

JOINT TOPOLOGY AND RADIO RESOURCE OPTIMIZATION FOR DEVICE-TO-DEVICE BASED MOBILE SOCIAL NETWORKS

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ABSTRACT

In this paper, we consider a joint topology and radio resource optimization for device-to-device (D2D) based mobile social networks. The considered social network is an interest based which is modeled as a d -intersection binomial random graph. The Radio network is also modeled as a random graph where an edge between any two distinct nodes is activated with a certain probability that is equivalent to the probability of exceeding a certain signal to interference ratio for that link. The entire network is then modeled as an intersection graph between the social and radio induced graphs. Thereafter, network topology is optimized such that enabled social edges satisfy certain network connectivity constrains under specific radio environment characteristics. Radio resource allocation is performed to maximize the radio resource utilization exploiting both social ties awareness among the network nodes and knowledge of channel gains among users' locations. We formulate our radio resource allocation problem as a semidefinite program over a graph representing the network topology. Simulation based numerical results are shown in terms of achieved link efficiency and optimized topology parameters.

Index Terms— Spectrum Cartography, Radial Basis Functions, Alternating Direction Optimization.

1. INTRODUCTION

Background and Motivation- One of the potential technologies to meet the huge data rate demands in next generations wireless networks is device-to-device (D2D) communications [1]. With D2D communications, devices in the proximity of each other can exchange data directly among each other through short-range low-power radio links instead of double hop communications through base stations.

On the other hand, instead of projecting social connectivity aspects only on the application layer, one of the recent trends in wireless communications is to utilize social relations and attributes among users information in wireless networking optimization under a framework called mobile social networks [2]. Mobile social networks can be combined with D2D communications where data is disseminated cooperatively within the network using D2D links and exploiting social awareness [3]. In this paper, we develop a mechanism for mutual inference between social and radio environment knowledge aiming at optimizing the network's performance in terms of connectivity. Moreover, we exploit this mechanism to address the problem of radio resource allocation for D2D based mobile social networks.

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Related work and contribution- Radio resource allocation for D2D communications is well documented in the literature [4–6]. Regarding mobile social networks, most of the previous work tackles the issue of data routing within the D2D based mobile social networks with a common assumption of having the radio resources already assigned [7].

Social aware radio resource allocation for D2D pairs is one of the advancing fields in mobile networking. However, most of the contributions there are in the context of coordination among D2D and cellular users exploiting social ties and communities, as in [8] and [9]. Both [8] and [9] assume predefined social characteristics of users and then the radio resource allocation is performed accordingly.

In contrast to the previous related work, the contributions of this paper are highlighted as follows:

1. We optimize a D2D based mobile social network topology by exploiting both social and radio environment information. The optimized network topology meets a specific connectivity constrain.
2. We formulate and solve a radio resource allocation problem among devices forming a D2D based mobile social network. The objective of our optimization problem is to maximize the radio resource utilization.

Even though modeling a network as a single graph representing an intersection between social and radio induced graphs is used to facilitate our main contributions in 1 and 2 above, it is still useful as a standalone contribution which to the best of our knowledge has not been done before.

Notation- Upper case bold letters are used to denote matrices such as $\mathbf{C} \in \mathbb{R}^{M \times N}$ while the element corresponding to the k^{th} row, l^{th} column of \mathbf{C} is denoted as C_{kl} . Column vectors are denoted by lower case bold letters as \mathbf{c} with c_k being the k^{th} entry of vector \mathbf{c} . We use $\text{tr}(\mathbf{C})$, \mathbf{C}^T to denote the trace and transpose of matrix \mathbf{C} , respectively. A diagonal matrix where the diagonal elements are the entries of vector \mathbf{c} is denoted as $\text{diag}(\mathbf{c})$. $|\mathcal{S}|$ is used to denote the cardinality of set \mathcal{S} . $\mathbf{1}$ and $\mathbf{0}$ denote the all-one and all-zero vectors, respectively. $\mathbb{P}[\alpha]$ is the probability that event α takes place.

Organization- The rest of this paper is structured as follows. Section 2 presents some necessary preliminaries on graph theory. Section 3 introduces the system model. In section 4, joint network's topology and radio resource optimization is explained. Our simulations are discussed in Section 5. Finally, Section 6 concludes the paper.

2. PRELIMINARIES ON GRAPH THEORY

Incidence and Laplacian matrices for undirected graphs- We define an undirected graph $G(\mathcal{N}, \mathcal{E})$ composed of the node set \mathcal{N} and the edge set \mathcal{E} , where $|\mathcal{N}| = N$ and $|\mathcal{E}| = M$. For each edge m , we define the column vector $\mathbf{a}^m \in \mathbb{R}^N$ where $a_i^m = 1, a_j^m = -1$ if the edge m connects nodes i and j , $a_k^m = 0, \forall k \neq i, j$. The incidence matrix [10] of graph G , $\mathbf{A} \in \mathbb{R}^{N \times M}$ is given by $\mathbf{A} = [\mathbf{a}^1 | \mathbf{a}^2 | \dots | \mathbf{a}^M]$ which indicates the available edges.

We define the weight vector $\mathbf{w} \in \mathbb{R}^M$ as $\mathbf{w} = [w_1 \ w_2 \ \dots \ w_M]^T$ where w_i is the corresponding normalized weight for the i^{th} edge. Therefore, the vector \mathbf{w} has to fulfill $\mathbf{0} \preceq \mathbf{w} \preceq \mathbf{1}$ and $\mathbf{1}^T \mathbf{w} = 1$. If we define $\mathbf{W} \in \mathbb{R}^{M \times M}$ as $\mathbf{W} := \text{diag}(\mathbf{w})$, then the Laplacian \mathbf{L} is given by:

$$\mathbf{L} = \mathbf{A} \mathbf{W} \mathbf{A}^T \quad (1)$$

Graph connectivity- A graph is connected if there exist at least one path in between any two distinct nodes. Graph connectivity is determined by the fact that the Laplacian is a positive semidefinite matrix [10] i.e. $\mathbf{L} \succeq 0$. Let us assume that the ascendingly ordered eigenvalues of \mathbf{L} are denoted by $\lambda_1, \lambda_2, \dots, \lambda_N$, then $\lambda_1 = 0 \leq \lambda_2 \leq \dots \leq \lambda_N$. In order to have a single connected graph, the second eigenvalue λ_2 , known as Fiedler eigenvalue or algebraic connectivity, has to satisfy $\lambda_2 > 0$ [11]. A graph is said to be k -connected if it stays connected after a removal of any $(k-1)$ edges [12].

A graph intersection- $G_c = G_a \cap G_b$ over the same set of nodes \mathcal{N} means that nodes $i, j \in \mathcal{N}$ have an edge in between in G_c if and only if they have an edge in between in both G_a and G_b .

Random Graph- Random graphs were introduced by Erdős and Rényi [13] and defined such that an edge between any two nodes exists with a specific probability independently of other edges. One of the key random graph models is a *binomial random d -intersection graph*, which is defined over \mathcal{N} as follows: Given a pool of items, each item is assigned to a given node with a specific probability independently of other nodes. Consequently, an edge between any two nodes is established if they have at least d items in common [12].

3. SYSTEM MODEL AND PROBLEM FORMULATION

3.1. Social network model

We consider a network consisting of a set \mathcal{N} of nodes randomly deployed in a geographical area, with each node indexed as i , $1 \leq i \leq N$ (i.e. $|\mathcal{N}| = N$). Social ties among nodes can be reflected by their common interest. Quantitatively, if each node i selects a set of objects \mathcal{X}_i from a pool of available objects \mathcal{P} where $\mathcal{X}_i \subseteq \mathcal{P}$ and $|\mathcal{P}| = P$, then two nodes i and j are socially connected if they select at least d common objects from the pool [14]. An example of such a pool could be for instance *Youtube* where the objects are all available videos.

The social networks can be modeled as an undirected random graph as follows. if we assume that each object in \mathcal{P} is selected with a probability ν , then our social network can be modeled as an undirected binomial random d -intersection graph denoted as $G_s(\mathcal{N}, d, \nu, \mathcal{P})$ which has a set of edges \mathcal{E}_s defined as

$$\mathcal{E}_s(d, \nu, \mathcal{P}) = \{(i, j) \mid |\mathcal{X}_i \cap \mathcal{X}_j| \geq d\} \quad (2)$$

For the sake of simplicity, the notation $G_s(\mathcal{N}, \mathcal{E}_s)$ will be used instead of $G_s(\mathcal{N}, d, \nu, \mathcal{P})$ and it will be referred to as the social

graph hereafter.

We denote the social tie between two nodes i and j as Δ_{ij} , which is defined as the ratio between the number of common objects in sets \mathcal{X}_i and \mathcal{X}_j and the total number of shared objects within the network. Δ_{ij} is therefore obtained by

$$\Delta_{ij} := \begin{cases} \frac{|\mathcal{X}_i \cap \mathcal{X}_j|}{\sum_{l=1}^N \sum_{\substack{m=1 \\ m \neq l}}^N |\mathcal{X}_l \cap \mathcal{X}_m|}, & i \neq j \\ 0, & i = j \end{cases} \quad (3)$$

Subsequently, we consider a symmetric matrix $\mathbf{\Delta} \in \mathbb{R}^{N \times N}$ to represent the social ties.

3.2. Radio network model

The nodes are assumed to have a knowledge of the radio environment maps (REMs) which are essentially spatial, frequency and time dependent maps that can be constructed for different radio parameters such as received signal power, channel gains and interference. In this paper, we consider channel gain based REMs, which can either be generated cooperatively by the nodes themselves or provided by another entity or sensing network. Thus, REM construction and dissemination are out of the scope of this paper and the reader is referred to [15, 16] on more elaborations on REMs constructions and applications. Accessing the REMs implies having a knowledge of a specific interference map at a given bandwidth centered at a certain frequency. Moreover, knowing the interference map, the attainable channel signal to interference ratios (SIRs) for a link between each pair of nodes is therefore revealed. If the link (i, j) between nodes i and j has a channel SIR denoted by γ_{ij} and modeled as a random variable (e.g. log normally distributed) and if a successful communication requires a minimum threshold γ_0 of the channel SIR, then the probability of failure of the radio link connecting nodes i and j denoted by μ_{ij} is $\mathbb{P}[\gamma_{ij} \leq \gamma_0]$. Hence, a radio environment induced random graph $G_r(\mathcal{N}, \mathcal{E}_r)$ is defined over the nodes set \mathcal{N} where nodes i and j have an edge in between $(i, j) \in \mathcal{E}_r$ with a probability μ_{ij} .

3.3. Combined social and radio network model

For a mobile social network, different objects are delivered to different nodes through D2D radio links, then the whole network can be modeled as a graph $G(\mathcal{N}, \mathcal{E})$ defined by

$$G(\mathcal{N}, \mathcal{E}) = G_s(\mathcal{N}, \mathcal{E}_s) \cap G_r(\mathcal{N}, \mathcal{E}_r) \quad (4)$$

Meaning that the intersection between the social network and the radio network induced graphs is interpreted as a network composed of the nodes set \mathcal{N} and the edges set \mathcal{E}_s , where each edge in \mathcal{E}_s is activated with a probability μ_{ij} . Let us denote the number of total edges as $M = |\mathcal{E}_s|$. Moreover, we re-index all the active radio links as $1 \leq m \leq M$ (i.e. there is a mapping $m \leftrightarrow (i, j)$) with their associated link failure probabilities as μ_m . Accordingly, we can obtain the probability that more than k links fail, denoted by ζ_k , as follows:

$$\zeta_k = \sum_{i=k}^M \sum_{j=1}^i \prod_{m=1}^i \mu_m^j \prod_{\hat{m}=i+1}^M (1 - \mu_{\hat{m}}^j) \quad (5)$$

where μ_m^j is the probability of link failure of the m^{th} link in the j^{th} realization, out of a total of $\binom{M}{i}$ of indexes of links (i.e. all possible combinations of selecting $i \geq k$ out of M links).

4. NETWORK PARAMETERS OPTIMIZATION

4.1. Social-radio network mutual inference

In order to assure objects dissemination across a social network, the graph $G(\mathcal{N}, \mathcal{E})$ has to be connected. As proved in [17], for our given binomial random d -intersection graph $G_s(\mathcal{N}, \mathcal{E}_s)$, the probability $\theta_{k,d}$ of k -connectivity, is given by

$$\theta_{k,d} = e^{-\frac{e^{-\omega}}{(k-1)!}} \quad (6)$$

where $\omega \in [-\infty, +\infty]$ such that:

$$\frac{1}{d!} \cdot \nu^{2d} \cdot P^d = \frac{\ln N + (k-1)\ln(\ln N) + \omega}{N}$$

This formula of $\theta_{k,d}$ holds under the condition $P \gg N$, which is imposed primarily for technical reasons but it is satisfied in most cases in practice as the size of objects pool is usually much larger than the number of users accessing it within a specic geographical area [14].

If the network has to be connected with a probability at least η , then our objective is to find the least number of links that assure this connectivity under the constrain that the minimum required common interest for establishing a social edge between any two nodes is kept as small as possible but not smaller than the original default value. Therefore, following is a formulation of topology optimization problem

$$\begin{aligned} & \underset{M, \kappa}{\text{minimize}} && M \\ & \text{subject to} && M = |\mathcal{E}_s|, \theta_{k,d} \geq \eta \\ & && \kappa \geq -d_{init}, \kappa = -d, \kappa \text{ integer} \end{aligned} \quad (7)$$

Where κ is a variable equivalent to $-1 \times d$ injected to be minimized which implies maximizing d and d_{init} is the initial value of d .

The optimization problem formulated in (7) is non convex as its feasible set is a discrete set, hence we solve it as follows.

1. We can find the maximum number k^* of links failing simultaneously with a probability $(1 - \eta)$ given by

$$k^* = k \text{ such that } \zeta_k \leq (1 - \eta) \quad (8)$$

Finding k^* by (8) uses the initial social network settings in terms of the minimum common interest d , which determines the topology and hence the number of links M .

2. Thereafter, the social network minimum common interest for establishing a social link needs to be updated to accommodate connectivity constrains in terms of η as:

$$d^* = \min(d_{init}, d \text{ such that } \theta_{k^*+1, d+1} < \eta \leq \theta_{k^*+1, d}) \quad (9)$$

It is important to note that modifying the minimum common interest as in (9) is a requirement to meet a network connectivity with probability η . Therefore, the minimum possible social edges among nodes are superimposed by determining the minimum required common interest for the connectivity constrain η .

3. After finding d^* , the incidence matrix $\mathbf{A}(d^*)$ of our optimized topology is found as social edges are identified. For a simplified notation, we use \mathbf{A} instead of $\mathbf{A}(d^*)$ hereafter.

4.2. Radio resource allocation

The entire available bandwidth B is divided into orthogonal M radio resource block RRB. Our ultimate goal is to maximize the utilization of the available bandwidth in terms of link efficiency constrained by the network connectivity. Next, we formulate analytically our objective with its associated constraints.

The entries of link weights vector \mathbf{w} have to be assigned proportionally to the volume of traffic they carry. If we assume that all objects have similar traffic volume then we can characterize carried traffic by the social ties matrix Δ ¹. Moreover, the assigned weights have to take into account also the differences in the SIRs across the links. Accordingly, we define our utility function $f(\mathbf{W})$ as:

$$f(\mathbf{W}) = \sum_{i=1}^N \sum_{j=1}^N \Delta_{ij} \underbrace{\log_2(1 + \bar{\gamma}_{ij})}_{\text{Link capacity}} \underbrace{(-L_{ij})}_{\text{Link weight}} \underbrace{(1 - \mu_{ij})}_{\text{Link activity}} \quad (10a)$$

where $\mathbf{W} = \text{diag}(\mathbf{w})$ and $L_{ij} = [\mathbf{A}\mathbf{W}\mathbf{A}^T]_{ij}$. If we define the matrix $\mathbf{\Gamma} \in \mathbb{R}^{N \times N}$ to be the link capacity matrix with components $\Gamma_{ij} = \log_2(1 + \bar{\gamma}_{ij})$ and the link activity probability matrix $\mathbf{\Xi} \in \mathbb{R}^{N \times N}$ where $\Xi_{ij} = (1 - \mu_{ij})$, then $f(\mathbf{W})$ can be written in a compact form as follows:

$$f(\mathbf{W}) = -\text{tr}(\Delta \mathbf{\Gamma} \mathbf{A} \mathbf{W} \mathbf{A}^T \mathbf{\Xi}) \quad (10b)$$

The minus sign in (10a) and (10b) comes from the fact that $\mathbf{A}\mathbf{W}\mathbf{A}^T$ has non-positive off diagonal entries.

Maximizing $f(\mathbf{W})$ may result in sparse \mathbf{w} , however network connectivity needs to be maintained which imposes constrains on \mathbf{W} that are derived as follows.

As introduced in Section 2, a connected graph has a Laplacian with a zero eigenvalue of multiplicity 1. This zero eigenvalue corresponds to the eigenvector $\frac{1}{\sqrt{N}}\mathbf{1}$. Henceforth, we use this property to impose constrains on the Laplacian $\mathbf{L} = \mathbf{A}\mathbf{W}\mathbf{A}^T$ that ensure a connected graph. As it is well known, there exist $(N - 1)$ eigen vectors that can be chosen to be orthonormal to $\frac{1}{\sqrt{N}}\mathbf{1}$, which can be obtained using Gram-Schmidt orthogonalization. Let us call these vectors $\mathbf{v}_1, \dots, \mathbf{v}_{N-1}$, then we can form the matrix \mathbf{V} as

$$\mathbf{V} \in \mathbb{R}^{N \times (N-1)} := [\mathbf{v}_1 | \mathbf{v}_2 | \dots | \mathbf{v}_{N-1}] \quad (11a)$$

The resulting projection matrix onto the complement space of $\frac{1}{\sqrt{N}}\mathbf{1}$ is:

$$\mathbf{V}\mathbf{V}^T = \mathbf{I} - \frac{1}{N}\mathbf{1}\mathbf{1}^T \quad (11b)$$

Furthermore, since $\frac{1}{\sqrt{N}}\mathbf{1}$ is in the null space of \mathbf{L} , then:

$$\mathbf{L}\mathbf{V}\mathbf{V}^T = \mathbf{L}\mathbf{I} - \mathbf{L}\frac{1}{N}\mathbf{1}\mathbf{1}^T = \mathbf{L}. \quad (11c)$$

In order to identify the characteristics of the Laplacian under connectivity constraints, we define the matrix $\mathbf{Q}_1 \in \mathbb{R}^{N \times N}$ as

$$\mathbf{Q}_1 := \mathbf{L} - t\mathbf{V}\mathbf{V}^T = (\mathbf{L}\mathbf{V} - t\mathbf{V})\mathbf{V}^T \quad (11d)$$

such that $\lambda_2 = t > 0$. Similarly, to facilitate network connectivity constrains, we introduce $\mathbf{Q}_2 \in \mathbb{R}^{(N-1) \times (N-1)}$ as:

$$\mathbf{Q}_2 := \mathbf{V}^T(\mathbf{L}\mathbf{V} - t\mathbf{V}) \succ 0 \quad (11e)$$

¹In case different objects have different traffic volume, each object can be weighted by its traffic volumes and incorporate that in the social ties matrix

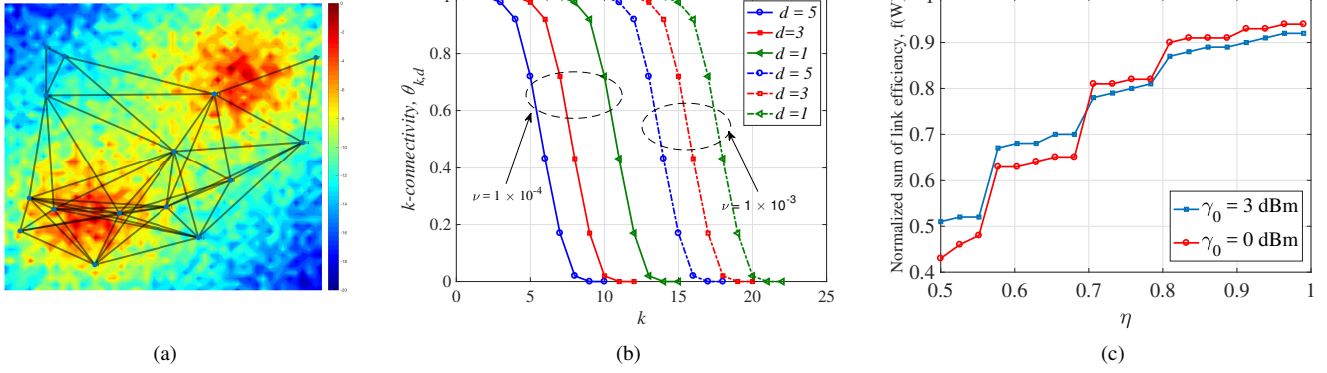


Fig. 1. (a) A realization of a simulated radio map incorporates a social network with the social edges. (b) k -connectivity probability for different values of k , minimum common interest for social edge establishment, d and the probability of object selection, ν . The pool size is 10000 objects. (c) Achieved network normalized average throughput for different values of η and γ_0 . The pool size $P = 10000$ objects, object selection probability is 1×10^{-4} .

$\mathbf{Q}_2 \succ 0$ always holds since $\frac{1}{\sqrt{N}}\mathbf{1}$ is not in the null space of \mathbf{Q}_2 . If $\mathbf{C} \in \mathbb{R}^{n \times m}$, $\mathbf{D} \in \mathbb{R}^{m \times n}$ and $n \geq m$, then it is well known that \mathbf{CD} and \mathbf{DC} share the same m eigenvalues while \mathbf{CD} has $(n - m)$ extra zero eigenvalues. Based on this property, (11d) and (11e), \mathbf{Q}_1 and \mathbf{Q}_2 have the same positive $N - 1$ eigenvalues with one additional zero eigenvalue of \mathbf{Q}_1 . This implies that

$$(\mathbf{L}\mathbf{V} - t\mathbf{V})\mathbf{V}^T \succeq 0 \quad (11f)$$

Subsequently, considering our objective (10b), the constraint set established by (11e) and (11f) and constraints on the Laplacian \mathbf{L} , we can formulate our optimization problem as:

$$\begin{aligned} & \underset{\mathbf{W}, t}{\text{maximize}} && -\text{tr}(\Delta\Gamma\mathbf{A}\mathbf{W}\mathbf{A}^T\Xi) \\ & \text{subject to} && \mathbf{V}^T(\mathbf{A}\mathbf{W}\mathbf{A}^T - t\mathbf{I})\mathbf{V} \succ 0 \\ & && (\mathbf{A}\mathbf{W}\mathbf{A}^T\mathbf{V} - t\mathbf{V})\mathbf{V}^T \succeq 0 \\ & && \mathbf{W} \succeq 0, \mathbf{W} \text{ diagonal}, \text{tr}(\mathbf{W}) = 1 \\ & && t > 0 \end{aligned} \quad (12)$$

which is a semidefinite programming (SDP) convex problem.

5. SIMULATIONS AND RESULTS

A setup of N LTE D2D-supportive nodes operate at 2.6 GHz distributed uniformly at random over an indoor area of $50m \times 50m$ is considered for simulations with two LTE indoor terminals as interferers. Hence, the 3GPP propagation model in [18] is used which results in a link failure probability as $mu_{ij} = 1 - \Phi((\gamma_0 - \bar{\gamma}_{ij})/(\sqrt{2}\sigma))$, where $\Phi(\cdot)$ is the cumulative density function (CDF) of Gaussian random variables. $\bar{\gamma}_{ij}$ is the mean SIR for the link between nodes i and j and $\sigma = 6$ dB is its log-normal distribution standard deviation. $\bar{\gamma}_{ij}$, σ and γ_0 are all measured in logarithmic scale. All terminals are assumed to transmit with a power of 30 dBm. Fig. 1(a) depicts the used REM where the normalized interference is shown. Moreover, Fig. 1(a) incorporates a realization of a social network into the radio map. The solid lines connecting different nodes are the social edges obtained from a d -intersection binomial random graph. In this particular example $d = 2$, $\nu = 1 \times 10^{-3}$ and $P = 10000$ objects. Simulations are carried out for a network consists of $N = 20$ nodes

The results of $\theta_{k,d}$ are shown in Fig. 1(b). As aligned with the theory, k -connectivity decreases with the increase of k as the more edges are randomly removed, the lower the probability of a graph to stay connected. Moreover, with the increase of the minimum common interest required for establishing a social edge, d , the k -connectivity decreases as the number of edges in the network decreases and thus removing edges influences more towards getting disconnected. Furthermore, the higher the probability of object's selection from a social pool, the higher the probability of connectivity as different nodes can establish more social edges which results in having more edges in the entire network.

Our achieved objective $f(\mathbf{W})$ which is the summation of link efficiency for all links found by $-\text{tr}(\Delta\Gamma\mathbf{A}\mathbf{W}\mathbf{A}^T\Xi)$ is shown in Fig. 1(c) for different values of η and γ_0 . Higher value of η implies connected network with higher probability and therefore higher normalized overall link efficiency. on the other hand, if a network requires a higher minimum link capacity γ_0 to activate the link at lower connectivity requirements, then higher average network throughput is achievable as maintaining connectivity is superimposed by adding more links to the network by means of decreasing the minimum common interest, d^* . However, at some point, if both network connectivity requirement and γ_0 are both high, then many social links will exist and several of them will have low efficiency as they carry few objects which decrease the entire network link efficiency.

6. CONCLUSIONS

A Framework for optimizing both network topology and radio resource in D2D based mobile social networks is developed based on random graphs and convex optimization over graphs. Network topology and radio resource are jointly optimized such that a minimum network connectivity is met while radio resource is maximally utilized.

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