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Research Article

New Delay-Dependent Stability Conditions for Time-Varying Delay Systems

Wei Qian[1] and Hamid Reza Karimi[2]

[1] *School of Electrical Engineering and Automation, Henan Polytechnic University, Jiaozuo, Henan 454000, China*
[2] *Department of Engineering, Faculty of Engineering and Science, University of Agder, 4898 Grimstad, Norway*

Correspondence should be addressed to Wei Qian; qwei@hpu.edu.cn

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This paper addresses the delay-dependent stability for systems with time-varying delay. First, by taking multi-integral terms into consideration, new Lyapunov-Krasovskii functional is defined. Second, in order to reduce the computational complexity of the main results, reciprocally convex approach and some special transformations are introduced, and new delay-dependent stability criteria are proposed, which are less conservative and have less decision variables than some previous results. Finally, two well-known examples are given to illustrate the correctness and advantage of our theoretical results.

1. Introduction

Time-varying delays are often source of the degradation of performance and the instability of practical control systems. In the last decades, the stability of time-delay systems was one of the hot issues in control theory, and many approaches are proposed to study this topic, such as the properly chosen LKFs [1–3], the delay-fraction approach [4–6], descriptor model transformation method [7], integral inequality lemma [8–10], free weighting matrices [11], and Jensen inequality [12, 13].

Recently, in order to further reduce conservatism of the stability results, some new methods were developed. In [14], by constructing novel LKF with matrices that depended on the time delays, stability conditions that depend on both the upper and lower bounds on delay derivatives were obtained. In [15], the convex analysis method was proposed, and in [16], reciprocally convex approach was used to consider the stability of time-varying delay systems. In [17], new LKF including quadratic terms multiplied by a higher degree scalar function was proposed and this method was further improved in [18], where the quadratic convex combination technique was used. In [19], some triple-integral terms were introduced to LKF, and in [20], some quadruple-integral terms were used in LKF to discuss the stability of time-varying delay systems. These methods reduced the conservatism of the stability results, but more decision variables were introduced; that is, the advantage in conservatism is obtained at the cost of high computational complexity. The above analysis naturally leads us to find possible solutions to this problem.

In this paper, we further discuss the stability of linear systems with time-varying delay. The main contribution of this paper lies in two aspects. First, a novel LKF which contains some multi-integral terms is introduced; second, the derivative of LKF is estimated by using reciprocally convex approach and some special transformations, which reduces the conservatism as well as the computational burden of the main results.

Throughout the paper, the used notations are standard. $\mathbf{R}^n$ denotes the n-dimensional Euclidean space, $\mathbf{R}^{n\times m}$ is a set of $n \times m$ real matrices, A^T is the transpose of A, $P > 0$ ($P < 0$) means symmetric positive (negative) definite matrix, $*$ in the matrix denotes the symmetric element, I is the identity matrix of appropriate dimensions, and $x_t = x(t+\theta)$, $\theta \in [-h, 0]$. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. Problem Formulations

Consider the following time-delay system:

$$\dot{x}(t) = Ax(t) + A_1 x(t - h(t)), \quad x(t) = \phi(t), \quad t \in [-h, 0], \tag{1}$$

where $x(t) \in \mathbf{R}^n$ is the state vector; the initial condition $\phi(t)$ is a continuously differentiable vector-valued function; $A, A_1 \in \mathbf{R}^{n\times n}$ are known real constant matrices; and $h(t)$ is the time-varying delay satisfying

$$0 \le h(t) \le h, \tag{2}$$

$$\dot{h}(t) \le d \le \infty, \tag{3}$$

where $0 < h$ and $d \ge 0$ are constants.

To obtain the main results, the following lemmas are needed.

Lemma 1 (see [16]). *Let $f_1, f_2, \ldots, f_N : \mathbf{R}^m \to \mathbf{R}$ have positive values in an open subset $\mathbf{D}$ of $\mathbf{R}^m$. Then, the reciprocally convex combination of f_i over $\mathbf{D}$ satisfies*

$$\min_{\{\alpha_i \mid \alpha_i > 0, \sum \alpha_i = 1\}} \sum_i \frac{1}{\alpha_i} f_i(t) = \sum_i f_i(t) + \max_{g_{ij}(t)} \sum_{i \ne j} g_{ij}(t) \tag{4}$$

subject to

$$g_{i,j} : \mathbf{R}^m \longrightarrow \mathbf{R}, \qquad g_{ij}(t) = g_{ji}(t), \quad \begin{bmatrix} f_i(t) & g_{ij}(t) \\ g_{ij}(t) & f_j(t) \end{bmatrix} \ge 0. \tag{5}$$

Lemma 2 (see [18]). *For any symmetric matrix X_0, X_1, X_2 and a vector ξ, let*

$$f(\alpha) = \xi^T X_0 \xi + \alpha \xi^T X_1 \xi + \alpha^2 \xi^T X_2 \xi \tag{6}$$

with $X_2 > 0$. Then for all $\alpha \in [\alpha_1, \alpha_2]$, one has $f(\alpha_1) < 0$ and $f(\alpha_2) < 0 \Rightarrow f(\alpha) < 0$.

3. Main Results

In this section, the stability of system (1) is investigated. Through constructing a novel LKF and estimating the derivative of it, new stability condition is provided.

Firstly, construct the following LKF candidate:

$$V(x_t) = \sum_{i=1}^{5} V_i(x_t), \tag{7}$$

where

$$\begin{aligned}
V_1(x_t) &= x^T(t) P x(t), \\
V_2(x_t) &= \int_{t-h(t)}^{t} \left[x^T(t) \;\; x^T(s)\right] Q \left[x^T(t) \;\; x^T(s)\right]^T ds, \\
V_3(x_t) &= \int_{t-h}^{t} \left[x^T(t) \;\; x^T(s)\right] R \left[x^T(t) \;\; x^T(s)\right]^T ds, \\
V_4(x_t) &= h \int_{-h}^{0} \int_{t+\theta}^{t} \left[x^T(s) \;\; \dot{x}^T(s)\right] M \left[x^T(s) \;\; \dot{x}^T(s)\right]^T ds\, d\theta, \\
V_5(x_t) &= 2 \int_{-h}^{0} \int_{\theta}^{0} \int_{t+\lambda}^{t} \dot{x}^T(s) U \dot{x}(s)\, ds\, d\lambda\, d\theta,
\end{aligned} \tag{8}$$

and matrices $P, U \in \mathbf{R}^{n\times n}$, $Q = \{Q_{ij}\ (1 \le i \le j \le 2)\}$, $R = \{R_{ij}\ (1 \le i \le j \le 2)\}$, $M = \{M_{ij}\ (1 \le i \le j \le 2)\} \in \mathbf{R}^{2n\times 2n} > 0$.

Next, let us define a vector as

$$\chi^T(t) = \left[x^T(t) \;\; x^T(t-h(t)) \;\; x^T(t-h) \;\; \int_{t-h}^{t-h(t)} x^T(s)\, ds \;\; \int_{t-h(t)}^{t} x^T(s)\, ds\right] \tag{9}$$

and $A_c = [A \;\; A_1 \;\; 0 \;\; 0 \;\; 0]$, block entry matrices e_i $(i = 1, 2, \ldots, 5)$ (e.g., $e_2^T = [0 \;\; I \;\; 0 \;\; 0 \;\; 0]$).

Taking the time derivatives of $V_i(x_t)$ along the trajectory of system (1) yields

$$\dot{V}_1(x_t) = \chi^T(t)\left(e_1 P A_c + A_c^T P e_1^T\right)\chi(t), \tag{10}$$

$$\begin{aligned}
\dot{V}_2(x_t) = {} & \left[x^T(t) \;\; x^T(t)\right] Q \left[x^T(t) \;\; x^T(t)\right]^T \\
& - \left(1 - \dot{h}(t)\right)\left[x^T(t) \;\; x^T(t-h(t))\right] \\
& \times Q\left[x^T(t) \;\; x^T(t-h(t))\right]^T \\
& + 2\left[Ax(t) + A_1 x(t-h(t))\right]^T \\
& \times \left[Q_{11} h(t) x(t) + Q_{12} \int_{t-h(t)}^{t} x(s)\, ds\right] \\
\le {} & \chi^T(t)\left\{[e_1 \;\; e_1] Q [e_1 \;\; e_1]^T - (1-d)\right. \\
& \left. \times [e_1 \;\; e_2] Q [e_1 \;\; e_2]^T\right\}\chi(t) \\
& + 2\chi^T(t)\left[A_c^T \;\; e_0\right] Q \left[h(t) e_1 \;\; e_5\right]^T \chi(t), \\
\dot{V}_3(x_t) = {} & \left[x^T(t) \;\; x^T(t)\right] R \left[x^T(t) \;\; x^T(t)\right]^T \\
& - \left[x^T(t) \;\; x^T(t-h)\right] R \left[x^T(t) \;\; x^T(t-h)\right]^T
\end{aligned}$$

$$\begin{aligned}&+2[Ax(t)+A_1x(t-h(t))]^T\\&\times\left[R_{11}hx(t)+R_{12}\int_{t-h}^{t}x(s)\,ds\right]\\&=\chi^T(t)\left\{[e_1\ \ e_1]R[e_1\ \ e_1]^T-[e_1\ \ e_3]R[e_1\ \ e_3]^T\right\}\chi(t)\\&+2\chi^T(t)\left[A_c^T\ \ e_0\right]R[he_1\ \ e_4+e_5]^T\chi(t).\end{aligned}\tag{11}$$

To the time derivative of $V_4(x_t)$, it can be calculated as

$$\begin{aligned}\dot V_4(x_t)&=h^2\left[x^T(t)\ \ \dot x^T(t)\right]M\left[x^T(t)\ \ \dot x^T(t)\right]^T\\&\quad-h\int_{t-h}^{t}\left[x^T(s)\ \ \dot x^T(s)\right]M\left[x^T(s)\ \ \dot x^T(s)\right]^T ds\\&=h^2\left[x^T(t)\ \ \dot x^T(t)\right]M\left[x^T(t)\ \ \dot x^T(t)\right]^T\\&\quad-h\int_{t-h(t)}^{t}\left[x^T(s)\ \ \dot x^T(s)\right]M\left[x^T(s)\ \ \dot x^T(s)\right]^T ds\\&\quad-h\int_{t-h}^{t-h(t)}\left[x^T(s)\ \ \dot x^T(s)\right]M\left[x^T(s)\ \ \dot x^T(s)\right]^T ds.\end{aligned}\tag{12}$$

To reduce the conservatism of the main results, the last two terms of $\dot V_4(x_t)$ were estimated by different methods. In [11], the free matrices were introduced. Reference [13] used convex combination by introducing an approximation on the difference between delay bounds. Reference [16] adopted reciprocally convex approach, which achieved performance behavior identical to approaches based on the integral inequality lemma but with much less decision variables. Therefore, in the following, by introducing appropriate matrix $N\in\mathbf{R}^{2n\times 2n}$ and using Lemma 1, we can estimate $\dot V_4(x_t)$ as

$$\begin{aligned}\dot V_4(x_t)&\le h^2\left[x^T(t)\ \ \dot x^T(t)\right]M\left[x^T(t)\ \ \dot x^T(t)\right]^T\\&\quad-\frac{h}{h(t)}\left(\int_{t-h(t)}^{t}\left[x^T(s)\ \ \dot x^T(s)\right]ds\right)^T\\&\quad\times M\left(\int_{t-h(t)}^{t}\left[x^T(s)\ \ \dot x^T(s)\right]ds\right)\\&\quad-\frac{h}{h-h(t)}\left(\int_{t-h}^{t-h(t)}\left[x^T(s)\ \ \dot x^T(s)\right]ds\right)^T\\&\quad\times M\left(\int_{t-h}^{t-h(t)}\left[x^T(s)\ \ \dot x^T(s)\right]ds\right)\\&\le\chi^T(t)\left\{h^2\left[e_1\ \ A_c^T\right]M\left[e_1\ \ A_c^T\right]^T\right.\\&\quad-[e_5\ \ e_1-e_2]M[e_5\ \ e_1-e_2]^T\\&\quad+[e_4\ \ e_2-e_3]M[e_4\ \ e_2-e_3]^T\\&\quad\left.+2[e_5\ \ e_1-e_2]N[e_4\ \ e_2-e_3]^T\right\}\chi(t),\end{aligned}\tag{13}$$

where $\left[\begin{smallmatrix}M & N\\ * & M\end{smallmatrix}\right]\ge 0$.

To the time derivative of $V_5(x_t)$, on the first step, the following transformation is used:

$$\begin{aligned}\dot V_5(x_t)&=h^2\dot x^T(t)U\dot x(t)\\&\quad-2\int_{-h}^{0}\int_{t+\theta}^{t}\dot x^T(s)U\dot x(s)\,ds\,d\theta\\&=h^2\chi^T(t)A_c^TUA_c\chi(t)\\&\quad-2\int_{t-h(t)}^{t}(s-t+h)\dot x^T(s)U\dot x(s)\,ds\\&\quad-2\int_{t-h}^{t-h(t)}(s-t+h)\dot x^T(s)U\dot x(s)\,ds,\end{aligned}\tag{14}$$

and then, in order to reduce the conservatism of the main result, the appropriate matrices $F_1,\ F_2\in\mathbf{R}^{n\times 5n}$ are introduced to establish the relations between different vectors of $\chi(t)$, so the last two terms in (14) can be estimated as

$$\begin{aligned}&-2\int_{t-h(t)}^{t}(s-t+h)\dot x^T(s)U\dot x(s)\,ds\\&\le-2\int_{t-h(t)}^{t}(s-t+h(t))\dot x^T(s)U\dot x(s)\,ds\\&\le2\int_{t-h(t)}^{t}(s-t+h(t))\chi^T(t)F_1^TU^{-1}F_1\chi(t)\\&\quad+4\chi^T(t)F_1^T\int_{t-h(t)}^{t}(s-t+h(t))\dot x(s)\,ds\\&=h^2(t)\chi^T(t)F_1^TU^{-1}F_1\chi(t)\\&\quad+4\chi^T(t)F_1^T\left[h(t)x(t)-\int_{t-h(t)}^{t}x(s)\,ds\right]\\&=h^2(t)\chi^T(t)F_1^TU^{-1}F_1\chi(t)\\&\quad+4\chi^T(t)F_1^T[h(t)e_1-e_5]^T\chi(t),\end{aligned}$$

$$
\begin{aligned}
&-2\int_{t-h}^{t-h(t)}(s-t+h)\,\dot{x}^T(s)\,U\dot{x}(s)\,ds\\
&\le 2\int_{t-h}^{t-h(t)}(s-t+h)\,\chi^T(t)\,F_2^TU^{-1}F_2\chi(t)\\
&\quad+4\chi^T(t)\,F_2^T\int_{t-h}^{t-h(t)}(s-t+h)\,\dot{x}(s)\,ds\\
&=(h-h(t))^2\chi^T(t)\,F_2^TU^{-1}F_2\chi(t)\\
&\quad+4\chi^T(t)\,F_2^T\left[(h-h(t))\,x(t-h(t))\right.\\
&\qquad\left.-\int_{t-h}^{t-h(t)}x(s)\,ds\right]\\
&=(h-h(t))^2\chi^T(t)\,F_2^TU^{-1}F_2\chi(t)\\
&\quad+4\chi^T(t)\,F_2^T\left[(h-h(t))\,e_2-e_4\right]^T.
\end{aligned}\tag{15}
$$

Therefore, combining (11)–(15), we can obtain

$$\dot{V}(x_t)\le\chi^T(t)\left[\Xi_0+h(t)\,\Xi_1+\Xi_2\right]\chi(t),\tag{16}$$

where

$$
\begin{aligned}
\Xi_0&=e_1PA_c+A_c^TPe_1^T+[e_1\;\;e_1]\,R[e_1\;\;e_1]^T\\
&\quad-[e_1\;\;e_3]\,R[e_1\;\;e_3]^T+[e_1\;\;e_1]\,Q[e_1\;\;e_1]^T\\
&\quad-(1-d)\,[e_1\;\;e_2]\,Q[e_1\;\;e_2]^T\\
&\quad+\left[A_c^T\;\;e_0\right]R[he_1\;\;e_4+e_5]^T\\
&\quad+[he_1\;\;e_4+e_5]\,R\left[A_c^T\;\;e_0\right]^T\\
&\quad+\left[A_c^T\;\;e_0\right]Q[e_0\;\;e_5]^T+[e_0\;\;e_5]\,Q\left[A_c^T\;\;e_0\right]^T\\
&\quad+h^2\left[e_1\;\;A_c^T\right]M\left[e_1\;\;A_c^T\right]^T\\
&\quad-[e_5\;\;e_1-e_2]\,M[e_5\;\;e_1-e_2]^T\\
&\quad+[e_4\;\;e_2-e_3]\,M[e_4\;\;e_2-e_3]^T\\
&\quad+[e_5\;\;e_1-e_2]\,N[e_4\;\;e_2-e_3]^T\\
&\quad+[e_4\;\;e_2-e_3]\,N[e_5\;\;e_1-e_2]^T\\
&\quad+h^2A_c^TUA_c-4F_1^Te_5^T+4F_2^T(he_2-e_4)^T,\\
\Xi_1&=\left[A_c^T\;\;e_0\right]Q[e_1\;\;e_0]^T+[e_1\;\;e_0]\,Q\left[A_c^T\;\;e_0\right]^T\\
&\quad+4F_1^Te_1^T-4F_2^Te_2^T,\\
\Xi_2&=h^2(t)\,F_1^TU^{-1}F_1+(h-h(t))^2F_2^TU^{-1}F_2.
\end{aligned}\tag{17}
$$

It can be found that the function $\dot{V}(x_t)=\chi^T(t)[\Xi_0+h(t)\Xi_1+\Xi_2]\chi(t)$ is a quadratic function on the scalar $h(t)$ and the coefficient of second order is $\chi^T(t)[F_1^TU^{-1}F_1+F_2^TU^{-1}F_2]\chi(t)\ge 0$ since $U>0$, which means the $\dot{V}(x_t)$ is a convex quadratic function for $h(t)$. Therefore, apply Lemma 2, if the following holds:

$$
\begin{aligned}
&\left[\Xi_0+h(t)\,\Xi_1+\Xi_2\right]_{h(t)=0}<0,\\
&\left[\Xi_0+h(t)\,\Xi_1+\Xi_2\right]_{h(t)=h}<0;
\end{aligned}\tag{18}
$$

then we have

$$\Xi_0+h(t)\,\Xi_1+\Xi_2<0\quad\forall h(t)\in[0,h],\tag{19}$$

which implies $\dot{V}(x_t)<0$, which means that system (1) is asymptotically stable. So, we give the main theorem of this paper as follows.

Theorem 3. *Given scalars $0<h$ and $\mu\ge 0$, system (1) with a time-varying delay satisfying (2) and (3) is asymptotically stable, if there exist $n\times n$ matrices $P>0$, $U>0$; $2n\times 2n$ matrices $Q>0$, $R>0$, $M>0$; $2n\times 2n$ matrix N; and $n\times 5n$ matrices F_1, F_2 such that the following LMIs hold:*

$$
\begin{aligned}
&\begin{bmatrix}M & N\\ * & M\end{bmatrix}\ge 0,\\
&\begin{bmatrix}\Xi_0 & hF_2^T\\ * & -U\end{bmatrix}<0,\\
&\begin{bmatrix}\Xi_0+h\Xi_1 & hF_1^T\\ * & -U\end{bmatrix}<0.
\end{aligned}\tag{20}
$$

Remark 4. In (14), the term $-2\int_{-h}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)U\dot{x}(s)ds\,d\theta$ also can be estimated by Jesen inequality as

$$
\begin{aligned}
&-2\int_{-h}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)\,U\dot{x}(s)\,ds\,d\theta\\
&\le-\frac{4}{h^2}\left[hx(t)-\int_{t-h}^{t}x(s)\,ds\right]^T\\
&\quad\times U\left[hx(t)-\int_{t-h}^{t}x(s)\,ds\right],
\end{aligned}\tag{21}
$$

but in this case, only relations between vectors $x(t)$, $\int_{t-h}^{t-h(t)}x(s)ds$, $\int_{t-h(t)}^{t}x(s)ds$ were established; the corresponding result would be conservative. Therefore, we converted the term to a quadratic term multiplied by a scalar function and used free matrices F_1, F_2 to establish the relations between $x(t), x(t-h(t)), x(t-h), \int_{t-h}^{t-h(t)}x(s)ds$, and $\int_{t-h(t)}^{t}x(s)ds$, which reduced the conservatism of the stability result.

Remark 5. In this paper, it can be seen that LKFs in [5, 11, 13, 16] are just reduced forms of LKF (7), so Theorem 3 is less conservative than those stability conditions. Moreover, the reciprocally convex approach and the LKF transformation in (14) are used to estimate the derivative of the LKF in this paper. Those are the main reasons why Theorem 3 is less conservative and has less decision variables.

Table 1: Admissible upper bound h for various μ in Example 7.

μ	0.0	0.05	0.1	0.5	3.0
Wu et al. [7]	4.47	3.98	3.60	2.00	—
He et al. [11]	4.47	3.98	3.60	2.04	—
Park and Ko [15]	4.47	4.01	3.66	2.33	1.86
Qian et al. [2]	4.60	4.12	3.70	2.33	1.86
Park et al. [16]	4.66	4.17	3.76	2.12	—
Kim [18]	4.97	4.35	3.86	2.33	1.86
Theorem 3	4.91	4.30	3.82	2.33	1.86

When the information of the time derivative of delay is unknown, just let $Q = 0$ in LKF (7); we can get the following result from Theorem 3.

Corollary 6. *Given a scalar* $0 < h$*, if there exist* $n\times n$ *matrices* $P > 0,\ U > 0$*;* $2n \times 2n$ *matrices* $R > 0,\ M > 0$*;* $2n \times 2n$ *matrix* N*; and* $n\times 5n$ *matrices* $F_1,\ F_2$ *such that LMIs* (20) *with* $Q = 0$ *are feasible, then system* (1) *with a time-varying delay satisfying* (2) *is asymptotically stable.*

4. Numerical Examples

In this section, the effectiveness of the obtained results in this paper is shown by two well-known numerical examples.

Example 7. Consider the linear system (1) with

$$A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \qquad A_1 = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}. \tag{22}$$

The purpose of this example is to compare the admissible upper bounds h which guarantee the asymptotic stability of the above system by different methods. Using the method in [12], when $d = 0.5,\ d = 0.9$, and $d > 1$, the upper bounds for the time delay are $h = 2.08,\ h = 1.66$, and $h = 1.66$; ours are $h = 2.33,\ h = 1.87$, and $h = 1.86$. Table 1 also lists the comparison of our results with some recent ones in [2, 7, 11, 15, 16, 18]. From Table 1, it can be seen that our results are less conservative.

Moreover, the maximum upper bounds h obtained by Theorem 3 are almost equal to those in [18], but the number of variables is $49n^2 + 5n$ in [18], and ours is just $21n^2 + 4n$. That is, the variables in [18] are around 2.2 times more than those in Theorem 3. Therefore, from both mathematical and practical points of view, our condition is more desirable than that in [18] even for some cases when the two methods give almost the same upper bound on the delay.

Example 8. Consider the linear system (1) with

$$A = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}, \qquad A_1 = \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}. \tag{23}$$

This example is used in many recent papers, such as [7, 15, 18]. For various μ, the maximum upper bounds on delay h by different methods are listed in Table 2. It can be seen that the result obtained in this paper is less conservative.

Table 2: Admissible upper bound h for various μ in Example 8.

μ	0.0	0.05	0.1	0.5	3.0
Wu et al. [7]	1.82	1.76	1.71	1.38	—
Park and Ko [15]	1.99	1.81	1.75	1.61	1.60
Kim [18]	2.52	2.17	2.02	1.62	1.60
Theorem 3	2.52	2.16	2.02	1.61	1.60

5. Conclusions

In this paper, the stability of time-varying delay systems has been discussed. Through constructing a novel LKF which contains multi-integral terms and using some new analysis methods, some delay-dependent stability criteria were derived. Compared with some previous stability conditions, the obtained main results in this paper have less conservatism and less decision variables. In the end, numerical examples were given to show the superiority of the obtained criteria and their improvements over the existing results.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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References

[1] X. Jiang and Q.-L. Han, "New stability criteria for linear systems with interval time-varying delay," *Automatica*, vol. 44, no. 10, pp. 2680–2685, 2008.

[2] W. Qian, L. Wang, and Y. Sun, "Improved robust stability criteria for uncertain systems with time-varying delay," *Asian Journal of Control*, vol. 13, no. 6, pp. 1043–1050, 2011.

[3] Z. Wang, Y. Liu, and X. Liu, "Exponential stabilization of a class of stochastic system with Markovian jump parameters and mode-dependent mixed time-delays," *IEEE Transactions on Automatic Control*, vol. 55, no. 7, pp. 1656–1662, 2010.

[4] F. Gouaisbaut and D. Peaucelle, "Delay-dependent robust stability of time delay systems," in *Proceedings of the 5th IFAC Symposium on Robust Control Design (ROCOND '06)*, pp. 453–458, fra, July 2006.

[5] X.-L. Zhu, Y. Wang, and G.-H. Yang, "New stability criteria for continuous-time systems with interval time-varying delay," *IET Control Theory & Applications*, vol. 4, no. 6, pp. 1101–1107, 2010.

[6] Z. Wang, Y. Wang, and Y. Liu, "Global synchronization for discrete-time stochastic complex networks with randomly occurred nonlinearities and mixed time delays," *IEEE Transactions on Neural Networks*, vol. 21, no. 1, pp. 11–25, 2010.

[7] M. Wu, Y. He, and J.-H. She, "New delay-dependent stability criteria and stabilizing method for neutral systems," *IEEE Transactions on Automatic Control*, vol. 49, no. 12, pp. 2266–2271, 2004.

[8] M. J. Park, O. M. Kwon, J. H. Park, and S. M. Lee, "A new augmented Lyapunov-Krasovskii functional approach for stability of linear systems with time-varying delays," *Applied Mathematics and Computation*, vol. 217, no. 17, pp. 7197–7209, 2011.

[9] X. Nian, H. Pan, W. Gui, and H. Wang, "New stability analysis for linear neutral system via state matrix decomposition," *Applied Mathematics and Computation*, vol. 215, no. 5, pp. 1830–1837, 2009.

[10] Z. Wang, Y. Liu, G. Wei, and X. Liu, "A note on control of a class of discrete-time stochastic systems with distributed delays and nonlinear disturbances," *Automatica*, vol. 46, no. 3, pp. 543–548, 2010.

[11] Y. He, Q.-G. Wang, L. Xie, and C. Lin, "Further improvement of free-weighting matrices technique for systems with time-varying delay," *IEEE Transactions on Automatic Control*, vol. 52, no. 2, pp. 293–299, 2007.

[12] C. Peng and Y.-C. Tian, "Delay-dependent robust stability criteria for uncertain systems with interval time-varying delay," *Journal of Computational and Applied Mathematics*, vol. 214, no. 2, pp. 480–494, 2008.

[13] H. Shao, "New delay-dependent stability criteria for systems with interval delay," *Automatica*, vol. 45, no. 3, pp. 744–749, 2009.

[14] E. Fridman, U. Shaked, and K. Liu, "New conditions for delay-derivative-dependent stability," *Automatica*, vol. 45, no. 11, pp. 2723–2727, 2009.

[15] P. Park and J. W. Ko, "Stability and robust stability for systems with a time-varying delay," *Automatica*, vol. 43, no. 10, pp. 1855–1858, 2007.

[16] P. Park, J. W. Ko, and C. Jeong, "Reciprocally convex approach to stability of systems with time-varying delays," *Automatica*, vol. 47, no. 1, pp. 235–238, 2011.

[17] M. N. A. Parlakçı, "Extensively augmented Lyapunov functional approach for the stability of neutral time-delay systems," *IET Control Theory & Applications*, vol. 2, no. 5, pp. 431–436, 2008.

[18] J.-H. Kim, "Note on stability of linear systems with time-varying delay," *Automatica*, vol. 47, no. 9, pp. 2118–2121, 2011.

[19] J. Sun, G. P. Liu, J. Chen, and D. Rees, "Improved delay-range-dependent stability criteria for linear systems with time-varying delays," *Automatica*, vol. 46, no. 2, pp. 466–470, 2010.

[20] W. Qian, S. Cong, T. Li, and S. M. Fei, "Improved stability conditions for systems with interval time-varying delay," *International Journal of Control, Automation, and Systems*, vol. 10, no. 2, pp. 1146–1152, 2012.