Fundamental Analysis of Triple Layer Area Decoupled Photovoltaic Modules

by

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ABSTRACT

The main objective of this work is to analyse the area decoupled triple layer photovoltaic module, by application of voltage matching under AM 1.5 light spectrum. This includes, calculation of detailed balance efficiency, calculation of optimal numbers of cells in each layer to achieve voltage matching comparisons with independently operated triple layer stack. Performance change under various different scenario is also observed. We have used MatLab programing software for all our analysis i.e., plots and calculations.

At first we investigated the optimal efficiency of independently operated cells of triple layer stack. Then applying area decoupling technique in a triple layer voltage matched module, we analysed the performance of such module under AM 1.5 spectrum. Upon comparisons the voltage matched module performed similarly to the independent stacks with only slight optimal efficiency difference, which occurred due to calculation round-off. If perfect area decoupling is achieved then it should be exact. Different sets of available semiconductor materials which are non-toxic are tried out in the analysis to find the best possible combination which gives optimal result. Lastly the performance of such optimized voltage matched area decoupled photovoltaic modules under other different spectra is studied. Which showed our area decoupled voltage matched module performed similarly to the independently operated stacks of cells under other different spectra too with no or only slight variation in efficiency.

The methods adapted to get the results are described in this report. The results show that the voltage matched tandem module, which is area decoupled gives detailed balance efficiency of 51.2 under numbers of cells in each stack at 32, 60 and 118, for top, middle and bottom layer respectively, with 60 silicon cells layer at middle layer. This tandem module performs similarly as the independently operated stacks under same condition. But change in spectrum affects its optimal performance due to spectral mismatch. If the area de-coupled module is optimized for the AM1.5 spectrum, it will be a little less efficient than an independently operated stack when the spectrum changes.

PREFACE

This report is the result of the Master Thesis ENE-500 (30 ECTS) course in MSc Renewable Energy degree to fulfill the requirements of final semester course content, at Faculty of Engineering and Science, University of Agder (UiA) in Grimstad, Norway. We have completed the main goal of fundamental analysis of area decoupled triple layer voltage matched photovoltaic module.

The work on project has been started from 05 January 2015 and ended on 09 June 2015.

I would like to express my gratitude to my project supervisor, Associate Prof. Rune Strandberg for his valuable assistance and supervision during thesis work. I would also like to thank our Studiekoordinator Stein Bergsmark for his constructive suggestions and support during the project work.

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CHAPTER 1 Introduction

With the increase in demand of energy from limited natural resources which is diminishing continuously with time, energy crisis is inevitable. Which has led to growing interest towards renewable energy resources. Amongst which solar energy technology has been one of the most interesting and potential field. Solar energy can provide both sustainable and environmental friendly energy need. This is basically divided into two main categories. Thermal solar, where electricity is final product from the heat energy utilized from solar energy and second the photovoltaics, providing electricity straight from solar energy. Photovoltaics accounts for a very small amount of energy production in total global electricity production. Installed photovoltaic capacity is as low as almost 1% of the total installed renewable energy at present. However, ongoing researches on this field for more economic and efficient technologies is promising. [3]

For photovoltaic capacity to be harvested more, there is a need of more efficient and economic alternatives to the current technologies. Up till now, photovoltaics can be divided into three generations of technology [2]. First generation consists of relatively expensive and less efficient cells, with only 15-20 percent efficiency. Second generation photovoltaic technology consists of sufficiently cheap cells to provide affordable electricity.
While third generations are advanced solar cell technologies with higher theoretical efficiencies at cheap price. The third generation cells utilize more advanced technology with higher theoretical efficiencies in comparison to first and second generation cells. Mostly used photovoltaics at current consists of first generation technology with use of highly abundant silicon crystals having property of being non-toxic. Despite high initial cost payback time is lowering with time and new technologies. With growing interest and development within photovoltaics there is an aim of reaching grid parity by 2020 in several countries which includes Australia, most of the Europe, Japan and USA. [11]

One the prospective field of new generations of advanced solar cell technology covers Tandem cell modules. Photovoltaic tandem cells consist of stacks of two or more solar cells, where the bandgap of the cells decreases from top to bottom in the stack. Detailed balance formalism is mostly used to analyse the theoretical efficiency limits of such cells.

Area de-coupled tandem cells consisting of multiple layer of cells in a module could be used to increase the efficiency of photovoltaic modules. Area decoupling enables us to differ the numbers of top cells in each layer in a multi junction module. Within each layers the cells are horizontally connected which can be either voltage matched or current matched in tandem module. These modules matches the efficiency of independently operated solar cells in stack. In one of the layers Silicon cells layer consisting 60 cells, which are commercially viable, could be used, while choosing various semiconductor materials at other layers. With the right choice of material and configuration the much higher efficiency with lower initial cost could be achieved with these tandem modules. Study of area de-coupled double tandem photovoltaic [12] forms the basis for this analysis of area decoupled triple layer photovoltaic module. All the new formulation analysis and calculations would be based on the previous work on the area de-coupled double tandem cells with use of MatLab software as main analysis tool for calculations and plots.

CHAPTER 2 Background and Literature Review

2.1 Theoretical Background

The backbone of study of photovoltaic solar energy conversion has always has been the single junction solar cells. Where the excited electrons moves from the valence band to the conduction band when the energy from the photon exceeds the bandgap. There is energy loss when these generated carriers collide with the lattice of the semiconductor until they thermalize to the energy states in the vicinity of conduction band. Moreover, the presence of significant number of photos having energy level below the band gap, passes through the cells without exciting any electron-hole pair. Thus if we could reduce the thermalization loss and utilize the photons with energy below bandgap, we could achieve much higher efficiency. Tandem modules are alternative solution which can reduce these two losses. Use of low pass filters in a tandem stack with decreasing bandgap from top to bottom in layers lead to absorption of photons by the cells which have bandgaps closer to that of incident photons. Thus, reducing the carriers thermalization losses. Also wider range of photon energies is utilized by the cells in carrier excitation. Energy conversion process can be further enhanced if more cells is used within the stack. Bandgaps of comparable photon energies can be matched which will induce reduction in thermalization in lattice and collection of unused photons [5].

2.2 Basic Detailed Balance Theory for single cell

Detailed balance is a technique to calculate the maximum efficiency of photovoltaic devices, proposed by Shockley and Queisser in 1961 [10] . Later extended in a paper published by Tiedje in 1984 [13].
Two Fundamental assumptions in detailed balance theory consists of assuming infinite carrier mobility of the carriers which allows the collection these carriers irrelevant to their place of generation within the cell. While second assumption states the complete absorption of all the photons which has energy higher than the band gap.

Basic calculation involves the particle flux for different configurations of the Plank's equation. The general form of the equation is:

$$\boldsymbol{\Phi}(E_a, E_b, \mu, T) = Cf \frac{2\pi}{h^3 c^2} \int_{E_g}^{\infty} \frac{E^2}{exp\left(\frac{E-\mu}{kT}\right) - 1} dE \tag{2.1}$$

Where T is the temperature of the cell and set at 300 K, h is Planck's constant, c is the speed of light, and k is Boltzmann's constant.

The general approach is to calculate the absorption flux and the flux emitted from the solar cell. The difference between these two named Generation Rate and Recombination Rate multiplied by q (elemental charge) gives the current from the solar cell. One photon excites one electron to the external circuit and in recombination, an electron de-excites while emitting a photon [6].

2.3 Absorption or Generation flux

The photovoltaic cell absorbs primarily from the sun and also from the regions of the sky. Considering the maximum concentration of the radiation using optics, the solar cell is illuminated by the radiation of the same temperature on the sun. While in a realistic approach under the conditions the concentration of radiation being less than maximum only portion of the radiation illuminates the cell while other portion is illuminated from the radiation source with the same temperature as the Earth. The maximum concentration is given by a value 46,200 based on the size of the sun disk in the sky. [6]

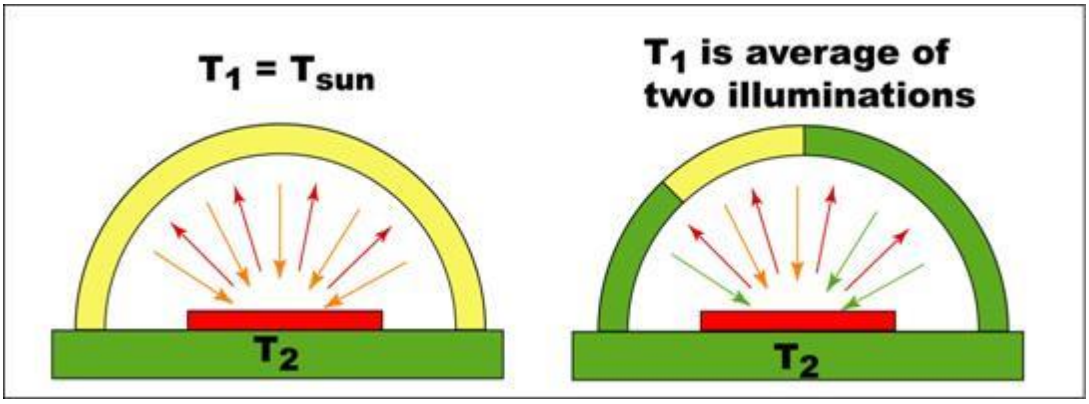

Figure 2.1: Schematic representation of detailed balance theorem [6]

On the left image light is collected from the entire hemisphere which corresponds to maximum concentration. For lower or no concentration the Sun can be seen as an arc giving lower particle flux [6].

The absorption flux from the sun is given by:

$$\boldsymbol{\Phi sun}\,(E_G, \infty, 0, Ts) = \mathrm{Cf}\frac{2\pi}{\mathrm{h}^3 c^2}\int_{E_g}^{\infty}\frac{E^2}{\exp\left(\frac{E}{kT_s}\right)-1}dE \qquad (1.2)$$

Where T_s is the temperature of the Sun.

And the absorption flux from the black body radiation of the earth is:

$$\boldsymbol{\Phi earth}\ (EG, \infty, 0, Tearth) = (1 - \mathrm{Cf})\frac{2\pi}{\mathrm{h}^3 c^2}\int_{E_g}^{\infty}\frac{E^2}{\exp\left(\frac{E}{kT_{earth}}\right) - 1}dE \tag{2.3}$$

Where $T_{earth}$is the temperature of the Earth.

The total absorption flux of the solar cell is:

$$\begin{aligned}\boldsymbol{\Phi_1} = \boldsymbol{\Phi sun}\ (EG, \infty, 0, Ts) - \boldsymbol{\Phi earth}\ (EG, \infty, 0, Tearth) \\ = \mathrm{Cf}\frac{2\pi}{\mathrm{h}^3 c^2}\int_{E_g}^{\infty}\frac{E^2}{\exp\left(\frac{E}{kT_s}\right) - 1}dE - (1 \\ - \mathrm{Cf})\frac{2\pi}{\mathrm{h}^3 c^2}\int_{E_g}^{\infty}\frac{E^2}{\exp\left(\frac{E}{kT_{earth}}\right) - 1}dE\end{aligned} \tag{2.4}$$

2.4 Emission Flux or Regeneration

The emission from the solar cell depends on the quasi-Fermi level separation (μ) of the solar cell [6]. Considering short circuit case where n is zero (μ=*mV)* The emission is calculated by:

$$\boldsymbol{\Phi_2}\ (E_G, \infty, 0, TE) = \mathrm{Cf}\frac{2\pi}{\mathrm{h}^3 c^2}\int_{E_g}^{\infty}\frac{E^2}{\exp\left(\frac{E-\mu}{kT_e}\right) - 1}dE \tag{2.5}$$

2.5 Calculation of efficiency for a fixed bandgap and black body

The power from the solar cell depends on the band gap and on the quasi-Fermi level separation. A quasi-Fermi level is chemical potential of electrons that describes the population of electrons separately in the conduction band and valence band, when their

populations are displaced from equilibrium [8]. For a given band gap, the quasi-Fermi level separation must be varied to find the maximum power point, i.e., where [6]:

$$\boldsymbol{P_{solarCell}} = q(\boldsymbol{\Phi_1} - \boldsymbol{\Phi_2})\mu \tag{2.2}$$

Where q is the elemental charge

at a maximum value. This is done by varying m from 0 to close to the open circuit condition (where $\varphi_1 = \varphi_2$), to find where the power is at a maximum [6].

The efficiency is given by:

$$\eta = \frac{\boldsymbol{P_{solarCell}}}{\boldsymbol{P_{sun}}} \tag{2.3}$$

At maximum efficiency

$$\eta = \frac{q(\boldsymbol{\Phi_1} - \boldsymbol{\Phi_2})\mu}{\sigma T_{sun}^4} \tag{2.4}$$

2.6 Efficiency as a function of band gap and Spectrum

To find the efficiency as a function of band gap, the above procedure is repeated for each band gap. The optimum is found for one particular band gap. [6]

The air mass coefficient (AM) defines the direct optical path length through the Earth's atmosphere, expressed as a ratio relative to the path length vertically upwards, i.e. at the zenith.. "AM1.5" is almost universal when characterizing terrestrial power-generating panels. For the calculation of efficiency under AM1.5 and other measured spectra, φ_1 is replaced by the summation of the photon flux of energies above the band gap while power from the sun is given by the summation of the power in the measured spectra [4].

2.7 Detailed Balance Theory of area decoupled tandem photovoltaic Module

Multi-junction or tandem solar cells consist of several cells stacked on top of each other. The cells in the stack are made of materials with different band gaps. The top cells have the largest band gap and absorb the high energy part of the solar spectrum. The cells below have smaller and smaller band gap and absorbs parts of the spectrum where the photons have lower energy. This structure gives cells with a higher theoretical efficiency than ordinary single band gap solar cells.
Until recently, the research on tandem cells has only considered stacks where all the cells have the same area. The types of stacks have then been divided into two-terminal stacks and multi-terminal stacks. In a two terminal stack, there are only two terminals connecting the cells in the stack to an external circuit. This has the consequence that the cells in the stacks becomes series-connected, and have to be current-matched, i.e. they all have to produce the same current. In multi-terminal stacks each of the cells in the stack are connected by two-terminals which enables each of the cells to be operated independently. This removes the requirement of current-matching, but the wiring and structure of the stack becomes more complex. Multi-terminal stacks have a higher maximum theoretical efficiency than two-terminal stacks, and more combinations of band gaps give a high theoretical efficiency in the multi-terminal stacks.

The concept of area de-coupled solar cells is a novel method that allows two-terminal stacks to reach the same theoretical efficiency as multi-terminal stacks. Area de-coupling can only be applied to modules, not single cells. An area de-coupled module consists of stacked layers of cells. The cells in each layer are series connected. The layers can then be either series-connected (current-matched) or connected in parallel (voltage matched).

Figure 2.2 (a): Independently operated triple layer photovoltaic cells

This (Figure 2.2 (a)) is termed as unconstrained tandem module where the cells being electrically isolated but are optically connected. The current and quasi-Fermi level of each cell is independent of the operation of all the other cells within the stack. To achieve the optimal efficiency limit bandgap of the cells in the module should be varied. The main drawback the unconstrained tandem stack is the necessity of large numbers of connections. Thus increasing the complexity of the structure as commercially available cables, connections, and inverters for PV-systems are designed for two-terminal systems.

As a solution these cells can be connected in a voltage matched or current matched module.

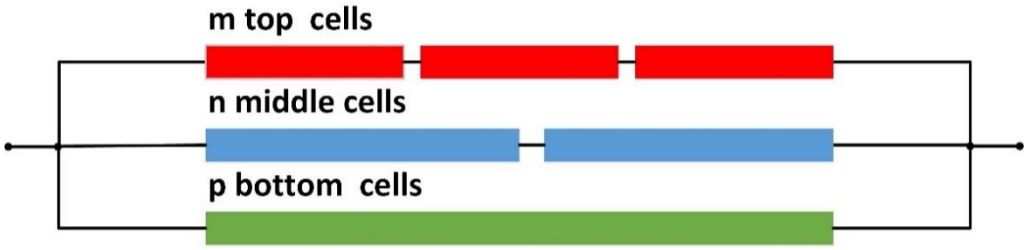

Figure 2.2 (b): Voltage matched area decupled triple layer photovoltaic module

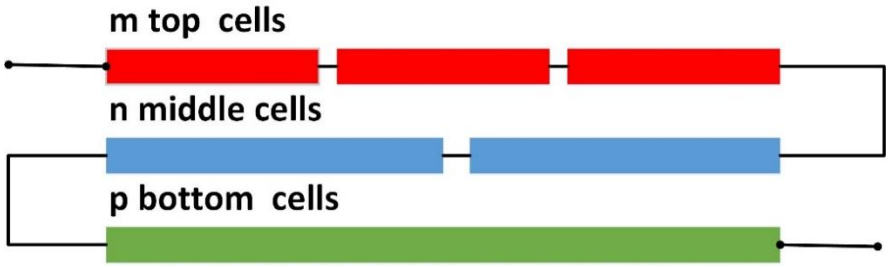

Figure 2.2: (c): Current matched area decoupled triple layer photovoltaic module

These module structures (Figure 2.2 (b), Figure 2.2: (c)) allows the cells in the different layers to have different areas, they are area de-coupled. The number of cells in each layer can be optimized to achieve perfect current- or voltage matching of the layers. It has been shown that area de-coupling allows two-terminal tandem modules to achieve the same theoretical efficiency as multi-terminal devices.

Existing work on area de-coupled cells [12] only covers double tandem modules, that is, stacks with two types of cells. In this project the work will be extended to triple tandem modules, where there are three layers of cells. The main objective of the project work is to analyze the triple junction area de-coupled tandem cells in terms of how much theoretical efficiency can be obtained with possibility of using various materials at each layer so that the construction would be optimal. i.e., We will calculate the number of cells required at each layer of a triple layer area decoupled module to obtain the voltage matching. Then keeping 60 cells silicon at one of the layer in triple layer module we will vary and try out different sets of bandgap combinations at other two layers which would give the maximum efficiency value. Corresponding number of cells and bandgap values for maximum efficiency will also be analyzed. In this work we implement detailed balance formalism where we calculate the current produced by the solar cell from the net absorption of photons with the use of MatLab software. The main objectives of the project work are:

- To investigate different configuration of area decoupled voltage-matched triple tandem modules that gives the highest possible theoretical efficiency for AM 1.5
- To find the optimal area and numbers of cells i.e. ratio of top, middle and bottom cells corresponding to the optimal efficiency
- To find out what combinations of band gaps that gives high efficiency if one of the layers in the structure consists of 60 silicon cells with bandgap of 1.11 eV of each silicon cell.
- To compare the performance of area decupled voltage matched triple layer model with that of the triple layer independently operated stack
- To analyze how spectral variation affects the overall efficiency
- To try out various abundant and nontoxic semiconductor materials in efficiency analysis in our tandem module to figure out the best possible combination that gives highest possible theoretical efficiency

Multi-terminal devices can easily adapt to changing conditions since each cell is operated independently. Area de-coupled cells have to be optimized to one particular set of conditions thus, may not behave optimally as condition changes. To investigate the spectral response of area de-coupled modules the analysis will be repeated with different spectra. Thus, the end result of this project work should show that the Triple junction area de-coupled photovoltaic cell should perform similarly to the independently operated solar cells in stack. Analysis of various combinations of abundant nontoxic semiconductors should result in a set of particular configuration of area decoupled voltage matched module with optimal number of cells at each layer for a particular semiconductor material with optimal bandgaps giving rise to maximum theoretical efficiency possible. These photovoltaic modules should have lower initial cost with higher lifespan of usage providing optimal output.

CHAPTER 3 METHODOLOGY

The efficiency limit for all the scenario of triple layer photovoltaic modules is based on work [1]. MatLab solution tool is used for all the simulation and plots. Similar previous study on the tandem modules (Strandberg, Detailed Balance Analysis of area de-coupled double tandem photovoltaic modules, 2014), forms the base for all the for implementation on triple junction area decoupled module.

Based on the previous studies of tandem cells [12] [1], formulation of triple junction tandem module can be done. Consider horizontal series connection in tandem modules allowing the number of cells in each three layers in triple tandem module differ from each other while maintaining the equal cell areas. Adjustment of number of top cells, middle cells and bottom cells independently allows us to current match the module. Further horizontal series connections allows for voltage matched module too if the three layers are connected in parallel. Dividing the module into an unlimited number of cells enables us to achieve perfect voltage or current matching by calculating the exact numbers of cells in each layers of the module. Operation of all the cells is considered at maximum power point in each layer, giving rise to efficiency of such tandem module equal to that of stack of independently operated cells. As typical PV module currently in use consists of 60 silicon cells, for all the analysis for area decoupled cells one of the layer in each tandem module is considered to consist of such sixty silicon cells. Numbers of cells in top, middle and bottom layers is denoted by m, n and p respectively. Layer consisting of sixty silicon cells is the assumed to be at the top, middle and bottom for various analysis. Different sets of bandgaps of solar cells is implemented in calculation in other two layers beside silicon cells layer, varying from a minimum to maximum limit, keeping the top cells bandgap the highest and in decreasing order towards the lower layers. For each of these conditions the optimal ratio and thus the number of cells in other two layers beside silicon is determined.

According to the variations in the connections of cells in tandem module detailed balance formalism is varied.

The detailed balance formulation used in the analysis of triple junction area decoupled tandem modules here has following assumptions:

(a) Recombination process within the device is radiative recombination only [9].
(b) All photons with energy greater than bandgap of cells are completely absorbed and rest are passed through.
(c) Quasi-Fermi levels are assumed constant throughout the device with their difference equal to the bias [7] [14].
(d) Each absorbed photon generates only a single electron–hole pair [5].

We consider tandem photovoltaic modules of triple layers, where the bandgap of solar cells are in decreasing order from top to bottom. It is assumed that the each cell absorbs all the incident photons with energies higher than the bandgap of the cell and possesses transparency to photons that have energies that are less than the energy of the bandgap.

Presence of low-pass energy filters is considered in each layers of Tandem module. These function as reflectors for light energy emitted by the cell onto itself while passing through the photons with energy lesser than the bandgap.

The flux of photons with energy between E_a and E_b emitted by a solar cell biased with a voltage V and obeying the above list of assumptions is given by

$$\boldsymbol{\Phi_{out}}(E_a, E_b, V, T) = C \int_{E_a}^{E_b} \frac{E^2}{exp\left(\frac{E - qV}{kT}\right) - 1} dE \tag{3.1}$$

where *T* is the temperature of the cell and set to 300 K in this work. The constant C equals $2\pi/h^3c^2$, where *h* is Planck's constant, *c* is the speed of light, and *k* is Boltzmann's constant.

As detailed balance modeling suggests that the each photon absorbed by the solar emits a electron to the external circuit. Triple tandem module with m series-connected top cells and a total module area A, produces current of

$$I_t = (\boldsymbol{\Phi_{in}}(E_t, \infty) - \boldsymbol{\Phi_{out}}(E_t, \infty, V_t)) * qA/m \tag{3.2}$$

Where q is the elementary charge, E_t is the bandgap of top cell and V_t is the voltage of each cell in the top layer. $\boldsymbol{\Phi_{in}}(E_t, \infty)$ is the incoming flux of photons with the energy between E_t and infinity. The latter is calculated from the solar spectrum, and, as mentioned above, the AM1.5G spectrum is used in this work.

For the middle layer containing **n** series connected cells, the current delivered is

$$I_m = (\boldsymbol{\Phi_{in}}(E_m, E_t) - \boldsymbol{\Phi_{out}}(E_m, E_t, V_m)) * qA/n \tag{3.3}$$

Where E_m is the bandgap of middle cell and V_m is the voltage of each cell in the middle layer. $\boldsymbol{\Phi_{in}}(E_m, E_t)$ is the incoming flux of photons with the energy between E_m and E_t.

For the bottom layer containing **p** series connected cells, the current delivered is

$$I_b = (\boldsymbol{\Phi_{in}}(E_b, E_m) - \boldsymbol{\Phi_{out}}(E_b, E_m, V_b)) * qA/p \tag{3.4}$$

Where E_b is the bandgap of bottom cell and V_b is the voltage of each cell in the bottom layer. $\boldsymbol{\Phi_{in}}(E_b, E_m)$ is the incoming flux of photons with the energy between E_b and E_m.

If the three layers are connected in parallel, the voltage matching implies that

$$V_p = mV_t = nV_m = pV_b \tag{3.5}$$

Where *m*, *n* and *p* are numbers of cells in each layer from top to bottom, and $V_{t,}V_{m,}V_b$ are voltages of top, middle and bottom cells respectively.

And total power produced by the module is then given by

$$P_p = V_p(I_t + I_m + I_b) \tag{3.6}$$

Where I_t, I_m and $I_b$are currents from top, middle and bottom layers.

While if the layers are connected in series current matching implies that

$$I = I_t = I_m = I_b \tag{3.7}$$

The total voltage of the module is

$$V_s = mV_t + nV_m + pV_b \tag{3.8}$$

The total power of the module is

$$P_s = I(mV_t + nV_m + pV_b) \tag{3.10}$$

CHAPTER 4 RESULTS

In this section we present results of all the analysis. The first part consists of the ideal detailed efficiency limits. We compute the efficiency limits of independently operated triple layer PV module consisting of silicon as one of the layer with bandgap 1.11 eV, by varying the position of silicon cell from bottom to top and also implementing different sets of bandgaps at the other two layers to achieve the maximum power point and thus the optimal efficiency limits.

Similar analysis is done for the Voltage matched area decoupled triple layer tandem PV module, while taking 60 cells silicon as one of the layer. In this case too position of silicon cells layer is varied from top to bottom while implementing different sets of bandgaps at the other two layers. Comparisons between the efficiency limit plots of the independently operated cells and tandem module is done.

Second part presents the calculation of number of cells within the two layers beside silicon layer its variation according to the different sets of varying bandgap combinations to achieve voltage matching. It shows how the variations in bandgaps and thus the number of cells within each layer impact on the efficiency limits.

In the third part of this study different sets of abundant semiconductor materials is implemented in the in our analysis thus giving us the idea which semiconductor materials could give realistic higher efficiency limits.
At last, we will show the results of how our photovoltaic tandem module which is designed for AM 1.5 would behave under different spectra.

4.1 Efficiency Limits for silicon at bottom, middle and Top

In this section we use single cell at each layer for independently operated cells with silicon with bandgap of 1.11ev placed at bottom. We try different combinations of the bandgaps by varying it for cells at top from 1.121 eV to 4 eV in a step of 0.001 eV, and for cells at bottom from 1.111 eV to 3.5 eV. Similar variations are implemented for area decoupled Tandem module where it is voltage matched. In this case 60 silicon cells is used in one of the layer of triple layer module. The position of the silicon cell in independently operated cells and 60 silicon cells in layer in case of Voltage matched are varied in position i.e., at bottom, middle and top to see its effect on the efficiency limits.

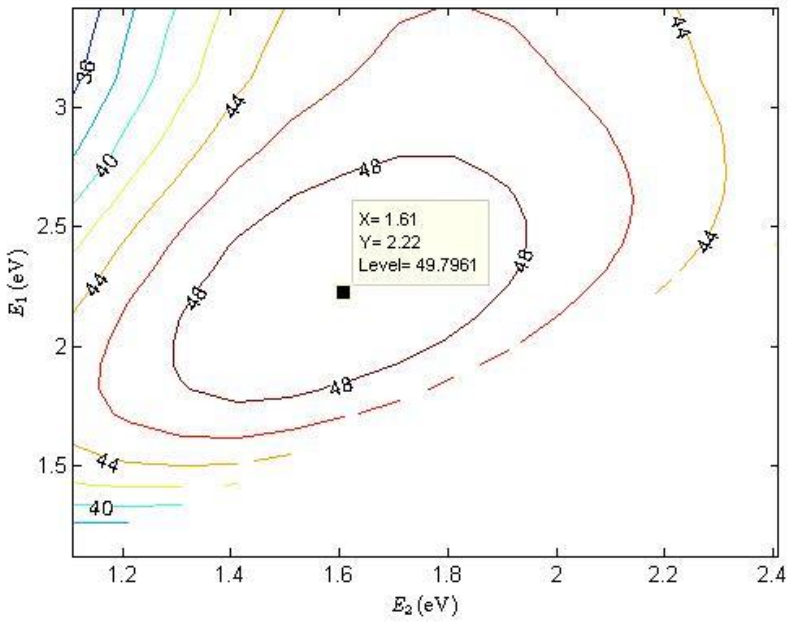

Figure 4.1 (a): Detailed balance efficiency plot for independently operated stack with silicon at bottom under AM 1.5

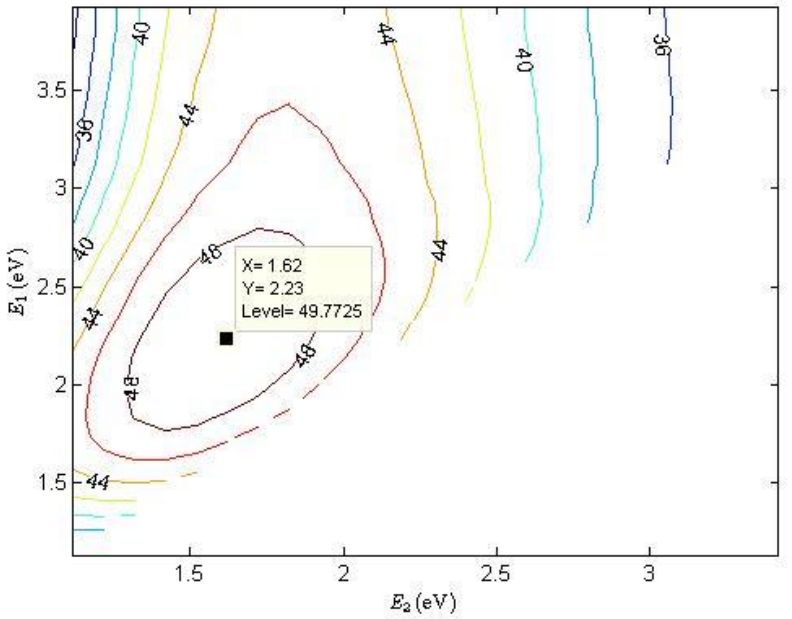

Figure 4.1 (b): Detailed balance efficiency plot for area decoupled voltage matched module with silicon at bottom under AM 1.5

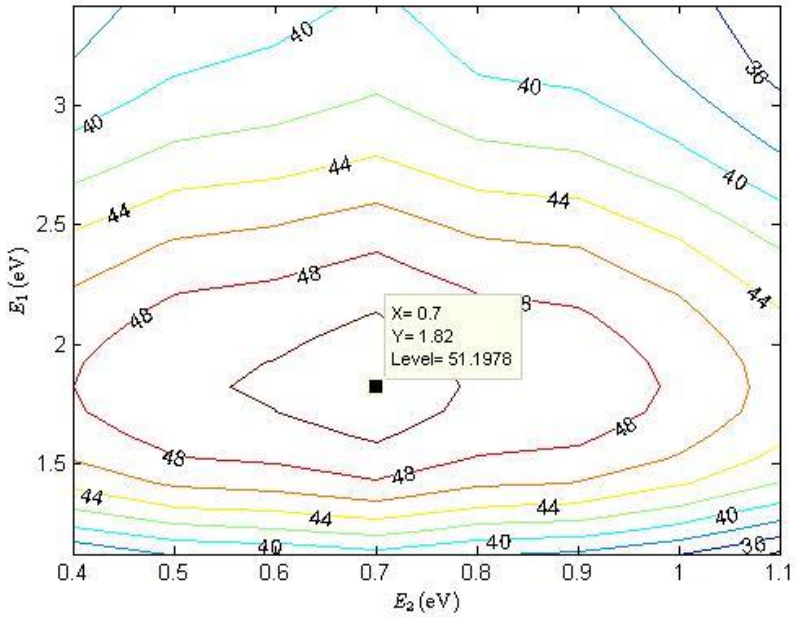

Figure 4.1 (c): Detailed balance efficiency plot for independently operated stack with silicon at middle under AM 1.5

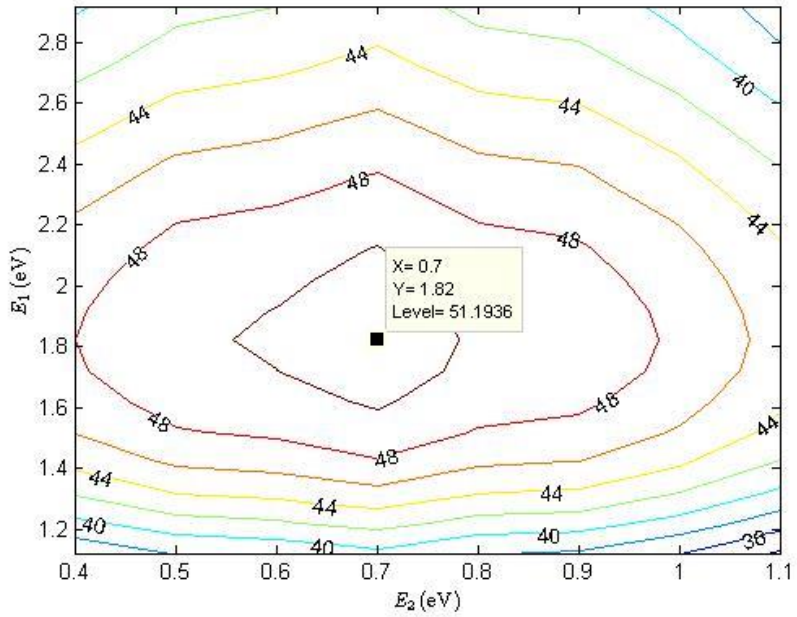

Figure 4.1 (d): Detailed balance efficiency plot for area decoupled voltage matched module with silicon at middle under AM 1.5

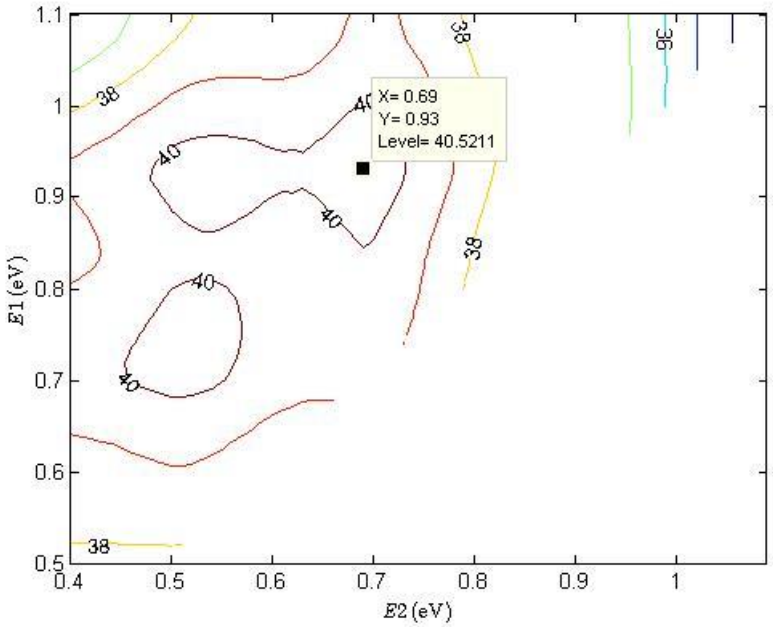

Figure 4.1 (e): Detailed balance efficiency plot for independently operated stack with silicon at top under AM 1.5

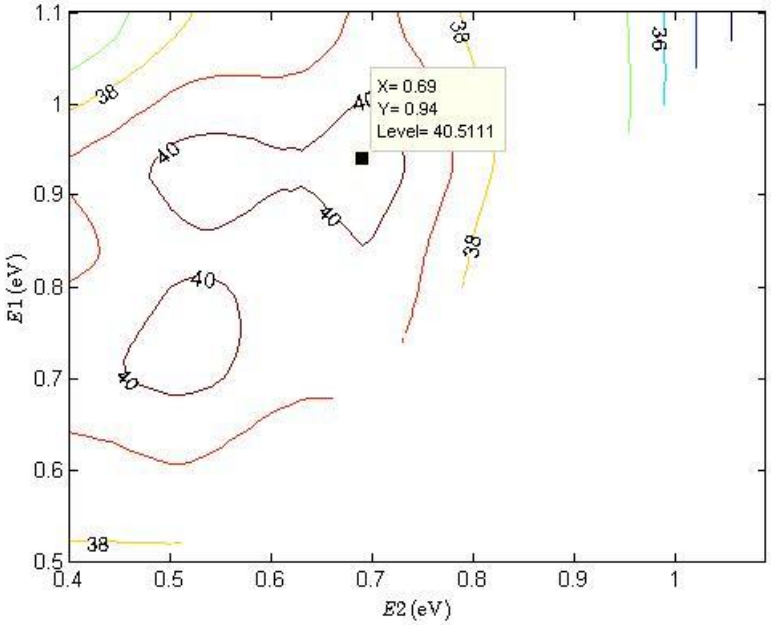

Figure 4.1 (f): Detailed balance efficiency plot for area decoupled voltage matched module with silicon at top under AM 1.5

From the plots (4.1(a),(b),(c),(d),(e),(f)) it is seen that the voltage matched photovoltaic module matches the efficiency limits of independently operated PV module in each scenario. In the case of silicon at the bottom the efficiency limit is 49.9 for both the cases For silicon at middle the efficiency limit is 51.2 and for the silicon at top the efficiency limit is 40.5
As seen when silicon is at middle we are able to get the highest efficiency limit amongst all.
For all the calculation the bandgap of preceding cell/s should possess higher bandgap, meaning all the calculation for preceding cells with lower bandgap is considered as invalid. As a result we could see empty spaces in the plots at the lower regions suggesting invalid data for the invalid cases.

Upon comparison it is seen that while having silicon at the middle layer the efficiency limit has a highest value, this is because we were able to implement a large variations of bandgap ranging from 0.4 eV to 4 eV, from bottom to the top cells. While having silicon cells at the bottom restricted the calculations for bandgap and thus the combinations from 1.11 eV to 4 eV mark . Similarly for silicon at top it could only calculate for efficiency limits for bandgaps and thus the combinations from 0.4 eV up to 1.11 eV only. For the maximum efficiency limit of 51.2, the corresponding bandgap of top cell is 1.82 eV and the bottom cell is 0.7 eV. For both silicon at bottom and silicon at top this combination of bandgap is not accessible.

4.2 Optimal number of cells in each layer of tandem module for maximum efficiency

The optimal efficiency in case of voltage matched tandem module with silicon cells layer at base corresponds to 25 of top cells and 36 number of middle cells. While for silicon cells layer at middle, Top cells relate to 32 numbers while bottom layers have 119 number of cells. Similarly for silicon at top cells equal 82 to and bottom cells equal to 130 gives the optimal efficiency.

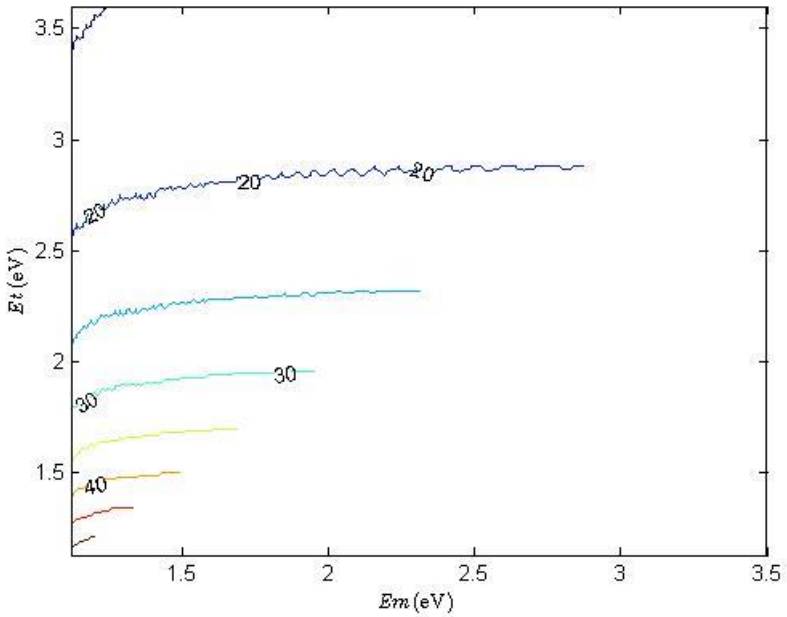

Figure 4.2 (a): Optimal number of cells in top layer for voltage matched tandem module with Silicon cells at bottom

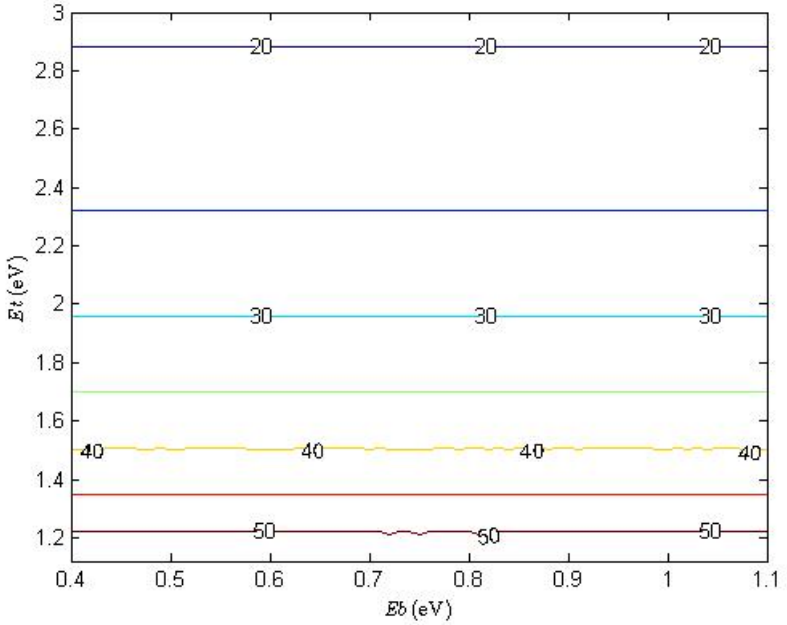

Figure 4.2 (b): Optimal number of cells in top layer for voltage matched tandem module with Silicon cells at middle

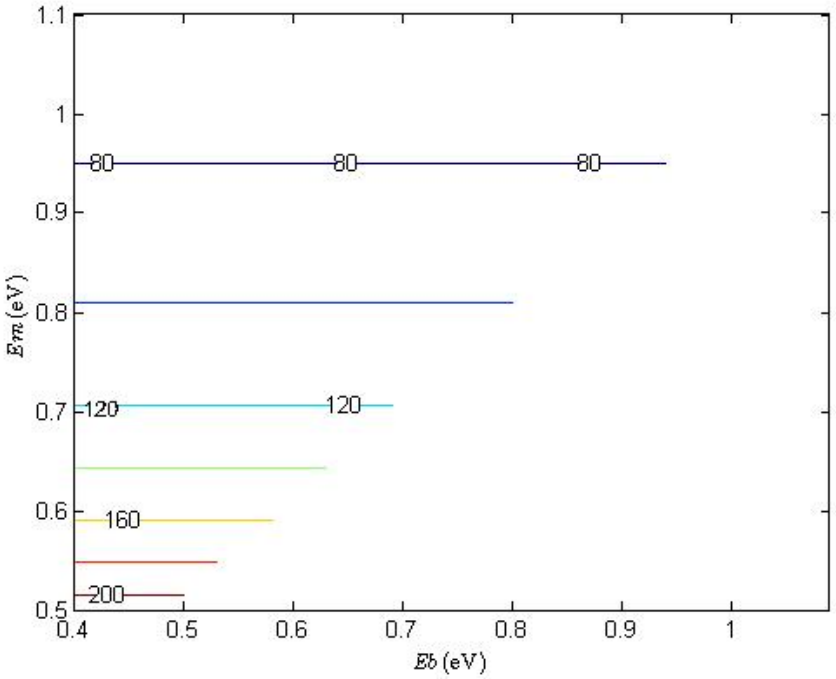

Figure 4.2 (c): Optimal number of cells in middle layer for voltage matched tandem module with Silicon cells at top

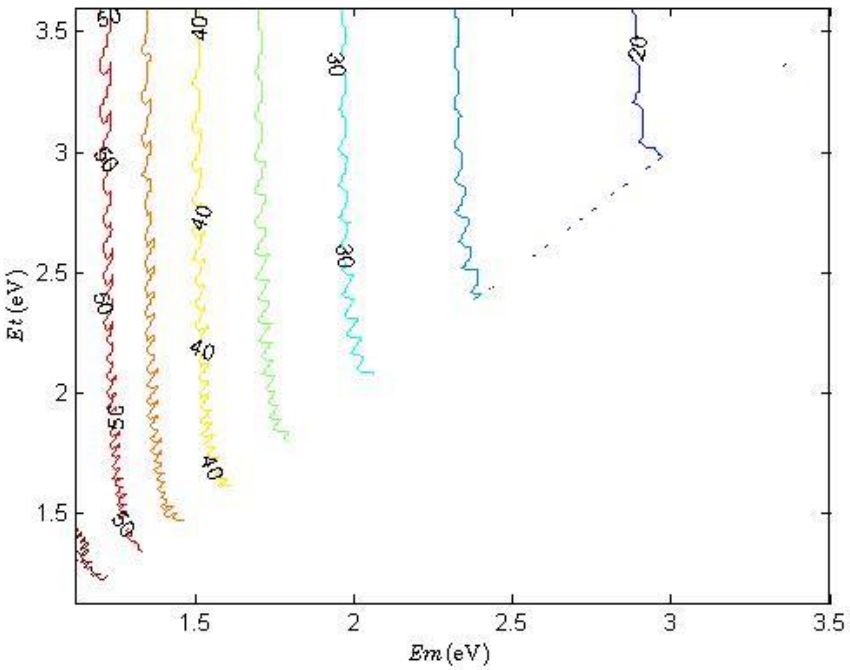

Figure 1.2 (d): Optimal number of cells in middle layer for voltage matched tandem module with Silicon cells at bottom

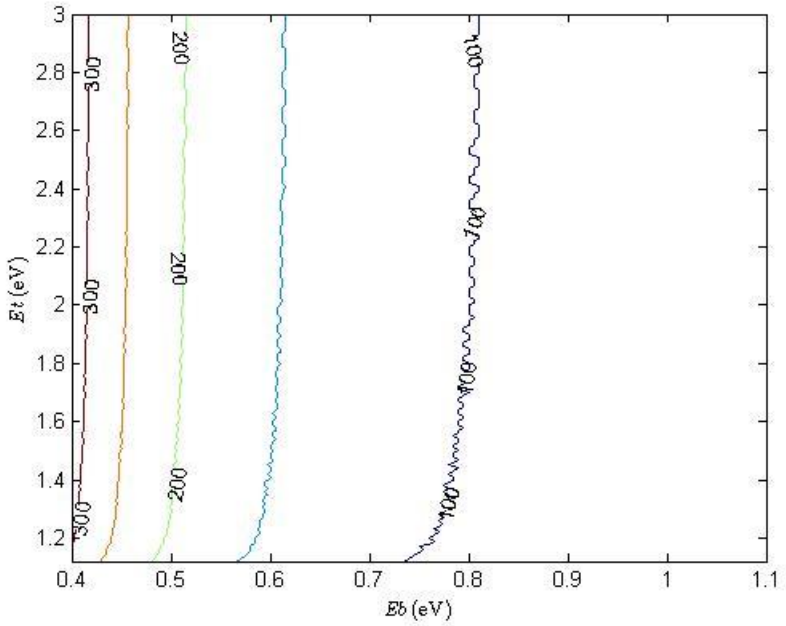

Figure 4.2 (e): Optimal number of cells in bottom layer for voltage matched tandem module with Silicon cells at middle

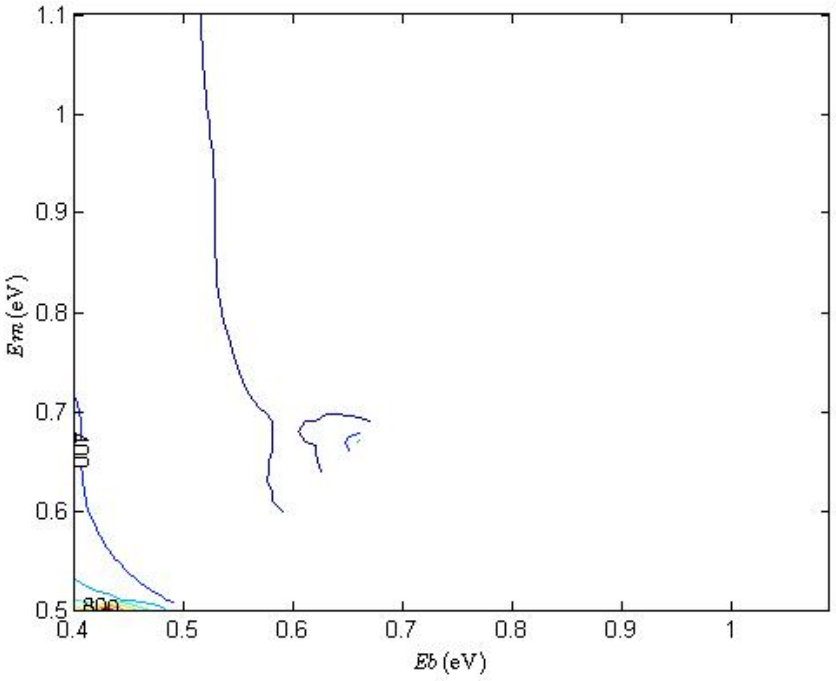

Figure 4.2 (f): Optimal number of cells in bottom layer for voltage matched tandem module with Silicon cells at top

As seen from the plots (Figure 4.2 (a),(b),(c),(d),(e),(f)) it is not completely smooth as the numbers of cells selected in each case for optimal efficiency is not the exact value as we cannot implement decimal numbers acquired from calculation in the application of exact numbers of cells. Thus some rounding up is done in calculation which has affected the plots.

As seen optimal number of cells in case of voltage matched module with 60 silicon cells at bottom layer, as the top band gap increases the number of cells in top band gap decreases for same numbers of middle cells. Now if the middle bandgap increases then for the same top cell voltage the number of top cells decreases.

Similar in the case of middle cells as the bandgap of middle cells increase, number of cells in the middle layer decrease and for same bandgap if the top bandgap increases the number of middle cell increase. All of this is due the requirement of voltage matching, the combinations of bandgap of cells and numbers must produce same voltage in each layer for voltage matching and thus the optimal efficiency at maximum power point. That means if top cells have high numbers of cell producing low voltage then there is requirement of either higher number of middle cells with lower bandgap.
As the preceding bandgap is always higher to the lower ones increment or decrement in numbers in upper cell directly correlates to increment or decrement in lower layer containing cells with lower bandgap and vice versa. This formulation follows in each of the analysis. The gaps in the plots appeared due the invalid combinations of bandgap and thus the numbers. Invalid here means at times when the preceding layer has cells with lower band gap than the proceeding ones then it omits calculation.

Table 4.2 (a) : Area decoupled Voltage matched Triple layer tandem PV module with 60 silicon cells at bottom layer

(Et) eV/ material	(Em) eV/ material	(Eb)eV/ material	Efficiency	(m)	(n)	(p)
2.2 ($ZnGeP_2$)	1.63 (a-Si:H)	1.11 (Si)	49.8	26	37	60
2.2 ($ZnGeP_2$)	1.64 (Cu_2ZnSn)	1.11 (Si)	49.8	26	37	60
2.2 ($ZnGeP_2$)	1.68 ($CuGaSe_2$)	1.11 (Si)	49.6	26	36	60
2.24 (GaP)	1.63 (a-Si:H)	1.11 (Si)	49.8	25	37	60
2.24 (GaP)	1.64 (Cu_2ZnSn)	1.11 (Si)	49.8	25	36	60
2.24 (GaP)	1.68 ($CuGaSe_2$)	1.11 (Si)	49.6	25	35	60
2.3 ($ZuSiP_2$)	1.63 (a-Si:H)	1.11 (Si)	49.7	24	36	60
2.3 ($ZuSiP_2$)	1.64 (Cu_2ZnSn)	1.11 (Si)	49.8	24	36	60
2.3 ($ZuSiP_2$)	1.68 ($CuGaSe_2$)	1.11 (Si)	49.6	24	35	60

Table 4.2 (b): Area decoupled Voltage matched Triple layer tandem PV module with 60 silicon cells at middle layer

(Et) eV/ material	(Em) eV/ material	(Eb) eV/ material	Efficiency	(m)	(n)	(p)
1.82 (CuP_3)	1.11 (Si)	0.5 (PbS)	49.7	32	60	131
1.82 (CuP_3)	1.11 (Si)	0.53(CuFeS_2)	50.0	32	60	207
1.82 (CuP_3)	1.11 (Si)	0.66 (Ge)	50.7	32	60	131
1.82($aSiC$)	1.11 (Si)	0.7 (SiGe)	51.2	32	60	119
1.82($aSi:H$)	1.11 (Si)	0.74 (Mg_2Si)	50.7	31	60	110
1.82 (ZnP_2)	1.11 (Si)	0.77(Mg_2Ge)	50.3	32	60	104

Table 4.2 (c): Area decoupled Voltage matched Triple layer tandem PV module with 60 silicon cells at top layer

(Et) eV/ material	(Em) eV/ material	(Eb) eV/ material	Efficiency	(m)	(n)	(p)
1.11 (Si)	0.9 (Cu_2Sn_3)	0.66 (SiGe)	40.2	60	85	140
1.11 (Si)	0.91(Cu_2Sn_3)	0.69 (SiGe)	40.3	60	82	140
1.11 (Si)	0.91(Cu_2Sn_3)	0.74(Mg_2Ge)	40.2	60	80	139
1.11 (Si)	0.93 (SiGe)	0.66 (Ge)	40.4	60	85	130
1.11 (Si)	0.93 (SiGe)	0.69 (SiGe)	40.5	60	82	129
1.11 (Si)	0.93 (SiGe)	0.74(Mg_2Ge)	40.4	60	80	130
1.11 (Si)	0.95 (FeS_2)	0.66 (Ge)	39.7	60	85	118
1.11 (Si)	0.95 (FeS_2)	0.69 (SiGe)	39.8	60	82	118
1.11 (Si)	0.95 (FeS_2)	0.74(Mg_2Ge)	39.8	60	80	117

We have chosen some semiconductor materials which are abundant and non-toxic which could be used in real tandem voltage modules to achieve the maximum possible efficiency limit (Doeleman, 2012). We have tried out various combinations of materials with bandgap close to that which gives optimal efficiency in our studies before. Upon comparing the results a tandem module with amorphous Silicon Hydride (a-Si:H) or amorphous Silicon Carbide (a-SiC), both highly stable and active component in photovoltaics [15] [16] with 1.82 eV, Silicon (Si) with 1.11 eV and Silicon Germanium (SiGe) which is already in active in photovoltaics [17] with 0.7 respectively at top, middle and bottom layer, seems to be the best possible combination, with which it is possible to reach efficiency limit of 51.2 theoretically.

Table 4.2 (d): Optimal efficiency and corresponding bandgap for independently operated stack for different configuration

Configuration	Efficiency	Et (eV)	Em (eV)	Eb (eV)
Si at bottom	49.87	2.23	1.64	1.11
Si at middle	51.2028	1.81	1.11	0.7
Si at top	40.5211	1.11	0.93	0.69

Table 4.2 (e): Optimal efficiency, corresponding bandgap and number of cells for area decoupled voltage matched module for different configuration

Configuration	Efficiency	Et(eV)	Em(eV)	Eb(eV)	m	n	p
Si at bottom	49.8464	2.23	1.64	1.11	25	36	60
Si at middle	51.1936	1.82	1.11	0.7	32	60	119
Si at top	40.5207	1.11	0.93	0.69	60	82	130

Upon comparisons from Table 4.2 (d),(e) we can see that the best optimal efficiency limit is achieved when the silicon cells layer is placed at the middle layer for both independently operated stacks and voltage matched tandem module.
In case of Independently operated stacks the maximum efficiency achieved is 51.2 with bandgap of 1.81, 1.11, 0.7 for top, middle at bottom respectively.
Similarly as seen from the table we are able to match the efficiency limit in case of voltage matched tandem module with that of independently operated stacks, with only 0.01 percent difference.
For voltage matched model the maximum optimal efficiency limit is achieved at bandgaps of cells at 1.82, 1.11, 0.7 for top, middle and bottom at numbers of cells at 32, 60 and 119 for respective layers from top to bottom.

4.3 Behavior of tandem module, designed and optimized for AM 1.5 under various light spectrum

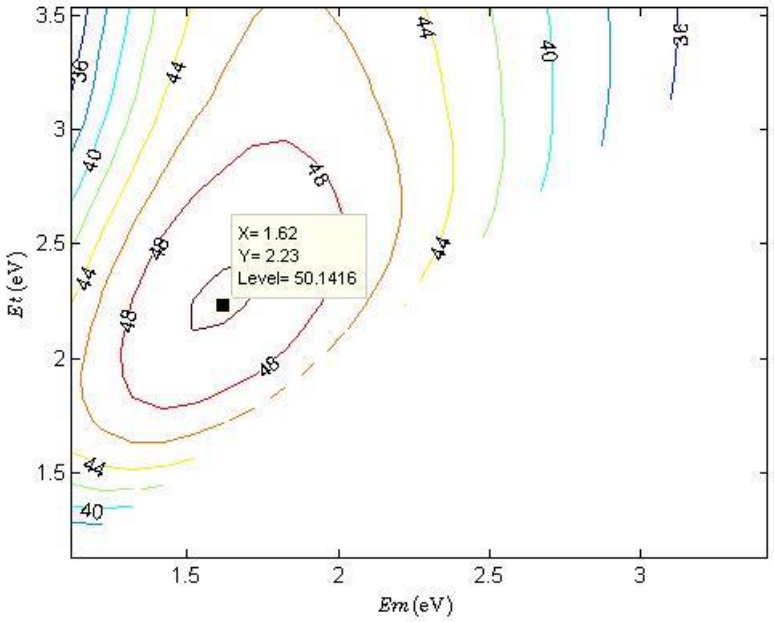

Figure 4.3 (a): Efficiency plot under AM 1.0 for Si at base for independently operated stack

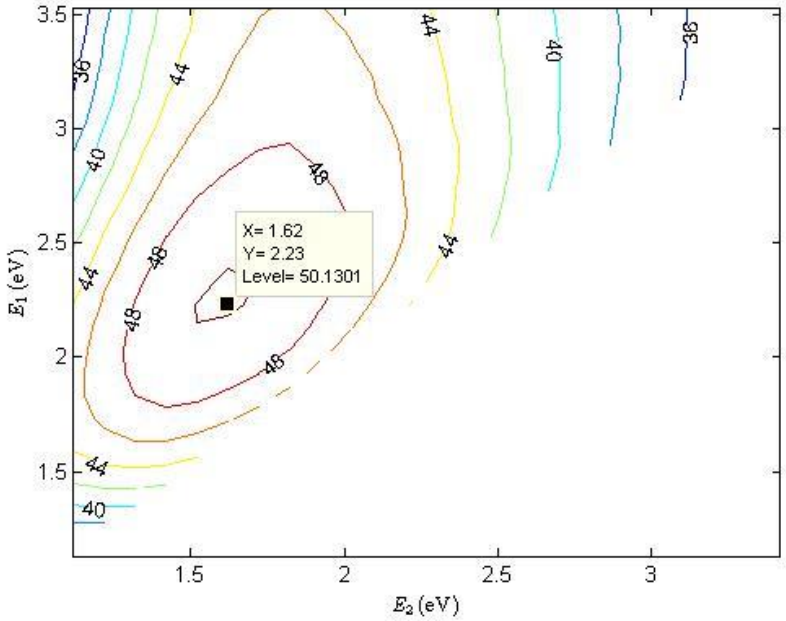

Figure 4.3 (b): Efficiency plot under AM 1.0 for Si at base for area decoupled module

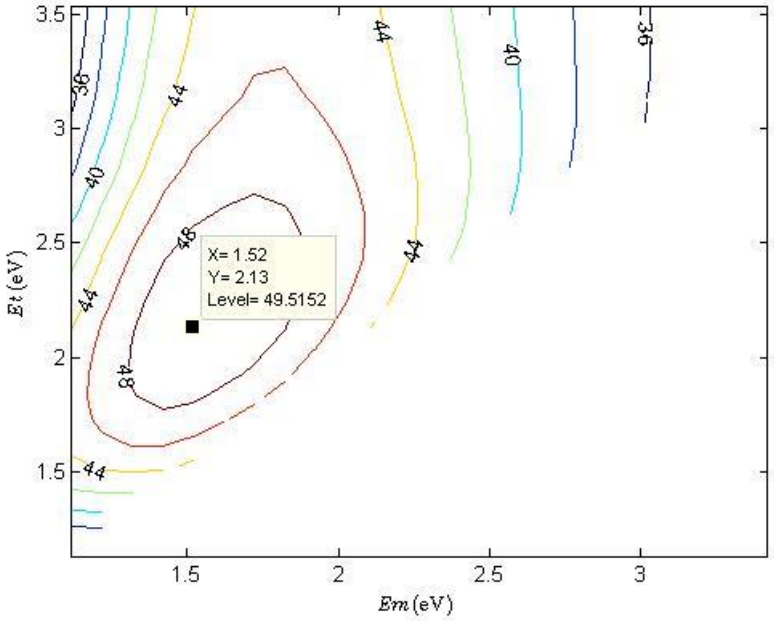

Figure 4.3 (c): Efficiency plot under AM 2.0 for Si at base for independently operated stack

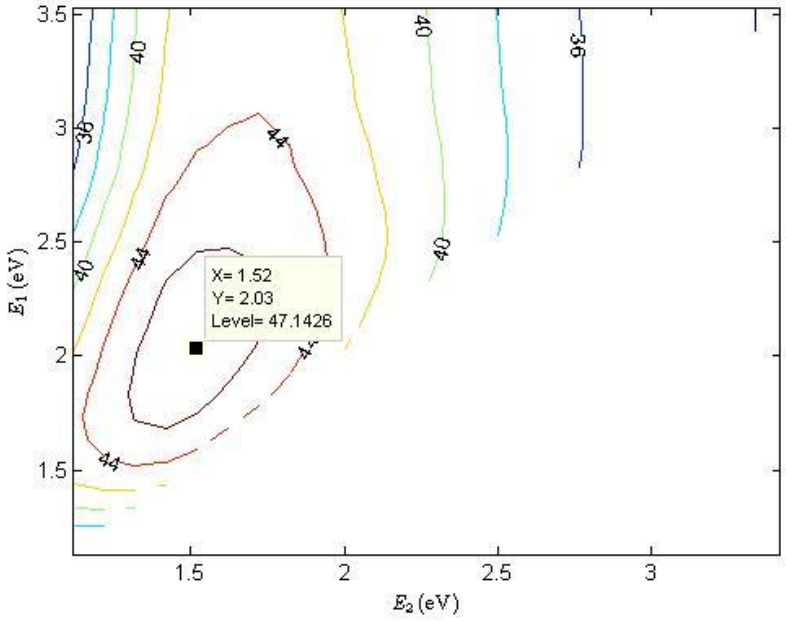

Figure 4.3 (d): Efficiency plot under AM 2.0 for Si at base for area decoupled module

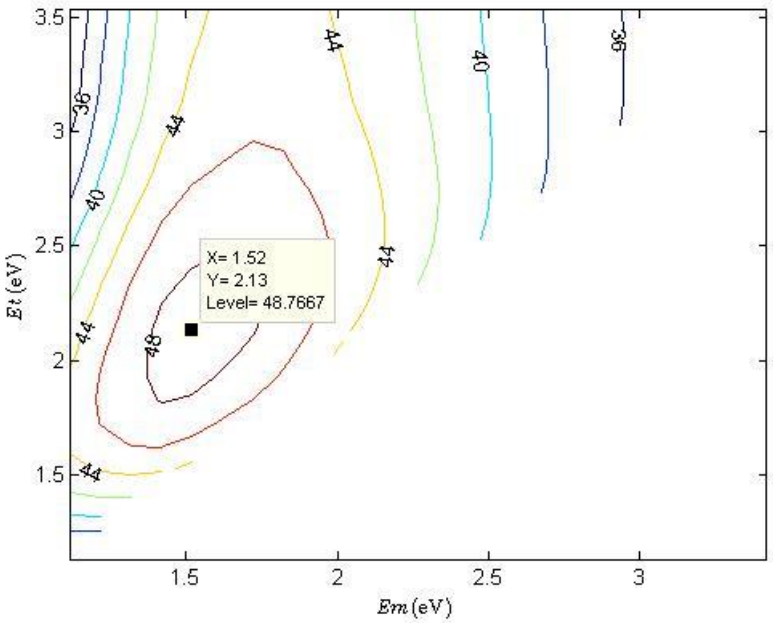

Figure 4.3 (e): Efficiency plot under AM 3.0 for Si at base for independently operated stack

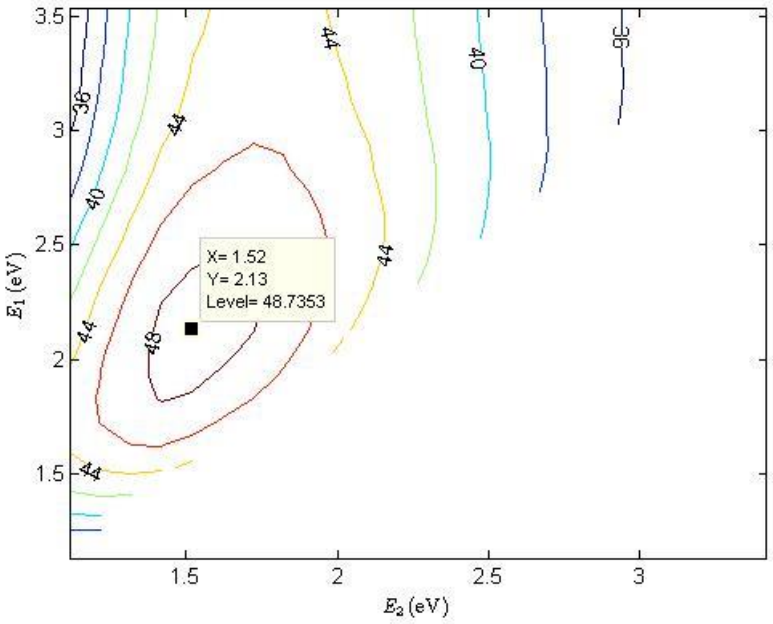

Figure 4.3 (f): Efficiency plot under AM 3.0 for Si at base for area decoupled module

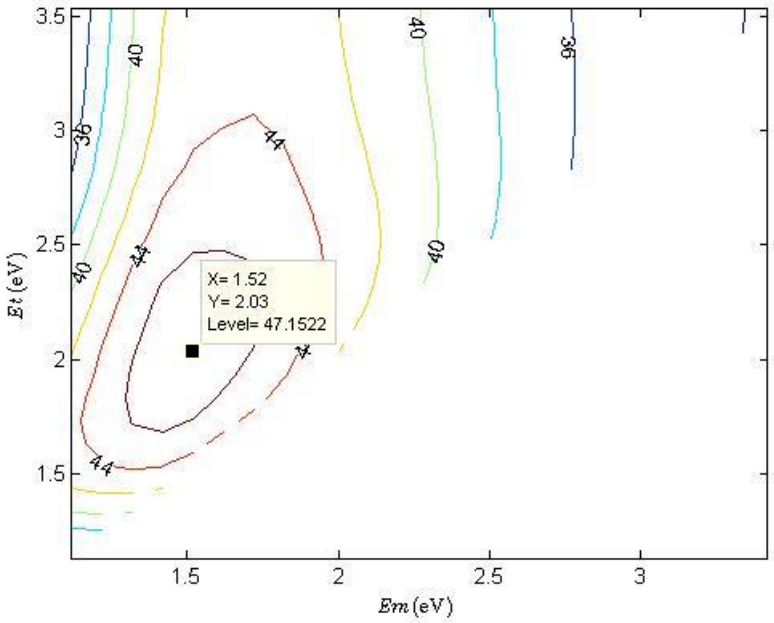

Figure 4.3 (g): Efficiency plot under AM 5.0 for Si at base for independently operated stack

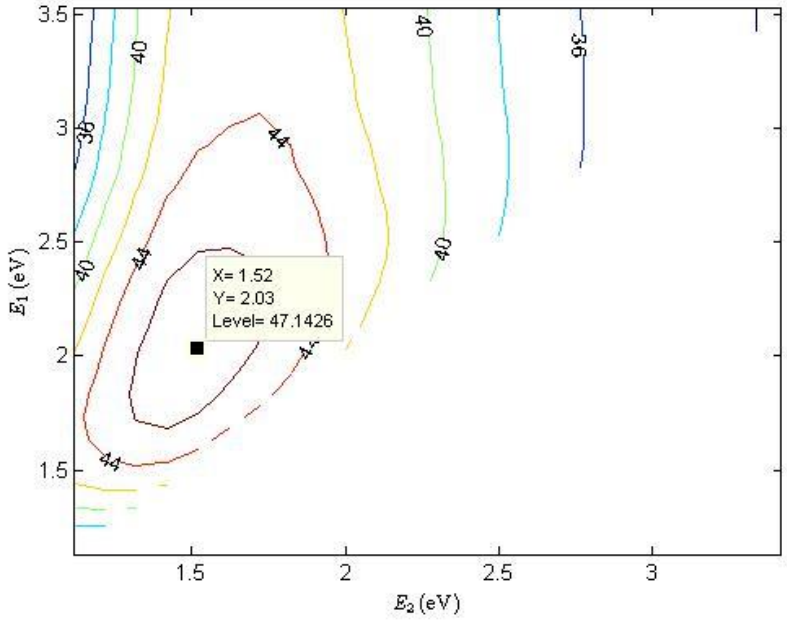

Figure 4.3 (h): Efficiency plot under AM 5.0 for Si at base for area decoupled module

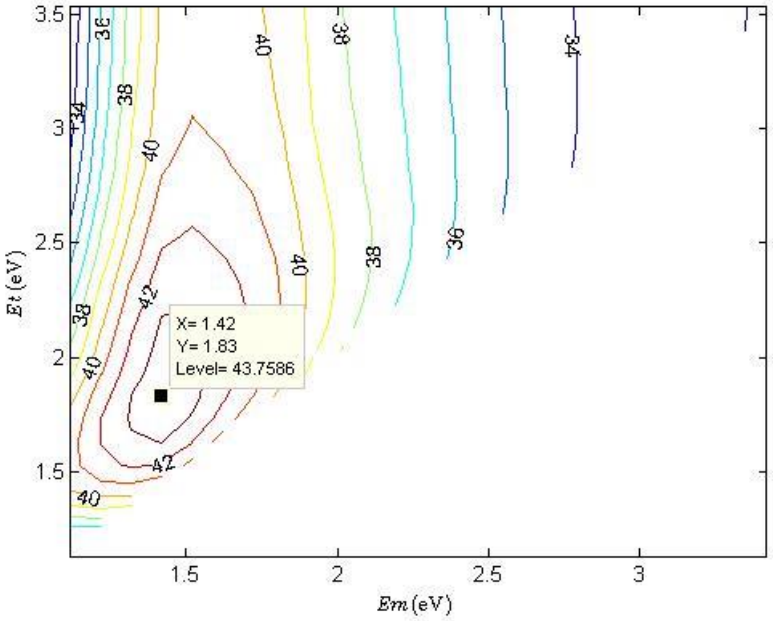

Figure 4.3 (i): Efficiency plot under AM 10 for Si at base for independently operated stack

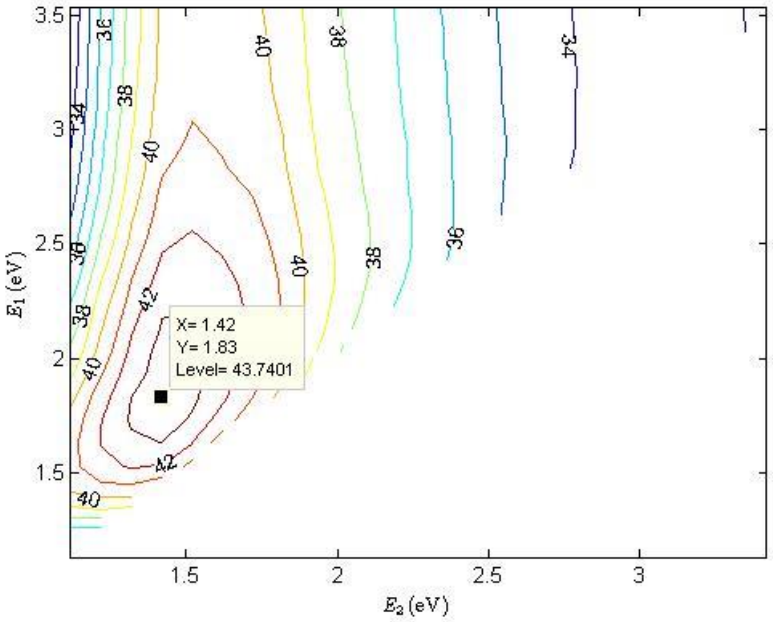

Figure 4.3 (j): Efficiency plot under AM 10 for Si at base for area decoupled module

Figure 4.3(a) to (j) show that the area decoupled voltage matched module optimized for AM 1.5 possess similar characteristics to that of the independently operated stack under other different spectra (AM 1.0, AM 2.0, AM 3.0, AM5.0 and AM 10) in terms of efficiency change in case of silicon cells at base layer. Same behaviors is also found in the

case of silicon cells at middle and silicon cells at top layers for area decoupled voltage matched module.

Table 4.3 (a): Efficiency comparisons under different spectra for triple layer independently operated cells and area decoupled triple layer voltage matched modules

Configuration	AM 1.0	AM 1.5	AM 2.0	AM 3.0	AM 5.0	AM10.0
Si at bottom for triple layer area decoupled module	50.1	49.9	47.8	48.7	47.1	43.7
Si at bottom for independently operated cells	50.1	51.2	49.5	48.8	47.2	43.8
Si at middle for triple layer area decoupled module	50.9	51.2	50.1	50.8	50.3	49.1
Si at middle for independently operated cells	50.9	51.2	51.1	50.8	50.3	49.2
Si at top for triple layer area decoupled module	39.4	40.5	40.5	41.0	41.7	42.8
Si at top for independently operated cells	39.5	40.8	40.5	41.0	41.8	42.8

From the results in Table 4.3 (a), for area decoupled voltage matched triple layer module, it is seen that the optimized module for AM 1.5 spectrum under some other spectrum e.g., for Si at base for AM 1.0 spectrum performs better and gives higher efficiency value than for AM 1.5. While for Si at middle case, better result is still at AM 1.5 with largest efficiency while for Si at top case, expect for spectrum AM 1.0 module has larger efficiency for every other spectra than that for AM 1.5.

While for the independently operated triple layer cells AM 1.5 holds the largest efficiency value in case of Si at bottom and middle layer scenarios. But for Si at top under AM3, AM5; AM10, It has larger efficiency values than that for AM 1.5.

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CHAPTER 5 Discussion and Conclusion

Application of area decoupling in triple layer module to voltage match the three layers enabled the module to perform similar to the independently operated stack of three layers of cells. Due to calculation round off perfect voltage matching is not acquired in calculations which resulted in negligible difference in maximum efficiency in some cases while comparing the voltage matched and indepdently operated modules.
Results showed the theoretical efficiency limit of 51.2 can be achieved with 32 cells of bandgap 1.82 eV, 60 cells of bandgap 1.11eV and 119 cells of bandgap 0.7 eV at top, middle and bottom layers respectively for area decoupled voltage matched module. This efficiency limit is equal to that of independently operated stack of 3 cells with 1.82 eV, 1.11 eV and 0.7 eV bandgaps at top, middle and bottom layers respectively. The comparison of overall efficiency plot for different bandgap combinations also follow the same behaviors in both cases. Which holds true in all three configuration, i.e. silicon cell or 60 silicon cells at top, middle or bottom layer.

When the area decoupled voltage matched tandem module which is optimized for AM 1.5 was theoretically tested under the different spectra (AM 1, AM 2, AM 3, AM 5 and AM 10), the results were again quite comparable to that of independently operated triple stack for corresponding AM configuration. Here, the efficiency difference is only slight and efficiency curve are quite similar in both cases.

Theoretical implementation of abundant, non-toxic semiconductor materials in the voltage matched module showed that amorphous silicon hydride (a-Si:H) or amorphous Silicon Carbide(a-SiC), with 1.82 eV bandgap of 32 cells, silicon (Si), with 1.11 eV bandgap of 60 cells and Silicon Germanium (SiGe), with 0.7 eV bandgap of 119 cells respectively at top, middle and bottom layer, is the best possible combination, which gives a theoretical efficiency of 51.2 for an area decoupled voltage matched triple layer photovoltaic module.

Thin film deposition technique offers a prominent step forward in achieving division in a module with numbers of various cells. While adapting to the current production process of wafer based silicon modules, deposition of thin films of cells on the top of silicon module with area decoupling might allow production of triple tandem modules [12].

All of the results obtained above are at fixed cell temperature of 300 K. Since both ambient and thus the cell temperature during operations vary continuously in real world scenario, meaning voltage matching will be affected with the shift in the conditions. The area of area decoupled modules needs be fixed for it to achieve voltage or current matching, if differed will not achieve the optimal results. Thus when the conditions change there is change in area too, making it not able to run the voltage of different cells at maximum power point. Which is the disadvantage of such modules compare to independently operated stacks of cells, which do not face such issue [12].

CHAPTER 6 Further work

Based on the previous studies on double tandem cells for both the voltage matched and current matched module [12], the voltage matched model has performed as before and if the current matched module in studied in similar way then it should perform similarly as in the case of double tandem module. i.e. with the same sets of limitations. However, if studied, may suggest different results also.
The number of oscillations in the plot can be minimized if number of silicon cell sets can be increased from 60 cells to 72 cells, which are already in use in some commercial purposes.

Appendices

A. MatLAb Codes

Generation Rate

```
function [G,inenergy]=gen(El, Eh,varargin)
%This function is used to calculate the generation rate. It returns the
%number of photons in the energy interval between El and Eh. If varargin is
%empty, the function will import data from the datafile indicated below. If
%varargin contains data it should be:
%gen(El, Eh, photon energy, energy in photon energy interval,
%number of photons in interval, total energy of spectrum)

if isempty(varargin)
    [photonenergy,energy,photoner,inenergy]=hentspektrum('AM1.5'); %Import of data
from datafile if varargin is empty
else
    photonenergy=cell2mat(varargin(1));
    energy=cell2mat(varargin(2));
    photoner=cell2mat(varargin(3));
    inenergy=sum(energi);
end
h=6.6260693E-34;
c=2.99792458E8;
q=1.60217653E-19;
k=1.3806505E-23;

%Finner matriseelementene som tilsvarer grensene for integrasjonen
Elindeks=find(min(abs(El-photonenergy))==abs(El-photonenergy));
if Eh==inf
    Ehindeks=length(photoner);
else
    Ehindeks=find(min(abs(Eh-photonenergy))==abs(Eh-photonenergy));
end

%Legger til bidraget fra grenseintervallene
G=photoner(Elindeks)*2*(photonenergy(Elindeks)+1/4*photonenergy(Elindeks+1)-
1/4*photonenergy(Elindeks-1)-El)/(photonenergy(Elindeks+1)-photonenergy(Elindeks-
1)); %Nedre grenseintervall
if Eh>=max(photonenergy)
    G=G+photoner(Ehindeks); %Øvre grenseintervall
else
```

```
    G=G+photoner(Ehindeks)*2*(Eh-
photonenergy(Ehindeks)+1/4*photonenergy(Ehindeks+1)-1/4*photonenergy(Ehindeks-
1))/(photonenergy(Ehindeks+1)-photonenergy(Ehindeks-1));%Øvre grenseintervall
end
for i=Elindeks+1:1:Ehindeks-1
    G=G+photoner(i);
end

end
```

Recombination Rate

```
function [R]=rec(El, Eh, V, T)
%This function calculates the photon flux from a cell that is emitting in
%the energy interval limited by El and Eh. This is equivalent to the net
%recombination rate.

h=6.6260693E-34;
c=2.99792458E8;
q=1.60217653E-19;
k=1.3806505E-23;
K=2*pi/(h^3*c^2);

El=El*q;
Eh=Eh*q;

recombination = @(E) K.*E.^2./(exp((E-q*V)./(k*T))-1);
R=quadgk(recombination,El,min(Eh,min(20*El,q*20)));



end
```

Read spectrum

```
function [photonenergy,energy,photoner,inenergy]=readspectrum(filname)
%Reads datafile with spectrum. hentspectrum('AM1.5') reads data from the
%file AM1.5.csv. Returns list of photon energies, the energy of the
%spectrum for these particular photonenergies, the number of photons in an
%energy interval as well as the sum of the energy in the spectrum.

q=1.60217653E-19;

display(['Importerer data fra ',filnavn,'.csv'])
```

```
data=importdata([filnavn,'.csv']); %Reads file

photonenergy=data.data(:,1);
energy=data.data(:,2);
photoner=data.data(:,3);
inenergy=sum(energy);

display(['The energy density of this spectrum is ',num2str(inenergy),' W/m^2'])

end
```

Plot spectrum

```
function []=plotspectra()
%Plots the differnt spectrum

[photonenergy,energy,photoner,inenergiAM1]=readspectrum('AM1');
[photonenergy,energy,photoner,inenergiAM15G]=readspectrum('AM1.5');
[photonenergy,energy,photoner,inenergiAM2]=readspectrum('AM2');
[photonenergy,energy,photoner,inenergiinnenergiAM1]=readspectrum('AM3');
[photonenergyAM5,energyAM5,photonerAM5,inenergiAM5]=readspectrum('AM5');
[photonenergyAM10,energyAM10,photonerAM10,inenergiAM10]=readspectrum('AM10
');

figure(101)
plot(photonenergyAM1, energyAM1,photonenergyAM15G, energyAM15G,
photonenergyAM2, energyAM2, photonenergyAM3, energyAM3, photonenergiAM5,
energyAM5,photonenergyAM10, energyAM10)
legend('AM1','AM1.5G','AM2','AM3','AM5','AM10')

figure(102)
plot(photonenergyAM1, energyAM1,photonenergyAM15G, energyAM15G,
photonenergyAM2, energyAM2, photonenergyAM3, energyAM3, photonenergiAM5,
energyAM5,photonenergyAM10, energy)
legend('AM1','AM1.5G','AM2','AM3','AM5','AM10')
end
```

Maximum power for independently operated stack

```
function
[Eff,P]=maxpoweri(Eg1,Eg2,Eg3,T,m,n,p,photonenergy,energy,photoner,inenergy)
%Returns the efficiency and power at the MPP for independently operated 3 layer stack.
%Eg1 is the band gap of the bottom cell. Eg2 is the band gap of the middle cell.
%Eg3 is the band gap of the top cell.
%T is the celltemperature.
% m is the number of top cells. n is the number of bottom
%cells. photonenergy is a list of photon energies (from readspectrum()).
%energy is a list of how much energy the spectrum carries in the energy
%intervals defined by the fotonenergi-list. photoner is the number of fotons
%in the energy intervals defined by the photonenergy-list. inenergy is the
%total energy of the spectrum.

q=1.60217653E-19;

Vmin=0;
Vmax=min([p*Eg1,n*Eg2,m*Eg3]);
V=(Vmax-Vmin)/2;

[G1,~]=gen(Eg1,Eg2,photonenergy,energy,photoner);
[G2,~]=gen(Eg2,Eg3,photonenergy,energy,photoner);
[G3,~]=gen(Eg3,inf,photonenergy,energy,photoner);
P=NaN;
teller=0;
while (Vmax-Vmin)>1E-4*V
  teller=teller+1;
  J1=q*(G1-rec(Eg1,Eg2,V/p,T));
  J2=q*(G2-rec(Eg2,Eg3,V/n,T));
  J3=q*(G3-rec(Eg3,inf,V/m,T));
  P=V*(J1/p+J2/n+J3/m);
  J1b=q*(G1-rec(Eg1,Eg2,V*(1+1E-5)/p,T));
  J2b=q*(G2-rec(Eg2,Eg3,V*(1+1E-5)/n,T));
  J3b=q*(G3-rec(Eg3,inf,V*(1+1E-5)/m,T));
  Pb=V*(1+1E-5)*(J1b/p+J2b/n+J3b/m);

  if P<Pb
     Vmin=V;
     V=(Vmax+V)/2;
  else
     Vmax=V;
     V=(Vmin+V)/2;
  end

  if teller>500
```

```
      Eff=NaN;
      P=NaN;
      break;
   end
end
Eff=P/innEnergi;

end
```

Maximum power for tandem module

```
function
[Eff,P]=maxpower(Eg1,Eg2,Eg3,T,m,n,p,photonenergy,energy,photoner,inenergy)
%Returns the efficiency and power at the MPP for voltage matched tandem module
%Eg1 is the band gap of the
%bottom cell. Eg2 is the band gap of the middle cell. Eg3 is the bandgap of
 % top cell. T is the cell
%temperature. m is the number of top cells. n is the number of bottom
%cells. fotonenergi is a list of photon energies (from hentspektrum()).
%energy is a list of how much energy the spectrum carries in the energy
%intervals defined by the photonenergi-list. photoner is the number of photons
%in the energy intervals defined by the photonenergy-list. Iinenergi is the
%total energy of the spectrum.

q=1.60217653E-19;

Vmin=0;
Vmax=min([p*Eg1,n*Eg2,m*Eg3]);
V=(Vmax-Vmin)/2;

[G1,~]=gen(Eg1,Eg2,photonenergy,energy,photoner);
[G2,~]=gen(Eg2,Eg3,photonenergy,energy,photoner);
[G3,~]=gen(Eg3,inf,photonenergy,energy,fotoner);
P=NaN;
teller=0;
while (Vmax-Vmin)>1E-4*V
   teller=teller+1;
   J1=q*(G1-rec(Eg1,Eg2,V/p,T));
   J2=q*(G2-rec(Eg2,Eg3,V/n,T));
   J3=q*(G3-rec(Eg3,inf,V/m,T));
   P=V*(J1/p+J2/n+J3/m);
   J1b=q*(G1-rec(Eg1,Eg2,V*(1+1E-5)/p,T));
   J2b=q*(G2-rec(Eg2,Eg3,V*(1+1E-5)/n,T));
   J3b=q*(G3-rec(Eg3,inf,V*(1+1E-5)/m,T));
```

```
  Pb=V*(1+1E-5)*(J1b/p+J2b/n+J3b/m);

  if P<Pb
     Vmin=V;
     V=(Vmax+V)/2;
  else
     Vmax=V;
     V=(Vmin+V)/2;
  end

  if teller>500
     Eff=NaN;
     P=NaN;
     break;
  end
end
Eff=P/inenergy;

end
```

Maximum power single case

```
function
[Eff,pow1,V,J]=maxpowersingle(Eg1,Eg2,T,photonenergi,energy,photoner,inenergy)
%Finds max efficiency for single cell that absorbs and emits in the energy
%interval Eg1 to Eg2. T is the cell temperature. photonenergy, energy,
%photoner and Iinenergy is data from the spectrum that is analysed.

q=1.60217653E-19;

Vmin=0;
Vmax=Eg1;
V=(Vmax-Vmin)/2;

[G,~]=gen(Eg1,Eg2,photonenergy,energy,photoner);

while (Vmax-Vmin)>1E-4*V
  R1=rec(Eg1,Eg2,V,T);
  R2=rec(Eg1,Eg2,V*(1+1E-5),T);
  pow1=q*(G-R1)*V;
  pow2=q*(G-R2)*V*(1+1E-5);

  if pow2>pow1
```

```
      Vmin=V;
      V=(Vmax+V)/2;
   else
      Vmax=V;
      V=(Vmin+V)/2;
   end
end
J=q*(G-R1);

Eff=pow1/inenergy;

end
```

Map of efficiency and number of cells for tandem module

```
function []=mnpnewb(T)
%Makes a map of the maximum efficiency as a function of the band gap for
%the voltage matched area decoupled tandem module.
%It also makes a map of the optimal m, n and p .
%Defining band gaps
Egb=1.11;
Egm=[1.11:0.1:2.5];
Egt=[1.12:0.1:3.5];
p=60;

%Import spectrum
[fotonenergi,energi,fotoner,InnEnergi]=hentspektrum('AM1.5');

Eff=zeros(length(Egt),length(Egm))+NaN;
m=Eff;
n=Eff;

for i=1:1:length(Egt)
    Egt(i)
    for j=1:1:length(Egm)
        if Egt(i)>Egm(j)


[Eff1,~,Vt,J1]=maxpowersingle(Egt(i),inf,T,photonenergy,energy,photoner,inenergy);

[Eff2,~,Vm,J2]=maxpowersingle(Egm(j),Egt(i),T,photonenergy,energy,photoner,inenerg
y);
```

```
[Eff3,~,Vb,J3]=maxpowersingle(Egb,Egm(j),T,photonenergy,energy,photoner,inenergy);

        Eff(i,j)=Eff1+Eff2+Eff3;
        m(i,j)=(Vb/Vt)*p;
        n(i,j)=(Vb/Vm)*p;
        m1(i,j)=round(m(i,j)-0.5);
        m2(i,j)=round(m(i,j)+0.5);
        n1(i,j)=round(n(i,j)-0.5);
        n2(i,j)=round(n(i,j)+0.5);

Eff_1(i,j)=maxpower(Egb,Egm(j),Egt(i),T,m1(i,j),n1(i,j),60,photonenergy,energy,photoner,inenerg);

Eff_2(i,j)=maxpower(Egb,Egm(j),Egt(i),T,m1(i,j),n2(i,j),60,photonenergy,energy,photoner,inenerg);

Eff_3(i,j)=maxpower(Egb,Egm(j),Egt(i),T,m2(i,j),n1(i,j),60,photonenergy,energy,photoner,inenerg);

Eff_4(i,j)=maxpower(Egb,Egm(j),Egt(i),T,m2(i,j),n2(i,j),60,photonenergy,energy,photoner,inenerg);
        Vector=[Eff_1(i,j),Eff_2(i,j),Eff_3(i,j),Eff_4(i,j)]
        max_effy(i,j)=max(Vector);
        if  max_effy(i,j)==Eff_1(i,j)
           a(i,j)=m1(i,j);
           b(i,j)=n1(i,j);
        elseif max_effy(i,j)==Eff_2(i,j)
           a(i,j)=m1(i,j);
           b(i,j)=n2(i,j);
        elseif max_effy(i,j)==Eff_3(i,j)
           a(i,j)=m2(i,j);
           b(i,j)=n1(i,j);
        elseif max_effy(i,j)==Eff_4(i,j)
           a(i,j)=m2(i,j);
           b(i,j)=n2(i,j);
        end
      else
        max_effy(i,j)=NaN;
        a(i,j)=NaN;
        b(i,j)=NaN;
      end
   end
end
%maxEff=max(max(Eff))
```

```
hold off
figure(1)
%trinn=1.0;
[C,h] = contour(Egm, Egt, max_effy*100);%, trinn);
set(h,'ShowText','on','TextStep',get(h,'LevelStep')*2);
xlabel('$E_2\,\mathrm{(eV)}$','Interpreter','latex')
ylabel('$E_1\,\mathrm{(eV)}$','Interpreter','latex')


figure(2)
%trinn=1.0;
[C,h] = contour(Egm, Egt, a);
set(h,'ShowText','on','TextStep',get(h,'LevelStep')*2);
xlabel('$E_2\,\mathrm{(eV)}$','Interpreter','latex')
ylabel('$E_1\,\mathrm{(eV)}$','Interpreter','latex')

figure(3)
%trinn=1.0;
[C,h] = contour(Egm, Egt, b);
set(h,'ShowText','on','TextStep',get(h,'LevelStep')*2);
xlabel('$E_2\,\mathrm{(eV)}$','Interpreter','latex')
ylabel('$E_1\,\mathrm{(eV)}$','Interpreter','latex')



end
```

Map of efficiency for independently operated stack

```
function []=mapi(T)
%Makes a map of the maximum efficiency as a function of the band gap
% for independently operated stack
%Defining band gaps
Egb=1.11;
Egm=[1.12:0.01:3.5];
Egt=[1.13:0.01:3.6];
m=1;
n=1;
p=1;

%Import spectrum
[photonenergy,energy,photoner,inenergi]=readspectrum('AM1.5');
```

```
Eff=zeros(length(Egt),length(Egm))+NaN;

for i=1:1:length(Egt)
   Egt(i)
   for j=1:1:length(Egm)
      if Egt(i)>Egm(j)


[Eff1,Pow1,Vt,J1]=maxpowersingle(Egt(i),inf,T,photonenergy,energy,photoner,inenergi)
;

[Eff2,Pow2,Vm,J2]=maxpowersingle(Egm(j),Egt(i),T,photonenergy,energy,photoner,ine
nergi);

[Eff3,Pow3,Vb,J3]=maxpowersingle(Egb,Egm(j),T,photonenergy,energy,photoner,inener
gi);

          Eff(i,j)=Eff1+Eff2+Eff3;

      end
   end
end
maxEff=max(max(Eff))
hold off
figure(1)
%trinn=1.0;
[C,h] = contour(Egm, Egt,Eff*100);%, trinn);
set(h,'ShowText','on','TextStep',get(h,'LevelStep')*2);
xlabel('$Em\,\mathrm{(eV)}$','Interpreter','latex')
ylabel('$Et\,\mathrm{(eV)}$','Interpreter','latex')
end
```

Check the results

```
function []=mnpnewcheck(T)
% To check if the results are correct
%Defining band gaps
Egb=1.11;
Egm=1.15;
Egt=2.5;
p=60;

%Import spectrum
```

```
[photonenergy,energy,photoner,inenergy]=readspectrum('AM1.5');

Eff=zeros(length(Egt),length(Egm))+NaN;
m=Eff;
n=Eff;

for i=1:1:length(Egt)
   Egt(i)
   for j=1:1:length(Egm)
      if Egt(i)>Egm(j)


[Eff1,~,Vt,J1]=maxpowersingle(Egt(i),inf,T,photonenergy,energy,photoner,inenergi);

[Eff2,~,Vm,J2]=maxpowersingle(Egm(j),Egt(i),T,photonenergy,energy,photoner,inenergi
);

[Eff3,~,Vb,J3]=maxpowersingle(Egb,Egm(j),T,photonenergy,energy,photoner,inenergi);

          Eff(i,j)=Eff1+Eff2+Eff3;
          m(i,j)=(Vb/Vt)*p;
          n(i,j)=(Vb/Vm)*p;
          m1(i,j)=round(m(i,j)-0.5);
          m2(i,j)=round(m(i,j)+0.5);
          n1(i,j)=round(n(i,j)-0.5);
          n2(i,j)=round(n(i,j)+0.5);

Eff_1(i,j)=maxpower(Egb,Egm(j),Egt(i),T,m1(i,j),n1(i,j),60,photonenergy,energy,photon
er,inenergi);

Eff_2(i,j)=maxpower(Egb,Egm(j),Egt(i),T,m1(i,j),n2(i,j),60,photonenergy,energy,photon
er,inenergi);

Eff_3(i,j)=maxpower(Egb,Egm(j),Egt(i),T,m2(i,j),n1(i,j),60,photonenergy,energy,photon
er,inenergi);

Eff_4(i,j)=maxpower(Egb,Egm(j),Egt(i),T,m2(i,j),n2(i,j),60,photonenergy,energy,photon
er,inenergi);
          Vector=[Eff_1(i,j),Eff_2(i,j),Eff_3(i,j),Eff_4(i,j)]
          max_effy(i,j)=max(Vector);
          if  max_effy(i,j)==Eff_1(i,j)
             a(i,j)=m1(i,j);
             b(i,j)=n1(i,j);
          elseif max_effy(i,j)==Eff_2(i,j)
             a(i,j)=m1(i,j);
```

```
            b(i,j)=n2(i,j);
        elseif max_effy(i,j)==Eff_3(i,j)
            a(i,j)=m2(i,j);
            b(i,j)=n1(i,j);
        elseif max_effy(i,j)==Eff_4(i,j)
            a(i,j)=m2(i,j);
            b(i,j)=n2(i,j);
        end
    else
        max_effy(i,j)=NaN;
        a(i,j)=NaN;
        b(i,j)=NaN;
    end
  end
end
%maxEff=max(max(Eff))
hold off
figure(1)
%trinn=1.0;
[C,h] = contour(Egm, Egt, max_effy*100);%, trinn);
set(h,'ShowText','on','TextStep',get(h,'LevelStep')*2);
xlabel('$E_2\,\mathrm{(eV)}$','Interpreter','latex')
ylabel('$E_1\,\mathrm{(eV)}$','Interpreter','latex')


figure(2)
%trinn=1.0;
[C,h] = contour(Egm, Egt, a);
set(h,'ShowText','on','TextStep',get(h,'LevelStep')*2);
xlabel('$E_2\,\mathrm{(eV)}$','Interpreter','latex')
ylabel('$E_1\,\mathrm{(eV)}$','Interpreter','latex')

figure(3)
%trinn=1.0;
[C,h] = contour(Egm, Egt, b);
set(h,'ShowText','on','TextStep',get(h,'LevelStep')*2);
xlabel('$E_2\,\mathrm{(eV)}$','Interpreter','latex')
ylabel('$E_1\,\mathrm{(eV)}$','Interpreter','latex')



end
```

Efficiency Map under different spectrum

```
function[]=map(T, spectrum)
%Makes 2D-plot of theoretical efficiency for modules with a particular
%configuration of top, middle and bottom cells. For example, if the optimal
%numbers of m,n and p has been calculated for the AM1.5 spectrum, then you can
%calculate how a module with this particular configuration will perform
%under another spectrum.

open a-b.fig %Open file with values of m/n-ratio
D=get(gca,'Children'); %Extract values from the figure
m=get(D,'ZData') %Store values
Egt=get(D,'YData')
Egm=get(D,'XData')
open b-b.fig %Open file with values of m/n-ratio
E=get(gca,'Children'); %Extract values from the figure
n=get(E,'ZData') %Store values

[fotonenergi,energi,fotoner,InnEnergi]=hentspektrum(spectrum); %Load spectrum

Eff=zeros(length(Egt),length(Egm))+NaN;

for i=1:1:length(Egt)
   Egt(i);
   for j=1:1:length(Egm)
      if Egm(j)<Egt(i)+1E-6

[Eff(i,j),~]=maxpower(1.11,Egm(j),Egt(i),T,m(i,j),n(i,j),60,fotonenergi,energi,fotoner,Inn
Energi) %Calculates efficiency with chosen cell structure.
      end
   end
   Egt(i);
   Egm(j);
   m;
   n;
end
Eff
maxEff=max(max(Eff))

%Plots the result
figure(1)
[C,h] = contour(Egm, Egt, Eff*100);%, trinn);
set(h,'ShowText','on','TextStep',get(h,'LevelStep')*2);
xlabel('$E_2\,\mathrm{(eV)}$','Interpreter','latex')
ylabel('$E_1\,\mathrm{(eV)}$','Interpreter','latex')
```

```
end
```

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