Non-standard Problems in an Ordinary Differential Equations Course

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We report first results from a teaching intervention in an ordinary differential equations (ODEs) course for engineering students. Our aim is to challenge traditional approaches to teaching of Existence and Uniqueness Theorems (EUTs) through the design of problems that students cannot solve by applying well-rehearsed techniques or familiar methods. We analyse how the use of nonstandard problems contributes to the development of students' conceptual understanding of EUTs and ODEs.

Keywords: existence and uniqueness theorems, design research, nonstandard problems, commognitive theory, mathematical discourse.

INTRODUCTION AND BACKGROUND TO THE STUDY

Although ODEs are an important topic in the engineering curriculum, students experience difficulties with mastering ODEs and with the very concept of a differential equation (Arslan, 2010). In our study, the lecturer, a mathematician, devised a set of nonstandard problems (see below, Problem 1 of 6) to challenge students' conceptual understanding of the EUTs. These problems formed an assessed piece of coursework.

1. (a) Verify that

$$y(x) = \frac{2}{x} - \frac{C_1}{x^2}$$

is the general solution of a differential equation

$$x^2 y' + 2xy = 2.$$

(b) Show that both initial conditions $y(1) = 1$ and $y(-1) = -3$ result in an identical particular solution. Does this fact violate the Existence and Uniqueness Theorem (EUT)? Explain your answer.

Figure 1. One of the problems in the study

We analysed how solutions changed and developed as students worked on the problems. Students' discussions in small groups were audio-recorded, transcribed, and then analysed using constructs from commognitive theory (Sfard, 2008). We are currently in the initial phase of the data analysis aimed at answering the following question: How nonstandard problems contribute to the development of students' mathematical discourse and further their conceptual understanding of fundamental notions and results in an ODE course?

RESULTS

For Problem 1a (P1a), students could use one of two methods: M1 (substitution) and M2 (integration). Working on the problems, several students changed their approach.

In the final script, only one student produced a correct and complete solution (M2) while 14 (of 19) students used M1 verifying that a given function is a solution (which is sufficient for the particular solution), but failed to explain why this solution is the general (hence incomplete M1). We conducted similar analyses for Problem 1b (P1b).

We present one extract (for P1b) as an example of our analyses of students' group discussions using commognitive constructs - narrative, routine, ritual, substantiation.

S12. The first idea was just to try to solve for C and I got the same constant, so that's OK. And I checked for asymptotes and I got one on x=0, so I noted that the equation is split to get two curves, at least, according to calculator we got it split about zero.

S11. So it's undefined at zero.

S12. Undefined at zero, so we get two different curves and both solutions work. We do not have a continuous curve which happens to intersect at these two points […]

S14. It's not continuous for $x = 0$?

S12. No. So if we take an interval from -3 to 1, it's discontinuous in this interval, so it's not a curve that happens to just hit these two points, it is two individual curves that have the same solution. So it's correct in just a tiny area.

S11. That was my argument as well. As the theorem states, there is a continuous interval but here it is split into two which contain two different t_0's.

S13. The theorem says, that there is a unique solution for every interval where the function is continuous. Since there are two intervals and there are two solutions, it does not conflict with the theorem. […]

Note that S12 is using two different visual realizations of solutions, first the algebraic representation and then the graph plotted by calculator. He shows that the realizations are not equivalent, they do not produce the same result. We see how the student demonstrates the ability to solve the problem by developing the realization tree and employing the mathematical object of "continuous solution" (discursive object). S11 is not so sure at the beginning, he is guided by S12 (considered "more experienced" interlocutor) and adopts the narrative offered by S12. S13 concludes by reformulating the expression "does not violate the theorem" as "does not conflict with the theorem".

We see how students worked to substantiate the narrative. This routine can be characterized as the exploration. Students gradually improved their abilities in developing and endorsing the EUTs narratives while working on all six tasks during the group discussions.

REFERENCES

Arslan, S. (2010). Do students really understand what an ordinary differential equation is? International Journal of Mathematical Education in Science and Technology, 41, 873-888.

Sfard, A. (2008). Thinking as communicating: human development, the growth of discourses, and mathematizing. Cambridge: Cambridge University Press.