

The introduction of Algebra

Comparative studies of textbooks in Finland, Norway, Sweden and USA

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This Master's Thesis is carried out as a part of the education at the University of Agder and is therefore approved as a part of this education. However, this does not imply that the University answers for the methods that are used or the conclusions that are drawn.

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Preface

It has been a pleasure and captivating working with eight different textbooks of four different nationalities. The connection to an international research project has given great inspiration for this study.

I would like to take this opportunity to express my appreciation of working with Professor Maria Luiza Cestari. Her positivity, encouragement and belief in my abilities allowed the exploratory nature of this study. Secondly, her flexibility to work with my schedule has made the finalization of this master thesis possible.

I would also like to express my gratitude to all the members of the VIDEOMAT project for providing a stimulating, academic and friendly environment of acceptance for a beginner researcher.

Lastly, I would like to thank my husband and my family for providing the opportunity to finish this education. Special thanks go to my mother who has devoted much of her time to help with the kids throughout the schooling.

Summary

This master thesis is a comparative analysis of textbooks from four different countries, Finland, Norway, Sweden and USA (California), concerning the introduction of algebra. The topic for this research was inspired and initiated in connection with the international project VIDEOMAT, which I have been involved with since its planning phase. Following the design of this project, two mathematics textbooks of two consecutive grades from the four different countries have been analyzed. The focus in these has been the 60 first tasks presented in the algebraic chapters.

The review of literature includes a look at the origins of algebra from a didactical perspective. This was found to be helpful for the understanding of the diverging and extensive material on modern algebra. In a problem solving context, algebra evolved and contributed to the field of mathematics a powerful symbolic language, which has opened many doors for new developments. The flip side of this in education is that students can no longer make further progress in mathematics without adopting fluency in this language. However, it has proven to be a great obstacle to countless of students. The abstract nature of this language is exemplified by the variable which always has the same representation in form of a letter, but has multiple meanings depending on the context in which it is found. This calls for a need to provide students with varied and meaningful experiences in algebra.

The aim of identifying the qualities of the introductory tasks and different perspectives of algebra reflected in the textbooks required the development of an original coding system. This has been generated inductively, where the initial stage was a descriptive set of codes that has developed and was finalized in the internet based research program Dedoose. The review of literature played a formative role in this process.

The study shows that the introductory tasks presented in the textbooks often have a purpose of developing technical skills necessary for work with algebra. An example of this are tasks focusing the sequences of operations, which prepare the students for equation solving. There was also found to be an emphasis on the equal sign as an attempt to deal with the misconception of using it as an operational sign. Only one textbook was found to present introductory tasks that provide a natural progression into the use of algebraic symbols embedded in meaning. This was done through the use of patterns which also have a motivational element for the learning of algebra. The frequent presence of tasks aimed at developing skills in interpretation and formulations of expressions and equations indicates a focus on algebra as a language early in the algebraic chapters.

The implications of this study are that opportunities for varied and meaningful experiences in algebra can be provided through tasks focusing the topics of problem solving, patterns and geometry.

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1 Introduction

In this study I wish to investigate how algebra is introduced and treated in textbooks from Finland, Norway, Sweden and USA (California). A surface review of the textbooks shows great variation in size, structure and amount of algebraic content. These are interesting parameters that I recognize as vital to describe in a comparison of textbooks from different cultures. However the main focus of the analysis is the introductory section and the tasks that are provided in the algebra chapters.

I have been working in a larger project called VIDEOMAT since its planning phase in the spring of 2011. It is a project aimed at comparing teaching methods in the four countries listed above, concerning the introduction of algebra. The research group decided to define introduction to algebra in school as the introduction of letters as variables. It was at first expected that this happens at a specific moment in the curriculum but it proved difficult to define. Textbooks were chosen as the main resource to base this decision on and each country investigated textbooks from 6-, 7- and 8-grade. In Norway I performed this research and wrote a résumé of the algebraic content of the most common textbooks. Many interesting features to school algebra and textbooks were discovered and my master thesis was an excellent opportunity to continue the investigation cross-culturally. In addition, this study will also have a potential to enrich the findings of the VIDEOMAT project.

In my own experience with school algebra I early on discovered that algebra was an efficient tool for problem solving and so a natural appreciation for the innovation of letters as variables developed. Later experiences have revealed other aspects of algebra, but they have neither been as positive nor overshadowed the moment of enlightenment and algebra remain foremost a problem solving tool to me. Through the school mathematics we meet algebra in other content area such as for example geometry and functions. As we advance in mathematic courses algebra is often part of all the mathematics we do. The society we live in has a growing demand of citizens that know and can use more complex mathematics and so algebra for all in elementary school has received increasing attention.

In the process of writing my master thesis I have been in contact with publishers from Finland, Sweden, Norway and USA. Both in Norway and Sweden there has been a push for more algebra in elementary school textbooks. In Norway the new curriculum of 2006 also have an added focus on algebra in K 5-7:

Algebra in school generalizes arithmetic by letting letters or other symbols represent numbers, which provides the opportunity to describe and analyze patterns and connections. Algebra is also used in connection with the main content areas *geometry* and *functions*. (Kunnskapsløftet)

Kriegler (2007) comments on the educational goal ‘algebra for all’ from California, that while there is no direct instruction on specific algebraic content, there is an apparent intention of a stronger focus on algebra in elementary and middle school mathematics.

There is no agreement in the international research community of what school algebra is as algebra has so many applications. Bell (1996) holds the view that algebra is not an ‘identifiable

course, separate from other branches of mathematics' (p. 167). Expressing it informally I will say that algebra can be seen as a tool, as a language and as a way of thinking. Perhaps algebra is all of this? For these reasons textbook authors have to make many decisions when writing an algebra chapter, probably more than any other chapter. And therefore this research on algebra chapters as a comparative study becomes all the more interesting.

A textbook analysis can be done with many different aims at mind. I decided early on that I wanted to capture the variation in algebraic content, more specifically how algebra is introduced and the characteristics of algebraic tasks. According to Pepin (2009) students spend much of their time in classrooms working with written material such as textbooks. Teachers mediate the textbook by choosing, structuring and assigning tasks. Pepin (op.cit) refers to the work of Doyle when she states that 'the tasks teachers assign to students influence how they come to understand the curricula domain and serve as a context for student thinking, not only during, but also after, instruction' (p. 107-108).

My aim with this study is to attempt to capture the profile and characteristics of textbooks in four different countries focusing on algebra including similarities and difference presented. The research questions are:

1. How is algebra introduced in the textbooks in form of the introductory tasks?
2. Which perspectives of algebra are approached in the tasks?

In order to answer these questions a coding system has been developed that identify the different aspects of the tasks. Early on in this study I envisioned a computer program that would let me codify them by their characteristics and by using analyzing tools to be able to develop categories. Jim Stigler, who is part of VIDEOMAT project, suggested to use Dedoose which is an Internet based research program developed at UCLA. I have been in contact with one of the main developers of this program and was told that they did not know of anyone that had used the program for textbook research. He also mentioned that certainly this program could be suitable for such an analysis. For this reason this study can be seen as an exploratory and experimental work with Dedoose and textbook research. Therefore I also have formulated a research question related to this program:

3. In which ways are Dedoose an effective tool to analyze textbooks?

2 Review of literature

This study has led me to investigate the origins and development of algebra as I have wrestled with a more general question related to what is algebra. There seems to be no agreement on the answer to this question. For many people algebra is simply calculations with letters. In this case algebra is only representation. And if we only think of equations where variables represent a specific unknown this picture fits. However if we think of a formula, it can be: $A = l \times w$, the letters does not only represent numbers but also a relationship between quantities that are common for all rectangles. Something happens as we make the shift of describing the area of a rectangle by using the quantities related to that specific rectangle to describing the same area with variables (letters). We go from the specific to the general. The use of letters as variables automatically moves us into a different world, from arithmetic (the realm of the specific) to algebra (the realm of generalization). We are no longer only describing and working with specific numbers, situations and problems but groups of them. And so algebra is not only a powerful tool of representation but also of generalization (as an inherent part of its nature) which much of the higher mathematics builds upon. The applications of algebra are many and to mention a few is abstract arithmetic, modeling, functions and problem solving.

To further compliment my quest and relevant to this study is the context of school. Much research has been done in this field, but not surprising, internationally there is neither an agreement on what school algebra is (or should be). But one can recognize an effort in the research literature to reshape school algebra, looking at it from an historical perspective. Many students have difficulty in passing from arithmetic to algebra. As a remedy some researchers has developed the idea of early algebra which students can be introduced to as early as first grade.

2.1 Algebra

Algebra is a wide term that includes many perspectives and approaches to teaching. I will present literature on the three perspectives that I found most applicable to my study: *algebra as a language*, *algebra as problem solving* and *algebra as generalizing*. These topics also follow a natural development historically. Algebraic thinking is sometimes used synonymous with algebra and has been the focus of many studies. This notion is perhaps more difficult to separate from other mathematics and is widely disagreed upon.

2.1.1 Algebra in history and its evolvment as a language.

The introduction of algebra in school as the VIDOMAT project has defined it; ‘the introduction of letters as unknowns and variables’ can be viewed as an emphasis on the perspective of algebra as a language. Rojano (1996) looks at the development of the algebraic language through history. Her concern is students understanding when the algebraic language is thought in school removed from the context and semantics in which it originated: ‘this Finished version of instrumental algebra with all its potential as a synthetic and formal language, is what teaching sometimes attempt to communicate prematurely to the student’ (p. 56). Rojano believes this cause a superficial understanding and is the reason why students seem to struggle connecting symbolic manipulation and its use in problem solving.

The algebraic language has evolved over a long period of time. Algebra from its beginnings in the ancient civilizations of Babylon and Egypt up until the *cossists* traditions (1500) of the Renaissance constituted a sophisticated way of solving arithmetic problems (op. cit). With only some exceptions such as Diophantus and Jordanus de Nemore the problems and their solutions were all expressed in words and is called rhetoric algebra. One of the points Rojano makes is that: 'during this extensive period, "the problem" posed and "the equation to solve it" are indistinguishable' (p. 56). Here is an example which is problem 26 from the Rhind papyrus:

A 'hau' and a quarter is altogether 15. Calculate with four, add 1/4 which is 1 and altogether 5. Divide 15 by 5 and get 3. Finally multiply 4 by 3 and get 12. The sought 'hau' is 12.

This method is called '*regula falsi*' and can be translated with '*guess and adjust*'. It works only on linear equations.

Syncopation of algebra (algebra with abbreviations) is seen as an important step towards a symbolic language (Rojano, 1996). The syncopated algebra can be exemplified by the notation used by Diophantus from Alexandria, about 250 AD. Here is an example from Diophantus written work *Arithmetica* with an explanation for the symbols by Breiteig.

Diophantus wrote for example,

$$K^Y \alpha \Delta^Y \iota Y \zeta \epsilon$$

for what we with our symbols would have written $x^3 + 13x^2 + 5x$. He uses Y and ζ as symbols for a number that can vary. These symbols do most likely represent the Greek word for numbers: arithmos. Further the symbol Δ , which is the same as the letter D , is used as an abbreviation for dunamis (exponent), especially for x^2 . K is an abbreviation for kubos, the cube. α means the number 1, and $K^Y \alpha$ symbolizes what we writes as x^3 . Breiteig (2005, p. 13, translation by me)

However, Diophantus does not present general methods for solving problems. Rojano believes that the most important contribution by Diophantus to algebra is that for the first time he makes it possible to separate the problem from the solution with the use of two representations; the rhetorical and the syncopated.

The written works of Jordanus de Nemore (ca. 1200) contains a general strategy. Instead of previous works that shows how to solve problems that has a specific numeric solution ($x + y + z = 228$) he shows in his written work *Numeris Datis* how to solve problems of the type $x + y + z = a$ (He did not use the symbols as shown here). His works is characterized by the use of letters to represent both the unknowns and the known quantities. His method of solving problems is based in a reduction method that transforms new problems to avail that they have the same structure as a previously solved problem.

Historical documents like the abacus texts (the first came in 1202) shows the limitations of the pre-symbolic algebra (Rojano, 1996). About 400 of these texts are known today and each contain more than 400 problems and their solutions in natural language. When we read them today with our knowledge of symbolic algebra we see that many problems that are identical in structure has been solved with a diversity of strategies. The limited ability to generalize methods clearly

separates the pre- and symbolic algebra (op. cit). Rojano also claims that the progress in symbolic representation from rhetoric to syncopated (Arithmetic and De Numeris Datis) happens as the mathematicians work on problems that are located at the edges of the scope of the representation available. It evolves through problem solving, were the problems at hand cannot be solved within the current available representation. The aim is not the symbols themselves but to solve problems.

And likewise the Vietan project, which is recognized as the beginnings of the construction of modern symbolic algebra, was motivated by ‘not leaving any problem unsolved’ (Rojano, 1996, p. 59). In Viètes representation we can see the clear link between geometry and the evolvment of symbolic algebra (op. cit, p. 60):

$$\frac{A \text{ CUBUS} - B \text{ SOLIDO } 3}{C \text{ in } E \text{ CUADRATIVA}}$$

which in modern notation would be:

$$\frac{X^3 - 3b}{cY^2}$$

And it is here we can see the limitation of working with the geometrical forms in mind as we would try to operate with volume, area and length in the same expression. Here we clearly see the need for leaving the concrete representation behind and move into an abstract world. Rojano (1996, p.61) expresses it beautifully: ‘Algebraic syntax gains a life of its own and becomes disconnected from the semantics of the problems from which it originated and, in a dramatic change of roles, becomes the problem-solving tool *par excellence*’.

Rojano also formulate the perspective of algebra as the language of mathematics as she comments on the construction of symbolic algebra (1996, p. 59: ‘Never before had humanity been capable of creating an autonomous, specifically mathematical language, in which it was not only possible to state problems and theorems, but also to express the steps of solutions and proofs’.

The conclusion is presented as lessons we can learn from history as we present algebra to the students:

- Problem solving can be a powerful tool or context for constructing new knowledge.
- The manipulation of symbols as an object of knowledge removed from semantic meaning is contrary to the historic development of algebra. Symbolic manipulation was only the means for a greater purpose.
- Acquiring algebra as a language should be built upon prior knowledge, methods and skills.
- Students should be allowed to experience the limitations of their current knowledge and skills before they are introduced to algebra.

From this article and a short study of the development of algebra we see that algebra was in the beginning methods for solving problems (equations), and the algebraic symbols developed in this quest. A part of this development is also an increasingly analytical approach to solving problems

as the focus is not only unknown and known quantities but also the relation between them. Charbonneau (1996) says that Viète uses mostly the term ‘analysis’ for his algebra and for the next century the terms algebra and analysis were synonymous. After the symbols had taken form, algebra also expanded from a problem solving tool to an ingenious language in which mathematics could be expressed and evolve.

2.1.2 Algebra as problem solving

Bednarz and Janvier (1996) explain that problem solving was not only the motor in the advancement of algebra but also made a major contribution to the teaching of algebra for centuries. The teaching of algebra involved for a long time a corpus of problems for which arithmetic and algebra offered different solution methods. Algebra was presented as a new and more efficient tool for problem solving. The students were also given problems that had not been solved arithmetically to provide a convincing argument for algebra. As teaching evolved the focus on the passage from arithmetic to algebra was left behind for a favoring of algebra as a language and manipulation of symbols. However problem solving remained a factor in the teaching of algebra.

Bednarz and Janvier deal with an important issue concerning the introduction of algebra, as the main focus of their study is on the nature of the problems students work with in arithmetic and in algebra. Their research aims to define their differences and evaluate if the problems make for a smooth passage from arithmetic to algebra or require a profound reconceptualization. They recognize three categories of tasks that are often provided in introductory algebra: problems of unequal sharing, problems involving a magnitude transformation and problems involving non-homogeneous magnitudes and a rate (op. cit, p. 118). The categories are developed based on the quantities involved and the relationship between them.

Compering the algebraic tasks with the tasks typical for arithmetic in problem solving, Bednarz and Janvier (op. cit.) found that the main difference is one of connection. Tasks in arithmetic are constructed so that one can easily establish a relationship between two known quantities, which are labeled ‘connected’. The arithmetic thinking established is one were the known quantities are operated on and the end result is the unknown. The information given in the algebraic tasks most often does not provide the links between known numbers and has to be solved with several elements in mind at the same time, labeled ‘disconnected’. One can say that an analytical approach, with an attention to the relationships between all the quantities is needed, to solve the problem.

As we have seen, algebra developed through problem solving, but is this a convincing enough argument for using this approach for teaching algebra in school? Bekken (2000) explains that in his two decades of teaching algebra that the ideas, concepts and methods the students had problems with in algebra often had been major obstacles in the past as algebra developed. To teach mathematics in the context of how it originated is called the genetic method and Bekken refers to a statement by H. M. Edwards made in 1977 to motivate this method:

The essence of the genetic method is to look at the historical origin of an idea in order to find the best way to motivate it, to study the context in which the originator of the idea was working, in order to find the “burning question” which he was striving to answer.

From a psychological point of view, learning the answers without knowing the questions, is so difficult that it is nearly impossible. The best way is to ignore the modern treatises until we have studied the genesis in order to learn the questions. (Bekken, 2000, p. 85)

2.1.3 Algebra as generalization

The book ‘Approaches to algebra: perspectives for research and teaching’ edited by Bednarz, Kieran and Lee (1996) presents four perspectives for teaching algebra: as generalization, as problem solving, as modeling and as functions. I present the argument for focusing algebra as generalization of which Mason (1996) is an advocator for.

Mason sees generalization as the heartbeat of mathematics. His argument is that if students become accustomed to generalize in the mathematics courses from the early grades then algebra would cease being the main obstacle in mathematics education for so many people. Mason describes generalizing as a natural algebraic way of thinking with roots in history all the way back to the ancient Babylonians. Generalization is even speaking as the words are general and not particular. Further he describes generalizing as (op. cit, p. 83): “detecting sameness and difference, making distinctions, repeating and ordering, classifying and labeling”. These activities express an attempt to minimize demands of attention. This is what Mason calls algebraic thinking and the roots of algebra. The focus he places on activity in describing algebra is also common with Kierans approach which is presented in 2.2.

Algebra was first a description of a certain method of solving equations. Mason feels that since then algebra has evolved from denoting a process (algebra) to and object (an algebra). He refers to Gattegno which believes algebra arose when people understood that they could operate on objects (numbers, shapes, expressions), and could operate on those operations. Therefore, Mason mentions, when you think of combining arithmetical operations you have begun to do algebra. Thinking of algebra in this fashion also connects school algebra to higher algebra.

Mason also writes of algebraic awareness as “necessary shifts of attention”, that make it possible to be flexible when interpreting written symbols (op. cit, p. 74):

- *as expressions and as value*
- *as object and as process*

These experiences of transitions can also be noticed in the developmental history of algebra

Lee (1996) also mentions generalization as an approach to algebra. He sees algebra as a language and as a set of activities belonging to an algebraic culture within the wider culture of mathematics. He makes the case for generalization as a central activity to algebra as from its early beginnings solution strategies for similar equations were invented. Students should therefore be introduced to generalization early on. Lee is also concerned with meaning making for the students as it often remains a puzzle to them why they should learn algebra. He separates the meaning making into two spheres, the general culture of society and the mathematical culture. Meaning making for algebra is more easily developed within the culture of mathematics and in the process of generalization the symbolic language of algebra appears as an efficient tool for expressing the generality.

The research by Lee includes findings in a small group of adults and from high school students (grade 10) as they engage in introductory algebra as generalization. The students in the small group found the work with patterns to be intriguing and captivating. I will sum up his findings as short points:

- The adult students were more successful in finding useful patterns and expressing those algebraically
- Both groups of students had problems
 - Seeing the intended patterns
 - Expressing the pattern verbally
 - Expressing the pattern symbolically
 - Exercising flexibility in seeing patterns as not all patterns are algebraically useful

The generalization perspective of algebra is closely linked with the ideas of algebraic thinking and early algebra which will be presented in 2.2.

2.1.4 The different meaning of a variable and an unknown

My experience in introducing algebra in school has made me aware of the difficulty the students have in making a shift from working with formulas and expressions to dealing with equations. After students have worked with a variable in the two different modes they often confuse the meaning of the variable in the different contexts. I therefore planned to focus some of the analyses of the tasks on how the textbooks handle this distinction, between a variable as a general number and as an unknown. This has not been possible because of the time restraint but remains an issue for investigation.

Radford (1996) shows, from a historical point of view, that the conceptual structure of an unknown and a variable is very different. His analysis of the two concepts appearance in history concludes that there are two main differences between them.

The first is the context, or in the goal or intentionality, in which they appear. Radford identifies two historical works, *On Polygonal Numbers* and *Arithmetica*, as the first appearances of the variable and the unknown. The main topic in *On Polygonal Numbers* is to establish relationships between numbers while *Arithmetica* is concerned with solving word problems. The variable appears in *On Polygonal Numbers* in a deductive manner as the “passage from a relational problem to one dealing with abstract set values calculations” is performed (op. cit, p. 51). In *Arithmetica* the problems refer to numbers representing magnitudes of different dimensions (squares, cubes etc.).

The second major difference between an unknown and a variable as they appear in history is that of representation. The variable is represented geometrical or as letters which are included in a geometrical representation. The unknown in *Arithmetica* is not represented geometrically.

As a conclusion Radford relates his findings to a modern teaching approach by asking some important questions. I will only include one (op. cit, p. 51): “Can current teaching methods introduce some appropriate distinction between the concepts of unknown and variable, using the ideas seen here?”

Another study by Janvier (1996) also deals with the different interpretations of letters in algebra: unknown, variable and placeholder. He claims that the unknown is a less complicated concept to come to grip with than that of a variable. An unknown represents a number and the students learn how to perform calculations to find it. The understanding of a variable, however, is not solely dependent on representation but also on the interpretation of the “solver”. One can use formulas like $A = \pi r^2$ or $d = v \times t$, without considering that the value for r and t can vary and that A and d varies respectively. The letters in the formulas can be seen simply as placeholders and a recipe for finding the area of a circle or the distance traveled. If a student has come to grips with the unknown as representing a specific number that he is to find, this concept may obstruct the understanding of a variable, particularly since both concepts have the same representation in form of a letter. The student has to consider the wider situation (an expression, an equation, a function etc.) to determine which concept is at play.

2.2 Algebra in school: algebraic thinking and early algebra

We have seen algebra evolve through history and at a certain point become a language that propels the growth of modern mathematics. Algebra has so many applications today that the meaning of the word algebra has to a certain extent become cultural and individual. The focus in the last decades on school algebra has brought about a movement to transform it. This is based in algebra as a way of thinking rather than algebra as a language and problem solving.

2.2.1 Early algebra

Lins and Kaput have written a chapter in *'The Future of the Teaching and Learning of Algebra'* (Stacey, 2004) concerning the issue of early algebra. According to them there are currently two separate ways to understand the meaning of early algebra. The first is the traditional one and it refers to early algebra as the first time students meet algebra in school, likely to happen when students are about 12-13 years old. This implies that algebra is treated as an own topic, letters are introduced as variables or as an unknown. The second understanding takes early algebra to refer to algebraic reasoning fostered at a much younger age, sometimes as early as seven years old. It does not mean to teach traditional algebra earlier but to develop algebraic thinking within other topics and so create a 'new algebraic world' from the beginning. This understanding is gaining ground based on research that has shown children able to do much more than previously believed. In the VIDEOMAT project and therefore also in this study the introduction of algebra is defined in the traditional perspective. It must be mentioned that the current curricula for Norway show clear intention for an introduction of algebra to children of younger age in the second meaning of early algebra. In the goals for 2.-, 4.- and 7.- grade under the description for the topic 'Numbers and Algebra' we find the same comment about algebra:

Algebra i skolen generaliserer talrekning ved at bokstavar eller andre symbol representerer tal. Det gjev høve til å beskrive og analysere mønster og samanhengar. Algebra blir òg nytta i samband med hovudområda *geometri og funksjonar*. (Kunnskapsløftet, <http://www.udir.no/grep>)

Algebra in school generalizes computing with numbers by letting letters or other symbols represent numbers. It provides the means to describe and analyze patterns and

connections. Algebra is also used in connection with the main areas *geometry* and *functions* (translation by the author).

Another study by Kieran (2004) presents a definition for algebraic thinking in the early grades, which is based on her generally accepted method of analyzing algebra done in 1996 and a comparison of different curricula. Her focus for the analysis is on activity which she divides in three levels:

1. **Generational activities** of algebra involve the forming of expressions and equations. Examples include: i) equations containing an unknown that represent problem situations, ii) expressions of generality arising from geometric patterns or numerical sequences and iii) expressions of the rules governing numerical relationships.
2. **Transformational activities** of algebra include collecting like terms, factoring, expanding, substituting, adding and multiplying polynomial expressions, working with equivalent expressions and equations.
3. **Global, meta-level, mathematical activities.** For these activities algebra is a tool, and they are not exclusive to algebra. They include problem solving, modeling, noticing structure, studying change, generalizing, analyzing relationships, justifying, proving and predicting. (Kieran, 2004, p. 141-142)

Meaning-building of the algebraic objects happens mostly through the **generational activities**. The **transformational activities** are concerned with maintaining equivalence. The **meta-level activities** provide a context and meaning for the use of algebra.

Instead of focusing on the differences in the curricula as Kendal and Stacey (2004) has done, Kieran (op. cit.) finds characteristics held in common. She compares the Davydov curricula (Russian), the Singapore primary mathematics curricula (6. Grade), Korean mathematics curricula (7. Grade), the U.S. *Investigations in Number, Data, and Space* K-5 curriculum. The most evident similarity between these curricula is the emphasis on relationships between quantities. But there are also a common focus on generalization, justification, problem solving, modeling, and noticing structure. Kieran concludes that the activities shared are indeed the global, meta-level activities. She then presents this definition of early algebra

Algebraic thinking in the early grades involves the development of ways of thinking within activities for which letter-symbolic algebra can be used as a tool but which are not exclusive to algebra and which could be engaged in without using any letter-symbolic algebra at all, such as, analyzing relationships between quantities, noticing structure, studying change, generalizing, problem solving, modeling, justifying, proving, and predicting. (Kieran, 2004, p.149)

2.2.2 Algebra in school

The complexity of algebra in school as a research topic has been focused by Kendal and Stacey (2004). The study is an effort to capture the thinking and reasoning behind school algebra in different parts of the world. The countries involved are Australia, Brazil, Canada, China, Czech Republic, England, France, Germany, Hungary, Israel, Italy, Japan, The Netherlands, the Russian

Federation, Singapore and the USA. The chapter is based on the research done by Sutherland in 2000, “*A comparative study of algebra curricula*”, among others (op. cit).

The organization of pupils and the structure of the mathematics content have consequences for the goals of algebra and the approach to teaching it. The educational jurisdictions are considered in terms of the duration of comprehensive schooling and streamlining of students. A conclusion made by Sutherland in Kendal and Stacey (2004) is that algebra taught in comprehensive classes focuses on generalization activities while streamlined classes have an emphasis on symbolic aspects of algebra. This has implication for who learns algebra and when. The exception is Japan who has mixed ability classes throughout high school but still places a strong focus on symbolic transformation. Two different structures of mathematics are recognized: the integrated mathematics and the layer-cake approach. This variation in structure makes the discussion of “what is algebra” and related “who is a teacher of algebra” very complex and difficult to conclude. Some views of algebra are presented:

- a) A way of expressing generality and pattern (strongly evident in British Columbia, England, Victoria, Singapore).
- b) A study of symbolic manipulation and equation solving (Brazil, France, Germany, Hong Kong SAR, Hungary, Israel, Italy, Russian Federation).
- c) A study of functions and their transformations (France, Hungary, Israel, Japan, the Netherlands, USA).
- d) A way to solve problems (usually word problems) beyond the reach of arithmetic methods (Czech Republic, France, Hungary, Italy, Japan, Hong Kong SAR, Singapore).
- e) A way to interpret the world through modeling real situations, precisely or approximately (Quebec, England, Netherlands, Victoria).
- f) A formal system, possibly dealing with set theory, logical operations, and operations on entities other than real numbers (Singapore, Hungary). (Kendal and Stacey, 2004, p.335)

Another issue presented is the variation of how these aspects of algebra are introduced to the students in the different countries. This is discussed under five subtitles: “Generality and pattern”, “Symbolism, formalism and abstraction”, “Other aspects of a formal approach”, “Functions and multiple representations”, “Using technology”.

So according to Kendal and Stacey (op. cit.) there is no “one way” to teach and approach algebra, nor is it possible to make a list of content that describes school algebra. The final conclusion of their study is that the content area of algebra is too large to fit a school curriculum and therefore choices must be made concerning content as well as the approaches to it.

2.2.3 Obstacles to learning algebra

In the study of Kieran (2004) that was referred to in 2.2.1 (Early algebraic thinking), as an introduction and as a verification of the need of early algebra, she presents the findings of Kilpatrick. Kilpatrick takes a closer look at the thinking developed by students in traditional

arithmetic programs in relation to algebraic thinking. Kieran sums up his findings by stating some of the adjustments needed for the transition to algebra:

1. A focus on relations and not merely on the calculations of a numerical answer;
2. A focus on operations as well as their inverses, and on the related idea of doing/undoing;
3. A focus on both representing and solving a problem rather than on merely solving it;
4. A focus on both numbers and letters, rather than on numbers alone. This includes:
 - (i) working with letters that may at times be unknowns, variables, or parameters;
 - (ii) accepting unclosed literal expressions as responses;
 - (iii) comparing expressions for equivalence based on properties rather than numerical evaluation;
5. A refocusing on the meaning of the equal sign. (Kieran, 2004, p. 140-141)

These are parameters that can be identified in the tasks that are analyzed.

I will sum up this section of review of literature on algebra by stating the position I'm taking in respect to the analysis in this study. There are many perspectives of algebra and I recognize all that are mentioned in this review of literature. However, for it to be labeled as algebra in this study it has to involve the symbolic language. This is also in concurrence with how the introduction of algebra is defined in the VIDEOMAT project. Developing "algebraic thinking" without the involvement of symbols I will think of as early algebra. The only exception I will make in the analysis of tasks is for the problem solving tasks, as I can only evaluate the potential and the pedagogical aim of a task and not the solution strategy chosen by students.

2.3 Textbook analyses

Research on algebra is extensive within the mathematics education community. Analyses of textbooks in mathematics have received less attention. However, Howson (1995) did a comparison of textbooks used in grade 8 in England, France, Japan, The Netherlands, Norway, Spain, Switzerland and USA in connection with TIMSS. He comments on the pedagogical structure of the texts and how the mathematical content is delivered. Especially interesting for this project is a table, which shows the number of algebra chapters in each textbook, that he presents (Howson, 1995, p. 50):

Table 2.1: Placing and Percentage of Chapters Devoted to Algebra

	No. of chapters	Algebra chapter numbers	Percentage of total
England	27	(Part 1) 12, (Part 2) 8, 10	11
France	17	4-6	18
Japan	8	1-4	50
Netherlands	13	3, 6, 10, 11, 13	38
Norway	14	9, 10, 13	21
Spain	29	1-29	100
Switzerland	9	3, 7	22
United States	14	4, 15	14

Howson also portrays the topics presented in each country within Arithmetic, Algebra, Geometry, Probability and Statistics. Here are the algebraic topics covered in the different countries (Howson, 1995, p. 69)

Table 3.2: Algebraic Topics

Topic	Country							
	E	F	J	Ne	No	Sp	Sw	U.S.
Formulae	●	●	●	●	●			●
Graphs for formulae	●	●	●	●	●			●
Linear equations	●			●	●	●	●	●
Equations and inequalities		●	●	●			○	
Linear (affine) maps ($y = mx + c$)		●	●		●	●		
Polynomials: addition and subtraction			●		○	●		
Remainder theorem						●		
Simultaneous linear equations			●			●		
Algebraic structure						●		
Quadratic equations						●		
Biquadratic equations						●		
Sets							●	●

E (England), F (France), J (Japan), Ne (Netherlands), No (Norway), Sp (Spain), Sw (Switzerland), U.S. (United States)
 ● Topics covered ○ Topics only lightly covered

However, this study does not show how the different topics within algebra are presented or the tasks the students are introduced to and therefore remain a surface study of textbooks. With the

scope of this research, where eight books are commented on and compared in their completeness, a more detailed analysis would be a tremendous task.

Textbook analysis is also part of a comparative study involving schools from China, Japan and the USA done by Stevenson and Stigler in 1992 (Stigler is also a member of the VIDEOMAT Project). Both textbooks in reading and in mathematics were analyzed. It was found that the mathematics textbooks in Japan and USA included about the same amount of concepts and skills while the difference in number of pages was considerable. The American textbooks tended to be very explicit while the Japanese textbooks are less detailed and the students must rely on the teacher for an elaboration on the mathematical content. The US textbooks provide many examples of particular tasks and show each step of the solution. In comparison the Japanese textbook writers invites the students to engage in problem solving to a larger extent by not providing detailed solutions. A result is that the students come up with different solutions strategies and this activates meaningful discussions in the classroom. Lastly, the Japanese textbooks are attended to in every detail while parts of chapters and also whole chapters are omitted by the teachers in the USA

In the Nordic countries there has been conducted some studies looking closely at mathematics textbooks. Brändström (2005) has looked at the tasks and their difficulty levels in three Swedish grade 7 textbooks. This study exemplifies better what I wish to do in my master thesis but is missing the strong connection to a specific content area and the international comparison.

Ponte and Marques have done a comparative study on proportion in school mathematics textbooks (2007). They focus on the approach to this content area of mathematics and on the tasks provided in the textbooks. The tasks are analyzed with respect to the cognitive demand, structure and context of the tasks following the PISA framework (OCDE, 2004). Similarities and differences are commented on. My master thesis will be similar to this study, only the topic is algebra, and instead of focusing the task analysis in the cognitive demand I wish to analyze them related to the algebraic content.

3 Methods

3.1 The empirical material: textbooks

The interest in analyzing algebra chapters from Finland, Norway, Sweden and USA (California) came in connection with the VIDEOMAT project. The idea was to analyze the textbooks used in the classrooms that are a part of this project. However, because of the early stage of the project as well as the timing of my master thesis writing, there is still some uncertainty that the exact textbooks are used in the specific classrooms. The textbooks from the other countries that are analyzed were suggested by members of the research group from the specific country. In Norway, the textbooks analyzed were used in the classrooms.

I am analyzing algebra chapters from textbooks used in two successive grades in each country to follow the design of the VIDEOMAT project. In an attempt to handle with the national differences and the difficulty in pinpointing the moment when the introduction of algebra takes place, it was decided that each country would conduct the research in two grades. The grades were selected on the basis of an investigation of 6th-, 7th- and 8th- grade textbooks of all countries involved.

In Norway we found that in grade 6 textbooks letters appear briefly in geometry (as variables) and in multiplication (as unknowns) but their appearance are not elaborated on. The focus is on geometry and multiplication, and not algebra. Two widely used 7th-grade textbooks revealed a large discrepancy in algebraic content (10 pages versus 4 pages). In addition, no definitions or rules were part of the algebraic experience provided in these textbooks. The 8th-grade textbook investigated has a larger amount of algebraic content, and rules and definitions are clearly stated. Finnish and Swedish textbooks seem consistently to be one year ahead of the Norwegian textbooks, even though children start elementary school in Norway one year before their peers in those countries.

To get data that is comparable it was decided that each country will collect data from two grades. Finland and Sweden will observe in 6th and 7th grade, where the students are 12 and 13 years of age. Norway will collect data in 7th and 8th grade where the students are of the same age as the other Nordic countries. In USA the data will be collected from 6th- and 7th- grade classrooms and here the students are one year younger, respectively 11 and 12 years.

Textbooks analyzed:

Finland	6. grade:	K. Asikainen, H. Fälden, K. Nyrhinen, P. Rokka, P. Vehmas, S. Törnroos & K. Westerlund. (2008). <i>Min matematik 6</i> . Helsingfors: Schildts Förlag.
	7. grade:	M. Heinonen, M. Luoma, L. Mannila; T. Tikka, H. Lindgrén, H. Mitts & C. Söderback. (2010). <i>Pi 7</i> . Helsingfors: Schildts förlag

Norway	7. gade:	Pedersen, B. B. (2005). <i>Abakus 6A</i> . Oslo: Aschehoug & Co. Pedersen, B. B. (2006). <i>Abakus 6B</i> . (2.edition). Oslo: Aschehoug & Co.
	8. grade:	Hjardar,E. (2006) <i>Faktor 1</i> . Oslo: Cappelen Damm.

Sweden	6. grade:	Carlsson, S., Liljegren, G., & Picetti, M. (2004). <i>Matte Direkt Borgen 6A</i> . Stockholm: Bonniers. Carlsson, S., Liljegren, G., & Picetti, M. (2004). <i>Matte Direkt Borgen 6B</i> . Stockholm: Bonniers.
	7. grade:	Carlsson, S., Hake, K. B., & Öberg, B. (2001). <i>Matte Direkt år 7</i> . Stockholm: Bonniers.

USA (California)	6. grade:	Bennett, J. M., Burger, E. B., Chard, D. J., Jackson, A. L., Kennedy, P. A., Renfro, F. L., Roby, T. W., Scheer, J. K., & Waits, B. K. (2008). <i>California course 1: numbers to algebra</i> . USA: Holt, Rinehart and Winston.
	7. grade:	Bennett, J. M., Burger, E. B., Chard, D. J., Jackson, A. L., Kennedy, P. A., Renfro, F. L., Roby, T. W., Scheer, J. K., & Waits, B. K. (2008). <i>California course 2: Pre-algebra</i> . USA: Holt, Rinehart and Winston.

3.2 The tasks analyzed

I have chosen to analyze the 60 first tasks in the algebra chapters from the textbooks listed above. Only tasks from the units in the textbooks have been included in this group of tasks. The two Swedish textbooks, *Matte direkt 6B* and *Matte direkt år 7*, have units with diverse levels of difficulty (color coded) presented after core units. One of these levels does not progress from the common material and I have therefore chosen to proceed analyzing the tasks from the advancing level of units. The Norwegian textbook, *Faktor 1*, have an additional book with tasks organized by the same units as the textbook. Because the textbook has so few tasks presented I have interpreted the tasks from the additional book to belong to the corresponding units. The tasks in this book are categorized in three levels of difficulty and I have analyzed the ones belonging to

the second category. I refer to chapter 4.1 for further details on the structural aspects of the textbooks.

In the American textbooks the material is very extensive and the chapters that has been analyzed from these books includes units which have not been considered to be part of an introduction to algebra and neither algebraic. In Numbers to algebra I bypassed unit *1-2 Exponents* and in Pre-algebra the unit *1-3 Integers and absolute value*.

I contacted the publishers of the textbooks and obtained permission to upload these tasks in an Internet based research program, Dedoose, where they have been codified. However, one textbook, The Norwegian 6th grade textbook *Abakus 7*, has not been a part of this codification as it does not have an algebra chapter. This textbook has been analyzed manually for tasks that include symbolic language, and in addition, tasks that have been recognized as early algebra.

3.3 The methodological approach

In order to answer the questions formulated in this study, the qualitative content analysis approach has been used. After the textbooks have been chosen, I focus particularly on the tasks in the algebraic chapters. First of all a preliminary table has been elaborated, where different components of the tasks have been explored (see Appendix 1). Secondly, these components have been applied as codes in the Internet based research program Dedoose. In using the codes I observed that some of them could be merged in a higher hierarchical level. The codes have also been reformulated based on the review of literature.

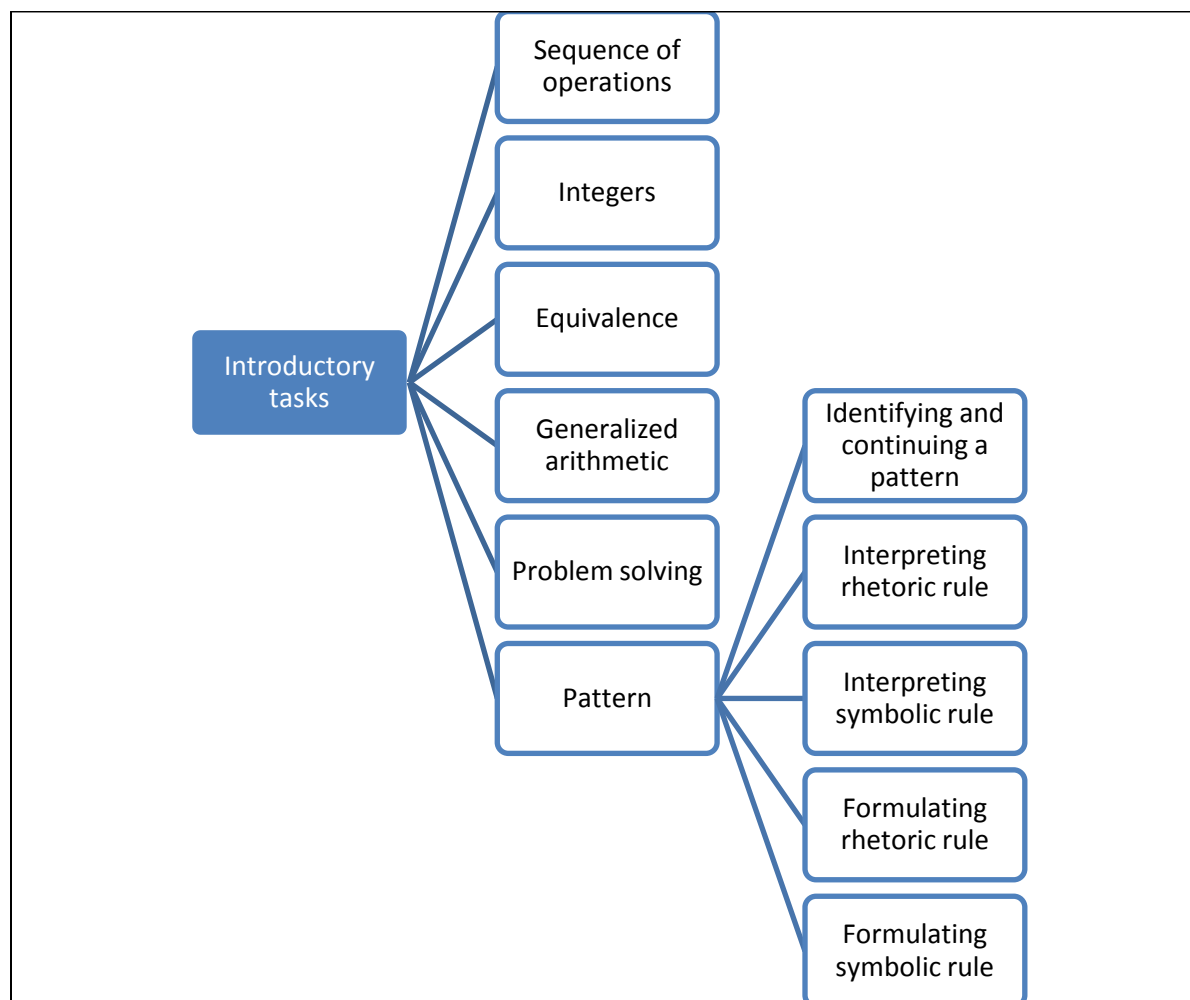
3.4 The coding system

The codes are developed as an effort to describe the tasks encountered in the textbooks. The codes have been modified and reduced as points of interests have emerged from the data. Reviewing literature and looking at algebra from a historical point of view have played formative and constructive roles in the coding scheme.

3.4.1 Codes describing the introductory tasks

A set of codes are developed to register how algebra is introduced in the textbook. Bell (1996) focusing at the problem-solving approach to algebra recognizes four formal approaches to introduce and develop school algebra: generalizing, problem solving, modeling and functions. Both formal and informal approaches have been found in the textbooks. The main code (root) for this part of the task analysis is “Introductory tasks” has six child codes. The textbook “Pi 7” works extensively with patterns in the introductory section and to capture the purpose of this section I have chosen to further codify these tasks with five sub codes.

Table 1. Codes for introductory tasks



The code “Sequence of operations” is an informal approach aimed at preparing the students for performing correct operations as they begin solving equations in the fashion of doing/undoing. “Integers” as a starting point to algebra are a formal way of talking about and working with operations on the whole numbers. It seems to be an introduction to negative numbers as well, often called the opposite number. The introductory tasks focusing on “equivalence” is a preparation for working with equations. The equal sign in arithmetic often take on a meaning of an operational sign for the students. Beginning algebra requires a correct understanding of this sign. “Generalized arithmetic” can be interpreted as a preparation for symbolic manipulation. The tasks aim at letting the students become familiar with the commutative-, the associative-, the identity and the distributive property. The code “Problem solving” is defined broadly as any task that cannot easily be solved with a known algorithm. The code “pattern” represents the approach to algebra through generalization.

3.4.2 Codes identifying the perspectives of algebra

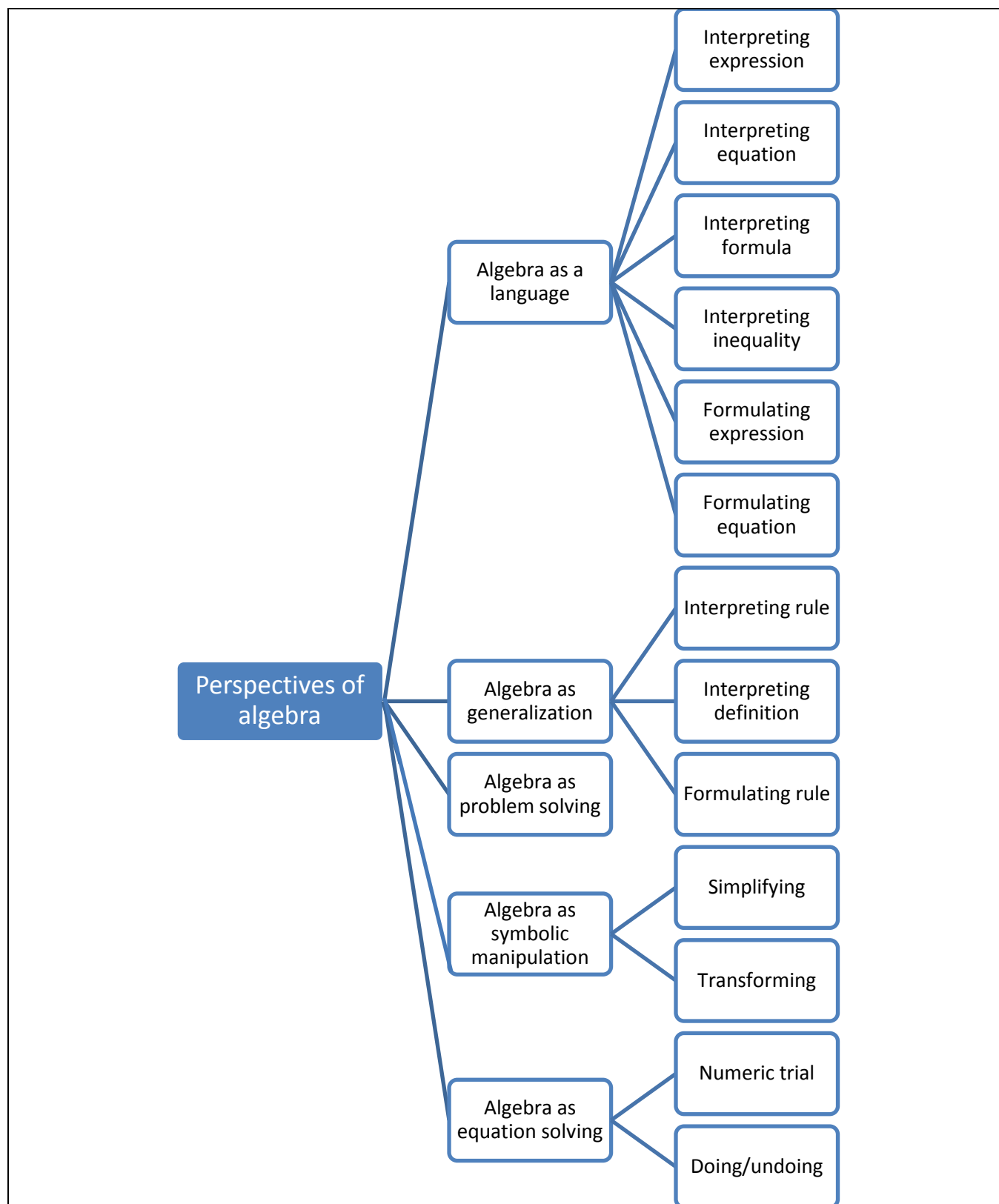
I have recognized five perspectives of algebra from the literature that is detectable in the textbooks analyzed: “Algebra as a language”, “Algebra as generalization”, “Algebra as problem

solving”, “Algebra as symbolic manipulation” and “Algebra as equation solving”. These perspectives are codes that are most often applied exclusively as the main pedagogical objective of the task is evaluated. However, there are a few tasks where two aims are evaluated to be equal and then two of these codes are applied.

Working with algebra in the activities of generalizing and problem solving can provide a meaning for the use of algebra. I will define “Algebra as generalization” much more narrowly than as done by Mason (1996). As generalization is an essential activity within mathematics as a whole, I want to make the distinction between the use of symbolic language to express or interpret generalities and the general mathematical activity of generalizing. Therefore this code will only be applied when there is an appearance of algebraic symbols. It is in the creation of a link between “Algebra as a language” (the language of generality) and the activity of generalization that a meaningful and rich context for the learning of symbolic algebra can be provided. “Algebra as problem solving” is a code that is applied to tasks that cannot be easily solved with a known algorithm and that focus on the relationship between quantities (analytical). Referring to Bednarz and Janvier (1996), see section 2.1.2, these tasks do not provide a direct connection between the known numbers but must be solved with the relationship between several quantities in mind at the same time.

In the development of algebra we have seen that the methods developed for solving equations were closely linked with problem solving. However in the context of school today these modes of working often appear completely separated. I have therefore added as an additional category of tasks, “Algebra as equation solving”, where the pedagogical focus is on strategies to solve equations. “Algebra as symbolic manipulation” is applied to tasks that simply ask the students to simplify or transform an expression with no other aim at heart.

Table 2. Codes identifying perspectives of algebra



To provide a better understanding of the codes I will present some tasks that can exemplify them.

Figure 1. Algebra as a language and Algebra as equation solving (Min matematik 6):

29. Skriv ekvationen och beräkna värdet på x . Kontrollera ditt svar.

a. Talet x multiplicerat med talet 4 är lika med 32.

c. Talet 5 multiplicerat med talet x är lika med 15.

e. Talet x multiplicerat med talet 14 är lika med 420.

b. Talet x multiplicerat med talet 6 är lika med 42.

d. Talet 13 multiplicerat med talet x är lika med 169.

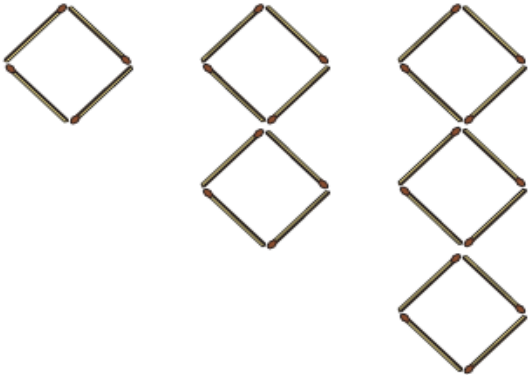
f. Talet 15 multiplicerat med talet x är lika med 240.

In this task we can detect a dual pedagogical aim, which are equal, of developing the algebraic language as well as equation solving, therefore both codes are applied. To further specify the task the child codes “Formulating equation” and “Doing/undoing”¹ is attached.

Figure 2. Algebra as generalization (Pi 7):

25. Hur många tändstickor finns det på bild nummer n ?

Bild 1 Bild 2 Bild 3



¹ The strategy for solving the equation is decided by the intentions put forth through examples in the textbooks.

The main pedagogical aim of this task is that of generalizing. The task is asking for an expression including the variable n . The solver must first recognize the pattern in the pictures and then describe the relationship between the picture number and the quantity of matches. In writing this expression the solver states a generalization of this relationship for all the pictures that would follow in this sequence. The root code “Algebra as generalization” is applied and further the child code “Formulating rule” is applied as well.

Figure 3. Algebra as problem solving (Min matematik 6)

49. På en läktare finns 55 personer. Av de vuxna är männen 5 fler än kvinnorna. Barnen är tre gånger så många som kvinnorna.

- a. Hur många barn finns det på läktaren?
- b. Hur många kvinnor finns det på läktaren?
- c. Hur många män finns det på läktaren?

Kontrollera ditt svar genom att addera antalet barn, kvinnor och män.

This task is codified with “Algebra as problem solving”. If we look at the problem there does not exist an easy connection between two objects, the solver has to operate with the relationships between all the objects at one time. The context is understood as a “Real life” problem. Algebra as a language can be an efficient tool in solving this problem, but the task does not suggest a solution method. If the task will be solved with algebra the solver has to create the representation (variables and equations) which will require some proficiency in this language on the student’s part. The pedagogical aim of this task is therefore viewed as developing skills in problem solving.

Figure 4, Algebra as symbolic manipulation (Faktor):

6.22 Trekk sammen.

- a) $3a + 4b + 2a + 3b$
- b) $3x + 2y + x + 4y$
- c) $4m + 2n - 3m + n$
- d) $8k + 5l - 7k - 3l$

The code “Algebra as symbolic manipulation” is applied to this task. The pedagogical aim is clearly that the students gain practice in how to simplify expressions.

A code (Context) is used to register another element of the semantic aspect of a task. The context is important as it connects variables and expressions to a familiar situation or object. The code “Real life” is used in the most inclusive meaning thinkable. It simply means that connections are made to something outside of the mathematical realm. If neither of these codes are used it means that the context is intra-mathematical (simply its own topic). Some of the books provide more

variation of context but because of the limitation of this study to only include the 60 first tasks, the contexts of geometry and real life are the only contexts detected in the textbooks.

3.4.3 Further characterization of tasks

To obtain more descriptive details of the tasks I have ten additional codes:

- Repetitive
- Multistep
- Formal mathematical language
- One variable
 - Figure or empty box
- Two or more variables
- Fractions
- Negative numbers
- Decimal numbers
- Exponentials

3.5 Reliability

The issue of reliability has been contemplated in this research. The categories developed are considered as stable as they either describe main aspects of algebra or introductory topics. The different perspectives of algebra are almost like different sides of a dice and so do not easily overlap. The introductory tasks have been separated into six topics that cannot be confused. The codes developed could be seen as more descriptive in nature than analytical. In addition, the fact that these codes have mainly been developed inductively through intensive work with the tasks has given the result that the codes fit very well with the material and few uncertainties occurred.

The codes have been designed so that the numbers of *child* codes add up to the quantity of the *root* code to which they belong (with the exception of “Introductory tasks”). This provided a great opportunity to discover coding mistakes. In addition, all the tasks have been reviewed and some recoded, a minimum of five times.

It would have been suitable to have a co-coder. This is also easily done in Dedoose, as another researcher could be given access and codify independently. However, because of the exploratory nature and the limitations of a 30 point master thesis this has not been done.

4 Analyses

This study is centered on the analyses of the 60 first tasks presented in the algebraic chapters. The main focus is on the introductory tasks and secondly the algebraic tasks. As an introduction to the analysis I will describe the general structure of the textbooks analyzed, which can give an indication of the view of instruction behind the making of the books.

4.1 The pedagogical structure of the textbooks

In this presentation and structural description of the textbooks I'm using the book of Howson (1995), *Mathematics textbooks: a comparative study of grade 8 texts* as a model.

All the books studied are aimed at the general population of students. The organization of the material in the textbooks will be described with respect to, chapters, units, introductory activities or examples, presentation of kernels (definitions, procedures, etc.) and exercises. In addition it will be commented on how the different textbooks deal with differentiation, review and key-answers.

4.1.1 Organization

Chapters

All the textbooks analyzed are divided in chapters which are then portioned into smaller units. The chapters and units are named by content in all the books except the Finnish textbook *Min matematik* where the chapters are simply numbered. In addition there is nothing marking the advancement from one chapter to another, and the transition from one content area to another is very subtle (in comparison, the other textbooks, with the exception of *Abakus 7*, use 2-4 pages to introduce a new chapter). The units in *Min matematik* are labeled by the content and they run from 1 to 86 independently of the chapters. All the books have pages with additional tasks and activities. I have chosen to only analyze the tasks that are part of the units in the textbooks, as the units appear to be the core material in all the textbooks.

To provide an overview of the organization of the textbooks related to the algebraic chapters I present this table:

Table 3. Place and percentage of chapters devoted to algebra

Nationality	Grade level	No. of chapters	Algebra chapter numbers	Percentage of total	Total no. p.
Finland	6	7 (8.chapter is review)	6	14	263
Norway	7	6	0	0	376
Sweden	6	10	9	10	320
USA	6	11	1, 11	18	601
Finland	7	3	3	33	312
Norway	8	7	6	14	292
Sweden	7	7	6	14	304
USA (California)	7	12	1, 3, 12	25	633

According to table 3 Finland and USA (California) devote more time to Algebra than the other countries. This is in spite that the students in California are one year younger than the students in the Nordic countries. Norway has a lesser focus on algebra than the other countries even though the students start school one year before their Nordic peers. The number of chapters in each book also varies greatly from 3 (*Pi 7*) to 12 (*Pre-algebra*) which might be a suggestion of a difference in amount of topics covered in a year. The California textbooks have almost the double amount of pages as the other textbooks. This is due partly to many organizational- and standard related pages. The books also offer a lot of additional activities and tasks. I also wish to comment that parts of chapters and also whole chapters may be left out from the teaching as the material is so extensive as explained by Stigler (1992)

Units

In this sub-chapter I will present some examples of the units in the textbooks. For the most part a unit from one textbook is shown as representative of similar units in other textbooks and therefore only four samples are given.

In the American (California) textbooks each chapter is divided in up to 11 units. These units have the same structure throughout the books. For the most part 1 unit consists of four pages where the first two pages consists of kernels, examples and two ‘think and discuss’ tasks, in that order. The remaining two pages of the unit are devoted to tasks. Here is a typical unit:

Figure 5. Numbers to algebra

1-1 Numbers and Patterns

California Standards
Preparation for AF2.0
Students analyze and use tables, graphs, and rules to solve problems involving rates and proportions.

Why learn this? You can use a pattern to find out how many football teams are in each round of playoffs.

The pattern in the table shows that the number of teams in each round of playoffs is half the number of teams in the previous round.

Football Playoffs				
Round	1	2	3	4
Number of Teams	64	32	16	8

EXAMPLE 1 Identifying and Extending Number Patterns

Reasoning Identify a possible pattern. Use your pattern to write the next three numbers.

Ⓐ 64, 32, 16, 8, , , , ...

$64 \quad 32 \quad 16 \quad 8$
 $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$
 $+2 \quad +2 \quad +2 \quad +2$

A pattern is to divide each number by 2 to get the next number.
 $8 \div 2 = 4$ $4 \div 2 = 2$ $2 \div 2 = 1$
 The next three numbers will be 4, 2, and 1.

Ⓑ 51, 44, 37, 30, , , , ...

$51 \quad 44 \quad 37 \quad 30$
 $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$
 $-7 \quad -7 \quad -7 \quad -7$

A pattern is to subtract 7 from each number to get the next number.
 $30 - 7 = 23$ $23 - 7 = 16$ $16 - 7 = 9$
 The next three numbers will be 23, 16, and 9.

Ⓒ 2, 3, 5, 8, 12, , , , ...

$2 \quad 3 \quad 5 \quad 8 \quad 12$
 $\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$
 $+1 \quad +2 \quad +3 \quad +4 \quad +5 \quad +6 \quad +7$

A pattern is to add one more than you did the time before.
 $12 + 5 = 17$ $17 + 6 = 23$ $23 + 7 = 30$
 The next three numbers will be 17, 23, and 30.

EXAMPLE 2 Identifying and Extending Geometric Patterns

Identify a possible pattern. Use your pattern to draw the next three figures.

Ⓐ A pattern is alternating squares and circles with triangles between them. The next three figures will be .

Ⓑ A pattern is to shade every other triangle in a clockwise direction. The next three figures will be .

EXAMPLE 3 Using Tables to Identify and Extend Patterns

Reasoning Make a table that shows the number of triangles in each figure. Tell how many triangles are in the fifth figure of a possible pattern. Use drawings to justify your answer. The table shows the number of triangles in each figure.

Figure	1	2	3	4	5
Number of Triangles	2	4	6	8	10

$+2 \quad +2 \quad +2 \quad +2$

Figure 4 has $6 + 2 = 8$ triangles. Figure 5 has $8 + 2 = 10$ triangles.

A pattern is to add 2 triangles each time.

Think and Discuss

- Describe two different number patterns that begin with 3, 6, ...
- Tell when it would be useful to make a table to help you identify and extend a pattern.

6 Chapter 1 Algebraic Reasoning

1-1 Numbers and Patterns 7

1-1 Exercises

California Standards Practice
Preparation for AF2.0

go.hrw.com
Homework Help Online
KEYWORD: MSJCAL11
Parent Resources Online
KEYWORD: MSJCA Parent

GUIDED PRACTICE

See Example 1 Identify a possible pattern. Use your pattern to write the next three numbers.

1. 6, 14, 22, 30, , , , ... 2. 1, 3, 9, 27, , , , ...

3. 59, 50, 41, 32, , , , ... 4. 8, 9, 11, 14, , , , ...

See Example 2 Identify a possible pattern. Use your pattern to draw the next three figures.

5. 6.

See Example 3 7. Make a table that shows the number of green triangles in each figure. Tell how many green triangles are in the fifth figure of a possible pattern. Use drawings to justify your answer.

INDEPENDENT PRACTICE

See Example 1 Identify a possible pattern. Use your pattern to write the next three numbers.

8. 27, 24, 21, 18, , , , ... 9. 4, 096, 1, 024, 256, 64, , , ...

10. 1, 3, 7, 13, 21, , , , ... 11. 14, 37, 60, 83, , , , ...

See Example 2 Identify a possible pattern. Use your pattern to draw the next three figures.

12. 13.

See Example 3 14. Make a table that shows the number of dots in each figure. Tell how many dots are in the sixth figure of a possible pattern. Use drawings to justify your answer.

Extra Practice
See page 192.

15. Start with 7; add 16 to each number to get the next number.

16. Start with 96; divide each number by 2 to get the next number.

17. Start with 50; subtract 2, then 4, then 6, and so on, to get the next number.

18. **Reasoning** Suppose the pattern 3, 6, 9, 12, 15... is continued forever. Will the number 100 appear in the pattern? Why or why not?

8 Chapter 1 Algebraic Reasoning

1-1 Numbers and Patterns 9

The Finnish textbook *Pi 7* has a similar structure. However, several kernels are presented and after each follow a number of examples. The units vary from four to eight pages long, but the kernels and examples are always in the first 2-4 pages and then there are 2-6 pages with only exercises. Often there are multiple kernels presented in one unit and for each kernel there are given examples.

Figure 6. *Pi 7*

196 3.1 Talföljder

Då vi räknar upp tal efter varandra får vi en talföljd. Talen i talföljden kallas element och vi skiljer dem åt med ett komma.

EXEMPEL
Talföljden 10, 20, 40 har tre element. Det första elementet är 10, det andra 20 och det tredje 40.

En talföljd kan vara ändlig eller oändlig. Vi visar att en talföljd är oändlig genom att skriva tre punkter i slutet.

EXEMPEL

- Talföljden 1, 2, 4, 8, 16 är ändlig.
- Talföljden 1, 2, 4, 8, ... är oändlig.
- Talföljden 2, 4, 6, ... 20 är ändlig.

Olika talföljder
Ofta bildas en talföljd efter någon bestämd regel. Vi kan t.ex. få nästa element i en talföljd genom att addera eller subtrahera föregående element med ett bestämt tal. Vi kan också få nästa element genom att multiplicera eller dividera föregående element med något tal.

EXEMPEL

- Nästa element i talföljden 0, 1, 2, 3, 4, ... får vi genom att addera föregående element med 1.
- Nästa element i talföljden 2, 8, 14, 20, ... får vi genom att addera föregående element med 6.
- Nästa element i talföljden 0, -2, -4, -6, ... får vi genom att subtrahera föregående element med 2 eller addera föregående element med -2.

EXEMPEL

- Nästa element i talföljden 1, 2, 4, 8, 16, ... får vi genom att multiplicera föregående element med 2.
- Nästa element i talföljden 1, -2, 4, -8, 16, ... får vi genom att multiplicera föregående element med -2.
- Nästa element i talföljden 10, 8, 4, 2, ... får vi genom att dividera föregående element med 2 eller multiplicera föregående element med 0,5.

Regeln för hur vi bildar en talföljd kan också vara att ett element beror av flera av de föregående elementen. En talföljd kan också bildas så att två olika talföljder skrivs in i varandra.

EXEMPEL

- Nästa element i talföljden 1, 1, 2, 3, 5, 8, 13, ... får vi genom att addera de två föregående elementen.
- Talföljden 1, 12, 3, 9, 5, 6, 7, 3, ... består av talföljderna 1, 3, 5, 7, ... och 12, 9, 6, 3, ... Regeln för den första talföljden är att addera föregående element med 2 och för den andra talföljden att subtrahera föregående element med 3.

UPPGIFTERNA
1-18

200 3.1 Talföljder

ÖVNINGSPROBLEM

- En talföljd är 2, 6, 10, 14.
 - Vilka tal är element i talföljden?
 - Vilket tal är det tredje elementet i talföljden?
 - Är talföljden ändlig eller oändlig?
- Vilka är de tre följande elementen i talföljden?
 - 1, 4, 7, 10, ...
 - 27, 25, 23, 21, ...
 - 55, 50, 45, 40, ...
- Vilka är de tre följande elementen i talföljden?
 - 0, 3, 6, 9, ...
 - 7, 5, 3, 1, ...
 - 9, -7, -5, -3, ...
- Hur många element har talföljden?
 - 2, 5, 4, 7, 1, 0, 8, 6, 3
 - 3, 2, 1, 0, -1, ...
 - 10, 20, 10, 20, 10
 - 7, 14, 21, ... 49
- Fortsätt följden med två bokstäver.
 - A, C, E, G, ...
 - T, Q, N, ...
 - A, C, F, J, ...
- Skriv ut de fyra första elementen i en talföljd där det första elementet är
 - 3 och varje nytt element är 5 större än det föregående
 - 42 och varje nytt element är 6 mindre än det föregående
 - 5 och varje nytt element är tre gånger så stort som det föregående
 - 48 och varje nytt element är hälften av det föregående.
- Bilda en talföljd av talen 2, 7, 3, 9, 10, 1, 5 och 8 så att varje element närmast efter
 - alltid är större än det föregående
 - alltid är mindre än det föregående.
- Bilda talföljden så att den kan spjällas upp i två talföljder som vardera uppfyller villkoret i a och så att elementen från de olika delarna uppfyller villkoret i b.
 - jämna naturliga tal i storleksordning
 - heltal i storleksordning
 - negativa heltal vars absolutbelopp är delbara med fem, i storleksordning.
- Skriv en talföljd med fem element där det mittersta elementet är 15 och
 - som bildas av heltal i storleksordning
 - som bildas av udda heltal i storleksordning
 - där varje element är 3 mindre än följande element.

Fortsätt talföljden med tre element.

- 1, 4, 9, 16, ...
 - 12, 7, 2, -3, ...
 - 2, 4, 8, 16, ...
- 1, -3, 9, -27, ...
 - 100 000, 10 000, 1 000, ...
 - 3, -3, 3, -3, ...
- 2, -4, 8, -16, 32, ...
 - 5, -2, 10, -3, 15, -4, ...
 - 5, -10, 15, -20, 25, ...
- En talföljd börjar 1, 3, 7, 15, ...
 - Skriv ut talföljdens femte, sjätte och sjunde element.
 - Skriv med ord regeln för hur talföljden bildas.
- Vilket element saknas i talföljden?
 - 1, 2, ..., 8, 16
 - 1, 2, ..., 7, 11
 - 1, 2, ..., 5, 8
 - 1, 2, ..., 4, 5
 Skriv också med ord regeln för hur talföljden bildas.

ÖVNINGSPROBLEM

- Vilken är regeln för hur tal- och bokstavsföljden bildas?
 - 1, B, 2, D, 3, F
 - A, 2, D, 4, I, 6, P
- Skriv som en talföljd. Är talföljden ändlig eller oändlig?
 - negativa heltal
 - tvåsiffriga jämna naturliga tal
 - primtal mindre än 20
- Vilka tal står x och y för?

1	2	6	x
2	3	5	y
- Vilket är följande element i talföljden?
 - 0, 0, 1, 1, 2, 4, 7, 13, ...
 - 1, 3, 3, 9, ...
 - 2, 5, 3, 7, 8, 10, 15, ...
- Hur många stjärnor finns det på den femte bilden?

Bild 1	Bild 2	Bild 3	Bild 4	Bild 5
☆	☆☆	☆☆☆	☆☆☆☆	☆☆☆☆☆
- Vilket är det tionde elementet i talföljden?
 - 2, 4, 6, 8, ...
 - 5, 10, 15, 20, ...
 - 1, 4, 7, 10, ...
- Det n:te elementet i en talföljd får vi med hjälp av regeln $n + 3$. Skriv ut talföljdens fem första element.

22. I tabellen har de fyra första elementen i en talföljd skrivits ut. Kopiera tabellen i ditt häfte och komplettera den.

element 1	element 2	element 3	element 4	element 5	...	element 10
5	7	9	11			

b)

element 1	element 2	element 3	element 4	element 5	...	element 10
2	5	8	11			

23. Hur många tändstickor behövs för att

- bild 4
- bild 5?

Bild 1 Bild 2 Bild 3

24. Hur många små kvadrater finns det på bild nummer

- 4
- 5
- 10?

Bild 1 Bild 2 Bild 3

25. Hur många tändstickor finns det på bild nummer n?

Bild 1 Bild 2 Bild 3

The other textbooks are divided into smaller units that appear less separated. In the Swedish textbooks a unit is often only one page with a headline, a kernel with examples and tasks. Here is an example:

Figure 7. Matte direkt

Uttryck med flera räknesätt

På sjörövarön handlas det med föremål som hittas i olika kistor. Priserna är i guldmynt. Se prislistan.

Om kostnaden kan skrivas $30 + 3 \cdot 50$ har du köpt ett armband och tre Halsband.

$30 + 3 \cdot 50$ kallas ett **uttryck** för kostnaden.


Du får betala:
 $30 + 3 \cdot 50 = 30 + 150 = 180$ guldmynt.

Om du köper 5 pärlor och 5 ädelstenar blir uttrycket för kostnaden:
 $5 \cdot (10 + 12) = 5(10 + 12)$

Man skriver oftast inte ut multiplikationstecknet framför en parentes. Kostnaden blir:
 $5(10 + 12) = 5 \cdot 22 = 110$ guldmynt

När man har flera räknesätt i en uppgift räknar man i den här ordningen:

1. Parenteser
2. Multiplikation och division
3. Addition och subtraktion



På sjörövarörens nota stod följande uttryck. Vad hade han sålt och hur många guldmynt fick han?

1 a) $2 \cdot 10 + 15$ b) $30 + 2 \cdot 50$ c) $5 \cdot 12 + 3 \cdot 10$
 2 a) $3(15 + 50)$ b) $2(10 + 15)$ c) $15 + 2(10 + 3)$

3 Skriv ett uttryck och räkna ut hur mycket du ska betala för

a) 3 ringar, 10 pärlor och 5 ädelstenar
 b) 4 Halsband, ett armband och 5 ringar

Räkna ut

4 a) $2 + 4 \cdot 5$ b) $8 \cdot 4 - 6$ c) $3 \cdot 7 + 6 \cdot 2$
 5 a) $\frac{12}{3} + 5$ b) $7 + \frac{36}{3}$ c) $\frac{14}{2} + \frac{50}{10}$
 6 a) $3(4 + 5)$ b) $2(6 - 3) + 5$ c) $3(4 + 2) - 2(8 - 3)$

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Uttryck med en variabel

En **variabel** är något som kan variera, något som kan ha olika värden. Ofta skriver man variabeln som en bokstav.

3 cm Omkrets: $(3 + 3 + 3 + 3) \text{ cm} = 4 \cdot 3 \text{ cm} = 12 \text{ cm}$




x Omkrets: $x + x + x + x = 4 \cdot x = 4x$ Man skriver inte ut multiplikationstecknet mellan en siffra och en variabel.

I den lilla kvadraten vet vi inte hur lång sidan är, den kan variera. Vi kallar den x. Kvadratens omkrets blir då $4x$.

Uttrycket för kvadratens omkrets är $4x$.

Omkretsen av **alla** kvadrater kan beskrivas med uttrycket $4x$. Värdet på x är längden på sidan. När du vill beräkna omkretsen av en speciell kvadrat sätter du in längden på sidan istället för x. En kvadrat som har sidan 5 cm har omkretsen $4 \cdot 5 \text{ cm} = 20 \text{ cm}$. Man säger att **värdet på uttrycket** $4x$ är 20 cm.


7 På sjörövarön har olika byar sina speciella "bymärken", gjorda av lika långa pinnar – ju längre omkrets (fler pinnar), desto mäktigare by. Vad blir omkretsen av bymärkena om längden på varje pinne är x?

a)  b)  c) 


8 Vad blir omkretsen av varje figur i uppgift 7 om x har värdet

a) 4 cm b) 7 cm

9 a) Skriv ett uttryck för omkretsen av dessa stjärnformade bymärken om längden på en pinne i märke A är 3y medan längden på en pinne i märke B är y.



A



B

Kom ihåg
 $3y = 3 \cdot y$

b) Hur stor blir omkretsen av bymärkena om y är 5 cm?




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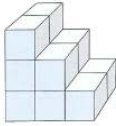
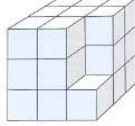
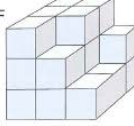
The units in the Finnish textbook *Min matematik* are similar only there are more tasks provided and the units are 3-4 pages long. The tasks are divided into; tasks following the kernel, homework and additional exercises. (In the Swedish textbooks, tasks for homework are in the back of the book). The Norwegian textbook *Faktor 1* have similar units only there are more examples provided and there are fewer exercises. This textbook comes with an additional exercise book.

The Norwegian 6th Grade textbook *Abakus 7* does not follow the structure of the other textbooks were kernels and examples are presented prior to the tasks. Often the units open with an exploratory task and sometimes a small kernel appear among the tasks. Very few examples are given and many units do not provide an example at all.

Figure 8. Abakus7A

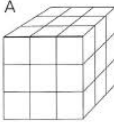
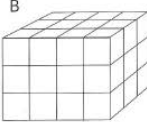
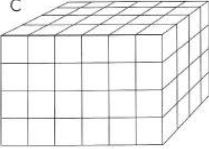
Volum

A  B  C 

D  E  F 

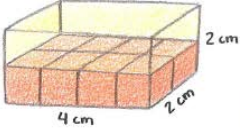
47 Hvor mange terninger har hver figur?
Diskuter hvordan du finner det ut.

En terning med sider 1 cm har volumet én kubikkcentimeter. Vi skriver 1 cm^3 .

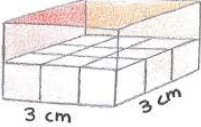
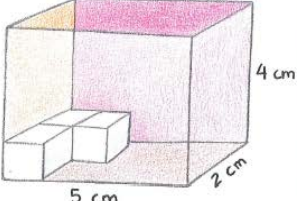
A  B  C 

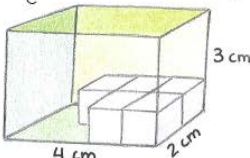
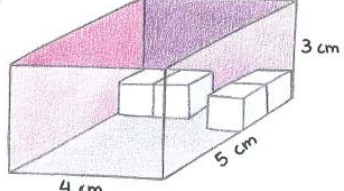
48 Blokkene er laget av terninger som er 1 cm^3 .
Hvor mange kubikkcentimeter er hver blokk?

49 Hvor mange kubikkcentimeter får plass i
a bunnen på esken
b hele esken



50 Hvor mange kubikkcentimeter er eskene?

A  B 

C  D 

Abakus 7 reflects a constructivist view of knowledge where the students are to “create” their own mathematical knowledge through inquiry and discovery. The other textbook in this analyzes display a more traditional view of learning where students adopt new knowledge through instruction and examples.

4.1.2 Differentiation

Differentiation in this subchapter is the pedagogical term for providing instruction that is appropriate for the individual student’s abilities.

The American textbooks and the Finnish *Min matematik* have no obvious hints of differentiation. However, the units in the American textbooks contain an average of 40 (roughly estimated) tasks where the first page are very basic and drill type of tasks while the second page often have more challenging tasks that are labeled similarly in each unit: Multi-Step, Hobbies, Science, Nutrition, What’s the error?, Write About It, Challenge. It is possible that the higher attaining students spend more time solving this type of problems while the lower attaining students have a thinner diet of more basic tasks. The same type of differentiation may apply to *Min matematik*, as the textbook provide an additional set of exercises in each unit which contain problem solving and

application tasks. This textbook also provides differentiation in the repetition tasks at the end of each chapter. A part of this repetition is a page with tasks that has the headline “Do you know this?”. After working with these tasks the students are asked to describe their proficiency with a letter; A (I need to practice more), B (I know this fairly well) and C (I know this very well). Then there is presented repetition tasks (5-7) labeled by the letters.


The Finnish textbook *Pi 7* and the Norwegian textbook *Factor* have differentiation in form of the tasks provided. The units in *Pi 7* have a large amount of tasks often covering 4-6 pages. The tasks are divided in three difficulty levels which are called: basic, deepening and application tasks. In the unit the deepening tasks are marked with a line and the application task are marked with a double line. The tasks analyzed are from all these difficulty levels. *Faktor1* have no differentiation of the tasks in the textbook with the exception of one or two tasks marked with a star. There is only an average of 5 tasks provided. However, the exercise book differentiates the tasks in three categories. The book is divided into chapters corresponding with the textbook, then into categories 1, 2 and 3 which are divided into units as the textbook. The tasks analyzed from *Faktor1* are also from the exercise book as I consider these tasks to belong to the units. The tasks gathered from this booklet are only from category 2.

The Swedish textbooks and the Norwegian textbook *Abakus 7* differentiate in form of units. The chapters in the Swedish textbooks are divided in a color scheme; green (ca. 10 p.), blue (ca. 5-6 p.) and red (ca. 5-6 p.). The green part of the chapter has units that all the students are to work with. After the green part there is a diagnostic test which designates the students to either the blue (mostly repletion) units or to the red (advancement) units. After codifying all the tasks in the yellow units I have chosen to codify tasks in the red units as these units progress in algebraic material. *Abakus 7* also have a similar color scheme; white (ca. 4 p.), red (ca. 8-10 p.), yellow (8-10 p.) and blue (ca. 6-8 p.). All the students are supposed to work with the white part of the chapter. The first two pages often have some exploratory tasks, other tasks and a kernel or two. The next two pages have the headline “Do you know this” and have the same function as the diagnostic test in the in the Swedish textbooks. If the student finds the tasks difficult he proceeds to the red units and if he does well he advances to the yellow units. There is also a “Test yourself” page at the end of the yellow and red unit. If it goes well the student can advance to the yellow or blue units respectively. If not, the student is told to talk to the teacher.

4.1.3 Review

In the American textbooks there is a small section of each unit that is called “Spiral Standards Review”. Here standards worked on in previous units and chapters are revisited. Example:

Figure 9. Numbers to algebra


SPIRAL STANDARDS REVIEW
← NS2.3, Prep for AF1.0, AF1.3, AF1.4

22. Multiple Choice Sunny Lawn Care charges \$25 per visit plus \$2 per cubic foot. Which equation models this situation?

(A) $y = x + 2$
 (B) $y = x + 25$
 (C) $y = 25x + 2$
 (D) $y = 2x + 25$

23. Multiple Choice Which is an equation that gives the values in the table?

x	3	4	5	6	7
y	8	11	14	17	20

(A) $y = 2x + 2$
 (C) $y = 2x + 6$
 (B) $y = 3x - 1$
 (D) $y = 3x + 1$

Simplify each expression. Justify each step. (Lesson 1-4)

24. $32 + 54 + 8$
 25. $5 \cdot (20 \cdot 32)$
 26. $4 \cdot 63 \cdot 25$
 27. $(108 + 51) + 9$

Simplify each expression. (Lessons 2-2 to 2-4)

28. $-30 \cdot (-4) + 8$
 29. $10 + 60 \div (-3)$
 30. $-48 \div 8 + (-2)$
 31. $15 + (-18) - 10$

The standards that are reviewed are written next to the headline. In addition at the end of each chapter there are two pages with tasks that has the headline “Mastering the Standards”. The pages are meant as a cumulative assessment of the chapter and also all the prior chapters. These “Spiral standard review” tasks are not a part of the task analysis even though they are present in each unit.

The Finnish textbook *Min matematik* does not include repetition of the prior units as the book progress but it has a review chapter at the end of the book. *Pi 7*, also Finnish, has repetition tasks in the middle of and at the end of each chapter. But the review is only from the prior units in that chapter.

The Swedish textbooks has intergraded a system of review in the homework tasks that are placed in the back of textbooks. The homework is sectioned and numbered and each section has tasks from several chapters. In addition the 6th grade textbook has a repetition chapter at the end of the book as well.

The Norwegian textbooks *Faktor 1* and *Abakus 7* does not provide any review of previous chapters. However, *Abakus 7* has a chapter at the end of the book called “Abamix” which provide tasks of a problem solving nature that are dependent of knowledge from many chapters.

4.1.4 Key-answers

The American textbooks provide key-answers for only the odd-numbered tasks at the end of the book. The reasoning behind this structure is perhaps that the students can’t easily cheat while

they are working with tasks, but can still have the opportunity to perform occasional checkups on the odd numbered tasks to see if they are solving the tasks in the right manner.


The 7th grade textbooks from Finland (*Pi 7*) and Sweden (*Matte direkt år 7*), and the Norwegian 8th grade textbook (*Faktor 1*) have key-answers to all the tasks in the back of the book. The textbooks for the grade below them in the respective countries (*Min matematik*, *Matte direkt 6B* and *Abakus 7*) do not. However, the Finnish textbook *Min matematik* does provide key-answers for some of the tasks and this is sometimes done in an ingenious and playful way. I will explain related to an example:

Figure 10. *Min matematik*

11. Lös ekvationerna enligt exemplet i undervisningsrutan. Skriv den bokstav som motsvarar ditt svar.

a. $5 + x = 9$	h. $x + 13 = 30$
b. $x + 7 = 10$	i. $x + 22 = 30$
c. $x + 7 = 15$	j. $20 + x = 36$
d. $x + 5 = 11$	k. $x + 12 = 44$
e. $9 + x = 13$	l. $39 + x = 45$
f. $21 + x = 24$	m. $46 + x = 51$
g. $29 + x = 34$	n. $x + 17 = 31$

3	4	5	6	8	14	16	17	32
E	N	Å	G	R	F	A	P	L



In writing the letters represented for the answers to all these equation you get a line of letters: NERGNÅPRALGÅF. Separated and read backwards you get the Swedish sentence: FÅGLAR PÅ EN GREN which means “birds on a branch”. This seems to be part of the theme of the book which is mathematics as engaging and teasers of the mind. This will be elaborated further in the next subchapter.

4.1.5 Brief comments

The Nordic textbooks analyzed are similar in size and range from about 260 – 380 pages with an average of 311 pages. This is in great contrast to the American textbooks which have about 600 pages and the pages are also of a bigger format of paper. The large amounts of pages in these textbooks that are related to the California standards and strategies for success have been mentioned earlier. Both the Norwegian and Swedish textbooks analyzed have small sections, in the first two pages of a new chapter, presenting goals for the chapter, which most likely have some relation to the national curricula. However, this cannot be compared with the detailed references and work with the standards in the American textbooks. The Finnish textbooks only present the mathematical content.

The biggest difference between size and structural aspects is found between the 6th grade textbooks of California (601 pages) and Finland (263 pages). It is also to be taken into account that the children in California are one year younger than their Finnish peers in 6th grade. Perhaps this reflects the politicization of the schools in America? It seems to me that the Californian textbook with all its formalities and relation to standards may have been written for the school boards who want to ensure funding for the schools, for the teachers to provide direction for the instruction and even for the involved and concerned parents, but I must allow myself to question if this book can have been written with eleven year old children in mind. In stark contrast is the playfulness of the Finnish textbook *Min matematik*. I refer to the section on key-answers and I will also display three tasks found in the algebra chapter which are representative of tasks that are scattered throughout the textbook (in the task analysis these tasks are codified as problem solving):

Figure 11. *Min Matematik*

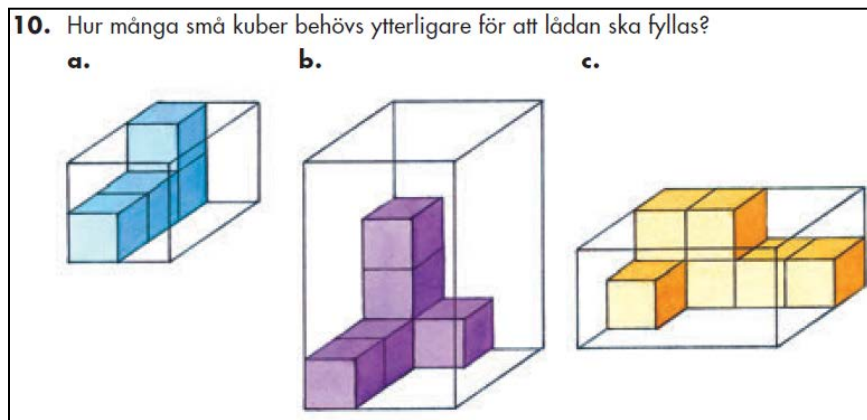


Figure 12. *Min matematik*

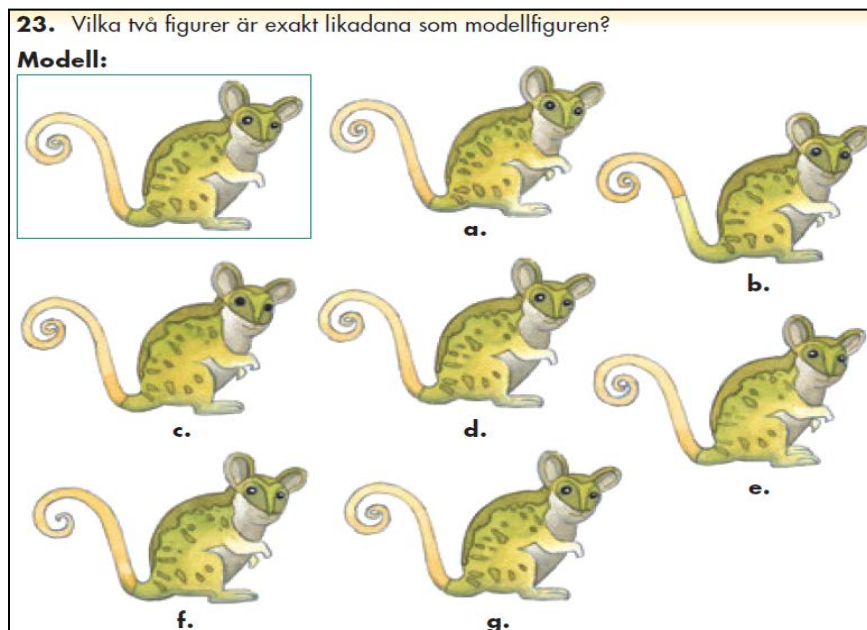
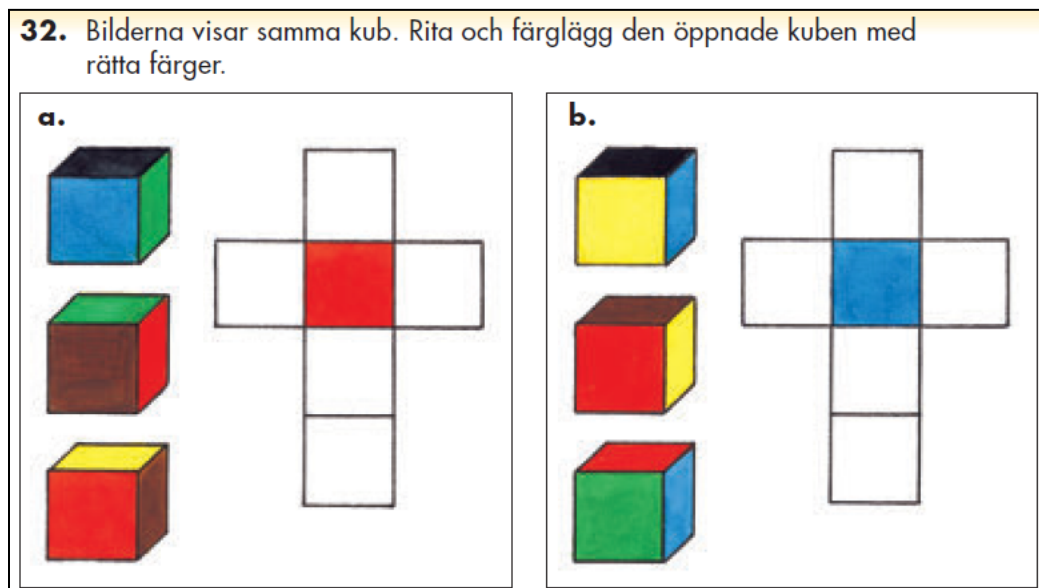


Figure 13. Min matematik



How the children engage with these very different textbooks becomes a burning question that is not within the scope of this study to answer. But perhaps it could be elaborated to some extent in the further work of the VIDEOMAT project.

The Nordic textbooks have more general titles while the American textbooks are labeled *Numbers to algebra* and *Pre-algebra*. Perhaps this reflects a different systemization and perspective on school mathematics. I have only recognized a few of the chapters in these books to be concerned with algebra as defined in this study.

4.2 Analysis of tasks

The analysis of tasks is based on the codification done in the computer research program Dedoose and the findings will be presented with charts created by this program. As an introduction to the “Analysis of tasks” I will present a comprehensive table of the code application on the ²seven documents uploaded in the program. As I proceed through the different aspects of the analysis of tasks I will break down the table into smaller parts, however, these smaller tables must be understood within the complete table. The Norwegian textbook *Abakus 7* has been analyzed manually and will be presented at the end of 4.2.1 as the tasks analyzed have much in common with the introductory tasks.

The analysis of tasks will be divided in two parts, which will each contain sections of comparisons between the different countries, in what is recognized as

² There are only seven because the Norwegian grade 6 textbook, *Abakus*, does not have an algebra chapter and cannot be codified on the same basis as the other textbooks. Tasks in *Abakus* is analyzed manually and without the use of codes, in respect to early algebra.

- 4.2.1 Introductory tasks: tasks that seem to have the intention of preparing the students for the introduction of algebra and/or are located at the beginning of the algebraic chapter.
- 4.2.2 Algebraic tasks: tasks that includes the use of letters.

The analysis of the algebraic tasks will manly focus on the perspectives of algebra, in addition the context of certain task will be commented on.

I have chosen to present a coding application table, created by Dedoose, as an introduction to the analysis of the tasks, because it provides an overview that has implications for the interpretation of the smaller segments that are used in the analyzes. The only codes that are not presented here are the codes I have labeled “Further characterization of tasks” as they do not influence how the rest of the table is interpreted. The colors used in the coding table are prescribed by Dedoose for the purpose of drawing attention to the frequency of applied codes.

The codes are applied so that the numbers of “child codes” that are used add up to the use of the “root code”. For example the root code “Algebra as generalization” has been applied to the textbook *Pi 7* 12 times, and we see that the child codes “Interpreting rule” and “Formulating rule” add up to 12. However, the child codes belonging to the root code “Introductory tasks” have not been applied in the same manner. If these elements are returned to later in the chapter I have registered that by using the same codes. Also some of the textbooks have small sections with tasks that have been recognized as introductory tasks in other textbooks and so I have seen a value in recording them by using the same codes.

An important element that I wish to draw attention to is the great variation of the number of tasks that have been recognized as introductory tasks. In the textbooks *Pi 7* and *Numbers to algebra 43* and 48 tasks have been labeled as introductory, while as a contrast only 3 tasks in *Pre-algebra* have gotten this label. This will influence the variation in the perspectives of algebra that is detected in the textbooks. I have limited this study to only include the 60 first tasks in the algebraic chapters and it is inherent that if 80% of these are introductory we will not get a comprehensive view of the algebra that is represented in the respective chapter. Also the great variation in size of these chapters means that for some textbooks 60 tasks cover a lot of the algebraic material while in other textbooks it is only a small sample. I have chosen to look at the first tasks as I find the introduction to a new topic in mathematics to be an important element in the work of instruction. Also the findings that emerge from these sections are informative of the perspectives on both mathematics and algebra that are presented in the textbooks. I will display and discuss in further detail the different segments of the table.

4.2.1 Introductory tasks

The introductory tasks in the algebra chapters analyzed often have a practical purpose. The most common of these are tasks working with “Sequence of operations” and “Equivalence” and these introductory tasks will be discussed under these labels. Other less common entry points will be discussed within the context of the textbook in which they are located. It is only the introductory tasks involving “Pattern” in the textbook *Pi 7* that seem to have a persistent objective in mind of providing a meaningful context for the learning of algebra. The textbook *Numbers to algebra* also provide tasks involving pattern but the link to algebra is harder to discern. The introductory tasks codified “Pattern” will be discussed under this label and within the context of the textbooks *Pi 7* and *Numbers to algebra*. Some of the textbooks have included several of these elements in the introductory section while others focus one and then provide other introductory elements as the chapter progress. All these tasks will be a part of the analysis of the introductory tasks.

Some of the categories of introductory tasks identified are used more frequently and the pedagogical reasoning for presenting the topic as an introduction to algebra is easily identifiable. These categories include “Sequences of operations”, “Equivalence” and “Pattern”. The tasks that belong to one of these will be analyzed by topic. Meanwhile, “Generalized arithmetic”, “Integers” and “Problem solving” are better understood within the context of the textbook in which they are found. The tasks from *Abakus 7* will also be analyzed as part of this chapter even if this book has not been included in the Dedoose coding system. In fact *Abakus 7* includes tasks which are very similar to the introductory tasks of the other textbooks. A comparison between the countries will be presented by the end of this chapter.

Table 5: Code application for introductory tasks.

	Introductory tasks	Sequence of operations	Integers	Equivalence	Generalized arithmetic	Problem solving	Pattern	Identifying and continuing a pattern	Interpreting rhetoric rule	Interpreting symbolic rule	Formulating rhetoric rule	Formulating symbolic rule
Pre-algebra (Ca)	3		17		4		1				1	
Pi7 (Fi)	43						43	19	9	2	5	8
Numb. to alg. (Ca)	48	17			15		16	11	1		4	
Min matem. (Fi)	10		10	3		6	2	2				
Matte Dir. år 7 (Sw)	6	6		5	1							
Matte Dir. 6B (Sw)	19	4		20								
Faktor 1 (No)	13	13										
Totals	142	40	27	28	20	6	62	32	10	2	10	8

The table above shows great differences in the number of introductory tasks in every textbook. Looking at the California textbooks we see that the algebra chapter in *Numbers to algebra* has the largest amount of introductory tasks while the algebra chapter in *Pre-algebra* has the least. Perhaps this reflects the understanding that algebra was introduced in 6th grade and is now developing and therefore there is no need for a second introduction to algebra. *Pi 7* have 43 tasks labeled as “Introductory”, which makes up for a whole unit called “Number sequenses”. In this unit we see how the work with pattern progress from simply recognizing patterns to be able to express and translate rhetoric rules and then symbolic rules. Therefore, some of the tasks in this section is also labeled as “Algebra as generalizing”. The textbook *Matte direkt 6B* has 19 introductory tasks which also make up a whole unit. In common with *Pi 7* this unit also works with algebra, only as a language and not as generalization. The textbooks with fewer introductory tasks like *Min matematik 6*, *Matte Direkt år 7* and *Faktor 1* have no involvement of algebra in these tasks with one exception: *Min matematik* has a task in the

introductory section that has been codified “Algebra as problem solving”. The introductory tasks in these textbooks also make up whole units, which are smaller in size. *Numbers to algebra*, however, have a large amount of introductory tasks but no involvement of algebra

Sequence of operations

The textbooks *Numbers to algebra*, *Matte direkt år 7*, *Matte direkt 6B* and *Faktor 1* all have introductory tasks labeled “Sequence of operation”. I interpret this introduction to be practical

and a preparation for solving equations. In these sections there is often a dual aim as some of the tasks require the students to build a numeric expression from a text. In *Faktor 1* the next unit is called “expressions with variables” which then appears like a second step in that the students are first asked to build expressions with numbers. Kieran (2004) recommends “a focus on both representing and solving a problem rather than on merely solving it” as being helpful in the transition to algebra. Here is an example of such a task:

Figure 14. *Faktor 1*

6.8 En gruppe på 27 elever skal på tur til Galdhøpiggen. De skal reise med buss og overnatte to døgn. Overnatting koster 80 kr per døgn per person og bussen koster 75 kr per person tur/retur. Sett opp og regn ut talluttrykket som viser hvor mye turen kommer på for hele gruppen.

The textbook *Matte direkt år 7* follows the same sequence. The other Swedish textbook *Matte direkt 6B* introduce the more advanced section (red) with “Sequence of operation” (these tasks are not labeled “Entry point”). There are not provided any task with text in this section and the next unit is called “More about equations”.

Equivalence

The first unit in the algebra chapter in the Swedish textbook *Matte direkt 6B* is named “Equivalence”. The kernel focuses on the equal sign as having the meaning of equivalence. The tasks in this section are all involving equations but they are called equivalences. There is a dual focus in this unit, both on the equal sign and on algebra as language (these tasks are also codified as “Algebra as a language”). The students are not asked to solve the equations but are asked what number the letter is representing. Here is an example:


Figure 15. Matte direkt 6B

Vilket tal ska stå i stället för bokstaven?

5 a) $3 + x = 8 + 2$ b) $8 + y = 12 + 6$

6 a) $7 + 11 = 8 + p$ b) $25 = 15 + q$

Ibland står det en bokstav i stället för en ruta.



The focus on the equal sign is emphasized by the placement of letters to the left in some equations and to the right in others and also that there are expressions to the right of the equal sign in the equations. The focus on the meaning of the equal sign is recognized by Kendal (2004) to be an important element in the transition to algebra as many students see the equal sign as operational. Also here there are several tasks where students are asked to translate between equations written in words and in symbols:

Figure 16. Matte direkt 6B

12 Välj den likhet som betyder: $x - 15 = 8$



a) Ett tal minus 8 är lika med 15. $x - 8 = 15$

b) Differensen mellan ett tal och 15 $8 - x = 15$
är lika med 8.

The textbook *Matte direkt år 7* focuses on equivalence before introducing equations (these are not labeled “Introductory tasks”), also represented by scales:

Figure 17. Matte direkt år 7

23 Hur många guldmynt innehåller påsen?

a)  b) 

För att få vara med och dela på guldpengarna måste du svara rätt på alla uppgifter.

Byt ut påsarna mot ett tal så att likheten stämmer.

24 a) $\text{👛} + 4 = 8$ b) $\text{👛} + 2 = 9$ c) $17 = 6 + \text{👛}$


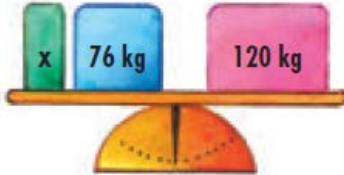

25 a) $\text{👛} - 10 = 35$ b) $12 = \text{👛} - 3$ c) $46 = 19 + \text{👛}$

26 a) $56 = 34 + 2 \text{👛}$ b) $14 + 3 \text{👛} = 41$ c) $5 \text{👛} - 25 = 75$

The Finnish textbook *Min matematik* has four tasks that are focused on equivalence. One task is located in the introductory section and is asking the students to insert correct signs in equations to make the statements true. The other three tasks are located in different units involving equation solving and focus “Equivalence” by the usage of scales:

Figure 18. Min matematik

14. Bilda en ekvation och beräkna värdet på x.

a.  b.  c. 

Pattern

From table 4.2.1 we see that the textbooks *Pi 7* (43 tasks) and *Numbers to algebra* (16 tasks) work extensively with patterns in the introductory section to the algebra chapter. While *Numbers to algebra* have three equal introductory sequences focusing first patterns, then sequences of

operation and at last generalized arithmetic *Pi 7* only focuses patterns as an introduction to algebra.

In a second moment it is interesting to see how these textbooks work with patterns and how algebra is introduced through these kinds of tasks. The simple numbers from the table tells us that both books present many tasks that only asks the students to identify and continue a pattern. The five and six first tasks in the two textbooks are of this nature. Here are examples from both books:

Figure 19. *Pi 7*

3. Vilka är de tre följande elementen i talföljden?

- a) 0, 3, 6, 9, ...
- b) 7, 5, 3, 1, ...
- c) -9, -7, -5, -3, ...

Figure 20. *Number to Algebra*

2 Identify a possible pattern. Use your pattern to draw the next three figures.

5. 

6. 

Pi 7 presents four tasks where the students have to interpret rhetoric rules next, and *Numbers to algebra* provide one. Here is one example of this kind of task:

Figure 21. *Pi 7*

6. Skriv ut de fyra första elementen i en talföljd där det första elementet är

- a) 3 och varje nytt element är 5 större än det föregående
- b) 42 och varje nytt element är 6 mindre än det föregående
- c) 5 och varje nytt element är tre gånger så stort som det föregående
- d) 48 och varje nytt element är hälften av det föregående.

Pi 7 also has more challenging tasks where the rules are related to groups of numbers (for example: even natural numbers). Both books keep following the same pattern in presenting these different types of tasks involving patterns. A few tasks were the students again are asked to

interpret and continue a pattern are presented and then some tasks asking the students to formulate a rhetoric rule. These tasks are different in formulation. Example:

Figure 22. Pi 7

13. En talföljd börjar 1, 3, 7, 15, ...

a) Skriv ut talföljdens femte, sjätte och sjunde element.

b) Skriv med ord regeln för hur talföljden bildas.

Figure 23. Numbers to algebra

? **27. What's the Error?** A student was asked to write the next three numbers in the pattern 96, 48, 24, 12, ... The student's response was 6, 2, 1. Describe and correct the student's error.

The American textbook does not involve the symbolic language in the work with patterns while *Pi 7* introduces letters as unknowns and then provide tasks where students are asked to interpret (2 tasks) and formulate symbolic rules (8 tasks):

Figure 24. Pi 7

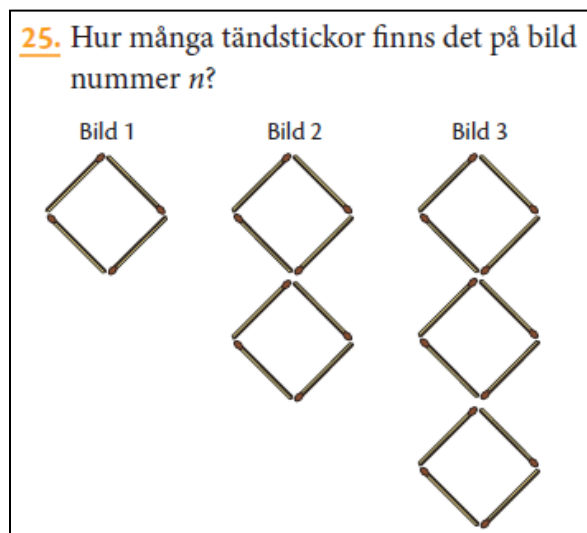
17. Vilka tal står x och y för?

1	2	6	30	x
2	3	5	11	y

Figure 25. Pi 7

21. Det n :te elementet i en talföljd får vi med hjälp av regeln $n + 3$. Skriv ut talföljdens fem första element.

Figure 26. Pi 7



I will make the argument that the possibility that children see the connection between patterns and algebra on the abstract notion of generalization is highly unlikely. The move from arithmetic to algebra is marked by the presence of letters in a subject that until this moment mainly have involved numbers. In most of the textbooks analyzed these letters simply appear without any explanation for their sudden presence. The children themselves are left to figure out the value of the symbolic language which may take years, or for some, will never happen. If we reflect back on history and how algebra was thought when it was a fairly new concept, it was thought in a manner that aimed at convincing the students of its usefulness.

Pi 7 provides an example for how algebra can be introduced in a meaningful way through the work with patterns. Expressing a pattern in the algebraic language is an efficient way of describing it and also for calculating the numbers that belong to the number sequence. The way in which the Finnish textbook progress through first expressing and formulating rhetoric rules also resonates with the evolvement of the algebraic language in history.

Lee (1996) found in his study that tasks involving patterns and generalization captivates many students that engage with them. These tasks go beyond drills to learn a certain algorithm as each pattern appears more as a puzzle to be solved. One of the great difficulties in this work that was discovered by Lee is that students can often see a pattern but it is not always one that is algebraically useful. And once students have discovered a pattern, they often have a hard time finding another.

Presentation of introductory tasks by textbook

Bellow the following topics “Generalized arithmetic”, “Integers” and “Problem solving” will be presented as they appeared in three of the textbooks.

Pre-algebra

Only three tasks are labeled “Introductory tasks” in the textbook “Pre-algebra”. The tasks are focusing on the mathematical language used in algebra and procedural knowledge:

Figure 27. Pre-algebra


Think and Discuss

- 1. Give an example** of a numerical expression and of an algebraic expression.
- 2. Tell** how to evaluate an algebraic expression for a given value of a variable.
- 3. Explain** why you cannot find a numerical value for the expression $4x - 5y$ for $x = 3$.

The first unit is labeled “Evaluating algebraic expressions” and I do not interpret these tasks to be introductory on the level with what has been recognized as introductory tasks in the other textbooks. However, units 3-6 in this chapter have a focus on integers. I chose not to codify unit 3 which deals with absolute value as there is no use of variables in that unit. I have codified unit 4 and 5 which have the respective headlines “Adding integers” and “Subtracting integers”. I evaluate these units to have a focus on calculating with negative numbers. In these units there are tasks involving variables and negative numbers. We can see a similarity here with the introduction to algebra in the Finnish textbook “Min matematik” which also focuses negative numbers.

Scattered in the chapter we find 4 tasks that have been interpreted as working with “Generalized arithmetic”, perhaps a more appropriate labeling would have been “Abstract arithmetic”. Here is one example:

Figure 28. Pre-algebra

 **39. Write About It** Paul used addition to solve a word problem about the weekly cost of commuting by toll road for \$1.50 each day. Fran solved the same problem by multiplying. They both got the correct answer. How is this possible?

This task focuses the relationship between multiplication and addition. The link to algebra in these tasks is an abstract perspective on arithmetic. There is also provided one task involving pattern:

Figure 29. Pre-algebra

★ 48. **Challenge** What is the sum of $3 + (-3) + 3 + (-3) + \dots$ when there are 10 terms? 19 terms? 24 terms? 25 terms? Explain any patterns that you find.

Numbers to algebra

In the textbook “Numbers to algebra” algebra is approached from several angles, “Pattern”, “Sequence of operations” and “Generalized arithmetic” which all represent whole units in the textbook. These segments appear to be more of a preparation for the learning of algebra than a way of introducing this new topic. There is also an additional unit, before letters are introduced as variables, which is about exponents. I have chosen to leave this unit out of the analyses. This textbook is the only one that works with the properties of numbers in a formal way. These tasks are labeled as “Generalized arithmetic”. Here is one example of these tasks:

Figure 30. Numbers to algebra

1 Tell which property is represented.

1. $1 + (6 + 7) = (1 + 6) + 7$	2. $1 \cdot 10 = 10$	3. $3 \cdot 5 = 5 \cdot 3$
4. $6 + 0 = 6$	5. $4 \cdot (4 \cdot 2) = (4 \cdot 4) \cdot 2$	6. $x + y = y + x$

We can see here that one task is presenting letters which can be interpreted either as variables or as placeholders. In this unit there are only two small tasks which involve letters. The properties are presented in kernels both with numeric examples and algebraically. I classify these tasks as arithmetic in nature. By stating a property the students are performing the act of applying a generalization but algebra as a language are not truly involved in these tasks (with the two exceptions). It is useful and necessary to know the properties of numbers to be able to conduct algebra as symbolic manipulation. It can also be said that there is a shift of focus from numeric calculations in the section on the “Sequence of operation” to a focus on relations in this unit, which is recognized by Kieran (2004) as a necessary adjustment for the transition to algebra.

Min matematik 6

The Finnish textbook “Min matematik 6” begins chapter 6 with a unit called “Integers”. The unit focuses calculation with whole numbers and also negative numbers. The number line is presented and numbers that are equally far from zero are called “opposite numbers”. It can appear as an introduction to negative numbers. This section also has tasks with text were the

students are asked to formulate a numeric expression. I cannot find any task in the rest of the chapter that includes negative numbers so in this respect the unit may seem a little oddly placed. However, there is clearly a focus on the relationship between numbers in working with the term “opposites”. The introductory unit also includes two tasks that have been labeled as “Problem solving” (defined in section 3.2.1). Here is one of them:

Figure 31. *Min matematik 6*

8. Lista ut vem som äger snäckorna.

a. b. c. d. e.

- Talet vid Saras snäcka är det motsatta talet till talet vid Sheilas snäcka.
- Jerrys tal får du då du subtraherar talet 7 från Paulas tal.
- Paulas tal får du då du adderar talet 3 till Saras tal.
- Johns tal är varken ett positivt eller ett negativt heltal.

This task cannot be easily solved by a known algorithm. If we look at the problem there does not exist an easy connection between two objects, the solver has to operate with the relationships between several objects at one time. This task is also codified as “Algebra as problem solving”. A task like this can provide a meaningful introduction to the use of letters as unknowns as the solution of the task is easier to represent, find and explain with the use of the algebraic language. If we look at table 4.2 we see that the code “Algebra as problem solving” has been applied 7 times so we can assume a purposeful meaning for the placement of this task. Two tasks have been labeled with the codes “Pattern” and “Identifying and continuing a pattern”, which are located in different units (not labeled “Introductory tasks”).

If we also take into account the four tasks focusing equivalence in this textbook, mentioned earlier under the headline equivalence, it can be said that this textbook has a varied repertoire of introductory tasks which are introduced as a mix in comparison with the textbook “Numbers to algebra” where these tasks are presented in separate units. The great effort that has been done to make “Min matematik” an engaging book for children as commented on earlier is also shown in the sequentially of tasks. There are rarely to similar tasks presented right next to each other.

Abakus 7

Abakus 7A and 7B do not have a chapter named algebra. I have searched these books thoroughly to find algebraic elements. I have only registered some use of the algebraic language in the geometry chapters. One of the goals for chapter 4 in *Abakus 7A* states that the aim is for students to learn how to “use formulas in the calculation of areas”. In addition, tasks that focus generalization are also indicated.

Algebraic content

In the yellow units of the geometric chapter found in *Abakus 7A* section we find the formulas for the area of a rectangle and the area of a triangle:

Figure 32. *Abakus 7A*

Areal = grunnlinjen · høyden
$A = g \cdot h$ ← Formel

Figure 33. *Abakus 7A*

Areal = $\frac{\text{grunnlinjen} \cdot \text{høyden}}{2}$
$A = \frac{g \cdot h}{2}$ ← Formel

At the same pages, task 63 and 67 ask the students to explain how the formulas are used to calculate the area of a rectangle (63) and a triangle (67). More tasks follow where the students are given information of lengths and perimeters and must use the formula to calculate the area. Even though the textbook does not mention that these letters indeed are variables, the students gain practice in using them and experiences with letters representing values that vary. However the letters may also only be interpreted by students as placeholders. Example:

Figure 34. *Abakus 7A*

64	Regn ut arealet til
a	et rektangel med sider 8 cm og 5 cm
b	et kvadrat med sider 7,5 cm

In *Abakus 7B*, yellow units, we find the formulas for the circumference and the area of a circle:

Figure 35. *Abakus 7B*

Slik kan vi regne ut omkretsen til en sirkel med diameter 5 cm:

Omkretsen = $\pi \cdot 5 \text{ cm} = 3,14 \cdot 5 \text{ cm} =$

65 Regn ut omkretsen til sirkelen. $O = \pi \cdot d$

Figure 36. *Abakus 7B*

Slik kan vi regne ut arealet til en sirkel med radius 5 cm:

Arealet = $\pi \cdot 5 \text{ cm} \cdot 5 \text{ cm} = 3,14 \cdot 25 \text{ cm}^2 =$

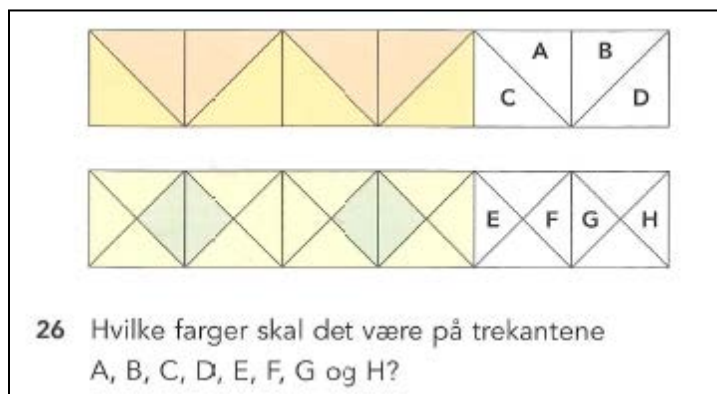
$A = \pi \cdot r \cdot r$, som vi skriver
 $A = \pi \cdot r^2$

Also here the students work with formulas and letters that represent values that vary.

Early algebra content

We also find examples of tasks that have been identified as “Introductory tasks” in the other textbooks. The tasks that have some element of generalization have been registered here. The first tasks works with generalization in the form of identifying and continuing patterns in figures. Here is one example from the geometry chapter:

Figure 37. Abakus 7A




One of the yellow units in this chapter has the headline “Pattern”:

Figure 38. Abakus 7A

Mønster

95 Hvor mange blå fliser blir det i det samme mønsteret hvis det er

a fire grå fliser c seks grå fliser
b fem grå fliser d sju grå fliser




96 Brukte du samme metode i alle oppgavene i 95? Hvordan tenkte du?

97 Hvor mange blå fliser trenger vi til

a 10 grå fliser c 60 grå fliser
b 20 grå fliser d 100 grå fliser

98 Hvor mange blå fliser trenger vi med dette mønsteret til

a to grå d 10 grå
b fire grå e 50 grå
c seks grå f 100 grå



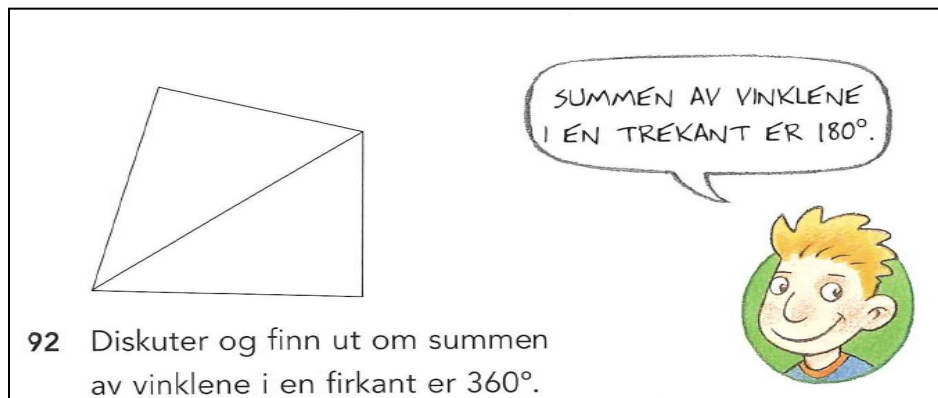
99 Tegn et mønster som gjentar seg, og lag oppgaver til det. Bytt oppgaver.

Task 95 can be solved by identifying the pattern and continuing the drawing. But task 97 and 98 must be solved by a transfer of representation from drawings to numbers and operations. The

grey and blue tiles are variable amounts that are in a relation with each other where the pattern describes this relationship. Chapter 6 (Abamix) in *Abakus 7A* also have tasks that involves patterns.

We also find more examples of tasks that involve the students in a generalization process:

Figure 39. *Abakus 7A*



The students are asked to find out and discuss whether the sum of the angles in a rectangle is 360.

In *Abakus 7B* the geometry chapter is introduced with an experiment aimed at moving from specific examples to a general rule that will be useful for finding the circumference of a circle:

Figure 40. *Abakus 7B*

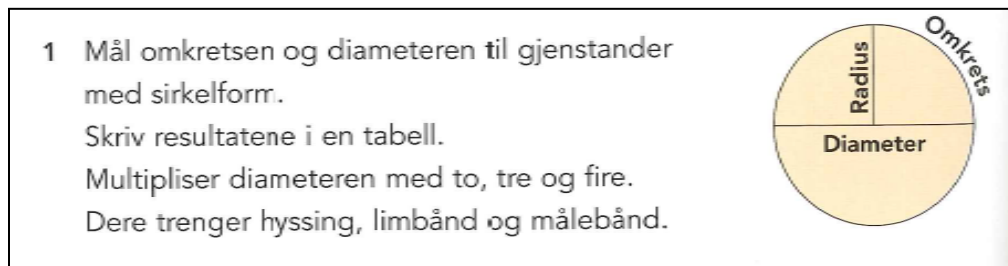
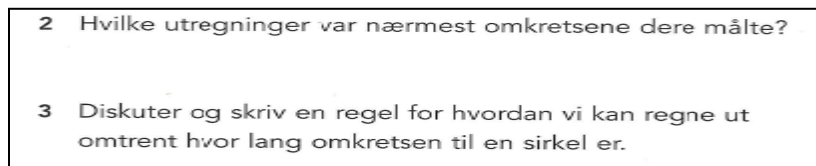


Figure 41. *Abakus 7B*



I believe it is fair to say that *Abakus 7* does not introduce algebra as a domain of knowledge. Some of the Norwegian 7th grade textbooks do, but the chapters and the experiences provided are

small in comparison with the other countries represented in this study. The analysis of *Abakus 7* is still valuable as it provides information of the experiences with early algebra and letters as variables that many Norwegian students have when they enter junior high school where algebra is introduced formally. The students that have engaged with *Abakus 7* may be familiar with the act of generalization both in the context of patterns and in geometry. The move to express these generalities with the algebraic language is perhaps a great opportunity to introduce algebra.

A comparison between the four countries related to the introductory tasks

The Swedish textbooks focus the introductory tasks on equivalence and sequence of operation. The 6th grade textbook begins the algebraic chapter with equations named equivalence and with a focus on the equal sign as having the meaning of equivalence. The more advanced section (color coded red) opens with tasks involving the sequence of operations. In the seventh grade textbook these tasks introduce the algebraic chapter and the unit that follows is named “Expressions with one variable”. Equivalence is focused later in the chapter and then in relation to equations. The two Swedish textbooks have an opposite sequentiality in the introduction of a variable and an unknown.

The Norwegian 8th grade textbook, *Faktor*, follows the same introduction as the Swedish grade 7 textbook but has no focus on equivalence as the tasks shift from involving expressions and variables to equations. The 7th grade textbook *Abakus 7* does not introduce algebra but provides tasks that are used as introductory tasks to an algebraic chapter in other textbooks and includes elements of early algebra.

The introductory tasks in the Norwegian and Swedish textbooks analyzed have a practical purpose. The tasks involving sequences of operations are simply drill in calculation skills which are useful in solving equations. The work with equivalence can be seen as an attempt to deal with the misconception of interpreting the equal sign as an operational sign. The correct understanding of the equal sign is essential to the work with equations.

The Californian textbook *Numbers to algebra* also has a unit on sequences of operation. In addition it also works with patterns. This book is the only one that works extensively with the properties of numbers. This is an interesting topic for introducing algebra as one looks at arithmetic, which has been the students’ main experience with mathematics until now, from an abstract viewpoint. The students can partake in generalization as these properties are expressed by letters. However, as with the unit on pattern, this textbook does not seem to make the connections between generalities and the algebraic language explicit. A meaningful introduction to algebra therefore eludes this textbook. The 7th grade textbook has been interpreted to not introduce algebra but only elaborate on this topic as it already has been introduced in the prior grade.

The Finnish textbook *Pi 7*’s introduction to algebra through the work with patterns has been elaborated on and is found to be a meaningful way to introduce algebra. The 6th grade textbook introduces the chapter by a unit on integers which includes a focus on relationship between numbers by the use of “opposites”. The introductory section also includes a problem solving task which could provide a meaningful way of introducing letters as unknowns. If we look at table 4.2 we see that this book is the only one that has a noticeable element of algebra as problem

solving (7 tasks). However, a consequent and purposeful intention for a meaningful introduction of algebra as in *Pi 7* cannot be claimed.

With algebra representing such a shift within school mathematics for many students, I cannot find the evidence that a great change in modes of working with mathematics takes place in the textbooks analyzed. Algebra comes and goes like any other topic in mathematics. From the title of the textbook *Numbers to algebra* it sounds like a revolution is about to take place. If the textbook really make this clear to the students throughout the book is a question that would require a full analysis of the textbook to answer. It appears like much work can be done in finding and exploring how to introduce algebra purposefully and with meaning. Perhaps the best opportunities offer themselves within the contexts of geometry, pattern and problem solving.

4.2.2 Algebraic tasks

This analysis has been limited by the unavoidable choice of only codifying the 60 first tasks that appear in the algebra chapter. A lot of work has been done developing these codes and I believe that the codification of all the algebraic tasks in the textbooks would have produced interesting findings. This work must be left for another study. This chapter will also have an focus on displaying some of the tools for analyses available in Dedoose.

The analyses of the algebraic tasks that I have been able to codify will focus in the most frequent aspect of algebra dealt with in these tasks: “Algebra as a language”. The other aspects of algebra will only be commented on in connection with algebra as a language

Table 6: Code application for algebraic tasks.

	Algebra as a language	Interpreting expression	Interpreting equation	Interpreting formula	Interpreting inequality	Formulating expression	Formulating equation	Algebra as generalization	Interpreting rule	Interpreting definition	Formulating rule	Algebra as problem solving	Algebra as symbolic manipulation	Simplifying	Transforming	Algebra as equation solving	Numeric trial	Doing/undoing
Pre-algebra (Ca)	32	26				6		1		1		1	2	1	1			
Pi7 (Fi)	11	6				5		12	4		8		6	6				
Numb. to alg. (Ca)	10	10											1		1	1	1	
Min matem. (Fi)	15				4		11					7				30	4	26
Matte Dir. år 7 (Sw)	39	16	3			18	2					2	6	6		6	6	
Matte Dir. 6B (Sw)	47	18	19			9	1					1				8	8	
Faktor 1 (No)	24	6		3		15						1	11	11		14	4	10
Totals	178	82	22	3	4	53	14	13	4	1	8	12	26	24	2	59	23	36

Looking at all the textbooks combined, most tasks have been codified as aiming at students developing “Algebra as a language”. Of the tasks that has been identified as algebraic 62% falls in this category. A complete analysis of all the algebraic tasks may reveal something different. From the American textbooks and also the Finnish textbook *Pi 7* only a small sample of the algebraic tasks has been codified while in the other textbooks either more or about half of these tasks have been codified. For the latter textbooks this table is somewhat more representable. 82 + 53 of the tasks codified “Algebra as a language” work with either interpreting or formulating expressions. “Algebra as generalization” is only introduced in *Pi 7* with one exception. Only *Min matematik* has enough tasks codified “Algebra as problem solving” to make this aspect of algebra visible. Several of the books have a few tasks that focus “Symbolic manipulation”. *Faktor 1* and *Min matematik* has a considerable amount of tasks devoted to “Algebra as equation solving”

Algebra as a language

It becomes clear from table 4.2.2 that introductory algebra in the textbooks analyzed often means a focus on algebra as a language. The main different modes in working with the algebraic language that have been registered in the textbooks can be described as

- Mode 1: Translating (formulating and interpreting) between rhetoric and symbolic expressions and equations
- Mode 2: Translating (formulating and interpreting) between text or geometric shapes and symbolic expressions and equations
- Mode 3: Evaluating expressions, formulas and equations for given values of variables

Mode 1: Translating (formulating and interpreting) between rhetoric and symbolic expressions and equations

All the textbooks analyzed have tasks in the first category. Here are three examples:

Figure 42. *Min matematik*

29. Skriv ekvationen och beräkna värdet på x . Kontrollera ditt svar.

a. Talet x multiplicerat med talet 4 är lika med 32.	b. Talet x multiplicerat med talet 6 är lika med 42.
c. Talet 5 multiplicerat med talet x är lika med 15.	d. Talet 13 multiplicerat med talet x är lika med 169.
e. Talet x multiplicerat med talet 14 är lika med 420.	f. Talet 15 multiplicerat med talet x är lika med 240.

3 7 8 13 16 17 30

Figure 43. *Matte direkt 6B*

31 a) 5 mer än a $x - 5$ $x + 5$ $a + 5$ $a - 5$
b) 5 mer än x

Figure 44. *Pre-algebra*

Write an algebraic expression for each word phrase.

11. 1 more than the quotient of 5 and n	12. 2 minus the product of 3 and p
13. 45 less than the product of 78 and j	14. 4 plus the quotient of r and 5
15. 14 more than the product of 59 and q	

The pedagogical aim with this kind of tasks is interpreted to be a drill in translation skills. Perhaps it is meant as a practice for and as a help to be able to solve word problems. These tasks brings a reminder to the historic development from a rhetoric- to a symbolic algebra, however the meaningful context in which it took place is completely void in these tasks. All that is left is two different types of representation. Reflecting back on the Finnish textbook *Pi 7's* work with patterns, where we saw a progress from rhetoric- to symbolic rules, these representations were embedded with meaning. And therefore the possibility for the students to develop a deep understanding of a variable through this progression was present.

Mode 2: Translating (formulating and interpreting) between text or geometric shapes and symbolic expressions and equations

The tasks in this category either have a geometric or a “real life” context. The context of the tasks has been part of the codification done in the textbooks. Dedoose has an analyzing tool which displays a code co-occurrence table:

Table 7: Code co-occurrence of context and “algebra as a language”.

	Geometry	Real life
Algebra as a language	35	55
Interpreting expression	10	27
Interpreting equation		3
Interpreting formula	3	
Interpreting inequality		1
Formulating expression	22	21
Formulating equation		3

Combining the information from tables 6 and 7, we see that out of 178 tasks that focus algebra as a language only 35 + 55 tasks are situated in a context beyond algebra as a topic. The tasks that provide a wider context work with the algebraic language in a different way than the tasks in the first categories as the variables involved represents more than an unknown or varying number. Here are some examples:

Figure 45: Matte direkt år 7

Talgåtor och ekvationer

Vilken ekvation i rutan hör ihop med talgåtan?

34 Om man lägger 8 år till min ålder får man 29 år.
Hur gammal är jag?

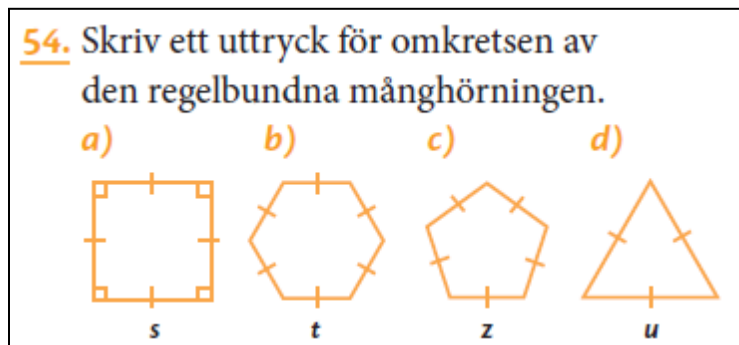
$x + 8 = 29$

$x - 8 = 29$

$x = 29 + 8$

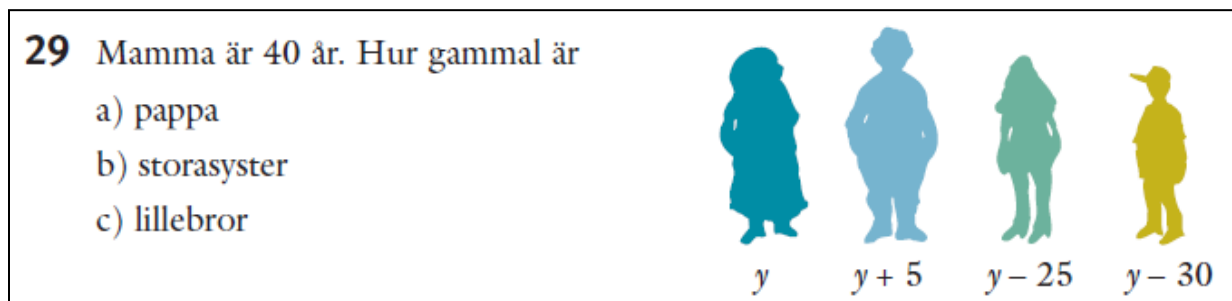
In this task the student has to do the translation between a rhetoric- and a symbolic equation like in the tasks in category 1. But in addition there is a meaning beyond numbers attached to the variable x and the student himself must make this connection to be able to solve the task.

Figure 46. Pi 7



An expression without a context does not provide any information. In this task the expressions formulated will be representative of a perimeter. In addition, the symbolic expression for the perimeter of the square does not only represent the specific square on the picture but all squares, the variable represent a side in a certain type of figure and the expression is a generality. Algebra can be experienced as useful and meaningful through working with this task. However, it is also possible, perhaps even likely, that the student without any direction will interpret the expression as specific and not general.

Figure 47. Matte direkt 6B



There are many tasks in the textbooks that have to do with the age of people. This is a familiar context for children, as age is very important to a child. My children often think of their age in relation to each other; “how old is my brother, when I’m 10” and so on. For this reason I believe these tasks can be meaningful to the students as the expressions represent ways of thinking that they are familiar with.

Mode 3: Evaluating expressions, formulas and equations for given values of variables

Some of these tasks are situated in a context beyond the algebraic topic and some are not. Here are two examples of these tasks:

Figure 48. Numbers to algebra

3 Evaluate each expression for the given values of the variables.

7. $\frac{8}{n} + 3m$ for $n = 2$ and $m = 5$ 8. $5a - 3b + 5$ for $a = 4$ and $b = 3$

I have interpreted this kind of tasks as focusing algebra as a language because they focus on the functionality of variables as replacement for numbers. In the first example this technicality is all the student need to be concerned with.

Figure 49. Numbers to algebra

1 The expression $12d$ represents the number of eggs in d dozen. Evaluate the expression for each value of d , and tell what the value of the expression means.

1. $d = 3$ 2. $d = 2$ 3. $d = 11$

Tasks like this example bring the experiences that a variable can represent varying numbers. In these tasks the variable and the expression also entail a concrete meaning. One need not only know the technicalities but also understand what the expression is representative of. This is often difficult to obtain and if we look closer at this task it becomes evident how confusing the use of letters can become. I found this task hard to read even if the meaning of the expression is spelled out in the first sentence. The letter d represents the number of dozens of eggs. This variable has been given the letter d which also could be seen as a shortage for *dozen*. If the solver gains this misunderstanding then it would be natural to replace the letter d with 12, and the task would be confusing.

Figure 50. Faktor 1

6.19 Formelen for omkretsen av et rektangel er $O = 2a + 2b$, der O står for omkretsen, a for lengden og b for bredden av rektangelet.

Regn ut omkretsen av rektangelet når

a) $a = 8$ cm og $b = 6$ cm

b) $a = 12$ cm og $b = 7,5$ cm

This task has a geometric context. The formula for the perimeter of a rectangle is general and the aim seems to be the experience of variables as numbers that vary. But the variables may as well be seen as placeholders as Janvier (1996) suggests.

Figure 51. Matte direkt 6B

9 I vilken likhet är $x = 10$?

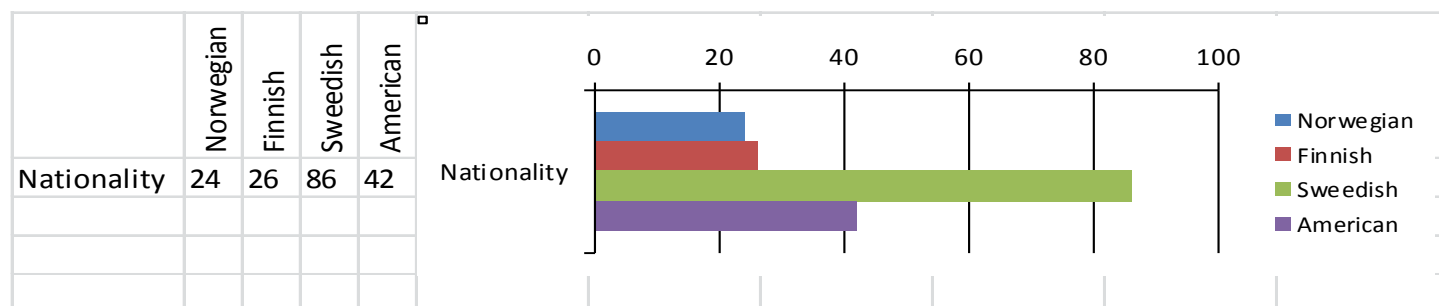
$x + 15 = 18 + 7$ $x + 5 = 20$ $10 = x + 20$

The last example of this mode of working with the algebraic language entails equations. Most tasks involving equations have been codified as “Equation solving”, but this task does not provide practice in solving equations. It belongs in this mode because the student is asked to replace a letter with a number. This task is also found in the introductory section and is codified as focusing “Equivalence” as well.

A comparison between the countries

There is a difference in amount of tasks focusing Algebra as a language. Dedoose creates a chart by the descriptor “Nationality” and the frequency of the code “Algebra as a language”:

Table 8: Frequency of the emergence of “Algebra as a language” by country



This chart displays that Sweden is more focused on Algebra as a language than any of the other countries. However, since only the 60 first tasks in the algebraic chapters are analyzed and there is great variation in the amount of introductory tasks in each textbook, this chart does not display a certain fact, and perhaps neither a tendency. In addition it must be mentioned that the Norwegian number registered only accounts for one textbook. I have chosen to display it because it shows one of the analyzing possibilities available in Dedoose. This chart can also be normalized and/or displayed in percentages. I believe this chart would have been more useful if all the tasks in the algebraic chapters had been analyzed. The chart does tell us that the attention given to “Algebra as a language” in the 60 first tasks of the algebraic chapter of the Swedish textbooks is quite undivided.

There are also qualitative differences. I will only mention the one that is most particular as the 3 categories above are common for all countries. The Finnish textbook *Pi 7* focuses the symbolic

language more formally than the other textbooks as it introduces the terms monomial and coefficients. Here is an example:

Figure 52. Pi 7

72. Rita av tabellen i ditt häfte och fyll i den.

Monom	Koefficient	Bokstavsdel
$5x$		
$-3a$		
$-y$		
0		
m		

Brief comments to this sub-chapter

The task from *Numbers to algebra*, involving **dozens** and the variable **d** discussed above, represent problematic elements in working with the algebraic language without combining it with algebra as problem solving or generalizing. Many of the tasks focusing algebra as a language appear as only portions of a context that can be difficult to make sense of, as the expressions are already made and the students are left to backtrack somebody else's mind. Additionally, these types of tasks lack motivation as they do not intrigue the mind as for example work with patterns or problem solving do. In allowing the students to formulate their own expressions and equations in a problematic context, these are more likely to be meaningful to them. Perhaps they would learn more from struggling through one task like that than solving many of these simpler tasks.

Table 6 shows that the textbooks focus "Algebra as generalization" and "Algebra as problem solving" the least. In fact the numbers that are there are mostly made up from The Finnish textbooks. Algebra as a language developed out of necessity and convenience as mathematicians through history worked with mathematical problems and especially equation solving. The symbolic language we have today developed over hundreds of years. In some ways introducing algebra as a fully developed language without connecting it to the contexts in which it originated may have implications for the depth of understanding of variables the students develop. I will conclude this paragraph with a quotation from Hiebert (1997) as he reflects on the nature of tasks that may build mathematical understanding (p. 18): "The task must allow the student to treat the situations as problematic, as something they need to think about rather than as a prescription they need to follow".

5. Conclusion

This study set out to investigate textbooks from four different countries, Finland, Norway, Sweden and USA, concerning the introduction of algebra. To be able to capture the algebraic content of these books I chose to focus on the tasks as these often are given most attention from students. My intention as I started out planning this study was to include all the tasks found in the algebraic chapters in the textbooks. It proved to be too ambitious for a 30 point master thesis and it was limited to involve only the 60 first tasks. I could not find any literature that offered a developed codification system for algebraic tasks that fit with the tasks I found in the textbooks I was analyzing. And therefore these codes had to be developed. Originally I started looking for a research program that would help me develop categories of tasks by codifying them descriptively (the descriptive codes are displayed in a table in Appendix 1). However, the final coding system developed both from the descriptive work with the tasks and from reviewing literature especially on the origins of algebra. The literature on algebra today is so extensive and diverse that I felt at loss for what algebra really was. The journey back in the history of algebra felt like finding gold and it has helped me make sense of the more modern views of algebra.

The textbooks from the four countries (two books of two consecutive grades in each country) have main differences in size, structure and content but there are also similarities. In addition there are also great differences between the two textbooks of one country which is the case in Finland and Norway. For the Swedish and American textbooks analyzed, the books for the consecutive grades are written by some of the same authors. I will first sum up the structural differences of the textbooks before I focus on answering the research questions concerning the algebraic content. Finally I will discuss Dedoose as an analyzing tool in textbook research.

Concerning the structure the most visible difference between the Nordic textbooks and the American (Californian) textbooks is the size. The American textbooks have an average of twice as many pages as the Nordic textbooks. Many pages are devoted to detailed work with the California standards and strategies for success which does not have their counterpart in the Nordic textbooks. The Swedish and Norwegian textbooks display short goals for the chapters, normally 3 or 4 short sentences, which most likely have some connection to the national curricula. The Finnish textbooks only contain mathematical content. The sheer size of the American textbooks and all the formalities that hints to the politicization of the American schools makes one question if the educational system has lost touch with their children.

Of the eight textbooks analyzed two books stand out in originality, the Norwegian grade 7 textbook *Abakus 7* and the Finnish grade 6 textbook *Min matematik*. *Abakus 7* have chapters and units like all the other textbooks but have very few kernels and examples. The introduction to a chapter is often done in form of inquiry tasks while all the other textbooks opens with kernels and then give examples before the tasks are presented. *Abakus 7* has been interpreted to display a constructivist view of learning. The Norwegian grad 8 textbook, *Faktor*, is in comparison a model of the traditional view of learning by instruction. *Min matematik* has the same structure as the majority of the books, but provides many tasks throughout the textbook that have the nature of a puzzle or problem solving. The key-answers are also given in form of a puzzle. The playful and intriguing effect of these elements in the textbook makes it appealing both to children and adults. In addition, *Min matematik* makes a unique effort to avoid boredom as seldom two tasks of the same nature are placed next to each other. The Finnish grade 7 textbook, *Pi 7*, is extremely

serious in comparison. Perhaps this reflects the change from elementary school to junior high school in Finland.

The algebra chapters in the textbooks analyzed vary in size and content but there are many similarities as well. The American 6th grade textbook has two chapters focusing algebra. The Finnish and Swedish textbooks of this grade have one chapter and the Norwegian textbook³ (grade 7) none. *Abakus 7* does have very little algebraic content but provides some tasks that have been identified as early algebra. The American 7th grade textbook has three algebraic chapters while the textbooks in the Nordic countries of the same grade level have only one. However, in the Finnish textbook the algebra chapter make up for one third of the entire textbook, while the algebra chapter in the Norwegian (grade 8) and Swedish (grade 7) textbooks only is one out of seven equal chapters.

Introduction of algebra in the textbooks

Dealing with the introductory tasks six categories are identified: “Sequence of operations”, “Integers”, “Equivalence”, “Generalized arithmetic”, “Problem solving” and “Pattern”. Most of these categories are interpreted to have a technical and rules learning purpose with the exception of “Problem solving” and “Pattern” which provide opportunities for a meaningful introduction of algebra. Some of the textbooks have several of these introductory elements placed in the introductory section, while others provide a few of these elements as the chapter progress. The algebra chapter in the American grade 7 textbook *Pre-algebra* has been interpreted to not have an introductory section.

The introductory tasks focusing on “Sequence of operations” have been interpreted to be a preparation for equation solving. In addition this section often includes tasks where the students are asked to formulate numeric expressions from text. This is done with a clear intention of preparing them for working with algebraic expressions. A focus on algebra as a language can in these cases be detected already in the introductory section. This is the case with the American textbook *Numbers to algebra*, the Swedish textbook *Matte direkt år 7* and the Norwegian textbook *Faktor 1*.

The focus on “Equivalence” is seen as an attempt to deal with the common misconception of the interpretation of the equal sign as an operational sign. A correct understanding of the equal sign is essential for solving equations. Two of the textbooks, *Matte direkt år 7* (Swedish) and *Min matematik* (Finnish) focus the equal sign by the use of old fashioned scales. While *Matte direkt 6B* works with the conception of the equal sign by placing expressions and variables on the right side as well as on the left side of equations.

Tasks involving “Pattern” are used purposefully as a meaningful introduction to algebra in the Finnish textbook *Pi 7*. Here we find a natural progression from identifying and continuing patterns to interpret and formulate rhetoric rules and then symbolic rules expressing patterns. This is the only textbook analyzed which introduce the symbolic language embedded in meaning. It also provides a motivation for the use of the algebraic language as it is efficient in expressing a pattern and also for calculating the numbers belonging to a pattern. The students are

³ The Norwegian textbooks are from one grade above the other countries, however the students are of the same age as the students in one grade below in the other Nordic countries.

engaging in generalization which is the central quality of the algebraic language. “Pattern” is also focused in the American textbook “Numbers to algebra” but no connection is made to the symbolic language.

The textbook “Numbers to algebra” is the only textbook found to focus on “Generalized arithmetic” in the introductory section. The textbook has a unit on the properties of numbers. The properties are introduced in a kernel with numeric examples and are generalized with the use of algebra. Most of the tasks, with the exception of two minor tasks, do not involve letters. These tasks are therefore interpreted as mainly serving the practical purpose of preparation for algebra as symbolic manipulation.

The Finnish 6th grade textbook, *Min matematik*, is unique in presenting two problem solving tasks in the introductory section. One of these tasks focuses on the relationship between several quantities where a connection between two of them cannot be easily made. Therefore has this task been labeled as algebraic in form and provides an opportunity for a meaningful introduction of variables as unknowns. However, the introductory section of this book is mainly devoted to “Integers” and is interpreted to be an introduction to negative numbers.

To conclude, only the Finnish textbook *Pi 7*, provide introductory tasks where the students may discover the effectiveness of algebraic representations. This kind of tasks, related to pattern, can also be considered as a motivating component to engage students in algebra.

Perspectives of algebra approached in the tasks

Tasks have been interpreted to be algebraic if they involve the use of letters as variables. They have been categorized as five main perspectives of algebra was perceived: “Algebra as a language”, “Algebra as generalization”, “Algebra as problem solving”, “Algebra as symbolic manipulation”, “Algebra as equation solving”.

The analysis has shown that “Algebra as language” is the dominant perspective in the referred textbooks⁴. These are tasks that foremost appear to set out to give a practice in either interpreting or formulating expressions and equations using the symbolic language. However, tasks that seem to focus on the functionality (how to replace a variable by a number) and/or the representational meaning of a variable -in either an expression or equation- have also been included in this category. Many of these tasks are very similar in all of the textbooks and therefore they have been distributed in three modes: translating (formulating and interpreting) between rhetoric and symbolic expressions and equations; translating (formulating and interpreting) between text or geometric shapes and symbolic expressions and equations; evaluating expressions and equations for given values of variables.

About half of the tasks recognized as “Algebra as a language” have been situated in a context beyond the topic of algebra (see table 7). The tasks that only involve numbers, letters and words describing them and their relationship are often limited to the opportunities for technical learning. In connecting expressions and equations to geometric figures or “real life” objects tasks

⁴ The findings described above are only representative for the 60 first tasks in the algebraic chapters and do not provide a comprehensive view of the perspectives of algebra present in the textbooks.

provide a concrete relation to the variable and therefore give an opportunity to develop a certain level of understanding. However, the concrete element is abandoned as progress in algebra is made according to Rojano (1996, see p. 6 in the present study). The variable in the context of generalization represents a more challenging concept for students to grasp, and from table 6 we see that only the Finnish textbook *Pi 7* offer an good opportunity to develop this understanding. The tasks belonging to the categories “Algebra as generalization” and “Algebra as problem solving” are the only ones recognized as having a motivational aspect for the leaning of Algebra. These tasks are almost exclusively found in the Finnish textbooks.

Dedoose as tool for textbook research

The experience in using Dedoose as an analytical tool has been very positive. It is an intuitive research program that does not require training. It saves a lot of time in the process of developing codes as the coding system is dynamic. It allows to rename the codes and also to merge the codes in higher hierarchal levels. The program can locate and present in one document all the tasks where one specific code has been applied. The program quantify how many times the individual codes have been used and provides many tools for analyzing. These can also be used effectively during the coding process. In my coding scheme the child codes often were designed to add up to the number of root codes. By using the code application table, I could check if the tasks had been codified accurately by controlling these numbers. If problems were discovered the tracing feature would locate the tasks implicated and the mistakes could quickly be eradicated. The analyzing tools present the results in graphs and tables that are exportable to excel.

One problematic element in using Dedoose for textbook analysis is the dependence on being able to upload them in the program. But once this is resolved it is a great tool to keep track of large amounts of text. It is a resource for comparative research in that by using different descriptors for the textbooks, the program can display (in this case) results either by title, grade level or nationality. Dedoose is an effective and helpful tool in developing and applying codes in textbook research, and once the coding is done the program saves time as the analyzing tools immediately display findings in useful charts.

6 Implications

The findings in this study shows that the introduction to algebra in the textbooks analyzed often are missing tasks that include components of motivation and conceptual structure. The introductory- and the algebraic tasks are mostly focused on developing technical skills. Algebra is a difficult language to learn, this is exemplified by the variable which always is represented by a letter but has multiple meanings dependable on the context in which it is located. A deep understanding of variables and the ability to perform the correct interpretation in different situations requires varied and conceptual experiences. The analysis shows that these topics can be helpful in meeting these needs:

- Pattern
- Problem solving
- Geometry

An inclusion of more elaborated and varied tasks in the units of the textbooks could provoke higher levels of reasoning and perhaps technical skills could be attained within a conceptual development.

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Appendix

All the documents of the codification done in Dedoose is available, but since this is very extensive (138 pages), I will only include the 10 first pages of the codification of tasks done in each textbook.

Appendix 1: Preliminary table

Main object of task	Numerical sequences	Geometrical sequences	Expression	Equivalence	Inequality	Function	
Variables	1 variable	2 variables	More variables				
Operations	Addition	Subtraction	Multiplication	Division	Exponents	Square root	
Kind of numbers	Integers	Negative numbers	Decimals	Fractions			
Function of letters	Letters as variables	Letters as unknowns	Letters provided	Letters absence			
Semiotic elements	Text	Picture	Geometric shapes	Table	Graph	Different symbols	
Activities asked to be done	Solving	Recognizing correct/incorrect solution	Determine false/true statements	Identifying pattern	Working directly with definitions	Making a table	Making a graph
Activities of reasoning	Routine solving	Simplifying	Formulating	Explaining	Justifying	Proving	Evaluating
Application of algebra	Generality and pattern	Symbol manipulation	A way of solving problems	Modeling	Functions and their transformations	A formal system	
Passage of context	Intra mathematical (same topic)	Intra mathematical (geometry)	Intra mathematical (functions)	Non mathematical (real life)			

Appendix 2: Pre-algebra document codified

✓ Evaluate Expressions

Determine whether the given expressions are equal.

- | | | | |
|--|--|--|--|
| 25. $(4 \cdot 7) \cdot 2$
and $4 \cdot (7 \cdot 2)$ | 26. $(2 \cdot 4) \div 2$
and $2 \cdot (4 \div 2)$ | 27. $2 \cdot (3 - 3)$
and $(2 \cdot 3) - 3$ | 28. $5 \cdot (50 - 44)$
and $5 \cdot 50 - 44$ |
| 29. $9 - (4 \cdot 2)$
and $(9 - 4) \cdot 2$ | 30. $2 \cdot 3 + 2 \cdot 4$
and $2 \cdot (3 + 4)$ | 31. $(16 \div 4) + 4$
and $16 \div (4 + 4)$ | 32. $5 + (2 \cdot 3)$
and $(5 + 2) \cdot 3$ |

Think and Discuss

1. Give an example of a numerical expression and of an algebraic expression.

Merknad [1]: Codes (24-28)
Introductory tasks
Mathematical language

2. Tell how to evaluate an algebraic expression for a given value of a variable.

Merknad [2]: Codes (28-32)
Introductory tasks
Mathematical language

3. Explain why you cannot find a numerical value for the expression $4x - 5y$ for $x = 3$.

Merknad [3]: Codes (33-36)
Introductory tasks
Interpreting expression
Mathematical language
Two or more variables
Algebra as a language

GUIDED PRACTICE

1 Evaluate each expression for the given value of the variable.

1. $x + 4$ for $x = 11$ 2. $2a + 7$ for $a = 7$ 3. $2(4 + n) - 5$ for $n = 0$

Merknad [4]: Codes (37-40)
Interpreting expression
One variable
Repetitive Weight: 1/5
Mathematical language
Algebra as a language

2 Evaluate each expression for the given values of the variables.

4. $3x + 2y$ for $x = 8$ and $y = 10$ 5. $5r - (12 + p)$ for $p = 15$ and $r = 9$

Merknad [5]: Codes (41-44)
Interpreting expression
Two or more variables
Repetitive Weight: 1/5
Mathematical language
Algebra as a language

3 If c is the number of cups of water needed to make papier-mâché paste, then $\frac{1}{4}c$ can be used to find the number of cups of flour needed. Find the number of cups of flour needed for each number of cups of water.

6. 12 cups 7. 8 cups 8. 7 cups 9. 10 cups

Merknad [6]: Codes (44-48)
Interpreting expression
One variable
Fractions
Task with context
Real life
Repetitive Weight: 1/5
Algebra as a language

INDEPENDENT PRACTICE

1 Evaluate each expression for the given value of the variable.

10. $x + 7$ for $x = 23$ 11. $7t + 2$ for $t = 5$ 12. $4(3 + k) - 7$ for $k = 0$

Merknad [7]: Codes (48-52)
Interpreting expression
One variable
Repetitive Weight: 1/5
Mathematical language
Algebra as a language

- 2 Evaluate each expression for the given values of the variables.
13. $4x + 7y$ for $x = 9$ and $y = 3$ 14. $4m - 2n$ for $m = 25$ and $n = 2.5$

Merknad [8]: Codes (52-56)
 Interpreting expression
 Two or more variables
 Repetitive Weight: 1/5
 Algebra as a language

PRACTICE AND PROBLEM SOLVING

Evaluate each expression for the given value of the variable.

19. $30 - n$ for $n = 8$ 20. $x + 4.3$ for $x = 6$ 21. $5t + 5$ for $t = 1$
 22. $11 - 6m$ for $m = 0$ 23. $3a - 4$ for $a = 8$ 24. $4g + 5$ for $g = 12$
 25. $4y + 2$ for $y = 3.5$ 26. $18 - 3y$ for $y = 6$ 27. $3(z + 9)$ for $z = 6$

Merknad [9]: Codes (56-60)
 Interpreting expression
 One variable
 Repetitive Weight: 3/5
 Algebra as a language

Evaluate each expression for $t = 0$, $x = 1.5$, $y = 6$, and $z = 23$.

28. $3z - 3y$ 29. yz 30. $1.4z - y$ 31. $4.2y - 3x$
 32. $4(y - x)$ 33. $4(3 + y)$ 34. $3(y - 6) + 8$ 35. $4(2 + z) + 5$
 36. $5(4 + t) - 6$ 37. $y(3 + t) - 7$ 38. $x + y + z$ 39. $10x + z - y$
 40. $4(z - 5t) + 3$ 41. $2y + 6(x + t)$ 42. $8txz$ 43. $2z - 3xy$

Merknad [10]: Codes (61-64)
 Interpreting expression
 Two or more variables
 Repetitive Weight: 4/5
 Algebra as a language

44. A rectangular shape has a length-to-width ratio of approximately 5 to 3. A designer can use the expression $\frac{1}{3}(5w)$ to find the length of such a rectangle with a given width w . Find the length of such a rectangle with width 6 inches.

Merknad [11]: Codes (64-68)
 Interpreting expression
 One variable
 Fractions
 Task with context
 Geometry
 Algebra as a language

45. **Finance** A bank charges interest on money it loans. Interest is sometimes a fixed amount of the loan. The expression $a(1 + i)$ gives the total amount due for a loan of a dollars with interest rate i , where i is written as a decimal. Find the amount due for a loan of \$100 with an interest rate of 10%. (*Hint:* $10\% = 0.1$)

Merknad [12]: Codes (68-72)
 Interpreting expression
 Algebra as a language
 Two or more variables
 Decimal numbers
 Task with context
 Real life

46. **Entertainment** There are 24 frames, or still shots, in one second of movie footage. To determine the number of frames in a movie, you can use the expression $(24)(60)m$, where m is the running time in minutes. Using the running time of *E.T. the Extra-Terrestrial* shown at right, determine how many frames are in the movie.



E.T. the Extra-Terrestrial (1982) has a running time of 115 minutes, or 6900 seconds.

47. **Choose a Strategy** A basketball league

California LINK
Entertainment

The photos above were taken in Palo Alto as part of Eadweard Muybridge's 1878 study on horses in motion. Muybridge's multi-camera techniques were a landmark in the history of cinematography.

47. **Choose a Strategy** A basketball league has 288 players and 24 teams, with an equal number of players per team. If the number of teams is reduced by 6 but the total number of players stays the same, there will be ___?___ players per team.

(A) 6 more (B) 4 more (C) 4 fewer (D) 6 fewer

E.T. the Extra-Terrestrial (1982) has a running time of 115 minutes, or 6900 seconds.

Merknad [13]: Codes (74-77)
Interpreting expression
One variable
Task with context
Real life
Algebra as a language

Merknad [14]: Codes (77-81)
Algebra as problem solving
Task with context
Real life

48. **Write About It** A student says that the algebraic expression $5 + x \cdot 7$ can also be written as $5 + 7x$. Is the student correct? Explain.

Merknad [15]: Codes (81-85)
One variable
Mathematical language
Algebra as symbolic manipulation
Simplifying
Task with context
Real life

49. **Challenge** Can the expressions $2x$ and $x + 2$ ever have the same value? If so, what must the value of x be?

Merknad [16]: Codes (85-89)
Mathematical language
Algebra as a language
Interpreting expression
One variable



SPIRAL STANDARDS REVIEW

50. **Multiple Choice** What is the value of the expression $3x + 4$ for $x = 2$?

- (A) 4 (B) 6 (C) 9 (D) 10

51. **Multiple Choice** A bakery charges \$7 for a dozen muffins and \$2 for a loaf of bread. If a customer bought 2 dozen muffins and 4 loaves of bread, how much did she pay?

- (A) \$22 (B) \$38 (C) \$80 (D) \$98

52. **Gridded Response** What is the value of $5(x - y)$ for $x = 19$ and $y = 6$?

Identify the odd number(s) in each list of numbers. (Previous course)

53. 15, 18, 22, 34, 21, 61, 71, 100

54. 101, 114, 122, 411, 117, 121

55. 4, 6, 8, 16, 18, 20, 49, 81, 32

56. 9, 15, 31, 47, 65, 93, 1, 3, 43

Find each sum, difference, product, or quotient. (Previous course)

57. $200 + 2$

58. $200 + 2$

59. $200 \cdot 2$

60. $200 - 2$

61. $200 + 0.2$

62. $200 \div 0.2$

63. $200 \cdot 0.2$

64. $200 - 0.2$



Writing Algebraic Expressions

Think and Discuss

1. Give two words or phrases that can be used to express each operation: addition, subtraction, multiplication, and division.

2. Express $5 + 7n$ in words in at least two different ways.

Merknad [17]: Codes (113-117)
Mathematical language

Merknad [18]: Codes (117-121)
Interpreting expression
Algebra as a language
One variable

GUIDED PRACTICE

- 1 Write an algebraic expression for each word phrase.
1. 5 less than the product of 3 and p
 2. 77 more than the product of 2 and u
 3. 16 more than the quotient of d and 7
 4. 6 minus the quotient of u and 2

Merknad [19]: Codes (121-125)
Formulating expression
One variable
Mathematical language
Repetitive Weight: 1/5
Algebra as a language

- 2 Write a word phrase for each algebraic expression.
5. $18 + 43s$
 6. $\frac{22}{r} - 37$
 7. $10 + \frac{y}{31}$
 8. $29b - 93$

Merknad [20]: Codes (125-129)
One variable
Fractions
Interpreting expression
Mathematical language
Algebra as a language
Repetitive Weight: 1/5

- 3 9. Mark is going to work for his father's pool cleaning business during the summer. Mark's father will pay him \$5 for each pool he helps clean. Write an algebraic expression to determine how much Mark will earn if he cleans n pools. Then evaluate the expression for 15, 25, 35, or 45 pools.

Merknad [21]: Codes (129-133)
Formulating expression
One variable
Mathematical language
Task with context
Real life
Multistep
Algebra as a language

- 4 10. Write a word problem that can be evaluated by the algebraic expression $x - 450$. Then evaluate the expression for $x = 1325$.

Merknad [22]: Codes (133-137)
Interpreting expression
One variable
Multistep
Algebra as a language

INDEPENDENT PRACTICE

- 1 Write an algebraic expression for each word phrase.
11. 1 more than the quotient of 5 and n
 12. 2 minus the product of 3 and p
 13. 45 less than the product of 73 and j
 14. 4 plus the quotient of r and 5
 15. 14 more than the product of 59 and q

Merknad [23]: Codes (137-141)
Formulating expression
One variable
Mathematical language
Repetitive Weight: 2/5
Algebra as a language

- 2 Write a word phrase for each algebraic expression.
16. $142 - 19t$
 17. $16g + 12$
 18. $14 + \frac{5}{d}$
 19. $\frac{w}{182} - 51$

Merknad [24]: Codes (141-145)
Interpreting expression
One variable
Fractions
Repetitive Weight: 1/5
Algebra as a language

- 3 20. A community center is trying to raise \$1680 to purchase exercise equipment. The center is hoping to receive equal contributions from members of the community. Write an algebraic expression to determine how much will be needed from each person if n people contribute. Then evaluate the expression for 10, 12, 14, or 16 people.

Merknad [25]: Codes (145-149)
Formulating expression
One variable
Multistep
Task with context
Real life
Mathematical language
Algebra as a language

- 4 21. Write a word problem that can be evaluated by the algebraic expression $372 + r$. Then evaluate the expression for $r = 137$.

Merknad [26]: Codes (149-153)
Interpreting expression
One variable
Multistep
Algebra as a language

PRACTICE AND PROBLEM SOLVING

Write an algebraic expression for each word phrase.

22. 6 times the sum of 4 and y
23. the product of 6 and y increased by 9
24. $\frac{1}{3}$ of the sum of 4 and p
25. 1 divided by the sum of 3 and g
26. half the sum of m and 5
27. 6 less than the product of 13 and y
28. 2 less than m divided by 8
29. twice the quotient of m and 35
30. $\frac{3}{4}$ of the difference of p and 7
31. 8 times the sum of $\frac{2}{3}$ and x

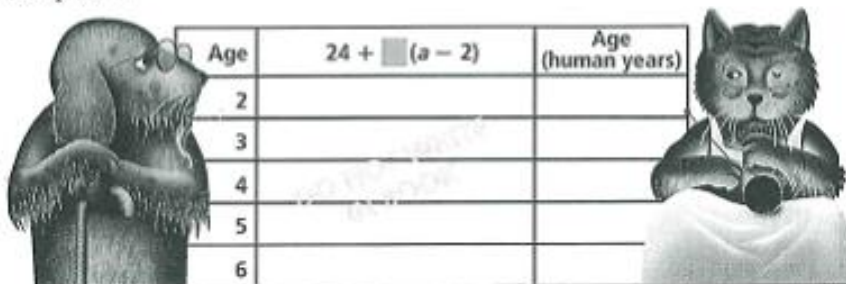
Merknad [27]: Codes (154-157)
Formulating expression
One variable
Fractions
Repetitive Weight: 3/5
Algebra as a language

Translate each algebraic expression into words.

32. $4b - 3$
33. $8(m + 5)$
34. $\frac{7}{8-x}$
35. $17\left(\frac{16}{w}\right)$

Merknad [28]: Codes (157-161)
Interpreting expression
One variable
Fractions
Repetitive Weight: 1/5
Algebra as a language

36. At age 2, a cat or a dog is considered 24 "human" years old. Each year after age 2 is equivalent to 4 "human" years. Let a represent the age of a cat or dog. Fill in the expression $[24 + \square(a - 2)]$ so that it represents the age of a cat or dog in "human" years. Copy the chart and use your expression to complete it.



Age	$24 + \square(a - 2)$	Age (human years)
2		
3		
4		
5		
6		

Merknad [29]: Codes (161-165)
Interpreting expression
One variable
Task with context
Real life
Algebra as a language
Multistep

37. **Reasoning** Write two different algebraic expressions for the word phrase " $\frac{1}{4}$ the sum of x and 7."

Merknad [30]: Codes (165-169)
Formulating expression
One variable
Fractions
Mathematical language
Algebra as symbolic manipulation
Algebra as a language
Transforming

38. **What's the Error?** A student wrote an algebraic expression for "5 less than the quotient of n and 3" as $\frac{n-5}{3}$. What error did the student make?

Merknad [31]: Codes (169-173)
Interpreting expression
One variable
Fractions
Task with context
Real life
Algebra as a language

39. **Write About It** Paul used addition to solve a word problem about the weekly cost of commuting by toll road for \$1.50 each day. Fran solved the same problem by multiplying. They both got the correct answer. How is this possible?

Merknad [32]: Codes (174-177)
Mathematical language
Task with context
Real life
Generalized arithmetic

- ★ 40. **Challenge** Write an expression for the sum of 1 and twice a number n . If you let n be any number, will the result always be an odd number? Explain.

Merknad [33]: Codes (177-181)
Interpret definition
One variable
Multistep
Algebra as generalization



SPIRAL STANDARDS REVIEW

← NS1.2, AF1.1, A

41. **Multiple Choice** Which expression means "3 times the difference of y and 4"?
- (A) $3 \cdot y - 4$ (B) $3 \cdot (y + 4)$ (C) $3 \cdot (y - 4)$ (D) $3 - (y - 4)$
42. **Multiple Choice** Which expression represents the product of a number n and 32?
- (A) $n + 32$ (B) $n - 32$ (C) $n \cdot 32$ (D) $32 \div n$
43. **Short Response** A company prints n books at a cost of \$9 per book. Write an expression to represent the total cost of printing n books. What is the total cost if 1050 books are printed?

Simplify. (Previous course)

44. $32 + 8 \div 4$

45. $24 - 2 \cdot 3 + 6 + 1$

46. $(20 - 8) \cdot 2 + 2$

Evaluate each expression for the given value of the variable. (Lesson 1-1)

47. $2(4 + x) - 3$ for $x = 1$

48. $3(8 - x) - 2$ for $x = 2$



1-3 Integers and Absolute Value

Think and Discuss

1. **Explain** the steps you would take to simplify $|5| - |-2|$.
2. **Explain** whether an absolute value is ever negative.

Think and Discuss

1. **Compare** the sums of $10 + (-22)$ and $-10 + 22$.

Merknad [34]: Codes (307-310)
Negative numbers

2. Describe how to add the following expressions on a number line: $9 + (-13)$ and $-13 + 9$. Then compare the sums.

Merknad [35]: Codes (310-314)
Negative numbers

GUIDED PRACTICE

1 Use a number line to find each sum.

1. $5 + 1$

2. $6 + (-4)$

3. $-7 + 9$

4. $-4 + (-2)$

Merknad [36]: Codes (314-318)
Repetitive Weight: 1/5
Negative numbers

2 Add.

5. $-12 + 5$

6. $7 + (-3)$

7. $-11 + 17$

8. $-6 - (-8)$

Merknad [37]: Codes (318-322)
Repetitive Weight: 1/5
Negative numbers

3 Evaluate each expression for the given value of the variable.

9. $t + 16$ for $t = -5$

10. $m + 7$ for $m = -5$

11. $p + (-5)$ for $p = -5$

Merknad [38]: Codes (322-326)
Interpreting expression
One variable
Negative numbers
Repetitive Weight: 1/5
Algebra as a language

4 12. Lee opens a checking account. In the first month, he makes two deposits and writes three checks, as shown at right. Find what his balance is at the end of the month. (*Hint:* Checks count as negative amounts.)

Checks	Deposits
\$134	\$600
\$56	\$225
\$302	

Merknad [39]: Codes (326-330)
Task with context
Real life
Negative numbers

INDEPENDENT PRACTICE

1 Use a number line to find each sum.

13. $5 + (-7)$

14. $-7 + 7$

15. $4 + (-9)$

16. $-4 + 7$

Merknad [40]: Codes (331-334)
Negative numbers
Repetitive Weight: 1/5

2 Add.

17. $8 + 14$

18. $-6 + (-7)$

19. $-8 + (-8)$

20. $19 + (-5)$

21. $22 + (-15)$

22. $17 + 9$

23. $-20 + (-12)$

24. $-18 + 7$

Merknad [41]: Codes (334-338)
Negative numbers
Repetitive Weight: 2/5

3 Evaluate each expression for the given value of the variable.

25. $q + 13$ for $q = 10$

26. $z + 21$ for $x = -7$

27. $z + (-7)$ for $z = 16$

Merknad [42]: Codes (338-342)
Interpreting expression
One variable
Negative numbers
Repetitive Weight: 1/5
Algebra as a language

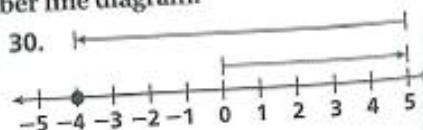
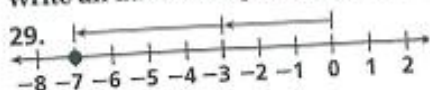
- 4 28. On Monday morning, a mechanic has no cars in her shop. The table at right shows the number of cars dropped off and picked up each day. Find the total number of cars left in her shop on Friday.

	Cars Dropped Off	Cars Picked Up
Monday	8	4
Tuesday	11	6
Wednesday	9	12
Thursday	14	9
Friday	7	6

Merknad [43]: Codes (342-346)
Task with context
Real life

PRACTICE AND PROBLEM SOLVING

Write an addition equation for each number line diagram.



Merknad [44]: Codes (347-350)
Negative numbers
Mathematical language

Find each sum.

31. $-9 + (-3)$

32. $16 + (-22)$

33. $-34 + 17$

34. $44 + 39$

35. $45 + (-67)$

36. $-14 + 85$

37. $52 + (-9)$

38. $-31 + (-31)$

Merknad [45]: Codes (350-354)
Negative numbers
Repetitive Weight: 2/5

Evaluate each expression for the given value of the variable.

39. $c + 17$ for $c = -9$

40. $k + (-12)$ for $k = 4$

41. $b + (-6)$ for $b = -24$

42. $13 + r$ for $r = -19$

43. $-9 + w$ for $w = -6$

44. $3 + n + (-8)$ for $n = 5$

Merknad [46]: Codes (354-358)
Interpreting expression
One variable
Negative numbers
Repetitive Weight: 2/5
Algebra as a language

45. Economics Refer to the data at right about U.S. international trade for the year 2004. Consider values of exports as positive quantities and values of imports as negative quantities.

	Exports	Imports
Goods	\$807,584,000,000	\$1,473,768,000,000
Services	\$338,553,000,000	\$290,095,000,000

Source: U.S. Census Bureau

- What was the total of U.S. exports in 2004?
- What was the total of U.S. imports in 2004?
- The sum of exports and imports is called the *balance of trade*. Approximate the 2004 U.S. balance of trade to the nearest billion dollars.



The number one category of imported goods in the United States is industrial supplies, including petroleum and petroleum products. In 2004, this category accounted for over \$412 billion of imports.

Merknad [47]: Codes (358-363)
Task with context
Real life

46. What's the Error? A student evaluated $-4 + d$ for $d = -6$ and gave an answer of 2. What might the student have done wrong? Give the correct answer.

Merknad [48]: Codes (363-367)
Interpreting expression
One variable
Negative numbers
Task with context
Real life
Algebra as a language

47. Write About It Explain the different ways it is possible to add two integers and get a negative answer.

Merknad [49]: Codes (368-371)
Negative numbers
Mathematical language
Generalized arithmetic

Merknad [50]: Codes (371-375)
Pattern
Negative numbers
Formulating rethoric rule

Appendix 3: Pi 7 document codified

UppgiFTerNA

19-32

200

1. En talföljd är 2, 6, 10, 14.

a) Vilka tal är element i talföljden?

b) Vilket tal är det tredje elementet i talföljden?

c) Är talföljden ändlig eller oändlig?

Merknad [51]: Codes (6179-6340)
Introductory tasks
Pattern
Multistep
Mathematical language
Formulating rhetoric rule

2. Vilka är de tre följande elementen i talföljden?

a) 1, 4, 7, 10, ...

b) 27, 25, 23, 21, ...

c) 55, 50, 45, 40, ...

Merknad [52]: Codes (6340-6452)
Introductory tasks
Pattern
Repetitive Weight: 1/5
Mathematical language
Identifying and continuing a pattern

3. Vilka är de tre följande elementen i talföljden?

a) 0, 3, 6, 9, ...

b) 7, 5, 3, 1, ...

c) -9, -7, -5, -3, ...

Merknad [53]: Codes (6452-6559)
Introductory tasks
Pattern
Repetitive Weight: 1/5
Negative numbers
Mathematical language
Identifying and continuing a pattern

4. Hur många element har talföljden?

a) 2, 5, 4, 7, 1, 0, 8, 6, 3 b) 3, 2, 1, 0, -1, ...

c) 10, 20, 10, 20, 10 d) 7, 14, 21, ... , 49

Merknad [54]: Codes (6559-6698)
Pattern
Repetitive Weight: 1/5
Mathematical language
Introductory tasks
Identifying and continuing a pattern

5. Försätt följden med två bokstäver.

a) A, C, E, G, __ ,

b) T, Q, N, __ ,

c) A, C, F, J, __ ,

Merknad [55]: Codes (6699-6816)
Pattern
Repetitive Weight: 1/5
Introductory tasks
Identifying and continuing a pattern

6. Skriv ut de fyra första elementen i

en talföljd där det första elementet är

a) 3 och varje nytt element är 5 större än det föregående

b) 42 och varje nytt element är 6 mindre än det föregående

Merknad [56]: Codes (6816-7142)
Introductory tasks
Pattern
Repetitive Weight: 1/5
Mathematical language
Interpreting rhetoric rule

c) 5 och varje nytt element är tre gånger så stort som det föregående

d) 48 och varje nytt element är hälften av det föregående.

7. Bilda en talföljd av talen 2, 7, 3, 9, 10, 1, 5 och 8 så att varje element närmast efter

a) alltid är större än det föregående

b) alltid är mindre än det föregående.

c) Bilda talföljden så att den kan spjälkas upp i två talföljder som vardera uppfyller villkoret i a och så att elementen från de olika delarna uppfyller villkoret i b.

8. Bilda en oändlig talföljd som består av

a) jämna naturliga tal i storleksordning

b) heltal i storleksordning

c) negativa heltal vars absolutbelopp är delbara med fem, i storleksordning.

9. Skriv en talföljd med fem element där det mittersta elementet är 15 och

a) som bildas av heltal i storleksordning

b) som bildas av udda heltal i storleksordning

c) där varje element är 3 mindre än följande element.

Fortsätt talföljden med tre element.

10. a) 1, 4, 9, 16, ... b) 12, 7, 2, -3, ... c) 2, 4, 8, 16, ...

11. a) 1, -3, 9, -27, ...

b) 100 000, 10 000, 1 000, ...

c) 3, -3, 3, -3, ...

12. a) 2, -4, 8, -16, 32, ...

b) 5, -2, 10, -3, 15, -4, ...

c) 5, -10, 15, -20, 25, ...

13. En talföljd börjar 1, 3, 7, 15, ...

a) Skriv ut talföljdens femte, sjätte och sjunde element.

Merknad [57]: Codes (7142-7485)
Introductory tasks
Pattern
Multistep
Mathematical language
Interpreting rhetoric rule

Merknad [58]: Codes (7485-7676)
Introductory tasks
Pattern
Repetitive Weight: 1/5
Negative numbers
Mathematical language
Interpreting rhetoric rule

Merknad [59]: Codes (7676-7895)
Introductory tasks
Pattern
Repetitive Weight: 1/5
Mathematical language
Interpreting rhetoric rule

Merknad [60]: Codes (7895-7991)
Introductory tasks
Pattern
Repetitive Weight: 1/5
Negative numbers
Identifying and continuing a pattern

Merknad [61]: Codes (7991-8063)
Introductory tasks
Pattern
Repetitive Weight: 1/5
Negative numbers
Identifying and continuing a pattern

Merknad [62]: Codes (8063-8145)
Introductory tasks
Pattern
Repetitive Weight: 1/5
Negative numbers
Identifying and continuing a pattern

Merknad [63]: Codes (8145-8294)
Introductory tasks
Pattern
Multistep
Formulating rhetoric rule

b) Skriv med ord regeln för hur talföljden bildas.

14. Vilket element saknas i talföljden?

a) 1, 2, __, 8, 16 b) 1, 2, __, 7, 11

c) 1, 2, __, 5, 8 d) 1, 2, __, 4, 5

Skriv också med ord regeln för hur talföljden bildas.

3.1 Talföljder

201

15. Vilken är regeln för hur tal- och bokstavsföljden bildas?

a) 1, B, 2, D, 3, F b) A, 2, D, 4, I, 6, P

16. Skriv som en talföljd. Är talföljden ändlig eller oändlig?

a) negativa heltal

b) tvåsiffriga jämna naturliga tal

c) primtal mindre än 20

17. Vilka tal står x och y för?

1	2	6	30	x
2	3	5	11	y

18. Vilket är följande element i talföljden?

a) 0, 0, 1, 1, 2, 4, 7, 13, ...

b) 1, 3, 3, 9, ...

c) 2, 5, 3, 7, 8, 10, 15, ...

19. Hur många stjärnor finns det på den femte bilden?

Merknad [64]: Codes (8294-8482)
Introductory tasks
Pattern
Repetitive Weight: 1/5
Multistep
Formulating rethoric rule

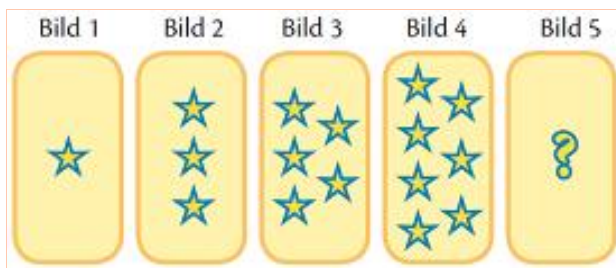
Merknad [65]: Codes (8501-8612)
Introductory tasks
Pattern
Repetitive Weight: 1/5
Formulating rethoric rule

Merknad [66]: Codes (8612-8753)
Introductory tasks
Pattern
Negative numbers
Mathematical language
Repetitive Weight: 1/5
Interpreting rethoric rule

Merknad [67]: Codes (8753-8788)
Pattern
Introductory tasks
Algebra as a language
Interpreting expression
Identifying and continuing a pattern

Merknad [68]: Codes (8788-8907)
Pattern
Repetitive Weight: 1/5
Introductory tasks
Identifying and continuing a pattern

Merknad [69]: Codes (8908-8965)
Pattern
Introductory tasks
Identifying and continuing a pattern



20. Vilket är det tionde elementet i talföljden?

a) 2, 4, 6, 8, ...

b) 5, 10, 15, 20, ...

c) 1, 4, 7, 10, ...

Merknad [70]: Codes (8965-9069)
 Pattern
 Repetitive Weight: 1/5
 Introductory tasks
 Identifying and continuing a pattern

21. Det n :te elementet i en talföljd får vi med hjälp av regeln $n + 3$. Skriv ut talföljdens fem första element.

22. I tabellen har de fyra första elementen i en talföljd skrivits ut. Kopiera tabellen i ditt häfte och komplettera den.

ele- ment 1	ele- ment 2	ele- ment 3	ele- ment 4	ele- ment 5	...	ele- ment 10
5	7	9	11			

Merknad [71]: Codes (9069-9181)
 Pattern
 One variable
 Introductory tasks
 Algebra as generalization
 Interpret rule
 Interpreting symbolic rule

Merknad [72]: Codes (9181-9314)
 Pattern
 Repetitive Weight: 1/5
 Introductory tasks
 Identifying and continuing a pattern

a)

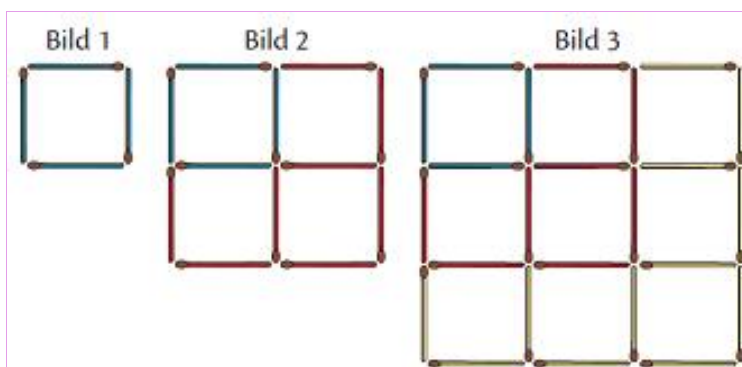
b)

ele- ment 1	ele- ment 2	ele- ment 3	ele- ment 4	ele- ment 5	...	ele- ment 10
2	5	8	11			

23. Hur många tändstickor behöver vi för

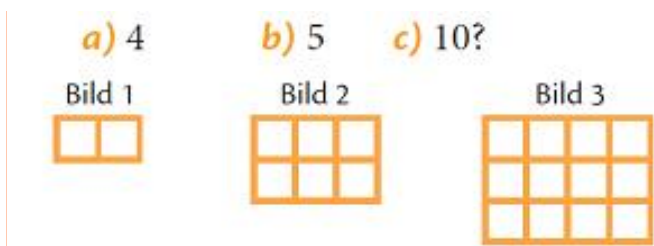
a) bild 4 b) bild 5?

Merknad [73]: Codes (9315-9380)
 Pattern
 Multistep
 Introductory tasks
 Identifying and continuing a pattern

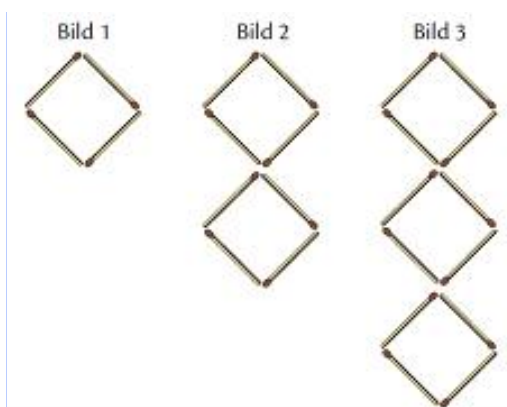


24. Hur många små kvadrater finns det på

bild nummer



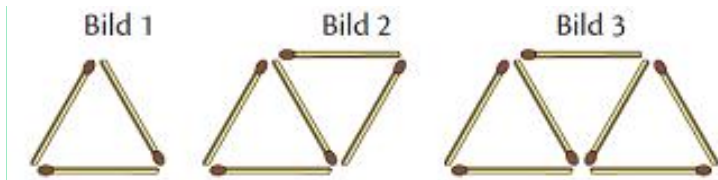
25. Hur många tändstickor finns det på bild nummer n ?



Merknad [74]: Codes (9400-9455)
 Pattern
 Multistep
 Introductory tasks
 Identifying and continuing a pattern

Merknad [75]: Codes (9456-9512)
 Pattern
 One variable
 Algebra as generalization
 Introductory tasks
 Formulating symbolic rule
 Formulate rule

26. Hur många tändstickor finns det på bild nummer n ?



Merknad [76]: Codes (9545-9601)
 Pattern
 One variable
 Algebra as generalization
 Introductory tasks
 Formulating symbolic rule
 Formulate rule

27. Skriv ut de fem första elementen i

en talföljd där element nummer n är

a) $n + 5$ b) $3 \cdot n$

c) $2 \cdot n - 5$ d) $5 \cdot n - 10$.

28. Element nummer n i en talföljd är $2 \cdot n$.

Vilket är element nummer

a) 10 b) 20 c) 100?

29. Vilket är talföljdens n :te element?

a) 100, 101, 102, 103, ...

b) 1, 5, 9, 13, ...

c) 0, 3, 8, 15, 24, ...

d) 3, 3, 3, ...

30. Vilket är talföljdens n :te element?

a) 3, 4, 5, 6, 7, ... b) 2, 4, 6, 8, 10, ... c) 3, 5, 7, 9, 11, ... d) 1, 4, 7, 10, 13, ...

31. Hur många tändstickor finns det på bild nummer n ?

Bild 1 Bild 2 Bild 3

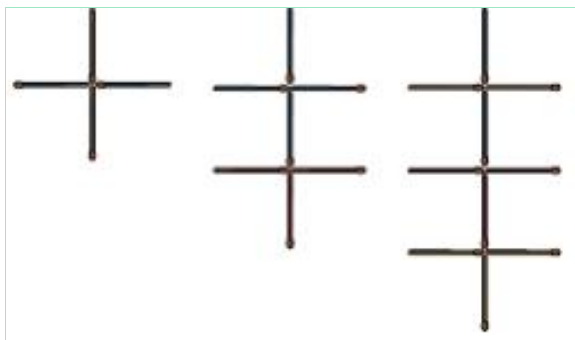
Merknad [77]: Codes (9610-9753)
 Algebra as generalization
 Pattern
 One variable
 Repetitive Weight: 1/5
 Mathematical language
 Introductory tasks
 Interpreting retoric rule
 Interpret rule

Merknad [78]: Codes (9753-9857)
 Algebra as generalization
 Pattern
 Repetitive Weight: 1/5
 Mathematical language
 One variable
 Introductory tasks
 Interpret rule
 Interpreting symbolic rule

Merknad [79]: Codes (9857-9976)
 Algebra as generalization
 Pattern
 One variable
 Repetitive Weight: 1/5
 Mathematical language
 Formulate rule
 Introductory tasks
 Formulating symbolic rule

Merknad [80]: Codes (9976-10100)
 Algebra as generalization
 Pattern
 One variable
 Repetitive Weight: 1/5
 Mathematical language
 Formulate rule
 Introductory tasks
 Formulating symbolic rule

Merknad [81]: Codes (10100-10229)
 Algebra as generalization
 Pattern
 One variable
 Introductory tasks
 Formulating symbolic rule
 Formulate rule

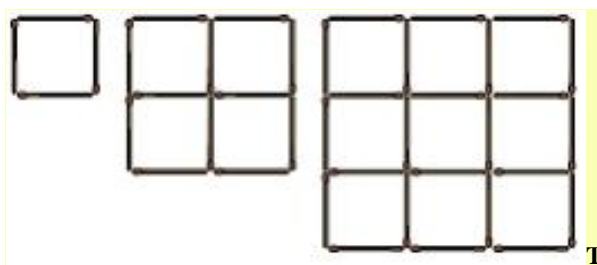


32. Hur många tändstickor finns det på den nionde bilden?

Bild 1

Bild 2

Bild 3



TANKENÖT

TANKENÄT

Det första elementet i en talföljd är 2. Ett element med jämnt ordningsnummer är 5 mindre än föregående element. Ett element med udda ordningsnummer får vi genom att multiplicera föregående element med

-1. Vilket är det 43:e elementet i talföljden?

H E m U p p g i F T E r

H E m U p p g i F T E r

Merknad [82]: Codes (10229-10374)
Pattern
Introductory tasks
Identifying and continuing a pattern

Merknad [83]: Codes (10397-10701)
Introductory tasks
Pattern
Interpreting rhetoric rule

203

33. Studera talföljden 17, 21, 25, 29, ...

a) Skriv ut de tre följande elementen.

b) Skriv med ord regeln för hur talföljden bildas.

34. Skriv ut de tre följande elementen i talföljden.

a) 25, 30, 35, 40, ... b) 101, 98, 95, 92, ... c) 5, 2, -1, -4, ...

35. Skriv ut de tre följande elementen i talföljden.

a) 3, 6, 12, 24, ... b) 2, 6, 12, 20, ... c) 1, -2, 4, -8, ...

Merknad [84]: Codes (10705-10838)
Pattern
Multistep
Introductory tasks
Formulating rhetoric rule

Merknad [85]: Codes (10838-10953)
Pattern
Negative numbers
Repetitive Weight: 1/5
Introductory tasks
Identifying and continuing a pattern

Merknad [86]: Codes (10953-11063)
Pattern
Negative numbers
Repetitive Weight: 1/5
Introductory tasks
Identifying and continuing a pattern

36. Skriv ut de tre följande elementen i talföljden.

a) 100, 90, 82, 76, 72, ...

b) 1, 2, 4, 8, 16, ...

c) 800, 400, 200, ...

37. Skriv ut de fyra första elementen i talföljden, där

a) det första elementet är 5 och vi får ett nytt element genom att multiplicera föregående element med 2

b) det första elementet är 10 och vi får ett nytt element genom att multiplicera föregående element med 2 och sedan subtrahera produkten med 6

c) det första elementet är 4 och vi får ett nytt element genom att multiplicera föregående element med 3 och sedan subtrahera produkten med 5

d) det första elementet är 1 och vi får ett nytt element genom att multiplicera föregående element med 2 och sedan addera produkten med 3.

38. Skriv ut de fem första elementen i talföljden där det n :te elementet är

a) $n - 2$ b) $-4 \cdot n$ c) $3 \cdot n - 2$.

39. Vilket är talföljdens n :te element?

a) 3, 4, 5, 6, ...

b) -2, -4, -6, -8, -10, ...

c) 2, 5, 8, 11, ...

d) 600, 300, 200, 150, ...

40. Hur många tändstickor finns det på bild nummer

a) 5 b) 10 c) 100?

Bild 1

Bild 2

Bild 3

41. Hur många tändstickor finns det på bild nummer n i föregående uppgift?

42. Hur många tändstickor finns det på bild nummer n ?

Bild 1

Bild 2

Bild 3

Merknad [87]: Codes (11063-11183)
Pattern
Repetitive Weight: 1/5
Introductory tasks
Identifying and continuing a pattern

Merknad [88]: Codes (11183-11768)
Pattern
Repetitive Weight: 1/5
Mathematical language
Introductory tasks
Interpreting retoric rule

Merknad [89]: Codes (11768-11892)
Repetitive Weight: 1/5
One variable
Pattern
Introductory tasks
Interpreting retoric rule
Algebra as generalization
Interpret rule

Merknad [90]: Codes (11892-12018)
Algebra as generalization
Pattern
One variable
Repetitive Weight: 1/5
Introductory tasks
Formulating symbolic rule
Formulate rule

Merknad [91]: Codes (12018-12170)
Pattern
Multistep
Introductory tasks
Identifying and continuing a pattern

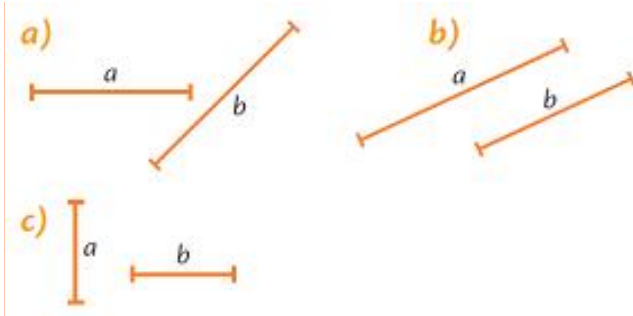
Merknad [92]: Codes (12181-12256)
Pattern
Algebra as generalization
One variable
Introductory tasks
Formulating symbolic rule
Formulate rule

Merknad [93]: Codes (12256-12386)
Pattern
One variable
Algebra as generalization
Formulate rule
Introductory tasks
Formulating symbolic rule



208

43. Vilket värde är störst, a eller b ?



Merknad [94]: Codes (12390-12431)
Repetitive Weight: 1/5
Algebra as a language
Two or more variables
Interpreting expression

Skriv som en produkt.

44. a) $3 + 3 + 3 + 3$

b) $(-2) + (-2) + (-2)$

c) $-4 + (-4) + (-4) + (-4) + (-4)$

Merknad [95]: Codes (12432-12531)
Negative numbers
Repetitive Weight: 1/5

45. a) $a + a + a + a + a$

b) $b + b$

c) $c + c + c + c + c + c + c$

Merknad [96]: Codes (12531-12594)
Repetitive Weight: 1/5
One variable
Algebra as a language
Interpreting expression

46. a) $-x - x$

b) $-y - y - y - y - y - y - y - y$

c) $-s - s - s$

Merknad [97]: Codes (12594-12656)
One variable
Repetitive Weight: 1/5
Algebra as symbolic manipulation
Simplifying

47. a) $b + b + b$

b) $-x - x - x - x$

c) $-a - a$

Merknad [98]: Codes (12656-12701)
Algebra as symbolic manipulation
Simplifying
One variable

48. Skriv av tabellen i ditt häfte och fyll i den.

Merknad [99]: Codes (12701-12754)
Mathematical language
Algebra as a language
Interpreting expression
One variable

Monom	Koefficient	Bokstavsdel
$3a$		
	-2	x
y		
	-1	z
$-4x$		

49. Förenkla, dvs. skriv på ett enklare sätt.

a) $6 \cdot x$ b) $5 \cdot (-a)$ c) $z \cdot 5$

d) $-1 \cdot y$ e) $1 \cdot (-m)$ f) $x + x$

50. Skriv som en produkt och förenkla.

a) $-x + (-x) + (-x) + (-x)$

b) $-b + (-b) + (-b) + (-b) + (-b)$

c) $+a + (+a) + (+a)$

51. Ange koefficient och bokstavsdel.

a) $6x$ b) $-3x$ c) y d) $-z$ e) $5a$ f) 4

52. Skriv monomet.

a) Det är en konstant -6 .

b) Det har samma koefficient som monomet $2x$ men bokstavsdelens är y .

c) Det har koefficienten -1 och bokstavsdelens är densamma som i monomet

$-2a$.

53. Beteckna den sammanlagda längden av sträckorna.

a)



b)

Merknad [100]: Codes (12755-12894)
One variable
Negative numbers
Repetitive Weight: 2/5
Algebra as symbolic manipulation
Simplifying

Merknad [101]: Codes (12894-13014)
One variable
Repetitive Weight: 1/5
Algebra as symbolic manipulation
Simplifying

Merknad [102]: Codes (13014-13091)
Mathematical language
Repetitive Weight: 1/5
Algebra as a language
Interpreting expression
One variable

Merknad [103]: Codes (13091-13283)
Mathematical language
Repetitive Weight: 1/5
Algebra as a language
Formulating expression
One variable

Merknad [104]: Codes (13283-13351)
Formulating expression
One variable
Task with context
Geometry
Repetitive Weight: 1/5
Algebra as a language

Appendix 4: Numbers to algebra document codified

Think and Discuss

1. Describe two different number patterns that begin with 3, 6, ...
2. Tell when it would be useful to make a table to help you identify and extend a pattern.

GUIDED PRACTICE

See Example 1

Identify a possible pattern. Use your pattern to write the next three numbers.

- | | |
|---------------------------------|-------------------------------|
| 1. 6, 14, 22, 30, , , , ... | 2. 1, 3, 9, 27, , , , ... |
| 3. 59, 50, 41, 32, , , , ... | 4. 8, 9, 11, 14, , , , ... |

Merknad [105]: Codes (24-28)
Introductory tasks
Pattern
Repetitive Weight: 1/5
Identifying and continuing a pattern

See Example 2

Identify a possible pattern. Use your pattern to draw the next three figures.

- | | |
|----|----|
| 5. | 6. |
|----|----|

Merknad [106]: Codes (28-32)
Introductory tasks
Pattern
Repetitive Weight: 1/5
Task with context
Geometry
Identifying and continuing a pattern

See Example 3

7. Make a table that shows the number of green triangles in each figure. Tell how many green triangles are in the fifth figure of a possible pattern. Use drawings to justify your answer.



Merknad [107]: Codes (32-36)
Introductory tasks
Pattern
Multistep
Task with context
Geometry
Identifying and continuing a pattern

INDEPENDENT PRACTICE

See Example 1

Identify a possible pattern. Use your pattern to write the next three numbers.

- | | |
|-----------------------------------|----------------------------------|
| 8. 27, 24, 21, 18, , , , ... | 9. 4,096, 1,024, 256, 64, , ... |
| 10. 1, 3, 7, 13, 21, , , , ... | 11. 14, 37, 60, 83, , , , ... |

Merknad [108]: Codes (36-40)
Introductory tasks
Pattern
Repetitive Weight: 1/5
Identifying and continuing a pattern

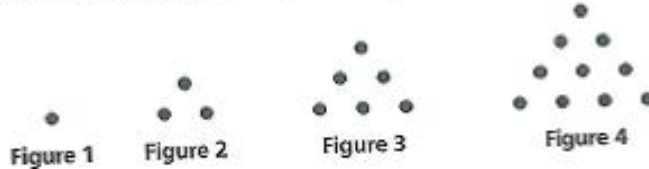
See Example 2

Identify a possible pattern. Use your pattern to draw the next three figures.

- | | |
|-----|-----|
| 12. | 13. |
|-----|-----|

Merknad [109]: Codes (40-44)
Introductory tasks
Pattern
Repetitive Weight: 1/5
Task with context
Geometry
Identifying and continuing a pattern

See Example 3 14. Make a table that shows the number of dots in each figure. Tell how many dots are in the sixth figure of a possible pattern. Use drawings to justify your answer.



Merknad [110]: Codes (44-48)
Introductory tasks
Pattern
Multistep
Identifying and continuing a pattern

PRACTICE AND PROBLEM SOLVING

Extra Practice

See page EP2.

Use the rule to write the first five numbers in each pattern.

15. Start with 7; add 16 to each number to get the next number.

16. Start with 96; divide each number by 2 to get the next number.

17. Start with 50; subtract 2, then 4, then 6, and so on, to get the next number.

Merknad [111]: Codes (48-52)
Introductory tasks
Pattern
Repetitive Weight: 1/5
Interpreting rhetoric rule

18. **Reasoning** Suppose the pattern 3, 6, 9, 12, 15... is continued forever. Will the number 100 appear in the pattern? Why or why not?

Merknad [112]: Codes (52-56)
Introductory tasks
Pattern
Formulating rhetoric rule

Identify a possible pattern. Use your pattern to find the missing numbers.

19. 3, 12, , 192, 768, , , ...

20. 61, 55, , 43, , , 25, ...

21. , , 19, 27, 35, , 51, ...

22. 2, , 8, , 32, 64, , ...

Merknad [113]: Codes (56-60)
Introductory tasks
Pattern
Repetitive Weight: 1/5
Identifying and continuing a pattern

23. **Health** The table shows the target heart rate during exercise for athletes of different ages. Assuming the pattern continues, what is the target heart rate for a 40-year-old athlete? a 65-year-old athlete?

Target Heart Rate	
Age	Heart Rate (beats per minute)
20	150
25	146
30	142
35	138

Merknad [114]: Codes (60-65)
Introductory tasks
Pattern
Task with context
Real life
Identifying and continuing a pattern

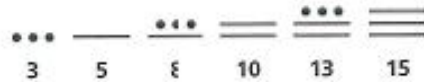
Draw the next three figures in each pattern.

24.

25.

Merknad [115]: Codes (65-69)
Introductory tasks
Pattern
Repetitive Weight: 1/5
Task with context
Geometry
Identifying and continuing a pattern

26. Social Studies In the ancient Mayan civilization, people used a number system based on bars and dots. Several numbers are shown below. Look for a pattern and write the number 18 in the Mayan system.



Merknad [116]: Codes (69-73)
 Task with context
 Real life
 Introductory tasks
 Pattern
 Identifying and continuing a pattern

27. What's the Error? A student was asked to write the next three numbers in the pattern 96, 48, 24, 12, The student's response was 6, 2, 1. Describe and correct the student's error.

Merknad [117]: Codes (73-77)
 Introductory tasks
 Pattern
 Task with context
 Real life
 Formulating rhetoric rule

28. Write About It A school chess club meets every Tuesday during the month of March. March 1 falls on a Sunday. Explain how to use a number pattern to find all the dates when the club meets.

Merknad [118]: Codes (77-81)
 Task with context
 Real life
 Introductory tasks
 Pattern
 Formulating rhetoric rule

29. Challenge Find the 83rd number in the pattern 5, 10, 15, 20, 25,

Merknad [119]: Codes (81-85)
 Introductory tasks
 Pattern
 Identifying and continuing a pattern



SPIRAL STANDARDS REVIEW

Prep for AF2.0

30. Multiple Choice Which rule best describes the pattern 2, 6, 18, 54, 162, . . . ?

- (A) Add 4. (B) Add 12. (C) Multiply by 3. (D) Multiply by 4.

31. Short Response What could be the next number in the pattern 9, 11, 15, 21, 29, 39, . . . ? Explain how you determined your answer.

Round each number to the nearest hundred thousand. (Previous course)

32. 4,224,315

33. 12,483,028

34. 8,072,339

Find each quotient. (Previous course)

35. $3,068 \div 26$

36. $4,680 \div 35$

37. $51,408 \div 136$

Think and Discuss

- 1. Describe** a relationship between 3^5 and 3^6 .
- 2. Tell** which power of 8 is equal to 2^3 . Explain.
- 3. Explain** why any number to the first power is equal to that number.

Identify a possible pattern. Use your pattern to write the next three numbers. (Lesson 1-1)

64. 100, 91, 82, 73, 64, ... 65. 17, 19, 22, 26, 31, ... 66. 2, 6, 18, 54, 162, ...

Think and Discuss

- Apply** the order of operations to determine if the expressions $3 + 4^2$ and $(3 + 4)^2$ have the same value.
- Give** the correct order of operations for simplifying $(5 + 3 \cdot 20) \div 13 + 3^2$.
- Determine** where grouping symbols should be inserted in the expression $3 + 9 - 4 \cdot 2$ so that its value is 13.

Merknad [120]: Codes (202-206)
Introductory tasks
sequence of operations
Multistep

GUIDED PRACTICE

See Example 1 Simplify each expression. Use the order of operations to justify your work.

1. $43 + 16 \div 4$ 2. $28 - 4 \cdot 3 \div 6 + 4$ 3. $25 - 4^2 \div 8$

Merknad [121]: Codes (206-210)
Introductory tasks
sequence of operations
Exponents
Repetitive Weight: 1/5

See Example 2

4. $26 - (7 \cdot 3) + 2$ 5. $(3^2 + 11) \div 5$ 6. $32 + 6(4 - 2^2)$

Merknad [122]: Codes (210-214)
Introductory tasks
sequence of operations
Exponents
Repetitive Weight: 1/5

See Example 3

7. **Career** Caleb earns \$10 per hour. He worked 4 hours on Monday, Wednesday and Friday. He worked 8 hours on Tuesday and Thursday. Simplify the expression $(3 \cdot 4 + 2 \cdot 8) \cdot 10$ to find out how much Caleb earned in all.

Merknad [123]: Codes (214-218)
Introductory tasks
sequence of operations
Task with context
Real life

INDEPENDENT PRACTICE

See Example 1 Simplify each expression. Use the order of operations to justify your work.

8. $3 + 7 \cdot 5 - 1$ 9. $5 \cdot 9 - 3$ 10. $3 - 2 + 6 \cdot 2^2$

Merknad [124]: Codes (218-222)
Introductory tasks
sequence of operations
Exponents
Repetitive Weight: 1/5

See Example 2

11. $(3 \cdot 3 - 3)^2 \div 3 + 3$ 12. $2^5 - (4 \cdot 5 + 3)$ 13. $(3 \div 3) + 3 \cdot (2 - 1)$
14. $4^3 \div 8 - 2$ 15. $(8 - 2)^2 \cdot (8 - 1)^2 \div 3$ 16. $9,234 \div [3 \cdot 3(2 - 1)]$

Merknad [125]: Codes (222-226)
Introductory tasks
sequence of operations
Exponents
Repetitive Weight: 2/5

See Example 3

17. **Consumer Math** Maki paid a \$14 basic fee plus \$25 a day to rent a car. Simplify the expression $14 + 5 \cdot 25$ to find out how much it cost her to rent the car for 5 days.

Merknad [126]: Codes (226-230)
Introductory tasks
sequence of operations
Task with context
Real life

18. **Consumer Math** Enrico spent \$20 per square yard for carpet and \$35 for a carpet pad. Simplify the expression $35 + 20(16)$ to find out how much Enrico spent to carpet a room with an area of 16 square yards.

Merknad [127]: Codes (230-234)
Introductory tasks
sequence of operations
Task with context
Real life

PRACTICE AND PROBLEM SOLVING

Simplify each expression. Use the order of operations to justify your work.

19. $90 - 36 \times 2$ 20. $16 + 14 \div 2 - 7$ 21. $64 \div 2^2 + 4$
22. $10 \times (18 - 2) + 7$ 23. $(9 - 4)^2 - 12 \times 2$ 24. $[1 + (2 + 5)^2] \times 2$

Merknad [128]: Codes (234-238)
Introductory tasks
sequence of operations
Exponents
Repetitive Weight: 2/5

Compare. Write $<$, $>$, or $=$.

25. $8 \cdot 3 - 2$ \square $8 \cdot (3 - 2)$ 26. $(6 + 10) \div 2$ \square $6 + 10 \div 2$
27. $12 \div 3 \cdot 4$ \square $12 \div (3 \cdot 4)$ 28. $18 + 6 - 2$ \square $18 + (6 - 2)$
29. $[6(8 - 3) + 2]$ \square $6(8 - 3) + 2$ 30. $(18 - 14) \div (2 + 2)$ \square $18 - 14 \div 2 + 2$

Merknad [129]: Codes (238-242)
Introductory tasks
sequence of operations
Repetitive Weight: 2/5

Reasoning Insert grouping symbols to make each statement true.

31. $4 \cdot 8 - 3 = 20$ 32. $5 + 9 - 3 + 2 = 8$ 33. $12 - 2^2 + 5 = 20$
34. $4 \cdot 2 + 6 = 32$ 35. $4 + 6 - 3 + 7 = 1$ 36. $9 \cdot 8 - 6 + 3 = 6$

Merknad [130]: Codes (242-246)
Introductory tasks
sequence of operations
Exponents
Repetitive Weight: 2/5

37. Bertha earned \$8.00 per hour for 4 hours babysitting and \$10.00 per hour for 5 hours painting a room. Simplify the expression $8 \cdot 4 + 10 \cdot 5$ to find out how much Bertha earned in all.

Merknad [131]: Codes (246-250)
Introductory tasks
sequence of operations
Task with context
Real life

38. **Consumer Math** Mike bought a painting for \$512. He sold it at an antique auction for 4 times the amount that he paid for it, and then he purchased another painting with half of the profit that he made. Simplify the expression $(512 \cdot 4 - 512) \div 2$ to find how much Mike paid for the second painting.

Merknad [132]: Codes (250-254)
Introductory tasks
sequence of operations
Task with context
Real life

39. **Multi-Step** Anelise bought four shirts and two pairs of jeans. She paid \$6 in sales tax.
- Write an expression that shows how much she spent on shirts.
 - Write an expression that shows how much she spent on jeans.
 - Write and simplify an expression to show how much she spent on clothes, including sales tax.



Merknad [133]: Codes (254-258)
Introductory tasks
sequence of operations
Task with context
Real life
Multistep

- 40. Choose a Strategy** There are four children in a family. The sum of the squares of the ages of the three youngest children equals the square of the age of the oldest child. How old are the children?
- (A) 1, 4, 8, 9 (B) 1, 3, 6, 12 (C) 4, 5, 8, 10 (D) 2, 3, 8, 16

Merknad [134]: Codes (258-262)
Introductory tasks
sequence of operations
Task with context
Real life

- 41. Write About It** Describe the order in which you would perform the operations to find the correct value of $[(2 + 4)^2 - 2 \cdot 3] \div 6$.

Merknad [135]: Codes (262-266)
Introductory tasks
sequence of operations

- 42. Challenge** Use the numbers 3, 5, 6, 2, 54, and 5 in that order to write an expression that has a value of 100.

Merknad [136]: Codes (266-270)
Introductory tasks
sequence of operations



SPIRAL STANDARDS REVIEW

AF

- 43. Multiple Choice** Which operation should be performed first to simplify the expression $18 - 1 \cdot 9 + 3 + 3$?

(A) Addition (B) Subtraction (C) Multiplication (D) Division

- 44. Multiple Choice** Which expression does NOT simplify to 81?

(A) $9 \cdot (4 + 5)$ (B) $7 + 16 \cdot 4 + 10$ (C) $3 \cdot 25 + 2$ (D) $10^2 - 4 \cdot 5 + 1$

- 44. Multiple Choice** Which expression does NOT simplify to 81?

(A) $9 \cdot (4 + 5)$ (B) $7 + 16 \cdot 4 + 10$ (C) $3 \cdot 25 + 2$ (D) $10^2 - 4 \cdot 5 + 1$

- 45. Multiple Choice** Quinton bought 2 pairs of jeans for \$30 each and 3 pairs of socks for \$5 each. Which expression can be simplified to determine the total amount Quinton paid for the jeans and socks?

(A) $2 \cdot 3(30 + 5)$ (B) $(2 + 3) \cdot (30 + 5)$ (C) $2 \cdot (30 + 5) \cdot 3$ (D) $2 \cdot 30 + 3 \cdot 5$

Identify a possible pattern. Use your pattern to write the next three numbers. (Lesson 1-1)

46. 56, 60, 64, 68, 72, ... 47. 5, 10, 20, 40, 80, ... 48. 70, 63, 56, 49, 42, ...

Find each value. (Lesson 1-2)

49. 8^6 50. 9^3 51. 4^5 52. 3^3 53. 7^1

Think and Discuss

1. Describe two different ways to simplify the expression $7 \cdot (3 + 9)$.
2. Explain how the Distributive Property can help you find $6 \cdot 102$ using mental math.

Merknad [137]: Codes (330-334)
Introductory tasks
Mathematical language
Generalized arithmetic

GUIDED PRACTICE

See Example 1 Tell which property is represented.

1. $1 + (6 + 7) = (1 + 6) + 7$
2. $1 \cdot 10 = 10$
3. $3 \cdot 5 = 5 \cdot 3$
4. $6 + 0 = 6$
5. $4 \cdot (4 \cdot 2) = (4 \cdot 4) \cdot 2$
6. $x + y = y + x$

Merknad [138]: Codes (334-338)
Introductory tasks
Repetitive Weight: 2/5
Generalized arithmetic

See Example 2 Simplify each expression. Justify each step.

7. $8 + 23 + 2$
8. $2 \cdot (17 \cdot 5)$
9. $(25 \cdot 11) \cdot 4$
10. $17 + 29 + 3$
11. $16 + (17 + 14)$
12. $5 \cdot 19 \cdot 20$

Merknad [139]: Codes (338-342)
Introductory tasks
Repetitive Weight: 2/5
Generalized arithmetic

See Example 3 Use the Distributive Property to find each product.

13. $2(19)$
14. $5(31)$
15. $(22)2$
16. $(13)6$
17. $8(26)$
18. $(34)6$

Merknad [140]: Codes (342-346)
Introductory tasks
Mathematical language
Repetitive Weight: 2/5
Generalized arithmetic

INDEPENDENT PRACTICE

See Example 1 Tell which property is represented.

19. $1 + 0 = 1$
20. $xyz = x \cdot (yz)$
21. $9 + (9 + 0) = 9$
22. $11 + 25 = 25 + 11$
23. $7 \cdot 1 = 7$
24. $(16 \cdot 4) \cdot 2 = (4 \cdot 16) \cdot 2$

Merknad [141]: Codes (346-350)
Introductory tasks
Mathematical language
Repetitive Weight: 2/5
Generalized arithmetic

See Example 2 Simplify each expression. Justify each step.

25. $50 \cdot 16 \cdot 2$
26. $9 + 34 + 1$
27. $4 \cdot (25 \cdot 9)$
28. $27 + 28 + 3$
29. $20 + (63 + 80)$
30. $25 + 17 + 75$

Merknad [142]: Codes (350-354)
Introductory tasks
Repetitive Weight: 1/5
Generalized arithmetic

See Example 3 Use the Distributive Property to find each product.

31. $9(15)$
32. $(14)5$
33. $3(58)$
34. $10(42)$
35. $(23)4$
36. $(16)5$

Merknad [143]: Codes (354-358)
Introductory tasks
Mathematical language
Repetitive Weight: 2/5
Generalized arithmetic

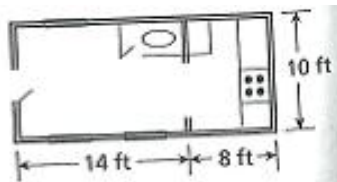
PRACTICE AND PROBLEM SOLVING

Write an example of each property using whole numbers.

37. Commutative Property 38. Identity Property
39. Associative Property 40. Distributive Property

Merknad [144]: Codes (358-362)
Introductory tasks
Mathematical language
Repetitive Weight: 1/5
Generalized arithmetic

41. **Architecture** The figure shows the floor plan for a studio loft. To find the area of the loft, the architect multiplies the length and the width: $(14 + 8) \cdot 10$. Use the Distributive Property to find the area of the loft



Merknad [145]: Codes (362-366)
Introductory tasks
Task with context
Real life
Geometry
Mathematical language
Generalized arithmetic

Simplify each expression. Justify each step.

42. $32 + 26 + 43$ 43. $50 \cdot 45 \cdot 4$ 44. $5 + 16 + 25$ 45. $35 \cdot 25 \cdot 20$

Merknad [146]: Codes (366-370)
Introductory tasks
Repetitive Weight: 1/5
Generalized arithmetic

Complete each equation. Then tell which property is represented.

46. $5 + 16 = 16 + \square$ 47. $15 \cdot 1 = \square$
48. $\square \cdot (4 + 7) = 3 \cdot 4 + 3 \cdot 7$ 49. $20 + \square = 20$
50. $2 \cdot \square \cdot 9 = (2 \cdot 13) \cdot 9$ 51. $8 + (\square + 4) = (8 + 8) + 4$
52. $2 \cdot (6 + 1) = 2 \cdot \square + 2 \cdot 1$ 53. $(12 - 9) \cdot \square = 12 \cdot 2 - 9 \cdot 2$

Merknad [147]: Codes (370-374)
Introductory tasks
One variable
Figure or empty box
Mathematical language
Repetitive Weight: 2/5
Generalized arithmetic
Algebra as equation solving
Numeric trial

54. **Sports** Janice wants to know the total number of games won by the Denver Nuggets basketball team over the three seasons shown in the table. What expression should she simplify? Explain how she can use mental math and the properties of this lesson to simplify the expression.

Denver Nuggets		
Season	Won	Lost
2001-02	27	55
2002-03	17	55
2003-04	43	39

Merknad [148]: Codes (374-378)
Introductory tasks
Task with context
Real life
Generalized arithmetic

55. **What's the Error?** A student simplified the expression $6 \cdot (9 + 12)$ as shown. What is the student's error?

$$\begin{aligned} 6 \cdot (9 + 12) &= 6 \cdot 9 + 12 \\ &= 54 + 12 \\ &= 66 \end{aligned}$$

Merknad [149]: Codes (378-382)
Introductory tasks
Generalized arithmetic
Task with context
Real life

56. **Write About It** Do you think there is a Commutative Property of Division? Give an example to explain your answer.

Merknad [150]: Codes (382-386)
Introductory tasks
Mathematical language
Generalized arithmetic

★ 57. **Challenge** Use the Distributive Property to simplify $\frac{1}{6} \cdot (36 + \frac{1}{2})$.

Merknad [151]: Codes (386-390)
Introductory tasks
Mathematical language
Generalized arithmetic



SPIRAL STANDARDS REVIEW

58. **Multiple Choice** Which expression is equivalent to $(24 + 8) + 12$?

- (A) $(24 - 8) - 12$ (B) $24 + (8 + 12)$
(C) $(24 + 12) + (8 + 12)$ (D) $(24 \cdot 12) + (8 \cdot 12)$

59. **Multiple Choice** Which number completes the equation

$$12 \cdot (20 + 6) = 12 \cdot 20 + \square \cdot 6?$$

- (A) 1 (B) 6 (C) 12 (D) 20

60. **Short Response** Show how to use the Distributive Property to simplify the expression $8(27)$.

Compare. Write $<$, $>$, or $=$. (Lesson 1-2)

61. 7^2 \square 50

62. 10^3 \square 300

63. 9^2 \square 6^3

64. 2^4 \square 4^2

Simplify each expression. (Lesson 1-3)

65. $25 + 5 - (6^2 - 7)$

66. $3^3 - (6 + 3)$

67. $(4^2 + 5) \div 7$

68. $(5 - 3)^2 + (3^2 - 7)$

Think and Discuss

1. Write each expression another way. a. $12x$ b. $\frac{4}{y}$ c. $\frac{3xy}{2}$
2. Explain the difference between a variable and a constant.

Merknad [152]: Codes (411-414)
Mathematical language
Two or more variables
Repetitive Weight: 1/5
Mathematical language
Algebra as symbolic manipulation
Transforming

GUIDED PRACTICE

1

The expression $12d$ represents the number of eggs in d dozen. Evaluate the expression for each value of d , and tell what the value of the expression means.

1. $d = 3$

2. $d = 2$

3. $d = 11$

Merknad [153]: Codes (414-418)
Interpreting expression
One variable
Task with context
Real life
Algebra as a language

2 Evaluate each expression for the given value of the variable.

4. $2x - 3$ for $x = 4$

5. $r \div 3 + n$ for $n = 6$

6. $5y^2 + 3y$ for $y = 2$

Merknad [154]: Codes (418-422)
Interpreting expression
One variable
Repetitive Weight: 1/5
Algebra as a language

- 3 Evaluate each expression for the given values of the variables.
7. $\frac{8}{n} + 3m$ for $n = 2$ and $m = 5$ 8. $5a - 3b + 5$ for $a = 4$ and $b = 3$

Merknad [155]: Codes (422-426)
Interpreting expression
Two or more variables
Repetitive Weight: 1/5
Algebra as a language

INDEPENDENT PRACTICE

- 1 The expression $\frac{q}{4}$ represents the number of dollars equal to q quarters. Evaluate the expression for each value of q and tell what the value of the expression means.
9. $q = 16$ 10. $q = 36$ 11. $q = 64$

Merknad [156]: Codes (426-430)
Interpreting expression
One variable
Fractions
Repetitive Weight: 1/5
Task with context
Real life

- 2 Evaluate each expression for the given value of the variable.
12. $5y - 1$ for $y = 3$ 13. $10b - 9$ for $b = 2$ 14. $p \div 7 + p$ for $p = 14$
15. $n \div 5 + n$ for $n = 20$ 16. $3x^2 + 2x$ for $x = 10$ 17. $3c^2 - 5c$ for $c = 3$

Merknad [157]: Codes (430-434)
Interpreting expression
One variable
Exponents
Repetitive Weight: 2/5
Algebra as a language

- 3 Evaluate each expression for the given values of the variables.
18. $\frac{12}{n} + 7m$ for $n = 6$ and $m = 4$ 19. $7p - 2t + 3$ for $p = 6$ and $t = 2$
20. $x - \frac{y}{4} + 20z$ for $x = 9$, $y = 4$, and $z = 5$

Merknad [158]: Codes (434-438)
Interpreting expression
Two or more variables
Fractions
Repetitive Weight: 1/5
Algebra as a language

PRACTICE AND PROBLEM SOLVING

Evaluate each expression for the given values of the variables. Justify each step.

21. $22p \div 11 + p$ for $p = 3$ 22. $q + q^2 + q \div 2$ for $q = 4$
23. $\frac{16}{k} + 7h$ for $k = 8$ and $h = 2$ 24. $f \div 3 + f$ for $f = 18$
25. $3t \div 3 + t$ for $t = 13$ 26. $9 + 3p - 5t + 3$ for $p = 2$ and $t = 1$
27. $3m + \frac{y}{5} - b$ for $m = 2$, $y = 35$, and $b = 7$

Merknad [159]: Codes (439-442)
Interpreting expression
Two or more variables
Fractions
Exponents
Repetitive Weight: 2/5
Algebra as a language

28. The expression $60m$ gives the number of seconds in m minutes. Evaluate $60m$ for $m = 7$. How many seconds are there in 7 minutes?

Merknad [160]: Codes (442-446)
Interpreting expression
One variable
Task with context
Real life
Algebra as a language

29. **Money** Betsy has n quarters. You can use the expression $0.25n$ to find the total value of her coins in dollars. What is the value of 18 quarters?

Merknad [161]: Codes (446-450)
Interpreting expression
One variable
Decimal numbers
Task with context
Real life
Algebra as a language

Merknad [162]: Codes (450-454)
Interpreting expression
One variable
Task with context
Real life
Algebra as a language

Appendix 5: Min matematik 6 document codified

157

- Till heltalen hör positiva heltal, negativa heltal och talet noll.

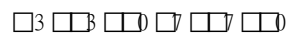
negativa heltal positiva heltal

- Tal som ligger lika långt från noll på tallinjen kallas motsatta tal.

Talen -3 och 3 är motsatta tal.

Talen 7 och -7 är motsatta tal.

- Summan av två motsatta tal är alltid noll.



1. Skriv det motsatta talet.

a. 6 b. 9 c. 15 d. 14 e. 11 f. 1

2. Räkna. Skriv den bokstav som motsvarar ditt svar.

a. $-10 + 5$ d. $-12 + 7 + 2$

b. $-7 + 7$ e. $-15 + 9 + 11$

c. $-10 + 20$ f. $-17 + 8 + 8$

-5	-3	-1	0	5	10
A	R	E	P	U	O

3. Räkna. Skriv den bokstav som motsvarar ditt svar.

a. 5 b. 7 d. $0 - 12$ g. $-1 - 8 - 6$

b. $0 - 14$ e. $-3 - 5$ h. $8 - 8 - 8$

c. $-7 - 8$ f. $-8 - 9$

-17	-15	-14	-12	-8
L	T	E	N	A

Merknad [163]: Codes (308-373)
Integers
Negative numbers
Repetitive Weight: 2/5
Introductory tasks

Merknad [164]: Codes (373-507)
Integers
Negative numbers
Introductory tasks
Repetitive Weight: 1/5

Merknad [165]: Codes (508-649)
Integers
Negative numbers
Introductory tasks
Repetitive Weight: 1/5

4. Välj rätt tecken $<$, $=$ eller $>$.

- a. $-5 + 8$ $-2 - 3$ d. $-14 - 7$ $-5 + 10$
b. $-9 + 6$ $0 - 3$ e. $-8 - 8$ $-20 + 4$
c. $-7 + 6$ $-1 - 1$ f. $8 - 12$ $-6 + 9$

Addition och subtraktion

52. Heltal

5 0 5 10 15

7 3 7

5. Beteckna och räkna. Skriv den bokstav

som motsvarar ditt svar.

a. Vilket tal får du om du först subtraherar

talet 21 från talet 12 och sedan

adderar talet 5 till differensen?

b. Vilket tal får du om du först subtraherar

talet 4 från talet -7 och sedan

subtraherar talet 9 från differensen?

c. Vilket tal får du om du först adderar

talet 12 till talet -13 och sedan adderar

talet 14 till summan?

d. Vilket tal får du om du först subtraherar

talet 7 från talet -2 och sedan

subtraherar talet 5 från differensen?

e. Vilket tal får du om du först adderar

talet -8 till dess motsatta tal och sedan

subtraherar talet 4 från summan?

f. Vilket tal får du om du till talet -12

adderar dess motsatta tal två gånger?

Merknad [166]: Codes (650-678)
Introductory tasks
Integers
Negative numbers
Repetitive Weight: 2/5

Merknad [167]: Codes (720-1431)
Introductory tasks
Integers
Repetitive Weight: 2/5
Mathematical language

-20	-14	-4	12	13
E	D	Y	S	N

Hemuppgifter

6. Räkna.

a. $4 + 1 = 5$ g. $20 - 8 = 12$ j. $18 - 20 = -2$

b. $9 - 5 = 4$ h. $6 - 7 = -1$ k. $4 + 6 = 10$

c. $1 - 2 = -1$ f. $0 - 6 = -6$ l. $8 - 2 = 6$

7. Beteckna och räkna.

a. Vilket tal får du om du först subtraherar

talet 7 från talet 2 och sedan

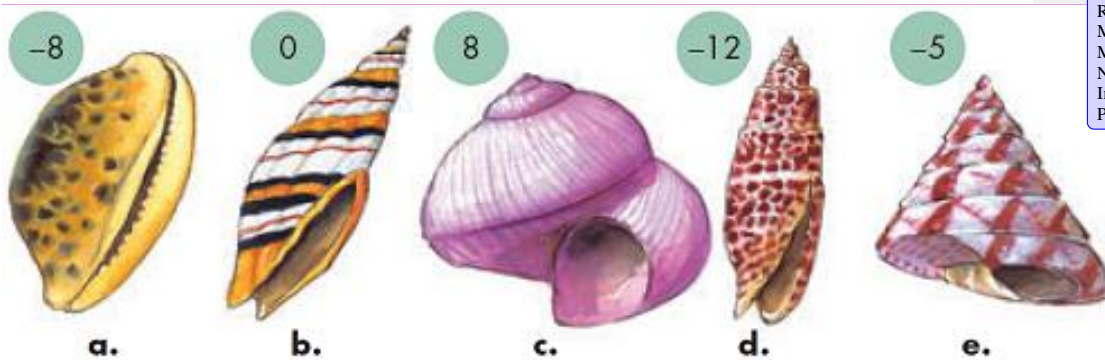
subtraherar talet 9 från differensen?

b. Vilket tal får du om du till talet -16

adderar dess motsatta tal och sedan

subtraherar talet 14 från summan?

8. Lista ut vem som äger snäckorna.



• Talet vid Saras snäcka är det motsatta talet till talet vid Sheilas snäcka.

• Jerrys tal får du då du subtraherar talet 7 från Paulas tal.

• Paulas tal får du då du adderar talet 3 till Saras tal.

• Johns tal är varken ett positivt eller ett negativt heltal.

Merknad [168]: Codes (1445-1601)
Negative numbers
Repetitive Weight: 3/5
Introductory tasks
Integers

Merknad [169]: Codes (1601-1850)
Negative numbers
Mathematical language
Repetitive Weight: 1/5
Introductory tasks
Integers

Merknad [170]: Codes (1850-2150)
Task with context
Real life
Multistep
Mathematical language
Negative numbers
Introductory tasks
Problem solving

9. Välj rätt tecken + eller -.

a. $2 \square 4 \square 3 \square 5 = 0$

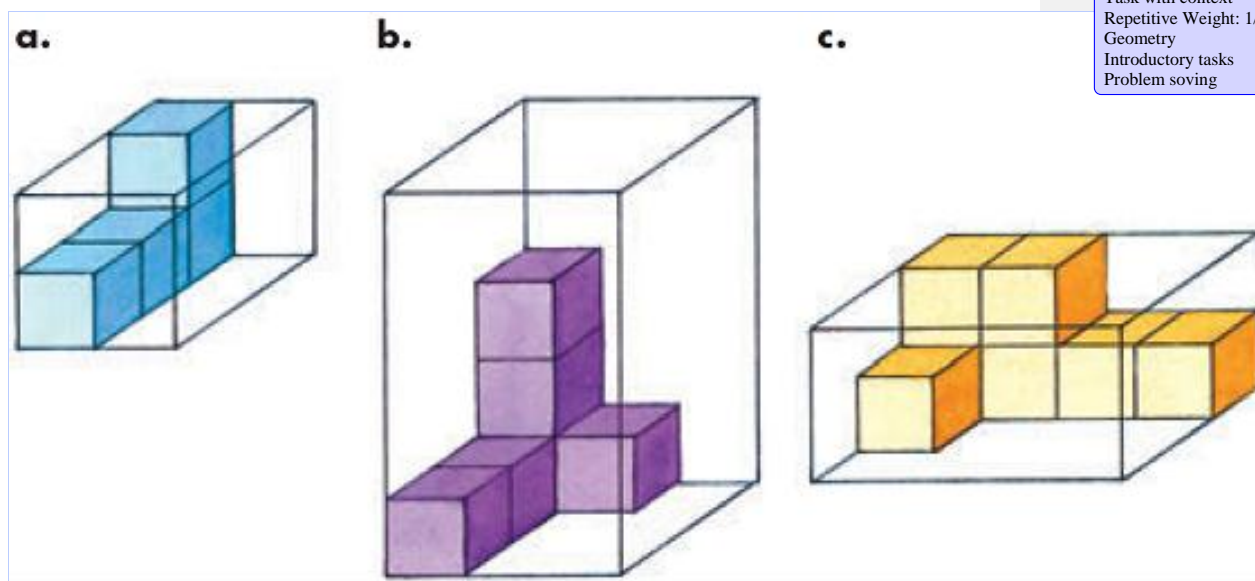
c. $0 \square 3 \square 6 \square 1 = -4$

b. $1 \square 2 \square 3 \square 4 \square 5 = -13$

d. $-8 \square 2 \square 4 \square 4 = -10$

Merknad [171]: Codes (2150-2183)
 Negative numbers
 Repetitive Weight: 1/5
 Introductory tasks
 Equivalence

10. Hur många små kuber behövs ytterligare för att lådan ska fyllas?



Merknad [172]: Codes (2184-2255)
 Task with context
 Repetitive Weight: 1/5
 Geometry
 Introductory tasks
 Problem solving

160

En ekvation består av två uttryck och ett likhetstecken mellan dem.

ekvation

$x \square \square 8 \square \square 5$

uttryck uttryck

Vi bildar en ekvation utgående från bilden och beräknar värdet på x.

$7 \text{ kg} \square \square x \square \square 12 \text{ kg}$

$x \square \square 12 \text{ kg} \square \square 7 \text{ kg}$

$x \square \square 5 \text{ kg}$

Kontroll: $7 \text{ kg} \square \square 5 \text{ kg} \square \square 12$

• Värdet på den obekanta termen x får du genom att subtrahera den andra

termen, d.v.s. 7 kg, från 12 kg.

• Du kan kontrollera lösningen av en ekvation genom att placera talet du fått

i stället för x i ekvationen.

Exempel x 4

x 9

x Kontroll: 5 4

53. Addition med den ena termen obekant

11. Lös ekvationerna enligt exemplet i undervisnings-

rutan. Skriv den bokstav som motsvarar ditt svar.

a. $5 + x = 9$

b. $x + 7 = 10$

c. $x + 7 = 15$

d. $x + 5 = 11$

e. $9 + x = 13$

f. $21 + x = 24$

g. $29 + x = 34$

h. $x + 13 = 30$

i. $x + 22 = 30$

j. $20 + x = 36$

k. $x + 12 = 44$

l. $39 + x = 45$

m. $46 + x = 51$

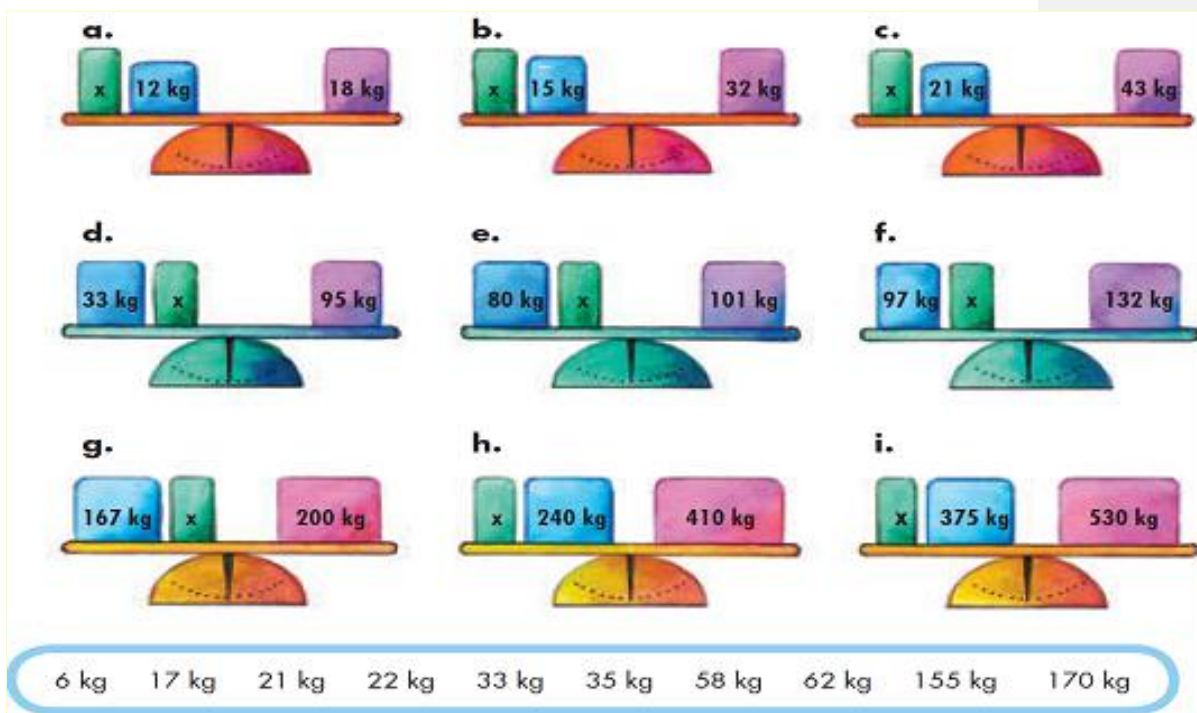
n. $x + 17 = 31$

3	4	5	6	8	14	16	17	32
E	N	Å	G	R	F	A	P	L

12. Bilda en ekvation och beräkna värdet på x. Kontrollera ditt svar.

Merknad [173]: Codes (2813-3123)
One variable
Repetitive Weight: 4/5
Algebra as equation solving
doing/undoing

Merknad [174]: Codes (3124-3196)
Equivalence
One variable
Repetitive Weight: 3/5
Algebra as a language
Formulating equation
Algebra as equation solving
doing/undoing



Hemuppgifter

13. Lös ekvationerna. Kontrollera ditt svar genom att placera talet du fått i stället

för x i ekvationen

a. $44 + 10x = 96$ $184 + 200 = x + 100$

b. $x + 76 = 90$ $255 + 420 = 1000 + 1471 + x$

14. Bilda en ekvation och beräkna värdet på x .



Merknad [175]: Codes (3210-3419)
One variable
Repetitive Weight: 2/5
Algebra as equation solving
doing/undoing

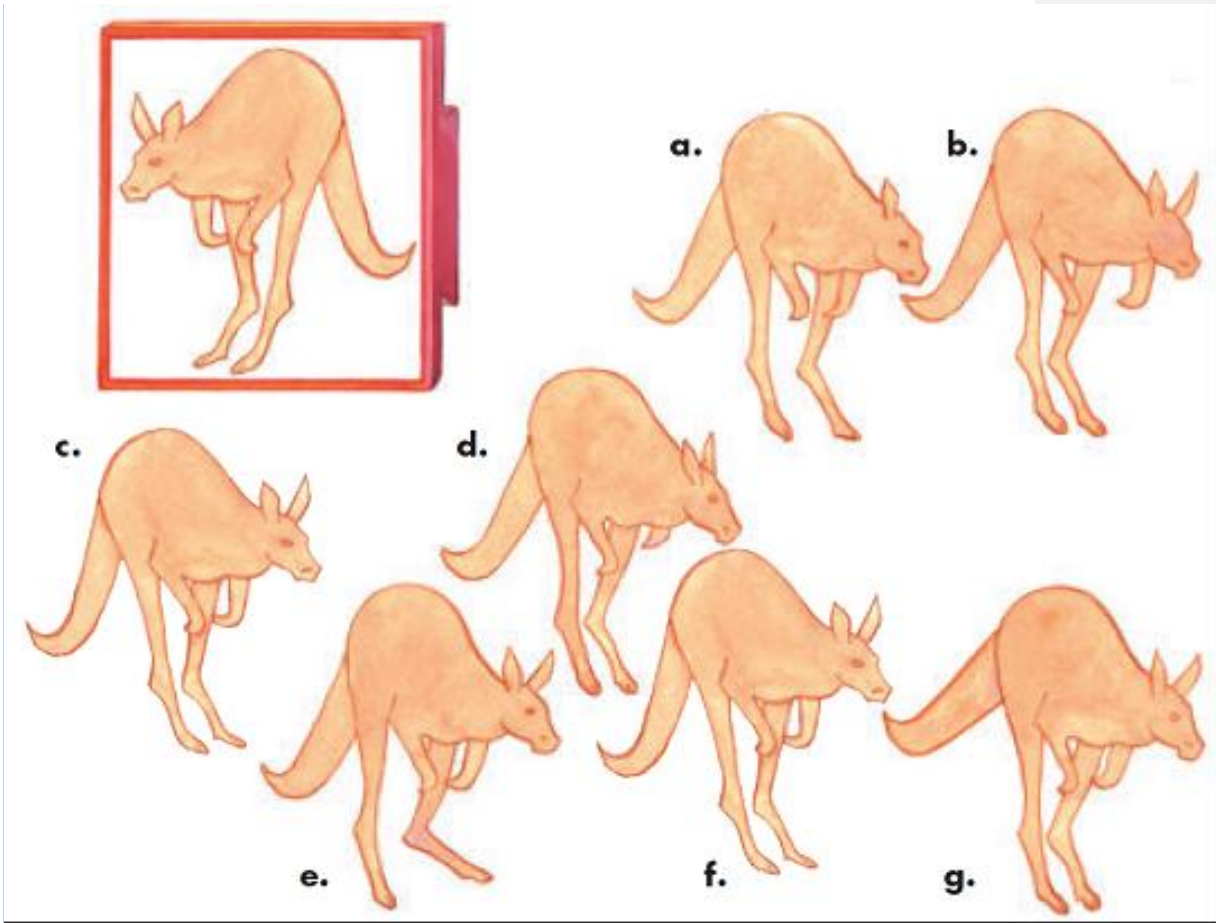
Merknad [176]: Codes (3419-3468)
One variable
Repetitive Weight: 1/5
Algebra as a language
Formulating equation
Algebra as equation solving
doing/undoing

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Tilläggsuppgifter

15. Vilka av bilderna är gjorda med stämpeln?

Merknad [177]: Codes (3491-3539)
Task with context
Geometry
Problem solving



16. Skriv de tre följande heltalen.

a. 13, 8, 3

b. 55, 30, 5

c. 29, 17, 5

d. 421, 365, 309

Merknad [178]: Codes (3540-3578)
Repetitive Weight: 1/5
Pattern
Identifying and continuing a pattern

17. Lista ut vilket heltal som passar i stället för bokstaven.

a. 22 18 k 31 90 608 249 18 153

b. 45 z 7 9 38 z 23 3 60

c. 919 k 55 307 600 e 24 e 600

Merknad [179]: Codes (3579-3801)
One variable
Repetitive Weight: 2/5
Algebra as equation solving
doing/undoing

18. Vilka tre på varandra följande heltal har summan

Merknad [180]: Codes (3801-3877)
Algebra as problem solving
Mathematical language

a. 24? b. 360? c. 450?

163

genom att addera 13 kg med 5 kg.

19. Lös ekvationerna. Skriv motsvarande bokstav i häftet.

a. 60 x =

b. 11 x = 6

c. x 28 9

d. 21 x =

e. 45 x = 0

f. x 19

g. x 19 69

h. 46 x = 41

5	15	18	38	57	88
A	G	E	O	P	J

Merknad [181]: Codes (4557-4734)
One variable
Repetitive Weight: 2/5
Algebra as equation solving
doing/undoing

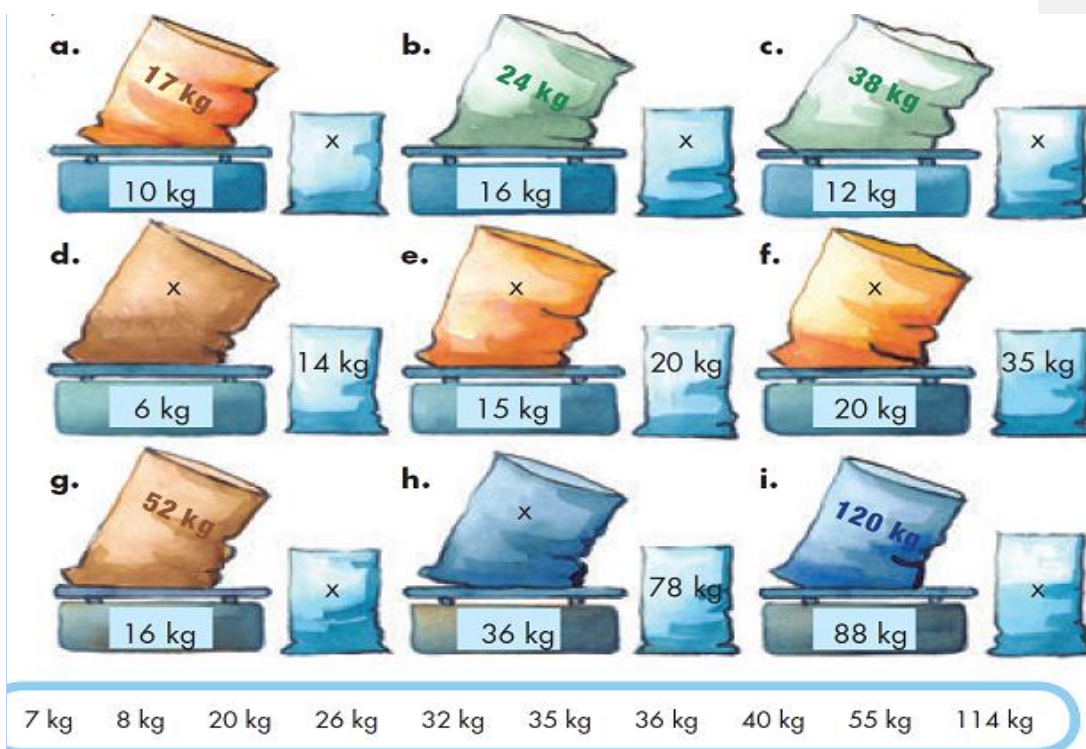
54. Subtraktion med den ena termen obekant

164

20. Det har tagits sand ur säcken på vågen. Bilda en ekvation och beräkna värdet

på x. Kontrollera ditt svar.

Merknad [182]: Codes (4782-4894)
Task with context
Real life
Repetitive Weight: 3/5
Algebra as a language
One variable
Formulating equation
Algebra as equation solving
doing/undoing



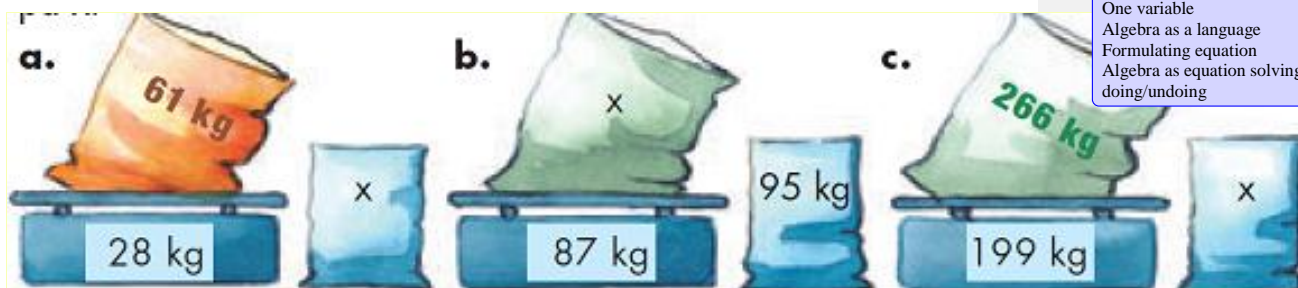
Hemuppgifter

21. Lös ekvationerna. Kontrollera ditt svar genom att placera värdet på x i ekvationen.

a. $13 - x = 7$ $734 = 70$ e. $117 - x = 98$

b. $23 - x = 14$ d. $x - 95 = 127$

22. Det har tagits sand ut säcken på vågen. Bilda en ekvation och beräkna värdet på x.



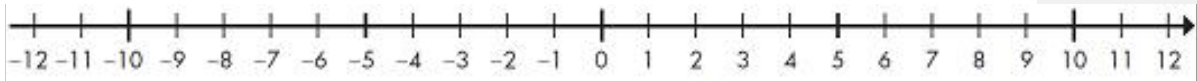
Merknad [183]: Codes (4909-5090)
One variable
Repetitive Weight: 1/5
Algebra as equation solving
doing/undoing

Merknad [184]: Codes (5090-5179)
Task with context
Real life
Repetitive Weight: 1/5
One variable
Algebra as a language
Formulating equation
Algebra as equation solving
doing/undoing

Tilläggsuppgifter

23. Vilka två figurer är exakt likadana som modellfiguren?

Merknad [185]: Codes (5198-5259)
Task with context
Problem solving



a. $x < 10$

c. $x > -9$

e. $-4 < x < 5$

g. $-7 < x < -1$

b. $x \leq -3$

d. $x \geq 7$

f. $-12 \leq x \leq -8$

h. $9 \leq x \leq 12$

Appendix 6: Matte direkt år 7 document codified



Lös följande talgåta! Försök att förklara varför resultatet alltid blir detsamma

vilket starttal du än väljer.

Lös talgåtorna på sidan 171 på liknande sätt.

172 Algebra och ekvationer

Uttryck med flera räknesätt

På sjörövarön handlas det med föremål som hittats i olika kistor. Priserna är i guldmynt. Se prislistan.

Om kostnaden kan skrivas $30 + 3 \cdot 50$ har du köpt ett armband och tre halsband.

$30 + 3 \cdot 50$ kallas ett **uttryck** för kostnaden. Du får betala:

$$30 + 3 \cdot 50 = 30 + 150 = 180 \text{ guldmynt.}$$

Om du köper 5 pärlor och 5 ädelstenar blir uttrycket för kostnaden:

$$5 \cdot (10 + 12) = 5(10 + 12)$$

Man skriver oftast inte ut multiplikationstecknet framför en parentes. Kostnaden blir:

$$5(10 + 12) = 5 \cdot 22 = 110 \text{ guldmynt}$$

När man har flera räknesätt i en uppgift räknar man i den här ordningen:

1. Parenteser
2. Multiplikation och division
3. Addition och subtraktion

På sjörövarens nota stod följande uttryck.

Merknad [186]: Codes (673-787)
Generalized arithmetic

Vad hade han sålt och hur många guldmynt fick han?

1 a) $2 \cdot 10 + 15$ b) $30 + 2 \cdot 50$ c) $5 \cdot 12 + 3 \cdot 10$

2 a) $3(15 + 50)$ b) $2(10 + 15)$ c) $15 + 2(10 + 3)$

3 Skriv ett uttryck och räkna ut hur mycket du ska betala för a) 3 ringar, 10 pärlor och 5 ädelstenar

b) 4 halsband, ett armband och 5 ringar

Räkna ut

4 a) $2 + 4 \cdot 5$ b) $8 \cdot 4 - 6$ c) $3 \cdot 7 + 6 \cdot 2$

5 a) $12 + 5$ b) $7 + 36$

3

c) $14 + 50$

2 10

6 a) $3(4 + 5)$ b) $2(6 - 3) + 5$ c) $3(4 + 2) - 2(8 - 3)$

173 Algebra och ekvationer

Uttryck med en variabel

En **variabel** är något som kan variera, något som kan ha olika värden. Ofta skriver man variabeln som en bokstav.



3 cm Omkrets: $(3 + 3 + 3 + 3)$ cm =

$= 4 \cdot 3$ cm = 12 cm

Man skriver inte ut multiplikationstecknet mellan en siffra och en variabel.

Man skriver inte ut multiplikationstecknet mellan en siffra och en variabel.

x Omkrets: $x + x + x + x =$



$= 4 \cdot x = 4x$

I den lilla kvadraten vet vi inte hur lång sidan är, den kan variera. Vi kallar den x . Kvadratens omkrets blir då $4x$.

Uttrycket för kvadratens omkrets är $4x$.

Omkretsen av **alla** kvadrater kan beskrivas med uttrycket $4x$. Värdet på x är längden på sidan. När du vill beräkna omkretsen av en speciell kvadrat sätter du in längden på sidan istället för x .

En kvadrat som har sidan 5 cm har omkretsen $4 \cdot 5$ cm = 20 cm. Man säger att **värdet på uttrycket** $4x$ är 20 cm.

7 På sjöörvarön har olika byar sina speciella "bymärken", gjorda av lika långa pinnar – ju längre omkrets (fler pinnar), desto mäktigare by. Vad blir omkretsen av bymärkena om längden på varje pinne är x ?

a) b) c)

Merknad [187]: Codes (1639-1721)
Introductory tasks
sequence of operations
Repetitive Weight: 1/5

Merknad [188]: Codes (1721-1801)
sequence of operations
Repetitive Weight: 1/5
Introductory tasks

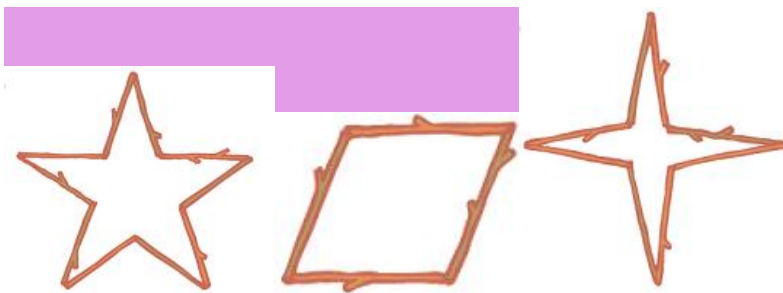
Merknad [189]: Codes (1801-1953)
Task with context
Real life
Repetitive Weight: 1/5
Introductory tasks
sequence of operations

Merknad [190]: Codes (1953-2029)
sequence of operations
Repetitive Weight: 1/5
Introductory tasks

Merknad [191]: Codes (2030-2168)
sequence of operations
Repetitive Weight: 1/5
Introductory tasks
Fractions

Merknad [192]: Codes (2168-2246)
sequence of operations
Repetitive Weight: 1/5
Introductory tasks

Merknad [193]: Codes (3231-3542)
One variable
Task with context
Repetitive Weight: 1/5
Geometry
Formulating expression
Real life
Algebra as a language



8 Vad blir omkretsen av varje figur i uppgift 7 om x har värdet

- a) 4 cm b) 7 cm

9 a) Skriv ett uttryck för omkretsen av dessa stjärnformade bymärken om längden på en pinne i märke A är $3y$ medan längden på en pinne i märke B är y .



Merknad [194]: Codes (3542-3638)
 Task with context
 Geometry
 Interpreting expression
 Algebra as a language
 One variable
 Repetitive Weight: 1/5

Merknad [195]: Codes (3638-3868)
 One variable
 Task with context
 Geometry
 Formulating expression
 Multistep
 Real life
 Algebra as a language

Kom ihåg

$$3y = 3 \cdot$$

b) Hur stor blir omkretsen av bymärkena om y är 5 cm?

174 Algebra och ekvationer

Uttryck med flera variabler



3 cm

a

4 cm

b

Omkrets: $4 + 3 + 4 + 3 \text{ cm} =$ Omkrets: $a + b + a + b =$

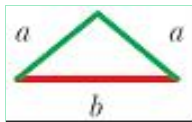
$2 \cdot 4 + 2 \cdot 3 \text{ cm} = 8 + 6 = 14 \text{ cm}$ $2 \cdot a + 2 \cdot b = 2a + 2b$

Omkretsen av den första rektangeln är 14 cm. I den andra rektangeln vet vi inte hur långa sidorna är. Här behövs två variabler eftersom sidorna har två olika längder.

Piratbossarna i byarna hade sina egna bomärken, gjorda av pinnar med olika färgade längder. Även här gällde att ju längre omkrets desto mäktigare piratboss.

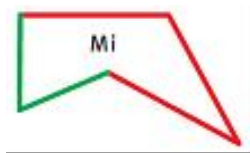
I0 Den "fattigaste" hövdingen hade en triangel som sitt bomärke

a) Skriv ett uttryck för triangelns omkrets.



b) Vilken omkrets har triangeln om

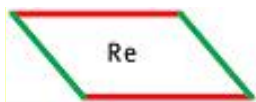
$a = 2 \text{ cm}$ och $b = 3 \text{ cm}$?



II De övriga hövdingarna Do, Re och Mi gör sina bomärken

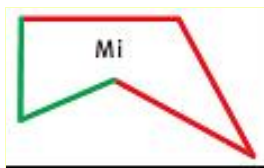
av likadana pinnar som den fattigaste hövdingen.

a) Skriv ett uttryck för omkretsen av deras bomärken.



Merknad [196]: Codes (4519-4699)
Task with context
Formulating expression
Two or more variables
Multistep
Real life
Algebra as a language
Geometry

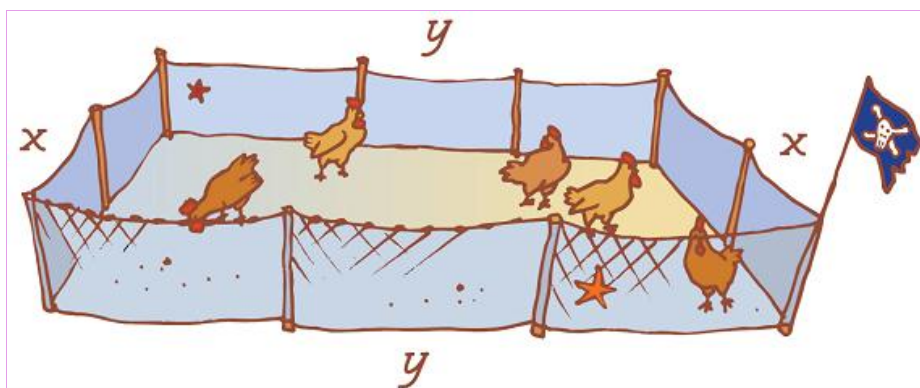
Merknad [197]: Codes (4734-5017)
Task with context
Geometry
Real life
Two or more variables
Formulating expression
Algebra as a language
Multistep



b) Vilken omkrets har bomärkena?

Vem var mäktigast?

12 Bilden visar en inhägnad för hönsen i en by. Alla inhägnader var rektangelformade och just denna hade en total längd på staketet som var 20 m. Vilka värden kan x och y ha? Ge två förslag.



Algebra och ekvationer 175

Tolka uttryck

På sjöörvarön bor familjen Robinson. Pappa Pirat är x år.

Mamma Madam är $(x + 3)$ år.

alltså tre år äldre än pappa.

Långfläta är $(x - 26)$ år, alltså 26 år yngre än pappa.

I år är sonen Lillpiran ($\frac{x}{4}$) år.

alltså en fjärdedel så gammal som pappa.

13 Hur gamla är familjemedlemmarna om pappa är 40 år?

14 Långfläta samlar snäckor i en låda. Hon har x st.

Vilket uttryck visar antalet snäckor om Långfläta

a) lägger till 7 snäckor

Merknad [198]: Codes (5017-5212)
Interpreting expression
Task with context
Two or more variables
Geometry
Real life
Algebra as a language

Merknad [199]: Codes (5256-5568)
Interpreting expression
One variable
Task with context
Real life
Algebra as a language

Merknad [200]: Codes (5568-5791)
One variable
Interpreting expression
Repetitive Weight: 1/5
Fractions
Algebra as a language

$$\begin{array}{ccc} x - 7 & x + 7 & x - 7 \\ \frac{x}{5} & 5 - x & x - 5 \\ 2 + x & 2 \cdot x & x + 2 \\ \frac{x}{6} & x \cdot 6 & \frac{6}{x} \end{array}$$

b) tar bort 5 snäckor

c) dubblar antalet snäckor

d) delar antalet snäckor med 6

15 Sköldpaddan är y år. Husmusen är 5 år yngre.

Hur kan du skriva Husmusens ålder? Välj rätt uttryck.

$5 + y$ $y - 5$ $5y$

Merknad [201]: Codes (5791-5943)
Interpreting expression
One variable
Task with context
Real life
Algebra as a language

16 Lillpiran är x år och hans värja är tre gånger

så gammal. Hur gammal är värjan? Välj rätt uttryck.

$x + 3$ $3x$ $\frac{x}{3}$

Merknad [202]: Codes (5943-6099)
Interpreting expression
One variable
Task with context
Real life
Algebra as a language

17 Långfläta har en kniv som är x cm lång.

Lillpirans kniv är 5 cm längre.

a) Vilket uttryck beskriver hur lång Lillpirans kniv är?

$5x$ $x + 5$ $x - 5$

b) Hur lång är Lillpirans kniv om Långflätas kniv är 10 cm?

176 Algebra och ekvationer

18 Lillpiran staplar guldmynt utan att räkna antalet mynt.

När han staplat färdigt säger han att det finns n st mynt i stapeln. Välj rätt uttryck till meningen.

$$\begin{array}{ccc} \frac{n}{2} & n - 2 & \\ n + 2 & 2 \cdot n & \end{array}$$

a) Stapeln ökas med två mynt

b) Lillpiran tar bort två mynt från stapeln

Merknad [204]: Codes (6373-6687)
Interpreting expression
One variable
Task with context
Real life
Repetitive Weight: 1/5
Fractions
Algebra as a language

c) Hälften av mynten tas bort

d) Lillpiran dubblar antalet mynt i stapeln

19 Talet eller variabeln a har värdet 6. Vilket värde får då

- a) $a + 7$ b) $a - 4$ c) $2a$

Kom ihåg!

$2 \cdot a$ skrivs som $2a$

Kom ihåg!

$2 \cdot a$ skrivs som $2a$

20 Låt x betyda talet 6. Vilket tal är då

- a) \underline{x}
2 b) $3x + 2$ c) $12 - 2x$

21 Mamma har ett bälte med x st ädelstenar. Långflätas bälte har bara hälften så många ädelstenar. Två av uttrycken beskriver antalet ädelstenar på Långflätas bälte. Vilka?

$2x$	$\frac{x}{2}$	$\frac{x}{0,5}$	$0,5x$
------	---------------	-----------------	--------

$x2x$

2



22 Tänk dig vilka som bor hemma hos dig. Beskriv din familj på samma sätt som familjen Robinson i exemplet. Kalla någon vuxens ålder för x år.

Algebra och ekvationer 177

Vi räknar ut det hemliga talet

Exempel

Merknad [205]: Codes (6687-6797)
Interpreting expression
One variable
Repetitive Weight: 1/5
Algebra as a language

Merknad [206]: Codes (6861-6962)
Interpreting expression
One variable
Repetitive Weight: 1/5
Fractions
Algebra as a language

Merknad [207]: Codes (6962-7139)
Interpreting expression
One variable
Task with context
Real life
Fractions
Decimal numbers
Algebra as a language

Merknad [208]: Codes (7148-7292)
One variable
Formulating expression
Task with context
Real life
Algebra as a language



I påsen finns likadana guldmynt som ligger på vågskålarna. Påsen väger så lite att den inte påverkar vågens utslag.

Hur många mynt måste finnas i påsen för att vågskålarna ska väga jämnt?



$$+ 8 = 20$$



$$= 12$$

23 Hur många guldmynt innehåller påsen?

a) b)






För att få vara med och dela på guldpengarna måste du svara rätt på alla uppgifter.




Merknad [209]: Codes (7572-7688)
 One variable
 Figure or empty box
 Repetitive Weight: 1/5
 Equivalence

Merknad [210]: Codes (7689-7906)
 One variable
 Repetitive Weight: 1/5
 Task with context
 Real life
 Figure or empty box
 Equivalence

Byt ut påsarna mot ett tal så att likheten stämmer.

24 a)  + 4 = 8 b)  + 2 = 9 c) 17 = 6 + 

25 a)  - 10 = 35 b) 12 =  - 3 c) 46 = 19 + 

26 a) 56 = 34 + 2  b) 14 + 3  = 41 c) 5  - 25 = 75

27 Hur många guldpengar fanns det i alla påsarna tillsammans i uppgift 24-26?

Fick du vara med och dela?

178 Algebra och ekvationer

Ekvationer



En ekvation innehåller en variabel

och ett **likhetstecken**. Variabeln

betecknas oftast med x .

Det som står till vänster om likhets-

tecknet och det som står till höger om

likhetstecknet är alltid värt lika mycket.

Du **löser** ekvationen genom att räkna ut

vilket tal som x står för.

Exempel

$x + 12 = 23$ (ett tal plus 12 är 23)

$x - 6 = 8$ (ett tal minus 6 är 8)

Merknad [211]: Codes (7906-7984)
Task with context
Real life
Equivalence
One variable
Figure or empty box
Repetitive Weight: 1/5

Merknad [212]: Codes (7984-8057)
One variable
Repetitive Weight: 1/5
Task with context
Real life
Figure or empty box
Equivalence

Merknad [213]: Codes (8057-8165)
Task with context
Real life

Appendix 7: Matte direkt 6B document codified

Mål

När du har arbetat med det här

kapitlet ska du kunna

- räkna med likheter
- lösa enkla ekvationer
- tolka och skriva uttryck med

variabler

94 Algebra

Algebra

Till 5 blå plattor behövs

det 6 vita plattor.

- Hur många vita plattor behövs till 10 blå plattor?
- Om du har 20 vita plattor, hur många blå plattor behöver du då till mönstret?

Borgen6Bkap9.qxd:Borgen6Bkap9.qxd 10-07-02 16.14 Sida 94

- Abu Jafar Muhammed ibn Musa al-Khwararizmi

hette en arabisk matematiker. Han levde i

Bagdad omkring år 800. Hans mest uppmärksammade

bok handlar om algebra och heter

Hisab al-jabr w'al-muqabala. Ordet algebra

kommer från boktiteln *al-jabr*. Från början

använde araberna bokstäverna i sitt alfabet som

tecken för talen. Det är alltså inte så konstigt att

algebra idag brukar kallas för bokstavsräkning.

- Arabiska siffror
- I arabisk text läser man från höger till vänster, det gäller också tal. Intressant är att siffran 2 från början var upp- och nedvänd. Det var först på 1100-talet som siffrornas utseende fick den form som de har idag.

Algebra 95

9 8 7 6 5 4 3 2 1

Borgen6Bkap9.qxd:Borgen6Bkap9.qxd 10-07-02 16.14 Sida 95

96 Algebra

Likheter

Här är ett exempel på en likhet. $4 + 5 = 6 + 3$

Det måste alltid vara samma värde på båda sidor om likhetstecknet.

Vilket tal ska stå i stället för rutan?

$$8 + \square = 20 + 4$$

I stället för ska det stå 16.

Det är lika

mycket på båda sidor

om likhetstecknet.

Vilket tal ska stå i stället för rutan?

1

$$\text{a) } 3 + \square = 8 + 2 \quad \text{b) } 8 + \square = 12 + 6 \quad \text{c) } 16 + \square = 35$$

2

$$\text{a) } 70 + 12 = 8 + \square \quad \text{b) } 53 + 20 = 45 + \square \quad \text{c) } 36 = \square + 5$$

Merknad [214]: Codes (1395-1442)
Introductory tasks
Equivalence
Repetitive Weight: 1/5
Interpreting equation
One variable
Figure or empty box
Algebra as a language

Merknad [215]: Codes (1442-1448)
Introductory tasks
Equivalence
Repetitive Weight: 1/5
Interpreting equation
One variable
Figure or empty box
Algebra as a language

3

a) $8 - \square = 11 - 6$ b) $10 - \square = 6 + 2$ c) $\square - 12 = 24$

Merknad [216]: Codes (1449-1455)
Introductory tasks
Equivalence
Interpreting equation
One variable
Repetitive Weight: 1/5
Figure or empty box
Algebra as a language

4

a) $12 - 7 = 15 - \square$ b) $14 + 12 = 40 - \square$ c) $27 = \square - 5$

Merknad [217]: Codes (1455-1462)
Introductory tasks
Equivalence
Repetitive Weight: 1/5
Interpreting equation
One variable
Figure or empty box
Algebra as a language

Vilket tal ska stå i stället för bokstaven?

5 a) $3 + x = 8 + 2$ b) $8 + y = 12 + 6$

6 a) $7 + 11 = 8 + p$ b) $25 = 15 + q$

7 a) $9 - x = 11 - 6$ b) $20 - y = 6 + 2$ c) $z - 12 = 36$

8 a) $12 + 7 = 35 - p$ b) $22 + 13 = 40 - q$ c) $50 = r - 5$

Merknad [218]: Codes (1462-1543)
Introductory tasks
Equivalence
Repetitive Weight: 1/5
Interpreting equation
One variable
Algebra as a language

Merknad [219]: Codes (1543-1578)
Introductory tasks
Equivalence
Repetitive Weight: 1/5
Interpreting equation
One variable
Algebra as a language

Ibland står det

en bokstav i stället

för en ruta.

Borgen6Bkap9.qxd:Borgen6Bkap9.qxd 10-07-02 16.14 Sida 96

Merknad [220]: Codes (1578-1631)
Introductory tasks
Equivalence
Repetitive Weight: 1/5
Interpreting equation
One variable
Algebra as a language

Merknad [221]: Codes (1631-1686)
Introductory tasks
Equivalence
Repetitive Weight: 1/5
Interpreting equation
One variable
Algebra as a language

Algebra 97

9 I vilken likhet är $x = 10$?

$x + 15 = 18 + 7$ $x + 5 = 20$ $10 = x + 20$

Merknad [222]: Codes (1814-1890)
Introductory tasks
Equivalence
Interpreting equation
One variable
Algebra as a language

10 I vilken likhet är $z = 16$?

$34 - z = 24$ $z - 4 = 8 + 4$ $30 - z = 24$

Merknad [223]: Codes (1890-1968)
Introductory tasks
Equivalence
Interpreting equation
One variable
Algebra as a language

11 Välj den likhet som betyder:

a) Ett tal ökas med 5 och

summan är lika med 12.

Merknad [224]: Codes (1968-2118)
Interpreting equation
One variable
Repetitive Weight: 1/5
Mathematical language
Algebra as a language
Introductory tasks
Equivalence

b) Summan av ett tal och 12 är 20.

$$x + 5 = 12$$

$$x + 5 = 20$$

$$x + 12 = 20$$

12 Välj den likhet som betyder:

a) Ett tal minus 8 är lika med 15.

b) Differensen mellan ett tal och 15

är lika med 8.

$$x - 15 = 8$$

$$x - 8 = 15$$

$$8 - x = 15$$

13 Skriv en likhet som betyder:

a) Jag lägger 8 till mitt tal och summan blir 15.

b) Jag minskar mitt tal med 8 och får talet 15.

Jag kallar

mitt tal för x .

Borgen6Bkap9.qxd:Borgen6Bkap9.qxd 10-07-02 16.14 Sida 97

98 Algebra

Vilket tal ska stå i stället för x för att

likheten ska stämma?

$$3 \cdot x = 15 + 6$$

Om x är 7 blir det 21 på båda sidor om likhetstecknet.

x är 7.

$$3 \cdot x$$

kan också skrivas

$$3x.$$

Vilket tal ska stå i stället för bokstaven

Merknad [225]: Codes (2118-2270)
Interpreting equation
One variable
Repetitive Weight: 1/5
Mathematical language
Algebra as a language
Introductory tasks
Equivalence

Merknad [226]: Codes (2270-2400)
One variable
Repetitive Weight: 1/5
Mathematical language
Algebra as a language
Introductory tasks
Equivalence
Formulating equation

Merknad [227]: Codes (2665-2791)
Interpreting equation
Repetitive Weight: 1/5
One variable
Equivalence
Algebra as a language

för att likheten ska stämma?

14 a) $3 \cdot x = 12 + 6$ b) $5 \cdot y = 30$ c) $20 + 4 = 6 \cdot z$

15 a) $2x = 14 + 4$ b) $4y = 28$ c) $29 - 8 = 7z$

16 I vilka två likheter är $x = 4$?

$6x - 4 = 20$ $30 = 7x$ $48 = 10x + 8$

Vilket tal ska stå i stället för bokstaven?

17

a) $\frac{12}{x} = 4$ b) $\frac{20}{y} = 5$ c) $\frac{15}{z} = 3$

18

a) $\frac{x}{2} = 10$ b) $\frac{y}{2} = 6$ c) $\frac{z}{4} = 6$

Vilken av likheterna betyder:

19 a) 4 gånger ett tal är lika med 24.

b) 8 gånger ett tal är lika med 24.

$4x = 24$ $24x = 4$ $8x = 24$

$\frac{30}{x} = 5$ $5x = 30$ $\frac{x}{5} = 30$

20 a) Ett tal dividerat med 5 är 30.

b) 30 dividerat med ett tal är 5.

c) 5 multiplicerat med ett tal är 30.

Merknad [228]: Codes (2791-2835)
Interpreting equation
One variable
Repetitive Weight: 1/5
Equivalence
Algebra as a language
Introductory tasks

Merknad [229]: Codes (2835-2906)
Interpreting equation
One variable
Equivalence
Algebra as a language
Introductory tasks

Merknad [230]: Codes (2906-2956)
Interpreting equation
One variable
Fractions
Repetitive Weight: 1/5
Algebra as a language
Introductory tasks
Equivalence

Merknad [231]: Codes (2957-2963)
Interpreting equation
One variable
Fractions
Repetitive Weight: 1/5
Algebra as a language
Introductory tasks
Equivalence

Merknad [232]: Codes (2964-3100)
Interpreting equation
One variable
Equivalence
Repetitive Weight: 1/5
Algebra as a language
Introductory tasks

Merknad [233]: Codes (3100-3212)
Interpreting equation
One variable
Repetitive Weight: 1/5
Mathematical language
Algebra as a language
Fractions
Introductory tasks
Equivalence

Ekvationer

I stället för **likhet** kan man säga **ekvation**.

När man löser en ekvation, räknar man ut vilket tal som gör att likheten stämmer. Talet som man söker, brukar man kalla för x .

Lös ekvationen $x + 8 = 14$

Vilket tal plus 8 är lika med 14?

6 plus 8 är 14.

Då är $x = 6$.

$$x + 8 = 14$$

$$x = 6$$

$$x = 5$$

$$= 8$$

$$x = 40$$

Lös ekvationen. Glöm inte att först skriva av ekvationerna.

$$21 \text{ a) } x + 17 = 25 \text{ b) } 23 + x = 32 \text{ c) } x + 12 = 40$$

$$22 \text{ a) } 45 - x = 10 \text{ b) } 82 - x = 75 \text{ c) } x - 8 = 46$$

Börja med
att skriva av
ekvationen.

Lös ekvationen.

$$23 \text{ a) } 8x = 40 \text{ b) } 4x = 36 \text{ c) } 5x = 45$$

$$a) \frac{x}{5} = 10 \quad b) \frac{x}{2} = 13 \quad c) \frac{32}{x} = 8$$

24

$$25 \text{ a) } x + 1,2 = 4,8 \text{ b) } 4,5 - x = 3,2 \text{ c) } 6x = 36$$

Merknad [234]: Codes (3584-3692)
One variable
Repetitive Weight: 1/5
Algebra as equation solving
Numeric trial

Merknad [235]: Codes (3692-3739)
One variable
Repetitive Weight: 1/5
Algebra as equation solving
Numeric trial

Merknad [236]: Codes (3775-3827)
One variable
Repetitive Weight: 1/5
Algebra as equation solving
Numeric trial

Merknad [237]: Codes (3827-3834)
One variable
Fractions
Repetitive Weight: 1/5
Algebra as equation solving
Numeric trial

Merknad [238]: Codes (3834-3882)
One variable
Repetitive Weight: 1/5
Decimal numbers
Algebra as equation solving
Numeric trial

$$\frac{150}{x} = 15$$

26 a) $125 + x = 420$ b) $x - 240 = 500$ c)

Lös ekvationen = 8

Vilket tal delat med 5 är 8?

40 delat med 5 är 8.

Då är $x = 40$.

$x5$

När man

löser ekvationer skriver

man likhetstecknen

under varandra.

Borgen6Bkap9.qxd:Borgen6Bkap9.qxd 10-07-02 16.14 Sida 99



Osman är 3 år äldre än Mohammed.

Leyla är 5 år yngre än Mohammed.

27 Hur gammal är Osman när Mohammed är

a) 10 år b) 15 år c) 30 år

28 Hur gammal är Leyla när Mohammed är

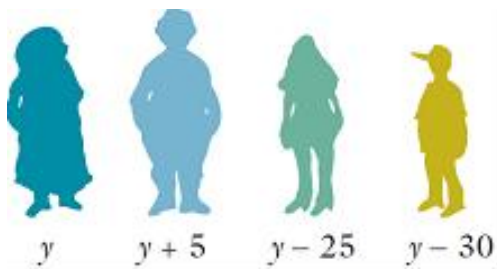
a) 10 år b) 15 år c) 30 år

Merknad [239]: Codes (3882-3926)
One variable
Fractions
Repetitive Weight: 1/5
Algebra as equation solving
Numeric trial

Merknad [240]: Codes (4138-4272)
Task with context
Interpreting expression
One variable
Repetitive Weight: 1/5
Real life
Algebra as a language

Merknad [241]: Codes (4272-4338)
Task with context
Interpreting expression
One variable
Repetitive Weight: 1/5
Real life
Algebra as a language

29 Mamma är 40 år. Hur gammal är



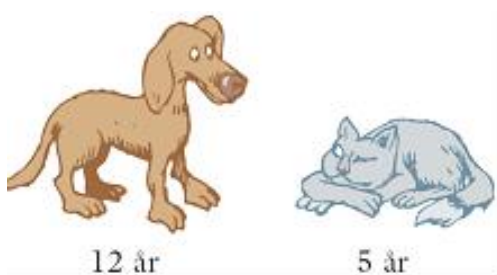
a) pappa

b) storasyster

c) lillebror

30 a) Vi kallar hundens ålder för y .

Hur kan vi då skriva kattens ålder?



$y - 5$ $y - 7$ $y - 12$

b) Kattens ålder kallas för x .

Hur kan vi då skriva hundens ålder?

$x - 7$ $x + 5$ $x + 7$

Men det

här är en ekvation

för den har ett

likhetstecken.

Merknad [242]: Codes (4338-4411)
Task with context
Interpreting expression
One variable
Repetitive Weight: 1/5
Real life
Algebra as a language

Merknad [243]: Codes (4411-4611)
Task with context
Interpreting expression
One variable
Repetitive Weight: 1/5
Real life
Algebra as a language

Skriv det uttryck som betyder

31

a) 5 mer än a

b) 5 mer än x

$$x - 5x + 5a + 5a - 5$$

32

a) 6 mindre än y

b) 8 mindre än y

$$y - 88 - y6 - yy - 6$$

33

a) 2 mer än z

b) 4 mindre än z

c) 4 mindre än a

$$z - 4z + 2a - 4$$

Skriv ett uttryck som betyder

34 a) 4 mer än x b) 8 mer än x c) 9 mer än y

35 a) 2 mindre än a b) 4 mindre än x c) 5 mindre än b

36 Låt x vara talet 52. Hur mycket är då

a) $x + 23$ b) $x - 30$ c) $100 - x$

Merknad [244]: Codes (5095-5180)
Interpreting expression
One variable
Repetitive Weight: 1/5
Algebra as a language

Merknad [245]: Codes (5180-5242)
Interpreting expression
One variable
Repetitive Weight: 1/5
Algebra as a language

Merknad [246]: Codes (5242-5313)
Interpreting expression
One variable
Repetitive Weight: 1/5
Algebra as a language

Merknad [247]: Codes (5313-5388)
One variable
Repetitive Weight: 1/5
Algebra as a language
Formulating expression

Merknad [248]: Codes (5388-5442)
Formulating expression
One variable
Repetitive Weight: 1/5
Algebra as a language

Merknad [249]: Codes (5442-5514)
Interpreting expression
One variable
Repetitive Weight: 1/5
Algebra as a language

37 Kalla mammans ålder för x .

Skriv ett uttryck för

a) pappans ålder

b) storebrors ålder

c) lillasysters ålder 40 år 38 år 10 år 7 år



40 år



38 år



10 år



7 år

Merknad [250]: Codes (5514-5650)
Formulating expression
One variable
Task with context
Real life
Repetitive Weight: 1/5
Algebra as a language

Appendix 8: Faktor 1 document codified

6

6.1 Regn ut.

a) $20 + 4 \cdot 7$ c) $9 \cdot 7 + 12$ e) $2 \cdot (5 - 1)$

b) $20 - 4 \cdot 3$ d) $10 \cdot (3 + 2)$ f) $26 - (4 \cdot 3)$

Merknad [251]: Codes (1583-1686)
Introductory tasks
sequence of operations
Repetitive Weight: 2/5

6.2 Regn ut.

a) $4 \cdot (3 + 10)$ c) $8 \cdot (24 - 14)$ e) $15 \cdot (15 - 12)$

b) $5 \cdot (12 - 4)$ d) $10 \cdot (16 + 1)$ f) $7 \cdot (42 - 33)$

Merknad [252]: Codes (1687-1802)
Introductory tasks
sequence of operations
Repetitive Weight: 2/5

Tall og algebra 1

6.3 Regn ut.

a) $3(4 + 5)$ c) $10(23 - 14)$ e) $100(34 - 29)$

b) $4(6 + 7)$ d) $7(35 - 27)$ f) $50(34 - 33)$

Merknad [253]: Codes (1820-1919)
Introductory tasks
sequence of operations
Repetitive Weight: 2/5

6.4 Regn ut.

a) $34 + (12 + 4)$ b) $45 - (50 - 20)$ c) $(4 + 4) + (23 - 8)$

Merknad [254]: Codes (1919-1989)
Introductory tasks
sequence of operations
Repetitive Weight: 1/5

6.5 Regn ut.

a) $2(6 + 3) - 9$ b) $56 + 6(21 - 19)$ c) $4(5 + 2) - 2(10 - 7)$

Merknad [255]: Codes (1990-2062)
Introductory tasks
sequence of operations
Repetitive Weight: 1/5

6.6 Regn ut.

Merknad [256]: Codes (2063-2078)
Introductory tasks
sequence of operations
Fractions
Repetitive Weight: 1/5

$$a) \frac{24}{3} - 6$$

$$b) \frac{24 - 12}{3} - 3$$

$$c) \left(\frac{21}{7} - 1 \right) + 2 \cdot 3$$

6.7 Sara kjøper 3 liter brus til 9,90 per liter, 2 poser potetgull til 14,90 per stk. og 3 hg sma°godt til 16 kr per hg.

Sett opp og regn ut talluttrykket som viser hvor mye hun ma° betale.

Merknad [257]: Codes (2079-2269)
Introductory tasks
sequence of operations
Task with context
Real life

6.8 En gruppe pa° 27 elever skal pa° tur til Galdhøpiggen. De skal reise med buss og overnatte to døgn. Overnatting koster 80 kr per døgn per person og bussen koster 75 kr per person tur/retur.

Sett opp og regn ut talluttrykket som viser hvor mye turen kommer pa° for hele gruppen.

Merknad [258]: Codes (2269-2550)
Introductory tasks
sequence of operations
Task with context
Real life

Utsikt fra

Galdhøpiggen

Tekstfarge plate (185,1)

Tall og algebra 185

?

Jeg er tre a° r eldre

enn deg.

Jeg er x a° r gammel.

Uttrykk med variabler

Hvis Bent er 10 a°r, hvor gammel er Hanna da?

Hanna er tre a°r eldre enn Bent.

Hvis vi tenker oss at x er et tall som sta°r for alderen til Bent, sa° er alderen til

Hanna

x + 3

Oppgaver

6.9 Skriv et uttrykk for

a) summen av x og 3 c) 10 mer enn y

b) differansen mellom x og 5 d) 12 mindre enn y

Merknad [259]: Codes (3965-4075)
Formulating expression
Repetitive Weight: 1/5
Mathematical language
One variable
Algebra as a language

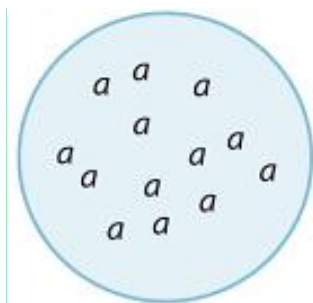
6.10 Lotte er x år. Skriv et uttrykk som viser hvor gammel

a) hun var for 5 år siden b) hun blir om 3 år

Merknad [260]: Codes (4075-4183)
Task with context
Repetitive Weight: 1/5
One variable
Real life
Algebra as a language
Formulating expression

6.11 Martin kjøper x kg epler i butikken. Eplene koster 20 kr per kilogram.

Lag et uttrykk som viser hvor mye Martin må betale.



Merknad [261]: Codes (4184-4313)
Formulating expression
One variable
Task with context
Real life
Algebra as a language

6.12 Lag et uttrykk for innholdet

i sirkelen.

Merknad [262]: Codes (4313-4363)
Formulating expression
One variable
Task with context
Geometry
Algebra as a language

6.13 Bestemoren til Herman kjører bil med en fart på 80 km i timen.

Lag et uttrykk som viser hvor langt hun kjører på x timer.

Merknad [263]: Codes (4364-4493)
Formulating expression
Task with context
One variable
Real life
Algebra as a language

6.14 Simen vil kjøpe x kg bananer til 12 kr per kilogram og en pose epler til 28 kr.

Lag et uttrykk for hvor mye Simen må betale i alt.

Merknad [264]: Codes (4495-4633)
Formulating expression
One variable
Task with context
Real life
Algebra as a language

6.15 Sara går x km til skolen og x km fra skolen hver dag.

Lag et uttrykk som viser hvor mange kilometer Sara går til og fra skolen på

a) én dag b) én uke

Merknad [265]: Codes (4633-4793)
Formulating expression
One variable
Multistep
Task with context
Real life
Algebra as a language

Oppgaver

6.16 Regn ut $12 - x$ når

a) $x = 4$ b) $x = 5$ c) $x = 6$

Merknad [266]: Codes (5681-5734)
Interpreting expression
One variable
Repetitive Weight: 1/5
Algebra as a language

6.17 Regn ut $15x$ når

a) $x = 2$ b) $x = 5$ c) $x = 1$

Merknad [267]: Codes (5734-5783)
Interpreting expression
One variable
Repetitive Weight: 1/5
Algebra as a language

6.18 Prisen for x liter bensin er $10x$.

a) Hva står tallet 10 for i formelen $10x$?

Regn ut hvor mye x liter bensin koster når

b) $x = 3$ c) $x = 4$

Merknad [268]: Codes (5783-5927)
Task with context
Real life
Algebra as a language
Interpreting expression
One variable

6.19 Formelen for omkretsen av et rektangel er $O = 2a + 2b$,

der O står for omkretsen, a for lengden og b for bredden

av rektangelet.

Regn ut omkretsen av rektangelet når

a) $a = 8$ cm og $b = 6$ cm

b) $a = 12$ cm og $b = 7,5$ cm

Merknad [269]: Codes (5927-6150)
Two or more variables
Mathematical language
Task with context
Geometry
Repetitive Weight: 1/5
Algebra as a language
Interpreting formula

Oppgaver

6.20 Trekk sammen.

a) $x + x + x + x + x$ d) $y + y + y + y + y + y$

b) $a + a + a$ e) $x + x + x$

c) $b + b + b + b$ f) $n + n + n + n + n + n + n$

6.21 Trekk sammen.

a) $2a + 5a$

b) $4x + 2x$

c) $6y + y$

d) $8n - 2n$

e) $5m - 3m$

f) $5x - 4x$

6.22 Trekk sammen.

a) $3a + 4b + 2a + 3b$

b) $3x + 2y + x + 4y$

c) $4m + 2n - 3m + n$

d) $8k + 5l - 7k - 3l$

Tekstfarge plate (192,1)

Tall og algebra ¹⁹²

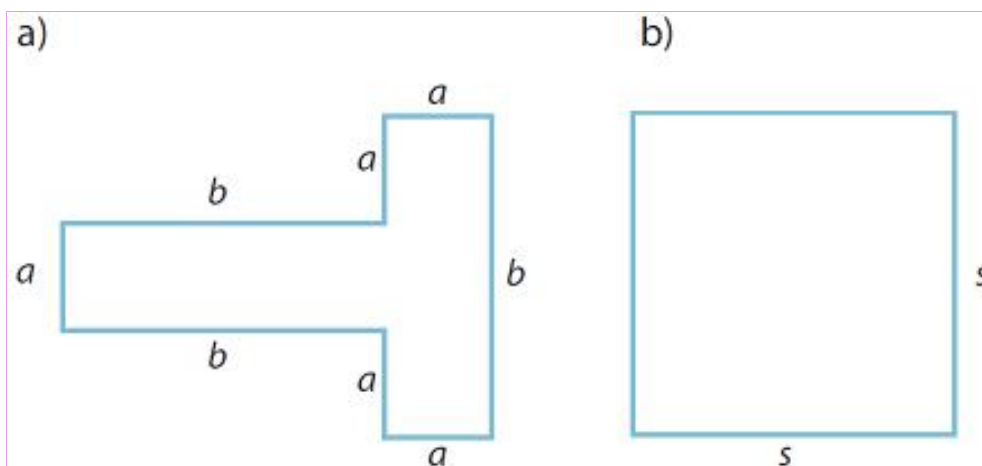
6.23 Skriv talluttrykk for omkretsen av figurene.

Merknad [270]: Codes (7291-7429)
One variable
Repetitive Weight: 1/5
Algebra as symbolic manipulation
Simplifying

Merknad [271]: Codes (7429-7514)
One variable
Repetitive Weight: 2/5
Algebra as symbolic manipulation
Simplifying

Merknad [272]: Codes (7514-7615)
Two or more variables
Repetitive Weight: 1/5
Algebra as symbolic manipulation
Simplifying

Merknad [273]: Codes (7660-7712)
Formulating expression
Two or more variables
Task with context
Geometry
Repetitive Weight: 1/5
Mathematical language
Algebra as a language
Algebra as symbolic manipulation
Simplifying



6.24 Trekk sammen.

a) $3x + 4y - 3x$

b) $-4a - 2b - 2a - b$

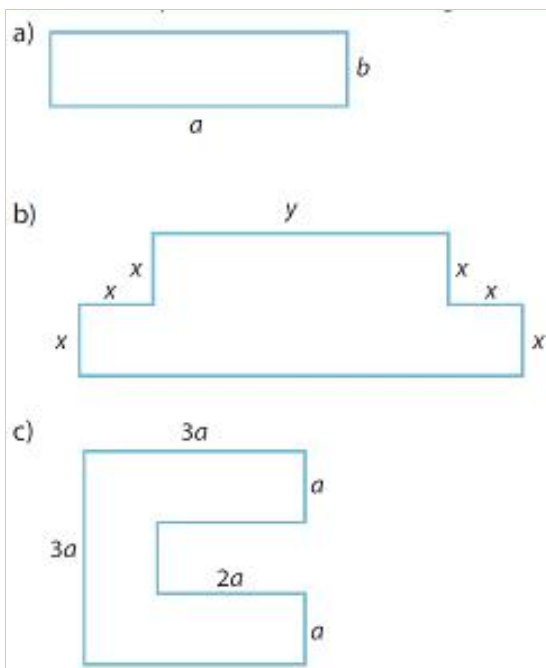
c) $3n - 3n + 2m - 2m$

d) $12x + 14x + 24x$

6.25 Skriv talluttrykk for omkretsen av figurene.

Merknad [274]: Codes (7713-7816)
 Two or more variables
 Repetitive Weight: 1/5
 Negative numbers
 Algebra as symbolic manipulation
 Simplifying

Merknad [275]: Codes (7816-7868)
 Formulating expression
 Two or more variables
 Task with context
 Geometry
 Repetitive Weight: 1/5
 Mathematical language
 Algebra as a language
 Algebra as symbolic manipulation
 Simplifying



6.26 Trekk sammen.

a) $4x - 2y - 4x + y + 2x$ c) $4x + y - 5x - y + 2y$

b) $2a - b - a + 2b + a$ d) $-2x + y + 3x - 2y + x$

Oppgaver

6.27 Hvilke tall passer i rutene?

a) $\square \cdot 5 = 15$ c) $3 + \square = 18$ e) $\square : 7 = 6$
 b) $\square \cdot 7 = 21$ d) $30 : \square = 5$ f) $23 - \square = 16$

6.28 Hvilket tall står x for i hver likning?

Merknad [276]: Codes (7869-7984)
 Algebra as symbolic manipulation
 Simplifying
 Two or more variables
 Negative numbers
 Repetitive Weight: 1/5

Merknad [277]: Codes (9052-9088)
 One variable
 Figure or empty box
 Repetitive Weight: 2/5
 Algebra as equation solving
 Numeric trial

Merknad [278]: Codes (9089-9137)
 One variable
 Repetitive Weight: 2/5
 Fractions
 Algebra as equation solving
 Numeric trial

a) $2 \cdot x = 12$ c) $\frac{x}{4} = 5$ e) $\frac{x + 1}{4} = 2$

b) $4 \cdot x = 16$ d) $\frac{36}{x} = 9$ f) $\frac{2x}{4} = 3$

6.29 Hvilket tall stå'r a for i hver likning?

a) $4 \cdot a = 20$

b) $4a = 24$

c) $a + 12 = 21$

d) $4a + 2 = 14$

e) $15 - a = 8$

f) $3a - 2 = 10$

Merknad [279]: Codes (9138-9269)
One variable
Repetitive Weight: 2/5
Algebra as equation solving
Numeric trial

6.30 Hvilket tall stå'r x for

i hver likning?

a) $x : 3 = 5$

b) $32 : x = 8$

c) $\frac{x}{5} = 4$

d) $\frac{36}{x} = 4$

e) $\frac{x + 1}{4} = 4$

f) $\frac{2x}{3} = 4$

Merknad [280]: Codes (9269-9317)
One variable
Repetitive Weight: 2/5
Fractions
Algebra as equation solving
Numeric trial

6.31 Herman tenker på et tall. Han multipliserer tallet med 4.

Deretter adderer han med 3. Da får han 23 til svar.

Hvilket tall tenker Herman på?

Merknad [281]: Codes (9318-9467)
Mathematical language
Algebra as problem solving

Oppgaver

6.32 Løs likningene.

Merknad [282]: Codes (10489-10597)
One variable
Repetitive Weight: 2/5
Algebra as equation solving
doing/undoing

a) $x + 13 = 31$ c) $x - 50 = 25$ e) $x + 10 = 8$

b) $14 + x = 23$ d) $x - 7 = 1$ f) $x - 15 = 0$

6.33 Løs likningene.

a) $x + 50 = 120$ c) $x + 40 = 30$ e) $x - 125 = 1$

b) $75 + x = 105$ d) $x - 100 = 175$ f) $x - 20 = -30$

6.34 Løs likningene.

a) $x - 3,5 = 7,5$ c) $3,9 + x = 4,1$ e) $x + 7 = 4 + 9$

b) $x + 2,3 = 8,1$ d) $x + 3 + 4 = 13$ f) $x + 7 - 3 = 6 + 4$

Oppgaver

6.35 Løs likningene.

a) $7x = 42$ c) $9x = 72$ e) $12x = 60$

b) $4x = 36$ d) $15x = 45$ f) $5x = 15$

6.36 Løs likningene.

a) $\frac{x}{6} = 3$

c) $\frac{x}{2} = 45$

e) $\frac{x}{15} = 4$

b) $\frac{x}{12} = 6$

d) $\frac{x}{3} = 13$

f) $\frac{x}{5} = 6$

6.37 Løs likningene.

a) $30x = 60$

c) $\frac{x}{6} = 7$

e) $21x = 84$

b) $9x = 108$

d) $\frac{x}{9} = 3$

f) $\frac{x}{9} = 12$

Merknad [283]: Codes (10597-10714)
One variable
Negative numbers
Repetitive Weight: 2/5
Algebra as equation solving
doing/undoing

Merknad [284]: Codes (10714-10843)
One variable
Decimal numbers
Repetitive Weight: 2/5
Algebra as equation solving
doing/undoing

Merknad [285]: Codes (11448-11538)
One variable
Repetitive Weight: 2/5
Algebra as equation solving
doing/undoing

Merknad [286]: Codes (11538-11561)
One variable
Fractions
Repetitive Weight: 2/5
Algebra as equation solving
doing/undoing

Merknad [287]: Codes (11562-11585)
One variable
Fractions
Repetitive Weight: 2/5
Algebra as equation solving
doing/undoing

6.38 Løs likningene.

a) $15 - 8 = x - 3$

b) $20 - x = 13$

c) $\frac{1}{4}x = 3$

d) $27 = 4\frac{1}{2}x$

Merknad [288]: Codes (11586-11644)
One variable
Repetitive Weight: 1/5
Algebra as equation solving
doing/undoing

Tall og algebra 2 ¹²²

Kategori 2

Talluttrykk

6.201 Regn ut.

a) $34 + 5 \cdot 7$

c) $8 \cdot 8 - 12$

e) $5 \cdot (2 + 10)$

b) $45 - 4 \cdot 6$

d) $9 \cdot 7 + 1$

f) $8 \cdot (14 - 6)$

Merknad [289]: Codes (18796-18813)
Introductory tasks
sequence of operations
Repetitive Weight: 2/5

6.202 Regn ut.

a) $5 + (4 \cdot 3)$

c) $25 + (1 \cdot 5)$

e) $20 - (3 \cdot 3)$

b) $12 + (5 \cdot 2)$

d) $24 - (3 \cdot 5)$

f) $30 - (5 \cdot 2)$

Merknad [290]: Codes (18814-18832)
Introductory tasks
sequence of operations
Repetitive Weight: 2/5

6.203 Regn ut.

a) $5(12 + 2)$

c) $7(10 + 1)$

e) $6(12 - 3)$

b) $5(10 + 3)$

d) $5(12 - 2)$

f) $10(11 - 2)$

Merknad [291]: Codes (18832-18849)
Introductory tasks
sequence of operations
Repetitive Weight: 2/5