UNIVERSITY OF AGDER

A model to analyse algebraic tasks solved by students

A comparative study from Finland and Norway

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This Master's Thesis is carried out as a part of the education at the University of Agder and is therefore approved as a part of this education. However, this does not imply that the University answers for the methods that are used or the conclusions that are drawn.

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Preface

I have spent a unique year studying here in Kristiansand, in Norway. When I started my Joint Nordic Master Degree Programme in Didactics of Mathematics (NORDIMA), I could never think it would be so magnificent, challenging and educational experience. During this year, I have recognized how much I can learn going outside of my comfort zone. I could never have imagined that someday I would write my master thesis in English and in the context of an international research, but now I have done it. It is an amazing feeling.

I am grateful to my supervisor Professor Maria Luiza Cestari for her encouragement and expert advice during this writing process of thesis. She has trusted my skills although this process has not been always so clear and easy for me. Also, I wish to thank the whole VIDEOMAT-group not only for my empirical material but also for the fresh spirit of doing the international research. Related to the Norwegian research group, I want to thank Unni, Jorunn and Amar for the productive and supportive way to work. In addition, warmly thanks to Ann-Sofi Röj-Lindberg related to the working with the Finnish material.

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Lastly, I want to say thanks to my family and my friends. You have been a great support during my year in Norway even though you have been so far away! Also my Norwegian school friends from the master student room have really earned their thanks. You are awesome! It has been wonderful to work (and spend) all days with you! I am not going to forget how there was always time for *en kopp kaffe* while working. Norway has taught me that life is not so rushed. *Tusen takk! En dag skal jeg komme tilbake til Norge. Det vet jeg.*

In Kristiansand 6th June, 2013

Vuokko Oikarainen

Abstract

Participating in the international VIDEOMAT-project, which is accomplished by four universities from Finland, Norway, Sweden and California (USA), this master thesis has been based on a comparative study about the introduction of algebra in Finland and Norway. The focus of the present study has been to compare the way students solve algebraic tasks: the procedures which they use while solving tasks and the mistakes they produce. Similarities and/or differences have been identified between those two countries.

The empirical material for this research has already been collected and it includes videotaped lessons, lesson graphs and students' written work. The focus is on the comparison of the students' written work. The empirical material comes from four classrooms both in Norway (7th and 8th grade) and in Finland (6th and 7th grade) when students are at the same age, 12–13 -year-old. This material have included 110 notebooks or tests of students: every student has solved approximately 11 tasks during the data collection. The micro analysis has been done and procedures and mistakes of all solution processes have been analysed by the model which has been created during the study. The findings have been presented as tables in which different students' procedures and mistakes of some analysed tasks are visible.

The findings have shown that the Finnish students are using more similar procedures among themselves while solving tasks than the Norwegian students. The identified mistakes are mostly related to the minus sign, equal sign and writing expressions. Related to the procedures, the Finnish students are supposed to present their working while solving the tasks, for example, with review of calculations. The Norwegian students have presented more atypical, but correct, solutions than the Finns. Both the Finnish and Norwegian students have difficulties with the minus sign and equal sign but in the context of writing expression the Finnish students have had difficulties to connect algebra and geometry while the Norwegian students had problems in writing an expression from the written instructions.

As a conclusion, the understanding of arithmetic operations is problematic in both countries. Students, for example, have presented mistakes related to the simply calculations with the minus sign. As a didactical implication, the connection between algebra and other fields of mathematics should be more strengthen to promote a deep and meaningful understanding of mathematics.

Key words: algebra, tasks, comparative study, Finland, Norway

Tiivistelmä

Tämä pro gradu –tutkielma on osa kansainvälistä VIDEOMAT-projektia, joka on toteutettu yhteistyössä neljän yliopiston kanssa Suomesta, Norjasta, Ruotsista ja Californiasta (USA). Tämä tutkimus on keskittynyt vertailemaan algebran opiskelua aloittamista Suomessa ja Norjassa, ja oppilaiden algebratehtävien ratkaisuja: minkälaisia menetelmiä oppilaat käyttävät ja millaisia virheitä he tuottavat. Yhtäläisyyksiä ja erilaisuuksia on havaittu kyseisten maiden välillä.

Tutkimuksen empiirinen materiaali on jo aiemmin kerätty ja se sisältää videonauhoitettuja oppitunteja, oppituntitiivistelmiä sekä oppilaiden kirjallisia tuotoksia. Tämä tutkimus keskittyy oppilaiden kirjallisiin tuotoksiin. Empiirinen materiaali on kerätty neljästä luokasta sekä Norjasta (7. ja 8. luokka) että Suomesta (6. ja 7. luokka) jolloin oppilaat ovat olleet samanikäisiä, 12–13-vuotiaita. Materiaali sisältää 110 oppilaiden vihkoa ja koetta: jokainen oppilas on ratkaissut keskimäärin 11 tehtävää tutkimuksen aikana. Tämän jälkeen menetelmät ja virheet ovat analysoitu mallilla, joka on luotu tutkimuksen aikana. Tulokset on esitetty tauluina, joissa joidenkin tarkemmin analysoitujen tehtävien menetelmät ja virheet ovat näkyvillä.

Tuloksista on havaittu, että suomalaiset oppilaat käyttävät keskenään enemmän samanlaisia menetelmiä kuin norjalaiset. Havaitut virheet liittyvät useimmiten miinusmerkkiin, yhtäsuurusmerkkiin ja lausekkeiden kirjoittamiseen. Suomalaisten oppilaiden edellytetään näyttävän työskentelynsä muuan muassa laskutoimitusten tarkistuksella. Norjalaiset oppilaat ovat taas enemmän esittäneet epätyypillisiä, mutta silti oikeita, ratkaisumenetelmiä suomalaisiin oppilaisiin verrattuna. Sekä suomalaisilla että norjalaisilla oppilailla on ollut vaikeuksia miinusmerkin ja yhtäsuurusmerkin kanssa, mutta lauseketta kirjoittaessa suomalaisten vaikeudet liittyvät algebran ja geometrian yhdistämiseen, kun taas norjalaisten lausekkeen kirjoittamiseen sanallisesta tehtävästä.

Johtopäätöksinä voidaan todeta, että aritmeettiset laskutoimitukset ovat ongelmallisia sekä Suomessa että Norjassa. Oppilailla on vaikeuksia esimerkiksi jo yksinkertaisissa, miinusmerkin sisältävissä laskuissa. Kehitysehdotuksena mainitaan että algebran tulisi olla enemmän yhteydessä matematiikan muihin osa-alueisiin tukien syvällisempää ja merkityksellisempää matematiikan ymmärrystä.

Asiasanat: algebra, tehtävät, vertaileva tutkimus, Suomi, Norja

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1 Introduction

This study focuses on the introduction of algebra in two different countries: Finland and Norway. The background is the VIDEOMAT-project which is accomplished by four universities from Finland, Norway, Sweden and California (USA). The participants are from the Åbo Academi University (Finland), the University of Agder (Norway), the University of Gothenburg (Sweden) and the University of California at Los Angeles (UCLA) (USA). The main goal of this project is to identify the different approaches used when algebra is introduced in 6^{th}, 7^{th} and 8^{th} grades on mathematics lessons. The content area of the introduction of algebra means the introduction of variables in expressions. The comparison is focused on Finland and Norway, not only because they are both Nordic societies, but according to the PISA- and TIMSS-results, students present different levels of knowledge. The comparison between these two countries could rise up some new knowledge about learning algebra in primary school from 6^{th} to 8^{th} grades.

Although my home country Finland has one of the best schools in Europe, I think we still have some challenges to keeping our knowledge of mathematics in a good level. According to new TIMSS-results (Kupari, Vettenranta & Nissinen, 2012), we are still good but, during the years, the results have decreased. Astala et al. (2006) have underlined how the good PISA-results do not tell the whole truth about the Finnish students' level of knowledge: the basis of concepts understanding is not learnt well enough for higher standards of mathematics. Mathematics is also needed in the context of higher learning and sciences: "a proper mathematical basis is needed especially in technical and scientific areas, biology included" (Astala et al. 2006, p. 9). Also the study of Näveri (2009) has shown how the results of mathematics tests in Finnish 9^{th} grade students have decreased from year 1981 to 2003. That is why it is necessary to develop our mathematics teaching practices to keep the performance in a good level in the future because mathematics is needed in society: "Knowledge of mathematics gives people the pre-conditions to make informed decisions in the many choices faced in everyday life and increases opportunities to participate in decision-making processes in society." (Skolverket, 2011, p. 59.) The meaning of school mathematics is not to teach some mathematical content out of the context but to give students good possibilities to solve everyday problems of their life. Algebra could be a useful tool for problem solving but it is usually seen only as simplifications and manipulations with variables in the context of school algebra. The school algebra has been felt as a disconnected part of mathematics and learning of it has been recognized to be difficult for students and teachers as well. This is the reason why I – as a researcher and a future mathematics teacher – want to survey what different aspects will rise up from different countries. What we could learn from each other? How could I be a better teacher in the future? Which issues could I pay attention to while teaching?

Many studies have already been done related to the learning and teaching algebra during last years, for example about the difficulties of learning and teaching algebra (Küchemann, 1981; Booth, 1988; Kieran, 1992; Herscovics & Linchevski, 1994; MacGregor & Stacey, 1997), the structural similarity between arithmetic and algebra (Linchevski & Livneh, 1999; Banerjee & Subramaniam, 2012), younger students' ability to use algebraic relations (Carraher, Schliemann, Brizuela & Earnest, 2006), algebraic thinking within arithmetic (Britt & Irwin, 2011) and the algebraic tasks of text books (Reinhardtsen, 2012). Despite those studies, there are still some areas not well known in the field of learning and teaching algebra. This work is concentrated on the comparison of written work of the students which is based on the tasks students are solving during mathematics lessons at school. This perspective is also beneficial for the whole VIDEOMAT-project because then it lets us to know how students are solving tasks and what kind of difficulties they may have.

The tasks are the main part of learning mathematics, so doing research about the students' working gives possibilities to know which are the most frequent procedures and mistakes presented while students are solving the tasks. As Booth (1988, p.299) has mentioned in the context of algebra: "One way of trying to find out what makes algebra difficult is to identify the kinds of errors students commonly makes in algebra and then investigate the reasons for these errors."

The aim of this study is to go through the written solutions of the Finnish and Norwegian students to get to know how they have solved tasks. The main focus is on the procedures and mistakes which students are doing while solving tasks. Of course, the international comparison is also part of this study. The written solutions of the students are only one part of mathematical learning but in this case, when we are focused on to the students' individual working on lessons, it is sufficient material to go through. In addition, one benefit of this study is that the tasks solved by students are connected to their own context: the tasks are in fact from their own text books and teachers' material. This is also a challenge of this study because the tasks are not always equal if we are comparing the two countries.

In the context of students' written work, my research questions are:

1. What kinds of procedures students use while solving tasks?
2. What kinds of mistakes students produce?
3. Which similarities and/or differences can be identified between Finland and Norway?

The aim of the study is not to give quantitative analysis of the solutions but to describe the richness of the empirical material which has been collected from Finland and Norway. The model to analyse the task has been created during this present study. Based on those findings about procedures and mistakes it is possible to consider what is needed to improve the teaching in Finland and Norway to get the students to better understand algebra. It is beneficial to know which things we have to pay attention to while we are teaching algebra at school.

Next, the structure of this study is introduced. At first, the earlier research about algebra is described: how algebra is seen in general and how it is seen at school. There is also possible to observe a gap between those two views. In addition, there are presented the challenges while teaching and learning algebra. Based on the fact that researchers have suggested many improvements of ideas, for example about the algebraic thinking, also some developmental studies about algebra are introduced.

After the review of literature, the methods are described related to the comparative study and the data collection. Then the analysis and the analytical approach, which has been created during this study, are described in detail. After analysis, the findings from the empirical material are introduced with illustrative tables and comments. In addition, the comparison among Finland and Norway is summarized. The discussion about the findings of this study is described with review of the earlier research and then the conclusion is also been done. In the end the pedagogical implications are portrayed.

The references and the appendices are located at the end of the paper. The appendices include the coding system and the coded tables of the procedures and the mistakes of the students.

2 Review of literature

This review of studies done earlier concentrates on the context of the school algebra: what algebra is, how it is seen at school and also which have been the challenges of learning and teaching algebra are described. Some development ideas from earlier research are introduced as well.

2.1 What is algebra?

Most people conceive algebra as calculations with letters, representing unknowns. They think there is some letter, usually *x*, for which they need to find a numerical value via different kind of procedures. As a contrast to this kind of mental image, Kieran (2011) underlines algebra being the way of thinking, not only literal symbols. This can illustrate the problem of school algebra: the way of thinking has a totally different kind of approach to algebra than manipulation of symbols. This manipulation is usually seen only as a part of teaching school algebra and it does not have contact to other parts of mathematics and students' everyday life. This could also be the reason why it is so difficult to motivate students to study algebra and why the knowing and understanding of algebra is continuously decreasing. As Subramaniam and Banerjee (2011, p. 89) mention: "algebra is a gateway to higher learning for some pupils and a barrier for others".

Historically, the development of symbolic algebra can be divided to three different stages. The first stage is the rhetorical one which has been developed before Diophantus (c. 250 AD). At that time, ordinary language has been used to solve problems without using unknowns. The second stage is related to Diophantus' introduction of a letter to represent the amount of unknown, but he has not had any general method to solve problems. The third stage has been in 1500s when Vieta has started to use letters both for the given and unknown quantity. This contribution has led to symbolic algebra. After that algebra has not been anymore only procedural tool but also has made possible for the symbolic forms to be used as objects. (Kieran, 1992.)

Kieran (1992) describes algebra having two natures: unknowns in equation solving and givens in expressing general solutions and as a tool for proving numerical relations are both represented using letters. Then letters are used both as descriptions of problem situations and as their solutions to symbolic representations and procedures. Radford (1996) defines that unknown is a certain number, but variable varies. For example number 5 can be unknown, but not variable. According to Malisani and Spagnolo (2009, p. 20), variable can define as a functional relation, "thing that varies".

Natural language is an important part of algebra. Filloy, Puig and Rojano (2008) mention how mathematical text usually consists of mathematical signs and vernacular language which are sharply isolated. They introduce the term Mathematical Sign System (MSS) and highlight the meaning of the term being "mathematical systems of signs", not systems of mathematical signs. It means that sign systems are also mathematical systems. The MSS of arithmetic helps to the introduction of algebra; some situations can be solved with algebra instead of arithmetic. That is why the building of this system is beneficial to start from arithmetic operations and expressions. "These [new] objects will signify not only numbers but also numerical representations, whether as individual items (e.g., unknowns), sets of numbers (e.g., coefficients of equations), an expression of relations between sets of numbers (e.g., proportional variation), or as functions, etc." (Filloy et al., 2008, p. 11.) According to Filloy et al. (2008), it is also important to structure the terms both their semantic and syntactic aspects before objects are said to be outlined and well defined. Then the whole meaning of mathematical expressions will open easier.

The notation of the equation can require some effort. Filloy et al. (2008) compare the meanings of *arithmetic* and *algebraic* equations. They describe the meaning of an equation $Ax \pm B=C$ (where A, B and are given numbers) C being an arithmetic equation because it is possible to solve with an arithmetic way. Instead, an algebraic equation $Ax \pm B = Cx \pm D$ (where A, B, C and D are also given numbers) has to be solved with an algebraic way because there is one unknown on the both sides of equation. Then students are not able to deduce with arithmetic calculations but they have to use algebraic expressions. An example of an arithmetic equation can be $2x + 5 = 11$ whereas an algebraic equation can be shown $3x – 1 = 2x + 2$. Kieran (1992) compares terms *procedural* and *structural* which have same kind of meanings than arithmetic and algebraic equations defined by Filloy, Puig and Rojano. *Procedural* means arithmetical operations which are done with numbers when no algebraic expression is used. Then term *structural* means that operations are done with algebraic expressions, not in the numerical way. Banerjee and Subramaniam (2012) have a different approach to arithmetic and algebraic expressions. Instead of the whole expression being categorized to be arithmetic or algebraic, they divide the expression to terms which could include variables or numbers and then the terms can be algebraic or arithmetic. For example, the arithmetic term can be like $–5$ while the algebraic $–5x$. The researchers underline that the notation and structure are having effort to learning algebra.

2.2 Algebra at school

The national curriculum is the basis of teaching. Comparing the Finnish and the Norwegian curricula it is possible to find the same kind of context of algebra but there are also some differences. Both curricula include for example the connection between school math and everyday life and the meaningful use of different kind of tools. Also Filloy et al. (2008) mention the same aspects about curriculum; the important issue is to give to students opportunities to solve their daily life situations. The Finnish curriculum includes only the teaching context and the achievable knowing level but the Norwegian has also the definition of the context. In this case, algebra has been defined in the Norwegian curriculum as "algebra in school generalises calculation with numbers by representing numbers with letters or other symbols" (Kunnskapsløftet, 2010, p.3).

According to the Finnish curriculum, students have to be able to use algebra in different ways in the end of the lower secondary school. The curriculum includes solving first degree algebraic equations, simplification of the algebraic expressions, operations of powers, forming the simple equations from everyday life problems and then solving those with using algebra or deduction, using pairs of equations and also evaluation and revision of the calculations. (Opetushallitus, 2004.) The similar aims have also been presented by the Norwegian curriculum (Kunnskapsløftet, 2010).

In this chapter the traditional school algebra is presented and also the challenges of learning and teaching algebra are described.

2.2.1 Traditional school algebra

At school, when the teaching of algebra starts, students feel it being a new, strange and unconnected part of mathematics (MacGregor & Stacey, 1997). Mason (2011) brings out how the contemporary school algebra focuses on learning rules and procedures instead of algebraic language and solutions of everyday life situations when algebra at school is losing its ultimate though. Traditional school algebra is also a watershed: some students learn to use symbols to express relationships but not everyone (also Subramanian & Banerjee, 2011). Learning of algebra is motivated saying “algebra is needed ‘later’ or is good for you” (Mason 2011, p. 561). Kieran (2004) and Carraher, Martinez and Schliemann (2008) mention that school algebra has been viewed only as the science of equation solving while Schoenfeld (2008) describes it being the science of patterns. Moreover, also Kaput (2008) highlights how only a narrow view of algebra has dominated whole school algebra, for example, symbolic manipulations.

Reinhardtsen (2012) has found that most of the first 60 algebraic tasks in the textbooks used in Finland, Norway, Sweden and California (USA) are introductory tasks with patterns and sequence of operations or tasks with use of algebra as a language. These kinds of tasks are for example interpreting or formulating expression. Author highlights how the algebraic tasks of the textbooks are mainly focused on developing technical skills. According to Malisani and Spagnolo (2009), students have not been used, for example, to invent their own problems from the algebraic expressions. Students have been supposed to invent a possible situation using the equation $6x–3y = 18$ but they have just carried out syntactic manipulation of the equation.

2.2.2 Challenges while learning algebra

Teaching and learning algebra has been a challenging part of the mathematical lessons. According the new PISA- and TIMSS-results (OECD, 2009; Kupari, Vettenranta & Nissinen, 2012), Finnish and Norwegian students underachieve in the field of algebra comparing to the other fields of mathematics. The PISA studies have also shown how the Nordic students find algebra and measurement being the most difficult fields of mathematics (OECD, 2009). Also Finnish National Board of Education has taken attention to this difficultness of algebra (Opetushallitus, 2008).

Why algebra is undergone so difficult? MacGregor and Stacey (1997) describe some reasons for this experience: unfamiliar notation system, no analogy with everyday life, other school subjects and other parts of mathematics but also non-purposeful teaching material. The knowledge of algebra can consist of misinterpretation, correction of which can take years. Writers also underline how algebra is usually disconnected to the other areas of mathematics. That is why students feel algebra being difficult to connect other areas and they also forget the expression of the algebraic notations while algebra is not used. Hassinen (2006) mentions students seeing algebra only as hierarchic structure where learning one thing helps to learn something from the next level. Näveri (2009) describes how students are using too procedural way of working and they do not always achieve the conceptual thinking. While learning algebra, the benefits to use algebra are hidden so students feel no sense to use it.

According to Filloy et al. (2008), students can have problems to acquire new habits and notations; it does not happen spontaneously but it needs the time for change. Confronting algebra at first time, the transition from arithmetic to algebra can feel difficult to students. As Booth (1988, p. 306) has described "some seemingly simple ideas are not always as simple for students as they may seem to adults". Filloy, Rojano and Solares (2010) bring out the term *a didactical cut* which appears when students confront the algebraic problem at first time and they have to build new meanings for the objects and the operations. Herscovics and Linchevski (1994) also mention that there is *a cognitive gap* between arithmetic and algebra and it is characterized by "the students' inability to operate with or on the unknown" (p.75). It is needed to stay conscious about this gap and elaborate some methodological strategies in order to introduce algebra in a meaningful way.

Fillot et al. (2008) explain some algebraic situations which students have when they are starting secondary education. They have three different problematic: the reserve of multiplication syndrome, the ambiguity of the notation of equality and also the translating between natural language and algebra. The reserve of multiplication syndrome will exist when students just use trial and error as a solving method. Solving equation $Ax = B$ with this method can generate mistakes because of too large numbers of *B.* Still the unknown, *x*, can make students to be conscious what to do, because "it is something that is not known" (Filloy et al., 2008, p. 12). The ambiguity of notation of equality will be seen firstly as an arithmetic equality when students try to connect the terms of the right side or read them as a single term before they perform any operation or give answer. Secondly, they can think both the equality of the left side and the right side as a whole, like $x + A$ is in entirety same as $B + A$, when the thinking is more visual than arithmetic. Thirdly, they will compare the equality of equation term by term when they can solve it very quickly but also have polysemy of *x*: they do not feel *x* being same variable but it varies even in same equation. "This [first] x ($x + \frac{x}{4} = 6 + \frac{x}{4}$) equals 6 and these [$\frac{x}{4}$] ($x + \frac{x}{4} = 6 + \frac{x}{4}$) can be any number." (Filloy et al., 2008, p. 15.) Also Malisani and Spagnolo (2009) have found the same: students can use the same letter to represent different variables. They do not have any control of used symbols. Authors highlight making sense of letters being one of the considerable problems: sometimes it is seen as unknown, sometimes as variable. Students can also have difficulties in translation which means reading and writing algebraic expressions (Filloy et al., 2008; Malisani & Spagnolo, 2009). Österholm and Bergqvist (2012) underline also the demands of reading skills while solving mathematical tasks. If the reading skills of student are weak then the understanding of mathematical word problems can be challenging because of long words and nominalization which means the use of a verb or an adjective as a noun. For example *he reads* has been written as *his reading*, when the meaning-making of the sentence is more complicated.

Kilpatrick, Swafford and Findell (2001) describe school arithmetic being so answer-oriented that students do not recognize the whole meaning of the equal sign, the representation of relations. As they have written the equal sign is seen by students as a left-to-right directional signal. According to Kieran (1992, p.393), students read the equal sign as *it gives*. For example Kilpatrick et al (2001) have described tasks about money and one student has formulated an own equation as 2,30 + 3,20 = 5,50 – 1,50 = 4,00. Then this student has seen the equation as string of equations, not as equivalence.

Students also use unusual operating with numerical parts of the equations and have problem specifically with the minus sign. Those students usually ignore the whole minus sign and operate it as plus sign. This operation is called as *detachment of minus sign* by Herscovics and Linchevski (1994). Ayres (2001) has also found that student confront difficulties while solving tasks with the minus sign, brackets and larger numbers. Booth (1988) has recognized students tending to have a numerical answer. They do not accept that the answer can be non-numerical in algebra. They also conjoin the algebraic terms; some students think $2a + 5b$ being $7ab$. If a means *apple* and b *banana*, students are writing that 2 *apples* and 5 *bananas* are 7 *apples-and-bananas*. They have not understood that $2a + 5b$ cannot be simplified.

Banerjee and Subramaniam (2012), Kaljula (2012) and Linchevski and Livned (1999) have focused on the students' mistakes within manipulative operations. Linchevski and Livneh (1999) mention students being inconsistent with the structure of the expressions and the order of the operations; they cannot use commutative law, distributive law and associative law. Students calculate straight from left to right or they conjoin addition at first, before multiplication or substraction. For example, during their study, fifty percent of students calculate $50 – 10 + 10 + 10$ as $50 – 30$. This same aspect has been found also by Booth (1988) who describes students thinking $18 \times 27 + 19$ having same answer than $27 + 19 \times 18$. According to Banerjee and Subramaniam (2012), students use the commutative law not only to addition and multiplication but also to subtraction and division. They also evaluate, without calculations, that $34 + 31$ is more than $34 + 29$ but they use the same strategy to evaluate that $34 – 17$ is more than $34 – 16$. Authors have also found the conjoining errors when students think $2 + 3 \times x$ being $5 \times x$. Some students have also problems with bracketed expressions. For example, one student explains that $24 – 13 + 18 \times 6$ and $24 – (13 – 18 \times 6)$ are both 'equal' and 'not equal' depending on student's way to deal with the brackets: does the student open the brackets or measure the inside of brackets at first. Kaljula (2012) has focused on mistakes which students have been doing while solving linear equations. The author has found that minus sign before a fraction or parentheses has not been taken into account. Students have presented mistakes also in arithmetic while dividing, multiplying, adding and subtracting but also while combining terms.

Fairchild (2001) has done study about the transition between arithmetic and algebra. Author has found that students with weak arithmetic knowledge have been able to deal with simple algebraic questions but then they have confronted difficulties while solving more complex algebraic tasks. As a conclusion, Banerjee and Subramaniam (2012) Linchevski and Livneh (1999) and Fairchild (2001) underline how the mathematical structure is necessary to be learned in the arithmetic system before going to the algebraic system.

2.2.2 Challenges of teaching algebra

The learning of algebra is, of course, also subject to the teaching of algebra. Kieran (1992) mentions the text books having much influence to algebra teaching. That is why she underlines that if changes are wanted in the way of teaching algebra then the natural way is to modify the text books. Törnroos (2004) has studied the relationship between the mathematical text books and the knowledge of students. He mentions that there are huge differences, especially related to algebra. For example, one of the text books does not present power with the letters, so the users of this book have a common mistake: they claim $n \cdot n \cdot n$ being same as $3n$. According to the Finnish National Board of Education (Opetushallitus, 2008), text books are also having influence to the exam results: there can be identified correlation between text books and exam results in the Finnish schools. The other things which influence to teaching are teachers' understanding of their students' cognitions and their behavior but also teachers' own awareness of effectiveness of their teaching (Kieran, 1992).

Some researchers have shown how much influence teacher have to the students' acquiring of the algebraic knowledge. MacGregor and Stacey (1997) describe how the particular teaching approaches will reflect to some misunderstandings of students. They underline how teachers have to be aware of the different kind of beliefs that students have and also to interfere those while teaching. The knowledge of context of teaching subject has an influence on teaching of algebra. Huang and Kulm (2012) have done a study about the teaching knowledge of mathematics teachers. They find that the participated teachers have a weak content knowledge about the algebraic areas they will need for teaching at school. This is revealed from the representations of quadratic functions and the content of function. Several mistakes have also been done while teaching, for example with solving equations and doing manipulations and reasoning. Authors highlight that teachers also need a possibility to develop their own algebraic knowledge. According to the National board of Education (Opetushallitus, 2008), the Finnish teachers would want to have some vocational education and training of mathematics.

The teaching of new area of mathematics will usually happen via different kind of examples which will facilitate learning of students. Zodik and Zaslavksy (2008) describe examples what teachers do in and for mathematics lessons. They identify examples and determine mathematical correctness of those examples. Three different kinds of mistakes, which have been identified, are too generalized examples, wrong counter-examples and non-existing cases. Teachers have different kind of strategies to do and show examples: some of them will work better than other and some of them could even be misleading. It is important to remember that the most remarkable aim of examples is to support the learning of students. While starting a new area of mathematics, students usually use the graphic model which teacher has presented with examples. Those models are also visible in the written work of the students.

2.3 Developing school algebra

Many researchers have not only presented the situation of todays' school algebra but also the development ideas about what should be done at school to get students learn algebra better. According to Banerjee (2011, p.138), research of algebra education has been mainly focused on *(a) making sense of the symbols in algebra and operations on them using various kinds of methods; and (b) the utility of connecting arithmetic to algebra.* In this chapter, I will present different kinds of views including, for example, envisioned school algebra, algebraic thinking and early algebra.

2.3.1 Envisioned school algebra

Mason (2011) has presented the ideal, *envisioned* algebra teaching, which will not be a topic but "a maniputable language for expressing relations and constrain" (Mason 2011, p. 560). This school algebra is used for modelling situations and then it will connect the material world and the symbolic world of mathematics. Filloy et al. (2008) also underline the meaning of modelling. Authors mention the sign system of algebra having lots of effort to express relations between variables but also the complete cycle of corresponding didactic paths:

1. Translation of "concrete" situations or situations expressed in natural language (word problems) to algebraic code;
2. Analysis of relations between variables, based on manipulation of the algebraic expressions produced (syntactic level); and
3. Interpretation of the "concrete" situation in the light of results of the work with algebraic syntax. (Filloy et al., 2008, p.38)

Writers underline how step (1) gives meaning to algebraic expressions and then steps (2) and (3) expressions will become meaningful by syntactic manipulation. The real life connections and co-operating will motivate the learners more than traditional letter manipulation alone. This kind of change is necessary if algebra is wanted to be learned with understanding. (Mason, 2011.)

Moreover, Hassinen (2006) has designed an own IDEAA (Idea-based Algebra) –model where the main goal is to connect *street math* and *math math* instead of using artificial *school math* based by Resnick's (1995) model. Author explains *street math* being mathematics which is needed everyday life. *Math math* is the mathematics which the mathematicians use and which is the real aim of school math. *School math* is this mathematics which is learnt at school. Hassinen (2006) have also described the central elements of IDEAA-model. First, it is necessary to make clear that algebra is needed. Secondly, the central ideas and aims of algebra can be clearly visible and describable. Thirdly, it is possible to discuss about algebra; it is used as natural language, familiar expressions and normal deduction. Lastly, the teaching of algebra is not concentrated on inert, technical skills. (Hassinen, 2006.)

2.3.2 Algebraic thinking

Algebraic thinking is an important issue while talking about the learning of algebra but its development takes time. Herscovics and Linchevski (1994) underline students usually do not have enough time for acquiring the new algebraic knowledge; they start to use it without understanding. According to Mason (2011), only a few students will achieve the level of algebraic thinking because of so traditional school algebra. Schoenfeld (2008) defines algebraic thinking to consist of purposeful and effective use of symbols which is meaningful and sensible. It can be either syntactic operating with the symbols or the using of contexts and relations with a meaningful way. Kieran (2004) present how the development of an algebraic way of thinking could happen:

1. A focus on relations and not merely on the calculation of a numerical answer,
2. A focus on operation as well as their inverses, and on the related idea of doing/undoing,
3. A focus both representing and solving a problem rather than on merely solving it,
4. A focus on both numbers and letters, rather than numbers alone. This includes:
    - (i) working with letters that may at times be unknowns, variables and parameters
    - (ii) accepting unclosed literal expressions as responses
    - (iii) comparing expressions for equivalence based on properties rather than on numerical evaluation;

5. A refocusing of the meaning of the equal sign. (Kieran, 2004, pp. 140-141.)

Those issues are meaningful while learning algebra and they also clearly demonstrate the benefits of using algebra.

2.3.3. Early algebra

Hihnala (2005), Filloy et al. (2008), Näveri (2009) and Banerjee and Subramanian (2012) underline how arithmetic knowledge is really needed to achieve the algebraic knowledge. According to the writers, this operativity is situated on the level of pre-algebraic knowledge which is a level between arithmetic and algebraic knowledge. Linchevski (1995) defines pre-algebra as arithmetic with the algebraic structures. The author mentions that the typical algebraic themes as variables and simplification of algebraic expressions, generalization, structure, equations and word problems can be found from the numerical presentation. The same kind of aim has also early algebra which has usually been employed in the recent studies. According to Carraher, Schliemann and Schwartz (2008), early algebra is not algebra early. It is not same as traditional school algebra, which is learnt earlier, but it is a conjunctive link between arithmetic and algebra. Authors have defined it:

1. Early algebra builds on background contexts of problems.
2. In early algebra formal notation is introduced only gradually.
3. Early algebra tightly interweaves existing topics of early mathematics. (Carraher, Schliemann et al., 2008, pp. 236–237.)

The main goals of both pre-algebra by Linchevski (1995) and early algebra by Carraher, Schliemann et al. (2008) are to make the transition from arithmetic to algebra easier to students. Then students will be able to use the algebraic thinking and as Schoenfeld (2008, p. 506) highlights "students will no longer find algebra to be a new and alien body of subject matter". Students may understand that there are also many benefits in using algebra instead of purely arithmetic operations.

Carraher, Schliemann, Brizuela and Earnest (2006) have found how 8-9-year-old students have the ability to use unknown to describe relations. For example, they have used the number line to support students to find the relations. Researches describe how students use letters to represent variables with meaningful way and they are also able to use algebraic expressions to represent functions. Also, those students are able to operate expressions without having had algebraic procedure teaching.

> --- 'N + 3 – 5 + 4' is equal to N + 2, and to be able to explain, as many children were able to, that N plus 2 must equal 2 more than what John started out with, what ever that value might be --- (Carraher et al., 2006, p. 108).

This study of Carraher at al. (2006) shows how younger students, who have not had any algebra teaching yet, are able to use and represent algebraic variables and relations. Also Britt and Irwin (2011) have had same kind of results about the ability of younger students. They have noticed that students are able to use algebraic structure of operations when they are solving arithmetic problems. Authors describe how they use $x + y = (x + a) + (y – a)$ with addition and $x - y = (x + a) - (y + a)$ with subtraction. They present also an example of how it works in practice: 47 + 25 = 50 + 22 = 72 (Britt & Irwin, 2011, p. 146). If this equation is written open, it can be 47 + 25 = (47 + 3) + (25 – 3) = 50 + 22 = 72. Authors prefer to have arithmetic numbers instead of algebraic letters which are not familiar for those students. As Banerjee (2011) has also explained that the world of algebra and algebraic thinking can open easier by using arithmetic context to identify, generalize and justify.

According to Linchevski and Livneh (1999), the learning of algebra is recommended to start from the structural properties of the expressions. It will develop students' *structural sense* so they are able to use an expression with creative and flexible way. In addition, the learning of the new abstract context will be facilitated if, at first, students learn the structure of expressions with familiar context of numbers. Banerjee and Subramaniam (2012) mention the use of structural similarity between arithmetic and algebraic expressions instead of precedence rules. Then the introduction of algebra could connect better to the arithmetic knowledge which has been learnt earlier; the awareness of structure helps students to understand better, which will facilitate understanding of rules and procedure. For example they advise students to mark one term, simple term (e.g. +12) or product term (e.g. $+3 \times 5$ or $+2 \times a$), as a rectangular box. This box can include simple or complex terms; the complex term includes product term and bracket term, like – (3 + 5). After using this kind of method researchers have observed that mistakes of students have decreased and students have been able to explain their procedures better than earlier both in arithmetic and algebra. They have understood better the equality of expressions and have used more flexible procedures.

2.3.4 Some more perspectives to the development of school algebra

In this chapter, some more studies related to the development of school algebra are presented. The study of Fairchild (2001) has found that students benefit when representation of arithmetic and algebraic expressions has been done by area and its geometric representation.

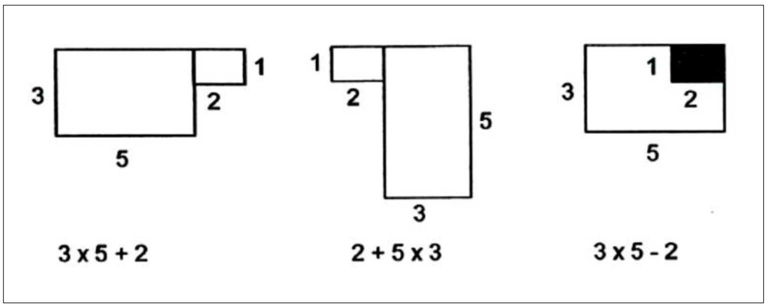


Figure 1: Arithmetic expressions as geometric pictures (Fairchild, 2001, p. 7)

Students who participated in the research were divided in two groups: to the experimental and the control group. The students also did two tests, both pre-tuition test and post-tuition test. Fairchild found that the experimental group got 33,6 % better results from the post-tuition test than from the pre-tuition test while the control group developed only 8,5%. This kind of geometric approach has also been presented by Filloy, Puig and Rojano (2008).

Due to the fact that the text books are also having a huge influence to mathematics teaching and learning (Kieran, 1992; Törnroos, 2004; Opetushallitus, 2008), the one way to develop teaching of algebra is to modify the text books. For example, Reinhardtsen (2012) has suggested that the tasks of text books should have more connection to geometry and problem solving instead to concentrate on technical skills.

Curriculum defines which the bases of learning mathematics at school should be. According to Kilpatrick (2011), it is not necessary to introduce new topics to getting better understanding in teaching and learning of algebra but rather new approaches to those topics which are presented in the curriculum. The meaning of curriculum is not to be a topic list but the list of experiences. The content of the Finnish and Norwegian curriculum is in fact connected to this view. In the curricula it is stated what students are supposed to know, but the transformation to practice is up to teachers in the Nordic countries. The teachers have possibilities to use new approaches and more creative ways to teach if they want to change their teaching: it is not limited by curriculum or other instructions.

3 Methods

The aim of this chapter is to present and describe the methodological approach of this study. At first, I will introduce the choice of methods, and secondly, the research design. Lastly, I will demonstrate the analysis, which is used in this study, in detail.

3.1 Comparative study

This is a comparative and cross-cultural study of two different countries: Finland and Norway. The whole VIDEOMAT-project deals with Finland, Norway, Sweden and California, USA but in connection with the limitation of this study (30 ECTS) only the two countries mentioned have been chosen for the analysis. Finland and Norway are interesting cases related to the contrast between the Nordic countries and the learning results of PISA and TIMSS: Finnish students have had the highest performance of all the European countries in both assessments (Savola 2008; OECD, 2009; Kupari, Vettenranta & Nissinen, 2012). The Finnish participants in the present study are from Swedish area of Finland and they have significantly lower results than Finnish-speaking students but their results are still higher compared to the other Nordic countries (Kupari, 2012).

Some international comparative studies about mathematics education have earlier been done. Johansen (2009) has studied the questions proposed by teachers to students in the mathematical classrooms in Finland and Norway. He has found that the Finnish teachers ask more “higher order” –questions than the Norwegian teachers. Savola (2008) has done a research between Finland and Iceland in the connection of video analysis. The aim of the study has been to analyse the effectiveness of video analysis but also the structure of mathematical lessons in both countries. The author has found that typical Finnish lesson has the structure called *Review – Lesson – Practice* while the Icelandic lesson has more *Independent learning* –strategy. In addition, Clarke, Keitel and Shimizu (2006) have edited a book about the comparative international studies in the context of mathematics classrooms in twelve countries including, for example, China, Japan, Sweden and the United States of America so this topic has been recently on view on the research field.

The comparative study has been defined by Bryman (2008) as design in which researcher will have contrasting cases using identical methods. Bryman (2008, p.58) describe how Hantrais (1996) has defined it more in detail: “The aim [of a comparative study] may be to seek explanations for similarities and differences or to gain a greater awareness and a deeper understanding of social reality in different national contexts”.

In this case, the comparative study is also cross-cultural because it is dealing with two different teaching and learning cultures in the Nordic countries. According to Cai and Moyer (2008), with international comparative studies it is possible to learn some developing ideas from other countries. Clarke, Emanuelsson, Jablonka and Mok (2006, p. 2) highlight that “international comparative research has an additional power: the capacity to reveal similarity within difference, structure within extreme diversity”. In the national context of the research many aspects can be left unrecognized. Hiebert et al. (2003) describes the benefits of comparative studies being:

1. Reveal one's own practices more clearly
2. Discover new alternatives
3. Stimulate discussion about choices within each country
4. Deepen educators' understanding of teaching (Hiebert et al., 2003, pp. 3–4.)

Clarke, Emanuelsson et al. (2006) add that the main goal of the international comparative studies is to be beneficial especially for all students in the different countries. Those ideas are easily connected to the VIDEOMAT-project because the aims of the whole project are, at first, the comparative analysis of teaching and learning algebra, and then secondly, the development of the teaching (Kilhamn & Röj-Lindberg, 2013). The empirical part of this master thesis is focused on the first part of the project.

3.2 The research design

Next, the design of this study is presented. It includes data collection, description of participants and teaching material and also some reflections about the reliability of this study.

3.2.3 The data collection

The empirical data of this research has been collected from Finland and Norway during the spring 2012. The entire material of the VIDEOMAT has already been collected and it includes videotaped lessons, teachers' interviews and lesson files and students' written work. This study concentrates to the students' written work due to the language issues.

In this case, the empirical material includes 110 students' work: notebooks or small tests. Those tasks students have solved during the mathematics lessons and, in addition, most of the tasks are from their own text books. The approximate amount of the tasks of one student is 11, so around 1200 tasks have been gone through during this study.

3.2.1 The participants

The participating students are aged 12-13 and from the public schools. The students from Finland are from Finnish-Swedish area so they are using Swedish at school. The Finnish students are at 6^{th} and 7^{th} grades but the Norwegian are at 7^{th} and 8^{th} grades because they have started their school one year earlier.

The Finnish 6^{th} grade classroom is taught by general education teacher. In addition, both the Finnish 7^{th} grade classroom and the Norwegian 7th and 8^{th} grade classrooms are taught by general education teachers who have mathematics specification. The Norwegian teachers have also qualification to teach sciences (physics, chemistry and biology) while the Finnish 7^{th} grade teacher has some studies about IT and, moreover, physics-chemistry subject, which is taught at Finnish primary school. Both Finnish teachers have also the master's degree in education.

The classrooms differ quite a lot among the countries. The Finnish 6^{th} grade classroom includes 27 students who are divided into a group of 13 and 14 students during the math lessons while the 7th grade has 21 students. The Norwegian 7th grades are both classes of 20 students and 8th grade classrooms have 23 and 25 students. The amount of collected notebooks and tests is smaller than the entire amount of students in each class.

3.2.2 The teaching material

The tasks, which students have solved, are mostly presented in their text books. In addition, some tasks solved by students are from teachers' own material. For example, the tasks solved by Finnish 7th grade students are inspired by *Pi 7* but not same tasks which are presented in the text book. The tasks, which students have solved during data collection, are focusing on the introduction of algebra.

The textbooks which are used in Finland and Norway are:

FINLAND	6.grade	Asikainen, K., Fälden, H., Nyrhinen, K., Rokka, P., Vehmas P., Törnroos, S. & Westerlund, K. (2008). *Min matematik 6*. Helsinki: Schildts förlag.
	7. grade	Heinonen, M., Luoma, M., Mannila, L., Tikka, T., Lindgrén, H., Mitts, H. & Söderback, C. (2010). *Pi 7*. Helsinki: Schildts förlag.
NORWAY	7. grade	Pedersen, B. B., Pedersen, P. I., Skoogh, L., Johansson, H. & Ahlström, R. (2007). *Abakus 7b*. (3. edition). Oslo: Aschehaug & co.
	8. grade	Hjardar, E. & Pedersen, J.-E. (2006). *Faktor 1*. Oslo: Cappelen Damn.

In this study, those books are only mentioned because the comparison between the algebraic tasks of those text books has already been done by Reinhardtsen (2012) who has compared, moreover, also some Swedish and US textbooks.

3.2.4 Reliability

According to Bryman (2008), the researcher has to consider that the empirical data is comparative. Although this empirical material is huge with 1200 analysed tasks, the nature of the research is more qualitative than quantitative. The aim of this study is to describe the richness of the students' written work. The tasks, which are compared, are different in both countries but the main goal of analysis is to find tasks with same nature. Then those tasks are compared.

In addition, dealing with international comparative study the translation has to be carried out competently (Bryman, 2008). In this study, the students' written solutions have been chosen to be compared because of language issues. The mathematical notations are quite same despite differing languages so the comparison is not dependent of translation. In addition, the natural language used while solving mathematics tasks has been recognized to be quite simple to understand without having to be a native language user. The data analyses are explained in detail in the next chapter.

3.3 The analysis

The aim of this part is to present the analytical approach to the data analysis. First of all, the steps of analysis will be introduced as well as the elaboration of the coding system including an example of the coding table. Then, the compared tasks from Finland and Norway will be presented.

This analysis has been created during the study. Due to the huge amount of the empirical material, the clear analysis has been needed. Five steps have been identified:

I: Reading the notebooks/tests of the students (checking over the original tasks)

II: Creating the coding system

a) *Procedures*
b) *Mistakes*

III: Creating tables according to the codes

IV: Comparison of the classes and the tasks

V: The final comparison between the Finnish and Norwegian students

In the following chapters, I will describe every step of analysis in detail.

3.3.1 Reading the notebooks of students

The material includes algebra tasks from 12 notebooks and 14 tests from Finnish students and 84 notebooks from Norwegian students.

Country	*Grade*	*Amount of classes*	*Amount of written work*
Finland	6	1	12
	7	1	14
Norway	7	2	38 (19 + 19)
	8	2	46 (22 + 24)

The material from Finnish 6^{th} grade class is quite small; there are only 12 notebooks of students from the classroom. The reason is that the class has been divided into two groups during the mathematics lessons and the material has been collected only from one group. The material from the Finnish 7^{th} grade is also small; only 14 tests have been collected. The number of the Norwegian notebooks is greater; 19 notebooks have been collected from both 7^{th} grade classes and 22 and 24 notebooks from 8^{th} grade classes. Therefore, the total amount of the students' written work from Finland and Norway is 110 notebooks and tests. The material from classrooms consists of notebooks with 1–10 pages expect Finnish 7^{th} grade which has a small test of one page. Approximate mean amount of tasks solved by one student during the data collection is 11.

The Norwegian material has been analysed first. During the analysis, the 7^{th} grade material has been recognized not to be comparable in this study because of the different nature of the tasks. Because only two Norwegian classrooms have been comparable then only two classrooms from Finland have been analysed. The Finnish empirical material of whole VIDEOMAT has been collected from three 6th grade classrooms and one 7th grade classroom. Thus, one 6th and one 7th grade classroom have been chosen to be compared with the Norwegian 8th grade classrooms. Tasks are connected to the context so while reading the tasks the checking over with original tasks has been done to understand the solution procedures and mistakes.

3.3.2 Creating the coding system

Many researches have already studied the procedures and the mistakes which students do while solving algebraic tasks. As mentioned and described earlier, Kieran (1992), Herscovics and Linchevski (1994) and Carraher, Martinez and Schliemann (2008) have focused on the solution process, Linchevski and Livneh (1999) on the choice of the first operation and Banerjee and Subramaniam (2012) on the structure. In addition, Booth (1988) has described the conjoining errors, MacGregor and Stacey (1997) difficulties which students have writing an expression, Linchevski and Livneh (1999) the order of operations, Ayres (2001) brackets and the minus sign and Banerjee and Subramaniam (2012) the commutative law. The findings of those studies have been used for the analysis and many new codes have been created to handle the present empirical material.

At first, the solution procedures have been analysed. The procedures have been divided in five categories according to answer, representation, calculation, substitution and others.

ANSWER
AA = answer absent
AP = answer partial
AO = answer only
AE = answer estimation
AD = answer doubtful

REPRESENTATION
RA = representation atypical
RD = representation with decomposition
RE = representation as expression/equation
RI = representation incorrect (answer correct)
RG = representation with grouping
RN = rounding the numbers
RR = representation of review
RS = representation of equation solving
RW = representation with words

CALCULATION
CD = calculations directly
CS = calculations separately
CR = calculations following rules

SUBSTITUTION
SD = making the substitution directly
SF = making the substitution to formula
SR = substitution with review
SE = substitution before expression

OTHERS
DP = drawing a picture
FM = finding a measure
FR = finding a rule
FS = finding similar picture

Figure 2: The identified procedures

Concerning mistakes, they have also been divided to five categories: unit of measure, operations, representation, forming equation and expressions and others.

UNIT OF MEASURE PROBLEMS

UA = unit of measure absent
UU = unit of measure unnecessary
UT = unit of measure transformation
UE = unit/ unit of measure erroneous

OPERATIONS

OB = operations with brackets
OC = operations connected
OE = operations of equation solving
OM = operation misinterpreted
OO = operation order
OP = operations with powers
OR = operations read from left to right
OS = operations with minus sign

REPRESENTATION OF TASK/ ANSWER

RC = representation with capital letters
RO = representation with wrong/without operation signs
RP = representation by a picture
RT = representation of task
RU = representation unclear
RV = representation of variable

FORMING EQUATIONS AND EXPRESSIONS

EE =writing an expression as an equation
EI = expression/equation incorrect

OTHERS

MC = miscalculation
MT = misinterpretation of task
MP = missing part of task
NA = no answer

Figure 3: The identified mistakes

The codes with details have been included in the appendices (Appendix 1, 2).

3.3.3 Creating a table according to the codes

The ready and detailed coding system has helped to make the tables for comparison. At first, every task has had one code for procedure but due to the fact that some tasks include smaller items (for example a,b,c) and students are not clear while solving tasks, some tasks have several codes. In addition, students have presented different kinds of mistakes during one task. In this respect, each task can have more than one code.

PROCEDURES - NORWEGIAN 8th GRADE - SCHOOL 2 - CLASS 2																			
Student	**Task**																		
	6.1	**6.2**	**6.3**	**6.4**	**6.5**	**6.6**	**6.7**	**6.8**	**6.9**	**6.10**	**6.11**	**6.12**	**6.13**	**6.16**	**6.17**	**6.18**	**6.19**	**6.20**	**6.21**
FA	CS	CS	CR	CR	CR	CD,CR	RN	RE		RE	RE			SD			SD	CD	CD
FB	CR	CR	CR				RE,RW		RE	RE	RE			SD			SF	CD	CD
FC	CR	CR	CR				RE		RE	RE	RE			SD			DP,SF	CD	
FD	CR	CR	CR	CR	CR		RE,RW			RE	RE			SF			SD	CD	CD,RD
FE	CR	CR	CR	CR	CR		RI			RE									
FF	CR	CR	CR				RE			RE									
FG	CR	CR	CR				RA			RE	RE			SF		RE		CD	CD
FH	CR	CR	CR	CR	CR	CR,RA	RN			RE	RE			SF			RI	RA	RA,CD
FI	CR	CR	CR	CR			RE			RE	RE			SF			SF	CD	CD
FJ	CR	CR	CR				RE			RE	RE			SD			RA	CD	CD
FK	CR	CR	CR						RE,RW										
FL	CR	CR	CR				RI			RE,RW	RE,RW								
FM	CR	CR	CR				RE,RW			RE	RE								
FN	CR	CR	CR				RE			RE	RE			SD			SF	CD	CD
FO	CR	CR	CR				RE,CD		RE	RE,RW	RE,RW			AO					
FP	CR						RI		RE,RW	RE									
FQ	CR	CR							RE	RE	RE	RE							
FR	CR,RI	CD	CD				RN		RE	RE	RE			SD					
FS	CR	CR	CR				RE												
FT	CR	CR	CR						RE										
FU	CR	CR	CR	CR	CR		RE			RE	RE			SF					
FV			CR				RE			RE	RE								
FW	CR	CR	CR	CR			RE		RE	RE	RE	RE	RE	SD	SD				
FX	CR,RI,AO																		
	CS	CS	CR	CR	CR	CD	RN	RE	RE	RE	RE	RE	RE	SD	SD	RE	SD	CD	CD
	CR	CR	CD			CR	RE		RW	RW	RW			SF			SF	RA	RD
	RI	CD				RA	RW							AO			DP		RA
	AO						RI										RI		
							RA										RA		
							CD												

Figure 4: Solution procedures of the Norwegian 8th grade

This table is just an example; the others have been placed in appendices (Appendix 3 – 14).

3.3.4 The comparison of the tasks and the classes

In the beginning of the task comparison, the interest has been to compare the tasks with several different kinds of procedures and mistakes. Those tasks could have been thought to be the most interesting cases. Due to the comparison between Finland and Norway, this has been recognized impossible because they have different tasks. As similar tasks as possible have been chosen from Finnish 6th and 7th grade and Norwegian 8th grade. The Norwegian 7th grade classes had to be left out from the final comparison because the nature of the tasks is so different compared to the other classes from Finland and Norway. After choosing tasks, five suitable categories have been found. Those categories are:

1. *Writing an arithmetic expression from a word problem (pre-algebra/ early algebra)*
2. *Writing (and solving) an algebraic expression from a word problem*
3. *Simplifying expressions with variables*
4. *Writing an expression from a geometric picture*
5. *Substitution to a formula*

Next, the tasks from both countries, which will be compared later, are introduced.

1. Writing an arithmetic expression from a word problem

In this category, tasks are constituted of word problems in which students have to write an arithmetic expression and also a numerical answer. According to Britt and Irwin (2011), it is important to support algebraic thinking, as creating the expressions, within arithmetic before algebraic expressions with variables and unknown. Also Banerjee and Subramanian (2012) highlight the meaning of arithmetic structure because it can also be the reason for problems with algebra.

Task from *Finnish* 6th Class (Teacher's material)

3. Beteckna och räkna. Ringa in rätt svar.

a. Vilket tal får du om du först subtraherar talet 5 från talet −8 och sedan subtraherar talet 7 från differensen?

Svar: ___________

b. Vilket tal får du om du först subtraherar talet 12 från talet 7 och sedan adderar talet 6 till differensen?

Svar: ___________

c. Vilket tal får du om du till talet −9 adderar talet 15 och sedan subtraherar talet 12 från summan?

Svar: ___________

d. Vilket tal får du om du till talet −6 adderar dess motsatta tal och sedan subtraherar talet 9 från summan?

Svar: ___________

−20 −9 −6 −3 1

Task from *Norwegian* 8th class (Faktor 1)

6.7 Sara kjøper 3 liter brus til 9,90 per liter, 2 poser potetgull til 14,90 per stk. og 3 hg smågodt til 16 kr per hg.
Sett opp og regn ut tallutrykket som viser hvor mye hun må betale.

An important issue to mention is that the same kind of task has been solved by students who are one year older in Norway than in Finland.

2. Writing (and solving) an algebraic expression from a word problem

The next step after arithmetic expression is creating the algebraic expressions according to the instructions. In these cases the students have to know the operations and also read the tasks very carefully. In the Finnish task, the students have to solve the tasks to get the value of the unknown and then check their solution with numbers available below. However, the Norwegian task only requires the expression. The interesting issue is whether students have problems with the correct order of the terms as MacGregor and Stacey (1993, 1994) have recognized.

Task from *Finnish* 6th class (Min matematik 6)

29. Skriv ekvationen och beräkna värdet på x. Kontrollera ditt svar.

a. Talet x multiplicerat med talet 4 är lika med 32.
b. Talet x multiplicerat med talet 6 är lika med 42.
c. Talet 5 multiplicerat med talet x är lika med 15.
d. Talet 13 multiplicerat med talet x är lika med 169.
e. Talet x multiplicerat med talet 14 är lika med 420.
f. Talet 15 multiplicerat med talet x är lika med 240.

3 7 8 13 16 17 30

Task from *Norwegian* 8th class (Faktor 1)

6.9 Skriv et uttrykk for
a) summen av *x* og 3
b) differansen mellom *x* og 5
c) 10 mer enn *y*
d) 12 mindre enn *y*

Also in this case, the Finnish students are one year younger than the Norwegian students.

3. Simplifying expressions with variables

The third category includes tasks which are related to a process of simplification. Students have to be able to deal with two different kinds of variables and the minus sign. In addition the Finnish students also have one numerical value. The difficulty of the minus sign has been studied earlier: Ayres (2001) have found students having systematically errors with the minus sign.

Task from *Finnish* 7th class (Teacher’s test)

4. Förenkla

a) $4k + 3r - 2k + 1 + r$

b) $-x - 6x - 2y - y + 3y$

Task from *Norwegian* 8th class (Faktor 1)

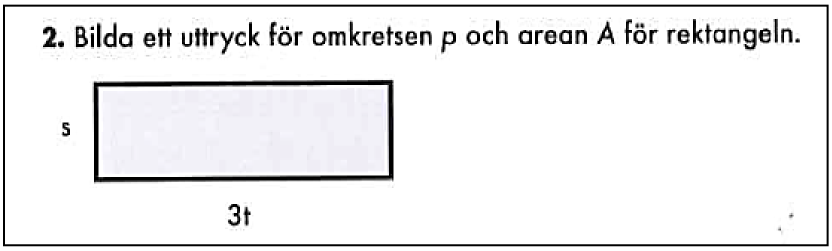
6.24 Trekk sammen.
a) $3x + 4y - 3x$
b) $-4a - 2b - 2a - b$
c) $3n - 3n + 2m - 2m$
d) $12x + 14x + 24x$

The tasks are similar by nature but the Finnish task is more difficult with a more complex structure while the Norwegian task has a simpler structure.

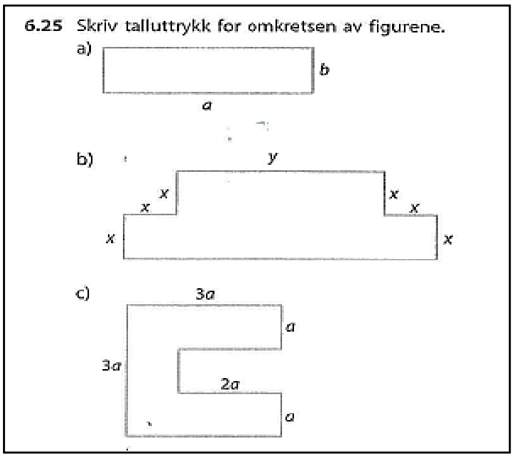
4. Writing an expression from a geometric picture

This category has tasks in which students have to create an algebraic expression from the geometric picture. For example, Banchoff (2008) has highlighted the meaningful connection between algebra and geometry. In this case, students are advised to write an expression for perimeter and in the Finnish task they are also supposed to write an expression for area, in this way increasing the complexity of the task (from one dimension to two dimensions) as shown in the example below.

Task from *Finnish* 7th class (Teacher's test)



Task from *Norwegian* 8th class (Faktor 1)



5. Substitution to a formula

The fifth category includes two tasks which issue is the substitution to the algebraic formula in the context of perimeter and area. The tasks differ slightly because in the Finnish task students have created the formula but in Norway the formula has been given.

Task from *Finnish* 7th class (Teacher's test)

3. Beräkna omkretsen och arean av rektangeln i föregående uppgift då $s = 1,5$m och $t = 2$ m.

Task from *Norwegian* 8th class (Faktor 1)

6.19 Formelen for omkretsen av et rektangel er $O = 2a + 2b$, der O står for omkretsen, a for lengden og b for bredden av rektangelet.
Regn ut omkretsen av rektangelet når
a) $a = 8$ cm og $b = 6$ cm
b) $a = 12$ cm og $b = 7,5$ cm

3.3.5 The comparison among countries

After choosing the tasks, the comparison has been done. The coded tables (4.1.3) have been gone through and one example of solution from each category has been chosen to be presented. At first, the procedures of each task will be introduced and then the mistakes as well. This comparison will be presented in the next chapter as findings.

4 Findings

The findings are presented in three parts according to the research questions: related to the procedures, related to the mistakes and then the final comparison between Finland and Norway. The procedures and mistakes of the students are demonstrated in tables. At first, the tasks are introduced and then the procedures/mistakes which are identified from the tasks are presented with the solutions of the students. In addition, the code of the student is also visible on the right side of the table. The meaning of the tables is to demonstrate the richness of the empirical material: quantitative analysis is not possible because the amount of the students differs according to the country.

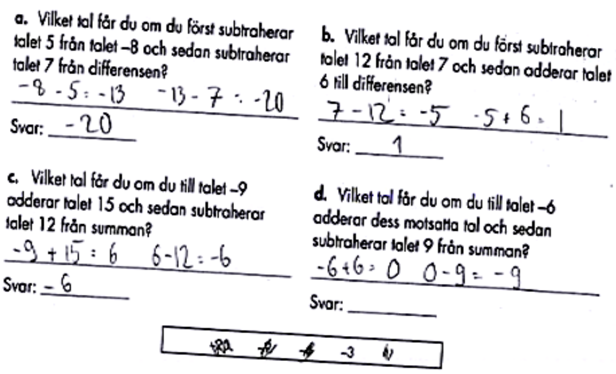
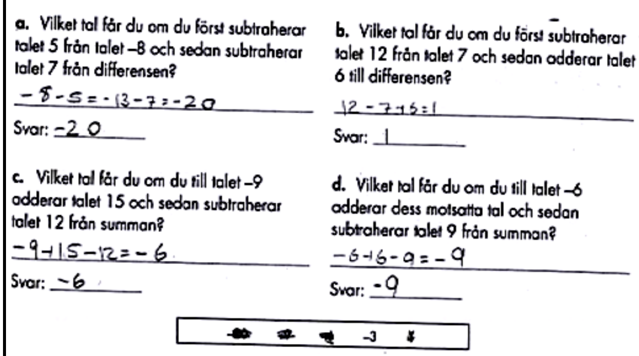
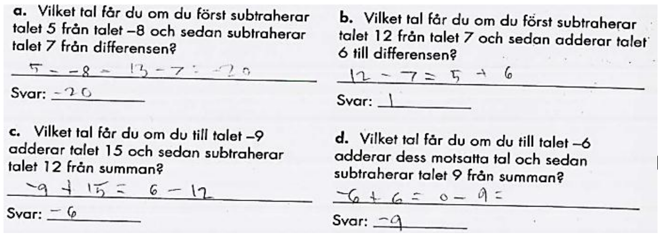
4.1 The identified procedures

The first research question is focused on the procedures which students use solving the tasks. The procedures are presented according to the categories introduced earlier and the most interesting or typical solutions have been chosen. Some solutions can have more than one procedure. In fact, some students have used different kinds of procedures while solving the tasks. Some of the tasks have several items, some students have solved these items in different ways.

4.1.1 Procedures while writing an arithmetic expression

Below the procedures of the tasks of the first category will be presented: at first, the Finnish procedures and then the Norwegian ones. The aim of the first category tasks is to write an arithmetic expression. The tasks differ slightly: the Finnish one is only dealing with numbers while the Norwegian one has also the currency unit.

Table 1: The procedures of the first Finnish task

3 *. Betekna och räkna. Ringa in rätt svar.*		
Procedure	***Solution***	
RE	 a. Vilket tal får du om du först subtraherar talet 5 från talet –8 och sedan subtraherar talet 7 från differensen? −8 − 5 = −13 −13 − 7 = −20 Svar: −20 b. Vilket tal får du om du först subtraherar talet 12 från talet 7 och sedan adderar talet 6 till differensen? 7 − 12 = −5 −5 + 6 = 1 Svar: 1 c. Vilket tal får du om du till talet –9 adderar talet 15 och sedan subtraherar talet 12 från summan? −9 + 15 = 6 6 − 12 = −6 Svar: −6 d. Vilket tal får du om du till talet –6 adderar dess motsatta tal och sedan subtraherar talet 9 från summan? −6 + 6 = 0 0 − 9 = −9 Svar: −3	CK
RE + RI	 a. Vilket tal får du om du först subtraherar talet 5 från talet –8 och sedan subtraherar talet 7 från differensen? −8 − 5 = −13 − 7 = −20 Svar: −20 b. Vilket tal får du om du först subtraherar talet 12 från talet 7 och sedan adderar talet 6 till differensen? 12 − 7 + 6 = 1 Svar: 1 c. Vilket tal får du om du till talet –9 adderar talet 15 och sedan subtraherar talet 12 från summan? −9 + 15 − 12 = −6 Svar: −6 d. Vilket tal får du om du till talet –6 adderar dess motsatta tal och sedan subtraherar talet 9 från summan? −6 + 6 − 9 = −9 Svar: −9 −3	CI
RI	 a. Vilket tal får du om du först subtraherar talet 5 från talet –8 och sedan subtraherar talet 7 från differensen? 5 − −8 = 13 − 7 = −20 Svar: −20 b. Vilket tal får du om du först subtraherar talet 12 från talet 7 och sedan adderar talet 6 till differensen? 12 − 7 = 5 + 6 Svar: 1 c. Vilket tal får du om du till talet –9 adderar talet 15 och sedan subtraherar talet 12 från summan? −9 + 15 = 6 − 12 Svar: −6 d. Vilket tal får du om du till talet –6 adderar dess motsatta tal och sedan subtraherar talet 9 från summan? −6 + 6 = 0 − 9 = Svar: −9	CF

As observed on the table above, the Finnish students have used three kinds of procedures to solve the task: representation as expression, incorrect representation and both. The right solution has been presented as one expression, as student CI has done item c), or two separated expressions (student CK) in all items. Many students, who have solved this task, have done it incorrectly using equal sign as *right to left* sign as student CF has done item c).

Table 2: The procedures of the first Norwegian task

6.7 Sara kjøper 3 liter brus til 9,90 per liter, 2 poser potetgull til 14,90 per stk. og 3 hg smågodt til 16 kr per hg.
Sett opp og regn ut tallutrykket som viser hvor mye hun må betale.

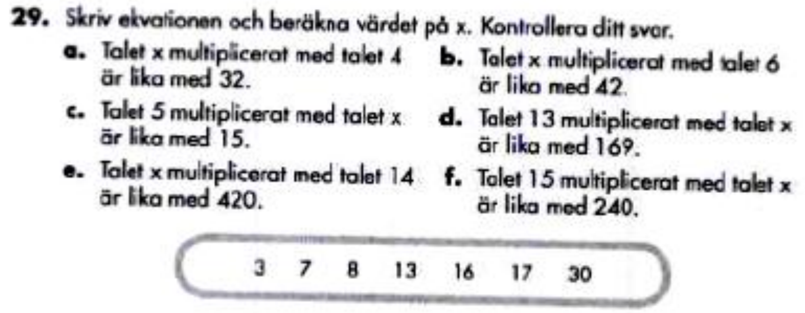
Procedure	***Solution***	
RE	6.7 | 3·9,90 + 2·14,90 + 3·16 29,7 + 29,8 + 48 = 107,5 kr	EC
RE + CD	3·9,90 + 2·14,90 + 3·16 107,50 kr	FO
RN	6.7) (3·10) + (2·15) + (3·16) = 108	EO
	3 l brus = 30 kr 2 p potetgull = 30 kr 3 hg smågodt = 48 kr 3·10 + 2·15 + 3·16 = 108 kr	FH
RE + RW	6.7 9,90·3 + 14,90·2 + 16·3 = 107,50 Hun må betale 107,50 kr!	FB
	(3·9,90) + (2·14,90) + (3·16) = 29,7 + 29,8 + 48 = 107,5 Hun må betale 107,5 kr.	FD
RI	6.7) 3·9,90 = 29,7 + 2·14,90 = 29,8 + 3·16 = 48 29,7 29,8 48 = 107,5	FP

The Norwegian students have used five kinds of categories: representation as expression, calculations directly, rounding the numbers, representation with the words and incorrect representation. Some of procedures are the same than those used in Finland, for example, the representation as expression. Some Norwegian students, for example EO, have rounded the numbers as it is usually done in everyday life. This is interesting, although this kind of way is not mathematically recommended. In addition, few students have written the answer separately with the words as students FB and FD have done. Also some incorrect representations are found, like connecting the calculations with the equal sign (student FP), although students still have got the right solution.

4.1.2 Procedures while writing an algebraic expression/equation

The tasks of the second category focus on writing algebraic expression or equation. The Finnish students are also supposed to solve the equation and control the right answer.

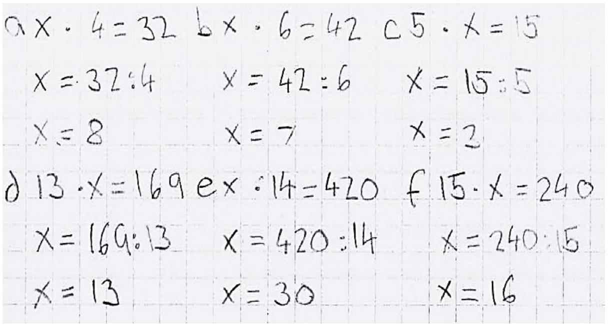
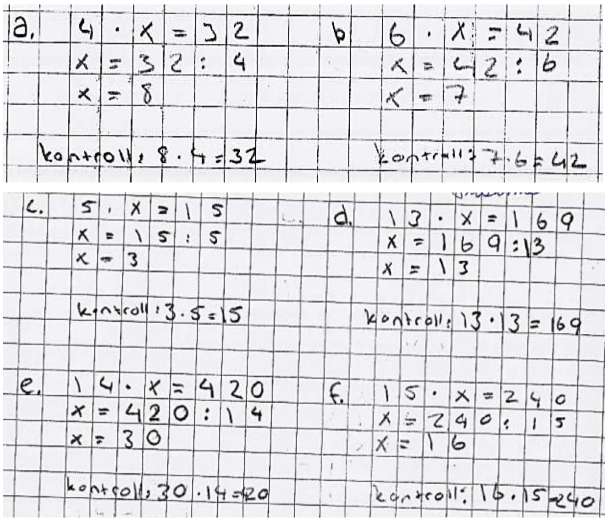
Table 3: The procedures of the second Finnish task



29. Skriv ekvationen och beräkna värdet på x. Kontrollera ditt svar.

a. Talet x multiplicerat med talet 4 är lika med 32.
b. Talet x multiplicerat med talet 6 är lika med 42.
c. Talet 5 multiplicerat med talet x är lika med 15.
d. Talet 13 multiplicerat med talet x är lika med 169.
e. Talet x multiplicerat med talet 14 är lika med 420.
f. Talet 15 multiplicerat med talet x är lika med 240.

3 7 8 13 16 17 30

Procedure	***Solution***	
RE + RS	 	AD
RE + RS + RR	 	AI

At this task the Finnish students present two kinds of categories. Due to the many-sided assignment, students have used two or three procedures which are representation as expression, representation of equation solving and representation of review. Most of the students have been able to write the equations and solve it and some has also controlled the solution with a separated calculation as student AI has done. Students are supposed to do this control and it makes them to feel sure with their calculations. One point which has been found from the material is that some students have done the same task twice: at first only with answer and then later with the calculations. It supports the idea that the Finnish students are supposed to control their answer substituting the *x* by its numerical value.

Table 4: The procedures of the second Norwegian task

6.9 Skriv et uttrykk for
a) summen av *x* og 3
b) differansen mellom *x* og 5
c) 10 mer enn *y*
d) 12 mindre enn *y*

Procedure	***Solution***	
RE	6,9) a) $x + 3$ b) $x - 5$ c) $y + 10$ d) $y - 12$	EQ
RE + EW	6.9 a) $x + 3$ = naboens alder b) $x - 5$ = Mammas alder c) $y + 10$ = Pappas alder d) $y - 12$ =	ET
	6.9) a) $x + 3$(Summen) b) $x - 5$(Differansen) c) $y + 10$ (pluss) d) $y - 12$(Minus)	FP

The Norwegian task is of the same kind but students only need to write the expression. They have two kinds of categories while solving the tasks: representation as expression and the same with the words. The interesting point is how some students have invented explanations for the expressions: student ET has explained the expression as the age of the neighbor/mother/father and student FP has described the used operations like addition and subtraction. Those students maybe try to make clear to themselves the meaning of expression; the expression is not just letters and numbers but it really means some relation between the variables.

4.1.3 Procedures while simplifying

The tasks of the third category are so similar by nature that students from both Finland and Norway have used the same procedures. The Finnish task is slightly more difficult than the Norwegian one. Students from both countries are supposed to simplify the algebraic expression.

Table 5: The procedures of the third Finnish task

4. Förenkla

a) $4k + 3r - 2k + 1 + r$

b) $-x - 6x - 2y - y + 3y$

Procedure	***Solution***	
CD	a) $4k + 3r - 2k + 1 + r = 2k + 4r + 1$ b) $-x - 6x - 2y - y + 3y = -7x + 0$	BF
RG	**4. Förenkla** a) $4k + 3r - 2k + 1 + r$ $4k - 2k + 3r + r + 1 = 2k + 4r + 1$ b) $-x - 6x - 2y - y + 3y$ $-7x + 0 = -7x$	BD
RG + AD	a) $4k + 3r - 2k + 1 + r$ $4k + 2k - 3r + r + 1 = ?$ b) $-x - 6x - 2y - y + 3y$ $3y - y - 2y + 6x - x = ?$	BM

In this case, students have to know how to deal with two variables and numerical value. The procedures, they have presented, have been calculations directly, representation with grouping variables and, in addition, the doubtful answer. Some students, as student BF, have calculated directly while some students, as student BD, have grouped the variables at first. The grouping of the variables is a way to help to perceive the similar components of the expression and then reduce the amount of the mistakes. In addition, some ones have done the grouping but they have still been doubtful about the solution, like student BM. Students may be unsure about the kind of answer for the simplification should be: is it just numbers or also variables.

Table 6: The procedures of the third Norwegian task

6.24 Trekk sammen. a) $3x + 4y - 3x$ b) $-4a - 2b - 2a - b$ c) $3n - 3n + 2m - 2m$ d) $12x + 14x + 24x$		
Procedure	***Solution***	
CD	6,24 a) $3x + 4y - 3x = 4y$ b) $-4a - 2b - 2a - b = -6a + -3b$ c) $3n - 3n + 2m - 2m = 0$ d) $12x + 14x + 24x = 50x$	EJ
RG	6,24 a) $3x + 4y - 3x$ $3x - 3x + 4y = 4y$ b) $-4a - 2b - 2a - b$ $-4a - 2a - 2b - b = -6a - 3b$ c) $3n - 3n + 2m - 2m$ $3n - 3n + 2m - 2m = 0$ 6,24 d) $12x + 14x + 24x = 50x$	EC

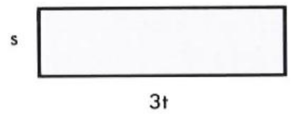
The Norwegian students have used the same procedures than the Finnish students: the calculations directly and grouping the variables. Although the Norwegian task is easier, the interesting point is how student EC has done the grouping by repeating just the same expression *3n – 3n + 2m – 2m* to be sure that it has grouped. The same student has also been unsure of the operation sign because some minus signs have been written in a reinforced way afterwards.

4.1.4 Procedures of writing an expression from the picture

The aim of the fourth category is to write the expression from the geometric picture using the given variables. This task will also examine the ability of the students to connect algebra and geometry: the knowledge to calculate the perimeter and the area is also needed.

Table 7: The procedures of the fourth Finnish task

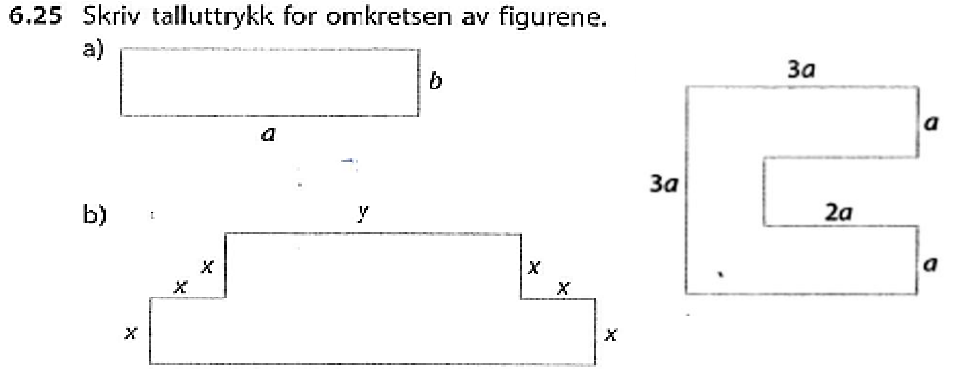
2. Bilda ett uttryck för omkretsen p och arean A för rektangeln.

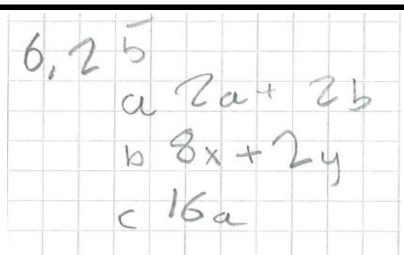
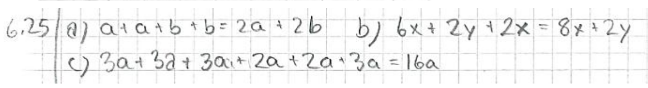
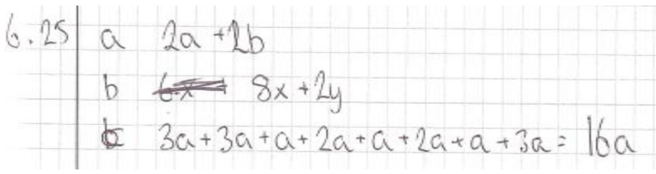
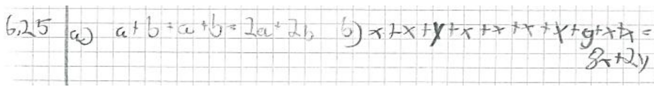
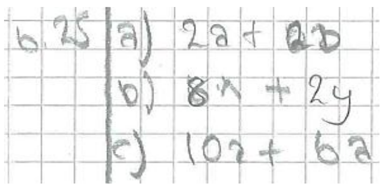


Procedure	***Solution***	
FR + RE	$P \cdot 6T + 2s$ $A = 3t \cdot S$	BJ
	$p = \overset{6t}{3T+3t} + \overset{2s}{S+S} = \underline{2s+6T}$ $A = 3t \cdot s = \underline{3sT}$	BK

The fourth category has been quite clear both in Finland and in Norway. The Finnish students have to find the formula of the area and the perimeter of the rectangle. The used procedures have been finding a rule and representation as an expression. The challenging part of the task is to remember to use *3t* instead of *t*. Some students have written only the expression but some have also the intermediate steps to make clear what they have thought and how they have built the expression. Student BK has also reorganized the variables according to the alphabetical order.

Table 8: The procedures of the fourth Norwegian task



Procedure	Solution	
RE		EO
		EJ
		EN
		ES
RE + RD		EC

The challenge of the Norwegian students has been finding the perimeter of the different kinds of objects. Students have presented two kinds of procedures: representation as an expression and, moreover, representation with decomposition of the answer. All quantities are not visible, when students have to perceive the object as a whole. The difference between solutions is whether student has written the intermediate steps or not. The point here is also how the students have written the intermediate steps; have they written variable by variable, as student EJ has done in item a), or just in order from one point to round the picture like students EN and ES have presented.

4.1.5 Procedures while making substitution

In the fifth category, it is visible how students are using the formulas and making substitutions. It tells us not only about their algebraic knowledge but also about their ability to connect algebra to the other fields of mathematics, as in this case geometry. Students are working with variables and they have to know which numbers are substituted by which variables.

Table 9: The procedures of the fifth Finnish task

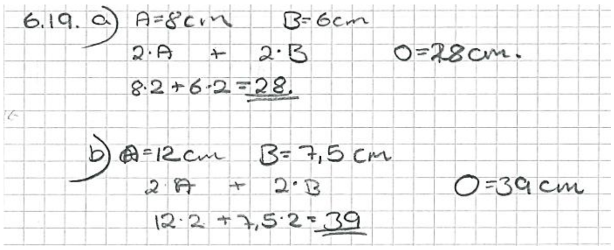
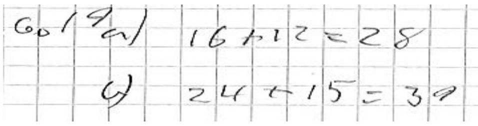
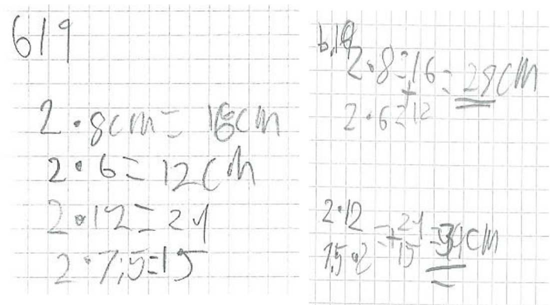
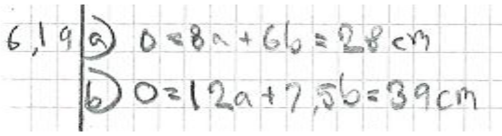
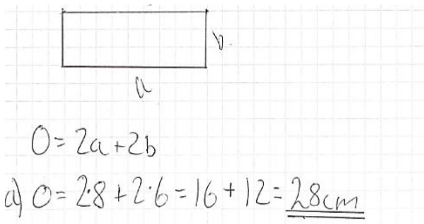
3. Beräkna omkretsen och arean av rektangeln i föregående uppgift då s = 1,5m och t = 2 m.

Procedure	***Solution***	
SD	$P = 2 \cdot 1{,}5 + 6 \cdot 2 = 15\,m$ $A = 1{,}5 \cdot 3 \cdot 2 = 4{,}5 \cdot 2 = 9\ m^2$	BD

The substitution of this Finnish task is dependent on the formula which students have created in the earlier task and it has done the task more difficult. Only one kind of solution has been found and it is substitution to formula directly without presenting the original formula. Mental calculations have also been done before writing the solution because $3t$ (when t = 2m) has been written as $6 \cdot 2$. The unit of measurement, m or m², has been presented only in the end of the solution.

Table 10: The procedures of the fifth Norwegian task

6.19 Formelen for omkretsen av et rektangel er $O = 2a + 2b$, der O står for omkretsen, a for lengden og b for bredden av rektangelet.
Regn ut omkretsen av rektangelet når
a) $a = 8$ cm og $b = 6$ cm
b) $a = 12$ cm og $b = 7{,}5$ cm

Procedure	***Solution***	
SF		FB
SD		ER
RA		ED
RI		ET
PH + SF		FC
AO		EO

Compared to the Finnish task, the Norwegian students present several different kinds of procedures to solve this task. Some students have substituted to the formula with intermediate steps (student FB) or have done the substitution directly (student ER). At this task, students have also created many atypical representations which are usually not used in mathematics as student ED has done. In addition, incorrect representations (student ET), pictures (student FC) and only answers (student EO) have been found from the material. The most interesting solution has been presented by student ED. This student has at first calculated the total length of sides *a* and *b* and then presented the final addition between those two calculations.

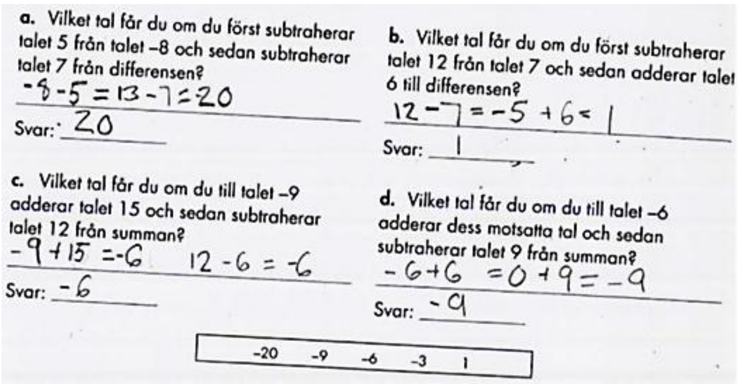
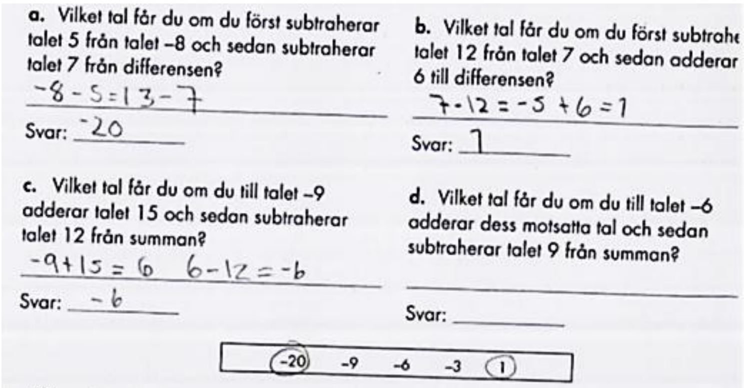
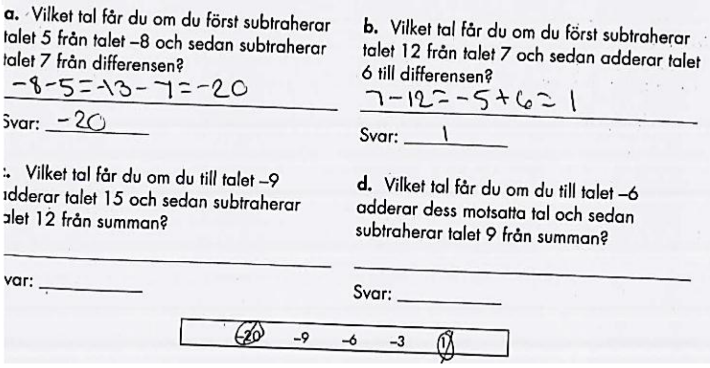
4.2 The identified mistakes

The second research question is about the mistakes students have done solving the tasks. The mistakes, which have appeared in those five tasks, will be presented in the same way as the procedures. In addition, many mistakes can be observed in one task because students can make different kinds of mistakes solving the task.

4.2.1 Mistakes while writing an arithmetic expression

Many mistakes are found from both Finland and Norway but the mistakes are different. The main reason can be, although the tasks are of the same nature, that the Norwegian task includes currency unit while the Finnish one includes only numbers.

Table 11: The mistakes of the first Finnish task

<table>
<tr><td colspan="3">3. Betekna och räkna. Ringa in rätt svar.</td></tr>
<tr><td>Mistake</td><td>Solution</td><td></td></tr>
<tr><td>OR + EI</td><td>
a. Vilket tal får du om du först subtraherar talet 5 från talet −8 och sedan subtraherar talet 7 från differensen?
−8 − 5 = 13 − 7 = 20
Svar: 20
b. Vilket tal får du om du först subtraherar talet 12 från talet 7 och sedan adderar talet 6 till differensen?
12 − 7 = −5 + 6 = 1
Svar: 1
c. Vilket tal får du om du till talet −9 adderar talet 15 och sedan subtraherar talet 12 från summan?
−9 + 15 = −6 12 − 6 = −6
Svar: −6
d. Vilket tal får du om du till talet −6 adderar dess motsatta tal och sedan subtraherar talet 9 från summan?
−6 + 6 = 0 + 9 = −9
Svar: −9
−20 −9 −6 −3 1</td><td>AC</td></tr>
<tr><td></td><td>
a. Vilket tal får du om du först subtraherar talet 5 från talet −8 och sedan subtraherar talet 7 från differensen?
−8 − 5 = 13 − 7
Svar: −20
b. Vilket tal får du om du först subtrahe[rar] talet 12 från talet 7 och sedan adderar 6 till differensen?
7 − 12 = −5 + 6 = 1
Svar: 1
c. Vilket tal får du om du till talet −9 adderar talet 15 och sedan subtraherar talet 12 från summan?
−9 + 15 = 6 6 − 12 = −6
Svar: −6
d. Vilket tal får du om du till talet −6 adderar dess motsatta tal och sedan subtraherar talet 9 från summan?
Svar:
−20 −9 −6 −3 1</td><td>AE</td></tr>
<tr><td>OR</td><td>
a. Vilket tal får du om du först subtraherar talet 5 från talet −8 och sedan subtraherar talet 7 från differensen?
−8 − 5 = −13 − 7 = −20
Svar: −20
b. Vilket tal får du om du först subtraherar talet 12 från talet 7 och sedan adderar talet 6 till differensen?
7 − 12 = −5 + 6 = 1
Svar: 1
c. Vilket tal får du om du till talet −9 adderar talet 15 och sedan subtraherar talet 12 från summan?
Svar:
d. Vilket tal får du om du till talet −6 adderar dess motsatta tal och sedan subtraherar talet 9 från summan?
Svar:
−20 −9 −6 −3 1</td><td>AA</td></tr>
</table>

Students have difficulties to write the expression in Finland. Firstly, the mistakes have been related to the incorrect expression, as student AC has done item c). Secondly, the equal sign has been seen as *left to right* sign as Kilpatrick, Swafford and Findell (2001) have defined this mistake. For example, student AA has got right answer but the way to present with the equal sign is not correct. In addition, the task may be quite misleading. Students have just done step by step and connected the steps with the equal sign. It is also possible to recognize from the solution of student AE how they can develop their working during one task: at first the solutions have been written incorrectly with equal sign but the last solution is already separated in two expressions. The interesting issue is also how the students AC and AE have forgotten the minus sign before number 13 while solving item a) but it can be found in front of the answer.

Table 12: The mistakes of the first Norwegian task

6.7 Sara kjøper 3 liter brus til 9,90 per liter, 2 poser potetgull til 14,90 per stk. og 3 hg smågodt til 16 kr per hg.
Sett opp og regn ut tallutrykket som viser hvor mye hun må betale.

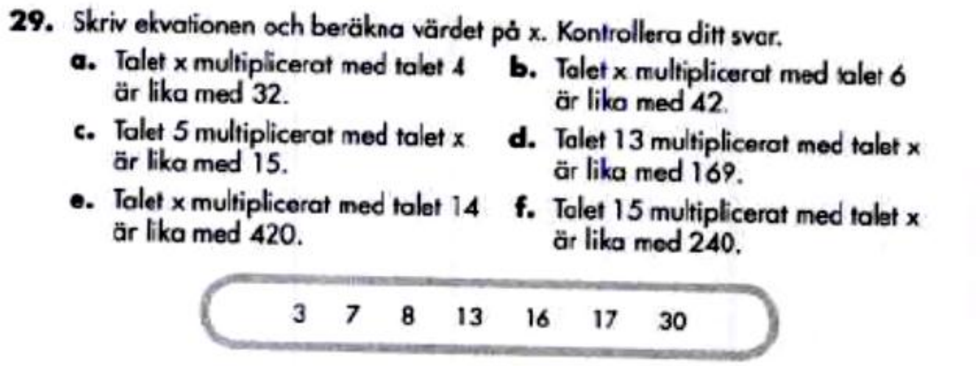
Mistake	***Solution***	
UA	6.7 3 · 9,90 + 2 · 14,90 + 3 · 16 = 107,50	FI
UA + MC	6.7 3 · 9,90 + 2 · 14,90 + 3 · 16 = 137,3 29,7 59,6 +49,0 =137,3	FF
UA + OC	6.7) 3 · 9,90 = 29,7 + 2 · 14,90 = 29,8 + 3 · 16 = 48 29,7 29,8 48 = 107,5	FP
RO	6,7) 9,90kr · 3 14,90kr · 2 16kr · 3 = 29,70 kr + 29,80kr + 48kr = 107,50 kr Sara må betale 107,50kr for varene	FL
RT	3 · 9,90 + 2 · 4,90 + 3 · 16 = 29,7 + 9,8 48 = 87,5	FC

The problems have been related to the unit of measurement and writing the expression in Norway. The most common error has been the missing unit, in this case the crowns. Miscalculations, incorrect representations of operations and of the original task have been found. The solutions of students FP and FL show how unaware students are with operation signs and the structure of the expression. Student FP has connected all expressions with plus-sign in the horizontal way what is incorrect, and after has done the addition separately in the vertical way. So the plus-sign has no meaning at all in this expression. The other student, FL, has left the plus-sign away totally but has still calculated all the expressions together.

4.2.2 Mistakes while writing an algebraic expression/equation

The second category focuses on writing an expression or an equation from the word instruction. While the Norwegian students write an expression, the Finnish students are supposed to write an equation and solve it.

Table 13: The mistakes of the second Finnish task


29. Skriv ekvationen och beräkna värdet på x. Kontrollera ditt svar.

a. Talet x multiplicerat med talet 4 är lika med 32.

b. Talet x multiplicerat med talet 6 är lika med 42.

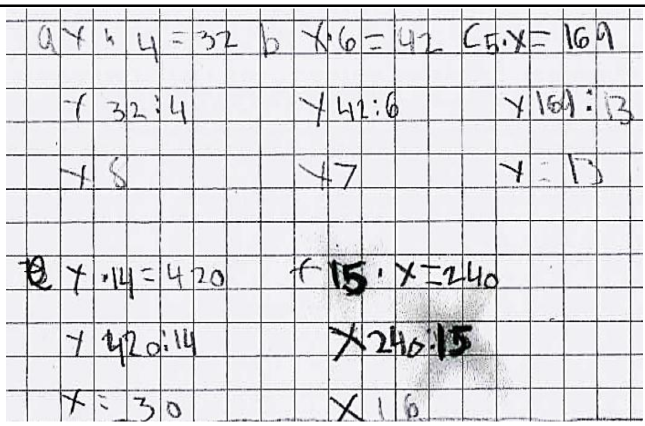
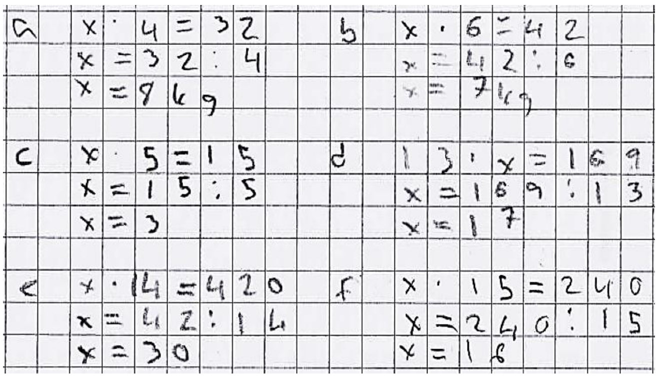
c. Talet 5 multiplicerat med talet x är lika med 15.

d. Talet 13 multiplicerat med talet x är lika med 169.

e. Talet x multiplicerat med talet 14 är lika med 420.

f. Talet 15 multiplicerat med talet x är lika med 240.

3 7 8 13 16 17 30

Mistake	Solution	
RU + RO		AF
UU + MC		AJ

At the second task, in Finland the appearing problems are unclear representation and careless calculations. For example, student AF has connected items c) and d). In addition, for some reason, student AJ has added unnecessary unit (kg) in the end of the solution although the task does not focus on the kilograms. The remarkable point is that students have not had any problems to write the equation from word instruction.

Table 14: The mistakes of the second Norwegian task

6.9 Skriv et uttrykk for
a) summen av x og 3
b) differansen mellom x og 5
c) 10 mer enn y
d) 12 mindre enn y

Mistake	*Solution*	
EI	6.9 a) $X+3$ b) $X-5$ c) $10+Y$ d) $12-Y$	EB
	6.9 a) $x+3$ b) $x-5$ c) $y+10$ d) $y+12$	EM
EE	6.9 a) X og 3 = ~~Min alder • 3~~ min alder + 3 = 16 år b) Min alder − 5 = 8 år c) $y+10=10$ d) $y-12=8$	EI
	6.9 a) $x+3=?$ b) $x-5=?$ c) $10+y=?$ d) $12-y=?$	EP
MT	6,9 a Det er det samme som $3+6$ b Det er det samme som $X-5$ c Det er det samma som $10+Y$	FK

The Norwegian students have had difficulties to write the expression according to the mathematical rules and the instruction, especially the items c) and/or d). The mistakes have been related to the incorrect expressions, writing an expression as an equation and misinterpretation of the task. The common mistake is that students have just written the variables on the same order as they have been written at the assignment without understanding. In addition, the meaning of the expression has been unclear to some of the students; they have invented some explanations or have written the expression as equation. Some of the solutions have been difficult to understand: for example FK has written *"It is the same as 3 + 6"* for items a) and the variable x is not visible at all.

4.2.3 Mistakes while simplifying

The task of the third category is focused on simplifying. The Finnish and Norwegian tasks are quite identical but both similar and different mistakes are found in the empirical material.

Table 15: The mistakes of the third Finnish task

4. Förenkla

a) $4k + 3r - 2k + 1 + r$

b) $-x - 6x - 2y - y + 3y$

Mistake	***Solution***	
OS	**a)** $4k + 3r - 2k + 1 + r$ $2k + 4k + 1$ **b)** $-x - 6x - 2y - y + 3y$ $7x + 6y$ V	BL
	a) $4k + 3r - 2k + 1 + r = 6k + 4r + 1$ **b)** $-x - 6x - 2y - y + 3y = -7x - 6y$	BH
OS + MC	**a)** $4k + 3r - 2k + 1 + r$ $6k + 4r + 1$ **b)** $-x - 6x - 2y - y + 3y$ $5x - 4y$	BC
	a) $4k + 3r - 2k + 1 + r$ $2k + 2r + 1$ b) $-x - 6x - 2y - y + 3y$ $-5x - 2y$	BE
MC	a) $4k + 3r - 2k + 1 + r$ $4k - 2k + 3r + r + 1 = 2k + 4r + 1$ b) $-x - 6x - 2y - y + 3y$ $-x - 6x - 2y - y + 3y = -7x - 4y$ V	BN
NA	a) $4k + 3r - 2k + 1 + r$ $4k + 2k - 3r + r + 1 = ?$ b) $-x - 6x - 2y - y + 3y$ $3y - y - 2y + 6x - x = ?$	BM

From the Finnish material it is possible to find many mistakes with the minus sign. Students are unsure of how to operate with it. In some cases, as student BL has done, the minus sign has just been ignored. On the other hand, student BH has dealt with plus-sign as minus-sign. Also some miscalculations and doubtful answers, which are expressed by an interrogation mark, are found.

Table 16: The mistakes of the third Norwegian task

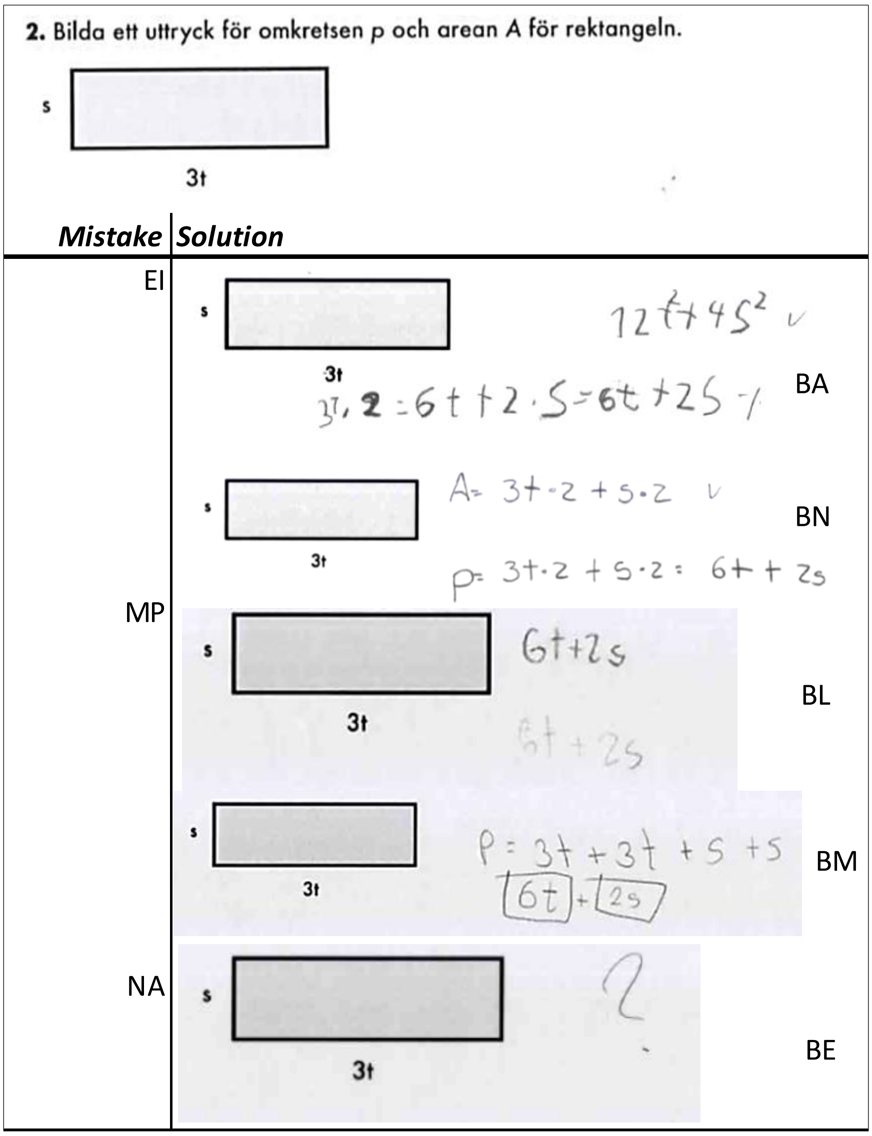
	6.24 Trekk sammen. a) $3x + 4y - 3x$ b) $-4a - 2b - 2a - b$ c) $3n - 3n + 2m - 2m$ d) $12x + 14x + 24x$	
Mistake	***Solution***	
OS	6.24 a) 4y b) -6a+B c) 0 d) 50x	EG
	6.24 a) 3x+4y-3x=6x+4y b) -4a-2b-2a-b= -4a-2a-2b-b = -6a+b c) 3n-3n+2m-2m=0n+0m d) 12x+14x+24x=50x	EM
	6.24 a) 3x+4y-3x 3x-3x+4y=4y b) -4a-2b-2a-b -4a-2a-2b-b=-6a+3b c) 3n-3n+2m-2m=0 d) 12x+14x+24x=90x	EP
RU	6.24 a 3x+4y-3x = 4y b -4a -2b-2a-b = -6a + -3b c 3n-3n+2m-2m = 0 d 12x + 14x + 24x = 50x	EN
RV + NA	a) 3x+4y-3x=4y b) -4a-2b-2a-b= c) 3n-3n+2m-2m=0n+0m d) 12x+14x+24x=50	EA

The Norwegian students have had problems with the minus sign as the Finnish students, for example EM has written $3x + 4y - 3x = 6x + 4y$. The other mistakes have been related to the unclear representation, the representation of the variables and the missing answer. The students have not always been systematic: sometimes they have changed both signs from minus to plus (student EP) but sometimes only one of the minus signs (student EM). The interesting point is that some students have problems with signs; they do not know how to operate in the situation where they have $+ - x$, so they have written both to the solution. In addition, some students have careless mistakes: for example, they have forgotten to write the variable.

4.2.4 Mistakes while writing an expression from the picture

The task of the fourth category is focused on writing an expression from the geometric picture. Students are supposed to write an expression both for the perimeter and the area in Finland but in Norway only for the perimeter.

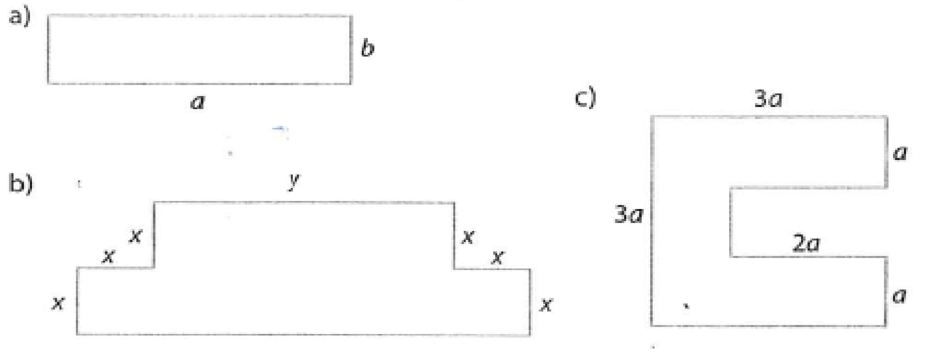
Table 17: The mistakes of the fourth Finnish task



This task has been recognized being quite challenging for Finnish students who have had many mistakes related to the incorrect expression, missing part of the task and the missing answer. They have done some careless errors and have forgotten to do the second part of the task; usually it has been the expression for the area of the rectangle. Students have also had difficulties in creating the formula of the perimeter and the area; some of them did not have any idea on how to do it. Then, for example, student BN has repeated the formula of the perimeter but only defined it as an area whereas student BL only has written the formula of the perimeter twice. Student BE has expressed their unawareness as a question mark.

Table 18: The mistakes of the fourth Norwegian task

6.25 Skriv talluttrykk for omkretsen av figurene.



Mistake	Solution	
MC	6.25 a) $2a+2b$ b) $2y+8x$ c) $13a$	EG
	6.25 a) $2a+2b$ b) $8x+2y$ c) $14a$	ER
	6.25 a) $2a+2b$ b) $y+x+x+x+x+x+x+y+x+x=2y+8x$ c) $3a+a+2a+a+3a+a=11a$	EP
MT	6.25 a) $a+b+b+a=2a+2b$ b) $x+x+x+x+x+x+y=6x+y$ c) $a+a+a+a+a+a+a+a+a+a+a=10a$	ET

In Norway, the students have had problems with calculating the variables and creating the expressions of the perimeter, especially at the item c). The possible reason is that they have only been careless because all the needed values are not visible. In addition, student ET has understood that it is necessary to calculate only the visible values.

4.2.5 Mistakes while making substitution

The fifth task has been rather challenging both in Finland and Norway; students have made many mistakes while solving them. The aim of the tasks is to substitute the given values to the formula.

Table 19: The mistakes of the fifth Finnish task

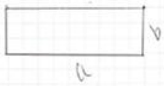
3. Beräkna omkretsen och arean av rektangeln i föregående uppgift då s = 1,5m och t = 2 m.

Mistake	*Solution*	
MT	omkrets = 1,5+1,5+2+2 = 7m Arean = 2m · 1,5m = 3,0m²	BJ
	1,5m · 2 = 3m 3t · 2 = 10 m = 13m	BE
MT + MP	p = 2·1,5m + 2·2m = 7m Svar: p = 7m	BK
MT + OR	**3.** Beräkna omkretsen och arean av rektangeln i föregående uppgift då s = 1,5m och t = 2 m. 7,5m·2 = 3,0m + 2m·2 = 3m + 4m = 7m 72m²	BA
MT + UE	2·1,5 = 3m 2·2 = 4cm 4+3 = 7cm	BC
MT + UA	S = 1,5+1,5+1,5+1,5 = 6 t = 2+2 = 4m	BH
MP	3m + 12m = 15m	BG
UA	p = 1,5+1,5+3·2+3·2 = 3+12 = 15 A = 1,5·6 = 9,0	BI
UE + OR	A = 1,5·2 + 3·2m·2 P = 1,5m·2 = 3m + 3·2m·2 = 3m + 12m = 15m	BN

Solving this task, Finnish students have made many mistakes. Some of them appeared because students have not been able to write a right expression to the earlier task (Task 2): those two tasks are connected to each other. In addition, some students have had a misunderstanding with the variables: they have used only *t* instead of 3*t*, as student BJ has done. Some students have written the expression from right to left (students BA and BN) and some other ones have suddenly used different unit of measurement (student BC) or totally forgotten the unit of measurement (student BI). Moreover, students BK and BG have calculated only the perimeter.

Table 20: The mistakes of the fifth Norwegian task

6.19 Formelen for omkretsen av et rektangel er $O = 2a + 2b$, der O står for omkretsen, a for lengden og b for bredden av rektangelet.
Regn ut omkretsen av rektangelet når
a) $a = 8$ cm og $b = 6$ cm
b) $a = 12$ cm og $b = 7{,}5$ cm

Mistake	***Solution***	
UA	6.19 a) $O = 8 \cdot 2 + 6 \cdot 2 = 16 + 12 = 28$ b) $12 \cdot 2 + 7{,}5 \cdot 2 = 24 + 15 = 39$	EJ
	6.19 a) $O = 2 \cdot 8 + 2 \cdot 6 = 16 + 12 = \underline{28}$ b) $O = 2 \cdot 12 + 2 \cdot 7{,}5 = 24 + 15 = \underline{39}$	FA
UA + MC	6.19  $O = 2a + 2b$ a) $O = 2 \cdot 8 + 2 \cdot 6 = 16 + 12 = \underline{28cm}$ b) $O = 2 \cdot 12 + 2 \cdot 7{,}5 = 24 + 5{,}5 = 29{,}5$	FC
	6.19 a) $O = 2a + 2b = 2 \cdot 8 + 2 \cdot 6 = 16 + 12 = \underline{28}$ b) $O = 2a + 2b = 2 \cdot 12 + 2 \cdot 7{,}5 = 22 + 15 = \underline{37}$	FI
MC	6.19 a) $8 + 8 = 16$ $6 + 6 = 12$ $= \underline{28}$ cm er omkretsen. b) $12 + 12 = 24$ $7{,}5 + 7{,}5 = 31$ $= \underline{55}$ cm er omkretsen	ES
MT	6.19 a) $8cm \cdot 2 = 16cm \cdot 2 = 32cm$ $6cm \cdot 2 = 12cm \cdot 2 = 24cm$ $32cm + 24cm = \underline{56cm^2}$	EP

The most common error of the Norwegian fifth task has been the absence of the unit of the measurement, as students EJ and FA have. Moreover, there are some miscalculations which usually exist with multiplications but also with addition. Some misinterpretations of the task have also been found, as seen from the solution of student EP. Instead of having once multiplied the given number once by two, this student has done it twice.

4.3 The comparison between Finland and Norway

The third research question focuses on the comparison between Finland and Norway: which similarities and/or differences they have. The procedures and mistakes of the five tasks both from Finland and Norway have been presented and at this part the comparison between those countries is done. Due to the huge amount of the empirical material, only some most interesting points are presented.

4.3.1 Comparison of procedures

In the tasks in which students are supposed to write arithmetic or algebraic expressions or equations, the Finnish students have usually used the typical mathematical procedures while Norwegian students have more atypical presentations. The first task (task 6.7 from Norway) is dealing with money so some students have rounded the numbers before writing the expression. In addition, some students from Norway and from Finland have also written separated written answer or the review of calculations. The interesting point is also how some Norwegian students have explained their expression with words, for example *x + 3 = naboens alder* (task 6.9 from Norway).

At the fourth category tasks the students are supposed to write the expressions from the geometric picture. Students have needed the formula of the perimeter and the area in Finland while in Norway only that of the perimeter. In both countries some students have written down every intermediate step before the final solution. Furthermore few students have decomposed the answer like *10a + 6a*.

The interesting task is the fifth task and its solutions. Students are supposed to make a substitution to the given formula. The Finnish task has been connected to the previous task, which has made this task more difficult because some students have also had problems with that previous task. The Norwegian students have had several different procedures while solving this task. The Finnish students have just substituted to formula while the Norwegian students have written the formula first or some students have even drawn the picture to their notebooks. Several atypical representations have also been identified from Norway; the answer is correct but the solution has not been done using mathematical norms. In addition, both some Finnish and Norwegian students have only written the final answer so it is impossible to know what they have been thinking.

As a conclusion, concerning procedures, Norwegian students have used more different kind of procedures while solving tasks. One explanation could be that the amount of the Norwegian students is higher.

4.3.2 Comparison of mistakes

Many mistakes have been identified from the empirical material and during this comparison the most common or otherwise interesting mistakes will be presented.

The most common, but not that much related to the algebra, have been the problems with the units. While solving the task, students have forgotten the units or have used wrong units.

Those mistakes have been more visible in Norway than in Finland, especially in the Norwegian tasks 6.7 and 6.19.

The other common mistake, which is related to the word problems, is the misinterpretation of the task. Many students both in Finland and Norway have not understood what they are supposed to do to get the task solved. For example, in Finnish task 3 some students have not understood that the task is connected to the previous one. Moreover, some students in Finland have forgotten to complete all parts of tasks 2 and 3.

Many mistakes have been made related to the arithmetic operations. The minus sign has been problematic both in Finland and in Norway: it is possible to recognize from the tasks 4 from Finland and 6.24 from Norway. Usually the Norwegian students have changed the minus sign to plus, but some Finnish students have also changed the plus sign to minus. In addition, some students in Norway have been uncertain on how to operate for example $+ - x$. Many miscalculations are also found both from Finland and Norway although the calculations are quite simple.

Writing expression and the structure of expression have provoked many mistakes. Students are uncertain about the meaning of the equal sign: it is seen as a sign of what to do next. This mistake has been common both in Finland and Norway. The interesting point is that the Norwegian students have had problems to write an algebraic expression from the written instruction while the Finnish students did not. The Finnish students have had more problems in connecting the written algebraic expression to their geometric knowledge about area.

Although Finnish and Norwegian students have different tasks, the same kinds of mistakes are visible. Only a few mistakes have been identified which are not found in the other country, for example, only the Finnish students have changed the plus sign to the minus sign. The main mistakes, like the minus sign, equal sign and writing expressions, can be identified easily from both Finland and Norway.

5 Discussion

The aim of this study has been to examine the procedures and the mistakes which students have done while solving algebraic tasks in Finland and Norway. Also the similarities and the differences have been identified. In this chapter, those findings are discussed with the review of the earlier researches. In the end, the different kinds of aspects of this research process have been presented.

According to MacGregor and Stacey (1997), mathematics is not seen to be connected to the everyday life. In this light, it has been interesting to see how some Norwegian students have rounded the numbers while solving the tasks which are dealing with currency unit (Table 2). Those students have connected the task to their life; they do as they are used to do in their everyday life. It is not the mathematically recommended way but it is an interesting observation which raises up the discussion between school math, math math and street math by Resnick (1995). Also the second task from Norway (Table 4) has some remarkable issues: students have explained the expression with words so they have been trying to make sense of the letters. Thus, they have kind of understood the expressions being relations.

The Finnish and Norwegian students usually have had same kind of procedures while solving tasks but the Norwegian students have more different kinds of solutions. It can also be dependent on the task. For example, the procedures of the first tasks differ because the Norwegian task offers more different kinds of changes to write the expression: the instruction of the task is not as guiding as the instruction of Finnish task. The students from Norway also have more atypical representations: the representation is correct but not presented according to the mathematical norms. Some of the solutions have demanded more deduction on what students have been thinking while solving the tasks (for example Table 10, student ED). According to Herscovics and Linchevski (1994), students usually have some logic behind their working although it can seem like only random operations. Also Carraher, Martinez et al. (2008) explain that one reason might be that students do not pay attention to the general formula.

Some Finnish tasks guide students to review their answer (Table 1 and 3) while it is not visible in the Norwegian tasks although the assessment of the validity has been mentioned both in Finnish and Norwegian curriculum and especially in the context of mathematics (Opetushallitus, 2004; Kunnskapsløftet, 2010). Also Kieran (2004) has explained how justifying and proving are part of the global, meta-level mathematical activities. It teaches student to trust themselves: to their deductions and solutions.

An interesting comparison between the tasks is how some students have written down each intermediate steps (Tables 5 & 6) while some students have written only answer to more complicated task (Table 10). From some Finnish tasks it has been found how students have done same tasks twice: at first they have had only the answer but then later in the notebook it is possible to find the new solution with calculations. This can be seen linked to the fact that Finnish students are supposed to show their solving process. However, the procedures, which both the Norwegian and the Finnish students have used, are more related to the individual way to do the task than to the instruction of the task. For example, students, who have written only the answer, have done it repeatedly. This is seen in the Appendix 3.

From the findings, it is possible to recognize how students have challenges to do the translation between natural language and algebraic notation as Filloy et al. (2008) have also described. The Finnish task (Table 1) can also be slightly misleading because some students have connected expressions with equal sign; they have just done what they are supposed to do. As task is translated to English: *Which number do you get if you first subtract the number 5 from the number –8 and then subtract the number 7 from the difference?* (Translated by author.) MacGregor and Stacey (1993) have mentioned one of the greatest difficulty in the beginning of algebra being making a connection between a mathematical situation and its formal description. This has also been recognized in the task of the second category in Norway (Table 14).

Although the connection between algebra and geometry has been seen important in school mathematics (Banchoff, 2008), students have had difficulties in creating an expression from the geometric picture, especially in Finland. In the context of algebra, students do not remember the geometric formulas (Table 17). The Norwegian students have had problems with perceiving the geometric pictures because all the values have not been written to the picture (Table 18). Students are not able to connect the other field of mathematics to the algebra: they concern their thinking to one narrow area at one time. This kind of thinking is contradictory with the curricula. According to the Norwegian curriculum, “algebra is also used in connection with the main subject areas geometry and functions” (Kunnskapsløftet, 2010, p. 3).

Many studies have already been done about students' difficulties with arithmetic operations at the context of algera (Herscovics & Linchevski, 1994; Linchevski & Livneh, 1999; Banerjee & Subramaniam, 2012). That is why it has not been a surprise how both the Finnish and Norwegian students have problems with arithmetic operations (Tables 15 & 16), especially with the minus sign. Sometimes they just have a detachment of the minus sign; they are dealing with the minus sign as if it would be the plus sign. In addition, some Finnish students have changed the plus sign to the minus sign which has not been visible in Norway. One remarkable issue, which shows how unsure students are with arithmetic operations, has been found from Norway: students let the final answer being as $+ - x$. They might not know how to simplify it or they might think it being the final answer.

Kilpatrick, Swafford and Findell (2001) and Kieran (1992) have described students’ difficulties to understand the meaning of equal sign: they usually understand it as l*eft to right* –sign which shows what to do next. This has also been found in the tables 11, 12 and 19, therefore the problem is visible in both countries. As mentioned earlier, some instructions (Tables 1 and 11) may also be misleading causing the students think the connection making with equal sign will be assumed. Some students also have challenges to understand what they are supposed to do while solving task, especially if the tasks are word problems. As Verschaffel, Greer and De Corte (2000) have mentioned, students confront word problems thoughtless; they are only doing some random operations without paying attention to the context (Table 14, student EB). In addition, Table 19 shows how the Finnish students have not understood the task well enough. The most common mistakes are that they have not connected the task on hand to the previous task as they are supposed to do.

Although students have the same kind of difficulties while solving the tasks, the Finnish tasks have been slightly more complicated than the Norwegian tasks. For example, while solving tasks of the third category the Norwegian students have two different variables which are more arranged but the Finnish students have also one number and the variables are more randomly arranged (Tables 5 and 6). Some Finnish 6th grade tasks are more complicated than Norwegian 8th grade task (Tables 3 and 4). But it is necessary to pay attention that the Norwegian 8th grade students are only one year older than the Finnish 6th grade students.

Because this study is focused on the written work, the findings of this study are only dependent on what students have written to the notebooks and tests. Thus, it is impossible to know what they have exactly been thinking while solving tasks. Nevertheless, the mistakes and procedures of the students can still be recognized in this material.

To make the reliability as high as possible, the micro-analysis has been completed. The analysis has been created during the study because I, as researcher, could not know how to do the analysis before going through the empirical material. That is why this study is also a model about analysing tasks. It has been hard work to analyse around 1200 task, but it has shown the huge variety of the procedures and mistakes which students have done. This study has also shown that it is possible to compare different tasks from different countries.

This international comparative study has had its own challenges. The tasks which have been compared are not the same in both countries. The first challenge of this study has been to find as similar type of tasks as possible. The tasks with same nature have been identified but the tasks are not similar. Some tasks could be slightly more difficult in another country but still the knowledge needed to solve the task is usually the same. Some differences are also found because the Norwegian students have not solved any equations during the data collection while the Finnish students did.

Although this issue of the different tasks has been a challenge during this study, it could be read as one of the benefits, too. Usually the international comparative studies are focused on same tasks in each country. Those tasks can be more familiar to some students than the others while in this study the tasks are from the students' own environment. This study has been focused on the ones which students have been solving in the classrooms on mathematics lessons. In this case, the similar data collection method is having more effect than the similarity of the tasks. At the next time, this kind of research could be even more demonstrative if the tasks which students are doing could be more similar but still familiar to all students. Then, the data collection should also have same kind of aim in both countries. For example, students would be advised to show their reasoning on paper, not only the answer.

6 Conclusion

In this chapter I will present the conclusion of this study trying to answer the research questions including the findings.

The research questions of this study have been:

1. What kinds of procedures students use while solving tasks?
2. What kinds of mistakes students produce?
3. Which similarities and/or differences can be identified between Finland and Norway?

Answering the first research question, the students are using several and different kinds of procedures while solving tasks. Usually there are some mainstream procedures according to the instruction, for example representation as an expression or an equation (see p. 65). The students also use some atypical procedures which are not recommended but which are not incorrect either (see p. 64).

Concerning the second question, while analysing the collected data, several mistakes have been identified. It is not a surprise that students have problems with the minus sign and equal sign. After the review of literature in this research topic many studies have pointed out the same mistakes. Also, writing expression from word problem has been found to be challenging to some students. One issue is also how students have difficulties in connecting algebra to other fields of mathematics, in this case to geometry.

Related to the third questions, both similarities and differences have been identified. In the context of procedures, the Finnish students usually have more similar procedures among themselves compared to the Norwegian students. The Finnish students also have shown their solving process, and the review of the solution, more in detail compared to the Norwegian students. The similarities can be found especially in the tasks of the third category: students from both countries have used nearly the same procedures. The mistakes the students have done are more similar than the procedures. Students from both Finland and Norway have made the same main mistakes with the minus sign, the equal sign and problems while writing the expression. In the context of writing expression, the Finnish students have more difficulties in connecting geometry and algebra while the Norwegian students have more problems in writing the expression from a written instruction.

7 Pedagogical implications

Many difficulties which students confront while learning algebra are connected to their arithmetic knowledge, for example to the minus sign and the equal sign. I would recommend, as also Banerjee and Subramaniam (2012) have suggested, that the analysis of the structure of the arithmetic and algebraic expressions should be supported. Then the introduction of algebra could also be easier for the students at schools in Finland and Norway. Moreover, algebra at school should be more meaningful for the students when solving the tasks.

From the Finnish material it has been possible to recognize the way how the students are used to representing their solution process. I would like to suggest that this justification of the student itself should be more visible in both countries. The important issue is that students could explain themselves what they have done and why they have done it. It should not be a teacher or an answer book who will certify the solution.

The analysed tasks from the classroom have mostly been from the text books. According to Törnroos (2004), the text books differ quite a lot: depending on the book, students can even have unequal learning possibilities. In addition, there is no continuum between the mathematics text book of the primary school and the lower secondary school because the text book series usually are different among those two school levels. I would suggest teachers to have more collaboration between primary school and lower secondary school. Even a shared text book serie to both schools could be important. Moreover, teachers should be encouraged to create own learning material or mix the available material to enrich their teaching and to decrease the powerful position of a certain text book.

There are some interesting issues, which could be studied in a more extensive and comprehensive way. Finland and Norway are quite similar societies, so it could be possible to do more research and development of teaching among those countries. One interesting issue could be the study of primary school mathematics focusing on how the teaching of mathematics is introduced in both countries during the first grades and what kind of differences could be identified in there. The collaboration between those two countries could be beneficial both to the Finnish and the Norwegian school system. Small countries cannot do everything alone: doing international research and collaboration it could be possible to have a better quality for the Nordic education in the future.

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9 Appendices

Appendix 1: Coded procedures

ANSWER

AA = answer absent

AP = answer partial

__6.22__ b) 3x + 2y + x + 4y

3x + 2y + x + 4y = 3x + x = <u>4x</u>
2y + 4y = <u>6y</u>

AO = answer only

__6.19__ Formelen for omkretsen av et rektangel er O = 2a +2b, der O står for omkretsen, a for lengden og b for bredden av rektangelet. Regn ut omkretsen av rektangelet når a) a = 8cm og b= 6cm

28 cm

AE = answer estimation

__9__ Omtrent hvor lang er omkretsen til b) ørene

o = ca 12 cm

AD = answer doubtful

__69__ b) Hva er arealet til kvadratet?

16?!

CALCULATION

CD = calculations directly

__6.1__ Regn ut. a) 20 + 4 · 7

20 + 4 · 7 = 48

__6.20__ Trekk sammen a) x + x + x + x + x

x + x + x + x + x = 5x

__6.21__ Trekk sammen a) 2a + 5a

2a + 5a = 7a

CS = calculations separately

6.1 *Regn ut d) 10 · (3 + 2)*
10 · (3 + 2) =
3 +2 = 5
10 · 5 = 50

6.1 *Regn ut. a) 20 + 4 · 7*
20 + 4 · 7 =
4 · 7 = 28
20 + 28 = 48

6.16 *Regn ut 12 · x, når b) x = 5*
12 · 5 = 60 (next to that has written 12 + 12 +12 + 12 + 12 = 60)

CR = calculations following rules

6.3 *regn ut a) 3(4 + 5)*
3 (4 + 5) = 3 · 9 = 27

6.5 *Regn ut. a) 2(6 + 3) – 9*
2 (6 + 3) – 9 = 2 · 9 – 9 = 18 – 9 = 9

REPRESENTATION

RA = representation atypical

6.16 *Regn ut 12 · x, når x = 5*
$\underline{12 \cdot 5}$ (usually student use this line as division sign, answer underlined twice)
60

6.19 *Formelen for omkretsen av et rektangel er O = 2a +2b, der O står for omkretsen, a for lengden og b for bredden av rektangelet. Regn ut omkretsen av rektangelet når a) a = 8cm og b= 6cm*

2 · 8 = 16
+ = 28 cm
2 ·6 = 12

8 + 8 = 16
6 + 6 = $\underline{12}$
= $\underline{28}$ cm er omkretsen

6.20 *Trekk sammen. a) x + x + x + x + x*
x5

6.6 *Regn ut. b) (24 – 12) : 3 – 3*
24 – 12 : 3 – 3 = 24 – 12 = 12, 12 : 3 = 4, 4 – 3 = 1

9 *Omtrent hvor lang er omkretsen til a) hodet b) ørene*

a) o = 8 cm ·3,14 = 25,12 cm b) o = 25,12 : 2 = 12,56 cm
(students has linked a and b and so on…)

The solution is connected, however the graphic presentation does not present the mathematical way of doing it.

RD = representation with decomposition

6.21 *Trekk sammen a) 2a + 5a*
2a + 5a = <u>a + a + a + a + a + a + a</u> 7a (answer underlined)

6.21 *Trekk sammen d) 8n – 2n*
8n – 2n = <u>n + n + n + n + n + n + n + n – n – n</u>

8n – 2n = n + n + n + n + n + n = 6n

RE = representation as expression/equation

6.10 *Lotte er x år. Skriv et uttrykk som viser hvor gammel a) hun var for 5 år siden*
x – 5

RI = representation incorrect (answer correct)

6.1 *Regn ut. a) 20 + 4 · 7*
20 + 4 · 7 = **4 · 7** = 28 + 20 = 48
20 + 4 · 7 = 4 · 7 = **28 + 20** = 48

6.7 *Sara kjøper 3 liter brus til 9,90 per liter, 2 poser potetgull til 14,90 per stk. og 3 hg smågodt til 16 kr per hg. Sett opp og regn ut talltrykket som viser hvor mye hun må betale.*
3 · 9,9 = 29,7 **+** 2 · 14,90 = 29,8 **+** 3 · 16 = 48
29,7 + 29,8 + 48 =107,5

6.5 *Regn ut. b) 56 + 6(21 – 19)*
56 – 6(21 – 9) = 56 + **6 · 2** = **12** + 56 = 68

RG = representation with grouping

6.22 *Trekk sammen. a) 3a + 4b + 2a + 3b*
3a + 4b + 2a + 3b = <u>3a + 2a</u> + <u>4b + 3b</u> = 5a + 7b

RN = rounding the numbers

6.7 *Sara kjøper 3 liter brus til 9,90 per liter, 2 poser potetgull til 14,90 per stk. og 3 hg smågodt til 16 kr per hg. Sett opp og regn ut talltrykket som viser hvor mye hun må betale.*
(3 · 10) + (2 · 15) + (3 · 16) = 108

RR = representation of review

__28__ Bilda en ekvation och beräkna värdet på x. Kontrollera ditt svar.
3 ·x = 21 kg
x = 21 : 3
x = 7

kontroll: 7 · 3 =21

RS = representation of equation solving

__27__ Läs ekvationerna. Skriv den bokstav som motsvarar ditt svar.
9 · x = 27
x = 27 : 9
x = 3 N

__19__ Läs ekvationerna. Skriv motsvarande bokstav i häftet.
x – 28 = 29
x – 28 + 28 = 29 + 28
x = 59 P

__20__ Det har tagits sand ur säcken på vågen. Bilda en ekvation och beräkna värdet på x. Kontrollera ditt svar.
38 – x = 12
38 – 26 = 12
x= 26 kg

RW = representation with words

__6.9__ Skriv et uttrykk for a) summen av x og 3
x er ukjent og 3 hva du må plusse med

SUBSTITUTION

SD = making the substitution directly

__6.16__ Regn ut 12 · x, når a) x = 4
12 · 4

SF = making the substitution to formula

__6.19__ Formelen for omkretsen av et rektangel er O = 2a + 2b, der O står for omkretsen, a for lengden og b for bredden av rektangelet. Regn ut omkretsen av rektangelet når a) a= 8 cm og b = 6 cm
O = 2a + 2b = 8 · 2 + 6 · 2 = 16 + 12 = 28

SR = substitution with review

__6.16__ Regn ut 12 · x når a) x = 4
12 · x = 48
12 · 4 = 48

SE = substitution before expression

__6.17__ Regn ut 15x når a) x = 2
15 · 2 = 30 = 15x x=2

OTHERS

DP = drawing picture

__13__ Tegn en cirkel med radius a) 3cm

FM = finding a measure

__8__ Finn radius og diamerer til sirklene som er a) hodet
Hodet: d = 8 cm r= 4cm

FR = finding a rule

__3__ Diskuterar og skriv en regel for hvordan vi kan regne ut omtrent hvor lang omkretsen til en sirkel er.
Du må gange diameter med 3,14

FS = finding similar (picture)

Appendix 2: Coded mistakes

UNIT OF MEASURE PROBLEMS

UA = unit of measure absent

> ***4*** *Omtrent hvor lang er omkretsen når diameteren er a) 5cm*
> 5 · 3,14 = 15,70
>
> ***6.11*** *Martin kjøper x kg epler i butikken. Eplene koster 20kr per kilogram. Lag et uttrykk som viser hvor mye Martin må betale.*
> x · 20 **kg** = x · 20 = 20x

UU = unit of measure unnecessary

> ***5*** *Omtrent hvor lang er radiusen når omkretsen er c) 12 m*
> 1200 cm : 3,14 **cm** = 376,8 cm

UT = unit of measure transformation

> ***5*** *Omtrent hvor lang er radiusen når omkretsen er c) 12 m*
> o = **120** cm   r = 19,10 cm

UE = unit/ unit of measure erroneous

> ***4*** *Omtrent hvor lang er omkretsen når diameteren er c) 8 m*
> Omkretsen er omtrent 15,75 cm når diameteren er 8cm

OPERATIONS

OB = operations with brackets

> ***6.15*** *Sara går x km til skolen og x km fra skolen hver dag. Lag et uttrykk som viser hvor mange kilometer Sara går til og fra skolen på b) én uke*
> x + x · 7 = 8x

OC = operations connected

> ***6.7*** *Sara kjøper 3 liter brus til 9,90 per liter, 2 poser potetgull til 14,90 per stk. og 3 hg smågodt til 16 kr per hg. Sett opp og regn ut talltrykket som viser hvor mye hun må betale.*
> 3 · 9,9 = 29,7 **+** 2 · 14,90 = 29,8 **+** 3 · 16 = 48
> 29,7 + 29,8 + 48 =107,5

OE = operations of equation solving

> ***14*** *76 kg + x kg = 120 kg*
> 76 + x = 120
> **76 – 120 = 44**
> 76 + 44 = 120   = 44 kg

19 f) x – 19 = 19
x – 19 = 19
-19 + 19 = 0, 19 + 19 =38
x = 38

OM = operation misinterpreted

6.2 *Regn ut. a) 4 · (3 + 10)*
4 · (**3 + 10**) = 4 · **30** = 120

6.5 *Regn ut. c) 4(5 + 2) – 2(10 – 7)*
4 (**5 + 2**) – 2 (10 – 7)
4 · **10** – 2 · 3
40 – 6 = 34

OO = operation order

6.5 *Regn ut. c) 4(5 + 2) – 2(10 – 7)*
4(5 + 2) – 2(10 – 7) = 7 – 3 · 2 · 4

6.1 *Regn ut. f) 26 – (4 · 3)*
26 – (**4 · 3**) = **12** – 26 = - 14

6.1 *Regn ut. b) 20 – 4 · 3*
20 – **4 · 3**   4 · 3 = **12** – 20 = -8

6.6 *Regn ut b) (21 : 7 – 1)+ 2 · 3*
21 : 7 – 1 + 2 × 3 = **21 : 7 = 3**, **3 – 1 = 2**, **2 × 3 = 6**

OP = operations with powers

5. *Förenkla a) $2x^2 + 3 - x - x^2 + 2x$*
4x – x – **2x** + 2x = 3x + 3

OR = operations read from left to right

5 *Omtrent hvor lang er radiusen når omkretsen er c) 24 cm*
24 : 3,14 **=** 7,64 : 2 **=** 3,82 cm

6 *Mål radiusene. Omtrent hvor lang er omkretsen til a) den blå sirkelen*
o = **2 + 2** = 4 · 3,14 = 12,56

6.1 *Regn ut. a) 20 + 4 · 7*
20 + 4 · 7 = **4 · 7** = 28 + 20 = 48

OS = operations with minus sign

6.26 *Trekk sammen a) 4x – 2y – 4x + y + 2x*
4x **– 2y** – 4x **+ y** + 2x = 2x **+ 3y**

REPRESENTATION OF TASK/ ANSWER

RC = representation with capital letters

> **6.20** b) *a + a + a*
> A + A + A = 3A

RO = representation with wrong/without operation signs

> ***6.7*** *Sara kjøper 3 liter brus til 9,90 per liter, 2 poser potetgull til 14,90 per stk. og 3 hg smågodt til 16 kr per hg. Sett opp og regn ut talltrykket som viser hvor mye hun må betale.*
> 9,90kr · 3___14,90kr · 2___16kr · 3 = 29,70kr + 29,80kr + 48kr = 107,50kr
>
> ***3*** *Betecka och räkna. Ringa ett rätt svar. b) Vilket tal får du om du till talet -6 adderar dess motsatta tal och sedan subtraherar talet 9 från summan?*
> -6 + 6 = 0 **+** 9 = -9

RP = representation by a picture

RT = representation of task

> **6.22** c) *4m + 2n – 3n + n*
> 4m + 2n **+** 3n + n

RU = representation unclear

> ***6.22*** *b) 3x + 2y + x + 4y*
> 3x + x + 2y + 4y = **4xy 6y**
>
> ***6.11*** *Martin kjøper x kg epler i butikken. Eplene koster 20kr per kilogram. Lag et uttrykk som viser hvor mye Martin må betale.*
> ***6.12*** *Lag et uttrykk for innholdet i sirkelen*
> 6.11) x · 20 =
> 6.12) a · 13 = 20x **og** 13a

RV = representation of variable

> ***6.21*** *Trekk sammen a) 2a + 5a*
> 7B
>
> ***6.21*** *Trekk sammen f) 5x – 4x*
> **1**x

FORMING EQUATIONS AND EXPRESSIONS

EE =writing an expression as an equation

> ***6.9*** *Skriv et uttrykk for c) 10 mer enn y d) 12 mindre en y*
> c) y + 10 = 10
> d) y – 12 = 8

EI = expression/equation incorrect

__6.9__ Skriv et uttrykk for d) 12 mindre en y
y + 12
12 – y

__3__ Betecka och räkna. Ringa ett rätt svar. b) Vilket tal får du om du först subtaherar talet 12 från talet 7 och sedan adderar talet 6 till differensen?
12 – 7 = - 5 + 6 = 1

OTHERS

MC = miscalculation = laskuvirhe

__6.19__ Formelen for omkretsen av et rektangel er O = 2a + 2b, der O står for omkretsen, a for lengden og b for bredden av rektangelet. Regn ut omkretsen av rektangelet når
b) a= 12cm og b = 7,5 cm
12 + 12 = 24
7,5 + 7,5 = 31
= 55 cm er omkretsen

__6.3__ 3 · (4 + 5)
3 · (4 + 5) = (4 + 5) · 3 = 28

MT = misinterpretation of task = tehtävän väärinymmärtäminen

__5__ Omtrent hvor lang er radiusen når omkretsen er a) 24 cm
Da er radiusen 12 cm når d er 24cm

MP = missing part of task

__2.__ Bilda ett uttrykk för omkretsen p och arean A för rektangeln.
(p is calculated, A not)

NA = no answer

Appendix 3: Procedures – Finland – School 1 – Class 1

PROCEDURES - FINNISH 6th GRADE - SCHOOL 1

Student	Task 11	12	13	14	15	16	18	19	20	21	22	23	27	28	29	30	31	e1_1	e1_2	e1_3	e2_1	e2_2
AA								RR	RE,RR	RR,RS	RE,RR		RS,RR	RE,RS,RR				CR	CR	RI		
AB			RR		FS	FR		RS	RS,RE	RS,RR	RS,RR		RS	RE,RS	RE,RS						RS	RS
AC	AO	(AO),RS	AO	AO				AO	AO				RR	RR				CD	CD	RI,RE		
AD								RS	RE,RS	RS	RS,RE		RS	RE,RS	RE,RS	RS	RE,RS	CD	CD	RI	RS	RS
AE	AO	AO	AO	AO				RS	AO	AO	AO,RS		RS	RE,RS				CD	CD	RI	RS	RS
AF							AO	(AO),RS	RS (AO)	AO	AO		(AO),RE,RS	RE,RS	RE,RS		RE,RS	CD	CD	RI	RS	RS
AG	RS	RS	RS	RE,RS	FS	FR		(AO),RS	RE,RS		RE,RS		RS	RE,RS	RE,RS				CD			
AH	AO	AO,CD	AO	AO				AO	AO	AO	AO		AO	RE,RS	RE,RS			CD			RS	RS
AI								RS	RE,RS	RS	RE		RS	RE,RS,RR	RE,RS,RR			CR	CR	RI,RE	RS	RS
AJ		AO	AO	AO,CD	AO			AO	AO	AO,RS	AO,RS	FS	AO,RS	RS,RE	RE,RS						RS	RS
AK	RR	RR,RE	RR	RR				RR	RR,RE	RR	RE,RR		RR	RE,RR	RE,RS	RR	RR	CD	CD	RE		
AL	AO	(AO),RE,RS	AO					AO	AO	AO	AO		RS	RE,RS				CD	CD			
	AO	AO	RR	AO	FS	FR	AO	RR	RE	RR	RE	FS	RS	RE	RE	RS	RE	CR	CR	RI	RS	RS
	RS	RS	AO	RE	AO			RS	RR	RS	RR		RR	RS	RS	RR	RS	CD	CD	RE		
	RR	CD	RS	RS				AO	RS	AO	RS		AO	RR	RR		RR					
		RR		CD					AO		AO		RE									
		RE		RR																		

Appendix 4: Procedures – Finland – School 4 – Class 1

PROCEDURES - FINNISH 7th GRADE - SCHOOL 4

Studens	**Task**				
	1	**2**	**3**	**4**	**5**
BA	RE	FR,RE	RE,AO	CD	CD
BB	RE	FR,RE	AO	CD	CD
BC	RE	FR,RE	RE	CD	CD
BD	RE	FR,RE	SD	RG	RG
BE	RE	AD	RA	CD	AD
BF	RE	FR,RE	RE	CD	CD
BG	RE	FR,RE	SD	CD	CD
BH	RE	FR,RE	RE	CD	CD
BI	RE	FR,RE	SD	RG	RG
BJ	RE	FR,RE	RE	CD	CD
BK	RE	FR,RE	RE	RG	RG,CD
BL	RE	FR,RE	OA	CD	CD
BM	RE	FR,RE	RE	RG,AD	RG,AD
BN	RE	FR,RE	SD	RG	RG
	RE	FR	RE	CD	CD
		RE	AO	RG	RG
		AD	SD	AD	AD
			RA		
			OA		

Appendix 5: Procedures – Norway – School 1 – Class 1

PROCEDURES - NORWEGIAN 7th GRADE - SCHOOL 1 - CLASS 1

Student	Task 3	4	5	6	8	9	12	13	14	15	16	17	18	62	63	64	65	66	67	68	69	70
CA	FR	SF		SD	FM	SD													FM,RW	FM	FM	
CB	RW,FR	SD	RI	SD	FM	SD								FR	SF	SF	SD		FM,RW,SD	FM	FM	DP
CC	FR,RW	SD	SD	FM,SD	FM	FM,RE,SD	FR,RW								RW	RW			FM,SF,RW	FM	FM,RE	
CD	FR	SD	SD	RI	FM	FM,SD		DP	DP	DP,RW	AO	AO							FM,RW	FM	FM	DP
CE	FR	SF	SF	SF,RE	FM	AO								DP,FR,RW	AE	AE	SD	AO	FM,RW	FM	FM	RW
CF	FR	SD	RI	SF	FM	SD,FM								DP,FR,RW	SD	AO	SD	RW	FM,AO,RW	FM	FM,AO	AO,RW
CG	FR,RW	SF,RW	SF,RW	SF	FM	FM, SD,RW		DP	DP	RW,DP,FR	RW	RW	RW						FM,SF,RW	FM	FM,RE	RW
CH																			RE,RW,AO	FM	AO	RW
CI	FR, RW	SD	RI	RI	FM	FM, AO								FR	RI	SD	SF	RI				
CJ	FR,RW	RW	RW	RW	FM	AO								DP, FR,RW	AO	RW	SF,RW		FM,RW	FM	RW	
CK	FR	SD	AO	SF	FM	FM,SD								DP, FR,RW	SF,RW,AE	RW	SD	RW	FM,SD,RW	FM	FM,RW	DP
CL	RW	SD						DP	DP										FM,,RW,FM	FM	RW,AD	RW
CM	RW,FR	SD	SF	SF	FM	FM,SF		DP	DP	DP, FR,RW	SF	SF	SF						FM,RW	FM	FM,RW	RW
CN	RW,FR	SD	RI	SD,RW	FM	FM,SD								RW,FR	RN,SD	RW,RN,SD		AO	FM,SF,RW	FM	FM,RW,SF	
CO								DP	DP	RI		AO	AO						FM,AO,RW	FM	FM,AO	DP
CP	RW	SD	SD	SD															FM,RW,AO	FM		
CQ	RW,FR	RW	RW,AO	RW	FM	FM,RW								DP,FR	RW,RN							
CR	FR,RW	SD	SD	SD	FM	FM,SD								FR,DP	RN,SD,RW	SD,RW	SD,RW	SD,RW	FM,SF,RW	FM	RW,SF,FM	DP,RW
CS	FR	SD	RI	SD,RW	FM	FM,SD,RW								FR,DP	SD	RW	RW,SD	AO				
	FR	SF	RI	SD	FM	SD	FR	DP	DP	DP	AO	AO	RW	FR	SF	SF	SD	AO	FM	FM	FM	DP
	RW	SD	SD	FM		FM	RW			RW	RW	RW	SF	DP	RW	RW	SF	RW	RW		RE	RW
		RW	SF	RI		RE				FR	SF	SF	AO	RW	AE	AE	RW	RI	SD		AO	AO
			RW	SF		AO				RI					SD	AO		SD	SF		AD	
			AO	RE		RW									RI	SD			AO		SF	
				RW											AO							
															RN							

Appendix 6: Procedures – Norway – School 1 – Class 2

PROCEDURES - NORWEGIAN 7th GRADE - SCHOOL 1 - CLASS 2

Student	Task 3	4	5	6	8	9	13	14	15	16	17	18	62	63	64	65	66	67	68	69	70	71	72
DA	FR,RW	RW	RW	FM,RW	FM,RW	RW							FR,DP	RW	RW	SD,RW	AO	FM,RW,SF,AE	FM	RW,SF			
DB	FR,RW	SF	RI	RI, AO	FM,RW		DP	DP	DP	AO	AO	AO											
DC	FR,RW	SD	SD	SD	FM	FM,SD							DP,RW,FR	SD	SD	SD	SD	FM,RW,SD	FM	FM,SD	FR,AO		
DE																		FM,AO,RW,AE	FM	FM,RW	AO		
DF	RW,FR	SD	RI,SD	FM,SD	FM	FM,SD	DP	DP	DP,RW,FR	AO	AO	AO						FM,RW,AO	FM	RW	FR,RW		
DG	FR,RW	SD	SD	SD,FM	FM	FM,SD							DP,RW,FR	AE	AE,RW	SD	AO	FM,RW,SD,AE	FM	FM,SD	SD,FR		
DH	FR,RW	SD	CD	CD	FM	FM,SD							DP,RW,FR	RW	RW	SD	AO	AO,FM,RW	FM	AO,FM	AO,FR		
DI	FR,RW	SD	SD	SD	FM	FM,SD	DP	DP	DP,RW,FR	AO	AO												
DJ	FR,RW	SD	RI,SD	FM,SD	FM	FM,RW,SD							DP,FR,RW	AE,RW	AE,SD	SD	SD						
DK	FR,RW	AO	SD	FM,SD	FM	RA												AO,FM	FM	RI,FM			
DL	FR,RW	SD	SD	FM,SD	FM	FM,SD							FR,DP	SD	SD	SD	SD	FM,RW,SD	FM	FM,SD	SD		
DM	RW,FR	SD	SD,AO	FM,SD,AO	SD,AO	AO							DP,RW	SD	SD	SD	AO	FM,AO,RW	FM	FM,AO	DP,RW		
DN	RW,FR	SD	RI	RI,SD	FM	FM,SD							DP,FR,RW	SD	SD	SD							
DO	FR,RW	SD	RI	RI,SD	FM	AO	DP	DP	DP,FR,RW	SD,RN	SD,RN	SD,RN						FM,RW,SD,AE	FM	FM,SD	DP		
DP	FR,RW	SF	SD	FM,SD	FM	FM, SD							DP,FR	SD	SD	SD						SD	FR
DQ	FR,RW	RW	CD	RW,CD,SF	FM	AE	DP	DP	DP,FR,SF	SF,AE	SD,AE	SD,AE						FM,SF,RW	FM	FM,SF		RW,SF	
DR	FR,RW	SD	SD	FM,SD	FM	FM, SD	DP	DP	DP,FR	SD	SD	SD,AE						FM,RW,SD,AO	FM	FM,SD	SD		
DS	SD	SD	RI	FM,SD	FM	FM,SD												FM,AO,RW	FM	FM,AO	RE		
DT	RW,FR	SD	SD	FM,RW,SD														FM,RW,AE	FM,RW	FM,SD			
	FR	RW	RW	FM	FM	RW	DP	DP	DP	AO	AO	AO	FR	RW	RW	SD	AO	FM	FM	RW	FR	SD	FR
	RW	SF	RI	RW	RW	FM			RW	SD	SD	SD	DP	SD	SD	RW	SD	RW	RW	SF	AO	RW	
	SD	SD	SD	RI	SD	SD			FR	RN	RN	RN	RW	AE	AE			SF		FM	RW	SF	
		AO	CD	AO	AO	RA			SF	SF	AE	AE						AE		SD	SD		
			AO	SD		AO				AE								SD		AO	DP		
				CD		AE												AO		RI	RE		
				SF																			

Appendix 7: Procedures – Norway – School 2 – Class 1

PROCEDURES - NORWEGIAN 8th GRADE - SCHOOL 2 - CLASS 1

Student	Task 6.1	6.2	6.3	6.4	6.5	6.6	6.7	6.8	6.9	6.10	6.11	6.12	6.13	6.14	6.15	6.16	6.17	6.18	6.19	6.20	6.21	6.22	6.23	6.24	6.25	6.26
EA	RI, CS	RI														SD	SD	RW		CD	CD	CD	RE	CD		
EB	CR	CR							RE	RE	RE	RE	RE	RE		SD	SD	RW,SD	SD	CD	RD	CD,RG				
EC	CR	CR	CR	CR	CR,RI	CR	RE	RE	RE	RE	RE	RE	RE	RW	RE	RA	RA	RE	RE	CD	CD	CD,RG	RD	CD,RG	RE,RD	CD
ED				RI					RE	RE,RW	RE	RE	RE	RE	RE	SF	SF	RW,RE	RA							
EE	AO	AA							RE	RE										CD	CD	AD				
EF											CR					SD	SD	RW		CD	CD	CD				
EG	CR	CR	CR						RE	RE	RE	RE				SD	SD	RW	RE	CD	CD	CD	RW	CD	RE	CD
EH	CR								RW,RE							SF, SD	SD	RW		CD	CD	CD				
EI	CR,RI	CR, RI	CR						RI	RE	RE	RE				SE	SE	RE,SD	RE,SF	CD	CD	CD				
EJ	CR	CR	CR	CR	CR	CR	RE		RE	RE,RW	RE	RE	RE	RE	RE	SF	SF	RW,SF	RE,SF	CD	CD	CD	RE	CD	RE	CD
EK	CR	CR	CR						RE	RE	RE	RE	RE		SD	SD	RW,SD	SD		CD	CD	RG				
EL	CR,CF	CR	CR													SD	SD	RW,SD	SF	CD	CD	RG	RE,DP			
EM	CR,CF	CR	CR	CR					RE	RE	RE					SD	SD	RW,SD	SF,RW	CD	CD	RG	RE	CD	RE	
EN	CR	CR							RE	RE	RE	RE	RE	RE	RE	SD	SD	RW,SD	SF	CD	CD	CD	RD	CD,AP	RE	
EO	CR,AO	AO					RN	RE	RE	RE	RE					AO	SD	RW,AO	AO	CD	CD		RE	RE		
EP	CR	CR	CR	CR	CR	CR	RE		EE	RE	RE	RE	RE	RE		SD	SD	RW,SD	RE	CD	CD	RG,CD	RD,RD	RG	RE	CD
EQ	CD	CD							RE	RE	RE	RE,RW				SR	SR	RW,AO		CD	CD	CD				
ER	CD	CD	CD						RE,RW	RE	RE					SD	SD	RW,SD	RE	CD	CD	CD	RE	CD	RE	CD
ES	CD,CR	CR,CS	CR						RE,AA	RE,RW	RE,RW	RE,RW				CS,SD	CS,SD	RE,RW	RA	CD	CD	CD	RE	CD	RE	
ET	CF,CR	CR	CR	CR	CR	CR			RE,RW	RE	RE	RE				SD	SD	RW,SD	RI	CD	CD	RG	RE	CD	RE	
EU																				CD	CD	RG				
EV	CR	CR	CR	CR					RE	RE	RE,RW	RE,RW	RE,RW	RE,RW	RE,RW	SD	SD	RW,SD	RE	CD	CD	CD	RE	CD		
	RI	RI	CR	CR	CR	CR	RE	RE	RE	RE	RE	RE	RE	RE	RE	SD	SD	RW	SD	CD	CD	CD	RE	CD	RE	CD
	CS	CR		RI	RI		RN		RW	RW	CR	RW	RW	RW	SD	RA	RA	SD	RE			RG	RD	RG	RD	
	CR	AA							RI		RW				RW	SF	SF	RE	RA			AO	RW	AP		
	AO	AO							AA							AO	RW	SF	SF				DP	RE		
	CF	CD														SR	SR	AO	RW							
	CD	CS														CS			AO							
																			RI							

Appendix 8: Procedures – Norway – School 2 – Class 2

PROCEDURES - NORWEGIAN 8th GRADE - SCHOOL 2 - CLASS 2

Student	Task 6.1	6.2	6.3	6.4	6.5	6.6	6.7	6.8	6.9	6.10	6.11	6.12	6.13	6.16	6.17	6.18	6.19	6.20	6.21
FA	CS	CS	CR	CR	CR	CD,CR	RN	RE		RE	RE			SD			SD	CD	CD
FB	CR	CR	CR				RE,RW		RE	RE	RE			SD			SF	CD	CD
FC	CR	CR	CR				RE		RE	RE	RE			SD			DP,SF	CD	
FD	CR	CR	CR	CR	CR		RE,RW			RE	RE			SF			SD	CD	CD,RD
FE	CR	CR	CR	CR	CR		RI			RE									
FF	CR	CR	CR				RE			RE									
FG	CR	CR	CR				RA			RE	RE			SF		RE		CD	CD
FH	CR	CR	CR	CR	CR	CR,RA	RN			RE	RE			SF			RI	RA	RA,CD
FI	CR	CR	CR	CR			RE			RE	RE			SF			SF	CD	CD
FJ	CR	CR	CR				RE			RE	RE			SD			RA	CD	CD
FK	CR	CR	CR						RE,RW										
FL	CR	CR	CR				RI			RE,RW	RE,RW								
FM	CR	CR	CR				RE,RW			RE	RE								
FN	CR	CR	CR				RE			RE	RE			SD			SF	CD	CD
FO	CR	CR	CR				RE,CD		RE	RE,RW	RE,RW			AO					
FP	CR						RI		RE,RW	RE									
FQ	CR	CR							RE	RE	RE	RE							
FR	CR,RI	CD	CD				RN		RE	RE	RE			SD					
FS	CR	CR	CR				RE												
FT	CR	CR	CR						RE										
FU	CR	CR	CR	CR	CR		RE			RE	RE			SF					
FV			CR				RE			RE	RE								
FW	CR	CR	CR	CR			RE		RE	RE	RE	RE	RE	SD	SD				
FX	CR,RI,AO																		
	CS	CS	CR	CR	CR	CD	RN	RE	RE	RE	RE	RE	RE	SD	SD	RE	SD	CD	CD
	CR	CR	CD			CR	RE		RW	RW	RW			SF			SF	RA	RD
	RI	CD				RA	RW							AO			DP		RA
	AO						RI										RI		
							RA										RA		
							CD												

Appendix 9: Mistakes – Finland – School 1 – Class 1

MISTAKES - FINNISH 6th GRADE - SCHOOL 1 - CLASS 1

Student	Task																					
	11	12	13	14	15	16	17	18	19	20	21	22	23	27	28	29	30	31	e1_1	e1_2	e1_3	e2_1	e2_2
AA										UA				MC							OR		
AB			MC,OE	OE		MT						UA											
AC		MC	AU																		EI,OR,		
AD									AA		UU								MC		OR,EI	OE	
AE		UA		UA						UA		UA			UA						OR, EI		
AF									RU,UU,MC	MC,UA	MC	UA			RO,UA	RU,RO		RO		OS	EI,OR	MC,OE	
AG		RT,UA	MC	MC,UA		MC	MC			UA		MC			UA								
AH		MC	MC,UA	UA	UA				MC	UA,MC		MC,UA			UA				MC			RI	MC
AI											RT	EI			UA						EI,OR	OE,MC	
AJ			UU	MC,UA						UA,MC		UA,MC		MC	EI,UA,MC	UU,MC						MC	MC
AK												MC							MC				
AL		UA								UA,MC	MC				UA								
	-	MC	MC	OE	UA	MT	MC	-	AA	UA	UU	UA	-	MC	UA	EC	-	RO	MC	OS	OR	OE	MC
		UA	OE	UA		MC			RU,UU,MC	MC	MC	MC			RU	RO					EI	MC	
		RT	AU	MC					UU		RT	EI			EI	UU						RI	
			UA						MC						MC	MC							
			UU																				

Appendix 10: Mistakes – Finland – School 4 – Class 1

MISTAKES - FINNISH 7th GRADE - SCHOOL 4 - CLASS 1

Student	Task				
	1	2	3	4	5
BA		EI	OR,MT	OS	RV,MC
BB		MP	MT,MP	OS	OS,OP, MC
BC	RO	MP	MT,UE	OS,MC	MC,OP
BD					
BE	EI	NA	MT	OS,MC	NA
BF		EI	MT		OP,MC
BG	RO		MP		OP
BH		EI	UA,MT	OS,MC	OP,MC,RV,OS
BI			UA	MC	OS,OP
BJ			MT		
BK			MT,MP		OS
BL	RV	MP	MT	OS	OS,OP
BM		MP	MT	NA	NA
BN	RO	EI	UE,OR	MC	RV,OS
	RO	EI	OR	OS	RV
	EI	MP	MT	MC	MC
	RV	NA	MP	NA	OS
			UE		OP
			UA		NA

Appendix 11: Mistakes – Norway – School 1 – Class 1

MISTAKES - NORWEGIAN 7th GRADE - SCHOOL 1 - CLASS 1

Student	Task 3	4	5	6	8	9	12	13	14	15	16	17	18	62	63	64	65	66	67	68	69	70
CA		UA		UA		UA																
CB			RV,UA	UA										RP	UU	MT	UA					
CC		UA	MT	UA	UA	UA										UA			UE			
CD		UA	MT	OR,UA,MC	UA	UA,MC				RP									UE		UE	
CE						MC											UA				UE	
CF		UA	OR,UA												UA		UA					
CG																			UE		UE	
CH																			MT		UE	
CI			OR	OR,MC										OR	UA,OR	UA	UA	UA				
CJ					UA	UA									MC,UA							
CK		UA	UT	OM		MC									UA	UT	UA		UE		UE	
CL		UA																	UA			
CM		UA		UA							OR	UA,OR	UA, OR								UE	
CN			OR																			
CO										MT		MC	MC									
CP	MT	UA	EI,UA	UA															UE			
CQ																						
CR				UA																		
CS		UA	UA, OR,RV	RV	UA	UA									UE,UA							
	MT	UA	RV	UA	UA	UA	-	-	-	RP	OR	UA	UA	RP	UU	MT	UA	UA	UE	-	UE	-
			UA	OR		MC				MT		OR	OR	OR	UA	UA			MT			
			MT	MC								MC	MC		OR	UT			UA			
			OR	OM											MC							
			UT	RV											UE							
			EI																			

Appendix 12: Mistakes – Norway – School 1 – Class 2

MISTAKES - NORWEGIAN 7th GRADE - SCHOOL 1 - CLASS 2

Student	Task 3	4	5	6	7	8	9	12	13	14	15	16	17	18	62	63	64	65	66	67	68	69	70	71	72
DA		UE	MT																	MT					
DB		UA	UA, OR	MT,UA							MT	MT,UA	MT,UA	MT,UA											
DC			UE	UT														UA,MT	MT	UE					
DE																				UA	UA	MT	UA		
DF		UA	UA,OR	RV,UA		UA	UA				EI	MT,UA								UA		UE			
DG		UE,UA	UE,UA	EI,MT																					
DH		UA	EI,UA	UA														UA	UA	UE					
DI		UA		RV,UA			UA				RP														
DJ		UA	UA,OR	UA		UA	UA								UA		UE,UA	UA	UA						
DK																				MT,UE		UE			
DL				MC																UE		UE			
DM		UA	UA	UA		UA, RI	UA										UA,EI	UA		UE					
DN			RV	RV,UA			UA									UA	UA,EI	UA							
DO		UA	UA,OR	RV,OR								UA	UA	UA											
DP																									
DQ			UA																						
DR		UA	RV,UA	UA								UA,EI	UA,EI	UA,EI											
DS		UA,MC	OR,UA	UA			UA													UE		UE			
DT			UA,RV	UA																UA		UA			
	-	UE	MT	MT	-	UA	UA	-	-	-	MT	MT	MT	MT	UA	UA	UE	UA	MT	MT	UA	MT	UA	-	-
		UA	UA	UA		RI					EI	UA	UA	UA			UA	MT	UA	UE		UE			
		MC	OR	UT							RP	EI	EI	EI			EI			UA		UA			
			UE	RV																					
			EI	EI																					
			RV	MC																					
				OR																					

Appendix 13: Mistakes – Norway – School 2 – Class 1

MISTAKES - NORWEGIAN 8th GRADE - SCHOOL 2 - CLASS 1

Student	Task 6.1	6.2	6.3	6.4	6.5	6.6	6.7	6.8	6.9	6.10	6.11	6.12	6.14	6.15	6.16	6.17	6.18	6.19	6.20	6.21	6.22	6.23	6.24	6.25	6.26
EA	OR															MC	NA						RV		
EB									EI					NA				UA			NA				
EC			MC											OB				UA			UE	MT			OS
ED				OB					EI				UA	OB			UE								
EE		NA																		UE	NA				
EF	NA																			RV,MC,UE	NA				
EG	RT	MC	NA						EI								NA	UA			UE		OS	MC	OS,RU
EH	NA								EI												RC				
EI									EE			RU			MC		UA	UA		UE	NA				
EJ					MC		MC,UA				UA			OB			UA	UA							OS
EK		MC															UA								
EL		OM	RT														UA	UA							
EM		OM							EI								UA						OS		
EN									EI									UA			UE		RU		NA
EO							UA	EI									UA								
EP	RT						UA		EE	AD							UA	MT					OS	MC	RU
EQ																			MC	MC,UE	RT,OS				
ER									EI								UA	UA			UE		RU	MC	UE,OS
ES			OM														UE	MC		UE	OS,RT		OS		
ET					RU												UA			MC,UE	RU,UE		OS	MT	
EU																			RC	MC,UE					
EV			MC						EI						MC		UA		RC,MC	RT, OS		MC			
	OR	NA	MC	OB	MC	-	MC	EI	EI	AD	UA	RU	UA	NA	MC	MC	NA	UA	MC	UE	NA	MT	RV	MC	OS
	NA	MC	NA		RU		UA		EE					OB			UE	MT	RC	RV	UE	MC	OS	MT	RU
	RT	OM	RT														UA	MC		MC	RC		RU		NA
			OM																	RT	RT				UE
																				OS	OS				
																					RU				

Appendix 14: Mistakes – Norway – School 2 – Class 2

MISTAKES - NORWEGIAN 8th GRADE - SCHOOL 2 - CLASS 2

Student	Task 6.1	6.2	6.3	6.4	6.5	6.6	6.7	6.8	6.9	6.10	6.11	6.12	6.13	6.16	6.17	6.18	6.19	6.20	6.21
FA								UA									UA		UE
FB									EI									RC	
FC			MC				RT										UA,MC		
FD																	UA		
FE		OB			OO,MC														
FF							MC,UA												
FG	MC	OM					UA										UA		
FH	MC					OB,OO											UA	RC	UE
FI		OM	MC				UA										UA,MC		UE
FJ							UA												NA
FK		NA							MT										
FL			OS				RO												
FM							UA												
FN	MC																		MC, OS,RC
FO																			
FP	OO						OC,UA												
FQ									EI										
FR	OO,OB		MC,NA				MC												
FS			MC																
FT	OS																		
FU							MC,UA												
FV							UA												
FW			OM,MC,OO,RT				UA												
FX																			
	MC	OB	MC	-	OO	OB	RT	UA	MT	-	-	-	-	-	-	-	UA	RC	UE
	OO	OM	OS		MC	OO	MC		EI								MC		NA
	OB	NA	NA				UA												MC
	OS		OM				RO												OS
			OO				OC												RC
			RT																