## UNIVERSITY OF AGDER

# Learning mathematics through mathematical modelling: A study of secondary school students in Nigeria 

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This master's thesis is carried out as a part of the education at the University of Agder and is therefore approved as a part of this education. However, this does not imply that the University answers for the methods that are used or the conclusions that are drawn.

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## Preface

With this thesis, I end my two years period as master's degree student at the University of Agder, Kristiansand, Norway and in the course of this programme, many people have supported me in different ways in writing and completing this work. However, I like to dedicate this study first to God almighty who strengthens me, without whom all efforts are naught. Special thanks to Associate professor Martin Carlsen, who has endured, guided, helped, encouraged, and supported me as my supervisor, in my pursuit of seemingly impulsive ideas. Thank you Martin! My thanks also go to the academic staff of the Department of mathematical sciences at the University of Agder as well as my fellow master's students for their valuable comments and suggestions during my master's thesis seminar presentation in autumn 2013. In particular, I will like to appreciate the supply of some articles as well as suggestions/contributions of Professor Pauline Vos all aimed at the improvement of this thesis. I must also not forget to thank my friend Ole Håvard Seland for his profound support and encouragement. You are one in a million. Last, but definitely not the least, I would like to thank my family for being available and helping out the best they could and for tolerating my absence from home for a long time. My thoughts go foremost to my loving and caring wife Elizabeth, whose patience, understanding, and support have been outstanding; I certainly have many things to thank you for Elizabeth, completing this programme is one of them. My final words go to my son David. Now I'm all yours!

Eric Oziegbe Omobude
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## Summary

The focus of this study is to investigate the impact that mathematical modelling activities may have in the learning process of mathematics amongst observed Nigerian secondary school students. I also wish to find out in what way mathematical modelling activities may help the observed Nigerian secondary school students to see the relevance of mathematics to real life. From a pedagogical perspective, I wish to examine if the application of modelling in the learning process of mathematics may be a vehicle for the enhancement and support of students' learning of mathematics as a subject. I started this thesis by looking at some past and present educational systems in Nigeria. This is to enable the readers to have an overview of Nigerian past and present educational systems.

The data collection process involved ten senior secondary students from one of the public (government funded) senior secondary school in Lagos, Nigeria. At the beginning, I designed ten mathematical modelling problems for the observed students. But before the students started working with the tasks, I interviewed them to get their opinions first about mathematics as a subject, secondly their opinions about the way they were currently learning mathematics and thirdly, their opinions about the connection between the mathematics they are learning in the classroom and the real life use of mathematics outside their classrooms. This pre-interview session was followed by some engagement of the observed students with the ten modelling problems that I prepared for them although I solved two of the problems for the students as examples. This process lasted for three days. In the end, I again interviewed the same observed group of students so as to get their feedback/opinions about the modelling activities they went through in the past three days. The emphasis at this point was to check the impact this modelling activities have had on the observed students' learning process.

In Nigeria, secondary school students are taught mathematics in an authoritarian and traditional way. By this, I mean teacher-centred and lecture-based learning approach that only emphasizes the solving of exercises that must lead to correct answers. The consequence is that students are rarely shown or asked to link any mathematical problem to real life situations (modelling). This impression results from my personal experience at different stages of the Nigerian secondary school mathematics education system. In an effort to investigate the students' learning process in mathematics and perhaps increase their interest in the discipline, alternative learning approaches that might offer different experience to students, may be introduced. My research follows this basis within the framework of Nigerian secondary school.

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## 1 Introduction

It is a common phenomenon for students to express difficulties with the learning of subjects like mathematics, even for those who take related science subjects. Such problems may become worse if the students are not able to visualize the practice of what they are studying and, usually, they study only for advancing forward in their programme. Students often find it difficult to learn mathematical concepts, methods, techniques, terminology, and results and to engage in mathematical activity, unless clear reference is being established to the use and relevance of mathematics to extra-mathematical contexts and situations (Niss, Blum, \& Galbraith, 2007).

### 1.1 My motivation/ background for study

As a student in Nigerian secondary schools, I experienced mathematics in a teacher-centred teaching approach, which emphasized learning regardless of context and applications while teachers stayed always strictly bound to the book. Even later, as a university student of mathematics, my experiences in class were hardly any different. Nowadays, considering such issues from the point of view of a prospective mathematics teacher, I feel the need to offer more stimulating and pleasant experiences to my students, in particular the less talented ones. My intention with this study has been to try a different teaching approach regarding the context of Nigerian secondary. Hopefully, my results may initiate discussion among Nigerian mathematics teachers and may inspire them to reassess their current practices.

The issue of my motivation arises from efforts made and sustained interest in the learning process of mathematics in Nigerian secondary schools. There are basically two types of motivational factors or variables. The intrinsic (self-impose) and the extrinsic (externallyimposed). Some mathematics education scholars like Davis (1964) referred to the extrinsic factor as the one that has some level of unfavourable effect on the learning process of mathematics. Regrettably, the present situation in Nigeria secondary school attach high level of importance to rewards and sanctions even when we are advised not to use a teacherimposed external reinforcement schedule to decide how a student answer questions or what the student thinks (Davis, 1964).

Consequently the most consistent source of motivation is from the student's intellectual curiosity. The student's inquisitiveness may lead him to several solutions regarding different tasks. His internal desire to discover some mathematical concepts, principles and even theories may as well lead to some solutions while success in earlier discovering may even lead to further taste of curiosity. Therefore, the student may have internal motivation as a result of being able to solve problems as well as a reward resulting from self-confidence as a result of accomplishment. We should however not be limited to motivational methods only. Polya (1965, p. 103) summarises it this way "The interest of the material to be learned should be the best stimulus to learning and the pleasure of intensive mental activity should be the best reward for such activity. Yet, where we cannot obtain the best we should try to get the second best or the third best". Thus we may reflect on both internal and external sources of interest in mathematics.

Motivational factors are crucial when it comes to the learning of mathematics. In this study, I will seek to motivate students learning of mathematics through mathematical modelling approach. According to Blomhøj (2004), mathematical modelling bridges the gap between students' real life experiences and mathematics. It motivates the students' learning of mathematics, gives direct cognitive support for the students' conceptions, and it places mathematics in the culture as a means for describing and understanding real life situations.

It might as well be important to emphasize that the increase in demand for professional mathematicians in various businesses and industries in recent times may also serve as a strong external motivation for students to learn mathematics since being a professional mathematician will to some extent guarantee a variety of rewarding careers. In addition, this industrial demand for professional mathematicians can further be strengthen with the teacher highlighting the application of mathematics in different endeavours. Mathematical refreshments may assist in relieving monotony. This may be in the form of puzzles, mosaics, paper folding, etc. Mathematics clubs in secondary schools may also provide a good atmosphere for motivating student interest. In this study, mathematical modelling is used as a possible mathematical refreshment.

### 1.2 Purpose of study/Research questions

This research study is an investigation into the role of mathematical modelling activities in the learning process of mathematics in Nigeria secondary schools.
The two research questions that form the basis for this study are:
(a) What impact will mathematical modelling activities have in the learning of mathematics amongst the observed Nigerian secondary school students?
(b) In what way will mathematical modelling activities help the observed Nigerian secondary school students to see the relevance of mathematics to real life?

### 1.3 Overview of the thesis

This thesis is an investigation into the role mathematical modelling activities might have in the learning process of mathematics in Nigerian secondary schools. I also try to see if the modelling activities could help Nigerian secondary school students to see the relevance of mathematics to real life situations. The main ideal here is to bring to the students' awareness the various areas of real life application of mathematics. This is to motivate students to have better learning attitude regarding mathematics.

This thesis comprises seven different parts: Introductory chapter, chapters 2, 3, 4, 5, 6 and 7. In chapter 2 I will give an overview of Nigerian secondary school mathematics curriculum. Here I examine the Nigerian secondary school system, the Pre-independence mathematics curriculum in Nigeria before 1960, the Post-independence mathematics curriculum in Nigeria and the Importance of mathematics in Nigerian Secondary School Curriculum. This section will be concluded by looking at the problems in the learning of mathematics in Nigerian secondary schools.

I begin chapter 3 by looking at the meaning of mathematical modelling. I will go ahead to look at mathematical modelling in the learning process of mathematics. In addition, I will consider some general mathematical modelling processes. Here some diagrammatic illustrations are used. I will conclude this chapter by looking at mathematical modelling in the Nigerian secondary school curriculum.

In chapter 4, I will explain the various research methods that I employ during this study. I will start by giving an overview of ethnography as one of the methods I use. I will go ahead to also describe other methods like observation method, interview and field notes. In this same chapter, I also consider the reasons for using observation as a method as well as how to use the interview method. Chapter 4 is concluded by explaining the entire procedure in this study.

Chapter 5 of this study gives detailed analyses of the entire field work regarding this study. I first consider the analysis of pre-interviews, followed by the analysis of the group work and then the analysis of the post-interviews. The concluding part of the chapter is the synthesis of the analyses. Chapter 6 is an overview of the discussion/concluding remarks. Here I talk about my findings vis-à-vis other previous studies relating to my topic as reflected in the different literature that were reviewed. I also come up with my conclusion based on my findings. Next to chapter 6 is chapter 7 where I consider the implications of my conclusion for further research. I also look at the pedagogical implications of my conclusion.

## 2 Nigerian secondary school mathematics curriculum

In this chapter, I will discuss issues relating to the Nigerian secondary school mathematics curriculum and modelling. Precisely, the Pre-independence mathematics curriculum in Nigeria before 1960 will be looked at in section 2.1. Here I will try to explain what the situation of Nigerian secondary school mathematics curriculum looked like during the colonial era. In section 2.2, I will discuss what happened to the mathematics curriculum after independence in 1960. Also, the importance of mathematics in Nigeria secondary school curriculum will be explained in section 2.3. I will deliberate on the problems in the learning process of mathematics in Nigerian secondary schools in section 2.4. Furthermore in this chapter, I will look at the meaning of mathematical modelling as given by different scholars in section 2.5. In addition, the need for mathematical modelling in the learning process of mathematics will be carefully examined in section 2.6 . This will be followed by a look at some modelling processes in section 2.7. Finally in this chapter, I will talk about mathematical modelling in the Nigerian secondary school curriculum.

### 2.1 Brief overview of Nigerian Secondary School System

Before the coming of the Christian missionaries which was followed by the introduction of western education, different tribal groups in Nigeria were already using various counting methods. Taiwo (1968) stated: "The Yorubas have developed a system of counting and have used a variety of human experience to promote practice and dexterity in enumeration. The Yoruba child is introduced early in life to counting by means of tangible substances, counting verses, folklore, theatres, and games, at home and on the farm." (p. 8-10). Other tribes like the Hausas and the Igbos also exhibited similar accomplishments connected to mathematics. Since Nigeria's independence in 1960, the nation has at various time and levels witnessed series of educational system aimed primarily at improving the quality of Nigerian education. The greatest importance attached to education in Nigeria was clearly highlighted in the National Policy on Education (FRN, 2004). The Federal Republic of Nigeria, in this policy, adopted education as the "par excellence" instrument for effecting national development.

The foundation of the entire educational system is built on the primary school and among the primary school subjects, mathematics has been regarded as a very core aspect of mathematics education at the basic education level (Awofala \& Awolola, 2011, p. 14). In Nigeria, the Federal Ministry of Education through the National Council on Education (NCE) has recently ratified and released the new basic education mathematics curriculum to schools. This curriculum is a product of the reviewed, reorganized, and re-aligned primary and junior secondary schools mathematics curricula to fit into the basic education programme. Obioma (2006) noted that the total development of the individual will depend to a large extent on the implementation of an appropriate curriculum. The emphasis is for the curriculum to make mathematics more of real life than abstract concept and as well advocate training and retraining of mathematics teachers in order to update their technology and competence so as to acquire more teaching skills (Ekwueme \& Meremukwu, 2010, p. 34). Experience with the teaching of mathematics in most Nigerian secondary schools showed that many mathematics teachers are under the pressure of external examination syllabus and school outline of work and this always put then in a hurry to complete the syllabus irrespective of the students' level of understanding of the concepts taught (Ekwueme, 2006, p. 51).

Within the context of this study, emphasis will be on the current educational system. The 9-34 system of education. This present 9-3-4 system is a modification of the 6-3-3-4 that was introduced in 1982. Some scholars like Omolewa (1986) and Omovo (2006), stated that the history of 6-3-3-4 system of education in Nigeria dated back to 8th September 1969 during the International Literacy Day when the then Federal Commissioner for Education, Wenike Briggs inaugurated a conference which formulated the ideas that led to the 6-3-3-4 programme. However, the current 9-3-4 system of education was introduced by the former president Olusegun Obasanjo under the universal basic education (UBE) on 30th September 1999. This system involves three levels of institutional learning processes:

- Primary and junior secondary school combined.--9 years.(6 and 3 years respectively)
- Senior secondary school level---3 years
- Tertiary level---4 years


### 2.1.1 Primary and junior secondary school combined.

Although nursery education in many instances forms the first stage of the learning process in Nigeria, the nursery stage is not officially recognised, hence it is not included in the 9-3-4 system and regrettably, many families cannot afford to send their toddlers to privately owned nursery schools which in most cases are very expensive. In Nigeria, children begin primary schools (elementary schools) at the age of six and spend the next six years of their lives learning at this level and graduate at the age of twelve. Upon completion of this first six years of learning, the pupils proceed to the junior secondary school level. The junior secondary school last for three years. At the end of the three years, students write the Junior Secondary School Certificate Examination (JSSCE) and the successful ones are awarded the junior secondary school certificate (JSSC). This is a prerequisite for admissions into the senior secondary school level.

In principle, Primary and junior secondary school education in Nigeria is free and compulsory in all government owned primary and junior secondary schools under the Universal Basic Education (UBE) programme. However, in reality, it may be hyperbolic to talk about free and compulsory primary education here because many parents still have to pay school levies imposed on pupils, buy school uniforms and so on. The saddest part of the reality on ground is that many parents have to send their children to private schools where they have to pay exorbitant fees and levies. This is basically due to the fact that the available government owned and supposedly free schools are not enough to accommodate every child and coupled with the fact that the standard in the government owned schools is a subject of national debate. However, education at this level in government owned schools is mainly financed by the government. Effective 1998, anybody wishing to teach at primary school level in Nigeria is required to possess the National Certificate in Education (NCE), which is awarded by Colleges of Education. However, due to lack of qualified teachers, holders of the Teacher's Grade 2 Certificates (TC 2) are still allowed to teach in some remote primary schools. Primary school pupils have to put on school uniforms throughout the country. Every school has its own uniform as a way of distinguishing its pupils from the other school.

### 2.1.2 Senior secondary school level

Successful completion of the junior secondary school evident by the ward of the junior school certificate (JSSC) is a precondition for this second level - the Senior Secondary School (SSS). This second level also lasts for three years. At the end of this period, students are made to take senior secondary school examination (SSCE) in order to obtain the senior secondary school certificate (SSSC).

The SSCE is usually conducted by two examination bodies, the West African Examination Council (WAEC) and the National Examination Council (NECO). Students are normally assessed in eight or nine subjects depending on the choice of the student. The curriculum usually made up of six core subjects and three electives. The core subjects are mathematics, English, one of the three Nigerian major languages (Yoruba, Hausa or Igbo), atleast one science subject, one social science subject and agricultural science. Private and public schools operate the same curriculum, but some private schools usually include the Cambridge international examination curriculum so as to enable their students register and take the International General Certificate of Secondary Education (IGCSE) in their final year since the IGCSE is more internationally recognized. Below is the grading system for both the WAEC and NECO bodies.

Table 1: WAEC and NECO grading system

| Grade | Description |
| :---: | :---: |
| A1 | Excellent |
| B2 | Very Good |
| B3 | Good |
| C4 | Credit |
| C5 | Credit |
| C6 | Credit |
| D7 | Pass |
| D8 | Pass |
| F9 | Fail |

Students intending to proceed to the university must as a general rule score a minimum of credit grade C6 or better in at least five relevant subjects including mathematics and English in order to be considered for admission into any university. Students who are able to meet these minimum requirements are further required to sit and pass another examination called the University Matriculation Examination (UME) which is organized by another examination body called the Joint Admission and Matriculation Board (JAMB). In UME, students take four subjects which must include English language and any other three that are relevant to the intended programme of study at the university. Total score for this examination is four hundred marks with each of the subject weighing one hundred marks.

The accepted pass mark for this examination is fifty percent ( $50 \%$ ), which means that a student must score at least two hundred for him to be considered to have passed the examination. However, the different universities independently set the cut off mark for different programmes, usually passed on the demand for a particular programme vis-à-vis the available space for that programme. This means that a university of high ranking will usually have high cut off marks while low ranking universities will have low pass marks. In recent years, some universities especially the highly ranked ones, still have moved a step further to
conduct a final internal screening and assessment of students, especially for the high demanding programmes like Medicine, Accountancy, Architecture, Geology and Engineering.

A parallel three year secondary educational programme that is run by some specialized schools is the vocational and technical education. This is also a three years programme that students who do not wish to proceed to university can take after their junior secondary school instead of going to the conventional senior secondary school. In this programme, students learn different vocational professions like Carpentry, Electrician, Plumbing, Mechanics, etc. The National Business and Technical Examination Board (NABTEB) is responsible for the conduct of all examinations in this line of study. Successful students in this examination are awarded the National Business and Technical Certificate. Below is the grading scheme for the National Business and Technical Certificate.

Table 2: NABTEB grading system

| Letter Grade | Percentage Score | Grade Point |
| :---: | :---: | :---: |
| A1 | $80 \%$ and above | 4.0 |
| A2 | $70-79$ | 3.5 |
| B | $60-69$ | 3.0 |
| C | $50-59$ | 2.5 |
| D | $40-49$ | 2.0 |
| E | $30-39$ | 1.5 |
| F | $20-29$ | 1.0 |
| G | $10-19$ | 0.5 |
| H | $0-9$ | 0 |

It should be emphasized that the English language is the medium of instruction in all Nigerian schools. Different secondary school students have to wear different uniforms as a way of easy identification especially outside the school. While there are predominantly mixed sex primary schools, there are however few single sex secondary schools whereby the boys attend only boys' school and the girls attend only girls' school. However, mixed secondary schools are now becoming a common sight.

### 2.1.3 Tertiary level

Tertiary education provides the final stage of formal education, and this usually takes a minimum of 4 years, and thus concluding the 9-3-4 educational system. It is worthy to mention that there are about four stages of education at the university level:

- Bachelor Degree
- Post graduate diploma/certificate
- Master's Degree
- Doctorate Degree

The first university in Nigeria was the University of Ibadan which was established in 1948. At the beginning of the 1970s, there were only six universities in Nigeria. This number later increased to thirteen (1979). The federal government of Nigeria passed a law in 1993 that authorized the establishment of private universities and the National University Commission maintained the register as well as the quality of learning in all the universities including the publicly owned ones. It is interesting to remark that in April 2014, Nigeria has a total of 129 universities. This is made up of 40 federal universities, 39 state universities and 50 private
universities. This is according to the official website of the National Universities Commission (NUC) http://www.nuc.edu.ng, 2014.

All the 129 universities in Nigeria are generally grouped as follows:

- The first generation (premier) universities: As a result of a great need for manpower development in Nigeria, five universities were founded between 1948 and 1965 to handle manpower training with a view to reducing the effect of professional shortage. Till date, these universities have continued to have a leading role in all aspects of manpower development, research and high standards of educational activities in Nigeria. The universities are fully funded by the federal government.
- The second generation universities: Between 1970 and 1985, twelve additional universities were established to take care of population explosion and the growing need of technological and scientific developments.
- The third generation universities: These are the ten universities that were created between 1985 and 1999 with a special mandate to produce the needed manpower in the area of modern agricultural development.
- State universities: The first state university was established in 1979. (Anambra state university of science and technology, ASUTECH). This is a fully state funded university. However as the increasing need for university education from qualified secondary school students became more and more dominant, state government had no choice but to invest in the establishment of state owned and funded universities.
- Private universities: In 1993, the federal government through a legislation announced that following some prescribed guidelines, private individuals and organizations can establish, run and fund their own universities. This led to establishment of Igbinedion University on May $6^{\text {th }} 1999$. Since then, Nigeria has witnessed a massive explosion in the number of private universities from all kinds of interest groups ranging from religious organizations to business moguls.

A bachelor degree in Nigeria usual takes four years of study, but in some professional programmes like Medicine, Pharmacy, Dentistry and Optometry, the duration is six years. All Engineering programmes last for five years. Some programmes can be taken as single or combined honours like a bachelor degree in mathematics and economics combined honour. Every student must submit an independent project/thesis/dissertation. This is a major requirement for the award of the degree. In addition to the bachelor degrees, universities also run programmes leading to the award of post graduate diploma (PGD) which is usually awarded after one year of additional studies after a bachelor degree. Also, two years master's degree programmes are available in all the universities with majority of the universities also running doctorate programmes.

The general grading system is based on a cumulative Grade Point Average (GPA) of the entire study duration with a maximum point of five and a minimum point of one.

Table 3: Nigerian university bachelor degree grading system

| Grade | Point |
| :---: | :---: |
| First Class | $4.50-5.00$ |
| Second Class Upper Division | $3.50-4.49$ |
| Second Class Lower Division | $2.40-3.49$ |
| Third Class | $1.50-2.39$ |
| Pass | $1.00-1.49$ |

Note that medical professional programmes like Medicine, Pharmacy, Dentistry, Nursing and other related programmes are usually graded either pass or fail as there are no first class doctors or pharmacists.


Figure 1: The Nigerian 9-3-4 system of education

### 2.2 Organization and Administration of the Nigerian Educational System

With the current 774 local government areas, 36 states and the federal capital territory structure of Nigeria, the educational organization and administration is also shared along this divide. The local government areas are solely responsible for the administration and organization of the primary schools within their domain. The state is solely responsible for the secondary schools within its boundaries although there are few secondary schools (Unity

Schools) they are organized and administered by the federal government. It is the responsibility of both the state government and the federal government to handle the organization and administration of the higher institutions. However, despite this divide in organization and administration, there is usually a central curriculum framework for all the schools. This is mainly to ensure quality control. There are several agencies and bodies that are saddled with various responsibilities at different levels of government. The table below shows the various administrative bodies and agencies as well as their responsibilities.

Table 4: Nigeria Educational Bodies/ Agencies and Their Responsibilities

| Administrative Bodies/Agencies | Responsibilities |
| :---: | :---: |
| Federal Ministry of Education | Coordinate all centralized educational policies like curriculum development and quality assurance |
| State Ministry of Education | Coordinate the various states educational activities |
| Local Government Education Authority | Responsible for all educational administration within the local government |
| National Educational Research and Development Council | Promote and coordinate all educational research activities in Nigerian |
| Joint Consultative Committee on Education | An independent body of educational scholars that serve as purely advisory body to both the federal and state government |
| National Universities Commission | This commission is responsible for the registration of all universities, accreditation of all the programmes in these universities, and ensuring over quality in all the universities in Nigeria |
| West Africa Examination Council | This council is responsible for conducting the senior school certificate examination |
| National Examination Council | The council also conducts an alternative examination at the senior secondary school level. This is to give the student the opportunity to make choice while at the same time removing the monopoly from one examination council. |
| Joint Admissions and Matriculations Board | This board conduct the universities matriculation examination |
| National Business and Technical Exanimation Board | This board is responsible for organizing all examinations leading to the award of the National Business and Technical Certificate |
| National Commission for Colleges of Education | This commission supervises all the activities going on at the colleges of education which are the main teachers' educational institutions in Nigeria |
| National Board for Technical Education | Supervision of all the activities going on at the polytechnics. |

### 2.3 Pre-independence mathematics curriculum in Nigeria before 1960

Before the arrival of the foreign powers, Nigeria was not a political unit. Different ethnic groups adopted their traditional ways to spreading knowledge - political, social and mathematical - from generation to generation. With the amalgamation of Northern Nigeria and Southern Nigeria by Lord Luggard in 1914, and a rise in contact between the first Europeans and Nigerians, some level of education for the natives became imperative. Just enough language and mathematics (arithmetic) to enable them act as escorts, messengers and interpreters in the white man's trade and expeditionary missions. Europeans started penetrating Nigeria from the south, they did not come not only as traders and men of adventures or explorers, but also as religious missionaries. Then it became necessary to establish formal Western Oriented education as they moved from South to the North. The Europeans needed the natives who could read, write and do some basic arithmetic calculations. Books used were all of foreign origin. Badmus (1977, p. 15) stated that early mathematics books used were Efficiency Arithmetic, a shilling Arithmetic and Larcombe Arithmetic. Before 1960, the only subjects compulsory in Higher Elementary Colleges were English and Arithmetic (Lassa, 1972, p. 2).

By early fifties, the idea of mathematics as subject in schools has developed in Christian Missionary Schools in the South but the subject was done in three different sections as arithmetic, Algebra and Geometry. These had different time slots in the timetable. The books generally used were Durell and Channon and Smith each in the three volumes and these were all foreign authored. It was in 1956 that mathematics became one single subject in the West African School Certificate Examinations. Although arithmetic remains as a separate subject to mathematics in Teacher Training Colleges. The move to make mathematics one was supported by the argument that the integral mathematics emphasizes the important process and concepts in mathematics, rather than routine calculation. It offers the possibility of having an alternative way of solving problem.

Therefore it strengthens the understanding of the problem. Additional mathematics which had topics from Pure Mathematics, Mechanics and Statistics, was for students who were mathematically inclined, while alternative B was for students commercially motivated. But despite the emphasis in mathematics at the secondary schools level, there was no such effort to prepare the pupils for mathematics at the primary levels. Primary school level mathematics remained arithmetic. Most of the teachers of mathematics in the Colleges were whites or a few Nigerian with the teacher grade two (TC ii) or "pivotal" teachers (i.e those who did grade two after passing the West African School Certificate).

After independence, there was need to train Nigerian technicians, engineers and teachers who will take over from the foreigners. There was population upsurge in schools. The mathematics curriculum as existed was no longer suitable to satisfy these influences especially in the South. In the Northern part of Nigeria, the Arabic education emphasized supreme obedience to Allah and constituted authority, and so there was no need for dominant western type of mathematics. Western education emphasized in the South freedom of thought and speech and logical reasoning through mathematics. After independence the imbalance in education between the North and the South became obvious, hence the need for change in the curriculum.

No secondary school student in Nigeria that is involved in the learning of mathematics at any stage can honestly say to himself that all is well with the learning of the subject. There are so many students today who leave the secondary school with distaste for mathematics. Even those who study other science related subjects like physics, chemistry or other subjects which require the application of mathematics often complain of the difficult task they face. There are many problems in the learning process of mathematics. The government, employers of labour, parents, students, teachers and other stakeholders have expressed great anxiety about the fact that large numbers of students, after secondary school course, are unable to perform many of the simple arithmetical and mathematical operations needed in their everyday life and work. One reason we can advance for this is that a majority of students never get detailed understanding of the real meaning of mathematical concepts. In most cases, they are only clever in the art of manipulating complicated sets of symbols, or they are confused by the unfriendly conditions that the present mathematical requirements in schools tend to place them. In the end, they cultivate a mutual approach - 'get the examination over', after which forget about mathematics. This is the present situation in the Nigeria secondary schools.

Mathematics is generally considered as a very difficult subject everywhere in the world even by the clever students in other subjects. But the severe problem in Nigeria is that very little or sometimes nothing is being done to change this situation. However, there has recently been certain feeling of worries among those concerned. The demonstration of dissatisfaction is on the increase by the day, although this has mainly been only on the pages of newspaper without any corresponding plan of solving this problem. Also, there are diverse viewpoints about the necessity of different types of transformations. It is the combined obligation of those responsible to bring about the required and necessary enhancement and modifications that is desired in the learning of mathematics in Nigeria secondary schools. The subject has to be simplified at all costs.

With the newly-approved syllabus introduced, it is now hoped that the Mathematics Association of Nigeria will wake up to the tasks of the day. Motivations should be given to those capable and interested in providing solutions to these problems at all times in terms of funds for workshops, seminars, conferences and useful researches and other forms of training. Governments at different levels and at different time have been expressed concern and interest in establishing quality education. It is believe that a comprehensive analysis of these problems and sincere stakeholders' commitments will help in reducing this problem. I will like to consider the situations as they exist in our schools today under the headings: historical, political, socio- cultural, economic, academic mathematical, pedagogical and psychological.

### 2.4 Post-independence mathematics curriculum in Nigeria

Although Nigerian post independent era started in 1960, but it was in 1969 that the first major restructuring in mathematics curriculum in Nigeria began. Let me quickly mention that secondary school mathematics curriculum development in Nigeria has witness basically three phases. The first phase started in 1969 when the then military administration created the Nigerian Educational Research Council (NERC) which was later renamed the Nigerian Educational Research and Development Council (NERDC). Part of the main objectives of this agency was to design a national mathematical policy frame work for Nigeria. The Nigerian post-independence mathematics curriculums witness a scientifically oriented boost. This was attributed to the launch of the first space satellite (sputnik 1) by Russia in 1957. It was generally and widely acclaimed that mathematics was the major asset in this human technological breakthrough.

Therefore, the concern stakeholders subsequently decided to make a shift from the traditional arithmetic curriculum to a more technologically driven curriculum. This new reformation led to the introduction of some new areas of mathematics like geometry, probability, complex number and a few others into the new mathematics curriculum for Nigerian secondary schools. In addition, there was also the creation of an independent sub project group called the African Mathematics Programme. This body was to constantly and regularly review and publish mathematics textbooks to be use in Nigerian secondary schools in line with the new mathematics curriculum. There were numerous and comprehensive workshops, seminars and training organized by the African Mathematics Programme which led to the production and launch of several mathematics textbooks for Nigerian secondary especially between the year 1969 and 1972.

However despite the good intention of this curriculum transformation, this first stage of the post-independence reform did not achieve much success. This was attributed to a number of factors, but obvious amongst these obstacles was the massive scarcity of competent teachers to handle especially the newly introduce areas of the curriculum. The situation was worse off in the Eastern region of the country haven suffered so much from the Nigerian-Biafra civil war between 1967 and 1970. This civil war led to the destruction of almost all existing infrastructures in the region like schools, teacher training colleges and a total disruption of all educational programmes and calendar in the entire eastern region.

The story of the Northern region was not in any way so different although the civil war was not really responsible for the setback in this region. This region mainly suffers from the existing pre independence educational gap that has always existed between the south and the northern region. As a result of the unsuccessful nature of this reform, there were widespread condemnations of this programme in the middle of the 1970s. Many criticized the programme and complain of the non-suitability of the reform in meeting the yearning and aspirations of the populace. Thus in 1977, the entire existing mathematics curriculum reform was abolished by the then commissioner for Education Dr Ali after holding a conference with the aim of setting up a new task force that will advise the government on the modality of improving or reorganising the hitherto organised mathematics curriculum. This could be seen as the beginning of the second phase. The new advisory task force was saddled with the following responsibilities:

- Evaluate the existing mathematics curriculum with a view to identifying grey areas that has resulted in the criticisms so as profile appropriate amendments.
- Give the government a solid and implementable proposal regarding the development of a new mathematics curriculum for Nigerian secondary schools.
- Evaluate the teacher materials for this curriculum so as to determine their suitability and perhaps recommend appropriate changes.

After series of nationwide consultations of all mathematics education stakeholders, the task force came up with the following recommendations:

- That inadequate publicity and lack of information regarding the motive behind the new curriculum was mainly responsible for the controversies.
- That because of the introduction of new topics into the new curriculum, the teaching of mathematics has become more difficult for mathematics teachers and hence the need for urgent retraining of all mathematics teachers in Nigerian secondary schools.
- That the above problem was further compounded by gross shortage of mathematics teachers especially in the east and northern region. The task force therefore recommend massive recruitment of competent mathematics teachers to take care of the shortfall.
- That most of the criticisms were based on personal sentiments and thus bias. It therefore recommends appropriate sensitization of the populace.

As a result of the recommendation of the task force, a working group was formed to work independently on the recommendations of the task force. This working group in collaboration with the Comparative Education Study and Adaptation Centre (CESAC) worked together to solve the problems associated with the secondary school mathematics curriculum. A good number of seminars and public hearings were conducted by CESAC and the working group all with a view to developing a more appropriate and acceptable secondary school mathematics curriculum. The National Council on Education (NEC) held a meeting in October 1977 in order to review the work of CESAC, and the working group and in 1978, the entire work of these two groups was made public for necessary and appropriate criticism during a conference that was held in Onitsha. The National Council on Education (NEC) accepted the reformations of these groups although full implementation of the new secondary mathematics curriculum did not take effect until 1981.

The third and present phase of the post-independence mathematical curriculum development started in 1999 with the introduction of the Universal Basic Education (UBE) by president Obasanjo administration. The major objective of this programme was to clearly separate the junior and senior secondary school and to also make the junior secondary school an integral part of the primary school thereby having nine years of basic and compulsory education. i.e, six year of primary and three years of junior secondary school. The ideal behind this programme led to a massive restructuring of the secondary school mathematics curriculum since the junior secondary will be more aligned with the primary school system instead of the secondary school system. The new senior secondary school mathematics curriculum is centred on Information and Communication Technology (ICT).

Topics in mathematics to will promote the advancement of ICT were introduced. Also, unlike the previous curriculum that is centred on examinations as the main and only means of assessment of student performance, the new curriculum now encompassed the use of project work and group work as alternative means of assessing students' performance. The government said this is in line with present global best practices. In addition, problem solving activities and quantitative reasoning were introduced. The present curriculum also tried to shift the teaching of mathematics from the traditional method of teacher-centre, chalk and board teaching method in which student learn to memorise processes and algorithms with a singular aim at arriving at the correct answer to a student centred process whereby students engage more in collaborative learning and working or realistic or real life mathematics. However, the implementation of this third phase did not commence until 2007 when it was eventually approved by the National Council on Education (NEC). The table in the next page shows some of the major differences between the old and the new secondary school mathematics curriculum as copied from (Adeneye, 2012, p. 18).

Table 5: Differences between the old and new secondary school mathematics curriculum. Copied from (Adeneye, 2012, p. 18).

| Old secondary school mathematics curriculum | New secondary school mathematics curriculum |
| :---: | :---: |
| No distinction between students' and teacher activities | Activities are broken down into students' activity and teacher activity |
| No sample evaluation item to guide the teacher | Evaluation guide consists of sample evaluation items to guide the teacher. |
| Topics are not aligned with quantitative reasoning tasks. | Every topic in the curriculum is associated with quantitative reasoning tasks to facilitate the development of problem solving and psychomotor skills. |
| Content is organized based on how teachers teach. | Content is organized based on how students learn. |
|  | There is more emphasis on how to develop positive attitude towards mathematics and on student motivation. |
| Teaching methods, techniques and strategies are not student-centred. | Teaching-learning activities prepared parallel to learning outcomes require student-centred methods, techniques and strategies |
| The existing primary and junior secondary schools mathematics curricula truncate the 9year continuous schooling. | The new basic education mathematics curriculum gives room for the 9 -year continuous schooling. |
| There are few sample activities that require the use of manipulations | Virtually all of the sample activities show how to use manipulations for students' construction of knowledge. |
| There are few examples of realistic mathematics | Daily use of mathematical knowledge is emphasized. |
| Curriculum content overload which does not encourage the use of those teaching and learning strategies that promote skills development. | Curriculum content is evenly distributed so as to encourage the use of facilitative teaching and learning strategies. |
| Academic nature of the old mathematics curriculum made it to lose touch of basic learning for life long survival. | The curriculum is interspersed with skills for lifelong survival. |
| No adequate representation of emerging issues. | Emerging issues such as HIV/AIDS are well represented. |



Figure 2: NERDC five levels of curriculum development.

### 2.5 The Importance of mathematics in Nigeria Secondary School Curriculum

Even though, the ideas and rules that form the foundation of all mathematical structure are discoveries of human thoughts, they are however not the thoughtless rubbish which might be bring together by a robot. There are motivations that support each product of Mathematics. Sometime that motivation takes the form of an inspiring potency, a need for a simpler and easier ways of dealing with a difficult situation. The motivations may as well have its root in history or in shared usage. The reason for learning a particular mathematical concept at a time and in a particular way may be explained in terms of its sensible match to the previous learning of the student.

Mathematics is one of the compulsory subjects that students must offer in senior secondary school, not minding whether such students are in science, commercial, arts or social science class. In Nigeria secondary school curriculum according to National Policy on Education (2004), there are core subjects as well as electives that students must offer. The core subjects are English Language, Mathematics, one of the Nigerian Languages (i.e. Hausa, Igbo and Yoruba), one of Physics, Chemistry or Biology; one of Literature in English, History or Geography and Agriculture or a vocational subject.

Also, there is a list of subjects tagged electives from which students have to offer three. These include Economics, Government, Commerce, Christian region knowledge, and Health Science. The students' anticipated career will determine which of the electives to offer. In summary, there are compulsory and elective subjects that students do offer because of their status in the curriculum, not minding whether they are in science, commercial, arts or social science class. To further their studies in institutions of higher learning especially in University, students are expected to have credit in five subjects. Credit pass in Mathematics is also required for students offering science and social science courses. This makes Mathematics one of the essential subjects for students' advancement.

The significance of Mathematics in producing resourceful graduates that are needed for economic development cannot be over-emphasized. This is why Setidisho (1996, p. 47) affirmed that Mathematics is a fundamental science that is necessary for understanding most of other fields in education. He stressed further that, it is glaring that no other subject forms such a strong force among the various branches of science. The Science Teachers Association of Nigeria (1992) referred to Mathematics as the central intellectual discipline of the technology societies. In his submission, Odusoro (2002, p. 17) affirmed that the knowledge of science remains superficial without Mathematics. It therefore means that, the position of Mathematics in secondary school curriculum in Nigeria is important for scientific development.

Majority of secondary school students often dread and show negative attitude towards Mathematics (Awofala, 2000, p. 23) and the trends of their achievement in the senior secondary school certificate examination is also a source of worry to the stakeholders Scholars have observed the fact that students' academic achievement is an output of educational system which cannot be examined in isolation of the inputs and process. Hence the World Bank in (1999) asserted that good quality of education requires efficient systems that would provide supportive learning environment, motivated staff with mastery of their subject matter, adequate access to resources, and students who are healthy and ready to learn. In the same vein, Obanya Okpala (1984, p. 7) submitted that, it is only a combination of quality inputs and quality processes that can produce quality outcomes.

However, the history of mathematics as well as its classification is essential to these connecting relations, which clarifies the mathematics that students learn. In some instances, careful examination of notions and ideologies may assist in providing students with connections and this should be entrenched in the student's learning processes. In the use of inductive or deductive processes of learning Mathematics, care should be taken not to overstress each of these processes. Students should be made to see where induction will fail and where it is the necessary and sufficient tool. While the use of induction that will lead to generalization may good for the junior secondary school students, we should be careful to note the limit of the process.

Mathematics is much more than arithmetic - the science of number and computation. It is far more than geometry - the study of shapes and sizes, and it is not sufficient with just algebra the language of symbols and relations. It involves more than statistics - the science of interpreting data and graphs and more than calculus - study of change, infinity and limits. But just why should this subject be learned by every student in Nigeria secondary school? In fact why should everybody learn mathematics? What are the benefits of dedicating so much resource (time, money, energy, etc.) to the learning of mathematics? What is the significance of this subject in the Nigerian secondary school curriculum? How does it enhance the worth and competence of the students? What is the place of mathematics in the Nigerian secondary school system of education?

All these questions apply to the need for mathematics education in our schools. Perhaps many curious students are interested in getting answers to these questions and finding answers to all or some of these questions may be the motivation the students need to do well in this subject. Most students wish to know why they need this subject in their curriculum. It is pertinent to mention that because the education of children and youths always takes place in a particular society, it is evident that any consideration of acceptance of any subject in school curriculum should take cognizance of the goals of education in that society. The Nigerian National Curriculum Conference (1969) had observed that the socio-economic development of the previous decade indicated a general progress but secondary education must remain a terminal education for majority.

Also, the National Policy on Education (1977, p.10) looks at secondary education as a terminal education for some. Hence, the two basic functions the secondary schools must serve are:
To prepare students for life, and to give those with the necessary background the opportunity to proceed to higher institutions (Nigeria, 1977, p. 10).

The basis for the selection of these objectives was to create an equilibrium and to accommodate the interests of the majority who may be unable to go on to the tertiary level, while the rest is effectively equipped to function as the centre of Nigeria's future manpower needs. Considering the evaluation methods of students learning mathematics in Nigeria secondary schools which are directed toward the cognitive achievements only, there is an apparent domination by the latter function of the secondary course over the one of preparation the students for life. Poor students' performance in mathematics has created major worries for mathematics educators in Nigeria. This is because, the learning of mathematics in Nigeria secondary schools have a significant role towards understanding the subject as well as science and technology related subjects .According to Adegboye (1991, p. 39), mathematics is one of the core subjects in both Junior and Senior Secondary School curricula in Nigeria. This
validates the claim that mathematics is indispensable in the development of technological development in Nigeria.

However, regardless of this apparent importance of mathematics in scientific and technological development of Nigeria, students' performance in mathematics in Nigerian secondary schools has not been encouraging (Adeyebe, 1993, p. 22). Contributing on under achievement, Fajemidagbe (1997) says attempts have been made by many researchers in mathematics education to deal with the problem of achievement in secondary school mathematics in Nigeria, but up till now there is more to be done. Korau (2006, p. 19) is of the opinion that several variables ranging from the students, the teachers, the textbooks, the curricula, school environment have been responsible for students' poor achievement in school Mathematics in Nigeria. The level of students' performance in mathematics determines the level of the students' difficulties in the learning process. For example, if a student's score in mathematics is very low, this undoubtedly indicates low level of the students' understanding of the learning process of the subject.

In spite of the collective acknowledgment of the importance of mathematics and the remarkable hard work being made by educationists, mathematicians, mathematics teachers and researchers towards improving the quality of the learning process of mathematics in Nigeria secondary schools, yet the students' performances are not encouraging. For instance in the research on areas of difficulties among JSS three students on Algebraic content, Galadima (1988, p.15) found, the performance of students to be generally low as a result of lack of proper understanding of the basic concepts and processes. Similarly, Inekwe (1997, p.41) while investigating algebraic and geometric reasoning difficulties affecting remediation at the secondary levels, found that, the general performance was below $30 \%$ mean score and more than $50 \%$ of the students have:

- Answer consciousness.
- Poor inductive reasoning ability, and
- Poor intuitive problem solving ability due to deliberate avoidance of non-numerical variable.

The National policy on education (1977) hopes that by instilling faith in student's ability to make rational decisions, the national objective may be achieved. It may be tempting to therefore infer that mathematics education is a worthwhile pursuit based on its inherent value. The ability for students to reason logically and think critically is one of the hallmarks of an educated student. Thus, the learning of mathematics could be an end in itself.

Mathematics is crucial to the understanding of the world we live in. It fortifies science, technology, medicine, economics and even government. In order to fully appreciate the importance of mathematics in Nigerian secondary school curriculum, let us first look at some definitions of mathematics as postulated by some scholars. Odilli (2006, p. 45) sees mathematics as a subject that helps students to form the habit of clarity, brevity, accuracy, precision and certainty in expression. Odilli went on to say that it is a body of knowledge, a collection of techniques and methods, and the product of human activity for solving problems. The Oxford Advanced Learners Dictionary (2001) defines mathematics as the science of size and numbers (of which arithmetic, algebra, trigonometry and geometry are branches). The New Encyclopaedia Britannica defines mathematics as the science of structure, order and relation that has evolved from elementary practices of counting, measuring and describing the shapes of objects.

Gerofsky, (2004, p. 34) defines "school mathematics" as an interconnected set of content knowledge including numbers and operations, algebra, geometry, measurement, and data analysis. Moreover, school mathematics comprises of cognitive process skills including the ability to use content knowledge and conceptual understanding to reason, solve routine problems, develop proofs, and effectively communicate, represent, and model mathematical ideas. But unfortunately many mathematics teachers still struggle to reply some frustrated students and perhaps some parents when asked if they seldom, if ever, use the mathematics they learn in schools. Tate and Malancharuvil-Berkes, (2006, p. 9) argued that although mathematicians, scientists, and engineers recognize that technological advances require a deep understanding of mathematics, many students especially at secondary school level can neither see the mathematics nor do they recognize when mathematics is used on a daily basis outside their classrooms. This is the situation despite many visible uses of mathematics all around us in our daily life around the world, e.g., bar codes that monitor inventory, global positioning systems (GPS), fast food restaurant cashier counters, and so on.

While teachers and even parents encourage students to use calculators and computers to aid them in the learning process of mathematics, rarely if at all are the students made to understand, recognize and appreciate the important mathematics that triggers the design, development, and maintenance of the technological supports they are using. Consequently, it becomes problematic to explain to students that these technological devices that they are using are dependent on the basic elementary concepts of mathematics (Hastings, 2007, p. 18). According to Kolawole and Oluwatayo, (2004, p. 5), the mathematics curriculum in Nigerian secondary schools is developed and structured around five major concepts: Number and Numeration, Algebraic processes, Geometry, Mensuration and everyday Statistics. In order for us to learn mathematics for the support of real life situations, the learners must be able to visualize mathematics beyond the classroom boundaries. This implies that the learners must be able to relate the classroom mathematics to the real world. But unfortunately, this is far from realization in the present day mathematics learning process in Nigerian secondary schools.

### 2.6 Problems in the learning of mathematics in Nigeria secondary schools

Even with numerous strategies by the government with regards to the improvement of mathematics learning, there are still hitches in the learning process of mathematics in Nigeria. Odilli (2006, p. 18) delineated some of these hitches as follows:

- Lack of curriculum integration.
- Shortage of mathematics teachers.
- Lack of instructional materials.
- Poor government policy.
- Poor classroom organisation by teachers.
- Lack of equipped mathematics laboratory for practical.
- Over population of students.
- Teachers' impatience and un-preparedness.
- Poor remuneration of teachers.

The major and reoccurring question is; what can we do to develop the teaching and learning of mathematics in Nigerian secondary schools, especially when all major stakeholders consider mathematics as a tool for self-reliance? Adequate manpower is required for any nation to develop economically, scientifically, technologically and even politically. This
manpower ranges from medical doctors, engineers, mathematicians, etc. However, we need a sufficient knowledge of mathematics in order to produce these self-reliant medical doctors, engineers and others. Calculations, addition, subtraction, multiplication, division, buying, selling, and some other activities are simple and essential procedures of mathematics which require colossal practice. The knowledge and skills in these procedures can be provided in an effective and systematic manner only by teaching mathematics in schools (Kulbir, 2006, p. 11).

But even with the believed usefulness of mathematics, it has remained one of the subjects that Nigerian secondary students have over the years dreaded, disliked and performed poorly. (Odilli, 2006, p. 22). Hence Kolawole and Oluwatayo (2004, p. 41.) stated that the more knowledge of mathematical notions with the corresponding knowledge of their application to real life seems to be declining. The assumption that mathematics is a fixed body of knowledge that offers clear-cut answers to numerically-based problems precludes recognition that mathematics is a creative and experimental tool that explicitly informs planning, organizing, and ethical decision making within specific constructs (Bakalar, 2006, p. 33). In addition to Odilli (2006, p. 29) delineated problems, the following factors were also considered.

### 2.6.1 Historical Problem

The inequality in the development of the learning process of mathematics in particular and of mathematics education in general in Nigeria secondary schools may be attributed to the nonearly introduction of Western Education in the North. This inequality of development of education in the country has created many educational problems in the country. The problem of uniform planning, the problem of uniform curriculum, the problem of over production of teachers of mathematics in the South, the quota system of admission into Colleges. It has been advocated that the federal government should make for provision of more competent teachers and necessary facilities especially in those states that are backward in mathematics. Another suggestion is the inter-state cooperation in employment of over produced staff. The mathematics teachers excessively produced in the south should be encouraged to take up employment in the North.

Another historical influence to our problems is the fact that the Colonial masters started a curriculum of Arithmetic in the country. And so, when 'new' or modern mathematics was introduced many people, including the products of Arithmetic were strongly against it. This is evident in the disagreements that transpired from the introduction and subsequent abolition of modern mathematics in the seventies. Nigeria is still feeling the problem till date.

### 2.6.2 Political Problems

Another horrible problem that has affected mathematics educational growth in Nigeria is politics. Nigeria is running a Federal system of government but the bulk of the revenue is centrally controlled. But this should not so since education is state (regionally) controlled in many areas. Drastic political decisions have in many instances affected the growth of mathematics education in Nigeria secondary school. The effects of the Federal Ministry of Education's decision on January 7th, 1977 to abolish over night the modern mathematics from schools. The effects of this one drastic political decision is properly summarized by Odili (1986, p. 34). This announcement led to a profound confusion as mathematics educators, state (regional) ministries of education, textbook publishers, and especially serious minded classroom teachers and students were caught by this shocking news.

Some unenthusiastically sustained whatever programmes they were using, while others scrambled to find twenty-five years old "traditional textbooks". One other drastic political decision, that shook educational foundation in Nigeria, was the introduction of the Universal Primary Education (UPE) without proper planning in the early seventies. This brought with it the population explosion in the primary schools without corresponding expansion in infrastructure, teachers, book and money in the secondary level to take care of or engage the primary outputs leading to a drastic fall in the standard of education. As a suggested solution, government should carry out proper planning and consultation at all levels before embarking on any educational policy or reform. Fundamental issues must be established and appropriate assessment done before the launching of the new programme. Another solution is to apply the method of piece-wise implementation of the programme. This method is being applied now in the philosophy of pilot schools being organized by the Nigeria Educational Research Development Council (NERDC).

### 2.6.3 Socio-cultural problems

The differences in ethnicity in the country created educational problems. Standards and priorities vary from ethnic group to ethnic group. One group may prefer Arabic, nomadic or cultural education while another group would introduce formal classroom western education. The results of these systems would definitely be different and this militates against the much desired unity and uniform development of the country. Some cultural differences bring about student disturbance. Student instabilities are also proliferated by religious differences. The consequences of the students' unrests are:

- Closure of schools. (strike)
- Loss of property, loss of life, loss of academic year and loss of manpower.

One of the social difficulties facing mathematics education in Nigeria is the type of job a mathematics graduate is likely to get in Nigeria in order earn a good living. The jobs that are open to a good mathematics graduate can barely give him a decent living. Whereas his business counterpart will likely be living in affluence. The mathematician cannot employ himself and he cannot practice self-employment like the engineer, medical doctor, nurses or a pharmacist. Hence, many students honestly feel that as mathematicians they have no future and therefore develop little or no interest in the learning of the subject. A solution to this is for the government to make the study of mathematics very lucrative, give scholarship to mathematics students and make the conditions of service for mathematicians as lucrative as those of medical doctors and other well remunerated professions.

### 2.6.4 Economic Problems

Economic difficulties are so tangible that in some states (regions), the enrolment in schools is less than one third of what it was in 1976. With the introduction of schools fees under the pretence of various names - education levy, equipment levy, education rate, Parent Teacher Association levy - many parents were compel especially during the economic crunch to withdraw their children from schools.

In the past decades, education use to have a large government budget allocation and foreign aids and oil revenues, and so finance was not the problem. However, for the present government because of instability in the country, foreign aids and foreign investments are not coming as fast and as largely as it should. And even the emphasis is not on foreign aids but on self- reliance by all Nigerians. What we need to do now is keep costs of our educational
planning low but the qualities high. In order to achieve this successfully, government should intervene and make conditions of service of workers correspond to the essential food prices.

### 2.6.5 Academic Problems

There is a group of problems, which I will like to refer to as academic problems. They include:

- Poor student motivation towards the learning of mathematics.
- Poor teacher quality.
- Teacher dis-satisfaction and indifference.
- Unavailability, expense or inappropriateness of textbooks.
- Non-application of appropriate curricular.
- Non-appreciation of mathematicians and mathematics teachers by the Nigeria society.

Many Nigeria students are defeated in mathematics from their first year in primary school. We can blame this on poor handling of the subject by many primary school teachers. After their first three years those teachers who inherit these students receive mostly students of poor quality in mathematics. In some cases most students become indifferent to the subject by the time they reach the junior secondary school. Interactions with those teachers who show some dedication to teaching mathematics at this level tell their stories of despair. Student indifference is complicated by poor teaching quality, especially at the primary school. The starting point in teaching any subject properly is knowing the subject. Unfortunately, many primary school teachers and some national certificate of education (NCE) and degree holders in our secondary schools do not know enough school mathematics. Some who know mathematics are not able to teach it well perhaps due to poor motivation on the job. The teachers sometimes demotivate the students by calling them idiots, fools and all kinds of names.

Then there is teacher dis-satisfaction arising from both the low status of teachers in the society and the poor treatment of teachers by those responsible for their welfare. For this and other reasons some of which are economic many teachers have become indifferent to the jobs and carry out their teaching responsibilities reluctantly. Moreover, the unfriendly work environment of teachers has to be pointed out as a problem for all teachers in general and mathematics educators in particular. The classroom is unexciting and uninspiring. Some basic equipment which may be used to enhance the learning of mathematics are not available. The textbook problem has many dimensions. First, a few of the popular mathematics textbooks now were written ahead of the curriculum and thus do not completely reflect the said curriculum. Second, the general economic condition makes some parents less able to purchase textbooks at a time when prices seem to increase daily. Third, some textbooks are not even available when needed.

It is also the case that in many schools if copies of the various curricula are available they will be kept away in the principal's office. Many who teach mathematics would neither have seen nor used the appropriate curriculum. The principals / headmaster waste much time on routine administrative issues and moving between their schools and ministries of education / school boards. In particular, mathematics and science teachers have complained of lack of financial and other support of their classroom and club activities.

The final problem which we may discus under this heading is that both mathematicians and mathematics teachers are not appreciated in the Nigerian society, indeed, the society does not
pay due regard to teachers. In the case of mathematics and science where teachers are few this lack of appreciation has had serious undesirable effects. Teachers who have the opportunity resign their jobs at short notice for greener pastures. Many of these problems have glaring solutions. For the quality of the students and the ultimate improvement of the learning process, mathematics should be taught as a specialist subject in secondary school. To increase the teachers' satisfaction and motivate them more, greater attention should be given to the working environment of the mathematics classroom teachers as well as their working conditions.

### 2.6.6 Mathematical Problems

The mathematical problem can be subdivided into three distinct points:

- the understanding of ideas.
- the acquisition of techniques, and
- the contents.

The situation in the field of understanding of ideas can be described as desperate. We talk of the need for the understanding of mathematics. Unfortunately, the present system of teaching mathematical information fails to involve this in most cases. Most students in third year of junior secondary school (JSS 3) could solve two simultaneous equations when written in this form:
$5 x+3 y=60$
$7 x+6 y=84$
But they could not solve the same problem put this way: Find two numbers such that five times the first added to three times the second is equal to 60 and also such that seven times the first added to six times the second is equal to 84. Again, a student may be familiar with all the procedures of resolving linear equations without having much idea what sort of thing a linear equation is. This means, a student may feel that he understands mathematics when in fact he does not. Such a student will not appreciate the interconnection of the various processes he knows. The teacher may also have the impression that a student understands something when in reality the student does not. As students have a fast technique of providing standard answers to standard questions and this gives a false impression of knowing a mathematical concept.

Concerning acquisition of techniques, it is tempting for us to have the impression that students are working hard to increase their competence in the achievement of new and better learning methods. Although some studies have been done by some scholars on alternative and better methods of learning mathematics as well as many books written of this subject matter, it is however unfortunate that many if not all of these developments have only been happening in the developed Western world.

With regard to the content of mathematics, the majority of the Nigerian secondary students are of the opinion that the mathematics curriculum is too loaded and lengthy. This assertion may be true especially with the use of the classified syllabus system. But this I believe has been improved with the current curriculum. It was argued by many mathematics educators in Nigeria that the old mathematics curriculum did not provide clues and instructions for the teachers. Hence, it was placing the teacher in a problematic situation while trying to use the curriculum. It is therefore imperative for the developers of the curriculum to point out areas of concentration for different topics in the curriculum. Areas that required learning aids and
devices like calculators and computers should be clearly indicated, while areas connecting to practical and perhaps group work should be indicated as well.

### 2.6.7 Pedagogical Problem

In looking at the pedagogical problems that are encountered in the learning process of mathematics in Nigeria, It is important to first establish the fact that despite the much publicised modern method of learning mathematics by the various stakeholders, a glimpse into the majority of Nigerian secondary school classrooms still show that the traditional teacher-centred and lecture-based learning approach. There are little or no group work and all emphasis are still geared towards solving problems that will lead to correct answers.

### 2.6.8 Learning Aids

The average Nigerian secondary school is characterized by inadequate mathematics learning aids. Students are rarely provided with any form of learning aids, thus making the students to continually see only the abstractness of mathematics.

### 2.6.9 Classroom organization and Management

The issue of classroom organization and management is another problem militating against the learning process of mathematics in Nigerian secondary schools. Some of the major factors responsible for this problem are over population, the sizes of the lecture rooms, inadequate furniture, as well as other facilities. On the average, a regular publicly owned Nigerian secondary school classroom especially in the densely populated cities like Lagos, Kano, and other big cities has not less than 40 to 50 students in a class. This can sometimes even go as high as 70 to 80 in some worse cases. The classroom spaces are so small and sometimes poorly ventilated. This undoubtedly makes it impossible for the teacher to create a one-on-one activity with any of the students. In most cases, the teacher is confronted with two major tasks. These are the tasks of arranging the students into appropriate learning groups as well as how to provide the students with the scarcely available learning materials.

### 2.6.10 Teacher's Competence

The problem of teacher's incompetency is a common phenomenon in most Nigerian secondary schools. The case is particularly bad in mathematics and science. It will be very difficult for an inadequately qualified teacher to properly deliver the content of the subject. Some of the teachers may have good examination results in the subject, but inadequate training regarding the delivery method might just be another setback. Proper training brings self-confidence to the teacher which will lead to proper delivery of the subject content and hopefully a strong motivation for the students. This problem is further compounded with the high level of work load which is as a result of oversize of the classrooms as earlier mentioned. However, in as much as I have emphasized motivation as being one of the problems in the learning process of mathematics in Nigerian secondary schools, it will as well be important to emphasize that, competent and inspiring teachers have significant role to play in this regard.

### 2.6.11 Individual Differences

The problem of individual differences in the learning process of mathematics is also another major problem especially at secondary schools level when student fully come to the realization of themselves academically. While some students believe that they are not good mathematically, other think they are just average while few think they are at the top. This is a challenge for curriculum developers as it poses a big task for developing mathematics curriculum that can take care of the various self-acclaimed levels of students' potentials. This situation we see here is where the students are naturally divided into three levels as slow
learners, average students and the academically talented students. This may be a reflection from students' performance in classroom work like logical reasoning, ability to manipulate figures and symbols, accuracy and speed with computations and a very high I.Q. This natural classification or segregation can lead to a very low self-esteem and thus strong demotivation for the students.

The questions that may arise now is how can every student be given about the same equal opportunity to display or show his potentials as fully as possible, whether he is a slow learner or an average learner or an academically talented student? Can we create different content for each category of students? However, some educational experts like Akinsola and Olowojaiye, (2008); Awofala and Sopekan (2013) and even psychologist are of the opinion that the basic content can be the same for all the groups, although emphasis may be placed on amount, presentational skills, organisational methods as well as individual attention. Furthermore, as a way of providing for the different group, each student may be given the opportunity to sometime choose what task he wants to work with. The teacher can correct the errors if any while serving as control. This may motivate the student to be more self-reliant and build confidence. The students' morale may be increase since they are given opportunity to do what they like and thus they are likely to progress at their own pace.

Some other reasons that may have contributed to the observed poor achievement in Mathematics in Nigeria secondary schools as submitted by scholars include; shortage of qualified Mathematics teachers (Ohuche, 1989, p. 43), poor facilities, equipment and instructional materials for effective teaching (Akpan, 1987, p. 43 \& Odogwu, 1994, p. 22), use of traditional chalk and talk methods (Oshibodu, 1988, p. 14 and Edwards \& Knight, 1994, p. 37), large pupils to teacher ratio (Alele-Williams, 1988, p. 21) and mathematics phobia/fright (Georgewill, 1990, p. 55), limited background preparation in Mathematics, lack of Mathematics teaching equipment and materials, fright and anxiety, low level of interest and some government policy (Abimbade, 1995, p. 19), lack of problem solving abilities (Abimbade, 1997), self-concept and achievement motivation (Akinsola, 2004, p. 44). It should be emphasized that these problems are not mutually exclusive and not exhaustive.

## 3 Mathematical modelling as a way of learning mathematics

In this chapter, I will begin by looking at some definitions of mathematical modelling as postulated by different scholars and this will be in section 3.1. In section 3.2, I will discuss mathematical modelling in the learning process of mathematics. This will be closely followed by section 3.3 where I will talk about the mathematical modelling process. This will further give way for me to look at the modelling process specifically within the classroom in section 3.4. Finally, this chapter will be concluded with section 3.5 where I will be talking about mathematical modelling in the Nigerian secondary school curriculum.

### 3.1 What is mathematical modelling?

Previous studies on student learning (Johnson \& Johnson, 1991, Ross \& Cousins, 1993, Marton \& Booth, 1997) advocate that students learn at different times and with different conditions, as well as adopt different approaches to learning. Although many scholars agree that modelling is a great idea by the scientific communities in mathematics and mathematics education (Blum et al., 2007; Lesh et al. 2007; Stillman et al. 2008), there is still no generally acceptable or distinctive definition of the term "mathematical modelling".

In recent time, mathematicians have cultivated the practice of separating the world into two parts: mathematics, on one hand and everything else on the other hand, and they sometimes refers to this other part as "the real world. There are as many definitions of mathematical modelling as there are authors writing about it. The differences between these definitions can usually be explained by the different scientific interests of their authors. Mathematical modelling is the process of using mathematical tools and methods to ask and answer questions about real-world situation (Abrams, 2012). Meanwhile, Heyman (2003) defines mathematical modelling as the applicability of mathematics; highlights its relation to the real world and again describes it as an easy way of presenting this relation. Also, Berry and Nyman (1998) see mathematical modelling as a translation of real life problems into mathematical problems, formulating mathematical models necessary for solving a problem and interpretation of the results.

In addition, Stevens (2000) stated that mathematical modelling is effective in helping the students to see mathematics as a source of production in their lives outside school and in creating a suitable environment for the improvement of their mathematical skills. For Dudley (2010), a modelling task is any mathematically-rich problem that engages students in mathematical thinking, drawing upon previously learned knowledge and supporting their understanding of the mathematical concepts currently being covered. Modelling tasks should challenge the students' curiosity, encourage both independent thinking and collaborative discussion, and provide significant mathematical ideas and themes. Furthermore, for Cheng (2001), mathematical modelling is a process of representing real world problems in mathematical terms in an attempt to find solutions to the problems. A mathematical model can be considered as a simplification or abstraction of a (complex) real world problem or situation into a mathematical form, thereby converting the real world problem into a mathematical problem. The mathematical problem can then be solved using whatever known techniques to obtain a mathematical solution.

This solution is then interpreted and translated into real terms. Breen (2010) suggests that the tasks should build on students' previous knowledge and encourage the formation of new ideas and concepts. Mathematical modelling plays a part in many diverse professions, like engineering, medicine, economics, geology, etc. It involves looking at real-world problem or situation, asking questions about the situation, formulating mathematical representations
(models) which depict the situation like equations, graphs, etc, and then extending these representations to learn new things about the situation. Some of these definitions have been extensively acknowledged in mathematics education in recent years. Equally, the term mathematical modelling may mean the process of model building, leading from a real situation to a mathematical model, or the whole applied problem-solving process, or again any manner of connecting the real world with mathematics.

In recent years, the term applications and modelling (or vice versa) is frequently used as an all-embracing expression for the various interrelations just mentioned. However, in this study, I will look at mathematical modelling in line with Swetz and Hartzler (1991) definition which says that "Mathematical modelling is a mathematical process that involves observing a phenomenon, conjecturing relationships, applying mathematical analyses (equations, symbolic structures, etc.), obtaining mathematical results, and reinterpreting the model." It could also involve the ability to apply the mathematical concepts learned in classrooms to real world application.

### 3.2 Mathematical modelling in the learning process of mathematics

Over the years, mathematics educators have acknowledged several causes of difficulties in learning mathematics. Students often fail to retain mathematical knowledge and are unable to apply previous knowledge to a new problem. Mathematical modelling is considered as a learning situation whereby students investigate through mathematics, situations from other areas. Hence, the significance of incorporating conditions from real life and other areas of knowledge in the classroom is to enabling students to interfere in this reality. In recent time, the relevance of mathematics in the society seems to be shifting. The more that technology impacts and influences our daily lives, the less mathematics is visible (Iverson, 2006; Noss, 2001; 2001; Skovsmose, 2005). While mathematicians, scientists, and engineers recognize that technological advances require a deep understanding of mathematics (Tate \& Malancharuvil-Berkes, 2006), societally, we do not explicitly "see" the mathematics nor do we perceive when mathematics is used on a daily basis (Empson, 2002; Gainsburg, 2006; Mudaly, 2007).

Mathematics educators have argued that the separation of content from application has adversely affected the learning of mathematics. Hiebert (1996) stressed that the learner learn facts and procedures but with few ties or connections to the context and application of knowledge. Mathematical modelling is not however a distinct method or tool for solving a particular mathematical problem, rather it is a process that permeates all topics in mathematics, sciences, social-sciences, etc. Hence, The American National Council of Teachers of Mathematics argued in 1989 that mathematics learners of all ages lacked the necessary basic literacy skills as well as higher order thinking skills and therefore made recommendations that students be educated to meet work place needs. This took effect in 1991 when students began to learn content while solving realist problems using mathematical modelling.

The use of computer, calculators and other modern technological gadgets are used to help students envision, discover, and manipulate a variety of mathematical ideas and symbol. But there are very few discussions about helping students identify the significant mathematics that triggers the design, development, and maintenance of the technological supports they are using. Thus, it becomes challenging to explain to students the mathematics behind the inventions, advances, and resourceful technological gadget they are using (Hastings, 2007). In addition, even when mathematics is explicitly used professionally or vocationally, the
mathematics taught in the K-8 classroom ("school math") often does not mirror the math used in occupational practices (Gerofsky, 2006; Masingila, 1996; Shockey, 2006; Tate \& Malancharuvil- Berkes, 2006). Students who are taught exclusively from traditional, teachercentred and textbook problems usually fail to understand the relevance of what they are learning. Most times, students form opinion of what mathematics is all about based on the tasks they are given. This perhaps clarifies why most students view mathematics as a set of rules or procedures on how to manipulate figures and move symbols around (Hiebert, 1997).

Several scholars have advanced various reasons as to the question of why modelling in the learning process of mathematics. According to Blomhøj (2004), mathematical modelling bridges the gap between students' real life experiences and mathematics. It motivates the students' learning of mathematics, gives direct cognitive support for the students' conceptions, and it places mathematics in the culture as a means for describing and understanding real life situations. Also, Jablonka (2003), opined that bringing to the classroom the mathematics used in working environment is one of the ways for associating the out of school mathematics with the curricular contents and, consequently, of showing the mathematical practical utility. The general purpose of building and making use of a model is to understand or tackle problems in some segment of the real world (Niss et al, 2007).

Using mathematical procedures within the context of real life activities allowed students to view mathematical processes as tools rather than the end result of their knowledge, thereby making them more likely to be able to adapt and use the procedures in other circumstances (Boaler, 1998). Several mathematics educators have claimed that students are unable to use the mathematical methods and rules that they learnt in school because they do not actually understand them. This lack of understanding could be linked to the way the students learn mathematics. Schoenfeld (1985) is of the opinion that the learning process that emphasised and concentrate on standard textbook questions encourage the development of procedural knowledge that is of limited use in non-school circumstances.

This and other related assertions have contributed to the increasing support for process-based or real life related mathematics. Campaigners of real life related mathematics are of the opinion that students should be given task that are connected to practical and real life as this will prompt the students to make their own decisions and draw the connection between mathematics and real life. In order to achieve this, teachers must be able to organise teaching activities that can raise the development of modelling ability in many educational frameworks. A modelling perspective to mathematical problem solving emphasizes the students' representational confidence with the use of mathematical concepts where the students have to make mathematical explanations of the problem context and data. When students translate, explain, draw diagrams, categorise, find relationships and make predictions, they are generally developing their conceptual systems or models through the mathematizing.

As they work with the rich contextual data, they would need to surface and communicate their mathematical ideas to clarify their thoughts and weigh the validity of their ideas. In other word, when students engage in model-stimulating activities, their "(internal) conceptual systems are continually being projected into the (external) world" (Lesh \& Doerr, 2003, p. 11). Thus, they make visible their sense-making systems of mathematical reasoning. As explanatory phases take place within the modelling process, numerous mathematical explanations of students are stimulated within each modelling stage. From this viewpoint, the modelling process is a non-trivial and thought-revealing problem-solving process. During the
last couple of decades, the introduction of mathematical modelling and applications are probably together with the introduction of information technology, the most prominent common features in mathematics curricula reforms around the world (Kaiser, Blomhøj \& Sriraman, 2006, p. 82). Mathematics can be used to "model", or represent, how the real world works.

Let us consider example 1 below (adapted from Cheng, 2001)
Example 1: The linear function (or graph of $y=m x+c$ )
A common way of introducing the linear function is to use the graph of a straight line. One would normally state that the graph of the linear function $y=m x+c$ is a straight line with gradient m and y -intercept c . This context-free way of learning is efficient and neat. However, it may be more interesting to see how such a graph and function can actually arise from a real practical situation. Consider the following situation where water flows from a tap into a measuring cylinder at a constant rate as showed in figure 2 below. Suppose we wish to construct a model to show how the water level changes with time so that we can predict how long it would take to fill the whole cylinder. The water level at various points in time can be read off the measuring cylinder. The data is recorded in the form of a graph as shown.


Figure 3: Representing rise in water level using a linear function (Cheng, 2001)
From the data, we can now try and guess the relationship between the water level, y, and the time after the tap is turned on, t , assuming that the initial water level is c . It is not hard to see that the water level, y , at any time t should be c plus some positive number, and this positive number should depend on t . Eventually, the model obtained should look something like

$$
\mathrm{y}=\mathrm{c}+\mathrm{kt} .
$$

By modelling this simple physical situation, the linear relationship could "come alive". The linear function is given some context and the graph actually represents something real and physical. Furthermore, the process of modelling would hopefully enable the learner to appreciate other related concepts. For instance, we get a steeper gradient of the graph when the rate of water flowing from the tap is increased. We can also consider another example to further make some revelations of how mathematical modelling can really aid student in the learning process of mathematics.

Example 2: How much space is inside this cardboard box?


> We know three measurements:
> $\mathbf{h}$ (height),
> $\mathbf{w}$ (width), and
> $\mathbf{L}$ (length),
> The formula for the volume of a cuboid is: Volume $=\mathrm{h} \times \mathrm{w} \times 1$

Figure 4: Volume of a box.
This implies that we have a very simple mathematical model of how much space the box in figure contains. Mathematical models can also be used to forecast future behaviour. For instance, an ice cream company keeps track of how many ice creams get sold on different days. By comparing this to the weather on each day they can make a mathematical model of sales versus weather. They can then predict future sales based on the weather forecast, and decide how many ice creams they need to make ahead of time.

There are several situations in which mathematical models can be used very useful in introductory education. Mathematical models can help students understand and explore the meaning of equations or practical connections. After developing a theoretical model of a physical system it is expected that we develop a mathematical model that will permit us to assess the numerical comportment of the system. Measureable results from mathematical models can easily be equated with observational data in order to ascertain the strengths and weaknesses of a model. Support for educational research involving modelling promises to continue (Niss et al, 2007) and might solve many significant questions associated with student learning of mathematics. In recent times, international research communities presented studies of modelling approaches in mathematics classrooms on a global scale and emphasised their impact on learning mathematics (Blum et al, 2007; Matos et al, 2001). In order to help students achieve learning objectives, teachers should be able to build and make stronger links between more formal, abstract mathematical concepts and real-world situations where mathematics plays a more applied role.

### 3.3 The mathematical modelling process

The process of modelling usually includes a multi-step process: Formulating the problem, building the mathematical model, processing the mathematics, interpreting the conclusions, and often revising the model before writing a report. The modelling process begins with the conceptualisation of some problem situations. This is followed by structuring, simplifying, and making this circumstance more specific. If suitable or applicable, data are gathered to offer detailed information on the problem of interest. These data often put forward the type of mathematical model that is suitable to handle the stated real-world problem. Through a process of Mathematization, the appropriate objects, relations, conditions, and assumptions
from the extra mathematical domain are then deciphered into mathematics, resulting in a mathematical model through which we address the identified problem.

Mathematical procedures are used to develop mathematical results related to problems arising from the interpretation of the real world problems. Such procedures consist of coherent inference from mathematical assumptions, application of theoretical results within mathematical topics, solving equations, execution symbolic operation or numerical calculations, assessing constraints, performing statistical testing, etc. The consequent mathematical outcome needs to be deciphered back into the extra-mathematical purview within which the original problem was found. The problem solver then validates the model by testing whether interpreted mathematical results are realistic and compatible in terms of the information given in the original problem. This is followed by model evaluation simply by checking if the solution is appropriate and purposeful. If there is an unacceptable or inadequate result from the test, the entire process is then repeated using a more improved and adapted procedure. However if the outcome is acceptable, the solution of the original real world is stated.


Figure 5: Mathematical Modelling Process (Blomhøj \& Jensen, 2003, p. 125).

A slightly different and perhaps simpler but somehow similar model in terms of ideology and interpretation is the one given by (Cheng, 2001) as depicted below.


Figure 6: Modelling Process (adapted from Ang, 2006, p. 64).
It is pertinent to mention that the above diagrammatic illustrations are by no means exhaustive, rather, they are grossly basic and simplified modelling processes. However, it is sufficient to observe that in mathematical modelling, the starting point is a real world problem or situation. It is obvious that in mathematical modelling, the emphasis is in solving a problem rather than looking for an answer that must exist.

Also, Giordano and Weir (1985) suggested the following six steps in the construction of a mathematical model:

- Identify the problem.
- Make assumptions;

Identify and classify the variables.
Determine interrelationships between the variables and sub models.

- Solve the model.
- Verify the model;

Does it address the problems?
Does it make common sense?
Test it with real-world data.

- Implement the model.
- Maintain the model.

Thus, when we approach the learning of mathematics through mathematical modelling, we are actually learning mathematical problem solving. We present mathematics in action, instead of as a confusing set of formulae scribbled on the chalkboard. We place mathematics in some context and focus on why mathematics exists in the first place. Moreover, many challenging and exciting skills are used in developing models and these have often been ignored in traditional school mathematics (Abrams, 2001).

### 3.4 Modelling process in the classroom

Although mathematical modelling and its applications are not completely new topics for mathematics instruction, different authors have defended its importance and the need for the topics to be integrated into the activities of a mathematics curriculum (Blomhøj, 2008). Niss (1989) suggested that the integration of mathematical modelling activities in the classroom may assist students in working together as a group in the class, thus increasing the applicability of classroom theories and their practical use.

According to Crouch and Haines (2004), many students usually experience difficulties in moving between the classroom mathematics and the real world mathematics. This is further corroborated by Galbraith and Stillman (2006) when they say that the transition from a real world problem statement to a mathematical model is one of the most difficult parts of the modelling cycle. The principal opinion from the educational perspective is to incorporate modelling in the learning process of mathematics. According to Kaiser, Blomhøj and Sriraman (2006, p. 82), the introduction of mathematical modelling and applications is perhaps together with the introduction of information technology the most prominent common features in mathematics curricula reforms around the world in recent time. In many Western nations, curricula reforms especially at secondary level have underscored mathematical modelling as a vital element in mathematics curriculum.

Mathematical modelling can be understood as the utilization of mathematics to resolve real problems. When applied in the classroom, this approach takes on special forms, depending on the educational context, the professionals involved, and the profile of the students, among other factors (Jussara, 2008). Bringing to the classroom the mathematics used in the working environment is one of the ways for associating the out of school mathematics with the curricular contents and, consequently, of showing the mathematical practical utility. But it is very difficult to relate the mathematical methods or procedures that are regularly used in work place to the classroom curriculum mathematics. Moreover, in most case, the linking of
classroom mathematics with the real-life requires great deal of effort and commitment on the part of the students (Jablonka, 2003). The process may also require sufficient time for additional research work and tasks outside the classroom.

Kline (1992) and Markarian (2000) stated that historical proofs from the earliest evolutions suggest that mathematics has been used to describe vital mankind phenomena, particularly between 1500 and 1770. Undeniably, the current applied mathematical situations have had an accelerated growth in all areas of knowledge. Therefore it becomes imperative to handle mathematical concepts relating to daily life in order to understand different social phenomena (Gómez, 1998; Aravena, 2001) such as the mathematics used in buying and selling, barcoding, etc. Skovsmose (1994, p. 52), went on to say that "mathematics has not only created ways of describing and handling problems, but it also becomes a main source for the reconstruction of reality."

### 3.5 Mathematical modelling in Nigerian secondary school curriculum

The Nigerian secondary school curriculum has witnessed many reforms both during the British colonial era and since independence in 1960. According to Gbamanja (1997), education has been regarded as the tool for achieving desirable change, and the curriculum as the instrument for the delivery of educational goals and objectives. Orji (2004) recognized the necessity for an appropriate and functional curriculum for the achievement of meaningful and sustainable development. He noted that a functional education is crucial for human capacity development, wealth creation, employment generation and value re-orientation. The Nigerian Educational System has experienced series of reforms tailored towards achieving national and global developmental goals. The National Policy on Education (Federal Republic of Nigeria, 2004) stipulated the aims/goals of education as building: a united, strong and self-reliant nation; a great and dynamic economy; a just and egalitarian society; a land of bright and full opportunities for all citizens, and; a free and democratic society.

However, these objectives cannot be achieved if the right curriculum is not in place. The new national mathematics curriculum for primary and junior secondary schools in Nigeria is focused on giving the children the opportunity to acquire mathematical literacy to function in an information age and cultivate understanding of the skills necessary for the changing technical world (Cecilia et al, 2013). According to Ekwueme and Meremukwu (2010), the current curriculum tries to make mathematics more of real life than abstract concept. This means that attempts are being made to introduce the concept of modelling into the new curriculum. However, despite the fact that the new mathematics curriculum in Nigeria secondary school seems to have some inbuilt elements of mathematical modelling, no such modelling activities or problems could be found in the various textbooks that are mainly used in the teaching and learning process at this level of education.

Ali (1986) highlighted that there exist problems in the learning of mathematics even to the teachers of mathematics in our secondary schools. Supporting this fact, Obodo (1993) stated that mathematics teachers at all levels of education find it difficult and uninteresting to teach for so many reasons. The reason given by the chief examiner of the West African Examination Council (WAEC) in his report for year 2001 was that teaching method contributed greatly to students' poor performances in mathematics examinations. The methods usually adopted by teachers seem not to make students problem solvers. The problem of poor quality of teaching method in mathematics can be minimized if teachers are well equipped with various types of problem-solving models. In recent years, research on students' achievement in mathematics has been the concern of many researchers in Nigeria.

Ogunkunle (2007) referred to researchers like Lassa (1994), Ojo (1986) and Steen (2003) as having documented students' poor achievement in mathematics. Outside the country, the concern is the same as indicated by Oakes (1990), Leder (1992), and Fennema and Hart (1994).

## 4 Research methods

In this chapter, I will look at the meaning/definitions of ethnography in section 4.1. This will be closely followed by a look at observation as a research method in section 4.2. In section 4.3, I will elaborate on the reasons for using observation as a means of data collection in research. Furthermore, in section 4.4, I will examine the interview as another tool or method in research. Section 4.5 will talk about how to use the interview method in research. In section 4.6, I will discuss the different types of interviews. Fieldnotes which is an essential part of any field work will be discussed in section 4.7. This chapter will be concluded with section 4.8 were I will looked at the procedure that I followed in the course of this research work.

### 4.1 Ethnography (micro-ethnography)

Ethnography is a social science research method. It relies heavily on up-close, personal experience and possible participation, not just observation, by researchers trained in the art of ethnography. It is a highly useful method for addressing a range of research questions. In particular, it can generate rich and detailed accounts of clinicians' professional and inter professional relationships. Understanding the foundations of ethnography and its key elements may assist readers when they come across reports that use this method, although ethnography has its roots planted in the fields of anthropology and sociology. Present-day practitioners conduct ethnography in organizations and communities of all kinds. Ethnographers study schooling, public health, rural and urban development, consumers and consumer goods, as well as other human arenas. While particularly suited to exploratory research, ethnography draws on a wide range of both qualitative and quantitative methodologies, moving from "learning" to "testing" (Agar, 1996).

Although the use of ethnography in research has some limitations like long period of observation and difficulty with the report writing, I have decide to use ethnographic method in this study because of the inherent superseding advantages like:

- It will enable me to capture the emotional behaviour of the students within different contexts.
- It will enable me to identify any inconsistencies between what the participant says and what they actually do.
- It will provide me with a rich source of visual data.
- It will provide me with extensive and in-depth findings about the participant.
- It will enable to possibly develop and explore new lines of inquiry since ethnography. relies on observation rather than some predetermined tests.

When used as a method, ethnography typically refers to fieldwork (alternatively, participantobservation) conducted by a single investigator who 'lives with and lives like' those who are studied, usually for a year or more (Van Maanen, 1996). Ethnography is a written description of a particular culture - the customs, beliefs, and behaviour - based on information collected through fieldwork (Marvin \& Orna, 2000). David Fetterman, (1998) opined that ethnography is the art and science of describing a group or culture. The description may be of a small tribal group in an exotic land or a classroom in middle-class. Fine (2003, p. 25) uses the term "peopled ethnography" to describe text that provides an understanding of the setting and that describes theoretical implications through the use of vignettes, based on field notes from observations, interviews, and products of the group members. He suggests that ethnography is most effective when one observes the group being studied in settings that enable him/her to "explore the organized routines of behaviour".

These ethnographers often work in multidisciplinary teams. The ethnographic focal point may include intensive language and culture learning, intensive study of a single field or domain, and a blend of historical, observational, and interview methods. Typical ethnographic research employs three kinds of data collection: interviews, observation, and fieldnotes. Ethnographic methods can give shape to new constructs or paradigms, and new variables, for further empirical testing in the field or through traditional, quantitative social science methods. Ethnography enhances and widens top down views and enriches the inquiry process, taps both bottom-up insights and perspectives of powerful policy-makers "at the top," and generates new analytic insights by engaging in interactive, team exploration of often subtle arenas of human difference and similarity. Through such findings ethnographers may inform others of their findings with an attempt to derive, for example, policy decisions or instructional innovations from such analyses.

Ethnographers take a detailed look at what is going on in a social setting. This is essential in educational research when studying complicated places like classrooms or trying to understand situations with many influences. Researchers tend to talk of 'ethnographic approaches', as there isn't a single method. An ethnographer in the classroom, for example, is likely to combine detailed observation over time with in-depth interviews; teachers and learners may also be involved in photography or by keeping field notes of their learning as part of the research. A central aspect of ethnography is that it is interested in participants' perspectives - What do learners think is going on? The ethnographer is also interested in the big picture of how activities fit into people's whole lives. Thus, he tries to ascertain the reasons behind why people choose to participate or not participate in formal learning activities, and whether or not they make an effort to do so. Such issues may only become evident over time.

Ethnography deals with the participant/observer/researcher/ethnographer immersing himself in a group for an extended period of time, observing behavior, listening to what is said in conversation between others and with the fieldworkers and asking questions as well as gathering further data through interviews and the collection of documents (Bryman, 2012 p . 432). However, because ethnography usually entails a long period of time in the field, it is possible to carry out a micro-ethnography which will focus on a relatively short period of time say couple of weeks to a few months (Wolcott, 1990, p. 90). In my own term, I described this as \{small/mini ethnography\}. Writing up ethnography is like narrating a story, except that the writer must clearly indicate the procedures employed in the construction of the account and must also provide evidence for the conclusions reached.

These evidence needs to be presented in a way which goes beyond ordinary illustration of the points made. The reader has to be given access to material that allows him/her to check the interpretations used by the observer. This may involve the presentation of substantial amount of field notes. In this study, other research techniques/materials that I will adopt are; (a) Structured interviews (b) Field notes (c) Recorded conversation during the lesson, test and interview (d) Lesson materials.

### 4.2 Observation

Observation is a method for systematically observing the behaviour of individuals in terms of a schedule of categories. It is a technique in which the researcher employs explicitly formulated rules for the observation and recording of behaviour (Bryman, 2012, p. 270). Other scholars have come up with some definitions of observation. Marshall and Rossman (1989, p. 32) define observation as "the systematic description of events, behaviours, and artefacts in the social setting chosen for study". Observations enable the researcher to describe existing situations using the five senses, providing a "written photograph" of the situation under study (Erlandson, Harris, Skipper, \& Allen, 1993, p. 64). Demunck and Sobo (1998, p. 14) described observation as the primary method used by anthropologists doing fieldwork. Fieldwork involves "active looking, improving memory, informal interviewing, writing detailed field notes, and perhaps most importantly, patience". Dewalt and Dewalt (2002, p. 22) said participant observation is the process enabling researchers to learn about the activities of the people under study in the natural setting through observing and participating in those activities.

Despite the fact that observation may be predisposed to the observer biasness, thereby undermining the reliability and thus the validity of the research data, I still decided to adopt observation in this research because it will enable me to get direct access to the phenomena under consideration. Observation will also, assist me to rely on first-hand information instead of depending on some kind of self-report. Thus avoiding all the problems associated with selfreport.

Fine (2003, p. 41), in part, defines "peopled ethnography" as being based on extensive observation in the field, a labour-intensive activity that sometimes lasts for years. In this description of the observation process, one is expected to become a part of the group being studied to the extent that the members themselves include the observer in the activity and turn to the observer for information about how the group is operating. He also indicates that it is at this point, when members begin to ask the observer questions about the group and when they begin to include the observer in the "gossip," that it is time to leave the field. This process of becoming a part of the community while observing their behaviours and activities is called participant observation.

We may wish to find out if teachers consistently favoured male or female students with more questions, more positive responses more attention or whatever. We could put some information together in a fairly straight forward way by counting aspects of teacher-student interactions and separating between male and female students. Perhaps, we could then intercede by training teachers to interact in a non-gender way and then go back to observe their teaching behaviours and similarly observe the interactions with a view to seeing if the change has been successful. This kind of observation can be based on a simple checklist, with the observer ticking against the name of each student whenever the teacher directly interact with him or her and then further indicating the kind of interaction as to whether it was a commendation, scolding, etc. This may help the observer to check the percentage positive or negative interactions for male and female students.

Some researchers may desire to depend on recording what is happening in the classroom by using videos, still photography, tape recorders, selective field notes or memory, or a combination of one or more of these. The trouble with all these methods is that the problem of what to do with all the data so collected is far more pressing than in the case of the structured observation schedules where the task is simply one of computation and statistical analysis.

One example of a structured observation schedule for classroom research is the Flanders Interaction Analysis Category System (FIAC) see Flanders, N, 1970, Analyzing Teaching Behaviour. This was used in an adapted fashion by the Oracle researchers looking at primary classrooms in the UK (Galton \& Simon 1980, p. 82).

Observation technique is a very important aspect of many research studies. Directly or indirectly, we all practice the arts of observation either in our personal or professional lives. Observation method has been used in a variety of disciplines as a tool for collecting data about people, processes, and cultures in qualitative research. We sometimes come to conclusions merely based on our observations and perhaps generate explanations, understanding and even predictions. Observation, for many years, has been a symbol of both anthropological and sociological studies. In recent years, the field of mathematics education has seen an increase in the number of qualitative studies that include observation as a way of collecting information.

Qualitative methods of data collection, such as interviewing, observation, and document analysis, have been included under the umbrella term of "ethnographic methods" in recent years. It is useful for us to improve on these skills and to further enhance that aspect of common sense that is of benefit. However, in research we need to go beyond the subjective and impressionistic. We need to know of and possibly eliminate unfairness. We need to be systematic and open about our procedures so that this can be publicly scrutinised so as to enable other readers of our work to check the premise on which we arrived at our conclusions. One might be interested in knowing what specific contribution observation can make to research in mathematics education. Let us look at some observation technique and at the contexts within which they are most applicable. Wolcott (1981) proposes four strategies for deciding what to look at and how to look:

- Observations by broad sweep.
- Observations of nothing in particular.
- Searching for paradoxes.
- Searching for the problem(s) facing the group.

The broad sweep approach usually has two outcomes: first, it makes the researcher aware of the need for selectivity; second, it makes the researcher aware of what really matters to him/her. The observations of nothing in particular approach is based on 'wait and see what jumps out' and is likened to watching for a blip on a radar screen that indicates unusual activity (You will know it when you see it!) The two 'searching' approaches (paradoxes and problems) are useful for fighting familiarity. Example of a paradoxical discovery is a study that shows that pupils drew more during ordinary lessons than they did during art lessons. Wolcott also mentions a fifth approach in which the researcher is trained in the techniques used by participants as a way to better understand events and knowing how to look. The issue of choosing where and when to look is a matter of systematic, principle, reflexive decisionmaking.

We may observe large groups of students with relatively freedom to mingle within broad categories (e.g. assembly; lunch sittings; communal singing). Another area is where small groups of students are in close proximity like classroom, corridors and staff rooms. The researcher should not neglect the playground and staff room. Finally, in terms of what to record, I am of the opinion that recording during observation should be carried out as modestly as possible, ideally noting verbatim speech or at least some key words/phrases that
will serve to jog the memory later. Nevertheless, ten minutes of good observation, well written up is worth an hour's notes lying forgotten in an unopened notebook. It is essential to constantly analyse and interpret data; or it may become so complex, convoluted and confusing that it fails to serve any purpose.

### 4.3 Why use observation to collect data?

Dewalt and Dewalt (2002, p. 72) believe that "the goal for design of research using observation as a method is to develop a holistic understanding of the phenomena under study that is as objective and accurate as possible given the limitations of the method. They suggest that participant observation be used as a way to increase the validity of the study, as observations may help the researcher have a better understanding of the context and phenomenon under study. Validity is stronger with the use of additional strategies used with observation, such as interviewing, document analysis, or surveys, questionnaires, or other more quantitative methods. When designing a research study and determining whether to use observation as a data collection method, one must consider the types of questions guiding the study, the site under study, what opportunities are available at the site for observation, the representativeness of the participants of the population at that site, and the strategies to be used to record and analyze the data (Dewalt \& Dewalt, 2002).

Participant observation is a beginning step in ethnographic studies. Schensul and Lecompte (1999, p. 46) list the following reasons for using observation in research:

- to identify and guide relationships with informants.
- to help the researcher get the feel for how things are organized and prioritized, how people interrelate, and what are the cultural parameters.
- to show the researcher what the cultural members deem to be important in manners, leadership, politics, social interaction, and taboos.
- to help the researcher become known to the cultural members, thereby easing facilitation of the research process and
- to provide the researcher with a source of questions to be addressed with participants.

Bernard (1994, p. 24) lists five reasons for including participant observation in cultural studies, all of which increase the study's validity:

- It makes it possible to collect different types of data. Being on site over a period of time familiarizes the researcher to the community, thereby facilitating involvement in sensitive activities to which he/she generally would not be invited.
- It reduces the incidence of "reactivity" or people acting in a certain way when they are aware of being observed.
- It helps the researcher to develop questions that make sense in the native language or are culturally relevant.
- It gives the researcher a better understanding of what is happening in the culture and lends credence to one's interpretations of the observation. Participant observation also enables the researcher to collect both quantitative and qualitative data through surveys and interviews.
- It is sometimes the only way to collect the right data for one's study.

Observation methods are useful to researchers in a variety of ways. They provide researchers with ways of checking for nonverbal expression of feelings, determine who interacts with whom, grasp how participants communicate with each other, and check for how much time is spent on various activities (Schmuck, 1997). Observation allows researchers to check definitions of terms that participants use in interviews by observing events that informants may be unable or unwilling to share. When doing so, the observer should be polite, sensitive, and carefully observe the situations that the informants may have previously have described in interviews, thereby making them aware of any distortions or inaccuracies if any in description provided by those informants (Marshall \& Rossman, 1995). Below are some advantages and disadvantages as inspired by the works of some scholars like Schmuck (1997) and Bryman (2012).

## Advantages of Observation

- It is a straightforward method of gathering data especially when dealing with human behaviour.
- It is a veritable source of getting reliable and accurate data.
- Data collected by means of observation aid accuracy of the research result.
- Observation assist in understanding both verbal and non-verbal responses of the participant more efficiently.
- With the aid of modern technology and gadgets, observation can be done over a very long period of time.
- Observation is not very tedious as the observer does not use so much physical strength in the process.
- Observation helps to easily identify any form of problem with the participants.
- Observation method may help the researcher to explore the effects of many explanatory variables at once.
- It gives the researcher access to the same places, people and events as the subjects.


## Disadvantages of Observation

- Observation does not allow the observer to study past problems.
- It also does not provide the researcher with other source alternatives as the observer has to completely rely of what he can observe.
- It is very difficult if not completely impossible to study attitude by mere observation.
- Observation does not give room for sampling.
- Observation is very time consuming as the observer or researcher has to wait for a very long period if he is to get detailed information.
- Observation alone does not provide complete answer or solution to any problem.


### 4.4 Interview

In the mid-1970s, some pioneer mathematics education researchers based their findings mainly on interview data (Erlwanger, 1973). This started a conversation among researchers on the aims and rationales for interview (Ginsburg, 1981). With the development of qualitative methods, interviewing has become one of the main tools in mathematics education research (Rina \& Orit, 1999). As the first step in analyzing interviews in mathematics education, focus should be on the stages of planning, specifically, on designing the interview questions. Concrete effort should be made to outline several features of interview questions and understand what guides the researchers in choosing the interview questions. One of the reasons why I decided to also include interview as part of my research methods in this study is to promote standardization of both the asking of questions and the recording of answers. This
is in line with Bryman (2012, p. 210). This is not to say that there are no limitations in using interview like the big issue of time consumption especially during transcribing.

Interview is one of a variety of forms of research, but it is the one that is most commonly employed in survey research (Bryman, 2012, p. 209). Interview is a managed verbal exchange (Ritchie \& Lewis, 2003). Therefore its efficacies greatly rely on the communication abilities of the interviewer (Clough \& Nutbrown, 2002). These include the ability to clearly structure questions, listen attentively, pause, probe and encourage the interviewee to talk freely. According to Frey and Oishi, (1995), an interview is a purposeful conversation in which one person asks prepared questions (interviewer) and another answers them (respondent). Although it does not fit all circumstances, interview is a very useful means of inquiry. Interviews are particularly useful for getting the story behind a participant's experiences. The interviewer can pursue in-dept information around the topic. Interview may also be useful as follow-up to certain respondents in order to further investigate their responses (McNamara, 1999).

In the past decade qualitative research methodologies have become not only acceptable but also predominant in mathematics education research. Within these, interviewing students has become a popular way of data-collection (Rina \& Orit, 1999). Researchers in mathematics education ask questions, get answers and then engage in attempts to analyze these answers. What kinds of questions are being asked? How do researchers choose these questions? What considerations influence researchers' choices? It is important to mention that while there are courses and books on the art and technique of interviewing, many mathematics educators, including myself, entered the field of interviewing with little or no specific practical training in this field.

Interviews can be used to collect facts, like information about people's age, gender, height, etc., but such questions are usually no more than opening items which pave the way for the main body. The greater part of interview questions try to find out information about attitudes, opinions, perspectives and meanings. Interviews are also in common use as a means of selection - for entry to school or college, getting a job or obtaining promotion. They are broadly used because they are a dominant means of both getting information and gaining insights.

Interviews are available in a range of styles, some of which are pre-packed such that they can be more or less picked off the shelf. Social scientists make similar use of tightly controlled pre-set interviews which have been conducted on sample groups to test their efficiency and accuracy before being tried out on larger populations. These structured interviews in their simplest form are sometimes like oral questionnaires used instead of the written form in order to obtain a higher response rate. At the opposite extreme in interview design are completely unstructured conversations between researcher and respondent, where the latter has as much influence over the course of the interview as the former. There is a half-way house, where the researcher designs a set of key questions to be raised before the interview takes place, but builds in reasonable flexibility about how and when these issues are raised and allows for a considerable amount of additional topics to be built in in response to the dynamics of conversational exchange. These are known as semi-structured interviews. This form is also widely used in education research.

### 4.5 How to use interview in mathematics education research

Interview is unquestionably the most dominant source of data in qualitative research. The one-on-one type of interview is most widespread, but sometimes group interviews and focus groups are also carried out. In most cases, the interviewer asks all the participants (interviewees) the same or very similar questions, but the order of the questions, the exact wording, and the type of follow-up questions may differ significantly. Good skills and experience are required to be a good interviewer as it is believed that skilful interviewing takes practice. It is very imperative that the researcher should first create a relationship with the interviewees because if they do not trust the interviewer, they will not open up and give their true state of mind, opinions, and intents. A far-reaching relationship is established over time as people get to know and trust one another. An important skill in interviewing is being able to ask questions in such a way that the interviewee have confidence that he or she can talk without restrictions. Some of the ways to improve interviewing proficiency include:

- videotaping your own performance during interview session,
- observing experienced interviewers, and
- assessing peers.

It is imperative that the researcher is open-minded, although this can sometimes be challenging in circumstances where the interviewee's opinions are reasonably at variance from those of the interviewer. The interviewer should be attentive to both spoken and nonverbal communications as well as be flexible in restating and following certain lines of questioning. The interviewer should use words that are unambiguous and meaningful to the interviewee and should be able to ask questions that can easily be understood by the interviewee. Above all, the interviewer has to be a good listener.

The use of a digital recording device is unquestionably the most common technique of recording interview data because it has the advantage of keeping the entire oral part of the interview for future analysis. Experience has showed that some respondents may be uncomfortable to talk while being recorded, this uneasiness usually fizzle out within a short period. However, one of the greatest disadvantages of recording interview is the problem associated with the malfunctioning of the recording device and sometimes some recording device may be very difficult and complex to operate. These problems can be very annoying, disappointing and frustrating if it occurs in the course of the interview, but it is more demoralizing if it happens subsequently when you are trying to replay and analyze the interview. It is therefore recommended that the interviewer should ensure that the recording device is in good condition and that he or she also understands the operational procedure of the device before embarking on the recording session.

In my attempt to review how to use interview in mathematics education research, I ask myself the following questions:
(a) Why should I use interviews rather than some other device for data collection? An overview of some of the underlisted advantages (strength) of interview prompted me to adopt for interview method in this research work.

- Flexibility - making possible changes in the order of questioning, the questions asked and the topics discussed.
- Probing - follow-up questioning seeking clarification or further explanation.
- Allowing for long and complex responses.
- Allowing for in-depth inquiry.
- Giving the interviewee the opportunity to check what is meant by a question.
- Allowing the researcher to probe the meanings interviewees give to their behaviour, ascertaining their motives and intentions.
- Giving the interviewee the chance to challenge the agenda set by the researcher, raising new issues and asking questions back.
(b) What questions should I ask?

When talking about whom to interview, like in every other methods of research, many participants as possible should be involved. But because of the time consuming nature of interview, the researcher may decide to select a particular group (observed group) in order to achieve the overall objective of the research. In many cases, the researcher may be interested in forming a group that is composed of participants that can give reliable responses to the intending questions. Unfortunately, this might not be the case in some cases as the researcher may have no choice other than to work with the people that are available (opportunity sample) (Woods, 2006). It is important for the researcher to indicate whether the observed group is opportunity sample or otherwise. In my study, I was only privileged to have an opportunity sample as I could not determine who should be part of the group and should not be part.

In looking at how to get access to the participants, two questions readily come to mind. The first is for the researcher to ask himself how he will establish a link between himself and the anticipated group. Secondly, the researcher must also consider if the anticipated group can provide the desired response with a view to achieving the overall object of the study. It is important to remember that irrespective of whatever group the researcher is dealing with, he must assure them of confidentiality, anonymity and even protection against prosecution. This will make the participant happy and more willing to give unbiased responses to the questions. It may help if the researcher gives a clear and precise description of the plan outline of the entire interview process to the group before the beginning of the interview session and also seek their willingness to participate in the interview (Wolcott, 1981).

After having identified the group to work with, what to ask them and how to access this group, the researcher may again be confronted with the issue of relating to the method or technique that should be adopted in order to achieve the best and most objective answers from the interviewees. As a basic way of surmounting this question, Woods (2006, p. 6), gave some suggestions especially in an unstructured interview situation and perhaps also in a semistructured interview situation. Woods suggested that the researcher should check for contradictions, imbalance, exaggerations or inconsistencies on the part of the interviewees. While asking for explanations is considered vital, the researcher will also need to search for the opinions of the respondents. Wood added that the researcher should try as much as possible to be a good and active listener, thereby showing the interviewee that great attention is given to what he is saying. This is to get the interviewee to focus on the subject matter. The researcher should as much as possible develop empathy with interviewees so as to win their confidence.

Interviews are generally easier for the respondents, especially if what are sought are opinions or impressions. Interviews are time consuming and can as well be resource intensive. The interviewer is considered a part of the measurement instrument. In the process of conducting interviews, it is proper for us to put in place certain conditions like: Explain the purpose of the interview to the interviewee, clarifying the layout of the interview, choosing a location with
little or no disturbance, addressing the terms of confidentiality, specifying how long the interview will take, permitting the interviewee to clear up any reservation about the interview, and organizing a method for recording the interview. e.g., take notes or tape recorder.

In this study, I decide to use interview as part of my data collection method because:

1. It provides the opportunity to generate rich data.
2. Language use by participants (interviewer and interviewees) was considered essential in obtaining the perception and values of the interviewees.
3. Background and interactive aspects were seen as important to understanding interviewee's opinions.
4. Data generated can be analyzed in different ways.
5. Interview promotes the standardization of both the asking of questions and the recording of answers.
(Bryman, 2012, p. 210).

### 4.6 Types of interview

There are different types of interview techniques that can be employed in any research study, like telephone interview, face-to-face interview, panel interview, group interview, etc. But within the context of this study, I will consider in broad terms three major types of interviews. These are:

1) Structured interview.
2) Semi-structured interview.
3) Unstructured interview.

In structured interview, there are requirements that must be followed. There are usually predetermined questions that must be strictly followed and the orders of the questions are usually fixed and specific with no opportunity for probing or deviating from the set agenda. The questions are structured such that both the questions and the answers either confirm or disconfirm the set hypothesis of the researcher. One of the major benefits of adopting this type of interview is that it reduces biasness to the barest minimum since the information is precise.

The semi-structured interview is a less rigid and less standardized interview technique. The interviewer would normally use a standardized set of questions which all the interviewees will be asked, but the researcher has the opportunity to explore some other areas outside the set down questions. This allows the interviewee to probe other related areas for relevant information through additional questions which is not possible with the structured interview. One of the benefits of using this type of interview is that it enables the interviewer to obtain larger and deeper data from the participants compared with the structured interview.

An unstructured interview is not ordered or conventionally arranged. This makes the unstructured interview the most relaxed and flexible type of interview. This therefore makes this type of interview to look more like a conversation rather than an interview session. Under this condition, the interviewees may be at liberty to talk about their personal biography in their own way although the interviewer can saliently put some guidelines in place so as to avoid total deviation from the focus research area.

According to Johnson (2001, p. 32), the subject matter is personal, intimate and emotional, and the objective is to achieve some kind of deep disclosure. Also Gubrium and Holstein (2001, p. 14) say that the emphasis in unstructured interview is to acquire deep knowledge and authenticity of people's life experiences. The unstructured interview is used when the interviewer wish to gather as much information as possible about the research topic. This may help to uncover vital information that may not be possible to get if the interviewer uses either the structured interview or the semi- structured interview.

However, while it is very difficult to say that one form of interview is better than the other, the researcher should therefore carefully consider his or her choice in line with the research question so as to achieve the overall goal of the whole exercise. In my study, I conducted a semi structured interview in order for me to possibly reveal the observed students' opinions regarding mathematics and mathematical modelling activities.

There are some advantages that may be associated with the use of interview as a method of collecting data during research. The interviewer may have the opportunity of getting data about complex and emotional issues. Also, interview gives the possibility of high level of adaptability as the interviewer can easily adapt the interview to suit the interviewee. In addition, interview gives the interviewee the opportunity to ask the meaning of a particular question.

However, interview method requires the interviewer to be highly skilled in the act of interviewing. Furthermore, interview has the disadvantage of non- standardization as it is very easy for the same question to be asked in different way at different time. This may lead to different response from different interviewee.

### 4.7 Fieldnotes

Although little attention has been paid to the practical details of note-taking, fieldnotes are however very important aspect of ethnographic research. Ethnographers must learn how to take useful and reliable notes regarding the details of their research. These fieldnotes will undoubtedly constitute a vital portion of the data on which later conclusions will be based. One might therefore be interested in knowing exactly how an ethnographer decides what to write about. The techniques a researcher adopt during fieldnotes taking in an observational study is very much of a personal resolution that is established over time as one becomes more and more experienced in note taking and observation.

Every field researcher adopts various techniques writing field notes. This is mainly according to whatever tactics that works best for any individual. For example one researcher may use brackets to indicate personal feelings and reflections on bits of data, while another field researcher may use the "comments" function colour to differentiate observations from reflections. Others might create two columns for their full field notes-one containing notes only about what was observed directly and the other encompassing reactions and impressions. It may therefore be tempting to conclude that there is no wrong way of writing field notes. The important message is that the researcher should adopt a technique that allows him or her to write accurately, as much detail as possible and to be able to differentiate between observations and reflections.

Chiseri-Strater and Sunstein (1997, p. 11) developed a list of what may be included in all fieldnotes:

1) Date, time, and place of observation.
2) Specific facts, numbers, details of what happens at the site.
3) Sensory impressions: sights, sounds, textures, smells, taste.
4) Personal responses to the fact of recording fieldnotes.
5) Specific words, phrases, summaries of conversations, and insider language.
6) Questions about people or behaviours at the site for future investigation.
7) Page numbers to help keep observations in order.

However, all field notes are generally made up of two parts:
Descriptive information: Here the researcher tries to precisely document the factual data like date and time as well as actions, behaviours, and conversations.
Reflective information: In this part, the researcher tries to take note of the interviewee's thoughts, ideas, questions, and concerns.

It is further recommended that field notes should be written as soon as possible after an observation is complete because the initial notes may be in ambiguous form and, except such notes are reconsidered as quickly as possible after the observation, vital details that may assist in the full interpretation the data may be lost. Therefore, immediately upon leaving any observation in the field, the researcher should take time to complete the brief notes he or she took while in the field. Even if you feel that the notes you've taken in the field are complete, you may be very surprised by how much more you will be able to remember once you sit down without disturbance and read through what you have jotted down. You will also have the opportunity to add your own reflections, or observations when you write up more complete notes. I may be reasonable to therefore say that one's field notes can never contain too much detail because writing as much as possible may help in avoiding hasty generalizing in your field notes. Also, it may be helpful to have some documentation of your first impressions and of the kind of details that later become so much a part of the everyday section.

### 4.8 Research Procedure

Day 1:
At day 1 of the field work during this research I conducted about 45 minutes interview with a group of 10 students. The essence of this interview was to get the opinion of the students about the current traditional, teacher-centre method of learning mathematics in Nigerian secondary school. The interview questions, students' response as well as the analysis of the interview are fully discussed in chapter 4.

Day 2:
At day 2, I solved two modelling tasks for the same group of students that I worked with in day 1. This session lasted for about 45 minutes. We then had 15 minutes break after which I gave the students another two modelling tasks to solve. The students were allowed to work in groups of two and three. This session lasted for 1 hour 30 minutes. The students activities were audio recorded and their worked solutions were also collected for further analysis.

Day 3: I repeated day 2 processes but with different modelling tasks.
Day 4: I further repeated day 2 processes and again with another set of different tasks.
Day 5: I interview the students again after completing the 3 days sessions. i.e day 2, 3 and 4 . This was to enable me to get the impression of the students about the entire modelling task that we went through in day 2 to day 4 .

As I earlier mentioned, the complete interview questions, analysis of the interview, analysis of the students' conversations during the group work, analysis of the students written answers as well as the analysis of the post interview are all in chapter 4 of the research work.

## 5 Analyses

In this chapter, I will review the analyses of all that happened during the field work. I will begin in 5.1 with the analysis of pre-interviews. This will be followed by the analysis of the group work in section 5.2 and then I will further discuss the post-interview analysis in section 5.3. This chapter will be ended with section 5.4 which is a synthesis of the entire analysis.

### 5.1 Analysis of pre-interviews

The field work aspect of this study started by conducting a pre-interview with the observed group (Ten senior secondary school students) in Nigeria. The essence of this pre-interview was to get the opinion of the students about the current traditional, teacher-centred method of learning mathematics in Nigerian secondary school. For the purpose of information confidentiality, I have decided to use the following acronyms to represent the ten students (observed group) that participated in the study.
Student 1: Ola
Student 2: Mgb
Student 3: Sal
Student 4: Bal
Student 5: Iwu
Student 6: Oye
Student 7: Yak
Student 8: Igb
Student 9: Owk
Student 10: Ekw
In the course of the pre-interview, the following questions were asked:

1. What is your opinion about mathematics?
2. Can you see any connection between the mathematics you learn in school and the outside world?
3. If you answer yes to question 2 above, can you give example(s) of how you use mathematics outside the classroom?
4. What is your opinion about the way(s) you are currently learning mathematics in school?
5. Why are you studying mathematics in school?
6. What do you like most about mathematics?
7. What do you like least about mathematics?
8. What would you tell a new student in class your class about what it takes to be successful in a mathematics class?
9. When and where do you see people using mathematics outside school?
10. How do you think you might use mathematics when you finish your secondary school education?

Below are the responses from five out of the ten students that were interviewed. The reason for the selection of these five students is because their responses also reflect the responses of the remaining five students. The complete version of the pre-interview responses is attached in appendix 2.

Question 1: What is your opinion about mathematics?
Ola: It is a very difficult subject because it requires a lot of thinking, memorizing and procedures.
Mgb: It's difficult to remember formulae.
Sal: I don't enjoy studying mathematics.
Bal: It's a good subject but very difficult to understand and remember.
Iwu: I think mathematics is a good subject although it is difficult.
Question 2: Can you see any connection between the mathematics you learn in school and the outside world?
Ola: No. It is hard to see. For me I only see some arithmetic like the one involving buying and selling like when making payment and either giving or collecting change. But I really do not think that is mathematics. So I can say maybe its primary school arithmetic.
Mgb : Not really but maybe as I heard, people say that engineers use mathematics a lot, but cannot see it.
Sal: No. I cannot.
Bal: No.
Iwu: Yes.
Question 3: If you answer yes to question 2 above, can you give example(s) of how you use mathematics outside the classroom?
Ola: Not applicable since the student's answer to question 2 was "no".
Mgb: Not applicable.
Sal: Not applicable.
Bal: Not applicable.
Iwu: I think architect use part of trigonometry and geometry when they are making their drawings. I also think the computer scientist and engineers are using some of the thing we are learning in school now.

Question 4: What is your opinion about the way(s) you are currently learning mathematics in school?
Ola: I think it's very difficult to learn mathematics because as I said before in my response to question 1 , learning mathematics requires a lot of memorizing and a lot of procedures to be remembered. It's more frustrating when I don't get the answer even after working and trying so hard to solve a particular task. I get demotivated when I don't get the answers to mathematics tasks.
Mgb : Well, I have not had the opportunity of knowing or testing another method of learning mathematics, so I cannot really say I prefer this method or the other one. This is the only method I know of for now. But as I said before, it's a difficult process.
Sal: I don't have choice because this is the only way we are being thought for now. But I would have preferred it's like chemistry or physics whereby we go to laboratory mix chemicals and see the reactions, colour change and others. But mathematics is not like that. So it's really difficult to learn so many things and formulae that you cannot see.
Bal: It's a difficult way but I do not know if there is another way.
Iwu: I really don't know if there is another way I can learn it, but I will prefer if some if not all of the concepts can be but into practical in the laboratory like physics and chemistry.

Question 5: Why are you studying mathematics in school?
Ola: Hmmmm (smile) Sincerely, I am studying mathematics in school because first its compulsory subject in secondary schools. So I cannot avoid it and secondly, it is a prerequisite for me to get into the university to study the programme I wish to study which is engineering.
Mgb: Because it is compulsory.
Sal: Because it's a prerequisite for me to be promoted to the next class.
Bal: Because it's a compulsory subject.
Iwu: Because it's a compulsory subject and I also want to be an engineer.
Question 6: What do you like most about mathematics?
Ola: When I solve tasks and I get the correct answers.
Mgb: When I remember the formula to solve a particular task, apply it to solve the task and I get the right answer, then I like mathematics.
Sal: The formulae when I remember them.
Bal: I don't think I like anything about mathematics. If I have my way I will not do the subject.
Iwu: The fact that everyone always arrive at the same answer.
Question 7: What do you like least about mathematics?
Ola: When I solve and I don't get the correct answer.
Mgb: When I don't get the current answers
Sal: Memorization.
Bal: Memorizing formulae.
Iwu: Long processes and memorization of so many formulae.
Question 8: What would you tell a new student in class your class about what it takes to be successful in a mathematics class?
Ola: I will advise him or her to practice the act of memorizing.
Mgb: Try to learn formulae.
Sal: I will tell him or her to learn how to memorize.
Bal: Mathematics is difficult and he should prepare and ready to memorize a lot of formulae and processes.
Iwu: Mathematics is not difficult if the student can remember all the formulae.
Question 9: When and where do you see people using mathematics outside school?
Ola: I can't really see anywhere. But, as I said before, may be primary school arithmetic in the process of buying and selling.
Mgb: I cannot see it anywhere.
Sal: None.
Bal: Maybe engineers and architects.
Iwu: Various calculations in the banks and also engineers.

Question 10: How do you think you might use mathematics when you finish your secondary school education?
Ola: I really do not know yet although I was told engineers use a lot of mathematics, but I am yet to see that. So I will wait to see when I get into engineering programme in the university. Then maybe I can see some engineering areas that I might be using mathematics.
Mgb : Hmmmmm. I am not sure yet.
Sal: I don't think I will use it because I want to be a lawyer.
Bal: I am not sure I will use it.
Iwu: When I start practicing as an engineer.
Table 6: Summary of pre-interview.

| Questions | Number of <br> students that gave <br> positive response | Number of <br> students that <br> gave negative <br> response | Number of <br> students that <br> are indifferent |
| :--- | :---: | :---: | :---: |
| 1. What is your opinion about <br> mathematics as a subject? | 2 | 8 | 0 |
| 2. Can you see any connection <br> between the mathematics you learn <br> in school and the outside world? | 3 | 7 | 0 |
| 3. If you answer yes to question 2 <br> above, can you give example(s) of <br> how you use mathematics outside <br> the classroom? | 3 | 7 | 0 |
| 4. What is your opinion about the <br> way(s) you are currently learning <br> mathematics in school? | 1 | 8 | 1 |
| 5. Why are you studying <br> mathematics in school? | 1 | 9 | 0 |
| 6. What do you like most about <br> mathematics? | 2 | 8 | 0 |
| 7. What do you like least about <br> mathematics? | 0 | 10 | 0 |
| 8. What would you tell a new <br> student in class your class about <br> what it takes to be successful in a <br> mathematics class? | 2 | 8 | 0 |
| 9. When and where do you see <br> people using mathematics outside <br> school? | 4 | 6 | 0 |
| 10. How do you think you might use <br> mathematics when you finish your <br> secondary school education? | 4 | 5 | 0 |

After a careful analysis of the student's responses and a thorough look at the summary table above, I was able to deduce the following:

Most of the students do not have a positive impression about mathematics as a subject. In fact, the statistics shows precisely that only one out of ten (response to question 4 as seen from table 6) of the observed students have a positive opinion about the way they are currently learning mathematics in Nigerian secondary schools. When I asked the observed students about their opinion with regards to the way they are currently mathematics, one of the students, Ola said "I think it's very difficult to learn mathematics because learning mathematics requires a lot of memorizing and a lot of procedures to be remembered. It's more frustrating when I don't get the answer even after working and trying so hard to solve a particular task. I get demotivated when I don't get the answers to mathematics tasks". Also another student from the observed group, Iwu has this to say "I really don't know if there is another way I can learn it, but I will prefer if some of the mathematical concepts can be but into practical in the laboratory like physics and chemistry. There is therefore urgent need to develop new approaches/methods of learning mathematics in Nigerian secondary schools with a view to changing the opinions of the students from the negative perception they currently have to a positive one.

Also, from table 6 , it is not difficult to see that majority of the students are having a great disconnect between the mathematics they are learning in the classrooms/schools and the mathematics used in real life. The majority claimed that they cannot see any connection between the mathematics they are learning in schools and the outside world. This is reflected in the observed students' responses to question 2 . Table 6 shows that only three out of ten of the observed students were able to manage to draw some connections between the mathematics they are learning in the classroom and the outside world mathematics or real-life mathematics. When I asked the observed students if they can see any connection between the mathematics they are learning in school and the outside world, one of the students, Mgb said "Not really but maybe as I heard, people say that engineers use mathematics a lot, but cannot see it. Therefore, new learning approaches like learning mathematics through modelling tasks and other activities may be introduced and incorporated into the curriculum so as to enable the students to engage in problems and activities that will show them the connections between their classroom mathematics and the real-life mathematics.

In addition, the pre-interview analysis also shows a situation whereby nine out of the ten observed students ( $90 \%$ ) said they are only studying mathematics in secondary school because it is compulsory. They went on to say that they cannot clearly see the need or importance for presently learning mathematics. In the course of the pre-interviews, I asked the observed students why they were studying mathematics in school, one of the students Sal said "Because it's a prerequisite for me to be promoted to the next class". Another student from the same observed group, Bal also corroborated it this way "Because it's a compulsory subject". This indicates that there is strong dislike for the subject by the observed students. However, this may be attributed to some of the reasons the observed students gave such as too many formulae, processes and algorithms to remember in mathematics as well as the fact that they cannot connect the mathematics they are learning in the class with the outside world. Thus, the bottom-line will be for the urgent introduction of modelling problems that can connect what the students are learning in the class with what they can see outside their classrooms.

Finally, while accuracy and precision should be encouraged in our mathematical processes, the emphasis on solving a task that must lead to the correct answer should be reduced. This is based on most of the students complain that they are demotivated to study mathematics whenever they solve any task and they are not able to arrive at the current answer. I tried to
get a reaffirmation of this assertion from the observed students when I asked them what they liked least about mathematics. One of the students, Ola said "When I solve mathematics problems and I don't get the correct answer". Hence, other mathematical activities that may encourage the learning of mathematics that may not necessarily lead to correct or incorrect answers should also be introduced. The other observations that I have, following the outcome of this pre-interview, will be discussed in chapter 6 of this study. This is to enable me to synchronize the pre-interview and the post-interview.

### 5.2 Analysis of group work

During the group work, I presented and went through the solutions to two modelling problems on the board for the students as examples after which I gave the observed students other modelling tasks to work with. The students were allowed to work together in groups of two or three. This process was repeated for three days. Below are the problems that were solved by me for the students as well as the ones they worked with on their own. It should be noted that problems number 1 and 5 were solved for the students as examples.

## Questions

1. Your school authority is considering optimizing the car park space within the school. The parking lots have already been painted and parking lines drawn in vertical side-by-side plan. Now, we wish to check to see if the existing plan has made maximum use of the car park space and if not, we wish to re-design the space to increase the number of lots. Assuming the maximum breadth per car lot in this arrangement is 1.5 m ,
(a) Sketch the existing design alongside a diagonal side-by-side parking plan that allows maximum breadth of 1.4 m car per lot.
(b) Determine the angle to will optimize the number of cars in the diagonal parking plan and hence the total number of cars that this plan can take. (Inspired by Cheng, 2010)
2. Your neighbour, an ornithologist, (a scientist that studies birds) has to leave for the weekend to do a research study. She has asked you to make sure her birdfeeder always has food in it so that the birds keep coming back throughout the day. Humans can't come around too often because it will frighten the birds and refilling too seldom will cause the birds to look elsewhere for food.
How often should you feed the birds so they keep coming back?
3. The Federal Communications Commission (FCC) needs to assign radio frequencies to seven new radio stations located on the grid as shown in the figure below. Such assignments are based on several considerations, including the possibility of creating interference by assigning the same frequency to stations that are too close together. In this simplified situation, we assume that broadcasts from two stations located within 200 miles of each other will create interference if they broadcast on the same frequency, whereas stations more than 200 miles apart can use the same frequency to broadcast without causing interference with each other. How can a vertex-edge graph be used to assign frequencies so that the fewest number of frequencies are used and no stations interfere with each other? What would each vertex represent? What would an edge represent? What is the fewest number of frequencies needed? (Inspired by James, Yuria \& Eric, 2010).
4. Making decisions can sometimes be quite difficult, especially when it's a decision about where you will spend the next couple of years of your life after your graduation from the secondary- we're talking about university, of course!
How can you choose the most suitable university for yourself?
5. A mathematician explained to oil company executives that their problem was very similar to the problem birds face in the winter when some birds need to find up to $40 \%$ of their body weight in food to survive a cold night. Often birds have to make a decision: do I stay where I am, where I know there are a few berries, or do I fly off and hopefully find a new bush with a bonanza of berries? Develop a mathematical model to explain this problem.
(Inspired by Tim, 2010)
6. Your grandmother will be arriving at the airport at 6:00 pm. You live 20 kilometers from the airport. The speed limit is 40 kilometers per hour. When should you leave to get her?
7. Your father is considering replacing his car and the major criteria for the selection of the type of car is fuel economy. Calculate which of the following would save more fuel? (a) Replacing a compact car that gets 34 miles per gallon (MPG) with a hybrid that gets 54 MPG.
(b) Replacing a sport utility vehicle (SUV) that gets 18 MPG with a sedan that gets 28 MPG.
(c) Both changes save the same amount of fuel.
(Adapted from Chang, 2008)
8. What will be the best location of a hospital in a small city, with no traffic?
9. Designing the layout of the kiosks in a school fair so as to raise as much money as possible.
10. The authority of your school is considering the construction of speed bumps within the college campus in order to calm the traffic. Decide the best location for the speed bumps.

Solution to question 1: (One of the examples that I solved for the students).


Figure 7: Illustration of diagonal parking diagram
From the diagram above, the relationship between the three variables, $\mathrm{Y}, \alpha$ and B can be established by the equation:

$$
\sin \alpha=B / Y
$$

## This implies B = Ysin $\alpha$

Since Y is fixed at a breadth of 1.5 m for the vertical side-by-side parking plan and we are told that the breadth for the diagonal side-by-side parking lot is 1.4 m maximum, it is easy for us to conclude that the number of car lots in the diagonal side-by-side will be more than the vertical side-by-side since the breath for the diagonal plan is less than the breadth for the vertical plan. This is of course trial. However, the major task here is to determine the angle that will enable us to get 1.4 m in the diagonal side-by-side parking plan. We can determine this angle by varying the angle so we can see the point that will yield 1.4 m . We can produce a table of values by varying the value of $\alpha$ as follows:

Table 7: Estimated angle to determine diagonal side-by-side- parking.

| A | B (in m) |
| :---: | :---: |
| 10 | 0.26 |
| 15 | 0.39 |
| 20 | 0.51 |
| 25 | 0.63 |
| 30 | 0.75 |
| 35 | 0.86 |
| 40 | 0.96 |
| 45 | 1.06 |
| 50 | 1.15 |
| 55 | 1.23 |
| 60 | 1.30 |
| 65 | 1.36 |
| 67 | 1.38 |
| 68 | 1.39 |
| $\mathbf{6 9}$ | $\mathbf{1 . 4 0}$ |
| 70 | 1.41 |
| 75 | 1.45 |
| 80 | 1.48 |
| $"$ | $"$ |
| $"$ | $"$ |

The information in the table above will be very useful to get the required information about the parking lot arrangement. After several iterations by varying the angle so as to determine the best angle that will yield the optimal result of 1.40 m , we arrived at an angle of 69 degrees. This means that for the diagonal side-by-side parking plan, the angle which is equal to $\alpha$ should be equal to 69 degrees. Only at this angle we can optimize the space using the diagonal side-by-side parking plan. Now, if we know the total distance of the available parking space which is the distance from M to N as showed in figure 7, then we can determine the number of parking lots that can be constructed by either plan. This will be a trivial arithmetic operation. This problem may enable the students to demonstrate their knowledge with respect to trigonometry, geometry, the sin function and the Pythagorean theorem.

At the end of this example, the students were amazed at the high level of correlation between the mathematics they were learning in the classroom and the reality of the various theorems and algorithms in real life. One of the observed students told me that he had never imagined that there is any mathematics involve in the construction of parking lots. Enthusiastically, another student shouted in loud voice saying "this is the type of mathematics we want". The observed students seemed to be motivated by this example to work on the other problems that I later gave them.

After this example, I asked the observed students to work on some modelling problems that I prepared for them. They were allowed to work in groups of two or three. Below two of the answers that were provided by two groups to problems 4 and 7. These ones were selected to
show and interpret some of the successes and difficulties that the students encountered. The complete version of all the students' work is in appendix 4.

Ola, Mgr ain sal
(44) Making decisions Can Sometimes be quite difficult, esperaily when $V^{t}$ 's decision about where you will spend the next couple of yean after your graduation from secondary school we ai e, taking of your the about university, of Carne!
Hour can you choose the most suitable university for yomself?
Answer:
Frost we assume thant the ne are ulnivasities labelled A -I, so we have:


Now we also assume the following suitability criteria
(1) Availability of my desiñed programm
(2) Cost of tirition
(3) Avaitabitity of sehotanship
(4) The rating of the university.

Let us now allocate 2 pants for each of these criterion.


Now: Thy table shows university $A$ am 1 having the highest score of 6 .
Pienefore we wit choose either A orD.

Figure 8: Solution of group 1 (Ola, Mgb and Sal$)$ to problem 4.

Now looking at the work of Ola, Kgb and Sal to problem 4, there are some issues that can be talked about. First, I observed that it was not so much of a big challenge for them to come up with assumptions. Secondly, it was not a very difficult situation for them to come up with a statistical or mathematical table to aid them in their illustrations.

However, one obvious difficulty or challenge that stands out is that they were not able to transit from making assumptions to formulating mathematical equations, thereby hindering their ability to solve any equation. This problem was a common phenomenon amongst the majority of the observed students as it was also evident from the solutions submitted by another group (Yak and Oye) to problem 7 as seen below:


Figure 9: Solution of group 2 (Yak \& Oye) to problem 7.
The transition from assumptions to the formulation of mathematical equations is a very critical part in any modelling process according to educational scholars like Blomhøj and Jensen (2004), Chang (2001) and others. In my quest to unravel the rationale behind this problem, I conducted a one-on-one interview with these students regarding these tasks. The students admitted that their problem lied in translating the problem into mathematical
equations. They said they were not familiar with this kind of tasks although they also admitted that they liked the directions of the questions because the entire tasks were all very practical and real-life based. The observed students further said they were confident that they could do better in the future if they continued to familiarize themselves with these kinds of tasks.

Furthermore, this problem of formulating equations was however not the only problem the observed students encountered although it was the most general one. Another problem was the formulation of assumptions. Some of the observed students were not able to come up with assumptions to some of the problems. Thus they were not able to begin with the problems. I also asked these students why they had this kind of challenge and they said these kind of modelling problems were very strange to them. I fully agree with the observed students' claim that these tasks were very unfamiliar to the students considering the present method (teachercentred and lecture-based learning approach that only emphasises the solving of exercises that must lead to correct answers) that the students go through in learning mathematics in Nigerian secondary schools.

### 5.3 Analysis of post-interviews

After the group work was concluded by the observed group of students, I again interviewed them so as to get their views and opinions about the processes and type of modelling problems they had been going through in the past three days. Thus, they were asked the following questions:

1. What is (are) your opinion(s) about the modelling tasks you have been going through in the last three days?
2. Has the modelling tasks and the entire processes help you to see mathematics from a different perspective?
3. What are the things you like about these types of questions?
4. What are the things you do like about these types of questions?

Below are the responses from five out of the ten students that were interviewed. My choice of these five students out of the observed ten students is based on the fact that I needed to be consistent with the same group whose answers/responses were presented and analyzed in section 5.1 during the pre-interviews session. In addition, their responses show a fare reflection of the entire observed groups' responses. The complete version of the postinterview responses is attached in appendix 3.

Question 1: What is (are) your opinion(s) about the modelling tasks you have been going through in the last three days?
Ola: It was a thought provoking process. They tasks made me think a lot.
Mgb: The questions are very practical and real life related. I think I will prefer mathematics tasks of these nature because it makes me to see the reality of mathematics.
Sal: They are wonderful questions.
Bal: They are good but I think they are too difficult for our level.
Iwu: Very practical.

Question 2: Has the modelling tasks and the entire processes helped you to see mathematics from a different perspective?
Ola: Hmmmmmm. I never imagine mathematics can be this practical. I love it.
Mgb: I never believed that mathematics can be so real like this.
Sal: Yes. I think this is the type of mathematics we should be doing because we can even apply it outside school and I am sure our parents will be very happy to see us solving real life problems like this at home using the mathematics we are learning in school.
Bal: Sure. Now I see that I can use my school mathematics to solve some problems at home like the tasks relating to the bird feeder. I have some birds at home and I will surely try it out next time at home.
Iwu: Yes, the knowledge from this will help me solve other real life problems.
Question 3: What are the things you like about these types of questions?
Ola: I like the practicality of the tasks.
Mgb : I like the practicality of the tasks.
Sal: The reality involve in the tasks.
Bal: They are very practical and I don't have to memorize any formula or algorithm.
Iwu: The tasks are very practical and real.
Question 4: What are the things you do like about these types of questions?
Ola: It makes me think too much.
Mgb: They make me think crazy.
Sal: Difficult to figure out how to start the task.
Bal: Hmmmm. Difficult to come out with assumptions that can help you develop your model. Iwu: They required a lot of thinking.

Table 8: Summary of post-interview.

| Questions | Number of <br> students that gave <br> positive response | Number of <br> students that gave <br> negative response | Number of <br> students that <br> are indifferent |
| :--- | :---: | :---: | :---: |
| 1. What is your opinion about the <br> modelling tasks you have been <br> going through in the last three <br> days? | 8 | 2 | 0 |
| 2. Has the modelling tasks and <br> the entire processes helped you to <br> see mathematics from a different <br> perspective? | 9 | 1 | 0 |
| 3. What are the things you like <br> about these types of questions? | 9 |  | 0 |
| 4. What are the things you do like <br> about these types of questions? | Not applicable | Not applicable | Not applicable |

Table 8 indicates that modelling tasks have been of help and motivation to the observed group of students. Eight out of ten students from the observed group affirmed that they have a positive judgement about the modelling problems that they went through in the entire process. When I asked the observed students during the post-interviews what are their opinions regarding the modelling problem they went through in three days, one of the students Mgb responded this way:
"The questions are very practical and real life related. I think I will prefer mathematics tasks of these nature because it makes me to see the reality of mathematics" This eight out of ten positive affirmation is an indication that modelling task may be of immense assistance in motivating students to have a positive attitude towards mathematics and the entire mathematics learning processes.

In addition, the post-interview also reveal that students were able to have a better connection between their classroom mathematics and what they see in real life. From the responses to problem 2 of the post-interview, the majority of the observed students (nine out of ten) admitted that they were able to easily connect the modelling tasks to the outside world without any difficulty. For the purpose of emphasis, I will represent some of the response to problem 2 of the post-interview. When I asked the observed students if modelling tasks and the entire processes have helped them to see mathematics from a different perspective, see what they have to say: Ola said, "Hmmmmmm. I never imagine mathematics can be this practical. I love it". Mgb responded by saying "I never believed that mathematics can be so real like this". Sal in his answer said "I think this is the type of mathematics we should be doing because we can even apply it outside school and I am sure our parents will be very happy to see us solving real life problems like this at home using the mathematics we are learning in school".

Another student, Bal added his voice by saying "Now I see that I can use my school mathematics to solve some problems at home like the tasks relating to the bird feeder. I have some birds at home and I will surely try it out next time at home". These and some other intriguing responses indicate that the modelling tasks really motivated the observed students in the learning process of mathematics. Sal went on to say that even their parents will be very happy to see them solving real-life problems like this at home using the mathematics they are learning in school. This statement connotes that even parents are not happy when their children cannot apply their classroom mathematics to real-life problems outside school.

### 5.4 Synthesis of the analyses

A combination and comparison of some vital questions that reflect and attempt to provide answers to the research questions both in the pre-interview and post-interview sessions, as well as the observed students' responses are show in tables 9 and 10 below.

Table 9: First summary of synthesis of analyses.

| Questions | Number of <br> students that gave <br> positive response | Number of <br> students that gave <br> negative response | Number of <br> students that are <br> indifferent |
| :--- | :---: | :---: | :---: |
| Pre-interview |  |  |  |
| 4. What is your opinion <br> about the way(s) you are <br> currently learning <br> mathematics in school? | 1 | 8 | 1 |
| 5. Why are you studying <br> mathematics in school? | 1 | 9 | 0 |
| Post-interview <br> 1. What is your opinion <br> about the modelling tasks <br> you have been going <br> through in the last three <br> days? <br> 2. Has the modelling <br> tasks and the entire <br> processes help you to see <br> mathematics from a <br> different perspective?$\quad 9$ |  |  |  |

From table 9, we can see that the opinions of the observed students changed almost entirely from the pre-interviews to the post-interviews. The statistics shows that during the preinterview of the observed group, only $10 \%$ gave a positive opinion about the way they are currently learning mathematics. This means that one out of every ten students liked the way they are currently learning mathematics. (Teacher-centred and lecture-based learning that only emphasizes the solving of tasks that must lead to correct answers). However, after going through some modelling problems, nine out of ten ( $90 \%$ ) made bold to say that they have a positive opinion about mathematics when they use some modelling problems in the learning process of mathematics.

Still with the same observed group, during the pre-interviews, nine out of ten (90\%), gave a negative response when asked why they were studying mathematics in school. They affirmed that they were only studying mathematics in secondary school because it was compulsory for them to do so since it is a prerequisite for them to advance to the next level of their study stages. But there was a clear departure from this as nine out of the ten observed indicated a positive response during the post-interview when they were asked if modelling tasks have helped them to better understand or learn mathematics.

Table 10: Second summary of synthesis of analyses.

| Questions | Number of <br> students that gave <br> positive response | Number of students <br> that gave negative <br> response | Number of <br> students that are <br> indifferent |
| :--- | :---: | :---: | :---: |
| Pre-interview |  |  |  |
| 2. Can you see any <br> connection <br> between the <br> mathematics you <br> learn in school and <br> the outside world? | 3 | 7 | 0 |
| 2. Has the <br> modelling tasks <br> and the entire <br> processes helped <br> you to see <br> mathematics from <br> a different <br> perspective? |  |  |  |

Looking at table 10 , it is clear to see that modelling problems helped almost all the observed students to see mathematics from a different perspective and thus assisting them to connect their classroom mathematics with some real-world activities. Again, we may try to seek inspiration and re-affirmation of this assertion from the response of one of the students Bal, when he said during the post-interview session that: "Now I see that I can use my school mathematics to solve some problems at home like the tasks relating to the bird feeder. I have some birds at home and I will surely try it out next time at home".

Still from table 10, we can see that the majority of the students cannot draw any connection between the classroom mathematics and the outside world or real life during the pre-interview session. This is evident from the fact that seven out of the ten observed students claimed that they cannot draw any connection between the mathematics that they are learning in school and their real life. Only three out of ten indicated that they can make some connections between their classroom mathematics and the outside world.

However, it is interesting to see that nine out of ten from the same group of observed students admitted during the post-interviews that they can clearly see and quickly draw connections between modelling problems and their real lives. The majority of the students said during the post-interviews that they were particularly happy with modelling problems and the entire modelling process because the problems were really connected to real-life situations and thus made them see the relevance of mathematics to real life. Many of the observed students also said they were very happy to see a situation where they do not have to memorize series of formulae, processes and algorithms. Only one out of the ten observed students said they were not comfortable with modelling problems because it required a lot of thinking.

## 6 Discussion and concluding remarks

In this chapter, I will discuss the results of my study with respect to previous studies that have been done in the past that are related to my study. Also, I will relate my findings to my research questions. This is to enable me to come up with possible answers to my research questions. Finally in this section, I will sum up and present some concluding remarks about this study.

I set out this study to come up with answers to the following research questions:
(a) What impact will mathematical modelling activities have in the learning of mathematics amongst the observed Nigerian secondary school students?
(b) In what way will mathematical modelling activities help the observed Nigerian secondary school students to see the relevance of mathematics to real life?

My findings from this study show that mathematical modelling problems may have the potential of being valuable with respect to the learning of secondary school mathematics. This is because, in this study, the observed students seemed to be motivated by the nature of the connections between classroom mathematics and real-life. Also, the case of non-abstractness of secondary school mathematics was showed. Some of the observed students affirmed that they never knew mathematics could be so real like the modelling problems they worked with. For example, some of the observed students said that they never imagined the car park problem could in any way be related to angles and sin functions that they study in the classroom. This was the assertions of the observed students when I solved example one on the black board. Although the connection between secondary school mathematics and the realworld was not very easy for the students at the beginning, it may be reasonsable to think that this is because the concept of mathematical modelling was relatively new to the students. This may support Galbraith and Stillman (2006) suggestion that the transition from a real world problem statement to a mathematical model is one of the most difficult parts of the modelling cycle. At the same time, mathematical modelling problems may be a veritable source of making students to see the relevance of mathematics to real life. This was demonstrated by the responses from the observed students during the course of this study. The modelling problems I introduced to the observed students seemed to make a positive impact in the learning process of mathematics among the observed students. In addition, the modelling tasks also helped them to see the relevance of mathematics to real life.

It is therefore interesting when Hiebert (1997) affirms that students who are taught exclusively from traditional, teacher-centred and textbook problems usually fail to understand the relevance of what they are learning. Hiebert goes on to conclude that most times, students form an opinion of what mathematics is all about based on the tasks they are given. My study follows the assertion of Blomh $\varnothing \mathrm{j}$ (2004) when he said that mathematical modelling bridges the gap between students' real-life experiences and mathematics. It motivates the students' learning of mathematics, gives direct cognitive support for the students' conceptions, and it places mathematics in the culture as a means for describing and understanding real-life situations. This was the case in this study as most of the students from the observed group attested to the fact during the post-interview session that the modelling tasks and processes that they went through during the course of this study made them see the relevance of mathematics to real life. The observed students further affirmed that modelling activities changed their perception about the abstractness of mathematics. In the course of the group work, the students seemed to gain better cooperation and understanding amongst themselves
as the suggestions from different member of the group contributed to the overall success of each group. It may therefore be proper to agree with Niss (1989) suggestion that the integration of mathematical modelling activities in the classroom may assist students in working together as a group in the class, thus increasing the applicability of classroom theories and their practical use.

My study also corroborates the claim by Jablonka (2003), who suggests that bringing to the classroom the mathematics used in working environment is one of the ways for associating the out of school mathematics with the curricular contents and consequently of showing the mathematical practical utility. My use of working environment modelled problems in this study was a catalyst for making the observed students to see and perhaps have an understanding of the practical utility of mathematics. This is indeed a confirmation of what Niss et al. (2007) say that the general purpose of building and making use of a model is to understand or tackle problems in some segment of the real world. In this study, some of the observed students were able to reasonably understand and tackled problems from the real word. This is reflected in the various assumptions and conclusion that the observed students came up with while working with the modelling problems. Some of the observed students were able to make assumptions to handle real-life problems.

However, the use of tasks that are not connected to real life in mathematics classrooms is also having some negative impact on the students learning attitude. This was revealed during the pre-interview session of this study with the observed students when many of them claimed that they could not see anywhere outside their classroom that mathematics is being used. This lack of connection they also said is a source of demotivation for them to learn mathematics. The students' opinions are partly in accordance with Schoenfeld (1985), who said that the learning process that is emphasized and concentrated in standard textbook questions encourage the development of procedural knowledge that is of limited use in non-school circumstances. Prior to this study with the observed students, they have only subjected to the concentration on standard textbooks questions, procedures and algorithms without any linkage to real-life problems. This is the one I earlier referred to as teacher-centred and lecture-based learning approach that only emphasizes the solving of exercises that must lead to correct answers.

In this study, some of the observed students were able to see mathematics from a different perspective. Some of the assertions during the post-interview indicate that some of the observed students now have a different and perhaps positive opinion about secondary school mathematics. This may be in agreement with Boaler (1998) suggestion that using mathematical procedures within the context of real-life activities will allow students to view mathematical processes as tools rather than the end result of their knowledge, thereby making them more likely to be able to adapt and use the procedures in other circumstances. This is corroborated by Skovsmose's (1994, p. 52) quote that says "mathematics has not only created ways of describing and handling problems, but it also becomes a main source for the reconstruction of reality".

In addition, this study also shows that group work during mathematical modelling activities may assist students in their mathematical learning process. This was evident from the group work activities of the students during this study. Contributions and different suggestions by various members of each group seemed to help the groups to make meaning of the mathematics and realize the mathematical meaning of the modelling problems. This possibly enables the observed students to see the relevance of mathematics in their daily life.

However, it is interesting to realize that some of the observed students were able to view mathematical modelling during and after the group work as a means of bridging the gap between classroom mathematics and the real-life. Some of the observed students confirmed this during the post-interviews. For example, when I asked one of the observed students during the post-interview session if modelling tasks and the entire processes has helped him to see mathematics from a different perspective, Bal answered and say "Sure. Now I see that I can use my school mathematics to solve some problems at home like the tasks relating to the bird feeder. I have some birds at home and I will surely try it out next time at home" This may be in line with Blomhøj (2004) assertion that mathematical modelling bridges the gap between students' real life experiences and mathematics and that it motivates the students' learning of mathematics, gives direct cognitive support for the students' conceptions, and it places mathematics in the culture as a means for describing and understanding real life situations.

Nevertheless, it may be important to emphasize that the learning process is an evolving and perhaps a slow system that may take a long period of time. But because of time constrain within the confine of this study, my observations were only within a relatively short period of time. The results may therefore vary when observation is done over a long period of time. Also, because the observed students were only exposed to limited amount and content of mathematical modelling problems and within a short period of time, my findings may also have a different outlook when compared with the students given a long time exposure to mathematical modelling problems.

Finally, one of the critical strength in this study is the combination of different research methods; Observations, interviews and field-notes all aimed toward achieving a reasonable result. However, some of the weaknesses in this study include small sample size and observation over a relatively short period of time. But for the limited time for this study, I would have preferred to observe larger population and over a relatively long period of time. This would have been in accordance with Delamont and Hmilton (1993) assertion that in the interest of objectivity, many systematic research studies feel compelled to survey large numbers of classrooms. They further argued that small samples may fail to provide statements relevant to the population at large.

## 7 Implications for instruction and further research

In this chapter, I will highlight the implications of my research findings for instruction in section 7.1. The implications for further research will be examined in section 7.2.

### 7.1 Pedagogical implications

The introduction of mathematical modelling as an approach for learning mathematics in Nigerian secondary schools may require that mathematics teachers have to be adequately trained to meet the challenges that may arise as a result of the introduction of mathematical modelling. Considering the fact that the introduction of mathematical modelling may be a new approach for the students and the teachers, the workload of the teachers and perhaps that of the students may be adopted in order to help both parties (students and teachers) to have sufficient time to understand the idea behind modelling processes and techniques.

In my own view, all the relevant Government agencies that are responsible for the regulation of teachers' quality will have to ensure a coherent training and retraining for mathematics teachers in Nigerian secondary schools. In addition, various economic incentives may be made available for the mathematics teaching profession so as to attract people to the job and thus reducing the workload on the existing teachers. Various mathematics teachers' training institutions may need to spend time in methodology of the subject content and curriculum development. This should be improved upon so as to enable mathematics teachers handle the multiple problems that may arise in the teaching of mathematics using mathematical modelling.

The organization of students into small groups within the class may also be of help in the process of introducing mathematical modelling as a means of learning mathematics. Therefore, there may have to be a shift from the existing authoritarian, traditional, teachercentred and lecture-based learning approach that only emphasizes the solving of exercises that must lead to correct answers. The students will have to be encouraged to work in group activities. This may help to motivate the students in their efforts to learn mathematics.

Therefore, secondary school mathematics curriculum should be defined and the procedures to be followed should be included so as to emphasize mathematical modelling as a challenging, but yet effective way of learning mathematics. Sufficient and reliable teaching aids and activities may help both the students and the teachers in the process of using mathematical modelling activities in Nigerian secondary school. The teachers may emphasize the application of each topic to real life.

### 7.2 Implications for further research

As a follow up of this study, I will suggest that the modelling activities and procedures that I used in this study be extended to a larger population and over a longer period of time. This may help to further substantiate or object my findings as data from larger sample tends to be more reliable. Also, the processes and procedures of this study may as well be replicated in different locations and for different levels. This will also help to get a wider perspective about my findings.

In addition, I will also suggest that additional data collection aids may be adopted. This may be in the form of other material like video camera during observation. This is because video recording may enable the researcher to repeatedly view an event or observations concerning any occurrence. This may help the researcher to get a second and perhaps a better and more
informed opinion about the whole processes. Therefore reducing the incidence of hasting or premature and perhaps inaccurate conclusions.

Finally, some of the observed students in this study complained that they were not familiar with mathematical modelling problems. This issue of non-familiarity with mathematical modelling problems may have been responsible for some of the difficulties the observed students encountered in the transition from real-life problem to mathematical models/equations. I observed from the group work that some of the observed students find it difficult to follow the modelling processes and stages as suggested by Blomhøj \& Jensen (2003) and Cheng (2001). Therefore, I am of the opinion that in further research, the observed students should be given more time to work with the mathematical modelling problems and familiarize themselves with the problems. This may reduce the problems associated with the students' unfamiliarity with mathematical modelling problems.

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## Appendix 1

Complete modelling problems.

1. Your school authority is considering optimizing the car park space within the school. The parking lots have already been painted and parking lines drawn in vertical side-by-side plan. Now, we wish to check to see if the existing plan has made maximum use of the car park space and if not, we wish to re-design the space to increase the number of lots. Assuming the maximum breadth per car lot in this arrangement is 1.5 m ,
(a) Sketch the existing design alongside a diagonal side-by-side parking plan that allows maximum breadth of 1.4 m car per lot.
(b) Determine the angle to will optimize the number of cars in the diagonal parking plan and hence the total number of cars that this plan can take. (Inspired by Cheng, 2010)
2. Your neighbour, an ornithologist, (a scientist that studies birds) has to leave for the weekend to do a research study. She has asked you to make sure her birdfeeder always has food in it so that the birds keep coming back throughout the day. Humans can't come around too often because it will frighten the birds and refilling too seldom will cause the birds to look elsewhere for food.
How often should you feed the birds so they keep coming back?
3. The Federal Communications Commission (FCC) needs to assign radio frequencies to seven new radio stations located on the grid as shown in the figure below. Such assignments are based on several considerations, including the possibility of creating interference by assigning the same frequency to stations that are too close together. In this simplified situation, we assume that broadcasts from two stations located within 200 miles of each other will create interference if they broadcast on the same frequency, whereas stations more than 200 miles apart can use the same frequency to broadcast without causing interference with each other. How can a vertex-edge graph be used to assign frequencies so that the fewest number of frequencies are used and no stations interfere with each other? What would each vertex represent? What would an edge represent? What is the fewest number of frequencies needed? (Inspired by James, Yuria \& Eric, 2010).
4. Making decisions can sometimes be quite difficult, especially when it's a decision about where you will spend the next couple of years of your life after your graduation from the secondary- we're talking about university, of course!
How can you choose the most suitable university for yourself?
5. A mathematician explained to oil company executives that their problem was very similar to the problem birds face in the winter when some birds need to find up to $40 \%$ of their body weight in food to survive a cold night. Often birds have to make a decision: do I stay where I am, where I know there are a few berries, or do I fly off and hopefully find a new bush with a bonanza of berries? Develop a mathematical model to explain this problem.
(Inspired by Tim, 2010)
6. Your grandmother will be arriving at the airport at 6:00 pm. You live 20 kilometers from the airport. The speed limit is 40 kilometers per hour. When should you leave to get her?
7. Your father is considering replacing his car and the major criteria for the selection of the type of car is fuel economy. Calculate which of the following would save more fuel? (a) Replacing a compact car that gets 34 miles per gallon (MPG) with a hybrid that gets 54 MPG.
(b) Replacing a sport utility vehicle (SUV) that gets 18 MPG with a sedan that gets 28 MPG.
(c) Both changes save the same amount of fuel. (Adapted from Chang, 2008)
8. What will be the best location of a hospital in a small city, with no traffic?
9. Designing the layout of the kiosks in a school fair so as to raise as much money as possible
10. The authority of your school is considering the construction of speed bumps within the college campus in order to calm the traffic. Decide the best location for the speed bumps

## Appendix 2

Students' responses during the pre-interview
Question 1: What is your opinion about mathematics?
Ola: It is a very difficult subject because it requires a lot of thinking, memorizing and procedures.
Mgb: It's difficult to remember formulae
Sal: I don't enjoy studying mathematics
Bal: It's a good subject but very difficult to understand and remember
Iwu: I think mathematics is a good subject although it is difficult
Oye: It is a difficult subject
Yak: It is good, but difficult to understand
Igb: It is very abstract
Owk: It is very difficult to understand why we do so many things in mathematics
Ekw: It is a good subject
Question 2: Can you see any connection between the mathematics you learn in school and the outside world?
Ola: No. It is hard to see. For me I only see some arithmetic like the one involving buying and selling like when making payment and either giving or collecting change. But I really do not think that is mathematics. So I can say maybe its primary school arithmetic.
Mgb: Not really but maybe as I heard, people say that engineers use mathematics a lot, but cannot see it
Sal: No. I cannot
Bal: No
Iwu: Yes
Oye: No
Yak:Yes
Igb: No
Owk: Yes
Ekw: No
Question 3: If you answer yes to question 2 above, can you give example(s) of how you use mathematics outside the classroom?
Ola: Not applicable since the student's answer to question 2 was "no"
Mgb: Not applicable.
Sal: Not applicable
Bal: Not applicable
Iwu: I think architect use part of trigonometry and geometry when they are making their drawings. I also think the computer scientist and engineers are using some of the thing we are learning in school now
Oye: Not applicable
Yak: I imagined that accounts use a lot of mathematics
Igb: Not applicable
Owk: I can see some application of mathematics with computer programmers
Ekw: Not applicable

Question 4: What is your opinion about the way(s) you are currently learning mathematics in school?
Ola: I think it's very difficult to learn mathematics because as I said before in my response to question 1, learning mathematics requires a lot of memorizing and a lot of procedures to be remembered. It's more frustrating when I don't get the answer even after working and trying so hard to solve a particular task. I get demotivated when I don't get the answers to mathematics tasks.
Mgb: Well, I have not had the opportunity of knowing or testing another method of learning mathematics, so I cannot really say I prefer this method or the other one. This is the only method I know of for now. But as I said before, it's a difficult process.
Sal: I don't have choice because this is the only way we are being thought for now. But I would have preferred it's like chemistry or physics whereby we go to laboratory mix chemicals and see the reactions, colour change and others. But mathematics is not like that. So it's really difficult to learn so many things and formulae that you cannot see.
Bal: It's a difficult way but I do not know if there is another way
Iwu: I really don't know if there is another way I can learn it, but I will prefer if some if not all of the concepts can be but into practical in the laboratory like physics and chemistry
Oye: I think it is good since you can do well if you can just memorize the formulae and processes without necessarily understanding what you are doing
Yak: It's a very abstract way
Igb: It is difficult
Owk: Too long processes and formulae to remember.
Ekw: Not motivating since it's very difficult to link it to anything outside the classroom
Question 5: Why are you studying mathematics in school?
Ola: Hmmmm (smile) Sincerely, I am studying mathematics in school because first its compulsory subject in secondary schools. So I cannot avoid it and secondly, it is a prerequisite for me to get into the university to study the programme I wish to study which is engineering.
Mgb: Because it is compulsory
Sal: Because it's a prerequisite for me to be promoted to the next class
Bal: Because it's a compulsory subject
Iwu: Because it's a compulsory subject and I also want to be an engineer
Oye: Because it's compulsory
Yak: Because I need it in my university study
Igb: I like it
Owk: Because it is mandatory
Ekw: Because everyone is doing it
Question 6: What do you like most about mathematics?
Ola: When I solve tasks and I get the correct answers
Mgb: When I remember the formula to solve a particular task, apply it to solve the task and I get the right answer, then I like mathematics
Sal: The formulae when I remember them
Bal: I don't think I like anything about mathematics. If I have my way I will not do the subject
Iwu: The fact that everyone always arrive at the same answer
Oye: Sometimes very easy when you remember the formula
Yak: Easy to know when you are right or wrong in most cases without waiting for the teacher to make any corrections

Igb: The formulae
Owk: Nothing really
Ekw: Hmmmmm..... Nothing
Question 7: What do you like least about mathematics?
Ola: When I solve and I don't get the correct answer
Mgb: When I don't get the current answers
Sal: Memorization
Bal: Memorizing formulae
Iwu: Long processes and memorization of so many formulae
Oye: Long formulae
Yak: To many little details to remember
Igb: Wrong answers after a long work
Owk: Abstractness of the subject
Ekw: Memorization
Question 8: What would you tell a new student in class your class about what it takes to be successful in a mathematics class?
Ola: I will advise him or her to practice the act of memorizing
Mgb: Try to learn formulae
Sal: I will tell him or her to learn how to memorize
Bal: Mathematics is difficult and he should prepare and ready to memorize a lot of formulae and processes
Iwu: Mathematics is not difficult if the student can remember all the formulae.
Oye: Learn to memorize
Yak: Mathematics is difficult
Igb: Learn a lot of algorithms
Owk: Be prepare for long processes that might sometimes not lead to correct answers
Ekw: It can be frustrating sometimes
Question 9: When and where do you see people using mathematics outside school?
Ola: I can't really see anywhere. But, as I said before, may be primary school arithmetic in the process of buying and selling.
Mgb: I cannot see it any where
Sal: None
Bal: Maybe engineers and architects
Iwu: Various calculations in the banks and also engineers
Oye: I cannot see it outside the classroom
Yak: I can't see it anywhere else
Igb: I think computer scientist use it
Owk: I think pilots and sailors mathematics
Ekw: I cannot see mathematics outside the school classrooms
Question 10: How do you think you might use mathematics when you finish your secondary school education?
Ola: I really do not know yet although I was told engineers use a lot of mathematics, but I am yet to see that. So I will wait to see when I get into engineering programme in the university. Then maybe I can see some engineering areas that I might be using mathematics.
Mgb: Hmmmmm. I am not sure yet.
Sal: I don't think I will use it because I want to be a lawyer.

Bal: Maybe if I am able to study to be an architect, then I think I will use it Iwu: When I start practicing as an engineer.
Oye: I do not think I will use
Yak: I am not sure of how and when I will really nedd to use the mathematics that I am learning now
Igb: I do not know yet
Owk: May be as computer scientist
Ekw: I am not sure I will use it

## Appendix 3

Students' responses during the post-interviews.
Question 1: What is (are) your opinion(s) about the modelling tasks you have been going through in the last three days?
Ola: It was a thought provoking process. They tasks made me think a lot.
Mgb: The questions are very practical and real life related. I think I will prefer mathematics tasks of these nature because it makes me to see the reality of mathematics
Sal: They are wonderful questions
Bal: They are good but I think they are too difficult for our level
Iwu: Very practical
Oye: The questions involve a lot of thinking
Yak: They are connected to what I can see
Igb: They are real life related
Owk: Difficult but very good
Ekw: They are practical questions
Question 2: Has the modelling tasks and the entire processes helped you to see mathematics from a different perspective?
Ola: Hmmmmmm. I never imagine mathematics can be this practical. I love it
Mgb: I never believed that mathematics can be so real like this
Sal: Yes. I think this is the type of mathematics we should be doing because we can even apply it outside school and I am sure our parents will be very happy to see us solving real life problems like this at home using the mathematics we are learning in school.
Bal: Sure. Now I see that I can use my school mathematics to solve some problems at home like the tasks relating to the bird feeder. I have some birds at home and I will surely try it out next time at home
Iwu: Yes, the knowledge from this will help me solve other real life problems
Oye: Yes. I now know that mathematics is real
Yak: Yes
Igb: Yes, because before now I didn't know that there is a great deal of connection between mathematics and real life
Owk: Yes.
Ekw: Yes
Question 3: What are the things you like about these types of questions?
Ola: I like the practicality of the tasks.
Mgb: I like the practicality of the tasks.
Sal: The reality involve in the tasks
Bal: They are very practical and I don't have to memorize any formula or algorithm
Iwu: The tasks are very practical and real
Oye: They are related to the things I can see
Yak: They are connect to what I know
Igb: The questions are based on realities
Owk: They questions do not make me memorize formulae and algorithms
Ekw: I did not have to memorize processes

Question 4: What are the things you do like about these types of questions?
Ola: It makes me think too much
Mgb: They make me think crazy.
Sal: Difficult to figure out how to start the task
Bal: Hmmmm. Difficult to come out with assumptions that can help you develop your model Iwu: They required a lot of thinking.
Oye: It's difficult to start them
Yak: You need to do a lot of thinking especially at the beginning
Igb: Difficult to determine the assumptions
Owk: They involve a lot of logics
Ekw: It requires plenty of thinking

Appendix 4
Students' solutions to some of the modelling problems.

Ola, mg and sal
(4) Making decisions Can Sometimes be quite difficult, esperaily when $V^{\prime}$ 's decision about where you will spend the rest couple of years after your graduation from secondary school- we ave, tacking of your the about univenity, of Carnie!
How can you choose the most suitable university for yomaself?
Answer:
Fist we assume the nt the we ae, universities labelled A -I, so we have:

| university | $A$ |
| :---: | :---: |
| university | $B$ |
| $L$ | $C$ |
| $L$ | $E$ |
| $L$ |  |
| $L$ |  |
| $L$ | $H$ |
| $L$ | $I$ |
| $L$ | $I$ |

Now we also assume the following suitability criteria
(1) Availability of my desiñed programm
(2) Cost of tirition
(3) Availability of sehobanship
(4) The rating of the university.

Let us now allocate 2 paints for each of these criterion.


Now: Thy table shows university $A$ oud ID having the highest score of 6 .
Pherefore we wat choose esther A or D.

1 yak and oye
(1) Answer to question 7
(a) We believe that it will be better to change from 34 miles per gallon (MPG) to 54 (MPG) because you achieve moor miles with Lesser gallon.
There is an increase of $59 \%$ increase if we change from 34 to 54
Ats But, if we change from $18\left(\mathrm{mPC} \mathrm{c}_{\mathrm{A}}\right)$ to $28(\mathrm{mPl})$ that means a decrease in achieved distance Per gallon
Therefore we recommend change from $54(\mathrm{mpl})$ to $34\left(M P e^{\prime}\right)$.

