Developing an identity as a secondary school mathematics teacher

A narrative case study of three mathematics teachers in their transition from university teacher education to employment in school

Kirsti Rø

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Doctoral Dissertation at University of Agder

University of Agder
Faculty of Engineering and Science
2018

Doctoral Dissertations at the University of Agder 185

ISSN: 1504-9272
ISBN: 978-82-7117-884-0

Printed by the Wittusen & Jensen
Oslo

Preface

Voices of others are part of this work, to whom I owe my thanks. First of all, I have had the pleasure to get to know three mathematics teachers in their early careers: Isaac, Nora and Thomas. Thank you so much for sharing your stories and your enthusiasm with me. I owe you my deepest gratitude.

Further, I want to thank my supervisors, Martin Carlsen and Frode Rønning, who have met me with great interest for my work and given me thorough feedback and support. Thank you for giving me freedom in my research and letting me follow the paths that I have found interesting.

Several people have been of invaluable help and support while writing this thesis. I want to thank my fellow Ph.D. colleagues at the University of Agder, for warmly welcoming me to a unique research community, for being the best company at conferences and doctoral courses, and for always letting me ventilate my joys and frustrations. I also want to thank my colleagues at the Department of Teacher Education at NTNU, for making the world's best workplace, for their patience, and for always making me laugh in the lunch breaks.

Alongside the writing of this thesis, I have been working part-time at the Department of Mathematical Sciences, NTNU. Thank you for giving me the opportunity to teach mathematics and being part of the mathematics community during my doctoral study. I want to thank Heidi Strømskag, for our conversations from which I always learn so much, and Kari Hag, for inviting me to social events with female mathematicians at Trondheim's best cafes.

I also want to thank my family and friends. Thank you for showing your interest in my work and being supportive, and for making me forget about it when I needed. A special thanks to my parents, for always having believed in me, for being so considerate, and for sharing your summerhouse when I needed a quiet place for writing up my thesis. Thank you, Erlend, for your patience, your presence and care.

Finally, I quote my favourite author of children's books, Astrid Lindgren, and her character Pippi Longstocking. Pippi has her own special way of doing everything, even going to school. Nevertheless, she expresses a great belief in own abilities, which I wish to pursue: "I have never tried that before, so I think I should definitely be able to do that!"

Kirsti Rø
Trondheim, Norway
September, 2017

Abstract

In this study, I report from analyses of secondary school mathematics teachers in their transition from university teacher education to employment in school. The process of becoming a mathematics teacher is studied in terms of identity and identity development, based on three prospective mathematics teachers' participation within and at the boundaries of communities of practice at the university and in school. Participants in the study are three prospective secondary school mathematics teachers from a five-year Master's programme including teacher education ("Lektorutdanning i Realfag"), and a one-year, post-graduate teacher education programme (PPU) in Norway.

Based on Wenger's (1998) social learning theory, I understand identity as negotiated experience of self when participating within and between communities of practice. Further, I adopt Ernest's (1991) model of educational ideologies and Belenky, Clinchy, Goldberger and Tarule's (1986) and Povey's (1995, 1997) categorisations of ways of knowing for describing prospective mathematics teachers' identification with practices of mathematics and mathematics teaching, and their negotiability within the related communities of practice. I also account for possible learning mechanisms that may occur from crossing boundaries of communities of practice.

Taking on a longitudinal case-study design with a series of interviews distributed across the transition, I present three comprehensive cases of prospective mathematics teachers' way into the profession. The cases are developed with methods from narrative analysis (Lieblich, Tuval-Mashiach, & Zilber, 1998; Polkinghorne, 1995), in which the teachers' accounts constitute evolving stories of becoming a secondary school mathematics teacher. By locating discontinuities in the teachers' accounts across the series of interviews, I have further identified critical events along their evolving stories. For each critical event, the associated accounts have been interpreted by means of the theoretical framework. Hence, I have searched for evidence of the prospective teachers' participation and non-participation in communities of practice and changes in their identification and negotiability when undergoing the transition from education to employment in school.

In a cross-case analysis, three teacher stories have been compared and contrasted, resulting in two dimensions for developing an identity as a secondary school mathematics teacher: *negotiating experience of self and mathematics*, and *negotiating experience of self and mathematics teaching*. The dimensions span secondary school mathematics teachers' ways of identifying with the discipline of mathematics and its teaching,

in addition to their negotiability or ways of knowing mathematics and mathematics teaching.

The study displays various reasons for becoming a secondary school mathematics teacher, various emotional relationships with the mathematics discipline, and situations of change and stagnation regarding the prospective teachers' expressed perspectives on mathematics and its teaching and learning. Further, the analyses provide means for articulating and discussing within university teacher education possible approaches to the mathematics teacher profession. The comprehensive cases can assist prospective mathematics teachers in coming to terms with their identity and to communicate it to others. Teachers' awareness of their identity can further empower negotiation of meaning and initiate anticipation regarding future practices of mathematics and mathematics teaching, and future ways of being a secondary school mathematics teacher.

Sammendrag

I denne studien rapporterer jeg fra analyser av matematikklærere (trinn 8 til 13) i overgangen fra matematikklærerutdanning ved universitet til yrkesdebut i skolen. Prosessen med å bli en matematikklærer studeres som identitetsutvikling, på bakgrunn av tre blivende matematikklæreres deltakelse i og mellom ulike praksisfellesskap på universitetet og i skolen. De tre deltakerne i studien har sin bakgrunn i femårig integrert lektorutdanning i realfag og i ettårig praktisk-pedagogisk utdanning.

Med utgangspunkt i Wengers (1998) teori for læring forstår jeg identitet som forhandlede erfaringer om selvet, på bakgrunn av en persons deltakelse i og mellom ulike praksisfellesskap. Jeg benytter Ernest (1991) sin modell for ulike kunnskaps- og utdanningssyn og Belenky, Clinchy, Goldberger og Tarule (1986), samt Povey (1995, 1997), sine kategorier av viten for å kunne beskrive matematikklærernes identifisering med ulike praksiser i matematikk og matematikkundervisning, og deres forhandlingsmuligheter i disse praksisene. Jeg gjør også rede for mulige læringsmekanismer som finner sted i grensekryssingen mellom ulike praksisfellesskap.

Studien er en longitudinell kasusstudie med en serie av intervju i overgangen fra utdanning til yrkesdebut. Jeg presenterer tre omfattende kasus av blivende matematikklærere og deres vei inn i matematikklæreryrket. Kasusene er utviklet ved hjelp av metoder fra narrativ analyse (Lieblich et al., 1998; Polkinghorne, 1995), hvor matematikklærernes beretninger danner fortellinger om det å bli en matematikklærer i ungdomsskolen og/eller videregående skole. Videre har jeg identifisert kritiske hendelser i fortellingene, ved å lokalisere uregelmessigheter eller brudd i lærernes beretninger. Beretningene tilknyttet hver kritiske hendelse er fortolket og analysert ved hjelp av studiens teoretiske rammeverk. Jeg har her søkt evidens for de kommende matematikklærernes deltakelse og fravær av deltakelse i ulik praksisfellesskap, og endringer i deres identifisering og forhandlingsmuligheter i fellesskapene.

Ved å sammenligne fortellingene i en cross-case analyse, har jeg kunnet arbeide fram to dimensjoner ved å utvikle en identitet som matematikklærer: *forhandling av erfaringer om seg selv og matematikk*, og *forhandling av erfaringer om seg selv og undervisning i matematikk*. De to dimensjonene spenner ut blivende matematikklæreres måter å identifisere seg med matematikkfaget på og dets undervisning, i tillegg til deres forhandlingsrom i matematikk og matematikkundervisningen.

Studien viser blivende læreres ulike grunner til å bli en matematikklærer, ulike følelsesmessige tilknytninger til matematikkfaget, og ulike situasjoner angående endring og stagnasjon i deres uttrykte perspektiver på matematikk, matematikkundervisning og -læring. Videre gir analysen

et middel i lærerutdanningen til å kunne sette ord på og diskutere ulike tilnærminger til matematikklæreryrket. De tre presenterte kasusene kan hjelpe kommende matematikklærere til å betrakte egen identitet og identitetsutvikling, og å formidle denne til andre. Videre kan matematikklæreres bevissthet om egen identitet legge til rette for meningsforhandling, samt initiere forventinger om fremtidig yrkespraksis og mulige veier for egen identitetsutvikling som matematikklærer i skolen.

Contents

1 Introduction

The research reported in this thesis concerns mathematics teachers' entrance into the teaching profession. More specifically, my focus is on the process of developing an identity as a secondary school mathematics teacher, when undergoing the transition from university teacher education to employment in school. The concept of identity has gained prominence within mathematics education research over the past two decades (Darragh, 2016; Losano & Cyrino, 2017; Lutovac & Kaasila, 2017), by providing an adjustable lens through which to study mathematics teacher learning (Lerman, 2000). The researcher can zoom out on the wider socio-political contexts for mathematics teachers' developing identities. Also, he or she can zoom in to the level of interactions between individuals or to an individual's relationship with mathematics. In this study, I adopt identity as a concept for better understanding the participative experiences of prospective secondary school mathematics teachers, as they move from subject studies in mathematics and university teacher education to a professional practice in school. Hence, I search for the teachers' meaning-making of themselves as actors of secondary mathematics teaching when negotiating shifting conceptions of what mathematics teaching is or may be (Beauchamp & Thomas, 2009). In this introductory chapter, I account for the background of the study, based on literature on mathematics teachers in transitions and my own professional history (Section 1.1). Further, I present in Section 1.2 the aim and research question of the study. In Section 1.3, I account for the context of the study, including the Norwegian teacher education and school system, before closing the chapter with an overview of the dissertation (Section 1.4).

1.1 Background

Becoming a mathematics teacher involves a shift from being a university student, who is learning and knowing mathematics for oneself, to becoming a mathematics teacher in school who is enabling others to know it (Rowland, Huckstep, & Thwaites, 2005). Alongside the transition, the character of the mathematics content changes. It goes from being a scientific discipline represented by the discourse taking place between mathematicians and mathematics students at the university, to becoming a school subject as part of general education, preparation for everyday life and as a basis for higher education. Hence, the process of becoming a mathematics teacher concerns changes in one's practices related to mathematics, its teaching and learning, and changes in the role of the discipline of mathematics.

Felix Klein, a German mathematician from the turn of the 19th century, described the transition between university and school as a double discontinuity (Klein, 1932): when going from school to university, and later returning to school as a teacher, and not seeing the connection between the mathematics content in either direction. Klein thus questioned the automatic transfer of advanced mathematical knowledge into relevant knowledge for teaching in school (Winsløw & Grønbæk, 2014). In response to the observed discontinuities, he engaged himself in developing and strengthening secondary school mathematics education. In addition, Klein advocated a selected content area of mathematics for German upper secondary teachers with good knowledge in mathematics, and pointed out the profit that could be drawn for teaching from the history of mathematics. This resulted in his book *Elementary mathematics from an advanced standpoint* (Klein, 1932).

Since Klein's days, a great amount of studies have been devoted to mathematics teachers' knowledge of mathematics for teaching (e.g. Ball, Thames, & Phelps, 2008; Davis & Simmt, 2006; Rowland et al., 2005), and their beliefs about mathematics and mathematics teaching (e.g. Phillip, 2007). Further, research on mathematics teachers' identities has expanded in recent years with the aim to better understand and support the needs of prospective mathematics teachers (Beijaard, Meijer, & Verloop, 2004; Ponte & Chapman, 2008). The increased number of studies on identity has coincided with what Lerman (2000) denotes as the social turn in mathematics education. According to Lerman, there has been a need for extending the unit of analysis from concerning individuals' acquired knowledge to including the social practices in which the individuals participate. Similarly, Ponte and Chapman (2008) state that in order to understand teacher learning, it must be studied within the multiple contexts where mathematics teachers take part, taking into account both the individual teachers and the social systems in which they participate. In line with Adler (2000), I thus consider mathematics teacher learning as a process of increasing participation in the practice of mathematics teaching, and through that participation, becoming knowledgeable in and about mathematics teaching.

A way of describing the differences between practices of university and school mathematics is to look at mathematics both from a mathematician's and a mathematics educator's point of view (Kilpatrick, 2008). To the mathematician, mathematics is the body of knowledge or an academic discipline that studies concepts such as quantity, structure, space and change. To mathematics educators, however, mathematics is a body of knowledge or an academic discipline with a certain nature, in addition to a field of practice. Because mathematics educators are concerned with how mathematics is learned, understood, and used, they take a comprehensive view on the discipline. It might then be a demanding task for a prospective secondary school mathematics teacher to link the academic's and the educator's view on mathematics when entering the profession. Accordingly, Adler, Ball, Krainer, Lin and Novotna (2005) have called for a greater understanding of how mathematics and teaching combine in teachers' development and identities. They claim in their survey of research in mathematics teacher education that "we do not understand well enough how mathematics and teaching, as inter-related objects, come to produce and constitute each other in teacher education practice" (Adler et al., 2005, p. 378). My study then aims for contributing to a growing research field on mathematics teacher learning, and more particularly, on the research field of secondary school mathematics teachers' identities and their development. Addressing the complex and mutual relationships that exist between prospective mathematics teachers, their peers, tutors, students and other actors of the institutions in which they are situated, secondary school teachers' identity appears as an important issue for investigation (Losano & Cyrino, 2017).

The study further relates to my own experience from university teacher education, as I have my educational background in a Master's programme of natural sciences with teacher education, and a specialisation in mathematics education. I can recall the feeling of insufficiency during school placement, when struggling to provide my students in upper secondary school with meaningful mathematics experiences. Further, I remember my growing expectations during teacher education of being able to practice mathematics teaching different from regular blackboard instructions. However, the expectations also grew in step with the fear of falling through when entering school. These personal experiences led to a Master's thesis concerning socialisation of novice mathematics teachers in their professional debut (Rø, 2010). I interviewed three newly qualified mathematics teachers, in order to sketch their mathematics teacher identities as part of their participation in communities of practice at their school of employment. In order to pursue my research interest of newly educated mathematics teachers' challenges and opportunities, the current study gave me the possibility to follow a group of prospective mathematics teachers over a longer period of time, and through a series of interviews. At the time of finishing this thesis, I am working as a teacher educator in mathematics. I then seek to understand better the process of becoming a mathematics teacher, and the role of teacher education for preparing our student teachers for their profession.

1.2 Aim and research question

The purpose of this study is to gain insight into the dynamics of mathematics teachers' learning, as prospective teachers move across different mathematics practices at the university and in school and unite their experience into a role as a secondary school mathematics teacher. I study the process of becoming a mathematics teacher in terms of identity development, when the mathematics teacher participates within and at the boundaries of communities of practice at the university and in school (Akkerman & Bakker, 2011; Wenger, 1998). The research question for the study is:

How do three prospective secondary school mathematics teachers' identities develop in the transition from university teacher education to school?

Based on Wenger's (1998) social learning theory, I understand identity as negotiated experience of self when participating within and between communities of practice. The term community of practice here refers to a set of relationships between people, who share some kind of competence through interaction, communication and negotiation of meaning. Assuming that a community of practice can emerge from people's shared histories of learning (Wenger, 1998), I take the prospective secondary school mathematics teachers' perspectives and investigate their participation in mathematics related communities of practice based on their accounts of entering the profession. Further, in order to adapt the theoretical framework to a study of *mathematics* teacher identities and not identities in its general sense, I combine Wenger's (1998) theorisation with accounts of educational ideologies in mathematics (Ernest, 1991), and categories of authoritative knowing in mathematics (Belenky, Clinchy, Goldberger, & Tarule, 1986; Povey, 1995, 1997). Ernest's (1991) model of ideologies and the related perspectives on the nature of mathematics, and its teaching and learning, provides a terminology for portraying the communities in which the prospective mathematics teachers participate. Further, the studies of Belenky et al. (1986) and Povey (1995, 1997) are helpful for describing the prospective mathematics teachers' ability and legitimacy to negotiate meanings of mathematics and mathematics teaching and learning within the given communities of practice.

In this study, I present three comprehensive cases or portraits of prospective secondary school mathematics teachers' way into the profession. It is based on their accounts of participating in mathematics practices at university and in school. One of the challenges for research on transition is to build methods that not only analyse the initial and the final state, but also take hold of the process of change itself, in all its complexity (Gueudet, Bosch, diSessa, Kwon, & Verschaffel, 2016). Hence, I aim for describing the learning trajectories of prospective secondary school mathematics teachers as they move into the practices of mathematics teaching in school. The longitudinal research design includes a series of interviews distributed across the three prospective mathematics teachers' last year in university teacher education and their first year as mathematics teachers in secondary school. Based on methods from narrative analysis, the accounts constitute evolving stories and related critical events of their narratively developing identities (Lieblich et al., 1998; Polkinghorne, 1995). The stories are further compared and contrasted through a cross-case analysis, in order to describe more generally the development of an identity as a secondary school mathematics teacher.

1.3 The Norwegian educational context

In the subsequent section, I present the Norwegian school system and the organisation of mathematics in school. Further, I present in section 1.3.2 the Norwegian teacher education system and the programmes in which the prospective teachers who participated in the study were enrolled.

1.3.1 The Norwegian school system and mathematics education

The Norwegian school system can be divided into three parts: compulsory primary school (age 6 to 13), compulsory lower secondary school (age 13 to 16); and upper secondary school (age 16 to 19). Although upper secondary school is not compulsory, the "Reform 94"[1] introduced a statutory right to a three-year upper secondary education for students who have completed primary and lower secondary education. An overview of the Norwegian school system is given in Table 1.1:

Table 1.1: The Norwegian school system

<table>
<tr><th>Grade</th><th>Age</th><th colspan="2">Educational programme</th></tr>
<tr><td>1-7</td><td>6-13</td><td colspan="2">Compulsory primary school</td></tr>
<tr><td>8-10</td><td>13-16</td><td colspan="2">Compulsory lower secondary school</td></tr>
<tr><td rowspan="2">11-13</td><td rowspan="2">16-19</td><td colspan="2">Upper secondary school</td></tr>
<tr><td>General education, 3 years</td><td>Vocational education,
2 years + 2 years of apprenticeship/ 2 years + 1 supplementary year</td></tr>
</table>

[1] "Reform 94" was a reform of the structure and content of upper secondary education, introduced in August 1994.

Mathematics in primary and lower secondary school is provided as one common core subject throughout the grades 1 to 7 and 8 to 10. Further, mathematics in upper secondary school is given as a set of selectable subjects. Students undergoing general education are obliged to study mathematics for at least two years, while students attending vocational education are obliged to take mathematics for one year. Students in vocational education can further take a supplementary year of general studies, in order to get qualified for higher education. This year will then include a supplementary mathematics course.

During their first year in upper secondary school, the students in general education can choose between Mathematics 1T and Mathematics 1P. The 1T curriculum is more theoretical, while 1P is more practical and regarded as the easiest of the two. Further, Mathematics 1T qualifies for Mathematics 2T, Mathematics R1 and Mathematics S1. The course 2T completes a theoretical education in mathematics; however, the course R1 and the additional course R2 provide mathematics for further studies within the mathematics, natural sciences and technology (STEM subjects). Alternatively, the course S1 and the additional course S2 provide mathematics for studies within economics and social sciences. Students in vocational education can choose between the courses Mathematics 1T-Y and Mathematics 1P-Y. The course Mathematics 1P-Y is regarded as the easiest of the two and corresponds to 3/5 of the curriculum of the general education mathematics course 1P. Possible mathematics trajectories in upper secondary school are shown in Table 1.2. Newly qualified mathematics teachers in Norwegian upper secondary school are likely to teach mathematics within both general and vocational education, since most schools offer both programmes.

Table 1.2: Possible mathematics trajectories in the Norwegian upper secondary school system

Grade	General education				Vocational education	
11	Alt. 1			Alt. 2	Alt. 1	Alt. 2
	1T			1P	1T-Y	1P-Y
12	Alt. 1a)	Alt. 1b)	Alt. 1c)	2P		
	2T	R1	S1			
13		R2	S2		Supplementary year	
					2T-Y	2P-Y

1.3.2 Norwegian teacher education

In this study, my focus is on the process of developing an identity as a secondary school mathematics teacher. Within the Norwegian educational context, this refers to teaching in both lower and upper secondary school, and thus, the grades 8 to 13. Until the academic year 2017/2018, prospective primary and lower secondary teachers in Norway have applied for a four-year undergraduate teacher education programme, either for the grades 1-7 (abbreviated to GLU 1-7) or the grades 5-10 (GLU 5-10). From the academic year 2017/2018, the GLU 1-7 and GLU 5-10 programmes are changed into five-year Master's programmes. For teaching in grades 8-13, student teachers attend either a five-year Master's programme including teacher education (for studies in mathematics and natural sciences named "Lektorutdanning i realfag", locally abbreviated to LUR), or a one-year post-graduate teacher education programme (PPU). Table 1.3 gives an overview of the teacher education programmes in Norway that qualify for mathematics teaching in school.

Table 1.3: Teacher education programmes in Norway that qualify for mathematics teaching in school

Teacher education programme	Abbreviation	Duration
Teacher education programme for grades 1-7	GLU 1-7	5 years
Teacher education programme for grades 5-10	GLU 5-10	5 years
Master's programme in mathematics and natural sciences, including teacher education (grades 8-13)	LUR	5 years
Post-graduate teacher education programme (grades 8-13)	PPU	1 year

Students undertaking the GLU 1-7 programme are all required to complete 30 ECTS credits of mathematics/mathematics education, while those undertaking the GLU 5-10 programme are required to take a minimum of 60 ECTS credits of mathematics/mathematics education if they want to qualify as mathematics teachers. However, these study programmes do not contain a university degree in mathematics and are not represented in my study. Instead, I investigate prospective secondary school mathematics teachers (grades 8-13) undertaking either the five-year Master's programme including teacher education (LUR), or the one-year, post-graduate teacher education programme (PPU).

At the university represented in my study, students attending the five-year Master's programme can choose the following combinations of subject studies: mathematics and physics, mathematics and chemistry, mathematics and biology, mathematics and computer science, and biology and chemistry. The student teachers choose to specialise in one of the two subjects, named Subject 1. The other subject is referred to as Subject 2. During the first four years of study, Subject 1 amounts to a minimum of 97,5 ECTS credits, while Subject 2 amounts to 60 ECTS credits. In their fifth year of study, the student teachers choose a Master's specialisation including an additional amount of 30 ECTS credits in Subject 1 (Master's level), in addition to writing a Master's thesis (30 ECTS credits). This specialisation can either be within a scientific discipline of the chosen subject (e.g. algebra or analysis in the case of mathematics) or within the teaching and learning of the subject (e.g. mathematics education). In parallel, the student teachers take a variant of PPU (60 ECTS credits) being distributed across several academic years. Certification for mathematics teaching in secondary school based on a five-year Master's programme can then be given on the basis of two alternatives, depending on whether mathematics is chosen as Subject 1 or Subject 2. The subject studies in mathematics are shared with other study programmes, such as the Bachelor's and the Master's programme of mathematical sciences. The Master's courses in both mathematics and mathematics education are provided by the Department of Mathematical Sciences, while the PPU programme is provided by the Department of Teacher Education.

Alternatively, students attending the one-year post-graduate PPU programme build on already completed university studies in mathematics. Certification for mathematics teaching in secondary school is then given on the basis of minimum 60 ECTS credits in mathematical sciences. In both the five-year Master's programme and the post-graduate teacher education programme, the students undergo longer periods of school placement in both lower and upper secondary school, and they take the same courses in general pedagogy and subject didactics[2]. The prospective teachers in my study have their educational background from both the five-year Masters' programme and the one-year post-graduate PPU programme, being provided at one Norwegian university. I have my educational background in the five-year Master's programme in mathematics and physics, with a specialisation in mathematics education.

[2] In Norwegian, the teaching and learning of a subject and the associated field of research is referred to as the subject's "didaktikk", here, directly translated to the subject's didactics. In this thesis, I denote the courses of the PPU programme concerning the teaching and learning of a given subject as courses in subject didactics (e.g. mathematics didactics). This is done to avoid confusion with the Master's courses in mathematics education, provided by the Department of Mathematical Sciences.

1.4 The structure of the dissertation

Subsequent to the introduction, I present in Chapter 2 the theoretical framework of the study. The chapter is divided into two parts. In Section 2.1, I present previous research concerning the concept of transition in mathematics education, and on mathematics teachers' knowledge, beliefs and identity. Further, I present in Section 2.2 the theoretical framework guiding my study. Chapter 3 is a presentation of the research methodology, in which I account for the operationalisation of the concept of identity and related methods for data collection and analysis. In Chapter 4, I describe the structure of the subsequent narrative analysis. The cases of Isaac, Nora and Thomas are presented in Chapters 5, 6 and 7 respectively. In Chapter 8, I make a thematic comparison of the three cases, in the form of a cross-case analysis. This leads in Section 8.3 to a synthesis of the findings and a more general description regarding developing an identity as a secondary school mathematics teacher. In Chapter 9, I discuss the findings in view of previous research and in relation to strengths and limitations of the research I have undertaken. In addition, I point at possible pursuits of research studies resulting from my study. I also reflect on potential implications for teacher education.

2 Theoretical framework

This chapter discusses the theoretical basis for analysing mathematics teacher identity development in the transition from university teacher education to secondary school. To provide a context for the study, I start the theoretical consideration with a review of previous research (Section 2.1). From the presentation of studies on transitions in mathematics education (Section 2.1.1) and on mathematics teachers' knowledge, beliefs and identity (Sections 2.1.2 to 2.1.4), I move towards a framework for studying individual mathematics teachers through their social settings. Here, I adopt Wenger's (1998) social learning theory of identity in community of practice as a general theorisation of learning and a superior framework for my study. In the Sections 2.2.1 and 2.2.2, I elaborate on the concept of communities of practice, by giving examples of practices and negotiated meanings of mathematics teaching and learning relevant for prospective mathematics teachers. I also account for the three dimensions constituting a community of practice: mutual engagement, joint enterprise, and shared repertoire. Further, I make in Section 2.2.3 an elaboration of the concept of identity. According to Wenger (1998), identity formation is a dual process of identification and negotiability. I therefore discuss the two components more thoroughly in Section 2.2.4 and Section 2.2.5 respectively. In Section 2.2.6, I briefly account for the development of a mathematics teacher identity as an emotional process. Since learning as identity development takes place not only within communities of practice, but also across their boundaries, I report on learning mechanisms when undergoing boundary crossing in Section 2.2.7.

Following Prediger, Bikner-Ahsbahs and Arzarello (2008), I consider the theoretical framework for this study to be dynamic, meaning that it is rooted in a social perspective on learning. However, it is developed and adjusted in order to answer the given research question (see Section 1.1). In order to adapt the framework to a study of mathematics teacher identity, and not identity in its general sense, I thus combine Wenger's (1998) theorisation with accounts of educational ideologies in mathematics (Ernest, 1991), and authoritative knowing (Belenky et al., 1986; Povey, 1995, 1997). In Section 2.3, I give some closing remarks regarding this combination of theorisations.

2.1 Previous research and implications for the study

In my account of previous studies, I start with considering the concept of transition in mathematics education research. The overview of relevant literature leads to a delimitation of the notion of transition that applies to my study. Further, I give an overview of research on learning to teach mathematics, in which I will move from acquisitionist perspectives towards a participationist perception of mathematics teacher identity. This brings on a review of research on the concept of identity in mathematics education. Following a Meadian tradition in which identity is considered as an action (Darragh, 2016), I further account for narrative approaches in research on mathematics teacher identity. The research review ends in Section 2.1.5 with a positioning of my study within the theoretical landscape of mathematics teacher learning and implications for the framework presented in Section 2.2.

2.1.1 Transitions in mathematics education research

I perceive the process of becoming a mathematics teacher as a learning process, which I investigate in the light of the transition from university teacher education to teaching in secondary school. Later in this chapter, I will account for this study's theorisation of mathematics teacher learning. Yet, as it frames the learning process for investigation, I start by reviewing the concept of transition.

In the introduction, I referred to Klein's (1932) notion of a *double discontinuity* between secondary school mathematics and university mathematics. The first discontinuity concerns the problems that students face when they enter university (e.g. Gueudet, 2008), while the second discontinuity concerns the return to school and the related transformation of university mathematics to school mathematics. Winsløw et al. (2009) have further distinguished three interrelated levels of Klein's double discontinuity: a personal, an institutional and an epistemological level. Transition at the *personal level* concerns the change of the prospective teacher's role within the current institutions, from being a student in a community of students to being a professional in a community of mathematics teachers. Transition at the *institutional level* concerns passing from a university context to a school context, with related changes of norms and other cultural assets. Alongside the transition is also a change at an *epistemological level*, when adapting mathematics to the conditions and requirements of teaching in school. For instance, future mathematics teachers tend to believe that the topics of university mathematics do not fit the demands of their later profession in school (Gueudet et al., 2016). Accordingly, Bergsten and Grevholm (2005) point at two possible divides which may occur in the teachers' knowledge: between disciplinary (mathematical) knowledge and pedagogical knowledge, and between the practical and theoretical parts of each of these. The former divide concerns challenges with transforming subject knowledge into pedagogical content knowledge (Shulman, 1986). I will come back to Shulman's work on knowledge for teaching in Section 2.1.2. The latter divide can be exemplified by teacher education seminars being practically oriented, when focusing on problems in mathematics classroom situations and giving tools for how to handle them. Yet, if no connections are explicitly made to theoretical knowledge regarding students' conceptions and understandings in mathematics, the student teachers may stay at what Bergsten and Grevholm (2005) denote as a punctual level in their teaching. Undergoing the transition from teacher education to school, the student teachers might draw on particular problems and techniques, and finding theoretical knowledge insignificant for their professional practice. In the current study of mathematics teachers' identities, I make a holistic approach towards teacher learning during transition in which all three of Winsløw et al.'s (2009) levels are considered as parts of their becoming. Prospective teachers' experiences of undergoing a personal transition are thus assumed to provide insight into institutional and epistemological possibilities and constraints as they enter the profession in secondary school.

Transitions are also discussed more generally in the mathematics education literature, due to the need for identifying crucial features and dynamics of movements between various mathematical practices. According to Abreu, Bishop and Presmeg (2002), the meaning of transition is by many social scientists taken for granted and assumed to be shared in discussions on mathematics learning across contexts. They therefore give a review of various definitions of transition in developmental psychology theories and sociological research, in order to account for their own understanding of the term. For instance, Abreu and her colleagues depart from what they denote as a common use of transition in traditional stage theories, in which development is considered as transitions from one stable stage to a more advanced and complex, stable stage. Becoming a mathematics teacher is then about replacing less-developed ways of thinking and acting in mathematics teaching by a more advanced form. Referring to Bronfenbrenner's (1979) perspective of human development-in-context, Abreu et al. (2002) build instead on the assumption that a developing person is a "dynamic entity that progressively moves into and re-structures the milieu in which it resides" (p. 13). Similarly, I perceive the prospective mathematics teachers as taking part in coexisting ways of knowing mathematics and mathematics teaching in respectively subject studies of mathematics, mathematics education courses and classroom teaching. Transitions in this study then concern their movements between what social groups count as legitimate knowledge regarding the teaching and learning of mathematics. Accordingly, I investigate the dynamics of the prospective mathematics teachers' learning, as they move across different mathematics practices at the university and in school and build their identity as a secondary school mathematics teacher.

Abreu et al. (2002) further refer to Beach' (1999) presentation of four categories of transition in education research:

1. Lateral transition – occur when an individual moves between two historically related activities in a single direction, such as moving from school to work. Participation in one activity is replaced by participation in another activity in a lateral transition.
2. Collateral transitions – involve individual's relatively simultaneous participation in two or more historically related activities, such as daily movements from school to home.
3. Encompassing transitions – occur within the boundaries of a social activity that is itself changing, and is often where an individual is adapting to existing or changing circumstances in order to continue participation within the bounds of the activity
4. Mediational transitions – occur within educational activities that project or simulate involvement in an activity yet to be fully experienced.

(Abreu et al., 2002, pp. 14-15)

According to Abreu and her colleagues (2002), all four types of transitions potentially involve a *construction* of knowledge or identity when undergoing a transition, rather than representing a simple application of what has been acquired in previous settings. Beach' (1999) categories are then suitable for describing the notion of transition which applies to my study. At first sight, the secondary school mathematics teachers' movement from a university to a school setting is in line with the first category of *lateral transition* and the above example of moving from school to work. Yet, the transition is not simply a replacement of participation in activities, since the prospective teachers are undergoing university teacher education including school placement. Hence, the study also concerns the prospective teachers' *collateral transition* between scholarly knowledge situated at the university, and the practice field of secondary school mathematics teaching. By choosing a longitudinal design and a repeated data collection during both university studies and school employment, I further attend towards an *encompassing transition*, or change in the activities themselves in which the mathematics teachers participate. For instance, undergoing the first year as certified mathematics teacher might involve the need to adapt to existing or changing circumstances at the school of employment. Consequently, encompassing transitions might lead to changes in the mathematics teacher's classroom practice, and further, changes in his or her identity.

In line with Abreu et al.'s (2002) understanding of transitions in mathematics education, Gueudet et al. (2016) describe sociocultural transitions as movements between groups in which mathematics practices are developed and shared. Like Abreu and her colleagues, they call attention to situated learning theories for investigating transitions in mathematics education. Following Wenger's (1998) conceptualisation of communities of practice, the process of transition corresponds with the process of becoming a member of a certain community, in the evolution from legitimate peripheral participation to full participation. Further, the mathematics discipline simultaneously belongs to several communities. The concept of boundary crossing then applies for investigating the transitions experienced by individuals when moving between the communities.

Making use of the concept of boundary crossing, Jansen, Herbel-Eisenmann, and Smith III (2012) investigate students' transitions from middle school to high school mathematics. They present a tool for analysing whether differences that students observe between settings are experienced as *discontinuities*. Here, they determine experience of discontinuity to be differences that are meaningful to students and that co-occur with a change in their attitudes. Referring to previous studies of researchers' observations of differences between practices, Jansen et al. (2012) emphasise the need to listen to the students' voices for capturing which differences matter to them, and to study the sort of learning they can experience as they make sense of these differences. Although their research focus is students' learning of mathematics, a similar need applies for mathematics teachers in transition. Hence, the concept of boundary crossing appears useful for investigating prospective teachers' learning of mathematics and mathematics teaching from their perspective, as they make sense of discontinuities between university and school.

By characterising the notion of transition in my study as both collateral and encompassing, I perceive Wenger's (1998) theorisation of identity and identity development as suitable. In addition, I follow Jansen et al. (2012), by investigating how differences between university and school play out in and are being shaped by the process of developing an identity as a secondary school mathematics teacher (Akkerman & Bakker, 2011). A further consideration of learning during boundary crossing will be given in Section 2.2. Nevertheless, the transition from mathematics teacher education into school employment is in the literature dominated by research on changes in beliefs and teacher knowledge (e.g. Brown & McNamara, 2011; Gueudet et al., 2016; Lerman, 2001). I will therefore give a further review of research on teacher learning and development in the subsequent section, by moving from acquisitionist perspectives towards a participationist framework for investigating mathematics teacher identity.

2.1.2 Research on mathematics teachers' knowledge and beliefs

Research on learning to teach mathematics has traditionally been conducted within the discipline of cognitive psychology (Kelly, 2006; Putnam & Borko, 2000). Here, teacher learning is typically described as an individual's acquisition of knowledge, change in knowledge structures or growth in conceptual understanding. Many studies regarding teacher knowledge refer to Shulman (1986) and his model of teachers' professional knowledge. As teachers need to know not only the subject content but also about teaching the content to students, Shulman hypothesised that teachers draw from seven domains of knowledge when planning and implementing instruction: *knowledge of subject matter*, *pedagogical content knowledge, knowledge of other content, knowledge of the curriculum, knowledge of learners*, *knowledge of educational aims* and *general pedagogical knowledge*. Acknowledging the complexity of the research domain of teachers' knowledge, and more specifically, mathematics teachers' knowledge, I will here be able to make only a brief overview of significant studies. Although the research domain is not directly relevant to the theoretical framework of my study, I still consider it to provide an informative background for the positioning of the study.

Adopting the notion of *pedagogical content knowledge* (PCK), Ball, Thames and Phelps (2008) have developed a model for *mathematical knowledge for teaching* (MKT) that represents the particular features of mathematical knowledge needed for exercising mathematics teaching. Their frame of reference is not university level knowledge, but rather the mathematics behind the curriculum for elementary school mathematics. In their framework, Shulman's (1986) PCK is understood as *knowledge of content and students* (KCS) and *knowledge of content and teaching* (KCT). Ball and her colleagues also distinguish between content knowledge in the academic discipline of mathematics and content knowledge in relation to teaching (Ball, Lubienski, & Mewborn, 2001). The former refers to what they denote as *common content knowledge* (CCK), while the latter is named *specialised content knowledge* (SCK). Based on their assumption that mathematical knowledge for teaching is situated in the context of teaching, the research of Ball and colleagues is built on the mathematics teacher's practice, meaning everything a teacher does to support his or her students' mathematics learning. This involves the teacher's interaction in the classroom, lesson planning, assessment of students' work, planning and correcting homework, and communication with parents.

Other researchers relating their work on mathematics teacher knowledge to Shulman's framework, are Rowland et al. (2005) with their empirically-based *knowledge quartet*. With help of the four dimensions of foundation, transformation, connection and contingency, they describe various ways in which mathematics content knowledge is part of the choices and actions of mathematics teachers in the classroom. *Foundation* is here about the teacher's possessed knowledge and understanding of mathematics, and beliefs about mathematics teaching and learning. *Transformation* is about how the foundation is transformed into pedagogical forms. Further, *connection* concerns choices and decisions made within parts of the mathematics content in terms of coherence of the planning and teaching across an episode, a lesson or series of lessons. Finally, *contingency* concerns contingent actions, meaning the teacher's responses to classroom events that were not anticipated in the planning.

In a recent Norwegian study, Nergaard (2017) uses the knowledge quartet in combination with Ball et al.'s (2008) MKT framework and Jaworski's (1994) concept of *the teaching triad* for investigating one experienced mathematics teacher's local knowledge for teaching the subject. The teaching triad concerns characteristics of investigative teaching in mathematics, comprising *sensitivity to students, mathematical challenge* and *management of learning* (Jaworski, 1994). Successful mathematics teaching is then recognised by harmony between the three elements of the triad. Nergaard defines local knowledge as the knowledge a teacher has developed in the process of her practice, also known as craft knowledge. Based on a substantial amount of lesson observations and conversations with the teacher, the study shows that there is more to the teaching of mathematics than what is elaborated in the MKT framework, such as a didactical dimension to *horizon content knowledge* (HCK).

The complex and constantly evolving research domain on mathematics teacher knowledge has contributed to better understanding of the nature of the knowledge, its complexity and importance for becoming a successful school mathematics teacher. According to Brown and Borko (1992), student teachers having a strong content preparation are more likely to be flexible in their mathematics teaching, as well as responsive to the students' needs. Contrarily, student teachers who lack adequate mathematics content knowledge are likely to lack confidence in their ability to teach mathematics well. Yet, one of the most difficult aspects of learning to teach mathematics is making the transition from one's personal orientation towards mathematics, to considering how to represent and organise the mathematics content to facilitate the students' understanding (Brown & Borko, 1992; Rowland et al., 2005). One important issue is therefore how mathematics teacher knowledge develops, whether it develops when practicing mathematics teaching, or if it also develops during subject studies and mathematics education courses at the university. According to Ponte and Chapman (2008), pre-service mathematics teachers have extensive knowledge about mathematics teaching and views on the nature of mathematics based on their experiences from own schooling. Hence, the mathematics teacher education needs to engage the teachers in learning opportunities that allow them to re-construct their initial knowledge of mathematics teaching. However, studies have shown that the knowledge pre-service teachers have developed about mathematics and its teaching and learning before entering teacher education tends to resist change (Brown & Borko, 1992; Ponte & Chapman, 2008). Purposeful and ambitious teacher preparation is therefore difficult.

Based on her assumption that pre-service teachers have needs not being met by the teacher education, Fuller (1969; Fuller & Brown, 1975) developed a theory of stages of teachers' concerns regarding self, the task of teaching and students' learning. The theory points out four stages of development in learning to teach: *pre-teaching concerns* about oneself as a (mathematics) student, *self-concerns* about one's survival and adequacy as a (mathematics) teacher, *task concerns* about the teaching situation (e.g. time pressure, amount of students to teach), and *pupil concerns* about the social and emotional needs of the students. Here, concerns regarding self are considered less desirable than concerns related to pupils. The progression of concerns are then explained by the general human tendency to be preoccupied with basic needs until they are satisfied (Brown & Borko, 1992). Although studies show changes in teachers' concerns as they progress through teacher education and into initial years of teaching, Fuller's (1969) framework cannot account for the complex networks of concerns that mathematics teachers have. According to Brown and Borko (1992), "teachers have concerns about self, task, and pupils in all phases of becoming a teacher; the strength of these concerns is what changes over time" (p. 230). They have therefore requested studies that investigate more closely individual teachers' concerns of mathematics teaching, and changes of the concerns, as the teachers move through their pre-service experiences and first years of mathematics teaching.

Complying with the request, Brown, McNamara, Hanley, and Jones (1999) investigated the ways in which primary student teachers conceptualised mathematics and its teaching, and how their views evolved as they progressed through an initial training course. From repeatedly interviewing student teachers during their one year at the teacher education programme, they found that many of them entered the programme with low levels of mathematics knowledge and considerable anxiety towards the subject. However, the college training fostered a more positive attitude to mathematics, although embedded in a pedagogically oriented frame.

According to Lerman (2001), research on mathematics teachers' concerns of mathematics and its teaching, and more general, their affective responses to the subject, is perhaps the predominant orientation in research on teachers and teacher education. Based on many years of experience as students in the mathematics classroom, prospective mathematics teachers have a well-developed view about mathematics, mathematics teaching and learning long before they enter the mathematics teacher education at the university (Jones, Brown, Hanley, & McNamara, 2000; Prescott, 2011). In line with this assertion, belief research has been proposed as an important approach to understand teachers' underlying reasons for their teaching practice (e.g. Skott, 2008). *Beliefs* are "generally understood as relatively stable, mental constructs that are subjectively true and the result of experiences gained over prolonged periods of time" (Skott, Larsen, & Østergaard, 2011, p. 30). The basic idea within the research domain is that by transforming a teacher's beliefs about mathematics and mathematics teaching and learning, it is possible to transform or develop his or her classroom practice.

Researchers have questioned whether changes in beliefs occur before or after changes in practice, or whether the effects work both ways between beliefs and practices (Phillip, 2007). Several studies show an inconsistency between beliefs and classroom practice (Wilson & Cooney, 2002), while others claim that the starting point in belief research should be that there are no contradictions or inconsistencies between teachers' beliefs and actions, and that researchers should strive to understand the teachers' perspectives more deeply in order to resolve these possible inconsistencies (Phillip, 2007). The inconsistency in results and the lack of a common definition has made belief research an object of criticism (e.g. Lerman, 2001; Lester, 2002; Skott et al., 2011). Among the problems that have been explicated is the methodological challenge when basing the research on a circular argument; that practice is explained by beliefs, and that beliefs can be inferred from observations of practice. Further, there is a problem of getting access to these deeply rooted and individual mental constructs that reside entirely within the individual mathematics teacher (Skott et al., 2011).

Research on mathematics teachers' beliefs and knowledge for teaching constitute cognitive approaches that have an important place in the landscape of theoretical perspectives. However, critique has been raised against these acquisitionist perspectives, as they refer to individual, mental constructions to be enacted by the teacher in the mathematics classroom (Ponte & Chapman, 2008; Putnam & Borko, 2000; Skott, Van Zoest, & Gellert, 2013). According to Ponte and Chapman (2008), teaching should instead be considered a holistic, participative activity, as mathematics teachers are engaged in practice not just with their knowledge and beliefs, but with all their being. Similarly, Hodgen (2011) argues that mathematics teacher knowledge is not simply applied within the context of teaching mathematics, but it is instead situated within the complex and social world of mathematics classrooms. In order to perceive knowledge as embedded in the practice of teaching, it should be referred to as the dynamic, contextualised and active process of *knowing* rather than the more static, abstract and passive notion of knowledge. Further, coming to know mathematics teaching is both intellectual and emotional, since becoming a somehow different learner and teacher of mathematics is inseparable from changing one's emotional relationship with the discipline (Hodgen & Askew, 2007).

Hodgen's (2011) social perspective on mathematics teacher knowledge is in line with what Lerman (2000) has denoted the *social turn* in mathematics education research. It is identified by the rise of theories viewing learning as participation in practices rather than as acquisition of knowledge and beliefs structures. Learning to teach mathematics should thus be understood as increasing participation in socially organised practices that develop the teachers' professional identities (Lerman, 2000, 2001; Peressini, Borko, Romagnano, Knuth, & Willis, 2004). By approaching mathematics teacher learning through the concept of identity, I can study how a teacher develops and positions him- or herself differently in various situations and in relation to other people, still maintaining a sense of self through time (Abreu et al., 2002; Akkerman & Meijer, 2011). Hence, I can investigate the dynamics of a mathematics teacher's learning, as he or she moves across different practices at the university and in school and unite his/ her experiences into a role as a professional teacher. However, the notion of identity used in educational research relates to a variety of theoretical explanations, and it often lacks a clear definition (Beijaard et al., 2004). In the subsequent section, I will therefore give a review of research concerning mathematics teacher identities.

2.1.3 Research on mathematics teacher identity

Identity in mathematics teaching has been explored from a range of theoretical approaches (Beauchamp & Thomas, 2009; Lutovac & Kaasila, 2017). It spans from categorising aspects of teacher identity in order to describe it and better understand the possible influences on teachers' practice, to view identity as a function of participation in different communities of practice. In their overview of research on (general) teacher identity, Beauchamp and Thomas (2009) find identity to be considered across the literature as dynamic and a constantly evolving phenomenon, being under the influence of a range of individual and external factors. They discuss a variety of issues that surface in the attempt of defining the concept of identity: the role of agency and emotion in shaping identity, the power of stories and discourse in understanding teachers' identity, and the contextual factors that promote or hinder identity development. Similarly, Lutovac and Kaasila (2017), refer to the lack of clarity in definitions in their overview of research on mathematics related teacher identity. They find studies on identity to address multiple themes, for instance theoretical models for defining identity, the contextual factors in teacher identity and its development, affective relationships with mathematics and changes in teacher identity, and the link between identity and teaching practices. However, studies focusing on social practices and structures within which teacher identities develop seem to predominate in the research field.

In her review of identity research in mathematics education, Darragh (2016) makes a distinction between two main paradigms: a psychological frame, in which identity is seen as acquisition, and a sociological frame, in which identity is perceived as action. The former paradigm is related to the work of Erikson (1968), and his understanding of identity as something that individuals have inside of themselves or something that becomes coherent and consistent. One example is Anderson's (2007) elaboration of *four faces* of identity, in which the *nature face* leads the discussion of identity towards attributes rather than modes of belonging within the other three faces. Also Gee's (2000) notion of *core identity* fits into this tradition. Although Gee generally perceives identity as flexible and socially constructed, the notion of core identity implies something internal to the individual which is transferred across contexts.

The latter paradigm relates to the work of Mead (1913/2011), who perceived identity as something a person does, and which is multiple, contradictory and socially constituted. Within this tradition is Wenger's (1998) account of identity, in terms of negotiated experience of self when participating in social communities. Hodges and Cady (2012) build on Wenger's social learning theory when investigating one mathematics teacher's identity and her efforts to integrate reform-based practices in her middle-grade classrooms. They emphasise how she came to participate in the mathematics classroom, in light of the priorities for mathematics instruction represented by the local district, the school, and professional development communities. Based on Wenger's (1998) notion of nexus of multimembership, the teacher's identity is then understood to be constituted by the ways in which she reconciles her participation in the various communities. Here, Hodges and Cady (2012) also refer to Gee's (2000) notion of core identity, in order to look for consistencies and inconsistencies in the teacher's characterisation of the contexts and her participation within them. The teacher's sense of autonomy for making mathematics instructional decisions, her value of communication and collaboration in learning mathematics, and her value of students' ownership of mathematical ideas appear as critical aspects of her core identity.

Also Van Zoest and Bohl (2008) combine theoretical constructs across the two paradigms outlined by Darragh (2016) when investigating secondary school mathematics teachers' learning to teach through practice. They take Wenger's (1998) social learning theory as the basis of their analytic framework. However, they argue that Wenger's conception of identity leaves something to be desired in terms of concrete reference to individual cognition and to the *mathematics* teacher profession. Based on Lerman's (1998) suggestion of changing the analytical focus from cognitive development to social participation, their mathematics teacher identity framework contains two interacting and overlapping components called aspects of self-in-mind and aspects of self-in-community. While the latter aspects relate to Wenger's (1998) communities of practice, the aspects of self-in-mind include teachers' knowledge, beliefs, commitments and intentions in mathematics. Here, the notion of teacher knowledge proceeds from the work of Shulman (1986).

In her investigation of mathematics teachers' learning from an in-service teacher education programme, Graven (2004) extends Wenger's (1998) four interrelated components of meaning, practice, identity and community with the notion of confidence. Although her study is not explicitly on mathematics teacher identity, Graven (2004) argues that the notion of confidence is pivotal in understanding and explaining mathematics teachers' learning. Her analysis of teacher interviews, questionnaires and classroom observations, shows that the notion of confidence appears as a *product*, in other words as a result from the teachers' learning, and as a *process*, in terms of an explanation for their learning. According to Graven (2004), confidence enabled the teachers in her study to move from being *teachers of mathematics* to becoming *mathematics teachers*. Alongside this change, the teachers moved from a periphery of mathematics education related communities of practice towards more central participation, identification and belonging in the communities.

Palmér's (2013) study of novice primary school teachers' professional identity development is an example of recent Nordic research within the field of mathematics teacher education. She follows seven novice primary school teachers from their graduation and two years onwards. Unlike the above-mentioned studies, Palmér draws entirely on a participatory framework when connecting Skott et al.'s (2011) concept of patterns of participation and Wenger's (1998) theorisation of identity. By conducting a case study with an ethnographic direction, including self-recordings made by the teachers, observations and interviews, Palmér claims to make visible the whole process of identity development, both the individual and the social part. With the individual mathematics teacher in the foreground, she investigates the teachers' patterns of participation regarding mathematics teaching and the sense of becoming as a teacher. This participation takes place in communities of practice during teacher education and the professional debut. Further, by placing the social dimension of identity development in the foreground, her analysis of current communities of practice leads to interpretation of how memberships affect the patterns of participation of individual teachers. The study reveals that the teachers' patterns of participation regarding the teaching of mathematics change when they become members in new mathematics teaching related communities of practice. However, the existence of such communities seems to be rare, and the possibilities to become members in those that exist are limited. As a consequence, none of the teachers in her study develops a professional identity as primary school *mathematics* teacher two years after graduation.

2.1.4 Research on mathematics teachers' narrative identities

So far, I have elaborated on the Meadian tradition by reporting on studies that adopt, partially or entirely, a participationist approach on identity. Taking this approach, mathematics teacher identity is thus perceived as an action rather than a psychological trait. Accordingly, I find the concept of narrative identity to be in accordance with an action perspective. Here, identity is related to the activity of communicating, meaning that teacher identities are represented by stories that people tell about themselves as learners in and teachers of mathematics (Beauchamp & Thomas, 2009; Beijaard et al., 2004; Darragh, 2016). Narrative research was first applied to educational research by Connelly and Clandinin (1990), who recognised the importance of narrative inquiry as a methodology when focusing on lived educational experiences. In a later work (Connelly & Clandinin, 1999), they report on interconnectedness of knowledge, context and identity in the stories of teachers and administrators. These stories are considered as personal, as they are shaped by the storytellers' knowledge, values, feelings and purposes. However, they are also collective, in the sense of being shaped by the broader social, cultural and historical context where the stories are lived out.

In their elaboration of an operational definition of identity, Sfard and Prusak (2005) delineate the concept of identity as collections of stories about persons that are "reifying, endorsable and significant" (p. 16). Hence, identifying oneself as a mathematics teacher is by Sfard and Prusak seen as a discursive activity, in which the stories themselves constitute a mathematics teacher identity. Further, identity development is connected to stories about the current state (current identity) and about states expected to be (designated identity). The teachers' stories give the reader a sense of what the narrators care about, and the conditions in which they carry out their work. They are therefore considered to represent teachers' growing understanding of their professional identities within changing contexts.

Sfard and Prusak's definition is well cited in the mathematics teacher identity literature (Darragh, 2016). For instance, Andersson (2011) has investigated one Swedish upper secondary school teacher's (Elin) narrated identities during a transformation of her mathematics teaching. The collected data comes from interviews and conversations during Elin's one year of teaching in two classes. Here, Andersson uses the plural form of identity, as she "equate[s] identities with stories about persons" (Sfard & Prusak, 2005, p. 14). Accordingly, she assumes that persons' identities change with the stories that they accept about themselves, as they engage with available discourses in a reform context. In her work of outlining the processes that supported or hindered Elin to engage in pedagogical development, Andersson (2011) reports on how Elin became aware of herself, her mathematics teaching and organisation, and her ways of interacting with the students. Hence, the narrated identities reveal why Elin struggled at times, and how she was constrained in becoming the mathematics teacher she wanted to become.

Also Bjuland, Cestari, and Borgersen (2012) adopt Sfard and Prusak's (2005) narrative framework in their study of the professional identity of one experienced primary mathematics teacher (Agnes). They analyse selected clusters of Agnes' narratives from various empirical situations over a 2-year period, including classroom teaching, collective semi-structured interviews and small-group discussions. The situations generate what Bjuland and colleagues define as *reflective narratives*, in which Agnes looks back and consciously reflects on her teaching and participation in a developmental and research project. In the analysis, they work out a set of identity indicators that give evidence of Agnes' professional identity as a mathematics teacher when engaging in collaborative inquiry processes with other teachers and didacticians[3]. In addition, they suggest the identity indicators to give empirical evidence of her professional identity development. In view of their findings, Bjuland et al. (2012) emphasise the need in mathematics education research to focus on identity as being continuously in a process of transformation, and not as a fixed position.

The importance of talk in research on teacher learning is further highlighted by Adler (1998), who claims that becoming a mathematics teacher involves learning to talk both within and about mathematics teaching and learning, rather than simply learning content knowledge. In line with Adler, Watson (2006) states that teachers' narratives provide a means by which they are able to integrate knowledge, practice and context within prevailing educational discourses. In other words, the activity

[3] In the study of Bjuland et al. (2012), the term didacticians refers to mathematics education researchers.

of constructing narratives is a way of 'doing' identity work. Her analysis of interviews with the teacher Dan suggests a number of resources from which his narratives are constructed: biographical resources, sources of professional knowledge, the context of his setting and wider educational contexts in which he takes part.

While Watson's (2006) study concerns teaching in its general sense, Kaasila (2007) applies narrative analysis of teacher interviews when investigating pre-service teachers' development into *mathematics* teachers. He perceives the teachers' views of mathematics as important parts of their narrative mathematical identities, including their view of themselves as learners and teachers of mathematics, their view of mathematics and its teaching and learning, and of the social context of learning and teaching mathematics. Based on a content analysis of the teacher Leila's story, a change in her views of mathematics becomes evident. Due to experiences of mathematics teaching during teacher education, Leila is turning negative memories of mathematics from own schooling into a positive action of student centricity in her mathematics teaching. Consequently, her view of herself as a mathematics teacher improves, and her mathematics teaching becomes more connected to her students' world of experience and their everyday life.

Among other relevant Nordic studies on general teachers' narrative identities, is Søreide's (2007) investigation of public narratives about teachers within the Norwegian elementary school system. In addition to interviews with female elementary school teachers, Søreide draws on texts from the Union of Education Norway[4] and public school policy documents. The analysis leads to a paramount identity construction within the text sources, in which the teacher is considered to be pupil centred, caring and including. This shared portray of the teacher might in turn affect how Norwegian teachers are able to construct their professional teacher identities.

One study including narrative analysis, however, not the notion of identity, is Bulien's (2008) investigation of student teachers' experiences from mathematics courses in Norwegian primary teacher education. The students' perceptions of teaching and learning mathematics, both prior to and during the compulsory course, are made visible through student narratives. Their experiences are further oriented towards four main areas of beliefs regarding mathematics, which to a small extent are changing during the course: beliefs about mathematics in general, beliefs about themselves as practitioners of mathematics, beliefs about teaching mathematics, and beliefs about how mathematics is learnt.

[4] "Utdanningsforbundet", in Norwegian

Another Nordic study on mathematics teacher education is carried out by Häll (2006). She follows student teachers from teacher education into their first years as upper primary and lower secondary school mathematics teachers. The longitudinal study sheds light on visions of mathematics teaching and learning stated by the novice teachers and in policy documents, and on what the novice teachers express as being the reality of mathematics in school. The student teachers in Häll's study want to teach mathematics as a creative and exploratory subject, containing laboratory elements, being detached from the textbook and having connections to everyday life. Simultaneously, they point at possible obstacles and difficulties for exercising this kind of teaching, such as assessment and time pressure. When entering school as certified mathematics teachers, there is a shift in their talk about what constitutes a good mathematics lesson, being more related to the climate in the mathematics classroom than to the mathematics learning taking place.

Similarly, Persson (2009) follows Swedish pre-service lower primary school mathematics teachers during teacher education and after graduation. She investigates the respondents' talk about mathematics and mathematics education, how this talk changes throughout and after teacher education, and how the talk relates to the teaching they perform. The results indicate that the language used by the student teachers is changing, and that they include terms from the national curriculum and the aims of the syllabus of the teacher programme when discussing mathematics teaching. Further, observations of their mathematics teaching after graduation show a clear relationship between the sort of mathematics teaching that they claim to perform and the sort of teaching they actually perform.

2.1.5 Towards a framework for investigating mathematics teacher identity

Having reviewed research on mathematics teachers' learning in terms of changes in their knowledge, beliefs and (narrative) identity, I turn to the position of my study within the theoretical landscape. It starts with an assumption regarding the researcher's perspective on the learning process of becoming a mathematics teacher. I have referred to Jansen et al.'s (2012) remark that previous studies on learners' transitions tend to emphasise the researchers' observations on differences between practices. Similarly, Palmér (2013) argues that studies on mathematics teachers often provide an external perspective, where the researcher observes and evaluates teaching in order to describe the teacher's mathematical knowledge and/or beliefs. However, in order to better understand mathematics teachers' challenges and opportunities when entering the profession, I find an internal perspective on mathematics teaching suitable. In other words, I search the individual's own meaning-making of him or herself as an actor of mathematics teaching, and the development of this meaning-making, in the transition from teacher education to school.

Beyond epistemological processes of coming to know content, e.g. mathematical knowledge for teaching, learning in terms of identity development is what Wortham (2006) denotes as an ontological process of learning that changes who the learner is. Thus, when a prospective teacher learns to teach mathematics, he or she does not only develop knowledge of concepts, skills and practices, but also perspectives on the nature of mathematics and understandings of him- or herself as a learner and teacher of the subject. The learning process thus includes questions like "who am I as a mathematics teacher" and "who do I want to become" (Beijaard et al., 2004, p. 122). Following Olsen (2008), I therefore consider mathematics teaching to be a complex personal and social set of embedded processes and practices that concern the whole person. In order to study mathematics teacher learning within and across practices, the notion of mathematics teacher identity becomes a conceptual tool for investigating how mathematics teachers negotiate meaning with others and how they regulate their participation according to reactions of others to them (Kelly, 2006; Ponte & Chapman, 2008).

Wenger's (1998) theorisation of identity in communities of practice enables me to study individual mathematics teachers through their social settings. Hence, by linking identity closely to social practices, the primary unit of analysis is neither the individual mathematics teacher, nor the current mathematics communities. Instead, it is an interweaving of the two, denoted as teacher-in-the-learning-community-in-the-teacher (Graven & Lerman, 2003; Lerman, 2000, 2001; Palmér, 2013). The first part, teacher-in-the-learning-community, acknowledges that the object of study is more than individual cognition and affect, since learning is the development of modes of participating with others in society (Wenger, 1998). Hence, a mathematics teacher's identity is constituted by the ways in which the teacher sees him- or herself in response to the actions of other community members towards him or her. The second part, learning-community-in-the-teacher, implies that participation develops identity in such a way that the practice becomes part of the individual. For instance, when moving from university to school, the prospective mathematics teacher adopt different stances towards mathematics, its teaching and learning, from which the identity develops. Accordingly, I study mathematics teachers' developing perspectives on mathematics and mathematics teaching and learning in light of their participation within and at the boundaries of communities of practice in university and school. A further consideration of the operationalisation of the concept of identity is given in Chapter 3 on methodology.

Wenger (1998) provides a general theorisation of learning and a superior framework for my study, and in the next section I will elaborate on an understanding of mathematics teacher learning within and between communities of practice. However, a problem to be addressed regarding the notion of identity is the tendency to emphasise the personal side of becoming a mathematics teacher in preference to the professional side of mathematics teaching (Beauchamp & Thomas, 2009). Accordingly, Adler et al. (2005) have uttered the need to understand better how the mathematics discipline and the teaching of mathematics combine in a teacher's developing identity. The prospective secondary school mathematics teachers in my study need to relate to and cope with mathematics both during university studies and in their profession. In order to describe their identity development as secondary school mathematics teachers, there is then a need for developing an analytical tool that makes the *mathematics* within the development visible. Hence, I will in the next section argue for combining Wenger's (1998) notion of identity with a dynamic interpretation of a person's negotiated perspectives on the nature of the discipline of mathematics, its teaching and learning. Here, I consider Ernest's (1991) description of educational ideologies in mathematics education, spanning from dualistic absolutism to relativistic fallibilism, as helpful terminology when accounting for the prospective mathematics teachers' identification with communities of practice and associated dynamic perspectives on the discipline.

My investigation on mathematics teacher identity share theoretical assumptions and methodological considerations with several of the aforementioned studies. In line with Häll (2006), Palmér (2013) and Persson (2009), I make use of a longitudinal design and follow a group of prospective mathematics teachers from their last year in teacher education into their first year as mathematics teachers in school. I also share with Hodges and Cady (2012) and Palmér (2013) the definition of identity development as formulated by Wenger (1998): "Building an identity consists of negotiating the meanings of one's experience of membership in social communities (p. 145)." However, the notions of transition and learning during boundary crossing are not mentioned explicitly in these studies. Admittedly, Van Zoest and Bohl (2008) combine Wenger's social theory of learning with Shulman's (1987) heuristic of teacher knowledge, and an understanding of knowledge and beliefs as cognitive in nature. However, unlike viewing identity as dynamic and in continuous development, they describe it as something the teachers "carry with themselves as they move from context to context" (Van Zoest & Bohl, 2008, p. 338). Consequently, their framework is not consistent with my investigation of how teachers make sense of their ongoing transition, due to their present situation of participating in communities of practice (Rø, 2015b). My study thus intends to contribute to a further theorisation of *mathematics* teacher identity and identity development, based on identification of crucial features and dynamics of prospective teachers' movements between various mathematical practices. Except for the studies of Andersson (2011) and Van Zoest and Bohl (2008), none of the other mentioned studies investigate prospective secondary school mathematics teachers, that is, mathematics teachers at the grades 8-13 who hold university degree in mathematics. The lack of research on secondary mathematics teacher education is confirmed by Ensor (2001). Additionally, according to Hammerness (2012), teacher education has only recently emerged as a topic of study in Norway, and overall, it is relatively underexamined.

Following the Meadian tradition, it is the mathematics teacher's activity of identifying oneself as a mathematics teacher, rather than to single out the one, true mathematics teacher identity, that is of my interest (Darragh, 2016; Sfard & Prusak, 2005). Like Andersson (2011), Bjuland et al. (2012) and Kaaslia (2007), I therefore choose a narrative approach for investigating the development of mathematics teacher identities. It is based on the assumption that peoples' accounts provide a window into the sort of learning they experience when moving from one setting into another (Jansen et al., 2012). These accounts are composed of recollections of the past and perceptions of the present, in line with Wenger's (1998) understanding of identity as a learning trajectory. Further, the teachers' self-authored accounts allow them to weave descriptions of feelings, attitudes and perspectives relevant to their mathematics teaching and learning experiences into their telling. A narrative approach to identity is thus in accordance with a holistic interpretation of mathematics teacher learning, as requested by Ponte and Chapman (2008). I will make a further discussion of storied identities in Chapter 3, when accounting for a narrative analysis of mathematics teachers' accounts.

2.2 Mathematics teacher learning within and between communities of practice

With his book *Communities of Practice: Learning, Meaning and Identity*, Wenger (1998) offers a theory of learning in which the primary unit of analysis is neither the individual nor social institutions, but instead *communities of practice*. A community of practice is defined through social participation, in which the members share some kind of competence through interaction, communication and negotiation of meaning. For the mathematics teacher, belonging to a community of mathematics teacher colleagues is a way of sharing the practice of mathematics teaching and negotiating the meaning of what mathematics teaching is or should be. Hence, the *community* lays a foundation for what the mathematics teacher does (*practice*), who the teacher is becoming (*identity*), and ways in which he or she creates *meaning* for the mathematics teaching. Wenger's (1998) theory explores the intersection of these four learning components: community, practice, meaning and identity. Together, they provide a conceptual framework for analysing learning as social participation, and the components are "deeply interconnected and mutually defining" (Wenger, 1998, p. 5). Figure 2.1 gives an overview of the four learning components:

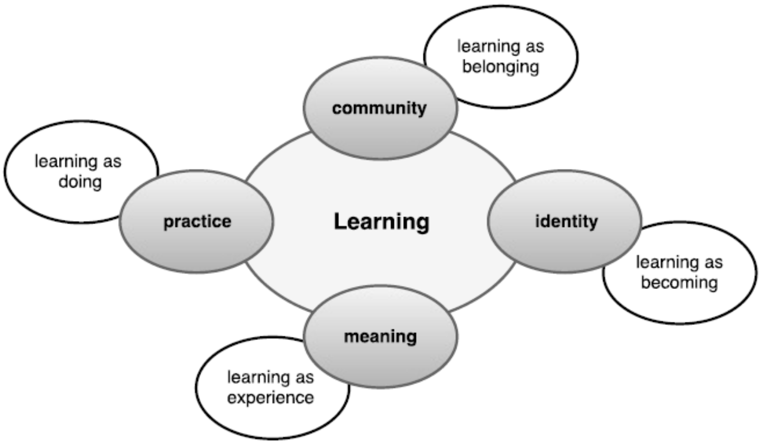

Figure 2.1: Components of Wenger's social learning theory (Wenger, 1998, p. 5)

As accounted for in the previous section, Wenger's theorisation enables me to study a prospective mathematics teacher identity as teacher-in-the-learning-community-in-the-teacher (Graven & Lerman, 2003; Lerman, 2000, 2001; Palmér, 2013). The first part, teacher-in-the-learning-community, places the social dimension of identity development in the foreground and thus addresses identity from the left part of Figure 2.1. It concerns the mathematics teacher's community memberships and the related mathematics teaching practices and negotiated meanings of mathematics and mathematics teaching and learning. Further, a focus on teacher-in-the-learning-community enables identification of the characteristics of current communities of practice and a study of their development over time, a detection of their boundaries and the relations with other communities. Developing an identity is then about evolving inbound memberships in communities, reconciling memberships across communities, and trying to establish continuity across boundaries.

By perceiving mathematics teacher identity from the right part of Figure 2.1 and placing the individual mathematics teacher in the foreground, I assume that a teacher who steps into a practice is a somehow changed teacher. By orienting towards the practice of a community, taking up new practices and marking a distance towards other practices, the teacher negotiates experience of self as a learner and doer of mathematics and mathematics teaching. Following Wenger (1998), (a mathematics teacher) identity is therefore strung between individual and social aspects of learning (mathematics, its teaching and learning). Individual aspects concern a prospective teacher's mathematical background, his or her dynamic perspectives on the nature of mathematics and mathematics teaching and learning, as well as considerations about him- or herself as a mathematics learner and teacher. These individual aspects are then continuously negotiated and reconsidered through participation in communities of mathematics and mathematics teaching at the university and at the school of employment.

In the subsequent sections, I will elaborate on a theoretical interpretation of mathematics teacher identity in transition, by approaching the notion of identity from a community perspective and from the individual mathematics teacher's perspective. I first turn towards possible communities of practice for prospective mathematics teachers to be engaged in at university and in school. This includes an account of university and school communities and their possible negotiated meanings of mathematics and mathematics teaching and learning and related mathematics practices. Next, I turn towards the process of identity formation, explained by Wenger (1998) as the dual process of identification and negotiability when participating in communities of practice. Identification in mathematics communities is elaborated on with help from Ernest's (1991) accounts of educational ideologies in mathematics, Andrews and Hatch' (1999) model of secondary teachers' conceptions of mathematics, and Hammersley's (2002) report on views on educational research. Further, negotiability is understood as authoritative knowing regarding mathematics and mathematics teaching (Belenky et al., 1986; Povey, 1997). This also brings about consideration of emotions when developing a secondary school mathematics teacher identity (Hodgen & Askew, 2007). In order to take into account learning as participation within and across community boundaries, I eventually discuss potential learning mechanisms when undergoing boundary crossing (Akkerman & Bakker, 2011).

2.2.1 Communities of mathematics, mathematics teaching and learning

Within the frames of Wenger's (1998) social learning theory, a prospective mathematics teacher's movement between university and school implies various forms of participation in communities of practice and the work of reconciling memberships across the community boundaries. The two institutions, university and school, represent different negotiated meanings of mathematics and mathematics teaching and learning, different ways of approaching the mathematics content and of organising mathematical activities. Making an overview of potential mathematics practices within secondary teacher education and lower and upper secondary school is then helpful.

Within the adopted theoretical framework, the mathematics practices within the subject studies at the university are understood to arise from potential communities of practice in which mathematicians, lecturers and/or university students participate (Wenger, 1998). The student teachers in my study take part in studies of mathematical sciences together with other undergraduate mathematics students, with an emphasis on understanding and specialisation in the discipline. The mathematical practices of a potential university community can then be described in line with Richards' (1991) definition of academic mathematics, as the discourse taking place within mathematics research and among mathematicians. Typical features are a technical language, the emphasis on making generalisations, conjectures and refutations, and a "subtle reliance on notions regarding the nature of proof" (Richards, 1991, p. 15).

Further, the practices of the mathematics community incorporate assumptions about the nature of mathematics, and in turn, assumptions about how mathematics should be taught to university students and how the students learn mathematics. This is claimed by for instance Thom (1973): "In fact, whether one wishes it or not, all mathematical pedagogy, even if scarcely coherent, rests on a philosophy of mathematics" (p. 204).

Ernest (1991) has described a range of perspectives on the nature of mathematics, spanning from a *dualistic absolutist* to a *relativistic fallibilist* view. The former sees the discipline of mathematics as certain, made up of absolute truths and structured into simple dichotomies such as right and wrong, true and false. Identifying with this perspective, mathematics may then be communicated to the students through unrelated routine tasks that involve the application of learned procedures. The latter position, on the other hand, combines an acceptance of multiple intellectual and moral perspectives with an understanding of knowledge as a social construction. Since mathematics is culture-bound, value-laden and based on human activity and inquiry, people need to actively engage with mathematics by posing as well as solving problems.

Based on his analysis of ideological groupings in the mathematics curriculum debate in Britain during late 1980s and early 1990s, Ernest (1991) presents five interest groups and their perspectives taken regarding the nature of mathematics and related objectives for teaching and learning mathematics. His model is based on a review of positions in the philosophy of mathematics, together with Perry's (1970) theory concerning the development of individuals' epistemological and ethical positions, as well as Williams' (1961) analysis of social groups and their influence on education. The five ideologies have later been discussed in relation the question of what it means to know mathematics (Ernest, 2004a, 2004b) and why mathematics should be taught in school (Ernest, 2000). A summary of possible perspectives on the nature of mathematics, its teaching and learning is given in Table 2.1 (see Ernest, 1991, pp. 138-139).

Table 2.1: Overview of social groups and educational ideologies in mathematics

Interest groups	Ideologies	Views of mathematics	Mathematical aims	Theory of teaching	Theory of learning
Industrial trainers	Dualistic absolutism	A set of truths and rules	'Back-to-basics', numeracy	Transmission of rules; drill and rote learning	Hard work, effort, practice
Techno-logical pragmatists	Multiplistic absolutism	Unquestioned body of useful knowledge	Useful mathematics to appropriate level and certification	Skill instructor, motivate through work-relevance	Skill acquisition, practical experience
Old humanists	Separated relativistic absolutism	Body of structured, pure knowledge	Transmit body of mathematical knowledge	Explain, motivate, pass on structure	Understanding and application in problem solving
Progressive educators	Connected relativistic absolutism	True and absolute; however, human, creative	Creativity, self-realisation through mathematics	Facilitate personal exploration, prevent failure	Activity, play, exploration
Public educators	Relativistic fallibilism	Social constructivism	Critical awareness and democratic citizenship via mathematics	Discussion, questioning of content and pedagogy	Questioning, decision making, negotiation

According to Ernest (1991), the industrial trainers and technological pragmatists represent utilitarian values of mathematics education and an emphasis on training a suitable workforce. However, while the former group favours basic skill training, the latter group is concerned with the acquisition and development of a broad range of mathematical knowledge and skills that prove worthwhile in employment. Hence, information technology capabilities, communication and problem solving skills are included. Yet, for both groups, mathematics is perceived as an absolutist discipline, being above all useful and applicable in other areas and disciplines. In contrast to utilitarian perspectives on mathematics, the old humanists consider pure mathematics to be worthwhile in its own right. In Ernest's model, this interest group is identified with the ideology of separated, relativistic absolutism. Hence, mathematics is considered rational and logic, neutral and value-free, with a clarity, purity and objectivity in reasoning. Teaching mathematics is then about communicating the structure of mathematics meaningfully. The progressive educators, too, relate to an absolutist philosophy of mathematics. However, they attend to the nature, interests and needs of the learner, represented by the typical slogan "save the child". Their aim is thus to enhance the mathematics learner in terms of self-esteem and as a confident agent in mathematics. The more radical group of public educators is concerned with education for all to promote democratic citizenship, including critical thinking in mathematics. They are the only interest group being identified with a fallibilist perspective on mathematics, in which the discipline is perceived as culture-bound, value-laden and based on human activity and inquiry.

According to Ernest (1991), a weakness of the model is its dependence on simplifying assumptions. For instance, the model assumes that a single ideology and interest group have maintained its distinctiveness over the course of time, and despite the large changes in knowledge, society and education. Especially, the model is developed within a British educational context, with its particular history and social characteristics. Despite its limitations, I yet assume that the practice of a mathematics community will, partially or largely, be in harmony with one or several of Ernest's (1991) presented ideologies in mathematics education and the related perspectives on the nature of mathematics, its teaching and learning. Hence, I find the model to be helpful in providing terminology when describing the characteristics of the communities in which the prospective mathematics teachers participate.

Another way of describing a community of university mathematics is to contrast its academic practices with those of other scientific communities. For some actors, mathematics can be perceived as a body of concepts, rules and methods being free of human bias. Thus, mathematics might hold a special status, compared to disciplines which demand a critical attitude and where scientific truth is perceived as imperfect and corrigible. This demarcation has by the physical chemist and writer C. P. Snow been denoted as *the two cultures* (Snow, 1963). In his lecture given at the University of Cambridge in 1959, Snow argued that the (natural) sciences and the humanities had through the years developed a tension to each other. The lecture was meant as a critique of the British educational system, with its one-sidedness and early specialisation. According to Snow, it made the students ignorant and hostile towards the other discipline regardless of which side they belonged.

Prospective secondary mathematics teachers also take part in practices situated in university teacher education. When undergoing the teacher education programme, PPU, either within the integrated five-year Master's programme or after having completed a university degree in mathematics, the student teachers in my study take part in lectures on general pedagogy and subject didactics. Another considerable part of teacher education is the approximately 14 weeks of school placement, divided between two semesters. Hence, mathematics goes from being a research domain and scientific discipline with its perceived nature and characteristics, to become a subject to be taught by the student teachers in secondary school. The mathematics student teachers thus enter what Snow (1963) would characterise as *the other culture* of social sciences. Here, practices of mathematics, mathematics teaching and learning arise from communities of practice in which lecturers at PPU, educational researchers, prospective mathematics teachers and/or tutors in school are members. An example of a community of practice is given by Palmér (2013). She reports on a *community of reform mathematics teaching*, in which the student mathematics teachers in her study became members during teacher education. It represents mathematics teaching practices characterised by student-focused teaching, being reality-based, varied and focused on problems and processes, in contrast to textbook focused, repetitive and abstract mathematics teaching. The student teachers also expressed a vision of teaching mathematics in line with the reform when entering school as certified teachers.

Further, the prospective mathematics teachers are exposed to mathematics teaching practices and related perspectives on the mathematics discipline, its teaching and learning, when entering school. According to Stadler (2009), mathematics practices in school is often associated with teaching where the students are listening to a short introduction made by the teacher at the blackboard, followed by individual work on tasks in the textbook. The tasks are solved by a certain method to get the one, correct answer. Mellin-Olsen (1990) denotes it as the *task discourse*[5], being a language and a practice that teachers tend to exercise in the mathematics classroom. He applies the metaphor of a journey, where the students to varying degrees are able to follow the teacher: they are either ahead of, behind, or have fallen off the wagon. The aim of the mathematics lesson is to clarify a set of mathematics conditions, so that the students are able to solve certain kinds of tasks. A consequence of the task

[5] "Oppgavediskursen", in Norwegian.

discourse is the students' tendency to focus on the correct answers, rather than finding and evaluating various ways of solving the tasks. Turning to Ernest (1991), the discipline of mathematics might then appear as a set of absolute and disconnected rules, formulas and algorithms to be applied on specific problems, which is in line with dualistic and/or multiplistic absolutism in Table 2.1. On the contrary, an inquiry process within a *landscape of investigation* (Skovsmose, 2001) invites students to answer and formulate questions such as "What if…?" and "Why is it so that…?" The aim of mathematics education within this perspective is critical thinking in mathematics, and in a long term perspective, the development of democratic citizenship. The students' active and critical engagement with the subject resonates with a view on mathematics as socially constructed. Hence, mathematics teaching through inquiry is related to Ernest's (1991) ideology of relativist fallibilism.

2.2.2 Community of practice as mutual engagement, joint enterprise and shared repertoire

Having accounted for possible mathematics practices and related perspectives on the mathematics discipline at university and in school, I now turn to Wenger's (1998) elaboration of the three dimensions which constitute a community of practice: mutual engagement, joint enterprise and shared repertoire (Figure 2.2).

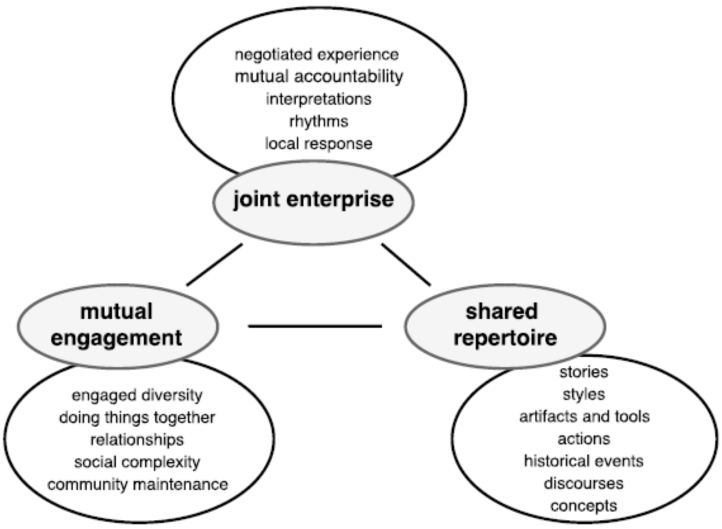

Figure 2.2: Dimensions of practice as the property of a community (Wenger, 1998, p. 73)

According to Wenger (1998), a community of practice exists because people are engaged in actions whose meanings they negotiate with one another. In a potential *community of mathematics teaching* at a secondary school, the participating mathematics teachers are *mutually engaging* in organising, planning, implementing and perhaps developing their mathematics teaching practice. This mutual engagement of negotiating what mathematics teaching is or should be like, what belongs to the community and what does not, constitutes in turn the community's *joint enterprise*. As an example, the members of the community of mathematics teaching might mutually engage in the joint enterprise of developing mathematics teaching through inquiry. This can further be reified into various resources, such as lesson plans, problem sets and collaborative project descriptions in mathematics, which then become parts of the *shared repertoire* of the community.

Further, the members of the mathematics teaching community negotiate the meaning of mathematics, its teaching and learning, based on what is communicated through the subject curriculum. However, public school policy documents are also related to negotiated practices within other, either overlapping or distinct, communities. According to Wenger (1998), a community of practice is therefore part of larger collections of communities which are connected to each other to a greater or lesser degree. The *boundary* of the community is therefore constitutive of what counts as participation and what does not. It is a demarcation or discontinuity between participants and outsiders. However, it simultaneously connects the community of practice with the rest of the world. Continuity across community boundaries can then be based on shared historical roots, related enterprises, shared institutional belonging, and shared artefacts and *boundary objects* such as the mathematics curriculum (Wenger, 1998). Another example of a boundary object is the lesson plan that student teachers are supposed to write and submit to the teacher educator and the tutor in school prior to their lessons during school placement. The paper may act as a boundary object when it bridges perspectives on mathematics teaching stated at the university with experience of classroom practice in school. However, the paper may also act as a disturbing element, being submitted only as a mandatory task.

Further, continuity can be established through boundary practices of *brokering*, when people introduce elements of one practice into another (Wenger, 1998). The newly qualified mathematics teacher may act as a broker when moving between communities at the university and in school, and bringing ideas of mathematics and mathematics teaching from one community into another. However, acting as a broker is challenging and uncomfortable. It requires enough legitimacy to influence the development of practice within a community, and enough courage to address conflicting interests. Entering school as a newly qualified mathematics teacher can then turn out as a confrontation between own expectations and expectations stated by teacher education, students in the classroom, colleagues or parents.

A community of practice can be designed, or it can emerge from people's shared histories of learning (Wenger, 1998). In the case of designed communities, it is nevertheless the members' mutual engagement and negotiation of the joint enterprise that constitutes the community of practice. Consequently, a community of practice is continuously developing due to its members' negotiation of meaning. Membership in a community is therefore not just a matter of belonging to an organisation or a category of for instance mathematics teachers. In my study, I am not designing or defining communities of practice on beforehand, in which I assume the research participants to belong by virtue of their educational background and professional career. Instead, I take the prospective mathematics teachers' perspectives and investigate their participation in mathematics related communities of practice based on their stories of entering the profession. Hence, I assume that the research participants' accounts of being university students and mathematics teachers in school provide evidence of mutual engagement, joint enterprises and shared repertoires regarding mathematics teaching and learning. The mathematics teacher's identity development from participating within and across the community boundaries is accounted for in the section to follow.

2.2.3 Mathematics teacher identity in communities of practice

The work of developing a shared mathematics teaching practice, for instance in a community of mathematics teachers in school, requires engagement and interaction among the associated actors, and acknowledgement of each other as participants. Simultaneously, each participant is negotiating ways of being a person in the community, e.g. being a mathematics teacher, a professional, a teacher colleague, and an employee. Hence, the existence of a community of mathematics teaching is also a negotiation of related identities. On this basis, Wenger (1998) defines identity as a negotiated experience of self when participating within and between communities of practice. Developing an identity as a mathematics teacher can thus be characterised as "increasing participation in the practice of teaching, and through this participation, (…) becoming knowledgeable in and about teaching" (Adler, 2000, p. 37). A gradual change in participation, from the periphery towards the centre of a community of practice, is by Wenger (1998) described in terms of engagement, imagination and alignment (see Figure 2.3).

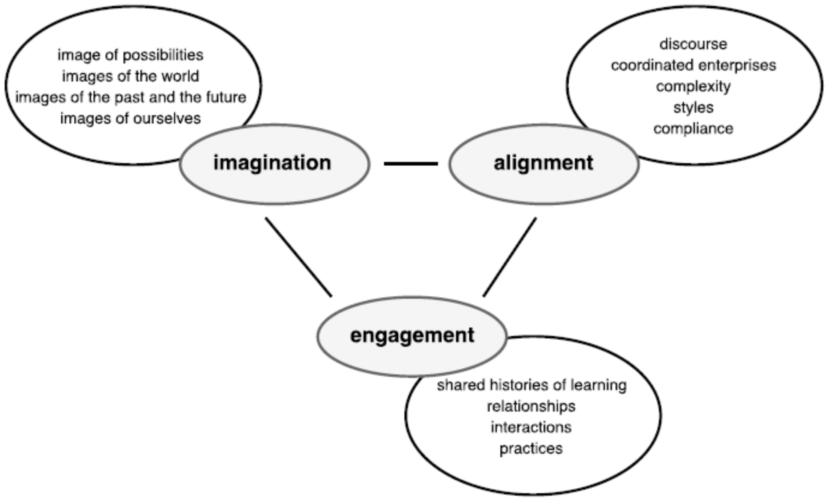

Figure 2.3: Modes of belonging (Wenger, 1998, p. 174)

As an example, a prospective mathematics teacher might *engage* with ideas of inquiry based mathematics teaching through involvement in communicative practices with other community members (e.g. teacher educators, tutors, fellow student teachers, students in the classroom, teacher colleagues) during teacher education or in school. Consequently, he or she takes part in ideas of mathematics teaching through *imagination*, by envisioning him- or herself as a teacher in a future classroom who is implementing the community's practice of inquiry based mathematics teaching. Doing what it takes to play part in the community, the prospective teacher also *aligns* with the conditions or characteristics of the community's practice. This can take place through reading and sharing relevant literature, implementing inquiry based activities in own mathematics teaching and being involved in professional development projects concerning inquiry based mathematics teaching. The term alignment has later been challenged by Jaworski (2006) in terms of *critical* alignment, a means of not just aligning with practice as established in the community, but of looking critically at that practice while aligning with it. Alignment can for instance be a critical process if a newly qualified mathematics teacher acts as a broker between previous communities of practice at the university and current communities of practice at the school of employment. In this situation, the teacher enters the mathematics classroom and questions the normal, desirable and harmonious state of mathematics teaching and learning (Jaworski, 2006). However, it is hard to operate against or challenge what is a harmonious, well-established practice. Mathematics teachers who start from critical perspectives on teaching might therefore be drawn into a school's mathematics practices in a way that makes it difficult to hold on to their imaginative ideals.

Having accounted for modes of belonging to communities of practice, I have considered the concept of identity to be constituted by what a person experiences as familiar, imaginable and what can be negotiated and made use of. However, Wenger (1998) also emphasises that a person identifies him- or herself through practices that he or she does *not* engage in. Hence, an identity is formed by both participation and non-participation in communities of practice. Assuming identity to be a mixture of being in and being out, Wenger distinguishes two cases of the interaction of participation and non-participation: *peripherality* and *marginality*. In the case of peripherality, the participation aspect dominates and leads to either full participation or a peripheral trajectory. In the case of marginality, however, the non-participation dominates and prevents full participation. This leads to either a marginal position or non-membership. Becoming a mathematics teacher is then about navigating oneself between various practices in university and school and the related negotiated meanings of mathematics, its teaching and learning. The teacher will become involved in some communities, and distancing him- or herself from others. For instance, he or she might experience belonging to communities of practice in which the discipline of mathematics is in the foreground of the negotiated enterprise, while the students' learning and their development remains in the background. Simultaneously, the prospective teacher might dissociate him- or herself from communities at university or in school in which general aspects of teaching and learning are in the foreground and the mathematics subject is absent. However, equally important for identity formation is the teacher's ability to shape the meanings that matter in the communities in which he or she belongs. Participation in communities of practice is thus a dual process of identification and negotiability, for which I will give a further elaboration in the Sections 2.2.4 and 2.2.5 respectively.

2.2.4 Identification in practices of mathematics teaching and learning

According to Wenger (1998), each of the three presented modes of belonging in communities of practice can be a source of the dual process of *identification* and *negotiability*. Identification concerns a person's investment in various forms of belonging to community practices, being both participative ("identifying with") and reificative ("identifying as"). Hence, identifying *with* a mathematics teaching practice or a group of actors of mathematics teaching is simultaneously a process of being identified *as* a special kind of mathematics teacher. In order to characterise the mathematics practices in which a prospective mathematics teacher exercises identification, I return to Ernest's (1991) model of educational ideologies in mathematics (see Table 2.1). As an example, a mathematics teacher can identify him- or herself with the aforementioned community of inquiry based mathematics teaching by aligning with a joint learning objective of facilitating students' mathematical confidence and social empowerment. These learning objectives are related to perspectives of the subject of mathematics as being a social construction; "tentative, growing by means of human creation and decision-making and connected with other realms of knowledge, culture and social life" (Ernest, 1991, p. 209). Through inbound participation in the community, the teacher's identity will then develop in correlation with the negotiated perspectives on the nature of the discipline.

According to Ernest (1991), perspectives on the nature of mathematics constitute the primary component of a person's *philosophy of mathematics* and underpin espoused theories of teaching and learning mathematics. These in turn have an impact on the mathematics teacher's practice, mediated however by the constraints and opportunities provided by the social context of teaching (Ernest, 1991).This system of beliefs, understood as mental objects that a mathematics teacher has or carries with him from one practice to another, is initially not compatible with Wenger's (1998) situated learning perspective. However, I understand a teacher's expressed perspectives of mathematics and mathematics teaching and learning to be dynamic, meaning that they are continuously negotiated through interaction with other participants within various communities of practice. These perspectives can thus be considered as related to the mathematics teacher's active interpretation of his or her role in a given community of practice, from where he or she adopt different stances towards the task of teaching mathematics.

The applicability of Ernest's model can further be founded on Wenger's (1998) term of reification. As part of the negotiation of meaning, reification is a way of giving form to a person's experience "by producing objects that congeal this experience into 'thingness'" (Wenger, 1998, p. 58). Reification of a community's practice can appear as artefacts and negotiated abstractions, symbols, concepts, stories and discourses on which the members can act. Becoming a participant in a community is therefore about growing into the practice in which one engages, including its reifications. The prospective mathematics teacher thus takes up new practices, including expressions of ideologies or perspectives on the nature of mathematics. Here, Ernest's (1991) model provides helpful terminology for characterising the possible views or discourses regarding the nature of mathematics to which a prospective teacher exercises identification. As previously argued, it also enables a mathematical profile to the prospective teachers' developing identities. Consequently, I do not seek to investigate perspectives on mathematics and mathematics teaching as properties of the individual mathematics teacher and underlying reasons for his or her teaching practice. Instead, I perceive these perspectives to represent characteristics of the social practices in which the prospective teacher participates and thus develop his or her mathematics teacher identity.

Comparable with Ernest's (1991) model is Andrews and Hatch' (1999) report on secondary teachers' conceptions of mathematics and mathematics teaching. They present four approaches to the discipline of mathematics taken by teachers that can be linked to the ideologies in Table 2.1: mathematics as an economic tool, mathematics as a service, mathematics as a life-tool, and mathematics as a diverse and pleasurable activity. Like the model of Ernest (1991), their review of teachers' conceptions is helpful when characterising the practices in which prospective teachers' exercise identification.

Mathematics as a personal *economic tool* has a narrow focus by presenting the subject as a means by which people can maintain their household accounts, and thus, gain pleasure from learning the subject. Although stressing mathematics as a pleasurable activity, its emphasis of basic skill training and the usefulness in daily life relates this conception to the ideology of industrial trainers in Table 2.1. Also, mathematics as a *service* resonates with Ernest's (1989) notion of instrumentalism, in which mathematics is considered a collection of disconnected facts, rules and skills that are used to support work in areas other than the discipline itself. Similar to the perspective of mathematics as an economic tool, it thus echoes absolutist perspectives on the nature of mathematics and the perspectives of the industrial trainers.

Mathematics as a *life-tool* stands in contrast to the service perspective, by acknowledging that mathematics is more than a tool in everyday problem solving. It is instead considered to empower learners through higher levels of understanding than what is gained from rote learning and the use of techniques. By still highlighting its utility, Andrews and Hatch (1999) relate this perspective to absolutism, however in combination with fallible components due to the demands for understanding and its empowering function. Compared to Ernest's (1991) model, it thus shares aspects with both the technological pragmatists and the public educators.

Taking the approach of a *diverse and pleasurable activity*, mathematics is offering a unique perspective on the world, in which people can gain enjoyment from its precision and the engagement with proofs. Hence, this conception shares aspects with the purist ideologies in Ernest's model, represented by the old humanist group. However, by acknowledging peoples own sense-making of the discipline and emphasising mathematics as problem-solving, Andrews and Hatch (1999) claim this perspective to be closer to fallibilism than absolutism. Consequently, I perceive the diverse and pleasurable activity with mathematics to be in accordance with the progressive educators' ideology in Ernest's (1991) model.

In addition to navigate among mathematics practices and related perspectives on the nature of mathematics, prospective mathematics teachers are exposed to talks and discussions about teacher education. For instance, the prospective teachers might take part in a discussion regarding the "ideal" teacher education programme and its role in improving mathematics teaching in school. Related to such a discussion are the possible ways of perceiving educational research and its contribution for the practice of mathematics teaching, to which the prospective teacher can exercise identification and non-identification. Hammersley (2002) reports on two dominating models for considering the contribution that research can make to policymaking or practice: the engineering and enlightenment models. According to the *engineering* model, educational research must provide applicable and technical solutions to problems of mathematics teaching in the way that natural science and engineering research are assumed to do. Implied in the engineering model is thus an absolutist perspective on knowledge, with the expectations that research-based evidence will give an answer to the question of "what works" in education (Biesta, 2007). In contrast, the *enlightenment* model represents fallible knowledge from which mathematics teaching practices can draw. Hence, research is one among several sources of knowledge. Prospective mathematics teachers should therefore develop the reasoning necessary to critically evaluate research on mathematics teaching and learning. Based on Hammersley's (2002) report, I assume that the prospective mathematics teachers in my study take part in negotiated ways of perceiving the role and aim of mathematics teacher education, from which they build their identity as a (student) teacher in mathematics.

2.2.5 Negotiability in practices of mathematics teaching and learning

Processes of identification with community practices define which meanings of mathematics matter to a prospective teacher. However, it does not determine the teacher's ability to negotiate these meanings. *Negotiability* then refers to the "ability, facility, and legitimacy to contribute to, take responsibility for and shape the meanings that matter within a social configuration" (Wenger, 1998, p. 197). As an example, the meanings that a mathematics classroom community (teacher and students) produce of mathematics, its teaching and learning, are not only local meanings. They are also part of a broader *economy of meaning* in which different meanings are produced in different locations and compete for the definition of what mathematics teaching and learning is or should be. Consequently, some meanings of mathematics teaching and learning achieve special status, being for instance those produced in the students' previous mathematics classroom communities or in their families. Further, the notion of *ownership of meaning* refers to the teacher's or his or her students' ability to take responsibility for negotiating the meanings of mathematics within the classroom community. An uneven ownership of meaning, in which some participants always produce proposals of meaning and others always adopt them, can result in peripheral participation or marginalisation for those only adopting meaning (Wenger, 1998). A possible consequence of students' dominating ownership can be a limited scope of action for the mathematics teacher to change or develop the reigning classroom practice. Another possible outcome is the students' resignation from the learning activities in mathematics, in which the mathematics teacher can experience an inability to act on the students' mathematics learning.

In order to describe negotiability in the context of a prospective mathematics teacher's identity development, I find it useful to draw on Belenky et al.'s (1986) study of women's *way of knowing*. Similar to Ernest (1991), their work builds on Perry's (1970) theory of intellectual and ethical growth. Perry's model consists of nine stages or positions from *basic duality* through various forms of *multiplicity* to *relativism*: *basic duality, full dualism, early multiplicity, late multiplicity, contextual relativism, pre-commitment, commitment, challenges to commitment*, and *post-commitment*. The initial positions involve a dualistic way of knowing: right or wrong, good or bad. However, intellectual development moves the person from this position of right answers for everything, towards the awareness that there are things that are not known yet, and further, that everyone has a right to his or her own opinion (multiplicity). At the position of relativism, the person becomes aware that theories are not truths but models or metaphors used to make sense of experience. Perry's model is developmental, meaning that people at the more advanced positions can understand earlier meanings; however, people within the earlier forms cannot understand the assumptions of more advanced positions.

Belenky et al. (1986) criticise Perry's (1970) work for being limited, as his study was based on observations of male college students only, at Harvard University. As an alternative, Belenky and colleagues (1986) used a composite female sample (135 women of various ages and sociocultural backgrounds) for creating a theory on the development of the individual as a knowledge maker. They developed five major epistemological categories for women's perspectives on knowing: silence, received knowing, subjective knowing, procedural knowing and constructed knowing. In the *silence* knowing stage, knowledge does not belong to the individual and cannot be vocalised. From the perspective of *received knowledge*, the knower is capable of receiving and reproducing knowledge from the all-knowing external authorities. However, she cannot create knowledge on her own. Contrarily, *subjective knowing* is a position of perceiving truth and knowledge as personal, private and subjectively known or intuited. Further, *procedural knowing* concerns knowing being invested in learning and instruction of objective procedures that can be applied for both obtaining and communicating knowledge. Finally, the stage of *constructed knowing* is recognised by individuals who perceive knowledge as contextual. Hence, they see themselves as creators of knowledge, and they value both subjective and objective strategies of knowing.

In my study, I do not investigate mathematics teacher identity from a gender perspective, for instance by comparing female and male identities and their development. Instead, I assume that the categories of Belenky et al.(1986) describe aspects of negotiability in mathematics related practices that apply to both female and male mathematics teachers. Starting from the work of Belenky and colleagues, Povey (1995, 1997) interviewed both female and male beginning teachers in mathematics and developed five categories of their ways of knowing in pedagogy and mathematics: silence, external authority, internal authority, and the author/ity of self and reason. Povey emphasises that ways of knowing in her study is not developmental, meaning that there is no clear sequential ordering of epistemological positions. Hence, becoming a mathematics teacher is not about undergoing a universal developmental pathway of knowing in mathematics and mathematics teaching. Instead, her categories are meant to illuminate the thinking of beginning teachers of mathematics, and further, their differences in action regarding the teaching of mathematics in school. Her work is helpful for aligning the notion of ways of knowing to my study of mathematics teacher identity.

Within Povey's (1995) categories, the state of *silence* is similar to the state described by Belenky et al. (1986). It is recognised by a teacher's disconnection with mathematics and its teaching, in which he or she is feeling "deaf" in terms of not learning from others and "dumb" because he or she lacks a voice. I perceive this state as in accordance with Wenger's (1998) description of marginalisation and absent negotiability. Since the mathematics teacher is struggling to maintain and develop a sense of self as a legitimate member of the profession, his or her negotiation of the current community's practice is also absent. This can for instance take place in unpleasant classroom situations, when being under judgement of what appear as hostile mathematics students. Further, the state of *external authority* is characterised by absolute and fixed knowledge given by "experts". The voice of an external authority is thus heard, however, the mathematics teacher lacks an inner voice to challenge the authority. One example is the mathematics teacher's possible reliance on external resources for his or her teaching, by acting in accordance with the tutor's or teacher educator's expectations. In addition, the discipline of mathematics is commonly perceived as being based upon the authority of others, who will make judgements about right or wrong (Povey, 1995). The lecturers at the university are examples of such authorities. Hence, received knowing can be a possible state of knowing for student teachers when undergoing mathematics subject studies at university.

On the contrary, the state of *internal authority* is recognised by the mathematics teacher's abstain from external authorities. The voice of authority is still heard and provides absolute answers, but now the authority is the self. Within this position, the mathematics teacher might find the purpose of teacher education to be an examination of a range of mathematics teaching styles and approaches, to see what fits him or her. Common for both external and internal authority are then the lack of negotiation or an uneven ownership of meaning within communities where the mathematics teacher participates. However, critical judgement and joint negotiation of meaning are present in the state of *author/ity of self and reason.* Here, both external sources of authority and the internal voice of the teacher are listened to and evaluated. A way of exercising this kind of authorship is to perceive and approach the discipline of mathematics as fallible, socially constructed and a subject for critique.

Belenky et al (1986) assume that the way a person perceives the nature of truth and reality shapes the way he or she sees the world and him- or herself as a participant in it. I join this statement, however, with the following rephrasing: that a person's partaking in ways of perceiving truth and reality also shape the ways that he or she negotiates him- or herself as a participant in it. Consequently, the ways of knowing, in other words the sense of negotiability in mathematics communities, are part of the mathematics teacher's identity and development. Further, I follow Povey (1997), who states that "individuals may have different ways of knowing about different aspects of their lives, as well as different ways of knowing about the same aspects of their lives at different times" (p. 333). For instance, silence can be experienced at moments of self-doubt or as an undertone in any of the other positions. Since the teacher's membership in any community of practice is only a part of his or her identity, a developing mathematics teacher identity thus needs to be viewed as a *nexus of multimembership* (Wenger, 1998). However, Wenger specifies that a nexus does not merge the specific learning trajectories a person forms within various communities into one; neither does the nexus decompose identity into distinct trajectories. Becoming a secondary school mathematics teacher requires therefore the work of r*ec-*

onciling different forms of memberships. Since reconciliation is conceived as a significant challenge when moving across community boundaries, I will in Section 2.2.7 elaborate on mathematics teacher learning during boundary crossing. However, I will first discuss affective aspects regarding negotiability in mathematics and mathematics teaching.

2.2.6 Mathematics teacher identity and emotions

According to Povey (1995), there are emotional aspects related to each of the presented ways of knowing. Due to a situation of feeling "deaf" and "dumb", the state of silence is associated with fear, outsiderness and isolation. Further, external authority is related to a dependency of others, and thus, it can lead to limited protection against the criticism of experts. Related emotions are therefore feelings of constraint and vulnerability. Separation and isolation are also present feelings in the state of internal authority, as the knower is not concerned with the states of mind of others. The perspective of author/ity of self and reason can lead to loneliness. However, by taking into consideration both the sources of external and internal authority, this state allows the knower to keep a distance from the critiques of others. The prospective teachers' accounts of exercising negotiability within mathematics practices can therefore reflect their emotional relationship with and sense of connectedness to the subject. Contrarily, emotional accounts of becoming a mathematics teacher can indicate the prospective teacher's sense of negotiability within the discipline, and thus, evidence of learning trajectories towards inclusion or exclusion in mathematics related practices.

The affective aspects regarding knowing in mathematics and mathematics teaching is also acknowledged by Hodgen and Askew (2007), who claim that learning and change in terms of identity development can be deeply emotionally threatening. Firstly, they claim, mathematics generates stronger emotive reactions than other school subjects. Secondly, changes in teachers' identities involve "letting go of what one has been at the same time as maintaining the more fundamental aspects of one's identity" (Hodgen & Askew, 2007, p. 474). This latter situation of becoming somehow different can be both attractive and repulsive. Nevertheless, their study of a primary mathematics teacher's shift from a disconnection with the subject to a position of authority suggests the importance of establishing a more positive relationship with the mathematics discipline itself in order to develop an identity as a mathematics teacher.

Related to the discussion of a connection between affect and ways of knowing is Bibby's (2002) study of teachers' experiences of shame in mathematics. She claims that beyond reporting on people's emotional states in mathematics, there is a need to take into account a composite mosaic of mathematical beliefs, including epistemological beliefs about the nature of the discipline. Shame is here understood as a reaction to other people's criticisms, being a signal to find out whether one is being excluded or submerged within a social group (e.g. the group of secondary school mathematics teachers). For instance, Bibby (2002) suggests a link between experiencing mathematics as an absolutist and fixed discipline, and feeling shame by fearing that the apparent objectivity of mathematics will reveal subjective inadequacy.

In my study, I do not investigate the prospective mathematics teachers' epistemological beliefs in the form of espoused theories for their mathematics teaching practices. Instead, I perceive their emotional accounts of learning and teaching mathematics to be intertwined with experiences of identification and negotiability in mathematics related practices. Hence, the affective aspects of learning are part of their continuous interpretation of who they are in relation to mathematics, its teaching and learning. Further, the transition from university to school might uncover changes in emotions over time, and thus, expressions of discontinuities between practices when developing an identity as a secondary school mathematics teacher. I will now turn to identity development from participation between communities of practice, denoted as boundary crossing.

2.2.7 Developing a mathematics teacher identity during boundary crossing

As previously explained, people define themselves through practices they engage in as well as practices they choose not to engage in, meaning that non-participation is as much a source of identity as participation (Wenger, 1998). This balance of participation and non-participation concerns how a mathematics teacher locates him- or herself within the landscape of mathematics teaching, what he or she cares about and neglect and with whom he or she seeks connections. Further, learning takes place across community boundaries, as the prospective mathematics teacher tries to create bridges between present and earlier practices (Wenger, 1998). Based on Wenger's notion of *boundary*, I explore the transition between mathematics teacher education and the professional debut in school in terms of *boundary crossing*. This means that the prospective mathematics teacher continuously synthesises what counts as legitimate knowledge within several communities, and simultaneously strives towards continuity by maintaining a sense of self through time (Abreu et al., 2002). Hence, by overcoming discontinuities, boundary crossing carries a potential to learn about mathematics practices and about one's own mathematics teacher identity.

In Akkerman and Bakker's (2011) review of research concerning boundary crossing, boundaries are defined as "sociocultural differences leading to discontinuities in action and interaction" (p. 133). The definition highlights that boundaries are not about sociocultural differences per se. Instead, boundaries are "real in their consequences" (op. cit., p. 152). Thus, unlike describing sociocultural differences between university and school, I am interested in how the differences play out in and are being shaped by the process of developing mathematics teacher identities. Akkerman and Bakker (2011) present potential learning mechanisms that may occur when crossing boundaries, two of them concerning processes of making sense of practices in multiple contexts. *Identification*[6] entails a renewed sense making of different practices and related identities, by encountering and reconstructing boundaries but not being able to overcome discontinuities. It takes place through the dialogical process of *othering,* in which one practice is defined in light of another. An example is a student teacher's contrasting of the meanings of mathematics teaching and learning stated by respectively lecturers at the university teacher education programme and tutors during school placement. In other words, it concerns identification of the challenging relationship between educational research and teachers' everyday practice. However, there is also a need for *legitimating coexistence* of what is perceived as differing meanings regarding mathematics teaching, since becoming a certified mathematics teacher implies interaction with mathematics practices in both settings.

Unlike identification, learning in terms of *reflection* results in an expanded set of perspectives and a new construction of identity, which in turn will have an impact on future practice. The learning mechanism consists of *perspective making* and, in turn, *perspective taking*. The former is making explicit one's interpretations of a particular issue, for instance by creating a narrative structure of one's experience with mathematics practices at the university. Next, perspective taking is about "looking at oneself through the eyes of other worlds" (Akkerman & Bakker, 2011, p. 145), and thus, seeing one's mathematics practices in a different light. In the case of learning as reflection, the prospective mathematics teacher is able to develop an identity as a mathematics teacher being compatible with both the setting of university teacher education and the school setting, even though differences in negotiated meanings of mathematics and mathematics teaching and learning are observed.

[6] Learning as identification, as defined by Akkerman and Bakker (2011), should not be confused with Wenger's (1998) notion of identification, acting in combination with negotiation.

2.3 Closing remarks

In this chapter, I have presented Wenger's (1998) social learning theory of identity in community of practice as a superior framework for my study. Further, I have argued for the need to extend the theoretical framework in order to make the mathematics part of developing a mathematics teacher identity more visible. I have presented Ernest's (1991) model of educational ideologies and Belenky et al.'s (1986) and Povey's (1995) categorisations of ways of knowing as helpful terminologies for describing identity formation in terms of identification and negotiability. I have also accounted for possible learning mechanisms that may occur from crossing boundaries of communities of practice (Akkerman & Bakker, 2011). In this summarising section, I will make some final remarks regarding the combination of theoretical elements for studying the phenomenon of mathematics teacher identity development in the transition from university teacher education to school.

In their discussion of ways to deal with the diversity of theories in mathematics education research, Prediger et al. (2008) distinguish two ways of characterising the notion of theory: a *static view*, and a *dynamic view*. The static view regards theory as "a human construction to present, organize and systematize a set of results about a piece of the real world, which then becomes a tool to be used" (Prediger et al., 2008, p. 166). Hence, a theory is given to make sense of a phenomenon, in some way. On the other hand, a dynamic view regards theory as a tool rooted in a philosophical background; yet, it needs to be developed in a suitable way in order to answer a specific research question. I follow the latter characteristic, meaning that the theoretical framework of my study is rooted in a social perspective on learning with its basic foundations and assumptions regarding mathematics teacher learning and the notion of transition. Here, Wenger's (1998) theorisation represents an established, coherent explanation of learning in social practices. However, I also argue for the need to adjust the framework in order to answer the given research question on the development of secondary school mathematics teacher identities (cf. Section 1.2). I therefore adopt what Prediger et al. (2008) denote as the strategy of *combining* theoretical approaches. In contrast to coordinating theoretical elements, a combination does not necessitate a complete coherence of the theoretical approaches in view. Hence, the aim is to develop a suitable analytical tool rather than to make a coherent and complete theory of mathematics teacher identity and identity development. The work of Ernest (1991), Belenky and collegaues (1986), and Povey (1995) are not in complete coherence with Wenger's (1998) social learning theory. However, I have argued for their suitability in describing the practices of mathematics, its teaching and learning, in which prospective mathematics teachers build their identity. I thus consider the studies as commensurable for studying mathematics teacher identity.

I stated in Section 2.1.5 that this study intends to contribute to a further theorisation of secondary school mathematics teacher identity and identity development. Based on the current consideration of the theoretical framework, I suggest the contribution to be a combination of theoretical approaches having a descriptive power regarding the process of becoming a secondary school mathematics teacher. In the subsequent analysis, the framework is adopted in a multiple case study of prospective mathematics teachers' entrance into the profession, in addition to a cross-case analysis of their narrated mathematics teacher identities. The aim is to get in-depth knowledge as well as developing a more general description of mathematics teacher identity in transition. I give a further consideration of the operationalisation of the concept of identity and the analysis process in the subsequent chapter on methodology.

3 Methodology

In this chapter, I outline the methodological basis for my research. Hence, I discuss how to learn about, analyse and interpret prospective mathematics teachers' learning to teach mathematics, denoted in my study as their developing identity when undergoing the transition from university teacher education to school. Aligning with Burton's (2002) position in methodology, I aim for describing not only "how" the research was done, but also accounting for "why", and thus, what possibilities and limitations my choices of methods bring about. I therefore introduce the chapter by a discussion in Section 3.1 on the operationalisation of the concept of identity. I start from literature on narratives in qualitative research, where both Sfard and Prusak (2005) and Elliott (2005) argue for a communicative approach to identity research. Further, I discuss in Section 3.2 issues of trustworthiness in narrative research and underlying epistemological assumptions. Here, I also account for possibilities for generalisation.

The discussion on trustworthiness gives direction for choices of methods for data collection and the sample of participants, as presented in Section 3.3. In Section 3.4, I account for ethical considerations in the study. Further, the narrative approach to identity leads in Section 3.5 to considerations of methods for analysis. Based on the work of Polkinghorne (1995) and Lieblich et al. (1998), I discuss various approaches, spanning from making coherent portraits of individual identities (narrative analysis) to highlighting common aspects of identity across a group of people (analysis of narratives). In my report on the analysis process in Section 3.5.2, I describe a combination of the two approaches. Hence, I combine an in-depth reading of the individual participants' stories with a cross-case analysis of comparing and contrasting their storied identities. The aim is to move towards a more general description of developing an identity as a secondary school mathematics teacher.

3.1 A narrative approach to mathematics teacher identity

Based on the theoretical framework of the study, I assume that a mathematics teacher's identity is participative, meaning that it is constructed through participation in social groups denoted by Wenger (1998) as communities of practice. Hence, in line with the Meadian tradition as accounted for in Section 2.1, I consider identity as an action rather than as acquisition (Darragh, 2016), or with Wenger's terms, as a constant becoming rather than an object in itself to be unveiled by the researcher. Although the idea of identity is pivotal in Wenger's (1998) theorisation of learning, he does not make an operational approach to the concept. However, by following Elliott (2005) and Sfard and Prusak (2005), I link identity to the activity of communication. According to them, *narratives* are providing the practical means by which a person can understand him- or herself as living through time, and thus, making identity accessible and investigable. The term narrative is here understood as a way of organising a sequence of events into a whole, so that the importance of each event can be understood through its relation to a whole (Elliott, 2005). Accordingly, people are subjects with a past, present and future, made whole by the coherence of the narrative plot with a beginning, middle, and end. Through such a lens, a prospective teacher can then make sense of his or her process of becoming a mathematics teacher by selecting elements of experience regarding mathematics, its teaching and learning, and patterning the chosen elements in ways that reflect stories available to the audience. The resulting narrative is not a reconstruction of the mathematics teacher's life, but a portrayal of how he or she perceives the process of becoming a secondary school mathematics teacher.

The temporal characteristic of narratives is in line with Wenger's (1998) interpretation of identity as a *learning trajectory*, when incorporating "the past and the future in the very process of negotiating the present" (p. 155). Further, Wenger claims that a sense of trajectory provides people ways of sorting out what matters and not, and thus, what contributes to the mathematics teachers' identities and what remains marginal. Similarly, Carter and Doyle (1996) emphasise that a narrative approach to the process of becoming a mathematics teacher highlights its negotiated nature. Mathematics teachers' identities are not simply formed by their life experience prior to, during, or after mathematics teacher education programmes. Rather, the teachers are active participants in interpreting their experience, "searching for and constructing images that capture the essential features of their understanding of the tasks they encounter in [mathematics] teaching" (Carter & Doyle, 1996, p. 134). Yet, despite that narratives provide a way of regarding mathematics teacher identity as an active construction of meaning, Wenger (1998) states that a person's identity cannot be equated with a narrated self-image:

> It is not equivalent to a self-image, it is not, in its essence, discursive or reflective. We often think about our identities as self-images because we talk about ourselves and each other – and even think about ourselves and each other – in words. These words are important, no doubt, but they are not the full, lived experience of engagement in practice. (1998, p. 151)

According to Wenger (1998), teachers' stories of self do not bring out their full, lived experience of engagement in practice when becoming secondary school mathematics teachers. Still, by following Jansen et al.

(2012), I assume that teachers can make sense of their experience of moving between settings in education and work by creating narratives, or what I henceforth will denote as *accounts*[7], composed of recollections of the past, perceptions of the present and images of the future. Unlike being merely recalls and reports, these self-authored accounts allow the mathematics teachers to present their experience of mathematics teaching and learning from their perspectives. It is done by weaving descriptions of emotions, attitudes and views being relevant to the experience into their telling (Chapman, 2008). Using the terms of Hodges and Cady (2012), it is then the mathematics teacher's report on who he or she is in relation to other actors, who the teacher wants to be and feels he or she has to be when going about work as a mathematics teacher that constitutes the identity to be investigated in this study. Since I search for a teacher's own interpretations or meaning-making of his or her experience when moving between contexts, the accounts are not made in comparison with observations of what actually transpired. Instead, I make observations of classroom teaching in order to stimulate thorough responses regarding mathematics teaching and learning in follow-up interviews. I will look more closely at the methods for data collection in Section 3.3.

In the study, the prospective mathematics teachers' accounts are collected from a series of interviews taking place during their last year at the university and their first year as a mathematics teacher in secondary school. The teachers' accounts are "not necessarily full-blown stories with requisite internal structure, but may be short accounts that emerge within or across turns in the interviews" (Gubrium & Holstein, 1997, p. 146). Following Lieblich, Tuval-Mashiach and Zilber (1998), I further assume that the accounts are constructed around a core of life events regarding mathematics teaching and learning, although allowing "a wide periphery for the freedom of individuality and creativity in selection (…) and interpretation of these 'remembered facts'" (p. 8). Essential is also the context within which the accounts were narrated, meaning that the stories are collectively shaped even if individually told (Sfard & Prusak, 2005). In other words, the accounts are in part personal stories shaped by the history and experiences of the teacher, and at the same time, they are collective stories shaped by cultural, historical and institutional settings in which they occur (Moen, 2006). The relationship formed between the mathematics teacher and me being the listener, as well as the mood of

[7] A mathematics teacher's narratives or oral stories, as collected through a series of interviews, are henceforth denoted as the teacher's *accounts*. This is done in order to avoid confusion with the teacher's stories resulting from *narrative analysis*, which I will elaborate on later in this chapter.

the teacher, are also parts of the telling. Consequently, I view the teachers' accounts as particular stories within what Lieblich et al. (1998) denote as polyphonic versions of their negotiated selves.

Moreover, the accounts provided in the interviews are only selected instances of the participant's life story, meaning that I only get specific, frozen images of a dynamically changing mathematics teacher identity. By conducting a series of interviews, I yet have access to a rich data material in which I can compare and contrast utterances across interviews, including the participant's reflection on his or her previous utterances. Based on a longitudinal research design, I then gain insight into an ongoing process of identity development, at given points of times over a prolonged period.

3.2 Trustworthiness in narrative research

Trustworthiness in narrative research concerns ensuring quality in the generated accounts on two levels. Firstly, it concerns the accounts told by the research participants, and thus, the quality of the chosen methods for data collection. Secondly, it concerns ensuring quality in the accounts told by the researcher in the associated analysis. Here, I report on the former issue, while the latter is included in Section 3.5 on narrative analysis.

According to Polkinghorne (2007), one threat particular to narrative research relates to the differences in people's experienced meaning and the stories they tell about this meaning. Hence, the storied evidence gathered in this study does not determine if events actually happened and which specific meanings the teachers made of the events as they occurred. Instead, I search for the meanings that the research participants continuously negotiate and share with me through a series of interviews, from which I search for narrative truths rather than historical truths (Polkinghorne, 2007). In order to handle the threat to trustworthiness, I need then to elaborate on the characteristics of the knowledge made accessible through the interviews. In other words, I clarify the epistemological assumptions underpinning the choice of methods for this study.

While research interviews on the one hand can be viewed as a pipeline for transmitting knowledge, the interviews conducted in this study is better understood as a site for a production of knowledge. Kvale and Brinkman (2009) use the metaphor of a miner and a traveller to elaborate on these two perspectives on knowledge developed from research interviews. The miner extracts knowledge being ideally unsoiled by the interviewer and any leading questions, and thus, he searches for the objective facts or the essential meanings of the interviewee's reports. The traveller, on the other hand, perceives the interview situation as jointly constructed by the interviewer and the interviewee. Possible meanings of the interviewee's accounts are then differentiated and unfolded through the traveller's interpretations and the narratives he creates in the reported research.

Aligning this study with the metaphor of a traveller, the prospective teachers' accounts do not capture a simple record of undergoing teacher education and conducting classroom mathematics teaching in the way that a video camera might do. Instead, I search for the negotiated meanings that the prospective teachers attach to their experience of becoming a secondary school mathematics teacher. According to Huberman (1995), the teachers' accounts are then ways of *re-presenting* meaning rather than merely *presenting* the meaning of becoming a secondary school mathematics teacher. By selecting the salient aspects of what he or she has experienced and ordering them into a coherent whole, the accounts serve as a means for taking distance from events in life, and thus, making them objects of reflection. Consequently, the quality of the methods for data collection will rely on the space left for the teachers to create thorough accounts, with rich descriptions, in which they can explore their feelings and experiences.

Polkinghorne (2007) points at sources for threats in narrative research and suggestions to comply with them. First, there are limitations in language to capture the complexity and depth of experienced meaning. Accordingly, not all the meanings one has about a situation is available in awareness and in the current interview setting. As an interviewer, I therefore need to allow time for participants to explore reflectively their meanings, by allowing silence and not interrupting them, and by asking follow-up questions to their stories when appropriate. Second, people are often resistant to reveal emotions and understandings to strangers, especially those that can create an undesirable self-portrait. However, by arranging a series of interviews over a longer period of time, I can create an atmosphere in which the teachers feel comfortable and where trust and confidence is established. The comfortable atmosphere can also be developed from empowering the participants to set the agenda, when inviting them to talk freely about their time in university teacher education and about learning and teaching mathematics at the beginning of the interview. Important is also acknowledging their reports, by nodding and showing interest to the stories they share.

By adopting a longitudinal design for the study, I can further attend to the continuity of narrative resources on which the research participants draw. However, rather than aiming for saturation in their narrated identity, the series of interviews can create what Thomson and Holland (2003) refer to as a kaleidoscope approach. It entails that every time I conduct an interview, and thus, gain insight into the prospective mathematics teacher's developing identity, I can find something rather different. What is different is based on accounts being composed of the same narrative resources as before, but in new configurations. Hence, the series of interviews can expand on already reported aspects of identity, confirm or on the other hand give evidence on change and development in ways that the teachers' perceive the process of becoming a mathematics teacher.

3.3 Methods for data collection

As I have stated in Section 3.1, I take an insider's position by investigating mathematics teacher identity development through prospective mathematics teachers' accounts. Hence, I am concerned with identifying accounts told by the prospective teacher, about him- or herself, and to me as a researcher through a series of interviews. Further, I search for accounts being categorised as *reflective* (Bjuland et al., 2012), meaning that the mathematics teachers "look back and consciously reflect about personal experiences" (p. 407) and relate those experiences to the present situation and their future. Based on the above discussion on trustworthiness, the tools for collecting mathematics teachers' accounts must therefore be able to reveal rich descriptions of their time in university, both subject studies and teacher education, and of their mathematics teaching in school. In the subsequent section, I account for the completion of the interviews, and the grounds for choosing research participants.

3.3.1 Interview methods

I report on a series of individual interviews with three secondary school mathematics teachers. They took place in different periods of the participants' last year at the university and their first year as certified mathematics teachers in school, and the interviews were audio recorded. All interviews were conducted based on an interview guide, having the same kind of composition of follow-up questions across the participants; however, the guide was not followed in detail. One example of an interview guide is given in Appendix A. The goal was to generate detailed accounts rather than brief answers or general statements, and to establish a climate that allowed for storytelling in all forms: from brief, tightly bounded stories to longer stories traversing temporal and geographical space (Riessman, 2008). Inspired by Elliott (2005), the interviews were therefore introduced by an invitation to talk freely about experiences from undergoing subject studies in mathematics, university teacher education, school placement, and later, experiences from being a newly qualified mathematics teach in school. An example of an invitation is: "Can you tell me about your educational background – about why you chose to study mathematics/why you chose to become a mathematics teacher?" During this initial session, I avoided interrupting the storyteller in order to give the participant time to elaborate on important episodes. The interview situation thus required me to give up control, despite having particular paths I wanted to follow (Riessman, 2008). In order to cover important parts of the participants' lives as learners and teachers in mathematics, the storytelling was followed by questions outlined in the interview guide. This questioning allowed me to go into details in important sections of the participants' introductory story. The follow-up questions concerned the participants' first mathematics teaching experience, memories of good and less good mathematics lesson, peaks and disappointments regarding mathematics teaching and learning, and expectations about the future. Since I partly have the same educational background as the participants, and from the same Norwegian university, I could more easily relate to their utterances and find appropriate follow-up questions. However, this might also have influenced my initial interpretation of their statements, and thus steered the conversation in a certain direction.

Another challenge was to encourage the participants to reflect on the teaching and learning of *mathematics*, and not only teaching and learning in its general sense. In the first three interviews with each of the participants, I therefore chose to include a mathematics task, see Appendix B. For the tasks, I chose the mathematical topic of functions, since it appears as one of the main areas of Norwegian lower and upper secondary mathematics curriculum (The Norwegian Directorate for Education and Training, 2010) and is therefore likely to be taught by beginning teachers. Further, the interview tasks were selected due to the criteria of being open-ended so that different ways of solving them could be discussed. In order to facilitate thorough responses and a safe environment for discussion, I handed out the tasks in advance of the interview session. The participants were during the interview asked to present brief solutions to the tasks and discuss possible student approaches when solving them, and to argue to what extent the problems are suitable in a secondary school mathematics classroom.

Prior to the last interview sessions, I conducted observation of the participants' mathematics teaching at their respective school of employment. In line with the interview tasks, the observation was carried out with the aim of generating detailed accounts regarding mathematics, mathematics teaching and learning. The observation was therefore followed by an interview, which generated data for analysis. See Appendix C for the interview guide template. During the observation of the mathematics lessons, I acted as a passive participant, meaning that I was present and visible to all actors but not interacting with them (Mertens, 2005). Further, I made notes of what was going on in the classroom guided by a list of keywords developed beforehand:

- Physical environment: How teacher and students are organised in the mathematics classroom
- Social environment: How people organise themselves in groups, patterns of interaction
- Activities: What the teacher and students are doing
  - Which mathematics activities are worked on?
  - How are the activities introduced?
  - How do the students respond to the activities?
  - Who are involved in the activities?
  - What is being said by the teacher and the students?
  - How does the lesson end?

In addition, I wrote down questions to be included in the follow-up interview, concerning discussions going on in the classroom, students' responses, and changes of plans during the lesson (see Appendix C). After the lesson was finished, I immediately wrote down a summary of my observation notes and re-structured the interview guide for the upcoming interview. In the upcoming analysis (Chapters 5, 6 and 7), I give summaries of the observed mathematics lessons; however, the analysis is based on data from the follow-up interview. For all the participants, the interviews were conducted the same day as the observation.

3.3.2 Participants

The selection of participants for the study is based on an interest in prospective mathematics teachers for "both their uniqueness and commonality" (Stake, 1995, p. 1). On the one hand, I hold what Stake denotes as an *intrinsic* interest in the case of mathematics teachers. In other words, my interest is in the case itself, and not to accomplish something other than an insight into the particular prospective mathematics teacher's situation. On the other hand, I go beyond the particular case in order to understand better the dynamics of mathematics teachers' learning during the transition between education and work, when I in Chapter 8 make a comparison of the individual cases. This is by Stake (2006) denoted as the multiple case study analysis, which I will elaborate on in the context of narrative analysis. Hence, the cases of the study need to be similar in some sense – they share a common characteristic or condition. The point of departure for selecting participants to the study was the label *prospective secondary school mathematics teacher*. In the Norwegian context, this usually means that the participants have a university background in mathematics, in addition to a one-year teacher education programme, PPU. I chose to approach student teachers in mathematics belonging to the same Norwegian university, however, belonging to two different study programmes. I thus conducted contact information to all last-year student teachers in mathematics belonging to either the five-year Master's programme of mathematics and natural sciences, including teacher education, or the one-year post-graduate teacher education programme, PPU. Since the latter category of students took part in lectures in mathematics didactics at the university, I visited the students to inform about my project. Further, I made a small survey which I sent out to all potential participants on e-mail. The survey contained the following questions:

1. What is your educational background? Please, list up all your university degrees in addition to PPU
2. What is your reason for studying mathematics/mathematics education?
3. What is your reason for studying teacher education?
4. How do you perceive yourself (yes/do not know/no)?
   a. I perceive myself as a mathematician
   b. I perceive myself as a teacher
   c. I perceive myself as a mathematics teacher
5. What are your future career plans?
6. Would you like to participate in the research project?
7. Other

The questions were meant to help me select participants with various motivations for becoming a mathematics teacher, and various educational specialisations: both within mathematics and mathematics education. One main challenge when finding appropriate participants to the project was, however, the responses given to question 5. Although there was a significant number of students taking the one-year post graduate teacher education programme, few of the students planned to work as a mathematics teacher after graduation. One of the respondents commented that he planned to start working in the business sector, and that PPU provided useful leadership training. Others planned to start working in business, and instead work as teachers in school later on in their career. A similar problem of career plans applied to the students belonging to the five-year Master's programme in mathematics and natural sciences, including teacher education. The search for participants resulted in a list of four student teachers, where two of them represented the five-year Master's programme with specialisation in mathematics education (Thomas and Sofie), and two belonged to the post-graduate teacher education programme (Isaac and Nora). An overview of the participants is given in Table 3.1, while a more detailed presentation is given in the upcoming analysis chapters (5-7).

Table 3.1: Overview of participants who were interviewed in the study

Name	Educational background	Other	After graduation
Isaac	Bachelor's degree in chemistry 60 ECTS credits in mathematics One-year post graduate teacher education programme (PPU)	Two years of teaching experience from upper secondary school before attending teacher education	Full-time mathematics and science teacher in upper secondary school (not permanent position)
Nora	Nearly a Bachelor's degree in physics 60 ECTS credits in mathematics One-year post graduate teacher education programme (PPU)	Dropped out of the five-year Master's programme including teacher education after two years of study. Switched to studies in physics, mathematics and PPU.	Part-time/on-call employee at a lower secondary school
Thomas	Five-year Master's programme in mathematics and natural sciences, including teacher education, PPU Master's degree in mathematics education		Full-time mathematics and science teacher in upper secondary school (not permanent position)
(Sofie)	Five-year Master's programme of mathematics and natural sciences, including teacher education, PPU Master's degree in mathematics education		Trainee, IT consultant company

I conducted a first interview with each of the four participants. However, during the spring semester of their last year in education, Sofie informed me that she had applied for and been employed in a trainee position at a consultant company. She was therefore taken out of the longitudinal investigation. However, I conducted a final interview with her during her first year of her trainee position. Since Sofie did not become a mathematics teacher, she is not one of the cases reported on in this thesis. An overview of the interviews conducted with each of the three remaining participants, and the length of each interview, is shown in Table 3.2.

Table 3.2: Overview of interviews with and observation of the participants

	Autumn year 1	Spring year 1		Autumn year 2		Spring year 2	
Isaac	December Interview 1 2 h, 5min			August Interview 2 1 h, 5 min	November Interview 3 1 h, 50 min	February Interview 4 Observa- tion 20 min	March Interview 5 1 h, 15 min
Nora	December Interview 1 2 h, 30 min	June Interview 2 2 h, 40 min		November Interview 3 2 h, 5 min		March Interview 4 Observa- tion 40 min	March Interview 5 Observa- tion 1 h, 15 min
Thomas		March Interview 1 2 h, 21 min	June Interview 2 1 h, 7 min	November Interview 3 2 h, 2 min		March Interview 4 Observa- tion 1 h, 38 min	

3.4 Ethical considerations

Any research that involves the participation of human subjects requires consideration of the potential impact of that research on those involved. Through the series of interviews, I entered into personal relationships with those I studied, which included direct contact with the participants during data collection and throughout the analysis and dissemination stages of the research. Further, I intruded into the lives of someone being in the middle of a demanding situation of finishing their university studies and preparing for entering the mathematics teacher profession, handling a host of responsibilities such as lesson preparations, classroom management and student assessment. In-depth interviews concerning the challenges of becoming a mathematics teacher may then have led the participants into stressful situations. Further, the preparation of a task for three of the interviews may have occupied a great amount of time for the participants, and is likely to threaten feelings of mastery. In order to create a comfortable atmosphere during the interviews, I followed Cooper and McIntyre (1996), who highlight empathy and unconditional positive regard as beneficial measures when creating circumstances that enable informants to express their personal view. The notion of empathy concerns the interviewer's ability to empathize with expressed views and showing that they are both understood and accepted. Unconditional positive regard is about showing interest in informants as individuals through verbal (e.g. humour, enquire for personal well-being) and nonverbal communication (e.g. eye contact, nodding). This may have given the participant a sense of comfort and security, in addition to minimizing defensive responses.

However, the choice of undertaking interviews with initial story telling did also bring some benefits. The conversations that took place during the interviews allowed the participants to mark out the territory in which intrusion was tolerated, and thus, gave them an opportunity to become active subjects within the research process (Elliott, 2005). Further, there is a possibility that the participants did benefit from being given the opportunity to reflect on and talk about their lives and experiences together with an interested listener. According to Huberman (1995), narrative in research allows the mathematics teacher

> (…) to escape momentarily from the frenzied busyness of classroom life – from its immediacy, simultaneity, and unpredictability – to explore his or her life and possibly to put it in meaningful order. (p. 131)

Prior to the data collection, I ensured that all participants understood the process in which they were to be engaged. By giving a voluntary informed consent, the participants confirmed that they understood and agreed to their participation without any duress. The informed consent included therefore a reason for the necessity of the prospective teachers' participation, how the data would be used (e.g. permission to report quotes and episodes from the classroom), and how and to whom it would be reported. This was carried out by giving the participants a consent form, including a brief introduction to the research project and the associated design, assurance of confidentiality, a request for permission to make audio recordings during the interviews and a clarification of their possibility to end the participation at any moment. See Appendix D for an English translation of the consent form. In relation to classroom observation, I contacted the head of department at the current school and asked for permission to observe the participant's mathematics teaching. Since no audio or video recording was made in the classroom, there was no need of signed consent from the students. The study is registered at the Norwegian Centre for Research Data (NSD)[8].

Further, I have the responsibility of ensuring confidentiality of all persons being part of the study. It involves protecting the privacy of the participants by ensuring that the data they provide is handled and reported on so that it cannot be associated with them personally. I have therefore given the participants fictive names, both when storing the in-

[8] Norsk Samfunnsvitenskapelig Datatjeneste

terview data, when writing out the interview transcripts and when reporting the results. Further, I have omitted the names of the practice schools during PPU and their schools of employment, and I have excluded the years of university education and the time of the participants' professional debut. The participants have read through the transcripts and the reported analysis, and they have been given the opportunity to omit sensitive information. However, this has not led to any changes in the dissertation.

3.5 Methods for analysis

The methods for analysis build on two approaches for analysing narrative data: *narrative analysis* and *analysis of narratives* (Polkinghorne, 1995). In the subsequent section, I account for the suitability of the former approach when making coherent portraits of individual mathematics teacher identities. Further, the latter approach is helpful for highlighting common aspects of identity across the individual cases. In Section 3.5.2, I report on the analysis process, in which I combine the two approaches into a multiple case study of three prospective secondary school mathematics teachers' entrance into the profession.

3.5.1 Narrative analysis and analysis of narratives

In order to make a multifaceted portrait of each of the participants based on their accounts, the interviews were analysed with methods from narrative analysis. Following Polkinghorne (1995), there is a distinction between narrative analysis and analysis of narratives. While the latter seeks to locate common themes among the accounts collected as data, and across cases or individuals, the former is about organising the data elements within the single case into a coherent developmental account (Polkinghorne, 1995). Polkinghorne bases his distinction on Bruner's (1985) two modes of thought or ways in which a person knows about the world: *paradigmatic cognition* and *narrative cognition*. The power of the paradigmatic mode of thought is to bring order to human experience by seeing individual aspects as belonging to a general category, and further, to identify relationships among the established categories. In contrast, narrative cognition operates by noticing the richness in each case and "the complexity of the situation in which an action was undertaken and the emotional and motivational meaning connected to it" (Polkinghorne, 1995, p. 11).

Polkinghorne's distinction can be aligned with Lieblich et al.'s (1998) model for classification of types of narrative analysis. Their model is based on two independent dimensions: *holistic* versus *categorical* approaches, and *content* versus *form*. The categorical approaches relate to Bruner's (1985) mode of paradigmatic cognition, in which the original story is dissected and sections or single words are organised into defined codes or categories. It is adopted when the researcher is primarily interested in a problem or a phenomenon shared by a group of people. In the holistic approach, which is related to narrative cognition, a person's story is taken as a whole, and sections of the text are interpreted in the context of other parts of the accounts. Further, a person's narrative entails information about what happened, and why, at what time, with whom, which constitute the content of the story. However, by analysing the form or structure of the story, the reader might also gain insight into how this person is constructing in the telling due to the dynamic characteristics of the plot, and by choice of vocabulary. Together, the two dimensions span out a four cell matrix, representing four modes of reading a narrative (Lieblich et al., 1998, p. 13):

Table 3.3: Four modes of analysing narratives

Holistic-content • The complete story of an individual is used and focus is on the content presented by it	**Holistic-form** • The plots or structure of complete stories are examined
Categorical-content • Separate utterances of the text are extracted, classified and gathered into categories	**Categorical-form** • Focus is on discrete stylistic of linguistic characteristics of defined units of the narrative

I view narrative analysis within the holistic-content mode and with its roots in narrative cognition as being in accordance with a holistic perspective on mathematics teacher identity (Akkerman & Meijer, 2011; Ponte & Chapman, 2008). As accounted for in Chapter 2 and in Section 3.1, I follow a Meadian tradition regarding identity research, where the participants' activity of identifying themselves as mathematics teachers through their series of accounts is of my interest. Instead of identifying aspects of the participants' accounts as instances of general identity categories, I am synthesising their accounts into a narrative explanation in the shape of three *case studies* of mathematics teachers' identity development. Here, I follow Stake's (1995) definition of an *instrumental case*, in which I examine each participant's accounts in depth to provide insight into the greater issue of becoming a secondary school mathematics teacher.

Narrative analysis refers in this study to my interpretation of the participants' accounts, in which I try to create coherence across themes and plots in what is individually told. Following Riessman (2008), the methods of narrative analysis grant the individual mathematics teacher's unity and coherence through time, and can therefore be considered a case-based method of analysis. Thus, I follow Ayres et al. (2003), who state that analysis of individual cases enables me to investigate “those aspects of experience that occur not as individual ‘units of meaning’, but as part of the pattern formed by the confluence of meanings within individual accounts” (p. 873).

Attending to the content of the participants’ accounts, the individual cases are investigated through the holistic-content mode referred in Table 3.3. This first level of analysis has resulted in three comprehensive portraits of prospective mathematics teachers, organised by what I denote as *emergent themes* in their telling. The cases further represent a longitudinal approach to analysis, by which I produce the portrait of each participant based on changes and continuities in his or her accounts over time. A more thorough elaboration on the analysis process and how these emergent themes are developed is given in Section 3.5.2.

Further, the three selected cases share the common topic *prospective secondary school mathematics teachers*. The cases are thus studied to gain information about a phenomenon, or what Stake (2006) denotes, a *quintain*, which is the “umbrella for the cases we will study” (p. 6). In the current study, the quintain is prospective mathematics teachers in the transition from university teacher education to professional life. The results from the narrative analysis will therefore constitute data for a *multiple case study* (Stake, 2006), based on an assumption that the comparison of mathematics teacher profiles provides greater insight of the topic than any single case study. Since this second level of analysis requires going beyond the individual cases, the study moves from the first to the second row of Lieblich et al.’s (1998) modes of analysis in Table 3.3, and towards *analysis of narratives*. Following Polkinghorne (1995), I then move from stories towards common elements across the cases and the related emergent themes, by making a thematic comparison of the three participants’ accounts. In Chapter 8, the comparison results in a more general description of mathematics teacher identity in the transition from university teacher education to school employment.

Regarding trustworthiness in case study analysis, Stake (2006) emphasises that both individual and multiple case studies consider particularisation more than generalisation. They produce context-dependent knowledge about certain mathematics teachers with a certain history. However, the reader is simultaneously prompted to move towards a broader commentary of the phenomenon of becoming a secondary school mathematics teacher. In this study, the findings of the individual cases have the shape of accounts being organised into emergent themes and related evolving stories, which then constitute processes of developing a mathematics teacher identity. The generalisability is then based on the contextual richness of each individual case. By further comparing and contrasting the individual cases in the subsequent analysis of narratives, the propositions regarding mathematics teacher identity can be saturated, and the specific features of each individual case are placed in the background. Hence, I seek to develop an interpretation of data that reflects each individual case and that applies equally well across all the three cases. Those themes that have explanatory force both in individual accounts and across the sample are then most likely to apply beyond the sample of prospective mathematics teachers (Ayres et al., 2003).

There are several layers of interpretation that come into play, being simplistically presented in Figure 3.1.

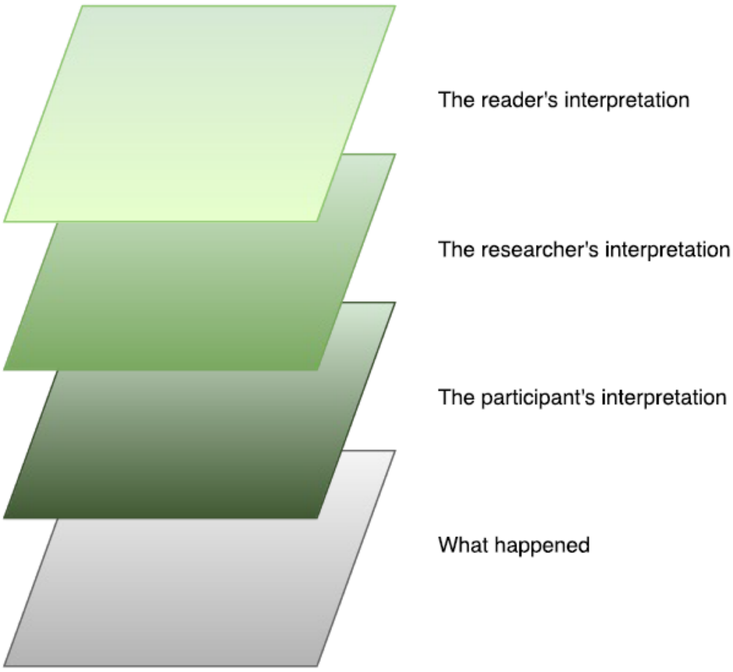

Figure 3.1: Levels of interpretation when investigating mathematics teacher identities

The ground level represents the episodes of mathematics teaching and learning in the prospective mathematics teacher's life. This "raw material" of life experience and mental images of what has happened in the past is not accessible to direct observation or interpretation[9]. Although it is not visible from Figure 3.1, the participant's interpretation of the past events takes place, according to Moen (2006), at two levels. The first level concerns the selection of episodes out of a complex situation, which implies that the event has already been infused with meaning. At

[9] One exception is the observation made of the participants' mathematics teaching at the end of the data collection period. However, it is the participants' accounts of the observed lessons that are objects of study, and not their teaching practice per se.

the second level, meaning is ascribed to the episodes by constructing a coherent account out of it, which can be presented to the listener. Since the events have happened before the participant starts to construct an account, the participant is aware of the ending and constructs the account from there. Further, this process of narrative identity formation is dynamic and occurs in a changing social and personal context throughout the period of study. A series of accounts with overlapping raw material is therefore available to me for processing.

The third level represents my storied interpretation of the participants' accounts, which has taken place through various processes at different times. First, it happens during the interview, when I choose to respond (or not) to what is told by the participant and thus steer the interview in some direction. Next, several rounds of interpretation take place: reading and re-reading the interview transcripts, being in dialogue with the theoretical framework and choosing to focus on selected accounts, organising the selected accounts into coherent stories, moving between interviews and comparing and contrasting similar utterances across interviews, and organising a representation of the results in the shape of a thesis. I will in the subsequent section elaborate on how the analytic process has been carried out.

3.5.2 The analysis process

Since the data collection took place in intervals, I made interview transcripts after each session in order to prepare for the upcoming interviews. For the first interviews, I made a detailed reproduction of everything that was recorded. Throughout the research process, I changed my strategy and wrote down in detail those passages which were evaluated to be of interest to the study. Further, I briefly summarised utterances about daily life and those not being related to the teaching and learning of mathematics in some sense. I also chose to omit sounds ("hm,", "eeh" etc.) and repetition of single words or phrases that did not contribute to the meaning or the coherence of the current accounts. This made the transcripts more readable. In the upcoming analysis and related interview excerpts, I have removed repeating filler words and sensitive information, marked by the brackets: (…). Further, I have added single words within square brackets, […], for clarifying utterances and creating coherence in the participants' accounts.

The greater narrative analysis took place after all interviews were conducted. I worked with one case at a time and wrote the analysis chapters in the same order. The reason for doing it case by case was to enable immersion into each individual participant, having the collection of cases more at the back of my mind. In order to be acquainted with the data material and the content and structure of what was told, I read the transcripts for each interview in several rounds. For every reading, I highlighted passages appearing as important for answering my research question, and I wrote keywords with my initial interpretation of what was told. I also marked contradictions and unfinished descriptions, as well as exceptions to my general impression of the narratives. Next, with help from the qualitative data analysis software NVivo, I divided the transcripts into segments; either episodes with a narrative structure where some sort of plot was unfolding, or segments where a certain topic was discussed. This was done interview by interview. Guided by initial key words and comparing and contrasting across segments, I gave the passages *labels*, due to their thematic focus. Further, the software helped me to sort the labels and related segments, to compare and contrast them, and developing new, either more specific or more general labels. An example of a segment from one interview with Isaac and related labels is given in Table 3.4.

Table 3.4: Example of segment in interview transcript with labels

Segment from transcript	Labels
I think PPU has been exciting, in general, I think it's been very fun to see the theoretical part of the pedagogy and the didactics, I think it has been valuable. I've liked that kind of studies, actually, much more than I had thought that I would like it, because I'm used to solving problems with two underlines underneath the answer, and it was more fun than I thought writing papers [laughing].	PPU as an upturn PPU as a transition from absolutist towards fallible knowledge

In addition to develop labels on the basis of the content or thematic focus of the interview segments, I wrote notes about the form of the segment when appropriate. Following Lieblich et al. (1998), this concerned identification of the topology of plots (progression, decline or stability), turning points, accounts of causality among events in life, accounts of evaluation of episodes, and choice of vocabulary ("it was the best/worst time of my life", "turning point", "crossroad", "progress", "stagnation" etc.).

The labels of the first interview of one participant were sorted into greater categories, or what I denote as *emergent themes* of the accounts, in order to display the linkage among the utterances within each label. The emergent themes developed from analysis of the first interview were brought into the analysis of the second interview and confronted by labels given to segments in the transcript. If necessary, new emergent themes were developed and the titles of the initial emergent themes were refined. A further refinement was made by comparing and contrasting accounts and the related labels across the cases of Isaac, Nora and Thomas. The labels shown in Table 3.4, extracted from the second interview with Isaac, were sorted under the emergent theme of *perspectives on mathematics and its role in mathematics teaching*. Other labels sorted under the same theme are *PPU as a resource for changing mathematics teaching* and *expectations of developing mathematics teaching through inquiry*. The emergent themes constitute what I henceforth denote as *evolving stories* of becoming a secondary school mathematics teacher.

By comparing and contrasting labels across the interviews, I further identified *critical events* along the evolving stories of each emergent theme. In other words, I located discontinuities in the teacher's accounts across the series of interviews and traced strong emotional involvement with mathematics practices at the university and in school. In each of the three upcoming cases (Chapters 5 to 7) and for each of the participants' emergent themes to be presented, I give an overview of the labels constituting the critical events along the related evolving stories (see e.g. Figure 5.2). In these overviews, the labels are linked to the interview excerpts being presented in the analysis. Thus, the overviews show how the critical events were identified, by comparing and contrasting labels both horizontally (within each interview) and vertically (across interviews) in the given figure, and why the given interview excerpts are selected for presentation. A further description of the structure of the narrative analysis is given in Chapter 4.

According to Webster and Mertova (2007), a critical event reflects the most memorable and impressionable stories, and can therefore only be identified after the event. What makes it critical is the impact the event has on the storyteller. The identification of critical events enabled me more easily to get at the core of what was important in the participants' accounts regarding becoming a mathematics teacher. Hence, it was useful for dealing with and reporting on the large amount of interview data. The evolving stories then represent the longitudinal approach to case study analysis by which I have created a portrait of Isaac, Nora and Thomas, made up of changes and continuities in their accounts over time. The analysis process for the individual case study analysis can be summarised into a series of steps, as shown in Figure 3.2. For simplicity, the analysis is represented as a linear process, although in reality it was more an iterative process of moving back and forth between the various phases.

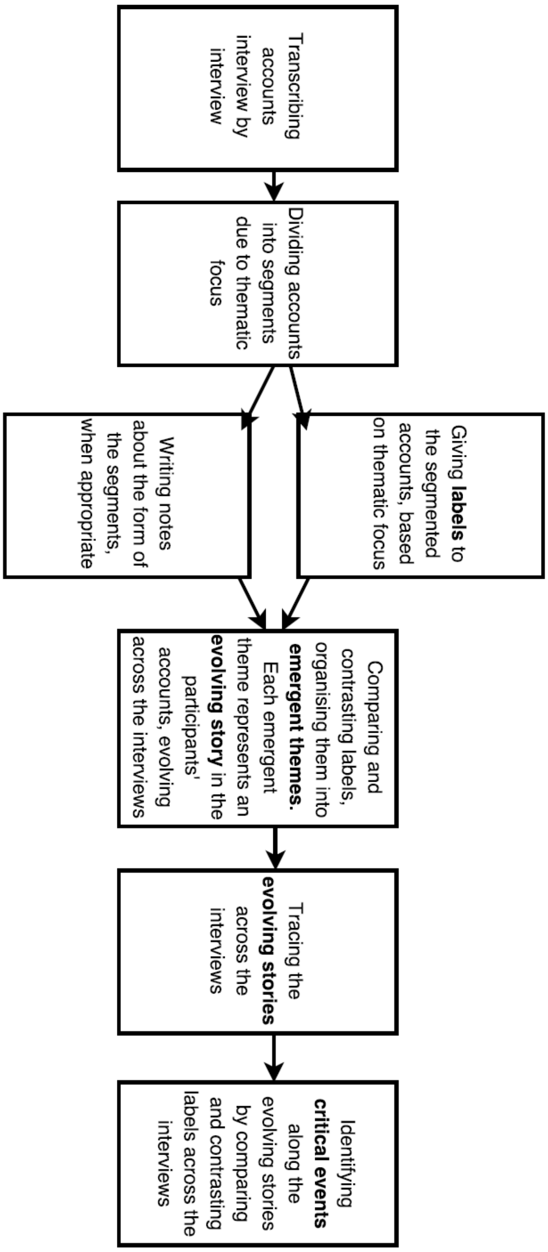

Figure 3.2: Overview of the analysis process

For each critical event, the associated accounts were interpreted by means of the theoretical framework. Hence, I searched for evidence of the prospective teacher's participation and non-participation in communities of practice based on Wenger's (1998) two components of identity: identification and negotiability. As elaborated on in Section 2.2.4, the notion of identification describes which meanings of mathematics and mathematics teaching and learning matter to a prospective teacher. I thus looked for the teacher's expressed philosophies of mathematics and mathematics education, as described by Ernest (1991) and summarised in Table 2.1. Further, negotiability refers to the teacher's possibility to contribute to and shape the meanings with which he or she identifies.

Here, I have used Povey's (1997) work of authoritative knowing and Belenky et al.'s (1986) ways of knowing, as discussed in Section 2.2.5. Making a longitudinal approach to the accounts, I have searched for changes in the participants' identification and negotiability when undergoing the transition from education to employment in school. Inspired by Jansen et al. (2012), I identified discontinuities in accounts of participation and non-participation in mathematics related practice, by comparing labels across interviews, marking out repeated accounts of turning points, and recognising changes in attitudes and perspectives regarding mathematics practices.

I also made a summary for each individual case, regarding the narrative progression across the interviews. By identifying the evolving stories as either progressive, regressive or stable narratives, I could give a more comprehensive picture of the teachers' developing mathematics teacher identity. The narrative progression is in each individual case related to the teacher's changes in or continuation of participation in identified communities of practice.

After writing out the complete analysis of all the three cases, I made a thematic comparison based on three common topics across the developed emergent themes and related evolving stories of critical events. The common topics were: (1) being a learner of mathematics; (2) undergoing university teacher education; and (3) being a teacher of mathematics. Here, the findings from the previously conducted individual case study analysis functioned as data for the cross-case analysis. Guided by the theoretical framework, the comparison lead to two shared dimensions of developing an identity as a secondary school mathematics teacher: negotiating experience of self and mathematics learning and negotiating experience of self and mathematics teaching. A more thorough account of this final step of analysis is given in Chapter 8.

4 Structure of the narrative analysis

In the subsequent narrative analysis of the three cases of Isaac, Nora and Thomas, I give a comprehensive report on their accounts of becoming a secondary school mathematics teacher (in Chapters 5, 6 and 7 respectively). For each of the cases, I give an initial introduction of the research participant, including information on educational background and the process of data collection. Further, each individual case is structured from the set of emergent themes being developed from the participant's accounts. Each emergent theme constitutes an evolving story, which develops across the interviews and is structured by a series of critical events. Isaac's case consists of three emergent themes and related evolving stories, while Nora's and Thomas' case consists of two emergent themes/evolving stories.

As accounted for in Section 3.5.2, the critical events are identified by comparing and contrasting labels and the related accounts across the interviews. In each of the three upcoming cases (Chapters 5 to 7) and for each of the participants' emergent themes to be presented, I therefore give an overview of the labels constituting the critical events along the related evolving stories (see e.g. Figure 5.2). In these overviews, I also link the identified labels to the interview transcripts being presented in the analysis. The labels were initially identified on the basis of particular statements, however, they were confirmed by comparing and contrasting accounts throughout the interviews. In addition to be linked with specific interview excerpts, the labels thus act as summaries of the participant's expressed reasoning regarding mathematics, its teaching and learning, and the process of becoming a mathematics teacher.

For each of the cases, the analysis is organised by first presenting the accounts that represent a critical event, and in the subsequent section, interpreting the content of the accounts by means of the theoretical framework. This analysis leads then to a characterisation of the given critical event, being represented as textboxes along the axes of evolving stories in Figure 5.1, Figure 6.1 and Figure 7.1 respectively, all three having the following structure:

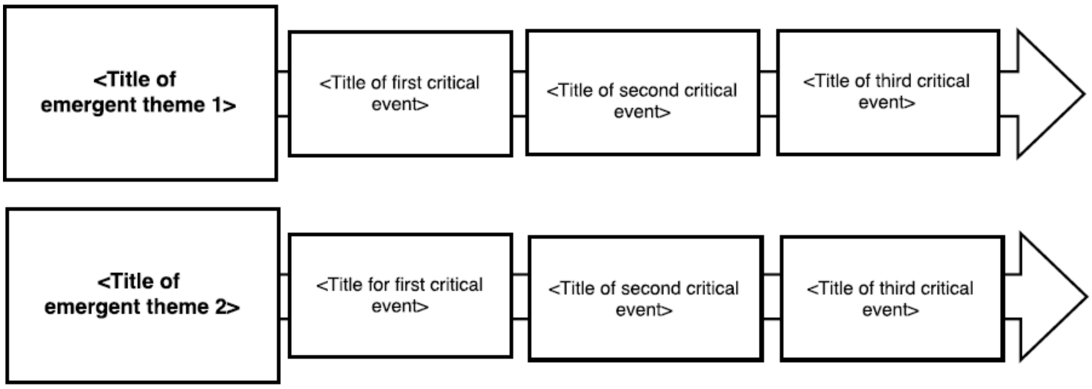

Figure 4.1: General structure of emergent themes and critical events, constituting axes of evolving stories of becoming a secondary school mathematics teacher

The structure of the report on the individual case study analysis is summarised in Table 4.1. Here, *X* refers to Chapters 5, 6 or 7, while *Y* refers to a section of Chapter *X*. Thus, Section *X.Y* represents an emergent theme in the participant's accounts. Each Section *X.Y* is further divided into reports on accounts and related critical events.

Table 4.1: Disposition of the individual case study analysis in Chapters 5, 6 and 7. *X* refers to the chapter; *Y* refers to the section of Chapter *X*.

X	The case of …		
	X.Y	Emergent theme (constituting an evolving story of critical events)	
		*X.Y.*1	Presentation of accounts related to the first critical event of the evolving story
		*X.Y.*2	First critical event: Analysis of the presented accounts
		*X.Y.*3	Presentation of accounts related to the second critical event of the evolving story
		*X.Y.*4	Second critical event: Analysis of the presented accounts

Each critical event, as in Section *X.Y.*2 and *X.Y.*4, is further portrayed as participation or non-participation in communities of practice. Here, I give illustrations in the shape of textboxes, representing the characterisation of a given critical event, which are connected to ovals, representing participation or non-participation in communities of practice (see Figure 4.2). These illustrations are then to be considered as an elaboration of the boxes along the evolving stories in Figure 4.1.

Figure 4.2: Example of illustration of critical event, with reference to participation or non-participation in communities of practice

For each of the cases, the narrative analysis leads to a summary of the developing identity as a secondary school mathematics teacher. As explained earlier, I change here the approach of the analysis, from studying in depth the content of the participant’s accounts, to focusing on the overall narrative progression of the evolving stories. The summary is also presented with an illustration, with reference to changes in participation or non-participation in communities of practice. A general illustration is given in Figure 4.3:

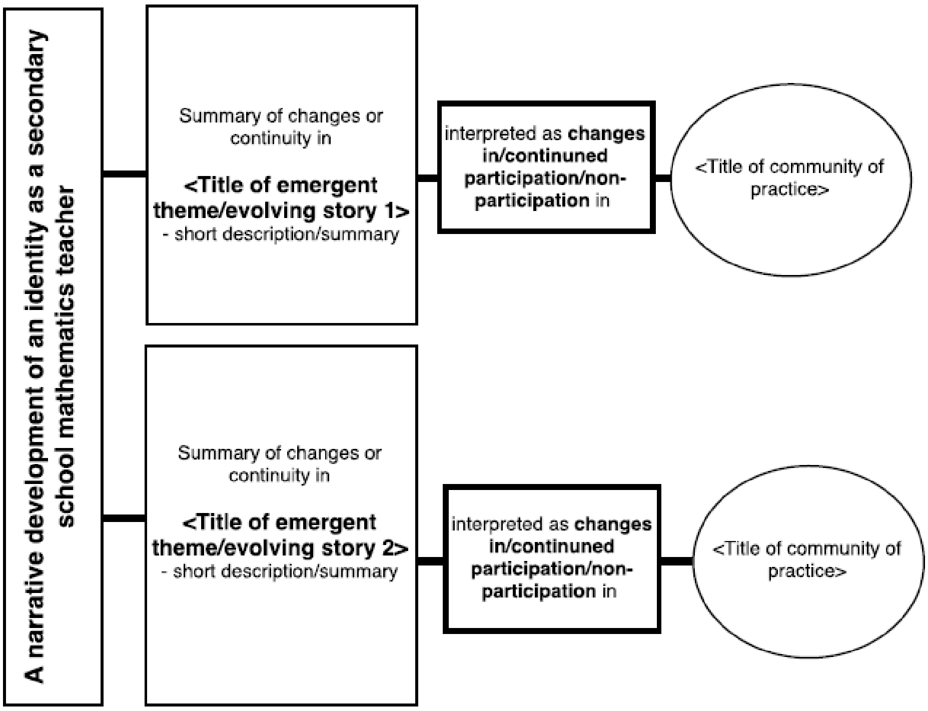

Figure 4.3: General illustration of a developing identity as a secondary school mathematics teacher

Throughout the report on the individual cases, I refer to interview transcripts by using the following notation:

I1_287 Nora: *I studied math and physics at upper secondary school, and I've found such subjects to be nice, it's really fun when you manage to solve a problem, knowing what you're doing, and can use it in other situations, and I enjoy myself when I do math nuts and things like that. But I've never been kind of run by, this is what I live and breathe for.*

Here, the notion I1 refers to Interview 1 (with Nora), while the number 287 is the line number in the interview transcript. Further, I report on sequences from observation of the participants' classroom teaching. I have chosen to present the sequences as an intermezzo within one of their evolving stories, leading further to a presentation of accounts from the follow-up interviews.

5 The case of Isaac

At the time of data collection, Isaac was a newly qualified mathematics and science teacher holding a Bachelor's degree in chemistry and 60 ECTS credits in mathematics from a Norwegian university. After having completed upper secondary school with specialisation in mathematics and the natural sciences, he first started on chemical engineering, which included four mandatory mathematics subjects in the areas of calculus, linear algebra and complex function theory. After one year of study, he switched to a Bachelor's degree programme in chemistry with the freedom of including mathematics in the study plan beyond the mandatory subjects. In order to achieve an approvable combination of subjects for being able to one day enter the one-year teacher education programme, PPU, he extended his study plan with subjects in number theory, discrete mathematics and geometry. His entrance into the teacher profession was provoked, however, by the necessity of taking a break from the university studies, due to his health condition. In order to try something different from university studies, he decided to take a temporary position at an upper secondary school as an uncertified mathematics and science teacher. The school, being located in the proximity of a Norwegian city, had about 50 employed teachers and 500 students, divided between the general education programme and the vocational programme of healthcare, childhood and youth development. Isaac taught in the main chemistry and mathematics for students attending the general education programme. Within the same two-year period, he also worked as a temporary lecturer at a university college, teaching chemistry for engineering students. The positive experiences from teaching mathematics and science in upper secondary school led to a desire of becoming a certified, full-time mathematics and science teacher, and he therefore decided to complete teacher education.

I approached Isaac to be a participant based on his responses to the questions that were sent out by e-mail to all student teachers in mathematics who were attending the teacher education programme, PPU, within one Norwegian university. In addition to report that he planned to work as a mathematics teacher immediately after graduation, Isaac ticked off "agree" on the statements "I perceive myself as a teacher" and "I perceive myself as a mathematics teacher". Five interviews were distributed across a period of one and a half years, where the first interview took place in December, at the end of his first semester at PPU. Further, the second interview was conducted in August the year after, when Isaac had completed PPU and just started in a one-year temporary position as a mathematics and science teacher in another upper secondary school. In terms of numbers of students and teachers, this school was twice as big as his first school of employment, including several vocational programmes in addition to the general education programme. It was located in the same area as his first school of employment, and it was considered new and modern in terms of architecture. Isaac taught Mathematics 1T, Mathematics R1 and science within the general education programme and Mathematics 1P-Y within the vocational education programme Technical and Industrial Production (TIP). The remaining three interviews were conducted during his year of employment at the school, the last one in March. An overview of the interviews and their lengths is given in Table 5.1.

Table 5.1: Overview of the interviews with Isaac

Year 1 Teacher education, PPU		**Year 2** Mathematics teaching in upper secondary school			
Autumn	Spring	Autumn		Spring	
Interview 1 December		Interview 2 August	Interview 3 November	Observation Interview 4 February	Interview 5 March
2h, 5 min		1h, 5 min	1h, 50 min	20 min	1h 15 min

The first interview was arranged in a meeting room at the University Library, being within reach of my office and his place of study. The other four interviews took place at Isaac's school of employment, in order to make it convenient for him to participate in the project. On the occasion of the fourth interview, I observed Isaac in one 45 minutes lesson in Mathematics 1T at the general education programme. A summary of the teaching sequence on linear modelling is given in Section 5.2.5, as a contribution to the evolving story of perspectives on mathematics and its role in mathematics teaching. The follow-up interview took place immediately after the lesson, while the fifth and last interview was conducted one week later. All five interviews were recorded on audiotape.

Based on a narrative analysis of the five interviews with Isaac, his building of a mathematics teacher identity is expressed through three emergent themes and related evolving stories concerning mathematics, its teaching and learning evolving over a period of one and a half years. They are *confidence in mathematics and mathematics teaching*, *perspectives on mathematics and its role in mathematics teaching* and *feedback available within the school environment*. The evolving stories represent the longitudinal approach to case study analysis by which I create a portrait of Isaac, made up of changes and continuities in his accounts over time. Each evolving story is in turn constituted by critical events, being identified from accounts of discontinuity in, or strong emotional involvement with, mathematics practices at the university and in school. Across Isaac's evolving stories, the critical events are related to him

- facing towards a future career as a certified mathematics teacher
- making a retrospective glance on the completed university teacher education
- working as a newly certified mathematics teacher

Figure 5.1 gives an overview of the three evolving stories and associated critical events. In the subsequent analysis, I explain each critical event in terms of Isaac's identification and negotiability in current communities of practice. Further, I give in Section 5.4 a summary of his narrative progression and corresponding learning mechanisms when becoming a secondary school mathematics teacher.

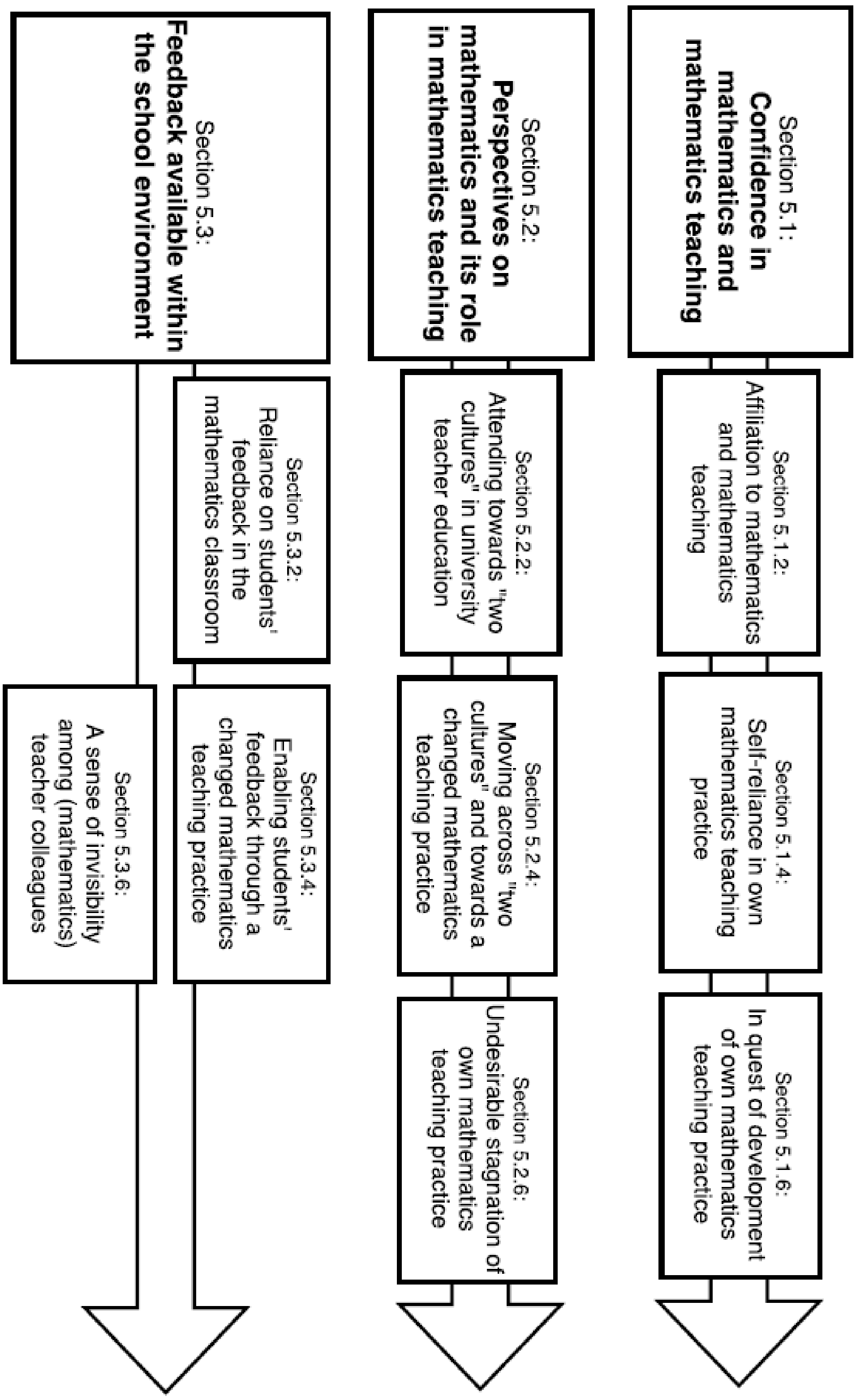

Figure 5.1: Emergent themes and related critical events in Isaac's accounts, constituting three evolving stories of becoming a secondary school mathematics teacher

5.1 Confidence in mathematics and mathematics teaching

The emergent theme of *confidence in mathematics and mathematics teaching* results from the narrative analysis of Isaac's accounts in the first interview, in which he voices a belief of being capable of both doing mathematics and exercising mathematics teaching. Based on descriptions of own schooling and his time at the university, the story of confidence in mathematics and mathematics teaching emerges from his expressed attraction towards the discipline as well as the surroundings' recognition of him as a successful student in mathematics and the natural sciences. Isaac's confidence in mathematics was further a door opener into the mathematics teacher profession. When leaving the university studies in order to try out secondary school mathematics and science teaching, the notion of confidence is apparent from his accounts of identifying with teacher colleagues who shared his desire for academic challenges. Further, Isaac describes his mathematics teaching as a kind of reflex from the backbone, in which he is able to exercise flexible and spontaneous teaching in collaboration with his students.

The accounts from the first interview pictures Isaac with a certain level of confidence in mathematics and its teaching, and in relation to that, a sense of affiliation to the discipline. However, the accounts represent as well a critical event in the evolving story of *confidence in mathematics and mathematics teaching*, due to the tension between Isaac's desire of exercising mathematics teaching and his surroundings' expectations of making an academic career. Further, the subsequent interviews show how confidence is part of his continued negotiation of self and mathematics practices during teacher education and when working as a certified mathematics teacher. As will be elaborated on in Section 5.1.4, the notion of confidence is in the second interview interpreted as Isaac's self-reliance in mathematics teaching during school placement. He marks a distance to his tutor's classroom management, being what he describes as bound by rules, and returns after graduation to the mathematics classroom with a faith in his own teaching style. Due to the opposition between Isaac's and the tutor's preferences for classroom management, the accounts of undergoing school placement represent a second critical event in the evolving story. In the final interview, Isaac voices confidence in mathematics and mathematics teaching by accounting for his need of seeking academic challenges within the mathematics teacher profession. Instead of expressing a general belief of being capable of exercising mathematics teaching, he now presents mathematics teaching as demanding, yet a desirable challenge that he is able to overcome with more classroom experience. The third identified critical event is thus related to a changed expression of confidence, based on Isaac's new perspectives on his first mathematics teaching experiences as an uncertified mathematics teacher.

An overview of labels belonging to the emergent theme *confidence in mathematics and mathematics teaching* is shown in Figure 5.2. The labels constitute three critical events along the evolving story, which I have identified by comparing and contrasting labels both horizontally and vertically in Figure 5.2. In this overview, I also link the identified labels to the interview transcripts being presented in the upcoming analysis. The labels were initially identified on the basis of particular statements, however, they were confirmed by comparing and contrasting accounts throughout the interviews. In addition to be linked with specific interview excerpts, the labels thus constitute summaries of Isaac's expressed reasoning regarding mathematics, its teaching and learning, and the process of becoming a mathematics teacher.

Confidence in mathematics and mathematics teaching

Labels related to the first critical event: **Affiliation to mathematics and mathematics teaching**	Expectations about making an academic career I1_1	Attraction towards school and university mathematics I1_10 I1_158	Mathematics teaching as a turning point in Isaac's career I1_291	The teacher profession being a desired challenge I1_312 I1_832	Teacher colleagues as ideals I1_291 I1_312	Absence of reality shock I1_94	Own mathematics teaching as a reflex from the backbone I1_736 I1_760
Labels related to the second critical event: **Self-reliance in own mathematics teaching**					Marking a distance towards the tutors' practice during school placement I2_23	A feeling of not owing the mathematics classroom during school placement I2_23 I2_48	Own mathematics teaching as flexible and adaptable I2_48
Labels related to the third critical event: **In quest of development of own mathematics teaching practice**				Mathematics teaching as demanding, yet a desirable challenge I5_123		Viewing previous mathematics teaching experiences in new light I5_209	The need for developing own mathematics teaching with help of further practice and experience I5_541

Figure 5.2: Overview of labels belonging to the emergent theme *confidence in mathematics and mathematics teaching*, constituting three critical events

5.1.1 Accounts of confidence in mathematics and mathematics teaching, constituting a first critical event

The first identified critical event of Isaac's evolving story of *confidence in mathematics and mathematics teaching* is related to his decision of entering the teacher education programme, PPU. The choice of becoming a certified mathematics teacher was made out of his desire of exercising mathematics teaching. However, it was also a choice made in spite of expectations of making an academic career. In the first interview, Isaac tells about his early interest in mathematics and the natural sciences, which led him into studies of chemistry and mathematics at the university.

I1_10	Isaac:	*[I have] always been into natural sciences, loved mathematics, physics and chemistry at upper secondary school, found out I wanted to study it. And I thought math and physics were maybe the easiest subjects, so I thought, when I'm going to study at university, I take what is most challenging, so I took chemistry. [I] started at chemical engineering, it has a lot of math, about half of the first year is math, and found out it was great fun, did not like chemical engineering that much, so I switched to a bachelor's degree in chemistry and a much freer curriculum. So I crammed it with as much math as possible, together with chemistry, because I thought it was very fun. And then I chose a math-heavy chemistry thing because I think it's fun to use math as a tool in science. Not just that, chemistry was fun too, but did not want some rubbish chemistry where you just sit and speculate [laughing], need to have numbers.*

He summarises his experience of studying mathematics and science in school with descriptions of appreciation and mastering, and his application for higher education was based on what he found was most challenging of the subjects. The choice fell on chemical engineering. However, at the university, Isaac discovers his main interest to be mathematics applied in the natural sciences. He therefore seeks a study programme with the freedom of including mathematics beyond the mandatory subjects, and which in a long term perspective can give him access to the one-year teacher education programme, PPU. Isaac crams his study plan for the Bachelor's degree in chemistry with mathematics subjects beyond what is required, and he chooses what he perceives to be a mathematical branch of chemistry. His preference for applications of mathematics in chemistry is further present in his concerns about the compositions of subjects in his certificate. Working largely with differential equations and their domains of solutions in his chemistry studies, Isaac has the experience of knowing mathematics beyond the 60 ECTS credits that is stated in his certificate.

I1_158 Isaac: *I find it very misleading that I only have 60 credits in mathematics, since I have theoretical chemistry, I have quantum mechanics, in which everything we do is solving second order differential equations (...) the only we thing we are doing is evaluating the domain of solutions of a set of equations and (...), yes, I have 60 credits in math, but I've got much more math than that [laughing]*

During his third year of study for the Bachelor's degree in chemistry, Isaac's health condition makes him leave the university in advance for mathematics and science teaching in an upper secondary school. The temporary position gives him the chance of fulfilling a wish from own schooling, of one day teaching the subjects to others.

I1_1 Isaac: *(...) already at lower secondary school [I] found it very fun to talk about subjects. (...) I said I could imagine becoming a teacher, but then I was told, no, people who are good at school do not become teachers. So, we do not think you should become a teacher [laughing]. So, I put it aside (...)*

Nevertheless, the story of Isaac's way into the mathematics teacher profession unfolds in a tension between what Isaac on the one side describes as an early interest of teaching or "talk[ing] about subjects" to others, and on the other side his surroundings' negative attitudes towards the teacher profession. Although accounting for positive experiences from teaching his classmates during own schooling, Isaac witnessed a negative feedback on his desire of one day becoming a teacher. The outcome was therefore to "put it aside". However, the meeting with his colleagues at the upper secondary school becomes a turning point in forming his opinion about the teacher profession. Working together with what Isaac describes as like-minded people, he can let go of the brand of being the smart guy, a brand which had haunted him since own schooling. The colleagues in the teacher common room, who are highly educated and competent teachers, make him feels like home.

I1_291 Isaac: *(...) they were like academically super smart, super clever, super engaged people, who were really idealistic towards the students, they kind of were so clear about, we're here for the students, and we're not here to get paid or we're not here because we could not enter something else, so we're here because we want the students to succeed, (...) And then I saw that clever people, they can become teachers, too (...) And it became a turning point in the way that, from, I thought this is something I can try out, then I saw kind of that this is a real career option (...). And I like very much not to feel like the smartest one in the room, and I did certainly not, it was a good feeling.*

I1_312 Isaac: *(...) it's been a challenge for me throughout schooling, always been good at school, it sounds horrible when I say it, but that's my experience, (...) my experience of never meeting challenges and always been the smart one or the clever one, I hate that brand. And I came to a place where people were smarter and cleverer than me, it feels like home.*

In his accounts of entering the teacher common room, Isaac marks a contrast between the fellowship and familiarity he experiences when being with his teacher colleagues, and the loneliness or differentness he felt when being appointed the smart one or the clever one during own schooling and the university studies. As opposed to the former feedback from his surroundings, the colleagues in the teacher common room give him the confirmation of clever people becoming teachers. They are a kind of teacher ideals or examples to follow, based on their idealism towards the students' learning and needs and their subordination of payment and career. Further, they represent a kind of knowledge or competence for Isaac to reach for, with the consequence of him not being appointed the "smartest one in the room", and thus, feeling like home. Based on his first meeting with the teacher profession, Isaac pictures himself as a future mathematics and science teacher, who actively takes part in a teacher community with the common desire for being academically challenged.

I1_832 Isaac: *I don't expect to find everything already prepared, no, I will take part in, kind of create this teaching staff myself, I will. And this great teaching staff consists of people in all ages, with all kinds of backgrounds, but they all have in common that they want to be academically challenged*

In addition to express attraction towards school and university mathematics, Isaac describes mathematics teaching as something that comes natural to him, like a reflex or a spontaneous reaction to the students' behaviour in the classroom. Unlike what he regards as common among student teachers, Isaac did not experience a "reality shock" when entering the mathematics classroom in the role of a teacher, in terms of confrontations between own expectations for teaching mathematics and limited possibilities of action in the mathematics classroom (see e.g. Veenman, 1984).

I1_94	Isaac:	*(...) I didn't get the smack in the middle of the face kind of experience that most teachers get [laughing]*
	Kirsti:	*No, did you have it before?*
	Isaac:	*Not really. It has always been so natural to me, eh, everyone complains about that there's so much to do as a teacher, eh, I'm naturally lazy, so I do not think so (…) I take shortcuts when I find them, I do what I have to do, and then I don't spend time on things I don't see the point of spending time on. And that makes me having plenty of time as a teacher, felt I had the energy needed. (…) When I work as a teacher it's more in the backbone, of course, I'm thinking, the mind is present, but I'm much more straightforward, meaning, it's not the kind of a heavy mental process as when I study, it's more like, fun collaboration with my students*

Isaac explains his natural talent or spontaneous teaching style by virtue of his personal characteristic of laziness. While others complain about the heavy workload when being a school teacher, Isaac experiences the opposite. The freedom he has in preparing lessons, the flexibility of structuring the workdays the way he prefers, makes him feel he has a kind of energy that is not present when being a university student. Further, Isaac makes a comparison between academic work (e.g. studying) and the work of teaching. While he describes the former as a heavy mental process in order to obtain insight and understanding, the latter is to him a more spontaneous reflex from the backbone, resulting from the teacher's interaction with the students in the classroom. Although initially identifying with the mental activity of studying the natural sciences, Isaac finds teaching even more joyful. The reason he gives for this change of interest is his positive experience of collaborating with the students in the classroom.

I1_124	Isaac:	*Maybe because the motivation is totally different. I find an incredible, sincere joy in teaching, meaning, I think it's fun to be together with the students. And that sincere joy I don't feel in the same way when I study. I like to learn things, but it's not always pleasurable sitting there and study for hours.*

In line with his characterisation of teaching from the backbone and of being a lazy teacher, Isaac's teaching plans are "fall[ing] down into [his] head", however, at the same time being thought through. Starting with the students' learning and their needs, and using the textbook merely as a guide, the structure and content for the mathematics lesson comes natural to him.

I1_736	Isaac:	*[I] observe that I cover about the same as the textbook, usually I close the textbook, and then I think through it carefully what I want the students to be left with. (...) so I build it from there, in my head only, could I exemplify this with an activity, or should I do it on the blackboard to show them how you represent it, [I] make such choices in my head, never write down anything (...).The teaching plan just falls down into my head, kind of. It's teaching on a whim [laughing], but it works very well because they're conscious choices, thought through.*
I1_760	Isaac:	*And it becomes so natural and organic, because it's like a result of the environment that I and my students create. It's not something you find on a stencil somewhere, made with an eye to someone else, meaning, it's a plan made with a view to my students, inspired by what we do together, with my experience and knowledge in the back of my mind.*

Through his descriptions of planning and implementing mathematics teaching, Isaac voices characteristics of independence, flexibility and spontaneity. Independence relates to his accounts of closing the textbook and creating own plans for the lessons based on the students' learning in mathematics and their needs. Unlike following readymade lesson plans, this requires a flexible understanding of the mathematics content, in addition to a range of ideas about how his students might be helped to learn the content. By stating that he never writes down anything, having it in his head only, he describes a spontaneity that is in line with his characteristic of being lazy and his appreciation of freedom in organising his workdays. Hence, Isaac talks about his mathematics teaching with the voice of an experienced, and thus, a confident teacher, who is able to make own decisions about how to best approach the professional tasks in the mathematics classroom.

5.1.2 First critical event: Affiliation to mathematics and mathematics teaching

The accounts from the first interview, which constitutes the emergent theme of *confidence in mathematics and mathematics teaching*, picture a student teacher with a sincere interest in mathematics and the natural sciences. Especially, Isaac voices enthusiasm for mathematics as an essential tool in other scientific disciplines. Although attending to a study programme in chemistry, he expresses belonging to a *community of mathematics as an applied science* at the university. Isaac's engagement in the community is visible by the way he is cramming his schedule with mathematics beyond what is required and by choosing a "mathematical" branch of chemistry. In addition, his belonging to the community leads to what Wenger (1998) denotes as identification in terms of reification, as he is identified by his surroundings as a successful student in mathematics and with the built-in expectations of making an academic career.

Isaac's participation in the *community of mathematics as an applied science* is further apparent from his expressed perspectives on the nature and the role of the discipline of mathematics. By stating that he "need[s] to have numbers" (I1_10) in order to find chemistry appealing, Isaac aligns to mathematics as the most certain of all knowledge, consisting of what can be regarded as unchallengeable truths. In addition, he accounts for mathematics as an important tool or an applicable discipline when stating "it's fun to use math as a tool in science" (I1_10) and by perceiving his mathematical background as exceeding the 60 ECTS credits in his certificate: "I have 60 credits in math, but I've got much more math than that" (I1_158). Here, Isaac portrays mathematics as existing in the main through its applications in other disciplines. In addition to its certainty, mathematics can be applied and utilised in a multiplicity of ways. Knowing mathematics is thus about mastering mathematical skills, procedures, facts and knowledge to be put into use in other subject areas.

In its most fundamental sense, Isaac's expressed perspective on the nature of mathematics relates to the position of multiplistic absolutism (Ernest, 1991). The absolutist view concerns certainty and the discipline's unique realm of pure knowledge, while multiplicity indicates the plurality of approaches or perspectives of the discipline's applications. Yet, by considering doing mathematics as the activity of developing and applying its instruments, Isaac runs the risk of overlooking the growth and development of mathematics as a discipline in itself. What can be left out is thus the ability of analysing, questioning and understanding the limits of validity of the applications, in addition to appreciating mathematics structure, its beauty and aesthetic, and its role in culture and society in general (Ernest, 1991).

Isaac's identification with the mathematics discipline appears in parallel with his expressed confidence in doing mathematics. In other words, he accounts for a sense of negotiability within the *community of mathematics as an applied science*. Following Wenger (1998), negotiability concerns to which degree a participant in a community of practice can make use of or assert as his the meanings that are negotiated by the community members. In Isaac's case, the mathematics discipline represents absolute knowledge; hence, its meaning is given by an external authority, which is heard and not subjected to independent critical judgement. Yet, Isaac expresses the ability to utilise the absolutist knowledge in effective ways, as he describes himself as a successful university student. Uttering a sense of ownership of meaning within the *community of mathematics as an applied science*, Isaac thus exercise what Belenky et al. (1986) denote as separate procedural knowing within the discipline. As distinct from received knowing, where one is listening to and returning words of authority, Isaac has a voice of reasoning when giving descriptions of "solving second order differential equations" and "evaluating the domain of solutions of a set of equations" (I1_158). However, the external authority of mathematics knowledge implies an impersonal way of knowing mathematics, meaning that ideas of what "feels right" is not taken under consideration: "[I] did not want some rubbish chemistry where you just sit and speculate [laughing], need to have numbers" (I1_10). Thus, unlike the position of constructed knowing, mathematics knowledge is not referred to by Isaac as socially constructed and contextual.

In addition to expressing belonging to a *community of mathematics as an applied science* at the university, Isaac accounts for a sense of familiarity with colleagues in the teacher common room at his first school of employment. Interpreted as participation in a *community of teacher colleagues*, Isaac aligns to the community of science and mathematics teachers who mutually engage in their students' learning and share the desire of being academically challenged. He engages in the joint enterprise of discussing teaching related topics and sharing mathematics problems during the lunch breaks, and he exercises imagination by picturing himself as a future mathematics teacher taking part in a similar community. Despite his surroundings' scepticism towards the desire of becoming mathematics and science teacher, Isaac's participation in the *community of teacher colleagues* leads to a sense of belonging to the profession, and in turn, an application for teacher education in order to get a permanent position.

From his experience of exercising mathematics teaching in the classroom, Isaac further identifies with the mathematics teacher profession by reifying himself as a "lazy" mathematics teacher. Isaac describes mathematics teaching as something that comes natural to him, like a reflex or a spontaneous reaction to the students' behaviour. The characterisation of teaching as natural indicates a perspective on teacher profession as partly innate, being carried out instinctively, and, according to himself, with success. According to Sfard and Prusak (2005), this reifying statement of being "naturally lazy" is an identifying technique in which he implements actions or incidents into properties of himself as the actor. In addition to talk about how he is *taking* shortcuts and *doing* effective teaching planning, Isaac implements the actions into *being* a lazy person or teacher. This initially negative quality is in addition turned into an advantage, by which Isaac draws attention to the way he thinks that he differs from most (student) teachers.

By describing his mathematics teaching practice as flexible and spontaneous, based on collaboration with his students, Isaac further appears as a negotiator within a classroom situated *community of secondary mathematics teaching*. In addition to identify with other actors of teaching, he voices the ability to contribute to and take responsibility for meanings of mathematics teaching and learning that is shared by the actors in the classroom. In the previously presented interview excerpts, Isaac states that he "take[s] shortcuts when I find them" (I1_94), not "spend[ing] time on things I don't see the point of spending time on" (I1_94) and that his teaching is based on "plan[s] made with a view to my students, inspired by what we do together" (I1_760). Here, he expresses the ability to produce meaning as well as to adopt his students' proposals of meaning regarding the teaching and learning of mathematics, when planning and implementing teaching in the mathematics classroom. However, his accounts of spontaneity and reflex-based teaching show as well a voice of internal authority. Unlike Isaac's previously described knowing in mathematics, knowledge about mathematics teaching derives from within, from that which feels right. In other words, his inner voice lets him know that she is on the right track in the mathematics classroom.

Based on utterances of identification and negotiability within the *community of mathematics as an applied science*, the *community of teacher colleagues* and the *community of secondary mathematics teaching*, Isaac reports on an identity of inclusion in practices of mathematics learning and teaching. At the entry of the evolving story of *confidence in mathematics and mathematics teaching*, the first critical event is characterised as *affiliation to mathematics and mathematics teaching.* In Figure 5.3, I link the critical event to three ovals representing current communities of practice in which Isaac exercises participation. His developing mathematics teacher identity evolves thus from inbound participation in communities of practice placed at both university and at his first school of employment.

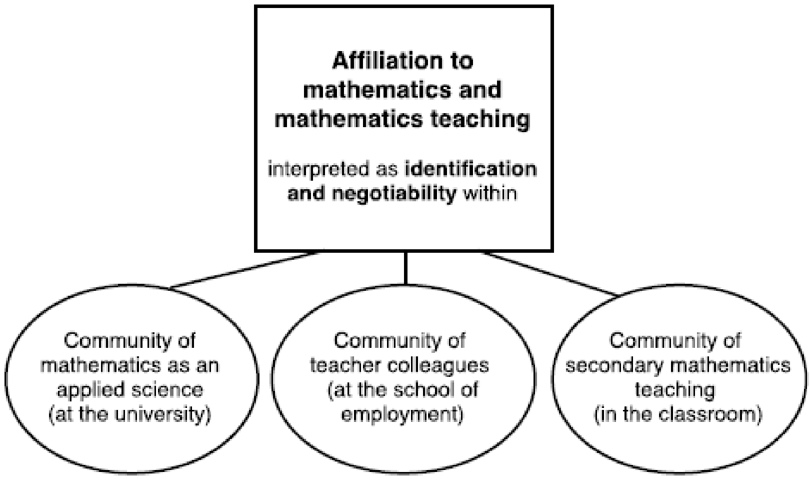

Figure 5.3: Confidence in mathematics and mathematics teaching, represented by the first critical event of affiliation to mathematics and mathematics teaching

5.1.3 Accounts of confidence in mathematics and mathematics teaching, constituting a second critical event

The second interview took place two weeks into Isaac's new job as a certified mathematics and science teacher in upper secondary school. It mainly concerned the newly completed teacher education programme, PPU, in which Isaac accounted for his experience of undergoing school placement. While he perceived the mathematics didactics lectures at PPU as enjoyable and motivating, the collaboration with the tutors in lower and upper secondary school appeared to him as intruding and frustrating. Due to the opposition between Isaac's and the tutors' preferences for classroom management in the mathematics lessons, the accounts of school placement make up the second critical event of the story of confidence in mathematics and mathematics teaching.

I2_23 Isaac: *(...) I didn't fit so well with my tutors (...). They had a rather different view on how things should work out in the classroom, meaning, the fluency in the classroom, than what I have, and then I had to try to fit with their model, and that worked only halfway. It might be I'm having a weird model, but it works very well when I'm in the classroom being allowed to manage it, (...) however, during school placement the tutors wanted me to do it more like they do it.*

Isaac describes the relationship with his tutor during school placement in lower secondary school in terms of not fitting with her way of teaching, having the consequence that he was forced into her more formal classroom model. It resulted in what Isaac characterises as bad lessons, in which he was "not allowed to own the classroom".

I2_48 Isaac: *I have a way of creating relationships with students which can appear unorganised, meaning, I'm found of the informal conversation, the informal contact with the students, and I'm maybe not so bound by rules, and to that she reacted very strongly, the woman being my tutor at that school, and forced me to be a much more bound by rules kind of leader. And then, I didn't master that situation in which I was supposed to be a very formal teacher, because that's not who I am, and then it became turbulent and very difficult to have good lessons, constructive lessons. Then there were many bad lessons (...).*

Kirsti: *What do you mean by bad lessons?*

Isaac: *Lessons where the fluency is bad, where there is a lot of noise, people not concentrating, in which the plan doesn't hit or catch anyone, because I'm not allowed to own the classroom, then I don't manage to own what is happening there. And that was very frustrating. However, now that I'm back and a master in my own classroom, then I enjoy it.*

Unlike the tutor's teaching style, being what he describes as "bound by rules"[10], Isaac appreciates the informal contact and conversations with his students. During school placement, Isaac was, however, forced to follow the tutor's set of rules for classroom management, which he did not fully master. His lessons became what he describes as noisy and chaotic, with unmotivated students. The presented excerpts delineates then a story of suffering adversity, despite Isaac's best of intentions of exercising free and informal mathematics teaching. Nevertheless, he goes back to the mathematics classroom at his new school of employment, still having faith in his own teaching style. Hence, the marked distance to his tutor's practice forms a portrait of Isaac as a confident mathematics teacher, in line with the emergent theme of *confidence in mathematics and mathematics teaching*.

5.1.4 Second critical event: Self-reliance in own mathematics teaching practice

Isaac's accounts of undergoing school placement can be interpreted as identity development through non-participation in a community of practice. Although Isaac and his tutor both are concerned with mathematics teaching in a lower secondary mathematics classroom, their collaboration lacks negotiation of the desired classroom environment, as well as negotiation of the mathematics teaching. Hence, their collaboration does not constitute a community of practice for them to be mutually engaged

[10] «Regelfast» in the original Norwegian transcript

in. Instead of taking up new practices with the consequence of reconstructing his identity as a mathematics teacher, the school placement experience results in a reinforcement of Isaac's identity, having been developed in previous settings. This is in line with learning at the boundary of a potential community of tutor and students in terms of identification (Akkerman & Bakker, 2011). Here, the discontinuity between former teaching experiences as an uncertified mathematics teacher and the school placement at PPU is recognised or identified, yet not overcome. Since Isaac's story of confidence in mathematics and mathematics teaching proceeds by a maintenance of his mathematics teaching practices, I interpret the critical event as a state of *self-reliance in own mathematics teaching practice*. It is illustrated in Figure 5.4 as Isaac's non-participation in a potential *community of tutor and students during school placement*.

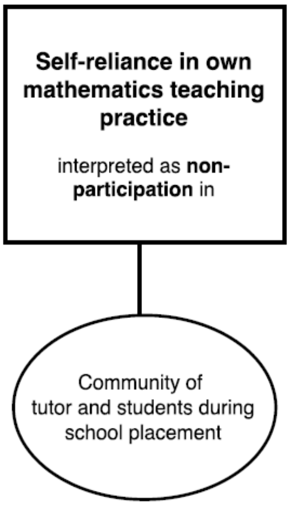

Figure 5.4: Confidence in mathematics and mathematics teaching, represented by the second critical event of self-reliance in own mathematics teaching practice

5.1.5 Accounts of confidence in mathematics and mathematics teaching, constituting a third critical event

In the first interview, Isaac accounted for experience of resonance in the teacher community at his first school of employment, regarding his desire of being academically challenged. This stood in contrast to the loneliness and differentness he felt when being appointed the smart and the clever student during own schooling and the university studies. In the last interview, he describes a similar affiliation to the mathematics teacher profession. However, instead of voicing a general belief of being capable of exercising good mathematics teaching, he now presents the profession as demanding, yet a desirable challenge he is able to overcome through more teaching experience. The changed expression of confidence also includes new perspectives on his previous mathematics teaching as an uncertified mathematics teacher. I therefore consider

Isaac's accounts of professional challenges when working as a newly certified mathematics teacher a critical event in his evolving story of *confidence in mathematics and mathematics teaching*.

When accounting for what makes mathematics teaching demanding, Isaac points to the challenge of engaging the students in terms of finding a way into the mathematics content that makes them take the bait. One example is teaching logarithms in the course Mathematics 1T. He describes the mathematics topic as brand new to the students, which makes it difficult to introduce it as meaningful to them. Yet, the didactics challenges of making meaningful approaches to the mathematics to be taught is according to Isaac a desirable part of the mathematics teacher profession. Although he does not always manage to make expedient approaches to the mathematics content, Isaac still expresses a motivation for searching the better approach, in order to overcome the current challenge.

I5_104	Isaac:	*(...) there are topics which I don't find the right entry gate for, in which I don't get to hook the students from the very beginning. (...) It's typically things they've never met before, things that are brand new to them, such as logarithms in 1T (...)*
I5_123	Isaac:	*It's heavy to do teaching in those periods, but I can't say I dislike it, because I like the challenge. (...) somewhere, there's a possibility I can find some kind of key, there exists a solution here. It's all about finding it. So that's exciting. I like academic challenges. That's why I became a teacher.*

Further, Isaac pictures the challenges of teaching mathematics as a necessity in order to further develop his teaching practice. Having taught mathematics for about one school year at his new workplace, Isaac sees his former school of employment in a different light. What in the first interview was displayed as teaching from the backbone, being natural to him or partly innate, is in the final interview explained on the basis of a special case of unusually clever students, and thus, a lack of required professional challenges.

I5_209	Isaac:	*(...) it made me a poorer teacher having so solid students (...) because I didn't get to train classroom management, I didn't get to train adapted education, I didn't get to train any of my pedagogy, and not the didactics either, it was too little resistance. It was a great path into the teacher profession, but today I'm very fond of the academic challenges. I'm happy about the challenges I've had, I then feel better as a teacher.*

This renewed perspective on the first school of employment demonstrates a shift in Isaac's story of confidence in mathematics and mathematics teaching. What initially served as a door opener into the teacher profession, in terms of identifying with competent colleagues and meeting desired academic challenges, is in the last interview displayed as being a basis for seeking new challenges with the purpose of developing his mathematics teaching practice. At the end of his first year as a certified mathematics teacher, Isaac therefore yearn for more practice in order to develop further as a mathematics teacher.

I5_541	Isaac:	*(...) I crave being in practice. I crave developing a bit more, get some more experience, reflect some more, meaning, I need that practice further on now. I feel it deep down in my teacher heart, that I need some more practice, and I look forward to it.*

5.1.6 Third critical event: In quest of development of own mathematics teaching practice

In the above interview excerpts, Isaac turns what he initially describes as demanding about mathematics teaching into a desirable challenge that he seeks in a profession. By also voicing a belief that he is capable of overcoming the didactic challenges in his mathematics teaching, I recognise his accounts to be part of the evolving story of *confidence in mathematics and mathematics teaching*. In line with his accounts in the first interview, Isaac identifies with the mathematics teacher profession based on his affiliation to the discipline and the quest for challenges in the mathematics teacher profession. However, in contrast to the first interview, he now emphasises the need for developing his mathematics teaching with help of further practice and experience. The critical event occurring in connection with Isaac's position as a newly certified mathematics teacher is therefore interpreted as being *in quest of development of own mathematics teaching practice.*

In line with his accounts from the time of entering teacher education, Isaac as a certified mathematics teacher appears as a negotiator within the *community of secondary mathematics teaching*. By stating that "somewhere, there's a possibility I can find some kind of key, there exists a solution here" (I5_104) when struggling with introducing new mathematics topics to his students, he voices the ability to shape the meanings of mathematics teaching and learning within the mathematics classroom. However, a new aspect regarding his expression of negotiability is his explicit desire for more practice experience in order to develop further his mathematics teaching. What he before described as a kind of innate ability of spontaneous and flexible mathematics teaching, independent of the tutor's guidance, is now considered to rely on his training and mastery of new challenges in the classroom. Along the story of confidence is then a critical event in the form of a change in Isaac’s expression of negotiability within the *community of secondary mathematics teaching*, as shown in Figure 5.5.

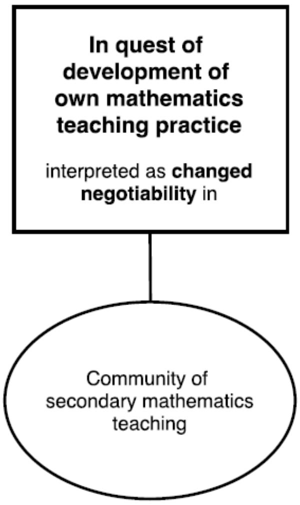

Figure 5.5: Confidence in mathematics and mathematics teaching, represented by the third critical event of being in quest of development of own mathematics teaching practice

The change concerns a movement away from negotiability merely based on internal authority or subjective knowing regarding the meaning of mathematics teaching and learning and towards author/ity of self and reason or constructed knowing (Belenky et al., 1986; Povey, 1997). Initially describing mathematics teaching as innate, Isaac tended to hold an “inner expert” who provided absolute answers regarding “good” mathematics teaching and which was known directly by him through his practice. Hence, he earlier considered teaching to be an intuitive reaction to what happened in the classroom, as something felt rather than actively pursued or constructed through critical judgement of voices of self and others. Consequently, an exchange of ideas, discussions and debate was less needed, which in Isaac’s accounts were exemplified by his lacking collaboration with the tutor during school placement. In the final interview, however, he evaluates his previous mathematics teaching experience as exceptional, in terms of having solid, autonomous and well-behaved mathematics students who gave him “too little resistance”. Holding a composite teaching experience, he accounts in the final interview for the need of meeting and overcoming resistance in order to develop further his mathematics teaching practice. In other words, the answer of what constitutes “good” mathematics teaching cannot be fully known by the Isaac’s internal authority. Instead, a continuous development of teaching through interaction with his students is possible and a necessary part of the teacher profession.

Another way of explaining this change of negotiability is to consider it as learning in terms of reflection during boundary crossing. Through Akkerman and Bakker's (2011) notion of reflection, a person develops new perspectives of differences between settings, and consequently, new perspectives on him- or herself as a learner and doer of mathematics teaching. When working as a certified mathematics teacher, Isaac recognises a discontinuity between first and second school of employment in terms of experience of difficulties and challenges in the mathematics classroom. He overcomes the discontinuity by seeing his previous experience of mathematics teaching in new light: What in the first interview was referred to as untroubled mathematics teaching ascribed his innate teaching qualities is in the final interview ascribed to the special circumstance of clever students. The result of the learning process is his changed expectations of development of his mathematics teaching, by considering himself as not being fully trained and thus being in the need for more practice. The quest for more practice takes the shape of a changed participation in the *community of secondary mathematics teaching*.

5.2 Perspectives on mathematics and its role in mathematics teaching

Isaac's accounts regarding *perspectives on mathematics and its role in mathematics teaching* are created around experience from own schooling and university studies in mathematics, from university teacher education (PPU) and from the mathematics classroom. His utterances in the first interview regarding mathematics are divided between his roles as a former and future secondary school mathematics teacher and a former university student in mathematics. Accounts concerning the first-mentioned role include Isaac's expressed perspectives on the challenging task of teaching mathematics to students. Being a teacher involves the necessity of adapting to the students' desired modes of working with mathematics and taking into account their varying aspiration for understanding the subject. Further, being conceived as a demanding subject, as absolute and exact and with either correct or incorrect solutions on its problems, Isaac reports on mathematics as holding a somehow unique position among school subjects. Regarding the objectives for teaching and learning mathematics in school, Isaac highlights the utility of mathematics within other disciplines, in addition to appreciation of its value in daily life and in the development of science and technology.

Isaac's expressed perspectives on the nature of mathematics and its teaching and learning is further informed by his accounts of mathematics in the context of being a university student. Here, Isaac gives descriptions of mathematics being an academic discipline in itself and being a part of the field of mathematics education. His initial reports on the nature of mathematics and its relation to other academic disciplines can be explored in terms of a dichotomy. Hence, when entering the teacher education programme, Isaac is attending towards two conflicting cultures of scientific practices at the university. The reported dichotomy makes up the first critical event of his evolving story of *perspectives on mathematics and its role in mathematics teaching*. However, when giving retrospective accounts of undergoing university teacher education in the second interview, Isaac voices a strengthened affiliation towards the practices of PPU. The second critical event is therefore based on his changed perspectives on university teacher education, and his desire of implementing mathematics teaching practices highlighted by PPU in his own classroom. Across the last three interviews, Isaac repeatedly accounts for ambitions of teaching mathematics in line with inquiry based practices. However, due to his struggles for realisation and a continued conflict with his students' regarding the meaning of mathematics teaching and learning, his accounts from being a certified teacher leads to a third critical event. The related analysis is partly based on accounts generated from observation of Isaac's mathematics teaching at his school of employment.

An overview of labels belonging to the emergent theme *perspectives on mathematics and its role in mathematics teaching* is shown in Figure 5.6. The labels constitute three critical events along the evolving story, which are linked in Figure 5.6 to the interview transcripts to be presented in the upcoming analysis. Similar to Figure 5.2, the labels were confirmed by comparing and contrasting accounts throughout the interviews. In addition to be linked with specific interview excerpts, the labels thus constitute summaries of Isaac's expressed reasoning regarding mathematics, mathematics teaching and learning.

Perspectives on mathematics and its role in mathematics teaching

Labels related to the first critical event: **Attending towards "two cultures" in university teacher education**	Consideration of students' needs in the teaching of mathematics I1_660	Demands for understanding when learning mathematics I1_660 I1_667 I1_1138	Knowing in mathematics as understanding "why" - not only "how" I1_454 I1_1138	Mathematics as part of students' life I1_376 I1_393	Mathematics as an applicable discipline I1_1138	Mathematics/natural sciences and social sciences as two cultures I1_609 I1_626 I1_667	Mathematics education and general pedagogy as two cultures I1_213 I1_642	Teacher education as a tool for conducting (mathematics) teaching I1_51	Teacher education as an advocate for the desired mathematics teaching practice I1_213 I1_460	
Labels related to the second critical event: **A movement across cultures and towards a changed mathematics teaching practice**					PPU as an upturn I2_12	PPU as a transition between two cultures I2_12 I2_90	PPU as a transition from a remote to an approachable scientific discipline I2_12 I2_90	PPU as a resource for changing own mathematics teaching I2_17 I2_136	Expectations of developing mathematics teaching through inquiry I2_136	
Labels related to the third critical event: **Undesirable stagnation of own mathematics teaching practice**			Knowing in mathematics as understanding "why" - not only "how" I3_479 I4_228 I3_510 I4_257 I4_128 I5_530		Inquiry based mathematics teaching as the desired teaching practice I3_144	Balancing time avilable and students' need for a task-based instruction I3_103	Own mathematics teaching as a settled structure I4_22		Exeperiences of stagnation in own mathematics teaching I3_453 I3_479 I5_477	Future ambitions for developing the mathematics teaching through inquiry I3_453 I3_510

Figure 5.6: Overview of labels belonging to the emergent theme *perspectives on mathematics and its role in mathematics teaching*, constituting three critical events

5.2.1 Accounts of perspectives on mathematics and its role in mathematics teaching, constituting a first critical event

In line with the story of confidence, Isaac's decision of becoming a certified mathematics teacher is a critical event for his evolving accounts of mathematics and its role in mathematics teaching. The choice of leaving the university studies implies a role change; from being a learner in university mathematics partially embedded in a study programme in chemistry, to become a learner in mathematics teaching, and thus, a teacher. Although Isaac characterises mathematics teaching as a reflex from the backbone, he also reflects on the challenge of differentiating the teaching with respect to students' needs and desires. While some students want thorough explanations and many examples at the blackboard, others want to jump right into the mathematics content, doing the examples and tasks by themselves.

I1_660	Isaac:	*(...) some want head-on, this is how it is, some want a thorough explanation of why we are talking about this in that way, some want a lot of examples, some want to do the examples by themselves, (...) it is just loads of problems with math because everyone have their own way of thinking math, and no one are satisfied no matter what I do at the blackboard, so, teaching at the blackboard in math is heavy work, since it gives the minimum benefit for the class as a whole. (...) And since math in general is perceived as demanding, it manifests itself in math more than in other subjects.*
I1_677	Isaac:	*(...) in math, the frustration is bigger, sitting there and trying to solve a task and not understanding what you are supposed to do, then one gets provoked. You become like, I don't understand a thing. What's this about? (...) In language subjects you will always be able to doodle something, even if you didn't quite get what you were supposed to do. In math you will go blank.*

Based on his utterances of students having "their own way of thinking math" and having "very different needs" when learning mathematics, Isaac problematizes the view on mathematics teaching as primarily concerning giving clear instructions and good explanations at the blackboard. To him, mathematics teaching goes beyond the mere presentation of the subject content, also taking into consideration the students' interests and needs when learning the subject content. Further, mathematics differs from other disciplines by being perceived as demanding and by generating strong emotive reactions such as frustration. One distinctive feature is the risk of "going blank" when trying to solve a mathematics problem. Hence, mathematics holds according to Isaac a somehow unique position or status, compared to other disciplines.

When accounting for what characterises a good mathematics teacher, Isaac highlights the need for making the students aware of the personal relevance of mathematics, in other words how mathematical thinking permeates everyday life and experience.

I1_376 Isaac: *(...) a good teacher is the one who makes them understand that, eh, math is not something stupid they have to do. A good teacher is basically one who's able to show them that math is everywhere. It's a part of their life, even if they never again will open a math book, math is part of their life.*

I1_393 Isaac: *It's more important than doing the procedures, it's more important than getting a good grade on the exam, it's more important than everything else, meaning, this understanding of math being both fun and useful, and to quote the ancient Greeks, making you a better person. If they manage to see that, then I'm very pleased.*

Appreciating mathematics as a central element in one's daily life is according to Isaac more important than being able to carry out the correct procedures on tests and exams. A good mathematics student manages therefore to look at mathematics beyond its numbers, rules and formulas, by considering it as a unique perspective on the world. In his discussion about one interview task on functions (Appendix B), Isaac stresses the need of making the students aware of the role of the concept of function in human culture, being an important contribution for describing and understanding the practical world.

I1_1138 Isaac: *I think the real challenge is to make the students understand that a function is an incredibly exciting mathematical construction, which has solved so many problems, and which, already the first time it was introduced created a world sensation, meaning that, the function is probably the most important contribution of mathematics in the practical world, meaning, making the students understand it, I think it's demanding. Because I don't think they want to. Students want to solve a task and get the correct answer. They enjoy getting that understanding, but they don't want to work for it.*

Based on his description of the function concept as "incredibly exciting", "a world sensation" and "the most important contribution of mathematics in the practical world", learning mathematics is to Isaac about learning utilitarian knowledge as well as appreciating the value of mathematics in daily life, science and technology. Considering mathematics as a means by which people can make sense of a complex world, Isaac's conception of the discipline further embodies demands for understanding. This is visible in his elaboration of what is meant by a good mathematics student. Here, he makes a distinction between being able to carry out a procedure with the focus on the *product*, in terms of getting the correct answer, and being able to understand *why* the procedure is appropriate, with the focus on the *process*.

I1_454	Isaac:	*One has to… try to understand what lies behind, meaning, thinking less about… do I have the right answer, but do I have the right method, maybe. Eh, being less curious about what's in the list of right answers and more curious about what is the right approach, I don't know, it's about, seeing the value of the process and… seeing the value in finding this procedure and understanding the procedure instead of following a recipe.*

In addition to account for mathematics, its teaching and learning from a former uncertified teacher's point of view, Isaac reflects on the meaning of the mathematics teacher profession based on his experiences from the first semester at PPU. He describes the teacher education programme as two-sided, by containing interesting and relevant lectures in the subject didactics courses and tedious lectures in general pedagogy. Searching for knowledge or expertise that can give him insight into how his teaching can improve, Isaac finds the general pedagogy insufficient for a professional study. On the contrary, the lecturer in mathematics didactics manages to make parallels between the literature and the mathematics classroom.

I1_51	Isaac:	*I'm very curious and desired to learn, so I think that, when I get the fill-up which I feel give me insight into how my work as a teacher can improve, when I get the fill-up which makes me feel that I can use it, then I find it very fun (...) But a lot of it is archaic, strange theories given in the thirties which don't fit into the today's modern reality, and then I get demotivated. I understand some of the arguments for learning about it, but I don't feel I get the fill-up that I need to become an engaged and motivated teacher. (...) I'm not in the kind of situation where I'm supposed to understand the historical development of pedagogy. I need a tool subject, a professional study, which gives me something for being a teacher. And then I think this academic discussion doesn't fit into a programme of a professional study. However, other parts have been fantastic. Especially, the didactics lessons are fantastic. (...) I don't feel like I'm learning outdated theories and scratching my head to see the relevance of it. I'm placed in a situation in which I'm able to directly see the parallel to the classroom.*

Isaac describes the desired teacher education programme with terms such as "a tool subject" and professional "fill-up" that he needs or can use as a mathematics teacher. This is opposed to archaic and outdated theories and discussions which he perceives to be irrelevant to his profession. In addition to problematize the usefulness of teacher education, Isaac also elaborates on the disparity of studying natural versus the social sciences. According to him, the disciplines differ greatly in structure and organisation of the study programmes and in the way of approaching the syllabus. While university mathematics represents a body of concepts and methods which he can "swallow raw", the teacher education syllabus demands a critical attitude, based on its understanding of scientific truth as imperfect and corrigible.

I1_609 Isaac: *To me it was very strange studying something outside the natural sciences. (...) Suddenly, we have two lectures per week, and here's a paper you can read if you like. It was kind of, is this everything? (...) And a totally different way of reading the syllabus and a totally different way of acting towards the syllabus, from being a formula which is like this, to become a paper you should have an opinion about (...) I can no longer swallow it raw, I have to be critical (...).*

I1_626 Isaac: *Now it's kind of, I feel and think, and, if we look at him, then maybe we can think that, however, no, if we look at him, then this is not true, but actually, neither of them [is true] (...) I want the two underlines, that I can put underneath the answer. If I can claim two things to be correct at the same time, then the laws of logic collapse, that's kind of* how it is

While Isaac alienates himself from the reading and discussions within the social sciences, he simultaneously expresses affinity towards the literature within the mathematics didactics course. Although said with a humoristic tone, Isaac thus delineates a dichotomy within teacher education. On the one side, the mathematics teacher educators represent the applied, practical educational research with the purpose of developing and improving mathematics teaching. On the other side of the dichotomy, the general educators represent uncertain and imprecise knowledge and educational research with no clear purpose.

I1_642 Isaac: *The math education papers seldom frustrates me, they are so pretty, they've got a nice purpose, and they make the effort of explaining or developing something, and then there's a nice conclusion at the end, so I feel kind of, I can take something nice and concrete out of it. (…) The pedagogical papers, especially those about classroom management and things like that, it's totally impossible to get something out of it. It's like, if we look at him, maybe that's smart, but if we look at her, maybe that's smart, (…) however, they say completely opposite things, can they both be true? (…) It's frustrating. In math, it's much more practical, meaning, we have looked at the implementation in hundred classrooms and we see that it works, or that it doesn't work. Then it depends on the class, but it works in between, and that is kind of OK, I can relate to that. And that is probably because they are mathematicians writing those articles, I'm convinced [laughing]. And then, they're social scientists writing the pedagogical papers [laughing].*

Nevertheless, during the lectures in the mathematics didactics course, Isaac engages in reading and discussing literature that puts into words many of the challenges he has met in his own mathematics teaching. Based on his desires for "removing the list of right answers", and by valuing the students' processes when doing mathematics, he associates with the literature on mathematics competences (Niss & Jensen, 2002) and landscapes of investigation (Skovsmose, 2003).

I1_213 Isaac: *I think that's been really good about the lessons in math didactics, making it so clear, all these problems, have learned about Niss' eight competences eh, and, just seeing that, so many different competences we have to develop in students in order to enable them to become skilled, (…) being just a teacher I feel that one only thinks about being a good leader (…) while the problems are much more complex the moment you put mathematics into it.*

I1_460 Isaac: *(…) after starting at PPU I have become a fan of Skovsmose, (…) inquiry based teaching is a fantastic thing, it really is. Eh… and that is not the only way, but I can see it does the right thing about removing the list of right answers.*

In his first semester at PPU, Isaac accounts for a sense of resonance in the mathematics didactics course, due to considerations of what constitutes good mathematics teaching and of the complexities of students' learning. Research in mathematics education appears thus as a useful tool for Isaac to develop and improve his teaching practice, by virtue of inquiry based mathematics teaching.

5.2.2 First critical event: Attending towards "two cultures" in university teacher education

Based on having worked as an uncertified mathematics teacher in upper secondary school, Isaac reports in the first interview on the distinctive characteristics of mathematics as a school subject. From his utterances of "not understand[ing] a thing" and having the risk of "going blank" (I1_677) when working on mathematics problems, his initial accounts have traces of an absolutist perspective on the nature of mathematics. In contrast to uncertain knowledge based on experience and observations of the world, mathematics knowledge is within this perspective viewed as absolute and objective, fixed and exact and with a unique structure. Consequently, mathematics might be perceived as a perfect, crystalline body of absolute truth, yet external, cold, hard and remote (Ernest, 1991). An absolutist perspective on mathematics can further be associated with mathematics teaching as instructing, meaning that the mathematics teacher focuses on the skill mastery and correct performance of the students (Ernest, 1989). However, learning mathematics in school is, according to Isaac, more than learning a set of separated rules and procedures and solving tasks with the purpose of getting the correct answer. In addition to portray mathematics as holding a unique position or status of absolutism, he accounts for the personal relevance of the discipline. By presenting mathematics as a substantial part of the students' life, Isaac's expressed perspectives on mathematics as a school subject are as well in line with Andrew and Hatch' (1999) notion of mathematics as a *life-tool*. In addition to the focus on every day usage, mathematics is considered to empower the learner "through higher levels of understanding than those acquired through rote learnt rules and techniques" (p. 216). Accordingly, being able to appreciate mathematics as part of one's daily life is more important than being able to carry out the correct procedures on a final exam.

Following Ernest (2004a), appreciation in mathematics is related to personal, cultural and social relevance of mathematics, beyond the useful or necessary mathematics for all or for some. It involves a broad understanding and awareness of its nature and value, within the whole realm of human culture, as well as understanding and being able to critique its social uses. In Isaac's case, appreciation of mathematics seems to circle about its role in the history and development of science and technology, and hence, its application in other disciplines. This is in the above interview excerpts exemplified by the concept of functions, and its importance for the development of the natural sciences. However, what is not part of Isaac's accounts is the appreciation of mathematics as a discipline in its own right, with its main branches and interconnected concepts, as well as its big ideas, such as infinity, symmetry, continuity, chaos and proof.

In addition to account for the characteristics of mathematics as a school subject, Isaac discusses what is meant by exercising good mathematics teaching. Here, he makes a distinction between doing mathematics with a focus on the product, or the one, right solution, and doing mathematics with a focus on the process. This distinction can be associated with literature on instrumental and relational understanding in mathematics (Skemp, 1976), which is part of the syllabus at PPU. Following Skemp, relational understanding in mathematics means knowing both *what* to do and *why*, as distinct from the instrumental understanding of knowing *what* to do, by simply applying a rule or algorithm in order to get the correct answer. Isaac's negotiation between good and less good mathematics teaching can then be summarised as follows:

- Good mathematics teaching goes beyond the mere presentation of the subject content, by taking into account the students' interests and needs when learning the subject content
- Good mathematics teaching highlights the utility of mathematics in other disciplines, as distinct from being a mandatory subject in school that "one has to do"
- Good mathematics teaching highlights the personal and cultural relevance of mathematics, as a substantial part of one's daily life as well as science and technology
- Good mathematics teaching emphasises understanding in terms of knowing what to do and why, not only knowing what to do

The negotiation takes place in the tension between Isaac's expressed perspectives on mathematics teaching and learning on the one hand, and what he regards as the students' common conceptions of the school subject on the other hand. While the students perceive mathematics as a set of disconnected rules or procedures to be carried out in order to get the correct answer, Isaac highlights the need of understanding *why* the procedures are correct. The division between his own and others' perspectives on teaching and learning mathematics can then be interpreted as a sense of belonging to practices of relational mathematics instruction. Here, Isaac imagines being a teacher who presents mathematics as a useful subject, having both personal and cultural relevance, and which requires understanding beyond rote learned procedures and algorithms.

From Isaac's accounts, it is unclear through which interactions, with whom (beside the students), and where the negotiation between good and less good mathematics teaching takes place. Hence, Isaac's accounts do not provide information about other actors of the practice of relational mathematics instruction. However, since his utterances on understanding can be linked with literature in the teacher education syllabus (e.g. Skemp, 1976), Isaac might take part in the negotiation through reading and discussing literature about mathematics teaching and learning during his first semester in teacher education. Consequently, he might align to conceptions about mathematics teaching and learning provided by the university teacher education. Another possibility is engagement in discussions about students' learning at the teacher common room at Isaac's first school of employment, with reference to common literature from teacher education on relational and instrumental understanding.

Regarding becoming what Isaac denotes as a good mathematics teacher, he further describes the desired teacher education programme with terms such as "a tool subject" and professional "fill-up" that he needs or can use as a future mathematics teacher. This he contrasts with archaic and outdated theories and discussions irrelevant to his profession. His distinction between useful, professional studies and irrelevant academic discussions brings along perspectives on the role of educational research and its relation to practice in school. Expecting PPU to provide useful information in order to improve his mathematics teaching, Isaac is to some degree expressing an instrumental view on the scholarly knowledge within teacher education, in line with Hammersley's (2002) engineering model. Educational research should then provide "specific and immediately applicable technical solutions to [teaching] problems, in the manner that natural science or engineering research is assumed to do" (p. 38). In contrast, the moderate enlightenment model holds that research is one among several sources of knowledge on which mathematics teaching practice can draw. Here, prospective mathematics teachers should develop the pedagogical reasoning necessary to critically evaluate research on mathematics education and its relationship to contextual concerns. Implied in the engineering model is an absolutist and unassailable perspective on knowledge, in contrast to the fallible knowledge produced within the moderate enlightenment model. Isaac's instrumental account on teacher education can therefore be seen in connection with his previous educational background in the natural sciences. As accounted for earlier, Isaac identifies with a *community of mathematics as an applied science* at the university and a perspective on mathematics as being an essential tool in other scientific disciplines. Consequently, his expectation regarding the scholarly knowledge provided at PPU might be about usefulness in rather direct and practical terms.

By describing the teacher education literature with terms of "feel[ing] and think[ing]" (I1_626), as opposed to "swallow it raw" (I1_609), Isaac draws further a dichotomy between the pure and absolute knowledge of mathematics and the natural sciences, and the fallible, nearly non-scientific knowledge of the social sciences. This dichotomy can be further understood through the words of Snow, from his lecture on the two cultures (Snow, 1963). According to him, the natural sciences and the humanities had through the years developed a tension to each other: “They have a curious distorted image of each other. Their attitudes are so different that, even on the level of emotion, they can’t find much common ground” (Snow, 1963, p. 12). Isaac’s accounts of entering the other culture of the one-year teacher education programme (PPU) give voice to his alignment to the *community of mathematics as an applied science* at the university, as well as his distance towards the social sciences. Yet, by expressing an affinity towards the discipline of mathematics education, and simultaneously a distance towards general pedagogy, he identifies with a *community of mathematics didactics* at PPU. From Isaac’s perspective, the members of the community are teacher educators and researchers having a mathematical background, implying they are dealing with the enterprise of an absolute and objective discipline holding a somehow unique position or status. By mutually engaging in problems regarding mathematics teaching and learning, they exercise according to Isaac useful and practical research for explaining and developing mathematics instruction.

Comparing the practices of the *community of mathematics didactics* and the former *community of mathematics as an applied science* at the university, Isaac accounts for a shared perspective on the nature of mathematics as an absolute and applicable discipline. His entrance into the *community of mathematics didactics* can therefore be understood as a movement across overlapping practices. In adddition, Isaac aligns with the joint enterprise of developing mathematics teaching through inquiry based practices. The literature on landscapes of investigation is in line with his perspectives on “good” mathematics teaching and the practices of relational mathematics instruction. Consequently, Isaac’s participation in the community can be understood on the basis of two different learning trajectories. On the one hand, the participation is a result of Isaac being a university student and identifying with mathematics as a university discipline with its certain nature and characteristics. This brings with it some expectations regarding the research within the discipline of mathematics education, in terms of its usefulness and ability to explain and develop mathematics instruction. On the other hand, the participation is a result of Isaac being a (student) teacher in mathematics, identifying with mathematics as a subject to be taught in line with practices of relational mathematics instruction. Here, Isaac sees the literature given in the mathematics didactics course as in line with his perspectives on meaningful mathematics teaching.

Being more than a single trajectory, Isaac's identity as a mathematics teacher can be considered a nexus of multimemberships, as he engages in different mathematics practices at university and school, holding different objectives for its teaching and learning. The nexus does not stand for a merger of Isaac's specific trajectories into one, comprehensive understanding of himself as a learner and actor of mathematics and mathematics teaching. The way he accounts for the nature of mathematics as a university discipline, in the role of a learner in mathematics, differs to some degree from the way he accounts for mathematics as a school subject, in the role of a teacher in upper secondary school. However, the trajectories become part of each other, when Isaac is reconciling the memberships and accounting for similar and overlapping practices. The learning trajectories thus share his perspective on both mathematics and mathematics education research acting as *tools*; the former as a tool to be applied in other scientific disciplines, the latter as a tool for developing and improving mathematics teaching by virtue of inquiry based mathematics teaching. In his first semester in teacher education, Isaac's affinity towards the disciplines of mathematics and mathematics education and his concurrent distance towards general pedagogy constitute then a critical event of *attending towards "two cultures" in university teacher education*. In Figure 5.7, I relate the critical event to Isaac's identification and negotiability within the *community of mathematics didactics* at PPU and the *community of mathematics as an applied science*. Simultaneously, his expression of "two cultures" relates to non-identification with practices belonging to a *community of general pedagogy at PPU.*

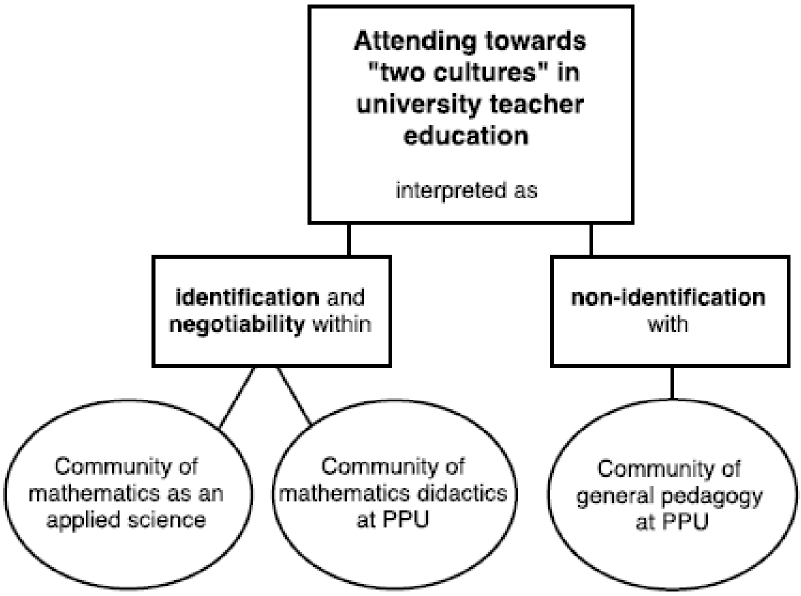

Figure 5.7: Perspectives on mathematics and its role in mathematics teaching, represented by the first critical event of attending towards "two cultures" in university teacher education

5.2.3 Accounts of perspectives on mathematics and its role in mathematics teaching, constituting a second critical event

Having just started in his new job as an upper secondary mathematics and science teacher, Isaac looks back at the one-year teacher education programme and describes it as a positive experience, despite his initial preference for studying the natural sciences with its "two underlines underneath the answers".

I2_12	Isaac:	*I think PPU has been exciting, in general, I think it's been very fun to see the theoretical part of the pedagogy and the didactics, I think it has been valuable. I've liked that kind of studies, actually, much more than I had thought that I would like it, because I'm used to solving problems with two underlines underneath the answer, and it was more fun than I thought writing papers [laughing].*

Isaac's description of the transition from chemistry and mathematics studies into teacher education is in line with his earlier stated dichotomy, between the pure and absolute knowledge of mathematics and the natural sciences, and the fallible knowledge of the social sciences. Eight months later, he looks back at PPU as something that turned out as more interesting and enjoyable than expected. Isaac elaborates on his changed view on teacher education with terms such as becoming part of the way of thinking and mastering the language of a new discipline.

I2_90	Isaac:	*After the first period (...), I didn't yet get the hang of it, mastering the discipline and its format properly (...), didn't get the message of what we were learning and doing, as well as I did in the second period. Because then I was part of the way of thinking, then I kind of felt I had an eye-opener about what, how to work with the discipline, how to understand the discipline.*
	Kirsti:	*What led to that eye-opener?*
	Isaac:	*It was largely that I got into the literature, and I suddenly understood what all these authors wanted to tell, I understood the language, the way of expressing, the way of thinking, and then at once it became much more interesting, meaning, I understood a lot more (...) because I was so unknown with the humanities, so then I was at least much more mature when entering the second period of school placement.*

In the second interview, Isaac tells about having an eye-opener into the language and the mind-set of the teacher education programme, where the lectures provide motivation for "chang[ing] the math classroom" and "break[ing] with the instinct of standing and chanting at the blackboard".

To make the change of his mathematics teaching possible, Isaac has signed up for an in-service course with the plan of exercising inquiry based mathematics teaching in own classroom and sharing his experience with the other course participants.

I2_17	Isaac:	*(...) it has motivated me to think now I'm going to change the math classroom (...). And I feel that I have a lot of commitment, many things I would like to do, because of things I've learned in for example the math didactics lessons.*
I2_136	Isaac:	*I signed up for a course (...), the topic is inquiry based teaching, I chose a math project, in which I will try to, to a greater extent, inquiry, landscape of investigation, and see how it works, and try to break the instinct of standing and chanting at the blackboard. (...) I think it's from all maths teaching I have ever experienced in my whole life. So I have a very strong picture of that is how math teaching takes place, the teacher stands at the blackboard chanting, and I want to break with that, and then I specifically want to try to do it with inquiry based work or maths problems. Then I have committed myself to it through this project, where we will meet teachers from other schools and present posters of what we have done and so on, so it becomes binding (...) So, that I expect, to be able to do it properly this time, this kind of thing, because I believe in it.*

Being on the threshold of a career as a certified mathematics teacher, Isaac accounts for a renewed perspective on PPU as well as a commitment to implement inquiry based mathematics teaching. Due to his strengthened affiliation towards the practices taking place during lectures and seminars at PPU, Isaac's retrospective accounts of undergoing university teacher education constitutes a critical event in his evolving story of *perspectives on mathematics and its role in mathematics teaching*.

5.2.4 Second critical event: Moving across "two cultures" and towards a changed mathematics teaching practice

Isaac's accounts in the second interview, of having "had an eye-opener" (I2_90) and of liking the teacher education studies "much more than I had thought I would like it" (I2_12), portray a less evident dichotomy between the natural and the social sciences at PPU. By describing a difference between first and second semester, from not "get[ting] the hang of it" and not "get[ting] the message", to understanding "the language, the way of expressing (…) and thinking", Isaac tells a story of overcoming a discontinuity in action and interaction when undergoing the teacher education programme. His accounts of a changed perspective on teacher education, its literature and reasoning, thus indicates an inbound movement within communities of practice at PPU. This differs from findings in Section 5.2.2, in which his description of two cultures at PPU led to participation in the *community of mathematics didactics,* however, with a marked distance to the general pedagogy. In the remaining section, I will explore in more detail how Isaac's accounts develop from the first to the second critical event.

In the first interview, Isaac describes PPU as two incompatible cultures of the natural and the social sciences, with the consequence of delineating a division between mathematics education and general pedagogy. Hence, he is solidifying his perspective on the discipline of mathematics as an unquestioned body of truths, being distinct from the imperfect and corrigible knowledge of the social sciences. In line with the process of identification during boundary crossing, Isaac is then constructing and reconstructing boundaries between the two cultures, rather than overcoming the discontinuities. In the second interview, Isaac tells about experiencing an eye-opener regarding the syllabus of teacher education programme. By explicating differences between the two cultures, however, with the result of expanding his perspectives on the practices of PPU, Isaac is overcoming the discontinuity by exercising reflection. According to Akkerman and Bakker (2011), the result of learning through reflection is a new construction of identity that informs future practice. In Isaac's case, the result is expectations of changing his mathematics teaching practice in the direction of inquiry based mathematics teaching.

Further, Isaac expressed in the first interview an interest in the mathematics education literature that dealt with many of the challenges he had met in his own mathematics teaching. Being in accordance with his negotiation of what counts as good and less good mathematics teaching, I argued for his belonging to the *community of mathematics didactics* and its joint enterprise of exercising mathematics teaching through inquiry. Teacher education was then considered by Isaac as an instrument or tool for his teaching, such as "see[ing] that it works, or that it doesn't work" in the mathematics classroom (see interview excerpt I1_642, Section 5.2.1). Isaac's belonging is strengthened in the second interview, where he calls attention to PPU as a resource for *changing* his mathematics teaching practice. Due to a less prominent dichotomy and a changed view on the importance of teacher education for own classroom practice, I interpret the *community of mathematics didactics* to be re-represented by a *community of inquiry based mathematics teaching*. The re-representation stresses as well Isaac's emphasis on inquiry based mathematics teaching as his desired professional practice. He identifies with the renewed community by exercising engagement and imagination: by sign-ing up for an in-service course at his new school of employment and imagining a continued and active membership, and by picturing himself as a future teacher who exercises mathematics teaching differently from the common blackboard instruction.

The re-representation of the *community of mathematics didactics* does not represent a prominent change in Isaac's perspectives regarding good mathematics teaching. Nevertheless, the process of overcoming discontinuities at PPU demonstrates a shift in perspectives on educational research in mathematics and its role in own classroom practice. After having completed the teacher education programme, I therefore identify a critical event of Isaac *moving across "two cultures" and towards a changed mathematics teaching practice*. In Figure 5.8, I link the critical event to his identification and negotiability within the *community of inquiry based mathematics teaching*.

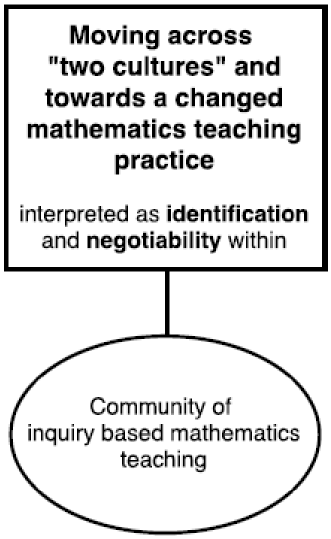

Figure 5.8: Perspectives on mathematics and its role in mathematics teaching, represented by the second critical event of a moving across "two cultures" and towards a changed mathematics teaching practice

5.2.5 Intermezzo: A teaching sequence on linear modelling and rate of change

In the intermezzo of Isaac's evolving story of *perspectives on mathematics and its role in mathematics teaching*, I give a summary of a teaching sequence led by Isaac in the course Mathematics 1T. It forms also an introduction to the subsequent analysis of Isaac's accounts on mathematics and its teaching from his first year as a certified mathematics teacher. The classroom observation took place in the middle of Isaac's second semester at his new school of employment, in order to generate detailed accounts regarding his mathematics teaching. In the next section, I therefore include excerpts from two interviews (I4 and I5) arranged in connection with the observation, in addition to comparable accounts from Interview 3.

5.2.5.1 The teaching sequence

Isaac introduces the lesson by writing a headline on the blackboard: *To determine the rate of change of a linear function*. He starts off with a review of linear functions, and writes the following example on the blackboard:

> One sunflower is 20 cm high. It grows about 5 cm per week. Find a function $h(x)$ that shows the height of the sunflower as a function of x weeks.

After having a short discussion in pairs, one student is asked to present his solution, which is the correct function $h(x) = 5x + 20$. Isaac continues with pointing out the concepts of constant term, slope, and variable from the given function expression. He then states that for a linear function, the rate of change equals the slope of the graph. Next, he presents the graphical representation of two linear functions in GeoGebra[11], as shown in Figure 5.9. He asks the students to find the function expressions for each of them, by discussing the solutions in pairs.

[11] GeoGegbra is free, interactive mathematics software for learning and teaching mathematics and science from primary school up to university level.

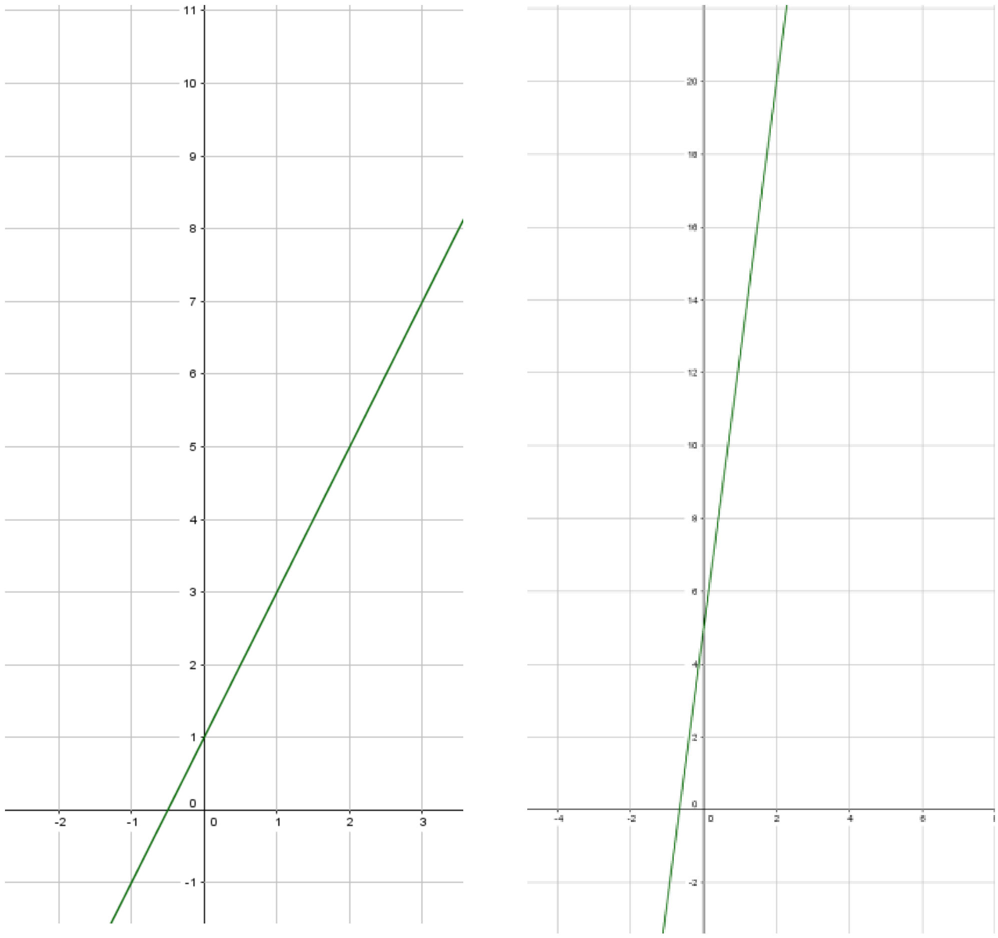

Figure 5.9: Two examples of linear functions in Isaac's lesson on linear modelling

After a while, one student suggests the function $f(x) = 7{,}5x + 5$ as a solution to the right hand graph. However, some other students are protesting. Dialogue 1 follows like this:

Dialogue 1:

Isaac:	*I can hear that not all of you are following the solution. Can you please explain in more detail how you found the rate of change?*
Student 1:	*I saw that it grew with 15 for every 2 x, so I divided by 2 on both sides, like an equation.*
	[Isaac rephrases the solution in order to clarify what is meant by dividing by 2 on both sides like an equation, and he asks whether the other students agree.]
Student 2:	*Can we always do it like this?*

Student 3:	*Yes, because there exist a formula with a delta-something.*
Isaac:	*That's right, there's such a formula, however, we will think about it in a different way. (…) I still don't have an answer to the question if we always can do it like this?*
Student 4:	*We can only do it for linear functions.*

Isaac further introduces another graphic example to the class, which shows the increase of population in the world for the years 1950 to 2010 based on data from the United Nations. The graph, being approximately linear, is shown in Figure 5.10.

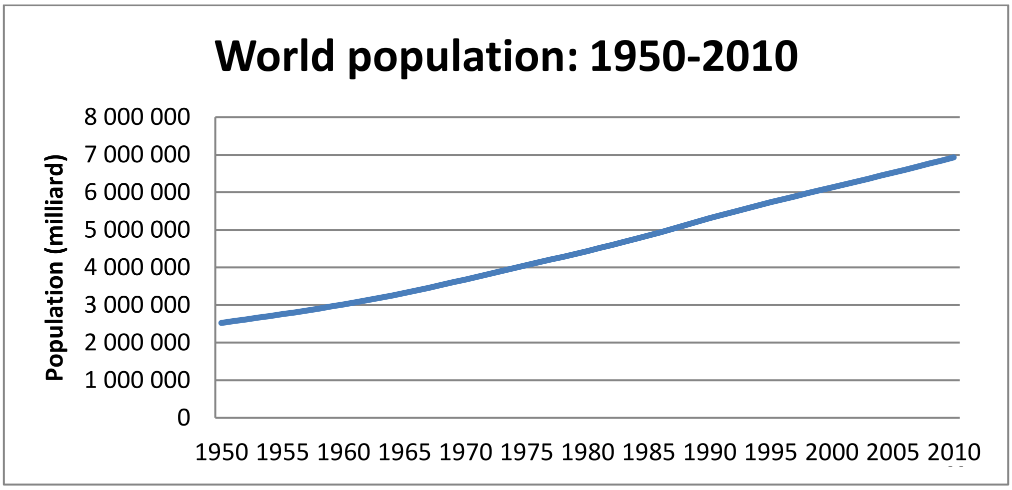

Figure 5.10: The world's population based on data from the United Nations, Department of Economic and Social Affairs

Isaac sets up the following task: *How can we find the rate of change of the graph? Find a linear function that describes the increase of the population*. He tells the students to set 1950 as year 0 and 2010 as year 60.One student presents the following linear model: $f(x) = 0{,}05x + 2{,}5$. She explains it by stating that it grows with 0,5 billion people in 10 years, and by dividing 0,5 with 10, she gets 0,05 billion people each year. Since the graph starts on 2,5 billion people, 2,5 becomes the constant term of the function. She then asks, in Dialogue 2:

Dialogue 2:

Student:	*Is it correct?*
Isaac:	*I'm not going to answer you whether it's correct or not. That's not how it works here.*

Isaac questions why it might be problematic to make a linear model for the ten first years of the graph. Then another student presents a solution in which he has found the average growth for 60 years. Isaac draws both linear models in GeoGebra and highlights the differences between them. He concludes that none of them is the one, correct linear model for the graph, since their different rates of change represent different areas of application in order to describe the increasing population in the world. He then reveals the continuation of the modelled graph, showing that it tends to flatten just below10 billion people. The conclusion is that none of the linear models are able to predict the future situation for the world's population.

5.2.6 Accounts of perspectives on mathematics and its role in mathematics teaching, constituting a third critical event

I met Isaac for a third interview three months into his first year as a certified mathematics teacher, in which he described experience of stagnation when trying to change his mathematics teaching according to his ambitions. Similar accounts of stagnation and conflict between him and his students regarding the meaning of mathematics and mathematics teaching and learning were given throughout the final interviews (4 and 5). Isaac's repeating descriptions of a desired mathematics teaching and his struggles for realising it constitutes a critical event in his evolving story of *perspectives on mathematics and its role in mathematics teaching*.

During his first year as a certified mathematics teacher, Isaac teaches mathematics and science at the general education programme (Mathematics 1T, Mathematics R1 and Natural Science), in addition to mathematics at the vocational education programme Technical and Industrial Production (Mathematics 1P-Y). In the mathematics course 1T (grade 11), Isaac and his students discuss what mathematics teaching and learning should be; whether mathematics is about learning procedures or formulas, or if it also includes giving rationales for one's solutions. According to Isaac, the students are not used to explain their solutions, which he believes is something they bring with them from lower secondary school. He is therefore facing a challenge of turning the teaching away from questions about "how" to solve a mathematics problem, towards questions about "why" a solution method is correct.

I3_479	Isaac:	*(...) I don't think they're used to it from lower secondary school, and I think it's very important to be able to explain a bit (...). Why is, why can you find the roots [of a second degree function] from the first-degree factors? (...) and then people start to answer kind of, yes, it must be the number next to the x. No, that's not why [laughing]. And then, the why is kind of, it requires a lot of work, but they are starting to get better. And I find that important. I try to teach for understanding, and I face a lot of resistance.*

In line with his emphasis on the process when learning mathematics, the desired mathematics lesson is according to Isaac a lesson in which he is making the frames for the activity and the students are formulating the subject content. One example is the introduction of the concept of vectors he had in the Mathematics R1 course. Here, he let the students observe colliding billiard balls and document their motions with photography. The mathematics task was to find an appropriate way to describe the motion of the colliding balls, in terms of their direction and velocity, in which the students themselves developed an initial theory of vectors.

I3_144	Isaac:	*(...) they discovered so much on their own. I managed to make the frames so that they could formulate the content, and that's not something I feel I manage very often, (...) often, to some degree I have to formulate the content, to make them reach what they are supposed to reach, but that time, I felt like I to a greater extent was only giving the frames and then they could discover the rest of it on their own, (...) then I experience success as a mentor.*

Isaac describes the lesson on vectors as successful in terms of acting as a mentor for the students' learning in mathematics. However, within other mathematics topics, he is struggling with finding the resources for developing new activities, both in terms of having enough time to implement it and to get the good ideas. His ambitions for developing his mathematics teaching through inquiry based practices are therefore not achieved.

I3_453	Isaac:	*I run dry of ideas when I'm supposed to find out how they can make the sign charts inquiry based. I've tried, but is has been a lot harder than I expected, to find these activities, so I haven't managed it. Some times I've managed it, as in the activity with the billiard balls, but not close to the amount that I wished for. I've done enough to meet the requirements of the in-service course, but I'm not able to do much more than that. I wanted a lot more. I have ambitions about doing more.*

Instead of being a trigger for changing his mathematics teaching, the in-service course on inquiry based teaching has went on in parallel with Isaac's mathematics teaching. Although he has met the course requirements, his expectations of changing the mathematics teaching has turned into a matter of priority regarding time available and the students' need for task-based instruction and volume training.

I3_103	Isaac:	*In mathematics, I find it demanding to do other things than solving tasks, and at the same time I think it's important to spend much time on solving tasks. So I've had experimental or inquiry based activities in math, but they're time consuming and I feel that the students suffer from volume training, so I feel I don't have time for being frisky.*

Regarding his future mathematics teaching, Isaac does, however, hold on to his ambition of changing his teaching practice. Despite having a "way to go to get the right tools for it", he imagines becoming a teacher who provides the students with a need to approach mathematics differently than seeking the one, correct answer on a task.

I3_510	Isaac:	*I want to become the teacher who basically does two things. First of all, makes the students understand* why*, not knowing, but understanding, that's my first ambition, and the second ambition is to be the one who makes you eager for knowledge, the one who makes you wanting to know, wanting to understand (...) and to reach that, I think I have to change the teaching in a way that students don't get away with rattling off or, they have to see the value in and the need of understanding. (...) I think I have a way to go to get the right tools for it.*

In the follow-up interview of the classroom observation, Isaac describes the lesson on linear modelling as typical for his mathematics teaching at the general education programme. He usually introduces new mathematics topics with a whole-class sequence and a discussion about the topic, in which the students are supposed to come up with the answers. Further, the lesson usually continues with the students solving tasks, individually or in pairs.

I4_22	Isaac:	*You see a lot, at least an attempt of dialogue, I try always to initiate a topic with a dialogue and discussions or group discussions about the topic, and then I try to make the students come up with the answer. (...) a few times there's a practical element in the middle, and then solving tasks at the end of the lesson. That's typically how I do it.*

However, one of the obstacles when introducing the concept of rate of change is the students' lack of understanding of the concept of functions. To some of the students, the mathematics function is a meaningless concept, being only a sketched line that is disconnected from other mathematics problems such as solving a linear set of equations based on the graphical representation.

I4_128	Isaac:	*They don't have a clue about functions, on any level. They don't understand what a function is. (...) It's something you can draw a line out of, and that's it. And it doesn't mean much. As an example, the thing about, where two graphs are crossing, that's where the equations are equal, that's all Greek to them.*

According to Isaac, mathematics in general is to these students a set of algorithms to be memorised. He perceives it as a result of not ever having had the need to learn mathematics differently during their time in school. Consequently, the students always seek to find the one solution method or path that gives the correct answer; a path which they never can move out of.

I4_228	Isaac:	*Mathematics is a set of algorithms to be memorised. Regardless of what I say, that's what math is, (...) and they have no faith in understanding being the key to anything, since, first, understanding and math, they are not compatible, they are two different things, and second, understanding doesn't solve anything. I don't think they've had an experience in which understanding has solved something that memorising can't solve.*
I4_257	Isaac:	*All the time, they have to find the route which one always have to walk, you must always walk along that path, and you can never move out of it.*

In the final interview, Isaac voices a similar limited ability for establishing a different mathematics practice. Although he endeavours to invite the students into mathematics activities with no one, correct answer to the problems being posed, some of the students seem to hold on to their previous developed learning strategies. These students find comfort in solving textbook tasks with clear instructions, and they simultaneously avoid solving mixed problems which requires a more flexible approach to the mathematics content. Into his second semester as a certified mathematics teacher, Isaac thus expresses lack of progress regarding his ambitions of teaching mathematics for understanding.

I5_477	Isaac:	*What I've tried to do is not giving them access to the procedures so easily, meaning, the algorithms. I have tried to limit it, quite simply, the possibilities to memorise, not giving them any choice. And then tried to give them some open tasks, meaning, not a, b, c, d, which leads to something, but instead give them only d. But they don't succeed. I don't succeed, even if I try. If I only give them the task d, then I get kind of, resigned, almost tearful looks (...) And then I have to hold their hands the whole way, and I can never take off the support wheels.*

I5_530 Isaac: *They manage to do the tasks under each section [of the textbook], since the section tells you which method to use. So they have this false security. (...) and then I have mixed problems at the end of each chapter which they never touch, since they don't know* which method to use.

5.2.7 Third critical event: Undesirable stagnation of own mathematics teaching practice

In the first interview, Isaac accounted for a disagreement between him and the students at his first school of employment, regarding knowing *what* to do and *why* when solving mathematics problems. The difference between his own and the others' perspectives on mathematics teaching and learning was initially related to Isaac's belonging to practices of relational mathematics instruction, and later, his participation in the *community of inquiry based mathematics teaching*. At his second school of employment, he struggles with introducing inquiry based mathematics practices to his students in the mathematics classroom. Here, the students' former memberships in communities of mathematics teaching and learning in school are as well part of the situation. Isaac's attempt of establishing a meaning of mathematics teaching and learning in the *community of secondary mathematics teaching* in the classroom is then an issue of negotiability. Following Wenger's (1998) notion of economies of meaning, the meanings that the students produce for mathematics understanding are not only local and delimited by the classroom. Instead, they are part of a broader economy in which different meanings compete for the definition of what mathematics teaching is or should be. Consequently, some meanings do achieve special status. In the general education classroom, the meanings of mathematics teaching and learning stemming from the students' previous mathematics practices seem to rule. This includes the students' imagination regarding mathematics learning, in which doing mathematics is perceived as undertaking a set of mathematics tasks leading to the required mathematics learning for passing a test or an exam. Hence, Isaac has limited ability to define what is appropriate regarding understanding in mathematics. Instead of establishing a new mathematics practice, the outcome is an ongoing conflict between him and the students regarding how mathematics should be taught. Isaac describes it as gaining "resigned, almost tearful looks" (I5_477) from the students when trying to limit their possibilities to memorise the subject content.

A further elaboration on negotiability within the classroom community can be done based on the presented teaching sequence and dialogues in Section 5.2.5. While working on the two linear functions in Figure 5.9, Student 2 is questioning a method for finding the rate of change for a linear function. Her question of “can we always do it like this” in Dialogue 1 can on the one hand be regarded as an opportunity to explore whether the method of finding the rate of change is valid beyond the given example of a straight line. On the other hand, it can be viewed as a request for a memorisable algorithm for calculating the rate of change for a straight line. When the third student mentions the formula $\frac{\Delta y}{\Delta x}$, Isaac does not accept it as a satisfactory answer and responds “we will think about it in a different way”. Here, Student 3 engages in the activity by calling attention to rate of change as being the number you get when you divide the delta *y* with the delta *x* for a straight line. This focus on the final *product* is somehow supported by the closed characteristic of the task, in which respectively 2 and 7,5 are the correct answers. Thus, Student 3 might be able to calculate the rate of change for a linear function, and as such be able to find the expressions for the linear functions in Figure 5.9, however, without considering rate of change more generally as the measure for the intensity of changes in varying quantities.

Isaac, on his side, engages in the activity by considering the *process* of finding the rate of change of the linear functions, for then to reach the conclusion that the rate of change is constant only for linear functions. This comes to the fore as he rephrases the question “can we always do it like this” in Dialogue 1 and further invites the students to investigate a variable rate of change for a non-linear graph on population growth. Compared to the previous tasks on the two linear functions, the linear modelling of population growth has a more open character, since there is no one, correct numerical answer to the problem. Hence, being able to correctly calculate a linear function expression is not sufficient for responding to the question of how to find a proper linear model for the graph. Yet, a conflict regarding how to approach the concept of rate of change appears in the Dialogue 2. Here, the student’s engagement concerns receiving confirmation on whether her linear model is correct, which Isaac will not give her.

The conflict regarding the meaning of understanding in mathematics can be understood on the basis of Isaac’s attempt to reconcile memberships in various communities of practice. This includes his continued membership in the *community of inquiry based mathematics teaching*, his and his students’ establishment of a *community of secondary mathematics teaching* in the classroom, and his students’ former memberships in communities of mathematics teaching and learning in previous schooling. Isaac voices in the final interviews and in the teaching sequence a limited ability to establish a shared meaning of knowing and doing mathematics in the classroom, in which the mathematics processes constitute the core practice. Following Palmér (2012), there is a tendency of prioritising the demands and the goals of the communities of practice in which one is a member by engagement. Unlike imagination and alignment, participation through engagement implies interaction among the community members, and hence, it makes up the ground for a negotiation of the joint enterprise. In Isaac's case, this concerns membership in the *community of secondary mathematics teaching* in the classroom, in addition to a potential membership by engagement in a community of colleagues[12]. Simultaneously, he continues to exercise participation by imagination and alignment in the *community of inquiry based mathematics teaching* from teacher education. In the classroom community, a task-based mathematics teaching and volume training turns out as the solution when facing the students' preferences for learning mathematics. Hence, the demands of the classroom community might pull Isaac away from his initial ambitions for exercising inquiry based mathematics teaching. Since active inquiry and professional development in mathematics teaching does not seem to be part of the joint enterprise of a community of colleagues, Isaac is left alone with finding the need and the resources for changing his mathematics teaching according to his ambitions. The third critical event in his evolving story of *perspectives on mathematics and its role in mathematics teaching* can therefore be characterised as *undesirable stagnation of own mathematics teaching practice*. In Figure 5.11, I present the critical event as Isaac's continued participation in the *community of inquiry based mathematics teaching*, concurrent with his limited negotiability within the *community of secondary mathematics teaching*.

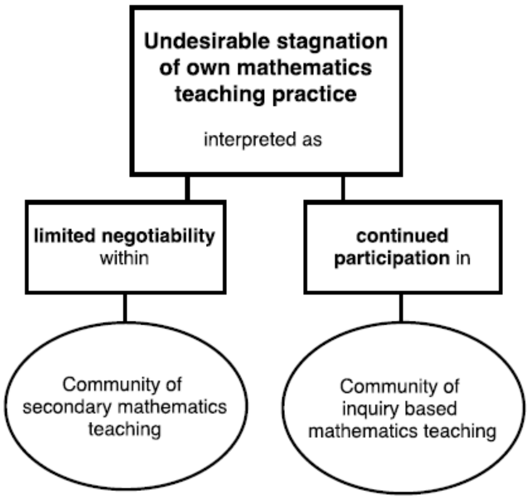

Figure 5.11: Perspectives on mathematics and its role in mathematics teaching, represented by the third critical event of an undesirable stagnation of own mathematics teaching practice

[12] Isaac's accounts concerning participation in communities of teacher colleagues will be further elaborated on in Section 5.3.

Isaac's sense of an undesirable stagnation in own mathematics teaching can further be considered as two-sided. The notion of *stagnation* relates, as explained above, to Isaac's limited negotiability in the *community of secondary mathematics teaching*. Further, the stagnation is considered as *undesirable* on the basis of Isaac's continued participation in the *community of inquiry based mathematics teaching*. Throughout his first year as a certified mathematics teacher, Isaac thus maintains his ambitions of becoming able to perform mathematics teaching in line with inquiry based practices.

5.3 Feedback available within the school environment

Throughout the five interviews, Isaac accounts for the pleasure of collaborating with his students in the mathematics classroom, due to their active engagement in the lessons and their direct response to his mathematics teaching practice. Based on narrative analysis of the first interview, his accounts of negotiating the mathematics teaching in the classroom community were therefore interpreted as part of the evolving story of *feedback available in the mathematics classroom*. Here, the students' feedback appears as a necessary condition for Isaac to be able to identify himself as a mathematics teacher. Accordingly, the first critical event is related to Isaac's emphasis on a student centred mathematics classroom. The story is extended by accounts from the third and fifth interview, concerning the lack of feedback in the vocational mathematics classroom and, as a consequence, Isaac's enforced change of his mathematics teaching practice. The second critical event is therefore based on accounts of a demanding mathematics teaching situation, due to the vocational students' resignation from his mathematics teaching.

When entering his second school of employment, feedback from teacher colleagues on his mathematics teaching practice constitutes an additional topic. However, the focus is now on the lacking opportunities for collaboration with his colleagues, and hence, absent feedback within the teaching staff. The outcome is a sense of invisibility. This stands in contrast to the *community of teacher colleagues* at his first school of employment, where identification with competent mathematics teachers made him change his mind regarding a future teacher career and apply for university teacher education. The strand of accounts concerning teacher collaboration was therefore interpreted as an evolving story of *feedback available within the teaching staff*. The story is based on the critical event of Isaac's unfulfilled expectations of a collaborative and progressive teacher community. Since both evolving stories of feedback concern negotiation of mathematics practices among actors of mathematics teaching and learning in school, I have chosen to gather them under the common evolving story of *feedback available within the school environment*. The two stories thus appear as parallel strands of critical events in Figure 5.1.

An overview of labels belonging to the emergent theme *feedback available within the school environment* is shown in Figure 5.12. The labels constitute three critical events along the evolving story, which are linked in Figure 5.12 to the interview transcripts to be presented in the upcoming analysis. The first two critical events relate to the strand *feedback available in the mathematics classroom*, and the third critical event relates to the strand *feedback available within the teaching staff*.

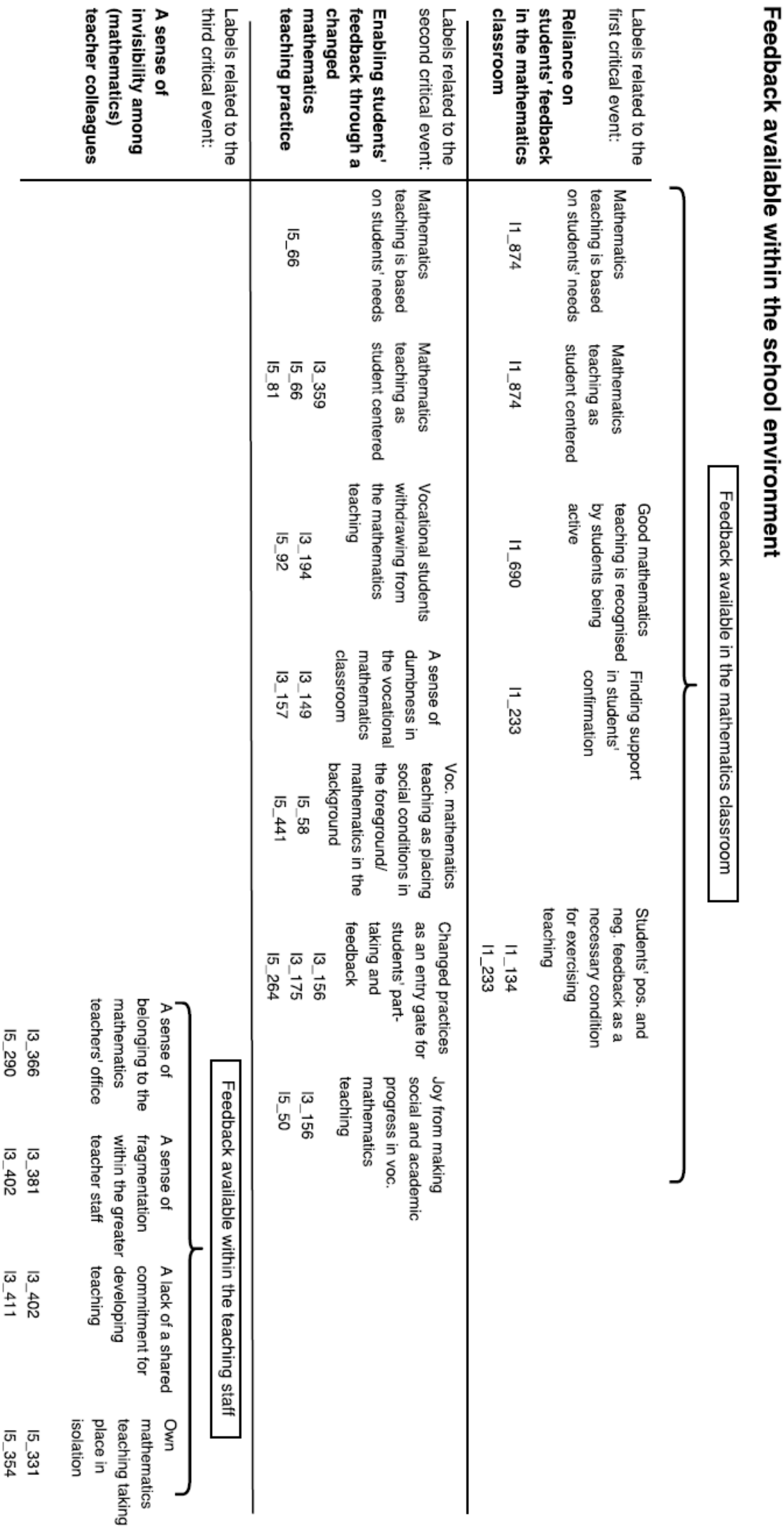

Figure 5.12: Overview of labels belonging to the emergent theme *feedback available within the school environment*, constituting three critical events

5.3.1 Accounts of feedback available within the mathematics classroom, constituting a first critical event

Isaac elaborates on the pleasure of collaborating with his students by comparing teaching experience from the secondary school with experience from being a part-time teacher in chemistry at a university college. Although the syllabus at the university college was on a higher academic level and therefore could imply a more interesting content to teach, Isaac missed the trustful environment he experienced with his students at school.

I1_134	Isaac:	*It was academically more challenging, but, to me, the social collaboration with the students is important, and there the [university college] students sat gruff with their books and wrote down everything I said, and when the lecture was over, they closed their books and walked out. I lacked the social contact, this direct thing you get with the students [in school]. (...) They come to me and say, this math lesson was very boring, could you do something else, or, now, this worked very well, and it was great you did that, you have to do it again. [I] don't know why the students feel they can be that open towards me, but I feel kind of, when I have that kind of dialogue with them, the environment is totally different. I don't get the [university college] students to say the same things.*

Isaac describes the university college students as sedentary listeners, who with a serious mine write their notes and leave the lecture room in silence. Hence, there is an undesirable distance between him and the students, which stands in contrast to the open and spontaneous feedback given by students at the upper secondary school. One example is the experience of being welcomed and appreciated by students who are considered by other teachers as challenging and hopeless in mathematics. Such episodes of positive response on his teaching practice constitute the peaks in Isaac's preliminary teacher career.

I1_233	Isaac:	*I have many peaks as a teacher. (...) when I walk into a classroom, (...) a class that I was warned against that this is a horrible class, by a teacher that usually has them, and the teacher says there is no hope, meaning, we must concentrate on making them pass in math. Then I walk into the class and I have a lesson and it's demanding. When I come back, (...) the day after, I hear the students say, yes! He is coming [back]! It's him coming [back]! (...) The class that was supposed to be hopeless has suddenly experienced mastering, for something I did, and that was a peak.*

Another kind of positive feedback is the students' engagement during the mathematics lessons. The desired teaching situation for Isaac is characterised by students who have no time to wait for being called on in a whole class teaching situation, bursting out their solutions on the mathematics tasks. A successful mathematics lesson is therefore recognised by active and enthusiastic students.

I1_690 Isaac: *(...) when I feel they voluntarily put up their hands or they don't even put up their hands, they have to burst out the answer (...), then I feel I've done something right. (...) I think that kind of engagement is worth its weight in gold. It leads to learning, it leads to joy, it leads to a merry mood. (...) And it's so fun when I see that I get to engage them (...) If I leave a lesson with the thought that my students have been engaged, then I feel good.*

Although Isaac accounts for a sincere joy of collaborating with his students in the mathematics classroom, their needs when learning mathematics have not always been the driving force in order to become a teacher.

I1_874 Isaac: *In the beginning I was just thinking about the presentation of the subject (...), that was my focus, it's fun to talk about subjects. (...) But now I've slowly realised that this is a less important aspect, this subject presentation. It's, the students are in focus, and the students who are in the centre (...) and that has been a great development, because I like to work with people. So I think it has been fun to discover that being a teacher is more about interpersonal things than about academic things (...) just saying that has been totally strange to me, it wasn't about students at that moment. (...) it was about what was being presented and not about the receiver. And today I find that back-to-front. Everything is about the receiver.*

By describing it as "a great development", Isaac tells about a progression related to his growing concerns regarding students' learning and their needs. It is made explicit by his accounts of differences between then and now, between his initial interest in the very presentation of the subject content and his recent realisation of having a main interest in the students' learning of the content. Having entered university teacher education, Isaac is appreciating the learner's leading part in teaching and, thus, the human dimension of his future profession. Due to the extensive accounts of a student centred mathematics teaching, Isaac's report on students' feedback in the mathematics classroom constitutes a critical event in his evolving story of *feedback available within the school environment*.

5.3.2 First critical event: Reliance on students' feedback in the mathematics classroom

Unlike exercising a unidirectional teaching at the university college, being responsive towards the students' feedback is according to Isaac at the core of classroom mathematics teaching. With theoretical terms, Isaac thus identifies himself as a mathematics teacher through engagement in the *community of secondary mathematics teaching,* in which the students' feedback constitutes a negotiated enterprise. When finding a way to carry out mathematics teaching in the classroom, the disagreements and discussions between Isaac and his students are a productive part of the community's enterprise. The very lack of both positive and negative feedback at the university college, and hence, the lack of a negotiated enterprise, makes it difficult to establish a community of practice in which both Isaac and the students can participate. Consequently, the chemistry lectures at the university college do not function as a resource for Isaac to identify himself as a teacher.

In addition to be part of the negotiated enterprise in the classroom community, the students' feedback also functions as confirmation on Isaac's mathematics teaching practice. Feedback is thus a resource for Isaac to exercise identification through imagination, by creating a picture of himself as a kind of mathematics teacher in the world of mathematics teaching. One example is the positive responses which Isaac characterises as peaks in his teaching career. He presents the peak episode as a romance, in which the heroic main character experiences challenges or threats and through a series of struggles emerges victorious (Murray, 2003). Acting the hero, Isaac undergoes the challenge of being a substitute teacher to a group of students with the only hope of passing the mathematics course. However, through the struggle of having a demanding lesson with them, he overcomes the adversity in terms of the "horrible class" and is welcomed back by the students the day after. By virtue of a romance, Isaac thus pictures himself as a mathematics teacher who, despite the challenge of hopeless students and the colleague's initial warning, succeeds in providing the students experience of mastering in mathematics. Consequently, he imagines being a mathematics teacher who is able to cope with even the most challenging students, and with situations in which other mathematics teachers would have given up.

Isaac's identification as a mathematics teacher further manifests itself through a learning trajectory or movement from concerns about presentation of the subject to concerns about the students' learning and their needs. According to Brown and Borko (1992), there are changes in mathematics teachers' concerns regarding themselves as teachers, the tasks of teaching mathematics and their students' needs, as they progress through mathematics teacher education and into initial years of teaching.

From a developmental perspective, concerns related to self are considered less mature and less desirable than the concerns related to the students' learning. Isaac's accounts indicate a development towards a higher level of concerns, which in turn brings up the question of how this change came about. One reason could be his expressed confidence in learning and teaching mathematics, as accounted for in previous sections. Giving the impression that he is competent in doing mathematics and mathematics teaching, Isaac might be capable of placing concerns about other people's learning of the subject in the foreground and concerns about his own role and competence in the background.

The growing concerns regarding students' learning in mathematics can further make a two-way relationship with potential feedback in the mathematics classroom community. On the one hand, the focus on the students' needs when learning mathematics enables Isaac to seek for and respond to the students' positive and negative feedback, which in turn creates opportunities for identifying himself as a kind of mathematics teacher. On the other hand, the very act of responding to the students' feedback, as part of the classroom community's negotiated enterprise, might enable Isaac to develop further his concerns regarding the teaching and learning of mathematics. The importance of feedback is also highlighted by Palmér (2013), as she accounts for the need of feedback from oneself and others in order to recognise oneself as a mathematics teacher. However, where Palmér states that memberships in communities of practice influence the teachers' possibilities of receiving feedback from others, the case of Isaac shows that responding to feedback is as well a necessary condition for establishing and developing memberships in classroom communities. Hence, students' feedback functions both as a premise for and a consequence of participation in a *community of secondary mathematics teaching*.

When facing towards a career as a certified mathematics teacher, Isaac delineates a profession in which the students and their learning are placed in the foreground. Hence, the kernel of the classroom enterprise is his and the students' negotiation of what mathematics teaching is and should be. Isaac's initial descriptions of a student centred mathematics teaching can then be related to identification and negotiability within the *community of secondary mathematics teaching*. Here, the students' feedback constitutes the negotiated enterprise, and therefore appears as a necessary condition for identifying oneself as a mathematics teacher. In other words, Isaac accounts for a *reliance on students' feedback in the mathematics classroom*, as shown in Figure 5.13.

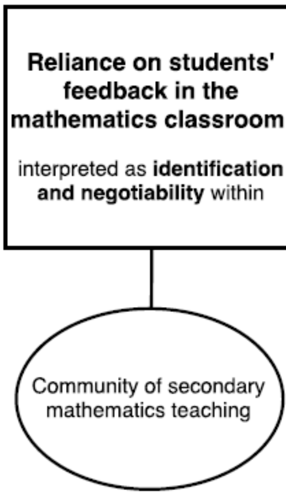

Figure 5.13: Feedback available within the school environment, represented by the first critical event of expressing a reliance on students' feedback in the mathematics classroom

5.3.3 Accounts of feedback available in the mathematics classroom, constituting a second critical event

Based on Isaac's appreciation of the students' active engagement in the mathematics classroom, the first interview showed how the students' feedback plays an important part in order for Isaac to identify himself as a mathematics teacher. His accounts of teaching mathematics at the general education programme at his second school of employment indicate a similar learning environment, based on students' response to his teaching practice. Isaac describes the classroom atmosphere as constructive and trustful, in which the students pay attention to his teaching by being critical and asking questions. Further, he highlights the meeting with the students as the main reason for enjoying mathematics teaching.

I3_348	Isaac:	*They give me the role I'm supposed to have as a teacher. That's been great. They listen to me, and at the same time they're putting question marks to what I'm doing, meaning, in the constructive way. (...) they don't take what I say on trust, and I find it extremely positive.*
I3_359	Isaac:	*(...) meeting the students is in general very fun, and that is why I like my job. Meaning, the academic part is clearly an interesting aspect of my job as well, but it's when I get to guide my students I'm having fun. It can be interesting sometimes making exciting and good teaching plans and things like that, but that's never as fun as when I'm in the classroom, carrying out the teaching plan at one with students.*

In the final interview, Isaac gives similar descriptions of a student centred classroom as the preferred one. A successful lesson is recognised by students who accept to take part in the mathematics learning activities that Isaac stages.

I5_66	Isaac:	*I try to have a very student centred classroom. It's a principle I believe in, the students are the focus, the students are the reason we are here, the students are the ones becoming better human beings. I'm supposed to have it nice too, but I'm in second line.*
I5_81	Kirsti:	*So, if I understand you correctly, it's fun to teach when you manage to have a student centred classroom?*
	Isaac:	*Yes, and if the students accept the invitation. That they take part, instead of resigning from what I try to gain, because, student centred is not the same as the students are dictating me. And they can resign from what I try to invite them into, and then it becomes a failed lesson.*

However, the mathematics teaching in the course Mathematics 1P-Y at the vocational education programme Technical and Industrial Production (TIP) represents a different situation. By reason of a colleague's sick leave, Isaac has to step into the class six weeks into the first semester, and he becomes their mathematics teacher the rest of the school year. Isaac describes the situation as demanding, in terms of the students' resignation from the mathematics activities and their denial of him as their mathematics teacher. Due to the students' lack of involvement and their unbalanced negative feedback on his mathematics teaching, the situation of entering the vocational mathematics classroom constitutes a critical event in Isaac's identity development.

I3_149	Isaac:	*I've struggled a lot with getting through to the students at the vocational studies, struggled with making them do anything, and actually acknowledge me as a teacher, on many levels; academically, interpersonally, personally, meaning, there are many levels of challenges here.*
I3_157	Isaac:	*Very often, I get a lot of resistance towards me as a person in that class. (...) It's about they suspecting me not to know mathematics, or that I don't get how to talk to other people, or that I think I'm better than them, so I just talk a completely different language which they don't understand, (...) and then a lot of flattery which are downright lies, such things that are kind of hurting because it so obviously wrong, kind of, you're the best teacher we've ever had, said in a way it's downright lies, and then it becomes very unpleasant.*

Based on his accounts of "struggl[ing] a lot with getting through to the students" and "struggl[ing] with making them do anything, and actually acknowledge me as a teacher", the vocational mathematics students seem to distance themselves from the mathematics subject, as well as from Isaac being their mathematics teacher. This unfolds in the classroom as victimisation, in which Isaac is treated as incompetent and unsuitable for being a mathematics teacher in school. Their withdrawing from the mathematics teaching causes also problems for their professional practice outside the mathematics classroom. At the workroom, the students avoid mathematical problems, and they struggle therefore with managing the tools properly.

I3_194	Isaac:	*The sad thing is that they're using mathematics as a tool at the workroom, a lot. But the teachers have to do it for them, since they are not able to solve the tasks they actually have to solve at the workroom. (...) the moment they realise they're doing mathematics, they give up trying. (...) so it's actually a problem to their professional practice, that they resist to work with math.*

In the final interview, however, Isaac explains the students' passivity in mathematics as a resignation from what they perceive as irrelevant and tedious. Mathematics as it is taught in the mathematics lessons reside outside their professional practice and have therefore little impact on their future life.

I5_92	Isaac:	*(...) it seems like very often they resign because mathematics basically isn't perceived as relevant and interesting. They don't want to take part in the community in that way. It matters very little to them, on all levels. They can't picture a situation in which they will regret that they didn't pay attention in math.*

In order to adapt his mathematics teaching practice to the students' behaviour, Isaac puts away the textbook and replaces the whole-class instructions with step-by-step tasks on worksheets. The tasks, which can be solved directly on the paper, are meant to gradually lead the students through the mathematics curriculum. In addition, Isaac brings pencils and other necessary equipment to every lesson. As a result of the changed teaching style, the students take to a greater extent part in the mathematics lessons, with the consequence of Isaac experiencing mastering.

I3_175	Isaac:	*(...) a lot less instruction, meaning almost no instruction at all, since it doesn't work, so I've put it into the tasks. And then I don't give textbook tasks, because they don't have the book or they don't understand the way the book asks questions (...), they don't understand how to look up the right page in the text-book, they have in general huge problems with using the text-book. So I've made special plans, in which I make special tasks on worksheets where they can write the answer directly on the paper, [the tasks] proceed gradually and works like a review of the content. [I] try to make tasks being so pedagogic that, in a way, they gradually go through the content, if they solve the task.*
I3_156	Isaac:	*Then suddenly, I got everyone to work, (...) and I didn't get so much personal resistance. (...) today they let me help them a bit, and we got a good dialogue, the students talked to me like I'm a human being, I don't know what I was before, but now I at least was a human being, it seemed like in their eyes, and then I experience an incredible mastery, now I reach them, now I get a kind of accept, having the possibility of actually doing my job, unlike before. It was a tremendous mastery!*

Isaac's experience of mastery in the vocational mathematics classroom do not only concern the progress of the students' learning in mathematics. As much part of the peaks and highlights are the experiences of social progress; whether the students respond to Isaac's initiatives and choose to participate in the classroom discussions. The best fundament for successful mathematics teaching is therefore clearly defined frames and a high degree of predictability in the lessons.

I5_50	Isaac:	*(...) then I walk on clouds after the lesson. Then I think, oh, they've learned something, I've had a great time, they've had a great time, this was just all winning. If I'm to conclude, I have the best time when I feel we have both social and academic progress in the lessons.*
I5_264	Isaac:	*What has made it work in vocational mathematics are clear expectations and frames, and what is going to happen is clear to everyone, hundred percent predictability, that works, new elements make chaos.*

In the final interview, Isaac characterises himself with terms of not acting like a *mathematics* teacher for the vocational students. He describes his role as mainly being a pedagogue, trying to make the students take interest in their own future. The students' mathematics learning serves mainly as a bonus.

I5_441	Isaac:	*The problems I've met and the role I've got there has exclusively been as a pedagogue and not as a didactician, I feel. (...) It's been about making them take interest in their own future (...) so I feel what concerns me in those lessons, is not about mathematics at all. I try to make them think school, and then I hope there's some mathematics while we're at it.*

Having taught mathematics at the vocational study programme for almost a year, Isaac aims at creating a safe learning environment in which he can have an impact on the students' general education. Besides educating them in mathematics, being a vocational mathematics teacher is mainly about challenging the students' mind-set and encouraging them to perceive reality from additional viewpoints.

I5_58	Isaac:	*I like to think that they are supposed to develop as holistic human beings, and if they're really into something, I hope I can make them think about it in a different way than when they discuss it with friends. I find it nice to be allowed to contribute to that part, at the same time as I contribute by making them better in mathematics.*

5.3.4 Second critical event: Enabling students' feedback through a changed mathematics teaching practice

Isaac's accounts of undergoing vocational mathematics teaching contain descriptions of students who are withdrawing from learning activities as well as questioning his role as mathematics teacher. Although holding a position of authority by virtue of being the teacher, Isaac's actual ability to define the meaning of mathematics teaching and learning in the vocational mathematics classroom is limited. By "not pictur[ing] a situation in which they will regret that they didn't pay attention in math" (I5_92), the students exercise imagination by assuming a meaning regarding learning mathematics. However, the meaning lies beyond their engagement in the mathematics classroom. In other words, the meanings of mathematics practices belong to someone else, being for instance Isaac as an academic, the students at the general education programme, or some other elite of mathematicians. The result is an absence of negotiation of the mathematics practices, and consequently, non-participation in a potential *community of secondary mathematics teaching*.

In order to enable the students' engagement in the mathematics lessons, Isaac makes changes to his classroom practice. The new kind of mathematics teaching with a step-by-step worksheets can be characterised as instrumental, as it "emphasises basic numeracy as knowledge of facts, rules and skills, without regard for meaningful connections within this knowledge" (Ernest, 1989, p. 21). Nevertheless, the individually undertaken tasks do constitute an enterprise for the students to more easily engage in. This can be related to the advantages of instrumental mathematics, discussed by Skemp (1976). According to Skemp, instrumental mathematics provides the student with rewards being more immediate and apparent than relational mathematics. If what is wanted is a page of right answers, instrumental mathematics can provide this more quickly and easily. Although providing a limited insight into the greater discipline of mathematics, the worksheets' low threshold does function as an invitation into a mathematics enterprise which the students can accept. Hence, the vocational mathematics classroom can hold a mathematics enterprise to be negotiated by Isaac and the students.

Isaac's establishment of a fragile negotiated enterprise is comparable with the *community of secondary mathematics teaching* being accounted for in Section 5.3.2. The previous analysis indicated that Isaac identifies himself as a mathematics teacher through engagement in the classroom community, in which the students' both positive and negative feedback constitutes the negotiated enterprise. In line with the results of interview 1, the presented accounts from interview 3 are therefore related to the evolving story of *feedback available in the mathematics classroom*. Here, the mathematics enterprise of individually solving tasks on worksheets makes up a basis for receiving feedback on the mathematics practices in the community, as well as feedback related to classroom management and the social environment in the class. Hence, the worksheets become an entry gate for Isaac to be able to identify himself as a mathematics teacher in vocational mathematics. The critical event of entering the vocational mathematics classroom can therefore be characterised by Isaac's *enabling students' feedback through a changed mathematics teaching practice*. This is pictured in Figure 5.14 as changes in negotiability within a potential *community of secondary mathematics teaching* in vocational education.

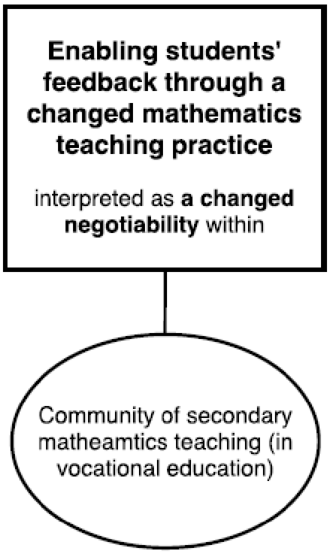

Figure 5.14: Feedback available within the school environment, represented by the second critical event of enabling students' feedback through a changed mathematics teaching practice

The negotiability in the vocational mathematics classroom is further related to Isaac's ability to shape the meanings of how he and the students are supposed to interact in the classroom. Hence, the establishment of a mathematics enterprise in the classroom community is constantly in interplay with the developing social conditions. The problems at hand at the vocational education programme, concerning the students' motivation for undertaking upper secondary teaching in general, are pushing the mathematics subject at the side-lines of Isaac's teaching. Hence, feedback in the vocational mathematics classroom does not primarily concern feedback on Isaac's *mathematics* teaching. Instead, he manages to build meaning for exercising his mathematics teacher profession on the basis of the social interaction with the vocational students. His developing identity as a mathematics teacher is thus framed by participation in the negotiated enterprise of creating a safe and predictable learning environment and by taking part in the vocational students' general education.

5.3.5 Accounts of feedback available within the teaching staff, constituting a third critical event

In the analysis of Isaac's accounts of teacher collaboration at his first school of employment, his expressed familiarity and fellowship with the colleagues was interpreted as part of his story of confidence in mathematics and mathematics teaching. This was based on Isaac's experience of resonance in the teacher community regarding his desire of being academically challenged. His accounts of teacher collaboration at his second school of employment have a different focus. Instead of picturing the teacher community as a resource for him when exercising the teacher profession, Isaac focuses on the lack of opportunities for collaboration, due to the organisation of the teaching staff and the design of the school building. As a consequence, his expectations about taking an active part in a progressive teacher community are not met. Isaac's accounts of insufficient professional collaboration constitute therefore a critical event within the story of *feedback available within the teaching staff*.

At his new workplace, Isaac is placed in an office together with seven science and mathematics teachers who teach some of the same mathematics courses. The rest of the teaching staff is organised likewise, however, they are placed all over the big school building. Due to the architectural design of the school building and the great number of teachers, they do not have a common room or a meeting place for eating lunch. Instead, the teachers are supposed to eat lunch together with their students in the bigger cantina, and to work at offices nearby their students' classrooms. According to Isaac, the school organisation leads to both affordances and constraints for his teaching practice. The affordances are better conditions for collaboration among teachers within each subject, such as the subject Mathematics 1T in the general education programme. The group of eight teachers at Isaac's office hold an open environment for sharing activities, tips, and experiences from teaching, which he benefits from.

I3_366	Isaac:	*The collaboration within each subject is better, since we are many who teach them (…) instead of being one or two teachers having 1T, we are six. (…) Then we get a group of teachers working, who can brainstorm together about good stuff, and that is very fun, so I like that a lot.*
I5_290	Isaac:	*I always ask if they've thought about something special or if they have some thoughts and experiences they would like to share. So, I'm the freeloader kind of in the group, but it seems like they're OK about sharing, and then sharing experiences and tips. (…) Meaning, I believe I can contribute with something, it's not that, but for the time being I'm more humble and want tips. I have clear thoughts of what I want, yes, but I ask, so it's a lot of collaboration going on, and that's very nice.*

Although Isaac accounts for a sense of community among him and the teacher colleagues at the office, he simultaneously expresses the need of belonging to a greater community of teachers at the school. However, as the teachers are located in smaller offices being spread all over the school building, there are fewer opportunities for spontaneous discussions in the hallway and around the lunch table. The school organisation is thus constraining collaboration beyond the single mathematics and science courses. Isaac misses the encounters with colleagues from other sections of the school, both mathematics and science teachers at other levels and educational programmes, and colleagues from the language and social studies.

I3_381	Isaac:	*(…) the circumstances at this school, they are not always right for, we may call it, the organic meetings, because we have kind of separate offices around the school, so we don't incidentally meet each other in the corridor, and there's no room for all the teachers at the lunch room, so there's no tradition for going there having lunch.*

In line with his former workplace, Isaac perceives the teachers at his new school of employment to be competent and dedicated in their work. There are teachers from the staff working towards the university and the Resource Centre for Science Education[13], as well as the Norwegian Centre for Science Education[14]. However, he rarely has the chance to meet them outside organised meeting activities, which oftentimes include all 30 science and mathematics teachers at the school. Isaac has therefore the impression that developmental projects in mathematics and science are done individually.

I3_402	Isaac:	*There are competent, dedicated people here, but they work separately. (…) they have their own projects, some working towards the university's Resource Centre for Science Education, some working towards the Norwegian Centre for Science Education, meaning, they do a great amount of good things towards the natural sciences, but we are not doing it together.*
I3_411	Isaac:	*We don't have the same commitment about things. At my former workplace, someone could bring a research paper or something like that, an article from a pedagogical magazine or something like that, and then, look what they have come up with, what do we think about this, and then we could have some fun about that. That hasn't happened here.*

Isaac gives a similar description of the school environment in the final interview, regarding the missing teacher common room and the lack of natural meetings points for initiating discussions. He refers to the ab-

[13] "Skolelaboratoriet", a regional centre offering courses for teachers in science and mathematics, both in-service and supplementary training. In addition, they develop teaching material, and they participate in research and development projects on science teaching in schools.
[14] «Nasjonalt senter for realfag i opplæringen (Naturfagsenteret)», a national resource centre for improving the quality of science education and increasing motivation and interest for the sciences at all compulsory school levels.

sence of a collective initiative for keeping themselves updated on research and ongoing debates, and consequently, a lack of initiative for making professional development.

I5_303 Isaac: *(….) there are very few natural meeting points. (…) From the previous school I'm used to, kind of, one suddenly meet each other in the hallway or at the common room, and then one starts a conversation which actually, in retrospect, has academic depth (…). This kind of natural reflection and spontaneous reflection I miss a lot more. (…) at the previous school there were people who had read academic papers or followed research, who had some commitment about the professional development, and who wanted to initiate reflections and discussions and so on. That I don't experience here.*

He further accounts for the consequences of the lacking collaboration, by describing a kind of loneliness in his profession. None of Isaac's colleagues know what is going on in the vocational mathematics classroom and can share with him the progress or his concerns regarding the students' learning and behaviour.

I5_331 Isaac: *As upper secondary teachers we are isolated in the classroom. We're alone behind closed doors, and here, you've got glass walls, but that's only for show, there's no one looking inside, so no one knows what's going on in the classroom and no one knows what starting point I had, and no one knows what I've managed to do about it (…). So I try to praise myself to others to get some recognition, but it doesn't work very well. (…) I don't have anyone to share with the feeling of having managed something (…).*

I5_354 Isaac: *It's been an important journey for me, been important for me to manage it, but it's always nice to have someone to share ups and downs with. But it's OK. I don't really feel lonely in the classroom, because I have a nice time with the students. But it's more those professional reflections which I miss, yes, the recognition, at least when I've managed it. Maybe also the professional feedback when I don't manage it so much.*

Although he is able to acknowledge to himself the achievement of having established a learning environment within the vocational classroom, Isaac still misses support and recognition from colleagues. At the end of his first year as a certified mathematics teacher, Isaac is this left alone with finding the needs and resources for developing his mathematics teaching.

5.3.6 Third critical event: A sense of invisibility among (mathematics) teacher colleagues

In the first interview, the descriptions of Isaac's sense of belonging to his teacher colleagues indicated a *community of teacher colleagues*, based on a joint enterprise of discussing teaching related topics and sharing mathematics problems during the lunch break. At Isaac's new school of employment, the group of teachers sharing office is replacing the former *community of teacher colleagues*, as they are mutually engaging in the planning, organisation and implementation of mathematics teaching in their common courses. Isaac positions himself as the freeloader of the collaboration, meaning he is primarily a receiver of the other teachers' tips and ideas. This indicates a peripheral participation based mainly on alignment, by which he has access to the shared repertoire in use. However, without taking part in the mutual engagement and a negotiation of the enterprise, Isaac runs the risk of not getting response on and finding the needed support for his working on his mathematics teaching practice. Due to the teachers' willingly exchange of materials and good ideas, the community might as well serve as a reproduction of already established practices, rather than being a community of reflective practitioners. Professional development seems therefore not to be part of the teacher collaboration at Isaac's office.

The lack of mutual engagement in a development of mathematics teaching seems also to apply to the greater teaching staff. Although some of Isaac's colleagues are involved in research and development projects at other institutions, there is no commitment to share and discuss the projects within the mathematics and science department. Hence, either the colleagues at Isaac's office, nor the other mathematics and science teachers at the greater department, make up a community in which developing mathematics teaching through inquiry is part of the negotiated joint enterprise. Isaac's former engagement in the *community of inquiry based mathematics teaching* is therefore not continued through teacher collaboration. Professional development is thus at the mercy of Isaac's participation in other communities of practice, in which the development of mathematics teaching is part of the negotiated enterprise. However, during his first year as a certified mathematics teacher, his professional practice seems to unfold mainly through participation in communities being situated in the mathematics classroom. Hence, the students serve as his main sparring-partner for negotiating meaning of mathematics teaching and learning. In the vocational mathematics classroom, this entails Isaac's struggle of establishing a learning environment, in which the subject of mathematics plays only the supporting role. In the course Mathematics 1T, it concerns an ongoing conflict regarding what is meant by understanding in mathematics, and as a consequence, experience of stagnation of practice. Although Isaac accounts for a sense of belonging to mathematics classroom communities, neither of them provides a scope for exercising professional reflection.

Based on accounts of absent forums for professional development, the collegial situation at Isaac's new workplace seems to prevent negotiation of enterprises concerning mathematics teaching rather than to enable it. Hence, the greater collegium of teachers does in a limited way constitute a resource for Isaac to identify himself as a mathematics teacher. Neither does it provide a resource for Isaac to develop the mathematics teaching in accordance with his ambitions of inquiry based mathematics teaching. The outcome is therefore *a sense of invisibility among (mathematics) teacher colleagues*. The condition of the lacking feedback within the school environment is shown in Figure 5.15, as limited identification and negotiability within the *community of teacher colleagues*.

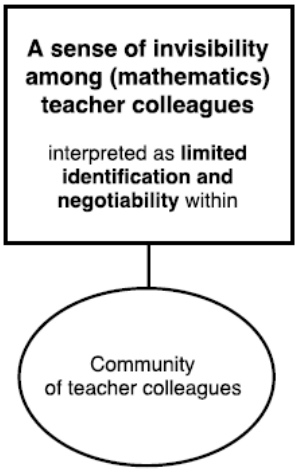

Figure 5.15: Feedback available within the school environment, represented by the third critical event of expressing a sense of invisibility among (mathematics) teacher colleagues

5.4 A summary of Isaac's stories of becoming a secondary school mathematics teacher

I5_567 Isaac: *(...) I had many thoughts about becoming a teacher, since I've met a lot of prejudice against teachers, everything from teachers telling me clever people do not become teachers, and aunts and parents being disappointed when I'm just going to be a teacher. Yes, so a lot of resistance. Well, I had a suspicion that it was going to be good, and it was. It was.*

The above excerpt is Isaac's very last utterance of the final interview. It depicts his journey into the mathematics teacher profession as a progressive narrative, despite challenges and difficulties along the way. This change of focus, from investigating the content of Isaac's accounts to look at its structure, introduces the next analytic approach of narrative analysis. In this section, I summarise the narrative progression, in terms of progressive, regressive or stable development across the critical events of Isaac's stories. The intention is to give a more comprehensive picture of his developing mathematics teacher identity. In addition to create expectations about narrative development, the series of critical events could also represent a "kaleidoscope" approach to identity, so that "each time you look you see something rather different, composed mainly of the same elements but in a new configuration" (Stanley, 1992, p. 158). In other words, although narrative progression or development may not be traced along the evolving story, the series of critical events still contribute to a rich and complex presentation of mathematics teacher identity in the transition from teacher education to employment in school.

Isaac's story of *confidence in mathematics and mathematics teaching* shows a steady situation from the first to the second critical event, by expressing an innate ability of exercising good mathematics teaching independent of the tutor's guidance. The transition from uncertified mathematics teaching to teaching during school placement constitutes thus a kaleidoscope representation of Isaac's expressed beliefs of being capable of doing mathematics and exercising mathematics teaching. Based on the theoretical framework, Isaac undergoes boundary crossing by identification, in which the discontinuity between former mathematics teaching and the periods of teaching practice at PPU is recognised, yet not overcome. It results in a reinforcement of Isaac's identity, being developed in previous settings. Progression is, however, apparent in the transition from the second to the third critical event. In the final interview, Isaac evaluates his first mathematics teaching experience as exceptional and providing him too little resistance. By seeing his previous mathematics teaching experience in new light, his transition from teacher education to school leads to learning in terms of reflection and expectations of developing own mathematics teaching through further practice. The building of a mathematics teacher identity in terms of altered confidence in mathematics and mathematics teaching is then based on changes in negotiability and, consequently, a changed participation in the *community of secondary mathematics teaching*.

In the story of *perspectives on mathematics and its role in teaching*, there is a progression from the first to the second critical event in terms of a shift in Isaac's perspectives on educational research presented at

PPU. What was initially considered as working on two incompatible cultures of the natural and the social sciences is after graduation referred to as having an eye-opener into the syllabus of the teacher education programme. By expanding his perspectives on the practices of PPU, Isaac is overcoming the discontinuity between subject studies and university teacher education by exercising boundary crossing by reflection. The result is his expressed expectations of changing his mathematics teaching practice in the direction of inquiry based mathematics teaching. However, the transition from the second to the third critical event, from university teacher education to school, shows undesirable stagnation. Due to limited negotiability within the mathematics classroom community and lacking engagement in practices of professional development, Isaac struggles with changing his teaching according to his ambitions. The process of developing a mathematics teacher identity, due to changes in one's expressed perspectives on mathematics and its role in teaching, is in Isaac's case composed of inbound participation in the *community of inquiry based mathematics teaching*. Accordingly, he accounts for a shift in perspectives on educational research in mathematics and its role for own classroom practice, as well as stating ambitions for his future teaching. However, the transition from education to work does not represent a prominent change in Isaac's perspectives regarding what is meant by good mathematics teaching. His accounts do neither indicate a changed professional practice.

Following Wenger's (1998) theorisation of learning, I perceive the evolving stories as representing mathematics teacher identity as negotiated experience of self and mathematics, its teaching and learning based on participation and non-participation in mathematics practices. However, the evolving stories are further interacting, meaning that Isaac's sense of confidence relates to his evolving perspectives on mathematics and its role in teaching. For instance, his expression of belonging to an absolute and applicable discipline during teacher education resonates with his initial confidence or expressed mastery in mathematics during subject studies and in own schooling. Further, Isaac's expressed confidence might enable inbound participation in practices of inquiry based mathematics teaching and, in turn, support his ambitions of changing his mathematics teaching. Isaac's negotiated experience of self and mathematics is as well framed by *feedback available in the school environment*, as it enables or restricts his sense of mastery and the need for developing his teaching practice. As an example, Isaac's expressed confidence in the first interview is likely to be framed by experience from the mathematics classroom in which the negotiation of the students' feedback contributes to feelings of mastery in mathematics teaching. Contrarily, the lacking feedback from teacher colleagues at his second school of employment prevents negotiation of mathematics and mathematics teaching. The absent collegial discussions concerning professional development lead then to the feeling of invisibility, in addition to stagnation regarding his classroom practice.

Attending towards the critical events of Isaac's story of feedback available in the school environment, there is a development from first to second event regarding *feedback available in the mathematics classroom*. In the first interview, Isaac places the students and their mathematics learning in the foreground of his teaching, making their positive and negative feedback a necessary condition for identifying himself as a mathematics teacher. However, when entering the vocational mathematics classroom, the situation has changed into students withdrawing from the mathematics learning activities and questioning his role as their mathematics teacher. In order to facilitate constructive feedback on the mathematics teaching, Isaac establishes a fragile mathematics enterprise through step-by-step worksheets with a low threshold. The outcome is his changed negotiability within the potential classroom community, concerning the students' academic as well as their social progress. Although the vocational mathematics teaching continues as demanding throughout the school year, Isaac recognises progression in shape of the establishment of a safe mathematics learning environment. His developing mathematics teacher identity includes then different appearances of feedback in the mathematics classroom: from being a necessary condition for performing identification as a mathematics teacher, to enforcing a change of practice and new meaning to the profession when it is lacking.

In parallel, Isaac gives a regressive story regarding *feedback available within the teaching staff* when entering school as a certified mathematics teacher. In contrast to his first school of employment, there is no commitment to share and discuss developmental projects and educational research among colleagues at his new working place. Instead, the students serve as Isaac's main sparring-partner for negotiating what mathematics teaching is or should be like, and as a main provider of confirmation on his progression in mathematics teaching. His developing mathematics teacher identity is therefore accompanied by marginal participation in the potential *community of teacher colleagues* at his second school of employment.

Taken together, the developmental process of Isaac's mathematics teacher identity is constituted by narrative progression in all his three evolving stories. In Figure 5.16, I give an overview by relating the narrative progression to Isaac's changes in participation in given communities of practice. As an example, I relate the alternation in his expressed *confi-*

dence in mathematics and mathematics teaching to a changed negotiability within the classroom situated *community of secondary mathematics teaching*. Further, the double arrows spanning between the given communities of practice represent the interplay of Isaac's various forms of participation in mathematics practices. Following Wenger (1998), a person's identity entails experience of multimembership, and thus, the work of reconciliation necessary to maintain one identity across community boundaries. Hence, Isaac's participation in one community of practice is not be considered as isolated from his participation in other current communities of practice.

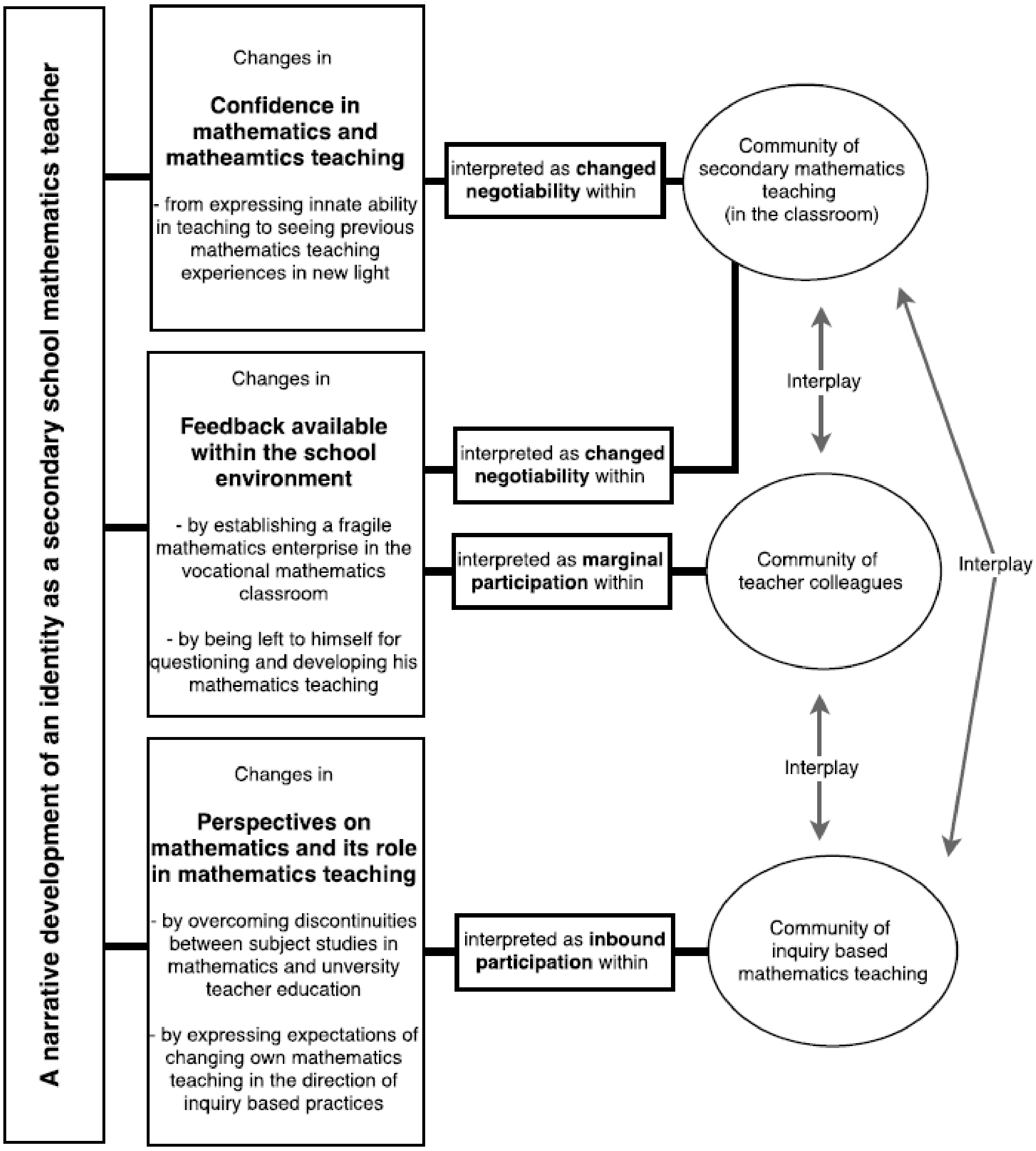

Figure 5.16: An overview of Isaac's developing identity as a secondary school mathematics teacher

6 The case of Nora

Nora's way into the mathematics teacher profession goes through various study programmes, subject studies and teacher education, and is characterised by undesirable changes in her study plans. She initially applied for a five year Master's programme in science and mathematics including teacher education (locally abbreviated as LUR), due to motivation problems with her original plan of becoming a music therapist. After one year of mathematics and physics studies within the LUR programme, she went through a personal crisis, which also had implications for her studies. Having failed on a series of exams, she came back to teacher education after one year of leave, and with a goal of finishing the five year teacher education programme. However, due to not having completed mandatory subjects, she was not approved for the first semester of teacher education and school placement. She therefore chose to leave the LUR programme, in order to complete a Bachelor's Degree in physics including 60 ECTS credits in mathematics. After having resumed exams from her first year of mathematics and physics studies, she was at the time of data collection a student at the one-year post-graduate teacher education programme (PPU), together with Isaac. Due to a non-approvable combination of subjects in physics, her only teaching subject was mathematics.

Similar to Isaac, I approached Nora to be a participant based on her responses to a small questionnaire. She reported that she planned to work as a mathematics teacher after graduation, and she ticked off "agree" on the statement "I perceive myself as a teacher". Further, she ticked off "do not know" on the statement "I perceive myself as a mathematics teacher", and she commented:

> *I know that I would like to work with people, and I like to be in a situation in which learning takes place and where students discover new things and are allowed to master something.*

Since she met the requirements of having a university background in mathematics and was likely to start working as a mathematics teacher after graduation, she was considered suitable for participating in the study on mathematics teacher identity in transition from university teacher education to school.

I present data from five interviews with Nora, distributed across a period of one and a half years. Similar to Isaac, the first interview took place at the end of Nora's first semester at PPU, just after she had completed her first period of school placement. The second interview was arranged half a year later, at the end of her second semester at PPU. After completing PPU, Nora was not offered a full-time position as a mathematics teacher. Instead, she chose to study physics part-time at the university in order to be approved for physics and science teaching in school. In parallel, she worked as a substitute teacher at a lower secondary school (grade 8 to 10) in which she was attending school placement in PPU. The school with its 425 students and 45 teachers was located near the centre of a Norwegian city. Nora taught mainly mathematics and science at the school, however, she was also given assignments in language subjects, religion and arts and crafts when needed. The remaining three interviews took place during her first year as a certified mathematics teacher, the last one in March. An overview of the interviews is given in Table 6.1.

Table 6.1: Overview of the interviews with Nora

Year 1 Teacher education, PPU		**Year 2** Substitute mathematics teaching in lower secondary school		
Autumn	Spring	Autumn	Spring	
Interview 1 December	Interview 2 June	Interview 3 November	Observation I Interview 4 March	Observation II Interview 5 March
2h, 30 min	2h, 40 min	2h, 5 min	40 min	1h, 15 min

On the occasion of the fourth interview, I observed Nora in two 1 hour lessons in mathematics on grade 9 at the lower secondary school. It took place during her 4 weeks long temporary position at the school, caused by a sick leave of one of the mathematics teachers. A summary of the observed lessons is given in Section 6.2.5, as a contribution to the evolving story of *perspectives on mathematics and its role in mathematics teaching*. The follow-up interviews (Interview 4 and 5) were arranged immediately after the lessons. The fifth and final interview did also function as a retrospective glance on the whole one and a half years long period in which I followed Nora.

Most of the interviews with Nora are lengthy (see Table 6.1), as she tended to freely talk about her past and present as a student teacher and a learner and teacher of mathematics. Due to the nature and format of the interview, she was in the beginning of each interview allowed to follow own paths in her storytelling, in which I ran the risk of generating accounts that did not deal with issues about mathematics teaching and learning. Her accounts tended to deal with issues about teaching and learning in general sense, as well as topics not being relevant for the stated research questions. Hence, the amount of interview excerpts presented in the subsequent analysis does not necessarily reflect the amount and length of her accounts making up the completed interview transcripts.

The analysis of the three cases in the thesis is presented chronologically. This means that the narrative analysis of Nora's accounts was carried out subsequent to the analysis of the interviews with Isaac. One exception is a delimited analysis of Nora's first interview, carried out for investigating mathematics teacher identity development in relation to the concept of boundary crossing (Rø, 2015a)[15]. Based on the narrative analysis, Nora's accounts constitute two emergent themes and related evolving stories: *mathematics as an attractive and repulsive discipline* and *perspectives on mathematics and its role in teaching*. While I developed the former story by making a grounded approach towards Nora's accounts in line with the analytic process described in Chapter 3, I chose the latter to be a continuation of the analysis of the case of Isaac. The reason for the transfer of a theme from one case to another was Nora's comparable accounts concerning the nature of mathematics and its teaching and learning. Hence, I found it useful to facilitate a further contrast and comparison across the cases by grouping commensurable accounts under similar emergent themes. In Section 6.3, I elaborate on how the presented interview excerpts constitute an evolving story of *perspectives on mathematics and its role in mathematics teaching*.

The structure of the upcoming analysis is similar to the case of Isaac, meaning that I report on the development of the emergent themes of Nora's accounts throughout the five interviews. Further, I account for critical events along the evolving stories, which in Nora's case concern the situations of

- facing towards a mathematics teacher career
- making a retrospective glance on the completed university teacher education programme, PPU
- working as a part-time substitute mathematics teacher

An overview of the two evolving stories and associated critical events is shown in Figure 6.1.

[15] In the given reference (Rø, 2015a), I use the name Benedicte on the same research participant that I here refer to as Nora. The change was made during the work on this thesis, due to the convenience of using a shorter name.

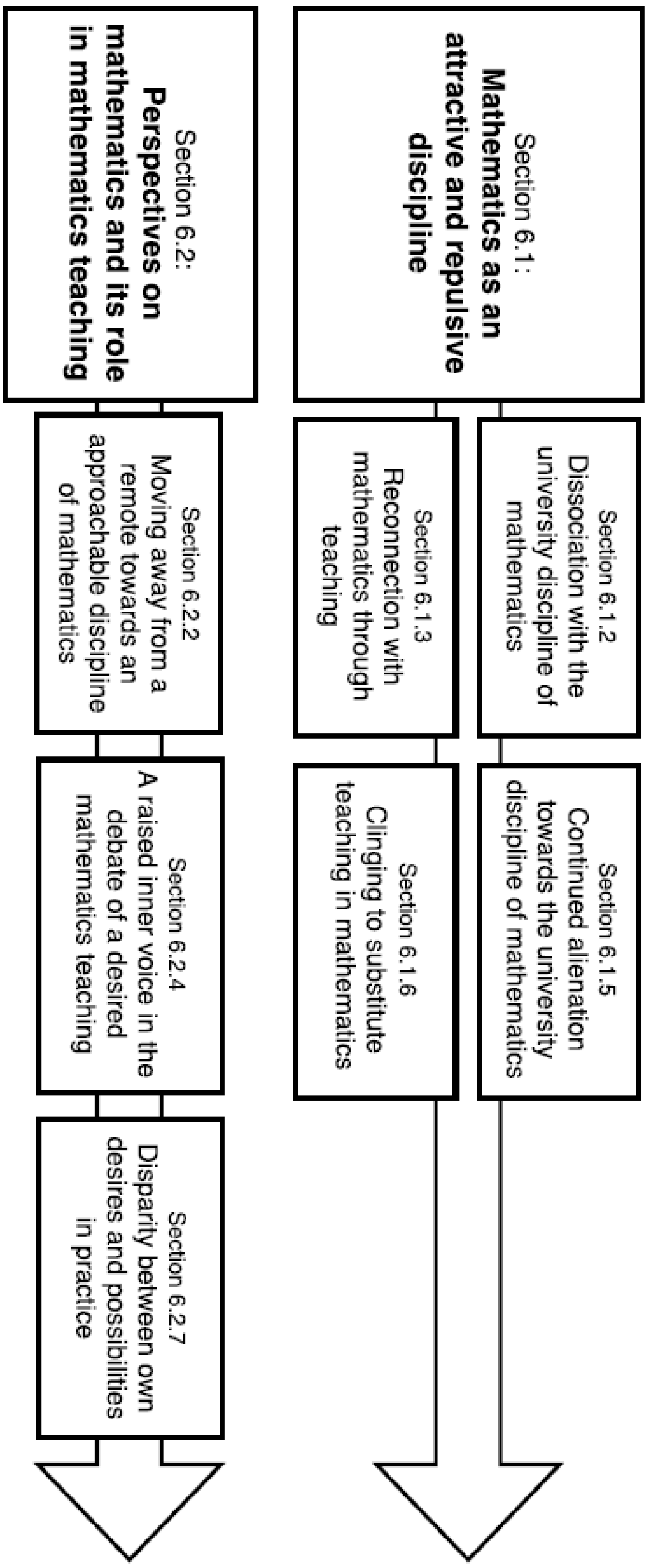

Figure 6.1: Emergent themes and related critical events in Nora's accounts, constituting two evolving stories of becoming a secondary school mathematics teacher

6.1 Mathematics as an attractive and repulsive discipline

Nora's way into the mathematics teacher profession starts with an initial interest in mathematics and its teaching; however, it continues through experience of defeat and undesirable changes in her educational plans when entering the university. Her accounts of studying mathematics include descriptions of incomprehension and alienation, but at the same time a desire of transforming defeat into mastery. Nora's positive relationship with the discipline of mathematics at lower secondary school concerned being capable of doing mathematics and helping peers doing mathematics. However, she accounts for a changed relationship with mathematics at the university, including experience of defeat when undertaking exams and a sense of alienation towards other performers of the scientific discipline. Her struggle of maintaining and developing a sense of herself as a legitimate member of the mathematics teacher profession during the subject studies at the university can therefore be understood as a story of *mathematics as a repulsive discipline.*

The story of dissociation with mathematics unfolds concurrently with accounts of approaching the activity of mathematics teaching, through the story of *mathematics as an attractive discipline* when undergoing PPU and when working as a substitute mathematics teacher. Nora's positive experience of mathematics teaching during school placement, based on the students' feedback on her teaching practice, leads to a feeling of mastery in her role as a mathematics teacher. Further, Nora accounts for a shift in her approach towards the discipline of mathematics, from an initial distance when undergoing the subject studies, into a sense of belonging to the discipline in the context of teaching. Alongside this shift is her reconsideration of PPU on the basis of her school placement in lower secondary school, from being irrelevant to becoming useful for developing her mathematics teaching. Accordingly, there is a change in Nora's emotional relationship with mathematics, with her sense of comfort when working with mathematics and mathematics teaching at PPU, and with her new initiative for becoming a mathematics teacher.

Since Nora's story of distancing herself from the university discipline of mathematics unfolds in parallel with her story of attraction towards the teaching of the subject, I interpret the strands to constitute a common emergent theme of *mathematics as an attractive and repulsive discipline*. The story is initiated by the critical event of Nora making the choice of becoming a secondary school mathematics teacher, despite her experience of difficulties and incomprehension during university subject studies. The story continues with two parallel episodes in her professional debut, of clinging to her substitute teaching in mathematics, while simultaneously experiencing a continued alienation towards the university discipline of mathematics. In the subsequent sections, I display each critical event in form of two parallel episodes. Hence, the accounts constituting the first critical event are presented in Section 6.1.1 and further examined in Section 6.1.2, as *dissociation with the university discipline of mathematics*, and in Section 6.1.3, as *reconnection with mathematics through teaching*. An equal structure applies for the second critical event.

An overview of labels belonging to the emergent theme *mathematics as an attractive and repulsive discipline* is shown in Figure 6.2. The labels constitute two sets of critical events along the evolving story. Since the story unfolds in two parallel strands, I have placed labels about mathematics as a repulsive discipline towards the left in Figure 6.2, and labels about mathematics teaching as an attractive discipline for teaching towards the right. The critical events have then been identified by comparing and contrasting labels both horizontally and vertically in the figure. Similar to the case of Isaac, the labels were initially identified on the basis of particular statements, however, they were confirmed by comparing and contrasting accounts throughout Nora's interviews. In addition to be linked with specific interview excerpts, the labels thus constitute summaries of Nora's expressed reasoning regarding mathematics, its teaching and learning, and the process of becoming a secondary school mathematics teacher.

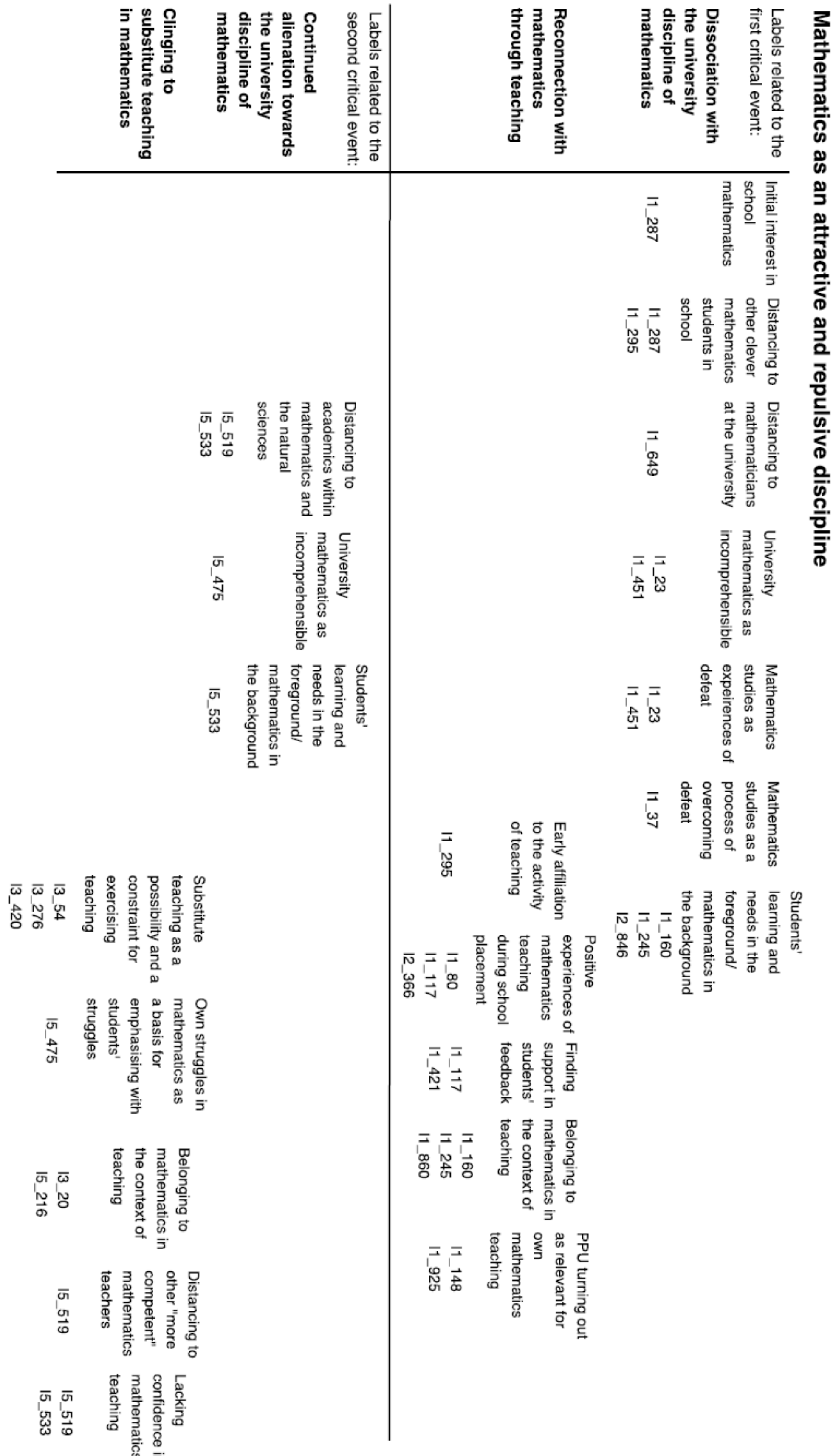

Figure 6.2: Overview of labels belonging to the emergent theme *mathematics as an attractive and repulsive discipline*, constituting two sets of critical events

6.1.1 Accounts of mathematics as an attractive and repulsive discipline, constituting a first critical event

Nora's accounts of becoming a mathematics teacher include descriptions of demanding mathematics studies at the university and mixed feelings towards the discipline of mathematics and mathematics teaching. Her initial interests of mathematics and its teaching stem from own schooling. In addition to enjoy puzzling with mathematics problems, she was considered clever in doing mathematics at lower secondary school and was therefore involved with helping fellow students during the lessons.

I1_287	Nora:	*I studied math and physics at upper secondary school, and I've found such subjects to be nice, it's really fun when you manage to solve a problem, knowing what you're doing, and can use it in other situations, and I enjoy myself when I do math nuts and things like that. But I've never been kind of run by, this is what I live and breathe for.*
I1_295	Nora:	*I think the thing of teaching the natural sciences has seemed exciting, because I did some of it myself when I was a student in lower secondary school, I was used as an assistant teacher since I worked quick and understood the math and it was OK, so I was used to be asked by others about what to do (...) And the other students in class that was equally clever always worked with upper secondary problems, and I wasn't really into that.*

Although Nora found the school subject of mathematics and its teaching to be attractive, the discipline of mathematics does not stand out as her main interest. This appears from her described differences between her and other performers of the discipline during own schooling. While other clever students at lower secondary school immersed themselves in mathematics at higher levels, Nora took interest in helping fellow students in mathematics. Since the mathematics subject itself was not something she "live[d] and breathe[d] for", becoming a mathematics teacher was not an obvious first choice when applying for higher education. Instead, teacher education turned out to be the least undesirable choice of higher education the moment her initial plan of becoming a music therapist failed.

I1_8	Nora:	*I had no motivation for anything at all, and suddenly I was in a bit of a fix concerning what to do next autumn, and it doesn't sound positive, me as a future teacher saying that, what I found to be least resisting when filling in that scheme, was putting me on teacher education in the natural sciences.*

Nora's first two years of mathematics and physics studies within the five- year Master's programme (LUR) proceeded into a series of failures on exams, which Nora describes as experience of defeat.

I1_23	Nora:	*I ended up having a really bad coping strategy, by realising that, if there were one thing that I could control, it was to fail on exams, so it became a vicious circle, leading to failure in eight subjects on a row (...) When I sat there and felt, I seriously don't know this, I don't know where to start, how to read the task, what should I extract of information, I just sat there and was so frustrated (...) I sat there and felt, I don't know this, and my best is not good enough, so it was kind of, a serious defeat.*
I1_451	Nora:	*To come there and realise this I don't know, and how much it affected me, by thinking, maybe I'm stupid, (...) since I'm not able to make it, I'm not worth anything. It's kind of a real defeat. It remained for a very long time, very deeply, very sore.*

The series of failures happened in the continuation of a personal crisis, which made it demanding for Nora to keep on with the university studies. She evaluates the episodes of failures as a kind of coping strategy for handling a demanding period of life, in which failing on exams made the university studies predictable. In order to overcome the adversity and "experience that the subjects didn't get me", she returned to the LUR programme after one year leave with ambitions of passing the exams and completing the study programme. However, new experiences of defeat happened when not being approved for school placement in year three of the teacher education programme, and later, when holding a non-approvable combination of subjects for physics teaching. As a plan B for her future career, Nora therefore completes 60 ECTS credits in mathematics in order to be allowed to enter the post-graduate teacher education programme, PPU.

I1_37	Nora:	*I decided, no, I want to complete it. I'm interested in it, and I want to experience that the subjects didn't get me, since I know I'm actually capable of making it. (...) And then I concluded I want to become a teacher. That's why I started on teacher education. But I'm not that interested in physics and mathematics that I want to write a Master's thesis in it. Since, during my period of study, I've seen that I'm more interested in what is happening between people and relations (...).*

Nora's way into the mathematics teacher profession is everything but straight forward. By referring to teacher education as the least resisting programme of study, and by pointing out physics as her favourite subject among the natural sciences, the mathematics teaching career is a result of educational studies that did not go as planned. Hence, a passion for the discipline of mathematics itself does not seem to be the driving force for Nora's choice of career. Instead, the activity of teaching stand out as appealing, as she accounts for an interest in the meeting with students and the establishment of a trustful relations with them in the mathematics classroom. Although Nora distances herself from what she regards as the mathematicians' inconceivable and excluding work, she is, however, in some sense attracted to their mind-boggling professional world. In order to widen her perspective on the discipline, considering that she is facing towards a mathematics teacher career, she joins the Friday seminars on mathematical curiosities at the university's mathematics department.

I1_649	Nora:	*(...) it's like an excluded little group who is doing something that we don't really know what's about, but they do it really good [laughing], so it's a little funny, I've been to some Friday seminars (...) I've started doing it the last years, since I notice this math teacher that has started to spire in me, it's fun to see what in the world are you doing [laughing], and do I follow it? A little, and then other times, no, I have no idea what it's about. But it's exciting to see that, math is much more than what I think about it.*

Nora's first teaching experience in mathematics took place during school placement at PPU, at a lower secondary school. She describes the collaboration with the tutor as positive, since he was clear on what he preferred for his mathematics lessons regarding structure and order and the students' involvement in the teaching activities. At the same time, Nora found a leeway of trying out activities and teaching methods from teacher education in the mathematics classroom.

I1_80	Nora:	*I was quite lucky with my tutor, I will say (...) he and I, we matched. He was quite strict and he knew what he thought was best, and he preferred ten minutes of summing up and theory, then tasks and closing. (...) So he was strict, and I found out that, ok, then I have to work within these frames and at the same time try to use drips and drops. And I was quite concerned with, that I had a good starting point, because he had worked with the class in that they were quite used to be questioned.*

During her practice teaching at grade 10 (age 15) at lower secondary school, Nora was supposed to teach the students how to multiply polynomials. In order to try out what she during the mathematics didactics courses had been introduced to as rich problems and investigative mathematics teaching, she based the teaching sequence on a Teaching Channel

Teams[16] video. It included an area model and a related grid method for expanding the polynomials, and the other way around, for factorising quadratic expressions into binomials. When working on what she denotes as an investigation of multiplication with brackets, Nora experienced positive feedback from the students in the shape of their increasing partaking in the activity. Hence, glimpses of progress in the students' work give her confirmation and support for further development of her mathematics teaching.

I1_117	Nora:	*And, in the beginning they were like, ah, I don't get it. But I let them talk in pairs, so then, I tried to create some verbal activity, because I thought I wanted to try this thing of approaching rich problems and approaching an, eh… investigation, even though it was a quite theoretical investigation, it was at least a start. And when I saw how it worked out, I became surprised, because they caught it. (...) and it was very nice to see that, at that point, they understood it, and they found, kind of understood how I worked.*
I1_421	Nora:	*You see it in the hidden, that there are things which have worked out, and it is an affirmation, a confirmation that I'm on the right track. And that's inspiration to move forward.*

Due to her main interest in students' learning and their needs, Nora elaborates on her approach to the mathematics teaching by placing the students in the foreground of her practice. To her, the mathematics teaching and learning in lower secondary school needs to build on trustful relations with the students, which she describes as being situated "in the backbone" of her teaching practice. The joy of mathematics teaching is therefore rooted in her attempts of creating a safe classroom environment for the students to be involved in. Consequently, her mathematical foundation from the university is placed in the background of her teaching practice.

I1_245	Nora:	*And it's pretty much in the backbone, thinking about the things with relations and trust, and I'm quite focused on it. That I think it's the key to reach the students. Of course, the thing about being academically solid (...) but to be able to establish a contact (...) that is kind of, a math teacher with a focus on relations and trust, as an approach for achieving learning.*

[16] Teaching Channel Teams is a nonprofit video-enabled professional learning platform developed in the United States, see www.teachingchannel.org

I1_160	Nora:	*And it's an everyday challenge (...), I am so happy I do not need to twist my head and think about solving triple integrals. But I get to twist my head in thinking how should I work with trustful relations in order to make a safe classroom so that it is OK to ask, it is OK to make mistakes, so I can hear how they are thinking to be able to make progress (...).*

When entering upper secondary school and the vocational mathematics classroom during the second period of school placement, Nora faces a situation which challenges her even further on presenting the mathematics subject as bearable and worthwhile. She describes the students in the building and construction technology class as unmotivated, lacking basic mathematics knowledge, and being in danger of failing in the subject. Her approach to the mathematics teaching is therefore to communicate "the subject in the easiest possible way".

I2_49	Nora:	*They were eight people, who didn't know the difference between the perimeter and the area. That was the starting point. It was kind of, a different, how to meet challenges, how to meet students who certainly don't have a clue.*
I2_214	Nora:	*(...) when they have so extremely little interest and motivation to work with the subject, it became a kind of different meeting with practice for me (...) when the students' level was at the mark 2 or below, then it was about working with communicating the subject in the easiest possible way so they could understand.*

Nora's description of the vocational mathematics classroom portrays a learning environment in which an extensive adaptation of the mathematics teaching is necessary in order to make the students take part in the mathematics activities. However, the students' temporary involvement in her mathematics teaching functions as confirmation and encouragement to continue her teaching practice.

I2_366	Nora:	*At least, in the building and construction technology class it's kind of, oh, let's do anything but math, and when you actually make them do something, hah, caught you in doing math there, that's been very fun. And I enjoy myself with people who don't really manage the subject very well, since that makes me become more challenged on the more relational level (...) I've got the chance to be challenged on finding out what to do, how can mathematics be presented in different ways, and that I've found to be very fun.*

I2_846 Nora: *So I'm kind of a special teacher, maybe I'm more a social-mathematician than a scientist, but this thing of being with the students and working with understanding, I find it to be so fun based on my practice experience in math that I want to continue with it.*

In continuation of her accounts of the first period of school placement, Nora emphasises her desire of making a low threshold to the mathematics content, so that students at all levels can be part of the teaching activities. The preference of presenting mathematics as simple, easy and understandable for everyone is thus enforced through her teaching practice in the vocational mathematics classroom. Here, the students' lack of motivation and their weak mathematics backgrounds make up a teaching situation for her to develop further an approach to mathematics teaching rooted in trustful relations and empathy with their struggles. In line with her student-centeredness, Nora continues to highlight a main interest in the students' basic mathematics knowledge and the trustful relations needed for providing them with positive experiences of learning mathematics.

Besides talking about her meeting with the tutor and students during school placement, Nora also gives accounts of the lectures in mathematics didactics at PPU. Repeatedly, she describes a shift in her relationship with mathematics practices at PPU in terms of "before and after" school placement. What she initially perceived as nonsense during the first lectures of the mathematics didactics course, has later turned out as an opportunity for viewing her classroom practice in a different light, and hence, given her new initiative to become a mathematics teacher in school.

I1_69 Nora: *I was sitting in the lectures, (...) thinking, yeah, sure! I will never have use for this. (...) Then, I decided, yes, I'm going to have practice in mathematics, and I can at least try to use some of the things they are talking about, and decide what I think about it, although, I'm critical (...) and I have to say, I have become kind of converted as a mathematics teacher.*

I1_148 Nora: *So I have kind of seen that suddenly, from throwing away the nonsense which was before practice, to say that, yes, I give it a chance, it has been so fun to see that, wow; this has given me new initiative, this interest and commitment for how to work in order to break down the mathematics to a level that everyone can reach, and working on how to explain it in various ways (...)*

I1_860	Nora:	*So, the thing of being able to develop myself in understanding math, it has during the practical parts of mathematics didactics lessons been very useful and in a way given me more insight in and belonging to mathematics, because I see now that I can be a part of it, too. (...) This I can handle, and this I can use.*
I1_925	Nora:	*I see a very clear division, before practice, this is just nonsense and the resistance, to become able to see more from what has happened, it is easier to relate it to something and you become prepared for what is being talked about [in the lectures]*

In parallel with descriptions of her altered perspectives on the university teacher education, Nora accounts for a changed emotional relationship with mathematics. She portrays mathematics within the context of mathematics didactics lectures to be more relaxed and less prestigious than mathematics within the previous subject studies at the university. Hence, entering the teacher education programme and undergoing school placement has contributed to a greater sense of safety when doing mathematics. She describes it with terms such as having a "calmer attitude", "having lower shoulders" and "being spared of the stress" of performing on the weekly exercises or a final exam.

I1_868	Nora:	*You have kind of a calmer attitude towards the math, since you're not evaluated the same way, and I note that I'm more susceptible (...) since I'm spared of the stress, (...) as a natural science student, you are going to (...) produce some kind of answer on an exam, knowing that it's the only thing that counts, while here, you've got the possibility to learn some mathematics while having lower shoulders, and relate it to something else.*

By approaching mathematics from a lower secondary teacher's perspective, not only from a university student's perspective, Nora describes a reunification with the discipline of mathematics. Based on positive experiences from the mathematics classroom and lectures at PPU, she can label herself a prospective mathematics teacher.

I1_337	Nora:	*(...) now that I have worked and been out in practice, the mathematics has in a way returned to me, and I have actually, I am able to acknowledge and smile and accept and say yes, I am a mathematics teacher and I am therefore kind of a mathematician even though it feels a bit strange to say that.*

Through her emotive accounts of moving away from an incomprehensible university discipline of mathematics and towards desirable teaching in school, the choice of becoming a mathematics teacher constitutes a critical event in Nora's evolving story of *mathematics as an attractive and repulsive discipline*. In the subsequent analysis, I split the critical event into two parallel episodes: *dissociation with the university discipline of mathematics*, and *reconnection with mathematics through teaching*.

6.1.2 First critical event: Dissociation with the university discipline of mathematics

Nora's relationship with mathematics and the teacher profession can be explained by means of her identity as a prospective mathematics teacher, being constituted by the interaction between participation and non-participation in mathematics practices. One example of this interaction stems from Nora's accounts of the time in lower secondary school. Here, she describes appreciation in mathematics; however, she locates herself in the distance from the other clever students who were working on upper secondary mathematics. Simultaneously, she expresses affiliation to the students who asked for her help during the mathematics lessons. In upper secondary school, she found mathematics and physics to be appealing, however, with the reservation of not being completely engrossed in it. This expression of restricting her participation in a potential mathematics community is further exemplified by her statement of not having the interest in studying for a Master's degree in physics or mathematics at the university. Instead, her main focus is on establishing trustful relations between people, for instance between her and the students in the mathematics classroom. As a consequence of the synergy of participation and non-participation, Nora thus positions herself towards the boundary of a *community of university mathematics*.

The marginal positioning comes further into sight by Nora's engagement and imagination within the practices of university mathematics. By stating that "I want to experience that the subjects didn't get me" (I1_37), Nora strives to be part of the discipline of mathematics by making tireless efforts of passing exams at the university. It includes descriptions of defeat, both when undergoing a series of failures on exams at the university and when not being accepted for school placement and physics teaching at PPU. She further reports on experience of incomprehension regarding mathematics, both when doing the exams and "don't know[ing] where to start [and] how to read the task" (I1_23). In addition to her accounts of inconceivability regarding university mathematics, Nora expresses alienation towards the mathematicians, being some kind of "wizards with magical mathematical abilities" (Picker & Berry, 2000, p. 89). She engages in the practices of university mathematics by joining the Friday seminars at the mathematics department, yet, the employees represent a separate and inconceivable group of scientists that Nora looks upon with wonder and amazement. She thus imagines herself as a prospective performer of mathematics teaching who is standing in the outskirts of the mathematicians' world.

Due to her marked distance towards the university discipline of mathematics, Nora's non-participation in the *community of university mathematics* might function as an identifying strategy, in which she protects herself from additional setbacks in mathematics. In other words, she distances herself from the discipline that has generated experience of defeat, by simultaneously expressing a sense of belonging to the practices of teaching and its interpersonal relations in which she appears as comfortable. In terms of Nora's developing identity as a mathematics teacher, the critical event of deciding to become a secondary school mathematics teacher can therefore be characterised by her *dissociation with the university discipline of mathematics*. In Figure 6.3, I relate the critical event to her marginal participation in the *community of university mathematics*.

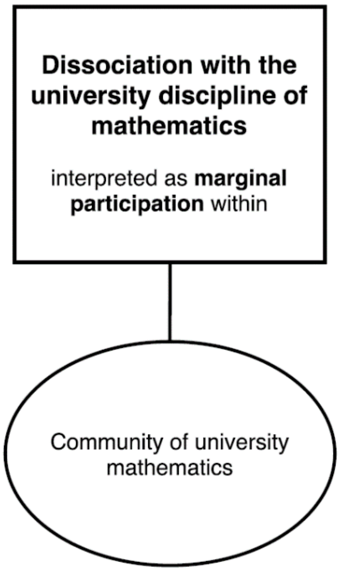

Figure 6.3: Mathematics as an attractive and repulsive discipline, represented by the first critical even of dissociation with the university discipline of mathematics

6.1.3 First critical event: Reconnection with mathematics through teaching

The story of mathematics as an attractive and repulsive discipline arises as well from accounts of Nora's attraction towards mathematics practices during school placement and the mathematics didactics lectures at PPU. In the mathematics classroom, Nora finds the students' engagement in the learning activities to make up a basis for gaining feedback on her teaching. In line with Isaac, responding to the students' feedback is to Nora a main component of exercising mathematics teaching in the classroom. In her case, the feedback is enabled through her alignment to the tutor's practice and his facilitation of mathematical discussions among the students. From her peripheral participation in the *community of tutor and students during school placement*, Nora thus negotiates her classroom practice through dialogues with the students when introducing them to new kinds of investigative mathematics activities. Her engagement in the community is therefore about feeling her way in the mathematics classroom, by using “drips and drops” from the teacher education programme or external resources such as videos from You Tube. Based on the students’ involvement and their positive and negative feedback on the mathematics teaching activities, she gains new initiative to further develop her mathematics teaching practice. In line with Isaac, the students’ feedback thus serves as confirmation on her mathematics teaching, with the consequence of recognising herself as a kind of mathematics teacher in lower secondary school. By labelling herself a special teacher and a “social-mathematician” in her second period of school placement, Nora voices belonging to mathematics teaching in which the students’ challenges and struggles with mathematics are dominating. Feedback in terms of students’ involvement in the vocational mathematics classroom makes her then recognise herself as a mathematics teacher who manages to adapt herself to and empathise with students’ insufficient competences and their various needs. Hence, the two periods of school placement become important resources for Nora to exercise identification through imagination, in terms of creating a picture of herself as a mathematics teacher in the world of mathematics teaching. Based on the presented accounts, Nora portrays a teacher who is able to establish trustful relations and a safe classroom environment for learning mathematics, despite students’ difficulties with the subject.

According to Hodgen and Askew (2007), developing an identity as a mathematics teacher, in terms of becoming a somehow “different” teacher and/or learner of mathematics, is inseparable from changing one’s emotional relationship with mathematics. Nora’s developing emotions concerning mathematics can be understood on the basis of several reports on change: from incomprehensible university mathematics to approachable mathematics in the context of learning and teaching; from alienation towards other mathematics performers into sense of belonging to the discipline in teacher education; from demanding expectations of performing well on exams to a more relaxed atmosphere for learning mathematics and its didactics; from experience of defeat to a new initiative for becoming a mathematics teacher. These accounts represent then a story of discontinuity in Nora’s action and interaction regarding mathematics when undergoing the first semester at PPU, which in turn indicates peripheral participation within a *community of secondary mathematics teaching*. Nora’s accounts do not provide information about a location for a community of mathematics teaching and its other members.

However, her reports on the mathematics didactics lectures indicates an existence of a community "out there" which is based on a mutual engagement of teaching mathematics as a problem-driven discipline. In other words, she accounts for a sense of community among mathematics educators, at first from the outsider's position of pointing towards *them* when "try[ing] to use some of the things they are talking about" (I1_69). Based on her two periods of school placement, Nora aligns to the practices of approaching the mathematics subject from a learner's perspective, in terms of exercising varied mathematics teaching and adapting the mathematics content to the students' different needs. In addition, she participates in the teaching practices by imagination, by acknowledging mathematics as her teaching subject and putting the label "mathematics teacher" on herself.

Nora's peripheral participation in the *community of secondary mathematics teaching* is related to her engagement in the *community of tutor and students during school placement*, in which she tried out "drips and drops" from the mathematics didactics lessons and received the students' positive feedback. In other words, the school placement functions as a basis for Nora to develop a different and more positive relationship with the discipline of mathematics, with the consequence of a strengthened interest in the discipline. The continuity along this developmental process is her genuine interest in students' learning (Rø, 2015a). The critical event of deciding to become a secondary school mathematics teacher is therefore characterised by Nora's *reconnection with mathematics through teaching*. In Figure 6.4, I relate the critical event to Nora's peripheral participation in the *community of tutor and students during school placement* and the *community of secondary mathematics teaching*.

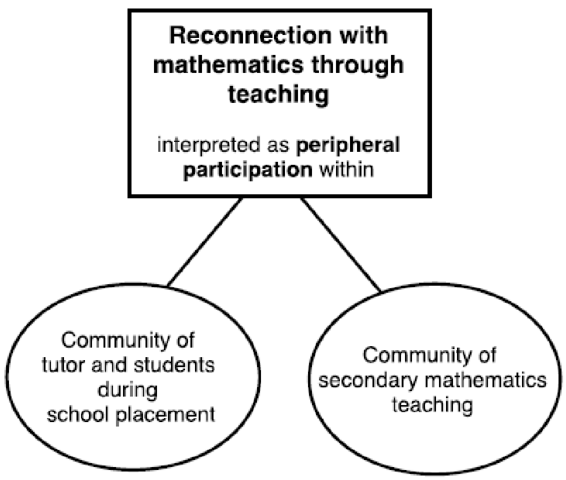

Figure 6.4: Mathematics as an attractive and repulsive discipline, represented by the first critical event of reconnection mathematics through teaching

The notion of reconnection with mathematics is inspired by Hodgen and Askew's (2007) story of a primary mathematics teacher's shift from initial disconnection with mathematics into her construction of a mathematics teacher identity. Like Nora, the primary teacher named Ursula became drawn to mathematics despite her initial avoidance of the subject. Yet, in Ursula's case, her development of a mathematics teacher identity resulted from participation in a professional development programme, in which she constructed a "strong and powerful image of a different mathematics teaching" (Hodgen & Askew, 2007, p. 482).

6.1.4 Accounts of mathematics as an attractive and repulsive discipline, constituting a second critical event

After completing the one-year teacher education programme, Nora starts to work part-time as a substitute teacher at the lower secondary school from her first period of school placement. The sporadic teaching is done in parallel with physics studies at the university, in order to become employed as a full time mathematics and physics/science teacher. Her job situation implies being called in the morning in order to work as a short run substitute teacher for one or more days. In some periods, she replaces a teacher for a couple of weeks, and in a variety of subjects. In addition to teach mathematics, she is assigned lessons in religion, arts and crafts and language subjects, based on what is needed in the teacher staff. Although Nora's unpredictable teaching situation leads to fewer opportunities for preparing her lessons well and being in charge of the current lesson plans, she perceives substitute teaching as a learning situation for developing her teaching practice. Hence, in order to adjust to the forthcoming teaching, she strives to be in the classroom well before the lesson starts. Further, the frequent meetings with new students make up a solid foundation for exercising classroom management.

I3_420 Nora: *Rarely, I come running into the classroom and then I complete the lesson. I rather come early, to consider, what will happen next (...) and thus, use it as a learning situation instead of just sitting there.*

Nora further describes the substitute mathematics teaching as an opportunity for receiving feedback concerning the students' difficulties with the subjects. Although her sporadic interaction with the students makes it difficult to establish solid and trustful relations, she still experiences less fear of exposing their lacking competences in mathematics.

I3_54	Nora:	*When they don't have their regular teacher, then they dare to maybe ask questions about other things, since the teacher has maybe went through it and then its quick work, [they] have not understood it, and then there's a new person and then it may be room for kind of, I didn't understand this, help me.*

However, being placed in another teacher's classroom also entails constraints for exercising one's mathematics teaching. Nora describes a limited scope of action based on the students' expectations of what mathematics teaching is or should be. In addition, she points to the need of showing loyalty to the absent teacher who is in charge of the mathematics teaching and its progress.

I3_276	Nora:	*As a substitute teacher, you cannot do things very different from what the regular teacher is doing, since then the students may show resistance and don't want to participate. Or you can create challenges for the teacher who will return, since, you don't do the fun stuff that the substitute teacher does, why can't we have the substitute teacher all the time?*

Nora's situation of entering the lower secondary school as a substitute mathematics teacher is similar to her former position as a student teacher during school placement. In both cases, she exercises mathematics teaching within a limited period of time, and within the frames of another mathematics teacher's practice. By not being the one in charge of the preceding and following mathematics teaching causes then freedom, as well as limitations, for exercising teaching. The freedom concerns the lack of responsibility for doing the final assessment, and thus, the privilege of not being responsible for the progress in relation to the curriculum. Contrarily, the limitation of following the plans and activities of others causes restrictions for making changes and introducing new approaches to the mathematics teaching. Nevertheless, in contrast to carrying out teaching she is not educated for, the mathematics lessons imply a sense of safety within a practice she aims for developing, and with that, a sense of belonging to the teaching of mathematics.

I3_20	Nora:	*What I've discovered when being at the school, I'm so math teacher [laughing], finally, give me something safe and nice I'm allowed to challenge myself in doing, instead of coming to a class and like, yes, now we're going to talk about Buddhism, run the debate! Kind of, I don't have a clue about what I'm doing, if you just have an opening and an end, then I'm satisfied with saving that lesson, while in mathematics, I actually get the chance to relax a bit, dare to try using what I got from PPU.*

In the final interview and her review of the past year as a substitute teacher in lower secondary school, Nora is further highlighting the appreciation of receiving the students' feedback on her mathematics teaching. Appearing in the form of students' mastering and progress, the feedback serves as confirmation on her success of explaining the mathematics content clearly to her students.

I5_216 Nora: *When I start to work on it, then it's so fun trying to find a way which is clear and to explain it to the students, and then it's just seeing that they start to do something, and just seeing some students follow it, then it's actually quite fun. In general, it's fun to be a math teacher*

When searching for the right angle from which to present the mathematics content, Nora points to her advantage of having the ability to empathise with students who struggle in learning mathematics. Based on her own experience of taking detours during her university studies, she assumes than she more easily can relate to the strained relationship some students have with the subject.

I5_475 Nora: *I think I have a huge advantage with not managing things like plain sailing, which so many others have done, following a path, become clever, finishing it, go straight to the next level. However, I've taken some detours on my way and experienced what I find difficult myself (...) and remember that not everyone understand everything so easily, that is a very good experience I bring with me in the meeting with students.*

By accounting for her ability to empathise with the students struggles in mathematics, Nora draws attention to the way she differs from what she regards as common among most mathematics teachers. However, in addition to pinpointing her advantages from having a difficult way into the mathematics teacher profession, she accounts in the final interview for a sense of uncertainty when "pretend[ing] to have control" as a mathematics teacher. Unlike what she assumes is the situation for her fellow students who were better in mathematics, Nora experiences to struggle with the thought of being good enough as a mathematics teacher.

I5_519 Nora: *Sometimes, I pretend to have control, since I struggle maybe a little with my math teacher confidence, compared to others I have studied with who were much better in math, and then I mean that they more quickly see the solutions and find the way to it, while I spent a lot of time at university on, I don't know what to do, I don't get the direction, and then I've often been thinking that those students are luckier, who have some of the other people I was together with at university [as their teacher] (...) and I'm steady enough to be a lower secondary school teacher or upper secondary school [teacher]. However, I don't have that confidence of a fully scientist.*

In addition to perceiving herself as a less confident practitioner in mathematics and the natural sciences, Nora accounts for less desire to learn new mathematics. Instead of acting as her main academic interest, mathematics is to Nora a means for getting to work with students and their mathematics learning in school. However, since her lack of interest in mathematics differs from what she regards to be the common practice among student teachers, Nora struggles with fitting into the common mathematics teacher category.

I5_533 Nora: *I perceive the natural sciences as a means for meeting the students, so, that I've felt a lot and still feel a lot, that I'm not the kind of a core scientist which enjoy to understand scientific articles (...) so, that feeling of me not seeing science in everything I meet, which I think the other people in my class were doing (...) I don't have that kind of science lust, and that has given me kind of, feeling a bit inferior as a teacher because of that, since I feel I don't fit into that box and the expectations I have about being a teacher (...) I struggle with fitting into that category myself.*

Due to her descriptions of belonging to as well as dissociating herself with the discipline of mathematics and its teaching in school, Nora's story of *mathematics as an attractive and repulsive discipline* evolves by the second critical event when entering school as a substitute mathematics teacher. Similar to the analysis of accounts from the first and second interview, I interpret the story to continue by two parallel episodes: Nora's *continued alienation towards the university discipline of mathematics*, and simultaneously, her *clinging to substitute teaching in mathematics*.

6.1.5 Second critical event: Continued alienation towards the university discipline of mathematics

Nora's accounts of entering her school of employment portray a newly qualified mathematics teacher having concerns regarding her teacher role. Following Wenger (1998), identifying with the mathematics teacher category requires the work of *imagination*, as it depends on the kind of picture of the world and oneself a person can make. Based on Nora's accounts of her fellow student teachers in mathematics, her picture of a mathematics teacher consists of someone having sincere interest in the discipline of mathematics and who is exercising a "kind of science lust" (I5_533). By stating that she does not fit entirely with this picture, Nora exercises imagination by opposition, meaning that she distances herself from what appears to her as the stereotypical mathematics teacher profession. Hence, the act of imagination leads to an identity of non-participation, and consequently, a continued marginal position within the *community of university mathematics*.

Further, by stating that she is steady enough to be a secondary school mathematics teacher, Nora simultaneously exercises imagination by association. In other words, she identifies herself with parts of the picture of a mathematics teacher, however, with a sense of inferiority. The notion of inferiority or being lower in status compared to other secondary school mathematics teachers is related to issues of power, which according to Wenger (1998) is inherent in social life. Following Wenger, "power is not construed exclusively in terms of conflict or domination, but primarily as the ability to act in line with the enterprises we pursue" (p. 189). In the case of Nora, it concerns the power to belong to a practice of mathematics teaching and to claim her place with the legitimacy of membership in mathematics teaching practices, based on her main interest in the students' learning and their needs.

However, the notion of power simultaneously contains the vulnerability of belonging to and identifying with mathematics communities, which in turn contribute to defining who she is or ought to be as a mathematics teacher. Implied in Nora's identification as a mathematics teacher are then her own and others' expectations of acting or behaving somehow differently. The outcome is a sense of uncertainty or what she denotes as a lacking confidence, rooted in her mathematics competences and academic interests. The critical event of entering school as a substitute mathematics teacher is therefore characterised by a *continued alienation towards the university discipline of mathematics*. It appears in the shape of a sustained marginal participation in the *community of university mathematics*, as shown in Figure 6.5.

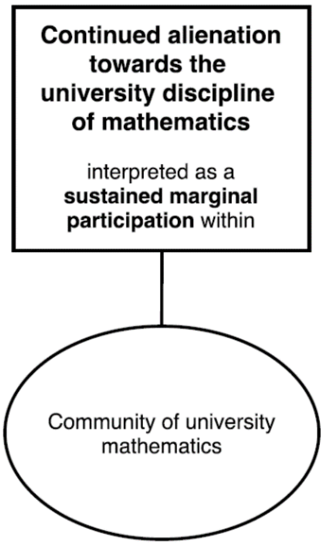

Figure 6.5: Mathematics as an attractive and repulsive discipline, represented by the second critical event of a continued alienation towards the university discipline of mathematics

6.1.6 Second critical event: Clinging to substitute teaching in mathematics

Similar to her experience of undergoing school placement, Nora's role as a substitute mathematics teacher can further be understood as being introduced to various *communities of students and their regular teacher*. In line with her accounts in the first interview, Nora expresses engagement in the classroom communities in terms of trying out mathematics teaching in search for the students' feedback. As during school placement, this feedback concerns the students' level of involvement and their progress in the current activities, in addition to their request for Nora's help when solving tasks. Hence, her collaboration with the students makes up a negotiated enterprise from which she can identify herself as a kind of mathematics teacher. Nora's accounts of her sense of familiarity within the mathematics classroom constitute therefore evidence of a continued belonging to mathematics through its teaching. Working as a substitute mathematics teacher makes her stick to the label "mathematics teacher", "I'm so math teacher" (I3_20) and leads to a consolidation of her existing mathematics teacher identity. Nevertheless, due to her temporary attendance and the lack of responsibility for the current lesson plans, Nora's ownership of meaning within the classroom community is limited. Although being sporadically engaged in negotiated enterprises of mathematics teaching, her partaking in the classroom communities is restrained by non-participation in a joint enterprise and shared repertoire being primarily negotiated by the students and their regular mathematics teacher. Consequently, Nora holds a marginal position in the *communi-*

ties of students and their regular teacher, meaning that her learning trajectory runs along the community boundaries. The possibilities for further development of her mathematics teacher identity, in the shape of an expanded set of perspectives on mathematics and its teaching and learning, are therefore restricted. Still expressing a sense of affiliation to secondary school mathematics teaching, the critical event of entering school as a certified mathematics teacher is characterised by Nora's expression of *clinging to substitute teaching in mathematics*. I have chosen the wording of clinging to mathematics teaching due to Nora's sense of holding on to mathematics as her main subject of interest, despite her lacking ability to influence the content and form of her teaching. As shown in Figure 6.6, the critical event is interpreted as Nora's marginal participation in the current *communities of students and their regular teacher*, by virtue of her limited negotiability. In parallel, Nora exercises a continued peripheral participation in the *community of secondary mathematics teaching*.

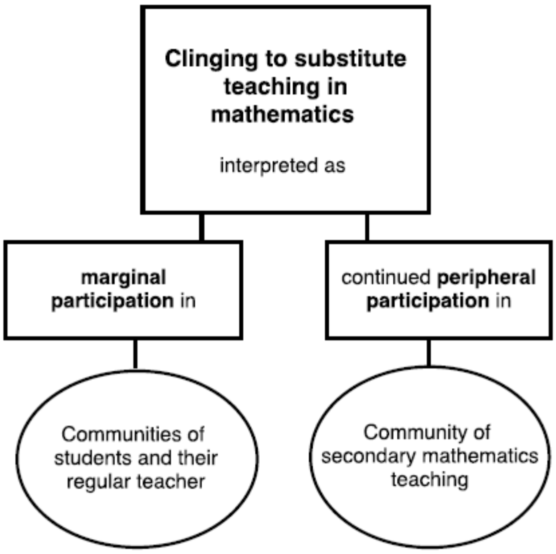

Figure 6.6: Mathematics as an attractive and repulsive discipline, represented by the second critical event of clinging to substitute teaching in mathematics.

6.2 Perspectives on mathematics and its role in mathematics teaching

In order to elaborate on Nora's expressed perspectives on mathematics and its teaching and learning, her accounts concerning the nature of the discipline were gathered under the theme *perspectives on mathematics and its role in mathematics teaching*. Throughout the interviews, she portrays mathematics as an absolute discipline, which in her case leads to experience of both immediate reward as well as feelings of discomfort when solving mathematics problems. In contrast stands the general pedagogy and mathematics didactics at PPU, which embody a fallible discipline, and hence, a shared negotiation of meaning. Accordingly, Nora reports on her dissociation with what she considers as cold and remote university mathematics, and her simultaneous attraction towards the social sciences at PPU, which appear to her as warm and welcoming. A recurring tone in Nora's accounts of becoming a mathematics teacher is therefore her emotional approach to the subject of mathematics, both when describing her mathematical background and when discussing the mathematics interview tasks. Hence, I interpret her story of *perspectives on mathematics and its role in mathematics teaching* in parallel with her accounts of emotions such as defeat, mastery, struggle, ability, alienation and belonging when she associates herself with the discipline. As a consequence, the evolving story provides a better insight into Nora's dissociation as well as reconnection with the discipline of mathematics.

Nora further accounts for the role of mathematics in her teaching, which I interpret to be related to her descriptions of experience of defeat when undergoing the subject studies in mathematics at the university and the following need to restore pride (see Section 6.1.1). As a kind of counter reaction to her experience of incomprehension and alienation towards university mathematics, her teaching of mathematics in school is expected to be inviting and manageable to students on all levels. The first critical event is thus constituted by accounts of her ambition of transforming a cold and remote university discipline of mathematics into an accessible and human school subject. Nora finds support for her approach towards the teaching of mathematics through the lectures in mathematics didactics at PPU. However, at the time of graduation, she makes a review of the teacher education programme with a raised critical voice regarding what has been communicated as the desired mathematics teaching. The second critical event is therefore made up of Nora's changed expressed perspectives on teacher education and its importance for own professional practice. The third critical event relates to Nora's accounts of own teaching practice, based on observation of her substitute mathematics teaching at the lower secondary school. Working sporadically and within the frames of the regular teacher's practice, Nora accounts for a divide between her desires for own teaching and the possibilities for performing it in the classroom.

An overview of labels belonging to the emergent theme *perspectives on mathematics and its role in mathematics teaching,* and which constitute three critical events, is shown in Figure 6.7.

Perspectives on mathematics and its role in mathematics teaching

Labels related to the first critical event: **Moving away from a remote towards an approachable discipline of mathematics**	Mathematics as either right or wrong I1_283 I1_835 I2_317 I2_814	Mathematics as a provider of immediate reward I1_567	Mathematics as a generator of feelings of discomfort I1_1302 I1_1495	Mathematics as useful in everyday life I1_134 I1_583 I1_639	Mathematics as cold and remote/social sciences as warm and welcoming I1_791 I1_835	Mathematics as, in actual fact, accessible to all students I1_639 I1_679 I2_814 I2_317	Teaching mathematics based on empathy with students who are struggling I1_165	Teaching mathematics by using a variety of activities and resources I2_238 I2_185 I2_433	
Labels related to the second critical event: **A raised inner voice in the debate of a desired mathematics teaching**	Educational research as a swinging pendulum I2_700	Discovering an own voice regarding mathematics teaching I2_513			A one-sided, categorical presentation of mathematics teaching at PPU I2_163 I2_661 I3_239 I3_291	Problematic relationship between theory and practice at PPU I3_239 I3_291			
Labels related to the third critical event: **Disparity between own desires and possibilities in practice**					Teaching mathematics within the frames of the students' regular teacher I4_93	Substitute teaching as a constraint for communicating mathematics as a coherent discipline I5_131 I5_301	Experiencing limited possibilities for responding to students' utterances I5_27	Good mathematics teaching recognised by active students. I5_71	Good mathematics teaching recognised by students applying mathematics in a variety of practical contexts I4_34 I4_47

Figure 6.7: Overview of labels belonging to the emergent theme *perspectives on mathematics and its role in mathematics teaching*, constituting three critical events

6.2.1 Accounts of perspectives on mathematics and its role in mathematics teaching, constituting a first critical event

As described earlier, Nora's way into the mathematics teacher profession is to some extend led by an early interest in the natural sciences. When accounting for her preferences for mathematics in own schooling, Nora points out the characteristic of having one correct answer to each problem or task. Being a future mathematics teacher, this distinctive characteristic makes it easier for her to relate to the students' learning and their progress. In addition, she finds mathematics pleasurable due to its immediate reward when solving a task correctly.

I1_283	Nora:	*I was very clear that, if I'm going to be a teacher, then I would like to teach physics and mathematics, since, you know you will have the two underlines underneath the answer, and then you can find out, OK, you haven't understood it, so then it's easy to find out what challenges you've got, instead of being the language teacher, sitting and evaluating whether the text is written good or bad.*
I1_567	Nora:	*(...) it's this really, actually, the short-term perspective. If I accomplish something and get satisfied, then it's fun.*

In contrast to gaining immediate reward, is that of working on mathematics problems which are perceived as difficult and challenging and which require greater perseverance. In her accounts of solving the interview task on quadratic functions in the context of a dog pen (Appendix B), Nora describes a relief when noticing a low threshold to the opening task of finding as many rectangles as possible with a fixed perimeter of 16 meters. She also finds comfort from solving the routine tasks in between the more demanding ones, such as sketching the graph of the quadratic function. Contrarily, the last question of finding an equation which describes the relationship between the area, A, and length, l, for any rectangle with perimeter 16 meters, was experienced as uncomfortable.

I1_1208	Nora:	*With this, I thought, this I can manage, so the thing about having a low threshold, I experienced it as very good as a teacher.*
I1_1302	Nora:	*I like a lot to evaluate and look at, kind of, what is the slope and the roots and so on in functions, but this thing of finding own [function] expressions, I think it's something being unfamiliar. That, I haven't done anything of that which I can remember in math before, and then it becomes kind of some resistance towards the new stuff. That I manage to come a long way in the beginning I think was good, and then that it was not so much resistance.*

I1_1495 Nora: *I was a bit relieved when I saw I'm just going to sketch the graph [laughing], since I started to kind of, ok, now, I've had the chance to grapple a bit, and then once more I can relax, and then get the chance to grapple with something again.*

Although she solved the task on quadratic functions correctly in advance of the interview, Nora expresses a sense of dislike when working on mathematical questions that require solutions beyond routine operations. Doing mathematics releases thus a range of rapidly fluctuating emotions, varying between appreciation and discomfort, confidence and uncertainty. A similar picture of mathematics, in terms of searching the one, correct solution on a task in order to receive immediate response on one's mathematics capabilities, suits the mathematics studies at the university. According to Nora, university mathematics is characterised by the presentation of undisputable mathematics knowledge during lectures, and the following activity of solving related problems on the weekly exercises. Contrarily, the lectures in general pedagogy and mathematics didactics at PPU require active engagement in discussions and a critical attitude to the subject content. Hence, mathematics in the context of university teacher education represents a more human discipline to be negotiated.

I1_791 Nora: *[In] mathematics, [it's like] now we're at a lecture, now we're solving the exercises, so then it's not kind of the talking which is so important in the pedagogy. (...) While in pedagogy it's very clear, do I agree with this? No, this I don't agree with it. So then you become challenged on viewpoints and knowledge on a different level.*

I1_835 Nora: *Of course, it's a change, in view of, now it's not so much focus on solving problems and check if it's correct and that the exercise is approved. There I see a difference. Now it's more talking about what is around.*

Nora's emotional accounts of mathematics are further related to her elaboration on how to approach mathematics in her own teaching. As a kind of counter reaction to her experience of incomprehension and alienation towards mathematics at the university, her mathematics teaching in school is expected to be inviting and manageable to students at all levels. Accordingly, Nora highlights the need of presenting the subject as useful in everyday life.

I1_134 Nora: *(...) something which interests me as a math teacher is how they in the last instance will see that math is used, every day.*

I1_583 Nora: *(...) the thing of knowing mathematics, it makes me strengthened in my everyday life. Just this thing of having control on a budget, knowing what's needed in order to have enough money in and out, as I'm responsible for my own economy (...) I'm not joking when stating that math is used in everyday life (...) it's a tool that can be used for own investigation without necessarily being very advanced, but also in small-scale questions.*

According to Nora, teaching mathematics in school is about making a low threshold to the mathematics content, by choosing basic examples to build on, so that everyone can be part of the mathematics activities. Hence, the mathematics classroom must provide the students with various practical contexts for encouraging the students' own methods of solution and formulations. From this point of view, "everyone knows some math", instead of being a subject accessible to the favoured few.

I1_639 Nora: *But then I think, you know math if you're able to use it in everyday life. So I think that everyone knows some math, instead of this (...) you know math only when you're in university and study it, so maybe I have kind of a low threshold for saying that you know math.*

I1_679 Nora: *How could I put it into a different context and work with that it should be a low threshold, so that everyone could try it out (...) there are challenges the whole time, you have to remember to lower yourself so incredibly many levels, so this thing of always knowing that you must use a basic example, more basic than what you think, and instead, to upgrade.*

Nora's preference of making the subject simple, easy and understandable for all students is further present by her expressed empathy with students' difficulties in mathematics. Due to own struggles during her subject studies at the university, Nora is able to relate to the students' problems and their frustration when learning mathematics in school.

I1_165 Nora: *(...) a little advantage for me, since I have during my studies been sitting there and knowing that I've failed, and I've really been there and not gotten it. I can't say I know how it's like, but still, I know this frustration when you find math boring because you don't understand it.*

A similar concern about students' various needs and their challenges when learning mathematics is present in Nora's accounts of entering the vocational mathematics classroom in her second period of school placement. In order to adapt to the students' difficulties, and yet, to present the subject as accessible to them, Nora deems it necessary to make the students work with the mathematics content beyond solving tasks in the textbook. While she at the lower secondary school could follow the pace of readymade teaching plans and work topic by topic in the textbook, the vocational students' problems with using the textbook makes it necessary for her to make own plans based on the curriculum.

I2_317	Nora:	*This thing of thinking about, what can I do today to make them learn something, it's been a hard nut to crack at that school. And like, when I was at lower secondary school, it was kind of easier when I had algebra as the topic, it was like, well, then we multiply brackets, and yes, that's what we do, but now it was more like, I spend the time more freely, and I was more concerned about following the curriculum than the textbook, based on the premises of the building and construction technology class. Since, that is what I found almost more appropriate than sitting and looking at the regular textbook saying this you should do, and when they don't even manage to read the first sentence, that doesn't make the subject more accessible.*

According to Nora, mathematics teaching in the vocational education programme should contain the students' discovery, play, discussions and explorations from a range of mathematics activities. Her emphasis on a safe learning environment, with several opportunities for exploring mathematics and developing one's autonomous mathematical thinking appears from her statements of "showing them the variety in mathematics" and "training some of the independent thinking".

I2_814	Nora:	*I want them to experience, showing them the variety in mathematics, that it's not just always being good at solving tasks which makes you clever in math. (...) and then meeting them with rich problems which have different possibilities (...) training some of the independent thinking. Since that's the difference, when I sit with a math book, this is how to do it, and then it's the list with right answers, OK, so then I finish it, and if I didn't manage, then I don't know why, and then you continue. (...) But bringing up that there are different mind sets which then make more people clever in math.*

In order to increase the students' interest and to facilitate greater involvement in the mathematics lessons, Nora refers to a range of mathematical activities in which the students "are supposed to meet mathematics not only by making calculations". During her periods of school placement, she has therefore used a variety of resources in her mathematics teaching, such as a readymade teaching plan of modelling, a geometry game and Kahoot quizzes.

I2_238	Nora:	*I tried modelling, too, which went down the drain [laughing]. I tried to let them shoot with rubber bands, I found a set of exercises, then they were supposed to find out how far they could shoot with the number of x rubber bands (...) you are supposed to meet mathematics not only by making calculations, but you have also this thing of expressing yourself in writing and orally and make presentations and communicate it.*
I2_185	Nora:	*I'd decided they were supposed to play the math game, it's kind of a ladder game with geometrical shapes, since we had worked with geometry, and then I thought I could use it as revising (...). I made the choice in order to see whether they could grasp it in a different way, than just that calculation, since calculating for them was very challenging.*
I2_433	Nora:	*Then I used Kahoot*[17] *as a repetition. (...) tried to make kind of, different ways of meeting the topic.*

Nora's accounts of aiming for inclusive and varied mathematics teaching in school are in some sense concurrent with her previously discussed dissociation with university mathematics and further reconnection with the discipline through teaching. In line with previous results regarding mathematics as an attractive and repulsive discipline, Nora portrays a cold and remote university discipline with a need for transformation into an accessible and human school subject. The current accounts give as well a deeper insight into Nora's expressed perspectives on the nature of the discipline and its distinctive characteristics as a teaching subject. Due to her elaboration of how to approach mathematics in own teaching, I perceive her current accounts to constitute an initial critical event in her evolving story of *perspectives on mathematics and its role in mathematics teaching*. The movement away from a remote towards an approachable discipline is in the subsequent section analysed on the basis of Nora's expressed authority for knowledge in mathematics and its teaching, and her preferences for a progressive mathematics teaching tradition.

6.2.2 First critical event: Moving away from a remote towards an approachable discipline of mathematics

When accounting for her reasons for becoming a mathematics teacher, Nora emphasises the subject's characteristic of providing immediate reward when working on tasks. By having "two underlines underneath the

[17] Kahoot is a free, game-based student response system for creating multiple choice surveys. For more information, see https://getkahoot.com/

answer", the discipline of mathematics differs from other school subjects, such as language and social sciences, by holding a unique position concerning its absolute truths. Hence, Nora expresses an absolutist perspective in which mathematics is regarded as certain, unquestionable and objective. The absolutist nature has further consequences for what is meant by doing mathematics. While mathematics may provide immediate reward, it contrarily generates feelings of deficiency if one does not succeed with solving a task. This is exemplified by Nora's emotive reactions when working on the interview task on quadratic functions and her sense of discomfort when facing a non-routine problem. Doing mathematics seems therefore in Nora's case to be about solving problems with only one correct answer, which in turn provides an immediate response and confirmation on her capabilities. Hence, mathematics is to her a static body of knowledge depending on external authority (Ernest, 1991; Povey, 1997). This authority is both the owner of certain mathematics knowledge and the judge of whether one's approach to the mathematics content is appropriate or not.

Bibby (2002) suggests a connection between experiencing mathematics algorithmically, procedurally and as a fixed discipline, and experiencing shame in mathematical contexts. Shame is here understood as a reaction to other people's criticisms, as a signal to find out whether one is being excluded or submerged within a social group (e.g. the group of secondary school mathematics teachers). In Nora's case, the situation of being asked to solve a mathematics problem in advance of the interview may lead to a sense of discomfort and shame if bringing up a thought of whether she is capable of reaching the one, correct answer in an elegant and effectively way. In other words, doing mathematics might for Nora lead to the fear that the apparent objectivity of mathematics will reveal subjective inadequacy in mathematics (Bibby, 2002).

A portrait of the unique position or status of the mathematics subject is further apparent in Nora's comparison of being a mathematics student at the university versus being a student teacher at PPU. Nora's accounts of the distinction between the discipline of mathematics and the social sciences are comparable with Isaac's accounts concerning the two cultures (Snow, 1963) at the university. As discussed in Section 5.2, Isaac drew a dichotomy between the pure, perfect and absolute knowledge of mathematics and the corrigible, imperfect and fallible knowledge of the general pedagogy. However, while he was initially attracted to a mathematics discipline free of human bias, Nora expresses a preference for the enterprises at PPU belonging to the social sciences. To her, the general pedagogy voices a different kind of authority, where meaning is negotiated. In contrast to the mathematics subject, which is either understandable or incomprehensible and therefore shows signs of being a cold, hard and remote discipline, the general pedagogy appears as humanistic and welcoming. Nora's re-connection with the subject of mathematics can then be explained on the basis of her negotiability within the *community of secondary mathematics teaching*. Mathematics in the context of teaching is according to Nora represented by knowledge which is socially constructed. By engaging in the practice in shape of acting critical in discussions at PPU, she is able to assert ownership to the negotiated meanings of mathematics teaching and learning. In contrast to the shared ownership is the negotiation of meaning in the *community of university mathematics* and the related subject studies. Due to Nora's voice of external authority, the meaning of mathematics is produced and owned by others, such as professionals at the mathematics department. This externally given meaning is further adopted by Nora in order to complete the weekly exercises and the final exam. However, her lack of ownership of meaning within the mathematics community results in a position towards the periphery.

Nora's accounts display further a student-centeredness in line with her earlier described interest of students' learning and their needs (see Section 6.1). Her marked distance to university mathematics is thus mirrored by her desire for making a low threshold to mathematics in school. In other words, Nora's experience of defeat and deficiency during her own mathematics studies are to be transformed into students' experiences of mastery and capability in her mathematics teaching. Her consideration of providing an appropriately structured environment for the students' learning of mathematics is in line with what Ernest (1991) denotes as the progressive tradition in mathematics education. It is based on an absolutist perspective on the discipline of mathematics, in which mathematical truth is viewed as absolute and certain. However, as the tradition is built on values of empathy, caring and human dimensions of situations, great emphasis is attached to the role of the individual in coming to know this truth. Hence, mathematics education is concerned with developing students as autonomous inquirers and knowers in mathematics, as well as fostering "confidence, positive attitudes and self-esteem with regard to mathematics" (Ernest, 1991, p. 191). One important feature is therefore students' self-expression in mathematics, concerning encouragement of their own methods of solution and formulations. This is exemplified by Nora's statement that "everyone knows some math" (I1_639), in contrast of being accessible to the favoured few.

The preference of making mathematics simple, easy and understandable for all students can further be related to the act of protecting or defending students against mathematics. According to Hodgen and Askew (2007), the belief that teachers' own difficulties with mathematics enable them to better empathise with and understand students' difficulties, might lead to protecting students from struggle. However, a focus on the experiences of the students when doing mathematics may lead to a student-centeredness which is opposed to mathematics-centeredness (Ernest, 1991). In other words, Nora's desire of presenting mathematics as useful and manageable for all students might be at the expense of developing mathematical concepts and structures to sufficient depth. Another possible outcome is overprotectiveness, in which students are shielded from the dissonance and conflict needed to provoke intellectual growth. Mathematics may as well appear as interesting and appealing to the students based on its complexity and difficulty. Further, if protection leads to emphasising mathematics as a step-by-step discipline which is made up of procedures, this can reproduce negative attitudes among the students, by being perceived as boring and irrelevant (Hodgen & Askew, 2007).

Within the progressive tradition, the related epistemological assumption is named empiricism, meaning that students' knowledge develop trough interaction with the world (Ernest, 1991). When being exposed to the appropriate experience, the full potential of the student's mathematical knowledge can be released. In Nora's case, the empiricism appears from her use of a variety of external resources in her vocational mathematics teaching, such as a readymade teaching plan of modelling, a geometry game and Kahoot quizzes. By letting the students "meet mathematics not only by making calculations" (I2_238), and making them "grasp it in a different way" (I2_185), Nora's accounts of her mathematics teaching can be related to the protective aspects of progressive mathematics education. In other words, the vocational students should be shielded from sources of negative experiences with mathematics, with Nora's terms being pure calculations. Although the attention towards the students' interests and needs represents strength of the progressive tradition, a great level of student-centeredness in mathematics teaching is also in danger of being at the expense of the awareness of the complexity of mathematics content involved. In Section 6.1, I related the student-centeredness to Nora's marked distance to university mathematics and her marginal participation in the *community of university mathematics*. Accordingly, her expressed approach to vocational mathematics teaching can be connected to own struggles, and thus, a desire of providing the students with a different and more positive impression of the subject.

Nora's description of an extensive use of external learning resources is also related to the notion of authority in her mathematics teaching. On the one hand, including a variation of mathematics activities within one's teaching practice can be an expression of participation through engagement and alignment in the *community of secondary mathematics teach-*

ing, by having adopted the practices of investigative mathematics teaching emphasised by PPU. Consequently, Nora might be moving towards central participation in which mathematics is considered a problem-driven discipline. On the other hand, an unbalanced adoption of resources for use in the mathematics classroom can be considered as receiving knowledge (Belenky et al., 1986), and thus, a sense of external authority for the meaning of mathematics teaching and learning. According to Belenky and her colleagues, learning by listening implies being open to take in what others have to offer, however, having less confidence in one's ability to speak. In Nora's case, her search for external resources leads to a range of ways to wrap the mathematics content in her vocational teaching. However, a thorough epistemological analysis of the mathematics content involved seems not to be part of her preparations. Hence, a teaching practice in which the mathematics content is considered useful, however as a kind of unrelated set of facts, rules and skills, may dominate. In terms of negotiability, Nora's ownership of the meaning of mathematics teaching and learning is thus limited. Consequently, she exercises peripheral participation in the *community of secondary mathematics teaching*.

Following Wenger (1998), the difference between marginal and peripheral participation in a community of practice can be explained by learning trajectories and their direction. In the case of peripherality, the participant either follows an inbound trajectory towards full participation or remains on a peripheral trajectory. Hence, participation aspects dominate and define non-participation as an enabling factor of participation. In the case of marginality, however, the non-participation aspects dominate and come to define a restricted form of participation, either towards non-membership or a marginal position in the community of practice. Due to Nora's marginal participation in the *community of university of mathematics* and her peripheral participation in the *community of secondary mathematics teaching*, I interpret her learning trajectories of her developing identity as *moving from a remote towards an approachable discipline of mathematics*. The critical event is shown in Figure 6.8.

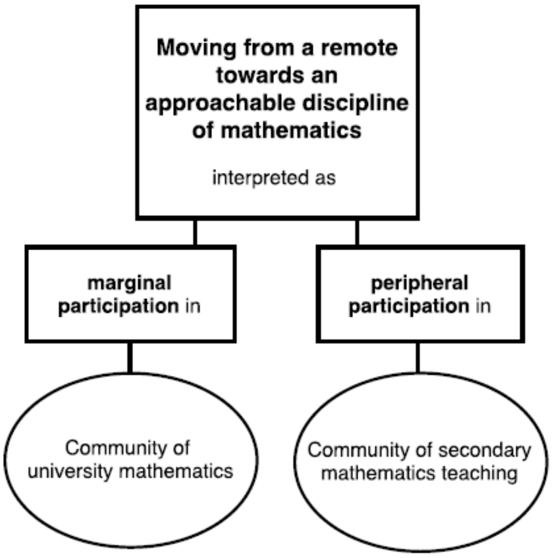

Figure 6.8: Perspectives on mathematics and its role in mathematics teaching, represented by the first critical event of moving from a remote towards an approachable discipline of mathematics

6.2.3 Accounts of perspectives on mathematics and its role in mathematics teaching, constituting a second critical event

In addition to account for her experience of undergoing school placement, Nora elaborates on the university teacher education programme, PPU, and its lectures and seminars on campus. Due to a raised critical voice towards teacher education and the mathematics didactics course, Nora's retrospective accounts of undergoing PPU constitute a critical event in her evolving story of *perspectives on mathematics and its role in mathematics teaching*. Yet, a positive aspect of teacher education is her experience of independence in her development as a mathematics teacher.

I2_513	Nora:	*The best thing was kind of experiencing the eye-opener that I can bring myself into it. (...) that the way PPU is showing us to be the right way, is maybe not the way I will follow in order to find out how I can become the best teacher and enjoy myself in that job.*

While expressing a sense of freedom in finding her own way of exercising teaching, Nora simultaneously refers to a communicated message regarding the teaching and learning of mathematics being transmitted in the curriculum and by lecturers at PPU. The message, concerning inquiry based mathematics teaching, is perceived as "the only right thing" when exercising mathematic teaching in the classroom. However, since it represents a so far unfamiliar perspective on mathematics teaching and learning for many of the student teachers, it causes also frustration.

I2_163 Nora: *The way we are presented to how to teach mathematics, it's typically, inquiry based mathematics teaching is the only right thing and the only way to do it. And there were someone, I think, who found this very challenging since they don't recognise themselves in it, and then they become insecure about how to teach since they don't manage it (...) so I'd wished he could just say it, from the beginning, why he highlight this, since that is what we need to practice, which can help us as math teachers to actually challenge ourselves to meet the teaching in a different way, than hiding behind the textbook.*

Although Nora considers herself as someone who manages to relate to the views on mathematics teaching and learning being stated in the mathematics didactics course, she still delineates a situation of being forced into a mind-set she does not necessarily fully agree with. The impression of a compelled adaptation exists in the shape of the student teachers' shared view on PPU as representing one, true learning perspective which they are expected to follow.

I2_661 Nora: *As long as you're answering what the tutor wants, and answering what the lecturers desire and expect, kind of guess what the teacher is thinking, then it's satisfying (...) making us think the same, instead of challenging us to see it differently (...) if you're a teacher who appreciates assessment with giving marks and you're more kind of an individual type, then you're kind of one who doesn't fit into this new mind set.*

Despite having discovered resonance for her desire of exercising varied mathematics teaching detached from the textbook, Nora tends to dissociate with what she depicts as the teacher education's given truths regarding good mathematics teaching. This is exemplified by her description of educational research as a swinging pendulum. Since the governing understanding of good mathematics teaching is depending on the trends within educational research, there are no teaching and learning perspectives being truly "better" than others.

I2_700 Nora: *Such as the learning perspective, it's a pendulum, very often, you're at the one extremity, that now it's just through language and interaction we are learning, and maybe then, [you will] return to, now it's just individual work and rote learning that counts, meaning, it's always kind of a swinging in what mind set one is focusing at. (...) meaning, it's only depending on what's the trend.*

In the third interview, and at the time of Nora's entrance into school as a substitute mathematics teacher, she describes a similar experience of having been fed with an unbalanced mind set in relation to mathematics teaching and learning. This mind set further stands in contrast to the reality she is facing in the lower secondary school.

I3_269	Nora:	*It was too much focus on doing inquiry based mathematics teaching. It was so much focus on it, that meeting the regular teaching situations which we have seen later, it's a complete crash. (...) So it's difficult in a way, to manage to hold on to that idea, of me wanting to do that kind of teaching, but then it has to be squeezed into what is the regular rhythm and thought.*
I3_291	Nora:	*So I find it a little uncomfortable that you in a way are fed to believe that, in this way, you create teaching, and then you meet the school and then this thing of, no, these new ideas, and that glow you've got, it's getting destroyed by the system. (...) I wish it could be more focused on, that it's OK with regular teaching, too (...) so we get more balance, but I'm happy we were kind of overfed with the other mind set so we are maybe aware of the variations and possibilities that exist, so I will not become rooted in an old fashioned blackboard instruction in mathematics.*

By describing the transition from mathematics teacher education into school as a "complete crash", Nora delineates a gap between the idealised world being portrayed in the mathematics didactics course at PPU and the reality that unfolds in the mathematics classroom. Based on her first teaching experience as a certified mathematics teacher, Nora thus accounts for a stagnation regarding her desire of confronting the regular blackboard and textbook-led mathematics instruction.

6.2.4 Second critical event: A raised inner voice in the debate of a desired mathematics teaching

In the above interview excerpts, Nora accounts for self-reliance, as well as both consensus and dispute, regarding ruling perspectives on mathematics teaching and learning stated by teacher educators at PPU. This sense of independency in balance with the need of adjustment when developing a mathematics teacher identity can be investigated on the basis of Nora's ownership of meaning within the *community of secondary mathematics teaching*. In the analysis of the first interview, I accounted for her alignment to the practices of approaching the mathematics subject from the learner's perspective and adapting the content to the students' needs. Further, I explained her membership in terms of her asserted ownership to the negotiated meaning of mathematics teaching and learning at PPU. In contrast to a sense of external authority and given meanings of mathematics within the *community of mathematics* during the subject studies, Nora experienced having an own, critical voice within the mathematics didactics course. However, in the above interview excerpts, she expresses a somehow different position regarding the ownership of meaning. Although having discovered resonance for her desire of exercising varied mathematics teaching detached from the textbook, she simultaneously dissociates herself from the teacher education's given truths of what constitutes good mathematics teaching. Hence, I interpret Nora's accounts of the lecturers' and tutors' ownership of truth as a description of an external authority she cannot fully accept. Instead, she asserts ownership of the meanings of mathematics teaching and learning by trusting an internal authority, which she considers to be somehow independent of the message communicated by PPU (Povey, 1997). This is visible from her statement "that the way PPU is showing us to be the right way, is maybe not the way I will follow" (I2_513).

Another example of a strengthened inner voice is Nora's portrait of educational research as a swinging pendulum. Instead of functioning as a space for questioning the nature of mathematics, its complexities and role in teaching, the mathematics teacher education represents to Nora a repertoire of teaching styles and approaches for discovering what suits her. An essential part of the community's repertoire is inquiry based mathematics teaching, being positioned by Nora towards the extreme of the pendulum. Although she identifies with the need of providing students with a rich mathematics learning environment, in which PPU functions as an inspiration for new ideas, inquiry based mathematics teaching does not represent to her a greater epistemological position to be negotiated. Since critical judgement of both internal voice and external authority is limited, Nora's participation within the *community of secondary mathematics teaching* seems to remain at the periphery.

I have earlier focused on Nora's tendency to move outwards for external resources when exercising teaching, and hence, in direction of received knowledge of the teaching of mathematics. This stands in contrast to her tendency of rejecting external authority in mathematics teacher education, and instead, relying on the authority of self. Such combination of external and internal authority could represent an inconsistency in Nora's developing mathematics teacher identity. However, I argue instead for a consistency in terms of Nora's limited negotiation of meaning within the *community of secondary mathematics teaching*. On the one side, Nora's peripheral participation due to the trust of an external authority is mirrored in a trial and error strategy for developing her mathematics teaching practice, based on her use of external teaching resources. Here, successful student learning tends to be evaluated by the students' engagement and enjoyment, rather than learning outcomes concerning their awareness of the complexities of the mathematics content involved. On the other side, Nora's peripheral participation related to the trust of internal authority is apparent from her reliance on personal qualities and characteristics, such as empathy, caring and patience. Appropriate personal qualities are undoubtedly valuable for becoming a successful mathematics teacher. Yet, an underlying expectation might be that the development of one's mathematics teaching happens from practical experiences in the classroom, rather than from being informed by scholarly knowledge within the university context. Despite the importance of mathematics teacher education for Nora to re-connect with mathematics and choose to become a teacher, her negotiation of its theoretical foundation seems to be less prominent when developing an identity as a mathematics teacher.

Nora further delineates a discontinuity between the idealised world being portrayed at PPU and the reality she meets in the mathematics classroom as a substitute mathematics teacher. This movement across community boundaries is related to her peripheral participation in the *community of secondary mathematics teaching* and her marginal position within the *communities of students and their regular teacher* at the school of employment. I have accounted for Nora's peripheral participation in the former practice based on a limited negotiation of meaning. Despite her alignment with a teaching practice that confronts what she perceive to be a reigning blackboard instruction in mathematics, Nora is not in a position of negotiating the community's joint enterprise. Hence, the participation does not result in broadened perspectives regarding mathematics and its role in mathematics teaching. Contrarily, her marginal position in the communities of students and their regular teacher leads to restricted engagement and an experience of being "squeezed into what is the regular rhythm and thought" (I3_269). Since the discontinuity between mathematics teacher education and substitute mathematics teaching is not overcome, Nora undergoes learning in terms of identification (Akkerman & Bakker, 2011). The learning potential resides in Nora's renewed sense-making of mathematics teaching and learning in lower secondary school, in light of her substitute mathematics teaching. Yet, the result is a reinforcement of her existing mathematics teacher identity, and a sense of difficulty regarding holding on to her idea of exercising inquiry based mathematics teaching.

Based on Nora's limited negotiability, I characterise the current critical event as *a raised inner voice in the debate of a desired mathematics teaching*. Although the development of an inner authority regarding mathematics teaching is a step forward in her teacher career, there are as well remnants of absolute perspectives in Nora's accounts. In others words, there are right answers regarding mathematics and its teaching, but "the fountain of truth (…) has shifted locale" (Belenky et al., 1986,

p. 54). Consequently, Nora continues to exercise participation at the periphery of the mentioned communities, which in turn restricts her development of an identity as a mathematics teacher, appearing as experience of stagnation in own teaching. In Figure 6.9, I relate the critical event to Nora's limited negotiability within the *communities of students and their regular teacher* and the *community of secondary mathematics teaching*.

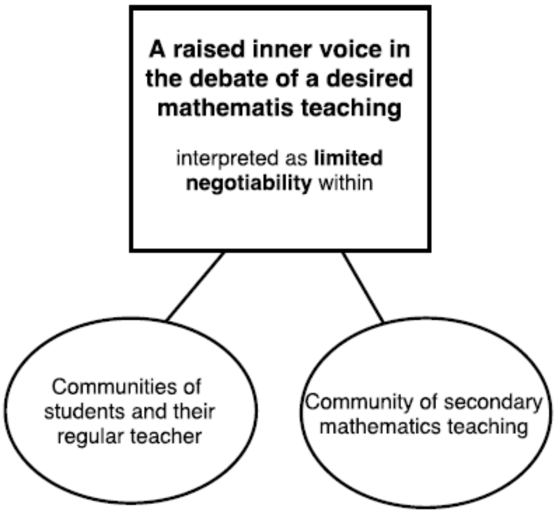

Figure 6.9: Perspectives on mathematics and its role in mathematics teaching, represented by the second critical event of a raised inner voice in the debate of a desired mathematics teaching

6.2.5 Intermezzo: Two teaching sequences on measurement

I met Nora for observation of her mathematics teaching in March, in connection with a four weeks long engagement of mathematics teaching at grade 9 in lower secondary school. The two 60 minutes teaching sequences on measurement constituted the first lesson in a project concerning the planning and building of a 1/75 scale model castle in cardboard. The classroom observations were related to the same teaching plan, however, carried out in two different student groups. The description of the project on measurement and its competence aims are shown in Appendix E. Due to the given criteria, the castle to be modelled was supposed to be enclosed by a 20 meters tall wall, containing a 4 meters broad and 7 meters tall gate. Further, it had to contain a knights' house with base of 40 square meters, a 1,5 meters tall cylindrical water well with diameter of 2 meters, a 25 meters tall tower with a quadratic base and sides of 4 meters, and a 10 meters tall flagpole with a 6x4 meters flag on top of it. The students were supposed to make a 1/50 scale drawing of the base of the knights' house. Further they were asked to find the area of the base of the entire castle, to calculate the surface area of the knights' house, to calculate the area of the gate in order to estimate the painting needed for its front side, and to calculate the volume of the water well and the tower. In addition, the following task was given at the end of the task sheet:

> Imagine that the circumference of the wall of the castle is 200 m, and that the base is quadrangular. What shape of the base do you think give the greatest area? Show your answer by examples.

In addition to build the 1/75 scale model castle in cardboard, the students were supposed to hand in a report including the necessary drawings and calculations. They were also asked to explain the concept of volume, and to list up other relevant mathematics concepts used in the project, with explanations.

6.2.5.1 The teaching sequences

In the first observed lesson, the measurement project is introduced briefly to the class by Nora's presentation of the task sheet and the related competence aims. She informs the students that they can find help and information to do the needed calculations in the textbook, in addition to use her as a consultant. Further, the class is divided into groups of four students each. Nora presents the equipment available for building the castle, and the students start immediately to make drawings and to cut the cardboard. One student asks about the surrounding wall of the castle: "20 meters tall, does it mean 20 centimetres?" Nora responds to the whole class: "One hint is that this task is about scale." In some groups, the students start to cut the cardboard before any drawings or calculations are made. In other groups, the students are divided into two parts, in which some are responsible for doing the tasks on paper, and the others are responsible for cutting the cardboard. Another group makes drawings and calculations together, and wait a long time before they pick up the equipment for building the castle. The following utterances can be heard within the various groups:

> Student 1: *Nora, you need to show us how we're going to do this, since no one understands it.*
>
> Student 2: [Said to the other group members] *You can do the tasks, and I will build the castle.*
>
> Student 3: [Said to the other group members] *I'm the one doing all the tasks here.*
>
> Student 4: *Look, the others have already started to cut the cardboard, so then we should start to do it, too.*

Nora is walking from one group to the next, responding to the students' questions. The lesson ends the moment another teacher enters the classroom and asks the students to bring with them their textbook for the upcoming lesson in Norwegian language.

A second introduction of the measurement project is made two days later, in another group of grade 9 students. Due to experiences of students' distraction when having access to the equipment for building the castle, Nora has decided not to hand out the cardboard and scissors until the second lesson. Further, the students in the first group had problems with using scale in the planning of the castle. She has therefore decided to introduce the project with a whole-class explanation of the notion of scale. Nora starts the second lesson by writing three tasks on the blackboard, with corresponding figures:

> Find the area of a circle with radius 2
> Find the volume of a [rectangular] prism with sides 2, 2 and 3
> Find the volume of a cylinder with radius 2 and height 4

She gives the students small paper sheets to write down their answers. Some students do the tasks quickly; others ask out loud whether someone remembers the formulas. After a couple of minutes, Nora asks the whole class whether someone can present their solution. One student answers $\pi \cdot r^2$ on the first question, with the explanation "that's the way you do it". Another student tries to give an explanation to the formula, by saying: "r^2 gives almost one fourth of the circle, and then one multiplies by 3 and the final fourteenth which gives the whole circle". Nora responds that she can understand some of the explanation and wonders whether the students know what the number π means. One student says it is roughly equivalent to 3,14. Nora responds that π is a number that equals the ratio of a circle's circumference to its diameter, and that she will bring a rope in the next lesson to show it to them. They continue with the other tasks on volume of a box and a cylinder. Nora explains that volume is related to three dimensions. First, you find the area of the base, defined by lengths in two dimensions, and then one can imagine that there are many such bases on top of each other that constitute the height of the box or the cylinder.

Next, she writes on the blackboard "Scale 1:100 000". She asks whether someone has seen it before. Some students respond that they have seen it on maps, where it means that 1 cm on the map relates to 100 000 cm in the terrain. She shows a map of Europe, with scale 1:500 000, and asks: "If a distance on the map is 12 cm, how long is the distance in the terrain?" One student answers that you can multiply 500 000 with 12. Nora responds that the number will be very big, so instead, one can write 500 000 cm =5 km, and then multiply 5 km with 12, which gives 60 km in the terrain. She then asks the following:

Nora: *If we know the distance in the terrain is 45 km, what is the related distance on the map, with the scale 1:100 000?*

Student: *It's the same. The scale is 1:100 000 cm equals 1 km. That gives 45 cm on the map. You just multiply with 45, so it's 45km on the right side, which gives 45 cm on the left side.*

Nora confirms that the solution is correct, and introduces the project with the 1/75 scale model castle. Since the building equipment will not be handed out until the next lesson, the students are supposed to do the preparations needed in order to make the castle fit with the given criteria. The students are divided into groups and they start to work on drawings of the castle. In some of the groups, the students work on ideas for the design of the castle, by drawing suspension bridges, moats, towers and flags. Nora encourages them to answer the questions stated in the task sheet, since they need these answers in order to make up the final report. At the end of the lesson she asks the students to clean up their desks, and she collects the papers with the students' solutions to the introductory tasks on area and volume.

6.2.6 Accounts of perspectives on mathematics and its role in mathematics teaching, constituting a third critical event

In this section, I present Nora's accounts from both follow-up interviews (Interview 4 and 5) for the classroom observations. In addition to generate accounts about the teaching sequences, the fifth interview functioned as a retrospective glance on the whole one and a half years period in which I followed Nora. Due to her expression of a divide between desires of own mathematics teaching and her possibilities for performing it in practice, I consider her current accounts to constitute a critical event in the story mathematics and its role in mathematics teaching.

Nora has received the above described project description from a teacher colleague, when searching for a practical assignment on the topic of measurement. She makes a few justifications to the original plan, concerning the report that the students are supposed to write from the project. In order to have a product to hand over to the regular teacher, as a basis for assessing the students' learning within the topic, she expands the report with questions regarding the meaning of the concepts of volume and scale.

I4_93	Nora:	*I thought I have to document what we've been through, and that's the idea with what I've added at the end, that they're supposed to write, what is a volume, try to describe it, whether they're left with an understanding of what they've been through in mathematics (...) what is scale, and to see that they've used it.*

Her intention with letting the students build a scale model castle in cardboard is to give them a practical experience in mathematics, being different from what she conceives as their usual textbook-led task solving. From making the necessary drawings and preparations when planning the design of the modelled castle, the students are given the possibility to use their mathematical competences, including using the given scales and calculating the area and volume of various geometrical shapes.

I4_34	Nora:	*(...) now they're going to use the mathematics, instead of just sitting there and doing tasks, since that is what they've been doing a lot together with the other teacher, so then the aim is to bring them out of their comfort zone (...) and then the next lesson becomes a work session (...) and then, if they work fast enough, the third lesson will be a final project work and then write down what they have calculated and to get to evaluate, afterwards, what they've used, what they've got from it..*
I4_47	Nora:	*The mathematical goal is this thing of giving them the experience of using scale, that it's not only a task in the textbook, and that thing of getting the chance to calculate volume (...) so, the idea is to let them work with volume, repeating this thing of calculating the area and to get to know scale and to use it.*

Since the building of the castle itself does not involve any mathematics operations, the project is in many groups organised by separating the calculations from the cutting of the cardboards. Hence, some students are responsible for finding the right measures, while other students are left with cutting and pasting. During the first implementation of the project, Nora observes this phenomenon. She therefore decides not to hand out any building materials in the other student group until their second lesson, so that all members of each group take part in the necessary preparations.

I5_71	Nora:	*I saw that in some groups, someone started to do the calculations and others started to draw and to be kind of, this is fun, then I don't need to do math. So then you saw that this split is maybe not so good, so then the question is how to make sure that everyone is doing the calculations, and this understanding of what is going on, that it's kind of a thought behind it.*

When working on the criteria for the model of the castle, Nora has further observed that many students struggle with transforming the measure of 40 square meters for the base of the knights' house into a correct 1/50 scale drawing. In both the observed lessons, she encourages the students to factorise the number 40, so they can determine the lengths of the base of the house (given a rectangular shape of the base and integer lengths) and in turn use the scale factor of 0,02 to find the correct measure.

I5_81	Nora:	*Almost everyone become speechless when they are asked to make a house with the base of 40 square meters. In every group, it's like, this I'll have to convert into square centimetres and then I'll have to use the scale, so what do we do when we measure it? That I've had to explain to most of the groups, and kind of asked like, but if we don't convert it, this thing of knowing what is a base (...) what is it that makes 40, some said five times eight (...) and then, OK, if you say it equals five times eight, then we have already moved from the second to the first.*

Based on her experiences of the students' problems of using the given scale factors, Nora chooses to introduce the second implementation of the measurement project with what she denotes as a set of warm-up tasks. In addition, she makes a short introduction to the notions of scale and scale ratio. Her intention is to make the students become focused on the mathematics content needed for being able to solve the practical task.

I5_50	Nora:	*When it's the warm-up tasks, it was this thing of them being more connected to the current topic, so, now they've started to think a bit, and that thought they can bring further into the lesson, so I thought it could be a kind of read thread.*

As previously described, the introduction tasks lead to a small discussion between Nora and a student, concerning why the formula of the area of the circle is correct. The student argues that the part r^2 of the formula gives a bit more than one fourth of the area of the circle. By multiplying with the number pi, one gets the area of the whole circle. Nora mentions the episode in the follow-up interview and points to the difficulty of addressing students' explanations in order to initiate whole-class discussions. Through dialogues with the students, she aims for developing their understanding of the mathematics content beyond their sheer memorisation of mathematics formulas. However, Nora finds it challenging to build on the students' contributions towards thorough mathematical reasoning.

I5_27	Nora:	*I want to have more discussions in the classroom and try to kind of initiate a conversation, and I did try it, but then I notice that I am not quite ready for it yet, I don't have the experience of getting to highlight more what they say and follow the threads they're bringing in, so that I thought about, that I wasn't totally satisfied with it. (...) I start and ask kind of, why do you say that it equals* $\pi \cdot r^2$*, kind of, what is the reason behind it, is it only because they know the formula or are there other things they've learned about the circle. And to be able to build further on that conversation. That is something I want to be better in doing and to be trained in. (...) I notice that I didn't follow it further, and then it was kind of, I accept the answer, and then continue.*

In her review of the teaching sequence on measurement, Nora further points to the general challenges of being a substitute mathematics teacher, in terms of not being involved in the preceding and following mathematics teaching. Due to her temporary attendance, she is set to teach mathematics as chapter by chapter in the textbook, in other words as a disconnected set of topics and ideas. Consequently, she finds it challenging to communicate mathematics as a coherent discipline.

I5_131	Nora:	*Like when I'm assigned this topic in mathematics, then I don't get to follow up the context, from what they've been doing before, how does it relate to what they're doing now, how is that supposed to be brought back to the other teacher they've had.*
I5_301	Nora:	*(...) how to bring in a topic from one chapter into another chapter and think that there's a whole, instead of, now it's functions, now it's volume, now it's area, meaning when it's so split up, and then I can't think kind of, in what way can I use the math more practically.*

6.2.7 Third critical event: Disparity between own desires and possibilities in practice

Nora's accounts of implementing the teaching activity on measurement in two grade 9 classes provide insight into her negotiation of mathematics teaching through participation in the *community of students and their regular teacher* and the *community of secondary mathematics teaching*. Negotiation of mathematics teaching in the former community concerns her expansion of the final report of the project, based on the need of having a product to hand over to the regular teacher. Further, negotiation related to Nora's classroom management appears from her organisation of the building material and her choice of postponing the building in the second student group. This decision was taken due to the division of labour within the smaller student groups, in which some of the students got away with only cutting the cardboard and not taking part in the mathematical tasks.

A third case of negotiation takes place in the intersection of both communities of practice, concerning Nora's guidance on the students' work on scaling and her choice of bringing in an introduction with warm-up tasks on measurement. When responding to the student's explanation of the area of the circle, there is a divide between Nora's expressed desires for her whole class teaching and the actual teaching that unfolds in the classroom. While the warm-up tasks represent mathematics as a set of disconnected rules or facts to be remembered, her review of the teaching sequence expresses a preference of not letting the students get away with merely memorising mathematics formulas. Hence, when seeking the students' reasoning of why the formula for the area of the circle is correct, she struggles with reacting to their explanations. Instead of recognising the student response as an explanation of why the formula *might* be correct, and not accepting it as a fully valid argument for the formula, she chooses to end the discussion and postpone the problem to the next lesson. Another case of a negotiation of the mathematics content is Nora's simplification of the mathematical problem of making a valid scale drawing of the knights' house. One question to be asked is how scaling is done in respectively two and three dimensions for any geometrical shape. In other words, the students could be given the opportunity to find out with what factor the area and volume is growing when a given shape is scaled with a factor n. However, in the described classroom episode, the students are instead bypassing the possibility of investigating multiplicative relations and the concept of scaling by factorising the base of the house and scaling the related lengths and width of the base.

I interpret the trend of inconsistency between Nora's actions and her described intentions to be related to her expression of external authority for the meaning of mathematics teaching and learning. In previous analysis, I have used the notion of external authority for describing Nora's tendency to move outwards and adopt external resources in her teaching. The lack of negotiation of meaning of mathematics and mathematics teaching was understood as an expression of peripheral participation in the *community of secondary mathematics teaching*. In her accounts of the present mathematics teaching, a similar situation seems to occur when she adopts the activity on measurement from a colleague. In line with a progressive perspective on mathematics and its teaching, Nora provides a practical context for the students to do mathematics. But although building a scale model castle involves the use of various mathematics formulas and operations, the activity unfolding in the classroom nevertheless strengthens the impression of mathematics being mainly memorisation of a set of disconnected formulas and procedures. In their work on the measurement project, the students are asked to plan and build a scale model by using two different scale ratios of respectively 1/75 = 0,01(3) and 1/50 = 0,02. In addition, they are asked to find a variety of areas and volumes of the modelled constructions, such as the volume of the water well and the surface area of the knights' house. However, these measurements are not needed for being able to build the castle. The mathematics activity of the project is therefore mainly based on repeatedly using a set of mathematics formulas, rules and procedures, which in turn are used for building a valid scale model. The consequence is a marginalisation of the mathematics content concerning multiplicative reasoning and scaling in one, two and three dimensions. A similar approach to the mathematics content is made in the warm-up tasks and the introduction of scaling in maps. Despite Nora's desire of moving away from rote learning and unconscious use of rules and formulas, her choice of examples bring a focus on application rather than understanding. Due to a limited ownership of the meaning of the mathematics content involved, in the shape of a lacking epistemological analysis of the subject content, Nora's leeway for exercising her desired mathematics teaching becomes narrowed.

In the analysis of the third interview with Nora, I interpreted her participation in the classroom community to be marginal. This was related to non-participation in the joint enterprise and shared repertoire of mathematics teaching and learning being mainly negotiated by the students and their regular teacher. In the above interview excerpt, the absent ownership of meaning appears as accounts of undesirable mathematics teaching and lack of opportunities for managing the subject as coherent. Due to the absence of inbound participation in mathematics communities where negotiation of the meaning of mathematics teaching and learning is part of the practice, the development of Nora's mathematics teaching practice is restricted. Consequently, it becomes a demanding task for Nora to close the gap between her desires for own mathematics teaching and her possibilities for performing it in practice. The related critical event of a *disparity between own desires and possibilities in practice* is in Figure 6.10 related to her continued limited negotiability in the *communities of students and their regular teacher* and *the community of secondary school mathematics teaching*.

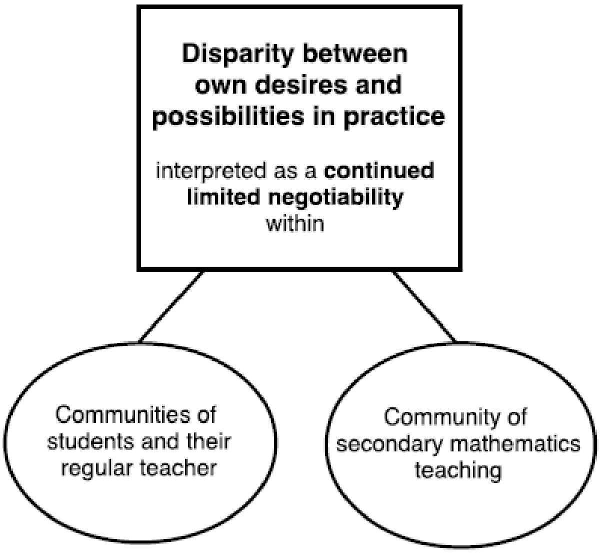

Figure 6.10: Perspectives on mathematics and its role in mathematics teaching, represented by the third critical event of a disparity between own desires and possibilities in practice

6.3 A summary of Nora's stories of becoming a secondary school mathematics teacher

In line with the case of Isaac, I review the narrative progression across the critical events of Nora's two evolving stories. The intention is to give a more comprehensive picture of Nora's developing mathematics teacher identity, and in the next round, to compare the narrative development of secondary school mathematics teacher identities across the three cases.

Nora's evolving story of *mathematics as an attractive and repulsive discipline* reveals a steady situation across the two critical events, in terms of marking a distance towards the university discipline of mathematics, yet expressing attraction towards mathematics in the context of secondary school teaching. Hence, Nora shows a sustained marginal participation in the *community of university mathematics*, and continued peripheral participation by alignment and imagination within the *community of secondary mathematics teaching*. However, her entrance into school as a substitute mathematics teacher indicates also regression, in the shape of limited negotiability within the classroom communities' enterprise of the teaching and learning of mathematics. Looking at the story as a whole, Nora's re-connection with mathematics through teaching at PPU can be considered powerful enough for initiating the development of a mathematics teacher identity. Yet, the lacking opportunities for negotiating the meaning of mathematics when working as a substitute teacher lead, for the time being, to stagnation regarding further professional development.

The story of *perspectives on mathematics and its role in mathematics teaching* shows initial progression, by Nora's movement away from a remote university discipline towards an approachable discipline for teaching in school. Through identification with what she regards as a warm, human and manageable mathematics enterprise, she exercises peripheral participation within the *community of secondary mathematics teaching*. Further, when undergoing the transition from teacher education to her professional debut in school, she demonstrates a raised critical voice and a distance towards the teacher education's given truths regarding good mathematics teaching. The expression of inner authority appears through Nora's delineated discontinuity between the idealised world of PPU and the reality of the mathematics classroom. Although a raised inner voice might indicate identity development by progression, Nora still exercises limited ownership of meaning regarding the theoretical foundation of PPU. Consequently, her peripheral participation in the *community of secondary mathematics teaching* is sustained. The transition from first to second critical event shows therefore boundary crossing by identification, resulting in reinforcement of Nora's existing mathematics teacher identity and an experience of being "squeezed into the system" (I3_269).

A similar situation of stagnation applies for the transition into the third critical event, when Nora is working part-time as a substitute mathematics teacher and accounts for a disparity between own desires and possibilities in practice. Being involved in mathematics teaching practices that are mainly negotiated by other actors, Nora is put in a marginal position within the *community of students and their regular teacher*. Since she does not exercise inbound participation in communities where mutual negotiation of mathematics teaching and learning is part of the practice, the development of Nora's mathematics teaching practice is restricted. The constancy of her mathematics teacher identity is shown by the continuation of her expressed perspectives regarding the nature of mathematics, its teaching and learning, being in line with the progressive tradition in mathematics education.

To sum up, Nora's evolving stories portray mathematics teacher identity development as a movement away from external authority in university subject studies, through a raised critical voice in teacher education, and towards both internal and external authority for mathematics practices in the professional debut. However, the stories show also experiences of stagnation and lacking opportunities for further development of one's mathematics teaching, by reason of Nora's position as a substitute teacher. An overview of her developing identity as a secondary school mathematics teacher is shown in Figure 6.11. I relate the steady situation of finding mathematics both repulsive and attractive to her sustained marginal participation in the *community of university mathematics* and her sustained peripheral participation in the *community of secondary mathematics teaching*. Further, Nora's sustained perspectives on mathematics and its teaching relates to her peripheral participation in the *community of secondary mathematics teaching* and her marginal participation in *communities of students and their regular teacher*. Similar to the case of Isaac, I perceive her participation in the communities to be in interplay, represented by the double arrows in the diagram.

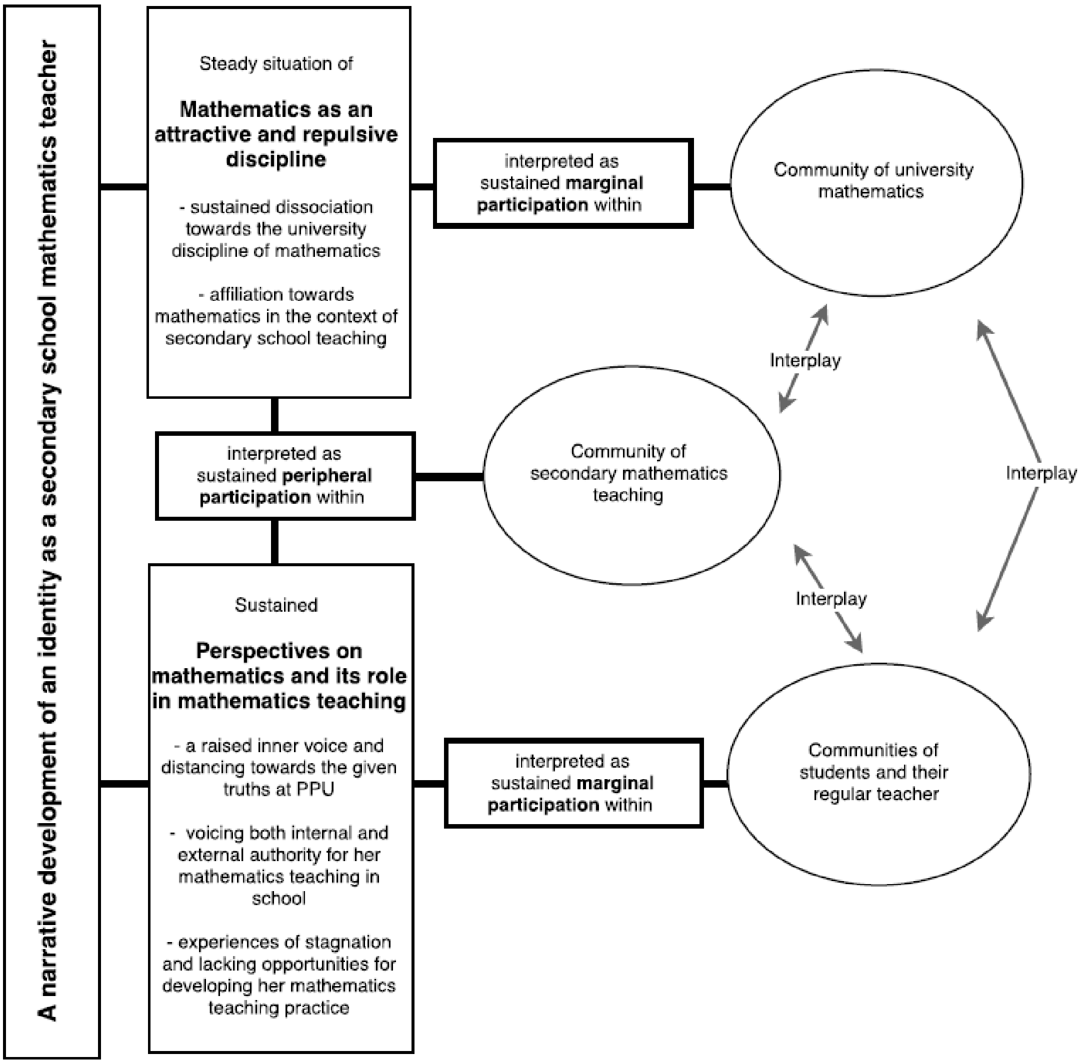

Figure 6.11: An overview of Nora's developing identity as a secondary school mathematics teacher

7 The case of Thomas

Unlike the previous cases, Thomas has his educational background from a five-year Master's programme in mathematics and natural sciences, including teacher education ("Lektorutdanningen i Realfag", locally abbreviated to LUR). At the time of data collection, he was in the final stages of the study programme, which had included

- subject studies in mathematics (97,5 ECTS credits)
- subject studies within biology, chemistry and physics (in total 67,5 ECTS credits)
- two semesters of teacher training, subject didactics courses and general pedagogy, equivalent to the one-year PPU programme
- courses within the specialisation in mathematics education (30 ECTS credits)
- a 30 ECTS credits Master's thesis within the field of mathematics education

Similar to Isaac and Nora, Thomas has completed the PPU programme, including its subject didactics courses. Here, the mathematics didactics course (15 ECTS credits) aims for student teachers' planning, implementation and evaluation of secondary school mathematics teaching based on research-founded knowledge. However, Thomas is the only participant taking a Master's degree in mathematics education. The courses within the Master's specialisation are provided by the Department of Mathematical Sciences. They are considered subject specific and research focused, as they prepare the student teachers for writing a Master's thesis within the field of mathematics education.

In line with the two previously presented cases, I contacted Thomas by e-mail and through the small survey that I had sent out to all last-year students in mathematics and mathematics education at the LUR programme. In addition to ticking off "agree" on the statements "I perceive myself as a teacher" and "I perceive myself as a mathematics teacher", he commented:

> *I started at the LUR programme because I wanted to become a teacher and because I feel that I am most passionate about the natural sciences. I guess that is what has motivated me throughout the whole course of study.*

Based on his educational background in both mathematics and mathematics education and the stated plan of working as a mathematics teacher after graduation, Thomas was considered suitable for participating in the study. He is also the participant holding the most ECTS credits in respectively mathematics and mathematics education. However, a late response to my request led to a later start-up for the interviews, which gave a shorter time span for the data collection period. While the first interviews with Isaac and Nora took place in the preceding autumn semester, Thomas' first interview was conducted in March, half-way into his final semester and his Master's project. Due to the structure of the LUR programme, Thomas had completed PPU and school placement before the data collection period. I therefore have access only to his retrospective accounts regarding teacher education. The second interview took place in June, after he had submitted the Master's thesis.

After graduation, Thomas got a full-time temporary position as a mathematics and science teacher at an upper secondary school. He taught the mathematics courses R1 and 2P within the general education programme, and the course 2P-Y within the supplementary year for vocational education programmes. The school of employment, being located in a neighbouring municipality to a Norwegian city, has about 500 students and 60 teachers. The third interview took place in November, during Thomas' first semester as an upper secondary school mathematics teacher. In the occasion of the fourth and final interview in March, in the subsequent semester, I observed Thomas during one 1-hour lesson in mathematics R1 on the topic of the derivative of exponential and logarithmic functions. A summary of the observed lesson is given in Section 7.2.3, as a contribution to the evolving story of *perspectives on mathematics and its role in mathematics teaching*. The follow-up interview was arranged immediately after the lesson, and functioned also as a retrospective glance on the whole one-year period in which I had followed Thomas. Table 7.1 gives an overview of time and length of the four conducted interviews and the observation.

Table 7.1: Overview of the interviews with Thomas

Year 1 Master's programme in mathematics education		**Year 2** Mathematics teaching in upper secondary school	
Spring		Autumn	Spring
Interview 1 March	Interview 2 June	Interview 3 November	Observation Interview 4 March
2h, 21 min	1h, 7 min	2h, 2 min	1h, 38 min

Based on narrative analysis of the interviews, Thomas' utterances were grouped into two emergent themes and related evolving stories: *mathematics as a professional mainstay* and *perspectives on mathematics and its role in mathematics teaching*. While the first theme is unique for the case of Thomas, the second theme is named similar to comparable emergent themes in the cases of Isaac and Nora. Although the participants gave differing accounts regarding the nature of mathematics, the three evolving stories of *perspectives on mathematics and its role in mathematics teaching* are objects for comparison in the cross-case analysis in Chapter 8.

Further, the structure of the analysis of Thomas' accounts is similar to the other two cases. I report on two emergent themes and their evolving stories throughout the four conducted interviews. Each of the two evolving stories contains two critical events, which correspond to the following situations in Thomas' life:

- facing towards a career as a secondary school mathematics teacher, before graduation
- working as a newly educated mathematics teacher in upper secondary school

Compared to the cases of Isaac and Nora, the interviews with Thomas are less comprehensive. Especially, the second interview is thin and does not have a prominent role in the upcoming analysis. One reason for its limited scope is that it took place at an unfortunate point of time, when Thomas was submitting his Master's thesis and therefore occupied with a demanding finishing stage. Further, Thomas' accounts are largely similar across the first and second interview (before graduation), and across the third and fourth interview (after graduation). The two critical events along each evolving story are therefore based on accounts given in respectively the first and second interview, and the third and fourth interview. An overview of the two evolving stories and associated critical events is shown in Figure 7.1.

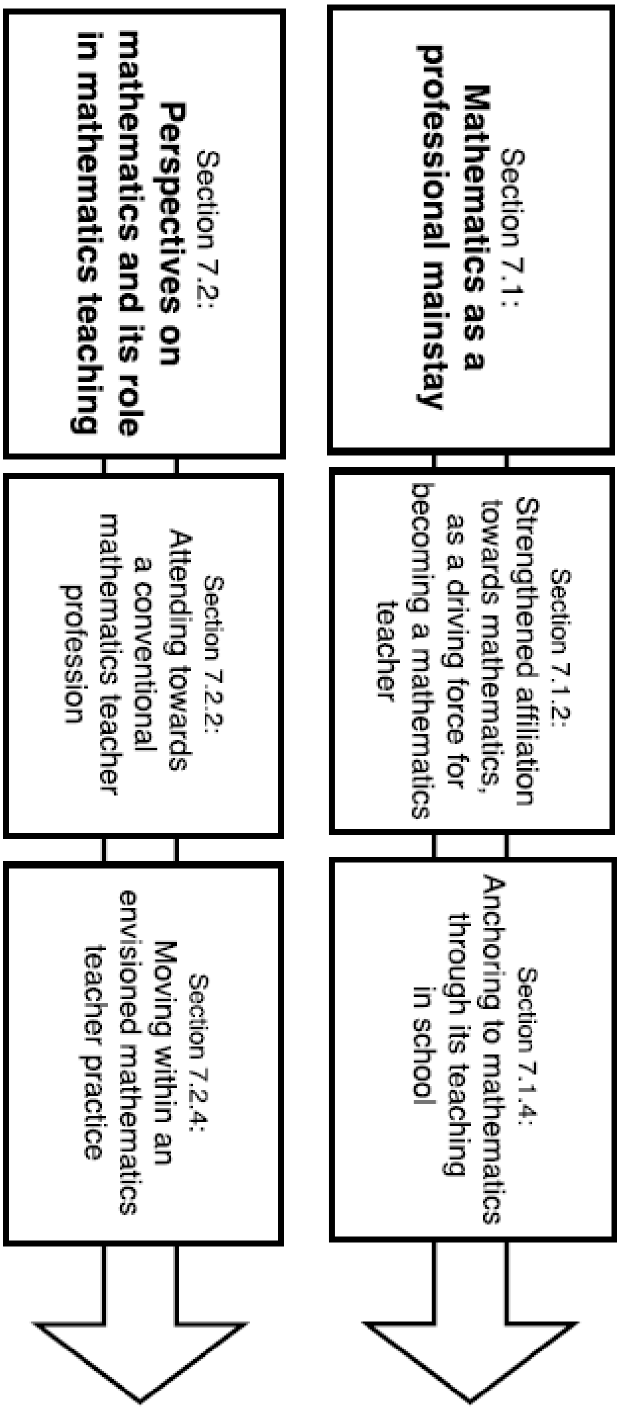

Figure 7.1: Emergent themes and related critical events in Thomas' accounts, constituting two evolving stories of becoming a secondary school mathematics teacher

7.1 Mathematics as a professional mainstay

Throughout the four interviews with Thomas, his accounts reveal a foregrounding of mathematics for his professional practice as a secondary school mathematics teacher. In the first interview, this foregrounding includes his descriptions of a general interest in mathematics and the natural sciences in own schooling. However, the main strand of his accounts concerns an increased belonging to the university discipline of mathematics based on what he refers to as a development of his academic maturity. Further, Thomas portrays a foregrounding of mathematics in terms of a sense of belonging to fellow students in the mathematical sciences. As will be elaborated on in the upcoming analysis, Thomas describes the academic environment among the mathematics students as supportive. Simultaneously, he delineates to some degree a distance to his fellow student teachers in mathematics, due to the general impression of their weaker academic background. In parallel with the concurrent attraction and dissociation towards actors of mathematics at the university, Thomas accounts for a belonging to mathematics education specialisation. However, he marks a distance to the educational practices at the teacher education programme, PPU. Thomas' accounts of becoming a mathematics teacher are thus depicting an affiliation towards the mathematics discipline, rather than a primary interest in the practice of teaching in its general sense. While he is steering his educational pathway towards a future mathematics career, by specialising within the field of mathematics education, he nevertheless portrays a mathematical profile of his profession.

The foregrounding of mathematics is further apparent after graduation, when Thomas accounts for his role as an employed mathematics teacher. By favouring upper secondary school mathematics teaching, and preferably the mathematics subjects for the natural sciences (Mathematics R1 and R2), he highlights the freedom of focusing on the mathematics subject and the current subject content in his teaching. Since the climate of cooperation among his colleagues is positive, as is the interaction with the students in the classroom, the main impression is a sense of contentment regarding the mathematics teacher profession.

Across all four interviews, Thomas points out the subject content of mathematics to be the core activity of his professional practice. Accordingly, I interpret the described foregrounding of mathematics to represent an evolving story of *mathematics as a professional mainstay*. As the transition from university to school can be described as falling into place, the related critical events to be presented show pursued practices of mathematics and its teaching rather than disturbance or discontinuity.

An overview of labels belonging to the emergent theme *mathematics as a professional mainstay*, constituting two critical events, is shown in Figure 7.2.

Mathematics as a professional mainstay

Labels related to the first critical event: **Strengthened affiliation towards mathematics as a driving force for becoming a mathematics teacher**	Initial interest in mathematics and the natural sciences in own schooling I1_2	Initial interest in the activity of teaching I1_2 I1_45	A turning point, from being academically immature to becoming confident in mathematics and its teaching I1_227 I1_248	Belonging to other university students in mathematics I1_330	Belonging to the Master's specialisation in mathematics education I1_33 I1_45 I1_170 I1_651	Distancing to other student teachers in mathematics I1_114	Distancing to lectures and seminars at PPU I1_38 I1_157 I1_185 I1_541	
Labels related to the second critical event: **Anchoring to mathematics through its teaching in school**	Placing the mathematics subject at the centre of the professional practice I3_464 I4_217	Affiliation to the upper secondary mathematics teacher role I3_424 I3_90 I4_691		Belonging to a supportive mathematics teacher staff I3_368 I4_700			A solid matheamtics background in the foreground of the mathematics teacher profession I3_305 I4_576	Scholarly knowledge from PPU remaining in the background of the mathematics teacher profession I3_294 I4_578

Figure 7.2: Overview of labels belonging to the emergent theme *mathematics as a professional mainstay,* constituting two critical events

7.1.1 Accounts of mathematics as a professional mainstay, constituting a first critical event

In the first interview, Thomas accounts for a growing desire of becoming a secondary school mathematics teacher during the five-year Master's programme in mathematics and mathematics education. As will be reported, the emergent sense of belonging to the mathematics teacher profession is related to what he describes as a development of his academic maturity. Thomas' retrospective accounts of undergoing the LUR programme constitute thus a first critical event in his evolving story of *mathematics as a professional mainstay*.

The decision of entering the teacher education programme was founded on his interests in mathematics and the natural sciences, being weighed together with other possible studies in engineering and architecture. Based on his positive experiences from coaching children and youths in sports, an educational approach towards mathematics seemed attractive.

I1_2	Thomas	*I've been in doubt, actually, for a long time, from the decision of becoming, wanting to study teacher education in the natural sciences, then it stood between kind of, more engineering and for example architecture (...)I like very much to work with people, and I have worked a bit as a coach (...) I find it very fun to see the progression and follow younger people, so that was maybe my main motivation for wanting to become a teacher, and then I'm really interested in the subject of mathematics, especially.*

Although Thomas found the mathematics studies to be appealing, the idea of wanting to become a schoolteacher in mathematics was not prominent during his first years of university studies. One milestone was therefore reached when he had to choose specialisation and the field of research for his Master's degree, being either within a mathematical topic or within mathematics education. A specialisation within a mathematical topic could lead him into a career outside school, and thus, provide flexibility to his professional future. Yet, he found the Master's courses in mathematics education to be of great interest and highly relevant for a teacher career. Hence, in parallel with choosing a specialisation within mathematics education, the idea of becoming a mathematics teacher became strengthened.

I1_33	Thomas	*(…) for a long time, it stood between a mathematical Master's degree and a Master's degree in mathematics education, but then I liked a lot the subjects in mathematics education at the Master's programme, after PPU, in particular (…) these subjects in mathematics education have given me a lot, and it's very interesting, so I'm happy with my decision of writing a Master's degree in mathematics education.*
I1_45	Thomas	*(…) many will still like to teach even though they take a mathematical Master's degree, but I feel I have some advantages having taken subjects in mathematics education. But at the time that I thought about writing a mathematical Master's thesis, I thought about trying out something outside teaching as well. (…) because then you have a sounder basis for application, but it has actually been teaching that I want to do (…)*

In addition to reflect on his way into the Master's studies of mathematics education, and thus, on gradually becoming attuned to a future teacher career, Thomas describes a maturity development: from being academically immature to becoming a confident mathematics student. Portraying it as a turning point, his changed approach towards the mathematics studies was realised by giving structure to his university studies and increasing the workload. In contrast to previous experiences of failure and not being able to live up to own and others' expectations, the act of "pull[ing] [him]self together" led to experiences of mastery.

I1_227	Thomas	*I had this period in the beginning of my education that was kind of, I was somehow in doubt, or I was maybe not fully mature for the education, which caused that I struggled a bit, dropped behind academically. (…) The turning point would actually be, from barely manage to complete it, to become more offensive, meaning, I made an effort, it was mostly about the workload and my structure which made up the turning point, I realised I can't do this forever, I can't go on retrying subjects, so I pulled myself together (…) and then I felt mastery (…)*
I1_242	Thomas	*To fail and to get bad results, it was a bad feeling towards me, but also towards, kind of, friends and family (…) They have expectations and hopes, too.*

One of the peaks during his time at the university was the moment of realising he could be a successful mathematics student and achieve good results on his exams. In addition to gain confirmation through his increased academic results, Thomas also got positive feedback from his students during school placement. The turning point was thus constituted by the change from being what he denotes as an unsuccessful student to becoming competent in both mathematics and its teaching.

I1_248	Thomas	*(...) being among the most successful ones [laughing], in a subject and on the exam, that gave me a lot, it was maybe in cryptography and in algebra. Otherwise, it gave me a feeling of mastery to teach [during school placement] when I received positive feedback. (...) the students told me, kind of, it was towards the end of the teaching period, and they wanted me to continue being their teacher. That was of course very fun to hear. So the turning point was kind of from being unsuccessful to feeling mastery.*

In parallel with accounting for an increased affinity to subject studies in mathematics as well as the mathematics teacher profession, Thomas expresses belonging to fellow mathematics students and their academic environment. This has been manifested through their shared study halls and the opportunities to ask elder students of help.

I1_330	Thomas	*I find the academic environment at the Master's programmes in mathematics to be very nice. I got a seat there quite early (...) there's a very short distance to elder students, or those being more experienced, so I got a lot of help there.*

However, in contrast to his sense of belonging to other Master's students in mathematics, Thomas delineates a divide between them and the other student teachers in mathematics. While he has been considered a clever student, in line with other successful Master's students in mathematics, the remaining group of student teachers has in general been perceived as struggling with the mathematics subject studies. Thomas explains the divide based on the packed study plan of the LUR programme, consisting of mathematics subject studies in combination with additional subject studies in the natural sciences, teacher education and school placement. Hence, the student teachers hold a different academic background, which might lead to disadvantages in the mathematics courses of higher level.

I1_114	Thomas	*During the last mathematics courses, I have felt kind of, being at the risk of using the wrong word, since it's kind of, not being stigmatised, but I felt that the group of student teachers was labelled. (...) you take subjects together with others having a broader academic background (...) I felt I had good control, while I experienced that other student teachers were struggling a lot. (...) I think, at least, that many of the student teachers felt that they were not the strongest ones, or they did not have the qualifications to be the most successful students in the group.*

Another divide is delineated between the Master's specialisation in mathematics education and the didactics courses during teacher education (PPU). Thomas perceives the former as relevant due to the presence of mathematical challenges and its clear connection to the field of practice. Hence, the Master's courses fruitfully unite opportunities for acting both mathematically and didactically.

I1_170	Thomas	*(...) maybe it's kind of a good mix of being a mathematician and a student teacher, during these mathematics education courses in the Master's programme. But that's because, I think that the lecturer is good at making demands and challenging us mathematically, too.*
I1_541	Thomas	*It's rather concrete. These things we are doing, they have a transfer value, even if we work with, for example, how to present examples in a mathematics class, we were working with coordinates and things like that, and that has transfer value to other things, other parts of mathematics (...) you are supposed to develop some thoughts about why you choose these examples, that is quite general, but he takes something rather concrete, it makes it quite easy to get into it. (...) At PPU, you worked kind of, you had to find a way to apply it.*

Contrarily, the mathematics didactics course at PPU were, according to Thomas, one-sided concerning inquiry based mathematics teaching and the implementation of rich problems. In addition to disagreeing with the stated uniform perspectives on mathematics teaching, Thomas refers to the learning activities during seminars as trivial, by characterising them as "playing with numbers". Accordingly, there were fewer opportunities to discuss possible ways to approach the mathematics content in own classroom teaching, with respect to students' learning and their difficulties.

I1_38	Thomas	*I think PPU was, I don't know, it was kind of a one-track teaching, I don't know if it was kind of the lecturer of the subject that was very into this one thing about teaching, rich problems, and he was very into this thing of students doing investigations (...) I don't know if I agreed with everything [laughing] that was taught, then.*
I1_157	Thomas	*I don't know exactly how to phrase it, but I do think it was too much of, kind of, playing with numbers (...) it was a lot about presentation, meaning, with rich problems, but it was not about how to approach it (...) how to plan teaching, what can be challenging (...) I felt it as a bit naïve, especially this thing about rich problems. I think it's very challenging to implement it successfully. It's challenging for the teachers.*

When comparing PPU with other parts of the LUR programme, Thomas further marks a distinction regarding expectations of performing at a high academic level. Unlike the demanding subject studies in mathematics and the Master's specialisation of mathematics education, the PPU programme did not bring with it expectations of being academically challenged. Instead, the expectations of making a good performance as a mathematics teacher was mainly set by himself. Thus, while the discipline of mathematics, its challenges and demands, is given a leading part in Thomas' educational pathway, the learning activities and related course content at PPU winds up in the background.

I1_185	Thomas	*(...) academically, it wasn't very challenging, really, at PPU. We had tasks that were quite free, so you could work with what you found to be most interesting within pedagogy and subject didactics. And the school placement can be very challenging, of course, but, personally, I felt like, in a way, I didn't meet any challenges beyond the pressure I put on myself, I didn't feel I had to convince anyone else than myself. Meaning, of course, you have a responsibility for being a good teacher during school placement, but that I feel, kind of, that pressure I put on myself, mostly.*

7.1.2 First critical event: Strengthened affiliation towards mathematics as a driving force for becoming a mathematics teacher

Thomas' accounts of undergoing university teacher education portrays a prospective secondary school mathematics teacher with a main interest in the discipline of mathematics. His struggles and accomplishments in the mathematics subject studies further appear as a driving force for his emerging affiliation towards the mathematics teacher profession. I argue in the subsequent analysis for the foregrounding of mathematics in Thomas' accounts, due to his participation and non-participation in mathematics related practices when being a student at the university.

A central topic of Thomas' accounts is his described development of academic maturity. I interpret the changed working habits and the related improvement of his exam grades as inbound participation in a *community of university mathematics*. Thomas then exercises identification and negotiability through engagement in the shape of pulling himself together and increasing the workload, and by searching out fellow mathematics students at the study hall. Further, his identification with practices of university mathematics depends on the kind of picture of university mathematics he builds, and how he can project himself into this picture. In terms of identification through imagination, Thomas expresses belonging to other competent Master's students in mathematics, by simultaneously dissociating himself from the general impression existing of student teachers in mathematics. His inbound participation in the community is thus steered by the aspiration of being recognised by himself and others as academically competent.

The foregrounding of the discipline of mathematics in Thomas' accounts is further based on his contrasting of parts of the LUR programme concerning mathematics education. While the Master's courses in mathematics education are considered to be of significance for his future practice and of great interest due to the mathematical focus, the comparable subject didactics courses at PPU is portrayed as disconnected and superficial. Despite his described discontent with the mathematics education practices at PPU, I nevertheless interpret Thomas to act as an inbound participant in a *community of secondary mathematics teaching*. His engagement is apparent from making the choice of specialising within the field of mathematics education, in preference of taking a Master's degree in pure mathematics. Consequently, Thomas is steering his educational pathway towards a future mathematics teacher career. In terms of negotiability, he is, however, dissociating himself from the practices of PPU by assuming that the stated meanings of what mathematics is or should be lie beyond his engagement in mathematics teaching practices. In other words, he chooses to distance himself from what he perceives as a biased presentation of inquiry based mathematics teaching. His lacking ownership of meaning within the practices at PPU is further expressed through the described absence of academic demands during teacher education. Hence, Thomas delineates absent mutual engagement, and thus, non-participation in the *community of university teacher education* at PPU.

Due to the portrayed turning point during Thomas' university education, I interpret the related critical event to be a *strengthened affiliation towards mathematics, as a driving force for becoming a mathematics teacher.* The strengthened affiliation manifests itself through better academic results and expressed belonging as well as dissociation from mathematics teaching related practices at PPU. Hence, Thomas' picturing of a future mathematics teacher career is based on his accounts of a strengthened belonging to the discipline itself, rather than a primary concern regarding the practice of teaching in its general sense. In Figure 7.3, Thomas' foregrounding of mathematics is shown as identification and negotiability within the *community of university mathematics* and the *community of secondary mathematics teaching*, in parallel with non-participation in the *community of university teacher education*.

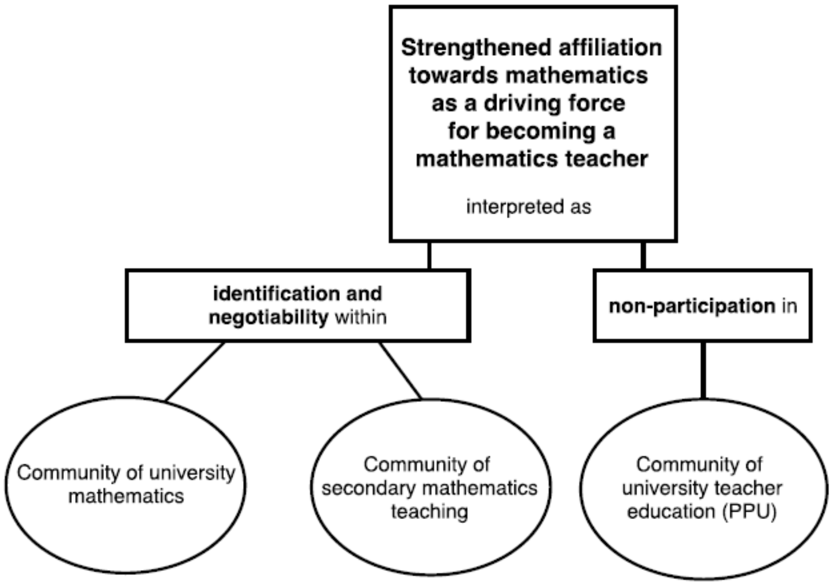

Figure 7.3: Mathematics as a professional mainstay, represented by the first critical event of strengthened affiliation towards mathematics during university teacher education

7.1.3 Accounts of mathematics as a professional mainstay, constituting a second critical event

The third and fourth interview with Thomas took place during his first year as a secondary school mathematics teacher. Through both interviews, Thomas accounted for his life as a newly qualified teacher, including challenges related to hectic working days and to students' differing needs when learning mathematics. Yet, the main impression appearing from his descriptions is nevertheless a sense of belonging to the mathematics teacher profession, due to the pleasure of engaging with mathematics through interaction with the students and his colleagues. Thomas' accounts of being content with the teaching of mathematics in school constitute then a second critical event of the story of mathematics as a professional mainstay.

Being encouraged to compare his present situation as a mathematics teacher with his previous life as a student teacher in mathematics, Thomas finds it difficult in the third interview to recall prior expectations about becoming a mathematics teacher. Similarly, he considers his memories of being a student teacher in mathematics to be fading away.

I3_349	Thomas	*When I was a student, I didn't think so much about how it would be to become a teacher. Then, the focus was mainly on the study. (...) but that's a bit the way I am as a person, too, that I take things as they come. A lot of it I feel is related to one's experience, that it's difficult to make a picture of how it is to be a teacher, when you haven't been there yet. And then the memories of being a student fade away when you're a teacher.*

Thomas describes himself as a person who lives in the present, who "takes things as they come". Hence, his previous life as a student teacher in mathematics is portrayed as untroubled by the future mathematics teacher profession. However, when he later in the third interview compares his present situation with previous expectations about becoming a mathematics teacher, Thomas reviews the teacher profession as better than expected.

I3_424	Thomas	*I think kind of, that the teacher profession has been better than expected, really. (...) I thought maybe that, it would be more demanding regarding being a leader. (...) In a way, I feel I have kind of a natural leadership role, and I may have imagined beforehand that I would have to fight more for it (...). I feel kind of that the students are listening to me now.*

One of Thomas' concerns about becoming a mathematics teacher was the classroom management. Yet, after three months of teaching mathematics in school, the leadership role has in a way come naturally. In addition to experiencing the students listening to him, Thomas describes the planning and implementation of his teaching, and the students' sometimes unexpected questions, as joyful.

I3_90	Thomas	*So far, I find the lessons to be… I think it's fun to teach math regardless of the level. (...) I like to prepare the math lesson and I think it's fun to teach. Now, I have kind of followed the same type of plans, it becomes a bit like, blackboard teaching, task solving kind of lessons. (...) Regardless of the class, one gets kind of interesting questions, and maybe one becomes now and then a bit surprised and under pressure. (...) Even if you know the content, you have to give an explanation which is understandable in a way. But that I find challenging. So, I think it has been very fun.*

In addition to giving positive accounts regarding the mathematics teaching, Thomas describes the first months of employment as a confirmation of his preference for teaching in upper secondary school. Based on reports from previous fellow student teachers, the mathematics teaching in lower secondary school appears to him as limited by obligatory paper work not concerning the teaching of mathematics. On the contrary, Thomas describes a great freedom in his work, which in turn enables him to mainly focus on the mathematics subject and the current subject content in his teaching.

I3_464	Thomas	*I think the teacher profession suits me well (...), I'm certainly not in doubt whether I want to teach at upper secondary school. (...) I'm really not that greedy for lower secondary school, either. (...) I have fellow students working in lower secondary school, and it seems like they are busier. (...) I feel like there is more obligatory work, or that you have some other fields of responsibility, too. Here, I feel I'm kind of free. It's a lovely feeling. And then I feel that the work I'm doing in the classroom is academically, or mathematically (...) and that is what I want to focus on in the lesson, the subject is at the centre.*
I4_691	Thomas	*I think in a way I have, I've been given confirmation that I really enjoy teaching. In a way, I knew that from school placement, that I enjoyed it, I feel kind of, it has been reinforced, this feeling that I have made the right decision.*

In line with his foregrounding of the mathematics subject for his professional practice, Thomas points at mathematics for the natural sciences, Mathematics R1, to be more interesting than the general course, Mathematics 2P. While the Mathematics R1 contains five hours of teaching per week, the Mathematics 2P course covers only three hours per week. Thomas describes the former course as generating more interesting questions from the students and giving them the chance of discussing *why* something is the way it is. In contrast, the latter course shows signs of only scratching the surface of the mathematics, being “just a little bit of everything”.

I4_217	Thomas	*I do think that the mathematics content in the R1 course is more exciting than the other subjects. At least in the 2P course which is a three-hour course, a lot is cut out. It becomes kind of just a little bit of everything. (...) I find that a bit boring. Meaning, I do think it's fun to teach (...) but one gets maybe more interesting questions from the other students, or, they have maybe a different basis which make them ask some other types of questions (...) many of them wonder maybe more about 'why', I think.*

Alongside his planning and implementation of the mathematics teaching, Thomas is collaborating with his colleagues on preparing the semester plans, including making homework assignments and written tests. Since they are a group of teachers sharing the responsibility for the courses R1 and 2P, Thomas and his colleagues find it helpful to regularly meet up for discussing the implementation of the current mathematics topic, what will be done in the subsequent lessons, and related students' difficulties. Thomas describes the climate of cooperation in positive terms, as friendly, welcoming and helpful, in which he is listened to by his fellow colleagues.

I3_368	Thomas	*I feel I have kind of a central role in everything we collaborate on. It's not like I'm a passive spectator, listening to the others that have been working here for thirty years. (...) I think that anyone who had worked here could have had the same feeling. Since they are very inclusive, at least those that I work with. (...) It is a lot of collaboration on plans, kind of consecutive, what have we been through, what do we have to go through, how did I explain this part, what tasks have you been working on, what have I given for home work. It is a lot on how to, yes, and about what is ahead. (...) and we discuss maybe individual students and their struggles and so on.*
I4_700	Thomas	*We work closely, we have several parallel groups in the math subjects and we have kind of a talk after each lesson, hearing a little bit about how it went and so on. (...) We collaborate on making the tests, listening to each other, do you think this task was good kind of. (...) We are all kind of different, however, at the same time we are all listening to each other's opinions. So I feel at least that this threshold is low. (...) I feel I have an active part. Even if I'm young, I'm listened to.*

In order to be prepared for the students' responses in the mathematics classroom and for giving them clear explanations, Thomas further highlights in both interviews the need for having a solid mathematical background. Although he may have had a different perspective on his educational background prior to his professional debut, he has now seen the need for being academically confident.

I3_305	Thomas	*(...) you do need a professional standing in mathematics. I notice that I need it a lot. (...) it's maybe easier to find understandable explanations to the students. I think it also helps you to more quickly answer questions, you have experienced the type of problems that the students meet (...) I have maybe changed my opinion, now when I've been working as a teacher, that I find it helpful to have had so much math.*
I4_576	Thomas	*But I think that kind of the professional standing in mathematics that you get from university, you have use for it even though not all the subjects are pointed towards teaching in upper secondary school, it is a mainstay.*

While highlighting the required university background in mathematics, Thomas refers, however, to the courses at the PPU programme as less important for being able to exercise good mathematics teaching. In both the third and the fourth interview, he mentions the idea of acknowledging the students' different needs as one of the important messages from PPU. However, according to Thomas, the idea of acknowledging students' needs is primarily based on practical experiences in the classroom or conventions, rather than on academic or scholarly knowledge. For him, the most significant part of PPU was thus the school placement, in which he focused on being structured in his teaching and exercising clear leadership.

I3_294	Thomas	*Occasionally, I think about what I was told, or what I remember from PPU, kind of, to acknowledge the student. But that is often something I'm not aware of, I try to do my best about it, but I feel kind of… there are other things from school placement that I've been thinking about, kind of, the structure and my role as a leader, those kind of things.*
I4_578	Thomas	*I do have some tips from the subject didactics course, maybe. But I feel there are many kind of unwritten laws, or, there are some rules at PPU, too, kind of acknowledging the student and… I feel that such things are related to the personality of the teacher (…) you can tell the teacher to acknowledge the student, but I don't think the student always feels he is acknowledged for that reason. You can learn the theory there, but that is something you need to experience when being in the classroom. Of course, maybe the school placement was what I learned the most of, at PPU. (…) The mathematics didactics course was mainly concerned with open tasks and inquiry based mathematics. And I do value it, however, at the same time, I don't know. (…) It's not something I ache for all the time, it takes a lot of time, and I feel kind of, since the everyday life is so stuffed up, I haven't used those tools so much.*

In line with considering the seminars, lectures and the related scholarly knowledge of PPU as less important for his professional practice, Thomas refers to the mathematics didactics course and its emphasis on inquiry based mathematics teaching as insignificant for his classroom teaching. One year into his mathematics teacher career, he instead expresses contentment with teaching mathematics as settled through practice, which is in accordance with experiences from school placement.

7.1.4 Second critical event: Anchoring to mathematics through its teaching in school

Rather than being different from or opposed to the first critical event, the second critical event of Thomas' story of mathematics as a professional mainstay represents a confirmation or continuation of an already described situation regarding his foregrounding of mathematics in teaching. In other words, Thomas points out the subject content of mathematics to be the core activity, rather than teaching in more general terms and related problem areas concerning students' needs. Further, he refers to the PPU programme as insignificant for being able to exercise good mathematics teaching. This is in accordance with previous findings about Thomas' strengthened affiliation towards mathematics during his university studies. Hence, I argue that the transition from university to work life is in Thomas' case characterised by falling into place, meaning that the *mathematics* teacher profession seems to come up to his expectations. The lack of discontinuity does, however, represent a critical event in Thomas' way into the mathematics teacher profession, as it reveals pursued practices rather than disruption. The transition is then based on his continued participation in the *community of secondary mathematics teaching*, which I further interpret to be rooted in previous participation and non-participation in communities of practice.

Thomas' participation in the *community of secondary mathematics* can firstly be explained from his negotiation of mathematics and its teaching with students in the classroom. When searching for well-structured and thorough explanations to the students' questions, he engages in the joint enterprise of whole-class teaching and task solving in which he favours the more advanced mathematics topics. Hence, he exercises identification with the practice of mathematics teaching in which the subject itself is in the foreground of the enterprise. Secondly, Thomas cooperates with his colleagues on the preparation and implementation of the mathematics courses R1 and 2P. By sharing material for the classroom teaching, as well as their experiences of the lessons, worries and concerns about students, the colleagues represent both an organisational and an emotional support for its members. Thomas' engagement is then characterised by him taking active part in the professional discussions, as he denotes himself a central member of the group. Although the colleagues collaborate on what they "have been through" and on what they "have to go through" in each mathematics lesson, a development of their mathematics teaching does not seem to be part of their joint enterprise. By feeling at home in upper secondary school, in which he has the freedom of placing mathematics at the core of his practice, Thomas thus exercises both identification and negotiability within an already established practice of mathematics teaching. The students' challenging and interesting questions, as well as his colleagues' supportive teamwork, functions as confirmation on his mathematics teaching and his choice of career.

The continued participation is further related to previous participation and non-participation in mathematics and mathematics teaching related communities. Regarding the *community of university mathematics,* I accounted in Section 7.1.2 for his aspiration of being recognised by himself and others as academically competent. In parallel, he expressed belonging to other competent Master's students in mathematics, by simultaneously dissociating himself from other student teachers in mathematics who represented poorer academic performances. Thomas continues along a similar narrative path in the third and fourth interview, in which he emphasises the need for his solid background in mathematics. In order to give thorough responses and to flexibly react to the students' various inquiries, he finds his university studies in mathematics to be helpful. Hence, I interpret Thomas to exercise imagination within the *community of secondary mathematics teaching* by portraying his professional practice to be in accordance with a main interest in the subject of mathematics. In other words, the continuity across university and work can be explained by his pursued identification with the discipline of mathematics in the school setting.

In parallel with pointing out the need for a solid mathematics background, Thomas gives accounts which bear the impression of his previous non-participation in the *community of university teacher education* at PPU. While he emphasises a strong mathematical background in order to exercise good mathematics teaching, the practice of teaching is based on classroom experience rather than scholarly knowledge provided at the teacher education programme. Consequently, PPU does not represent to him academic or scholarly knowledge having a considerable impact on his professional practice. Instead, Thomas describes a situation in which he follows the track of mathematics teaching stemming from school placement, where he was focused on structured presentation of the subject content and classroom management. The rooting in previous communities thus contributes to Thomas' foregrounding of the discipline of mathematics in two ways: by emphasising his belonging to the (university) discipline of mathematics, and by distancing himself from teaching related practices held by a teacher education community. Thomas' contentment in exercising secondary school mathematics teaching is then interpreted as a critical event of *anchoring to mathematics through its teaching in school,* illustrated in Figure 7.4.

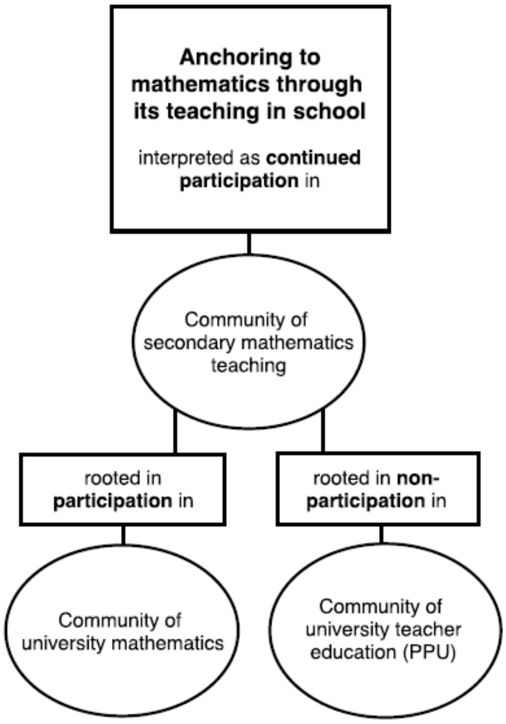

Figure 7.4: Mathematics as a professional mainstay, represented by the second critical event of anchoring to mathematics through its teaching

In addition to accounting for a strengthened belonging to the discipline of mathematics throughout education and his professional debut, Thomas simultaneously conveys perspectives on the nature of mathematics and its role in mathematics teaching. In the subsequent section, I will therefore elaborate on aspects of the mathematics discipline that is foregrounded in Thomas' narratively developing mathematics teacher identity.

7.2 Perspectives on mathematics and its role in mathematics teaching

Thomas' accounts from the first and second interview regarding his *perspectives on mathematics and its role in mathematics teaching* can be considered to evolve along three narrative paths: accounts of mathematical appreciation; accounts of mathematical argumentation; and accounts of own mathematics teaching. Following Ernest (2004a), appreciation of mathematics is about personal, cultural and social relevance of the discipline, rather than immediate social utility from an economic perspective. Hence, it covers an awareness of how mathematics is central to all aspects of our daily life and experience, such as commerce, economics and communication technology. Further, appreciation involves an understanding of the main branches and concepts of the mathematics discipline itself, by having a sense of their interconnections and interdependencies. In line with Ernest (2004a), Thomas' accounts concern the awareness of the applicability of mathematics in problem solving, in the shape of being an extensive and rich tool box. In addition, he portrays mathematics as a cohesive discipline, in which one gains enjoyment from the discipline's precision. In the upcoming analysis, Thomas thus describes a pleasurable university discipline, due to both its permeation in other areas and its inner complexity.

Regarding the narrative thread of mathematical argumentation, Thomas' accounts show to some degree a distinction between mathematics as a university discipline and mathematics as a subject to be taught in school. Regarding university mathematics, he highlights in the subsequent analysis the role of argumentation among mathematicians. According to him, mathematical enterprises take place within certain communities, meaning that one's reasoning needs to be verified by colleagues. Similarly, Thomas takes part in and values discussions with fellow mathematics students. However, he finds school mathematics to deal mainly with task solving and learning a set of techniques. Accordingly, Thomas makes a divide between knowing *how* to solve a problem and knowing *why* the method is correct. A good mathematics student is recognised by being able to make the latter approach to mathematics.

I argue further that there is a divide between Thomas' accounts of mathematics as a field of study and mathematics as a subject for teaching in upper secondary school. While the former practice is described as dynamic processes of investigating and arguing within mathematics, the latter is presented as rather static methods of presenting and explaining the subject content to the students. Hence, Thomas' story of *perspectives on mathematics and its role in mathematics teaching* is introduced by him making an approach towards a conventional mathematics teacher profession. This contrasts the possible situation of desiring to change or develop the reigning practice of mathematics teaching in school to aim for converging university and school mathematics. After graduation, Thomas' accounts regarding own mathematics teaching deal with the challenge of finding the right balance between blackboard instruction and individual task-solving. The analysis of Thomas' postgraduate perspectives is partly based on accounts generated from observation of his teaching. Due to his concerns about the varying pace of the teaching and the students' result-orientation in their learning, I argue that Thomas' views are similar to Mellin-Olsen's (1990) *task discourse*[18]. Further, the described teaching practice seems to coincide with previous expectations regarding own teaching, about explaining some mathematical aspects

[18] "Oppgavediskursen", in Norwegian

clearly to the students so that they can solve the upcoming tasks correctly. The story of *perspectives on mathematics and its role in mathematics teaching* seems then to evolve along a continued narrative path, rather than representing discontinuity in Thomas' practices of mathematics and mathematics teaching.

An overview of labels belonging to the emergent theme *perspectives on mathematics and its role in mathematics teaching*, constituting two critical events, is shown in Figure 7.5.

Perspectives on mathematics and its role in mathematics teaching

	Accounts of mathematical appreciation			Accounts of mathematical argumentation				Accounts of own mathematics teaching			
Labels related to the first critical event: **Attending towards a conventional mathematics teaching profession**	Mathematics as an extensive and rich toolbox I1_366 I1_373 I1_379	Mathematics as the appreciation of investigating and discovering relations across topics I1_358 I1_388	Mathematics as immediate response and confirmation I1_354	Knowing mathematics is about being able to argue mathematically I1_401 I1_528	University mathematics is characterised by mathematical discussions I1_438	Being a mathematics teacher is about arguing to students why something is correct I1_450	Knowing in mathematics as understanding "why" - not only "how" I1_388 I1_528	Own mathematics teaching as traditional teaching I1_211 I1_628	Own mathematics teaching is based on using the text book I1_653 I2_81	Good mathematics teaching is characterised by students taking part in mathematical discussions I1_628	Good mathematics teaching is structured and well explained I1_628 I1_653
Labels related to the second critical event: **Moving within an envisioned mathematics teaching practice**	Good mathematics teaching puts into use a wide spectrum of the discipline I4_448	Good mathematics teaching emphasises the discipline's cohesiveness I4_494					Good mathematics teaching as problem-oriented rather than result-oriented I4_38 I4_179	Own mathematics teaching as balancing blackboard instruction and students' task solving I4_1 I4_152	Own mathematics teaching is about finding the right pace of a mathematics lesson I3_35 I3_140	Students appreciating a traditional mathematics teaching I3_176	Own mathematics teaching follows a given structure I3_176 I4_362

Figure 7.5: Overview of labels belonging to the emergent theme *perspectives on mathematics and its role in mathematics teaching*, constituting two critical events

7.2.1 Accounts of perspectives on mathematics and its role in mathematics teaching, constituting a first critical event

In the previously discussed findings on mathematics as a professional mainstay, I argued for Thomas' foregrounding of mathematics during his university studies and when facing towards a future teacher career. His sincere interest in the university discipline is further apparent from his expressed appreciation from learning and doing mathematics. One reason for finding mathematics exciting and fascinating is its applicability in problem solving, both within and outside the discipline. When succeeding in solving a problem, the mathematics subject provides immediate response on one's achievements, and consequently, a sense of mastery.

I1_354	Thomas	*It's kind of an easy way into a sense of mastery (...) it's so easy to recognize the sense of mastery, when you're able to solve a task or when you start to understand things, that causes an enormous sense of mastery.*
I1_366	Thomas	*You have in a way a tool box that you can use for solving tasks, or not tasks, but you can kind of solve problems, with different techniques. And then I think it's cool that mathematics is often applicable. Or a great deal of mathematics is applicable. That there's mathematics behind many things.*

In line with its applicability and the analogy of being a tool box in problem solving, Thomas gives examples of the usefulness of university mathematics. Based on recent experiences from own subject studies, he highlights numerical analysis with its approximations and cryptography dealing with the security of cryptosystems. Hence, mathematics is appealing due to its permeation in both life and work.

I1_373	Thomas	*(...) for example, numerical analysis, with all its analysis that you are using, that you can make good approximations about something you really don't know so much about, I don't know, it appears to me as mysterious sometimes. But when you understand that it's not mysterious, you have these points, take for example interpolation, which is actually quite simple. It's just cool that you only have those tools that you can use for so many things.*
I1_379	Thomas	*Like in cryptography, the lecturer we had who was very into applications, many of the lectures concerned the applications of the subject. Everything wasn't relevant for the exam, but he wanted us to see what cryptography really is about.*

In addition to the usefulness and the extensive applications of mathematics, appreciation of the discipline also relates to the act of gaining greater insight into the main branches and concepts of mathematics and discovering their interconnections. What appeared in the beginning of his university studies as separate subjects, has during Thomas' time of study emerged as interdependent divisions of the discipline.

I1_358	Thomas	*It's this thing about relations, when you have so many different kinds of (university) subjects and you see the relations between them. You thought in the beginning that there wasn't any relation, and then there are quite clear relations, I find it very exciting.*

Thomas further accounts for the distinctive characterisation of mathematics on the basis of argumentation. Regarding mathematics as a university discipline, argumentation takes place among mathematicians in such a way that one's reasoning needs to be verified by colleagues. Hence, mathematical activities take place within certain communities, and for Thomas' part, it is helpful to discuss mathematics with his fellow students.

I1_401	Thomas	*It's a way of arguing, maybe? It's the same as, kind of, in the social sciences, that if you're arguing, it demands some weight behind what you're arguing with. That's kind of how I think about a mathematician, that's a lot about, maybe in a way explaining so that at least other mathematicians can verify it. (...) I think at least that this thing of arguing in mathematics is what characterizes it, or arguing mathematically.*
I1_438	Thomas	*I guess many like to work alone, too, but there are many who, at least here, among the students, there are many who like to discuss, to have someone to discuss mathematics with, and to get better, or, they feel like they can push their limits.*

Although mathematics as a university discipline can be characterised by its ways of arguing, Thomas describes mathematics in upper secondary school as being mainly about solving tasks and learning to use a set of methods. Due to the lacking demands for making valid mathematical arguments, students tend to do mathematics without understanding, for instance when using and "becom[ing] addicted to the calculator". Hence, the school subject appears to him in a lesser degree as a cohesive discipline, and not as much oriented towards relations between its different parts.

I1_528	Thomas	*In upper secondary school, you become addicted to the calculator somehow, while here, at the university, you don't work with, there are many exams were you don't need the calculator. (...) there are not like very great demands regarding mental calculations, either, but there are kind of different working techniques, here. I think maybe that one should learn more to argue mathematically in upper secondary school.*
I1_388	Thomas	*(...) like doing mathematics, it doesn't follow that you know it, or that you know mathematics if you are able to perform mathematics. You can get the correct answer without understanding what you're working with. Also, this thing of being able to see relations between different parts of mathematics, that's about the understanding in mathematics (...) if you have a broad understanding, then you can do the mathematics, too, of course, but it's not necessarily the other way around.*

Similar to a mathematician, one could regard the professional practice of a mathematics teacher as a way of arguing. However, the teacher's job is, according to Thomas, to convince the students about the relevance and importance of the mathematics content being taught. He further portrays the mathematics teacher profession by the tasks of clearly explaining, simplifying and structuring the subject content to the students. Consequently, the core practice of a mathematics teacher is to put forth the mathematics content without taking away or transforming the very essence of the mathematics concepts involved. Due to the cohesiveness of the discipline, this demanding task of simplifying and structuring the subject content is more urgent in mathematics than in other school subjects.

I1_450	Thomas	*(...) compared to the mathematics researcher, you're able to argue correctly about things. But as a teacher, you have to argue correctly, too, but then you have to simplify things, you have to know what's important, let's say in functions, what's important about the definition of functions (...) you have to be able to simplify without taking away what's important or the essence in what you're teaching. (...) A good teacher does at least make the students have faith in these things being important, or that it seems logic. Compared to other teachers, I think there are kind of greater demands for being able to convince your students. (...) I don't think maybe, in Norwegian for example, that there are any demands about simplifying things (...) I think maybe there are greater demands for what is needed to be there in math, since everything is so related to each other.*

The qualities of explaining well is further highlighted in Thomas' report on good mathematics teachers from own schooling. By choosing explanatory models that linked the classroom mathematics with experiential realties of the students, Thomas' lower secondary school teacher managed to present the subject as connected to real life.

I1_653 Thomas *The guy at lower secondary school had many nice models when he taught, when it was this classic blackboard instruction, I remember he managed to find figures and things like that, which we could associate with, we got a relation between real things and the paper (...) At upper secondary school, I felt they were very good at explaining, and they were structured (...) I had the impression that all my teachers at upper secondary school were very clever. It's especially about being good at explaining.*

Turning then towards his role as a future secondary school mathematics teacher, Thomas emphasises predictability in his lessons, in terms of following a set structure of initial whole class instruction, individual task solving and a closing summary. This safe and well known teaching style was also the starting point for his teaching during his school placement.

I1_211 Thomas *In upper secondary school, it was much like, we stuck with the safe, familiar teaching, you can say, or, it was kind of a presentation, then solving tasks. Otherwise, we had students' experiments [in the science lessons], and these experiments were also already put within frames, so we followed a very, kind of, safe path.*

With the intention of making his mathematics lessons well-structured and orderly, Thomas prefers to use the chronology of the current text book as a basis for his teaching planning. Similar statements regarding the use of the mathematics textbook are given in the first and the second interview. However, successful mathematics teaching is also characterised by students taking an active part in whole-class discussions. In order to facilitate fruitful discussions, Thomas highlights the teacher's need to marvel together with the students on current mathematics problems.

I1_628	Thomas	*I'm fond of being the old fashioned one, I think it's important with a part at the blackboard, at first, (...) so I want to, kind of the standard lesson I think would be like, introducing them to a topic, and then kind of very a, b, c, take an example at the blackboard (...) and then let them work on their own, and then there would be a summary (...) Hopefully, there's some discussion in the classroom. I find it important, I believe it's important for learning, that the students are able to express themselves. That you question a lot and wonder about things as a teacher.*
I1_653	Thomas	*I have kept myself to the text book, I've used it as a basis, and I'm not a very structured person, but I try to plan how to present things, at the blackboard and how I formulate things. And then I don't write down many key words, I write it very shortly, maybe, but it's very general.*
I2_81	Thomas	*(...) I do envisage that, in a way, it might be restricted to the textbook in the beginning (...) if it will be a lot of work to do, then it is easy to just lean on the text book (...) then I rather spice it up once in a while [laughing].*

Being on the threshold of a mathematics teacher career, I interpret Thomas' accounts of a future teaching practice to constitute a critical event in his evolving story of *perspectives on mathematics and its role in mathematics teaching*. In the subsequent section, I thus elaborate on Thomas' participation in the community of university mathematics and the community of secondary mathematics teaching, due to associated perspectives on mathematics as respectively a university discipline and a school subject.

7.2.2 First critical event: Attending towards a conventional mathematics teacher profession

Thomas' expressed perspectives on mathematics and mathematics teaching can be perceived as a continuation of his accounts of a sincere interest in, and a foregrounding of, mathematics for his teacher profession. In other words, the analysis to be presented might be regarded as an elaboration of the evolving story of *mathematics as a professional mainstay*. However, I will in the current section expand on what Ernest (1991) denotes as personal philosophy in mathematics. This includes Thomas' expressed views of the nature of mathematics and related theories of the teaching and learning of mathematics at university and in school. A discussion on Thomas' uttered philosophy will in turn enlighten the characteristics of the mathematics practices in which he exercises identification.

As explained earlier, Thomas' accounts of the discipline of mathematics and mathematics teaching from the first interview can be considered to evolve along three narrative paths: accounts of mathematical appreciation; accounts of mathematical argumentation; and accounts of own mathematics teaching. The first path is related to the awareness of the applicability of mathematics in problem solving, both within and outside the discipline. Examples from the transcripts are Thomas' descriptions of mathematics as a tool box and the usefulness of recent university subjects concerning numerical analysis and cryptography. A consequence of its unique contribution in problem solving is also the subject's immediate response on one's achievements. Further, mathematical appreciation is in Thomas' accounts related to gaining insight into the nature of mathematics, described by him as a cohesive discipline with "quite clear relations" (I1_358). The descriptions of an enjoyable discipline, a useful, however, a complex logical system, is in line with Andrews and Hatch' (1999) notions of mathematics as being a life-tool and a diverse and pleasurable activity. Following their division of secondary school mathematics teachers' conceptions of the discipline, mathematics as a life-tool concerns the empowerment of the individual through higher levels of understanding than the mere use of rote learned techniques. Hence, knowing mathematics is helpful in everyday life, in addition to offering a unique perspective on the world. Further, the conception of mathematics as diverse and pleasurable activity includes gaining enjoyment from the precision of mathematics. One example is Thomas' fascination regarding numerical analysis and the related tools for approximations and interpolation. Related to the two conceptions on mathematics are then elements of both absolutism and fallibilism. The former philosophical stance implies that mathematics is considered an immutable body of objective and certain knowledge, being useful due to its universal validity (Andrews & Hatch, 1999; Ernest, 2004b). However, according to Thomas, doing mathematics implies as well own sense-making in relation to problem-solving activities, which relates to the latter philosophical stance.

The second thread of Thomas' accounts, being about the distinctiveness of mathematical argumentation, can be related to what Ernest (1991) denotes as a relativistic absolutist stance on the nature of mathematics. From this point of view, of what Ernest denotes as the purist ideologies, the discipline of mathematics is perceived as a body of pure, objective knowledge, based on reason and logic, not authority (Ernest, 1991). A plurality of interpretations and frames of reference for the mathematics content is acknowledged, though, its truth depends on inner structure of mathematics, being logic and proof. Hence, mathematics should be considered by emphasising the structure, conceptual level and rigour of the subject. These characteristics correspond to Thomas' highlighting of argumentation within the practices of mathematicians, where they are supposed to "explain so that at least other mathematicians can verify it" (I1_401). Accordingly, students in secondary school should learn to argue mathematically and be "able to see relations between different parts of mathematics" (I1_388). However, while purist ideologists deny any connection between pure mathematics and its applications, Thomas recognises the utility of mathematics beyond the discipline itself. Hence, his expressed perspectives on mathematics, its teaching and learning are not uniformly in line with what Ernest (1991) classifies as formal absolutism.

Nevertheless, a purist ideology is apparent in the third thread of Thomas' accounts, regarding his expressed theories of mathematics teaching. He describes himself as an old-fashioned teacher whose main task is to explain the mathematics content in a structured, meaningful and understandable way to the students. Hence, Thomas communicates a view of mathematics teaching being in line with the ethos of "teaching mathematics", as opposed to "teaching children" (Ernest, 1991, p. 177). Within the purist tradition, the classroom mathematics teaching should further be enriched with additional problems and activities, in which the teacher is adapting the structured textbook approach to the current learning situation. In line with Thomas' emphasis on whole class discussions, the students are then expected to come up with different approaches and methods when applying their knowledge in problem solving.

The elaboration of Thomas' expressed perspectives on the nature of mathematics and mathematics teaching can be considered as characteristics of the community of practices in which he exercises participation. Regarding the previous defined *community of university mathematics*, Thomas exercises identification through engagement by investing himself in the university subjects in mathematics and in the relations with fellow students through mathematical discussions. From Thomas' viewpoint, the community's practice is then characterised by the joint enterprise of mathematical argumentation within a cohesive discipline.

Further, Thomas' accounts of his own mathematics teaching can be interpreted as identification with the *community of secondary mathematics teaching*. During school placement, Thomas' engagement in practice is characterised by following what he denotes a safe path, by keeping himself to a standard lesson structure and the chronology of the textbook. His perspectives on mathematics teaching can as well be related to an alignment with mathematics teaching traditions from own schooling, in which a good mathematics teacher was recognised by a well-structured instructions containing clear mathematical explanations. According to Lerman (1990), an absolutist view on mathematics implies that the teacher is the possessor of mathematical knowledge which the students must gain. Since the knowledge is certain, as are the methods used to solve mathematical problems, teaching becomes a conveying of that mathematical knowledge and those methods. Hence, teachers tend to transmit their knowledge in the well-defined and replicable ways that the teachers used before them (Andrews & Hatch, 1999). In terms for authorities for knowing the teaching of mathematics, Thomas' previous teachers act as external authorities, whose voices he seems to accept and utilise. Following Belenky et al. (1986), Thomas' way of knowing mathematics teaching is then in line with separate procedural knowing, where meaning is taken as given and dictated by external authority.

Regarding participation in communities of practice, I argue that Thomas exercises participation in the *community of secondary mathematics teaching* by imagination. It is due to Thomas' picturing of himself as a future mathematics teacher who commences a well-known practice. His participation thus stands in contrast to imagine a desire to change or develop the reigning mathematics teaching practice in school. Further, his simultaneous participation in the *community of university mathematics* and the *community of secondary mathematics teaching* can be considered as encountering and reconstructing boundaries, rather than overcoming discontinuities between university and school mathematics. Regarding university mathematics, he expresses both absolutist and fallible perspectives when emphasising the need for valid argumentation and for own sense-making from problem solving activities within a cohesive discipline. However, when it comes to mathematics teaching in school, Thomas highlights in the main absolutist characteristics of conveying mathematical knowledge and related methods in structured and well-defined ways to his students. Hence, while Thomas accounts for experiences of mathematics learning at the university as a dynamic and active process of investigating relationships and conveying arguments, the teaching of mathematics in school is to him about rather static methods of directing and presenting content via thorough explanations. His way of *attending towards a conventional mathematics teacher profession* is then interpreted as a first critical event in the evolving story of *perspectives on mathematics and mathematics teaching*, as shown in Figure 7.6. The figure illustrates that Thomas' negotiation of meaning regarding mathematics and mathematics teaching at university and in school takes place in coexisting memberships in two communities of practice. Here, Thomas' expressed perspectives related to his learning of mathematics are represented by participation in the *community of university mathematics*. His perspectives related to the teaching of mathematics in school are, on the other hand, represented by participation in the *community of secondary mathematics teaching*.

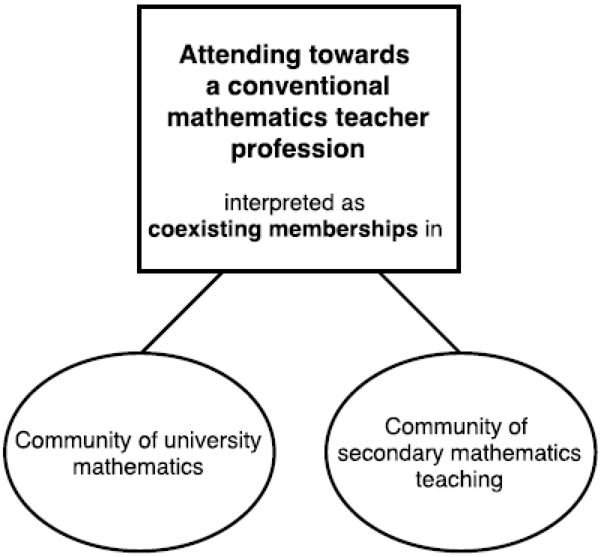

Figure 7.6: Perspectives on mathematics and its role in mathematics teaching, represented by the first critical event of attending towards a conventional mathematics teacher profession

7.2.3 Intermezzo: A teaching sequence on the derivatives of exponential and logarithmic functions

The classroom observation of one lesson in Mathematics R1 took place in March, during Thomas' second semester as an upper secondary school mathematics teacher. Accounts from the follow-up interview are presented in the subsequent section, constituting a second critical event of Thomas' story of perspectives on mathematics and its role in mathematics teaching.

7.2.3.1 The teaching sequence

The lesson is a review of a previously examined topic, with the main activity of solving former exam tasks on the derivatives of exponential and logarithmic functions. However, it is introduced by Thomas demonstrating the derivative of the functions $f(x) = e^x$ and $f(x) = \ln(x)$ at the blackboard. Thomas says that the natural exponential function is rather special, since, at any point of the function graph, the slope of the tangent line equals the value of the function at that point. Further, he sketches the graph of the logarithmic function, $f(x) = \ln(x)$ and emphasises that the derivative of the function is always positive. Although the graph flattens out, the function is increasing.

Next, Thomas demonstrates an example from the text book: For the function, $f(x) = xe^x$, find its local maxima/local minima. He refers to the product rule for derivatives, writing it on the blackboard. Further, he differentiates the function with reference to the product rule, without any contribution from the students, and gets $f'(x) = e^x + xe^x$. Thomas now asks the students what he can do to find the extreme points of the function *f*. One student suggests that he can put e^xoutside the brackets, and Thomas agrees. Hence, he writes the following expression at the blackboard: $f'(x) = e^x(1 + x)$. Thomas says further that, in order to find out where the derivative equals zero, he must observe when $(1 + x)e^x$ equals zero. He asks the students, for which value of *x*, e^x equals zero. A couple of students raise their hands, and one of them says: "I don't think e^x ever equals zero". Thomas responds: "No, good! e^x will never become zero. So, then it's $1 + x$ that can become zero." Another student raises her hand and says that the expression will equal zero for $x = -1$. Thomas draws a sign chart and shows that $f(x)$ has a global minimum in $x = -1$. Further, he finds the related function value with help from the calculator, and says: "Since they ask for the point, we have to find the *y*-value."

During the whole blackboard instruction, the class is quiet, and the students pay attention to what Thomas is explaining. After the introduction, they start working, individually or in pairs, on a selection of previously given exam tasks. The tasks are similar to those shown in Figure 7.7:

Example 1:	Example 2:
Find the derivatives of the following functions: a) $g(x)=3\ln(x^2-1)$ b) $h(x)=\frac{2x^2}{e^x}$	The function *f* is given by $f(x) = 4x^2 \cdot e^{-x}$ a) Show that $f'(x) = 8x \cdot e^{-x} - 4x^2 \cdot e^{-x}$. Draw the graph of *f'*. b) Use the graph of *f'* to find any extreme points on the graph of *f*.

Figure 7.7: Examples of previously given exam tasks on the derivative of exponential and logarithmic functions.

Thomas walks around in the classroom and helps the students who ask him for help. Some students ask Thomas whether they have solved the tasks correctly. They discuss the use of the chain rule and the product rule, and combinations of the two. The lesson ends with Thomas stating that the time is up, and that they will go through some of the exam tasks next week.

7.2.4 Accounts of perspectives on mathematics and its role in mathematics teaching, constituting a second critical event

In the subsequent section, I present accounts from the third and the fourth interview, both taking place during Thomas' first year as a secondary school mathematics teacher. The fourth interview was arranged in connection with observation of one of Thomas' lessons in Mathematics R1. However, his accounts throughout his first year as a secondary school mathematics teacher are comparable and considered to constitute a joint critical event in the evolving story of *perspectives on mathematics and its role in mathematics teaching*. As will be elaborated on in the subsequent section, the critical event concerns Thomas' way of *moving within an envisioned mathematics teaching practice*. Here, Thomas' described classroom practice can in the main be characterised as blackboard- and task-driven. His upcoming accounts of mathematics and its role in his own secondary teaching follow in general the narrative path from before graduation.

In the follow-up interview of the classroom observation, Thomas weighs the pros and cons of the session on derivatives of exponential and logarithmic functions. He highlights the students' progression in their use of the differentiation rules, although the pace of their work was slower than expected. When it comes to the structure of his teaching and the students' needs to practice mathematics through task solving, Thomas points to the challenge of finding the right balance between whole-class instruction and individual task solving. While the former activity is necessary for contextualising and explaining the current mathematics content, the latter is an essential learning opportunity in terms of letting the students applying the mathematics content.

I4_1	Thomas	*I feel at least that they learned something, at least that they were a little bit more secure about what we worked with, on those derivative rules. But I had maybe envisaged that we had time to get through, work at more than they did. But that I take as a symptom for the necessity to take kind of that review, that they were insecure. (...) The way it proceeded, it was, I took a small introduction and some repetition, because it was a while since they've had, there was some lessons that were lost.*
I4_152	Thomas	*I could have made them more aware of what I wanted from the lesson and I could have made a clearer conclusion in order to link together what we've been working on. (...) I find it difficult. It's something that's at the expense of other things, such as, I have more blackboard instruction and then less time for work[ing with tasks], what's best. And that's something I'm satisfied with, that they got time to work on their own.*

Nevertheless, the observed lesson is according to Thomas representative regarding the structure of his mathematics teaching. As reported by Thomas, an ordinary mathematics lesson is characterised by an introduction of the current mathematics content in combination with one or more assigned tasks related to what he has presented at the blackboard. A rule of thumb is then to divide the lesson into equal parts of respectively whole-class instruction and individual tasks solving.

I4_362	Thomas	*I introduce most of the math lessons by going through the subject content. The length of the review varies a lot, yes, and then I maybe give them a task which we've talked about at the blackboard, and then we maybe finish up. But in general, it's kind of, there are some tasks afterwards.(...) But I try to, kind of fifty-fifty, at least give them some time for doing it by themselves.*

In comparison to science teaching, mathematics teaching is characterised by Thomas as more structured and to a greater extent split into the two parts of teacher-directed teaching and task solving. Although he acknowledges the need to vary his teaching in science, the feedback from the students indicate that they appreciate a blackboard- and task-driven mathematics teaching.

I3_176	Thomas	*In science, it has been more classroom discussions. There, I haven't structured the lessons the same way as I do it in math. In math, I have kind of moved on with it, or, I try to not talk through the whole lesson, but it's more like separated than in science. I try to vary the teaching in science, and that's maybe helpful in math, too. However, I've had the individual student conversations and it seems like they appreciate the traditional way of teaching.*

However, Thomas faces challenges with the pace of his teaching and the related blackboard instruction. In addition to experience lessons which are progressing more slowly or faster than expected, students tend to follow his whole-class teaching with varying tempo. Since he is teaching both general education courses in mathematics and mathematics within the natural science specialisation, he struggles with accelerating his teaching from one lesson to the next. Due to the varying pace and the students' differing progression, there is then a need to adjust the time schedule from one week to the next.

I3_35	Thomas	*I've found it difficult to adapt the pace in the different [mathematics] classes, in comparison with being in school placement. When for instance I had 1T in this and that lesson, and then, I now have for example, first a group in the supplementary year of general studies[19] and then I have R1 afterwards, or the reversed order, and then there is a huge difference in the tempo for the blackboard instruction (...) so it's difficult to accelerate, kind of.*
I3_140	Thomas	*I follow the bigger semester plan, but there are some changes to it when something goes more slowly than expected and vice versa, something goes faster, the students catch things quickly and we can continue. (...) If they don't follow you at all, then there's no point in barging in, really. (...) If we then have moved forth more slowly or more quickly than planned, I just change it from week to week, from one lesson to the next, on the time schedule.*

In addition to accounting for challenges with the students' varying progression and the pace of the mathematics lessons, Thomas describes his students as being preoccupied by the right answer on a task rather than being concerned with the current mathematical topic or problem. One indication of their focus on the result of their task solving is their tendency to ask Thomas for clarifications on errors in their calculations. In their struggle of finding the correct answer, the students then appear to him as uncertain about their work in mathematics. In contrast, students showing a solid understanding in mathematics would be able to correct these mistakes themselves.

I4_38	Thomas	*They are kind of result-oriented. That they want, I don't give them any answers, by purpose, like I think, when they do tasks in the book, it's often like there's a long polynomial division or tasks with a sign error. And then they put up their hands to make the teacher find the error. If you're confident, in a way, that you've understood it, then, yes, you can maybe weed out the miscalculations over time. Volume training is also important.*
I4_179	Thomas	*They are maybe concerned about learning the procedures and concerned about the results on the tests and if the tests count, but they don't see the longer… don't focus on the task itself, but more on the results. They become very concerned with how to achieve the high grades. I feel, at least, that it can be kind of cramming maths sometimes. I try to pull them away from it.*

[19] Students in vocational education can take a supplementary year of general studies, in order to get qualified for higher education. They choose between the subjects 1P-Y and 1T-Y.

The students' result-oriented approach to mathematics is further apparent from their striving for good grades. Accordingly, the students focus on learning mathematical procedures for solving tasks that are likely to show up on a future test or exam. Thomas, on his side, wishes the students to focus on the current mathematical problem, by valuing the mathematics subject and its various applications. Examples of highlights from his mathematics teaching are then the lessons in which he has arranged for using a wider spectrum of the discipline. One successful lessons concerned the activity of performing algebraic generalisations from shape patterns.

I4_448	Thomas	*I had a lesson in which we looked at sequences and patterns. (...) It was in 2P and 2P-Y* [20] *(...) about this thing of using different representations and see relations between them. I find it exciting, since you can use concepts of area and circumference and so on to describe rate of change, or whether the growth is linear. (...) And then I felt we used kind of a wide spectrum of the mathematics. Both kind of doing calculation of area and looking for patterns in figures, expressing the pattern mathematically. I find this part to be nice for the students at 2P and 2P-Y.*
I4_466	Thomas	*(...) it worked out for many of the students. I thought that was fun watching. And this thing of going from something special to finding a general expression which works for everything. I think they found it kind of... I got the feeling that they thought it was cool.*

Thomas describes this activity of "going from something special to finding a general expression" as both exciting and manageable for the students in the mathematics courses 2P and 2P-Y. He highlights the opportunity for them to experience the cohesion of the mathematics subject and to share his appreciation of puzzling out the patterns.

I4_494	Thomas	*I think I succeeded in angling the teaching so they could see the relations with what they've learned earlier and how it can be applied. (...) I find it fun doing such things myself (...) Not all the patterns are self-evident, so it becomes kind of like a puzzle.*

During his first year as a secondary school mathematics teacher, Thomas portrays, in the main, a teaching practice being similar to what he before graduation characterised as the safe and familiar way of teaching mathematics. Instead of being formed and developed by impact of PPU,

[20] The subject 2P-Y is part of the supplementary year within vocational education, as a substitute for the general education subject 2P.

Thomas' mathematics teaching is targeting what he has experienced as good mathematics teaching in own schooling. His way of organising the mathematics teaching is also cherished by his students. However, due to their preoccupation with future test results and exams, Thomas appreciates the moments in which he manages to communicate mathematics as a cohesive and pleasurable discipline. Hence, the two-parted practice of making blackboard instructions and guiding task solving is occasionally replaced by some more varied and complex activities. Nevertheless, I expand in the subsequent section on Thomas' portraying of mathematics teaching as mainly task-driven. I argue that a prominent feature of his accounts is the way of talking about school mathematics as making a journey through a series of tasks, with stopovers and a variable pace.

7.2.5 Second critical event: Moving within an envisioned mathematics teaching practice

Looking back at Thomas' accounts from before graduation, his future ambitions concerned becoming a teacher who explains the mathematics content in a meaningful and understandable way to his students. In the first interview, he brought up his own teacher in lower secondary school, who managed to choose explanatory models that linked the classroom mathematics with experiential realities of his students. After graduation, Thomas portrays mathematics teaching to be about finding the right balance between blackboard instruction and individual task solving. Since tasks represent an important means of providing the students with learning opportunities, the role of the whole-class-instruction is to explain some mathematical aspects so that the upcoming tasks can be solved correctly. Hence, Thomas' described mathematics teaching practice seems to coincide with his previous expectations, yet with a foregrounding of task solving in the learning and teaching of school mathematics.

Since Thomas' accounts of mathematics and its role in mathematics teaching are to some extent coinciding with previous accounts, the critical event of *moving within an envisioned mathematics teaching practice* can be interpreted as a continued participation in the *community of secondary mathematics*. This is illustrated in Figure 7.8.

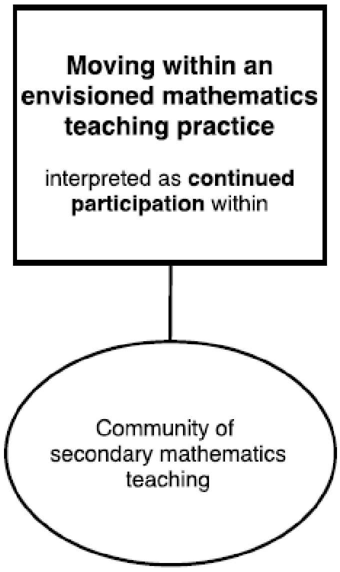

Figure 7.8: Perspectives on mathematics and its role in mathematics teaching, represented by the second critical event of moving within an envisioned mathematics teaching practice

However, in order to distinguish the second critical event from the first, and to elaborate on his continued participation in the community of practice, there is a need to highlight the characteristics of Thomas' accounts in the third and fourth interview. Following Wenger (1998), a community of practice can be recognised by its shared discourse which reflects a certain perspective on the world, or in this case, a certain perspective on mathematics and its role in teaching. For Thomas, I argue that his continued participation in the *community of secondary mathematics teaching* can be characterised by his particular way of talking about school mathematics and its teaching. More specifically, I argue that his accounts are in line with what Mellin-Olsen (1990) denotes as the task discourse. Mellin-Olsen applies the metaphor of a journey in his elaboration of the discourse: the school subject of mathematics tends to be described as a journey to be carried out in a certain period of time. Since one is talking about time and distance, one is likely to talk about speed, or in Thomas' case, the pace of a lesson. A certain speed is required in order to complete the journey, and the students might fall behind if they are not able to keep the pace. Consequently, the teacher faces a challenge of making the amount of the subject content to fit the time available in the mathematics lesson.

The metaphor of making a journey through blackboard presentations and sets of tasks is present in Thomas' descriptions of own teaching, as he accounts for "going through the subject content" (I4_362) in a regular lesson. Further, Thomas evaluates the observed lesson in Mathematics R1 based on the students' progression and the expected pace of their task work. Especially, his teaching subjects in mathematics are characterised by their pace and the difficulty of accelerating from one lesson to the next. Deviation from the expected progression makes it necessary to adjust the time schedule, and hence, to adjust the pace of his mathematics teaching.

Another hallmark of the task discourse is the students' tendency to focus on finding the correct answers as quickly as possible. According to Mellin-Olsen (1990), the journey metaphor brings with it expectations of keeping pace with the teacher's plans for the journey. The goal of the journey is usually the final exam. Consequently, the students ask the teacher for help, instead of taking time to find and evaluate various approaches and solutions on their own. The mathematics teacher therefore faces two problems: the problem of students making a halt when they meet a task they do not immediately know how to solve, and the problem of lacking methods for making the students reflect on tasks and their solutions. Thomas gives accounts in line with Mellin-Olsen's characterisations, when describing his students as result-oriented. Instead of showing an interest for the current mathematics problem, the students pay attention to the procedures and the correct solutions of tasks that are likely to be given on their future exam. Hence, they are in the main concerned with achieving good academic results.

The characteristics of Thomas' accounts regarding classroom mathematics teaching can further be related to Ernest's (1991) range of perspectives on the nature of mathematics. In line with an absolutist view, Thomas portrays the school subject of mathematics as a somehow static and immutable body of objective knowledge. By foregrounding the role of tasks in teaching, he describes the school subject of mathematics as consisting of absolute components to be carefully explained by the teacher, and further, to be applied by the students through solving specific problems. The role of the mathematics teacher is then to transmit mathematical knowledge and to verify that learners have received that knowledge through their task solving. Although Thomas appreciates the lessons in which he manages to communicate mathematics as a cohesive and pleasurable discipline, the main impression made by his accounts is, nevertheless, descriptions of a task-driven teaching. Hence, the mentioned lesson on algebraic generalisation of shape patterns appears as variation from the usual routine, rather than representing a main target for his teaching.

An absolutist perspective on mathematics and its role in teaching is further in line with Thomas' accounts from the first and second interview. As explained in Section 7.2.2, Thomas in the role of a prospective teacher described his future teaching of mathematics as based on static methods of directing and presenting content via thorough explanations.

Since mathematical knowledge and its methods were considered as certain, the teaching of mathematics was portrayed as conveying the knowledge and methods in what he characterised as a structured and old-fashioned way (see I1_628 in Section 7.2.1). By continuing along an existing narrative path after graduation, Thomas then appears to negotiate the meaning of mathematics and mathematics teaching in school within the frames of previously founded practices. The students in the mathematics classroom take the role as bargainers within the *community of secondary mathematics teaching*, by responding to the pace of the lessons, the balance of whole-class instructions and related task solving, and by aiming for the one, correct answer on tasks. Here, the students' former memberships in communities of mathematics teaching and learning in school are also part of the negotiation, as they appear to "appreciate the traditional way of teaching" (I3_176). Consequently, Thomas' continued participation in the *community of secondary mathematics teaching* concerns alignment with mathematics teaching practices stemming from own schooling and school placement during teacher education, and where development or a change of his reigning mathematics teaching practice is not a prominent part of the negotiation.

7.3 A summary of Thomas' stories of becoming a secondary school mathematics teacher

Similar to the two previously presented cases of Isaac and Nora, I review the narrative progression across the critical events of Thomas' two evolving stories. Hence, I change the approach of the analysis, from presenting the content of Thomas' accounts as evolving stories based on a series of critical events, to focusing at the stories' progression, as either progressive, regressive or stable narratives of mathematics teacher identity development.

Thomas' story of *mathematics as a professional mainstay* evolves from the situation of facing towards a future mathematics teacher career during his final year in teacher education, to become an employed full-time upper secondary school mathematics teacher. Before graduation, he gives retrospective accounts concerning a turning point during his university studies, in relation to a development in his academic maturity. I have argued that the described turning point, in the shape of a strengthened affiliation towards the mathematics discipline, serves as a driving force for becoming a mathematics teacher. Hence, Thomas' imagination of a future teacher career is based on a reinforced belonging to the mathematics discipline itself, rather than a primary concern regarding the practice of teaching. Nevertheless, the evolving story reveals in the main a confirmation or continuation of mathematics and mathematics teaching related practices throughout the four interviews. Hence, the transition from university studies to school teaching is portrayed through a stable narrative, where continuity is represented by Thomas' sustained interest in and foregrounding of the discipline of mathematics. When working as an upper secondary school mathematics teacher, the foregrounding of mathematics is apparent from Thomas' described freedom of placing mathematics at the core of his teaching practice. In parallel with expressing a constant belonging to the learning and teaching of mathematics, Thomas also maintains a marked distance towards the teacher education programme of PPU.

On the basis of Wenger's (1998) social learning theory, the narrative stability of Thomas' story of *mathematics as a professional mainstay* can be described by his continued participation in the *community of secondary mathematics teaching*. Before graduation, Thomas' participation by engagement concerns the steering of his educational pathway towards a future teacher career, by choosing his Master's specialisation within the field of mathematics education. The inbound participation is further pursued after graduation, in the shape of negotiating the meaning of mathematics, its teaching and learning, with his students in the mathematics classroom. In addition, Thomas exercises an active involvement in the mathematics teacher collaboration. A common theme in his engagement is his preference for the more advanced mathematics subjects. The narrative stability is therefore related to Thomas' sustained practices based in the *community of university mathematics*. It concerns a pursued participation by imagination, where Thomas' projects himself into a picture of an academically competent mathematics teacher. Simultaneously, his accounts display a sustained non-participation in the *community of university teacher education* at PPU. Here, Thomas presents the community's meanings of mathematics and its teaching to lie beyond his engagement at PPU, as he finds the practice of teaching to be based on own classroom experience rather than the programme's scholarly knowledge of general pedagogy and mathematics education.

Like his foregrounding of mathematics for his professional practice, Thomas' story of *perspectives on mathematics and its role in mathematics teaching* reveals continuity in the transition from university to school. Before graduation, his accounts outline a divide between perspectives on university mathematics as a subject for study and perspectives on school mathematics as a subject for teaching. While the former is characterised by problem solving and argumentation within a cohesive discipline, the latter concerns conveying mathematical knowledge and organising task solving in a clear and structured way. However, when entering upper secondary school as an employed mathematics teacher, Thomas' accounts reveal continuation in the shape of his pursued emphasis on a blackboard- and task-based mathematics teaching. The narrative stability is further apparent by the consistency between his expectations of own future teaching and the reality he is meeting in the mathematics classroom. As I have argued in Section 7.2.5, Thomas is moving within an envisioned mathematics teaching practice, where a foregrounding of tasks is characteristic for his classroom practice. In terms of participation in communities of practice, Thomas is thus exercising a continued participation within the *community of secondary mathematics teaching*, in interplay with previous engagement in the *community of university mathematics*. Due to the continuation of mathematics and mathematics teaching related practices, Thomas' identity as a secondary school mathematics teacher is a case of reinforcement rather than changes or development in identification and negotiability in community of practices. Figure 7.9 gives an overview of his narratively developing identity.

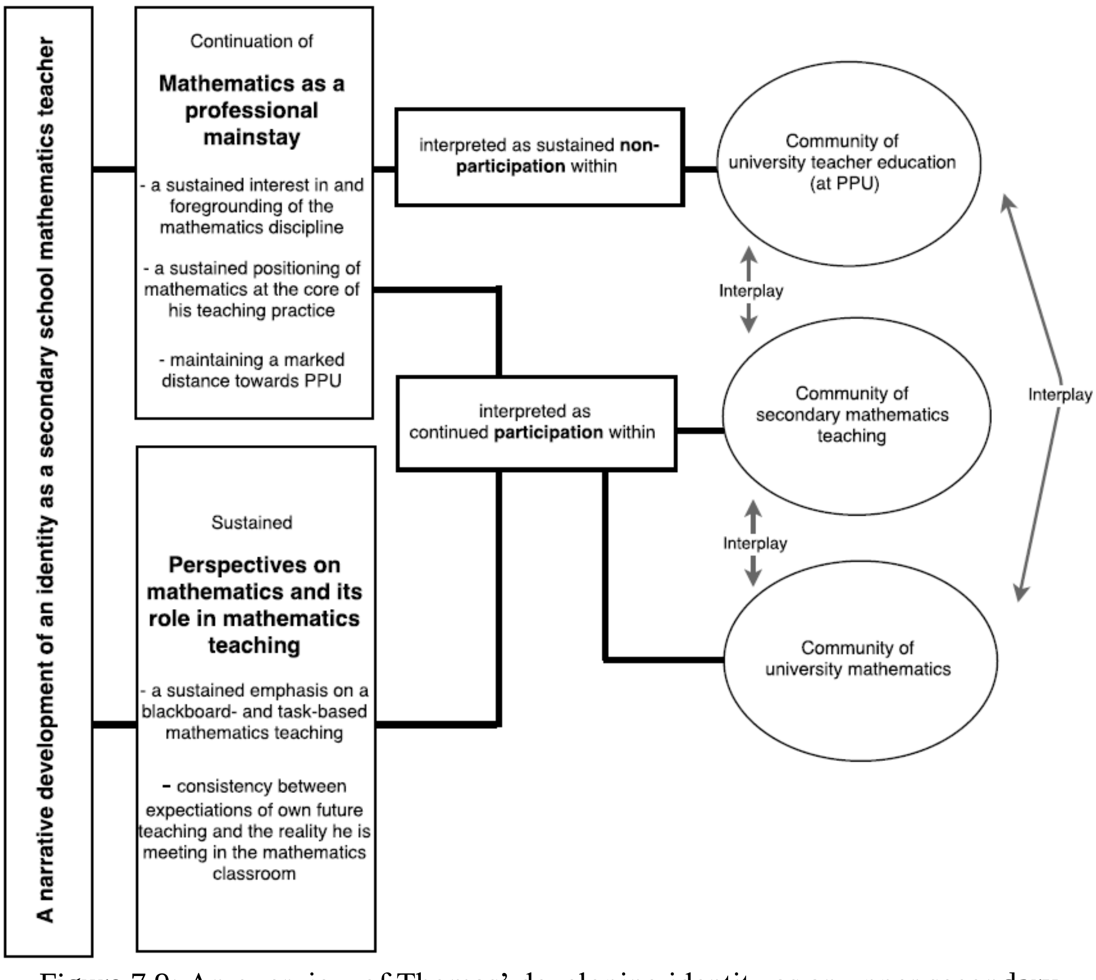

Figure 7.9: An overview of Thomas' developing identity as an upper secondary school mathematics teacher

The continuation of *mathematics as a professional mainstay* is linked to Thomas' continued participation within the *community of secondary mathematics teaching*. Next, I perceive this continued participation to be related to his sustained non-participation in the *community of university* *teacher education* at PPU and a sustained participation within the *community of university mathematics*. Accordingly, the continuation of Thomas' *perspectives on mathematics and its role in mathematics teaching* bonds to his continued participation in the *community of secondary mathematics teaching*. I further assume Thomas' participation in the communities of practice to be in interplay, as represented by the double arrows between the ovals in the diagram.

8 Cross-case analysis

In this chapter, I make a thematic comparison of the three participants' accounts. I seek similarities and differences across the cases, by discussing three common topics that occur in the participants' stories: being a learner of mathematics, undergoing university teacher education, and being a teacher of mathematics. These common topics should not be considered as main findings or categories of the cross-case analysis. Instead, they structure the comparison and enable me to move across the participants evolving stories. The intention of the cross-case analysis is then to move from individual cases and the related findings, being the participants' evolving stories and their narrated evidence of participation and non-participation in communities of practice, towards a more general description of mathematics teacher identity in the transition from university teacher education to school employment. Based on interpretation of the thematic comparison by means of the theoretical framework, I present two dimensions of developing an identity as a secondary school mathematics teacher. The dimensions, being synthesised in Section 8.3, are denoted *negotiating experience of self and mathematics learning* and *negotiating experience of self and mathematics teaching*. They are refined by the research participants' expression of ways of knowing mathematics and mathematics teaching, based on Belenky et al.'s (1986) categorisation. Further, the dimensions mirror the participants' expressed perspectives on the nature of mathematics, its teaching and learning. Hence, I also relate the compared accounts to Ernest's (1991) report on educational ideologies in mathematics.

8.1 Being a learner of mathematics

The topic of being a learner of mathematics is common to the three cases, as both Isaac, Nora and Thomas give accounts of own schooling and of undergoing mathematics studies at the university. Isaac's story of *confidence in mathematics and mathematics teaching* concerns his attraction towards both school and university mathematics and his expressed belief of mastering the discipline. It is based on his described interest for mathematics in school and of his enthusiasm for mathematics as an essential tool in other disciplines at the university. In addition, others recognise him as a successful student in mathematics, with the built-in expectations of making an academic career. One characteristic of Isaac as a learner of mathematics is thus his identity of inclusion in practices of mathematics, more specifically through participation in the *community of mathematics as an applied science* at the university. His belonging to the community appears from the act of cramming his university schedule with mathematics and by voicing a perspective on the discipline being in line with Ernest's (1991) notion of multiplistic absolutism. Further, Isaac voices negotiability within the community in line with what Belenky et al. (1986) denote as separate procedural knowing. Since the mathematics discipline represents absolute knowledge, its meaning is given by an external authority; however, Isaac expresses the ability to utilise the absolute knowledge in effective ways.

Isaac's identification with the discipline of mathematics is also present in his evolving story of *perspectives on mathematics and its role in mathematics teaching*. In the story's beginning, Isaac delineates a dichotomy between the pure, absolute discipline of university mathematics and the fallible, nearly non-scientific social sciences found at PPU. His affiliation towards the mathematics discipline is yet a door opener into the mathematics teacher profession, as he accounts for identification with competent teacher colleagues and for meeting desired academic challenges.

Nora, too, describes an initial interest in school mathematics from her time in secondary school. However, her accounts of studying mathematics at the university are characterised by incomprehension and alienation towards the discipline. Making tireless efforts of passing university exams in order to be accepted for school placement at PPU, Nora struggles with perceiving herself as a legitimate member of the *community of university mathematics*. Her marginal participation in the community is also apparent from her alienation towards other performers of the university discipline, such as the mathematicians at the department's Friday seminars. As a learner of mathematics at the university, Nora thus describes *mathematics as a repulsive discipline*.

Her evolving story of *perspectives on mathematics and its role in mathematics teaching* gives further insight into her dissociation with the subject. Based on her accounts of mathematics as a provider of immediate reward when working on tasks, but on the other hand, feelings of discomfort when not being able to solve a problem, Nora depicts an absolute, cold and remote discipline depending on external authority. Similar to Isaac, Nora draws a dichotomy between the certain, unquestionable and objective mathematics discipline, and the corrigible, imperfect and fallible knowledge of general pedagogy. However, unlike Isaac, she identifies with the latter discipline and its human face. Her developing identity as a mathematics teacher is therefore *not* founded on a well-built interest in the discipline of mathematics. Instead, mathematics becomes the means for her to pursue an interest in the practice of teaching.

In the case of Thomas, I have argued for a foregrounding of mathematics in his accounts of becoming a mathematics teacher. Central to his evolving story of *mathematics as a professional mainstay* is his developing academic maturity during his university studies in mathematics. As a learner of mathematics, Thomas describes actions of pulling himself together and increasing the workload for his mathematics subject studies. Further, he expresses identification with other competent students in mathematics. In contrast to Nora's distancing from the mathematicians' world, Thomas thus accounts for inbound participation in the *community of university mathematics* during his university studies. He is drawn towards a teacher career in secondary school; however, he gives a mathematical profile to his teacher profession.

Similar to Isaac, Thomas describes mathematics as an extensive and rich toolbox, being applicable beyond the discipline itself. However, Thomas highlights even more the discipline's basis on reason and logic, where truth depends on inner structure and proof. In his story of *perspectives on mathematics and its role in mathematics teaching*, he thus depicts university mathematics as a relativistic, absolutist discipline, being characterised by mathematical argumentation. By highlighting learner's own sense-making in problem-solving activities, his accounts hold also instances of fallibilist perspectives.

The three cases give insight into prospective mathematics teachers' varied ways of belonging to mathematics, when considering their role as learners of the discipline. Accordingly, they depict different reasons to enter the mathematics teacher profession. While Isaac and Thomas express affiliation towards mathematics, and consequently, towards mathematics teaching in school, Nora depicts a detachment towards university mathematics. On the other hand, she accounts for a sincere interest in the practice of teaching. Adopting the vocabulary of Wenger (1998), I denote these different relationships with mathematics as the work of *negotiating experience of self and mathematics* when becoming a secondary school mathematics teacher. The notion of self refers here to the three cases as learners of mathematics, based on their accounts of undergoing subject studies in the discipline. Hence, the negotiation of self and mathematics can be associated with considering the following questions: How do I perceive the discipline of mathematics? How do I perceive myself as a learner within the discipline? According to Wenger, a person's identity exists then in the constant work of negotiating the self through participation within and between communities of practice. Hence, the negotiated experience of self and mathematics are, in the three presented cases, made through identification or non-identification, and negotiation of meaning within respectively the *community of mathematics as an applied science* (Isaac) and the *community of university mathematics* (Nora, Thomas).

Based on the participants' accounts regarding being learners of the discipline of mathematics, I can relate their mathematics teacher identity to a spectrum of accounts of the discipline, as shown in Figure 8.1. The double-headed arrow spans the participants' various ways of identifying with mathematics as a scientific discipline: from being described as an absolutist discipline (Nora and Isaac) towards absolutist reports with instances of fallibilism (Thomas). The spectrum thus displays Ernest's (1991) range of ideologies of mathematics education, from dualistic absolutism towards relativistic fallibilism. In parallel are the participants' expressed negotiability regarding mathematics, in other words their ways of knowing the discipline. Here, the arrow spans from Belenky et al.'s (1986) notion of received knowing, and thus, mathematics knowledge depending on external authority (Nora), to voices of reasoning and procedural knowing (Thomas and Isaac). Ways of knowing that are not apparent from the research participants' accounts, namely silence and subjective knowing, are enclosed in brackets. Accordingly, I place the narrated mathematics teacher identities of Nora, Isaac and Thomas along the spectrum. They appear in the diagram as silhouettes, where their various depictions of mathematics are related to their participation and non-participation in communities of practice at the university.

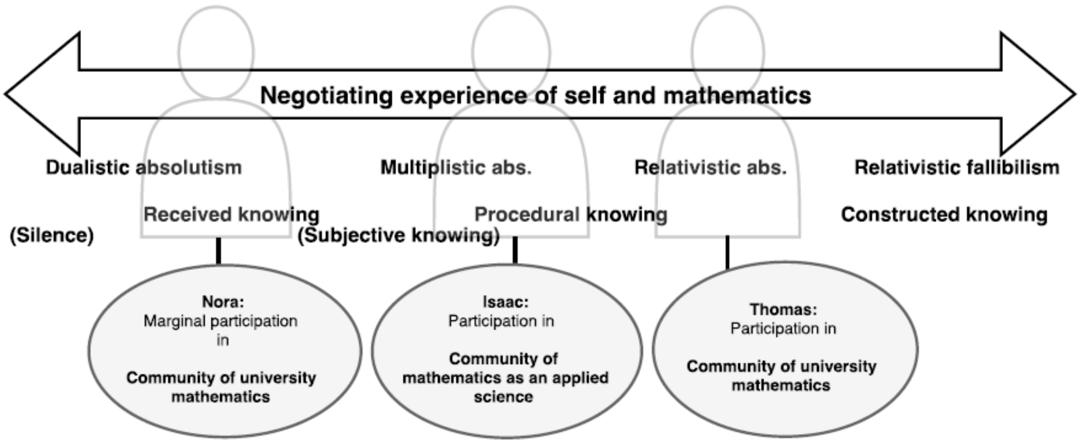

Figure 8.1: Developing an identity as a secondary school mathematics teacher, represented by the work of negotiating experience of self and mathematics

According to Figure 8.1, the range of ideologies of mathematics education is coinciding with the ways of knowing in mathematics. As an example, Nora's descriptions of receiving and reproducing mathematics knowledge coincide with her portray of an absolute, cold and remote university discipline. The concurrence along the spectrum might not be surprising, as both Ernest (1991) and Belenky et al. (1986) relate their work to Perry's (1970) theory of intellectual and ethical growth and his notions of dualism, multiplicity and relativism (see Section 2.2.5). Nevertheless, the cross-case analysis reveals a consistency between the three cases' expressed philosophies of mathematics, and the ways they portray themselves as knowers of the discipline. This is in line with an assumption of Belenky and colleagues, that the way a person perceives the nature of truth and reality shapes the way he or she sees the world and him- or herself as an actor in it.

8.2 Undergoing university teacher education

Another common topic across the three cases is their accounts of undergoing the university teacher education programme, PPU. In his story of *confidence in mathematics and mathematics teaching*, Isaac reports on undergoing school placement. His accounts display non-participation in a potential *community of tutor and students*, due to the lack of negotiating the mathematics teaching with his tutor. Further, Isaac gives a comprehensive report of undergoing university teacher education in his evolving story of *perspectives on mathematics and its role in mathematics teaching*. During his first semester at PPU, he describes the desired teacher education programme with instrumental terms, such as being a useful tool to improve his teaching. However, he accounts for a divide between what he finds to be a useful mathematics didactics course, and what appears to him as irrelevant academic discussions within general pedagogy. His accounts represent later a shift in perspectives on the role of PPU, from considering teacher education to represent two incompatible cultures of the natural and the social sciences, to viewing its syllabus in a new light. Isaac's changed perspectives are also visible from his expressed expectations of changing his mathematics teaching in direction of inquiry based practices. Accordingly, he exercises inbound participation in the *community of inquiry based mathematics teaching*. Alongside Isaac's change of perspectives is his shift in ways of knowing mathematics teaching. Hence, he moves from expressing innate abilities or subjective knowing, to voicing a need for further training, and thus, doing critical judgement of both external and internal authorities on knowledge regarding the teaching of mathematics.

Isaac is the only research participant displaying a narrative development across the interviews regarding expressed perspectives on mathematics teaching and university teacher education. Nora, for her part, gives retrospective accounts in her first interview on altered perspectives during her first semester at PPU. In addition to accounting for a growing attraction towards mathematics teaching during school placement, she describes a shift from initially perceiving the mathematics didactics course as nonsense, to giving her new initiative to become a mathematics teacher. However, across the interviews, she gives stable reports on a limited ownership of meaning, and consequently, a sustained peripheral participation in the *community of secondary mathematics teaching*. The limited ownership of meaning relates to Nora's inner authority or subjective knowing regarding knowledge of mathematics teaching. Examples are her portrait of educational research in mathematics as a swinging pendulum, and the statement that "the way PPU is showing us to be the right way, is maybe not the way I will follow" (I2_513).

Thomas, too, gives stable accounts of undergoing university teacher education, which I have interpreted as sustained non-participation in the *community of university teacher education*. His dissociation with the community's practices appears from his contrasting of parts of the LUR programme. While the Master's courses in mathematics education appear to Thomas as significant for his future mathematics teaching, the subject didactics courses at PPU are disconnected and superficial, giving what he considers a biased presentation of inquiry based teaching. Thomas' continued display of a non-participation in the *community of university teacher education* corresponds then to his expression of an internal authority or subjective knowing regarding the scholarly knowledge of general pedagogy and mathematics education at PPU.

The three reports on undergoing university teacher education show different learning trajectories of becoming a mathematics teacher, as they represent different impacts of PPU on the participants' accounts. Hence, their accounts constitute a second dimension of developing an identity as a secondary school mathematics teacher, being the work of *negotiating experience of self and mathematics teaching*. The notion of self refers to the prospective mathematics teachers being learners, not only of the mathematics discipline, but also of the teaching of mathematics. Based on the research participants' accounts, the negotiation of experience of self and mathematics teaching are made through non-participation within the *community of tutor and students* (Isaac) and the *community of university teacher education* (Thomas), peripheral participation within the *community of secondary mathematics teaching* (Nora) and inbound participation in the *community of inquiry based mathematics teaching* (Isaac). Further, the three cases show various ways of knowing regarding mathematics teaching, as relying on an internal authority for own teaching (Nora and Thomas), or as making critical judgement of both external and internal authorities on knowledge of mathematics teaching (Isaac). However, the work of negotiating experience of self and mathematics teaching is further based on carrying out mathematics teaching in school. The following section therefore completes the elaboration of the second dimension, by making a closer look at the participants' accounts of working as secondary school mathematics teachers.

8.3 Being a teacher of mathematics

Isaac initially describes mathematics teaching as something that comes natural to him, like a spontaneous reaction to the students' behaviour. By voicing an ownership of meaning regarding mathematics teaching and learning in the classroom, he thus appears as a participant within the *community of secondary mathematics teaching*. However, by characterising mathematics teaching as somehow innate, Isaac voices internal authority or subjective knowing regarding mathematics teaching. Yet, his accounts are changing when he enters secondary school as a certified mathematics teacher. In the final interview, Isaac evaluates his first mathematics teaching experiences as exceptional and providing him with too little resistance. He reports on challenges of engaging his students in mathematics practices, by making meaningful approaches to the mathematics topics being taught. At the same time, he finds such challenges essential for developing further his mathematics teaching practice. Isaac's altered accounts represent therefore changes in negotiability, from holding an internal authority to making critical judgements in order to improve his mathematics teaching. Consequently, he exercises a changed participation in the *community of secondary mathematics teaching*.

When undergoing PPU, Isaac imagines becoming a teacher who presents mathematics as a useful subject and which requires understanding beyond rote learned procedures and algorithms. Finding Skovsmose's (2003) literature on landscape of investigation to be in line with what he regards as good mathematics teaching, Isaac further aligns with the enterprise of developing mathematics teaching through inquiry based practices. Hence, he exercises participation by imagination and alignment in a *community of inquiry mathematics teaching*. Due to limited negotiability in the mathematics classroom and lacking possibilities for collegial engagement on developing the reigning teaching practices, Isaac struggles with implementing a mathematics teaching consistent with his ambitions. Nevertheless, his accounts of the desired mathematics teaching have common features with the teaching being emphasised by public educators or social change ideologies (Ernest, 1991). According to their educational philosophy, the teaching of mathematics must involve discussion and cooperative group work, problem posing and investigative activities, as its aim is students' empowerment and democratic citizenship.

Traces of Isaac's perspectives on mathematics teaching are also present in his story of *feedback available within the school environment*. He places the students and their mathematical learning in the foreground of his teaching, in which their positive and negative feedback is a necessary condition for identifying himself as a mathematics teacher. When facing the vocational students' withdrawing from mathematics, he finds himself enforced to make changes in his mathematics teaching, in the direction of basic numeracy and step-by-step worksheets. In parallel, Isaac gives a regressive story regarding feedback available within the teaching staff at his second school of employment. The students therefore serve as his main sparring-partner for negotiating the meaning of mathematics teaching.

In the case of Nora, the notion of feedback does not constitute an evolving story in itself. However, feedback still appears as important for her in order to reconnect with mathematics during school placement, and for wanting to become a secondary school mathematics teacher after graduation. In her story of *mathematics as an attractive and repulsive discipline*, Nora thus portrays peripheral participation in the *community of tutor and students*. Further, the students' feedback provides Nora with new initiative for further developing her mathematics teaching, in which the students' challenges and struggles with mathematics make up the focus of her attention. Hence, Nora portrays herself as a teacher who is able to establish trustful relations and a safe classroom environment for her students in the mathematics classroom. Accordingly, she exercises participation in the *community of secondary mathematics teaching*.

I have argued that Nora's accounts of mathematics teaching are in line with the progressive tradition in mathematics education. It relates to an absolutist perspective on the discipline of mathematics, however, with values of empathy, caring and human dimensions of mathematics. Associated with the progressive tradition is further the epistemological assumption of empiricism, meaning that students' knowledge develops through their composite interaction with the world. Nora displays empiricism by her way of using a variety of external recourses in her teaching, such as digital learning resources and mathematics games. Although including a variation of activities can be an expression of Nora's adoption of investigative mathematics practices from teacher education, an unbalanced adoption of resources is also an expression of received knowledge and external authority for the meaning of mathematics teaching and learning. Consequently, Nora's negotiability regarding mathematics teaching and learning appears as limited, leading to peripheral participation in the *community of secondary mathematics teaching*.

When entering school as a substitute mathematics teacher, Nora accounts for a restrained partaking in practices of mathematics teaching at her school of employment. Due to her temporary attendance and lack of responsibility for the current lesson plans, she holds a marginal position within *communities of students and their regular teacher*. Consequently, there are restricted possibilities for development of her mathematics teaching, and thus, her mathematics teacher identity.

Unlike Nora, Thomas reports on mathematics teaching in which the subject content of mathematics is the core activity. This is apparent from his favouring of the more advanced mathematics topics in upper secondary school, and by his emphasis of a strong mathematical background for being able to exercise good mathematics teaching. Further, by portraying himself as a kind of old-fashioned mathematics teacher, whose main task is to explain the mathematics content in a structured and understandable way, he aligns with purist ideologies of mathematics education. In line with an absolutist perspective on the mathematics discipline, the teacher is then a possessor of mathematical knowledge that the students must gain. Since the knowledge is certain, as are the methods used to solve mathematical problems, teaching is, according to Thomas, about conveying that mathematical knowledge and those methods. Accordingly, Thomas holds previous mathematics teachers as external authorities for his teaching. By accepting their practices and voices of reason, Thomas exercises a procedural way of knowing mathematics teaching in school.

After graduation, Thomas continues along a similar narrative path, however, with an emphasis of a task-based teaching in line with Mellin-Olsen's (1990) notion of the task discourse and the journey metaphor. Here, the students take the role as negotiators, by responding to the pace of Thomas' lessons, the balance of whole-class instruction and task solving, and by aiming for the one, correct answer on tasks. Further, Thomas appears to negotiate the meaning of mathematics teaching in school within the frames of previously founded practices. Hence, Thomas' continued participation in the *community of secondary mathematics teaching* concerns alignment with mathematics teaching practices stemming from own schooling and school placement, in which previous teachers appear as authorities. A development or a reform of his reigning mathematics teaching practice seems not to be part of the negotiation.

Based on the participants' different accounts, I can relate their mathematics teacher identity to a spectrum of expressions regarding mathematics teaching. In Figure 8.2, I thus present the dimension of *negotiating experience of self and mathematics teaching* as a double-headed arrow spanning the participants' various ways of identifying with the teaching of mathematics. The dimension can be associated with the following questions: What are the characteristics of mathematics teaching? How do I perceive myself as a prospective teacher in mathematics? The individual case study analysis reveals perspectives of purist ideologies for mathematics teaching in line with old humanists (Thomas), perspectives of progressive educators (Nora) and of public educators (Isaac). Further, the spectrum shows various degrees of negotiability or ownership of meaning regarding the teaching of mathematics. Nora reports on both internal and external authorities for her mathematics teaching, and thus, what I have argued is a limited negotiability within the current communities of practice. Thomas, too, reports on limited negotiation of meaning regarding the development of his mathematics teaching, as he follows the examples of previous teachers in own schooling. He simultaneously marks a distance towards PPU, in the form a sustained non-participation in the *community of university teacher education* at PPU. Isaac, on his side, portrays a movement away from subjective knowing towards efforts of integrating what he knows intuitively and what other resources from university teacher education know. This is apparent from his inbound participation in the *community of inquiry based mathematics teaching*.

Like in Figure 8.1, I place in Figure 8.2 the narrated mathematics teacher identities of Thomas, Nora and Isaac along a spectrum of perspectives of mathematics teaching and ways of knowing. The individual cases appear in the diagram as silhouettes, where their various positions link to their participation and non-participation in communities of practice.

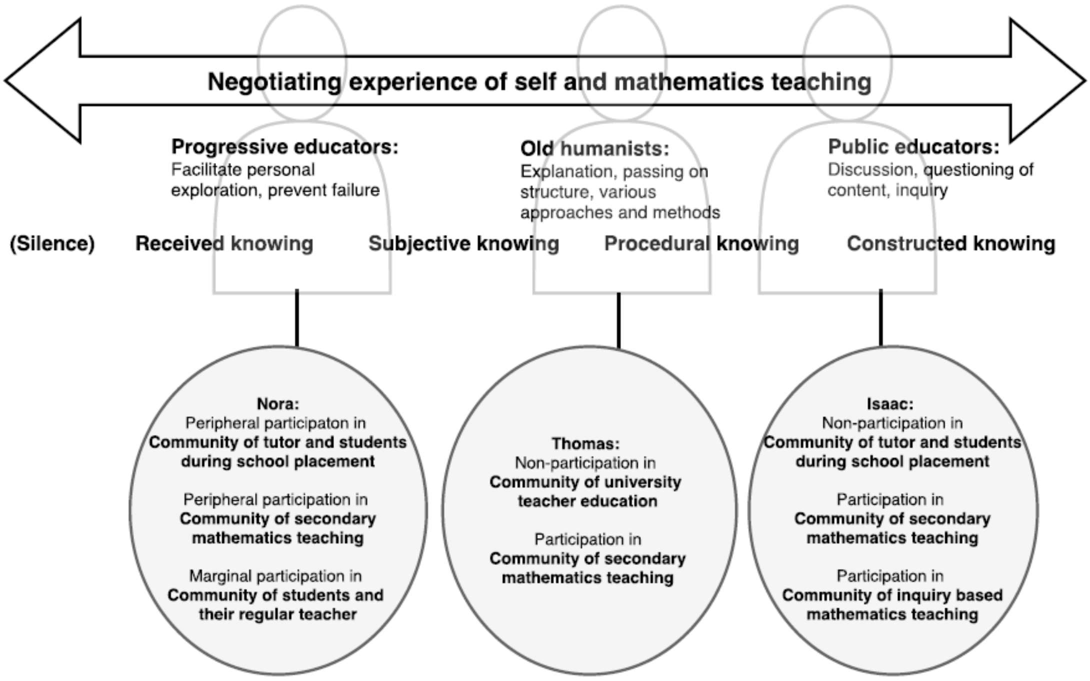

Figure 8.2: Developing an identity as a secondary school mathematics teacher, represented by the work of negotiating experience of self and mathematics teaching

The parallel representation of identification and negotiability is based on the assumption that the ways teachers perceive mathematics teaching is associated with the ways they perceive themselves as participants or knowers in it. For instance, perceiving mathematics teaching in line with the old humanists, as conveying mathematics in well-defined and replicable ways, is associated with a restricted scope of action for negotiating mathematics teaching towards a changed teaching practice. Similarly, identification with mathematics teaching in line with the public educators, as aiming for social change, relates to extensive negotiability among both students and the mathematics teacher. However, looking at Figure 8.2, the cross case analysis reveals some inconsistencies between Nora's expressed educational philosophies and the way she describes herself to act as a knower of mathematics teaching. Here, Nora's accounts of progressive education, in terms of facilitating the students' exploration and preventing failure, appears coincidental with her accounts of a limited negotiability in her own mathematics teaching. The inconsistency between Nora's desired teaching and her scope of action for implementing it can be associated with the gap discussed in belief research, between teachers' beliefs regarding mathematics teaching and their actual classroom practice (Wilson & Cooney, 2002). With Wenger's (1998) terms, the inconsistency concerns then the two-sidedness of a person's identity: While the notion of identification defines which meanings of mathematics teaching matter to Nora, it does not determine her ability to negotiate these meanings in current communities of practice. Her accounts of identifying with connected relativistic absolutism corresponds to a position between Thomas and Isaac, in other terms between the old humanists' separated relativistic absolutism and the public educators' relativistic fallibilism. However, by accounting for a limited scope of action for carrying out her mathematics teaching, and thus, for a limited ownership of meaning, I locate Nora's narrated identity towards the left end of the spectrum.

Isaac holds the position towards the right end, as he is moving towards constructed knowing by critically taking into account meaning communicated in university teacher education. In addition, he is aiming for an ideal mathematics teaching with terms of discussion, problem posing and inquiry. Thomas holds the middle position, due to his descriptions of mathematics teaching as carefully explaining and passing on structure to his students. In addition, his position represents a reliance on external authorities, being his previous mathematics teachers. However, unlike Nora, he exercises an inbound participation in the *community of secondary mathematics teaching*, by negotiating his mathematics teaching with the students and having an active involvement in the mathematics teacher collaboration.

8.4 Synthesis: Developing an identity as a secondary school mathematics teacher

Based on the thematic comparison and explanation of the two dimensions of becoming a mathematics teacher, I synthesise the cross-case analysis into a unified presentation in Figure 8.3. According to the diagram, developing an identity as a secondary school mathematics teacher is then about navigating in a landscape formed by the work of negotiating experience of self and mathematics, and negotiating experience of self and mathematics teaching.

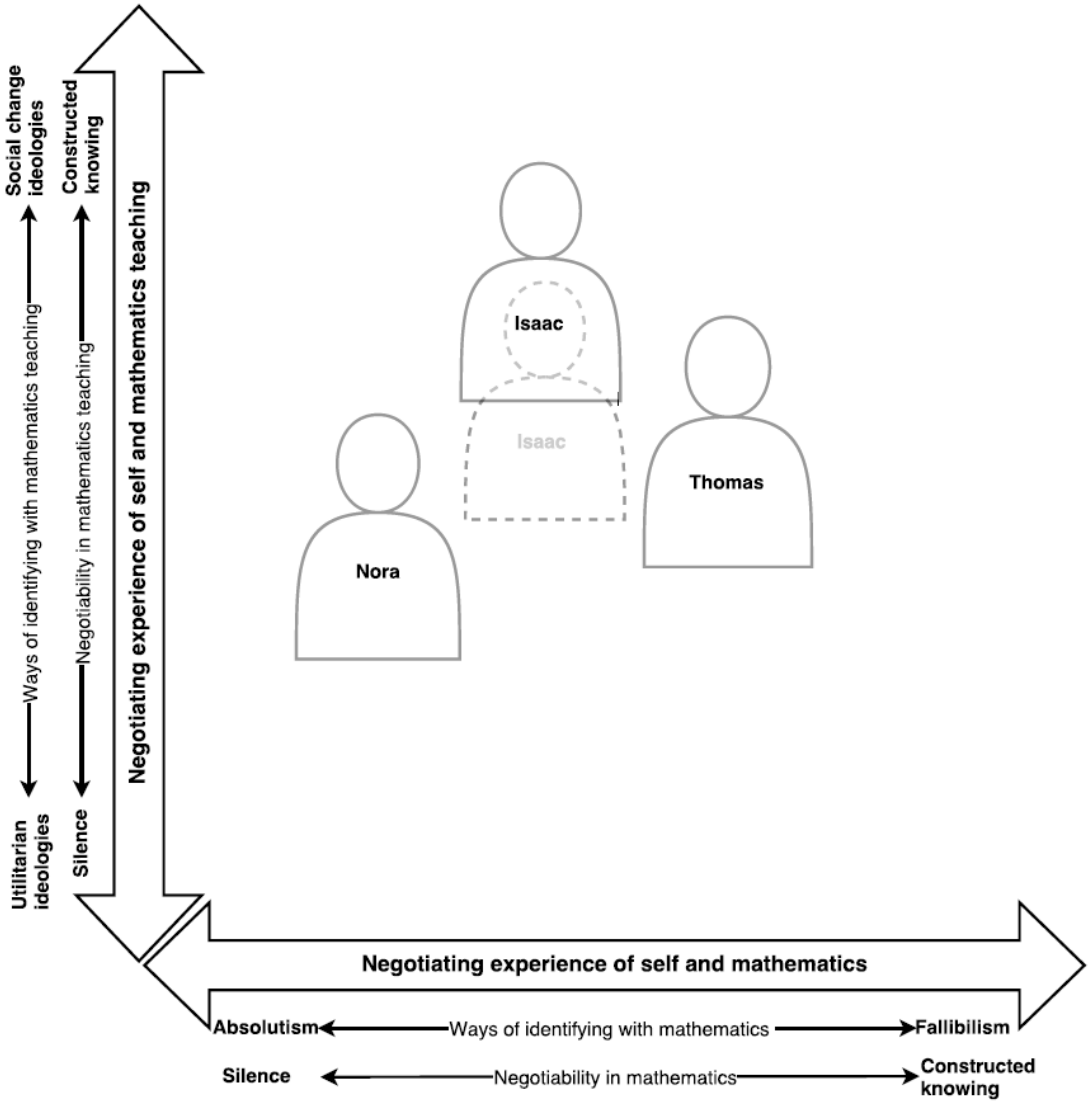

Figure 8.3: A synthesis of developing an identity as a secondary school mathematics teacher

In continuation of Figure 8.1 and Figure 8.2, I place the silhouettes of Isaac, Nora and Thomas into the diagram. However, regardless of a prospective mathematics teacher's accounts of entering the profession, I assume that his or her narrated identity relates to a location within the span of the two dimensions illustrated in Figure 8.3. The location corresponds to the teacher's participation and non-participation in communities of practice in university teacher education and school. Further, both axes represent the two components of identity defined by Wenger (1998): identification and negotiability. The notion of identification describes which meanings of mathematics (horizontal axis) and mathematics teaching (vertical axis) matter to a prospective teacher. In order to characterise the practices in which teachers exercise identification, the axes span educational ideologies in mathematics. For the horizontal axis, the ideologies is represented by philosophies of mathematics, spanning from absolutism to fallibilism. In parallel, degrees of negotiability refers to their possibility to contribute to and shape the meanings of identification, denoted as ways of knowing in respectively mathematics and mathematics teaching.

Mathematics teachers' identities are dynamic, as they are continuously negotiated through interaction with other participants in communities of practice. Identity development in terms of boundary crossing can then be portrayed as maintained or changed locations in the diagram of Figure 8.3. Over time, a maintained position refers to boundary crossing in terms of Akkerman and Bakker's (2011) notion of identification, meaning a renewed sense-making of practices and reconstruction of boundaries when moving from university to school. Consequently, the mathematics teacher reinforces his or her identity. On the other hand, boundary crossing by reflection causes movement along the axes, as one is seeing mathematics related practices in a different light. By overcoming discontinuities between communities of practice, boundary crossing by reflection leads to changes in identification and negotiability, and thus, a changed identity when undergoing the transition.

Returning to the individual cases of identity development on which I can apply the diagram, Isaac displays a narrative development regarding his expressed confidence in mathematics and mathematics teaching and his perspectives on mathematics teaching and his role as a mathematics teacher. I have argued for a movement from his initial reports on innate ability, and thus, subjective knowing in mathematics teaching, towards the need for doing critical judgement of both external and internal authorities in order to develop his teaching practice. The movement corresponds to his changed negotiability, and thus, his changed participation within the *community of secondary mathematics teaching*. Accordingly, his developing mathematics teacher identity involves moving upward along the vertical axis of negotiating experience of self and mathematics teaching, as illustrated in Figure 8.3. Isaac's changed location relates also to his inbound participation in the *community of inquiry based mathematics teaching*. It concerns overcoming discontinuities between subject studies and university teacher education, and expecting to change his teaching in the direction of inquiry based practices. However, throughout the data collection period, Isaac's experiences of stagnation when negotiating his teaching with the students is restricting the movement along the vertical axis.

Nora's and Thomas' narrated identities do not display any movement along the axes in Figure 8.3. I have argued that Nora reports on a shift, from external authority in university subject studies, through a raised critical voice in teacher education, and towards both internal and external authority for mathematics practices in the professional debut. However, her evolving stories also reveal a sustained, limited negotiability regarding meanings of mathematics and mathematics teaching provided in the current communities of practice. Due to her accounts of stagnation, Nora's position therefore remains towards the lower left corner in the diagram throughout the data collection period. Similarly, Thomas accounts for continuation, yet, in form of a sustained interest in and foregrounding of the mathematics discipline and maintained distance towards practices at PPU. Hence, he exercises continued participation in the *community of university teaching* and the *community of secondary mathematics teaching*, continued marginal negotiability in the *community of university teacher education*. By also communicating continued perspectives on mathematics teaching, in terms of a sustained emphasis on a task-based mathematics teaching practice, Thomas maintains his position in Figure 8.3 throughout the data collection period.

9 Discussion

The research question for this study was: *How do three prospective secondary school mathematics teachers' identities develop in the transition from university teacher education to school?* The individual case study analysis has displayed comprehensive portraits of three prospective secondary school mathematics teachers' narrated identities and their development when entering the profession. Further, the cross-case analysis has resulted in two dimensions for describing more generally the development of an identity as a secondary school mathematics teacher: *negotiating experience of self and mathematics*, and *negotiating experience of self and mathematics teaching*. In this final chapter of the thesis, I will discuss the findings and their implications for mathematics teacher education and further research. I do this by discussing in Section 9.1 some emerging issues regarding becoming a secondary school mathematics teacher in view of previous research. Further, in Section 9.2, I account for theoretical contributions of the thesis, in addition to pointing out questions for future research. In Section 9.3, I discuss the strengths and limitations of the study. This part is structured on the basis of Lincoln and Guba's (1985) four criteria for ensuring trustworthiness in qualitative research: credibility, transferability, dependability and confirmability. Finally, I reflect on possible implications for educational practice in Section 9.4.

9.1 Reflections on emerging issues regarding becoming a secondary school mathematics teacher

Based on the narrative analysis of three research participants' accounts, I have in this study aimed for capturing prospective secondary mathematics teachers' "perspectives and situated complexities of their work" (Chapman, 2008, p. 16). In other words, I have reported on what the teachers recognise as significant in their stream of experience (Carter & Doyle, 1996), regarding the teaching and learning of mathematics at university and in secondary school. Hence, I do not seek to answer the stated research question by concluding on identity development in the form of stages in mathematics teacher learning being generalizable to all prospective mathematics teachers. Instead, my study reveals what is learned in various practices of mathematics teaching and learning, and what kind of learning experiences that are potentially available in university teacher education and in secondary teachers' professional debut.

Based on a comprehensive description of three teachers' realities, this study displays various reasons for becoming a secondary school mathematics teacher, various emotional relationships with the discipline of mathematics and possible consequences, and conceivable situations of change and stagnation regarding the teachers' expressed perspectives on mathematics and its role in teaching. Hence, I discuss in the current section emerging issues regarding becoming a secondary school mathematics teacher in view of previous research on mathematics teacher learning. First (in Section 9.1.1), I account for the prospective teachers' sense of confidence in and personal relationship with mathematics, and their related reasons for becoming a mathematics teacher. Second (in Section 9.1.2), I discuss the images of secondary school mathematics teaching being available for the research participants, and how the discipline of mathematics is part of their imagination. Finally (in Section 9.1.3), I argue how feedback can function as confirmation and support, as well as initiating anticipation and looking ahead at future practices or future ways of being within the mathematics teacher profession.

9.1.1 Prospective secondary teachers' relationship with mathematics and reasons for entering the profession

An underlying premise for studying mathematics teachers in the transition from education to work is that their relationship with mathematics, due to their role as learners of the discipline, is of significance for their developing mathematics teacher identity. Other studies have reported on primary and middle school mathematics teachers' problematic relationship with the discipline (e.g. Brown et al., 1999; Kaasila, 2007) and their effort to strive away from their own schooling in mathematics (e.g. Palmér, 2013). In this study, all the three prospective teachers account for positive experiences with mathematics from own schooling. However, their relationships with mathematics have turned out differently during university studies. While Isaac and Thomas represent identities of inclusion in mathematics related communities of practice at the university, Nora reports on a strongly emotional and repulsive relationship with the discipline, and consequently, a marginal participation in the *community of university mathematics*. She also accounts for problems with completing her education. Yet, similar to the cases of primary school teachers presented in Kaasila's (2007) study, Nora has transformed her negative memories of learning mathematics into positive action in the mathematics classroom by exercising care and empathy towards her students.

I have previously referred to the study of Hodgen and Askew (2007), who critically discuss what they regard as a common belief: that teachers' difficulties with mathematics enables them to better empathise with and understand students' struggles. Yet, according to Hodgen and Askew, a caring approach to mathematics learning runs the risk of protecting the students from the discipline, by focusing on simple and easy mathematics with step-by-step procedures. Nora accounts for a similar case, by expressing a desire for presenting mathematics as a useful and manageable discipline for all students in line with what Ernest (1991) denotes as the progressive tradition in mathematics education.

In response to the common belief regarding teachers' previous struggles, Hodgen and Askew (2007) emphasise the necessity for prospective teachers to confront and challenge their negative attitudes towards mathematics. Such a confrontation is necessary, they claim, for the teachers to reconnect with the discipline and, thus, to develop a mathematical voice. In this study, the case of Nora exemplifies lacking opportunities for establishing a more positive relationship with mathematics when entering the teacher profession. Although Nora is reconnecting with the discipline through her positive experiences from school placement during PPU, her situation is nevertheless a case of limited possibilities for further negotiating the meaning of mathematics. Since she is only working part-time as a substitute mathematics teacher after graduation, Nora has limited opportunities to confront and process her affective relationship with mathematics, for instance through a continual collaboration with her mathematics teacher colleagues. In other words, she does not enter a space for questioning the subject, its teaching and learning. Instead, being a substitute teacher leads to a position at the periphery of the *community of secondary mathematics teaching.* Accordingly, Nora accounts for stagnation regarding further development of her mathematics teaching and a sense of uncertainty and a lacking confidence in her role as a mathematics teacher.

The central role of confidence in teacher learning has been emphasised by Graven (2004), in her study of mathematics teachers' learning during an in-service teacher education programme. She argues that confidence is pivotal in understanding and explaining mathematics teachers' learning, and thus, it should be considered as closely interwoven with the work of identifying oneself and being identified by others as a mathematics teacher. Despite Nora's strongly emotional account for studying university mathematics, and further, a sense of uncertainty in her substitute teaching, her desire for becoming a mathematics teacher is yet remarkable. Nora's reconnection with mathematics through teaching at PPU appears as the main driving force or explanation for her entrance into the profession, and thus, her identification with the *community of secondary mathematics teaching*. Hence, her expression of confidence seems mainly to be related to the mathematics classroom and the students' feedback on her mathematics teaching. Accordingly, she portrays herself as a teacher who is able to establish trustful relations and a safe classroom environment for her students. Although Hodgen and Askew (2007) call for caution regarding teachers' difficulties in mathematics as being a driving force for choosing the profession, Nora's case still demonstrates the considerable strength of this force. Hence, a corresponding awareness among teacher educators is essential in order to challenge and confront prospective secondary teachers' perspectives on the mathematics teacher role, and to facilitate their emotional development regarding the discipline of mathematics.

In contrast to Nora's situation is Isaac's expressed belief of being capable of both doing mathematics and exercising mathematics teaching. Although this study does not involve any assessment or measuring of the teachers' knowledge for mathematics teaching, the presented accounts still give a reason to believe that Isaac holds a stronger mathematical background. I thus assume that his sense of confidence enables him to set aside potential worries of his own competence in mathematics and to focus instead on the current opportunities and challenges of his mathematics teaching. This can be supported by the previously referred statement of Brown and Borko (1992), that student teachers having a strong content preparation are more likely to be flexible in their mathematics teaching, as well as responsive to the students' needs. Contrarily, student teachers who lack adequate mathematics content knowledge are likely to lack confidence in their ability to teach mathematics well. Isaac is as well the only research participant of the three who is moving towards constructed knowing in mathematics teaching (shown in Figure 8.3.), by taking critically into account meaning communicated in university teacher education in addition to own experiences from his classroom teaching.

9.1.2 Ways of imagining the secondary school mathematics teacher profession

In this study, I have assumed that the discipline of mathematics has been a prominent part of the teachers' developing identities. Hence, developing an identity as a secondary school mathematics teacher is somehow different from developing a teacher identity in other subjects. In view of that, I have applied a theoretical framework with the aim of making the *mathematics* within the developing mathematics teacher identity visible. In order to describe identity as identification and negotiability in practices of mathematics and mathematics teaching, and not practices of teaching in the general sense, I found the work of Ernest (1991) and Povey (1995, 1997) helpful. I will discuss further the theoretical contributions of the study in Section 9.2.

Based on accounts of learning and teaching mathematics, this study reports on three prospective mathematics teachers who all identify themselves as mathematics teachers. Although Nora expresses a dissociation with university mathematics, she still refers to mathematics as her subject of teaching in secondary school. The three cases portray, however, varied ways of belonging to mathematics related communities of practice, as displayed in Figure 8.3 and discussed in the former section. Yet, the common identification with mathematics teaching can be contrasted to Palmér's (2013) finding, regarding the identity development of primary school teachers in mathematics. She concluded that none of her teachers had developed a professional identity as a primary school mathematics teacher. Although they were teaching mathematics in school, the teachers did not think of themselves as *mathematics* teachers.

According to Lutovac and Kaasila (2017), research shows differences between the identities of specialist and non-specialist mathematics teachers. Especially, primary teachers do often not relate to the discipline, and they do therefore not identify themselves as mathematics teachers but instead as teachers of mathematics. In Section 2.1, I claimed that the prospective secondary school mathematics teachers in my study have had to relate to and cope with mathematics both during university studies and in their profession. In particular, they have chosen mathematics as one of two subjects for teaching when entering PPU, which have made them qualified for employment in secondary school. However, their accounts of being a learner and a teacher of the discipline reveal different foregrounds or imaginations for developing a mathematics teacher identity, with consequences for their negotiation of meaning in communities of practice.

I have argued that Thomas communicates a view of mathematics teaching as "teaching mathematics" rather than "teaching children". Hence, he imagines being a certain kind of secondary school mathematics teacher and having related possibilities for his teaching, in which the discipline itself is at the core of his practice. His engagement in the negotiated enterprise of the classroom situated *community of secondary mathematics teaching* and the *community of teacher colleagues* is then in interplay with his imagination of what is included in his professional practice and what is not. In contrast is the case of Nora and her reconnection with mathematics through teaching. Her imagination of future mathematics teaching is characterised by a foregrounding of the students' needs from struggling with learning mathematics, which makes a different frame for her engagement in communities of practice. Nevertheless, the analysis by means of the theoretical framework has made it possible to pursue these different foregrounds in terms of ways of identifying and negotiating mathematics and mathematics teaching when entering the profession.

According to Palmér (2013), the image of a good mathematics teacher has for all her respondents been a prominent part of the practices during teacher education. However, this image does not seem to have been united with their image of a primary school teacher when entering the profession. She thus argues that primary teacher education needs to integrate the increased amount of content knowledge into their education programme, so that both images can be unified into a holistic image of a primary school *mathematics* teacher. For the prospective teachers in my study, their image of a secondary school teacher has different conditions for growth. While Thomas has his educational background from a five-year Master's programme with a specialisation in mathematics education, Isaac and Nora have completed subject studies in mathematics before making the decision to enter PPU. However, all three of them have completed similar versions of PPU in terms of lectures, seminars and school placement. Hence, they may have taken part in similar discussions regarding what counts as good mathematics teaching, being initiated by actors within the university teacher education programme. The cross-case analysis then shows various ways of embracing the offered images of good mathematics teaching and integrating it into their negotiated imagination of secondary school mathematics teaching. For instance, Isaac expresses ambitions for own mathematics teaching being in line with practices communicated at PPU. Thomas, however, accounts for the mathematics education courses as being at the outskirts of his professional practice. This despite the fact that Thomas has completed the integrated teacher education programme, in which the disciplinary subjects and PPU run in parallel. In the subsequent section, I argue that the prospective secondary teachers' imagination of the secondary school mathematics teacher profession is framed by ways of negotiating feedback from others on their professional practice. Hence, responding to others' reactions regarding mathematics teaching and learning can be a way of refining the images of what counts as good mathematics teaching, and further, enabling identity development.

9.1.3 The role of feedback for the development of mathematics teacher identities

In addition to prospective teachers' relationship with mathematics and their sense of confidence, the negotiated process of becoming a mathematics teacher lies upon the feedback available on one's professional practice. In connection with Isaac's story of feedback available within the school environment (Section 5.3), I have argued that feedback is both a consequence of participation by engagement in communities of practice, and a premise for establishing and further developing memberships. Hence, responding to feedback from others on one's mathematics teaching enables negotiation of meaning regarding mathematics, its teaching and learning, and can lead to a refinement of one's images of effective mathematics teaching. I here take a closer look at how negotiation of feedback relates to the work of identifying with the mathematics teacher profession and to imagining new directions for own mathematics teaching. In other words, I discuss the role of feedback for developing a mathematics teacher identity.

Although feedback appears as an emergent theme in only one of the cases, all three of the prospective teachers report on students' responses being important for their mathematics teaching. In the case of Nora, feedback from her mathematics students is necessary for reconnecting with mathematics during school placement, and for wanting to become a secondary school mathematics teacher after graduation. For Thomas, the students' challenging and interesting questions in the mathematics lessons function as confirmation of his teaching and his choice of career. Hence, handling students' responses to the mathematics classroom practice seems in the three presented cases to be essential for identifying with the secondary school mathematics teacher profession. The emphasis on students' role and their learning for directing the mathematics teaching practice is also in line with results from previous research. According to Jones et al. (2000), students' reactions and performance are seen by prospective teachers as the most important indicator of their effectiveness in becoming teachers. Similarly, Palmér (2013) reports on how assessment was used by her respondents in order to receive confirmation of their students' learning, and consequently, of the effectiveness of their mathematics teaching.

The multiple case study further shows examples of feedback available within the teaching staff, and thus, the related possibilities for negotiating mathematics teaching and learning with mathematics teacher colleagues. Isaac's accounts of invisibility among his colleagues at his second school of employment exemplify limited possibilities for being engaged in exchanges of perspectives on the profession. Although he reports on a sense of belonging to the group of teachers at his office, neither they nor the greater community of teachers provide a scope for negotiation in terms of professional reflection. Consequently, Isaac accounts for an undesirable stagnation of own mathematics teaching practice. Nora, in her position as a substitute mathematics teacher, accounts for involvement in mathematics teaching practices that are mainly negotiated by other actors. I have argued that the collegial situations for Isaac and Nora offer a limited resource for them to develop their mathematics teaching in accordance with their ambitions. Hence, further professional development is at the mercy of their participation in other communities of practice, in which the development of mathematics teaching is part of the negotiated enterprise. Hodges and Cady (2012) make a similar argument in their study of a novice middle-grades mathematics teacher's identity and her efforts to implement reform-based practices. Analysing how multiple professional communities of practice influenced Katie's identity, the authors emphasise her continued opportunities to engage in communities that valued and encouraged approaches to mathematics instruction that were consistent with reform documents in mathematics education.

For Thomas, his collaboration with the fellow mathematics teachers functions as an organisational and an emotional support, in which he takes an active part. Similar to Isaac's situation, the collaboration includes a sharing of experiences from mathematics lessons and worries and concerns about students, in addition to a distribution of material for upcoming lessons. However, unlike Isaac, Thomas does not report on lacking opportunities for engagement and of stagnation in his mathematics teaching. Instead, the teacher staff represents a supportive teamwork, functioning as a source for identification within an already established practice of mathematics teaching. This is in line with previous research and prospective teachers' reference to comfort in personal relations with colleagues and students as evidence of their transition into a mathematics teacher role (Jones et al., 2000).

Yet, Isaac and Thomas display different roles of feedback for developing a mathematics teacher identity. The students' feedback is for Isaac a source for negotiating meaning regarding the teaching and learning of mathematics. In particular, his negotiation of feedback in the vocational mathematics classroom leads to identity development in terms of changes in his community membership and new meaning for his professional practice. Isaac also refers to his teacher colleagues as a possible source of feedback, having experienced an enriching collaborative environment at his first school of employment. Thomas, on his part, displays feedback from both students and colleagues as merely a source for confirmation and acknowledgement of his mathematics teaching practice. In parallel, what has been emphasised during school placement is a structured presentation of the subject content and classroom management rather than questioning aspects of mathematics and consequences for its teaching. The response from others on Thomas' professional practice is then a basis for identification; however, it does not lead to negotiation of meaning of mathematics, its teaching and learning. Hence, the feedback is not a source for expanding Thomas' imagination. The absent negotiation results in identity development in terms of sustained or continued community memberships, and a sense of falling into place in an already envisioned mathematics teacher role. Consequently, Thomas can choose to distance himself from the practices of PPU and, thus, to omit the related perspectives from his image of what constitutes effective mathematics teaching.

According to Wenger (1998), a person's identity depends on the kind of picture of the world and of him- or herself this person can build. Due to the current discussion, I argue that prospective mathematics teachers' picture building or imagination is framed by ways of negotiating feedback from others on one's professional practice. Feedback as a source for negotiation of meaning can initiate reflection regarding the current mathematics teaching practice, as well as initiating anticipation and looking ahead at future practices or future ways of being. Consequently, it can initiate identity development in the form of moving along the axes in Figure 8.3. However, if feedback mainly functions as confirmation and support, imagination in the form of taking new perspectives is less likely. Instead, feedback becomes a source for reconstruction of the current mathematics teacher identity and a maintained position in the diagram. In order to support prospective teachers in developing their mathematics teaching and making a desired movement towards constructive knowing in Figure 8.3, university teacher education must then give student teachers the opportunity to explore their identity through articulation and discussion with other actors. Without such articulation, the negotiation of feedback and alternative perspectives on one's mathematics teacher identity is difficult. Further, the hard work of engagement in the form of negotiating feedback from others is essential in order to take into consideration alternative views and practices and to exercise extended imagination. With an enriched image of the discipline of mathematics, the prospective mathematics teacher can identify with a new vision of teaching, so that identity development becomes possible.

9.2 Theoretical contributions of the thesis and implications for future research

Based on my account of previous research (Section 2.1), I have sought to contribute to a further theorisation of mathematics teacher learning in the transition from university teacher education to employment in school. Using the theoretical framework of identity as participation in communities of practice, I have aimed for identifying the crucial features and dynamics of prospective teachers' movements between various mathematical practices. I have assumed that mathematics teacher learning is more than coming to know content, skills and practices, being rather an epistemological and ontological process that somehow changes who the person is. From this starting point, Wenger's (1998) theorisation of identity has been helpful for investigating how individual mathematics teachers negotiate meaning when participating in communities of practice, and how their participation changes according to reactions of others to them. In line with Carter and Doyle (1996), I have thus claimed that teachers are not simply formed by their life experiences prior to, during or after university teacher education. Rather, they are highly active participants who negotiate their experiences, and further, create images of themselves and their challenging tasks and dilemmas in mathematics teaching. Similarly, Sfard and Prusak (2005) claim that "human beings are active agents who play decisive roles in determining the dynamics of social life and in shaping individual activities" (p. 15). Developing a teacher identity thus brings with it "a sense of agency, of empowerment to move ideas forward, to reach goals or even to transform the context" (Beauchamp & Thomas, 2009, p. 183) in which the mathematics teaching takes place. Although agency is not a term used by Wenger (1998), his theorisation of identity as identification and negotiability has provided an analytical tool for investigating mathematics teachers' ability and legitimacy to contribute to and take responsibility for the meanings that matter regarding mathematics, its teaching and learning.

Identification with practices of mathematics, its teaching and learning, is an important first step towards change and development of mathematics teachers' practices. Based on Ernest's (1991) model of educational ideologies, I have described which meanings of mathematics (horizontal axis in Figure 8.3) and mathematics teaching (vertical axis) matter to a prospective secondary school mathematics teacher. Further, the longitudinal case study design has enabled me to trace changes in the prospective teachers' identification from participation in communities of practice. Here, Isaac appears to be the only participant moving towards social change ideologies for mathematics teaching. However, in order to explain further the dynamics and possibilities for changing positions in the diagram of Figure 8.3, negotiability has appeared as an important construct. Based on Belenky et al.'s (1986) and Povey's (1995) theorisations of ways of knowing, I have described the prospective teachers' possibilities to contribute to and shape the meanings of identification. Hence, the interplay between identification and negotiability has made it possible to display tensions between prospective teachers' desires for own mathematics teaching and their experiences of possibilities in practice.

The analysis thus documents coherence as well as inconsistency regarding the work of identification and of negotiability. An example of coherence between expressed identification and negotiability regarding mathematics teaching is Thomas' identification with the old humanist tradition and his simultaneously expressed procedural knowing. His accounts of mathematics teaching and learning display the teacher as a possessor of mathematical knowledge that the students must gain. Since the knowledge of mathematics is certain, mathematics teaching is mainly about explaining this knowledge carefully to the students. Accordingly, Thomas accepts his previous mathematics teachers' practices of structured and good explanations, from which he exercises a procedural way of knowing in mathematics teaching. Due to the coherence, considerable disruption in the form of changed conditions for his mathematics teaching (e.g. inbound participation in a professional development programme) is thus needed in order to provoke identity development and a movement along the axes of Figure 8.3. In the case of Isaac, however, there is some inconsistency between his increased identification with social change ideologies and mathematics teaching through inquiry and his sense of negotiability within the mathematics classroom and the teacher staff. Hence, Isaac exercises imagination in terms of a changed mathematics teaching practice. Yet, further participation by engagement within a reform-oriented enterprise is needed in order to implement the desired changes into practice.

Although the three cases display various identities and development within Figure 8.3, further research is needed in order to establish the value of the analytic process beyond this study. One question remaining unanswered is whether ways of identifying with and negotiating the discipline of mathematics correspond to desired ways of identifying with and negotiating mathematics teaching. In other words, can researchers and teacher educators expect patterns of positions along the two axes of Figure 8.3 as prospective teachers undergo the transition from university teacher education to employment in school? Further, can one expect similar patterns of positions for in-service secondary school mathematics teachers? I have studied prospective mathematics teachers holding a university degree in mathematics. What will the patterns of positions in Figure 8.3 look like for prospective and in-service primary and lower secondary school mathematics teachers, undergoing the teacher education programmes for the grades 1 to 7 and the grades 5 to 10 (GLU 1-7 and GLU 5-10) in Norway? Another question is what enables movement along the horizontal axis, since none of the three presented cases represents such movement. More precisely, which mechanisms of participation in communities of practice at university and/or school enable a movement towards constructed knowing and identification with fallibilist philosophies of mathematics?

9.3 Strengths and limitations of the study

In order to discuss further the strengths and limitations of the study, I draw on the criterion of trustworthiness for evaluating the quality of the study (Bryman, 2012; Guba & Lincoln, 1994; Lincoln & Guba, 1985). According to Lincoln and Guba (1985), trustworthiness is made up of four sub-criteria, each of which has an equivalent criterion in quantitative research and the terms of validity and reliability: *credibility* (parallels internal validity), *transferability* (parallels external validity), *dependability* (parallels reliability), and *confirmability* (parallels objectivity).

9.3.1 Credibility

The credibility criterion has to do with the extent to which the readers of the thesis find the claims and conclusions, made in my case about mathematics teacher identity, believable (Lester & Lambdin, 1998). A strategy for ensuring credibility is then to make the research process visible, allowing systematic scrutiny. In writing this thesis, the aim has been to provide the reader with "thick descriptions" (Geertz, 1973) of the accounts underlying the analysis, and the interpretations made and methods used for transforming the accounts into findings. In order to facilitate respondent validation (Bryman, 2012), I have shared the transcripts and analysis with the three participants. This was done in order to ensure correspondence between the prospective mathematics teachers' expressed perspectives and my interpretations and findings. However, the respondent validation did not lead to any changes in the thesis. Further, the study has been subject to peer critique, taking place through presentations at conferences and seminars[21], and through peer reviewed publications (Rø, 2015a, 2015b). Peer critique has also been provided by my supervisors.

The criterion of credibility also concerns whether the research design and the chosen methods are suitable for investigating what is stated in the research question (Kvale & Brinkmann, 2009). In this study, I have aimed for capturing an always-in-motion process of identity development, based on the prospective teachers' work of identifying with and negotiating meanings regarding mathematics and mathematics teaching and learning. I have chosen to focus on their accounts rather than their classroom practice. Hence, I have privileged what they share about their educational background and professional practice, rather than what I could have observed from their practice. A possible threat to credibility is that the prospective teachers' accounts are not reports of experiences. Rather, the teachers have made sense of and therefore inevitably distorted those experiences (Elliott, 2005). Further, the prospective teachers were somehow forced to reflect on their experiences regarding mathematics teaching and learning, meaning that their accounts were produced specifically for me in the interviews. Yet, this opportunity to make sense of the process of becoming a mathematics teacher, by selecting elements of experience and patterning them in ways that reflect stories available to the audience, can be considered a strength of narrative-based research. According to Polkinghorne (2007), the storied descriptions that people give about the meaning they attribute to life events is the best evidence

[21] Conferences: NORMA 14 (The Seventh Nordic Conference on Mathematics Education); CERME 9 (The Ninth Congress of European Research in Mathematics Education). Seminars: First year seminar at the University of Agder (2014), with critical friend Prof. Maria Luiza Cestari; 90 % seminar at the University of Agder (2016), with opponent Prof. Ulla Runesson Kempe.

available to researchers about people's experience. In addition, Elliott (2005) argues that an interview setting is not the only kind of interaction in which individuals would expect to give an account of their educational background and choice of profession. In Section 3.2, I have reported on how I in the preparations for data collection sought to comply with validity threats for the data gathered. Nonetheless, the narrative analysis has revealed different stories and learning trajectories of entering the mathematics teacher profession, evolving along different patterns in the transition from university teacher education to school. The cases have thus shown developing stories about how one can be prepared to engage in the mathematics teacher profession, with a richness regarding the prospective mathematics teachers' meaning-making of mathematics, its teaching and learning. Through what Lieblich et al. (1998) denote a comprehensiveness of evidence, the presented evolving stories are thus ways of capturing the complexity, specificity and interconnectedness of the phenomenon of becoming secondary mathematics teachers.

Another possible threat to credibility is the limited time span for data collection, which covered only the very beginning of the mathematics teachers' professional debut. The situation of being thrown into unexpected duties beyond time-consuming lesson preparations and teaching has been described in the literature as a sense of chock (e.g. Arfwedson, 1983), which might diminish as the mathematics teachers enter their second year of service. A consequence is a limitation to the outcome of the study, as not representing the professional debut for mathematics teachers in a more general sense. Yet, the first year of being a mathematics teacher in secondary school lays the basis of the following socialisation process. Having conducted two (Thomas) and three (Isaac and Nora) interviews during their first year of teaching in school, I have had the possibility to detect changes occurring in the very beginning of the participants' career.

9.3.2 Transferability

Transferability is about whether the findings of the study are transferable to other situations. As explained in Section 3.5, this study considers particularisation more than generalisation. Hence, the narrative analysis has produced context-dependent knowledge about certain mathematics teachers with a certain history. The transferability or generalisability is then based on the contextual richness of each individual case. By further comparing and contrasting the individual cases in the cross-case analysis, the specific features of each individual have been placed in the background. Hence, I have aimed for developing an interpretation of data that reflects each individual case and that applies equally well across all the three cases. This is based on the assumption that those aspects having explanatory force both in individual accounts and across the cases are most likely to apply beyond the sample of prospective mathematics teachers. Bassey (1999) denotes this as “fuzzy generalizations”:

> A fuzzy generalization carries an element of uncertainty. It reports that something has happened in one place and that it may also happen elsewhere. There is a possibility but no surety. There is an invitation to ‘try it and see if the same happens to you’. (p. 52)

In order to provide a basis for the reader to make a judgement about transferability, I have in the thesis aimed for thick descriptions of the research process, and the connectedness to theory and previous research on mathematics teacher learning.

9.3.3 Dependability

A trustworthy study also rests on dependable instruments, including a critical evaluation of the interviewer. I have claimed in Section 3.3 that a possible threat is related to my prior understanding or bias, as I am, to a greater or lesser extent, sharing the same educational background as the research participants. This might have influenced the initial interpretation of their statements during the interview session, and could thus have funnelled the conversation in a certain direction. Another possible outcome is that I have overlooked significant qualities in the teachers’ accounts. However, the shared educational background has also been an advantage, as I more easily could relate to the prospective teachers’ utterances. Further, the study has depended on the prospective teachers’ willingness to tell about their educational background and their professional debut in the interviews, and to share their feelings and thoughts. I have therefore been preoccupied with creating an atmosphere in which the teachers have felt comfortable talking with me. Our common educational background has been helpful in order to show interest in their utterances and for communicating understanding and acceptance regarding their stories.

According to Lincoln and Guba (1985), one technique of establishing dependability is adopting an auditing approach. This entails ensuring that complete records are kept of all phases of the research process, so that they can be examined by auditors or peers. The auditing also includes assessing the degree to which theoretical interpretations can be justified. As accounted for in Section 9.3.1, the research process described and interpretations made in this thesis have been open to critical peer review. Further, I have aimed for providing sufficient examples from the interview data to allow readers to judge the strength of the interpretations made.

9.3.4 Confirmability

Being the qualitative parallel to objectivity, confirmability is concerned with ensuring that the research undertaken has not allowed personal values or theoretical inclinations to influence the findings derived from the analysis (Bryman, 2012). Similar to the criterion of dependability, an auditing approach is an effective technique for establishing confirmability.

A strength of this study is that the transcript data presented can be reanalysed by other researchers. However, I could have provided greater transparency regarding the observations made, by using audio or video recording of the teaching sequences and making detailed transcripts of the lessons. Yet, the observations were carried out with the aim of generating detailed accounts regarding mathematics, mathematics teaching and learning, and they did therefore not constitute the main source of data for the study.

9.4 Possible implications for educational practice

This thesis has reported on an analytic process of describing secondary school mathematics teachers' identities and their development when undergoing the transition from university to school. It has resulted in a theoretical analysis of mathematics teachers' navigation in a landscape formed by the work of negotiating experience of self and mathematics, and negotiating experience of self and mathematics teaching. In this final section, I make some considerations regarding the contribution of the study to the educational practices within university teacher education.

The challenges are many as prospective mathematics teachers make their way through subject studies in mathematics, the university teacher education programme and into their initial practice in school. This study makes a contribution to the practice field by raising consciousness about the complex negotiation of meaning that prospective secondary school mathematics teachers undergo. Hence, the study has the potential to inform future designs of teacher education programmes when the focus is directed towards prospective teachers' emerging understandings of themselves as actors within mathematics and mathematics teaching. From the discussion in Section 9.1, the findings indicate the importance of facilitating prospective mathematics teachers' emotional development regarding the discipline of mathematics, and of giving them the possibility to explore and confront their expressed perspectives on the mathematics teacher profession.

Further, the theoretical analysis of developing an identity as a secondary school mathematics teacher, as synthesised in Figure 8.3, provides a means for articulating and discussing within university teacher education possible approaches to the mathematics teacher profession. First, the three comprehensive cases can assist prospective mathematics teachers in coming to terms with their identity and to communicate it to others. Second, the awareness of one's own learning trajectory and positioning in the landscape portrayed by Figure 8.3 can empower further negotiation of meaning and initiating anticipation regarding future practices or future ways of being a secondary school mathematics teacher. I claim that a student teacher in mathematics who is capable of arguing for own approaches to mathematics, its teaching and learning, would be better prepared for negotiating and making use of the possibilities for development offered by communities of practice within their professional debut.

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11 Appendices

Appendix A: Interview guide (Interview 1)

Appendix B: Three interview tasks

Appendix C: Template for follow-up interview after observation

Appendix D: Consent form

Appendix E: A task on scaling from the observation of Nora

Appendix A: Interview guide (Interview 1)

Introduction
The first part of the interview is what I call a narrative interview. This means that you are supposed to tell me about your time as a mathematics student and student teacher at the university, and about you present situation. I would like to hear as much as possible regarding your experiences from university. If you like, you can talk about specific episodes and events that have been important to you. When your story is finished, I will ask you some further questions written down in my interview guide. In addition, I have asked you to prepare a mathematics task. We will discuss the task at the end of the interview session.

Your name and other names on persons, schools etc. will be replaced by pseudonyms in both the interview transcription and the thesis.

Invitation to make a narrative
Can you tell me about your educational background – about why you chose to study mathematics/why you chose to become a mathematics teacher? You can start the story from your first year at university, and continue until your present situation.

Follow-up questions/reflection

1. Can you tell more about why chose to become a mathematics teacher?

2. In which situations do you perceive yourself as a
   a) Mathematics students
   b) Mathematics student teacher
   c) Student teacher

3. I would like you to think about three episodes related to mathematics/subject studies/PPU/school placement which stand out as
   a. A highlight
   b. A low point
   c. A turning point

   Can you describe as thoroughly as possible what happened?
   What did you think or feel in that situation?
   Why does the episode stand out as important?

4. Do you have any persons or group of people who have had positive/negative influence on your relationship with mathematics? How/why?

5. What motivates you to learn mathematics?

6. What does it mean to know and do mathematics?

7. What characterises a good mathematics teacher?

8. What characterizes a god mathematics student?

9. Can you highlight parts of your education which have been demanding? Why?

10. Can you highlight parts of your education which you have liked the most/least? Why?

A. For participants undergoing PPU at the time of the interview

11. Can you tell about your first weeks at teacher education (PPU)?
    a. How will you describe teacher education if you compare it to other parts of you educational background?
    b. Can you describe your first meeting with the mathematics classroom, being a student teacher?

12. Can you tell more about how it is like to teach mathematics in school?
    a. What is most challenging?
    b. What makes it enjoyable?

13. Can you tell about a mathematics lesson that did not go as planned?
    a. Why did it not go as planned?
    b. What characterises such a mathematics lesson?
    c. What characterises the students' activity
    d. What characterises your role?

14. How do you plan your mathematics teaching?
    a. Who do you ask for help?

15. How is mathematics teaching, compared to teaching in other subjects?
    a. Which similarities/differences are there?

B. For participants undergoing the five-year Master's programme

16. Can you tell about your first meeting with the mathematics classroom, being a student teacher in mathematics?

17. Can you tell about a mathematics lesson that did not go as planned?
    a. Why did it not go as planned?
    b. What characterises such a mathematics lesson?
    c. What characterises the students' activity
    d. What characterises your role?

18. How did you plan your mathematics lessons during school placement?
    a. Who did you ask for help?

19. Can you tell more about your mathematics teaching? If I were to observe your mathematics teaching during school placement, what would I observe?
    a. What is your role, being the mathematics teacher?
    b. How would you describe the students/classroom atmosphere?

20. What is most challenging about teaching mathematics?
21. What makes it enjoyable?

22. How is mathematics teaching, compared to teaching in other subjects?

    a. Which similarities/differences are there?

About the future

23. You have talked about your past and present as a student teacher in mathematics. Now I would like you to imagine your future as a mathematics teacher in school. What is the most positive/negative future for you?
    a. Which ambitions do you have for your future?
    b. What do you fear?

24. What are your thoughts and expectations about becoming a mathematics teacher?
    a. What will be most challenging?
    b. How do you want your mathematics teaching to be like?

25. Can you compare your first year in university education and your thoughts about your future at that time, with your present thoughts about future – what is similar/different?

Questions about the interview task

26. What was your first impression of the task?

27. How did you solve the task?
    a. Are there other ways of solving the task?

28. What is the topic of the task?
    a. Which mathematical concepts are involved?

29. Which mathematics competences are necessary to solve the task?

30. To which students do the task fit, and why?

31. What is challenging about the task?
    a. Which challenges can the students face?
    b. Which mistakes can the students make?

32. In what way could you have used the task in your own mathematics teaching?
    a. Which changes would you make to the task?

At the end of the interview

You have told me a lot about yourself as a student in mathematics and a student teacher in mathematics. Is there anything you would like to comment on? Do you have any questions?

I will write down this conversation, using pseudonyms on you and other persons, schools etc. which have been mentioned the interview.

If you have any questions, please do not hesitate sending me an e-mail about it.

Appendix B: Three interview tasks

Task 1: The dog pen[22]

Imagine you have 16 meters of fence to make a dog pen. To keep things simple, you will make the pen rectangular, but at the same time give the dog as much area as possible to play in. How should you use the 16 meters of fencing to maximize the area?

a) How many rectangles with a fixed perimeter of 16 meters can you find? Make a sketch of each rectangle, and label the dimensions and the area.

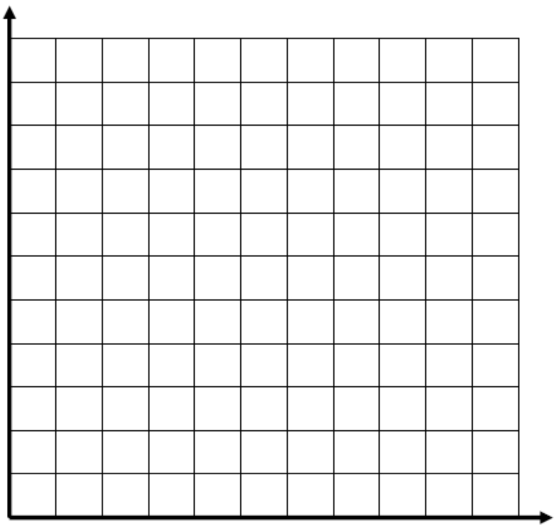
b) Complete the table below for every rectangle with a perimeter of 16 meters and integer side lengths.

Lengde ***l***	**Bredde** ***b***	**Areal** ***A***

c) Write down at least three patterns that you notice in your table.

d) Create a graph of your data where length is the independent variable and area is the dependent variable. Label each axis, and use appropriate scales.

[22] Obtained with some minor changes from Fredrick Peck, Freudenthal Institute US, and the mathematics teaching resources RME Quadratic Units. Available at: http://rmeintheclassroom.blogspot.com



e) What is the domain of the function?

f) Based on the graph, how should you arrange the fencing to give the dog the largest possible area? Why is it so?

g) Write an equation to describe the relationship between the area, A, and length, l.

Task 2: The point of no return[23]
Imagine that you are the pilot of the light aircraft, which is capable of cruising at a steady speed of 300 km/h in still air. You have enough fuel on board to last four hours. You take off from the airfield and, on the outward journey, are helped along by a 50 km/h wind which increases your cruising speed relative to the ground to 350 km/h. Suddenly you realise that on your return journey you will be flying into the wind and will therefore slow down to 250 km/h.

a) What is the maximum distance that you can travel from the airfield, and still be sure that you have enough fuel left to make a safe return journey?

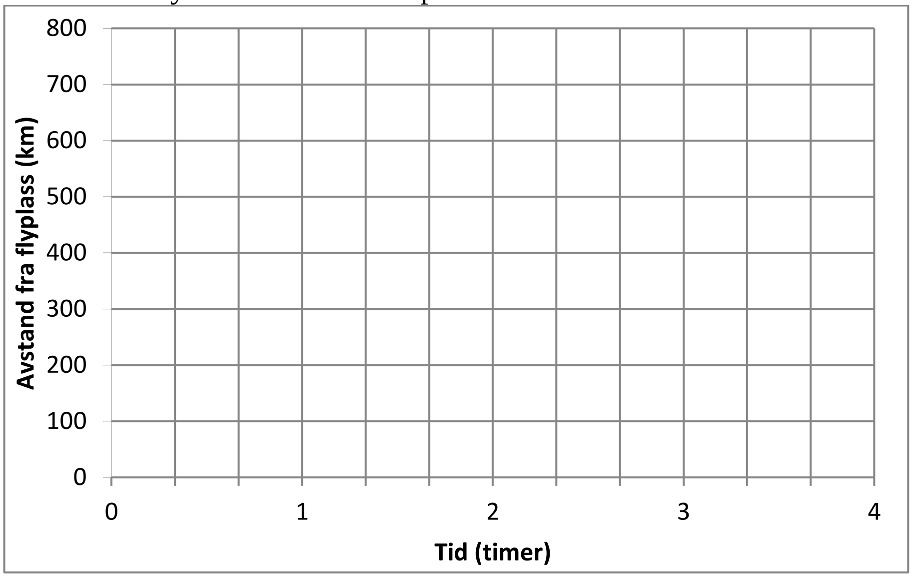
b) Investigate these 'points of no return' for different wind speeds. What pattern do those ‘points of no return’ make? Can you explain why they make this pattern?

Some hints
It can be helpful to draw a graph to show how your distance from the airfield will vary with time.

- How can you show an outward speed of 350 km/h?

[23] Obtained with some minor changes from the Shell Centre for Mathematical Education and the book “Language of functions and graphs” by Malcolm Swan, University of Nottingham. Available at: http://www.mathshell.com/publications/tss/lfg/lfg_teacher.pdf

- How can you show a return speed of 250 km/h?



Use your graph to find the maximum distance you can travel from the airfield, and the time at which you should turn round.

On the same graph, investigate the 'points of no return' for different wind speeds. What kind of pattern do these points make on the graph paper? Can you explain why?

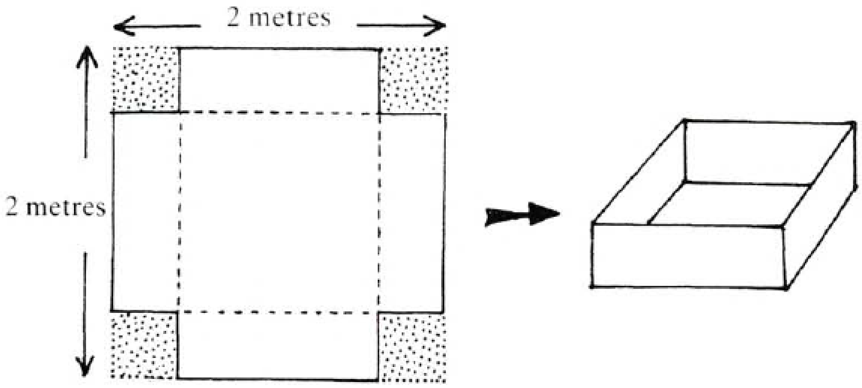
Suppose the wind speed is *w* km/h, the 'point of no return' is *d* km from the airfield and the time at which you should turn round is *t* hours.

- Write down two expressions for the outward speed of the aircraft, one involving *w* and one involving *d* and *t*.
- Write down two expressions for the homeward speed of the aircraft, one involving *w* and one involving *d* and *t*.
- Try to express *d* in terms of only *t*, by eliminating *w* from the two resulting equations. What will the graph of the expression look like? What information can you read from the expression regarding the various 'points of no return'?

Task 3: Designing a water tank[24]

A square metal sheet (2 metres by 2 metres) is to be made into an open-topped water tank by cutting squares from the four corners of the sheet, and bending the four remaining rectangular pieces up, to form the sides of the tank. These edges will then be welded together.

[24] Obtained with some minor changes from the Shell Centre for Mathematical Education and the book "Language of functions and graphs" by Malcolm Swan, University of Nottingham. Available at: http://www.mathshell.com/publications/tss/lfg/lfg_teacher.pdf



- How will the final volume of the tank depend upon the size of the squares cut from the corners? Describe your answer by:

  a) Sketching a graph that shows the relation between the volume of the water tank and the lengths of the sides of the cut squares (without making a algebraic formula)

     Hint: Imagine cutting very small squares from the corners of the metal sheet. In your mind, fold the sheet up. Will the resulting volume be large or small? Why?
     Now, imagine cutting out larger and larger squares.What are the largest squares you can cut? What will the resulting volume be?

  b) Explaining the shape of your graph in words

  c) Trying to find an algebraic formula for the relation between the volume of the water tank and the lengths of the sides of the cut squares

- How large should the four corners be cut, so that the resulting volume of the tank is as large as possible?

Appendix C: Template for follow-up interview after observation

1. Can you tell me about your immediate reflections regarding the mathematics lesson?

2. What was the aim of the lesson?

    a. What made up the basis for the teaching planning?
    b. What was the mathematical aim of the lesson?
    c. Were there any other aims?
    d. How is the lesson related to previous and upcoming lessons in mathematics?

3. What would you highlight as positive about the lesson?

4. What did not go as planned?

5. Do what extent would you say that you followed your plan for the lesson?

6. If you were to do the lesson once more, what differences would you make?

7. Can you describe the group of students?
    a. How is the group compared to other student groups?
    b. What characterizes the group of students?
    c. What characterizes "good" mathematics students in the group?

8. Questions from the observation (examples only):

- Why did you change the scale for the castle? (Nora)
- Student: "I don't understand how I can find *y* when I only know the *x*" What was your response to the student, and why? (Isaac)

Appendix D: Consent form

Participation in a doctoral study at UiA
My name is Kirsti Rø, and I am a Ph.D. research fellow in mathematics education at the University of Agder (UiA). In my doctoral study, I investigate mathematics teachers' experience of undergoing the transition between university teacher education and professional debut in school. The project has the following working title: *Mathematics and mathematics teachers in the transition between university teacher education and secondary school.*

The purpose of the study is to gain better insight into the developing process when prospective mathematics teachers move from studies at the university to employment in secondary school. In that connection, I seek participants having a Master's degree in mathematics/mathematics education from [*name of university*], including a one-year teacher education programme (PPU). I would like to follow you and the other participants in the period from your last year in education into your first year as a mathematics teacher in school; a period of 1,5 years. During this period of time, I will conduct a series of four to five interviews with each participant. In addition, it might be of interest to make observations of your teaching at the end of the period, as a basis for a follow-up interview. All interviews will be audio-recorded. In addition, video recording of your mathematics teaching might be of relevance.

During the interviews, I would like you to talk about your experience of undergoing the five-year Master's programme in mathematics/mathematics education and the professional debut in school. In addition, parts of the interviews will include discussions about mathematics tasks. In advance of each interview sessions, you will therefore receive a task by e-mail, along with some questions. During the interview, we will talk over the task and its possible solutions. The preparations should not exceed one hour. During your first semester as a certified mathematics teacher in school, it could be of interest to make observations of your mathematics teaching. The observation will serve as a basis for an interview. Data from interviews and observations will be reported on in the doctoral thesis in the shape of interview excerpts and observation descriptions.

All information gathered through audio and video recordings will be treated confidentially. Your name, and the names of the university, practice schools and school of employment, will therefore be replaced by pseudonyms. The data material will only be available for the undersigned; however, parts of the material will be discussed in collaboration with two supervisors.

The project period is of three years, until 2016. Audio and video recordings will be stored safely at the University's servers until 2030, and later be deleted. The project is registered at the Norwegian Centre of Research Data (Norsk Samfunnsvitenskapelig datatjeneste).

Participation in the research project is voluntary, and you might at any time withdraw from the study without giving any reason for it. If you choose to withdraw, all data will be deleted.

Please do not hesitate to contact the undersigned if you have questions regarding the study. You can contact Kirsti Rø by e-mail […], or by phone […]. In addition, supervisor and associate professor Martin Carlsen is available for questions on email […] and phone […]

Yours sincerely,
Kirsti Rø
Ph.D. research fellow
Faculty of Engineering and Science

Consent
I agree to participate in interviews and observations, and I agree that the data material will be used in the doctoral study on the terms described above.

____
(Signed by the participant, date)

Appendix E: A task on scaling from the observation of Nora

Make a medieval castle in scale 1:75 from cardboard (58x33x42cm) and solve the following tasks about the castle

First, get an overview of the conditions for the construction. Discuss them in the group before you start to build, cut, and make the necessary calculations. Everyone in the group have responsibility for making the decisions about the castle and for doing the tasks.

The castle

You choose the shape of the castle, but the surrounding wall must be 20 m high. What is left from the cardboard will be used for building other parts of the castle. Inside the wall, it shall be:

- A knights' house with base 40 m^2
- A water well, being 1,5 m tall and having a diameter of 2 m
- A gate, having a width of 4 m and height of 7 m
- A tower, being 25 m tall and having a square base with sides 4 m
- In the tower, you must make a 10 m high flag pole and a flag of 6x4 m

Task 1:

Find the area (the base) of the castle from the way you have designed it. Make a drawing that shows how you plan to make the castle

The knights' house

Task 2:

The knights live in a house with base 40 m^2. It has two floors. Make a drawing of the base of the house in scale 1:50, and build the house (in scale 1:75, so that it fits the castle). Find the surface of the house.

The gate

The castle has a gate with a width of 4 m and a height of 7 m. The gate has a semicircle shape on top. See figure.

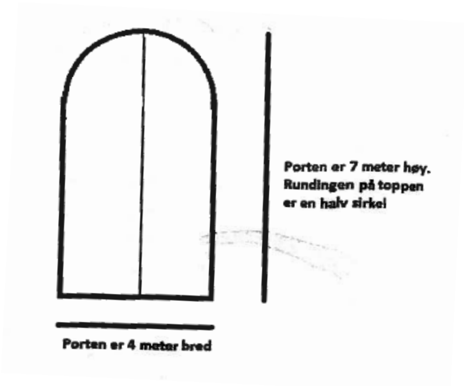
Task 3:

Find the area of the gate.

The gate needs painting.

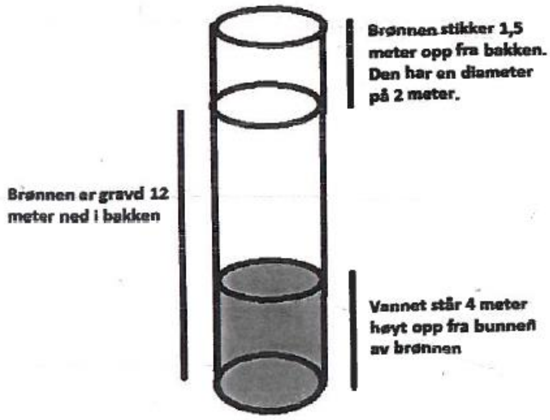
Task 4:

Find how many liters of painting you need to paint the front side of the gate, if 1 liter of painting covers 4 m^2 of the gate.



The water well
The castle will have a cylindrical well. It is buried in the ground; however, a cylindrical brick wall being 1,5 m tall and having a diameter of 2 m stands up from the ground.

Task 5:
Find the volume of the water well when it is buried 12 m down in the ground (see figure). How can you find the surface of the cylinder? Explain with your own words.



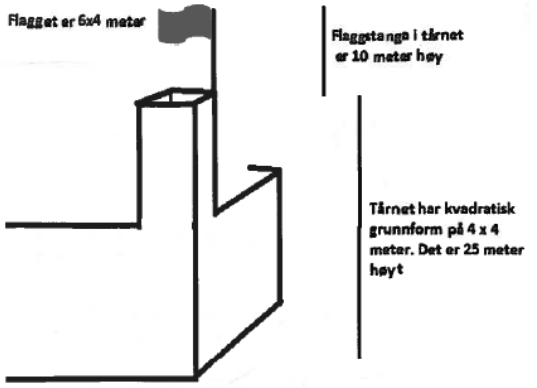
The water level is 4 m.

Task 6:
How many liters of water is it in the water well?

The tower
You can make as many towers as you like, but one tower must have the following shape:

- It must be a prism
- The tower must have a quadratic base, 4x4 m
- It must be 25 m tall

Task 7:
How can you find the volume of the tower?



The flag
You must build a flagpole inside the tower. It must stand 10 m up from the tower.
You must make one flag for the flagpole. It will be 6x4 m.
The flag must have a symbol of a parallelogram.

Task 8:
Find the area of the flag and the area of the parallelogram.

Task 9:
Imagine that the circumference of the wall of the castle is 200 m, and that the base is quadrangular. What shape of the base do you think give the greatest area? Show your answer by examples.

Final work
In addition to build the castle in cardboard, you will make a final report from the project, including answers to all the tasks. You must show the calculations you have made and the approaches you have used for solving the tasks. You must also write down what is meant by volume, and which concepts you have used, with explanations.

From the curriculum

Measurement:

The aims for the education are that the students shall be able to

- Make estimates about and calculate length, circumference, angle, area, surface, volume and time, and use and change scale
- Select appropriate units of measurement, explain relations and convert between different units of measurement, use and evaluate instruments and methods for measurement in practical situations and discuss precision and uncertainty.
- Explain the number π and use it in calculations of circumference, area and volume.