Math Ed Res J (2012) 24:239–256
DOI 10.1007/s13394-012-0040-9

ORIGINAL ARTICLE

# First-year engineering students' use of their mathematics textbook - opportunities and constraints

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Received: 15 June 2010 / Revised: 30 September 2011 / Accepted: 30 September 2011 /
Published online: 13 June 2012


**Abstract** The role of the mathematics textbook at tertiary level has received limited exposure in previous research although it is likely that students work individually and that some of this work depends on the use of the textbook. The aim of this study was to investigate the process of approaching the textbook from epistemological, cognitive, and didactical perspectives. The focus was on identifying and discussing the opportunities and constraints in the process. The study was an explorative case study and the participants were first-year engineering students taking a basic calculus course. The data were collected through questionnaires, observations, and interviews. Results showed that the textbook was used to a very low degree and mainly perceived as a source of tasks. Different opportunities and constraints are pointed out and some didactical implications are suggested. The results and discussion indicate that a need for greater awareness about the use of mathematical textbooks in meaningful ways at tertiary level.

**Keywords** Mathematics textbook · Calculus · Engineering students · Constraints

## Introduction

Over the past two decades there has a rapid increase in the amount of research into mathematics education at tertiary level (Holton 2001; Niss 1998). However, issues associated with the use of the mathematics textbook at this level have received limited exposure in previous research. It is expected that students work individually

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more frequently at the tertiary level than in upper secondary school. As Wood (2001) stated:

> Students have trouble coping with large amounts of new material in a short time. Academic staff seem unapproachable and there may be little support for students with difficulties. Students are expected to do much of the work by themselves. (p. 93)

Some of the students' assumed individual work may rely on the use of the textbook and on how the book is used. Because of this it is reasonable to ask: How do first-year students perceive and approach the mathematics textbook? What opportunities are offered and what difficulties can arise?

This study aims to explore first-year engineering students' use of the calculus textbook by identifying the support and difficulties they experience when they start to study the concept of the derivative. According to Artigue (2001), one of the goals of research on mathematics learning and teaching at tertiary level is "to improve the understanding of students' difficulties and the dysfunction of the educational system" (p. 207). The results of this study might expand the understanding of what really happens by giving some insight into students' activities. It might not only make both teachers and students more conscious of the possible problems and help them face these problems more effectively, but also indicate how to take advantage of the existing opportunities. Finally the results may be interesting to authors of tertiary-level textbooks. More knowledge about students' perception of the textbook may inform decisions about introduction and treatment of concepts. The following research questions were posed:

1. What characterises first-year engineering students' approaches to mathematics textbooks?
2. What possible opportunities and constraints influence the ways textbook are approached by students?

## Three perspectives on the process of approaching the textbook

The process of approaching the textbook is complex. The student with her previous knowledge, experience, and ideas about mathematics and learning mathematics makes the first evaluation and decisions about further use of the textbook. The process takes place in the context of a certain didactical environment with a given curriculum, and is influenced by the extent to which the textbook content is explicit, and the teacher's vision of how the textbook should be used. Considering the process of learning, Artigue (1994, p. 32) describes the following types of constraints:

1. The *epistemological nature* linked to the mathematical knowledge at stake, the characteristics of its development, and its current way of functioning
2. The *cognitive nature* linked to the population targeted by teaching
3. The *didactical nature* linked to the institutional functioning of the teaching

These three perspectives were found valuable when discussing the opportunities and limitations arising when the students approach the textbook. Within the epistemological perspective the focus will be on how presentation of mathematical knowledge in the textbook and students' ideas about learning may have implications for the ways in which the book is perceived. Within the cognitive perspective the main focus will be on identifying students' cognitive barriers with emphasis on their previous knowledge. Within the didactical perspective the main focus will be on how the textbook is embedded in the calculus course and how the students are expected to use the textbook.

### The epistemological perspective

The nature of mathematical knowledge at tertiary level differs from that at secondary level. The knowledge is based more on the formal definition and formal proofs of theorems related to the main concepts (Tall 1991). Raman (2002) conducted an epistemological analysis of how pre-calculus, calculus, and analysis texts treated the notion of continuity. She concluded that the texts send conflicting messages about status and purpose of mathematical definitions. Results obtained from analysis of the textbook used by the students in this study (Randahl and Grevholm 2010) showed that the book promotes formal mathematics.

The concept of a derivative in real-variable calculus is clearly mathematical and student understanding of it at tertiary level is linked to their knowledge of the limit concept. The absence of practical contexts or situations which could point out the necessity of extending the existing student knowledge and the strict, pure mathematical contexts can contribute to the fact that students see the textbook as hard to use (ibid; p. 23). The epistemological perspective refers also to students' ideas about mathematics and learning mathematics. Students generally consider mathematics as "a collection of procedures to be used in order to solve some typical questions given in some crucial exams" (Vinner 2007, p. 4). Many engineering students are not primarily interested in mathematics but admit that mathematics is important in engineering contexts. At the same time the students think that they have to learn only concrete and applied mathematics and not abstract and pure mathematics (Kummerer 2001).

### The cognitive perspective

This study takes a constructivist approach to students' learning. The focus is on the learner as an individual who constructs her own knowledge often in an interaction with the others and within an institutional environment. By taking the constructivist perspective on learning it is assumed that the textbook's presentation and treatment of the concept in the textbook are not commonly perceived, with each student makes making her own interpretation. Students using the same textbook can make totally different interpretations of the text. Students' previous knowledge and earlier experience are important when they approach the textbook with the aim to learn mathematics. The calculus course for first-year students is considered to be quite difficult. According to Eisenberg (1991) there is quite a big difference between mathematics in high school and calculus: New concepts need to be learned quickly, and there are many generalisations, abstractions, and formalisations. Tall (1991) emphasised what he called the "cognitive expectations":

> The move from elementary to advanced mathematical thinking involves a significant transition: that from describing to defining, from convincing to proving in a logical manner based on those definitions. This transition requires a cognitive reconstruction which is seen during the university students' initial struggle with formal abstractions as they tackle the first years of university. (p. 20)

Previous research has pointed towards a gap that exists between *concept image* and *concept definition*, and it has been argued that this can be a problem when students learn mathematics (Tall and Vinner 1981; Juter 2006). The *concept image* is all the mental pictures, properties, associations, and processes related to a given concept. It can change with new situations and experiences. Only a part of the concept image may be evoked at a particular time and that part is called an *evoked concept image*. The *concept definition* is a "form of words used to specify the concept" (Tall and Vinner 1981, p. 152).

Students can form their concept image through different examples of the concept. It is expected that the students entering the calculus course have concept images based on earlier experiences and that these interact with the more formal definitions that are presented. The concept of the derivative should not be new for the students. According to the goals stated in the curriculum for the course in upper secondary school, the pupils should have knowledge about average and instantaneous rates, be able to find approximate values for instantaneous rate by calculation, and be able to recognise and interpret examples of instantaneous rate in practical problems.

## The didactical perspective

Generally, the curriculum describes the mathematical knowledge that is to be learnt by students. It usually specifies learning goals, content, methods, and assessment procedures (Tietze 1994). The overall goal for engineering education in Norway is centrally stated in the *Rammeplan* [1](2005) in the following way: "To educate engineers who combine theoretical and technical knowledge with practical proficiency and who take the responsibility for interaction between technology, environment, individuals and society" (p. 3). The university colleges at which future engineers are educated formulate more subject-specific *core curricula*, with goals, content, assessment forms, and a list of literature. For example the university college where the study was conducted defined the following learning goals for mathematics courses

1. to ensure a sound theoretical foundation that can be aptly applied to engineering subject matter
2. to contribute to giving the students a solid basis/foundation for further specialisation and post-qualifying education
3. to ensure the same quality standards as in international education programmes

[1] *Rammeplan*=studyplan

4. to ensure that students are able to work with professional literature based on mathematics, and that students develop a language conducive to communication in the technical-scientific environment

Both the study plan and the specific core curriculum express the importance of theoretical foundations and strongly emphasise the applicability of mathematics. It is expected that the future engineers will acquire sufficient mathematical knowledge and skills to enable them to identify, analyse, and resolve engineering problems.

## The multiple-perspectives methodology

### The setting for the study

The engineering students involved in the study reported here took the *Mathematics 1* course during the first year and *Mathematics 2* later. *Mathematics 1* comprised calculus and linear algebra, but in this study only the calculus part is considered. The mathematics course is compulsory and the normal minimum prerequisite is the completion of an advanced mathematical course in upper secondary school. Some of the students receive the required "study competencies" by taking an intensive mathematical introduction course at the university college. Because of these different preparatory experiences the level of students' previous knowledge can vary greatly.

The calculus section covers topics from differential and integral calculus such as functions, the concept of derivative, rules for differentiation, applications of derivatives, integration, and differential equations. The textbook used by the students in the study reported here was recommended by the teacher for the course. There were many available calculus textbooks, most of them in English, that covered the basic concepts named in the curriculum. The year when the study was carried out the textbook *Calculus* by Adams (2003, 2006) was used. According to the author, this text was designed "for general calculus courses, especially those for science and engineering students" (p. xv).

The investigation took place at one of the university colleges in northern Norway. All 90 participants were first-year engineering students who were taking the compulsory *Mathematics 1* course that comprises calculus and algebra. The textbook, *Calculus – a complete course*, written by Robert A. Adams, was used in the calculus part of the course.

### Forms of data: multiple perspectives

The data for the study were collected by a questionnaire given to the students, an interview with the teacher, observations of lectures and task-solving sessions, interviews of three students, and some informal conversations with students (mainly during task-solving sessions).

The questionnaire. At the beginning of the calculus course the students were administered a questionnaire consisting of clearly mathematical questions and questions about students' ideas about learning mathematics and learning sources. The main aim was to obtain insight into students' previous knowledge and to get some idea of students' assumed choice of learning sources and their ideas about learning mathematics. For example, in one section of the questionnaire the students were asked to make sense of the definition of the derivative and use it further. Students were required not to state the formal definition of the derivative, but only to explain how they understand it and to use it to prove an easy rule.

The following excerpts from the questionnaire (translated from Norwegian) offer the reader an idea of the kinds of information sought through the questionnaire:

Question 1 The more "formal" definition of the derivative was introduced in the upper secondary school.

We repeat it here:

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Explain how you understand this expression and then use it to show that $(x^2)' = 2x$

Question 2 a) The following function $f(x) = x^2 + 3$ is given. The derivative f′(x) to f(x) will be 2x.

Explain what the derivative $f'(x) = 2x$ tells us about the function f.

b) We can find the value of the derivative for a particular x; for example for x=4 we get $f'(4) = 2 \times 4 = 8$ . What does the number 8 tell us?

Question 3

*What do you assume to be most important / helpful when you learn calculus:*

- ☐ *Textbook*
- ☐ *Lecture notes*
- ☐ *Help/discussion with the teacher*
- ☐ *Cooperation/discussion with other students*
- ☐ *Other*

*Which?* ______________________________

*Question 4*

*What do you consider to be most important when you learn mathematics?*

- ☐ *Understanding*
- ☐ *Own interest*
- ☐ *To see how mathematics can be used*
- ☐ *To get right answer*
- ☐ *Formulas and methods*
- ☐ *Modelling*
- ☐ *Other reasons*

*Which?* ______________________________

Interview with the teacher. The interview with the teacher was conducted in the beginning of the term. The purpose of this interview was to discover reasons for the choice of the particular calculus book, and to gather information on the teacher's experiences in using the textbook.

Interviews with students. The choice of three student interviewees was based mainly on observations of the classes and on some informal talks with the students. During class observations one of the interviewees seemed to refer to the textbook frequently, but the other two interviewees rarely referred to the text. All three interviewees performed quite well in tests and during the task-solving sessions. One of the students was female, the two others were male and one of them was from an Asia nation.[2] The student with foreign background was speaking Norwegian at a level which made it possible to follow the lectures and participate in task-solving sessions.

During the interviews with the students, the interviewees responded to a set of questions about mathematics and about the textbook that was used. They were also asked questions about one of the tasks in their textbook. The interviews were audio-recorded and transcribed. The early questions during the interview inquired about interviewees' attitudes to mathematics, for example: Why, in your opinion, do we have mathematics in engineering education? Are you interested in mathematics? Do you like mathematics? Have you had problems in mathematics before?

The questions about using the textbook were as following: Do you use/not use the book during the course? What are the reasons for this? What expectations of the book do you have? What are you looking for? How do you perceive the book: difficult, easy? If difficult, what is difficult? If you do not use the textbook, what do you use: other books, lectures notes?

Observations of lectures and task-solving sessions. Observations of lectures and task-solving sessions took place over a period of six weeks, the aim being to find out more about how the textbook was used by the teacher and the students. For each observation there were about100 students in the lecture hall, and during the group-work sessions most of the students worked in groups of three or four people. However, some students worked alone. During the observations the writer paid particular attention to the extent to which teacher followed textbook approaches during the lectures, and to any references she made to the textbook. Of interest was whether the students were encouraged to use the textbook and, if they were, how. A specific question was: Were the exercises that the students were asked to do taken from the textbook? If they were not, then, where were they from?

During the task-solving sessions the writer was interested in whether the students used their textbook when attempting the tasks. If the answer was "Yes," then how did they use the textbook? Did the students help each other while working in small groups? What kind of questions did they ask each other, and where did they get their answers from? Were sources other than the textbook consulted?

[2] The university college traditionally enrols many students from Asia.

## Other methodological considerations

A common concern across all of the data collection techniques was the possibility of loss of objectivity (Lester and Lambdin 1998; Bryman 2004; Golden 2006; Schoenfeld 2007). Could we trust the results? Did the students tell the truth? The choice of methods, of course, depended on the research questions that needed to be answered.

By building into the research design a variety of sources of data it was expected that triangulation of analysis would be facilitated. However, as with all methods, there are advantages and limitations. It was important to consider how data gathered from any one vantage point verified or contradicted information obtained from other vantage points (Wellington 2000; Golden 2006). In this study the questionnaire, observations, interviews, and informal talks were the main methods of data collection. The aim of the questionnaire was to obtain insight into students' previous knowledge and to discover whether the students intended to use the textbook and also what they considered as important when learning mathematics. Analyses of responses to the questionnaire, together with observations, were expected to make it easier to choose students for possible interviews. Interviews might possibly give more information and a better picture of how the book was perceived by the students.

There were several potential limitations associated with the use of the questionnaire. Questionnaire responses revealed only what the responding students wanted us to know. One can assume that they were honest with their answers. But there can be a gap between stated and actual behaviour. With respect to the students' answers to questions on pure mathematics, if no answer was given to a question it would be difficult to know whether the students were unable, or unwilling, to answer the question. Golden (2006) warned that students' responses to questionnaires or interviews may be affected by their desire to present themselves as responsible, especially to researchers who belong to the college staff. In this study the students were encouraged to reflect on any problems they had with perceiving the book as a possible learning source. It has been confirmed by observations that they experienced some difficulties when using the textbook.

The researcher's role as an observer was also considered. One of the issues I faced was whether I should participate in group discussions and help the students with the tasks. In this case participation had to be considered in terms of the possible influence on results and the conclusion. I considered the possibility that it would be easier to be a participant observer, because it might have offered me more contact with the students. But if I had done that, there would have been the possibility that the "natural setting" would have been disturbed by my presence (Leder and Forgasz 2002). Another possible problem was that the students would want quick hints and direct help on how to solve the tasks and the "*how to*" might have become more important than the "why." Being "outside the groups" might help to have more focus on what I would observe.

Another issue was concerned with observations of the lectures. The teacher knew in advance that the research would be about textbooks. How might this information have influenced her behaviour? Would she refer more often than she usually did to the textbook during the lectures? This thought caused me to reflect on the possibility of presenting a more general aim of the study (for example, "Students' difficulties with calculus learning") to those who would be generating the data.

The above reflections and considerations commented on possible limitations of the study. The limitations were identified for two main reasons: first, they made the writer more aware of limitations when the data were being analysed; second, the conclusions reached could be considered in the light of the limitations, which could influence research designs and methodologies for future related studies.

## Results

The data comprised responses to the questionnaire (50 students answered), transcripts of interviews with the teacher and 3 students, written notes on observations of 20 lectures and 10 task-solving sessions, and informal conversations with students.

### Responses to the questionnaire

Students' responses to the mathematical part of the questionnaire were analysed in terms of their concept images of the derivative concept. Both observations and interviews were transcribed and the outcomes were categorised taking into account the epistemological, cognitive, and didactical perspective. Then responses were analysed in terms of *support* and *difficulty*. In order to find and discuss possible connections, these results were also related to the textbook (Randahl and Grevholm 2010) and to the curriculum.

In response to the question about what learning sources students primarily used when studying, 55 % of the students indicated lectures notes and 35 % indicated the textbook. In response to the question about what is really important when learning mathematics, 78 % chose "understanding" and 14 % answered "correct answer."

Analyses of students' responses to the mathematical tasks on the questionnaire suggested that most students entered the calculus course with poor knowledge of related mathematical content.

Regarding responses on task 1a (making sense of the definition)

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x};$$

- 30 of the 50 students who submitted responses to the questionnaire did not answer this particular question;
- 16 students stated that they could not remember this, or they had never seen anything like it before; and
- 4 students made an attempt to answer, mostly providing explanations of the symbols used in the definition.

Regarding task 1b: using the definition to derive the formulae and *to show* why $(x^2)'=2x$

- none of the 50 respondents answered, or commented on the question

Regarding responses on task 2a, asking for an explanation of what the fact that the derivative $f'(x) = 2x$ *says in relation to the function* $f(x) = x^2 + 3$

- 18 of the 50 questionnaire respondents did not answer this question;
- 15 students gave an incorrect answer that referred to maximum or minimum turnings points;
- 11 gave answers related to the notion of slope;
- 4 gave answers relating to the concept of "rate of change"; and
- 2 commented that it meant that the *f* was differentiable.

Regarding responses on task 2b, which was concerned with the meaning of

$$f'(4) = 2 \times 4 = 8$$

- 18 of the 50 questionnaire respondents did not answer the question;
- 5 students gave answers related to the slope of the tangent at a point;
- 4 students gave an answer related to "rate of change";
- 14 students indicated nothing more than a value of $f'(x)$ had been found; and
- 9 students gave an answer that was unrelated to the concept of a derivative.

Seen from a cognitive perspective, the responses to the questionnaire suggested that the students had poor previous knowledge of differential calculus at the beginning of their course. They were unable to make sense of the definition, and showed no evidence of being able to use it to justify a basic rule. They had difficulties with interpreting meanings of derivative function concepts. When it is recalled that 50 students did not respond to the questionnaire, it is possible that the actual overall situation was worse than that suggested by the above analysis.

Interview with the teacher

According to the teacher, the textbook used in the calculus course had been used for the course for only a few years. Mathematical errors in the text previously used and student complaints about difficulty level were the main reasons given for the change.

According to the teacher two issues were of particular interest during discussions on which textbook should be adopted as the new text for the course: first was clarity of the presentation and treatment; and second was the number of tasks required of students. Before the final decision, two different books were considered but finally *Calculus* by Adams (2006) was chosen. The deciding factor was that the other mathematicians had been satisfied when using Adams textbook with their courses. The following excerpt from the interview with the teacher is pertinent:

> I have looked at and considered many textbooks. You cannot judge them before you have used them, you know. Many had huge amount of tasks, diagrams. I could not recommend them. So finally we [the staff] considered two books and

> after some recommendations from other mathematicians we decided to use this one. [*Calculus* by Adams (2006)]. The book seemed to be well arranged and offered a lot of different tasks to engage students.

When talking about experiences from the year before, the teacher admitted that many students had perceived the new book as difficult, especially its level of formal mathematical language. Because of this, she spent much time explaining the subject matter during the lectures:

> We teach engineering students, not mathematics students, you know. So they have very poor understanding of the mathematical language. This is very difficult for them. So I have to take it on the blackboard.
> The applications are important to them….and I have to use additional examples, not only from the book. You know, the meaning with the textbook is that the student should read it…work with it by herself. But it is not possible. The textbook is huge but it includes many things that are not of our concern.

### Observations of lectures and task-solving sessions

The lectures had a clear and careful structure. The teacher used her own notes and wrote everything on the blackboard. She followed the sequence of topics in the textbook, but she did not always use the definitions or examples from the book. It was obvious that she prepared the presentation using several sources (other calculus textbooks, for example). She was a capable and experienced lecturer with very good subject knowledge. She had a friendly attitude to the students and they were willing to ask her questions during or after the lectures. The students made lecture notes. Many students had the textbook on the desk but they looked at it only when the teacher made direct references to the book. For example the teacher said:

> This definition […..] and other examples you can also find in the book. You can look at this later [at home].
> [*Many students opened the book and checked if the reference was correct*]

At the end of every lecture the teacher wrote on the blackboard a list of exercises recommended for the next task-solving session. Approximately 90 % of the tasks were selected from the textbook. The students knew the content plan of the lectures but they were not encouraged to read the text in the book in advance. During the task-solving sessions most of the students were sitting in groups with 3–4 people. Students appeared to be quite motivated during the task-solving sessions. Many students spent most of their time studying examples from the book, trying to apply them to obtain correct answers, but they were uncertain how to start to solve the problems. They were turning over the pages in the textbook, and it seemed that they were not very familiar with the book. Some of students were working in a special way; they started with the first exercise in the exercise section in the book (even if it is not the "homework"). Students were asked for the reasons why they chose to work in this way. They answered that mathematics was difficult for them and they had to start with the easy exercises and go forward to the more difficult ones. But there were many exercises in each section (approximately 60), the time was limited, and the work was not finished. The result was that students were very frustrated at the end of the sessions. Many students were waiting to get the help from the teacher. Here is one observed episode:

> Two students, who were sitting together, were trying to work on the task.
> Student 1: ….what shall we do here?
> Student 2: I am not sure…, look in the book, maybe we can find something similar…
> [*one of the students looks in the book, the other looks at lecture notes. They turn over the pages; no one looked at the pages with theory. It takes approximately 12 min. They do not talk to each other*]
> Student 1: Ok……maybe this one….no, not similar…
> Student 2: Maybe look at the answers…
> [*He looked at the answer section in the textbook*]
> Student 1: … no, nothing,…..only short answer….
> Student 2: we have to ask the teacher…..
> They try to contact the teacher. He is quite busy with helping the other students. Student 1 and student 2 are sitting and waiting. They do not try to work any more. At last the teacher came to them and asked what the problem was. They said that they had tried to work with the exercise but it was too difficult. The teacher explained the problem, drew the situation from the exercise on the paper and gave some hints to solve the problem. The students started to work; they did not talk any more.

The outcomes of the observations were mainly of didactical nature and they gave a picture of how the textbook was used by the teacher during the lecture and by the students during the task-solving sessions.

Interviews with the students

Two of the students were Norwegian; the third one was from Asia. All three agreed that the mathematics was important for future engineers. The three interviewees regarded the tasks as particularly important. The following comments were typical:

> All engineers have to study mathematics. But not so much theory - I mean definitions and theorems. I cannot see how it can help us to understand ….mathematics.
> You know, the tasks are very important for engineers. It is all about the tasks. We spend most of the time working on them.

The Norwegian students perceived the textbook as very difficult to use. Both of them liked mathematics and one of them was doing very well at the tests. During secondary schooling they had become accustomed to reading mathematics textbooks. Because of these experiences they both selected the textbook as one of the main learning sources when they answered the questionnaire. But according to them the book they had to use for this course was too difficult. One of the students had tried to read the text in the beginning of the term but that been a frustrating experience. Here is the relevant excerpt from the interview:

> Student: I read the first chapter [Preliminary] and the two following chapters in the book before I gave it up. Now I use it only when I have some problems. It is much easier to use the lecture notes.
> Interviewer: What kind of problems?
> Student: When I had problems with the tasks.

Another interviewee stated:

> I look through examples….beyond that it is so much talk…the book has so much text, it makes it so difficult. And so compact…I use it only if it is absolutely necessary, for example when the exercises given by the teacher are from the book…

Regarding differences between textbooks at secondary and tertiary levels, one student said:

> It was easier to find what I looked for, it was easier to understand, easier to read, with easier examples.
> And it was in Norwegian, because of these things, [it] was easier.

Excerpt from the interview:

> Interviewer: When you have problems, what do you do: ask anybody to help, use the book or the lecture notes?
> Student: I do this [use the book and the lecture notes] when I sit alone, but at school it is easier to ask for help because one can get an explanation.
> Interviewer: What about explanations in the book?
> Student: They are not so good,….I mean the teacher explains better.

Both of the Norwegian interviewees said that the book was not much used in lectures. In their opinion it was possible to perform well at the tests by using the lecture notes only. They said that by studying the lecture notes they got the necessary understanding of the calculus. The student from Asia had quite a different opinion about the textbook. First of all she was used to reading the book before the lecture.

The following was a comment made by the Asian student in an informal talk during a task -solving session:

> I have to read it in advance. In this way I can think more deeply when I follow the lectures. By doing this I find it easier to follow what the teacher is writing and to know what he means.

She explained that this was the way she had got used to working when studying mathematics in Asia. She was adamant that for her this was the only way she could obtain an understanding of important calculus concepts. She perceived the theory in the textbook as important and wished that she had more time to read through all the definitions and examples.

> I read the theory first, and after this I read examples to understand better the theory. The theory is most important, you know, so I read it first and after this the examples, and tasks… and once more the theory….The book is good, it is well organised. There are a lot of examples, tasks, …and theory. But it is in English. Sometimes…I have difficulties with the language.

## Discussion

The aim of this study was to find out what characterises first-year engineering students' approaches to using the calculus textbook. The assumption was that in the process of approaching the textbook some opportunities and limitations were present and that they could be recognised. The process was considered from epistemological, cognitive, and didactical perspectives.

Analysis of the data indicated that the students preferred to use the lecture notes rather than the textbook. During the term the students seemed, increasingly, to ignore the textbook except in relation to the tasks offered by the book.

Why did the students lose interest in the textbook? One of the reasons seemed to be that the formal treatment of the concepts in the textbook was too difficult for the students. Other researchers have also reported that ways in which mathematics is presented in textbooks can present real difficulty for students. Kajander and Lovric (2009), for example, when considering the source of students' misconceptions, suggested that more attention should be paid to the presentations of mathematical concepts in textbooks. According to Dreyfus (1991) mathematics is often presented to the students as

> the finished and polished product, even though historical mathematics was created through error, intuitive formulations, etc. This way of presenting may work well for students who major in mathematics, but it can be difficult for students majoring in science, engineering and taking mathematics as a required service subject. (p. 27)

Lakatos (1976) drew attention to the "deductivist style" of presenting mathematics:

> This style starts with a painstakingly stated list of axioms, lemmas and/or definitions. The axioms and definitions frequently look artificial and mystifyingly complicated. One is never told how these complications arose. (p. 142)

And Alsina (2001) stated:

> Mathematics courses present positive results, solved problems, bona fide models. Students become convinced that mathematics is almost complete, that theorem proving is just a deductive game, that errors, false trials, and zig - zag arguments, which play such a crucial role in human life, have no place in

> the mathematical world. Unfortunately, in some ways many textbooks have inherited the cold-journal style. This style of presentation kidnaps the 'human nature' of mathematical discoveries, the mistake that were made, the difficulties and the need for simplification. (p.5)

The following question arises: Did the first-year students using the textbook have any particular expectations of the textbook?

Sosniak and Perlman (1990) pointed out the relation between cognitive demands and students' prior knowledge so far as textbook usage was concerned:

> The cognitive demands of textbooks cannot be analysed without paying attention simultaneously to the prior knowledge and experience of the student who will use the book and the uses to which the book will be put. (p. 440)

Lakatos (1976) noticed:

> Some textbooks claim that they do not expect the reader to have any previous knowledge, only a certain mathematical maturity. This frequently means that they expect the reader to be endowed by nature with the ability to take a Euclidean argument without any unnatural interest in the problem-background, in the heuristic behind the argument. (p.142)

This study showed that first-year engineering students when starting the calculus course experienced serious difficulty not only in making sense of textbook definitions but also in using them. Their textbook introduced important concepts through formal definitions, and hence students found that it was not much help trying to use the textbook as an aid to understanding. The formal language used by the textbook was clearly perceived by the students as something that made the theory incomprehensible for them. It seems to have been the case that the gap between the students' previous knowledge and the expectations represented in the presentation of mathematical knowledge in the textbook was too large. The above discussion resonates with Zevenbergen's (2001) statement that having "access to the formal language of instruction and text, students' progress is enhanced or impeded depending on their levels of familiarity and competence in the language of instruction" (p. 15).

Taking a didactical perspective raises the question of the role of the textbook in the calculus course given by the particular educational institution. The explicitly defined goals of the core curriculum might be regarded as possible affordances in the process of approaching the textbook. They gave the students opportunity to define their own aims with the course and to relate them to the textbook. But because the goals of the curriculum were not clearly related to the use of the textbook by the teacher, the students found that they could easily ignore the book. Additionally it was necessary to take into consideration how mathematics textbooks were used in secondary mathematics classes. In fact, when they had been at school, most of the students in the present study had not made much use of mathematics texts. Apparently that is not uncommon, for as Sosniak and Perlman (1990) wrote:

> The textbook is seldom used as the source for insight into strategies for the solution of the problem or for explanation or for clarification of the concepts underlying the problems students are asked to solve. Occasionally the students report being expected to read the narrative portion of the mathematics text ('the sides of the pages'), but it is true only in a small number of instances. More importantly, perhaps students rarely are encouraged to study this narrative seriously. Instead, they count on teachers to 'explain it correctly'. (p. 429)

In the interviews it was confirmed that little experience with reading mathematics text in the textbook at secondary level was one of the reasons that the tertiary students found it difficult to use their mathematics textbook.

This study also called attention to the different ways in which the mathematics textbook is embedded as a teaching tool at primary, secondary, and tertiary levels. Whereas school teachers of mathematics, especially at the primary level, tend to depend very much on the textbook (Johansson 2006), at the tertiary level teachers often perceive themselves as experts in mathematics. In the present study, the teacher gave lectures which she had prepared, and the lectures she gave were interpreted by the students as representing their teacher's own knowledge.

However, this perceived expertise of the teacher sometimes caused didactical problems. The strong focus on lectures and lecture notes meant that the students did not see much value in studying their textbook. If the lecturer had made more references to the textbook during the lectures this might have encouraged students to make greater and more effective use of their textbook. As it turned out, the textbook seemed to be perceived by both the teacher and most students as merely a source of tasks.

During the process of choosing the textbook, the staff focused mostly on clarity of the presentation and on number of tasks. The textbook used in the present study had a large number of different exercises which, if carefully used, could have given the teacher opportunities to individualise the presentations in class. But students worked on mainly drill tasks and they were most interested in the short-term goal of getting correct answers. When they got correct answers they began to believe that they would succeed in mathematics and they did not really need to know the theoretical parts in order to achieve their goals. If the students had been directed to the theoretical sections more often, and if assessment tasks had a stronger theoretical orientation, then this might have persuaded students that it was necessary to read and comprehend the text.

Certainly, the textbook included exercises that were not merely drill tasks. According to the textbook's author, Adams (2006), "other exercises are designed to extend the theory developed in the text" and therefore enhance the students' " understanding of the concepts of calculus" (p. xiii). But the students did not work on these non-drill tasks because they did not see the point of trying to work through the theoretical sections in the book. They thought they did not have time to waste on what they perceived as unhelpful theory. Because so many students were asking for help, the teacher did not refer to the book but usually told them how the task should be solved.

## Didactical implications

The findings of the study point to the need for further research into the role of the mathematical textbook at tertiary level. The textbook is intended to be a learning and teaching tool, and, thus, more awareness is needed of student difficulties with the textbook. Perhaps, mathematics education researchers have focused too much, and too narrowly, on learning problems.

There is not the same attention paid to learning theories in the delivery of university mathematics as there is in the teaching of the subject at lower levels. Greater consciousness of factors that should influence the choice of textbook for mathematical courses at tertiary level is necessary. Different textbooks should be evaluated, taking into account different approaches to presentation and treatment of mathematics concepts, students' previous knowledge, and curriculum goals.

In particular, the question of how the textbook should be embedded in the learning and teaching context in order to achieve the goals stated by curriculum needs to be considered. A short account about how the textbook is intended to be used should be given during the first lectures in a semester. More frequent references to the textbook should be made in lectures, especially in relation to showing, explaining, and discussing how important concepts are treated by the textbook author. This could help students to realise the opportunities afforded by the book.

More exercises that encourage use of the book (especially those that require reading the text) should be proposed in the task-solving sessions. Further specific research, probably some *design research* studies, about using of the textbook at tertiary level is needed. It could help to recognise and utilise the potential of the mathematics textbook as a learning and teaching tool.

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