Integral Control Action in Precise Positioning Systems with Friction

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Abstract: For high precision positioning systems a fast and accurate settling to the reference state is most significant and, at the same time, challenging from the control point of view. Traditional use of an integral coaction in feedback can attain a desired reference tracking at steady-state motion, but can fail in case of precise positioning. Most crucial is that this is independent on how accurate the integral control part is tuned. This paper addresses the feedback control action in precise positioning systems with friction. Analyzing the closed-loop control dynamics with nonlinear friction in feedback it is shown why the integral action cannot efficiently cope with Coulomb friction which becomes time-varying at motion onsets and reversals. The latter leads to the reduced control performance expressed in desired immediate stop at the reference position. The nature of presliding friction as functional of positioning control error, in vicinity to the reference position, and not as function of the time argument, is postulated as main disturbing factor that limits efficiency of the integral control coaction. The conclusions drawn in performed analysis are also reinforced by the demonstrated numerical examples of a controlled motion with nonlinear friction.

Keywords: Friction, motion control, precise positioning, integral control, pre-sliding

1. INTRODUCTION

Nonlinear friction in presliding can limit the performance of feedback control system when high-precision positioning is required. High precision motion control systems should mostly comply with a set of requirements concerning the response/settling times and trajectory/settling accuracies, see Iwasaki et al. [2012]. While a short response time, i.e. fast transients, can be achieved by a model-based feed-forwarding and other two-degrees-of-freedom control techniques, see e.g. Umeno and Hori [1991], Araki and Taguchi [2003], the settling time and accuracy appear as most crucial in view of the continually increasing requirements posed on the precise positioning systems.

Fast settling response is mostly suffering from nonlinearities, such as friction, in the mechanical systems, see e.g. Armstrong et al. [1994], Al-Bender and Swevers [2008]. Well known, the nonlinear friction in positioning control can lead to the stick-slip motion and so-called hunting limit cycles, see Hensen et al. [2003]. An undercompensated and overcompensated friction in 1DOF mechanical systems have been analyzed e.g. in Putra et al. [2007], while considering PD position control with an additional model-based friction compensation. The focus has been put on the static friction and its transition to the gross sliding, the so-called Stribeck effect. It has been shown that either stable limit cycles or a final stop with non-zero steady-state errors occur in case of an overcompensated correspondingly undercompensated friction. An analysis of PID control performance in presence of static friction also leads back to the former work by Armstrong and Amin [1996]. The settling performance of precise positioning control have been recently addressed e.g. in Maeda et al. [2013], Ruderman and Iwasaki [2015, 2016b].

Either directly associated with frictional phenomena or not, but almost all studies on the settling behavior in positioning control recognize the importance and difficulties related to the integral control part and its tuning for improvement of settling time with a required steady-state accuracy. Recently, the synthesis of variable gain integral controllers for linear motion systems has been addressed in Hunnekens et al. [2015], while exposing the real-world settling behavior in a micrometer range when using the integral control action. Another explicit study by Ruderman and Iwasaki [2016b] demonstrated a slow (creeping) settling response at micro-positioning, that cannot be efficiently handled by integral control coaction and, apparently, represents a signature of nonlinear friction in vicinity to the controlled motion stop. Even thought the PID based motion control systems are widely accepted and the analytic methods and heuristics for theirs control gains tuning are well developed, see Åström and Hägglund [2001], Ang et al. [2005], the high precision positioning systems appear to be not quite satisfactory with the existing solutions. The achievable and above all well-deterministic settling times, for the required steady-state accuracies, demand for alternative control strategies. These should be able to cope with disturbing nonlinearities on a micro-positioning scale and that in a robust manner.

The goal of this paper is to analyze the action of an integral feedback control part in motion control systems with nonlinear Coulomb friction in presliding. The presliding distance is considered in a close vicinity to the

reference position. While an exactly controlled stop of relative motion is required, certain transient overshoots and oscillations (even if infinitesimally small) occur as practically unavoidable around the reference position. This leads to an initially large, even going towards infinity, and then highly-varying stiffness at each motion reversal. That appears to be weakly manageable and independently on how accurate the feedback control gains are tuned. In the following, we analyze the closed control loop dynamics with standard PID feedback regulator and general first-order motion system with nonlinear friction. We demonstrate weak convergence of the controlled position towards the reference value, when being within presliding distance, and draw some assumptions about reachability of an exact reference state. Finally, we reinforce our analysis by two numerical examples which argue on non-efficiency of the feedback control action in precise positioning.

2. PROBLEM STATEMENT

Independent of type of motion control system the positioning problem can be scaled down to the moving mass (or generally inertia) as schematically illustrated in Fig. 1. The homogenous mass block is on a contact surface
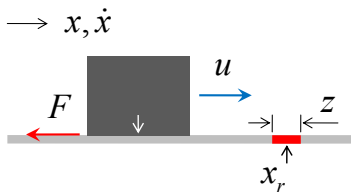

Fig. 1. Motion system with reference position and friction

and the generalized actuation force u is counteracted by friction force F. The shown 1DOF motion system can approximate both rotational and translational motion so that the generalized coordinates x are assumed. Further we assume a constant normal load so that the frictional coefficient remains constant, or at least independent of the motion state. For the sake of simplicity, and without loss of generality, we will consider a unity mass so that the motion dynamics can be easily described by

$$\ddot{x}(t) + F(\dot{x}, t) = u(t). \quad (1)$$

The kinetic friction is generally assumed as a nonlinear function of relative velocity and time. Here, the time-variance can incorporate all transient effects of dynamic friction and varying conditions parameters as well. Thus we do not need to mapped in correspondingly restricted our consideration a particular friction phenomena at this stage. Further on we will, however, focus on the presliding friction phenomenon for which the time-variance of friction force manifests itself in the varying Coulomb friction coefficient at motion onsets and motion reversals. This nonlinear friction behavior occurs within the so-called presliding distance denoted in relative coordinates by z. The presliding distance characterizes a motion range between the last motion reversal (or stop) and the so-called gross sliding where the friction amplitude can be assumed constant for a constant instantaneous relative velocity. In light of high precision positioning systems and advanced resolution of the sensors used for feedback, the presliding distance comes into the foreground of analysis and design.

A standard positioning task is in attaining the given reference x_r (see Fig. 1) within a possibly short time after starting deceleration from the steady-state motion. Here we can note that a transient overshoot or even short transient oscillations at settling are not as crucial as the residual positioning error and associated settling time. Therefore, the problem is to find a stable and realizable control $u(t)$ which could guarantee attaining $x = x_r$ within an acceptable settling time t_s. It is well known that the PD controlled mechanical systems can successfully cope with this in case of a linear (viscous) friction only, i.e. $F = D\dot{x}$. That case the design procedure is straightforward and the control loop response can be arbitrary shaped by the proportional and derivative control gains, assuming the mass and viscous damping parameters are known. To cope with unknown constant disturbances the integral control action can be further included which results in a standard PID control, that is widely accepted in an uncountable number of applications, including positioning systems and precise machinery. Often, the integral control part is attempted to be tuned for compensating also for the nonlinear Coulomb friction. This can succeed for reference tracking at steady-state motion, i.e. motion without stops and reversals, but can fail in case of precise positioning.

In the following, we will assume a standard PID control, as a common solution integrated in most systems of the control engineering practice. We then analyze the performance of integral control part when compensating for kinetic friction at the precise positioning.

3. PID CONTROL OF MOTION WITH FRICTION

We consider the PID position control with proportional, integral, and derivative feedback gains K_p, K_i, K_d correspondingly. The initial integrator state is set to zero. Substituting the control law into eq. (1) and taking the time derivative results in the closed control loop dynamics

$$\dddot{x} + K_d\ddot{x} + K_p\dot{x} + K_i x + \dot{F}(\dot{x}, t) = K_d\ddot{x}_r + K_p\dot{x}_r + K_i x_r. \quad (2)$$

One can easily see that for steady-state motion, i.e. motion with a constant velocity for which $\dot{F} = 0$ can be assumed, eq. (2) reduces to

$$\dddot{x} + K_d\ddot{x} + K_p\dot{x} + K_i x = K_p\dot{x}_r + K_i x_r. \quad (3)$$

For that one, the control loop response can be arbitrary shaped by determining the control gains, for which computation various methods of the linear control theory are available, see e.g. Franklin et al. [2009]. However, once a positioning control task is assumed and $\dot{F} = 0$ is not longer valid in vicinity to the final reference position, the control system dynamics from eq. (2) should be reconsidered again, while its r.h.s. (right-hand side) can be replaced by $K_i x_r$ only. Next, we are to analyze the linear (viscous) friction case often assumed in the control practice, following by the general nonlinear friction case which inherently lead to degradation of the controlled positioning performance.

3.1 Linear viscous friction

As mentioned in Section 2, the linear (viscous) friction can be represented by $F(\dot{x}, t) = D\dot{x}$ where D is a positive viscous friction coefficient. Correspondingly, the friction time derivative results in

$$\frac{d}{dt}F(\dot{x}, t) = D\ddot{x},$$

and the closed-loop dynamics from eq. (2) results in

$$\dddot{x} + (K_d + D)\ddot{x} + K_p\dot{x} + K_i x = K_i x_r. \qquad (4)$$

It is evident that the viscous friction additionally increases the system damping, otherwise achieved by an appropriate K_d selection. Therefore the final positioning accuracy $x(t)|_{t>t_s} = x_r$ can be ensured by tuning the control gains.

3.2 Nonlinear Coulomb friction in presliding

In case of the nonlinear Coulomb friction in presliding, the friction force is $F(\dot{x}, t) = F(\dot{x}, z)$, where $\dot{z} = \dot{x}$ according to presliding friction and distance map, as schematically shown in Fig. 2. Taking the total friction derivative with respect to the time results in

$$\dot{F}(\dot{x}, z) = \frac{\partial F}{\partial \dot{x}}\ddot{x} + \frac{\partial F}{\partial z}\dot{x}. \qquad (5)$$

Since the nonlinear Coulomb friction does not depend on the velocity amplitude, the first r.h.s. term in eq. (5) can be further neglected. Also note that the linear viscous friction contribution is already captured as described before in Section 3.1, so that $\partial F/\partial \dot{x} = D$ becomes less relevant for our consideration further on.
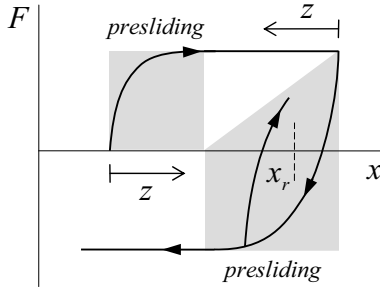

Fig. 2. Presliding friction with motion onset and reversal

Substituting $\dot{F} = \partial F/\partial z\, \dot{x}$ into eq. (2) results in the closed-loop control system dynamics

$$\dddot{x} + K_d\ddot{x} + \left(K_p + \frac{\partial F}{\partial z}\right)\dot{x} + K_i x = K_i x_r. \qquad (6)$$

Here one can see that the characteristic polynomial on the l.h.s. (left-hand side) of eq. (6) becomes state-dependent

$$s^3 + a_1 s^2 + a_2(z)s + a_3 = 0, \qquad (7)$$

where

$$a_1 = K_d, \ a_2 = K_p + \frac{\partial F}{\partial z}, \ a_3 = K_i.$$

For analyzing stability of the closed-loop control system one can apply the Routh's criterion to eq. (7) by requiring all element in the first column

$$\left[1, K_d, K_p + \frac{\partial F}{\partial z} - \frac{K_i}{K_d}, K_i\right]^T \qquad (8)$$

of the Routh array to be positive. Since $\partial F/\partial z \geq 0$, which is apparent from Fig. 2, the Routh's criterion can be easily fulfilled by requiring solely

$$K_p - \frac{K_i}{K_d} > 0, \qquad (9)$$

provided all feedback control gains are positive. Even thought this way the stability of the closed-loop control system with nonlinear friction can be shown, an open issue remains the transient dynamics towards the final

positioning $x(t) = x_r$ for $t < t_s$. Important to note is that two boundary cases occur in relation to the transient friction dynamics $\dot{F} = \partial F/\partial z\, \dot{x}$. The first one $\partial F/\partial z \to \infty$ characterizes the initial presliding stiffness at each motion onset and motion reversal. Note that this is equally a boundary case for discontinuous Coulomb friction approach $F = F_c \mathrm{sign}(\dot{x})$, which is still widely used up to date in the system and control theory, see e.g. Ruderman [2015], Ruderman and Iwasaki [2016a] for recent analysis. An initial stiffness towards infinity in presliding constitutes one of the main challenges for the high precision positioning control, where an acceptable error band lies far below than the average presliding distance. The second boundary case $\partial F/\partial z = 0$ characterizes the gross sliding, in which the Coulomb friction is saturated at the constant level. In this case, the closed-loop dynamics in eq. (6) reduces to the linear one, and no frictional impact occurs on the controlled motion system. However, a positioning process generally assumes at least one motion reversal or onset in vicinity to the reference position (compare with Fig. 2), so that $\partial F/\partial z \to \infty$ inherently reenters the dynamics of the closed control loop.

For demonstrating the impact of $a_2(z)$ we consider the system given by eq. (7) and linearized about several operation points $K = \partial F/\partial z$ when applying the root locus of Evans. For doing this, rewrite eq. (7) as

$$1 + K\frac{b(s)}{a(s)} = 0, \qquad (10)$$

where $b(s) = s$, and the polynomial coefficients of

$$a(s) = s^3 + a_1^* s^2 + a_2^* s + a_3^*$$

are

$$a_1^* = K_d, \ a_2^* = K_p, \ a_3^* = K_i.$$

As an illustrative numerical example assume the poles of the closed-loop control system $\lambda_{1,2,3} = [-20, -30, -40]$ which result in $a_{1,2,3}^* = [90, 2600, 24000]$. Note that the poles configuration of characteristic polynomial $a(s)$ is fully controllable by the design parameters K_p, K_i, K_d. The root locus diagram is shown in Fig. 3. The poles
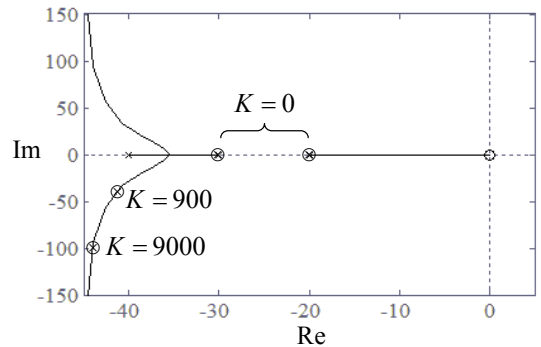

Fig. 3. Root locus diagram for $K = \partial F/\partial z$

migration is indicated in dependency on $K = \partial F/\partial z$, and that starting from $\partial F/\partial z = 0$ and going towards $\partial F/\partial z \to \infty$. The step responses of the linearized system at $\partial F/\partial z = [0; 1,000; 1,000,000]$ are shown in Fig. 4 (a)-(c) correspondingly. One can see that higher the presliding stiffness $\partial F/\partial z$ is, slower is the transient response of the closed-loop control towards the unity reference step. From
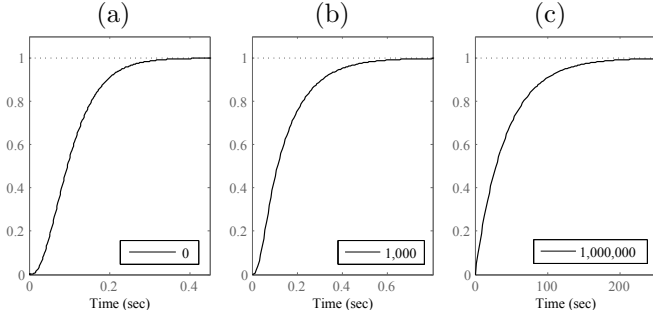
Fig. 4. Step responses of linearized systems for $\partial F/\partial z = 0$ (a), $\partial F/\partial z = 1,000$ (b), $\partial F/\partial z = 1,000,000$ (c)

the root locus diagram and step responses, shown in Figs. 3, 4 correspondingly, it can be seen that the slower dominant pole, which migrates towards zero, mainly affects the settling time of the controlled position response. Therefore, neglecting the higher-order dynamics a simplified system can be derived from eq. (6), while explicitly taking into account $\partial F/\partial z$ as the main factor which influences the positioning response. When the bidirectional relative displacements, i.e. transient overshoots/oscillations in vicinity to the reference position, remain within presliding distance, the overall first-order coefficient on the l.h.s. of eq. (6) can be assumed as a positive nonlinear function

$$\varepsilon(|x_r - x|^{-1})$$

of the inverse absolute positioning error. This results in the first-order nonlinear dynamic system

$$\varepsilon\left(|x_r - x|^{-1}\right)\dot{x} + x = x_r. \quad (11)$$

with a state varying inertia $0 < \varepsilon < \infty$. Note that K_p and K_i control gains from eq. (6) are equally incorporated into ε approximation, in particular due to the fact that within presliding distance $\partial F/\partial z \gg K_p$. The nonlinear differential equation (11) can be hardly solvable in a closed analytic form. Therefore, the following general assumptions only can be made which, however, guarantee existence of an approximative solution. The latter can be afterwards determined numerically.

(i) The system given by eq. (11) is stable since $\varepsilon > 0$ for all positioning errors $x_r - x$.
(ii) For $|x_r - x| \to 0$ the system inertia $\varepsilon \to \infty$ so that the convergence $x(t) \to x_r$ can appear in time $t_s \to \infty$.

Assumption (ii) states that, theoretically, no final state $x = x_r$ can be reached within finite time, independently on how the selection of feedback control gains is made.

3.3 Numerical example

To demonstrate the convergence of the controlled position response developed in Section 3.2, and in particular to prove the Assumption (ii) made above, consider two numerical examples. Both are implemented in Matlab/Simulink using the third-order discrete time solver with a sampling time set to 0.001 sec. The first example simulates the step response of the system given by eq. (11) with assumed

$$\varepsilon = \frac{1}{K_i}\left(K_p + \frac{B}{|x_r - x|}\right), \quad (12)$$

where B is the scaling factor characterizing the progress of variable stiffness in presliding. The second example simu-

lates the step response of the system given by eq. (6) while $\partial F/\partial z$ is computed by an implemented presliding friction model. The latter assumes that the area of presliding hysteresis loops increases proportionally to the 2nd power of presliding distance, so that

$$F \sim z\left(1 - \ln(z)\right).$$

For more details on this modeling approach see Ruderman and Iwasaki [2014], while the relationship between n-th power of presliding distance and hysteresis area built by presliding friction loops has been originally established in Koizumi and Shibazaki [1984]. Most important here is that $\partial F/\partial z \to \infty$ at $z \to 0$. The model is parameterized by the Coulomb friction constant assumed to be $F_c = 1$ and scaling factor S which determines the length of presliding distance z until the Coulomb friction saturates at $\pm F_c$. In both numerical examples the feedback control gains are selected so that the linearized systems from by eq. (6) and eqs. (11,12), i.e. these without variables stiffness that means with $B = 0$ and $\partial F/\partial z = 0$, provide similar step responses as demonstrated in Fig. 5 (a) and (b).
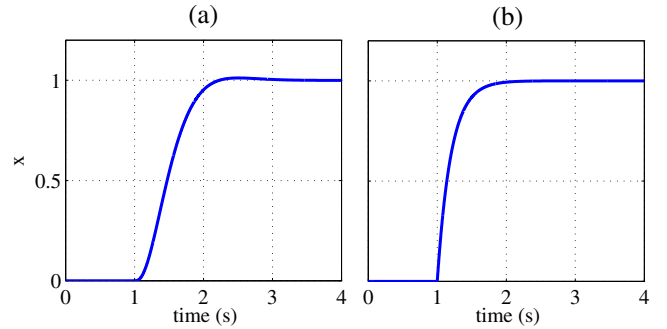

Fig. 5. Step responses of the linearized systems from eq. (12) with $B = 0$ (a), and eq. (6) with $\partial F/\partial z = 0$ (b)

For the first numerical example, various scaling factors associated with a variable, presliding-related stiffness are assumed $B = [0.01, 1, 100]$. The convergence of the step error $x_r - x$ is shown logarithmically in Fig. 6 over the relatively large (comparing to zero steady-state error at step response shown in Fig. 5 (a)) time of 200 sec. The convergence shapes are similar for all considered B values and neither reach zero final state.
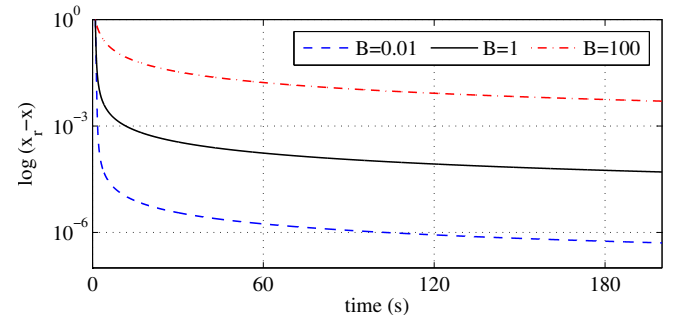

Fig. 6. Convergence of the step error $x_r - x$ (logarithmic) of the system (12) for various scaling factors B

For the second numerical example, various scaling factor of presliding distance are assumed $S = [1000; 10000; 100000]$. Here also the convergence of the step errors is shown

logarithmically in Fig. 7 over the same runtime of 200 sec (compare with Fig. 5 (b)). One can see that while for $S = 1,000$ the position error is continuously decreasing, even thought without reaching zero in a reasonable time, the higher S values result in an aperiodic error pattern around some average value. Note that due to a logarithmic representation the absolute error value $|x_r - x|$ is taken. Recall that higher scaling factor S means shorter presliding distance until the Coulomb friction saturates at the constant level $\pm F_c$.
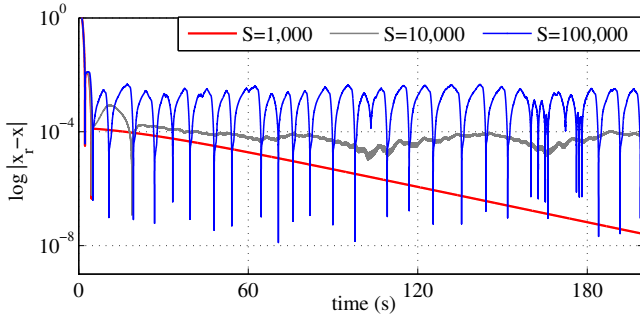

Fig. 7. Convergence of the step error $x_r - x$ (logarithmic) of the system (6) for various scaling factors S

Both numerical examples, i.e. an approximation given by eqs. (11-12) and the full-order closed-loop control system given by eq. (6) with the modeled friction, argue in favor of the analysis accomplished above and enforce the Assumption (ii) derived from that.

4. CONCLUSIONS

This paper has addressed the issue of integral control coaction used in the feedback control systems for precise positioning in presence of the nonlinear presliding friction. The standard PID feedback control has been assumed and the closed-loop control dynamics has been derived with respect to the varying Coulomb friction at motion onsets and reversals. It has been analyzed and shown with numerical examples that a final zero steady-state positioning error can be hardly reachable, independent on how accurate the feedback control gains are tuned. The main conclusions which can be deduced from the recent study are: (i) the variable presliding stiffness, and in particularly those going towards infinity when the velocity sign changes, is the principal challenge for fast and accurate precise positioning control; (ii) when using any arbitrary determinable PID feedback control gains the settling time of reaching an exact reference position can go towards infinity. The demonstrated analysis should motivate to seeking for the alternative and robust control strategies. These should be able to improve the control performance when applied to the real-world positioning mechanisms with inherent nonlinear frictional phenomena.

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