of our knowledge, the system of underlay CRNs with SWIPT and two-way cooperation has not been studied before. In this paper, consideraing the interference from the primary networks to the secondary networks, we investigate the system where the primary network consists of one pair of primary transceivers and the secondary network comprises two sources which exchange messages via an AF relay node. The relay is not equipped with the power source and is assumed to utilize the power splitting (PS) protocol to harvest energy and decode the information from the received signal.

The contributions of this work are summarized as follows:

Firstly, we present a novel model of underlay CRNs, where primary users and secondary users coexist in a time-slotted mode. For the purpose of maintaining the primary QoS and assuring the transmission throughput of the secondary networks, the two-way relay cooperation on the secondary system is adopted.

Secondly, according to the proposed model, we derive the outage probability of the secondary networks and investigate the throughput of both primary networks and secondary networks. It is indicated that ergodic sum-rate is maximized in the underlay CR case, when interference power distribution parameter is half across the SU terminals. Thereafter, we also analyze the energy efficiency of the whole system according to aforementioned deduction.

Thirdly, we consider that there are many candidate relays in the secondary system, and we select the best relay to cooperate data transmission. We formulate the relay selection as an optimization problem. Thereafter, we use an interior point method which is based on the penalty function to approach the global optimal result.

Finally, numerical results confirm our theoretical derivation and demonstrate that the proposed scheme guarantees the high-quality transmission for both the primary and secondary networks without extra relay energy consumption.

The rest of this paper is organized as follows. In the section 2, we introduce related studies on relay cooperation and energy harvesting. Section 3 describes the system model and the transmission scheme. The performance of energy-harvesting based two-way relay cooperation in the underlay CRNs is analyzed, and the outage probability is derived in Section 4. Section 5 investigates single relay selection scheme. Numerical results are obtained and analyzed in Section 6 before we conclude this work in Section 7.

2. RELATED WORK

2.1. Relay Cooperation

Relay cooperation had attracted a lot of studies. In [13], authors investigated performance evaluation of cooperative communication systems by considering outdated channel estimate with only AF relaying mode. In [14], authors analyzed the performance of cooperative spectrum sharing in single-carrier relay systems which took the peak interference power at the PU and the maximum transmission power at the SU into account. The capacity of underlay cognitive multi-hop relaying over independent and non-identically distributed generalized-K fading channels was analyzed [15]. In [16], authors proposed a differential chaos shift keying cooperative communication system with two users, which had an orthogonal sub-channel in broadcast phase and cooperative phase through orthogonal Walsh code sequences as its multi-access phase. A comprehensive analysis of the incremental-best-relay cooperative diversity, which exploited limited feedback from the destination terminal, was introduced [17]. Cooperative communication with single relay selection was a simple but effective communication scheme for power-constrained networks. A novel selective single-relay cooperative scheme, combining selective-relay cooperative communication with physical-layer power control, was proposed [18]. In [19], author presented end-to-end performance of two-hops wireless communication systems with non-regenerative relays over flat Rayleigh-fading channels.

Although one-way relay cooperation improves performance in the wireless communication, the transmission performance will also be compromised due to the reduced transmission time slot. Therefore, two-way relay becomes a more effective method for cooperative transmission, which had attracted attention in academica [20-23]. In traditional two-way relay, exchanging different information between two nodes took place in only two phases to accomplish the transmission instead of four phases in traditional one-way relaying. In the first phase, the users simultaneously transmitted their signals to the relays. Subsequently, in the second phase, the relays broadcasted the signals to the users [20]. The authors in [21] investigated the performance and the spectral efficiency of the two-way relay transmission and compared them with the traditional one-way relaying transmission. A cooperative CRN was studied [22], where two PUs exchanged information with the help of a SU that was equipped with multiple antennas and in return, the SU superimposed its own messages along with the primary transmission. The energy efficiency problem was investigated [23] for underlay cognitive multiuser two-way relay networks, which jointly optimized the power allocation of secondary users and the beamforming matrix of the cognitive relay.

2.2. Energy Harvesting

A green paradigm for the underlay coexistence of the PUs and the SUs in energy harvesting CRNs was studied [24], wherein the battery-free SUs captured both the spectrum and the energy of the PUs to enhance spectrum efficiency and green energy utilization. In [25], authors presented an underlay multiple-input-multiple-output (MIMO) CRNs including a pair of primary nodes, a couple of secondary nodes, and an eavesdropper, where the secondary transmitter was powered by the renewable energy harvested from the primary transmitter in order to improve both energy efficiency and spectral efficiency. In that paper, authors investigated the green CRNs where each SU was solely powered by an energy harvester which extracted energy from RF signals of a primary transmitter [26], they also considered the feasibility of energy harvesting underlay CRNs with the primary system employing power control. Primary and secondary nodes distributed randomly in R^2 as homogeneous Poisson point processes were considered [27]. In [28], authors studied the two-hop underlay cognitive relay networks with an energy harvesting relay, and proposed a modified simple time-switching protocol in which energy was transmitted in sub-slots until the relay node was sufficiently charged. The achievable sum rate of a wireless-powered multi-cell/multi-user CR massive MIMO system with underlay spectrum sharing was investigated [29].

The natural energy sources used in these articles are time-varying and unreliable because of the time variations of harvested energy. Compared with other ambient energy harvesting techniques, the WPT technologies can provide the networks with stable energy supplies. The WPT provides a novel solution to the painstaking power-charging issue in cooperation with CRNs. Presently, there are two typical WPT technologies, which are respectively based on coupled magnetic resonances and RF signals. However, energy transfer based on magnetic resonances is usually activated by near field induction from more powerful nodes (e.g., sink and vehicles). Since RF signals carry not only energy but also information, harvesting energy from RF signals has recently spurred an upsurge of research interests on simultaneous wireless information and power transfer (SWIPT).

Although this power transfer technique is still at its initial stage, there have been some studies on it [30, 31, 32, 33, 34]. In paper [30], authors first proposed SWIPT, and assumed that the receiver was able to decode information and to harvest energy independently from the same received signal without any loss, which was not realizable due to practical circuit limitations. Consequently, two practical receiver architectures for SWIPT was designed [31], namely, (1) time switching (TS), which switched between decoding information and harvesting energy at a time, and (2) power splitting (PS), which splitted the signal into two streams with adjustable power ratio for decoding information and harvesting energy separately. By dual utilization of RF signals for information and energy transfer, SWIPT enabled wireless systems to achieve sustainable operation at a lower cost [32]-[34]. The SWIPT was applied to the wirelessly powered sensor networks, where each node had two circuits, which operated on energy harvesting mode and information decoding mode separately. The maximization of energy efficiency by taking advantage of SWIPT technique was investigated in wirelessly powered sensor networks [35]. Therefore, the WPT was reliable and stable, and energy harvesting based on WPT was appropriate for low-power applications, such as sensor networks, compared with conventional energy harvesting technique based on ambient energy sources [36].

3. SYSTEM MODEL AND TRANSMISSION SCHEME

3.1. System Model

In this paper, we consider a new two-way relay cooperation scheme in the underlay CRNs based on energy harvesting, which consists of the primary networks (PNs) and the secondary networks (SNs). The PNs are composed of one pair of primary transmitter (PT) and primary receiver (PR), and the SNs are composed of three nodes. We denote them as S_1, S_2 and the relay R respectively

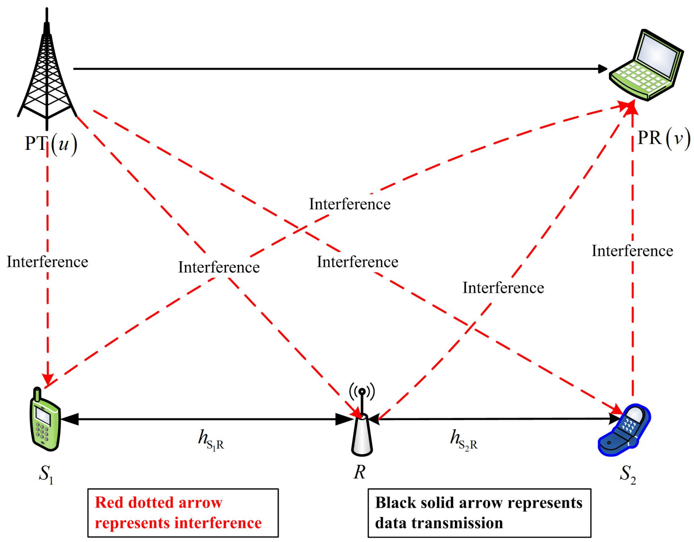

Figure 1: System model with two-way relay cooperation in the underlay CRNs

as illustrated in Fig. 1. In the underlay CRNs, the SNs which can coexist with the PNs must meet certain interference constraint, that is to say, interference for PNs caused by SNs should be less than a certain threshold value. The PNs hold the licensed spectrum and the primary users can communicate with each other, while the SNs do not have the licensed spectrum and the secondary transmission power must not exceed one fixed threshold value if it seeks to transmit its own data. Suppose that there exists a secondary user who agrees to act as the relay to assist the secondary transmission. The secondary transmission power can then be reduced sharply and at the same time the transmission rate of the secondary system can be improved. Furthermore, all nodes are assumed to be equipped with a single antenna and work on the half-duplex mode here [37].

The secondary networks can be considered as sensor networks or internet of things, because they are all known as low-power consumption, energy-constrained networks. Then they need to operate energy harvesting from the ambient natural resources or the radio frequency signals for providing energy supply and maintaining network operation in order to prolong network lifetime.

We assume that the primary users have stable state grid energy supplies in the primary networks. It is assumed that only S_1 and S_2 have stable state grid energy supplies in the secondary networks, while there is no energy provided for relay node R which means the relay node R needs to harvest energy from natural surrounding resource in transmission process among PU, S_1, and S_2.

In the secondary transmission, we assume that the transmission time is divided into equal timeslots, each of which has a duration T, and the transmission of each timeslot includes two phases that are called as multiple access (MA) phase and broadcast (BC) phase. During MA phase, both two secondary users S_1 and S_2 transmit their information with the same lower power to the relay R simultaneously in order to avoid reducing performance of the PNs. In the BC phase, the relay node R employs the harvested energy to broadcast the resulting signals along with information prepared for S_1 and S_2 in phase BC. We assume that the duration of phase MA and phase BC are equal as $\frac{T}{2}$.

The channel between transmitter u and receiver v is assumed to undergo independent Rayleigh block fading with channel gain $h_{u,v}$. It remains invariant during one fading block duration, and changes from one slot to another slot. The channel power gain is exponentially distributed with mean $|h_{u,v}| = d_{u,v}^{-\frac{\alpha}{2}}$, where $d_{u,v}$ is the distance and α is the path loss exponent. Thus $h_{u,v}^2 \sim \mathcal{CN}(0, \lambda_{u,v})$ denotes complex Gaussian distribution with mean 0 and variance λ for u and v. Suppose that the channel between u and v is symmetric, i.e., $h_{u,v} = h_{v,u}$ [33].

3.2. Transmission Model and Energy Harvesting

Firstly, we describe the secondary transmission model. In the underlay CRNs, only when the interference from the secondary networks to primary networks is less than a certain threshold value, i.e., the secondary transmission cannot impair the QoS of primary link, can the secondary data transmission be carried out. In order to improve the secondary throughput and reduce the transmission power of the secondary system, we present a promising and novel scheme which is the two-way relay cooperative scheme.

In the MA phase, both S_1 and S_2 send message to the relay R simultaneously, and we consider that the PNs also impact the SNs. Then the received signal in the relay R can be formulated as

$$y_R = \sqrt{P_{S_1}} h_{S_1,R} x_{S_1} + \sqrt{P_{S_2}} h_{S_2,R} x_{S_2} + \sqrt{P_P} h_{PT,R} x_{PT} + n_R, \tag{1}$$

where x_i is the unit-power transmitted information intended for node i ($i = S_1, S_2$), and P_{S_1}, P_{S_2} and P_P represent the transmission power of S_1, S_2 and PT respectively. They are the fixed constant during the whole timeslot. x_{PT} denotes the unit-power transmitted information intended for the PT. And $h_{S_1,R}, h_{S_2,R}$ and $h_{PT,R}$ denote the channel gain between S_1 and R, S_2 and R, PT and R, respectively. $n_R \sim \mathcal{CN}(0, \sigma_R^2)$ denotes the additive white Gaussian noise (AWGN) at the relay node R.

According to paper [31], the relay R employs power splitting to scavenge energy from radio frequency signal. The received signal can be divided into two portions, one for scavenging energy while the other for transferring signal. Therefore, the signal for scavenging energy is formulated as

$$\begin{aligned} \sqrt{\delta} y_R =& \sqrt{\delta}\sqrt{P_{S_1}} h_{S_1,R} x_{S_1} + \sqrt{\delta}\sqrt{P_{S_2}} h_{S_2,R} x_{S_2} \\ &+ \sqrt{\delta}\sqrt{P_{PT}} h_{PT,R} x_{PT} + \sqrt{\delta} n_R, \end{aligned} \tag{2}$$

where $\delta \in (0, 1)$ represents the part of signal power splitting for energy harvesting. Consequently, we can obtain the scavenging energy at the relay R as follows

$$E_R = \frac{1}{2} T \varphi \delta \left(P_{S1} |h_{S_1,R}|^2 + P_{S_2} |h_{S_2,R}|^2 + P_{PT} |h_{PT,R}|^2 \right), \tag{3}$$

where $\varphi \in [0, 1]$ denotes the energy conversion efficiency. At high SNR, we can neglect the energy of the noise. Therefore, we assume that all harvested energy is used for transmission of the following phase. Then the transmission power at the relay R can be expressed as

$$P_R = \frac{E_R}{(1/2)\, T} = \varphi \delta \left(P_{S_1} |h_{S_1,R}|^2 + P_{S_2} |h_{S_2,R}|^2 + P_{PT} |h_{PT,R}|^2 \right). \tag{4}$$

Similarly, the signal received at the primary receiver PR in MA phase can be given by

$$y_{PR} = \sqrt{P_P} h_{PT,PR} x_{PT} + \sqrt{P_{S_1}} h_{S_1,PR} x_{S1} + \sqrt{P_{S_2}} h_{S_2,PR} x_{S_2} + n_{PR}, \tag{5}$$

where $n_P \sim \mathcal{CN}(0, \sigma_P^2)$ denotes the AWGN at PR. Note that (5) still includes interference caused by the secondary transmission. By treating the secondary transmission messages as noise, the signal-to-interference-plus-noise ratio (SIN-R) of PR to decode x_{PT} is

$$\gamma_{PR}^{MA} = \frac{P_P |h_{PT,PR}|^2}{P_{S_1}|h_{S_1,PR}|^2 + P_{S_2}|h_{S_2,PR}|^2 + \sigma_{PR}^2}. \tag{6}$$

Correspondingly, in the first phase, the transmission rate of the PN is denoted by

$$R_{PR}^{MA} = \log(1 + \gamma_{PR}^{MA}).$$

In the BC phase, for AF cooperation scheme, the relay R amplifies the received signal and forwards it to the S_1 and S_2 with transmitted power P_R. The information broadcasted at relay R during BC phase is

$$x_R = G\sqrt{1-\delta}\sqrt{P_R}y_R,$$

where the normalization factor of power amplifying gain G of the relay R is

$$\begin{aligned} G &= \frac{1}{\sqrt{1-\delta}\sqrt{P_{S_1}|h_{S_1,R}|^2 + P_{S_2}|h_{S_2,R}|^2 + P_P|h_{PT,R}|^2 + \sigma_R^2}} \\ &\approx \frac{1}{\sqrt{1-\delta}\sqrt{P_{S_1}|h_{S_1,R}|^2 + P_{S_2}|h_{S_2,R}|^2 + P_P|h_{PT,R}|^2}}. \end{aligned} \tag{7}$$

Therefore, substituting (4) and (7) into x_R, the transferring signal at the relay R can be be deduced, as follows

$$x_R = \sqrt{\varphi\delta}y_R.$$

The received signal at the S_1 in BC phase is specified [38] as

$$\begin{aligned} y_{S_1}^{AF} =& h_{S_1,R}x_R + \sqrt{P_{PT}}h_{PT,S_1}x_{PT} + n_{S_1} \\ =& h_{S_1,R}\sqrt{\varphi\delta}\left[\sqrt{P_{S_1}}h_{S_1,R}x_{S_1} + \sqrt{P_{S_2}}h_{S_2,R}x_{S_2} + \sqrt{P_P}h_{PT,R}x_{PT} + n_R\right] \\ &+ \sqrt{P_P}h_{PT,S_1}x_{PT} + n_{S_1} \\ =& \sqrt{(\varphi\delta)\,P_{S_1}}|h_{S_1,R}|^2 x_{S_1} + \sqrt{(\varphi\delta)\,P_{S_2}}h_{S_1,R}h_{S_2,R}x_{S_2} \\ &+ h_{S_1,R}\sqrt{\varphi\delta}\sqrt{P_P}h_{PT,R}x_{PT} + h_{S_1,R}\sqrt{\varphi\delta}n_R + \sqrt{P_P}h_{PT,S_1}x_{PT} + n_{S_1}, \end{aligned} \tag{8}$$

where $n_{S_1} \sim \mathcal{CN}(0, \sigma_{S_1}^2)$ denotes the AWGN at S_1. Because S_1 has perfect knowledge about its own message, the received signal from R at the S_1 can be written, after cancelling self-interference, as,

$$\begin{aligned} y'_{S_1} =& \sqrt{\varphi\delta P_{S_2}} h_{S_1,R} h_{S_2,R} x_{S_2} + h_{S_1,R}\sqrt{\varphi\delta}\sqrt{P_P} h_{PT,R} x_{PT} + h_{S_1,R}\sqrt{\varphi\delta} n_R \\ & + \sqrt{P_P} h_{PT,S_1} x_{PT} + n_{S_1}. \end{aligned} \tag{9}$$

Finally, the signal to interference plus noise ratio (SINR) of S_1 to decode x_{S_2} message is thus given by

$$\gamma_{S_1} = \frac{\varphi\delta P_{S_2} |h_{S_1,R} h_{S_2,R}|^2}{\varphi\delta P_{PT} |h_{S_1,R} h_{PT,R}|^2 + |h_{S_1,R}|^2 \varphi\delta\sigma_R^2 + P_{PT} |h_{PT,S_1}|^2 + \sigma_{S_1}^2}. \tag{10}$$

Similarly, the received signal at the S_2 in BC phase is specified [38] as

$$\begin{aligned} y_{S_2}^{AF} =& h_{S_2,R} x_R + \sqrt{P_{PT}} h_{PT,S_2} x_{PT} + n_{S_2} \\ =& h_{S_2,R}\sqrt{\varphi\delta}\left[\sqrt{P_{S_1}} h_{S_1,R} x_{S_1} + \sqrt{P_{S_2}} h_{S_2,R} x_{S_2} + \sqrt{P_P} h_{PT,R} x_{PT} + n_R\right] \\ & + \sqrt{P_P} h_{PT,S_2} x_{PT} + n_{S_2} \\ =& \sqrt{(\varphi\delta)\, P_{S_1}} h_{S_1,R} h_{S_2,R} x_{S_1} + \sqrt{(\varphi\delta)\, P_{S_2}} |h_{S_2,R}|^2 x_{S2} \\ & + h_{S_2,R}\sqrt{\varphi\delta}\sqrt{P_P} h_{PT,R} x_{PT} + h_{S_2,R}\sqrt{\varphi\delta} n_R + \sqrt{P_P} h_{PT,S_2} x_{PT} + n_{S_2}, \end{aligned} \tag{11}$$

where $n_{S_2} \sim \mathcal{CN}(0, \sigma_{S_2}^2)$ denotes the AWGN at S_2. Because S_2 node also has perfect knowledge about its own message, the received signal from R at the S_2 can be written, after cancelling self-interference, as

$$\begin{aligned} y'_{S2} =& \sqrt{(\varphi\delta)\, P_{S1}} h_{S1,R} h_{S2,R} x_{S1} + h_{S2,R}\sqrt{\varphi\delta}\sqrt{P_P} h_{PT,R} x_{PT} \\ & + h_{S2,R}\sqrt{\varphi\delta} n_R + \sqrt{P_P} h_{PT,S2} x_{PT} + n_{S2}. \end{aligned} \tag{12}$$

Therefore, the SINR of S2 to decode x_{S1} message is formulated by

$$\gamma_{S_2} = \frac{\varphi\delta P_{S_1} |h_{S_1,R} h_{S_2,R}|^2}{\varphi\delta P_{PT} |h_{S_2,R} h_{PT,R}|^2 + |h_{S2,R}|^2 \varphi\delta\sigma_R^2 + P_{PT} |h_{PT,S_2}|^2 + \sigma_{S_2}^2}. \tag{13}$$

Because of the relay cooperation in the secondary networks, the interference from the secondary system to the primary system also is changed, and then the received signal in the BC phase at the PR can be expressed as

$$\begin{aligned} y_{PR}^{BC} =& \sqrt{P_{PT}} h_{PT,PR} x_{PT} + h_{R,PR} G \sqrt{1-\delta} y_R + n_{PR} \\ =& \left(\sqrt{P_{PT}} h_{PT,PR} + h_{R,PR} \sqrt{\varphi\delta} \sqrt{P_{PT}} h_{PT,R} \right) x_{PT} \\ & + h_{R,PR} \sqrt{\varphi\delta} \left(\sqrt{P_{S_1}} h_{S_1,R} x_{S1} + \sqrt{P_{S_2}} h_{S_2,R} x_{S_2} + n_R \right) + n_{PR}. \end{aligned} \tag{14}$$

As a result, the SINR of PR to decode its desired message x_{PT} with mutual interference from relay R in the BC phase can be expressed as

$$\gamma_{PR}^{BC} = \frac{P_{PT} \left(|h_{PT,PR}|^2 + |h_{R,PR} h_{PT,R}|^2 \varphi\delta \right)}{|h_{R,PR}|^2 \varphi\delta \left(P_{S_1} |h_{S_1,R}|^2 + |h_{S_2,R}|^2 + \sigma_R^2 \right) + \sigma_{PR}^2}. \tag{15}$$

Correspondingly, in the secondary phase, the transmission rate of the PN is given by

$$R_{PR}^{BC} = \log(1 + \gamma_{PR}^{BC}).$$

In the AF relay cooperation scheme, the channel coefficients $h_{PT,PR}$, $h_{R,PR}$, $h_{S_1,R}$, $h_{S_2,R}$, h_{R,S_1} and h_{R,S_2} are assumed to be independent to each other. The mobility and positioning of the nodes are incorporated into the channel statistic model. The channel coefficients are assumed to be known at the receivers.

4. PERFORMANCE ANALYSIS

In this section, we analyze the performance of the secondary system. For secondary users S_1 and S_2, we derive the exact expression of outage probability. Therefore, we investigate the trade-off between the ergodic capacity for information transfer and harvested energy for power transfer at the secondary user S_i, where $i = 1, 2$.

4.1. Outage Probability

In the underlay CRNs, the transmission power of the SUs and the relays must be adapted so that the interference occurring at the PR is below a threshold Q, which is the maximum tolerable interference level to guarantee the communication of the primary system. Then, the transmission power of S_1, the transmission power of S_2 and that of R need to fulfil certain conditions as follows

$$\frac{Q}{P_{S_1}|h_{S_1,PR}|^2 + P_{S_2}|h_{S_2,PR}|^2} \le 1, \qquad P_R \le \frac{Q}{|h_{R,PR}|^2}. \tag{16}$$

Subsequently, the harvested energy in the first phase is used to transfer the signal to both S_1 and S_2, and the transmission power of the relay R is under strict interference power and energy causality constraints as follows

$$P_R = \min\left\{P_R^{EH}, \frac{Q}{|h_{R,PR}|^2}\right\}. \tag{17}$$

An outage event arises when the transmission rate of user S_i $(i = 1, 2)$, is below the given target rate. Accordingly, the outage probability for each user in the secondary networks is given as

$$\mathcal{P}_{S_i}^{out} = \Pr\left(R_{S_i} < R_{S_i}^{target}\right) = \Pr\left(\gamma_{S_i} < \gamma_{S_i}^{target}\right), \tag{18}$$

where $R_{S_i}^{target}$ denotes the target transmission rate of the S_i $(i = 1, 2)$.

For the sake of simplicity, we also assume that transmitted power of the secondary users S_1 and S_2 is equal, that is to say $P_{S_1} = P_{S_2} = P_S$. Without loss of generality, we presume that the antenna noise power at all receiver is equivalent [39]. Then, we have $\sigma_R^2 = \sigma_P^2 = \sigma_{S_1}^2 = \sigma_{S_2}^2 = \sigma_0^2$. Therefore, the SINR of S_2 to decode the intended x_{S_1} at the secondary user S_2 and the SINR of S_1 to decode the intended x_{S_2} at the secondary user S_1 are thus rewrited as respectively

$$\gamma_{S_1} = \frac{\varphi\delta P_S |h_{S_1,R}h_{S_2,R}|^2}{\varphi\delta P_P |h_{S_1,R}h_{PT,R}|^2 + |h_{S_1,R}|^2 \varphi\delta\sigma_0^2 + P_P |h_{PT,S_1}|^2 + \sigma_0^2}, \tag{19}$$

$$\gamma_{S_2} = \frac{\varphi\delta P_S |h_{S_1,R}h_{S_2,R}|^2}{\varphi\delta P_{PT} |h_{S_2,R}h_{PT,R}|^2 + |h_{S_2,R}|^2 \varphi\delta\sigma_0^2 + P_P |h_{PT,S_2}|^2 + \sigma_0^2}. \tag{20}$$

In accordance with above two equations, we have following propositions.

Proposition 1: Let $a = \frac{P_P}{\varphi\delta}\gamma_{S_1}^{target}, b = \frac{\sigma_0^2}{\varphi\delta}\gamma_{S_1}^{target}, c = P_P\gamma_{S_1}^{target}, d = \sigma_0^2\gamma_{S_1}^{target}$, and denote $\gamma_{S_i}^{target} = 2^{R_{S_i}^{target}} - 1$ $(i = 1, 2)$. Then according to the probability density function and character of exponential distribution function, we obtain the following outage probability for second users S_1,

$$\mathcal{P}_{S_1}^{out} = 1 - H_1 + H_2 - H_3, \tag{21}$$

where

$$H_1 = \frac{P_S}{c\lambda_z + P_S} \exp\left(-\frac{d}{P_S}\right), \tag{22}$$

$$H_2 = \frac{\lambda_{\mathrm{y}} P_S}{(\lambda_{\mathrm{y}} P_S + \lambda_{\mathrm{z}} c)} \exp\left(-\frac{d}{\lambda_{\mathrm{y}} P_S}\right), \tag{23}$$

and

$$\begin{aligned} H_3 = \int_0^\infty \int_0^\infty & \exp\left(\frac{cz+d}{P_S \lambda_y}\right) \sqrt{4P_S\left(aw+b\right)} \\ & \times \mathcal{K}_1\left(\sqrt{\frac{4\left(aw+b\right)}{\lambda_y^2 P_S}}\right) f\left(z\right) f\left(w\right) dzdw. \end{aligned} \tag{24}$$

In (24), $\mathcal{K}_n(\cdot)$ is the modified Bessel function of the second kind with order n defined in [40]. It is noted that it is intractable to derive a closed-form expression of H_3 in Proposition 1. Nevertheless, it is not difficult to approximate expression of H_3. However, we ignore the mathematical derivation process due to space limit.

Proof: The proof is given in the Appendix A.

In a similar way, we derive the outage probability $P_{S_2}^{out}$ for secondary user S_2.

Proposition 2: Let $l = \frac{P_P}{\varphi\delta}\gamma_{S_2}^{target}, m = \frac{\sigma_0^2}{\varphi\delta}\gamma_{S_2}^{target}, n = P_P\gamma_{S_2}^{target}$, $k = \sigma_0^2\gamma_{S_2}^{target}$, and for second users S_2 in the secondary system, we obtain the following formulation of the outage probability,

$$\mathcal{P}_{S1}^{out} = 1 - Q_1 + Q_2 - Q_3, \tag{25}$$

where

$$Q_1 = \frac{P_S}{n\lambda_z + P_S} \exp\left(-\frac{k}{P_S}\right), \tag{26}$$

$$Q_2 = \frac{\lambda_{\mathrm{y}} P_S}{(\lambda_{\mathrm{y}} P_S + \lambda_{\mathrm{z}} n)} \exp\left(-\frac{k}{\lambda_{\mathrm{y}} P_S}\right), \tag{27}$$

and

$$\begin{aligned} Q_3 = \int_0^\infty \int_0^\infty & \exp\left(\frac{nz+l}{P_S \lambda_y}\right) \sqrt{4P_S\left(lu+q\right)} \\ & \mathcal{K}_1\left(\sqrt{\frac{4\left(lu+q\right)}{\lambda_y^2 P_S}}\right) f\left(z\right) f\left(u\right) dzdu. \end{aligned} \tag{28}$$

in (28), $\mathcal{K}_n(\cdot)$ is the modified Bessel function of the second kind with order n defined in [40].

Proof: Similar to the notations in Appendix A, define $X = |h_{S_1,R}|^2, Y = |h_{S_2,R}|^2, Z = |h_{PT,R}|^2, U = |h_{PT,S_2}|^2$. The rest of proof is similar to Proposition 1. Because of space limit, we ignore the detailed derivation process.

4.2. Ergodic Sum-rate of Secondary Networks

In this subsection, we discuss the influence of the secondary transmission power to the PU system. Subsequently, we can derive the throughput of the secondary networks for two-way AF relaying channels with energy harvesting in the underlay CRNs.

We assume that there is no energy consumption in receiving signal. All of the harvested energy should be applied to forward the SU massage. Because the SU operates in the underlay mode, the transmitting power of the SU and the maximum tolerable interference limit value Q must be satisfied. The power constraints are applied to the SU terminals S_1 and S_2, and relay R. The power allocation strategy considering interference limit is

$$\frac{Q}{P_{S_1}|h_{S_1,PR}|^2 + P_{S_2}|h_{S_2,PR}|^2} = 1, \qquad P_R = \frac{Q}{h^2_{R,PR}}. \tag{29}$$

Therefore

$$P_{S_1} = \epsilon \frac{Q}{|h_{S_1,PR}|^2}, \qquad P_{S_1} = (1-\epsilon)\frac{Q}{|h_{S_2,PR}|^2}, \tag{30}$$

where ϵ is the interference power distribution parameter.

For the underlay CRNs, the interference constraint of the PU must be met. Thus the performance of secondary system is limited by the transmission power rather than the maximum available power at S_1, S_2, or R. Then, in this case, due to $\max\{P_{\max}, \frac{Q}{|h_{R,PR}|^2}\} = \frac{Q}{|h_{R,PR}|^2}$, the outage of secondary users is the lowest. Therefore, we optimize the interference power distribution parameter ϵ so that we can maximize the ergodic sum-rate for the system under consideration, which is given by

$$R^S = \mathbb{E}\big[\log_2(1+\gamma_{S_1})\big] + \mathbb{E}\big[\log_2(1+\gamma_{S_2})\big]. \tag{31}$$

The optimization problem for maximizing ergodic sum-rate with interference power distribution parameter ϵ is therefore expressed as

$$\max_{\epsilon} R^S = \frac{\partial \mathbb{E}\big[\log_2(1+\gamma_{S_1})\big]}{\partial \epsilon} + \frac{\partial \mathbb{E}\big[\log_2(1+\gamma_{S_2})\big]}{\partial \epsilon}. \tag{32}$$

According to the equations above, we find that only the numerator term contains ϵ and $1-\epsilon$. Besides, due to the monotonous increasing nature of logarithmic function, we can readily conclude the concavity of ergodic sum-rate. The optimized value of interference power distribution parameter ϵ that maximizes sum-rate is found to be $1/2$ regardless the value of other system parameters, which means that the sum-rate is always maximal when the interference power distribution is half between S_1 and S_2.

5. SINGLE RELAY SELECTION SCHEME

In this section, we consider multi-relays scheme. As shown in Fig. 2, we assume that there are M potential relays which assist the SUs to transmit data in the cooperation CRNs. For the underlay mode, we need to select the best relay as the SU relay node which can use AF scheme to forward messages. The ith candidate relay node is denoted by R_i $(i = 1, 2, \cdots, M)$. Channel estimation should be performed before selecting the best relay so that the R_i $(i = 1, 2, \cdots, M)$ is aware of the channel state information of the all links between SUs and relays. Furthermore, we assume that these M candidate relay nodes are capable of harvesting energy from the ambient environment.

5.1. Transmission Model and Channel Capacity

Similar to the single relay case, we consider the same channel state information during each timeslot. Let h_{uv} denote Rayleigh channel power gains of the link between node u and node v. The channel conditional coefficient keeps static in one slot, but changes independently slot by slot. Subsequently, we also assume that the energy harvesting is stationary and ergodic at all relay nodes. We assume that the energy harvesting process has accomplished and each relay can harvest energy E_{R_i} at the beginning of data transmission.

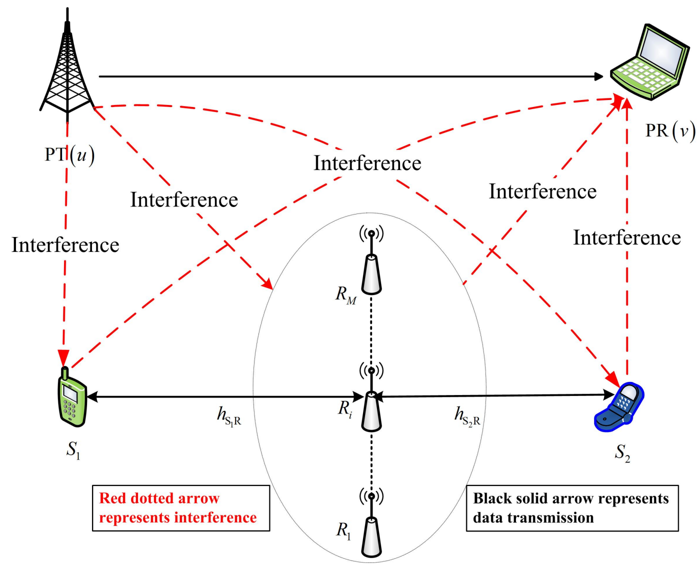

Figure 2: System model with multi-relays two-way cooperation in underlay CRNs

For the secondary transmission, there are two phases: MA phase and BC phase. In the MA phase, S_1 and S_2 send data to the relay R_i $(i = 1, 2, \cdots, M)$, and thus the signal received at the relay R_i can be expressed as

$$y_{Ri} = h_{S_1,R_i}\sqrt{P_{S_1}}x_1 + h_{S_2,R_i}\sqrt{P_{S_2}}x_2 + h_{PT,R_i}\sqrt{P_P}x_{PT} + n_{R_i}, \tag{33}$$

where h_{S_1,R_i}, h_{S_2,R_i} and h_{PT,R_i} are the channel power gains between S_1 and relay R_i, between S_2 and relay R_i, between PT and relay R_i respectively. P_{S_1}, P_{S_2} and P_P are the transmission power of the secondary system and PT respectively. n_{R_i} is the additive white Gaussian noise at the relay.

In the BC phase, the selected relay will forward signal to the SUs. In this paper, we use the AF cooperative strategy. The amplifying power normalization factor G of the relay R is given in (7). Based on the analysis, the SINR of S_1 and S_2 are given as $\gamma^i_{S_1}$ and $\gamma^i_{S_2}$ which are similarly as (10) and (13) respectively. When the bandwidth is B, the throughput is denoted as respectively

$$\mathcal{C}^i_{S_1} = B\frac{T}{2}\log(1 + \gamma^i_{S_1}), \tag{34}$$

$$\mathcal{C}^i_{S_2} = B\frac{T}{2}\log(1 + \gamma^i_{S_2}). \tag{35}$$

5.2. Relay Selection Scheme

In this section, our main objective is to select the best relay R to assist data transmission of the secondary system by forwarding secondary signal. The point to select the best relay is to find the relay which can make the secondary system achieve the maximum revenue. We define the revenue as transmission rate, which relects channel gain and harvesting energy. In other words, we determine that a relay is the best one when it has better channel gain and harvests more energy to assist the secondary transmission. To simplify the procedure, we assume that the N relays satisfying the requirement should be firstly found out.

For the relay R_i, the corresponding utility function U_i is the revenue which is transmission rate of the secondary system, which includes channel gain and harvesting energy. It can be shown as

$$U_i = \rho(\mathcal{C}^i_{S_1} + \mathcal{C}^i_{S_2}) + \kappa \frac{P_R}{E_{R_i}}, \tag{36}$$

where ρ and κ are the preference price of the transmission rate and harvesting energy utilization efficiency respectively. In this subsection, we will jointly optimize the channel gain and the transmission power of the relay R_i with forwarding system signals so that we can select the best relay. As a result, the optimization problem is given by

$$\begin{aligned} \max_{\{h_{u,v}, P_R\}} U_i &= \max\left(\rho(\mathcal{C}^i_{S_1} + \mathcal{C}^i_{S_2}) + \kappa \frac{P_R}{E_{R_i}}\right), \qquad (37)\\ s.t \quad & B_1: \ \frac{Q}{P_{S_1}|h_{S_1,PR}|^2 + P_{S_2}|h_{S_2,PR}|^2} \leq 1,\\ & B_2: \ P_R = \min\left\{\frac{E_{R_i}}{\tau}, \frac{Q}{|h_{R,PR}|^2}\right\},\\ & B_3: \ \mathcal{C}^i_{S_1} > \mathcal{C}_{th}, \mathcal{C}^i_{S_2} > \mathcal{C}_{th}. \end{aligned}$$

The constraint B_1 refers to the limitation regarding the interference power from the secondary system to the primary system. The constraint B_2 represents the power of the relay for forwarding the SUs' signals, and it must be less than harvesting energy and interference power. The constraint B_3 denotes that the transmission capacity should meet the demand of the secondary system after selecting the relay. As a consequence, the ith relay is selected only if it will receive the maximum utility because of the optimal channel gain and the harvesting energy, i.e.

$$U_i^* = \max_{\{h_{S_j}, R_i, P_R\}} U_i, \quad i = 1, 2, \cdots, N; j = 1, 2. \tag{38}$$

It is quite clear that the objective function is convex on the account of logarithmic monotonicity of the capacity and the linear of the harvesting energy. By using the traditional lagrangian duality optimization (LDO) with Karush-Kuhn-Tucker (KKT) conditions, we can easily resolve that optimization problem in polynomial time. Particularly, we need to conduct some mathematical operations with linear equations and inequalities to receive the optimum value. Specifically, these Lagrange multipliers need to be determined by iterative algorithm, for example sub-gradient method. Furthermore, inequality constraints from B_1 to B_3 and the fact that the closed-form resolution cannot be obtained inspire us to resort to the method based on revenue function. This method is very effective to solve the convex optimization with nonlinear constraints by using limited iterations and avoiding solving equations. According to the aforementioned steps, we can acquire the optimal solution of the problem (37). And then the revenue of each potential relay R_i is known. Subsequently, each potential relay provides feedback its own information consolidation about channel gain and harvesting energy to the SUs, and the SUs find the optimal relay R_i to decode the forwarding signals.

6. NUMERICAL RESULTS AND DISCUSSIONS

6.1. Numerical Results

In this section, we evaluate the proposed model using analytical and simulation results. The primary transmission power is 10 dB, and the interference limit from the SUs and the relay is 0.5 dB. The secondary targeted rate is 0.3 bit/s/Hz. There are 8 potential relays which may assist to transmit message and deploy randomly between S_1 and S_2. The step value d is 0.05, the allowed

Algorithm1: Interior point revenue function based on channel gain and harvesting energy

1. Initialization:

Each potential relay R_i receives channel gain and the harvesting energy.

2. Derive interior point revenue function based on certain coefficients:

1) Set up the initial value of the revenue function variables $C^{(0)}$ and the allowed error $\mu > 0$.

2) Select an initial point of h_{S,R_i} and P_{R_i}, and denote them as $Z^{(0)} = (h_{S,R_i}^{(0)}, P_{R_i}^{(0)})$ and let $n = 1$.

3) for n=1 to the limit number of iterations

Using $\mathbf{Z}^{(n-1)}$ to derive the optimal solution coefficient, the unrestricted problem of $\max_{\mathbb{A}} \phi(\mathbf{Z}, \mathcal{C}^{(0)})$, where $\mathbb{A}$ is the feasible domain of optimal problem.

if $||\mathbf{Z}^*(\mathcal{C}^{(\mathbf{n})}) - \mathbf{Z}^*(\mathcal{C}^{(\mathbf{n-1})})|| \leq \mu$ and $\frac{\phi(\mathbf{Z}^*(\mathcal{C}^{(\mathbf{n})}))}{\phi(\mathbf{Z}^*(\mathcal{C}^{(\mathbf{n-1})}))} \leq \mu + 1$

then the iteration is over and $\mathbf{Z}^*(\mathcal{C}^{(n)})$ is the optimal solution.

else $\mathcal{C}^{(n+1)} = d\mathcal{C}^{(n)}$, where d is the step size.

$n = n + 1$;

end if

end for

3. Relay selection:

Each potential relay R_i achieves its optimal utility value. Then it sends this value to the SUs. The SUs select the one which has the maximum revenue and decode signals from this R_N to forward signals.

error μ is 0.01. The transmission timeslot is normalized to 1. The bandwidth of each sub-channel is normalized to 1. Both the distances from the S_1 to relay and from S_2 to relay are equivalent, 10 m. Numerical results are presented to analyze and verify the accuracy of the derived analytical expressions. In the simulation, we consider that both SUs have the same transmission power.

In Fig. 3, it shows the changes in the relationship between outage probability and the SINR of the SU. Since the SINR is directly proportional to the transmission power under the invariable channel gain, we can find that with the increase of the transmission power of the SU, the outage probability significantly decreases. We can also find that when the power within the scope of interference threshold adds up to 30 dB, the outage probability becomes very small. This is to say the secondary system can successfully transmit data as high as possible. At the same time, when selecting the best relay, the outage probability of the successful data transmission can also be improved.

Fig. 4 shows that there are certain impacts on performance (outage probability) of the PU from SINR of the SU (transmission power) under different schemes. From the Fig. 4, we know that analytical results for the outage probability closely match with the numerical results. Therefore, the correctness of our analytical model is rational. We can discover that the outage probability of

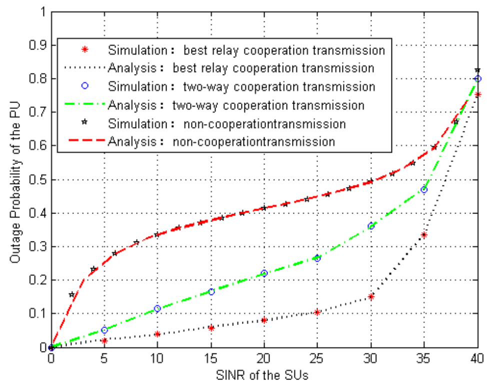

Figure 4: Outage probability of the PU vs. SINR of the SU

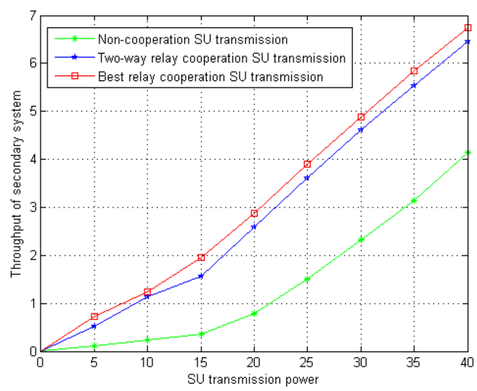

Figure 5: Secondary system throughput vs. SU transmission power

the PU increases with the increase of the SINR of the SU. When the transmission power increases to a certain value, the performance of the PU is seriously affected. We also can find that the proposed scheme is preponderance with the other schemes.

Fig. 5 plots the throughput vs SU power allocation and compares throughput with three different transmission modes. This plot also indicates the influence of SU transmission power. Clearly, the throughput increases when we enlarge the SU transmission power. The larger the transmission power becomes, the more the channel SINR and throughput are under the interference range. In

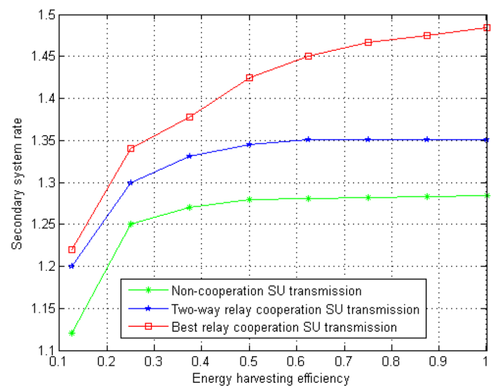

Figure 6: Effect of relay R's energy harvesting efficient

addition, one can see that the cooperation transmission mode is always better than non-cooperation mode. Similarly the best relay cooperation transmission can also improve the performance of the secondary system.

Fig. 6 shows the effect of relay R's energy harvesting, where the proposed joint energy harvesting with relay selection and two-way cooperation scheme outperforms the non-cooperation scheme and the random selection scheme. We note that the growth in energy harvesting efficiency improves the secondary system rate in all cases with or without cooperation as expected. In a slot, the energy harvesting efficiency limits the transmission power from the relay to SUs even if the relay has a better channel to SU. Thus the gain achieved by the cooperation scheme is reduced. However, as the efficiency of energy harvesting grows, the relay R can obtain more energy to keep the latter data forward for the cooperation and achieve significant rate gain over no energy cooperation scheme. If the energy harvesting efficiency is sufficiently large to accommodate the harvested energy, there is a very limited additional advantage. Due to the interference limitation from the relay R to the PU, the secondary system rate becomes greater, and the relay R's energy harvesting efficiency also increases. At last, it is obvious that the secondary system rate in our proposed scheme is higher than the other schemes.

6.2. Further Discussion

Based on our above study, we can extend our proposed scenario to the multiple secondary users' case. With the increasing number of SUs, two challenges will arise. One is that the interference between the SUs transmissions will be generated so that the power and band allocation will be more complicated. The other one is that the interference from the SU system to the PU system will increase and we can formulate the process of the relay cooperation and the relay selection as a matching problem in the secondary networks.

7. CONCLUSIONS

In this paper, we investigate a two-way relay cooperation scheme for underlay CRNs with energy harvesting, where AF relay can harvest energy from radio frequency signal and assist the secondary transmission. The secondary system with relay-assisted is in a two-way transmission mode. The outage probability of the secondary system is derived and the ergodic sum-rate of the secondary system is analyzed. In addition, we consider multi-relays selection scheme and we employ the interior point method based on penalty function to find the optimal relay. Numerical results show that the proposed scheme gives higher throughput for the secondary networks than other strategies.

Acknowledgement

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Appendix A

PROOF OF PROPOSITION

Substituting equation (10) into equation (18), we can rewrite $P_{out}^{S_1}$ as follows

$$\begin{aligned}
&\mathcal{P}_{out}^{S_1}\\
&=\Pr\left[\frac{\varphi\delta P_{S_2}|h_{S_1,R}h_{S_2,R}|^2}{\varphi\delta P_{PT}|h_{S_1,R}h_{PT,R}|^2+|h_{S_1,R}|^2\varphi\delta\sigma_R^2+P_{PT}|h_{PT,S_1}|^2+\sigma_{S_1}^2}<\gamma_{S_1}^{t\arg et}\right]\\
&=\Pr\left[X\left(P_{S_2}Y-\left(P_{PT}Z+\sigma_R^2\right)\gamma_{S_1}^{t\arg et}\right)<\gamma_{S_1}^{t\arg et}\left(\frac{P_{PT}W}{\varphi\delta}+\frac{\sigma_{S_1}^2}{\varphi\delta}\right)\right]. \qquad (39)
\end{aligned}$$

Let $a=\gamma_{S_1}^{t\arg et}\frac{P_{PT}}{\varphi\delta}, b=\gamma_{S_1}^{t\arg et}\frac{\sigma_{S_1}^2}{\varphi\delta}, c=P_{PT}\gamma_{S_1}^{t\arg et}, d=\sigma_R^2\gamma_{S_1}^{t\arg et}$, $X=|h_{S_1,R}|^2, Y=|h_{S_2,R}|^2, Z=|h_{PT,R}|^2, W=|h_{PT,S_1}|^2$

Then, therefore the $P_{out}^{S_1}$ can be reformulated as

$$\begin{aligned}
\mathcal{P}_{out}^{S_1}&=\Pr\left[X\big(P_{S_2}Y-(cZ+d)\big)<aW+b\right]\\
&=\Pr\left[X<\frac{aW+b}{P_{S_2}Y-(cZ+d)}\middle|Y>\frac{(cZ+d)}{P_{S_2}}\right]\\
&\quad+\Pr\left[X>\frac{aW+b}{P_{S_2}Y-(cZ+d)}\middle|Y<\frac{(cZ+d)}{P_{S_2}}\right]\\
&=\Pr\left[X<\frac{aW+b}{P_{S_2}Y-(cZ+d)}|Y>\frac{(cZ+d)}{P_{S_2}}\right]+\Pr\left[Y<\frac{(cZ+d)}{P_{S_2}}\right]. \qquad (40)
\end{aligned}$$

Now let us resolve the first and second in the RHS of the above equation as Ψ_1 and Ψ_2 respectively. So, we have

$$\Psi_1=\int_0^\infty\int_0^\infty\int_{\frac{(cz+d)}{P_{S_2}}}^\infty\int_0^{\frac{aw+b}{P_{S_2}y-(cZ+d)}}f(x)\,f(y)\,f(z)\,f(w)\,dxdydzdw, \qquad (41)$$

$$\Psi_2=\int_0^\infty\int_0^{\frac{(cz+d)}{P_{S_2}}}f(y)f(z)\,dydz. \qquad (42)$$

Therefore, we have

$$\begin{aligned}
&\mathcal{P}_{out}^{S_1} \\
&= \underbrace{\int_0^\infty \int_0^\infty \int_{\frac{(cz+d)}{P_{S_2}}}^\infty \left\{1 - \exp\left(-\frac{aw+b}{P_{S_2}y - (cz+d)}\right)\right\} f\left(y\right) f\left(z\right) f\left(w\right) dydzdw}_{\Psi 1} \\
&\quad + \underbrace{\int_0^\infty \left\{1 - \exp\left(-\frac{(cz+d)}{P_{S_2}}\right)\right\} f\left(z\right) dz}_{\Psi_2}. \qquad (43)
\end{aligned}$$

Subsequently, we resolve Ψ_1 and Ψ_2 separately

$$\begin{aligned}
\Psi_2 &= \int_0^\infty \left\{1 - \exp\left(-\frac{(cz+d)}{P_{S_2}}\right)\right\} f\left(z\right) dz \\
&= \int_0^\infty \left\{1 - \exp\left(-\frac{(cz+d)}{P_{S_2}}\right)\right\} \frac{1}{\lambda_z} \exp\left(-\frac{z}{\lambda_z}\right) dz \\
&= 1 - \int_0^\infty \exp\left(-\frac{(cz+d)}{P_{S_2}}\right) \frac{1}{\lambda_z} \exp\left(-\frac{z}{\lambda_z}\right) dz \\
&= 1 - \frac{1}{\lambda_z} \exp\left(\frac{d}{P_{S_2}}\right) \int_0^\infty \exp\left\{-\left(\frac{c}{P_{S_2}} + \frac{1}{\lambda_z}\right) z\right\} dz \\
&= 1 - \frac{P_{S_2}}{(\lambda_z c + P_{S_2})} \exp\left(-\frac{d}{P_{S_2}}\right), \qquad (44)
\end{aligned}$$

and

$$\begin{aligned}
\Psi_1 &= \int_0^\infty \int_0^\infty \int_{\frac{cz+d}{P_{S_2}}}^\infty \{1 - \exp(-\frac{aw+b}{P_{S_2}y - (cz+d)})\} f(y)f(z)f(w)dydzdw \\
&= \underbrace{\int_0^\infty \int_0^\infty \int_{\frac{cz+d}{P_{S_2}}}^\infty f\left(y\right) f\left(z\right) f\left(w\right) dydzdw}_{I_1} \\
&\quad - \int_0^\infty \int_0^\infty \underbrace{\int_{\frac{cz+d}{P_{S_2}}}^\infty \exp(-\frac{aw+b}{P_{S_2}y - (cz+d)}) f(y)dy}_{I_2} f(z)f(w)dzdw, \qquad (45)
\end{aligned}$$

where

$$
\begin{aligned}
I_1=&\int_0^\infty\int_0^\infty\int_{\frac{cz+d}{P_{S_2}}}^\infty \frac{1}{\lambda_{\mathrm{y}}}\exp\left(-\frac{y}{\lambda_{\mathrm{y}}}\right)\frac{1}{\lambda_{\mathrm{z}}}\exp\left(-\frac{z}{\lambda_{\mathrm{z}}}\right)\frac{1}{\lambda_{\mathrm{w}}}\exp\left(-\frac{w}{\lambda_{\mathrm{w}}}\right)dydzdw\\
=&\int_0^\infty \frac{1}{\lambda_{\mathrm{w}}}\exp\left(-\frac{w}{\lambda_{\mathrm{w}}}\right)dw\int_0^\infty \frac{1}{\lambda_{\mathrm{z}}}\exp\left(-\frac{z}{\lambda_{\mathrm{z}}}\right)\int_{\frac{cz+d}{P_{S2}}}^\infty \frac{1}{\lambda_{\mathrm{y}}}\exp\left(-\frac{1}{\lambda_{\mathrm{y}}}\right)dydz\\
=&\int_0^\infty \frac{1}{\lambda_{\mathrm{w}}}\exp\left(-\frac{w}{\lambda_{\mathrm{w}}}\right)dw\int_0^\infty \frac{1}{\lambda_{\mathrm{z}}}\exp\left(-\frac{z}{\lambda_{\mathrm{z}}}\right)\exp\left(-\frac{cz+d}{\lambda_{\mathrm{y}}P_{S_2}}\right)dz\\
=&\int_0^\infty \frac{1}{\lambda_{\mathrm{z}}}\exp\left(-\frac{z}{\lambda_{\mathrm{z}}}-\frac{cz+d}{\lambda_{\mathrm{y}}P_{S_2}}\right)dz\\
=&\frac{1}{\lambda_{\mathrm{z}}}\exp\left(-\frac{d}{\lambda_{\mathrm{y}}P_{S_2}}\right)\int_0^\infty \exp\left(-\left(\frac{1}{\lambda_{\mathrm{z}}}+\frac{c}{\lambda_{\mathrm{y}}P_{S_2}}\right)z\right)dz\\
=&\frac{\lambda_{\mathrm{y}}P_{S_2}}{(\lambda_{\mathrm{y}}P_{S_2}+\lambda_{\mathrm{z}}c)}\exp\left(-\frac{d}{\lambda_{\mathrm{y}}P_{S_2}}\right).
\end{aligned}
\tag{46}
$$

Let $P_{S_2}y-(cz+d)=m$, then $y=\frac{m+(cz+d)}{P_{S_2}}$, therefore we plug it into the integrand function expression of I_2 and denote it as following

$$
\begin{aligned}
I_2=&\int_{\frac{cz+d}{P_{S_2}}}^\infty \exp\left(-\frac{aw+b}{P_{S_2}y-(cz+d)}\right)\frac{1}{\lambda_{\mathrm{y}}}\exp\left(-\frac{y}{\lambda_{\mathrm{y}}}\right)dy\\
=&\int_0^\infty \exp\left(-\frac{aw+b}{\lambda_{\mathrm{y}}}\frac{1}{q}-\frac{q+(cz+d)}{P_{S_2}\lambda_{\mathrm{y}}}\right)dq\\
=&\exp\left(\frac{cz+d}{P_{S_2}\lambda_{\mathrm{y}}}\right)\int_0^\infty \exp\left(-\frac{aw+b}{\lambda_{\mathrm{y}}}\frac{1}{q}-\frac{q}{P_{S_2}\lambda_{\mathrm{y}}}\right)dq\\
=&\exp\left(\frac{cz+d}{P_{S_2}\lambda_{\mathrm{y}}}\right)\sqrt{4P_{S_2}\left(aw+b\right)}K_1\left(\sqrt{4\frac{aw+b}{P_{S_2}\lambda_y^2}}\right).
\end{aligned}
\tag{47}
$$

Finally, we combine all the current equations, we can obtain the outage probability of the S1 as following

$$
\begin{aligned}
\mathcal{P}_{out}^{S_1}=&1-\frac{P_{S_2}}{c\lambda_z+P_{S_2}}\exp\left(-\frac{d}{P_{S_2}}\right)+\frac{\lambda_{\mathrm{y}}P_{S_2}}{(\lambda_{\mathrm{y}}P_{S_2}+\lambda_{\mathrm{z}}c)}\exp\left(-\frac{d}{\lambda_{\mathrm{y}}P_{S_2}}\right)\\
&-\int_0^\infty\int_0^\infty \exp\left(\frac{cz+d}{P_{S_2}\lambda_{\mathrm{y}}}\right)\sqrt{4P_{S_2}\left(aw+b\right)}\\
&\times K_1\left(\sqrt{4\frac{(aw+b)}{\lambda_y^2P_{S_2}}}\right)f\left(z\right)f\left(w\right)dzdw.
\end{aligned}
\tag{48}
$$

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