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Achievement of 9th and 11th Grade Students in Algebra and Numbers

Research Based on Data from the KUL – LCM Project

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Preface

To write this work was both challenging and rewarding to me. I am very grateful to receive the opportunity to combine my main interests in mathematics and didactics and write this thesis.

First of all I wish to thank my supervisor Trygve Breiteig for his honest and constructivist criticism, advice and support. I wish to thank also Barbro Grevholm, who helped me get started on the project and who supervised together with Trygve Breiteig my early work. It has been an honour and a privilege to have worked with both of them.

Being a student at Agder University College I wish to thank personally Hans Erik Borgersen, Leiv Storesletten, Rolf Nossum, Siri Bjorvand and Gro Blomgren who guided my studying and to Maria Luiza Cestari for guiding me in the MERG project. I wish to thank also Elna Svege and Veslemøy Johnsen for helping me when I was in need. I want also to thank all the teachers from the schools participating in the KUL project who collected the data, analysed in this study. I would like also to extent my thanks to Hildegunn Espeland for giving me files with data from the previous years of the project, practical information and for helping me with advices. I want to thank Odd Helge Mjellem Tonheim who brought this project closely to my attention. I also extend my thanks to my colleagues master students at the Agder University College, for their interest and support. Very special thanks to my family for their help and emotional support.

Kristiansand, May 2007

Emilia Log

Summary

The project Learning Communities in Mathematics (LCM) is a project established by the Agder University College (HiA) in 2004 and supported by the Research Council of Norway. The aims of the project include to design and study mathematics teaching development through communities of inquiry between teachers from the school participating in the project and didacticians from HiA who research the outcome of the project.

Part of the LCM project is a longitudinal study of students' achievements in 4th, 7th, 9th and 11th grade. Groups of students in those grades, from seven schools participating in the project are given diagnostic tests at the beginning and at the end of the school year. The tests are designed to fit the goals of the mathematical curriculum for those grades. Those tests were first given to groups of students in the fall of 2004 and in the spring of 2005. The same tests were used again during the school year 2005 – 2006, but with new groups of students in 4th, 7th, 9th and 11th grade participating.

The study reported in this thesis, focuses on the students' results on the tests given in 9th and 11th grade, the school year 2005 – 2006. The necessary data for this study was coded and analysed. This data included:

- Data 9th grade

9th grade students' results to a test performed at the beginning and at the end of the school year 2005 – 2006: *Test 1- the total number of students is 167; Test 2- the total number of students is 94.*

- Data 11th grade

11th grade students' results to a test performed at the beginning and at the end of the school year 2005 – 2006: *Test 1- the total number of students is 227; Test 2- the total number of students is 126.*

The big drop out of students is due to some misunderstanding - the second test was not performed in some of the classes in certain schools.

The main aims of this study are first of all concerning the development during the school year 2005 – 2006 and second concerning the longitudinal perspective of the project:

- What are the students' achievements shown in the results on the tests? What is the development revealed in the students' results for the school year?

- How can the students' results be compared with the results of the other studies in the LCM project?

- What comparison can be made of the students' results on selected tasks and the results on similar tasks from studies performed before the curriculum L97 took place?

The study assesses to what extent the students in 9th grade of lower secondary school and 11th grade in upper secondary school acquired knowledge and skills that are essential for mathematical literacy. In this study the achievements of 9th and 11th grade students in algebra and number for the school year 2005 -2006 are compared with the achievements of the groups of students participating in the previous school year 2004 -2005. In addition for some of the tasks a comparison is made with results from other projects.

8. Table of contents

1. Introduction

It became interesting to me to study additionally mathematics and didactics after I came to Norway. It is very unusual for a person with my background to do a research in mathematics education in Norway. Coming from Bulgaria, I studied mathematics following intensive instructional program. I have already a master degree in Computer Science from Sofia University, Bulgaria, so that helped me to work with databases and to organise and compare different type of data in an effective way. Three years ago I moved to live to Norway about for family reasons, because my husband is a Norwegian. My perspective is both that of a researcher and of a teacher in mathematics. As researchers we expect the students to give answers that are mathematically correctly written and as teachers we want to praise the students for their thinking, although their explanations might have some weaknesses. I am concerned about how the students learning of mathematics can be improved and I believe that doing this study of the students' test performance would help teachers to improve their future classroom activities.

I became interested to participate in this longitudinal study of the students' algebraic knowledge and abilities in numbers after I participated in the MERG project (Mathematical Education Research Groups). My observations of a class of 8th grade students showed that the students experienced significant problems in solving complex type of tasks, requiring application of different abilities. The students were studying perimeter and area of a figure. When I looked at the students' notebooks it became evident that the majority of the students had chosen to work only on simple tasks related to application of the learnt formulas and procedures and less than half of the students tried to solve some more difficult tasks. The observations showed that many students experienced problems when they worked in small groups. There were some students who seemed to experience difficulties to do simple mental calculations and so they had major problems to understand the reasoning of the other students in the groups. Although the teacher of the class tried to help such students, they were not able to follow the teacher's reasoning, because that was also connected to abilities to do simple mental calculations.

The results and the impact of international comparative studies are discussed by a variety of researchers. There is an ongoing discussion about what are the reasons for the students' difficulties, and different studies investigated closely the students' problems and their possible causes. The results of the big international study Program for International Student Assessment (PISA) in mathematics are discussed in the mathematical community in Norway. The items used in PISA were aimed to measure the application of mathematical knowledge to problems within a real-life context. The analyses of the results showed that the Norwegian school system had not been successful when it comes to give students mathematical literacy (Kjærnsli, Lie, Roe, & Turmo, 2004). The results showed that as overall the Norwegian students' presentation in mathematics was below the average for all countries participating in the study. The performance of Norwegians 15 year olds in mathematical literacy and problem solving was lower than the average performance of students in most of the OECD countries (Organization for Economic Cooperation and Development countries).

The study Trends in International Mathematics and Science Study (TIMSS) is a large international study conducted at grades 4th and 8th. The National Centre for Education Statistics (NCES) published a report in 2004, concerning the results of the TIMSS study 2003. The analyses of the results from the TIMSS study reveal that the Norwegian students seem to lack some basic knowledge and skills, necessary for learning mathematics (Grønmo, Bergem, Kjærnsli, Lie, & Turmo, 2004). Norwegian students score lower than the international mean in both grades (4th and 8th grade), lowest on numbers and algebra / patterns, especially low on items requiring exact calculations, but on the other hand, the Norwegian students score high on data representation (higher than international mean). The low performance of the Norwegian students in international comparisons as PISA and TIMSS may be associated to the fact that in comparison to other countries there is less time for mathematics in lower secondary school in Norway, as noted by Grevholm, Fuglestad, Bergsten, Botten, Holden & Lingjefjärd (2004).

This study is a part of the KUL-LCM project (Kunnskap, Utdanning og Læring- Learning Communities in Mathematics), organised by the Agder University College and financed by Norges Forskningsråd. The study reported in this paper uses quantitative techniques and is a part of the longitudinal analyse of the students' learning of Algebra and Numbers in 9th and 11th grade.

Jaworski & Goodchild (2006) presented a theoretical paper related to this research project and discussed some important issues. The project is based on creating *community of inquiry*:

> The LCM project emerges from a vision of an activity system whose motive is to engage, collaboratively, didacticians, teachers and students in developing and researching the teaching and learning of mathematics through process of inquiry.
> (Jaworski & Goodchild, 2006, p.8)

This study follows the analyses done by Andreassen (2005) and Espeland (2006) and is using as additional data the results found in those studies. This was necessary in order to make comparisons for the purposes of the longitudinal project – to organise the collected data in comparisons in order to provide information that can be used as a guide for the teaching and learning process in school.

The data with results for the testing of the 9th grade and 11th grade students in the fall of 2004 was analysed by Andreassen (2005), reporting the results for the whole test with all tasks done by the students in 4th, 7th, 9th and 11th grades. The next year Espeland (2006) used part of the data created in the pilot analyses of Andreassen. Espeland (2006) focused on analyses of the 9th grade and 11th grade students' results, received after the students were tested in the beginning and the end of the school year 2004 and 2005. For the purpose of her analyses she selected a group of 202 students from the 11th grade and 74 students from the 9th grade who did both tests at the beginning and at the end of the school year 2004 – 2005 and made a detailed report for those groups of students.

Aims of the study

This study is a part of a longitudinal project Learning Communities in Mathematics (LCM) and is based on data collected in the previous years of the project and on the new results from tests performed in 9th and 11th grades at the beginning and the end of the school year 2005 - 2006. One of the main aims of the study is to describe the development shown in the students' results during the school year 2005 – 2006. The presented analyses of the students' results is organised in several groups, because the data collected in this project contained different type of results. Since I have the opportunity to compare the results of the new groups of students and the previous groups of students participating in the project, I can relate those groups of results and describe the differences and the similarities. In addition I consider a closer look to specific tasks from the tests and investigate how the students solved those problems.

The aims of the study are:

- What are the students' achievements shown in the results on the tests? What is the development revealed in the students' results for the school year?

- How can the students' results be compared with the results of the other studies in the LCM project?

- What comparison can be made of the results on selected tasks and the results on similar tasks used by studies performed before the curriculum L97 took place?

This work is part of ongoing longitudinal research on students' achievement in mathematics within the KUL-LCM project at HiA. Therefore I adopt some instruments, analysis methods, and terms that are developed in previous studies in the project. For the sake of convenience and for easy comparison with the results of the other studies, part of this project, I will use some Norwegian words in the presentation of tables, figures and as names of groups of students.

I hope that this study would be of help for all participants in the project and the results would be further analysed and discussed by other studies.

2. Literature review

This chapter treats the theoretical background of the study, the researcher's perspective on early algebra learning and includes a discussion of relevant mathematical topics. First are described current views related to mathematical proficiency and mathematical competences, followed by an overview of the concepts of diagnostic teaching and diagnostic tests, and a presentation of the report Evaluation of Reform 97. Next are presented the current views on the learning and teaching of early algebra, and perspectives on the students' difficulties in the transition from arithmetic to algebra. There are discussed also some issues related to cognitive perspectives on knowledge and learning and some issues related to patterns.

Since the test used in this study combines a variety of tasks, it was necessary to discuss also a variety of issues related to algebra, word problems, problem solving and study of patterns. That is why it is important to take into consideration and review a broad variety of issues when analysing the students' responses to the tests used in this study.

2.1 Mathematical competencies

There are continuing debates about what does it meant to learn mathematics successfully and how it is best learnt. Those are extremely important issues in mathematics education.
It is central to address the following question: What does it mean to master mathematics?

2.1.1 Mathematical proficiency

Conceptualisation of successful mathematics learning called *mathematical proficiency* was proposed by a working group of experts led by Jeremy Kilpatrick, who worked together for 18 months in a project, established as the *Committee on Mathematics Learning* by the National Research Council (NRC) in 1998. This project was requested by the National Science Foundation's Directorate for Education and Human Resources and the U.S. Department of Education's Office of Educational Research and Improvement. The committee reviewed and synthesized relevant research on mathematics learning, from pre - kindergarten to grade eight. The group published a report called "Adding it up: helping children learn mathematics" with editors Jeremy Kilpatrick, Jane Swafford and Bradford Findell (Kilpatrick, Swafford & Findell, 2001).

As part of the report, the group presented a conceptualisation of successful mathematics learning called *mathematical proficiency*. The five different, interwoven strands of mathematical proficiency were:

- *conceptual understanding* – comprehension of mathematical concepts, operations, and relations
- *procedural fluency* – skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- *strategic competence* – ability to formulate, represent, and solve mathematical problems
- *adaptive reasoning* – capacity for logical thought, reflection, explanation, and justification
- *productive disposition* – habitual inclination to see mathematics as sensible, useful and worthwhile, coupled with a belief in diligence and one's own efficacy

(Kilpatrick et al., 2001, p.116)

The term mathematical proficiency was chosen in order to describe what is necessary for a student to learn mathematics successfully. The authors argued that the instructional programs should concentrate on all five strands of mathematical proficiency. The strands shouldn't be viewed separately, but as being interdependent and interwoven - this notion reflects the findings of variety of cognitive studies which provided general support for the ideas contributing to these five strands.

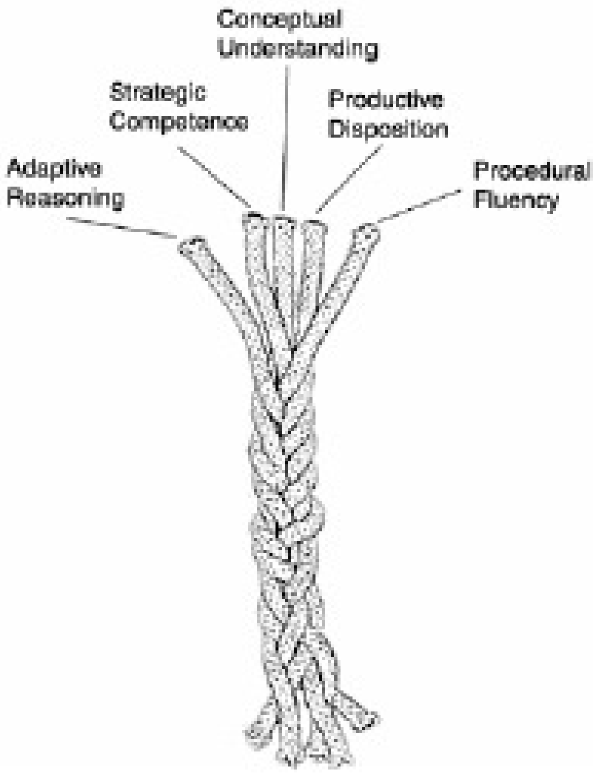


Figure 2.1.1.1: Mathematical proficiencies (Kilpatrick et al., 2001, p.117)

Conceptual understanding

Conceptual understanding refers to ability to join together mathematical ideas and know more than separated facts and methods. Students with conceptual understanding are able to systematize their knowledge in a logical way and learn new ideas by connecting them to the previous knowledge. In addition the learner is able to link related concepts rather than relying on simple memorization. That provides bases to remember new knowledge easily and reconstruct it without difficulties when it is forgotten.

Students may be capable to understand mathematical ideas, but not necessarily capable to explain their understanding verbally. For example, situations in class when students are asked to explain their methods of solutions verbally - many students might experience problems although they were able to understand the relations between the concepts and their mathematical representations.

An important indicator of conceptual understanding is ability to embody mathematical situations in different manner, which means to be able to use different mathematical representations according to the different purposes. Conceptual understanding refers to knowledge that has been learnt with understanding. The learner is successful in avoiding mistakes and capable to check the results for errors.

Procedural fluency

Procedural fluency is related to knowledge of mathematical procedures and ability to use them properly and precisely when that is necessary. Procedural fluency is especially important when students learn basic skills – for example how to solve equations or simplify algebraic expressions, how efficiently to operate with decimal numbers and fractions. Students need to be able to do simple computations on their own, both mentally and using paper and pencil.

Procedural fluency is very important for deep understanding of mathematical ideas and problem solving and in order to understand that mathematics is well structured science – students don't learn procedures isolated but to solve whole class of related problems. If students perform procedures without understanding, they might apply incorrect procedures and then it might be difficult for the students to learn the proper one, or it might be difficult for the students to join activities which help them to understand the reason underlying certain procedures. Procedural fluency and conceptual understanding are in very close connections – without sufficient practice of procedures students have problems to apply problem solving skills.

Strategic competence

Strategic competence stands for abilities to formulate mathematical problems, know how to represent them mathematically, and abilities to apply problems solving skills. Strategic competence can't be achieved without conceptual understanding and procedural fluency – there are joint connections and mutual relations between all strands of mathematical proficiency. Mathematical representation of a problem involves the use of numerical, graphic, symbolic or verbal description of it.

In order to represent a problem mathematically, students need first to understand the problem situation and build mental images of the key components. It's important also that students avoid using "number grabbing" methods by looking just at the surface of the problem and fail to recognize structural similarities between problems that differ in context. Instead, students need to focus on the problem's nature and be able to make a meaningful mathematical representation of the problem. It is essential to focus on the key elements of the problem and ignore the irrelevant features. To understand a problem, it is useful to make some drawings, diagrams, construct equations or use other problem solving strategies if that is necessary.

Students become proficient problem solvers by being flexible and knowledgeable to solve non routine problems. The students who are able to solve non routine problems are flexible to use a variety of different strategies as reasoning, guess-and-check or algebraic methods. Non routine problems require productive thinking and ability to invent a way to solve the problem. The students do not know the solution method in advance and they need to create a novel solution method. The opposite of non routine problems are routine problems. For such problems the students know what kind of solution method to apply in advance, and the students need only to reproduce this method.

Adaptive reasoning

Adaptive reasoning denotes ability to think logically about relations among concepts and situations, justify and give a proof to a problem.

> Many conceptions of mathematical reasoning have been confined to formal proof and other forms of deductive reasoning. Our notion of adaptive reasoning is much broader, including not only informal explanation and justification but also intuitive and inductive reasoning based on pattern, analogy, and metaphor… Analogical reasoning, metaphors, and mental and physical representations are "tools to think with", often serving as sources of hypothesis, sources of problem-solving operations and techniques, and aids to learning and transfer. (Kilpatrick et al., 2001, p.129)

Students need to apply adaptive reasoning when they justify and explain their solutions. The term justify is used in the sense of "provide sufficient reason for" – the proofs can be both formal and informal, but it's important to be logically complete. Students make their reasoning clear when they know how to justify and explain their ideas, for example if students need to learn algorithm, they need experience in explaining and justifying it with many different problems in order to understand it.

Productive disposition

Productive disposition refers to the complex relationship between attitudes towards the subject of mathematics, beliefs about one's own ability and achievements. The productive disposition and the other strands of mathematical proficiency are in relations of mutual support. When students develop better understanding of the subject of mathematics they develop also better self confidence and more positive attitudes towards mathematics.

Students who see themselves as being very good in mathematics tend to have high levels of achievements. If students tend to perceive themselves as not very good in mathematics, that is an obstacle for the learning process.

Many studies investigated the differences in achievement between boys and girls and the influence of the factors ethnicity and social class. Kilpatrick et al. (2001) gave as example the findings of a variety of international studies which found a connection between high achievement and positive attitudes towards mathematics or some tendencies certain groups of students to perceive themselves as not good in mathematics. Different research documented stereotype threat that might account for the differences between girls and boys and between ethnic groups. In addition the role of the teacher of mathematics is very important. Teachers need to support the student's development and encourage students to maintain positive attitudes towards mathematics.

2.1.2 Mathematical competences - the Danish KOM project

The Danish KOM project

The Danish KOM project (KOM: Competencies and the Learning of Mathematics) was requested by the Danish Ministry of Education and aimed to make a proposal for in-depth reform of the Danish mathematics education (Niss, 2003). The KOM report was produced by a working group of 12 experts and was published in 2002 (Niss et al., 2002). Niss (2003a) described the KOM project as an *analytical development project*, which aimed to provide recommendations for reform in mathematics education in Denmark and to build up a framework for mathematics, independent of specific topics and specific levels.

Niss (2003a) described some of the central issues for the project. The following questions were of primary importance:

> • To what extent is there a need for innovation of the prevalent forms of mathematics education?
> • Which mathematical competencies need to be developed with students at different stages of the education system?
> • How do we measure mathematical competence?
> (Niss, 2003a, p.4)

Mathematical competency

Among the aims of the KOM project was to describe the Danish curricula in a way so that one can determine what it implies to master mathematics, using the notion of a *mathematical competency*:

> *Mathematical competence* then means the ability to understand, judge, do, and use mathematics in a variety of intra- and extra-mathematical contexts and situations in which mathematics plays or could play a role. Necessary, but certainly not sufficient, prerequisites for mathematical competence are lots of factual knowledge and technical skills, in the same way as vocabulary, orthography, and grammar are necessary but not sufficient prerequisites for literacy. (Niss, 2003a, p.7)

The report defined eight central mathematical competencies divided in two groups. The first group was defined as *being able to ask and answer questions in and with mathematics* included the mathematical competencies*: mathematical thinking, problem solving, modelling* and *reasoning* competencies. The second group *being able to deal with mathematical language and tools* consisted of the *representation, symbol & formalism, communication* and *aids & tools c*ompetencies.

1. Thinking mathematically

The competence *thinking mathematically* refers to abilities for conceptual understanding, such as understanding and managing the *scope* and *limitations* of a given *concept*, *extent* the scope of a *concept* by *abstraction*; and *generalising results* to big groups of objects; differentiate between the various *kinds of mathematical statements* such as definitions, theorems, conjectures; abilities to *pose questions* and *know the kinds* of answers.

2. Posing and solving mathematical problems

The competence *posing and solving mathematical problem*s is associated with having problem solving abilities such as being able to *identify, formulate* and *specify* different types of mathematical problems - pure and applied problems or open-ended or closed problems, and *solve* such problems. Students need to apply problem solving competence in order to think in a more organised way when solving mathematical problems, such as to ask what is known, what needs to be done and find a solution strategy.

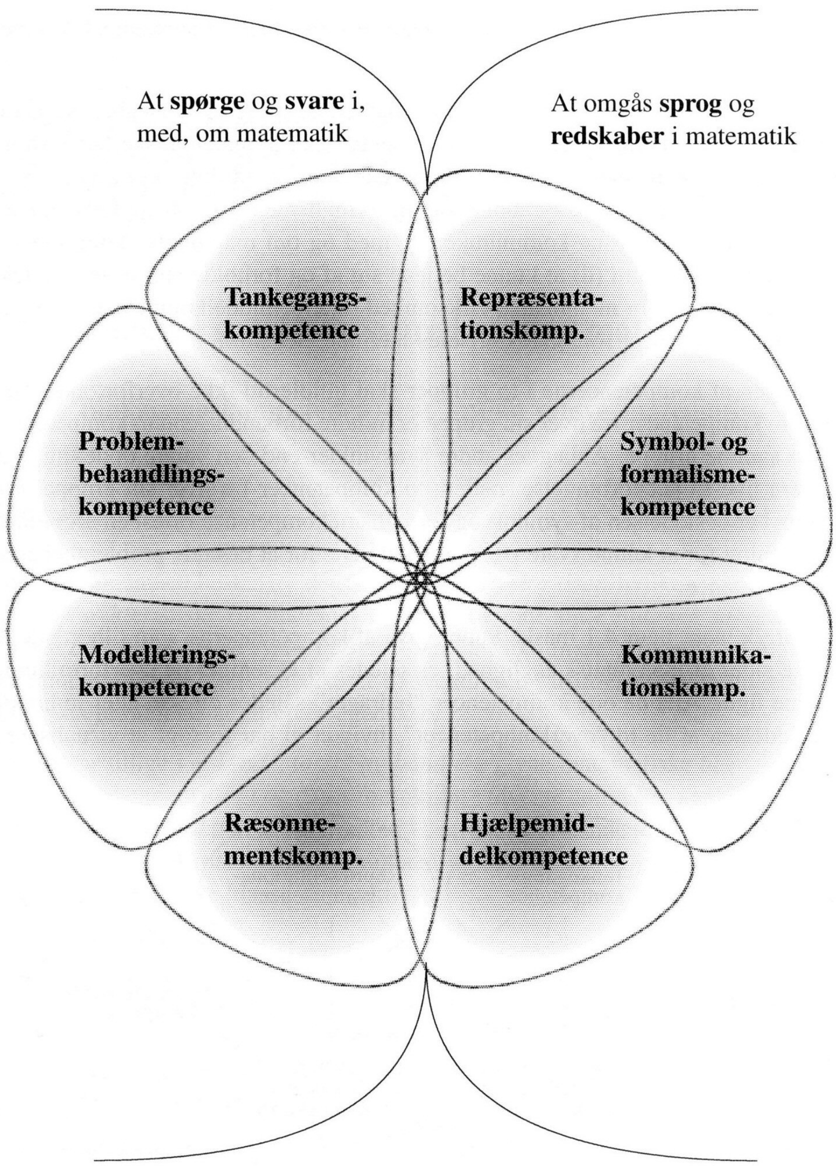


Figure 2.1.2.1: Mathematical competencies (Niss et al., 2002)

3. Modelling mathematically

Modelling mathematically competence referred to the abilities to understand and use mathematical modelling and consists of variety of abilities associated with (1) analysing and decoding of *existing models* and abilities associated with (2) *performing active modelling in a given context* such as - being able to structure the situation to be modelled, present a *mathematising* (or describe the situation in mathematical way), resulting in a mathematical model, *analyse* and *criticise* the model, *interpret* the results, *validate* the model, *apply monitoring* of the entire modelling process.

4. Reasoning mathematically

The reasoning competence referred to the abilities to understand, to identify, and to create mathematical arguments and proof. It included also abilities to follow and understand chains of arguments, to distinguish main arguments from details, abilities to create formal and informal mathematical arguments, abilities to distinguish the differences between a mathematical proof and other kinds of mathematical reasoning.

5. Representing mathematical entities

The representation competence referred to the abilities to understand and use different types of representations of mathematical objects, phenomena and situations. It included abilities to recognise and use different representations of the same entity, know the advantages and limitations and use flexibly the different representations of the same object.

6. Handling mathematical symbols and formalisms

This competence referred to the abilities to understand, to decode and to interpret mathematical symbols and formalisms. It included understanding of the syntax and semantics of the formal mathematical systems, abilities to transform a statement from natural language to symbolic language and abilities to manipulate symbolic statements and expressions.

7. Communicating in, with, and about mathematics

The communication competence referred to the skills to express oneself in verbal, visual, or written form, and implied different levels of precision. This competence contains also skills to understand variety forms of written, visual and oral mathematical statements.

8. Making use of aids and tools

This competence referred to the skills for flexible use of various tools and aids (including information technology tools), and to be aware of their advantages and limitations.

Three central insights concerning mathematics

In addition to those eight *mathematical competences*, Niss et al. (2002) identified three central *insights* concerning mathematics as a discipline. Those insights were described as being in close relationships to the eight *mathematical competences*, but not derived from them (Niss, 2003a). The *insights* are related to mathematics as a whole and are described as being *insights* into:

- The actual application of mathematics in other subjects and fields of practise that are of scientific or social significance;
- The historical development of mathematics, internally as well as externally; and
- The special nature of mathematics as a discipline. (Niss, 2003a, p.219)

The *mathematical competences* and *insights* are described as comprehensive, overarching, detached of specific content and educational level (Niss, 2003a).

In addition the mathematical competences and insights can be used for two different purposes. One of them is a *normative purpose* - they can be used for the normative purposes of the curriculum and for the normative purposes of the desired outcome of the students learning. The second use is a *descriptive purpose* – to characterize teaching practise and student learning, or to compare curricula.

Characteristics of the mathematical competences

Niss et al. (2002) considered that the mathematical competences are closely related and in some cases overlapping. For example the competence in building of models is developed from a wide range of abilities, which specifically relate to the act of modelling. Some of the skills applied in solving modelling tasks are considered also essential for the problem solving competence.

A description of mathematics by the means of competencies is related to how the subject of mathematics is perceived, how it should be taught and how it should be learnt (Niss et al., 2002; Niss, 2003a). The eight mathematical competencies are strictly belonging to the sphere of mathematics (Niss et al., 2002).

In addition Niss (2004) emphasized the important role of *intuition* and *creativity* as closely related to the eight mathematical competences and noted that:

> Intuition is on the agenda in most of the competences, for instance when we speak of the kinds of questions and the kinds of answers that are characteristic of mathematics, of developing heuristic reasoning, and of making use of different representations. Creativity can be seen as the amalgamation of all the performance sides of the eight competencies.
> (Niss, 2004, p.186)

Niss et al. (2002) provided additional characteristics:

> - En kompetence kan *udøves* i forhold til et givet stof, dvs. komme i spil og til udtryk i omgangen med dette stof.
> - En kompetence kan *udvikles*, dvs. skabes eller konsolideres, ved omgang med et givet stof.
>
> (Niss et al., 2002, p.113)

The nature of mathematics

Niss et al. (2002) recognised mathematics as a subject, related closely to other subject domains. Many aspects of learning mathematics are of particular importance for real life. Niss (2003b) was concerned by the purpose of learning mathematics and the nature of mathematics. Mathematical literacy was viewed more or less the same as the mastery of mathematics. Mathematics was characterized as a field possessing "*a five-fold nature*". Niss viewed the mastery of mathematics as far more than ability to operate with purely mathematical topics:

> We may adopt a broader – partly sociological, partly epistemological – perspective and perceive mathematics as a field possessing a five-fold nature: as a pure, fundamental science; as an applied science; as a system of tools for societal and technological practice ("cultural techniques"); as an educational subject; and as a field of aesthetics.
> (Niss, 2003b, p.216)

Mathematical literacy is not cultivated only in the mathematical classrooms – there are other important sources for development of mathematical literacy, including other subjects at schools.

2.2 The KIM project, diagnostic teaching and diagnostic tests

2.2.1 Aims of the KIM project

The Quality in Mathematics Education project (KIM) was a large research and development project which started in 1992 and was supported by the Norwegian Ministry of Education (Brekke, 2002; Gjone, 2004). The project involved two institutions – Telemark Research and the Centre for Teacher Education at the University of Oslo.

The project was based on the main principles of diagnostic assessment and focused on the development of diagnostic test items, pre-service teaching materials, and in-service materials for teachers in order to be used by teachers as foundation to aid the conceptual development of students.

The objective of the KIM project was to develop diagnostic tests that can systematically identify misconceptions and misunderstandings that may hinder further learning of mathematics.

The main aims of the KIM project included:

- Utvikle en integrert prøve- og utdanningspakke som kan brukes av lærere som ledd i intern vurdering.
- Utvikle prøvemateriell av diagnostisk karakter som kan danne utgangspunkt for konkrete undervisningstiltak innenfor ulike deler av faget.
- Kartlegge holdninger og forestillinger elever har til matematikk og undervisningen i faget.
- Beskrive hele spekteret av elevprestasjoner innenfor ulike områder av faget, ikke bare minimum kompetanse.

(Brekke, 2002, preface)

The students' problems in mathematics can be seen as results from basic misconceptions as for example “multiplication makes bigger”. The KIM project was built on the principles for diagnostic teaching of mathematics where the wrong answers of students are viewed as an important source of information.

Diagnostic tasks have been developed for the main areas of mathematics in primary education and some of the most important topics in secondary education. The areas of school mathematics, for which materials were produced, were number and calculations, functions, measurement, algebra, and geometry. Detailed written guidance was made to aid the use of the tests and to help the teaching of students with particular difficulties.

2.2.1 Mathematical competences

Brekke (2002) described a conceptualised model for mathematical competencies. The key question was: ”Hva er kunnskap, og hvordan utvikler elevenes ideer og begreper seg?” (p.3). In order to capture in a framework what it means for anyone to learn mathematics, Brekke presented descriptions of five basic components of mathematical competence: (1) *Factual knowledge* (Faktakunnskap), (2) *Skills* (Ferdigheter), (3) *Conceptual structures* (Begrepsstrukturer), (4) *General strategies* (Generelle strategier), and (5) *Attitudes and beliefs* (Holdninger).

Brekke (2002) provided basic characteristics of each component of mathematical competence and gave lots of examples as illustration. *Factual knowledge* concerns the factual information that is necessary to do mathematics. *Skills* provide to students the opportunity to use factual information and are “veletablerte prosedyrer i flere steg”. Understanding is demonstrated when students uncover the *conceptual structures* of the solved problems. The *general strategies* refer to the students’ abilities to choose proper methods and procedures to solve unknown problem. The *general strategies* include broad range of abilities as to represent or generalise problems, test hypothesis, pose questions, use the mathematical language, control and analyse results. *Attitudes and beliefs* influence how teachers and students relate to the subject of mathematics.

2.2.2 Diagnostic problems

The *diagnostic problems* are used in order to investigate what misunderstandings and misconceptions students developed in the process of learning. Diagnostic tasks are closely connected with identifying students’ *misconceptions*. In school mathematics, the concept of *diagnostic problem* is related to conceptual knowledge. The main aims of the use of *diagnostic problems* are:

> - Å identifisere og framheve misoppfatninger som elevene har utviklet, også uten at det trenger å ha vært noen formell undervisning i det en vil undersøke,
> - Å gi læreren informasjon om løsningsstrategier elevene bruker for ulike typer av oppgaver,
> - Å rette undervisningen mot å framheve misoppfatningene, for på den måten å overvinne dem og de delvise begrepene,
> - Å utvikle elevenes eksisterende løsningsstrategier,
> - Å måle hvordan undervisningen har hjulpet elevene til å overvinne misoppfatningene ved å bruke de samme oppgavene både før og etter undervisningssekvensen.
>
> (Brekke, 2002, p.16)

Diagnostic tasks are a helpful way for an investigation of the outcome of the learning process in relation to what is the students’ individual understanding of a certain mathematical concept. Diagnostic tasks are used in order to identify the students’ difficulties. There is a difference between ordinary problems and diagnostic problems.

When students solve a typical problem, it’s possible that they provide a correct answer without having proper understanding, so such problems can’t be used to collect information concerning the students’ understanding or misconceptions of a particular concept. The diagnostic problems are specially designed, they can be also given to students before a concept is studied in class, and they are not considered to play the functions of normal test problems.

Misconceptions

Misconceptions are a central issue in mathematics education. Analysing the various misconceptions held by students with regard to a particular mathematical concept, the author noted that there is a difference between misconceptions and mistakes. Mistakes are accidental, for example they can be a result of a lack of concentration. Misconceptions are not accidental – they can be explained as a result of a particular way of thinking.

There is always a particular reason for the existence of some misconception, as for example the interference of previous knowledge related to a different subject or to everyday life experience.

Some examples given for common misconceptions are:

- Det lengste desimal tallet har alltid størst verdi.
- En kan ikke dele et lite tall med et stort.
- Multiplikasjon gjør alltid svaret større.
- En kan bare dividere med hele tall.
- 3 : 6 og 6 : 3 gir samme svar.
- Divisjon gjør alltid svaret mindre.

(Brekke, 2002, p.11)

2.2.3 Diagnostic tests

Diagnostic tests can be used for variety of purposes. Typically, the diagnostic tests include many problems which generate incorrect answers from students. It's necessary that the students are informed in advance about the reasons to do such tests. Teachers can obtain diagnostic information through careful investigation of the students' work. The diagnostic tests can be viewed to be a collection of diagnostic tasks, which provide important information. They can be used in different ways. For example they can be used as a written test or they can be employed while a teacher organises a discussion with the students.

2.2.4 Diagnostic teaching

Diagnostic teaching is a type of teaching methodology based on collecting information concerning the students' understanding of specific concepts and the misconceptions related to those concepts.

Brekke (2002) pointed that some research studies had proved students' learning to be very effective when students are both exposed to cognitive conflicts and after that the students' misconceptions are resolved through discussion. This practice is different from more traditional methods of teaching as memorisations and practice of routine procedures. Traditional teaching is based mainly on the use of textbooks in class - the use of textbooks' problems is suitable for individual work in class and the mathematical problems are made to fit this model of individual work in class.

The diagnostic teaching is based on a method of systematic diagnosis of the students' misconceptions and misunderstandings. The diagnostic teaching involves the following phases:

1. Identifisere misoppfatninger og delvis utviklede begreper hos elevene.
2. Tilrettelegge undervisningen slik at eventuelle misoppfatninger eller delvise begreper blir framhevet. En kaller dette å skape en kognitiv konflikt.
3. Løse den kognitive konflikten gjennom diskusjoner og refleksjoner i undervisningen.
4. Bruke det utvidede (eller nye) begreper i andre sammenhenger.

(Brekke, 2002, p.19)

Diagnostic problems are necessary in order to find the students' misconceptions, the understanding of the studied concepts and the students' strategies of solution. Next the misconceptions and misunderstandings are brought into class and students are exposed to cognitive conflicts. An intensive discussion is organised as a way to help students recognize their problems. The students' reflections are important to resolve the cognitive conflict:

> På tilsvarende måte er refleksjon over hvordan nye ideer eller en utvidelse av et begrep er bundet sammen med de erfaringer en har på feltet fra før, et sentralt punkt i denne delen av arbeidet.
> (Brekke, 2002, p.19)

Brekke (2002) noted that different studies proved that when this method of teaching is used, students develop conceptual understanding effectively.

2.3 Evaluation of Reform 97

The report *Endringer og utvikling ved R97 som bakgrunn for videre planlegging og justering: matematikkfaget som kasus* (Alseth, Breiteig, & Brekke, 2003) focused on the evaluation of the subject of mathematics, as a part of the program Evaluation of Reform 97, organised by Norges Forskningsråd and requested by Kirke-, utdannings og forskningsdepartementet. The report focused on analyses of the development, implementation and the effects of the national curriculum L97 in relation to the subject of mathematics.

The report was a result of a cooperative project between Telemarksforsking – Notodden (TFN) and Høgskolen i Agder (HiA). Data was collected through analyses of the mathematical curriculum in L97, analyses of textbooks, classroom observations, interviews with teachers, and testing of big groups of students in 4^{th}, 7^{th} and 9^{th} grade.

The analyses showed that the national curriculum L97 was in close connection to the latest research in mathematics education. The central areas for L97 were practical applications of mathematics, conceptual development, investigations and communications. L97 emphasized the need of pupils to develop abilities and insights into the subject of mathematics that they can use in a variety of context and that the students should develop understanding of the subject beyond factual knowledge. Mathematics connections with everyday life are considered to be very important.

The report indicated that the observed teachers were well informed about the content of the curriculum, but the class observations showed that their teaching did not correspond to the intentions. Many teachers were working in much the same way as they always had done. There only minor changes and the teaching remained traditional, although teachers had good knowledge about the ideas of the new curriculum. The study suggested that teachers lacked practical knowledge how to apply the aims of the curriculum in class. The observed teachers based their teaching practise mostly on textbooks. The teachers asked for more resources and ideas that could help them in the classroom.

One of the main aims of the study was to analyse the changes in the mathematical abilities of the students after the introduction of the new curriculum L97. The study compared the students' performance before and after the introduction of L97. Groups of students in 4^{th}, 7^{th} and 9^{th} grade performed specially designed tests. The results of those groups of students were compared with the test results of groups of students in 3^{rd}, 6^{th} and 8^{th} grade who solved the same problems in 1994 and 1995 before the curriculum reform L97 took place. The comparison of the collected data from those tests showed that the new groups of 7^{th} and 9^{th}

grade students were less prepared to solve computational tasks and showed lower performance on the tests. The authors of the report pointed out that the biggest decrease in the results was on the test problems related to application of procedural knowledge and that 'De samlede resultatene viser en nedgang. Det er en klar nedgang når det gjelder tall og tallregning'(Alseth et al., 2003, p.193). Very big changes were observed for the tasks requiring abilities to compute with fractions and with decimal numbers.

2.4 Algebra

2.4.1 Development of algebra – historical remarks

The historical development of algebra was a focus of a variety of studies. The limitations for writing and the lack of book printing naturally led to abbreviations of words (Radford, 1997). The algebraic notation was invented by François Viète in 1591 and was of major importance for the rapid development of mathematics (Van Amerom, 2002). Van Amerom (2002) presented an overview of the different periods in the history of Algebra. There were three main periods in the development of algebra, according to the different forms of notations used:

- *rhetorical phase*
- *syncopated phase*
- *symbolic phase*

The *rhetorical phase* was a period lasting from ancient times until around 250 AD. In early algebra the focus was mainly on solving word problems. It was typical for this period that the problems were posed and solved using words. According to Van Amerom the problems were mostly arithmetical, but there were cases that could be viewed as requiring algebraic thinking. The unknown was a *magnitude* articulated in words, for example "heap", "length", "area", "thing", "root", "cosa", "res" or "ding(k)", and the solution was provided in "terms of instructions and calculations". Van Amerom (2002) noted that: "These calculations indicate that unknowns were treated as if they were known and reasoning about an undetermined quantity apparently did not form a conceptual barrier" (p.38). The methods *regula falsi* or Rule of the False Position was used by the Babylonians to solve problems. Linear equations were common in Egypt. Babylonians knew how to solve linear, quadratic and specific cases of cubic equations.

The *syncopated phase* began around 250 AD with the first introduction of shortened notations by Diophantus. The unknowns were represented by symbols not in words. Diophantus used abbreviations to denote powers of numbers, relations and operations in a systematic way. Although Diophantus demonstrated "a pursuit of generality of method", he was concerned about to find a (single) solution for the problems. The *syncopated notation* was not suitable for the level of generality. There were known a variety of general methods of solving indeterminate, quadratic and cubic equations. However there were limitations, because the language was not suitable to represent the given numbers in the problem, and it was difficult to write down the procedures.

The *symbolic phase* of algebra began in 1591 when Viète proposed a new system for denoting the unknown. The new way of symbolization was related to the use of the signs and symbols separated from that what they represent. Van Amerom (2002) claimed that the earlier use of unknown was to find its value, but in “the new symbolic algebra the unknown served a higher purpose, namely to express generality, since *x* could stand for an arbitrary number” (p.42).

2.4.2 Variables

Many researchers explored students understanding of variables. Studies investigated what mistakes students commonly make in basic algebra and tried to identify the reasons for those mistakes. The students’ difficulties in learning algebra are associated with the different uses and interpretations of algebraic variables (Usiskin, 1988). Küchemann (1981) used quantitative analyses of the students’ results from the CSMS project (Concepts in secondary mathematics and science) and described six different categories of interpretations of the algebraic letters which were found in the students’ responses, and identified four levels of students’ use of variable. Herscovics & Linchevski (1996) argued that many students were not able “to operate spontaneously with or on the unknown”.

2.4.2.1 The different uses of variable

According to Usiskin (1988) the different uses of variables are connected with the different conceptions of algebra - algebra as generalized arithmetic, algebra as generalizers of patterns, algebra as a study of relationships, algebra as a study of structures. Usiskin pointed that historically different definitions were used to describe variables. The conception of variable changed over time. Usiskin (1988) presented different examples for earlier and more modern definitions of variable and argued that “Trying to fit the idea of variable into a single conception oversimplifies the idea and in turn distorts the purposes of algebra.”(p.10).

Usiskin distinguished between four different uses of variables: variable as (1) a *pattern generalizer*, (2) *an unknown* or *constant*, (3) *an argument* or *parameter*, and (4) *an arbitrary symbol.*

- In the conception of algebra as *generalized arithmetic*, variables are used as *pattern generalizers*. The key instructions for students are *translate* and *generalize*. In this conception students generalise arithmetical patterns by translating and generalizing – students find relations between numbers and describe the properties of the patterns by generalizing.

- In the conception of algebra as generalizers of patterns, variables are used either as *unknowns* or as *constants*. The key instructions for students are *simplify* and *solve*. For example students simplify equations, using different procedures in order to find the unknowns.

- In the conception of algebra as a study of relationships among quantities, a variable is an *argument* or a *parameter*. The key instructions for students are *relate* and *graph*. Usiskin (1988) noted that “Under this conception, a variable is an argument (i.e. stands for a domain value of a function) or a parameter (i.e. stands for a number on which other numbers depend).“ (p.14).

- In the conception of algebra as a study of structures, variable is an *arbitrary symbol*. The key instructions for students are *manipulate* and *justify*. Under this conception algebra involves the studying of structures as groups, rings, vector spaces, integrals.

Usiskin (1988, p.16) noted that in this conception the variable becomes 'an arbitrary object in a structure related by certain properties'.

2.4.2.2 Students' levels of variable use

Students' difficulties with the use of variables were associated to the use of variables as specific unknowns, as generalized numbers, and as varying quantities (Küchemann, 1981). Küchemann (1981) analysed the students' results in a longitudinal testing called CSMS project and described six different categories of interpretations of the algebraic letters which were found in the students' responses. The six categories of students' responses were: (1) letter evaluated, (2) letter not used, (3) letter used as an object, (4) letter used as a specific unknown, (5) letter used as a generalised number and (6) letter used as a variable.

Separately from those categories, Küchemann (1981) distinguished four hierarchical "levels of understanding" as determined by the special uses of the variables. The first two categories level 1 and level 2 indicated elementary uses of variables such as ignoring the letters, giving arbitrary value and using letters as names of objects. Students categorised as being on level 3 and 4 were able appropriately to use variables. Level 3 included the use of variable as specific unknowns and generalized numbers, while level 4 related to the use of variables in functional relationships. Küchemann related the four different levels of variables use as corresponding to the Piagetian sub-stages (below late concrete, late-concrete, early-formal and late-formal).

2.4.2.3 Mathematical structures in algebraic expressions

Research (Booth, 1988; Sfard, 1991) focused on the students' problems with respect to the mathematical structures in algebraic expressions. Studies referred to the mathematical structures in algebraic expressions and described the *name-process dilemma* (Booth, 1988) and the *process-object duality* (Sfard, 1991).

The name-process dilemma refers to the statement that algebraic expression may represent both procedure and relationship. Booth (1988) pointed that students experienced difficulties with the nature of the algebraic "answers" and reported that Davis had earlier done such research and described the *name-process dilemma*. As example can be given the expression "n+3", it can be both "instruction" (3 to be added to the variable n), and an "answer" of performed addition (the number which is 3 bigger than n).

The process-object duality

Sfard (1991) used the terms *operational* and *structural* processes when presenting the theory of the *process-object duality*. The *operational* and *structural* processes are described as:

> These two approaches, although ostensibly incompatible, are in fact complementary. It will be shown that the processes of learning and of problem-solving consist in an intricate interplay between operational and structural conceptions of the same notions.
> (Sfard, 1991, p.1)

Sfard (1991) distinguished between *process* conception and *object* conception. According to Sfard *structurally* denoted the conception of abstract notation as *objects*, and *operationally* denoted the conception of abstract notation as *process*. The *process* conception precedes the *object* conception. The *process* conception of a *mathematical concept* can be viewed as being on a lower level of mathematical abstraction than its conception as an *object*.

Sfard described the processes of *interiorization, condensation*, and *reification.*

> On the grounds of historical examples and in the light of cognitive schema theory we conjecture that the operational conception is, for most people, the first step in the acquisition of new mathematical notions. Thorough analysis of the stages in concept formation leads us to the conclusion that transition from computational operations to abstract objects is a long and inherently difficult process, accomplished in three steps: *interiorization, condensation*, and *reification*. (Sfard, 1991, p.1)

Sfard and Linchevski (1994) proposed that algebra is first process-oriented and only later the process become treated as structures or as mathematical objects. For example students initially treat expressions as computations. After some time students recognize expressions as objects, which is necessary for higher order operations.

2.4.3 Students' difficulties in the studying of algebra

MacGregor (2003) discussed why so many students had little understanding in algebra. The obstacles students face when learning and using algebra are many. Lots of students believe that algebra is not helpful in their future lives and do not see any sense in learning it at school, so they do not want to be part of "the algebra culture". Many students in secondary school do not have proper foundation in arithmetic when they begin to learn algebra. MacGregor referred to the Piaget's stage theory of cognitive development, as Inhelder & Piaget suggested in 1958 that the majority of students below 14 years were not able to work with abstract mathematical objects. MacGregor claimed also that the students with weak reading skills (as for example students with dyslexia) had problems to understand mathematical text and algebraic notation.

Breiteig & Venheim (2005) emphasized that there are different algebraic rules and conventions for symbolizing and for manipulation of expressions, and noted that the teachers should be sure that the students pay enough attention to them, because many students hold misconceptions as some of those that the authors pointed: "Noen tenker kanskje at $2a^2$ er det same som $2a \cdot 2a$, og at $5 + 3 \cdot a$ det same som $8 \cdot a$. Hvorfor er det så mange som tenker at $(a^2 + b^2)$ er lik $a^2 + b^2$?" (p.21). Further they discussed the importance for conceptual understanding, when students learn to work with algebraic formulas and expressions:

> Ofte har disse problemene blitt oversett i skolens algebraundervisning, mens det har blitt fokusert mye på å forenkle eller omforme uttrykk og på å utføre beregninger der variablene skal erstattes med oppgitte tallverdier. Det er fortsatt et mål at elevene skal beherske slike omforminger, men vi må sørge for at alt dette arbeidet har et sikkert fundament ved at begrepene som symbolene står for, virkelig er formet hos eleven.
> (Breiteig & Venheim, 2005, p.21)

2.4.3.1 Differences between the studying of algebra and arithmetic

The focus of algebraic activity

Researchers (Schliemann, Carraher & Brizuela, 2007) recognised that students' difficulties reflect how they had been taught in school. In studying arithmetic there is the emphasis in obtaining numerical answers to problems involving particular numbers.

Those problems were analysed by Booth (1988) and described as differences of the focus of the activities. Many students have difficulties in basic algebra, because they assume that what is required is to find a numerical answer. In arithmetic it is important to find a numerical answer, but in algebra the focus of the activity is to express in general form derivation of procedures and relations.

The use of notation and convention in algebra
Recording of statements in algebra needs exact precision, whether in arithmetic it is important to perform the necessary operation correct and inadequacies in the recording makes little difference if the computation is correct (Booth, 1988).

Misunderstanding of arithmetical conventions
Students' misconceptions are connected with incorrect view of arithmetical representation (Booth, 1988). The use of parenthesis is an important issue for the learning of algebra - students write for example p x $a + m$, instead of p x $(a + m)$, because they ignore the need for parenthesis. Many students also believe that the value of an expression does not change even if the order of computation is varied – such students think that if the order of operations is changed than the value of the expression remains the same (Booth, 1988).

(Vinje-Christensen, 2005) used ethnographic methods to observe algebra lessons of a class of 9th grade students in Norway. He found that students' difficulties in early algebra were connected to students' difficulties in arithmetic. For example students had problems with multiplication of a term with a bracket, or brackets with a minus sign in front, in simplifying algebraic expressions, which caused big problems and difficulties in the learning of algebra.

2.4.3.2 The algebraic language

Students experience difficulties with the interpretation of symbols (Booth, 1988). Booth suggested when students are introduced to the conjoined term that represents multiplication (for example 3n), that the term be written in expanded form (for example n x 3) at least over the introduction period.

Van Amerom (2003) investigated how algebraic notation becomes instrumental to mathematical reasoning. The study found that symbolizing was a major obstacle for students. Students in 5th, 6th and 7th grade participated in the study. She gave evidence that students may successfully apply formal algebraic strategy when solving equations with one unknown, but still use very informal way of symbolizing their reasoning. The students' difficulties in transition from arithmetic to algebra were recognised by much earlier research. As an example, Van Amerom pointed that in the 1960s Freudenthal considered the algebraic syntax as consisting of big number of rules, partially contradicting the everyday language and the language of arithmetic.

2.4.3.3 Cognitive obstacles for the learning of algebra - interpretation of algebraic variables

Küchemann (1981) related students' responses on algebra problems to the Piagetian levels of cognitive development and characterised the students' difficulties as due to shortcomings in their reasoning.

MacGregor & Stacey (1997) presented evidence that the origin for the students' difficulties in understanding algebraic notation could be described as:

- Intuitive assumptions and sensible, pragmatic reasoning about an unfamiliar notation system;
- Analogies with symbol systems used in everyday life, in other parts of mathematics or in other school subjects;
- Interference from new learning in mathematics;
- Poorly designed and misleading teaching materials

(MacGregor et al., 1997, p.1)

Reading variable letters as labels

Many researchers explored students understanding of variables. Students give different meaning to the letters (see 2.4.2.2 Students' levels of variable use) - the study of Küchemann (1981) concluded that the majority of students with age between 13 to 15 years are not able to interpret algebraic letters as generalised numbers or specific unknowns. Students at level 1 and level 2 of variables use don't have proper understanding of variables. Problems were caused by the difficulties of the students to use the algebraic letters as variables instead of ignoring the letters, replacing them with numerical values or regard them as labels. Those difficulties can lead to the possible danger of meaningless symbol manipulation.

Booth (1988) argued that lots of students experience many problems in developing meaning for variable and suggested careful introduction of variables. Students develop misconceptions if they are given examples stating "*a* represents the number of apples", then students may interpret 3*a* as "3 apples", not as "3 times the number of apples". In arithmetic m is used for meters and c for cents, 3m means "3 meters" in arithmetic, so students might be confused, reading variable letters as labels. For example the statement "$a = l$ x w" is used in arithmetic to denote the verbal statement "area = length x width", and in algebra that is why it's hard for some students to read such statement as relationships between variables.

2.4.3.4 The students' use of informal methods

The use in arithmetic of informal methods, such as informal problem-solving methods, influences students' ability to construct and interpret general algebraic statements (Booth, 1988). According to Booth when the goal is that students learn to use formal methods, students need to develop understanding of the importance of the formal methods - students may be able to use informal methods as well, but the need for the use of informal methods should be discussed and the students need to recognise the value and the possible limitations of such methods.

Extensive research of the students' use of informal methods in the transition period from arithmetic to algebra was described by Van Amerom (2003). Her investigation focused closely on whether informal, pre-algebra methods can minimize the students' difficulties in the transition period between arithmetic to algebra. The case of solving word problems was investigated in details and the students' difficulties were described and analysed.

2.4.3.5 Discussion - early introduction of algebra?

Schliemann et al. (2007) demonstrated the potential of early algebra activities in helping students develop and use algebra notations and tools to solve problems in class, but didn't explore the limits of children's capabilities regarding algebra. Schliemann et al. posed as question – under what circumstances is it useful (or not) to introduce algebraic notation?

2.5 Cognitive perspectives on knowledge and learning

Kilpatrick (1985) and Mason & Spence (1999) discussed the importance of the *self-awareness* for the learning process. The idea for *self-awareness* comes from the cognitive view that students are able to learn better mathematics when they become more conscious of what they are doing. Kilpatrick (1985) distinguished the processes of *reflection* and *recursion* as integrated elements in the process of developing *self – awareness*. According to Mason and Spencer, knowledge is dependant on one's "state of awareness" rather than on an object. For them knowledge is a "snapshot of a state of knowing" which is changing constantly, it is not fixed or absolute.

2.5.1 The ideas of reflection and recursion

Kilpatrick (1985) viewed *self-awareness* to be essential for the teaching and learning mathematics more effectively. The terms *reflection* and *recursion* could be "used metaphorically as strands on which to thread some ideas about *self-awareness* as it relates to mathematics education" (Kilpatrick, 1985, p.2). Kilpatrick provided a very extensive review of the different cognitive research and related cognitive theories of learning. He recognized that there were two related movements in psychology, called "a resurrection of the concept of consciousness and a recognition of the importance of executive procedures to guide thinking" (Kilpatrick, 1985, p.6). Further he referred to the works of Dewey, Piaget, Freudenthal, Vergnaud and other scholars who investigated closely the process of learning. *Reflection* and *recursion* are used as methods to promote better learning. Kilpatrick emphasized the need of reflection and recursion for learning:

> Both reflection and recursion, when applied to cognition, are ways of becoming conscious of, and getting over, one's concepts and procedures. To turn a concept over in the mind and to operate on a procedure with itself can enable the thinker to think how to think, and may help the learner learn how to learn.
>
> (Kilpatrick, 1985, p.6)

To encourage reflection is a challenge to teacher educators and teachers. Kilpatrick suggested that improvement of learning depends on students' and teachers' *self-awareness* of the need "to turn their cognitions back on themselves".

2.5.2 *Knowing-to* act in the moment

Mason & Spence (1999) focused their attention on how students use knowledge effectively in their work "The importance of knowing-to act in the moment". They investigated closely the question - what enables students to know how to apply their skills in a novel context. Mason and Spence took the position that the traditional educational practice to train mastery of skills by repetition until perfect "is insufficient in itself". Such practice didn't guarantee that students would be able to act successfully in the moment when it is necessary to solve a non routine problem. Students could be successful in solving routine problems of the type on which they have been trained, but they could experience major difficulties with more general or less familiar problems. “They are mostly at sea” and they didn’t appear to apply the new knowledge what they had learned.

They distinguished what they call *knowing-to* from *knowing-about* the subject, and they used those terms for the purpose of their investigation. *Knowing-to* is active, practical knowledge present at the moment, it can be used or called upon when required and it enables people to act creatively. *Knowing-about* the subject included *knowing-that* (factual knowledge), *knowing-how* (technique and skills) and *knowing-why.* They described the interplay and interdependency between the different forms of knowledge:

> Once the moment of knowing-to takes place, knowing-how takes over to exploit the fresh idea; knowing-that forms the ground, the base energy upon which all else depends and on which actions depend; knowing-why provides an overview and sense of direction that supports connection and link making and assists reconstruction and modification if difficulties arise en route. Knowing-how provides action, things to do, changing the situation and transforming it, and providing the various knowings with fresh situations upon which to operate. (Mason & Spence, 1999, p.146)

The authors investigated closely why *knowing-to* is very important for the learning process. They argued that *knowing-to* requires some degree of awareness in the moment: “Knowing is not a simple matter of accumulation. It’s rather a state of awareness, or preparedness to see in the moment. (Mason et al., 1999, p.151)”. The lack of *knowing-to* blocks students to act innovatively in the moment. Students become stuck on a problem, not knowing what else to do. If students are not aware of being stuck they can’t intentionally call to mind some strategy.

The teacher’s role of guiding the learning process requires an extra degree of awareness. Teachers actively support the students learning. There are powerful techniques that help teachers to work on knowing-to act. The authors claimed that important factors for successful learning is the use of some process as specialising, generalising, working backwards that activates the actions to ‘come to mind’. They stated that exams give little indication of whether the *knowing-to* can be used or called upon when needed.

2.6 Word problems

Word problems are an important part of the mathematics program in school. Historically word problems played an important role for the development of mathematics (Verschaffel, De Corte & Greer, 2000). They were used in the ancient times by the Egyptians, Babylonians, Chinese, Indians and Greeks.

There can be given different examples of ancient word problems. The Greek mathematician Diophantos (~250) composed his *Arithmetica* as containing a collection of solved problems (Mason, 2002). There were originally 13 books, but 7 books were lost early and only 6 books have survived. *Arithmetica* concerns computational arithmetic needed to solve practical problems. His problem ‘given the sum and differences of two numbers, to find those numbers’ is among his most famous problems. Mason (2002) noticed that the work of

Diophantos is remarkable for using only the context of numbers and for 'stating the problems in general but resolving them in particular, using one as-yet-unknown'.

Different functions of word problems

Verschaffel et al. (2000) distinguished five different functions of word problems: (1) *application function* – in order to practise situations from everyday life, (2) *motivation function* – in order to motivate students to develop confidence that once they have learnt the techniques, they can apply them also out of school, (3) *selection function* – in order to evaluate the mathematical abilities of students, (4) *thought–provoking function* – in order students to develop heuristics skills and problem – solving abilities, (5) *concept–formation function* – in order to provide context for exploration and construction of new mathematical concepts.

2.6.1 Characteristics of word problems

Verschaffel et al. (2000) attempted to investigate closely the use of word problems in the book "Making sense of word problems". According to the authors, verbal numerical problems as the problem "What do you get if you subtract 3 from 8?" should not be included in the group of word problems. Typically, word problems are presented to students as brief texts describing some situation, some quantities are given and others are not and the solver is asked to give a numerical answer to a specific question. Word problems can be defined as having the characteristic element "the use of words to describe a situation". Verschaffel et al. (2000) stated that:

> Word problems can be defined as verbal descriptions of problem situations wherein one or more questions are raised the answer to which can be obtained by the application of mathematical operations to numerical data available in the problem statement. (Verschaffel et al., 2000, p. ix)

Components in a word problem

Following previous studies, Verschaffel et al. (2000) considered the following components in a word problem:

- *the mathematical structure* – quantities, mathematical operations
- *the semantic structure* – the way in which an interpretation of the text directs to specific mathematical relationships
- *the context* – what the problem is about
- *the format* – how the word problem is formulated and presented

Classification of word problems

Addition and subtraction word problems can be classified in four groups – those groups included problems with the same type of action: "(a) joining, (b) separating, (c) part-part-whole relations, and (d) comparison relations" (Kilpatrick et al., 2001, p.184). Within each group there can be made additional classification according to which quantity is the unknown – for example unknown is the initial quantity, unknown is the quantity of the result, unknown is the change of the quantity, unknown is the total quantity. Multiplication and division problems included in curriculum are: the number of sets, the number in each set, the total number, rate problems, multiplicative comparison problems, array and area problems, and Cartesian products (Kilpatrick et al., 2001, p.183 - 184).

Verschaffel et al. (2000) considered that word problems can be classified in terms of the branch of mathematics involved, for example arithmetic, algebra, geometry, logic, probability. Another distinction can be made between pure and applied tasks or between tasks as "being labelled" or "not labelled" as problems, depending on how they are perceived by the learner and whether they can be solved with standard techniques or not.

2.6.2 The goals of using word problems in class

The curriculum L97

The concept of "mathematics in everyday life" is central for the curriculum L97. L97 emphasized the need to relate studying of mathematics in class to everyday life: "Pupils should learn to use their mathematical knowledge as a tool for tackling assignments and problems in everyday life and in society (RMERC, 1999, p.178). The aims of L97 for the lower secondary school included:

> Grade 9
> Pupils should have opportunity to
> - register, formulate and work on problems and assignments relating to social life, such as employment, health and nutrition, population trends and election methods
> - work on questions and tasks relating to economics, e.g. wages, taxes, social security and insurance
> - experience simple calculations relating to trade in goods, using such terms as costs, revenues, price, value added tax, loss and profit
> - use mathematics to describe and process some more complex situations and small projects (RMERC, 1999, p.180)

Mathematics connections with everyday life are emphasized by L97. The learning of mathematics has many aspects. Mathematical knowledge is essential for the studying of other school subjects. For example, for grades 8-10 of lower secondary school, L97 stated that mathematical knowledge is supposed to be useful, and mathematical tasks should be approached in a variety of ways:

> Pupils should learn to use their mathematical knowledge as a tool for tackling assignments and problems in everyday life and in society. When dealing with a relevant theme or problem area, pupils will be able to collect and analyse information using the language of mathematics, to develop results using methods and tools they have mastered, and try out their approaches on the matter in question. Pupils should know about the use of IT and learn to judge which aids are most appropriate in the given situation.
> (RMERC, 1999, p.178)

Research proposals

Research (Kilpatrick et al., 2001; Mason, 2002; Verschaffel et al., 2000; Costello, 1991; Mosvold, 2006) identified the goals of using word problems in mathematics education. The word problems can be used to study mathematics in a more meaningful way, to provoke students to think more deeply than just at the surface level of arithmetical operations, to develop new mathematical concepts and skills, to motivate students, to train students to think creatively, to develop problem-solving skills, to learn new concepts, to assess the mathematical abilities of students. Students should not only be able to perform arithmetical calculations, but also need to be able to apply computational skills in appropriate context, as practice of everyday life situations.

Mason (2002) pointed that word problems are important for the learning of algebra. He proposed that students are taught to be 'aware of generality' and that students learn to generalise with the help of word problems. He considered that using generalisation of word problems makes students aware of "algebra as a science of generalisation" and to expose students to the significance of algebra.

2.6.3 Effective use of word problems

Historical perspectives

How word problems can be taught in a more effective way, was the focus of investigation for Mason (2002). He argued that the pedagogical potential of word problems had been underused over the centuries and even abused. Mason pointed at the significance of word problems for the learning of mathematics and investigated the use of word problems in the past. Historically, word problems were not taught in an effective way - word problems had been often viewed by students and teachers as "the hard bit", left at the end of the chapters, they had been perceived as application of some general mathematical technique and students had not been given a chance to see the similarities, to state and determine general classes of problems, to extend the problems and to attempt to generalise the problems in different ways (Mason, 2002).

Contemporary approaches

Mathematics can be more meaningful to children and mathematics instruction more effective with the use of tasks carefully designed and embedded in a realistic context (Verschaffel et al., 2000; Mosvold, 2006; Costello, 1991).

Research indicated that word problems are more meaningful to students when used as applications of mathematics to solve problem situation in realistic context, as for examples the studies described by Verschaffel et al. (2000) and by Van Amerom (2002). Kilpatrick et al. (2001) proposed that children solve word problems by modelling the situation – modelling the actions and relations described in the problem.

A teaching experiment related to realistic mathematical modelling

Verschaffel et al. (2000) discussed the results of an exploratory teaching experiment called 'The learning environment project'. The experiment aimed to establish classroom culture in which word problems are used as 'exercises in realistic mathematical modelling'. Verschaffel et al. suggested that word problems are regarded as "application problems" and as "simple exercises in mathematical modelling". The learning environment experiment used realistic problems designed as stories accompanied by questions. The participating students needed to use real world knowledge in order to learn 'to pay attention to the complexity involved in the realistic mathematical modelling'. The study proposed students to be presented word problems from the modelling perspective.

Example of a problem used in this experiment is the problem "A soldier's day":

> A soldier's day
>
> 300 soldiers must be transported by jeep to their training site. Each jeep can hold 8 soldiers. How many jeeps are needed?
>
> At the training site, the soldiers are brought to a hangar. This hangar is filled with a large number of heavy crates that need to be moved to another hangar. These crates are so heavy that it requires 8 men to carry each one. How many crates can be moved at the same time by the 300 soldiers?
>
> Back in the barracks, all the soldiers are very hungry. The cook has prepared 300 litres of hotpot. He needed 8 big pots of the same size, all completely full, to make the hotpot. How many litres of hotpot does one pot contain?
>
> In the evening, the soldiers have to participate in a military parade. They have to form rows of 8. How many soldiers are left over after forming as many rows as possible?
> (Verschaffel et al., 2000, p.88)

The study described the teaching experiment as involving a control group of students following normal instruction and a second group of students following a special program consisting of teaching/learning units of 2 ½ hours each. The model of a teaching unit was at first students to work on a set of realistic problems in small groups, after that followed whole-class discussions. Next the students worked again in groups on a set of new problems and once more the students had the opportunity to discuss the results in a whole-class discussion. At the end of the teaching unit each student received one task as an individual assignment and after that the students' individual reactions to the given assignment were also discussed.

Realistic problems

Mosvold (2006) discussed what realistic problems look like, whether the problem is meaningful to students or is presented in artificial way, whether the context relates to issues from the students' everyday life or real life in a broad-spectrum. A challenge for the teacher is not to give ready explanations and formulas, but stimulate students to discover things for themselves. Mosvold gave examples of how projects and small-projects can be organised so that students learn by "reinventing":

> If the learning activities are organised as projects (or small projects), and/or if pupils are guided through a process where they reinvent the mathematical theories, it is believed that they will also be able to use their knowledge in different contexts to a stronger degree than if they have only been solving textbook tasks in a traditional way.
> Mosvold, 2006, p.255)

However, Mosvold (2006) noted that "repetitions and hard work" are also important elements of teaching, because doing projects requires hard work by the pupils.

If a word problem is described in familiar settings it is easier for students to recognize the necessary operations to solve it, because it is easier to find relationships in known situations

(Mosvold, 2006; Costello, 1991). Costello (1991) argued that there are tasks which can be solved without students having previous knowledge of the context, but he noted that such tasks should be well described and distinguished the students' ability to visualise the problem's settings as important to make sense of it.

To practice 'seeing through the particular to the general'

Mason et al. (2005) offered word problems to be used in order to stimulate students to make their own by varying the language or the quantities. Another strategy is to ask students to change the problem's structure: change what should be found and what data is needed. The idea is if students try several variations of a problem using different numbers, this may help them "begin to get a sense" of how they solved this group of problems. It is useful to help students practice 'seeing through the particular to the general'. Students can be offered several similar tasks and than can be asked to generalise to a wide class of problems all of which can be solved by the same method:

> Find two numbers with a total of 50 and a difference of 10.
> Now find another way of solving the same problem … and another.
> Does a model or diagram help?
> Try the same problem with different numbers, such as a total of 47,358 and a difference of 3,57. Which method do you prefer now?
> (Mason et al., 2005, p. 202)

To generalise the solution

A simple task can be presented to students and then they can be asked to generalise the solution by using unknowns instead of specific numbers (Mason et al., 2005). Students can be presented a simple task and then asked to try different solution methods. For example such a simple task can be:

> A farmyard contains both chicken and sheep. The farmer knows there are 26 heads and 74 legs. How many chickens and how many sheep are in the yard?
> (Mason et al., 2005, p. 173)

After that students can try to ignore the specific numbers and find an approach for the general case where there are H heads and F feet. In this way students develop skills on deciding which type of solution methods gives flexibility of thinking in the general case. Mason et al. (2005) suggested that the students are asked: 'Which approaches lend themselves most easily to generalisation?' This method helps students develop *awareness of the modes of representations* – students must develop 'skills in knowing' which type of representation to select, and to know more about the strengths and weaknesses of the different representations.

2.6.4 Cognitive model of algebra word problem solving

Chaiklin (1989) considered that the *process of algebra word problem solving* involves processes of *problem comprehension* and *equation solving*. The *comprehension process* is considered to consist of four processes: (1) reading the problem; (2) forming a mental representation – the information is interpreted into objects with associated properties; (3) organizing the relations among the objects; and (4) representing the relations as equations. The *equation solving process* includes transformations of the equations in order to find an answer.

- *The problem comprehension process*

The *problem comprehension process* can be approached as *direct–translation problem solving* or as *principle–driven problem solving.* The *direct–translation problem solving* consists of solving the problem phrase-by-phrase, at each phrase the student tries to convert it into algebraic notation. This approach has severe limitations, because students might not notice contradictions in the problems statements. The *principle-driven problem solving* refers to approach where the solver constructs integrated representation. The solver is able to find contradictions in the problem statement and has greater understanding of the problem. Cognitive studies investigated closely how knowledgeable students solve problems and found evidence that students can form conceptual relations among parts of a problem and the problems are interpreted into a structure of schematic relations.

- *The equation solving process*

Chaiklin (1989) referred to studies of the process of solving equation and considered that the *equation solving* involves the ability to apply both *strategic* and *procedural knowledge*. The strategic knowledge involves setting goals which procedures are necessary to be performed. Chaiklin referred to studies which systematically investigated students' errors and found that students' mistakes are mostly systematic and reflect the students' beliefs about what needs to be done. The less able students needed longer acquisition time to learn to solve equations compared with higher-ability students.

2.6.5 Composing word problems

Examples of tasks

- *To make similar problems*

Students can be asked to compose a problem similar to a problem which they solved in advance. For example Van Amerom (2002) used open-ended problems to test the students' reasoning abilities and abilities to make assumptions when information is missing. The students were also invited to make their own problems and solve them after they had already solved similar problems in a test. The problems were given to students in 5th and in 7th grade. One of the tasks given to students was the task "How far away from school?":

> **How far away from school?**
> Danny lives 4 times as far from school as Michael.
> 1 Can they ride to school together?
> 2 Draw a map to show how they could ride from home to school.
> 3 If Michael lives 1 km away from school, how far away from school does Danny live? And how far away from Michael does Danny live in that case?
> 4 One afternoon Danny says to Michael: 'It takes me 5 minutes longer to ride to your home than it takes to ride to school.' Can you say anything now about how far apart their homes lie? And about their location?
> 5 Now make up some questions about how far away you and your biking friend live from school. Write down the answers, too.
> (Van Amerom, 2002, p. 119)

Another open-ended task inviting students to make their own story was the task "Family riddle".

> **Family riddle**
> Father is 5 times as old as his daughter Pam, and 4 times as old as his son Robert.
> 1 Who is older, Pam or Robert?
> 2 How old can Robert, Pam and Father be?
> 3 How old will Robert be at least? And how old can he be at most?
> 4 Now make your own family riddle and solve it.
> (Van Amerom, 2002, p. 119)

- *To compose several problems related to the same expression*

It is interesting to ask students to construct different stories with different answers to the same numerical problem. Students need to be taught that mathematics problems may have more than one correct answer - in an experimental study students were asked to compose several problems related to the same arithmetical expression (Verschaffel et al., 2000). Such type of problem was used in a previous study by Verschaffel and De Corte in 1997 and was presented as an example by Verschaffel et al. (2000). The students were presented a worksheet containing the task:

> Invent stories belong to the numerical problem 100 : 8 = … such that the result is, respectively, 12, 13, and 12,5.
> Good luck! (Verschaffel et al., 2000, p.89)

Review-Respond-Reflect Model for composing of a word problem

Pugalee (2005) offered a method called *Review-Respond-Reflect Model* for composing of a word problem. This method aims to engage students in a meaningful construction of a problem. Research found that it could help students grasp the mathematical and linguistic structure of the word problem. This method is proposed in order to help students develop better modelling skills and conceptual understanding: "Such activities reinforce conceptual understanding by providing students with an opportunity to think about the application of mathematical concepts and ideas" (Pugalee, 2005, p.172).

This model for composing a word problem provides a mechanism to support students' mathematical reasoning and reflection on their own learning. The model has three phases:

(i) *Review phase* – At first it's necessary that students find key concepts, ideas and information in the task. Students find what they know and initial evaluation of the difficulty of the task. A key question is: "What do I know?"
(ii) *Respond phase* – This phase requires that students develop a plan of action and identify the procedures and processes. A key question is: "What plan or processes is necessary?"
(iii) *Reflect phase* – In this phase students are engaged in revising their work. Students focus their attention on whether the outcome is reasonable and they are critical to their work. A key question is: "Is my outcome reasonable?"

2.6.6 Strategies for solution of word problems

Students can be encouraged to solve word problems by the use of a variety of strategies. For example problem-solving strategies can be applied when solving word problems if the procedure of solution is not known in advance.

Verschaffel et al. (2000) systematized the solution process of realistic problems designed as modelling activity and used the term *mathematical modelling* to describe the solution process. The authors identified five phases:

- understanding the described situation
- construct a mathematical model that describes it
- work through the mathematical model to find computational outcome
- interpret the outcome
- evaluate the results in comparison with the original situation
- communicate the findings

Mason (2002) proposed that when students solve word problems to be invited for example to make another similar problem in context and numbers, create another problem using the same numbers and operations but in a new context, vary the problem composition by giving different information, look at what values would make the problem realistic and finally generalise the problem by substitution of the numbers with parameters. Costello (1991) suggested as helpful the strategies to visualise the situation, discussion in a class or in a group, draw a picture or a diagram to represent the situation and replace "awkward numbers by easier ones".

Some word problems are considered to require only application of procedural knowledge – such problems are typically more simple and don't require many steps of solution and use of innovative methods. The essential steps for such problems are to understand the problem question, decide what calculation to do and simply doing it (Costello, 1991).

2.6.7 Perspectives on students' difficulties

Costello (1991) described a variety of difficulties that students experience when solving word problems and associated the level of difficulty of a word problem as related to: the context of the problem, the language and the readability of the problem, and the size and complexity of the numbers.

Lean, Clements & Campo (1990) reported a quantitative study with two large groups of students participating – Australian students with native English and bilingual students from Papua New Guinea. The students were asked to solve arithmetic word problems posed in English. The study found that students from the two countries used similar strategies and made similar mistakes, and that the semantic structure of the questions was the main factor determining the difficulty for the students.

2.7 Perspectives on problem solving

2.7.1 Research recommendations

Framework for teaching of problem solving and proof strategies

A framework for designing the learning of problem solving and proof strategies was proposed by Bell, Burkhardt, Crust, Pead & Swan (2004). The framework was in agreement with the results of previous research on the teaching of problem solving strategies and contained six phases:

> -*Diagnosis* of performance on a range of relevant tasks.
> -Identification and *naming* of the strategies needed.
> -Student work on sets of *tasks* which *focus*, initially, each on one aspect of the desired strategy.
> -Help in the form of a *reference sheet listing the strategic steps* to be taken.
> -Reflection on the way in which the strategies apply in the actual problem.
> -A sequence of further tasks, to be dealt with in a similar way, in which the recognition of the needed strategy and how to apply it become progressively more difficult.
> (Bell et al., 2004, p.130)

The authors recommended that the range of tasks in which a teacher intends to develop competence should be "as broad as possible", because strategies studied in one context do not easily transfer to another.

The Competent problem – solving model

Verschaffel et al. (2000) pointed that word problems are mostly used in class as routine applications that doesn't imply the use of reasoning skills or any higher-level thinking ability. The study discussed the results of an experimental project conducted in 1999 called Learning Environment which aimed students to acquire 'an overall strategy for solving mathematics application problems' and 'positive attitudes and beliefs with regard to the solution, the learning, and the teaching of mathematical application problems' (Verschaffel et al., 2000, p.96).

A problem – solving model containing five steps was central for the project. The proposed five steps of the problem – solving model are:

> Step 1: Build a mental representation of the problem
> Heuristics: Draw a picture
> Make a list, a scheme, or a table
> Distinguish relevant from irrelevant data
> Use your real–world knowledge
> Step 2: Decide how to solve the problem
> Heuristics: Make a flowchart
> Guess and check
> Look for a pattern
> Simplify the numbers
> Step 3: Execute the necessary calculations
> Step 4: Interpret the outcome and formulate an answer
> Step 5: Evaluate the solution
> (Verschaffel et al., 2000, p.97)

2.7.2 A model for the assessment of students' growth in mathematical problem solving

Lester & Kroll (1991) took a serious look at the evaluation practices used by teachers in class and developed a model for the assessment of students' growth in mathematical problem solving. They considered problem-solving performance as a function of at least five factors –

(1) knowledge acquisition and utilization, (2) control, (3) beliefs, (4) affects, and (5) socio-cultural contexts. Data from observations, interviews and analyses of written work can be used for the aims of the assessments.

Four groups of assessment techniques

They proposed four groups of assessment techniques: (1) Observation and questioning students (informal observation and questioning; structured interviews), (2) Collecting written assessment data from students (student self-reports, student self-inventories), (3) Using holistic scoring techniques (analytic holistic scoring; focused holistic scoring; general impression scoring), (4) Using multiple choice and completion tests.

Further they discussed the advantages and disadvantages of each technique, but did not try to provide some sort of comparison across the techniques. As suggestion they recommended that those assessment techniques can be used by teachers as help when they design and implement their own assessment techniques. Teacher's choice of evaluation technique should be based on certain factors as purpose of evaluation, teacher's experience, time available, number of students, or type of problem-solving skill being evaluated.

For example, teachers can use informal observations and questioning technique while students work in small groups to find out what strategies students understand and use, whether they check their work efficiently and how they feel about solving a particular type problem. The structured interviews are performed with one or to students, they are audio- or videotaped for a detailed analysis. It is useful to collect data directly from students using student self-reports in order to assess the students' affects or beliefs. The students self-inventories is a more structured self report – it is for example an attitude or belief survey and is used mostly in combination with other evaluation techniques. The three holistic scoring techniques focus on the students' written work. For example the analytic scoring methods are useful when it is necessary to give a student feedback about their presentation in some key categories of problem solving, it is useful when it is necessary to provide the students diagnostic information, but it takes a lot of time.

The technique focused holistic scoring is needed as a quick, consistent technique in order to identify students' strengths and weaknesses, and it can be for example end-of-semester problem-solving examination. The technique general impression scoring is used in classroom for short assignments or quizzes and it doesn't provide important diagnostic information. Lester & Kroll (1991) considered the technique using multiple choice and completion tests to be "the least satisfactory means for assessing students' progress in mathematical problem solving" (p.64), but to be widely used, easy to apply, and better suited for measuring problem-solving performance than assessing attitudes and beliefs. They recommended the use of tests which incorporate questions intended to exploit the broad range of cognitive processes involved in problem-solving, and has in addition the students' comments to individual items.

2.8 Patterns

Orton (1999) found that it was not easy to define the concept of pattern in mathematics. The difficulties are caused by the variety of different meanings of this word. For example the notion of *pattern* can be viewed in a broad sense as "a particular disposition or arrangement of shapes, colours or sounds with no obvious regularity" (p.vii). Groups of patterns can be called for example *shape patterns, geometrical patterns, number patterns, graphical patterns, repeating patterns* or *tessellations. Number patterns* are considered to be sequences of number which has regularity. For example number patterns are the sequences of numbers (1, 3, 5, 7, 9, 11,...) or (11, 22, 33, 44, 55,…) or the Fibonachi numbers (1,1,2,3,5,8,13,21,…).

2.8.1 The purpose of studying patterns in class

Patterns are considered to be very important for the teaching and learning of mathematics, they make possible that students learn mathematics in a more meaningful and enjoyable way and in addition patterns enhance the learning effect and help students remember better (Orton, 1999). Mason et al. (2005) viewed patterns to provide important sources of generalisations – for example generalisations are used in order to detect and express number patterns, or students can be asked to continue a picture sequence of shape patterns. The purpose of studying patterns in mathematics is usually connected with the search for order and regularity (Orton, 1999). Stacey (1989) argued that the use of patterns in class is an important strategy for mathematical problem solving.

Küchemann (2005) proposed that children at level 1 and 2 of variables use are in need for appropriate teaching methods in order to "ease the transition to formal operational thought". He suggested that children are presented sequences of figure patterns and are asked to recognise properties of the patterns. It is important that children learn to find the relationships between the sequential patterns. Such tasks are challenging for children and provide necessary experience to find numerical patterns related to the figure patterns, so children face the necessity of representing the relationships "economically and unambiguously". Küchemann (2005) recognised that "the strength of such problems is that the configuration are easily defined … and easily constructed" (p.119) and that the studying of patterns in class helps students develop appropriate understanding of variables.

Breiteig & Venheim (2005) noticed the importance of patterns, especially in the introduction period of studying of algebra. For example number patterns can be presented to students with the aim to stimulate students to develop conceptual understanding by investigation of the numbers, by activities to continue the patterns or describe the pattern. The authors took the position that:

> Elevenes første møte med algebra bør inkludere det å studere tallmønstre. De kan for eksempel knyttes til geometriske figurer som vokser etter et mønster. Elevene kan finne og lage uttrykk for tallene, og tolke og sammenlikne slike uttrykk. Vi bør legge opp til en utvikling fra retorisk algebra – eventuelt med støtte i figurer eller i konkretiseringer – via elevenes egne algebraiske uttrykk og til den vanlige, symboliske uttrykksformen. Disse ulike måtene å uttrykke seg på bør får leve side om side inntil elevene blir fortrolige med det algebraiske symbolspråket.
> (Breiteig & Venheim, 2005, p.17)

2.8.2 Activities and tasks

Gibbs (1999) suggested variety of ideas for the use of pattern in classroom, but did not discuss the pedagogical potential of the proposed activities. For example simple activities in class are to create patterns with paper, design patterns with different materials or more advanced activities are to relate pattern in shape to number pattern and try to generalise the number pattern.

- *Activities for creating patterns*

For example simple activities in class are to create patterns with paper. Children can create nesting patterns, overlapping patterns or tessellations. Nesting patterns are for example patterns of a set of squares, which nest inside each other. Overlapping patterns are patterns of polygon made from paper, which can be further investigated for their properties. Tessellations can be made of overlapping similar objects as paper circles or cards. Original tessellations can be made using grid paper, as for example students can learn to continue on grid paper a started tessellation

- *Traditional activities*

Such patterns are practical activities in class, as creating patterns using different material as bead work, dramis, cross-stitch patterns, string games. Such activities demand creativity and exploration using artefacts. The creation process of a pattern involves the use of symmetry and regularity as part of the process of designing the structure.

- *Linking pattern in shape and pattern in number and developing generalizations*

Gibbs (1999) proposed activities of relating patterns of shapes to pattern of numbers. Those activities require finding the number pattern that can be associated to a given pattern of shapes. Such tasks require more exploration, because students need also to generalise the number pattern that they find - students need to identify an algebraic expression with one variable that describes the number pattern in a general form. Such activities include finding generalization that is (1) linear relations of the form $x+c$ or $x-c$; (2) linear relations of the form ax; (3) linear relations of the form $ax+c$ or $ax-c$; (4) quadratic relations of the form $n(n+c)$ or $n(n-c)$; (5) quadratic relations of the form $n(an+b)$ or $n(an-b)$; (6) quadratic relations of the form $n(n+1)/2$.

2.8.3 Studies related to patterns

2.8.3.1 Children's perception of geometrical patterns

An extensive review of research related to perception of geometrical patterns was described by Jean Orton (1999a). Geometrical pattern are studied in class in connection with symmetry, transformations and tessellations. Jean Orton described different theories about the children's perception of geometrical patterns, discussed the students' difficulties and misunderstandings. In addition the issue of whole or parts was presented. Orton made a review of the different cognitive theories about pattern recognition - Template matching, Feature analyses, Symmetry of the brain, Connectionist model and the Kosslyn's theory of imagery.

Early studies of shape recognition

Jean Orton (1999a) related to the earlier research of Piaget concerning the child's knowledge of shape. Piaget conducted a lot of experiments. He considered that the child's knowledge of shape is embedded in the early experience with concrete objects and progressively develops with the time according the different stages of cognitive development.

Levels of spatial development - van-Hiele model

The different levels of thinking of students when learning geometry, was the focus of the research of Dina and Piere van–Hiele (Fuys, Geddes, & Tischler, 1988). Van-Hiele described different levels of spatial development. They were concerned about their students' difficulties in geometry and developed this model to help their teaching. Van-Hiele proposed a model of explaining the different level of thinking of the students when solving geometry problems. At level 1, children distinguish figure as a whole and don't distinguish the relationships between parts of a shape. At level 2, children can notice some relationships as for example that a rectangle has four right angles. Level 3 is theoretical level and at level 4 students are able to do deductive reasoning and abstract thought. They found that students learning geometry follow five levels of thinking – a learner can't achieve one level of thinking without passing through the previous levels.

Levels of pattern recognition

Jean Orton (1999a) discussed the students' difficulties and misunderstandings and reported a study of the testing of a group of 300 students in grades 5^{th}, 7^{th}, 9^{th} and 11^{th}. They were asked to solve a variety of tasks related to pattern recognition and tasks related to different type transformations of a pattern as rotation, symmetry and mental transformations. The test items didn't involve algebraic description of a given pattern. Orton recognised *three different stages of development* in terms of items of the pattern recognition tests. Those stages were described in relations to the different students' abilities to recognise patterns and transformations of patterns.

2.8.3.2 The use of pattern in class

An exploration of the learning and teaching of pattern was offered in the book *Pattern in the teaching and learning of mathematics* (Orton, 1999a). *Pattern in Mathematics Research Group* was a project organised by the University of Leeds. The outcome of the project was presented in a group of articles which described the different use of patterns in school, offered activities in class, explored different perspectives of the students' learning and discussed theoretical issues (Orton, 1999a).

Students' strategies for analysing linear and quadratic sequences

Hargreaves, Threlfall, Frobisher & Shorrocks-Taylor (1999) focused their attention on the students' abilities to make generalization about linear sequences and quadratic sequences. Different type tasks, requiring analysis of linear and quadratic sequences were presented to children between 7 and 11 years. The study found that most children had strategies for analysing linear and quadratic sequences and many students were able to use what they find to generalise the sequences. The children's strategies for analysing linear and quadratic sequences included:

- looking for difference between terms;
- looking at the nature of the differences;
- looking for differences between the differences
- looking at the nature of the numbers, usually odd and even;
- looking for multiplication tables;
- combining terms to make other terms.

(Hargreaves et al., 1999, p.82)

Hargreaves et al. (1999) argued that young students shouldn't be asked only *to continue* a sequence, because this leads only to children's focus on finding the difference between two terms. Tasks which require "*to solve*" (to provide a rule for a given sequence) and "*to sort*" (a child has to find if a sequence of numbers is a pattern or not) proved to provide best context for work with number sequences. The *sort tasks* and the *solve tasks* were most suitable for broadening the children's strategies and for promoting flexibility in the use of strategies. The study suggested that students should be provided a very big variety of pattern structures and type of tasks as a way to encourage children to be more persistent and be able to use a variety of strategies.

Number patterns

Zazkis & Liljedahl (2002) found that the students' ability to express generality verbally did not depend on algebraic notation. Zazkis et al. (2002) reported a study witch aimed to analyse the attempts of pre-service school teachers to generalise visual number patterns. The study found a gap between the students' ability to express generality verbally and the students' ability to use algebraic notation. Zazkis et al. (2002) argued that some students were able to think algebraically, although they lacked abilities to use standard algebraic symbolism, and noted that:

> Furthermore, when our participants demonstrated both algebraic thinking and the ability to use algebraic notation, they lacked synchronization between the two. Therefore neither the presence of algebraic notation should be taken as an indicator of algebraic thinking, nor the lack of algebraic notation should be judged as an inability to think algebraically.
> (Zazkis et al., 2002, p.400)

Linear patterns

Stacey (1989) reported an investigation study related to linear patterns, presented as figures of expanding ladders and trees. Stacey asked 8-13 years old students to look at a few sequences of linear patterns and to generalise the sequences. The students followed a traditional curriculum and did not have special instruction with focus on problem solving methods or use of patterns. The students were shown pictures of ladders and Christmas trees and a task with number pattern. The pictures represented linear patterns because the number elements (matches, Christmas lights) for the n^{th} pattern could be expressed in a linear way as an + b.

Stacey noted that the problems which required linear generalisation were challenging for those students, although seeing a pattern was not a problem for them. But to investigate the task carefully, with deeper understanding of the nature of mathematical generalisations, was hard for most of the students.

Methods of solutions of the Ladder task

For example the Ladder task was given as a task in a problem solving test to 9-11 years old students. The students were asked to generalise and explain how they found the solutions. They were given two ladder patterns with 2 and 3 rungs built with matches and were asked to find how many matches were necessary to make the same sort of ladder with 4, 5, 111, 112, 20 and 1000 rungs. Stacey analysed the students' responses and found that the primary students used mainly four methods when finding the number matches for 20 rungs and the number matches for 1000 rungs. The first method was a *counting method*. The second method was *difference method*- students multiplied number rings by the common difference 3. The third method was *whole – object method* - students assumed that the number of matches is proportional with the number of matches for a smaller ladder. The fourth method was *linear method* – students were able to recognise that both multiplication and addition were involved and were able to find a linear model for the n^{th} rung as an + b. A similar task was the Christmas trees task: the students had to determine the number of lights in a Christmas tree of a given size.

The problem solving behaviour

Stacey compared the students' answers in comparison with similar tests' results of a small group of students, called "experienced students" who followed special instruction in problem solving. The experienced students did not have lessons with special focus on generalising linear patterns, although they solved in class some tasks related to generalising other types of patterns. Stacey (1989) observed the regularity of the students' responses and noted that the students' written solutions gave limited amount of information about their problem solving strategies and tactics, but that certain conclusions could be drawn. The inexperienced students tended to grab at "easy relationships" and didn't work from the simple cases. Most students assumed that to "explain" their solutions required to show their calculations. There was not "spontaneous use of algebra to express the generalisations", although many students indicated that they found generality in the specific numbers. The experienced students tended to examine the data "more completely" although some of them made technical mistakes and could not find the right formula. They were more consistent in using linear models when answering to all questions. When they explained their solutions, those students more often related the answers to spatial patterns and number patterns. They seemed to be "more aware of the nature of the pattern to be found", they were "aware of the data" and in addition those students were "more aware that the one relationship applied for all values".

Students' difficulties

Stacey (1989) found that the observed big group of students used common solution methods that were the same in all age groups and across all three tasks. There was observed "inconsistency of choice of mathematical model" across the three tasks (students were using different models when answering to different generalising questions). Stacey noted that many students were "attracted of simple rule" and didn't attempt to check their answers. For example they used incorrectly direct proportion model for the harder questions, although at first they used correct linear model for the easier questions. The direct proportion method implied incorrectly that the n^{th} element is determined as n multiplied by the common difference. Stacey noted that students lacked ability to check their answers.

3. Methods

This study is a part of the KUL-LCM project (Kunnskap, Utdanning og Læring- Learning Communities in Mathematics), organised by the Agder University College and financed by Norges Forskningsråd. In this project the mathematical researchers work in cooperation with the teachers from the participating. The participating classes were classes from 4th grade, 7thgrade, 9th grade and 11th grade in seven different schools in one town. The schools in the longitudinal study were one combined primary and upper level of compulsory school, two primary schools, two upper level of compulsory school and two upper-secondary schools. This study obtained the necessary approval from the Statistics Norway (Statistisk Sentralbyrå) and The Data Inspectorate (Datatilsynet).

3.1 The tests for the 9th and 11th grade

The tests for the 9th grade and 11th grade were designed to test the mathematical knowledge and abilities of the students in relation to the aims of the L97 curriculum. Both tests given to 9th and 11th grade students are presented in the appendix part of this thesis.

3.1.1 The background of the tests

Selection of the tasks

Andreassen (2005) provided information about how the problems were selected. The problems were specially selected for the students from the four grades participating in the study - 4th, 7th 9th and 11th grade. All problems were very carefully selected and the process of designing the tests took significant time, because it was necessary to ensure the quality of all tests. According to Andreassen (2005, p.21) the tasks were selected after several group discussions among different persons involved in the selection process and the problems were collected from a variety of sources: some tasks were used in KIM project (Brekke, 1995), Kassel-Exeter project (Hinna, 1996; Burghes, 1999), Third International Mathematics and Science Study (Brekke, Kobberstad, Lie & Turnmo, 1998), or some tasks were used also in the study Evaluering av Reform 97 (Alseth et al., 2003). The method of selection of the tasks makes it possible to compare students' results from different projects. In order to be able to investigate the growth in students' understanding over time, some test problems were included in the tests for different grades.

Editorial changes in the tests

The changes were small – a few tasks were removed, a few tasks had small changes in the text, compared with the tests which were performed in the fall 2004, and a few new tasks were added. Both tests for 9th grade and for 11th grade, used in the new study 2005 – 2006 were: (1) the same tests as those used in the spring of 2005 and (2) had almost all tasks that were included in the tests used in 2004 fall. The changes for 9th and 11th grade tests concerned the following tasks:

- 9th grade

The first test used in the fall of 2004 fall had a few tasks which were not included in the test in 2005 – 2006. Those were the tasks 1, 22c, 22d, 23e, 27d, 28, 29 and 32. Espeland (2006) reported that the changes were done, because the students tested in the fall of 2004 were short of time and needed to get more time to work in order to complete all tasks of the test. In addition the task 23e was changed.

- 11th grade

The new test used in the fall of 2005 and spring 2006 had a few tasks which were not included in the test used in the fall 2004. Those new tasks were 12d and 12e. Some tasks had changes in the text - those were tasks 9e, 11 and 14. Espeland (2006) gave information concerning the changes in the test – the new problems were added because the observations of the students who were solving the tests showed that the students had enough time, in addition the changes in the text of some problems were done after a discussion with the teachers concerning what fitted best to the aims of the project.

Types of test problems

Some of the test problems were multiple choice problems for other of the problems the students had to find alone the mathematical operation needed or had to write the answer of the problem, without giving an explanation how they got their answers. There were also some open ended tasks, where the students were asked to provide their methods for solution or were asked to construct a new task. For example, some open ended tasks were designed to test the way students apply problem solving skills to a given word problem - to find the solution of the problem given by concrete numbers, to explain and justify their solution and finally to generalise the solution for the same problem but in the general case, using variables. Other open ended tasks were for example those tasks where the students were asked to construct their own word problems which described a given arithmetical expression.

Administration of the tests

The tests were administrated in the same way for all classes. This way of testing was applied during all years of the longitudinal study. Here is presented an overview of how the tests were performed:

- The students had 45 minutes to work.
- The tests were conducted in the normal mathematical hours for the classes
- The teachers of the classes were the only one present at the tests.
- The students didn't receive grades for their answers.
- The students were not allowed to use calculators or other materials.

3.1.2 The test for 9th grade

Overview

The test for 9th grade contains problems, designed to assess the mathematical knowledge and abilities of the students in some important topics in algebra and arithmetic.
This is an overview of the areas in which the students' mathematical knowledge and abilities were tested:

- Percents
- Addition and subtraction of fractions
- Multiplication and division of integers
- Multiplication and division with decimals
- Positional system, decimal numbers

- Choice of operation for a word problem
- Converting a given fraction to a decimal number
- Converting a given fraction, described in words, to a decimal number
- Measurement tasks – to find decimal numbers on a given scale
- Estimation of the value of a simple arithmetical expression
- Finding the value of an algebraic expression when the values of the variables are given
- Addition of algebraic expressions
- Comparison of decimal numbers
- Simplification of algebraic expressions
- Skills in working with simple algebraic expressions
- Equations and the role of a variable as a general number
- Abilities to construct a word problem with mathematical context describing a given arithmetical expression
- Problem solving skills related to solution of a word problem with given pattern - ability to find the solution to a problem with concrete numbers and ability to find the solution to the generalised problem
- Solution of word problems – abilities to construct an arithmetical expression related to the problem's question, abilities to find the solution and abilities to explain how a word problem was solved

3.1.3 The test 11th grade

Overview

This is an overview of the areas in which the students' mathematical knowledge and abilities were tested:

- Addition and subtraction of fractions
- Multiplication and division of integers
- Multiplication with decimals
- Positional system of decimal numbers
- Choice of operation for a given word problem
- Converting given fraction to a decimal number
- Converting given fraction (described in words) to a decimal number
- Comparison of decimal numbers
- Solution of word problems – ability to construct an arithmetical expression related to the problem's question
- Solution of word problems – ability to choose an algebraic expression related to the problem's context
- Simplification of algebraic expressions
- Addition of algebraic expressions
- Skills in working with simple algebraic expressions
- Finding the value of an algebraic expression when the values of the variables are given
- Equations and the role of the variables as general numbers
- Solution of algebraic equations with one unknown
- Ability to interpret the role of the constants in a given linear function (the context of the task is described in words)

- Abilities to represent and analyse geometrical situations and structures using algebraic formulas and symbols
- Abilities to find a solution, explain, and generalise the solution for open ended task requiring application of problem solving skills

3.2 Participants

This study focuses on the groups of 9th and 11th grade students who did the tests in the beginning and the end of the school year 2005 – 2006. Because this study is a part of a longitudinal project, the classes and the schools were selected in advance. Most of the students repeated the test, but there were a few classes with students who couldn't do the test in the spring.

3.2.1 9th grade participants

3.2.1.1 Groups of all students

Skole	Gruppe	Elevgruppe Antall elever Test 1	Elevgruppe Antall elever Test 2
32	1	23	22
33	1	22	-
33	2	20	-
33	3	19	-
33	4	23	20
34	1	60	52

Table 3.2.1.1.1: 9th grade - information for the groups of all students (Test 1 and Test 2) - schools, classes and number students

A group of 167 students performed the first test in the fall of 2005 and 94 students did the second test in the spring of 2006. The following table 3.2.1.1.1 presents information for both groups of students – the first group of students, 167 students (Test 1 - fall 2005), and the second group of students, 94 students (Test 2 - spring 2006). The table gives the information - from which schools and classes were the groups of 167 students and 94 students, and how many students were in each class participating. In this table the schools and the classes were called with anonymous numbers. The table provides the number students in each class.

- The classes were from three schools which participated in the longitudinal project. Those schools were schools numbers 32, 33 and 34.
- The first group of 167 students (Test 1) was a group with students coming from six different classes in those schools.
- The second group of 94 students tested in the spring is a smaller group, because three of the classes from one of the schools participated in the fall 2005, but didn't join in the spring 2006. The testing of some students was not conducted due to a fault.

Gender and average age

Elevgruppe	Test år	Alle Elever	Antall gutter	Antall jenter	Gjennomsnitt Alder
Alle Test 1	2005 høst	167	93	72	14 år 2 mnd
Alle Test 2	2006 vår	94	47	47	14 år 10 mnd

Table 3.2.1.1.2: 9th grade, all students Test 1(2005 fall) and all students Test 2 (2006 spring) – gender information and average age of the students. Note: Two students, who did Test 1 didn't provide gender information.

There were more boys than girls when the students were tested for a first time in the fall, but when the test was performed for a second time there were equal numbers girls and boys.

3.2.1.2 A group of 92 students – Test 1 & Test 2

A group of 92 students, who did the test in the beginning and at the end of the school year, was selected from the groups of 167 students and 94 students. This group of 92 students is very interesting to analyse – since one of the aims of this study is to look at the results of this group of students at the beginning and at the end of the same school year (2005 – 2006) and to describe the development shown in the students' results on the tests.

Elevgruppe	Skole	Gruppe	Antall elever
92 elever Test 1 & Test 2	32	1	22
	33	4	19
	34	1	51

Table 3.2.1.2.1: 9th grade, 92 students (2005 – 2006) - schools, classes, number students in each class who did both tests

The table 3.2.1.2.1 presents information about the group of 92 students, who performed both tests.

- Those 92 students came from three different schools – schools number 32, 33 and 34.
- The students were coming from three different classes in those schools.

Gender and average age

Elevgruppe	Test år	Gjennomsnitt Alder	Antall gutter	Antall jenter
92 elever Test 1 & Test 2	2005 høst	14 år 2 mnd	45	47
	2006 vår	14 år 10 mnd		

Table 3.2.1.2.2: 9th grade, 92 students (2005 – 2006) – gender information and average age of the students

It is very important also to look at the number of girls and number of boys in the classes.
The number of girls and boys in this group of students was almost the same. A few students didn't provide information about their age.

3.2.2 11th grade participants

3.2.2.1 Groups of all students

The participants in the 2005 – 2006 school year consisted of 227 students in the fall of 2005 and 126 students in the spring of 2006. Here is presented information about the two groups of students who performed Test 1 - the first test, fall 2005 and Test 2 – the second test in the spring 2006. The table gives information about from which schools and groups were those groups of students.

Skole	Gruppe	Elevgruppe	
		Antall elever Test 1	Antall elever Test 2
36	1 mxa	25	28
36	2 mxb	28	29
36	3 mxc	24	20
36	4 mxd	30	23
36	5 mxe	26	26
36	2 myb	17	-
37	1 mxa	24	-
37	2 mxb	27	-
37	1 mya	26	-

Table 3.2.2.1.1: 11th grade - information for the groups of all students who did Test 1 and Test 2 - schools, classes and number students in the classes

- The schools which participated in the longitudinal project were schools numbers 36 and 37.
- The 227 students tested in the fall were from nine different classes in those schools
- The 126 students who did the second test were from five classes in one of the schools. The group of students who did the test in the spring was much smaller group of 126 students. Some of the classes didn't take the second test in the spring, due to a fault. All 126 students were taking mxa course in mathematics. The two mya classes which participated in the fall didn't perform the test in the spring.

Gender and average age

The following table gives gender information and average age for the groups of all students who did Test 1 and all students who did Test 2.

Elevgruppe	Alle elever	Test år	Gjennomsnitt Alder	Antall gutter	Antall jenter
Alle Test 1	227	2005 høst	16 år 3 mnd	97	130
Alle Test 2	126	2006 vår	16 år 11mnd	53	72

Table 3.2.2.1.2: 11th grade, all students Test 1(2005 fall) and all students Test 2 (2006 spring) – gender information and average age of the students in the group.
Note: One student, Test 2 (2006 spring) didn't provide gender information.

Both years the majority of the students were girls and that is an important characteristic of both groups. It should be noted here that the previous school year 2004 – 2005 in the longitudinal project there were tested two groups of students with more boys than girls.

3.2.2.2 A group of 113 students – Test 1 & Test 2

A group of 113 students, who did the same test at the beginning and at the end of the school year 2005 - 2006 was chosen in order to analyse the development of those students through the school year shown in the two tests. It's very interesting to study in details the students' results from the tests at the beginning and at the end of the same school year.

Elevgruppe	Skole	Gruppe	Antall elever
113 elever Test 1 & Test 2	36	1 mxa	22
	36	2 mxb	27
	36	3 mxc	18
	36	4 mxd	23
	36	5 mxe	23

Table 3.2.2.2.1: 11th grade, 113 students (2005 – 2006) - schools, classes and number students in the classes who took both tests

- The participating 113 students were from five different classes in one of the schools.
- This group of 113 students is smaller when being compared with the group of all students tested in the fall of 2005. The reason was that one class from school 36 and three classes from school 37 didn't participate in the spring of 2006. This means that there was a more narrow selection of students.

Gender and average age

Here is presented a table 3.2.2.2.2 with gender information and average age of the 113 students who did both tests. There were more girls than boys in this group of 113 students – the number of the girls was 45% higher than the number of the boys.

Elevgruppe	Test år	Gjennomsnitt Alder	Antall gutter	Antall jenter
113 elever Test 1 & Test 2	2005 høst	15 år 10 mnd	46	67
	2006 vår	16 år 6 mnd		

Table 3.2.2.2.2: 11th grade, 113 students Test 1 & Test 2, 2005 -2006 – gender information and average age of the students in the group

3.3 Instruments and Procedures

3.3.1 Reports for the schools

At the end of November all participating schools in the longitudinal study received letters with information, gathered after the students' results were coded and analysed. Presenting such information was very important. The schools involved in this study were provided information for each class, organised in tables. Those tables gave the average, minimum and maximum points' result for the group of all students in the class. The table contained also the points' results for each student - on the tests in 2005 and 2006 or only on the first test in 2005 if the class didn't participate in the spring.

- Diagrams

The information was presented in the same way as it was done in the previous years of the project. The diagrams for the big groups of all students who did the same test were included. For 9th grade those were diagrams for the students from all schools participating in the study – diagrams of results for every test item for the group of 167 students, who did the first test and diagrams for the group of 92 students who did both tests. For 11th grade those were diagrams for the students from all schools – the diagrams of results for every test item for the group of 227 students, who did the first test and diagrams for the group of 126 students who did the second tests and the diagrams for the group of 113 students who did both tests.

- Tables

Additional information for the 9th and 11th grade students was presented in tables - the average, minimum and maximum points' results for the groups of all students who did both tests, all students who did the first test and all students who did the second test. Other information for the 9th and 11th grade students was given as gender comparisons. It was presented as tables which provided the average, minimum and maximum points' result for the boys and the girls in the different groups of students included in the study.

3.3.2 The coding of the students' results

3.3.2.1 Coding system

The coding system for the 9th and 11th grade was created by Andreassen (2005) in order to code the results given by all students in 4th, 7th, 9th and 11th grade tested in the fall of 2004. Irene Andreassen coded all data with students' results for the test in the fall of 2004. Hildegunn Espeland focused on the analyses of the students' results for 9th and 11th grade students and she coded the students' results from the spring of 2005.

In the current study we use the coding system applied by Andreassen and Espeland. All answers given by the students were coded with:

Code 0 - no answer
Code 1- correct answer
Code 2 and 3 - partly correct answer
Code 11 - incorrect answer

Detailed description of the coding system for the 9th and 11th grade tests is given in the appendix of this thesis.

3.3.2.2 Validity and reliability

For the purposes of this study, I coded all students' results. Initially I made a personal consulting with Espeland several times, in order to discuss some details of the coding system that was used, and that was than for the sake of consistency.

Quality of coding
A major concern when doing such study is to do the coding of the data and all necessary procedures and operations with the highest quality possible in the given circumstances. It was very helpful that Espeland created databases containing the original students' answers, except the answers for some open ended tasks. So I could use the information given there in order to keep the same standards of coding the answers for the new databases. I used also this model of work and first I created databases containing the original students' answers, as given by the students on the test. After that those databases were converted in new databases containing only the coded students' answers.

Some procedures
Each database with original students' answers and the corresponding database with answers converted to code 0, 1, 2, 3 and 11 were compared many times. That was necessary in order to achieve validity of the coding and also to compare where the answers were coded correctly. The procedures of converting the original results into coded results and comparing the results took long time, because it was important to do it with maximum precision and quality.

Coding of open ended tasks
Very special attention was paid to the open ended tasks where the students were asked to explain their way of thinking. Coding the open ended tasks was the most demanding and time consuming part of the coding process. It was necessary to apply the same way of classification of the students' answers as used in the previous studies. It was very useful to get from Espeland additional word and pdf files containing all students' answers including the incorrect one for the tasks 21a, 21b (in the test for 9^{th} grade) and the tasks 15a, 15b, 15c (in the test for 11^{th} grade). It was important to get this information for the purposes of the coding of those tasks. Such tasks were for example tasks 30 and 31 for 9^{th} grade where some of the students gave as explanation of their way of thinking answers using drawings or schemes.

First of all the students were expected to use correctly the mathematical language. If a student provided a rhetorical explanation that was making sense such answer was also coded with 1 or 2, following the model of coding used by Andreassen and Espeland. The rhetorical explanations given of the students were compared if possible with the similar rhetorical explanations given by the students in the previous years of the longitudinal study, in order to keep the same model of coding. When coding the open ended tasks I gave code 1 for a student's answer only if the student gave a mathematically correct answer. For example some 11^{th} grade students got the correct answers for task 15a, but failed to give a mathematically correct explanation how did they found their answers, although they used a correct mathematical strategy for finding the solution. For example they had difficulties with the use of the parenthesis or didn't use correctly the equation sign.

3.3.3 Computer programs

The programs used for conducting this study were EXCEL and the statistical program SPSS. The program EXCEL was used when the databases with all students' answers were created and when it was necessary to create databases, containing only coded students' answers. Databases created with EXCEL can also be transferred by simple copy and paste operation into files used by SPSS. The program SPSS was used in order to do analyses of the frequencies for each test item included in the databases. First were analysed the databases with the coded data – the results from the SPSS analyses for the groups of 9^{th} and 11^{th} grade students who did both tests are given as information in the attachment of this paper. Another type analyses was when it was necessary to classify the different types of incorrect students' answers for the different test problems. Such analyse of the frequencies of the different type answers was also done with SPSS. The results were tables created by the program that were used for additional sorting of the results with the EXCEL program, in order to match the same type results from the years 2005 and 2006 and to organise them in tables. Such tables were done with EXCEL and they are included in the attachments of the thesis. All diagrams necessary for the analyses of the students' results were done with the help of EXCEL.

4. Analyses

4.1 Introduction - Organisation of the analyses

Here is presented an overview of chapter Analyses. In chapters 4.2 and 4.3 would be presented a more wide and detailed analyses. The analyses of the results is organised in two main groups – 9^{th} grade results and 11^{th} grade results.

4.1.1. 9^{th} grade comparisons – summary

Main structure

Here is presented the structure of the analyses of the 9^{th} grade results. There are three 9th grade comparisons, related to different types of students groups:

- **First comparison - Trends in the students' progress during the year**

The first comparison concerns the results of the group of 92 students who did both tests. Additionally, in order to the group of 92 students was also divided in three subgroups in order to analyse.

- **Second comparison - A comparison with the previous year**

The second comparison focuses on the group of all 167 students who did the test in the fall 2005 compared with the group of all 90 students who did the test in the fall of 2004.

- **Third comparison - Trends in the students' progress**

The third comparison presents the group of 92 students who did both tests in the beginning and the end of the school year 2005- 2006, compared with the group of 74 students who did both tests in the beginning and the end of the school year 2004- 2005.

Next would be presented summaries of those three comparisons.

4.1.1.1 Trends in the students' progress during the year

Summary

This comparison concerns analyses of the students' results on the first test (2005 fall) when compared with the students' results on the second test (2006 spring) for the group of 92 students who did both tests. The main aim of this comparison was: What was the development shown in the students' results for both tests in the beginning and the end of the school year?

Organisation of the students' results

In order to find additional information, the students' results for both tests were sorted, using the SPSS and EXCEL programs in order to produce different tables containing every test problem and the corresponding solution frequencies. The results were different type tables, done with the following procedures:

- Tables with solution frequencies, giving for every test problem only the different types of students' answers, coded with 0, 1, 2, 3 or 11

There were created tables for the first and the second test, containing every test problem and the corresponding solution frequencies – those tables contained only the different types of students' answers, coded with 0, 1, 2, 3 or 11. The tables were created with the help of the SPSS program.

- Tables with solution frequencies organised separately (for the first test in 2005 and separately for the second test in 2006) and containing every test problem and the corresponding solution frequencies including all different types wrong answers given by the students

There were created tables containing every test problem and the corresponding solution frequencies, but this time were used the database with all different types wrong answers. The results were tables with the frequencies for every test problem for 2005 and 2006. Since the students gave a very big variety of incorrect answers for each item it was necessary to make additional sorting and grouping.

- Tables presenting the solution frequencies for both tests after manual matching the results for 2005 and 2006 - for every problem, the results on the first test (2005) and on the second test (2006) were matched and displayed in one table

It was necessary to make another sorting – for every task it was important to organise the students' results for 2005 and 2006 only in one table. That was done in order to have tables which give information what type incorrect answers the students gave in the fall compared with the students' answers in the spring. The similar type answers for the results in 2005 and the results in 2006 were matched and displayed in tables for every problem, except the open ended task 21. This matching was done manually, because it was necessary to organise the data in a new way – find which type of answers were similar and presenting them in the same group.

Three subgroups

The group of 92 students was additionally divided in three subgroups, in order to analyse the development shown in the students' results in more details. Those subgroups were called Nederste, Midterste and Øverste groups 9th grade students. The ranking of the students' results on the first test was used as a base for making the groups. In addition those groups were selected to have similar number of students.

4.1.1.2 A comparison with the previous year

Summary

This comparison concerns the results for all 167 students who did the test in the fall 2005 compared with the results for all 90 students who did the test in the fall of 2004. Why is such comparison necessary? The students who performed the test in the beginning of the school year were a big group of 167 students. A much smaller group of 94 students did the test in the spring of 2006, because some of the classes didn't participate. Since the group of 167 students is a large group of students it is important to compare the results with another similar group of students – that is the group of all 90 students who did the first test in the fall of 2004, with results for this test reported by Andreassen (2005).

Tasks not included in this comparison

In order to eliminate the differences between the two groups of compared students, it was necessary to have the same number of tasks. Problems 1, 23e, 22c, 22d, 27d, 28, 29 and 32 from the test done by the students in the fall of 2004 were not included in the comparison. In the spring of 2005 the test was reduced with those tasks, as reported by Espeland (2006). The reason was that the test, done by the students in the fall of 2004, required a lot of time and many students didn't have enough time to finish it. So it became necessary to cut a few of the tasks in order to give more time for the students to work. In the fall of 2006 the test didn't have additional changes so the 167 students received the test as it was presented in the spring of 2005 (without tasks 1, 23e, 22c, 22d, 27d, 28, 29 and 32, maximum possible score 57 points).

4.1.1.3 Trends in the students' progress

Summary
This comparison focuses on the results of the group of 92 students who did both tests in the beginning and the end of the school year 2005- 2006, and the results of the group of 74 students who did both tests in the beginning and the end of the school year 2004- 2005. This study is a part of a longitudinal project. One of the main aims of this study is to compare the new students' results reported in the analyses part of this thesis with the previous students' results, reported by Espeland (2006) and Andreassen (2005). This comparison concerns analyses of the two test results of all students who did both tests (2005 - 2006) and the test results of all students who did both tests the previous school year (2004 - 2005).

The main aim of this comparison was: What was the development shown in the students' results for both tests for the group of 92 students compared with the group of 74 students who performed the tests the previous school year?

4.1.2 11th grade comparisons - *Summary*

Main structure
Here is presented the structure of the analyses of the 11th grade results. This study focuses on the analyses of the results of the following comparisons of 11th grade students:

- **First comparison - Trends in the students' progress during the year**

The first comparison concerns the results of the group of 113 students who did both tests in the beginning and the end of the school year 2005 – 2006. In addition this group was divided in three subgroups.

- **Second comparison - A comparison with the previous year**

The second comparison is between the group of all 227 students who did the test in the fall 2005 compared with the similar group of all 236 students who did the test in the fall of 2004.

Next would be described those two comparisons.

4.1.2.1 Trends in the students' progress during the year

Summary
This comparison includes the results for the group of 113 students who did both tests in the beginning and the end of the school year 2005- 2006. It's interesting to analyse what

similarities and differences could be found in the students' responses related to the same test tasks when the data from the test in the fall of 2005 and the spring 2006 is compared.

A group of 113 students who did both tests was selected. Those 113 students were selected from the group of 227 students who did the test in the fall 2005 and the group of 126 students who were tested next spring. It's important to point that the group of 126 students tested in the spring of 2006 was half the size compared with all 227 students tested in the fall of 2005. So the big group of all 227 students, tested in the fall, and the smaller group of 126 students who did the same test in the spring were two different sample groups and couldn't be compared in the analyses of the results. All 113 students were taking mxa course in mathematics. The students' results for both tests were sorted using the SPSS and EXCEL programs, in order to produce different tables for each test item and the corresponding solution frequencies.

Organisation of the students' results

The same model of procedures as described in details for the 92 students from 9th grade was followed. The results were presented in the following tables:

- Tables with solution frequencies - giving for every test problem only the different types of students' answers, coded with 0, 1, 2, 3 or 11. The tables were created with the help of the SPSS program.

- Tables with solution frequencies separately for the first test in 2005 and separately for the second test in 2006- containing every test problem and the corresponding solution frequencies including the different type wrong answers. The tables were created with the help of the SPSS program.

- Tables presenting the solution frequencies for both tests after manual matching the results for 2005 and 2006 - for every problem, the results on the first test (2005) and for the second test (2006) were matched and displayed in one table. The tables were done using the EXCEL program, with manual matching of the different type results.

Three subgroups

This group of 113 students was further divided in three subgroups – Nederste, Midterste and Øverste subgroups of students (further called groups). To do this dividing was used the method applied by Espeland (2006). In order to do so, the students' points results on the first test were counted and then the group of 113 students was sorted according the received test's score, starting form the lowest students' score. The three subgroups Nederste, Midterste and Øverste groups were formed with the main aim - to analyse and compare the results for those subgroups.

4.1.2.2 A comparison with the previous year

Summary
The results of all 227 students who did the test in the fall 2005 were compared with the results of all 236 students who did the test in the fall of 2004. Because this study is a part of a longitudinal project, it's possible to compare the group of 11th grade students who did the test in the fall of 2005 with the similar group of 237 students who did the test in the fall of 2004.

The data for the group of 237 students tested in 2004 was taken from an SPSS file prepared by Andreassen.

Elevgruppe - 236 elever, høst 2004		
Skole	Gruppe	Antall elever
36	1 mxa	22
36	2 mxb	22
36	3 mxc	25
36	4 mxd	24
36	5 mxe	29
36	1 mya	14
36	2 myb	17
37	1 mxa	42
37	1 mya	41

Table 4.1.2.2.1: All 236 students, fall 2004 – schools, classes and number students in every class

Elevgruppe - 227 elever, høst 2005		
Skole	Gruppe	Antall elever
36	1 mxa	25
36	2 mxb	28
36	3 mxc	24
36	4 mxd	30
36	5 mxe	26
36	2 myb	17
37	1 mxa	24
37	2 mxb	27
37	1 mya	26

Table 4.1.2.2.2: All 227 students, fall 2005 – schools, classes and number students in every class

Tasks not included in this comparison

Since it was necessary to compare the tests results for the both groups at equal bases some tasks were not included in this analyses. Tasks 12d and 12e were not included in the comparison - the students did them in the fall of 2005, but the first groups of students tested in 2004 (fall) didn’t solve those tasks. Task 9e was also not included, because it was changed from $3a+a^2+a^3$ to $3a+a^2+a^2$. Espeland (2006) also didn’t include those tasks 12d, 12e and 9e when she compared the group of 202 students from 11th grade who did the test in the beginning and the end of the school year 2004 – 2005. The text of problem 11a and 11b has been reformulated in the test done by the students in 2005, but the tasks were included in the comparison, because the changes were minimal and made in order to make the mathematical problem clear to the students.

It’s important to point out that this comparison contains results only for the test items included in the test for both compared groups of students, with maximum possible score for the test 48 points after the changes.

4.2 9^{th} grade comparisons

4.2.1 Trends in the students' overall progress during the year

4.2.1.1 The results of the group

In this part of the analyses we present the development in the results for a group of 92 students who did both tests. This group was selected in order to compare the students' results for both tests and describe the development during the school year shown in the students' responses. It is important to find for which tasks the students made significant progress and for which tasks the results on the second test show decline. In fact the students' responses provide a very rich source of information. A detailed study of the collected data can be used to analyse the main achievements of those students. The following table was made using as a source data the points sum for every student in the group of 92 students and is related to the results for both tests. It shows the minimum, maximum, average and the standard deviation for this data for the first and the second test.

Elevgruppe	Antall	Test år	Minimum	Maksimum	Gjennomsnitt	St.avvik
Elever - Test 1 & Test 2	**92**	2005 høst	2	44	18,8	9,7
		2006 vår	1	47	23,9	12,2

Table 4.2.1.1.1: Results in points for the group of students who did both tests.
Note: The maximum score a student can get for the test is 57 point.

The students received on average 18,8 points (test 1) and 23,9 points (test 2). The average score in the spring is 27% higher compared with the average score in the fall 2005. The average results for the students are much lower than the possible maximum score a student could receive on the test. Both tests were difficult for the majority of the students. The students got on average 5 points higher results on the second test. This increase is better compared with the increase of 4,1 points for the group of 74 students analysed by Espeland (2006).

The following diagrams give the distribution of the points' sums for all students. It can be observed that on the second test there were more students who got higher points sums. The maximum score in the group was 44 points in the fall of 2005 and 47 points in the spring of 2006. No one of the students could show a result very close to the maximum score of 57 points.

- 79% of those students increased their score on the second test
- 21% of the students had a lower score or didn't change their score on the second test. If we consider also that a group of 10% of the students got only one point increase of the results on the second test, there is a group of 31% of the students with similar results or lower results on the second test.
- 54% of the students with points change below the average of 5 points for the group

The second test is about eight months after the first one, so the students could benefit a lot from getting a lot of mathematical instruction. They became more mature with the natural process of growing. But we do not have evidence why those students did not improve their results.

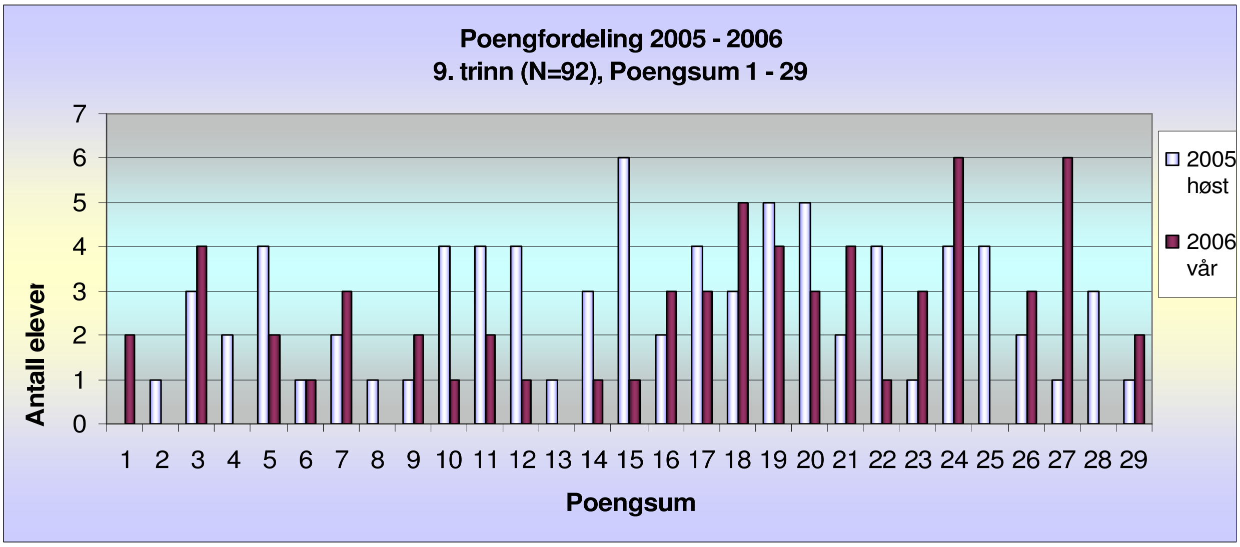


Figure 4.2.1.1.1: 9th grade (N=92) - Students' points sums – 1 to 29 points

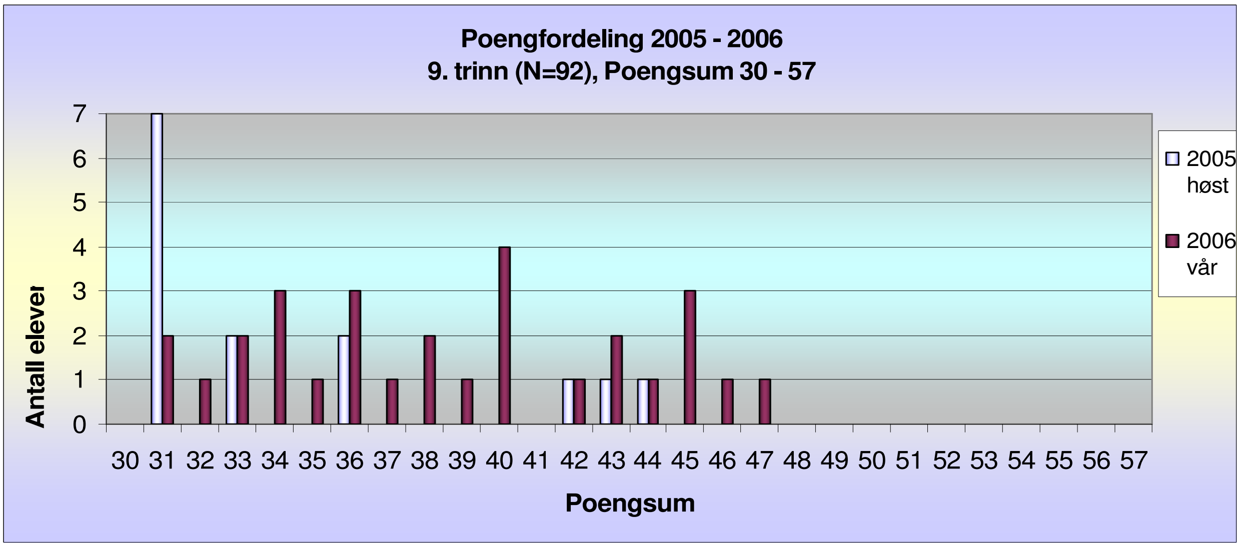


Figure 4.2.1.1.2: 9th grade (N=92) - Students' points sums – 30 to 57 points.

It is observed a positive development in the students' results, although some of the test items were very difficult for the students. There was a big increase of the group of students with scores above 30 points and a big decrease of the group of students with minimal scores. The comparison of the students' results for both tests show:

- 15% of the students (test 1) and 31% of the students (test 2) received scores above 30 points.
- The group of students with scores below 20 points was reduced from 56% of the students (test 1) to 38% of the students (test 2).
- The group of students with scores between 30 and 39 points increased from 12% of the students (test 1) to 17% of the students (test 2).
- 3% of the students (test 1) and 14% of the students (test 2) received scores between 40 and 49 points

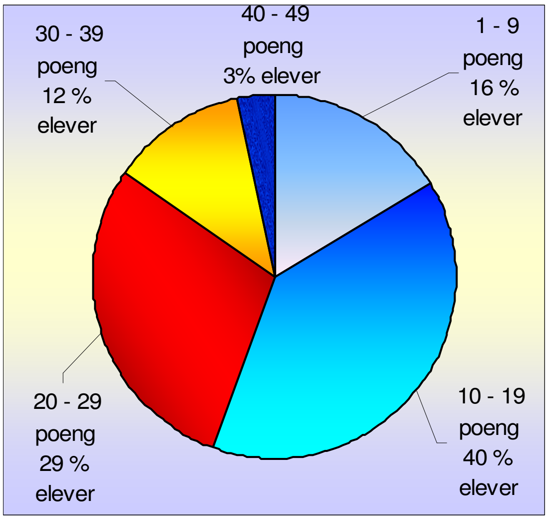


Figure 4.2.1.1.3: 9th grade (N=92) – test 1, 2005, Distribution of the students' points sums

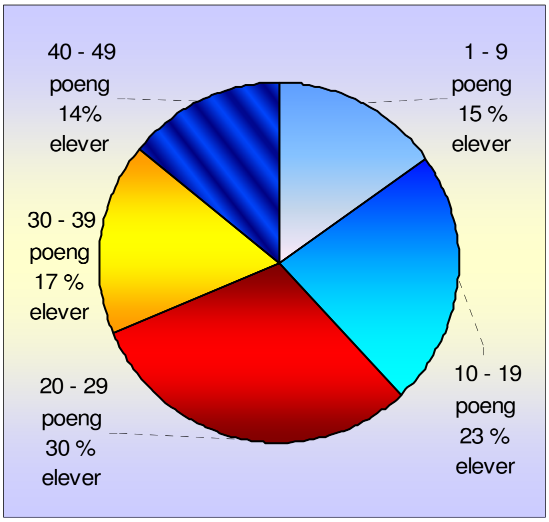


Figure 4.2.1.1.4: 9th grade (N=92) – test 2, 2006, Distribution of the students' points sums

4.2.1.2 Solution of different tasks

The main intentions for this part of the analyses are to describe the development of the students' results for both tests. First the results for some selected tasks will be analysed in more details. It is useful to present the analyses of the results on the tasks in selected groups:

(1) Test items with high results on both tests
(2) Test items with low results on both tests
(3) Test items with big increase of the results on test 2
(4) Test items with lower results on test 2 in comparison with test 1

Summary of the students' results on all test items

The following figures present the summary of the students' results on all test items. The solution frequency for every test item was calculated and the data is presented as comparison of those results for both tests.

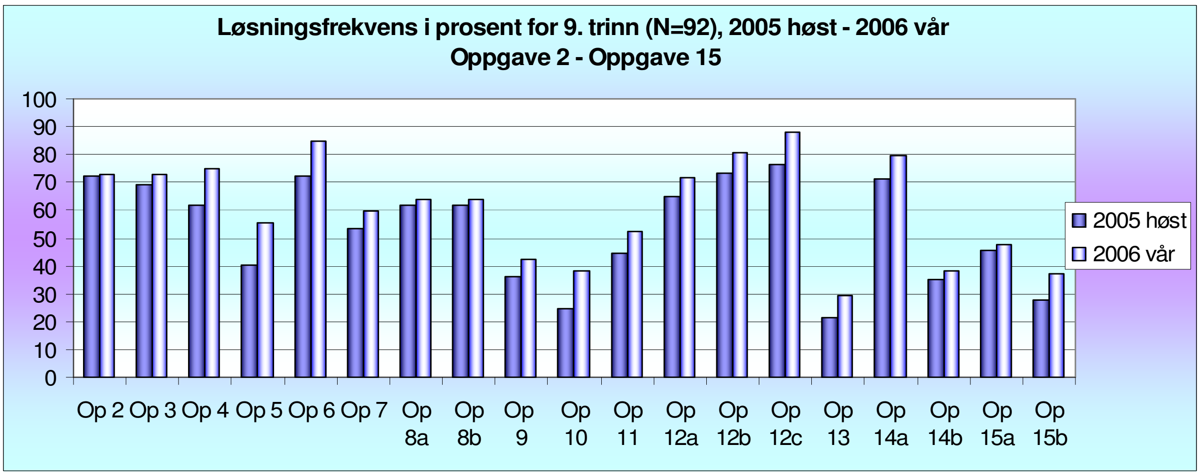


Figure 4.2.1.2.1: 9th grade (N=92) - Solution frequency in percents for tasks 2-15b, test 1(2005 fall) and test 2 (2006 spring). Note: Task 1 was not presented to the students.

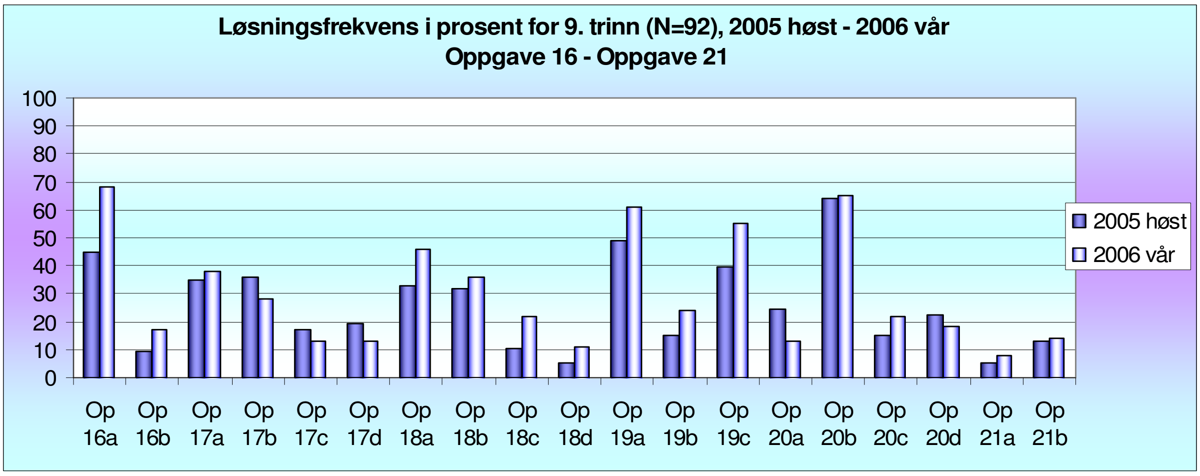


Figure 4.2.1.2.2: 9th grade (N=92) - Solution frequency in percents for tasks 16a-21b, test 1(2005 fall) and test 2 (2006 spring).

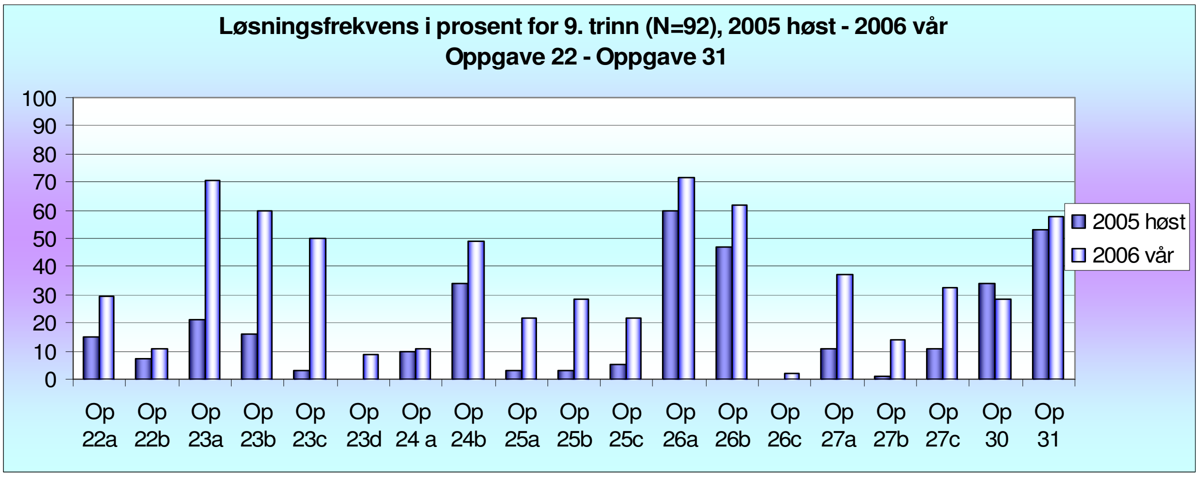


Figure 4.2.1.2.3: 9th grade (N=92) - Solution frequency in percents for tasks 22a-31, test 1(2005 fall) and test 2 (2006 spring). Note: Tasks 28 and 29 were not presented to the students.

A closer study of selected tasks

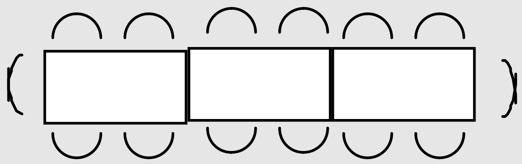
Next we would have a closer look on the students' solutions of concrete tasks. The tasks 26a, 26b, 26c are related to a figure pattern. They give information how the students describe the pattern in two concrete cases and in the general case. Tasks 21a, 21b are related to a construction of a word problem describing an arithmetical expression and are open ended tasks.

Exploration of a figure pattern

Task 26 is a sequence of three tasks, which progressively become more difficult to solve. The first two tasks can be dealt in a similar way. The task 26a asks how many chairs are necessary to construct a pattern with 5 tables (the 5th term of the sequential pattern), task 26b asks the same question for 7 tables (the 7th term) and task 26c is related to the general case with *n* tables (the *n* th term).

Students work initially on a set of two tasks which focus on finding a mathematical model that describes the pattern. They need to find a strategy of solution for both tasks, where the second task is more difficult than the first one. A problem – solving model containing five steps was proposed by Verschaffel et al. (2000). Costello (1991) and Mason (2002) suggested variety of helpful strategies for solution of word problems. It is desired to observe the given pattern, give some new insight and then construct a model that explains the situation. The students need to search for the conditions which describe the model. It is useful fist to solve examples to which the given figure pattern can be applied. Students need to turn back to their experience in solving tasks that involve generalisations. Then they need to reflect on how the concept of generalisation can apply in the given situation. This involves self-awareness of how the solution of the last task might follow from the solution of more simple tasks.

Oppgave 26 Et konfirmasjonsbord er satt sammen av småbord, og rundt er det satt stoler.



a Hvor mange stoler blir det plass til om vi har 4 småbord?..............................

b Enn om vi har 7 småbord?
....

c Dersom vi har *n* småbord, hvor mange stoler blir det da?...............................

Sequential figure pattern

Students are given a figure with a sequence of three tables and chairs around the tables. There are four chairs for every table, except the first table and the last table which have in addition one more chair put at the short side. Students need to notice that when the numbers of tables are increased with one, it is always necessary to add four chairs.

The tasks 26a, 26b, 26c are related to a given *figure pattern* presenting three tables put next to each other, surrounded by chairs. We can consider a figure pattern with elements tables and chairs. This pattern can be considered to be the third pattern in a *sequence of figure patterns*: the first term of the sequential pattern has one table with 6 chairs, the second term has two tables and 10 chairs, the third term of the patterns (presented in the task) has three tables and 14 chairs, the nth term has n tables and the number of chairs is $4.n + 2$. As a sequential figure pattern the table pattern is similar to the task 'Christmas tree lights' analysed by Stacey (1989). Two *linear number patterns* are related to the sequential figure pattern. The first number pattern is 1, 2, 3, 4, 5, … and presents the number of tables. The second number pattern is 6, 10, 14, 18, 22, 26, 30, 34, … and is related to the number of chairs necessary for a figure pattern containing 1, 2, 3, 4, 5, 6, 7, 8, … tables.

Students are more successful if they have previous experience of the described activity - the context acts as a model for the calculation, giving it some meaning (Mosvold, 2006). Costello (1991) also recognised the importance of previous experience, but argued also that students are able to solve unfamiliar problems if the students had developed abilities to visualise the problems' settings.

Oppgave 26a: Hvor mange stoler blir det plass til om vi har 4 småbord?..............................

Strategies for solution

Task 26a can be easily solved by counting the chairs necessary for four tables. This strategy was used for example by students who made help drawings in order to complete the pattern with one more table. Many students used additional drawing of tables to solve the task 26a and 26b. In order to complete the pattern with one more table it is necessary to observe that four more chairs should be added.

Table 4.2.1.2.1: 9th grade, Task 26a - solution frequency for Test 1 (2005 fall) and Test 2 (2006 spring)

Oppgave 26a	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	10	8
18	**61**	**72**
16	8	4
17, 19, 20	4	10
Andre svar	17	6

Another strategy is to observe the structure of the pattern - that there are four chairs for each table added, and count how many chairs are necessary for a pattern with three tables and add number 4 to it.

Most students were able to interpret the problem question and find how this figure pattern can be continued and how many chairs altogether would the new pattern of four tables have. Table 4.2.1.2.1 shows the students' results for task 26a – the results show that groups of 61% of the students (test 1) and 72% of the students (test 2) of the students were able to answer correctly to this question. In addition groups of 29% of the students (test 1) and 20% of the students (test 2) provided incorrect numerical answers.

Students' difficulties

The students who answer 16 chairs focus on the four chairs aside every table, but probably forgot that the first and the last table have in addition one more chair. When a student simply answers 14 chairs, they directly count the number of chairs shown in the picture, ignoring or not understanding that the question is related to continue the pattern with additional table. Groups of 4% of the students (test 1) and 10% of the students (test 2) answered 17, 19 or 20. This type of answer implies that the student possibly noticed that the patterns is defined by four chairs for each table, but the student ignored, forgot or did not notice that the first and the last table have additionally one more chair. It is possible also that the students counted incorrectly the number of chairs, using a help drawing. Verschaffel et al. (2000) pointed the necessity that students follow a model for problem solving and suggested that after the students execute the necessary calculations, interpret the outcome, and formulate an answer, they need finally to evaluate their answers in order to verify whether they found it.

Verschaffel et al. (2000) collected a lot of empirical data related to research of students' solutions of word problems and described cases of students' lack of sense making when solving word problems. The students inability to use real-world knowledge and sense-making of a given word problem were related to a number of factors as for example "stereotyped and straightforward nature" of the most word problems that students usually solved in class or lack of metacognitive skills to carefully read and analyse the problem situation (Verschaffel et al., 2000).

Oppgave 26b: Enn om vi har 7 småbord? ....

Most students who solved Task 26a were able to successfully solve this task as well. Groups of 48% of the students (test 1) and 62% of the students (test 2) could find the answer. This task is more difficult compared with the previous one. Orton and Orton (1999) noticed that the ability to continue a pattern comes before the ability to find the general term.

It is possible to use different strategies for solving the task as a strategy focusing on the structure of the pattern and counting strategy.

Table 4.2.1.2.2: 9th grade, Task 26b - solution frequency for Test 1 (2005 fall) and Test 2 (2006 spring)

Oppgave 26b	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	17	14
30	**48**	**62**
26	5	3
28	4	
29	4	1
31	3	1
32	2	3
42	5	3
Andre svar	10	12

Focusing on the structure of the pattern

Another strategy is to do calculations focusing on the structure of the pattern. It is necessary to observe the structure of the pattern and notice that an extended pattern of 7 tables needs 4 chairs for each table and add two more chairs. This strategy involves abilities for more analytical type of thinking, and more training in problem solving.

Through experiences with similar tasks students can develop better understanding how a problem can be extended to a new problem (Mason, 2002). Previous experience helps students to determine how to continue the pattern with additional elements. Previous experience with patterns in a variety of problem contexts would be very helpful in interpreting relationships among the elements of a pattern (Stacey, 1989). The students' attention is drawn to important characteristics of the figure pattern. The previous experience helps students to recognise linear relationship defining the pattern when the change is constant.

Counting strategy

A simple strategy of solution is to complete the pattern with four more tables and then count the chairs. This strategy can be effective in determining the answer, but it is possible that the experience of drawing and counting does not bring in focus the structure of the pattern. This strategy has certain limitations. If students are more engaged to find quickly the answer to the task, they respond to the question with less attention on the structure of the pattern. If a student used a help drawing to complete the pattern with more tables, it is possible to ignore, forget or not notice that additional chair is necessary at the end table to complete the pattern.

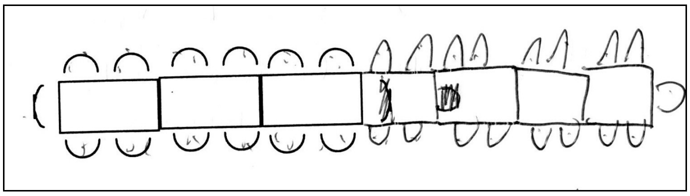


Figure 4.2.1.2.4: Student's drawing of four more tables

The students need to engage more on how the pattern is constructed and observe the pattern in order to develop conceptual understanding. It is also important to use the problem solving strategies looking back and to evaluate the received answer. Some students could not provide the correct answer for example because of incorrect counting, and because they did not check their answers. It is very likely that some students prolonged the pattern with four more tables and forgot to add one chair to the short side of the last table. So they found the chairs necessary for a different type of pattern – without additional chair at one of the sides or both of the sides. This way of thinking is based on focus of the difference between two consecutive table patterns - 4 chairs are necessary to prolong the pattern with one more table. Stacey (1989) noted that inexperienced students lacked ability to check their answers: "Inconsistency is a key feature – students grab at relationships and do not subject them to any critical thinking" (p.163).

Oppgave 26c Dersom vi har *n* småbord, hvor mange stoler blir det da?...............................

Generalisation of patterns is considered to be a very useful activity, especially when students pass a transition period between arithmetic and algebra. Hargreaves et al. (1999) analysed the students' abilities to make generalizations of linear number sequences and quadratic number sequences. The study found that most children between 7 and 11 years had strategies for analysing linear and quadratic sequences and many students were able to use what they found to generalise the sequences. Küchemann (1981) recognised that the studying of patterns in class helps students develop appropriate understanding of variables. Mason et al. (2005) noticed that "Every learner who starts school has already displayed the power to generalise and abstract from particular cases, and this is the root of algebra" (p.2). In addition specialising and generalisation are not just "things to do" to make tasks interesting - they lie at the heart of mathematical thinking, both when exploring, and when trying to make sense of some assertion or conjecture. Zazkis et al. (2002) found that there was a tendency among researchers "to separate algebraic symbolism from algebraic thinking" and argued that students need opportunities to engage in situations that help them to develop algebraic thinking "without the constraints of formal symbolism".

Table 4.2.1.2.3: 9th grade, Task 26c - solution frequency for Test 1 (2005 fall) and Test 2 (2006 spring)

Oppgave 26c	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	71	65
***n*.4+2**		**2**
"*n*=*x* bord.4 +2"		**1**
tallverdi (eks. 4, 40; 120, 11; 15;)	19	10
Bokstav (*N; m; x; å; b*)	9	17
"*n*.4"; "4.*n*"		3
bokstavuttrykk ("bord=stol.4(+2)"; "*n*.6")		2
"ingen"	1	

The problem requires first to observe the regularity in the task, and than to describe the nature of the regularity precisely enough. The difficulty is to find a class of cases over which this pattern applies. To investigate the task closely students need to apply problem solving abilities. The problem solving strategy that can be used is specialising for concrete cases and generalising for the problem in the general case.

Strategies for solution

The starting point lies in the problem statement – it is an example of a pattern. Students have to assume that this concrete and observable pattern can be extended to a sequence of patterns. A successful response to this problem depends on the recognition that there are infinite numbers of possibilities which can be described with a general case. The difficulty is to form an argument how a sequence of patterns can be defined.

The investigation of the problem opens the door to explorations using a variety of strategies. Relationships among elements in the pattern provide opportunities for students to examine how the pattern can be described algebraically in the general case:

- There are four chairs for every table and in addition a chair at every end of the table pattern, so for n tables there are $4n+2$ chairs.
- There are five chairs for the first and last table and there are four chairs for every table between, so for n tables there would be (n–2) tables with four chairs, and the total number of chairs would be $2 \cdot 5 + 4(n\text{-}2)$.
- Another opportunity is to count the chairs around the tables – one chair at each end and at the both long sides the chairs are 2+2+2+ ... 2, so many times two chairs as the number of tables. Then the total number of chairs would be $(1+n \cdot 2) \cdot 2$.
- Another possibility is to construct a number pattern 6, 10, 14, 18, 22, 26, 30, 34, … which describes the number of chairs necessary for the sequential figure pattern with 1, 2, 3, 4, 5, 6, 7, 8, … tables. A necessary question to ask is "What is the rule describing the number sequence 6, 10, 14, 18, 22, 26, 30, 34, … ? " and observe that the difference between two elements in the sequence is 4 and the number 2 is added for each element in the sequence, than the n th element in the sequence is $4n+2$. This strategy of thinking is more likely to be used if the students had previous experience with generalisations of linear number patterns.

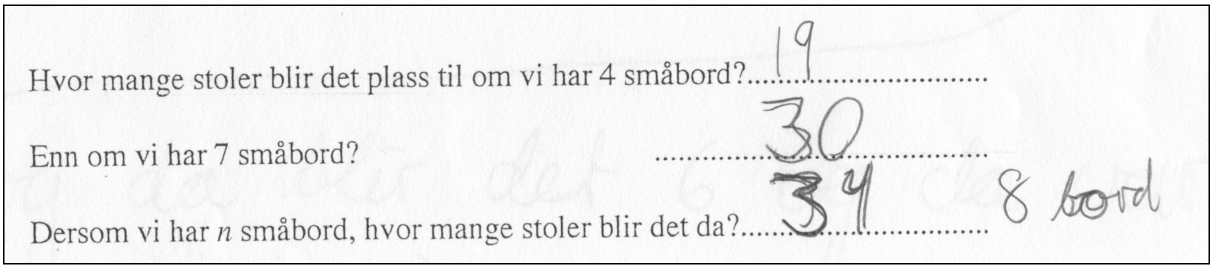
Only three of the students tested in the spring were able to find an expression describing the pattern. Two of the students found the formula $4n+2$. It is interesting that the third student used syncopated notation and gave as an answer "*n*=*x bord*.4 +2". The student received a point for this answer, although the answer is partly correct. Such answer showed that the student was able to generalise the pattern, although the student experienced problems to notice what role had the variable n.

Comparison of the results with the previous year

The comparison of the results indicated that the group of 92 students experienced a lot more obstacles in generalisation of the pattern compared with the group of students tested in the previous year. The results for the group of 74 students who solved the same task 26c the previous year show that 4% of the students (2004 fall) and 14% of the students (2005 spring) were able to generalise the pattern (Espeland, 2006). On the other hand the comparison of both groups shows that the results on the task 26a were very similar. For task 26b the group of 92 students performed a lot better in the spring – 62% of the students (test 2) solved task compared to 49% of the students from the group of 74 students (tested in 2005 spring).

Students' difficulties

Most students did not give answers to the problem 26c. Some of those students responded with a question demonstrating uncertainty - "Hva står *n* for?", "Hva er *n*?". Such students had problems to determine that the number of possible tables can be a general number instead as a specific number. Some students gave a numerical answer to the question - 20% of the students (test 1) and 9 % of the students (test 2). Such students could not use the variable n as a general number and instead replaced it with concrete numbers. So the answer they gave was the number of chairs for a pattern with concrete number of tables. Some students responded to the question stating an answer that was a letter: *N; m; x; å;* or *b*. It can be noticed that the students recognised that in case of *n* tables the number of chairs is not a concrete number, but could not find a concrete formula to describe the *n*th term. An understanding of the meanings and uses of variables develops gradually (Van Amerom, 2002). The ability to relate symbolic expressions to problem contexts is related to the student's initial understanding of the meanings and the uses of variables. The study of Küchemann (1981) found that the majority of 14 -15 years old students were not able to use the variables as general numbers and that most middle-grades students would need considerable experience with algebraic problems before they would develop proper understanding of the meanings and the uses of variables. In order to express generality students need to be able to use the variable *n* as a general number. The figure below shows the answers of one student to the tasks 26a-c. The student could find the solution of the task 26b, but failed to see a correct solution in the general case.


Hvor mange stoler blir det plass til om vi har 4 småbord?.......19.......................

Enn om vi har 7 småbord? ....30...........

Dersom vi har *n* småbord, hvor mange stoler blir det da?.......34.................. 8 bord

Figure 4.2.1.2.5: Student's answers for tasks 26a-c

For the question 26c the student provided the solution for the 8^{th} term of the sequential figure pattern and noticed next to the solution that it is related to 8 tables. The student could find the answer for concrete cases - 7 tables and 8 tables. The student intended to solve the task 26c, but could not use the variable *n* as a general number and could not describe the general case. These findings are similar to those of other researchers.

Orton (1999a) discussed the students' levels of pattern recognition and described three different stages of development in terms of items of the pattern recognition tests. Those stages were in close relation to the students' abilities to recognise patterns and transformations of patterns. The children's strategies for analysing linear and quadratic sequences included looking for difference between consecutive terms, looking at the nature of the differences, looking for differences between the differences, looking at the nature of the numbers, usually odd and even. The study suggested that students should be provided with a very big variety of pattern structures and type of tasks as a way to encourage children to be more persistent and be able to use variety of strategies.

The study of Hargreaves et al. (1999) found that young students shouldn't be asked only *to continue* a sequence, because this leads only to children's focus on finding the difference between two terms. Tasks which require "*to solve*" (to provide a rule for a given sequence) and "*to sort*" (a child has to find is a sequence of numbers a pattern or not) proved to provide best context for work with number sequences.

Stacey (1989) observed the regularity of the students' responses to three pattern tasks and noted that the students' written solutions gave limited amount of information about their problem solving strategies and tactics, but that certain conclusions could be drawn. The students inexperienced in problem solving tended to grab at "easy relationships" and did not work from the simple cases. Most students assumed that to "explain" their solutions required to show their calculations. There was not "spontaneous use of algebra to express the generalisations", although many students indicated that they found generality in the specific numbers. The students with experience in problem solving and pattern tasks tended to examine the data "more completely" although some of them made technical mistakes and could not find the right formula. They were more consistent in using linear models when answering to all questions. When they explained their solutions, those students more often related the answers to spatial patterns and number patterns. They seemed to be "more aware of the nature of the pattern to be found", they were "aware of the data" and in addition those students were "more aware that the one relationship applied for all values".

The students' problems to construct an algebraic description of the pattern in the general case can be related to the obstacles that the beginner learners of algebra experience with developing proper understanding of the meanings and the uses of variables. Herscovics (1989) identified obstacles induced by instruction, obstacles of an epistemological nature and obstacles connected with the learner's process of accommodation. He considered the Piaget's theory of *equilibrium* to provide a suitable framework for analyses of the students' difficulties. From Piagetian perspective, the acquisition of knowledge is a process of constant interaction between the learner and the environment.

The tasks related to construction of word problems

The process of construction of a word problem describing a mathematical expression is an open ended activity, requiring application of problem solving skills. Constructing a problem related to a given arithmetical expression is not an elementary task and involves careful planning, implementation and evaluation. First students need to read carefully the problem and find what it states – what are the key concepts, ideas and information. It is necessary to understand well the problem's statement. The task to construct a problem is not a routine task. The students are given an opportunity to construct a mathematical problem according to their own interests. Students build better conceptual understanding when they are involved in construction and investigation activities (Mason 2002, Mason et al., 2005). Research found that it helps students grasp the mathematical and linguistic structure of the word problem and "helps students to understand the relationships between problems or tasks and the contexts or situations that model them" (Pugalee, 2005, p.172). Pugalee (2005) viewed construction of a problem to be an activity helping students develop better conceptual understanding and proposed *Review-Respond-Reflect* method for composing word problems as it was already described in the literature review (see chapter 2.6.5 Composing word problems).

The problem's statement is a source of investigation – students need to focus attention on what real life situation to describe, what question to pose, what words to use, is it desirable to change the question with another one, is it necessary to vary the problem context, could it be better to search for other possibilities. Pupils need to be presented different opportunities to explore mathematical ideas and develop better conceptual understanding - explorations help pupils become more sophisticated and more proficient in mathematics (Kilpatrick et al, 2002).

An important part is to develop a plan and concentrate on generating ideas (Verschaffel et al., 2000; Pugalee, 2005). Students know what sort of a result to expect, so they may need to modify their initial statements in order to get a valid result. They can apply problem solving strategies as for example looking back to their previous experience, specialising, conjecturing, solve a similar problem by replacing the "awkward numbers by easier ones", or try to visualise the situation described (Costello, 1991; Mason et al., 1991). It is important to acknowledge if being stuck (Mason et al., 1991). Awareness of being stuck helps students recognise the need to change the problem solving strategy. When being stuck students need to learn to respond positively, and try to find new possibilities.

A meaningful construction of a problem requires students also to be critical to their results and use reflection as an important strategy to control the outcome of the construction process. The process of reflection does not mean simply to think back what happened (Mason et al., 1991; Kilpatrick, 1978). It is important to reflect in a disciplined, intentional and systematic way. Reflection is important to "stimulate pupils to integrate ideas", because the involvement in an activity often absorbs all their attention. When working in a class, it is important that students reflect upon their experience in organised small groups or whole class discussions (Brekke, Rosén, & Grønmo, 2000).

Oppgave 21a Lag din egen fortelling som passer til disse regnestykkene:

$\boxed{4 : 0{,}5}$

Table 4.2.1.2.13: 9th grade, Task 21a - solution frequency for Test 1 (2005 fall) and Test 2 (2006 spring)

Oppgave 21a	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	48	43
Riktig målingsdivisjon og realistisk kontekst	**3**	**7**
Riktig målingsdivisjon (kode 2)	**2**	**1**
Urealistisk kontekst	9	12
Subtraksjonsfortelling	2	2
Regnefortelling til det inverse regneuttrykket	11	18
Multiplikasjonsfortelling	3	2
Andre svar	18	11
fortelling (eks. "Hva er 4:0,5.")	4	4

The group of students who constructed a correct story was 5% of the students (test 1) and 8% of the students (test 2). Many students did not write a story. Some of the students provided a story for the reverse expression (0,5 : 4) or described a story using unrealistic context.

In the fall of 2005 three students out of 92 students could construct a problem related to the expression, which were given code 1, as for example the stories:

(1) "Det koster 0,5 kr for en tyggis. Hvor mange kan Elise kjøpe for 4 kr?"

(2) "Per hadde 4 kr. Så kjøpte han små ting til 0,5 kr. pr. ting. Hvor mange ting kunne han kjøpe?"

In addition two students (first test) described a context which relates to the story, but did not pose a question - one of the stories was:

"*Ola hadde 4 l. saft, og posjonerte det ut i 0,5 l. flasker*".

In the spring of 2006 (second test) 8% of the students could describe a story related to the expression, in addition one student made a story which was coded with code 2, because the problem was not explained clearly enough and the described context had a second meaning, not related to the expression. Examples of students' answers, describing a story related to the expression are:

(1) "Per har 4 kr, hvert eple koster 0,5 kr hvor mange epler får han?"
(2) "Ada hadde 4 liter vann. Hun hadde flasker som rommet 0,5 liter. Hvor mange flasker trengte hun?"
(3) "Mel skal fylles i esker, med 0,5 kg i hver. Hvor mange esker trenger man til 4 kg?"

Students' difficulties

Many students (49% - test 1; 50% - test 2) made a story not related to the expression 4 : 0,5. Such problems can be classified as involving the operations addition, subtraction, multiplication or division of the numbers 4 and 0,5. The problems students had to construct a story can be related to the structure of the expression 4 : 0,5. It involves the decimal number 0,5 and division operation. Division word problems that can be related to it can not be for example common problems for dividing a group of objects between a group of people.

Many students constructed word problems not related to the expression 4 : 0,5. Commonly the students used as a context shopping of apples, bananas, cola or candies:

(1) "Asle kjøper 4 0,5 L colaflasker. Han skal dele med 3 venner. Hvor mange liter får de hver?"
(2) " Lise skal kjøpe epler, hun skal ha 4 epler og 4 epler koster 0,5 kr hva koster ett?"

Many students tried to construct a problem as a direct translation of the expression into everyday language and failed to recognise that the stories did not describe the problem:

(1) "4 personer skal dele en halv pizza. Hvor mye får hver av dem?"
(2) "fire venner fant 0,5kr. de ville dele, så det gjorde de..."

There were also stories not describing a mathematical word problem:

"Det var en gang ei jente som hadde en prøve. Hun skulle prøve og regne ut 4:0,5 men hun skjønte ingen ting!"

There were examples of suspension type problems, as the problem:

(1) "Bob Kåre gikk til butikken for å kjøpe bleier. Han kjøpte 4 pakker bleier og hadde et barn som bare trengte en halv bleie. Hvor mange bleier brukte barnet til Bob Kåre?"

Many students (46% - test 1; 43% - test 2) did not attempt to construct a story and did not give any answer. Some of them commented that they could not find a meaningful way to construct a problem related to the task.

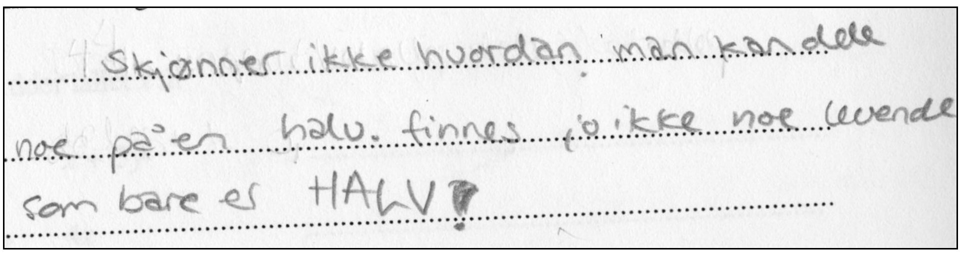

skjønner ikke hvordan man kan dele
noe på en halv. finnes jo ikke noe levende
som bare er HALV?

Figure 4.2.1.2.6: Student's comments for task 21a

There were some students who made stories which can not be considered to be mathematical problems. Examples for such stories are:

(1) "Johny Baluba har 4 epler og skal dele de på en halv person."
(2) "4 Bala Garbaer gikk langs veien, så kom det en halv Roger og ville ha dem, hvor mange fikk han?".

Oppgave 21b Lag din egen fortelling som passer til disse regnestykkene:

$$\boxed{5{,}25 \cdot 3{,}28 = 17{,}22}$$

Groups of 58% of the students (test 1) and 61% of the students (test 2) did not make a word problem related to this expression. 13% of the students (test 1) and 14% of the students (test 2) were able to construct a relevant mathematical problem, although most problems were not realistic enough. Most problems given by the students were connected with shopping of products. They were related to the expression and were given code 1, or code 2, although the students had problems constructing the problems in a very realistic way. Some students experienced problems, because the numbers 3,28 and 5,25 are decimal numbers and are not suitable for describing quantity of bananas, apples, nuts, or candies. In addition it is problematic in every day life to find objects with a price 3,28 kr or to talk about someone working 3,28 hours.

Examples for problems that were constructed in a meaningful way were connected with finding area and distance:

(1) "Petter målte en aker som hadde bredde3,28m og lengde 5,25m. Hva var arealet? 17,22"
(2) "En sykkelist rydder 5,25 km i t, hvor mange km kom han etter 3 timer og 28 min"

Students' difficulties

For example some problems can be considered to be artificial, although they are related to the given expression. The students were given one point for such answers, although the problems were not described in a realistic way. There can be given some examples:

(1) "Jon har fått seg jobb, han får 5,25 kr i timen. Han jobber 3,28 timer. hvor mye tjener Jon?"
(2) "5,25 nøtter lå på bordet, og Røkke kjøpte 3,28 ganger så mange. Hvor mange kjøpte Røkke."
(3) "Per fik 5,25 bananer av en ape til jul. Han fikk også 3,28 ganger så mange bananer av mor. Hvor mye bananer fikk han av mor?"
(4) "Knut hadde 5 og en kvart kg mel. Men han trengte 3,28 ganger mer. Hvor mye mel må han ha?"
(5) "Anne kjøper en tyggis for 5,25 og siden det var tillbud kjøper hun 3,28 tyggiser til det ble17,22."
(6) "Jeg kjøpte druer som kosta 3,28 kr/kg. Derfor kjøpte jeg 5,25 kg=17,22 kr."

Groups of 25% of the students (test 1) and 29% of the students (test 2) constructed stories not related to the expression. Here are some examples for problems that described other expressions, or not well defined problems; most of those word problems do not have a realistic context:

(1) "Hvor mye må du betale til sammen hvis du kjøper epler til 5,25 kr og bananer til 3,28 kr og ganger de to prisene sammen?"
(2) "3 venner jobbet i en butikk. Lønna var 5,25 kr. Til hver av dem. De fikk 17,22 kroner til sammen."
(3) "5,25kg mel trenger man til en brød deig. Men hvor mye vann om det til sammen skal veie 17,22kg."
(4) "Du er perfeksjonist og skal bake en kakke du skal ha 5,25 dl mel ganget 3,28 du blander det sammen og for 17,22 på vekta."
(5) "Per hadde 5,25 kr. Lise hadde 3,28 kr. Til sammen hadde de 17,22."
(6) "Per fant 5,25 kr. så fant han 3,28 kr da hadde han 17,22 kr til sammen ..."

Some students did not try to make a word problem, but just related a story:

"17,22 er lik 3,28 ganger 5,25 det er riktig. "

Other students related stories to the expression, but did not describe the context in a serious way. For example some students made stories as:

(1) "5,25 liter vann og gange det med 3,28 l vann blir til sammen 17,22 liter"
(2) "Det var en gang 5,25 som ville gange seg sammen med 3,28. Kan du hjelpe da å finne svaret?"
(3) "Det jobber 5,25 på en jobb. De tjener 3,28 kr i timen. Da blir det til sammen 17,22 kr."

Some of the students' difficulties can be related to their previous experiences. Some students found it complicated to use decimal numbers to construct word problems. Limited previous experience with solving word problems can interfere in the process of constructing word problems. Alseth (1998) argued that most students experience problems, because of the limited experience with division problems - that can be an obstacle for students to construct meaningful problems. Alseth discussed results from the KIM study (Tall og tallregning) concerning a task requiring students to construct a mathematical problem to the expression 18 : 4,5 = 4. The study found that 12% of the 7th grade students and 26% of the 9th grade students could provide a relevant story. When students work with division problems in school, typically they are presented mostly problems where some objects are divided in groups, so students think in a similar way when they need to construct a problem related to division. It is necessary that students have opportunity to solve a big variety of tasks (Alseth, 1998).

Costello (1991) argued that the level of difficulty of a word problem is related to the size and the complexity of the numbers. Brekke et al.(2000) discussed students' results from the KIM project for a task requesting students to construct a word problem related to the algebraic expression $3a + 2a = 5a$. The participants were groups of students in 6th, 8th and 10th grade. A close investigation of the students' responses showed that only 2%, 5% and 7% of the corresponding groups of students could write a relevant word problem. About one third of the students in each group did not answer to this test item. One of the reasons for this was that students were not used to work with such type problems (Brekke et al., 2000). Usually students work in the "opposite way": they are given as a problem to analyse particular context and it is necessary to translate the problem's situation into algebraic language.

The activity of constructing a word problem requires focus, attention, persistence and involvement. It is possible that some students failed to recognise that they need to search among different possibilities, and to do an evaluation of the received result. It is possible also that some students were not attracted to the activity of constructing a task and responded rather mechanically, not being interested to put additional efforts in search for possible solution. Another aspect of the task is that maybe some students found it to be too difficult. It is likely that they gave up after they tried mentally or on paper some possibilities, and came to the point when they felt being stuck and could not continue because they were not focused enough or not persistent enough to search for other strategies of solution.

The importance of the construction of word problems is recognised by variety of researchers. Mason et al. (1991) considered that it is essential students to be interested in the activity. Mason et al. (2005) suggested students to be stimulated to make their own problems when they solve a particular word problem by varying the language or the quantities in the word problem - this may help them recognise how they can solve a whole group of word problems, and may help them see the generality through the particular. Mosvold (2006) noticed that a challenge for the teachers is to stimulate students to discover things for themselves and be active learners.

Test items with high results on both tests

Now we would like to go closer into some groups of tasks from the tests. It is interesting to find the tasks with high increase of the students' results on the second tests. Those tasks were solved by at least 80% of the students on one of the tests. The test items with very high solution frequency for both tests are Op 2, Op 3, Op 4, Op 6, Op 12a, Op 12b, Op 12c, and Op 14a. On those test items the average solution frequency is 71% of the students (test 1)and 79% (test 2). Op 2, 3, 4, and 6 are problems from the beginning part of the test. The task Op 2 asks the students to find one fourth of 60g. To solve task Op 4 students need to find 25% of 40 km.

Oppgave 3 Finn lengden på gardinstoffet Mari må kjøpe, når det skal være fem gardiner, og hvert skal være 1,05 meter.

Task Op 3 is a multiplication word problem, requiring a simple calculation to solve it. All numbers needed to solve the problem are exactly stated in the problem. This is a problem describing a real life situation. To find the answer, students need to know how to multiply decimal numbers with whole numbers. This task was unproblematic to solve – most of the students (71% of the students (test 1) and 73% of the students (test 2)) could find that they need to multiply the two numbers given in the problem and could do the calculation correctly.

Oppgave 6 Temperaturen forandrer seg fra –5 ˚C til +8 ˚C.
Hva er stigningen i temperatur?

Table 4.2.1.2.4: 9th grade, Task 6 - solution frequency for Test 1 (2005 fall) and Test 2 (2006 spring)

Oppgave 6	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	9	8
13 grader	**74**	**85**
14 grader	8	2
12 grader	3	2
3 grader	3	1
Andre svar	3	2

To solve the task the students need to analyse the described situation and find that they need to subtract the two given temperatures. Students need to know to subtract negative numbers. The groups of 74% of the students (test 1) and 85% of the students (test 2) could find the right answer. This task was also easy to solve for most of the students. Some of the students used counting strategies – they wrote down all numbers between (-5) and (+8) or they used help drawings of thermometers.

Oppgave 12a Sett *ring rundt* det største tallet og *kryss over* det minste tallet:

0,625 0,25 0,3753 0,125 0,5

Many students answer correctly to the problem posed in task 12a. The students need to mark the biggest and the smallest number among a group of five decimal numbers. All the five decimal numbers are smaller than 1. The students need to look at the numbers and be able to compare them in order to find the minimum and the maximum. The difficulty is that the decimal parts of those numbers are chosen to have different numbers of digits in the decimal part.

Table 4.2.1.2.5: 9th grade, Task 14a (minimum) - solution frequency for Test 1 (2005 fall) and Test 2 (2006 spring)

Oppgave 12a (minste tall)	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	11	6
0,125	**57**	**65**
0,625	1	3
0,25	9	4
0,3753	15	13
0,5	7	8
Andre svar	1	1

Table 4.2.1.2.6: 9th grade, Task 12a (maximum) - solution frequency for Test 1 (2005 fall) and Test 2 (2006 spring)

Oppgave 12a (største tall)	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	12	10
0,625	**59**	**66**
0,25	3	2
0,3753	7	8
0,125	1	3
0,5	13	11
Andre svar	4	

Groups of 33% of the students (test 1) and 29% of the students (test 2) made an error decision when finding the minimum. Groups of 28% of the students (test 1) and 24% of the students (test 2) could not find correctly the maximum. Table 4.2.1.2.5 shows that the most common mistake was when students found that the minimum was the number 0,3753. Many students thought that the maximum number was the number 0,5. These types of wrong answers suggest that some students had misconceptions as that a decimal number is the smallest when it has the biggest number digits after the decimal point or that a decimal number is the biggest when it has the smallest number digits after the decimal point.

This task was used in the KIM project. The participating students were asked to find the minimum. 79% of the 8th grade students found the correct result, 10% of the students answered 0,3753 and 7% of the students answered 0,5 (Brekke, 1995).

Oppgave 12b Sett *ring rundt* det største tallet og *kryss over* det minste tallet:

3,521 3,6 3,75

Table 4.2.1.2.7: 9th grade, Task 12b (minimum) - solution frequency for Test 1 (2005 fall) and Test 2 (2006 spring)

Oppgave 12b (minste tall)	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	20	17
3,521	**70**	**76**
3,6	9	5
3,75		1
Andre svar	2	

Table 4.2.1.2.8: 9th grade, Task 14a (maximum) - solution frequency for Test 1 (2005 fall) and Test 2 (2006 spring)

Oppgave 12b (største tall)	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	14	13
3,75	**65**	**74**
3,521	9	7
3,6	11	7
Andre svar	1	

Task 12b is a similar task to 12a – this time students need to compare three numbers with very close values of the decimal part. Most students did not have any problems to find both the minimum and the maximum. The results on the second test are better. Small group of students 11% of the students (test 1)and 6% of the students (test 2)could not find the minimum correctly. The most typical wrong answer for a minimum was 3,6- the decimal number with least digits after the decimal point. 21% of the students (test 1) and 14% of the students (test 2) had problems to find the maximum.

This task was used in the KIM project. The participating students were asked to find the maximum. Groups of 88% of the 8th grade students found the correct result, 6% of the students answered 3,521 and 5% of the students answered 3,6 (Brekke, 1995).

Oppgave 12c Sett *ring rundt* det største tallet og *kryss over* det minste tallet:

4,09 4,7 4,008

Table 4.2.1.2.9: 9th grade, Task 12c (minimum) - solution frequency for Test 1 (2005 fall) and Test 2 (2006 spring)

Oppgave 12c (minste tall)	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	19	16
4,008	**71**	**78**
4,7	7	1
4,09	2	3
Andre svar	2	1

Table 4.2.1.2.10: 9th grade, Task 12c (maximum) - solution frequency for Test 1 (2005 fall) and Test 2 (2006 spring)

Oppgave 12c (største tall)	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	8	7
4,7	**77**	**86**
4,09	9	4
4,008	5	3
Andre svar	1	

Most students could solve task 12c. The three decimal numbers were chosen to have different number of digits in the decimal part and to be close to the number 4.

This task was used in the KIM project. The participating students were asked to find the maximum. 94% of the 8th grade students found the correct result, 4% of the students answered 4,09 (Brekke, 1995).

Oppgave 14a Les av på følgende skalaer og skriv riktig desimaltall i ruta.

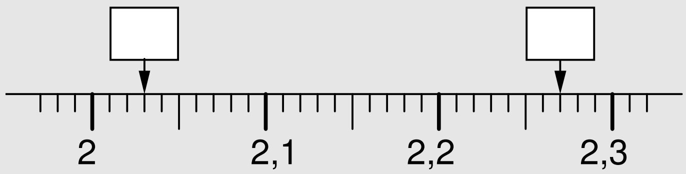


Table 4.2.1.2.11: 9th grade, Task 14a (route 1) - solution frequency for Test 1 (2005 fall) and Test 2 (2006 spring)

Oppgave 14a (rute 1)	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	12	12
2,03	**67**	**74**
2,3	9	4
2,4		2
3	5	3
2	3	
Andre svar	3	4

Table 4.2.1.2.12: 9th grade, Task 14a (route 2) - solution frequency for Test 1 (2005 fall) and Test 2 (2006 spring)

Oppgave 14a (rute 2)	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	15	14
2,27	**52**	**61**
2,07	7	7
2,7	2	1
2,207	10	10
7	5	2
Andre svar	9	5

Task 14a is a measurement task. The students are asked to find two numbers on a ruler. It is a "broken ruler" - it shows a section of a non standard unit of measurement. Students need to analyse the figure and make interpretation of the given information. They need to construct proper mental models in order to find the two positions on this ruler. Such activity help students develop better conceptual understanding in geometry.

The tables above show the different answers that the students gave. It is interesting to notice that 13 students (test 1) and 15 students (test 2) answer correctly 2,03 for the first number and gave as a second answer 2,07 or 2,207. This task was used also in the KIM project - 71% of the 8th grade students could find the first number, 67% of the students found correctly the second number (Brekke, 1995). Brekke argued that students experienced problems connected with the symbolising of decimal numbers and zero as place holder.

Instructional activities need to help students develop conceptual understanding when studying geometry (Bjuland, 2002; Mason et al., 2005). In a very extensive study Strutchens, Martin & Kenney (2003) analysed NAEP results over the years and contributed the difficulties of the students to the fact that 'students lack experience with attributes and unites of measurement before they begin to use standard measuring devises and formulas' and recommended that students get more experiences that form conceptual understanding of the different types of measurement.

Test items with minimal results (0 – 11 solution frequency points) on both tests

Some of the mathematical problems were very difficult to solve. Tasks Op 18d, Op 21a, Op 22b, Op 23d, Op 24 a, Op 26c were test items with very low solution frequency for both tests. For those test items the average solution frequency was 4,5% of the students (test 1)and 8,5% (test 2). Op 21a is an open ended task requiring construction of a word problem. Op. 23d requires students to simplify the expression $2y \cdot y^2$, Op 24 - students have to answer whether $x + y + z = x + p + z$ *is always true, never true, or sometimes true*, Op 26c is a task involving generalisation of a figure pattern requiring students to find a formula for the n^{th} term.

Oppgave 18d Skriv svaret som desimaltall.

0,6 : 0,2 = ……………………………

Table 4.2.1.2.13: 9th grade, Task 18d - solution frequency for Test 1 (2005 fall) and Test 2 (2006 spring)

Oppgave 18d	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	63	53
3	**5**	**11**
0,3	25	23
0,03	1	3
Andre svar	5	10

This task was among the most difficult tasks to solve. The majority of students (63% - test 1 and 53% - test 2) did not try to calculate the answer. Only 5% of the students (test 1) and 11% of the students (test 2) were able to find the answer correctly. 30% and 33% of the students tested in the fall and the spring made mistakes. The students had problems to find the correct place of the decimal point in the result. The majority of students who tried to solve the expression answered 0,3. This wrong answer suggests that most students experience problems when dividing two decimal numbers. Maybe students experience problems, because it is not clear to them what rule they need to apply for the decimal point in the result. Many students had the misconception that the division of two numbers always makes the result smaller (Brekke, 1995).

Oppgave 22b Skriv riktig tall i ruta

$14 : \boxed{} = 0{,}25 \cdot 14$

Table 4.2.1.2.14: 9th grade, Task 6 - solution frequency for Test 1 (2005 fall) and Test 2 (2006 spring)

Oppgave 22b	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	52	53
4	**8**	**11**
0,25	5	4
2	4	5
Andre svar	30	26

This task was another very difficult task to solve. The students need to find the value of an unknown number in the left side of the arithmetical expression. More than half of the tested students did not provide their solutions. Very small groups of students - 8% of the students (test 1) and 11% of the students (test 2) found the answer. The variety of the wrong answers is big. If students do not develop conceptual understanding to support the procedural knowledge, the rules can easily be forgotten.

Test items with big increase of the results on test 2

Increase between 15 and 19 solution frequency points

Test items with very high solution frequency on both tests are Op 5, Op 19c, Op 22a, Op 24b, Op 25a, Op 25c, and Op 26b. The group of all students showed improvement of the results on those seven test items with increase between 15 and 19 solution frequency points. Every test problem in this group is a test problem, which was solved by at least 14 more students in the spring. The tasks in this group include simple algebraic tasks, task related to figure pattern, estimation of expression with decimal numbers, addition of fractions.

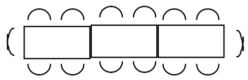
Oppgave		Frekvens i prosent	
		2005 høst	2006 vår
Op 5	$\frac{1}{2}+\frac{1}{4}=?$	40	55
Op 19c	Sett ring rundt det tallet du mener er *nærmest svaret*. Du trenger ikke regne ut svaret. 0,73 · 46,2 | 0,03 0,3 3 30 300	39	55
Op 24b	$2a + 3 = 2a - 3$ Dette ☐ er alltid sant ☐ er aldri sant ☐ *kan* være sant, nemlig når	34	49
Op 25a	Adder tallet x til $x + 3y$	3	22
Op 25c	Adder tallet x til 7	5	22
Op 26b	Et konfirmasjonsbord er satt sammen av småbord, og rundt er det satt stoler.  Enn om vi har 7 småbord?	47	62

Table 4.2.1.2.15: Group of tasks

Increase above 20 solution frequency points

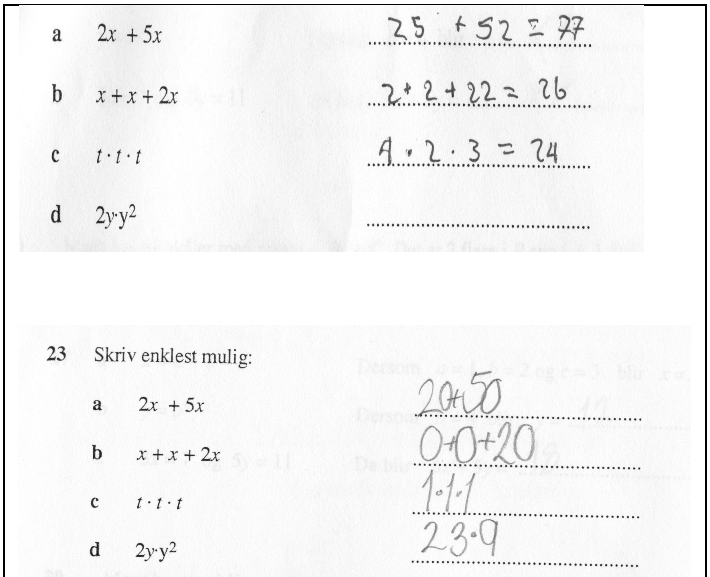
The group of all students showed very significant improvement of the results for seven test items. For test items Op 16a, Op 23a, Op 23b, Op 23c, Op 25b, Op 27a, and Op 27c the increase was between 21 and 49 solution frequency points. Every test problem in this group is a test problem, which was solved by at least 18 more students in the spring. Most of the tasks in this group are simple algebraic tasks.

Oppgave		Frekvens i prosent	
		2005 høst	2006 vår
Op 16a	Skriv riktig tall i rutene 574 = 5·100 + ☐ ·10 + 4 · 1	45	68
Op 23 a	$2x + 5x$	21	71
Op 23b	$x + x + 2x$	16	60
Op 23c	$t \cdot t \cdot t$	3	50
Op 25b	Adder tallet x til $4x$	3	28
Op 27a	$x = a + b - c$ Dersom $a = 1$, $b = 2$ og $c = 3$ blir $x =$........................	11	37
Op 27c	$3x = 7$ og $5y = 11$ Da blir $3x + 5y =$	11	33

Table 4.2.1.2.16: Group of tasks

Tasks 23a, 23b, 23c

The student' responses to the tasks 23a, 23b, 23c gave evidence for a big improvement in the students' abilities to simplify algebraic expressions.


a $2x + 5x$ 25 + 52 = 77
b $x + x + 2x$ 2 + 2 + 22 = 26
c $t \cdot t \cdot t$ 4 · 2 · 3 = 24
d $2y \cdot y^2$

23 Skriv enklest mulig:
a $2x + 5x$ 20+50
b $x + x + 2x$ 0+0+20
c $t \cdot t \cdot t$ 1·1·1
d $2y \cdot y^2$ 23·9

Figure 4.2.1.2.7: Students' answers for tasks 23a-d

In the fall it was quite common for the students to look upon the variables as objects rather than general numbers, as it is demonstrated in the students' responses to the tasks given as examples in the figure. The examples presented here show some cases of students' inability to use the variables as general numbers. Instead the students ignored the variables and replaced them with concrete numbers.

The study of Küchemann (1981) investigated closely the role of the variables in algebra and found that students give different meaning to the letters. If students are at level 1 and level 2 of variables use, they do not have proper understanding of variables and are not able to interpret algebraic letters as generalised numbers or specific unknowns.

Test items with lower results on test 2 in comparison with test 1

The group of 92 students showed lower results for test 2 for test items Op 20a, Op 20d, Op 17b, Op 17d, Op 17c, and Op 30. The students' results on those test items were lower between 4 and 11 solution frequency points.

Oppgave 20a Sett ring rundt *alle* regneuttrykkene som passer til regneoppgaven:

24 halsbånd pakkes i eske. Om 24 halsbånd veier 3 kg, hvor mye veier da ett halsbånd?

24 · 3 24 : 3 3 : 24 3 · 24 24 – 3 3 + 24

This is a simple division word problem with real life context. The students were given six alternatives to choose the answer.

25% of the students (test 1) and 13% of the students (test 2) solved the task. The decrease in the solution frequency is very high - 47% less students could solve the task when the whole group was tested for a second time.

Table 4.2.1.2.17: 9th grade, Task 20a - solution frequency for Test 1 (2005 fall) and Test 2 (2006 spring)

Oppgave 20a	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	11	12
3:24	**25**	**13**
24: 3	25	42
3 : 24 og 24 : 3	19	22
24 · 3 og 3 · 24	7	2
Andre svar	14	9

It is not possible to know why such small number students could solve the task in the spring. Espeland (2006) discussed in details the students' choice of operation and investigated closely the students' misconceptions.

Many students (19% (test 1) and 22% (test 2)) mark as answers two expressions - the expressions 3 : 24 and 24 : 3. This fact suggests that those students possibly believed that division is commutative. Another big group of students related the expression 24 : 3 as an answer(25% of the students (test 1) and 42% of the students (test 2)) and possibly used a direct translation of the problem into mathematical language or experienced problems to understand the problem's context. Verschaffel et al. (2000) described the students limitations to deal with simple word problems. The study found that part of the students' problems were related to the fact that many students do not try to make genuinely sense of the problems. Instead they "grab" the numbers and do operations with them in a mechanical way without trying to understand well the problem's context.

Tasks 17a – 17d

Tasks 17a – 17d consist of four test items related to converting fractions into decimal numbers or to symbolise fractions. For tasks 17b, 17c, and 17d the students showed lower results in 2006. For task 17a (to convert 3/10 into a decimal number) there was a small increase of the results - 36% of the students (test 1) and 38% of the students (test 2) solved the task.

The students experienced significant difficulties with the four tasks. 28% of the students (test 1) and 34% of the students (test 2) did not try to answer to any of the four problems. 25% of the students (test 1) and 22% of the students (test 2)students answered incorrectly to all tasks they tried to solve. 10% of the students (test 1) and 14% of the students (test 2) answered correctly only to one of the tasks. Only 10% of the students (test 1) and 7% of the students (test 2) were able to write all four answers correctly. Only 8% of the students (test 1) and 5% of the students (test 2) solved three of the tasks without a mistake. The following figures show to examples of students' answers who solved all four tasks incorrectly.

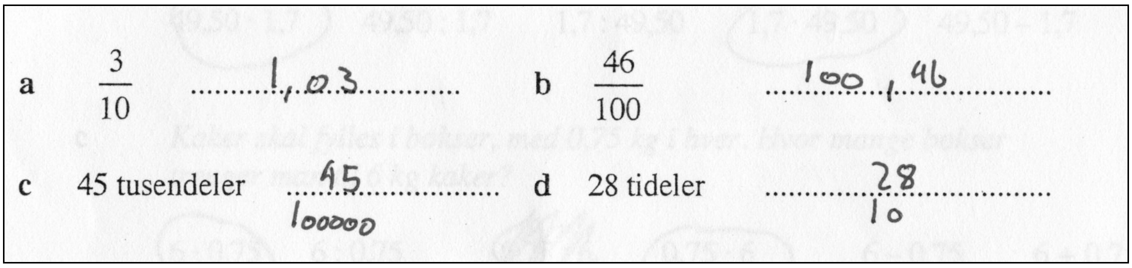
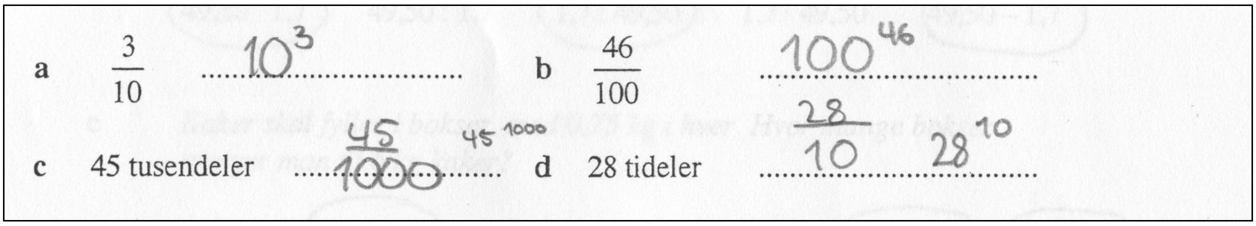


Figure 4.2.1.2.8: Student's answers for tasks 17a - 17d



a $\frac{3}{10}$ ....10^3.................. b $\frac{46}{100}$ ....100^{46}..................

c 45 tusendeler ....$\frac{45}{1000}$ 45^{1000}.......... d 28 tideler ....$\frac{28}{10}$ 28^{10}..................

Figure 4.2.1.2.9: Student's answers for tasks 17a - 17d

Both figures give examples of students struggling to solve the tasks 17a -17d. Many students had problems to convert fractions and find their equivalent decimal numbers. The misconceptions found in the students responses indicate that some students looked upon decimal numbers as a pair of numbers or looked upon fractions as a pair of numbers that can be placed together with the decimal point in certain order as a way to represent a decimal number. Other students experienced problems and wrote 3/10 as for example 0,3333. Many students attempted to solve the four tasks, but could not provide correct answers, because of problems to find the place of the decimal point in the result.

Oppgave 17b Skriv som desimaltall

$\frac{46}{100}$ ....

Table 4.2.1.2.18: 9th grade, Task 17b - solution frequency for Test 1 (2005 fall) and Test 2 (2006 spring)

Oppgave 17b	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	35	46
0,46	**37**	**28**
0,046	2	9
46,1; 46, 100	10	5
46; 46,0; 46,00	4	3
Andre svar	12	9

This task was also very difficult task for the majority of the students. 37% of the students (test1) and 28% of the students (test 2) were able to convert this fraction into a decimal number. 22% less students solved this task correctly when tested for a second time.

Significant number of students (35% (test 1) and 46% (test 2)) did not answer to this problem. The most common wrong answer was 46,1 for test 1 and 0,046 for test 2. Students' answers like 46,1 or 46,100 suggest that those students interpreted the fraction sign as a decimal point. The students who answered 0,046 or 0,0046 could not place the decimal point correctly. It is interesting to notice that almost all students who answered 46,100 to this task answered also 3,10 to task 17a (to convert 3/10 into a decimal number).

Oppgave 17c Skriv som desimaltall

45 tusendeler ....

The students experienced problems to place correctly the decimal point. Many students gave for example answers 0,45 or 0,0045 or 4,5 and could not find the right place of the decimal point in the number.

Table 4.2.1.2.19: 9th grade, Task 17c - solution frequency for Test 1 (2005 fall) and Test 2 (2006 spring)

Oppgave 17c	Frekvens i prosent	
	2005høst	2006 vår
Ikke besvart	39	46
0,045	**17**	**13**
0,0045	14	20
0,45; 0,450; 0,4500	3	2
00,45; 000,45	2	2
4,5; 4,500	8	3
45/1000	4	1
Andre svar	12	13

Only 17% of the students (test 1) and 13% of the students (test 2) were able to write the fraction as a decimal number. The results show that many students (39% (test 1) and 46% (test 2)) did not give an answer.

The students who answered 45/1000 to task 17c answered also 28/10 to task 17d. Most of those students could not solve correctly also the previous tasks 17a and 17b. The students who answered 0,0045 to this task answered 0,28 in most of the cases to the task 17d. Those observations show that the students' mistakes are systematic and are related to limitations of the students abilities to represent and convert decimal numbers and fractions.

Oppgave 17d Skriv som desimaltall

28 tideler ....

Table 4.2.1.2.20: 9th grade, Task 17d - solution frequency for Test 1 (2005 fall) and Test 2 (2006 spring)

Oppgave 17d	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	40	46
2,8	**20**	**13**
0,28	22	29
0,028	4	1
28,0 og 28	4	3
28,10	4	1
Andre svar	5	6

The task was difficult for most of the students. Only 20% of the students (test 1) and 13% of the students (test 2) gave correct answers to the problem. 32% less students were able to solve the task when tested for a second time.

The majority of the students (40% (test 1) and 46% (test 2)) did not write any answer. The most students who gave wrong answer wrote the decimal number 0,28. Again the students demonstrated problems to find the correct place of the decimal point. Those problems are related to some difficulties to understand the concept of place-value, in the case of decimal numbers.

4.2.1.3 Dividing the students in three subgroups

In this part of the analyses we would like to look closer to the students' results when the group of 92 students is used to form three subgroups. The group of 92 students was divided in three subgroups – Nederste, Midterste and Øverste groups. Because this study is a part of a longitudinal project, it was necessary to use the same names as in the previous year of the project. Every student was placed in one of those groups according to the received score for test 1 (fall 2005), and that was already explained in details in the chapter on methods. The 92 students were divided in three subgroups with almost equal number of students. It was important to analyse smaller groups of students with less differences among them. The comparison of the students' results for the three subgroups shows that there were major differences between the students in the different groups.

The students were divided in three subgroups in order to:

- gather additional information for each subgroup
- focus on the specific problems the students have in this group
- have a group of students with less differences
- compare the students' results for the three subgroups

Elevgruppe		Antall	Test år	Minimum	Maksimum	Gjennomsnitt	St.avvik
Elever - Test 1 & Test 2		92	2005 høst	2	44	18,8	9,7
			2006 vår	1	47	23,9	12,2
Elever Test 1 & Test 2	Nederste tredjedel	31	2005 høst	2	14	8,5	3,8
			2006 vår	1	27	12,5	7,6
	Midterste tredjedel	31	2005 høst	15	22	18,4	2,4
			2006 vår	3	34	23,1	6,6
	Øverste tredjedel	30	2005 høst	23	44	29,8	5,7
			2006 vår	19	47	36,7	7,5

Table 4.2.1.3.1: 9th grade, Statistical information for the groups of 92 students, Nederste, Midterste and Øverste groups of students.

There has been a positive development in the results of the three groups of students. The average score for the Nederste group increased from 8,5 points (test 1) to 12,5 (test 2), but was still very low. The maximum score in this group was only 14 points on the first test, but was almost a loton the second test. The average score for the Midterste group was close to the average scores for the group of all students. Those students received on average 18,4 points (test 1) and 23,1 points(test 2). For the Øverste group of students , there was a big increase in points of the average students' scores – from 29,8 points (test 1) to 36,7 points (test 2).

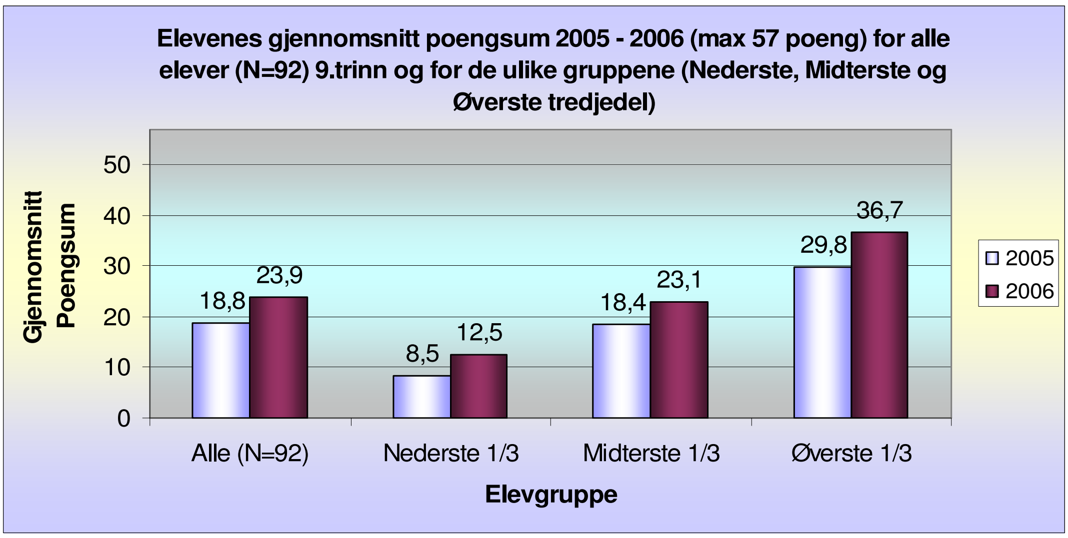


Figure 4.2.1.3.1: 9th grade, Average points sums for the groups of all students and Nederste, Midterste and Øverste groups for test 1 (2005) and test 2 (2006).

Comparison - increase of the students' points

The table and the figure below give comparison for all 92 students and for the Nederste, Midterste and Øverste group of students . Such comparison can give us a lot of information, because the students' results in 2005 and 2006 vary a lot and it's interesting to look also for more detailed information. Looking at the students' tests scores we can try to find more information, dividing the students in different categories. After that it is easier to compare the results in each category and look for more information concerning the students' performance.

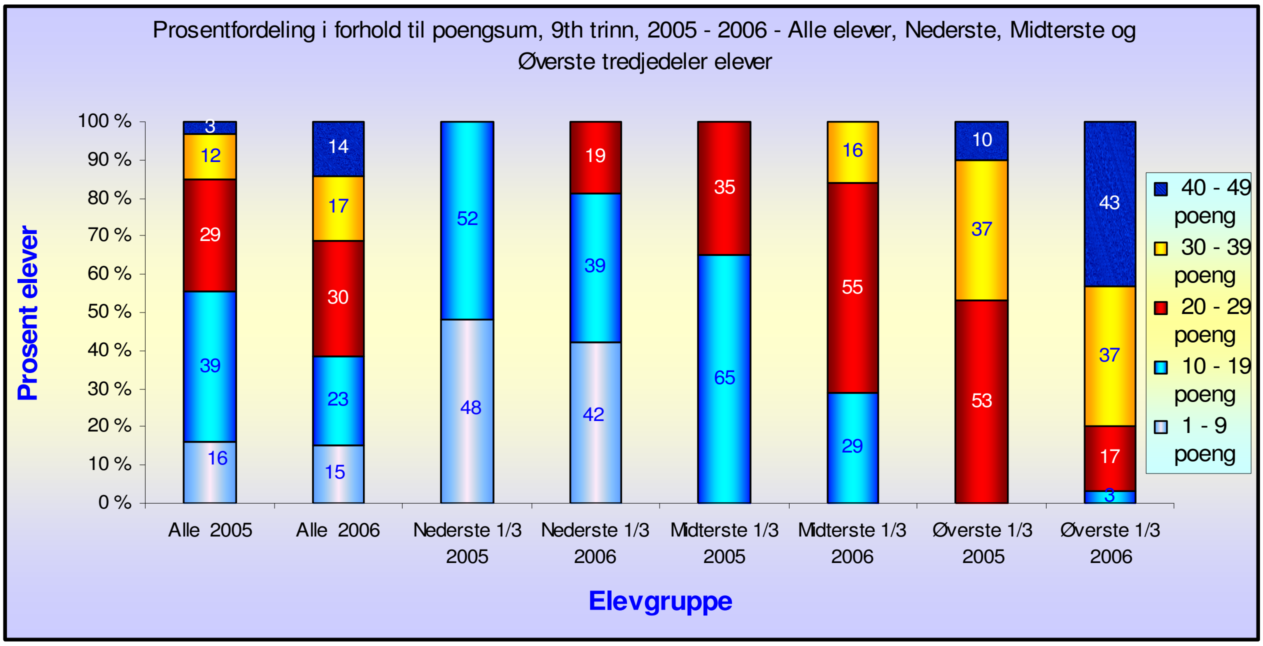


Figure 4.2.1.3.2: 9th grade, 2005 - 2006, Distribution of the points sum in percents – all students, Nederste, Midterste and Øverste groups.

The information displayed in the figure 4.2.1.3.2 and the table 4.2.1.3.2 can help us to see that there was a positive development in all groups of students. However, there are observed big differences among the three subgroups. There is a big increase of the groups of students with scores above 30 points and a big decrease of the students with a score below 20 points. In addition there were few students who achieved scores in the category 40-49 points. There were no students with a score close to the maximum for the test.

Elevgruppe	År	1 – 9 poeng	10 - 19 poeng	20 - 29 poeng	30 - 39 poeng	40 - 49 poeng	50 - 57 poeng
Alle (N=92)	2005	16	39	29	12	3	0
Alle (N=92)	2006	15	23	30	17	14	0
Nederste	2005	48	52	0	0	0	0
Nederste	2006	42	39	19	0	0	0
Midterste	2005	0	65	35	0	0	0
Midterste	2006	0	29	55	16	0	0
Øverste	2005	0	0	53	37	10	0
Øverste	2006	0	3	17	37	43	0

Table 4.2.1.3.2: 9th grade, 2005 - 2006, Distribution of the points sum in percents – all students, Nederste, Midterste and Øverste groups.

Investigating this data more closely, we can find more results for the three groups of students.

Nederste group

- The group of students with scores below 20 points was reduced from 100% of the students (test 1) to 81% of the students (test 2).
- The group of students with scores between 20 and 29 points increased from 0% of the students (test 1) to 19% of the students (test 2).

Midterste group

- The group of students with scores below 20 points was reduced from 65% of the students (test 1) to 29% of the students (test 2).
- The group of students with scores between 20 and 29 points increased from 35% of the students (test 1) to 55% of the students (test 2).
- The group of students with scores between 30 and 39 points increased from 0% of the students (test 1) to 16% of the students (test 2).

Øverste group

- The group of students with scores between 40 and 49 points increased from 10% of the students (test 1) to 43% of the students (test 2).
- The group of students with scores below 30 points was reduced from 53% of the students (test 1) to 20% of the students (test 2).

Increase of the students' points in percents

The students in every group - 92 students, Nederste, Midterste and Øverste groups were first sorted according the student's test score in 2005. For every student it was found how many percents the student's points score in 2006 was higher or lower compared with the first score in 2005. After that the students were divided in five categories. Those categories were as shown on the table:

- students with negative change in the results
- students who didn't change their results
- students who increased their results between 1 and 49%
- students who increased their results between 50 and 99%
- students who increased their results with 100% or more

Elevgruppe		Antall elever	Endring Poengsum: negative	Endring Poengsum: 0	Endring Poengsum: 1% - 49%	Endring Poengsum: 50% - 99%	Endring Poengsum: 100% eller mer
Alle elever	antall elever	92	12	7	46	18	9
	frekvens	100	13	8	50	20	10
Nederste tredjedel	antall elever	31	6	4	6	8	7
	frekvens	100	19	13	19	26	23
Midterste tredjedel	antall elever	31	3	2	20	4	2
	frekvens	100	10	6	65	13	6
Øverste Tredjedel	antall elever	30	3	1	20	6	0
	frekvens	100	10	3	67	20	0

Table 4.2.1.3.3: Distribution of the students' results in groups, according to the change of the students' scores on the second test, the data presents the results for 92 students, Nederste, Midterste and Øverste groups of students.

Students who increased their score

A group of 72 students increased their score in the spring. How many students had very significant increase of the results?

- How many students increased their score with 10 points or more?

10 points or more increase had a group of 26 students. Half of those students were from the Øverste group, followed by 7 students from Midterste and 6 students from Nederste group. The group of 6 students from Nederste group is very interesting. They had an average score 9 points in the fall and 23 points in the spring, an increase of the second result with 202%. The other students from the Midterste increased their average score from 18 to 31 points (73% increase) and the students from the Øverste group from 28 points to 41 points (41% increase). So we can ask why those 6 students scored so high on the second test? Those 6 students are probably students who didn't show their real abilities on one of the tests, because such high increase of the results doesn't look natural in the period of a few months. For example there's a case student who increased the score from 7 points in the fall to 26 points in the spring. Such increase of 202% is a lot higher than the rest of the students in this group or compared with the average increase of the results for all 72 students with positive increase – on average 53%. So it is possible that those 6 students in some way didn't show their normal performance.

This group of students had an increase of 91% of their score in the spring – increase three times higher than the average increase 27% for all 92 students. This group of 26 students had an average increase of 13,1 points on the second test – a result much higher than the 5,1 point's average increase for all of the 92 students. The students' average result in the fall was 20,8 points or a result very close to the average for all 92 students. But in the spring their average points score was 33,9 points or 10 points higher than the average results for all students 23,9 points.

- How many students increased their score between 50% and 99%?

18 students of all 92 students increased their points score in the spring between 50% and 99%. Almost half of the students – 8 students come from the Nederste group, 6 students from Øverste group, followed by 4 students from Midterste group. Those were students who received on average 17,3 points score in 2005, a score only 1,5 points lower than the average for all 92 students. But the 18 students increased their score in the spring on average with 64% to 27,8 points – higher than the average for all students, but quite close to the result for all 72 students with increase in the results.

- How many students increased their score with more than 100%?

Such high increase of 100% or more achieved 9 students - 7 students from Nederste group and 2 students from the Midterste group. The two students from the Midterste group obtained the lowest score for the Midterste group of students – both students received 15 points on the first test, only a point away from the group of the Nederste students. Considering also that the Nederste group is a group of 31 students with the lowest score on the first test with score between 2 and 14 points, those two students from the Midterste group can easily be compared with the 7 students from the Nederste group. So the group of 9 students, with a big variation in the results – from 100% increase to the maximal increase is 533%. The average result in the fall was only 8,5 points, but those 9 students increased their score on the second test with 186% on average. The average score for this group was already 21,2 points, 70% higher than the 12,5 average score for the Nederste group and very close to the average score for all 92 students. The data shows that 7 of the students had an increase higher than 10 points, the other two students were belonging to the lowest achievement students. Such increase of both points differences with at least 10 points and overall high percentage of the increase looks very unlikely in real life. We can ask the question how realistic can this be, does such big improvement in the students' performance on the same test can happen only because the students worked hard on the tests or there are other reasons.

Students who did not change their results on the tests

They were 8% of all students - 4 students from the Nederste group, 2 students from the Midterste and 1 student from the Øverste group. Those students didn't get a high score - on average 14,1 points for both tests.

Students who decreased their score in the spring of 2006

This is a group of 12 students - 6 students are from the Nederste group, 3 students from the Midterste and 3 students from the Øverste groups. Their average score decreased with 41,4% in the spring, ranging from decrease of students' results with 2 points to decrease of 12 points. This group of students had on average a sum of 15 points on the first test and only 10,3 points on the second test. In 2006 those students had very low average results compared with the results for all 92 students in 2006. The two lowest results belong to a student from the Midterste group, with a score decreased from 15 on the first test to 3 points on the second test and another student from the Nederste group who received 12 points and 3 points. From the test results only we can not say why those students couldn't show improvement on the second test. Some interviews with them can give more information about the reason.

Nederste group of students

This is a group of 31 students who scored lowest on the first test. Their average points score was very low for both tests – 8,5 points (test 1) and 12,5 points (test 2). The minimum result in the group was 2 points (test 1) and 1 point (test 2). In 2005 all 31 students scored below the average score for the students who did both tests and in the spring of 2006 only 3 students received results above the average results for all students.

Elevgruppe	Antall	Test år	Minimum	Maksimum	Gjennomsnitt	St.avvik
Nederste tredjedel	31	2005 høst	2	14	8,5	3,8
		2006 vår	1	27	12,5	7,6

Table 4.2.1.3.4 : 9th grade, Statistical information for Nederste group.

The students' results in 2005 were more than twice lower than the average results for all 92 students. Half of the students had a score between 2 and 9 points in the fall, and in the spring the students with the lowest scores 1 – 9 points were still the majority in the group. In 2006 the average points score for the whole group was still about twice smaller compared with the average score for all 92 students. As a change can be noticed that in the group there were already 19% of the students who received a score between 20 and 29 points in 2006.

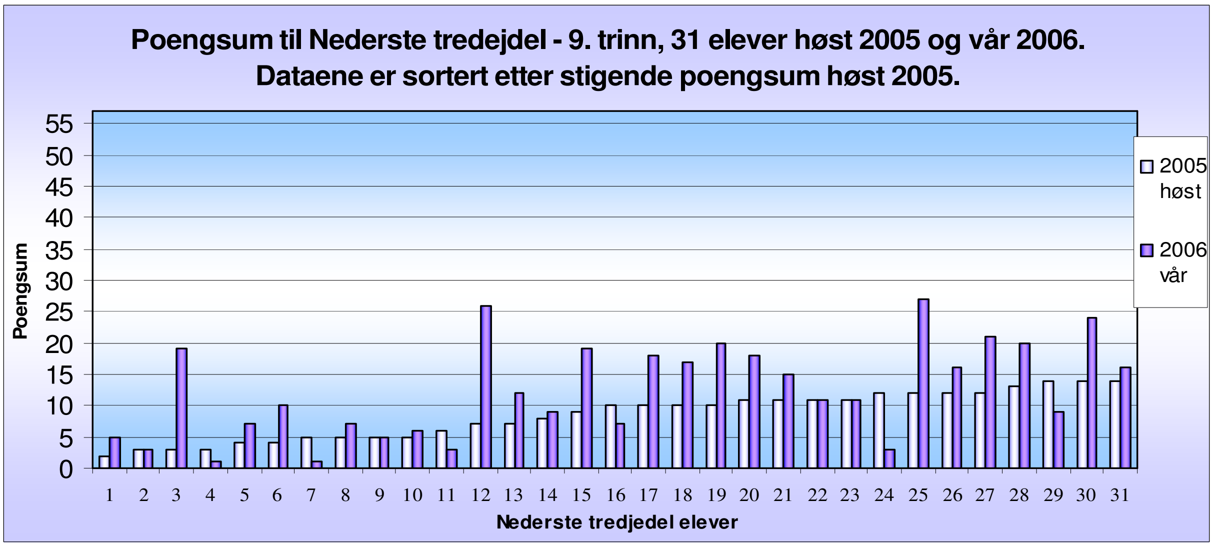


Figure 4.2.1.3.3: 9th grade, Nederste group - points sums for every student in the group (both tests).

The increase / decrease of the points' scores of the students on the second test

In this group there were some variations of the points scores of the students on the second test - there were 6 students who got lower points score in the spring (on average 4,3 points lower score), 4 students did not change their score and 21 students had an increase of their score (on average 7 points increase), 15 of all 31 students increased their spring score with more than 50%, including 6 students with increase of more than 100% of the score. Those 6 students had on average 223% increase of the score.

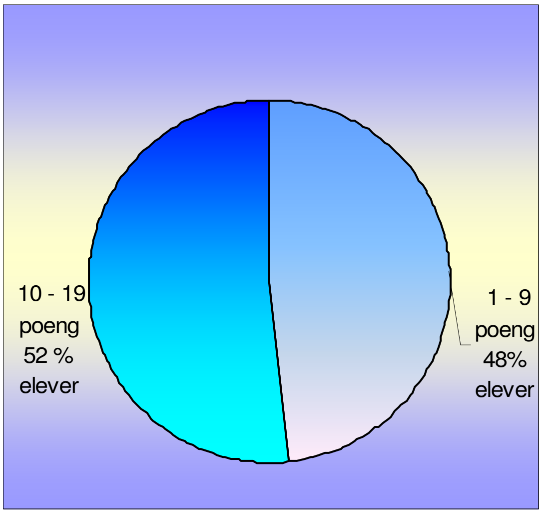


Figure 4.2.1.3.4: 9th grade, Nederste group (N=31) – test 1, 2005, Distribution of the students' points sum

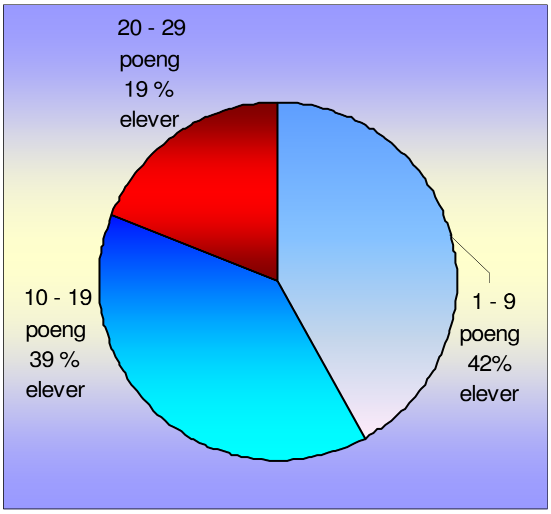


Figure 4.2.1.3.5: 9th grade, Nederste group (N=31) – test 2, 2006, Distribution of the students' points sum

Summary of the students' results on all test items

The following figures present a summary of the students' results of the Nederste group on all test items. The solution frequency for every test item was calculated and the data is presented as comparison of those results for both tests.

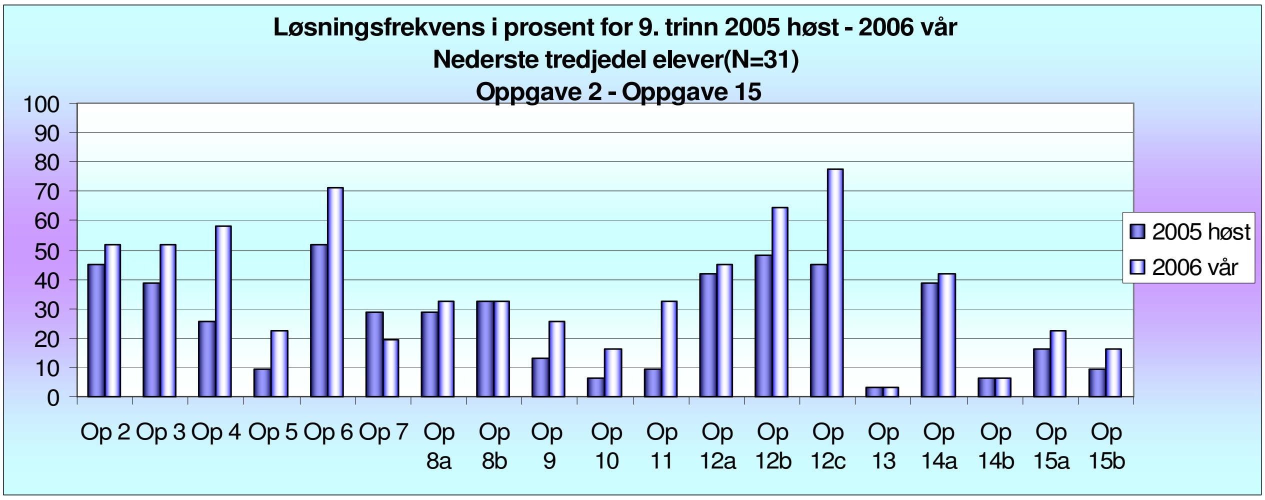


Figure 4.2.1.3.6: 9th grade, solution frequencies for test items 2 – 15b, Nederste group of students (both tests).

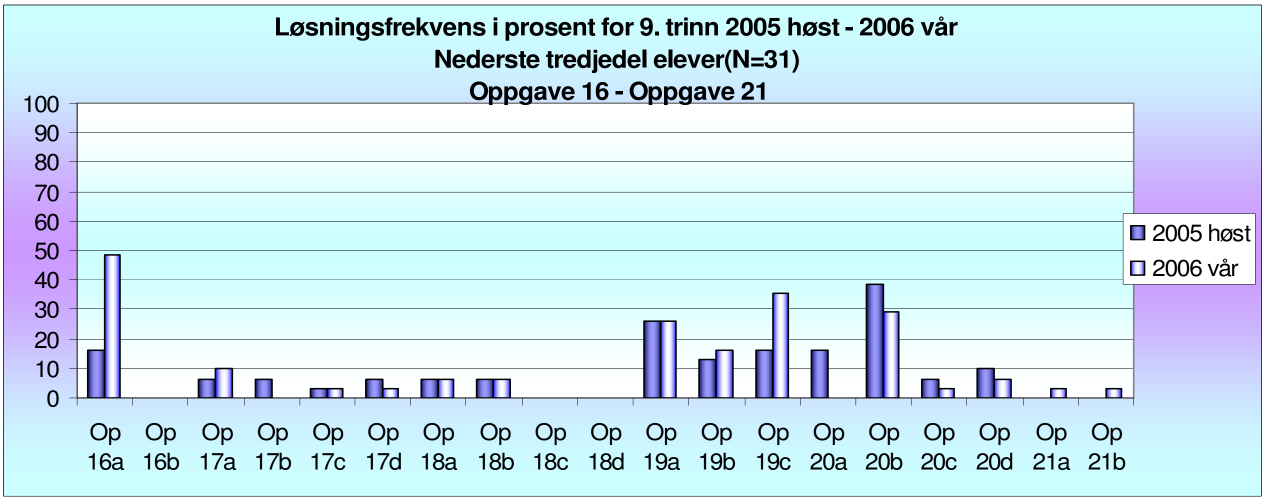


Figure 4.2.1.3.7: 9th grade, solution frequencies for test items 16a – 21b, Nederste group of students (both tests).

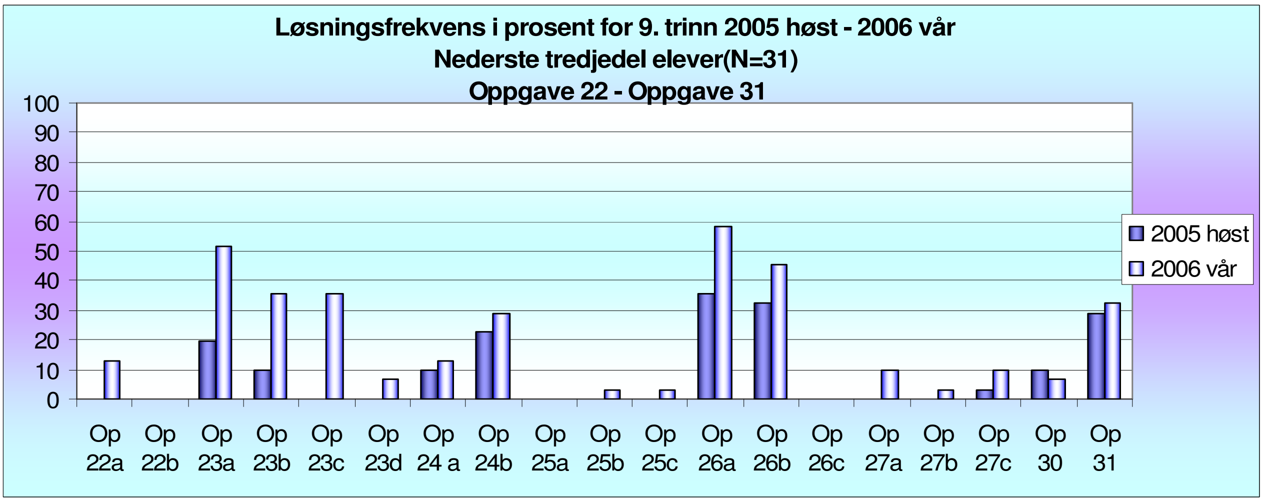


Figure 4.2.1.3.8: 9th grade, solution frequencies for test items 22a – 31, Nederste group of students (both tests).

Many of the test items were quite hard for this group of students. For example no one of the students was able to solve 6 of the problems - Op 16b, Op 18c, Op 18d, Op 22b, Op 25a, and Op 26c.

In addition 19 of the problems were solved by not more than 10% of the students in the group, 12 of the problems were solved by less than 30% of the students, and only eight test items were solved by at least half of the students from the Nederste group on one of the tests.

Group of test items with big improvement of the results on the second test

Increase of 20 solution frequency points or more

In this group of problems are included the problems with increase of the solution frequency on the second test higher than 20 solution frequency points. The Nederste group of students showed very significant improvement in the results for 10 test items. Every test problem in this group is a test problem, which was solved by at least 6 more students by the Nederste group in the spring. Some of the tasks are simple algebraic tasks, estimation of arithmetical expression, task asking the students to find the number of elements in the next term of a figure pattern, task involving simple calculation with a fraction, task requiring comparison of decimal numbers. It is interesting to notice that the big group of all 92 students increased their results for all those tasks as well, but had very high improvement only for four of those tasks (Op 16a, 23a, 23b, 23c) and in addition three other tasks.

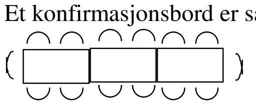
Oppgave		Frekvens i prosent	
		2005 høst	2006 vår
Op 4	Hva er 25% av 40 km?	26	58
Op 6	Temperaturen forandrer seg fra –5 °C til +8 °C. Hva er stigningen i temperatur?	52	71
Op 11	$\frac{2}{5}$ av en masse er 20 gram. Hva er massen?	10	32
Op 12c	Sett *ring rundt* det største tallet og *kryss over* det minste tallet: 4,09 4,7 4,008	45	77
Op 16a	Skriv riktig tall i rutene **a** $574 = 5 \cdot 100 + \square \cdot 10 + 4 \cdot 1$	16	48
Op 19c	Sett ring rundt det tallet du mener er *nærmest svaret*. Du trenger ikke regne ut svaret. $0{,}73 \cdot 46{,}2$ | 0,03 0,3 3 30 300	16	35
Op 23a	$2x + 5x$	19	52
Op 23b	$x + x + 2x$	10	35
Op 23c	$t \cdot t \cdot t$	0	35
Op 26a	Et konfirmasjonsbord er satt sammen av småbord, og rundt er det satt stoler.  Hvor mange stoler blir det plass til om vi har 4 småbord?...........................	35	58

Table 4.2.1.3.6: Group of tasks

Group of test items solved by more than half of the students

The eight test items presented in the table below were solved by more than half of the students from the Nederste group on the second test. For five of the tasks the increase of the number students who gave correct answers was very significant. For example the results show that in the spring the task Op 4 (Hva er 25% av 40 km?) was solved by a lot more students in the group, the task Op 23a (simplify the expression $2x + 5x$) was solved by 19% of the students (test1) and 52% of the students (test 2).

Oppgave		Frekvens i prosent	
		2005 høst	2006 vår
Op 2	Hva er en kvart (en firedel) av 60 gram?	45	52
Op 3	Finn lengden på gardinstoffet Mari må kjøpe, når det skal være fem gardiner, og hvert skal være 1,05 meter.	39	52
Op 4	Hva er 25% av 40 km?	26	58
Op 6	Temperaturen forandrer seg fra –5 ˚C til +8 ˚C. Hva er stigningen i temperatur?	52	71
Op 12b	Sett *ring rundt* det største tallet og *kryss over* det minste tallet: 3,521 3,6 3,75	48	65
Op 12c	Sett *ring rundt* det største tallet og *kryss over* det minste tallet: 4,09 4,7 4,008	45	77
Op 23 a	$2x + 5x$	19	52
Op 26a	Et konfirmasjonsbord er satt sammen av småbord, og rundt er det satt stoler. Hvor mange stoler blir det plass til om vi har 4 småbord?...........................	35	58

Table 4.2.1.3.7 : Group of tasks

Group of test items with decrease in the results on the second test

The students in the Nederste group had lower results for 8 of the test items - Op 7,Op 17b, Op 17d, Op 20a, Op 20b, Op 20c, Op 20d, Op 30. The solution frequencies for those items are presented in the table below. It is interesting to notice that five of those items were word items. The students experienced problems also with converting fractions into decimal numbers and presenting percents as fraction. The biggest decline in the results was for the word problem Op 20a. As a comparison the group of all 92 students also had decline in the results for five of the items - Op 17b, Op 17b, Op 20a, Op 20d and Op 30.

Oppgave		Frekvens i prosent	
		2005 høst	2006 vår
Op 7	Uttrykk 20% som en brøk.	29	19
Op 17b	Skriv som desimaltall $\frac{46}{100}$	6	0
Op 17d	Skriv som desimaltall 28 tideler	6	3
Op 20a	*24 halsbånd pakkes i eske. Om 24 halsbånd veier 3 kg, hvor mye veier da ett halsbånd?*	16	0
Op 20b	*1 kg pølser koster 49,50 kr. Per kjøper 1,7 kg. Hvor mye koster det?*	39	29
Op 20c	*Kaker skal fylles i bokser, med 0,75 kg i hver. Hvor mange bokser trenger man til 6 kg kaker?*	6	3
Op 20d	*Anne kjøper bananer i en butikk til 13,50 kr per kg. Hvor mye kan Anne kjøpe for 10,50 kr?*	10	6
Op 30	Marit har tre skåler med nøtter *A, B* og *C*. Det er 2 flere i *B* enn i *A*. I skål *C* er det ganger så mange nøtter som i skål *A*. I alt er det 14 nøtter. Hvor mange nøtter er det i hver av skålene?	10	6

Table 4.2.1.3.5: Group of tasks

Midterste group of students

This is a group of 31 students who received scores very close to the average scores for the group of all students. The average score in the group was 18 points (test 1) and 23 points on the second test. It is interesting to notice that the minimum score in the group was only 3 points on the second test. Some of the students had a big decrease of the results in the spring- we can observe this, but we can not know why this happened.

Elevgruppe	Antall	Test år	Minimum	Maksimum	Gjennomsnitt	Standardavvik
Midterste tredjedel	31	2005 høst	15	22	18,4	2,4
		2006 vår	3	34	23,1	6,6

Table 4.2.1.3.8: 9th grade, Statistical information for the Midterste group.

The variations in the students' scores were minimal on the first test, but there are more differences among the students on the second test.

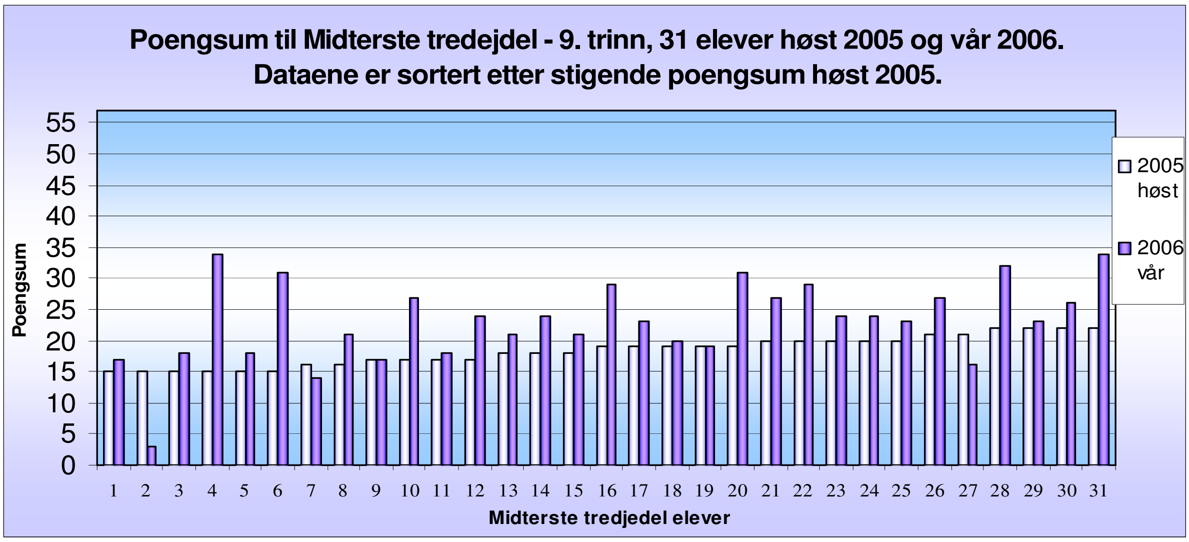


Figure 4.2.1.3.9: 9th grade, Midterste group - points sum for every student in the group (both tests).

The increase / decrease of the points' scores of the students on the second test
In this group there were some variations of the points scores of the students on the second test - there were 3 students who got lower points score in the spring (on average points lower score), 2 students did not change their score and 26 students had a positive increase of their score varying from 1 to 19 points increase (on average 6 points increase). 7 students increased their spring score with more than 50%, including 2 students with increase of more than 100% of the score.

The situation in the group improved in the spring – already 55% of the students had a score between 20 and 29 points and the group of students with score between 10 and 19 points reduced dramatically - from 65% of the students (test 1)to 29% of the students (test 2). There were also 16% of the students (test 2) who received scores between 30-39 points.

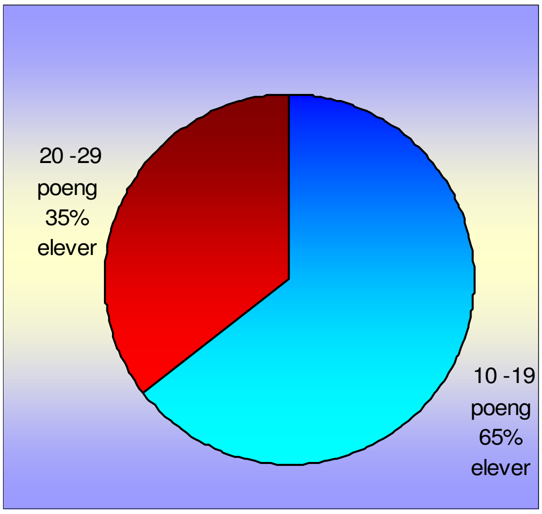


Figure 4.2.1.3.10: 9th grade, Midterste group – test 1, 2005, Distribution of the students' points sum

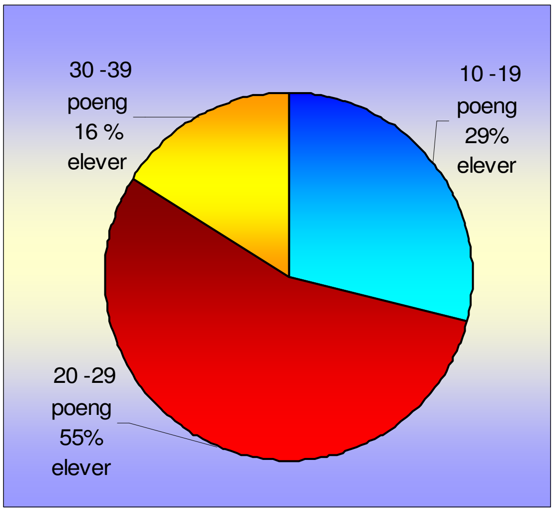


Figure 4.2.1.3.11: 9th grade, Midterste group – test 2, 2006, Distribution of the students' points sum

Summary of the students' results on all test items

The following figures present a summary of the students' results on all test items, included in both tests. The results show that most difficulties the students had with the tasks Op 17c, Op 26c, Op 16b, Op 24 a, Op 21a, Op 22b, Op 23d, Op 27b. As a total 25 of the tasks were solved by less than a third of the students in the group and 22 of the tasks were solved by at least half of the students on one of the tests.

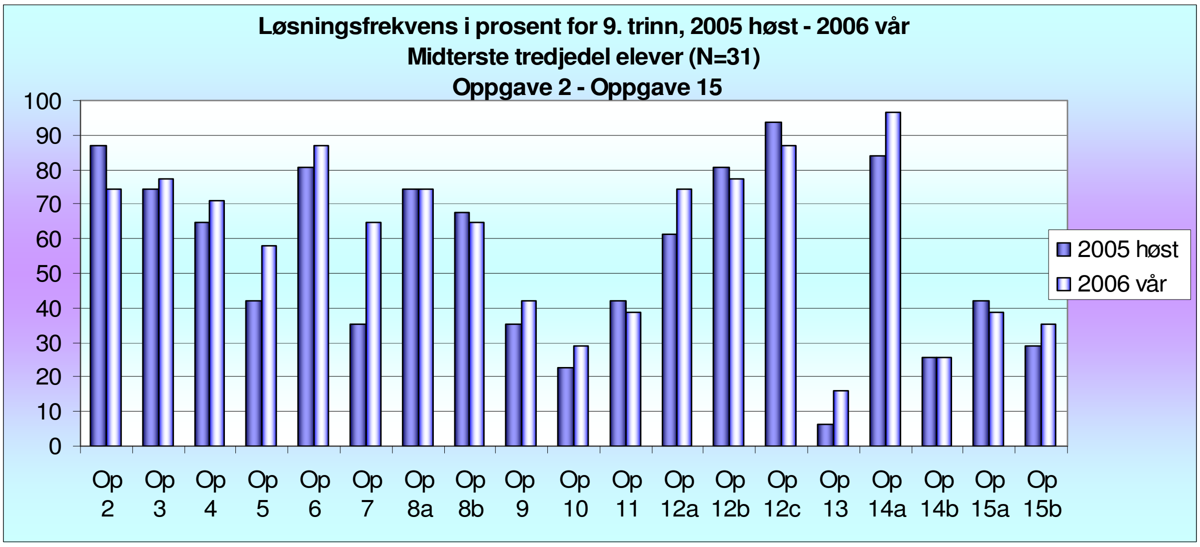


Figure 4.2.1.3.12: 9th grade, solution frequencies for test items 2 – 15b, Midterste group of students (both tests)

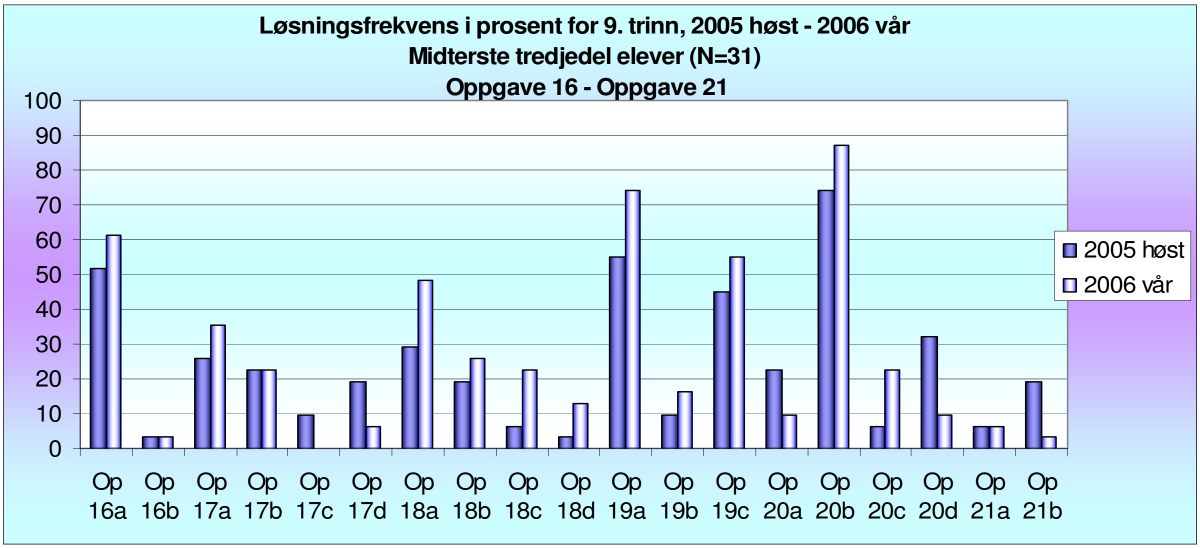


Figure 4.2.1.3.13: 9th grade, solution frequencies for test items 16a – 21b, Midterste group of students (both tests).

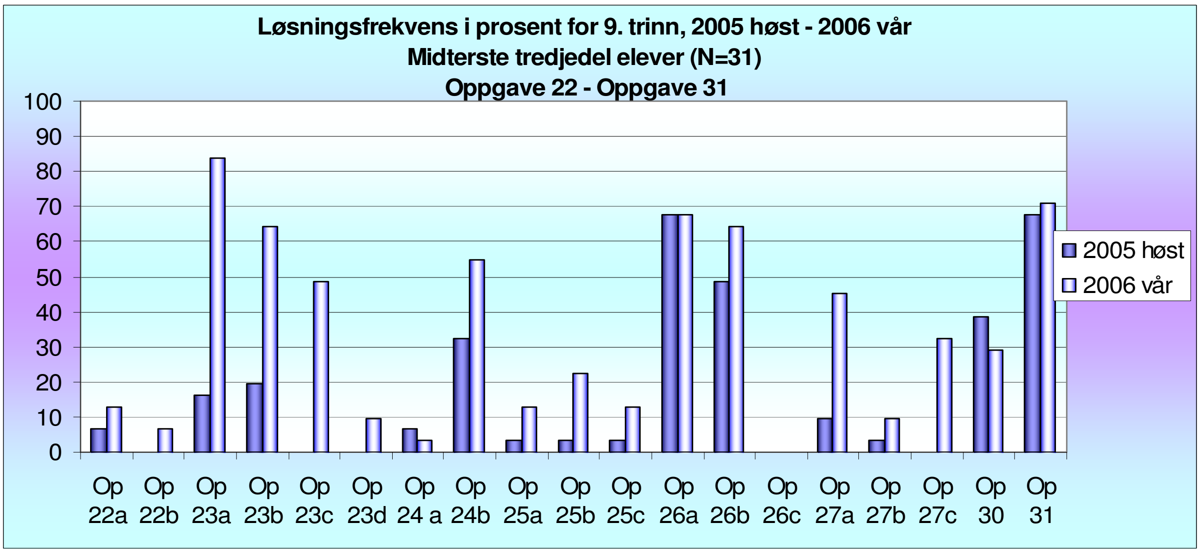


Figure 4.2.1.3.14: 9th grade, solution frequencies for test items 22a – 31, Midterste group of students (both tests).

Group of test items solved by more than half of the students

22 of the test items were solved by at least half of the students on one of the tests. This group of test items are: Op 3, Op 2,Op 4, Op 5, Op 6, Op 7, Op 8a, Op 8b, Op 12a, Op 12b, Op 12c, Op 14a, Op 16a, Op 19a, Op 19c, Op 20b, Op 23a, Op 23b, Op 24b, Op 26a Op 26b, Op 31.

For six of the tasks Op 3, Op 4, Op 6, Op 16a, Op 19c, Op 31the increase was minimal – between 3 and 10 solution frequency points. For the tasks Op 5, Op 12a, Op 14a, Op 16a, Op 19a, Op 19c, Op 20b, Op 26b the increase was between 13 and 19 points.

In the figure below are presented the test items solved by at least 80% of the students on one of the tests.

Oppgave		Frekvens i prosent	
		2005 høst	2006 vår
Op 2	Hva er en kvart (en firedel) av 60 gram?	87	74
Op 3	Finn lengden på gardinstoffet Mari må kjøpe, når det skal være fem gardiner, og hvert skal være 1,05 meter.	74	77
Op 6	Temperaturen forandrer seg fra –5 ˚C til +8 ˚C. Hva er stigningen i temperatur?	81	87
Op 12b	Sett *ring rundt* det største tallet og *kryss over* det minste tallet: 3,521 3,6 3,75	81	77
Op 12c	Sett *ring rundt* det største tallet og *kryss over* det minste tallet: 4,09 4,7 4,008	94	87
Op 14a	Les av på følgende skalaer og skriv riktig desimaltall i ruta. 	84	97
Op 20b	*1 kg pølser koster 49,50 kr. Per kjøper 1,7 kg. Hvor mye koster det?*	74	87
Op 23a	$2x + 5x$	16	84

Table 4.2.1.3.9: Group of tasks

Group of test items with decrease in the results on the second test

For 13 of the test items the students from the Midterste group showed lower results on the second test. Those test items were Op 2, Op 8b, Op 11, Op 12b, Op 12c, Op 15a, Op 17c, Op 17d, Op 20a, Op 20d, Op 21b, Op 24 a, Op 30. For five of those items - Op 11, Op 12b, Op 15a, Op 24 a, Op 8b the results were only 3 solution frequency points lower. In the table below are presented the test items with more than 5 solution frequency points decline in the results.

The biggest regression in the students' results were for the word problem 20d "*Anne kjøper bananer i en butikk til 13,50 kr per kg. Hvor mye kan Anne kjøpe for 10,50 kr?*"- the results for this problem were 23 solution frequency points lower on the second. The problem 21b asking students to construct a word problem related to the expression $\boxed{5,25 \cdot 3,28 = 17,22}$ also showed big decrease in the results - the results on the second test were 16 solution frequency points lower. The students from this group experienced also problems with converting numbers told in words into decimal numbers.

Oppgave		Frekvens i prosent	
		2005 høst	2006 vår
Op 2	Hva er en kvart (en firedel) av 60 gram?	87	74
Op 12c	Sett *ring rundt* det største tallet og *kryss over* det minste tallet: 4,09 4,7 4,008	94	87
Op 17c	Skriv som desimaltall 45 tusendeler	10	0
Op 17d	Skriv som desimaltall 28 tideler	19	6
Op 20a	*24 halsbånd pakkes i eske. Om 24 halsbånd veier 3 kg, hvor mye veier da ett halsbånd?*	23	10
Op 20d	*Anne kjøper bananer i en butikk til 13,50 kr per kg. Hvor mye kan Anne kjøpe for 10,50 kr?*	32	10
Op 21b	Lag din egen fortelling som passer til disse regnestykkene: $\boxed{5,25 \cdot 3,28 = 17,22}$	19	3

Op 30	Marit har tre skåler med nøtter *A, B* og *C*. Det er 2 flere i *B* enn i *A*. I skål *C* er det 4 ganger så mange nøtter som i skål *A*. I alt er det 14 nøtter. Hvor mange nøtter er det i hver av skålene? Vis eller forklar hvordan du tenkte.	39	29

Table 4.2.1.3.10: Group of tasks

Group of test items with big improvement of the results on the second test

Increase of 20 solution frequency points or more

A group of 10 of the test items were solved additionally by at least 6 more students when the students were tested for a second time. The algebraic tasks 23a, 23b, and 23c were the tasks with the biggest increase of the number of students who solved them in the spring. The biggest increase in the results was for task Op 23a ($2x + 5x$ ....) solved by 16% of the students (test 1) and 84% of the students (test 2).

	Oppgave	Frekvens i prosent	
		2005 høst	2006 vår
Op 7	Uttrykk 20% som en brøk.	35	65
Op 18a	$6 \cdot 0{,}5 =$	29	48
Op 19a	Sett ring rundt det tallet du mener er *nærmest svaret*. Du trenger ikke regne ut svaret. 13 : 4,32 | 0,03 0,3 3 30 300	55	74
Op 23a	$2x + 5x$	16	84
Op 23b	$x + x + 2x$	19	65
Op 23c	$t \cdot t \cdot t$	0	48
Op 24b	$2a + 3 = 2a - 3$ Dette ☐ er alltid sant ☐ er aldri sant ☐ *kan* være sant, nemlig når	32	55
Op 25b	Adder tallet x til $4x$	3	23
Op 27a	$x = a + b - c$ Dersom $a = 1$, $b = 2$ og $c = 3$ blir $x =$..........	10	45
Op 27c	$3x = 7$ og $5y = 11$ Da blir $3x + 5y =$	0	32

Table 4.2.1.3.11: Group of tasks

Øverste group of students

The students in this group showed the highest results on the first test. It is reasonable to expect that those students also had the highest results on the second test, but some students had lower results in the spring. As it is shown in the table below the average score in the group increased from 30 points in the fall to 37 points in the spring.

Elevgruppe	Antall	Test år	Minimum	Maksimum	Gjennomsnitt	Standardavvik
Øverste tredjedel	30	2005 høst	23	44	29,8	5,7
		2006 vår	19	47	36,7	7,5

Table 4.2.1.3.12: 9^{th} grade, Statistical information for Øverste group

It is interesting to notice that the number of students with the highest results above 40 points increased very significantly - from 3 students (test 1) to 13 students (test 2). However the students with highest results still did not receive score very close to 57 points - the maximum score that a student could get for the test. Considering also the very low results in the Nedreste group of students it seems that the test was difficult for the majority of the students, there is also the possibility that some students did not have enough time to work.

The figures displayed below show information about the students' scores for both tests.

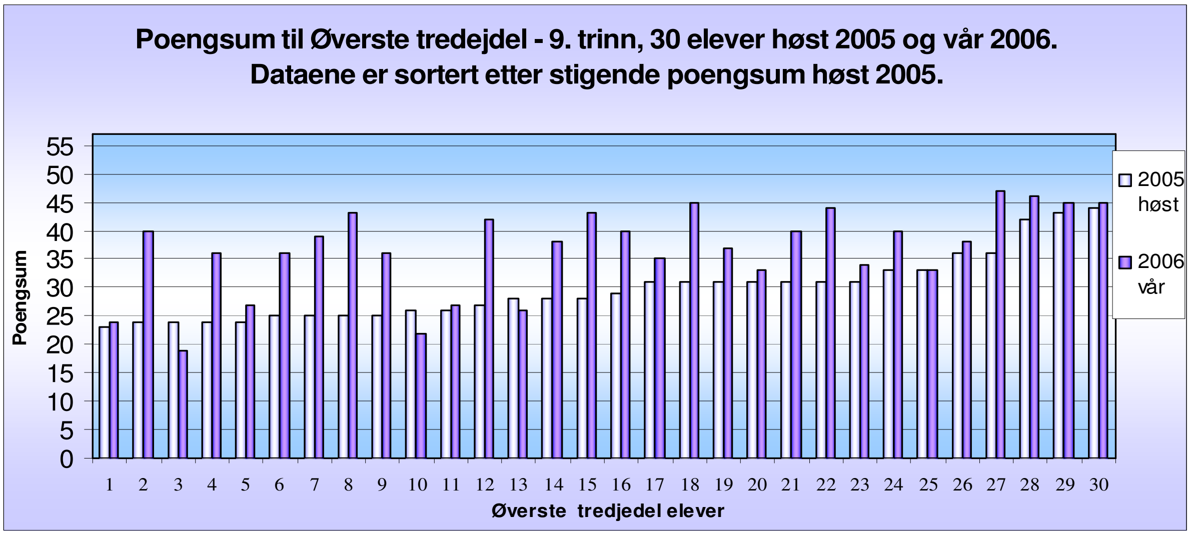


Figure 4.2.1.3.15: 9th grade, Øverste group - points sum for every student in the group (both tests).

The increase / decrease of the points' scores of the students on the second test

As it is shown in the figure there were some variations of the points scores of those students - there were 3 students who got lower points score in the spring, 1 student did not change the score and 26 students increased their score with between 1 to 18 points (on average 8 points increase), 6 students increased their spring score with more than 50%.

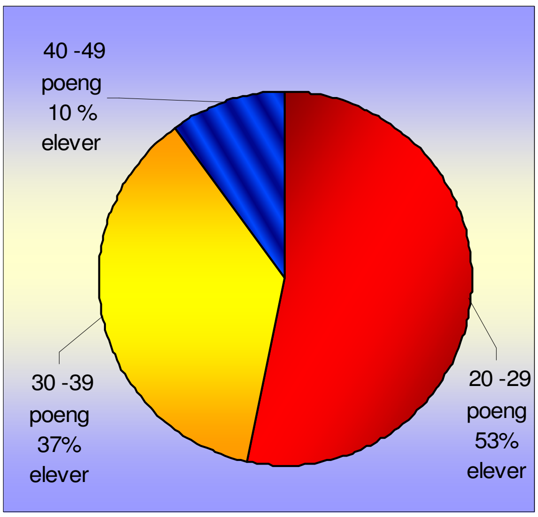


Figure 4.2.1.3.16: 9th grade, Øverste group (N=30) – test 1, 2005, Distribution of the students' points sum

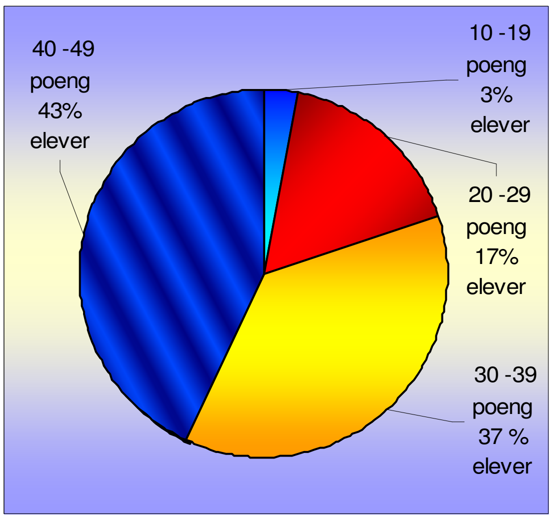


Figure 4.2.1.3.17: 9th grade, Øverste group (N=30) – test 2, 2006, Distribution of the students' points sum

Data analyses show that there was improvement in the spring – already 43% of the students had a score between 40 and 49 points. In addition fewer students in the group had a score between 20 and 29 points - about half of the students had such score in the fall, but this group of students was reduced in the spring to only 17% of all students.

Summary of the students' results on all test items

The figures with the solution frequencies for each problem give additional information for the students' presentation on the tests.

The majority of the students had very high results for the algebraic tasks included in the second test, but there were some exceptions as the problem Op23d asking students to simplify $2y{\cdot}y^2$ (solved by 0% of the students (test 1), and 10% of the students (test 2)), and Op 27b ($y = b^3$, Dersom $b = 4$ blir $y = \ldots$) solved by 0% of the students (test 1) and 30% of the students (test 2)). Very low results in the group are on the task 24a ($x + y + z = x + p + z$), solved by only 14% of the students (test 1)and 17% (test 2). The results for Op 21a (Lag din egen fortelling som passer til disse regnestykkene: 4 : 0,5) also were not good enough. The students showed in general very high results on almost all arithmetical tasks, but had major difficulties to calculate correctly Op 18d (0,6 : 0,2 =) and Op 22b (14 : □ = 0,25 · 14).

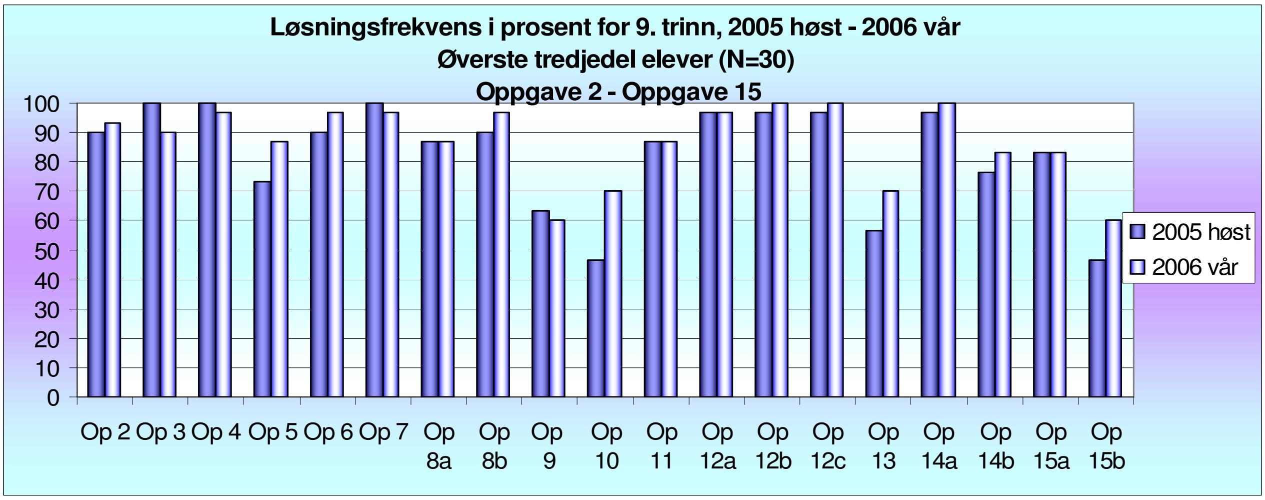


Figure 4.2.1.3.18: 9th grade, solution frequencies for test items 2 – 15b, Øverste group of students (both tests).

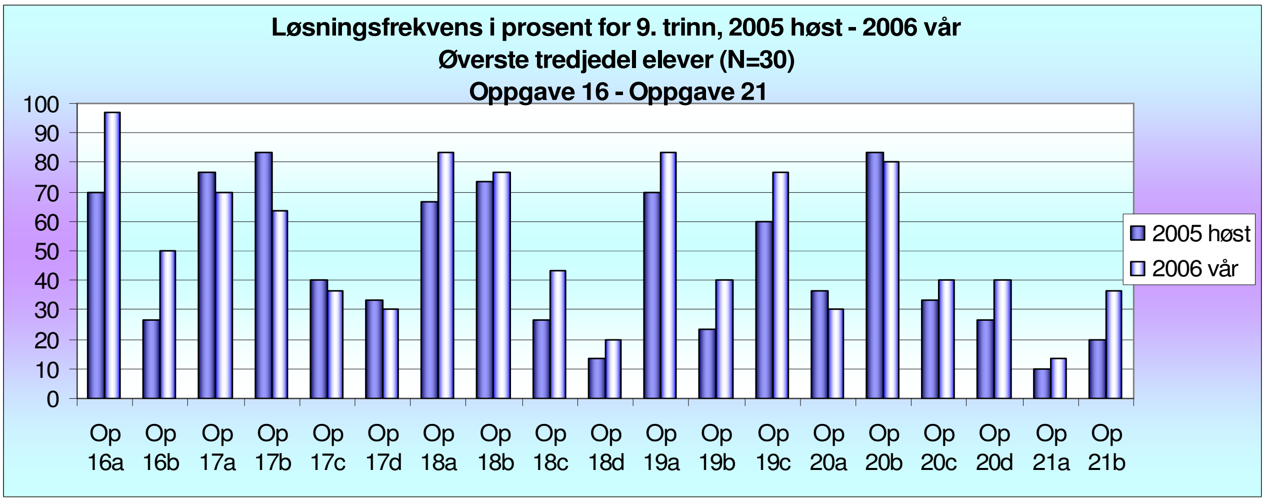


Figure 4.2.1.3.19: 9th grade, solution frequencies for test items 16a – 21b, Øverste group of students (both tests).

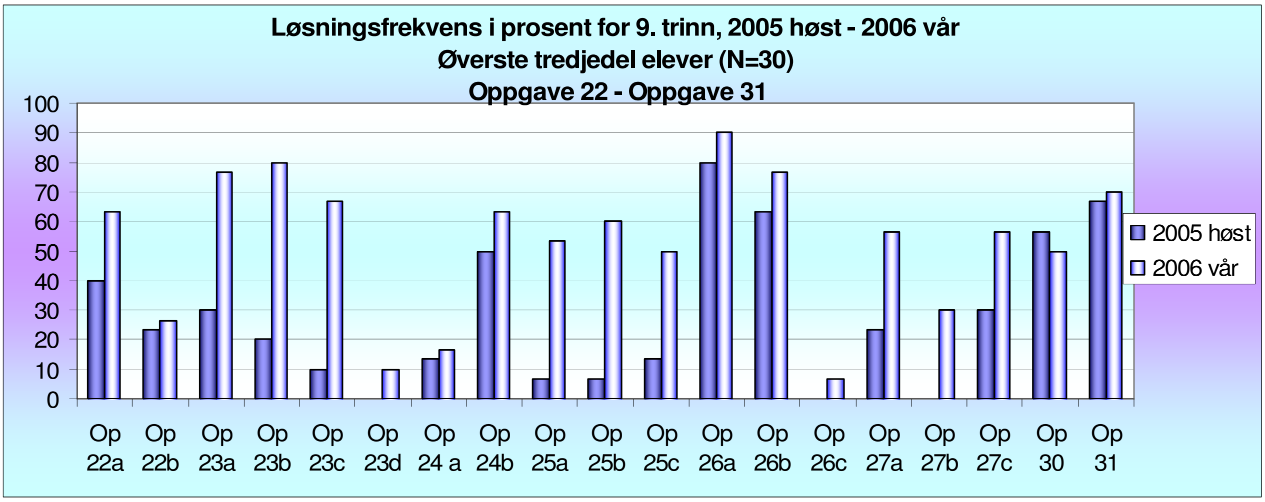


Figure 4.2.1.3.20: 9th grade, solution frequencies for test items 22a – 31, Øverste group of students (both tests).

Group of test items with big improvement of the results on the second test

Increase of 20 solution frequency points or more

For 13 of the problems the Øverste group of students showed increase in the results higher than 20 solution frequency points on the second test. Most of the problems in this group are simple algebraic problems as to simplify algebraic expressions, to find the value of algebraic expression. It is interesting that in the group of problems with big increase were also four arithmetical tasks. The biggest increase was for the algebraic problems Op 23a, Op 23b, Op 23c, Op 25a, and Op 25b – for all those items the increase was between 46 and 60 solution frequency points.

Oppgave		Frekvens i prosent	
		2005 høst	2006 vår
Op 10	$\frac{1}{2} - \frac{1}{3} = ?$	47	70
Op 16a	Skriv riktig tall i rutene **a** $574 = 5 \cdot 100 + \square\, 10 + 4 \cdot 1$	70	97
Op 16b	Skriv riktig tall i rutene $5{,}74 = 5 \cdot 1 + 7 \cdot \square + 4 \cdot \square$	27	50
Op 22a	$14 : 2 = \square \cdot 14$	40	63
Op 23a	$2x + 5x$	30	77
Op 23b	$x + x + 2x$	20	80
Op 23c	$t \cdot t \cdot t$	10	67
Op 25a	Adder tallet x til $x + 3y$	7	53
Op 25b	Adder tallet x til $4x$	7	60
Op 25c	Adder tallet x til 7	13	50
Op 27a	$x = a + b - c$ Dersom $a = 1$, $b = 2$ og $c = 3$ blir $x =$........................	23	57
Op 27b	$y = b^3$ Dersom $b = 4$ blir $y =$	0	30
Op 27c	$3x = 7$ og $5y = 11$ Da blir $3x + 5y =$	30	57

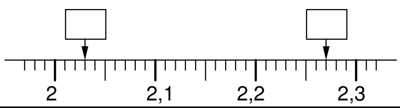
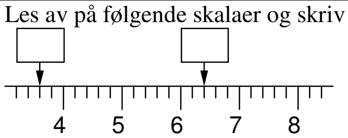
Table 4.2.1.3.13: Group of tasks

Test items solved by more than half of the students

42 of the test items were solved by at least half of the students in the Øverste group. The students showed very good skills in solving simple algebraic and arithmetical problems with some exceptions.

Very interesting task are the tasks Op 26a, Op 26b, Op 26c related to a figure pattern (Et konfirmasjonsbord er satt sammen av småbord, og rundt er det satt stoler.). The results for the Øverste group show that Op 26a (Hvor mange stoler blir det plass til om vi har 4 småbord?) was solved by 80% of the students (test 1)and 90% (test 2), Op 26b (Dersom vi har 7 småbord, hvor mange stoler blir det da?.*)* was solved by 63% of the students (test 1) and 77% of the students (test 2), but Op 26c requiring students to find a solution in the general case was not solved by the students in the fall and was solved by only 7% of the students in the spring.

There were 21 test items solved by at least 80% of the students on one of the tests – the solution frequencies for those test items are presented in the next table.

Oppgave		Frekvens i prosent	
		2005 høst	2006 vår
Op 2	Hva er en kvart (en firedel) av 60 gram?	90	93
Op 3	Finn lengden på gardinstoffet Mari må kjøpe, når det skal være fem gardiner, og hvert skal være 1,05 meter.	100	90
Op 4	Hva er 25% av 40 km?	100	97
Op 5	$\frac{1}{2}+\frac{1}{4}=?$	73	87
Op 6	Temperaturen forandrer seg fra –5 ˚C til +8 ˚C. Hva er stigningen i temperatur?	90	97
Op 7	Uttrykk 20% som en brøk.	100	97
Op 8a	En bestemt type penner koster 15 kr for hver. **a** Hvor mange kan du kjøpe for 200 kr?	87	87
Op 8b	En bestemt type penner koster 15 kr for hver. **b** Hvor mye vekslepenger får du da tilbake?	90	97
Op 11	$\frac{2}{5}$ av en masse er 20 gram. Hva er massen?	87	87
Op 12a	Sett *ring rundt* det største tallet og *kryss over* det minste tallet: **a** 0,625 0,25 0,3753 0,125	97	97
Op 12b	Sett *ring rundt* det største tallet og *kryss over* det minste tallet: 3,521 3,6 3,75	97	100
Op 12c	Sett *ring rundt* det største tallet og *kryss over* det minste tallet: 4,09 4,7 4,008	97	100
Op 14a	Les av på følgende skalaer og skriv riktig desimaltall i ruta.  	97	100
Op 14b	Les av på følgende skalaer og skriv riktig desimaltall i ruta.  	77	83
Op 15a	Sett ring rundt det tallet som ligger nærmest i størrelse til 0,16 0,1 0,2 15 0,21	83	83

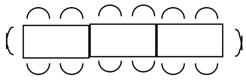
Op 16a	Skriv riktig tall i rutene **a** 574 = 5·100 + $\square$ ·10 + 4 · 1	70	97
Op 18a	6 · 0,5 = ………………………………	67	83
Op 19a	Til venstre i rammen står et regnestykke. Sett ring rundt det tallet du mener er *nærmest svaret*. Du trenger ikke regne ut svaret. 13 : 4,32 | 0,03 0,3 3 30 300	70	83
Op 20b	*1 kg pølser koster 49,50 kr. Per kjøper 1,7 kg. Hvor mye koster det?*	83	80
Op 23b	$x + x + 2x$	20	80
Op 26a	Et konfirmasjonsbord er satt sammen av småbord, og rundt er det satt stoler.  **a** Hvor mange stoler blir det plass til om vi har 4 småbord?...............................	80	90

Table 4.2.1.3.14: Group of tasks

Test items with negative change in the results

For 11 test items the Øverste group of students showed lower results on the second test. Those were the test items Op 3, Op 4, Op 7, Op 9, Op 17a, Op 17b, Op 17c, Op 17d, Op 20a, Op 20b, Op 30, for six of those items the declines of the solution frequencies were only 3 solution frequency points. In the table below are shown the problems with decline bigger than 5 solution frequency points. The biggest decline is for problem 17b (Skriv som desimaltall $\frac{46}{100}$) as it is shown in the table 83% of the students (test 1) and 63% of the students (test 2) solved this problem. It is noticeable also that the problem "24 halsbånd pakkes i eske. Om 24 halsbånd veier 3 kg, hvor mye veier da ett halsbånd?" was difficult also for the Øverste group of students and was solved by less than a third of the students in the spring.

Oppgave		Frekvens i prosent	
		2005 høst	2006 vår
Op 3	Finn lengden på gardinstoffet Mari må kjøpe, når det skal være fem gardiner, og hvert skal være 1,05 meter.	100	90
Op 17a	Skriv som desimaltall $\frac{3}{10}$	77	70
Op 17b	Skriv som desimaltall $\frac{46}{100}$	83	63
Op 20a	*24 halsbånd pakkes i eske. Om 24 halsbånd veier 3 kg, hvor mye veier da ett halsbånd?*	37	30
Op 30	Marit har tre skåler med nøtter *A, B* og *C*. Det er 2 flere i *B* enn i *A*. I skål *C* er det 4 ganger så mange nøtter som i skål *A*. I alt er det 14 nøtter. Hvor mange nøtter er det i hver av skålene?	57	50

Table 4.2.1.3.15: Group of tasks

4.2.2 A comparison with the previous year

4.2.2.1 Results of the group of 167 students

This part of the analyses focuses on the results of the group all 167 students who did the first test. The main aims are to present the summary of the students' results and to do a comparison of the results of this group of students with the previous year results. The analyses do not present details of the results for the different tasks, because it was necessary to put more efforts in the analyses of the development of the results during the school year for the students who did both tests.

Elevgruppe	Antall	Test år	Minimum	Maksimum	Gjennomsnitt	St.avvik
Alle Elever - Test 1	**167**	2005 høst	1	44	19	9,7

Table 4.2.2.1.1: 9th grade, Results in points for the group of all students, test 1, fall 2005.
Note: The maximum score a student could receive for this test was 57 points.

The table presented here shows that the group of 167 students had average points sum of 19 points. Considering that the maximum score a student could get for the test was 57 points, such average result is low. The minimum result in the group was very low. The maximum result in the group was 44 points and was not very close to 57 points.

4.2.2.2 Summary of the students' results on all test items

The diagrams displayed below show the solution frequencies for every test item for the two groups of students. The students received code 1 if they gave correct answers or code 2 and code 3 if their answers were partially correct.

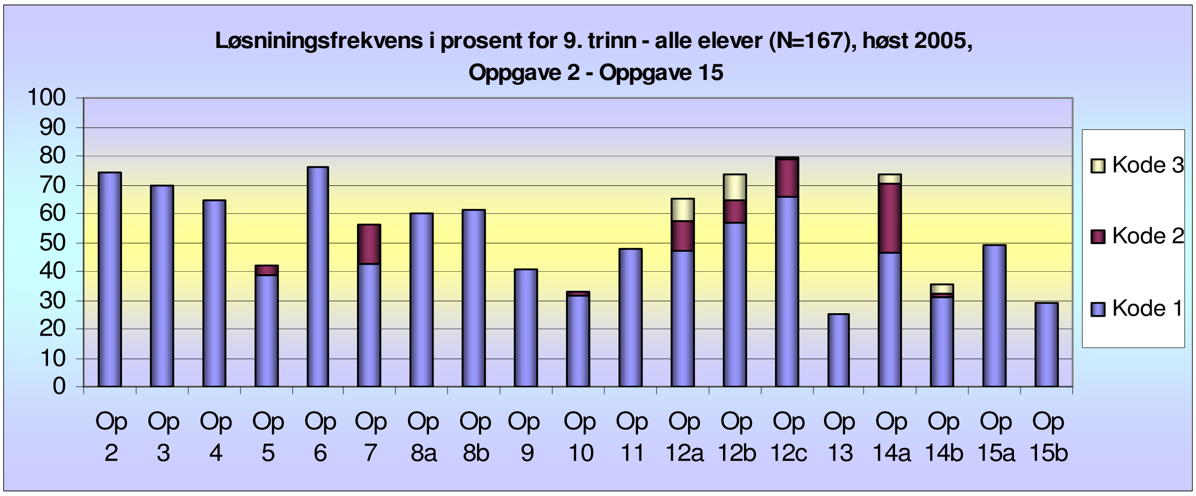


Figure 4.2.2.2.1: 9th grade (N=167) - Solution frequencies in percents for tasks 2- 15b, test 1(2005 fall)

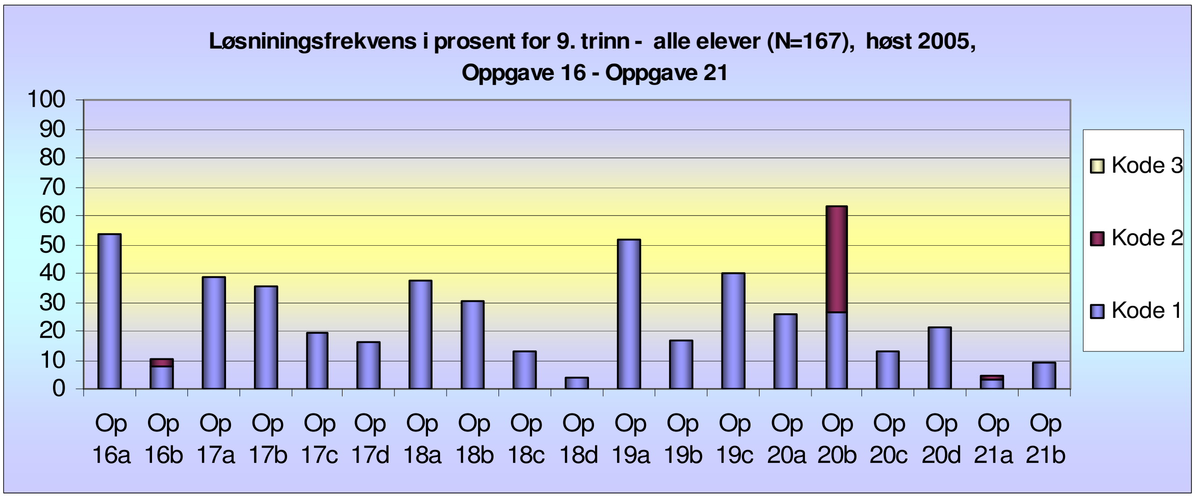


Figure 4.2.2.2.2: 9th grade (N=167) - Solution frequencies in percents for tasks 16a - 21b, test 1(2005 fall)

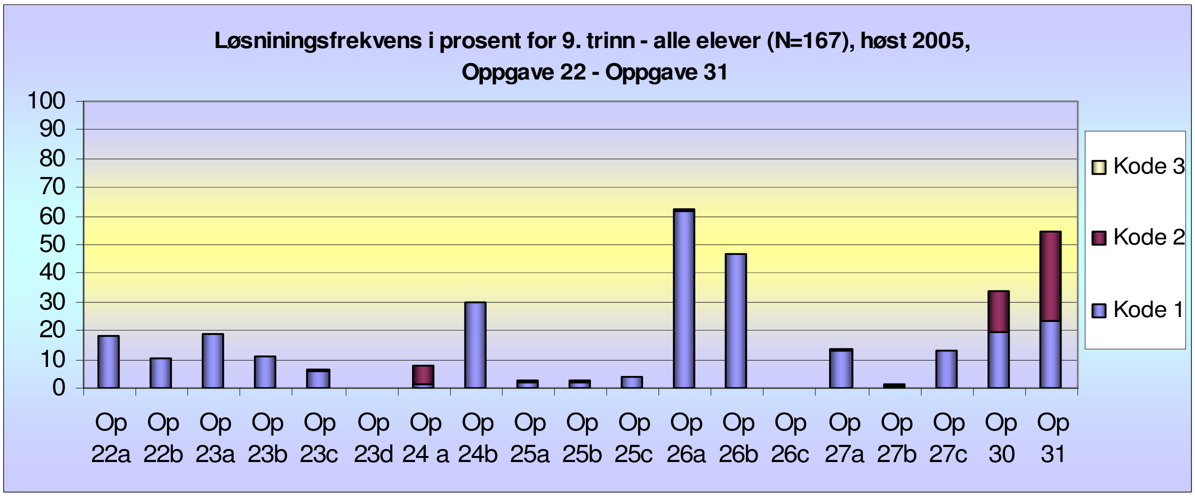


Figure 4.2.2.2.3: 9th grade (N=167) - Solution frequencies in percents for tasks 22a - 31, test 1(2005 fall)

4.2.2.3 Comparison of the results with the previous year results

In this report the results for all 167 students tested in the fall of 2005 are compared with the previous year results for the group of all 90 students (fall 2004) reported by Andreassen (2005). Since those groups of students took the test early in the beginning of the school year, we can expect that many students were affected by the time of the test.

The table below shows statistical data that concerns both groups of students – the minimum, maximum and the average points sum are given for both groups of students.

Elevgruppe	Antall	Test år	Minimum	Maksimum	Gjennomsnitt	St.avvik
Alle Elever - Test 1	**90**	2004 høst	1	49	20,1	10,4
Alle Elever - Test 1	**167**	2005 høst	1	44	19	9,7

Table 4.2.2.3.1: 9th grade, Results in points for the group of 90 students, test 1, fall 2004 and the group of 167 students, test 1, fall 2005. Note: Maximum score - 57 points for both groups of students

The group of all students tested in 2004 had on average points sum of 20,1 points, a result similar to the average points sum for the compared group of 167 students. Those average results for both groups of students are relatively low, taking into account that the maximum score a student could receive on the test was 57 points. We should not forget also that the students were tested in the beginning of the school year and we can expect that some of the students were still not focused enough.

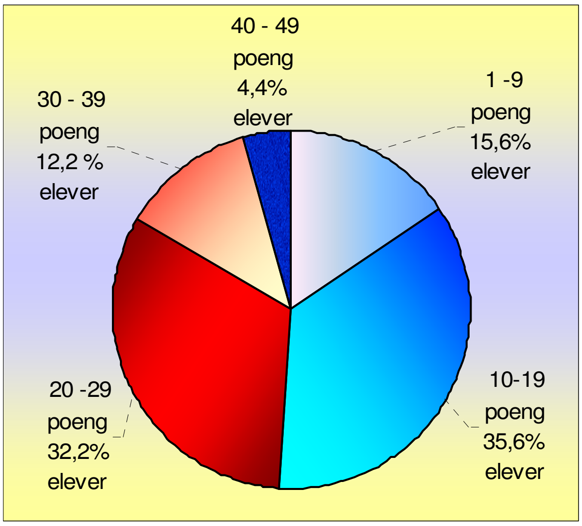


Figure 4.2.2.3.4: 9th grade (N=90) – test 1, 2004 fall, Distribution of the students' points sums in subgroups.

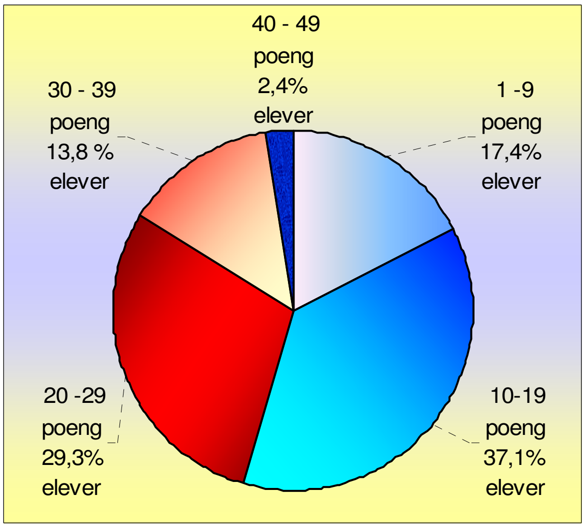


Figure 4.2.2.3.5: 9th grade (N=167) – test 1, 2005 fall, Distribution of the students' points sums in subgroups.

It should be noted that because problems 1, 23e, 22c, 22d, 27d, 28, 29 and 32 from the test in the fall of 2004 were not included in the comparison, the points score for the 90 students (tested in 2004) was counted only for the test items done by both groups.

Elevgruppe	År	1 - 9 poeng	10 - 19 poeng	20 - 29 poeng	30 - 39 poeng	40 - 49 poeng	50 - 57 poeng
Alle (N=90)	2004 høst	15,6	35,6	32,2	12,2	4,4	0,0
Alle (N=167)	2005 høst	17,4	37,1	29,3	13,8	2,4	0,0

Table 4.2.2.3.2: 9th grade, 90 students (2004 fall) and 167 students (2005 fall): Distribution of the students' points sums in percents.

The two figures 4.2.2.3.4 and 4.2.2.3.5 show that there were not main differences between the two big groups of students. In 2004 the biggest subgroups of students were the students with score between 10 and 19 points (35,6% of all 90 students) and the students with score between 20-29 points (32,2% of all 90 students). The next year the majority of students were also from those subgroups - 37,1% of all 167 students received between 10 and 19 points and 29,3% of all students had scores between 20 and 29 points. Both years the students with results above 30 points were small subgroups of students - less than 17% of the students were the groups of students who got the higher results.

Summary of the results for the test items, solved by both groups

Here are presented diagrams with the results of the comparison between both groups of students. The results for every test problem for the first group of 90 students were compared with the results for every test problem for the second group of 167 students in order to find the differences and the similarities. The data for the group of 90 students tested in 2004 was taken from an SPSS file prepared by Andreassen. This database was necessary to change, because some of the test items the students received on the test in 2004 were not included in the data for analyses. The problems 1, 23e, 22c, 22d, 27d, 28, 29 and 32 from the test in the fall of 2004 were not included in the comparison, so the maximum score is 57 points for both groups of students

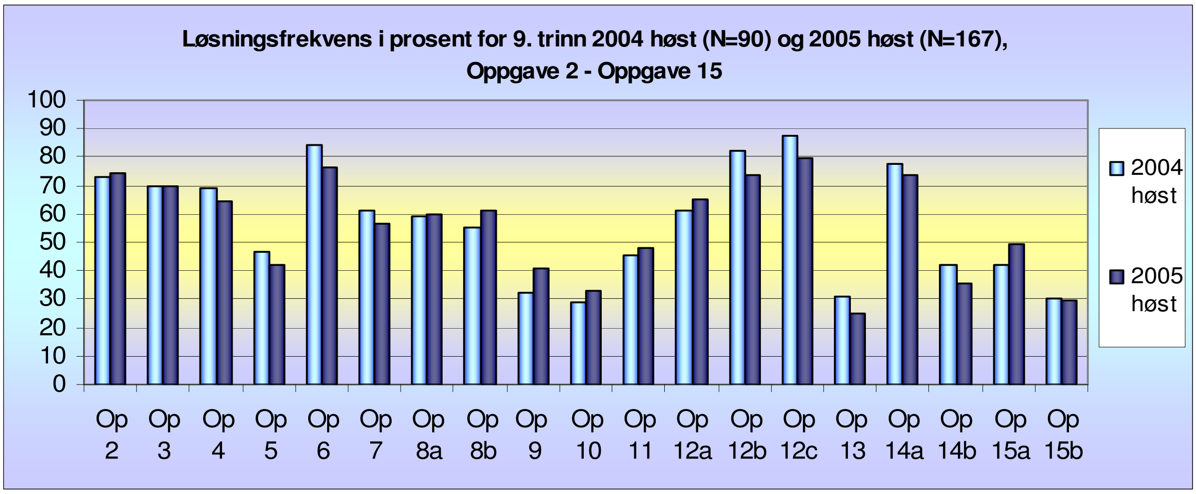


Figure 4.2.2.3.1: 9th grade, solution frequencies for test items 2 – 15b for the group of 90 students (first test - fall 2004) and the solution frequencies for the compared group of 167 students (first test – fall 2005).

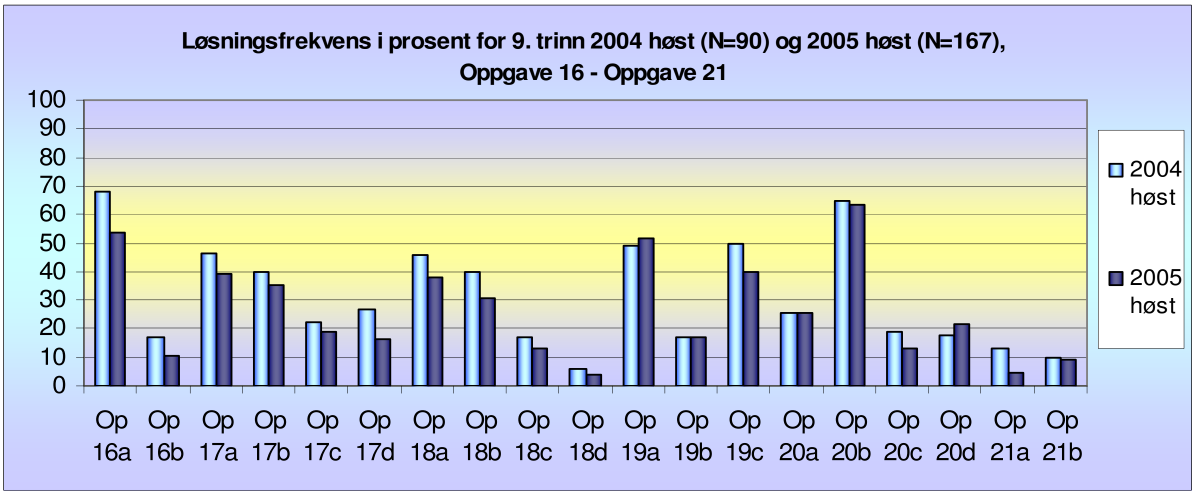


Figure 4.2.2.3.2: 9th grade, solution frequencies for test items 16a – 21b for the group of 90 students (first test - fall 2004) and the solution frequencies for the compared group of 167 students (first test – fall 2005).

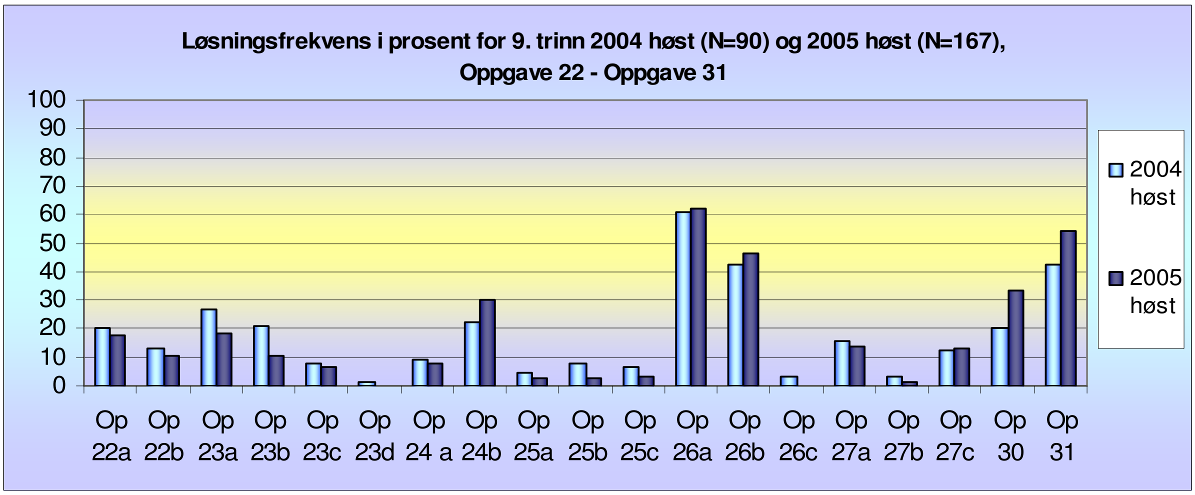


Figure 4.2.2.3.3: 9th grade, solution frequencies for test items 22a – 31 for the group of 90 students (first test - fall 2004) and the solution frequencies for the compared group of 167 students (first test – fall 2005).

Here would be presented some more details about the performance of both groups of students. The comparison of the results shows that:

- The group of167 students tested in 2005 had better results for 18 test items
- The group of 90 students tested in 2004 showed higher results for 39 of the tasks

Test items with better results for the group of 167 students

The group of all 167 students (fall 2005) showed better results for 18 of the 57 tests items, compared with the results for the 90 students tested the previous year 2004. Those were problems from different parts of the test and included the following tasks 2, 8a, 8b, 9, 10, 11,12a, 15a, 19a, 19b, 20a, 20d, 24b, 26a, 26b, 27c, 30, 31. Only problem Op1a had solutions frequency above 70 solution frequency points for both groups of students and the problems 8a, 8b,12a, 26a had solutions frequency above 50 for both groups students. The other 13 tests items had solution frequency below 50 solution frequency points for both groups of students. Some of the tasks with the highest difference in the results for both groups were: (1) task Op 9 ($60 \cdot 450 = ?$), (2) task Op 20d (*Anne kjøper bananer i en butikk til 13,50 kr per kg. Hvor mye kan Anne kjøpe for 10,50 kr?*, (3) task Op 24b (evaluation of $2a + 3 = 2a - 3$), (4) task Op 30 (Hvor mange nøtter er det i hver av skålene?), and (5) task Op 31 (5 liter bær veier 10 kg. Hvor mye veier 6 liter bær?).

Test items with lower results for the group of 167 students

The group of 167 students showed lower results for 39 of the test items, compared with the group of 90 students. Those were tests problems 3, 4, 5,6, 7, 12b, 12c, 13, 14a, 14b, 15b, 16a, 16b, 17a - 17c, 18a- 18d, 20b, 20c, 21a, 21b, 22a, 22b, 23a- d, 24a, 25a-c, 26c, 27a, 27b. Those were mostly problems solved by less than half of the students in both groups; only 10 of those items had solution frequencies above 50 solution frequency points for the group of 90 students. The comparison of the results shows that the biggest difference in the results for both groups were on the items Op 16a ($574 = 5 \cdot 100 + \square \cdot 10 + 4 \cdot 1$), followed by the problems Op 17d (Skriv som desimaltall 28 tideler....) and the problem Op19c (estimation of the answer for the expression $0{,}73 \cdot 46{,}2$).

The most difficult tasks

There were some tasks which were very difficult for most of the students and were solved by less than 10% of the students in both groups. The tasks 18d, 21b, 23c, 23d, 24a, 25a, 25b, 25c, 26c, 27b from the second section of the test were among the most difficult for both groups of students. Less than 10% of the students from the two groups gave acceptable answer. The most difficult were the task Op 23d (to simplify the algebraic expression $2y.y^2$), task Op 26c - "konfirmasjonsbord" requiring generalising and problem solving skills, and task Op 27b (to find the value of $y= b^3$, if b=4) and Op 25a (to construct algebraic expression) - less than 5% of the students could generate correct solutions on those tasks.

Between 10% and 20% of the students in both groups could solve the eight test items – 16b, 18c, 19b, 20c, 20d, 21a, 22a, 22b. The tasks 16b, 18c, 19b, 22a and 22b were arithmetical tasks, 20c and 20d were word problems (multiple choice tasks), task Op 21a asked the students to make their own word story that describes the arithmetical expression "4:0,5".

4.2.3 *Trends in the students' progress*

This comparison concerns analyses of the two test results of the group of 92 students (2005 - 2006) and the results of the group of 74 students who did both tests the previous school year (2004 - 2005). The main aim of this comparison is to analyse the development shown in the students' results for both tests for the two groups of students and to find the main differences and similarities. In this part of the analyses is presented a comparison of the solution frequency points for all tasks done by both groups (called as group A – the group of 74 students (2004 – 2005) and group B – the similar group of 92 students (2005 – 2006)).

Additional notes - age and gender information

The two groups of students, included in this comparison, have the same average age when tested in the fall and in the spring. It is important to notice that both groups have relatively small number of students. The 92 students were already described as a group of students. The group of 74 students was analysed by Espeland (2006). All of them performed the same test at the beginning and at the end of one school year. The average age of those 74 students was 14 years and 2 months when tested in the fall of 2004 and 14 years and 10 months when the students were tested in the next spring. The 74 students were 37 boys and 36 girls (one student didn't provide gender information). So there were not gender differences compared with the second group of 92 students (47 boys and 45 girls).

4.2.3.1 The results of the groups

The table 4.2.3.1.1 shows statistical data concerning both groups of students. The table presents the minimum, maximum, average and the standard deviation for the students' points sum for the first and the second test for both groups of students.

Elevgruppe	Antall	Test år	Minimum	Maksimum	Gjennomsnitt	St.avvik
Elever - Test 1 & Test 2	**74**	2004 høst	1	49	21,1	10,9
		2005 vår	0	53	25,2	13,5
Elever - Test 1 & Test 2	**92**	2005 høst	2	44	18,8	9,7
		2006 vår	1	47	23,9	12,2

Table 4.2.3.1.1: Results in points for the groups of 74 students (2004 – 2005) and 92 students (2005 – 2006) who did both tests. Note: The maximum points a student can get on the test is 57 points.

The average points sums for the group of 74 students were 21,1 points (test 2004) and 25,2 points (test 2005). The compared group of 92 students had average points sums 18,8 points (test 2005) and 23,9 points (test 2006). The differences in those results are minimal, but those results are slightly lower for the second group B. We observe also that the maximum scores for both tests for group A and are slightly higher in comparison with the maximum results for group B.

Additional information concerning the distribution of the group of 74 students is given in the following figures. The five subgroups show percents students in the whole group with points sums: (i) 1- 9 points, (ii) 10 -19 points, (iii) 20 – 29 points, (iv) 30 – 39 points, (v) 40 – 49 points, and (vi) 50 – 57 points.

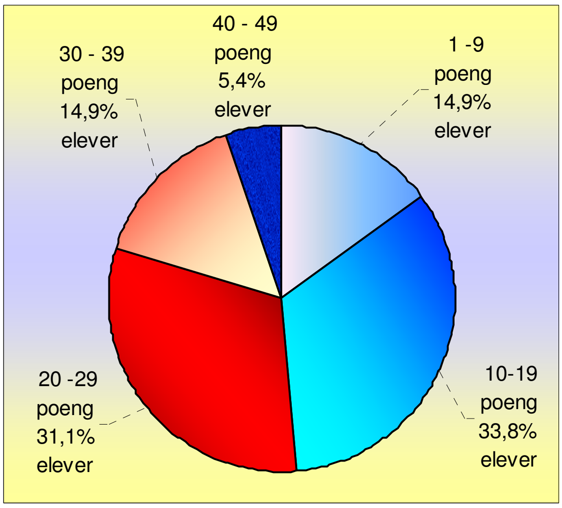


Figure 4.2.3.1.1: 9th grade (N=74) – test 1, 2004 fall, Distribution of the students' points sums in subgroups.

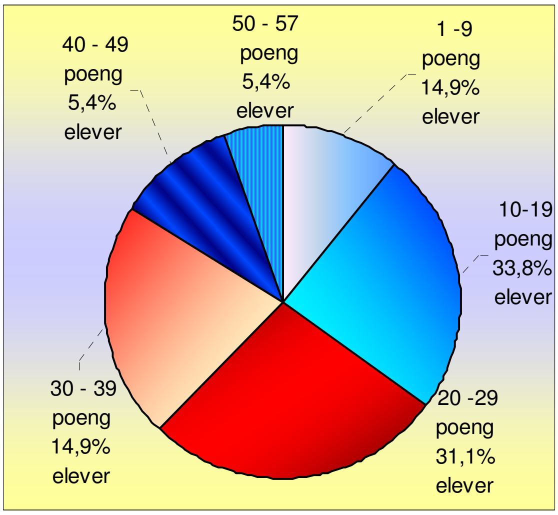


Figure 4.2.3.1.2: 9th grade (N=74) – test 2, 2005 spring, Distribution of the students' points sums in subgroups.

In both groups from the 9th grade there has been a positive development. There was a big increase of the students with scores above 30 points and a big decrease of the groups of students with minimal scores. The results for the second group of 92 students, displayed in the figure 4.2.3.1.3 and figure 4.2.3.1.4, were already presented in the analyses of the results of the group. There are observed some differences between the two groups A and B in relation to how many students are in each of the corresponding subgroups, but the differences are minimal.

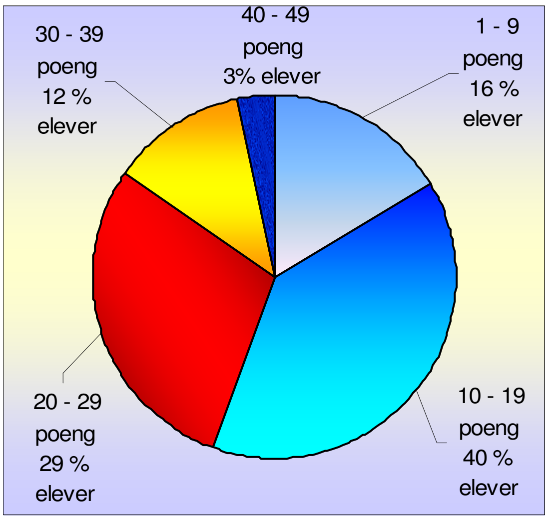


Figure 4.2.3.1.3: 9th grade (N=92) – test 1, 2005, Distribution of the students' points sums in subgroups

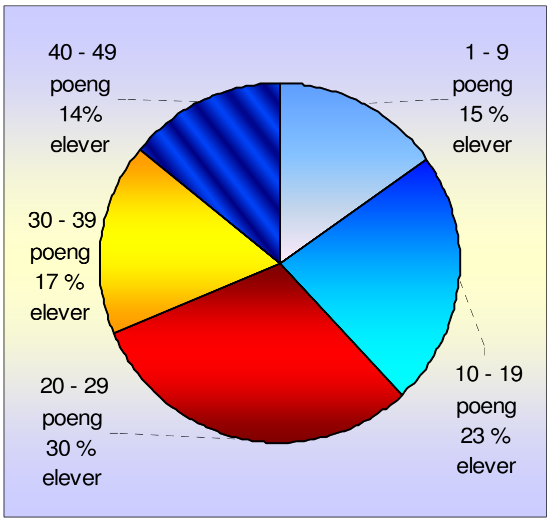


Figure 4.2.3.1.4: 9th grade (N=92) – test 2, 2006, Distribution of the students' points sums in subgroups

- Group A – 74 students

The figures 4.2.3.1.1 and 4.2.3.1.2 show that: 20% of the students (test 1) and 36% of the students (test 2) received points sums above 30 points; the group of students with points sums below 20 points was reduced from 49% of the students (test 1)to 35% (test 2).

- Group B – 92 students

The figures 4.2.3.1.3 and 4.2.3.1.4 show that: 15% of the students (test 1) and 31% of the students (test 2) received points sums above 30 points; the group of students with points sums below 20 points was reduced from 55% of the students (test 1)to 38% (test 2).

Elevgruppe	År	1 – 9 poeng	10 - 19 poeng	20 - 29 poeng	30 – 39 poeng	40 – 49 poeng	50 – 57 poeng
74 elever	2004 høst	15	34	31	15	5	0
	2005 vår	11	24	27	22	11	5
92 elever	2005 høst	16	39	29	12	3	0
	2006 vår	15	23	30	17	14	0

Table 4.2.3.1.2: 9th grade, 74 students (2004 – 2005) and 92 students (2005 – 2006): Distribution of the students points sum in percents, results for the first and the second test.

Additional information 9th grade

The table 4.2.3.1.3 shows statistical data concerning both groups of students A and B combined with statistical data regarding the corresponding three subgroups (Nederste, Midterste and Øverste tredjedel) in both groups. The table presents the minimum, maximum, average and the standard deviation for the point sums of the students in each group for the first and the second test. This information shows a comparison of data presented in the previous parts of the analyses and data from the previous school year, described by Espeland (2006).

Elevgruppe		Antall	Test år	Minimum	Maksimum	Gjennomsnitt	St.avvik
Elever - Test 1 & Test 2		74	2004 høst	1	49	21,1	10,9
			2005 vår	0	53	25,2	13,5
Elever Test 1 & Test 2	Nederste tredjedel	25	2004 høst	1	16	9,6	4,5
			2005 vår	0	26	12,9	7,6
	Midterste tredjedel	25	2004 høst	16	27	20,6	3
			2005 vår	22	41	24,2	7,9
	Øverste tredjedel	24	2004 høst	27	49	33,6	6,2
			2005 vår	20	53	39,1	9,3
Elever - Test 1 & Test 2		92	2005 høst	2	44	18,8	9,7
			2006 vår	1	47	23,9	12,2
Elever Test 1 & Test 2	Nederste tredjedel	31	2005 høst	2	14	8,5	3,8
			2006 vår	1	27	12,5	7,6
	Midterste tredjedel	31	2005 høst	15	22	18,4	2,4
			2006 vår	3	34	23,1	6,6
	Øverste tredjedel	30	2005 høst	23	44	29,8	5,7
			2006 vår	19	47	36,7	7,5

Table 4.2.3.1.3: Results in points for the groups of 74 students (2004 – 2005) and 92 students (2005 – 2006) who did both tests. Note: The maximum points a student can get on the test is 57 points.

4.2.3.2 Comparison of the solution frequency points for all tasks

In this section of the analyses we present a summary of the students' results for both groups of students. The data is presented as comparison of the solution frequency results for every test item for both tests for group A and group B. After that the analyses of the results focuses on the results of both groups organised in the following comparisons:
(1) Comparison – results first test
(2) Comparison – results second test
(3) Test items divided in six groups, according to the change of the solution frequencies for every test item

Summary of the students' results on all test items

The development of the results for each test item is shown in the following diagrams. For every task are presented the solution frequencies for the group of 74 students (test - fall 2004 and test - spring 2005) and the solution frequencies for the compared group of 92 students (both tests).

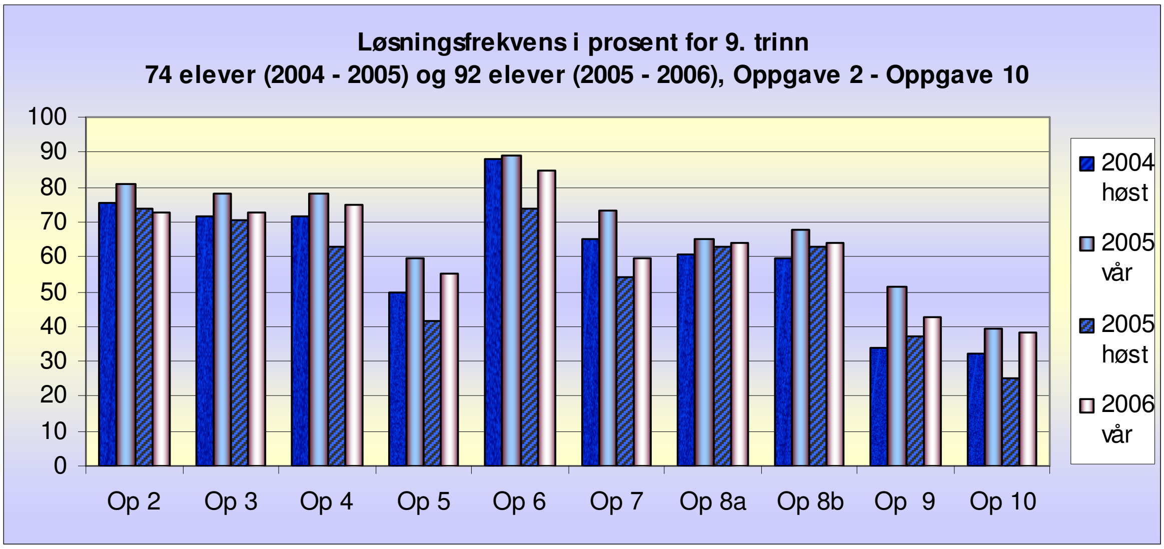


Figure 4.2.3.2.1: 9th grade, solution frequencies for test items 2 – 10 for the group of 74 students (both tests - fall 2004 and spring 2005) and the solution frequencies for the compared group of 92 students (both tests – fall 2005 and spring 2006).

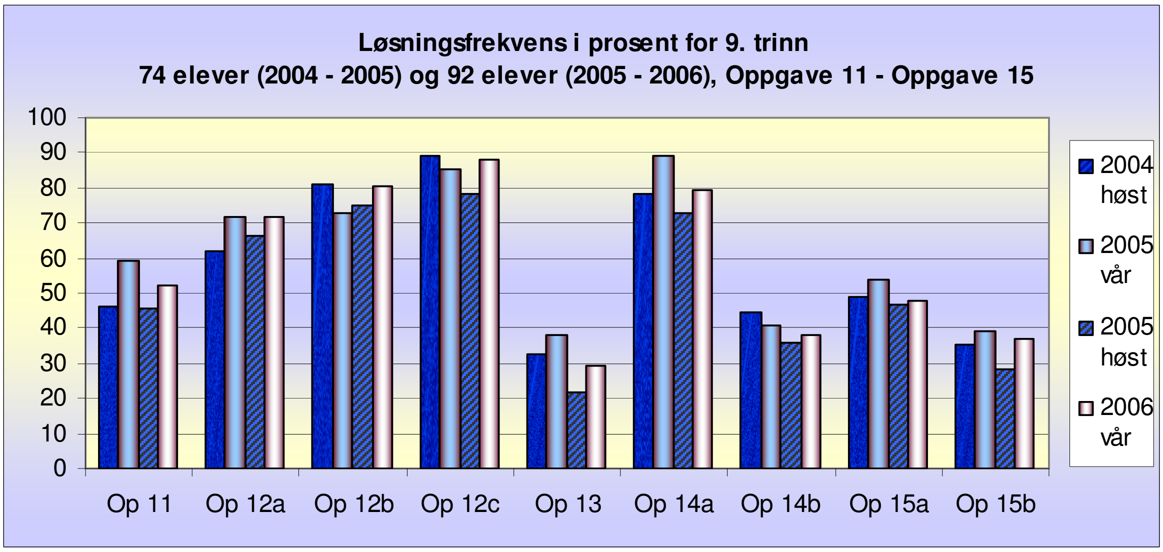


Figure 4.2.3.2.1: 9th grade, solution frequencies for test items 11 – 15 for the group of 74 students (both tests - fall 2004 and spring 2005) and the solution frequencies for the compared group of 92 students (both tests – fall 2005 and spring 2006).

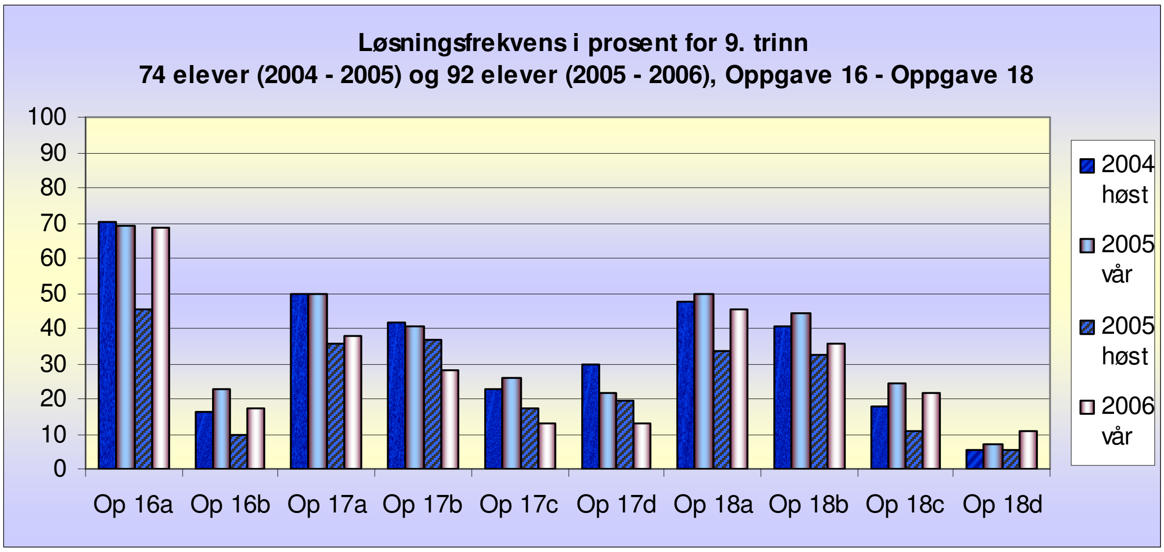


Figure 4.2.3.2.2: 9th grade, solution frequencies for test items 16a – 18d, 74 students (both tests - fall 2004 and spring 2005) and the solution frequencies for the compared group of 92 students (both tests – fall 2005 and spring 2006).

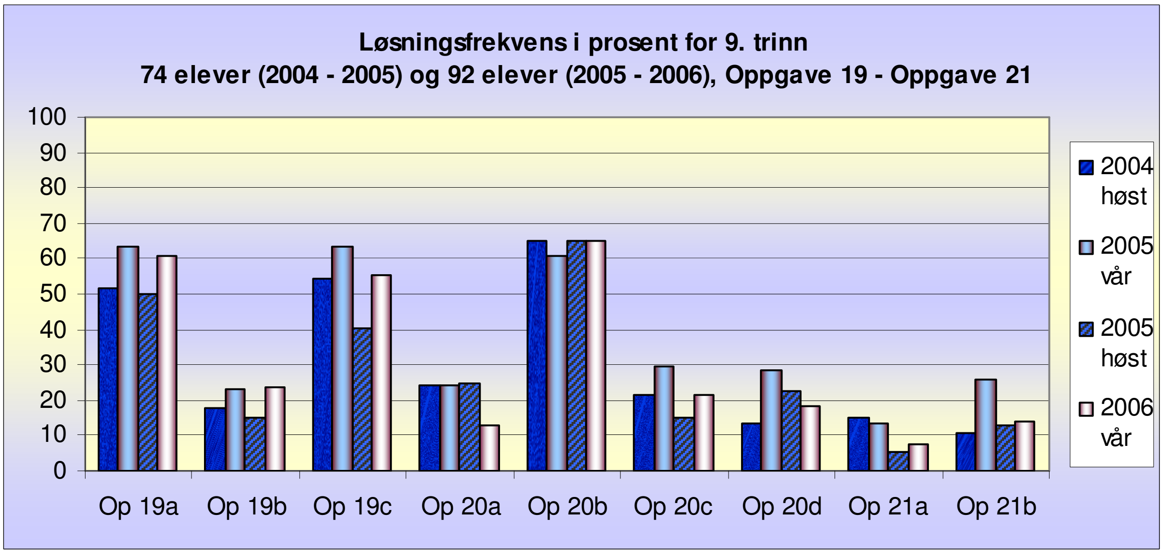


Figure 4.2.3.2.3: 9th grade, solution frequencies for test items 19a – 21b for the group of 74 students (both tests - fall 2004 and spring 2005) and the solution frequencies for the compared group of 92 students (both tests – fall 2005 and spring 2006).

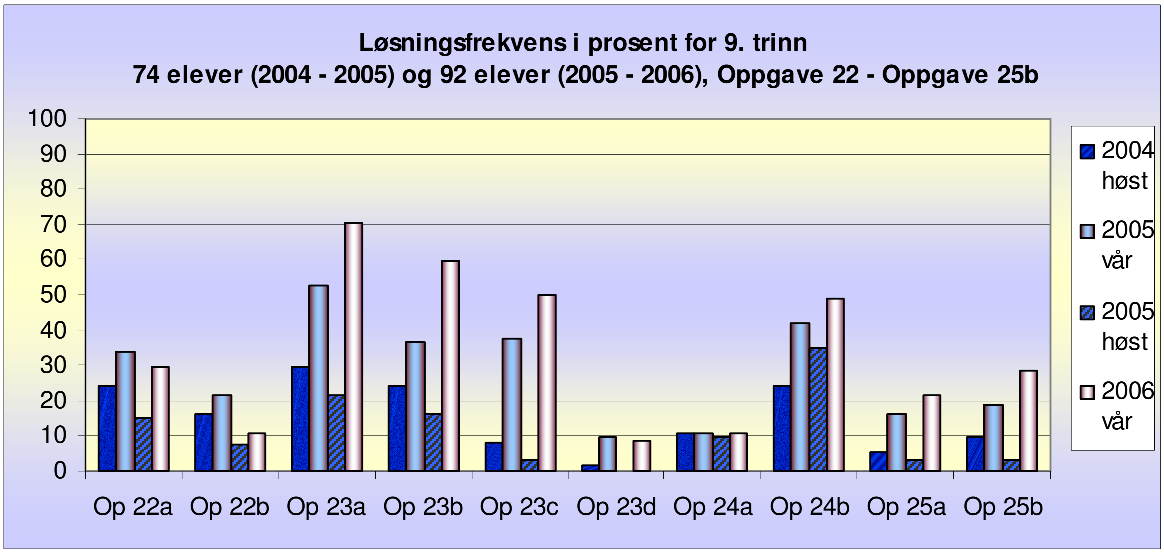


Figure 4.2.3.2.4: 9th grade, solution frequencies for test items 22a – 25b for the group of 74 students (both tests - fall 2004 and spring 2005) and the solution frequencies for the compared group of 92 students (both tests – fall 2005 and spring 2006).

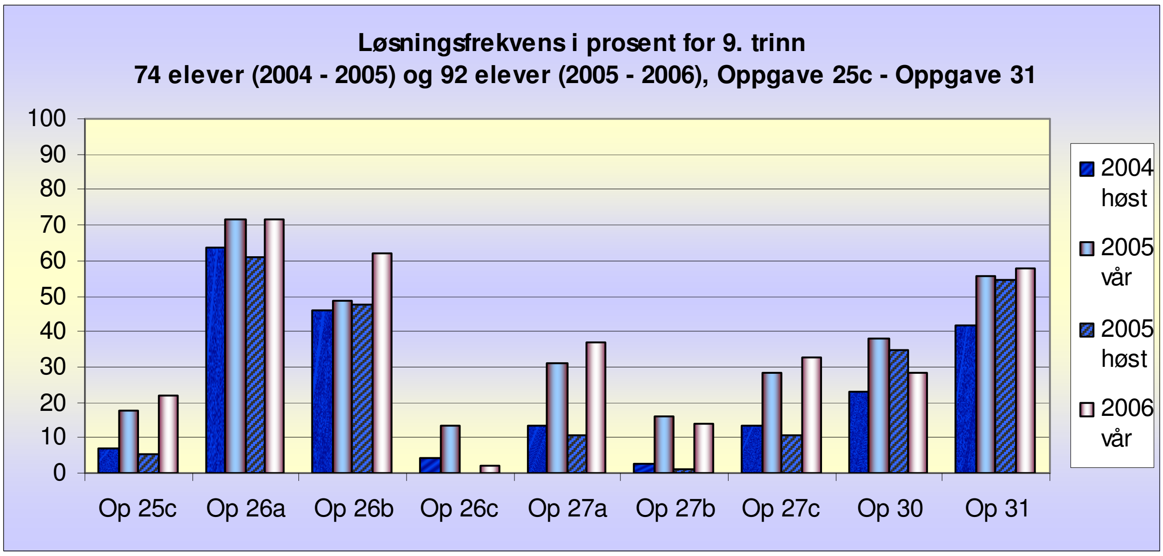


Figure 4.2.3.2.5: 9th grade, solution frequencies for test items 25c – 31 for the group of 74 students (both tests - fall 2004 and spring 2005) and the solution frequencies for the compared group of 92 students (both tests – fall 2005 and spring 2006).

Comparison – results first test

1. The group A showed better results in comparison with the group B on totally 44 of the test items on the first test. For 14 of those 44 test items the difference between the two groups is minimal - between 0 and 2,6 solution frequency points.

- *Test items with results (test 1) for group A better than for group B : higher between 25 and 10 solution frequency points*

The group of 74 students showed higher results - between 25 and 10 solution frequency points for 9 test items: Op 6, Op 7, Op 12c, Op 13, Op 16a, Op 17a, Op 17d, Op 18a, and Op 19c. On those 9 test items the students from group B had on average a score lower than the score of group A with 14 points (test 1) and 9 points (test 2). Those tasks were mostly related to understanding of numbers as for example operations with fractions, comparison of decimal numbers, operations with decimal numbers, estimation of expression with decimal numbers. On almost all of those tasks the group A showed higher results also on the second test.

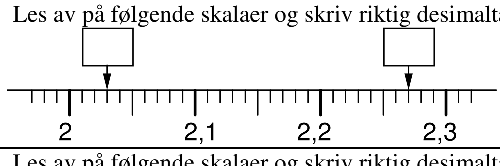
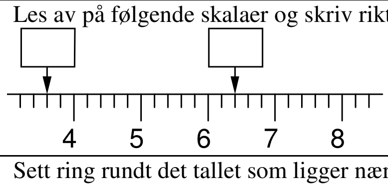
Oppgave		Frekvens i prosent			
		LCM 9. trinn 74 elever		LCM 9. trinn 92 elever	
		2004 høst	2005 vår	2005 høst	2006 vår
Op 6	Temperaturen forandrer seg fra –5 ˚C til +8 ˚C. Hva er stigningen i temperatur?	88	89	74	85
Op 7	Uttrykk 20% som en brøk	65	73	54	60
Op 12c	Sett *ring rundt* det største tallet og *kryss over* det minste tallet: 4,09 4,7 4,008	89	85	78	88
Op 13	Finn et tall med to desimaler som ligger mellom 15,755 og 15,762	32	38	22	29
Op 16a	Skriv riktig tall i rutene $574 = 5\cdot 100 + \boxed{}\cdot 10 + 4\cdot 1$	70	69	46	68

Op 17a	Skriv som desimaltall $\frac{3}{10}$	50	50	36	38
Op 17d	Skriv som desimaltall 28 tideler	30	22	20	13
Op 18a	Skriv svaret som desimaltall. $6 \cdot 0,5 =$ ………………………………	47	50	34	46
Op 19c	Til venstre i rammen står et regnestykke. Sett ring rundt det tallet du mener er *nærmest svaret*. $0,73 \cdot 46,2$ | 0,03 0,3 3 30 300	54	64	40	55

Table 4.2.3.2.1: Solution frequencies for test items with results (test 1) for group A better than for group B: higher between 25 and 10 solution frequency points.

- *Test items with results (test 1) for group A better than for group B: higher between 9,9 and 5 solution frequency points*

The group of 74 students showed higher results - between 9,9 and 5 solution frequency points for 18 test items: Op 4, Op 5, Op 10, Op 12b, Op 14a, Op 14b, Op 15b, Op 16b, Op 17c, Op 18b, Op 18c, Op 20c, Op 21a, Op 22a, Op 22b, Op 23a, Op 23b, Op 25b. On those 18 test items the students from group B had on average a score lower than the score of group A with 7 points (test 1) and 2 points (test 2). Those tasks were mostly related to understanding of numbers as for example operations with fractions, comparison and operations with decimal numbers, measurement, expressions with whole and decimal numbers, and to simplify algebraic expressions. There were also two word problems. For four of those tasks the group B showed higher results than group A on the second test. On the tasks 23a and 23b (simplification of algebraic expressions) the results on the second test were much higher than those of group A.

Oppgave		Frekvens i prosent			
		LCM 9. trinn 74 elever		LCM 9. trinn 92 elever	
		2004 høst	2005 vår	2005 høst	2006 vår
Op 4	Hva er 25% av 40 km?	72	78	63	75
Op 5	$\frac{1}{2}+\frac{1}{4}=?$	50	59	41	55
Op 10	$\frac{1}{2}-\frac{1}{3}=?$	32	39	25	38
Op 12b	Sett *ring rundt* det største tallet og *kryss over* det minste tallet: 3,521 3,6 3,75	81	73	75	80
Op 14a	Les av på følgende skalaer og skriv riktig desimaltall i ruta. 	78	89	73	79
Op 14b	Les av på følgende skalaer og skriv riktig desimaltall i ruta. 	45	41	36	38
Op 15b	Sett ring rundt det tallet som ligger nærmest i størrelse til 2,08 209 2,9 2,05 2,1 20,9	35	39	28	37
Op 16b	Skriv riktig tall i rutene $5,74 = 5 \cdot 1 + 7 \cdot \square + 4 \cdot \square$	16	23	10	17

Op 17c	Skriv som desimaltall 45 tusendeler	23	26	17	13
Op 18b	Skriv svaret som desimaltall. 3 : 6 = ...…………………………………	41	45	33	36
Op 18c	Skriv svaret som desimaltall. 3 : 0,5 = ………………………………	18	24	11	22
Op 20c	Sett ring rundt *alle* regneuttrykkene som passer til regneoppgaven: *Kaker skal fylles i bokser, med 0,75 kg i hver. Hvor mange bokser trenger man til 6 kg kaker?* 6 · 0,75 6 : 0,75 0,75 : 6 0,75 · 6 6 – 0,75 6 + 0,75	22	30	15	22
Op 21a	Lag din egen fortelling som passer til disse regnestykkene: $\boxed{4 : 0,5}$	15	14	5	8
Op 22a	Skriv riktig tall i ruta 14 : 2 = $\boxed{}$ · 14	24	34	15	29
Op 22b	Skriv riktig tall i ruta 14 : $\boxed{}$ = 0,25 · 14	16	22	8	11
Op 23a	Skriv enklest mulig: 2x + 5x	30	53	22	71
Op 23b	Skriv enklest mulig: $x + x + 2x$	24	36	16	60
Op 25b	Adder tallet x til 4x	9	19	3	28

Table 4.2.3.2.2: Solution frequencies for test items with results (test 1) for group A better than for group B: higher between 9,9 and 5 solution frequency points

- *Test items with results (test 1) for group A better than for group B: higher between 4,9 and 0,1 solution frequency points*

The group of 74 students showed higher results - between 4,9 and 0,1 solution frequency points for 17 test items: Op 2, Op 3, Op 11, Op 15a, Op 17b, Op 19a, Op 19b, Op 23c, Op 23d, Op 24a, Op 25a, Op 25c, Op 26a, Op 26c, Op 27a, Op 27b, Op 27c. On those 17 test items the students from group B had a very minimal difference with group A : 2 points (test 1) and 1 points (test 2). Those tasks were a mixture of different type problems - problems related to understanding of fractions, estimation of expression, problems involving algebraic expressions, tasks related to figure pattern and simple word problems. For seven of those tasks the group B showed higher results than group A on the second test.

Oppgave		Frekvens i prosent			
		LCM 9. trinn 74 elever		LCM 9. trinn 92 elever	
		2004 høst	2005 vår	2005 høst	2006 vår
Op 2	Hva er en kvart (en firedel) av 60 gram?	76	81	74	73
Op 3	Finn lengden på gardinstoffet Mari må kjøpe, når det skal være fem gardiner, og hvert skal være 1,05 meter.	72	78	71	73
Op 11	$\frac{2}{5}$ av en masse er 20 gram. Hva er massen?	46	59	46	52
Op 15a	Sett ring rundt det tallet som ligger nærmest i størrelse til 0,16 0,1 0,2 15 0,21 10	49	54	47	48
Op 17b	Skriv som desimaltall $\frac{46}{100}$	42	41	37	28
Op 19a	Til venstre i rammen står et regnestykke. Sett ring rundt det tallet du mener er *nærmest svaret*. Du trenger ikke regne ut svaret. 13 : 4,32 | 0,03 0,3 3 30 300	51	64	50	61
Op 19b	Til venstre i rammen står et regnestykke. Sett ring rundt det tallet du mener er *nærmest svaret*. Du trenger ikke regne ut svaret. 7,5 : 0,24 | 0,03 0,3 3 30 300	18	23	15	24

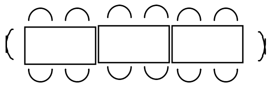
Op 23c	Skriv enklest mulig: t · t · t	8	38	3	50
Op 23d	Skriv enklest mulig: $2y{\cdot}y^2$	1	9	0	9
Op 24a	$x + y + z = x + p + z$ *Dette* ☐ er alltid sant ☐ er aldri sant ☐ *kan* være sant, nemlig når	11	11	10	11
Op 25a	Adder tallet *x* til *x* + 3*y*	5	16	3	22
Op 25c	Adder tallet *x* til 7	7	18	5	22
Op 26a	Et konfirmasjonsbord er satt sammen av småbord, og rundt er det satt stoler.  Hvor mange stoler blir det plass til om vi har 4 småbord?.............................	64	72	61	72
Op 26c	Et konfirmasjonsbord er satt sammen av småbord, og rundt er det satt stoler. Dersom vi har *n* småbord, hvor mange stoler blir det da?................	4	14	0	2
Op 27a	$x = a + b - c$ Dersom $a = 1$, $b = 2$ og $c = 3$ blir x=......................	14	31	11	37
Op 27b	$y = b^3$ Dersom b = 4 blir **y = ...**	3	16	1	14
Op 27c	3x = 7 og 5y = 11 Da blir 3x + 5y = ...	14	28	11	33

Table 4.2.3.2.3: Solution frequencies for test items with results (test 1) for group A better than for group B: higher between 4,9 and 0,1 solution frequency points

2. The group B showed higher results in comparison with the group A on totally 13 of the test items on the first test. For 6 of those 13 test items the difference between the two groups is minimal - between 0 and 2,2 solution frequency points.

- *Test items with results (test 1) for group B better than for group A: higher between 0,1 and 5 solution frequency points*

The group of 92 students showed higher results - between 0,1 and 5 solution frequency points for 9 test items: Op 8a, Op 8b, Op 9, Op 12a, Op 18d, Op 20a, Op 20b, Op 21b, Op 26b. On those 9 test items the students from group B had minimal difference from group A: (+ 2) points (test 1) and (-2) points (test 2). Those tasks were mostly word problems – related to pattern, construction of task, simple word problems. For four of those tasks the group B showed higher results than group A on the second test – the best improvement of the result was for task 26.

Oppgave		Frekvens i prosent			
		LCM 9. trinn 74 elever		LCM 9. trinn 92 elever	
		2004 høst	2005 vår	2005 høst	2006 vår
Op 8a	En bestemt type penner koster 15 kr for hver. Hvor mange kan du kjøpe for 200 kr?	61	65	63	64
Op 8b	En bestemt type penner koster 15 kr for hver Hvor mye vekslepenger får du da tilbake?	59	68	63	64
Op 9	60 · 450 = ?	34	51	37	42
Op 12a	Sett *ring rundt* det største tallet og *kryss over* det minste tallet: 0,625 0,25 0,3753 0,125 0,5	62	72	66	72
Op 18d	Skriv svaret som desimaltall. 0,6 : 0,2 = ……………………………	5	7	5	11
Op 20a	*24 halsbånd pakkes i eske. Om 24 halsbånd veier 3 kg, hvor mye veier da ett halsbånd?* 24 · 3 24 : 3 3 : 24 3 · 24 24 – 3 3 + 24	24	24	25	13
Op 20b	*1 kg pølser koster 49,50 kr. Per kjøper 1,7 kg. Hvor mye koster det?* 49,50 · 1,7 49,50 : 1,7 1,7 : 49,50 1,7 · 49,50 49,50 – 1,7	65	61	65	65

Op 21b	Lag din egen fortelling som passer til disse regnestykkene: $\boxed{5{,}25 \cdot 3{,}28 = 17{,}22}$	11	26	13	14
Op 26b	Et konfirmasjonsbord er satt sammen av småbord, og rundt er det satt stoler. **b** Enn om vi har 7 småbord?	46	49	48	62

Table 4.2.3.2.4: Solution frequencies for test items with results (test 1) for group B better than for group A: higher between 0,1 and 5 solution frequency points

- *Test items with results for group B better than for group A: higher between 5,1 and 10 solution frequency points*

The group of 92 students showed higher results - between 5,1 and 10 solution frequency points for 1 test items: Op 20d. This problem is a simple word problem – the results on the second test were with negative change for group B.
Note: This test item is presented together with other test items in the table 4.2.3.5.

- *Test items with results (test 1) for group B better than for group A: higher between 10,1 and 15 solution frequency points*

The group of 92 students showed higher results - between 10,1 and 15 solution frequency points for 3 test items: Op 24b, Op 30, Op 31. We include in the following table the results on task Op 20d. In this group are two word problems, where the students were asked to explain how they found the answer. The students were given a point for solving those items also when their answers were coded with code 2. So students who could provide the answers, but who had problems to explain their solutions or did not provide a solution method received also a point. It should be taken in consideration that the group A improved significantly their scores on those three test items on the second test. On the second test group B scored better than group A for two of the items on the second test, but the performance for task 30 was not good.

Oppgave		Frekvens i prosent			
		LCM 9. trinn 74 elever		LCM 9. trinn 92 elever	
		2004 høst	2005 vår	2005 høst	2006 vår
Op 20d	Sett ring rundt *alle* regneuttrykkene som passer til regneoppgaven: *Anne kjøper bananer i en butikk til 13,50 kr per kg. Hvor mye kan Anne kjøpe for 10,50 kr?* 13,50 · 10,50 10,50 : 13,50 13,50 : 10,50 13,50 – 10,50 13,50 + 10,50	14	28	23	18
Op 24b	$2a + 3 = 2a - 3$ Dette ☐ er alltid sant ☐ er aldri sant ☐ *kan* være sant, nemlig når	24	42	35	49
Op 30	Marit har tre skåler med nøtter *A, B* og *C*. Det er 2 flere i *B* enn i *A*. I skål *C* er det 4 ganger så mange nøtter som i skål *A*. I alt er det 14 nøtter. Hvor mange nøtter er det i hver av skålene? Vis eller forklar hvordan du tenkte:	23	38	35	28
Op 31	5 liter bær veier 10 kg. Hvor mye veier 6 liter bær? Vis hvordan du tenkte:	42	55	54	58

Table 4.2.3.2.5: Solution frequencies for test items with results for group B better than for group A: higher between 5,1 and 15 solution frequency points

Comparison – results second test

3. The group A showed better results in comparison with the group B on totally 38 of the test items on the first test. For 9 of those 38 test items the difference between the two groups is very minimal - between 0 and 2,6 solution frequency points.

- *Test items with results (test 2) for group A better than for group B : higher between 15 and 10 solution frequency points*

The group of 74 students showed higher results - between 15 and 10 solution frequency points on 8 test items: Op 7, Op 17a, Op 17b, Op 17c, Op 20a, Op 21b, Op 22b, Op 26c. On those test items the students from group B had on average a score lower than the score of group A with 6 solution frequency points (test 1) and 12 points (test 2). Those tasks were related to understanding of numbers and required operations with fractions, operations with decimal numbers, or solving of word problems. On almost all of those tasks the group A showed higher results also on the first test. Tasks 26c and 21b are among the most difficult problems in the test and are related to abilities in all mathematical competences. Task 26c is related to generalisation of figure pattern – students need to provide the solution in the general case when there are n tables as elements. This task was very hard for both groups, but group B showed much lower results in comparison with group A. Task 21b required the students to construct a word problem related to arithmetical expressions. The number of students from group A, who could solve this task on the second test, was much higher. In comparison the students from group B showed small improvement when solving this task.

Oppgave		Frekvens i prosent			
		LCM 9. trinn 74 elever		LCM 9. trinn 92 elever	
		2004 høst	2005 vår	2005 høst	2006 vår
Op 7	Uttrykk 20% som en brøk.	65	73	54	60
Op 17a	Skriv som desimaltall $\frac{3}{10}$	50	50	36	38
Op 17b	Skriv som desimaltall $\frac{46}{100}$	42	41	37	28
Op 17c	Skriv som desimaltall 45 tusendeler	23	26	17	13
Op 20a	Sett ring rundt *alle* regneuttrykkene som passer til regneoppgaven: *24 halsbånd pakkes i eske. Om 24 halsbånd veier 3 kg, hvor mye veier da ett halsbånd?* 24 · 3 24 : 3 3 : 24 3 · 24 24 – 3 3 + 24	24	24	25	13
Op 21b	Lag din egen fortelling som passer til disse regnestykkene: $\boxed{5,25 \cdot 3,28 = 17,22}$	11	26	13	14
Op 22b	Skriv riktig tall i ruta 14 : $\square$ = 0,25 · 14	16	22	8	11
Op 26c	Et konfirmasjonsbord er satt sammen av småbord, og rundt er det satt stoler. Dersom vi har *n* småbord, hvor mange stoler blir det da?...............................	4	14	0	2

Table 4.2.3.2.6: Solution frequencies for test items with results (test 2) for group A better than for group B : higher between 15 and 10 solution frequency points

- *Test items with results (test 2) for group A better than for group B : higher between 9,9 and 5 solution frequency points*

The group of 74 students showed higher results - between 9,9 and 5 solution frequency points for 15 test items: Op 2, Op 3, Op 9, Op 11, Op 13, Op 14a, Op 15a, Op 16b, Op 17d, Op 18b, Op 19c, Op 20c, Op 20d, Op 21a, Op 30. This is a big group of tasks for which the students from group B showed on average results lower than the results of group A with 3 solution frequency points (test 1) and 8 points (test 2). Those tasks were related to calculations with fractions, decimal numbers, measurement, estimations, construction of a word problem or solving word problems. For 12 of those tasks the group A showed higher results also on the first test. The construction of a word problem related to the expression 4 : 0,5 (Task 21a) was the most difficult task for both groups of students. The students in group A showed much better results on this task in comparison with group B.

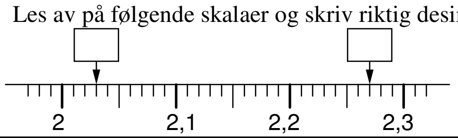
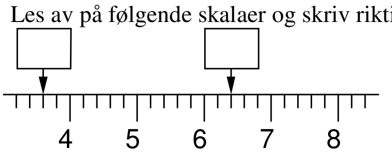
Oppgave		Frekvens i prosent			
		LCM 9. trinn 74 elever		LCM 9. trinn 92 elever	
		2004 høst	2005 vår	2005 høst	2006 vår
Op 2	Hva er en kvart (en firedel) av 60 gram?	76	81	74	73
Op 3	Finn lengden på gardinstoffet Mari må kjøpe, når det skal være fem gardiner, og hvert skal være 1,05 meter.	72	78	71	73
Op 9	$60 \cdot 450 = ?$	34	51	37	42
Op 11	$\frac{2}{5}$ av en masse er 20 gram. Hva er massen?	46	59	46	52
Op 13	Finn et tall med to desimaler som ligger mellom 15,755 og 15,762	32	38	22	29
Op 14a	Les av på følgende skalaer og skriv riktig desimaltall i ruta. 	78	89	73	79
Op 15a	Sett ring rundt det tallet som ligger nærmest i størrelse til 0,16 0,1 0,2 15 0,21 10	49	54	47	48
Op 16b	Skriv riktig tall i rutene $5{,}74 = 5 \cdot 1 + 7 \cdot \square + 4 \cdot \square$	16	23	10	17
Op 17d	Skriv som desimaltall 28 tideler	30	22	20	13
Op 18b	Skriv svaret som desimaltall. 3 : 6 = ...	41	45	33	36
Op 19c	Til venstre i rammen står et regnestykke. Sett ring rundt det tallet du mener er *nærmest svaret*. Du trenger ikke regne ut svaret. $0{,}73 \cdot 46{,}2$ | 0,03 0,3 3 30 300	54	64	40	55
Op 20c	*Kaker skal fylles i bokser, med 0,75 kg i hver. Hvor mange bokser trenger man til 6 kg kaker?* $6 \cdot 0{,}75$ 6 : 0,75 0,75 : 6 $0{,}75 \cdot 6$ 6 – 0,75 6 + 0,75	22	30	15	22
Op 20d	*Anne kjøper bananer i en butikk til 13,50 kr per kg. Hvor mye kan Anne kjøpe for 10,50 kr?* $13{,}50 \cdot 10{,}50$ 10,50 : 13,50 13,50 : 10,50 13,50 – 10,50 13,50 + 10,50	14	28	23	18
Op 21a	Lag din egen fortelling som passer til disse regnestykkene: $\boxed{4 : 0{,}5}$	15	14	5	8
Op 30	Marit har tre skåler med nøtter *A*, *B* og *C*. Det er 2 flere i *B* enn i *A*. I skål *C* er det 4 ganger så mange nøtter som i skål *A*. I alt er det 14 nøtter. Hvor mange nøtter er det i hver av skålene? Vis eller forklar hvordan du tenkte:	23	38	35	28

Table 4.2.3.2.7: Solution frequencies for test items with results (test 2) for group A better than for group B : higher between 9,9 and 5 solution frequency points

- *Test items with results (test 2) for group A better than for group B : higher between 4,9 and 0,1 solution frequency points*

The group of 74 students showed higher results - between 4,9 and 0,1 solution frequency points for 15 test items: Op 4, Op 5, Op 6, Op 8a ,Op 8b, Op 10, Op 14b, Op 15b, Op 16a, Op 18a, Op 18c, Op 19a, Op 22a, Op 23d, Op 27b. On those 15 test items the students from group B had minimal difference with group A: average result lower with 7 points (test 1) and 3 points (test 2). Those tasks were three simple word problems and tasks related to understanding of numbers, and simple algebra tasks. For eight of those tasks there was much smaller difference between the two groups on the second test – the students from group A managed to improve their results and showed already close results to those of group A. For example on the tasks Op.6, Op 16a and Op18a group B showed much lower results than group A on the first test, but the second results were close to the results of group A. The most difficult task for both groups of students were tasks 23d (to simplify the expression $2y{\cdot}y^2$) and the task 27b ($y = b^3$, Dersom $b = 4$ blir $y =$ ....), which indicate that both groups of students could not calculate well with variables in higher potencies. The results for task 10 (to calculate $\frac{1}{2} - \frac{1}{3} =$) were also very low for both groups of students.

Oppgave		Frekvens i prosent			
		LCM 9. trinn 74 elever		LCM 9. trinn 92 elever	
		2004 høst	2005 vår	2005 høst	2006 vår
Op 4	Hva er 25% av 40 km?	72	78	63	75
Op 5	$\frac{1}{2} + \frac{1}{4} = ?$	50	59	41	55
Op 6	Temperaturen forandrer seg fra –5 ˚C til +8 ˚C. Hva er stigningen i temperatur?	88	89	74	85
Op 8a	En bestemt type penner koster 15 kr for hver. Hvor mange kan du kjøpe for 200 kr?	61	65	63	64
Op 8b	En bestemt type penner koster 15 kr for hver Hvor mye vekslepenger får du da tilbake?	59	68	63	64
Op 10	$\frac{1}{2} - \frac{1}{3} = ?$	32	39	25	38
Op 14b	Les av på følgende skalaer og skriv riktig desimaltall i ruta. 	45	41	36	38
Op 15b	Sett ring rundt det tallet som ligger nærmest i størrelse til 2,08 209 2,9 2,05 2,1 20,9	35	39	28	37
Op 16a	Skriv riktig tall i rutene $574 = 5{\cdot}100 + \boxed{} \cdot 10 + 4 \cdot 1$	70	69	46	68
Op 18a	Skriv svaret som desimaltall. 6 · 0,5 = ……………………………	47	50	34	46
Op 18c	Skriv svaret som desimaltall. 3 : 0,5 = ……………………………	18	24	11	22
Op 19a	Til venstre i rammen står et regnestykke. Sett ring rundt det tallet du mener er *nærmest svaret*. Du trenger ikke regne ut svaret. 13 : 4,32 | 0,03 0,3 3 30 300	51	64	50	61
Op 22a	Skriv riktig tall i ruta $14 : 2 = \boxed{} \cdot 14$	24	34	15	29

Op 23d	Skriv enklest mulig: $2y \cdot y^2$	1	9	0	9
Op 27b	$y = b^3$ Dersom $b = 4$ blir $y =$	3	16	1	14

Table 4.2.3.2.8: Solution frequencies for test items with results (test 2) for group A better than for group B : higher between 4,9 and 0,1 solution frequency points

4. The group B showed higher results in comparison with the group A on totally 19 of the test items on the first test. For 5 of those 19 test items the difference between the two groups is very minimal - between 0 and 2,2 solution frequency points.

- *Test items with results (test 2) for group B better than for group A: higher between 0,1 and 5 solution frequency points*

The group of 92 students showed higher results - between 0,1 and 5 solution frequency points for 10 test items: Op 12a, Op 12c, Op 18d, Op 19b, Op 20b, Op 24a, Op 25c, Op 26a, Op 27c, Op 31. This is group of tasks for which the students from group B showed results very close to the results for group A for both tests, with some small exceptions (task 12c and task 31). Those tasks were related to calculations with decimal numbers, estimation, comparison of decimal numbers, figure pattern and three algebraic tasks. For six of those tasks group A showed higher results on the first test. The most difficult for both groups of students were tasks related to decimal numbers Op 18d (calculate 0,6 : 0,2), Op 19b (estimation of 7,5 : 0,24), and the algebraic tasks Op 25c (Adder tallet *x* til 7), and Op 27c (3x = 7 og 5y = 11 Da blir 3x + 5y = ...).

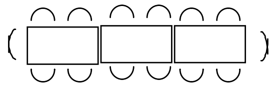
Oppgave		Frekvens i prosent			
		LCM 9. trinn 74 elever		LCM 9. trinn 92 elever	
		2004 høst	2005 vår	2005 høst	2006 vår
Op 12a	Sett *ring rundt* det største tallet og *kryss over* det minste tallet: 0,625 0,25 0,3753 0,125	62	72	66	72
Op 12c	Sett *ring rundt* det største tallet og *kryss over* det minste tallet: 4,09 4,7 4,008	89	85	78	88
Op 18d	Skriv svaret som desimaltall. 0,6 : 0,2 = ………………………………	5	7	5	11
Op 19b	Til venstre i rammen står et regnestykke. Sett ring rundt det tallet du mener er *nærmest svaret*. Du trenger ikke regne ut svaret. 7,5 : 0,24 | 0,03 0,3 3 30 300	18	23	15	24
Op 20b	*1 kg pølser koster 49,50 kr. Per kjøper 1,7 kg. Hvor mye koster det?* 49,50 · 1,7 49,50 : 1,7 1,7 : 49,50 1,7 · 49,50 49,50 – 1,7	65	61	65	65
Op 24a	$x + y + z = x + p + z$ ☐ er alltid sant ☐ er aldri sant ☐ *kan* være sant, nemlig når	11	11	10	11
Op 25c	Adder tallet *x* til 7	7	18	5	22
Op 26a	Et konfirmasjonsbord er satt sammen av småbord, og rundt er det satt stoler.  Hvor mange stoler blir det plass til om vi har 4 småbord?..............................	64	72	61	72
Op 27c	3x = 7 og 5y = 11 Da blir 3x + 5y = ...	14	28	11	33
Op 31	5 liter bær veier 10 kg. Hvor mye veier 6 liter bær? Vis hvordan du tenker.	42	55	54	58

Table 4.2.3.2.9: Solution frequencies for test items with results (test 2) for group B better than for group A: higher between 0,1 and 5 solution frequency points

- *Test items with results (test 2) for group B better than for group A: higher between 5,1 and 10 solution frequency points*

The group of 92 students showed higher results - between 5,1 and 10 solution frequency points for 5 test items: Op 12b, Op 24b, Op 25a, Op 25b, Op 27a. On those 5 test items the students from group B had minimal difference with group A: average result lower with 1 points (test 1) and 7 points (test 2). Those tasks were three simple algebraic problems and tasks related to comparison of numbers. The performance of both groups of students for the algebraic tasks was not high enough, but group B showed much higher improvement on those algebraic tasks in comparison to group A.

Oppgave		Frekvens i prosent			
		LCM 9. trinn 74 elever		LCM 9. trinn 92 elever	
		2004 høst	2005 vår	2005 høst	2006 vår
Op 12b	Sett *ring rundt* det største tallet og *kryss over* det minste tallet: 3,521 3,6 3,7	81	73	75	80
Op 24b	$2a + 3 = 2a - 3$ Dette ☐ er alltid sant ☐ er aldri sant ☐ *kan* være sant, nemlig når	24	42	35	49
Op 25a	Adder tallet x til $x + 3y$	5	16	3	22
Op 25b	Adder tallet x til 4x	9	19	3	28
Op 27a	$x = a + b - c$ Dersom $a = 1$, $b = 2$ og $c = 3$ blir x=.......................	14	31	11	37

Table 4.2.3.2.10: Solution frequencies for test items with results (test 2) for group B better than for group A: higher between 5,1 and 10 solution frequency points

- *Test items with results (test 2) for group B better than for group A: higher between 10,1 and 24 solution frequency points*

The group of 92 students showed higher results - between 10,1 and 24 solution frequency points for 4 test items: Op 23a, Op 23b, Op 23c, Op 26b. On those four test items the students from group B showed much better improvement of their results in comparison with the students from group A - although group B scored on average 5 points (test 1) lower than group A, the results for group B on the second test were already on average 17 points higher.

Oppgave		Frekvens i prosent			
		LCM 9. trinn 74 elever		LCM 9. trinn 92 elever	
		2004 høst	2005 vår	2005 høst	2006 vår
Op 23a	Skriv enklest mulig: 2x + 5x **..................................**	30	53	22	71
Op 23b	Skriv enklest mulig: $x + x + 2x$	24	36	16	60
Op 23c	Skriv enklest mulig: t · t · t	8	38	3	50
Op 26b	Et konfirmasjonsbord er satt sammen av småbord, og rundt er det satt stoler. Enn om vi har 7 småbord?	46	49	48	62

Table 4.2.3.2.11: Solution frequencies for test items with results (test 2) for group B better than for group A: higher between 10,1 and 24 solution frequency points

Test items divided in six groups, according to the change of the solution frequency

What was the increase of the number of students who solved each task included in the test when the students were tested for a second time? We can ask such question for the group of 74 students, tested in 2004 – 2005 and for the group of 92 students tested in 2005 – 2006. It is possible to do such analyses by categorising all test items, according to their change of the solution frequency – we need to relate such analyses to the tests' results for the group of 74 students and to the tests' results for the group of 92 students.

We can consider *change of the solution frequency for a test item* to be the difference between the solution frequency for the test item on the second test and the solution frequency for the same test item on the first test (for the same group of students). What was the change of the solution frequency for every test item for the first group of 74 students tested in 2004 – 2005 and how those results could be compared with the change of the solution frequency for every test item for the second group of 92 students tested in 2005 – 2006?

Type of groups
Every test item can be included in one of the following 6 groups, according to the change of the solution frequency:

- Group 1 – Includes test items with negative change in the results

Every test problem in Group 1 is a test problem, which was solved by fewer students when the students were tested for a second time compared with the number of students when the first test was performed.

- Group 2 - Includes test items with minimal change: change of 0 – 5 solution frequency points

Every test problem in Group 2 is a test problem, which was solved by 0% to 5% more students in the group when the students were tested for a second time. Only a few more students solved such tasks when they were tested in the spring.

- Group 3 - Problems with moderate change: change of 5,1 – 10 solution frequency points

Every test problem in Group 3 is a test problem, which was solved by between 5,1% to 10% more students in the group when the students were tested for a second time. More students solved such tasks when they were tested for a second time, but the change was not big.

- Group 4 - Problems with change of 10,1 – 15 solution frequency points

Every test problem in Group 4 is a test problem, which was solved by between 10,1% to 15% more students in the group when the students were tested for a second time. More students solved such tasks when they were tested for a second time, and the increase of the number of students who solved such tasks was above average.

- Group 5 - Problems with high change: change of 15,1 – 20 solution frequency points

Every test problem in Group 5 is a test problem, which was solved by between 15,1% to 20% more students in the group when the students were tested for a second time. Much more students solved such tasks when they were tested for a second time, and the increase of the number of students who solved such tasks was very good.

- Group 6 - Problems with the highest change: change higher than 20 solution frequency points

Every test problem in Group 6 is a test problem, which was solved by more than 20% of students in the group when the students were tested for a second time. Significant number of students solved such tasks when they were tested for a second time, and the increase of the number students who solved such tasks was maximal for the tested group of students.

Sorting the test items in six groups

It is interesting to look at the change of the solution frequency for every test item included in both tests performed at the beginning and the end of a school year by the same group of students. Using some sorting procedures with the available data for the group of 74 students or the group of 92 students we can find the following data:

- 74 students, Group 1: Problems with negative change in the results

The 74 students tested in 2004 – 2005 showed lower results for 8 of the test items: Op 12b, Op 12c, Op 14b, Op 16a, Op 17b, Op 17d, Op 20b, Op 21a.

- 92 students, Group 1: Problems with negative change in the results

The 92 students tested in 2005 – 2006 showed lower results for 7 of the test items: Op 2, Op 17b, Op 17c, Op 17d, Op 20a, Op 20d, Op 30.
Two of the test items were common – 17b and 17d.

- 74 students, Group 2: Problems with minimal change: change of 0 – 5 solution frequency points

The 74 students tested in 2004 – 2005 showed minimal change of 0 – 5 solution frequency points for the results of 10 test items: Op 6, Op 8a, Op 15b, Op 17a, Op 17c, Op 18a, Op 18b, Op 18d, Op 20a, Op 24a, Op 26b.

- 92 students, Group 2: Problems with minimal change: change of 0 – 5 solution frequency points

The 92 students tested next school year had minimal change for the results of 9 problems: Op 8a, Op 8b, Op 14b, Op 15a, Op 17a, Op 18b, Op 20b, Op 21a, Op 21b.
Two of the test items were common - 8a, 18b.

- 74 students, Group 3: Problems with moderate change: change of 5,1 – 10 solution frequency points

The 74 students tested in 2004 – 2005 showed moderate change of 5,1 – 10 solution frequency points for the results of 21 test items: Op 2, Op 3, Op 4, Op 5, Op 7, Op 10, Op 12a, Op 13, Op 15a, Op 16b, Op 18c, Op 19b, Op 19c, Op 20c, Op 22a, Op 22b, Op 23d, Op 25b, Op 26a, Op 26c.

- 92 students, Group 3: Problems with moderate change: change of 5,1 – 10 solution frequency points

The 92 students tested next school year had minimal change for the results of 14 test items: Op 7, Op 9, Op 11, Op 12a, Op 12b, Op 12c, Op 13, Op 14a, Op 15b, Op 16b, Op 18d, Op 19b, Op 20c, Op 23d.
Five test items were common - Op7, Op13, Op 16b, Op19b, Op 20c.

- 74 students, Group 4: Problems with change of 10,1 – 15 solution frequency points

The 74 students tested in 2004 – 2005 showed change of 10,1 – 15 solution frequency points for the results of 12 test items: Op 11, Op 14a, Op 19a, Op 20d, Op 21b, Op 23b, Op 25a, Op 25c, Op 27b, Op 27c, Op 30, Op 31.

- 92 students, Group 4: Problems with change of 10,1 – 15 solution frequency points

The 92 students tested in 2005 – 2006 showed change of 10,1 – 15 solution frequency points for the results of 12 test items: Op 4, Op 5, Op 6, Op 10, Op 18a, Op 18c, Op 19a, Op 22a, Op 24b, Op 26a, Op 26b, Op 27b.
Two of the test items were common: Op 27b, Op 19a.

- 74 students, Group 5: Problems with high change: change of 15,1 – 20 solution frequency points

The 74 students tested in 2004 – 2005 showed change of 15,1 – 20 solution frequency points for the results of 3 test items: Op 9, Op 24b, Op 27a.

- 92 students, Group 5: Problems with high change: change of 15,1 – 20 solution frequency points

The 92 students tested in 2005 – 2006 showed change of 15,1 – 20 solution frequency points for the results of 3 test items: Op 19c, Op 25c, Op 25a.

- 74 students, Group 6: Problems with the highest change: change higher than 20 solution frequency points

The 74 students tested in 2004 – 2005 showed change higher than 20 solution frequency points for the results of 2 test items: Op 23a, Op 23c.

- 92 students, Group 6: Problems with the highest change: change higher than 20 solution frequency points

The 92 students tested in 2005 – 2006 showed change higher than 20 solution frequency points for the results of 7 test items: Op 16a, Op 23a, Op 23b, Op 23c, Op 25b, Op 27a, Op 27c.
Two of the test items were common: Op 23a, Op 23c.

4.3 11th grade

4.3.1 Trends in the students' overall progress during the year

4.3.1.1 The results of the group

This section of the analyses focuses on the results of the group of 113 students who performed both tests. It is necessary to analyse the students' results on the tests and describe the development during the school year. For which test item the students showed better results on the second test? For which tasks the students made a significant progress? Which of the test items were difficult for the students? The following table presents the minimum, maximum, average and the standard deviation for the students' points on the tests.

Elevgruppe	Antall	Test år	Minimum	Maksimum	Gjennomsnitt	St.avvik
Elever - Test 1 & Test 2	113	2005 høst	4	42	26,2	7,3
		2006 vår	10	45	30,5	7,3

Table 4.3.1.1.1: Results in points for the group of students who did both tests

There has been a positive development in the results of most students. The average score for the group increased from 26,2 points (test 1) to 30,5 points (test 2).

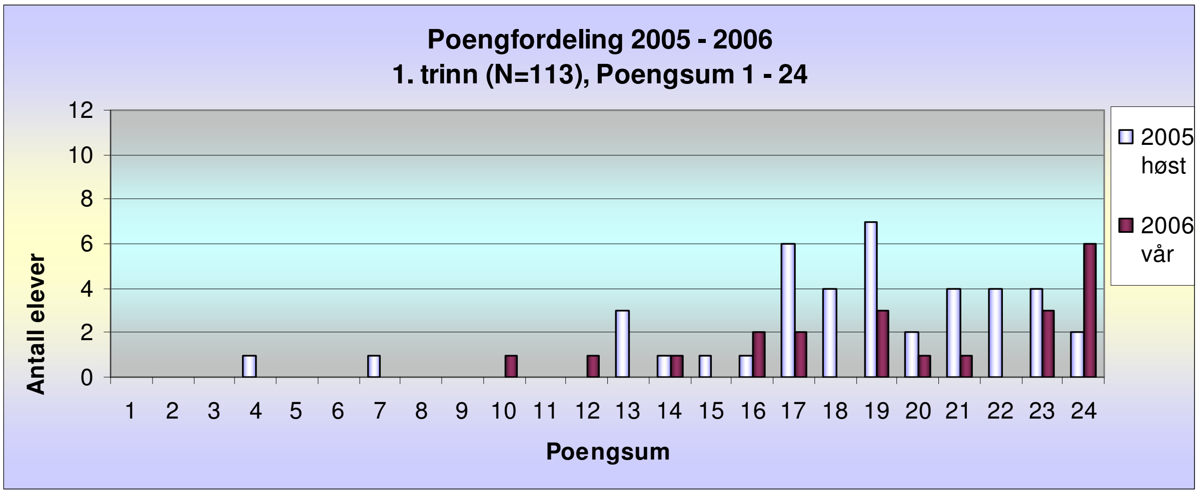


Figure 4.3.1.1.1: 11th grade (N=113) - The students' points sums – 1 to 24 points

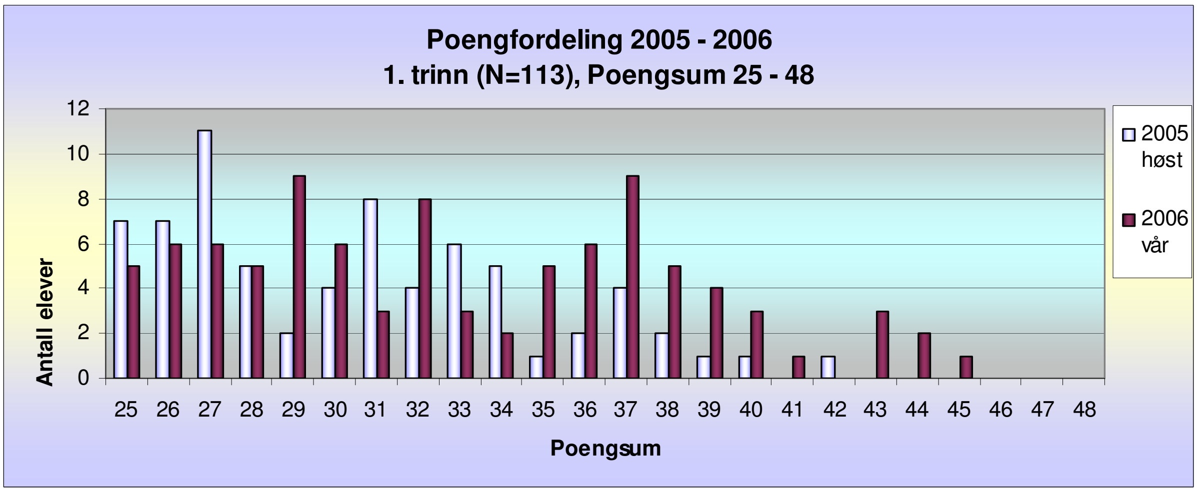


Figure 4.3.1.1.2: 11th grade (N=113) - the students' points sums – 25 to 48 points.

The maximum possible score on the test was 48 points and there were some very successful students who received scores close to this maximum. The average increase of the students' scores was 4,3 points. The students' scores varied from 4 to 42 points in 2005, and from 10 to 45 points. Some students increased their result on the second test with 1 or 2 points, but there were some students who got increased of their score with 26 points on the second test.

It is interesting to notice that: (i) 77% of the students increased their score on the second test; (ii) 23% of the students had lower score on the second test or did not change it, if we count also the students who got only one point increase in the score we get a big group of (iii) 31% of the students who didn't improve or improved with the minimal one point their result in the spring, (iv) 43% of the students improved their results for test 2 with more points than the average 4,3 points. Additional information concerning the distribution of the students' points sum is given in figures 4.3.1.3 and 4.3.1.4.

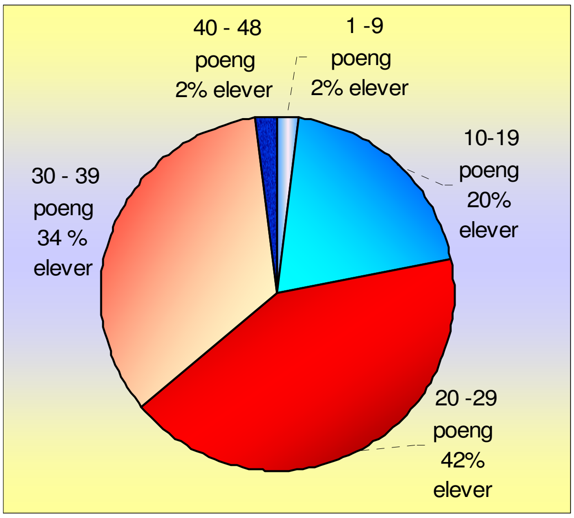


Figure 4.3.1.1.3: 11th grade (N=113) – test 1, 2005, Distribution of the students' points sums

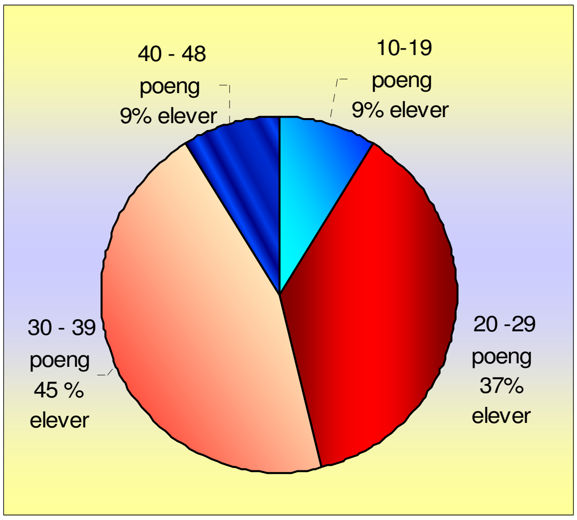


Figure 4.3.1.1.4: 11th grade (N=113) – test 2, 2006, Distribution of the students' points sums

Elevgruppe	År	1 - 9 poeng	10 - 19 poeng	20 - 29 poeng	30 - 39 poeng	40 - 48 poeng
Alle	2005	2	20	42	34	2
Alle	2006	0	9	37	45	9

Table 4.3.1.1.2: Distribution of the students' points sums in percents of students in five groups, 11th grade (N=113), 2005-2006.

There was a positive development – there was a big increase of the group of students with scores above 30 points and a big decrease of the group of students with minimal scores.

- 36% of the students (test 1) and 54% of the students (test 2) received scores above 30 points.
- The group of students with scores below 20 points was reduced from 22% of the students (test 1)to 9% (test 2).
- The group of students with scores between 30 and 39 points increased from 34% of the students (test 1) to 45% of the students (test 2).
- 9% of the students (test 2) received scores close to the maximum 48 points.

4.3.1.2 Solution of different tasks

In this section of the analyses we present a summary of the students' results on all test items - the data is presented as comparison of the solution frequency results for each test item on the tests. Further follows a close investigation of selected tasks. After that the analyses of the results focuses on four groups oftest items organised in the following groups:

(1) Test items with high results on the tests
(2) Test items with low results on the tests
(3) Test items with big increase of the results for test 2
(4) Test items with lower results for test 2 in comparison with test 1

Summary of the students' results on all test items

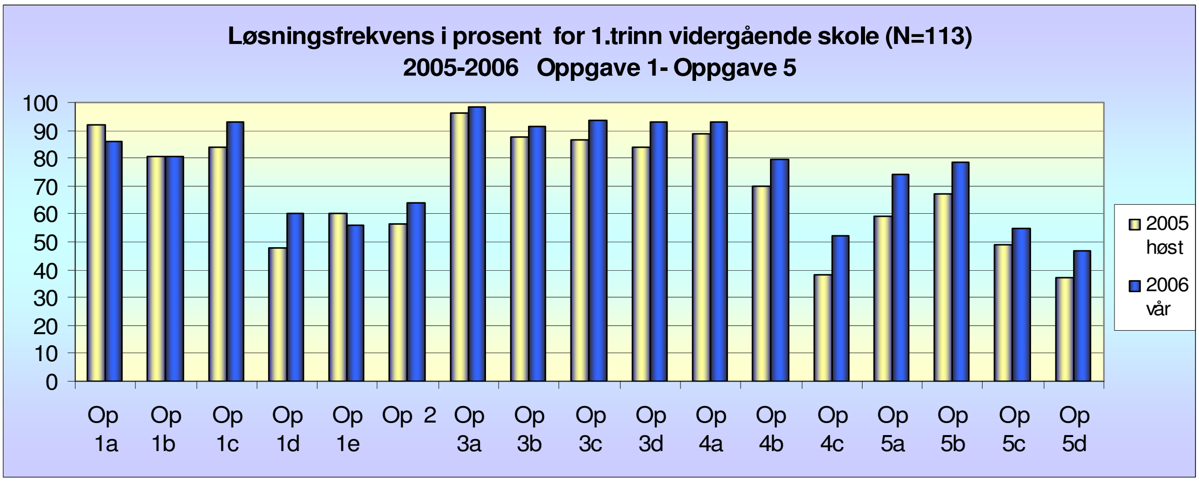


Figure 4.3.1.2.1: 11th grade, solution frequencies for test items 1a – 5d, 113 students (both tests - fall 2005 and spring 2006)

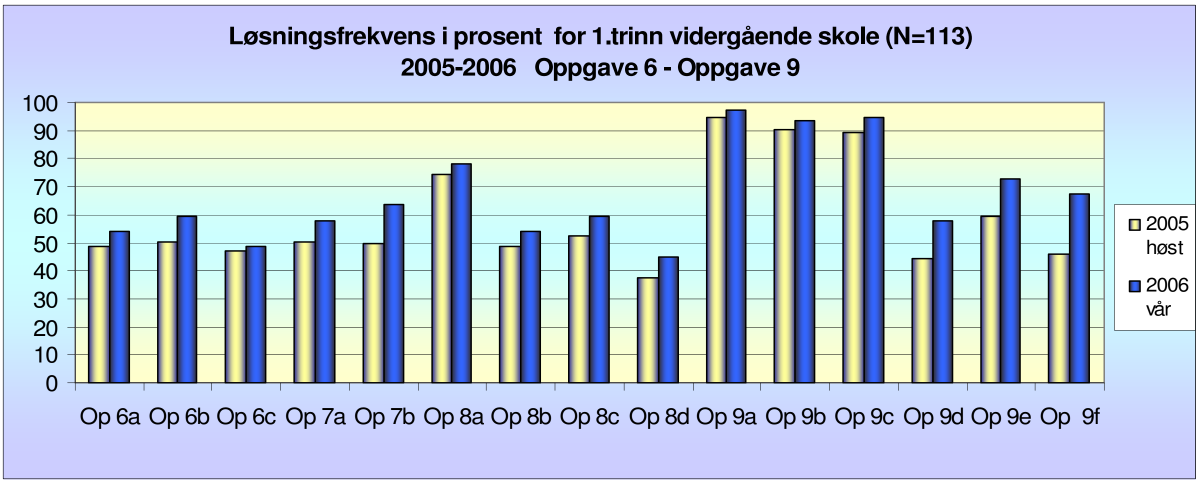


Figure 4.3.1.2.2: 11th grade, solution frequencies for test items 6a – 9f, 113 students (both tests - fall 2005 and spring 2006)

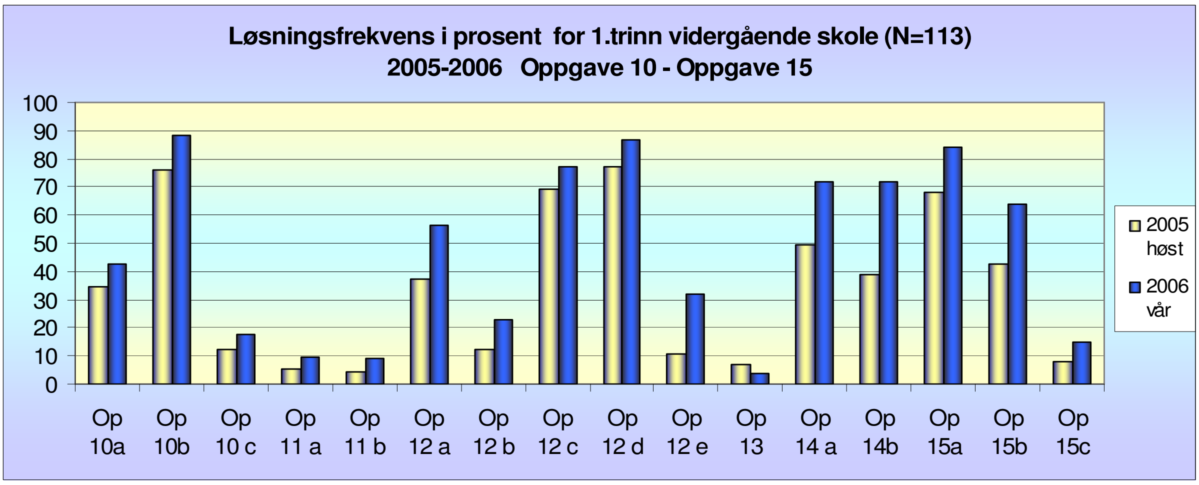


Figure 4.3.1.2.3: 11th grade, solution frequencies for test items 10a – 15c, 113 students (both tests - fall 2005 and spring 2006)

A closer study of selected tasks

To represent and analyse geometrical situations and structures using algebraic symbols

We would select some tasks to go deeper into the problem and describe the way students communicated their findings. The test items Op 11a and Op 11b serve multiple purposes. The tasks provide opportunities for students to build content knowledge, opportunity to apply problem solving strategies and to make connections among geometrical concepts and their algebraic representations.

A major goal of the curriculum is to prepare students with sufficient knowledge and tools that enable them to approach and solve problems more difficult than they have studied. It is expected that students are able to explain and justify their reasoning, and have sufficient experience in articulating their findings clearly enough. L97 emphasizes the importance the importance of problem solving and the view that mathematics is a sense-making discipline rather than a school subject in which a set of rules for working exercises are given by the teachers to be memorized.

Oppgave 11 For å finne volumet V av en sylinder, har vi denne formelen: $V = \pi r^2 h$.
Her er r radius i grunnflata , og h er høyden av sylinderen.

a Sett at vi har en sylinder vi kaller S, med et volum y. Sett at vi lager en ny sylinder vi kaller T, med dobbelt så stor radius og dobbelt så stor høyde som sylinderen S. Hva blir volumet av denne sylinderen T?

b Sett at S har radius r og høyde h. Vi vil nå lage en ny sylinder U. Den skal ha dobbelt så stor radius som S, men likevel ha samme volumet som S. Hvor stor må høyden i U da være?

Thetest items Op 11a, Op 11b describe geometrical context, but to find the answers the students need to use the language of algebra. Those tasks provide the opportunity students to develop experience in making mathematical connections across different content areas. The described problem is complex and challenging – students need to apply mathematical ideas from the area of geometry and support their thinking using as a tool algebraic representation.

Students need to have variety of abilities - to interpret the situation, represent it symbolically, analyse what is given, select appropriate methods of solution, manipulate the algebraic expressions and when they identify the solution they need to judge whether the result is plausible. It is essential to apply abilities for conceptual understanding, abilities to reason mathematically, representation abilities, abilities to manipulate symbolic statements, ability to express mathematical ideas in a written form.

Students need to learn how to write mathematically and that is an essential element in their process of learning in the classroom. David Pugalee (2005) was interested in an investigation of the relationship between language and mathematics learning and viewed writing as a powerful tool in developing students' mathematical literacy (defined by Kilpatrick et al. (2001)). He considered that writing can be "an amazing learning tool" for the learning of mathematics and it supports "deeper mathematical understanding". Writing is considered to be a language tool that helps to form meaning and build better understanding. The writing process has three phases – planning, composing and revising. Writing has different purposes and goals – students need to write for variety of purposes and goals, as for example to inform, describe, and persuade. Writing has different formats – the use of multiple formats is very important. Such formats are for example recounting, explaining, describing procedures and methods, arguing, reporting. Writing has different audiences – for example writing for oneself, for the teachers, for professionals, for informal audiences. Writing should be done often – students need multiple opportunities to write mathematically. He related to previous research on problem solving and reported the findings that students who wrote descriptions of their problem-solving processes were significantly more successful in comparison to students who used strategy think aloud while solving the problems. Combination of both written and oral communication in mathematics has "tremendous untapped potential".

Booth (1988) argued that many students experience problems developing meaning for variable. The previous knowledge students constructed when dealing with arithmetical tasks can interfere in the process of solution of algebraic tasks and that is why it is hard for some students to read such statement as relationships between variables. For example the statement "$a = l$ x w" is used in arithmetic to denote the verbal statement "area = length x width". When students solve tasks related to finding area of a figure in arithmetic, they normally need to apply such formulas in order to find concrete numerical answer. The analyses of Booth (1988) revealed that in algebra many students experience difficulties to operate with formulas, because students have problems to read such statement as relationships between variables.

Oppgave 11a Sett at vi har en sylinder vi kaller *S*, med et volum *y*. Sett at vi lager en ny sylinder vi kaller *T*, med dobbelt så stor radius og dobbelt så stor høyde som sylinderen *S*. Hva blir volumet av denne sylinderen *T*?

At first the students need to investigate the task carefully and consider what is given in the task and look for a proper strategy to find it. It is necessary to provide algebraic representation of the volume of the new cylinder T and manipulate the algebraic formula in order to evaluate the result and find a relation with the volume of the cylinder S. Students are asked to find the volume of the cylinder T – the problem states that it is given that a cylinder called S has volume *y*, and a cylinder called T has double radius and double height compared with the cylinder S. Mason & Spence (1999) noted that important factors for successful learning is the use of some process as specialising, generalising, working backwards that activates the thinking process.

Recording of statements in algebra needs exact precision and that is a main problem for the students. They need to use their abilities to construct correct symbolic expressions and ability to manipulate the expressions in a systematic way. The activity of solving the task is too long and too complex to be solved mentally. The intention is that students produce their own schematic notations to support and explain their reasoning. This is a geometry problem that requires reasoning about varying and unknown quantities.

Table 4.3.1.2.1: 11th grade, Task 11a - solution frequency for Test 1 (2005 fall) and Test 2 (2006 spring)

Oppgave 11a		Frekvens i prosent	
		2005 høst	2006 vår
Ikke besvart		24	18
Kategori 1	**"V=8y"; "T=8y"**	**1**	**2**
Kategori 2	**”8 ganger så stort som volumet av sylinderen S”; "Volumet blir 8 ganger så stort"**		**2**
Kategori 3	**"V=PI(2r) ^{2}.2h" ; " T=Pi.4r^{2}.2h" ; "T=2PI.2r^{2}.2h"**	**3**	**6**
Kategori 4	**"Volumet av sylinderen T = volumet av sylinderen S.2^{3}"; "Volumet til S.2^{3}"**	**2**	
Kategori 5	eks. "Pi.2r^{2}.2h"; "PI.r^{2}.2.h.2"; "4y"; "4 ganger så stor"	17	22
Kategori 6	eks. "V= PI.r^{2}.h.2" ;"2y"; "Volumet blir det dobbelte";	19	19
Kategori 7	To forskjellige varianter i et svar: (eks. "T=2y T: V=PI. 2r^{2}.2h")	7	6
Kategori 8	eks. "y^{2}" ; "2y^{2}"; "S/T=y/y^{2}"; "V=PI.r^{4}.h^{2}"; V(T)= (PI.r^{2}.h) ^{2}	7	15
Kategori 9	Andre svar	20	10

First of all the students need to have proper understanding of the concept volume of a cylinder and what does it mean to find the volume of a cylinder described in more complex settings. It is essential to determine a correct formula for the cylinder T. This activity was a major problem for most of the students who tried to solve this task.

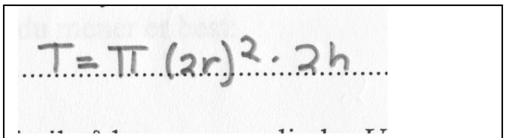


Figure 4.3.1.2.4: Student’s answer in Category 3

In the table 4.3.1.2.1 is given information about the type of answers for the volume of T, given by the students. The students’ answers show that to construct a correct formula for the volume of T was not easy.

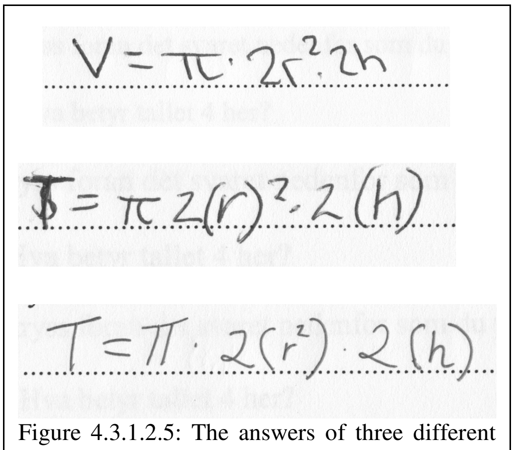


Figure 4.3.1.2.5: The answers of three different students in Category 5

A lot of the students had problems set the formula of the volume correctly. For example 17% of the students (test 1) and 22% of the students (test 2) gave an answer in Category 5 ("Pi.$2r^2.2h$"; "Pi.$r^2.2.h.2$"; "4*y*"; "4 ganger så stor"). Such answer can be attributed to limited knowledge how to set up a formula, using brackets for the elements of the formula which are in second order.

Other examples can be given with the answers in *Category 6* (eks. "*V*= Pi.$r^2.h.2$" ;"2y"; "Volumet blir det dobbelte"), given by 19% of the students (test 1) and (test 2). Such students had possibly problems to understand the problem situation and to use in their notations 2*r* in the formula – it is possible that those students could not express symbolically “double radius” in a formula for a volume.

Some students experienced problems to deal with the task, because they did not have proper understanding of the role of the variables to represent varying and unknown quantities. Such students gave a numerical answer. One of the students made the note: ”Hva er målene til sylinder S???”.

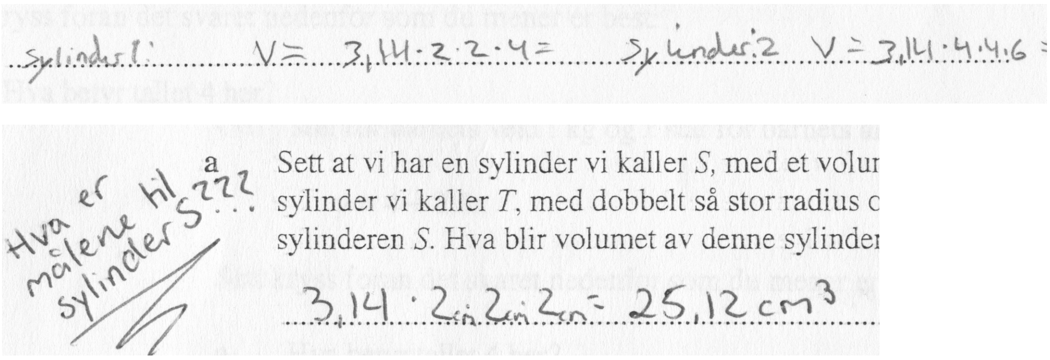


Figure 4.3.1.2.6: The answers of two students - numerical answers, Category 9

The students were not asked to give explanations of their results. Some of the students used the free space on the back of their tests to make notes. The example given below presents a sequence of notes made by a student who found the answer to the task by specialising.

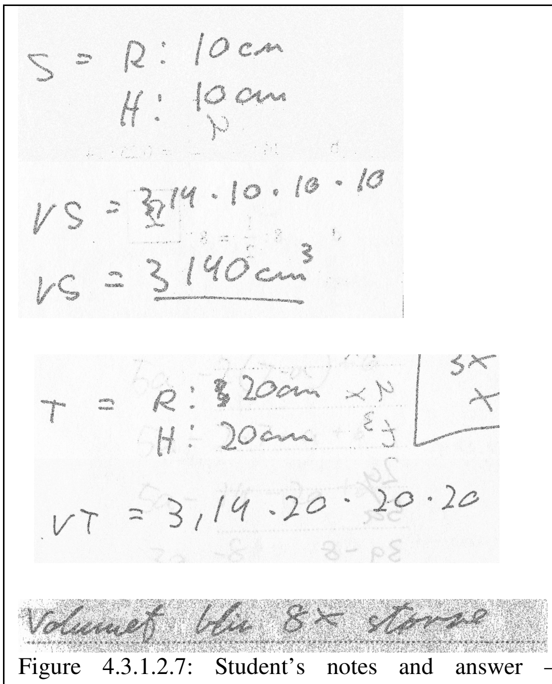


Figure 4.3.1.2.7: Student’s notes and answer – specialising strategy of solution, Category 2

The student used concrete numbers for *r* and *h* and found the volumes of the sylinders S and T. Comparing the results the student noticed the relation between the two numbers and concluded that the “Volumet blir 8 x større.” This student received a point for this answer, although it is not clear whether the student was able to solve the task in the general case.

Informal strategies as guess and check can be used, but the students are supposed to apply formal algebraic thinking that enables them to find the solution. Students demonstrated in other tasks that they are able to perform some simple substitution in algebraic formulas or find the value of an expression, but they experienced major problems in solving an investigative task. They can deal with concrete numbers, but had problems to apply correctly the algebraic language.

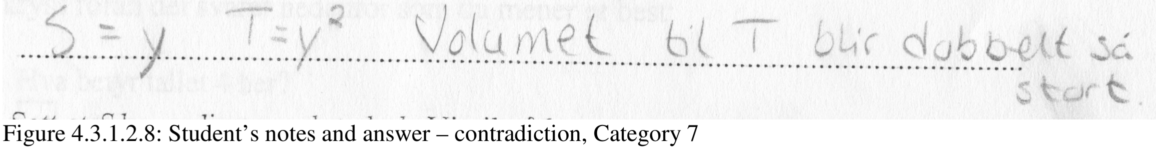


Figure 4.3.1.2.8: Student’s notes and answer – contradiction, Category 7

Students were not able to express properly what they found to be the answer – one student for example stated both that T= y^2 and "Volumet til T blir dobbelt så stort" , and in addition there were a few similar cases with contradictions in the students' notations. Those students did not have enough skills in using properly algebraic notation or were not able to monitor their writing. Such problems created major obstacles for the students to explain properly what answers they found.

Oppgave 11b For å finne volumet V av en sylinder, har vi denne formelen: $V = \pi r^2 h$.
Her er r radius i grunnflata , og h er høyden av sylinderen.

Sett at S har radius r og høyde h. Vi vil nå lage en ny sylinder U. Den skal ha dobbelt så stor radius som S, men likevel ha samme volumet som S. Hvor stor må høyden i U da være?

The context of task 11b is related to the context of the previous task. It is given again that the cylinder S has radius r and a height h, an in addition is described a new cylinder U, with double radius compared with cylinder S and volume, the same as the volume of the cylinder S. The students need to apply formal algebraic reasoning and find the height of the new cylinder U in relation with the height of S. The information given in the problem statement how the variables are related, enables the students to find the solution of the problem.

Table 4.3.1.2.2: 11th grade, Task 11b - solution frequency for Test 1 (2005 fall) and Test 2 (2006 spring)

Oppgave 11b		Frekvens i prosent	
		2005 høst	2006 vår
Ikke besvart		44	28
Kategori 1	**"1/4h" ; "0,25h" ; " fire ganger så liten" ; "4 ganger mindre enn høyden i S" ; "1/4 av høyden i S" osv.**	**1**	**8**
Kategori 2	**"H=H/4" ; "1/4 av S" ; "U=h.1/4"**	**3**	**1**
Kategori 3	**"u= Pi. (2r) ^{2}.h/4"**	**1**	
Kategori 4	"U=2r^2.(1/2)h.PI" ; " V=PI.2r^2. 0,5h" ;	4	11
Kategori 5	"1/2h" ; "0,5h" ; "halvparten av h i S" ; "høyden i U=1/2høyden i S"	28	33
Kategori 6	" Dobbelt så høy som S"; "hU=hS.2" ; "2.h" ; "Pi.r^2.2.h"	3	3
Kategori 7	Andre svar	17	17

The correct answers were given by only 5% of the students (test 1) and 9% of the students (test 2). We do not have concrete information for the strategies used by the students - the students are not asked to explain how they found the answer.

Some students were able to find the answer, but were not consistent enough when symbolizing the answers. Such answers were in category 2- those students gave the following answers for the height of U: "H=H/4" ; "1/4 av S" ; "U=h.1/4". Those answers were given a point, but the notation used by the students had certain limitations. It is necessary to use different names for the name of a cylinder and for its volume, but for example some students used the name of the cylinders U and S when writing a formula for their volumes. One student provided an answer in Category 3 ("u= Pi. (2r) ^{2}.h/4") and did not give explicitly the height of U. The student used incorrectly the name of the cylinder U when writing a formula for its volume.

Some students used specialising as a strategy to find the height of U. It is not clear how many students did so, because they were not asked to provide their strategy of solution.

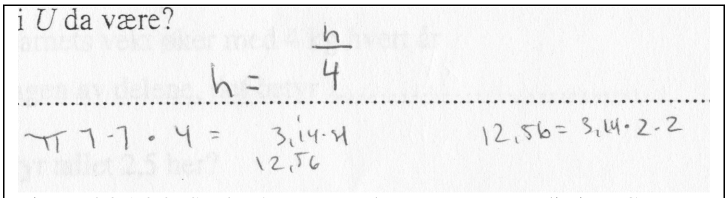


Figure 4.3.1.2.9: Student's notes and answer – contradiction, Category 2

It is visible from the notes made by some students that they operated with numerical values for the radius and the height of S. They used concrete values for the unknowns, substituted those values in the formulas for the volumes of both cylinders and found the values for the volumes of the two cylinders. Comparing the two numbers, they were able to find the height of U.

An example of specialising is given in the figure; this student received a point for finding the answer, although the answer was not found by using manipulation of the formulas of the two cylinders. So those students were able to state a correct answer for the height of the cylinder U, but did not operate with variables. The students specialised with concrete numbers and made a conjecture for the height of the cylinder U, but were not able to apply appropriate algebraic techniques and solve the task in the general case.

Many students gave an answer in Category 5: "1/2*h*" ; "0,5*h*" ; "halvparten av *h* i S" ; "høyden i U=1/2 høyden i S" or Category 4: "U=2r^2.(1/2)*h*.PI" ; " V=PI.2r^2. 0,5*h*".

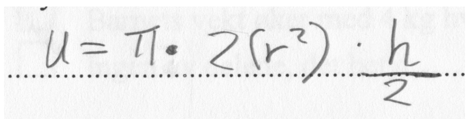


Figure 4.3.1.2.10: Student's answer – Category 4

The students provided a formula for the volume of U, but failed to apply it correctly. They experienced problems to symbolize correctly – for example they represented the formula of the cylinder using 2(r^2) or 2r^2 instead of $(2r)^2$. Such type of symbolizing leaded to the incorrect conclusion that the height of the new cylinder is *h*/2. It can be noticed also that some students used syncopated notation as a student who gave the respond "høyden i U=1/2 høyden i S". It was common also to use the name of the cylinders U in the formula for its volume. Those difficulties can be attributed to the limited experience in solving geometrical problems that some of the students might have.

Comparison of the results with the previous year

The comparison of the results indicated that the group of 113 students experienced similar problems compared with the group of 206 students tested in the previous year. The results for 2005 spring show that the majority of 218 students who solved the same tasks, tried to solve the tasks, but gave incorrect answers – 151 students (69%)gave incorrect answer on task 11a and 114 students (52%)gave a wrong answer for 11b. Only a very small number students answered to the question correctly - 4 students (2%) found the answer for problem Op 11a and 8 students (4%) found the answer for problem Op 11b (Espeland, 2006). The results for 2004 fall are similar, with little increase of the correct answers. The tested group of students was 236 students – 198 students (84%) gave incorrect answers on task 11a and 150 students (64%) gave a wrong answer for 11b. The students who solved the tasks were 9 students (4%) for Op 11a and 18 students (8%) for Op 11b (Andreassen, 2005).

Students' difficulties

The students' results on both tests show an increase in the results, but still the solution frequencies on the tasks were very low. The students' difficulties can be related to the level of complexity of the tasks and the limited experience students had in finding algebraic representation of geometrical concepts. Op11a and Op 11b are complex tasks. To examine them the students need careful investigation. It is necessary students to apply sophisticated methods of reasoning. The students should find correct algebraic formula for volume of a cylinder, they need to use effectively their problem solving abilities and determine the right procedures that would help them to find the solution. Since the students were not asked to provide a method of solution, we can only analyse the answers they gave and have limited source of information.

The analyses of the results reveal that a major difficulty for the students was to find the right algebraic representation of the geometrical concepts. Most students tried to construct a formula for the volume of the new cylinders, but experienced problems in doing it correctly. Some students forgot to use brackets for the terms in second order and provided a different formula. Other students experienced more problems and were not able to represent the given information in correct algebraic form. They demonstrated combination of limited abilities to communicate their results and limited conceptual understanding in geometry settings, which leaded to problems to analyse properly the geometrical situations and to use algebraic symbols.

Test items with high results on the tests

Next we examine the development of the students' results for thetest items solved by at least 70% of the students on one of the tests. The analyses of the results showed that 22 of the tests items were solved by more than 70% of the students. This is a big group of tasks, so we would like to provide first more details about the items solved by at least 90% of the students (test 2)and by at least 70% - 89% of the students (test 2).

Problems solved by at least 90% of the students (test 2)

For some of the items the results on both tests were very high. Such items were solved by about 90% of the students or more on the second tests. The analyses of the results shows that the test items with the highest solution frequencies for the second tests are Op 1c, Op 3a -d, Op 4a, Op 9a-c. Almost all students solved those tasks - on average 89% of the students (test 1) and 94% of the students (test 2).

Oppgave **9** Skriv enklere dersom det er mulig:

a $2x + 5x$ ....

b $x + x + 2x$ ....

c $t \cdot t \cdot t$ ....

Task 9 consists of 6 different algebraic expressions that need to be simplified. Almost all students were able to simplify the expressions in the three items 9a, 9b, and 9c. Students need to be fluent when they have to do symbol manipulations, because such skills are critical when they need to represent and analyse mathematical problems using algebraic symbols. The other test items 9d, 9e, 9f involve more complex algebraic expressions and many students experienced problems to find a proper way to simplify them.

Table 4.3.1.2.3: Task 9a - solution frequency

Oppgave 9a	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	2	1
7x	**95**	**97**
2x+5x	1	
Andre svar	3	2

Table 4.3.1.2.4: Task 9b - solution frequency

Oppgave 9b	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	3	
4x	**90**	**94**
2x+2x	1	
Andre svar	7	6

Table 4.3.1.2.5: Task 9c - solution frequency

Oppgave 9c	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	2	
t^3	**89**	**95**
3t	7	3
Andre svar	2	2

Next are presented the tasks 3a – 3d. They are simple tasks aiming to assess the students' abilities to find the values of simple algebraic expressions. Those tasks require computational fluency and abilities to use the variable through representing quantities in variety of situations.

3 Fyll ut hele tabellen:

x	$4x$	$\frac{x}{2}$	x^2
2	8	1	4
5			
		12	
			16
	2		

In task 3a the value of the variable x is given. In tasks 3b, 3c and 3d it is known the value of one of the expressions and the students need to find the value of the variable in order to find the value of the other algebraic expressions. Previous experience to solve problems in which the students use tables to represent and examine functions and pattern of change can be very helpful. The students are given one point for a task if they give correctly at least one of the three answers.

Students' solutions of tasks 3a – 3d:

- 25% of the students (test 1) and 31% of the students (test 2) gave 12 correct answers (no mistakes) for the four tasks 3a – 3d
- 21% of the students (test 1) and 30% of the students (test 2) gave 11 answers correct and one incorrect answer for the four tasks 3a – 3d
- 25% of the students (test 1) and 17% of the students (test 2) gave 10 correct answers and two incorrect answers for the four tasks 3a – 3d
- 5% of the students (test 1) and 7% of the students (test 2) gave 9 correct answers and three incorrect answers for the four tasks 3a – 3d

The following tables provide detailed information on the tasks 3a and 3b:

Table 4.3.1.2.6: Task 3a - solution frequency

Oppgave 3a	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	4	2
Riktig tall i alle rutene	**81**	**87**
Feil i en av rutene	**13**	**9**
Feil i to av rutene	**3**	**3**
Feil i tre av rutene	0	0

Table 4.3.1.2.7: Task 3b - solution frequency

Oppgave 3b	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	6	5
Riktig tall i alle rutene	**41**	**45**
Feil i en av rutene	**41**	**41**
Feil i to av rutene	**6**	**5**
Feil i tre av rutene	8	5

The tables show that most students did not have problems with the first problem 3a - 81% of the students (test 1) and 87% of the students (test 2) found all of the three values of the algebraic expressions. 16% of the students (test 1) and 12% of the students (test 2) experienced problems to find one or two of the values – the most problems students experienced with finding the value of x^2. In task 3b the value of the expression $x/2$ is given. Less than half of the students - 41% of the students (test 1) and 45% of the students (test 2) found all of the three answers in this task. 41% of the students (test 1 and test 2) provided two of the answers correctly, 6% of the students (test 1) and 5% of the students (test 2) found one of the answers. The most problematic for students was to find the value of x^2, especially because the value of x is 24 and many students experienced problems to find the square of this number.

Table 4.3.1.2.8: Task 3c - solution frequency

Oppgave 3c	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	8	3
Riktig tall i alle rutene	**80**	**89**
Feil i en av rutene	**5**	**3**
Feil i to av rutene	**2**	**2**
Feil i tre av rutene	7	4

Table 4.3.1.2.9: Task 3d - solution frequency

Oppgave 3d	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	14	7
Riktig tall i alle rutene	**49**	**54**
Feil i en av rutene	**27**	**35**
Feil i to av rutene	**8**	**4**
Feil i tre av rutene	4	1

Task 3c is a problem where the value of the expression x^2 is given. Most students could find all the three answers in the task.

Task 3d was a more difficult task to solve, because the value of the variable x is not a whole number. Most students were able to find the value of x and the value of $x/2$. About half of all students - 49% of the students (test 1) and 54% of the students (test 2) were able to find all three numbers. The problems were mostly connected with finding the value of x^2, when having as a value for x the decimal number 0,5 or the fraction 1/2 - 12 students (test 1) and 10 students (test 2) gave as an answer 1; other students simply left this field empty .

Problems solved by at least 70% - 89% of the students (test 2)

Another group of test items with very high solution frequencies were 13 test items Op 1a, Op 1b, Op 4b, Op 5a, Op 5b, Op 8a, Op 9e, Op 10b, Op 12c, Op 12 d, Op 14a, Op14b, Op 15a. Those tasks were solved on average by 68% of the students (test 1) and 79% of the students (test 2).

Oppgave 1 a $\frac{1}{2}+\frac{1}{4}=$

Table 4.3.1.2.10: 9th grade and 11th grade, Task - solution frequency for Test 1 (2005 fall) and Test 2 (2006 spring)

Oppgave 1a	Frekvens i prosent		Frekvens i prosent	
	LCM 9.trinn	LCM 9.trinn	LCM 1.vgs	LCM 1.vgs
	2005 høst	2006 vår	2005 høst	2006 vår
Ikke besvart	14	15	1	1
3/4	**37**	**41**	**83**	**81**
6/8; 9/12; 2/16	**4**	**14**	**9**	**4**
1/2; 2/4; 4/8; 4/6	12	10	4	2
Andre svar	33	20	3	12

This task is a simple task for finding the sum of two fractions. The table 4.3.1.2.10 shows the results for the 11th grade students in comparison with the 92 students in 9th grade participating to both tests. The majority of the 11th grade students could do the operation correctly (92% of the students (test 1) and 85% of the students (test 2)). There is some decline in the number of 11th grade students who solved the task in the spring.

Oppgave 1b $\frac{1}{2}-\frac{1}{3}=$

Table 4.3.1.2.11: 9th grade and 11th grade, Task 1b - solution frequency for Test 1 (2005 fall) and Test 2 (2006 spring)

Oppgave 1b	Frekvens i prosent		Frekvens i prosent	
	LCM 9.trinn	LCM 9. trinn	LCM 1.vgs	LCM 1.vgs
	2005 høst	2006 vår	2005 høst	2006 vår
Ikke besvart	47	33	2	4
1/6	**25**	**38**	**81**	**80**
1000/6000				**1**
5/6	3	8	11	4
1; 1/1; 6/6	9	4	2	2
2/3; 4/6	4	1	1	
Andre svar	12	16	5	10

This task is another simple task for finding the difference of two fractions. The table shows the results for the 11th grade students in comparison with the 92 students in 9th grade students participating to both tests.

The results for the 11th grade students show that most students were able to find the answer - 80% of the students (test 1) and 81% of the students (test 2). The most common error was to give as an answer 5/6 which is the sum of the two fractions.

Oppgave 10 b $a + b\cdot 2 = 2b + a$ Dette

☐ er alltid sant ☐ er aldri sant ☐ *kan* være sant, nemlig når

Table 4.3.1.2.12: 11th grade, Task 10b - solution frequency for Test 1 (2005 fall) and Test 2 (2006 spring)

Oppgave 10b	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	3	1
alltid sant	**76**	**88**
aldri sant	18	8
kan være sant	3	3
Andre svar	1	

The students need to compare the two algebraic expressions in the left and the right sides of the equal sign and notice that they are equal, because of the commutative law for addition and multiplication of terms. Most students recognised that the given expression is always true – 76% of the students (test 1) and 88% (test 2). The group of students who said that it is never true was reduced from 18% of the students (test 1) to 8% of the students (test 2).

The tasks Op 14a and Op14b are presented in the other section of the analyses -group of test items with big increase of the results for test 2. The solution frequencies on the other tasks in the group *Problems solved by at least 70% - 89% of the students (test 2)* are presented in the following table.

Oppgave		Frekvens i prosent	
		2005 høst	2006 vår
Op 4b	Sett ring rundt det tallet som ligger nærmest 2,08 i størrelse 209 2,9 2,05 2,1 20,9	70	80
Op 5b	Skriv som desimaltall $\frac{46}{100}$	67	79
Op 8a	$14 : 2 = \square \cdot 14$	74	78
Op 12 c	$3x = 7$ og $5y = 11$ Hva er da $3x + 5y$?	69	77
Op 12 d	Finn tallet x om $x = a + b - c$ og $a = 1, b = 2$ og $c = 3$	77	87
Op 15a	Even tenker på to tall. Summen av dem er 19. Differensen mellom dem er 5. Finn tallene.	68	84

Table 4.3.1.2.13: Group of tasks

Test items with minimal results on the tests

In this group are the test items Op 11 a, Op 11 b, Op 13. Those tasks were solved by less than 10% of the students on the tests. Tasks Op 11 a, Op 11 b are very complex tasks that require analytical and representational abilities. Those tasks were presented in details in the first part of the analyses. The students need to identify the essential relationships in the described situations and use symbolic algebra to represent and explain the mathematical relationships. Finding the relationships is easier if the students are given to work with concrete numbers. Those tasks are cognitively more demanding, because the students need to work in the general case. Another difficulty is to know how to represent the situation described rhetorically using algebraic symbols. Previous experience with similar type tasks can give a proper base for exploration of the tasks.

13 På en skole er det 10 elever for hver lærer. Hvilke av disse uttrykkene forteller dette?
L = antall lærere, E = antall elever. Sett ring rundt det eller de riktige uttrykk.

$10L = E$ $10E = L$ $L = 10E$ $E = 10L$ $10L + E$ $11LE$

This task was among the most difficult tasks to solve. Only 7% (test 1) and 4% (test 2) of the students managed to answer the question. The majority of students tried to find the answer, but failed to recognise the right expressions: 86% of the students (test 1) and 93% of the students (test 2) were not able to find the answer.

The generation of appropriate algebraic equations can be viewed as translation from one symbol system - natural language to another - algebra. Herscovics (1989) used this perspective to analyse the cognitive obstacles students experienced with such type of task. The possible cognitive obstacles are related to the interference of the natural language, poor understanding of mathematical variable and the notion of proportionality and were described by Herscovics. The problem that he analysed was very similar - it was related to the expression $6S=P$.

Table 4.3.1.2.14: 11th grade, Task 13 - solution frequency for Test 1 (2005 fall) and Test 2 (2006 spring)

Oppgave 13	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	7	4
$10L = E$ *og* $E = 10L$	**2**	**2**
$10L = E$ *eller* $E = 10L$	**5**	**2**
$10E = L$ og $L = 10E$	44	66
$10E = L$	21	9
$L = 10E$	16	10
$10E = L$ og $L = 10E$ og $11LE$	2	6
Andre svar	3	2

Herscovics considered that beginner students have limited knowledge how to use the new symbol system and its specific semantic structure. The cognitive problems involved into this translation process persist with time and even scientific oriented college students experience those problems. Herscovics presented results of three earlier studies. Tests given to groups of engineering students contained the problem "Write an equation using the variables S and P to represent the following statement: "There are six times as many students as professors at this university." Use S for the number of students and P for the number of professors."(p.64). In addition three other studies used similar problems to this one. In all cases the majority of wrong answers were the reversals: $6S = P$ instead of $S = 6P$.

The interference of the natural language

It was interesting to investigate the types of thinking that could lead to such mistakes (Herscovics, 1989). Analyses of videotaped interviews of individual students found two basic sources for mistakes: a *syntactic type* of thinking and a *semantic type*. Students reply with a *syntactic translation* when they assume that the sequence of words maps directly into a corresponding sequence of literal symbols in an algebraic equation. Students respond with a *semantic translation* when they link the equation to the meaning of the given problem. Students made some drawings and showed that they were aware of the relative sizes of the two sets, but nevertheless gave the wrong answer $6S = P$.

Poor understanding of mathematical variable

Herscovics reported that additional misunderstanding were found by Clement in1982- the letter S was not perceived as a variable which represented the number of students, the equal sign in the reverse equation was not perceived to express equivalence, but was used to express comparison or association.

The notion of proportionality

The incorrect answers were also associated to be connected with the fact that the solutions of such type of problems involve proportional reasoning. For example some students constructed used terms in algebraic statements as $6S + P$. It was found that various levels of proportional reasoning competence could also explain the students' problems.

Test items with big increase of the results for test 2

Increase between 15 and 19 solution frequency points

In this group are the test items Op 5a, Op 12a, Op 15 a, for which the results in the spring were much better. Those tasks were solved additionally by between 15% and 19% of all students. The task 5a requires students to write $\frac{3}{10}$ as a decimal number- this task was solved by 59% of the students (test 1) and 74% of the students (test 2). Task 15a is related to finding the values of two numbers with given sum 19 and difference 5 and was solved by 68% of the students (test 1) and 84% of the students (test 2).

12 Hva må x være dersom

a $123 + 2x = 195 - x$ ....

Solving equations with one unknown is a traditional algebraic topic. This test item requires performing routine procedures in order to find the solution of a given linear equation with one unknown. A possible reason for the students' difficulties could be that the solution of this equation required several steps. Most of the incorrect solutions are spread along a big variety of numbers that the students gave as a solution. It's difficult to classify the students' mistakes only looking at the incorrect responses and not having the method of the students' solutions provided.

Research suggests the need for structural understanding of algebra – students need to develop abilities to operate with algebraic expressions as entities or objects (Sfard, 1991). It is the treatment of algebraic expressions as objects and according to structural rules that causes the main difficulty. There are variety of strategies used by children to solve an equation as for example use of counting techniques, undoing or working backwards, trial and error substitution, transforming and performing the same operation on both sides (Kieran, 1992). Chaiklin (1989) referred to studies of the process of solving equation and pointed out that the equation solving involves the ability to apply both strategic and procedural knowledge. The strategic knowledge involves setting goals which procedures to execute. Chaiklin referred to studies which systematically investigated the students' errors and found that students' mistakes are mostly systematic and reflect the students' beliefs what to be done. The lower ability students needed longer acquisition time to learn to solve equations compared with higher-ability students.

Table 4.3.1.2.15: 11th grade, Task 12a - solution frequency for Test 1 (2005 fall) and Test 2 (2006 spring)

Oppgave 12a	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	29	20
24	**35**	**52**
72/3	**4**	**4**
72	6	2
Andre svar	27	22

The comparison of the test results show that 38% of the students (test 1) could solve this equation compared to 56% of the students tested in the spring. Many students tested in the fall of 2005 found this item as difficult – 29% of the students did not attempt to solve the equation, 33% of the students gave an incorrect answer. Those groups of students were smaller in the spring of 2006 – 20% of the students did not try to solve the task and 24% of the students experienced difficulties to find the solution.

Increase above 20 solution frequency points

In this group are the test items Op 9f, Op 12 e, Op 14 a, Op 14b, Op 15b
for which the results on the second test in the spring were much better. Each test problem in this group is a test problem, which was solved by at least 22 more students in the spring. Task Op 9f is related to simplification of the algebraic expression $2y \cdot y^2$ and was solved by 46% of the students (test 1) and 67% of the students (test 2). The task 15b asks students to explain

their strategy of solution to the problem “Even tenker på to tall. Summen av dem er 19. Differensen mellom dem er 5. Finn tallene.” 42% of the students (test 1) and 64% of the students (test 2) demonstrated abilities to explain and justify their solution methods. Many students provided an algebraic solution to the problem. They used two variables or one variable and constructed a system of equations or one equation and showed an algebraic method of solution. Some of the students explained they used the strategy “guess and check” and they also received a point for this method of solution.

Oppgave 14 a Vekta av et barn avhenger av alderen. For et barn fra 2 til 10 år kan vi bruke følgende sammenheng som en slags normal:
Om y står for vekt i kg og x står for barnets alder i antall år, så er:

$$y = 4 + 2{,}5x$$

Sett kryss foran det svaret nedenfor som du mener er best:

Hva betyr tallet 4 her?

☐ Barnet er 4 år
☐ Barnet er 4 kg ved fødselen
☐ Barnets vekt øker med 4 kg hvert år
☐ Ingen av delene, det betyr……………………..

The task gives the opportunity to test the students understanding of the role of the parameters in a given linear function. The tasks 14a and 14b appear at the end of the test and happen to be between other difficult tasks – tasks 13 and 15.

Table 4.3.1.2.16: 11th grade, Task - solution frequency for Test 1 (2005 fall) and Test 2 (2006 spring)

Oppgave 14a	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	6	4
Barnet er 4 kg ved fødselen	**50**	**72**
Barnets vekt øker med 4 kg hvert år	27	18
Barnet er 4 år	9	2
"Ingen av delene, det betyr..." med kommentarer	7	4
"Ingen av delene, det betyr..." uten kommentarer	1	1
Andre svar	1	

At the beginning the problem states that the weight of a child from 2 to 10 years is dependant on the child’s age and then it is presented in the form of a given linear function $y = 4 + 2{,}5x$, where y is introduced as the variable for the child’s weight and x is the variable for the child’s age. To solve the problem question on task 14a, the students needed to interpret the meaning of the constant 4 in the given function - the students had to find what this constant represents in the function. The task is a multiple choice task, so the students were given a number of possible answers. The students had to choose only one answer that correctly explains the role of this constant in the linear function.

Groups of 50% of the students tested in 2005 and 72% of the students tested in 2006 answered correctly to the problem question. Those results indicate that the students, who were tested at the end of the school year, were much better in finding what role the constants play in the linear function. This finding suggests that most students understood the real life situation presented in the task and were able to make meaning of the symbolical representation of the problem. The students' responses that the child's weight increases with 4kg every year were the largest group among the students who gave incorrect answers. This type incorrect response was given by 27% of the students in 2005 and 18% of the students in 2006. Those students could not figure out that the constant 4 had the role of a given constant and since it was fixed, so it could not contribute to the increase of the child's weight from year to year. The possible explanation could be given by the answer that states that 4 had the meaning the weight of the child at birth. Another reason for the students' difficulties could be that the definition of the problem describes the relation with the condition child's age from 2 to 10 years, and does not specify the weight of the child out of this range.

Herscovics (1989) reported the results of a study which tested whether relating the variables to more relevant situations might reveal better students' understanding. The problem was presented to a big group of students as a given formula $W = 17 + 5A$, which related the weight of young boys to their age. The students were asked to analyse the formula, and find how much more the boy should weight with every year and the results showed that 64% of the students answered correct, but surprisingly 30% of 17 years old students answered 22, simply adding 17 and 5.

Oppgave 14b Vekta av et barn avhenger av alderen. For barn i skolealder, kan vi bruke følgende sammenheng som en slags normal:
Om y står for vekt i kg og x står for barnets alder i antall år, så er:

$$y = 4 + 2{,}5x$$

Sett kryss foran det svaret nedenfor som du mener er best:
Hva betyr tallet 2,5 her?

☐ Barnet er 2,5 år
☐ Barnet er 4 + 2,5 år, altså 6,5 år
☐ Barnet veier 2,5 kg når det blir født
☐ Barnets vekt øker med 2,5 kg hvert år
☐ Ingen av delene, det betyr……………………..

Table 4.3.1.2.17: 11th grade, Task 14b - solution frequency for Test 1 (2005 fall) and Test 2 (2006 spring)

Oppgave 14b	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	9	3
Barnets vekt øker med 2,5 kg hvert år	**39**	**72**
Barnet er 2,5 år	27	17
Barnet var 2,5 kg når det blir født	12	7
Barnet er 4 + 2,5 år, altså 6,5 år	4	1
"Ingen av delene, det betyr..." med kommentar	6	1
"Ingen av delene, det betyr..." uten kommentarer	2	
Svar med kommentar	1	

In order to make meaning of the context described in the task the students need to have adequate understanding of the role of the variables and the constants in a given linear function.

In 2006 the students performed considerably better on this task compared with the previous year. 39% of the students could answer the problem's question in 2005 compared with 72% of the students tested in 2006.

The most common error was the answer that the number 2,5 in the function means that the child is 2,5 years old. A less common error, shown by the students, was that the child was 2,5kg when it was born. All students who had problems with the task seem to have difficulties understanding the role of the variable x in the linear function. When the value of the variable x (number years of the child) increases with 1 than the value of the variable y (child's weight) increases with 2,5 , showing that the child's weight increases with 2,5kg each year. All incorrect students' answers indicate that those students probably did not have proper understanding of linear algebraic functions in general and had substantial problems to interpret the role of the variables and the constants in a given function. Those students showed inability to make meaning of what the constants in a given algebraic function represent.

The test's results revealed that the students who had difficulty with this task were a lot smaller group when the second test was performed. So this could be a sign that the students had very productive time when they were working with linear functions during the school year.

Oppgave 12e Hva er da verdien av uttrykket $3b^2 – abc$ når $a = 3$, $b = -1$ og $c = 5$.

$3b^2 – abc =$……………………………………

Table 4.3.1.2.18: 11th grade, Task 12e - solution frequency for Test 1 (2005 fall) and Test 2 (2006 spring)

	Frekvens i prosent	
Oppgave 12e	2005 høst	2006 vår
Ikke besvart	27	15
18	**11**	**32**
-12	9	7
2	5	
Innsatt tall, men ikke regnet ut		5
andre svar bokstavuttrykk	5	
andre svar tallverdier	43	42

A solution of this problem requires students to substitute the variables in the expression with their values and compute the answer. Only 11% of the students tested in the fall could do correctly the calculations required by the task. The percentage of correct responses increased a lot in 2006 – already 32% of the students could do the necessary operations properly, but still the task was difficult for the majority of the students. It's difficult when looking at the incorrect answers to systemize all students' mistakes, because the students' wrong answers are a very diverse group. It seems that many students had problems when calculating with negative and positive numbers - when a negative and a positive number are multiplied, the result is negative, but that is not clear to all students.

Increase between 10 and 15 points

For test items Op 1d, Op 4b, Op 4c, Op 5a, Op 5b, Op 5c, Op 7b, Op 9d, Op 9e, Op 10b, Op 12 b the increase was between 10 and 15 solution frequency points. Each test problem in this group is a test problem, which was solved by at least 11 more students in the spring.

Oppgave 12b Hva må x være dersom

$$\frac{x+1}{x+4} = \frac{4}{5}$$

In order to solve this task algebraically the students needed to apply more complex method of solution. The students were not asked to show a method of solution, so we can't know how many students were able to solve the task using algebraic methods. It's difficult to find the solution of the equation only using the strategy guess and check, but it's possible that some of the students tried with many different values for x and could arrive at the solution x=11 in this way. There are evidence that some students used guess and check strategy – they made some notes about that. Those students substituted the unknown x with different values and checked whether the answer is equal to the desired result.

A group of 12% of the tested students in the fall identified the correct answer compared to 23% with the same group of students tested in the spring. The groups of students who didn't attempt to solve the task were very big – 66% of the students tested in 2005 and 55% of the students tested in 2006.

Table 4.3.1.2.19: 11th grade, Task - solution frequency for Test 1 (2005 fall) and Test 2 (2006 spring)

Oppgave 12b	Frekvens i prosent	
	2005	2006
Ikke besvart	66	55
11	**12**	**23**
3	5	4
"x=3 x=1" ; "x=3 og 1"	2	
"3/1"	1	3
1	1	2
"3 og 1"	1	4
andre svar tallverdier	6	9
svar bokstavuttrykk	6	

The most common wrong answer was to assign value 3, or both 3 and 1, or only 1 to the unknown x. Such type of answers suggested that those students did not have enough conceptual understanding and enough fluency in performing procedures for solving equations.

Brekke (2000) reported the results of 10th grade students who had to solve the equation (x+1)/(x+3)=3/4. The study found that the majority of those students found the solution after trying with different numbers, only one of ten students found the correct answer by the use of algebraic methods for solution. Most students who gave incorrect solutions tried to substitute x with 2 or gave as answer "2 and 1". The low number of students who could solve this equation algebraically showed that few students in 10th grade had real understanding of the routines for solving an equation (Brekke, 2000). The solution frequencies for the other tasks with increase between 10 and 15 points are presented in the following table.

Oppgave		Frekvens i prosent	
		2005 høst	2006 vår
Op 1d	70 · 0,3 =	48	60
Op 4b	Fyll ut hele tabellen: ($\frac{x}{2}$ =12)	70	80
Op 4c	Fyll ut hele tabellen: (x^2 =16)	38	52
Op 5a	Skriv som desimaltall $\frac{3}{10}$	59	74
Op 5b	Skriv som desimaltall $\frac{46}{100}$	67	79
Op 5d	Skriv som desimaltall 28 tideler	37	47
Op 7b	*Et terrengløp går i ei løype som er 5,6 km. Hvor mange engelske mil er det? (Ei engelsk mil er 1,609 km).*	50	64
Op 9d	$2y \cdot y^2$	44	58
Op 9e	$3a + a^2 + a^2$	59	73
Op 10b	$a + b \cdot 2 = 2b + a$ Dette ☐ er alltid sant ☐ er aldri sant ☐ *kan* være sant, nemlig når	76	89
Op 12 d	Finn tallet x om x = a + b - c og a = 1, b = 2 og c = 3	77	87

Table 4.3.1.2.20: Group of tasks

Test items with lower results for test 2 in comparison with test 1

Tasks Op 1a, Op 1e, Op 13 were test items with lower solution frequency on the second test. For those test items the average solution frequency was (test 1) and (test 2). Op 1a is a simple task of calculation with fractions ($\frac{1}{2}+\frac{1}{4}=$), and Op1e is another task requiring a simple computation ($60 \cdot 450 =$). For those two tasks the results on the second test are 6 and 4 solution frequency points lower in comparison with the results for test 1. Op 13 requires students to find an equation related to the problem " På en skole er det 10 elever for hver lærer." and was already presented in details – only 7% of the students (test 1) and 4% of the students (test 2) were able to find the right answer.

4.3.1.3 Dividing the students in three subgroups

In this section we analyse additional information, addressing the students' development when we consider three subgroups. The group of 113 students was divided in three subgroups – Nederste, Midterste and Øverste groups. Every student was placed in one of those groups according to the received score on the first test, and some additional details were presented in the chapter Methods.

The students were divided in three subgroups in order to collect additional information for each subgroup and focus on the specific problems of the students in each group. It was important to analyse smaller groups of students with less differences among them. The comparison of the students' results for the three subgroups shows the major differences between the students in the groups.

Elevgruppe		Antall	Test år	Minimum	Maksimum	Gjennomsnitt	St.avvik
Elever - Test 1 & Test 2		**113**	2005 høst	4	42	26,2	7,3
			2006 vår	10	45	30,5	7,3
Elever Test 1 & Test 2	**Nederste tredjedel**	**37**	2005 høst	4	23	17,8	4,1
			2006 vår	10	38	24,3	6,5
	Midterste tredjedel	**38**	2005 høst	23	30	26,4	1,7
			2006 vår	23	40	30,4	4,5
	Øverste tredjedel	**38**	2005 høst	30	42	34,1	3,1
			2006 vår	26	45	36,6	4,8

Table 4.3.1.3.1: 11th grade, Statistical information for the groups of all students, Nederste, Midterste and Øverste groups of students.

There are significant differences among the three subgroups. In general there is a big increase of the groups of students with scores above 30 points and a big decrease of the students with a score below 20 points. In addition there were some students who got scores close to the maximum on the test.

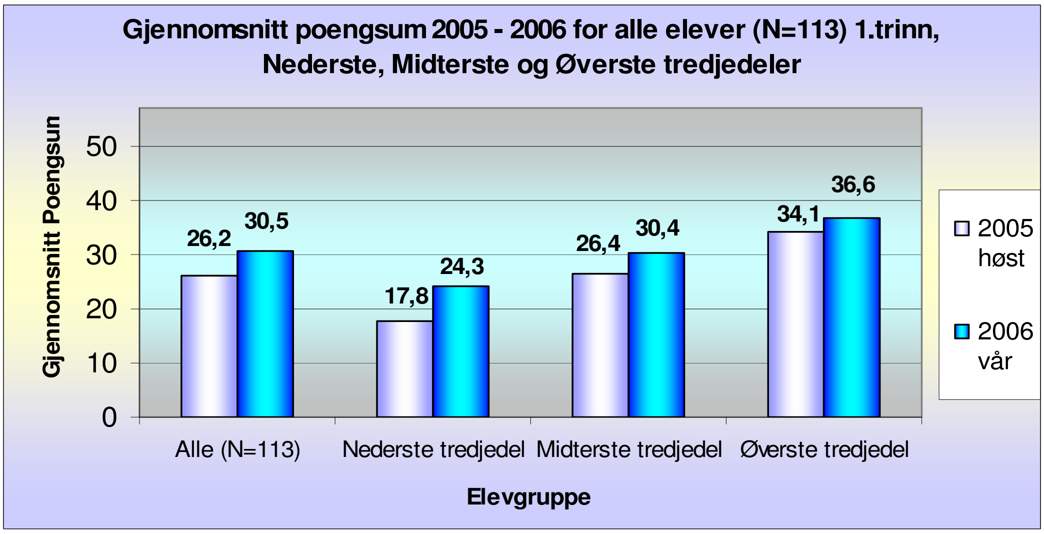


Figure 4.3.1.3.1 : 11th grade, Average points sums for the groups of all students and Nederste, Midterste and Øverste groups for test 1 (2005) and test 2 (2006).

The analyses of the data showed that there was a positive development in all groups of students. It can be observed that:

Nederste group
- The group of students with scores below 20 points was reduced from 69% of the students (test 1)to 27% of the students (test 2).
- The group of students with scores between 20 and 29 points increased from 35% of the students (test 1) to 51% of the students (test 2).

Midterste group
- The group of students with scores between 20 and 29 points decreased from 97% of the students (test 1) to 49% of the students (test 2).
- The group of students with scores between 30 and 39 points increased from 5% of the students (test 1) to 51% of the students (test 2).

Øverste group
- The group of students with scores between 40 and 48 points increased from 5% of the students (test 1) to 24% of the students (test 2).
- The group of students with scores between 30 and 39 points was reduced from 95% of the students (test 1) to 63% of the students (test 2).
- The group of students with scores between 20 and 29 points increased from 0% of the students (test 1)to 13% (test 2).

The table and the figure below give comparison of all 113 students, Nederste, Midterste and Øverste group of students.

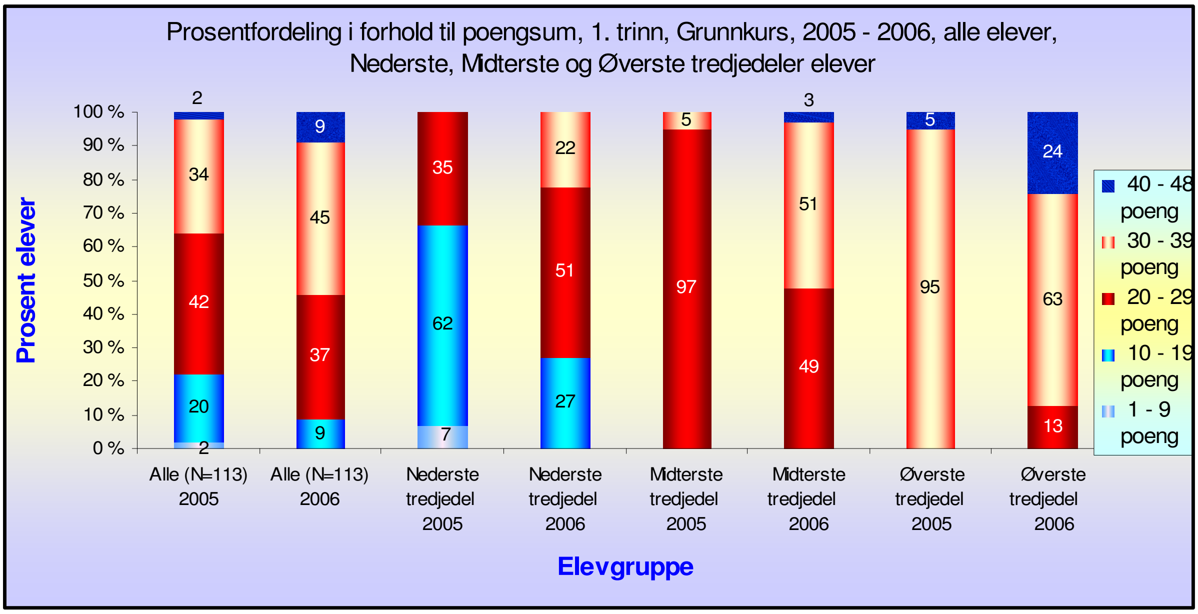


Figure 4.3.1.3.2: 11th grade, 2005 - 2006, Distribution of the points sum in percents – all students, Nederste, Midterste og Øverste groups.

Comparison – increase / decrease of the students' points

- 87 students (77% of the students) increased their result in the spring. High percentage of students who didn't change or decreased it 29 students (23% total) (6% didn't change it , 17% had a decrease in the result.
- The big majority of those students who had a decrease or zero change were from the Midterste or Øverste group (21 students).
- The students who increased their results a lot in percentage were 11 students and they were all from Nederste group.
- The students who increased their results with more than 10 points biggest increase were 11 from Nederste, 6 from Midterste and only 3 from Øverste groups. The students from Øverste group already performed high to both tests and it's a lot harder to improve their score – they had minimal improvement in points.

Elevgruppe		Alle	Endring Poengsum: negative	Endring Poengsum: 0	Endring Poengsum: 1% - 49%	Endring Poengsum: 50% - 99%	Endring Poengsum: 100% eller mer
Alle elever	antall elever	113	19	7	76	8	3
	frekvens	100	17	6	67	7	3
Nederste tredjedel	antall elever	37	5	0	21	8	3
	frekvens	100	14	0	57	22	8
Midterste tredjedel	antall elever	38	6	3	29	0	0
	frekvens	100	16	8	76	0	0
Øverste	antall elever	38	8	4	26	0	0
Tredjedel	frekvens	100	21	11	68	0	0

Table 4.3.1.3.2: 11th grade, comparison for 113 students, Nederste, Midterste and Øverste group of students.

The students in every group - 113 students, Nederste, Midterste and Øverste groups were first sorted according the individual student's test scores in 2005. For every student was counted how many percents the student's points score in 2006 was higher or lower compared with the first score in 2005. After that the students were divided in five categories. Those categories were as shown on the table:

- students with negative change in the results
- students who didn't change their results
- students who increased their results between 1 and 49%
- students who increased their results between 50 and 99%
- students who increased their results with 100% or more

The table above shows for every group of students the number students in each category and the corresponding frequency in the group.

Such comparison can give us a lot of information, because the students* results in 2005 and 2006 vary a lot and it's interesting to look also for more detailed information. Looking at the students' tests scores we can try to find more information, dividing the students in different categories. After that it is easier to compare the results in each category and look for more information concerning the students' performance.

Students who increased their score

We can look closer at the group of 87 students who increased their score in the spring. How many students increased of their score with 10 points or more? How many students had increase of 50% -99% of their score in the spring? How many students had double or even higher increase of the results?

How many students increased of their score with 10 points or more?

10 points or more increase had a group of 20 students. Half of those students 11 students were from the Nederste group, followed by 6 students from Midtesrte and 3 students from Øverste group. As a whole this group increased their score with 13 points from 19,5 in 2005 to 32,8 points in 2006. The 9 students from the Midterste and Øverste groups increased their results with 38% on average from 28,7 points to 39,5. The group of 11 students from Nederste group is a very interesting group of students. They increased they score from 15,4 in 2005 to average 30,3 points in 2006, where 10 of those students have on average 84% increase, and one student who increased his score with 650% - from 4 points to 30 points. So we can ask why one student scored so high on the second test?

How many students had increase of 1% - 49% of their score in the spring?

76 students (67% of all 113 students) increased their points score in the spring between 1% and 49%. Big number students – 29 students were from the Midterste group, 26 students from Øverste group, followed by 21 students from Nederste group. This is a group with big variations in the students scores – between 7 and 42 in the fall and between 10 and 45 in the spring. The average score for this group was 26,8 in 2005 and 31,7 for 2006, values very close to the average values for the group of all 113 students.

Other interesting data can be collected when looking at the students with high percentage increase of the points score in 2006.

How many students had increase of 50% -99% of their score in the spring?

This is a small group of students 8 students – all from Nederste group, they increased their score from 16,6 points in the fall to 29,7 points in the spring or 79% on average. The maximum increase in this group made a student from the Nederste group – from 19 points in 2005 to 37 points in 2006 – almost double, with a very big increase of 93%. Such increase of the points differanse for this student (overall high percentage of the increase) looks very unlikely.

What students got increase of the score with more than 100%?

Such high increase of 100% or more achieved 3 students - all from Nederste group.
The average result in the fall was only 12 point, 5 points below the average for the Nederste group in the fall, but those 3 students increased their score on the second test with 285% on average. Two of the students were students who made double increase of his score from 19 to 38 points and from 13 to 27 points, the third student is the student from the Nederste group who got increase from 4 to 30 points.
The average score for this group was already 31,7 points in 2006. Those three students have quite a high increase of their results that doesn't look plausible and it is questionable whether they showed their real abilities to both tests.

Students who did not change their results on the tests

They were 7 students - 3 students from the Midterste and 4 student from the Øverste group. Those students didn't get a high score - on average 33 points on the tests, their scores vary from 24 points to 39 points.

Students who decreased their score in the spring of 2006

This is a group of 19 students – most of them - 8students were from the Øvertste group, 6 students were from the Midterste and 5 students from the Nederste groups. Their average score decreased with 10,6% in the spring, ranging from 1 points less to 9 points less in the spring. This group of students had on average 27,5 points on the first test and 24,7 points on the second test. In 2006 those students didn't have much lower average results compared with the results for all 113 students in 2006.

From the test results only we can't say why those students couldn't show improvement on the second test. Some interviews with them can give more information about the reason. It is possible for example - lower motivation, sabotage of the second test, emotional up and down, physical problem, cheating or some other reason. But in reality such a big decrease in the students' performance for both test is not natural and we can question should such students' results be taken seriously.

Nederste group of students

This is a group of 37 students who achieved the lowest results on the first test. Their average points score was 17,8 points (test 1) and 24,3 points (test 2). In 2005 all 37 students scored below the average score for the group of all students who did both tests and in the spring of 2006 only 4 students received results above the average results for all students.

Elevgruppe	Antall	Test år	Minimum	Maksimum	Gjennomsnitt	St.avvik
Nederste tredjedel	37	2005 høst	4	23	17,8	4,1
		2006 vår	10	38	24,3	6,5

Table 4.3.1.3.3: 11th grade, Statistical information for Nederste group.

This group of students has a very big difference in the students' abilities in mathematics. In 2005 the minimum score in the group was only 4 points and the maximum 23 points. In 2006 the difference is even bigger – minimum 10 points and maximum 38. The average score of the students in the group increased from 17,8 to 24,3 points – an increase of 37% more than twice higher than the average increase of the students score in percents for the group of all 113 students. The majority of the students in the group 21 students increased their score between 1% and 49%, but there's a group of 5 students who had a decrease of the score on average with 17,8%. A big number of students 8 students increased their score between 50% and 99% and 3 students had increase of more than 100%. Four of those students with high increase of the score in percents had also very high increase in points.

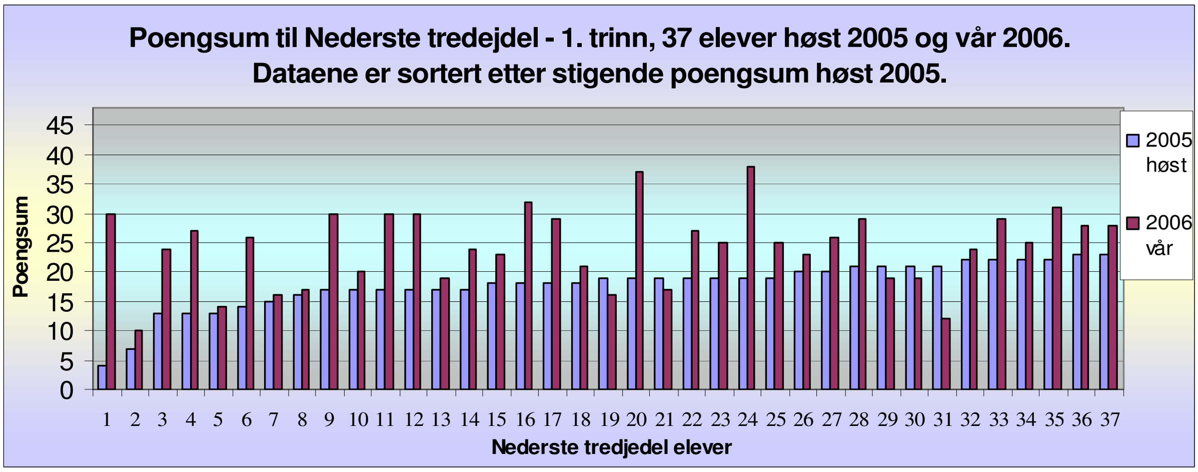


Figure 4.3.1.3.3: 11th grade, Nedertse group - points sum for every student in the group (both tests).

Nederste tredjedel is a group of 37 students. They were selected after a procedure of sorting the points results for every student on the first test. The points (2005 fall) for every student in the whole group of 113 students were sorted from the lowest to the highest score points. The first 37 students after this sorting of the data became the group of the Nederste tredjedel students. The standart deviation is smaller for the first test, since the students were selected according the results in 2005.

In 2005 the average result on the first test is 17,8 points, which is about one third below the average score of 26,2 points for all of the 113 students. Most students -31 students from the Nederste group showed lower points results than the average of 26,2 points for all 113 students, only 6 students in this group received higher than the average points score for the all 113 students.

In 2006 the average points score for the students in the Nederste group is 24,3. So on average in 2006 the students from the Nederste group had results 36% higher than the average points result for the group in 2005. Still the largest part of this group 33 students had lower points score than the average points score - 30,5 points for all 113 students. In 2006 the average points score for the Nederste group is 6,2 points lower than the all students average result, so in the spring this difference is smaller compared with the fall (8,5 points difference).

From 37 students in the Nederste group, 32 students showed positive change in their results in 2006. The students had better results in the spring of 2006 with average increase of 8 points in 2006, compared with the fall 2005. The remaining 5 students in the group had a decline in their results - on average 3,6 points on the second test.

In Nederste group there are some students with much higher results compared with the rest of the students in the group in 2006. 11 students (30% of the students in the group) got points results at least 10 points higher than in 2005. Those students had on average 15 points higher score than in 2006. The majority of the students - 10 of those 11 students had a result 84% higher on the second test, one student showed an increase of 650%- from 4 points in 2005 to 30 points in 2006. This increase in the results in 2006 with 84% is much higher than the increase of the avarage points score for all of In 2006 the 113 students who took both tests got on average small increase of their average score - 16,4% increased compared with the average points score in 2005.
When analysing this statistical information concerning this group of 11 students with very high increase of the results in 2006, we can’t know only looking at the tests why those students showed such a big improvement in the results. Only qualitative interviews might help to find the exact reasons for this increase of the results. One of the possible reasons can be that the students are human beings with their emotional up and downs and the emotional status, physical condition, lack of concentration, lack of motivation might affect significantly the student’s performance on a test.

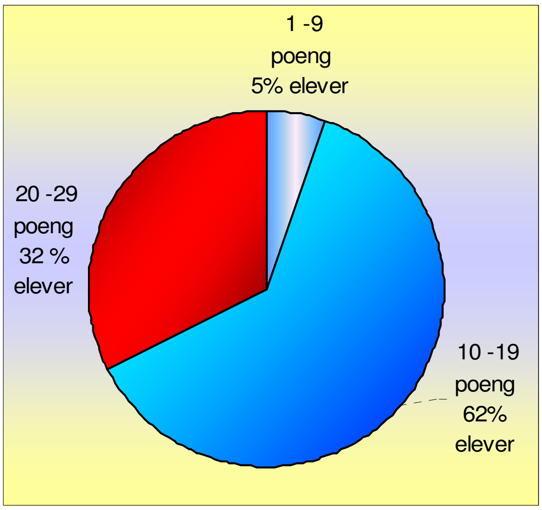


Figure 4.3.1.3.4: 11th grade, Nederste group – test 1, 2005, Distribution of the students’ points sum

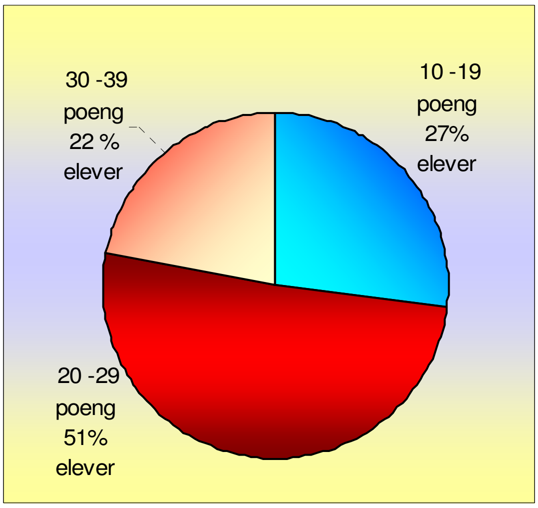


Figure 4.3.1.3.5: 11th grade, Nederste group – test 2, 2006, Distribution of the students’ points sum

The figures present information how many percents students had score points between 1 and 9 points, 10 and 29 points, 30 and 39 points, and 40 and 48 points.

In the beginning of the school year (the fall of 2005), 5% of the Nederste group of students received points scores between 1 and 9 points, 62% got between 10 and 19 points and 32% of the students – between 20 and 29 points. So the group of students with the results between 10 and 19 points is the largest and it is almost twice bigger than the group of 20-29 points – the students with the highest results in the Nederste group.

At the end of the school year 27% of the students had a points score between 10-19, the group of students with points scores between 20-29 points were the largest – it was already 51% and the rest of 22% of the students showed the highest results between 30 and 39 points. The students with scores between 20-29 points in 2006 were more than twice smaller group compared with the same group of students in 2005.

Summary of the students' results on all test items

The following diagrams show the solution's frequency for all of the test items on the tests – Test 1(2005) and Test 2(2006).

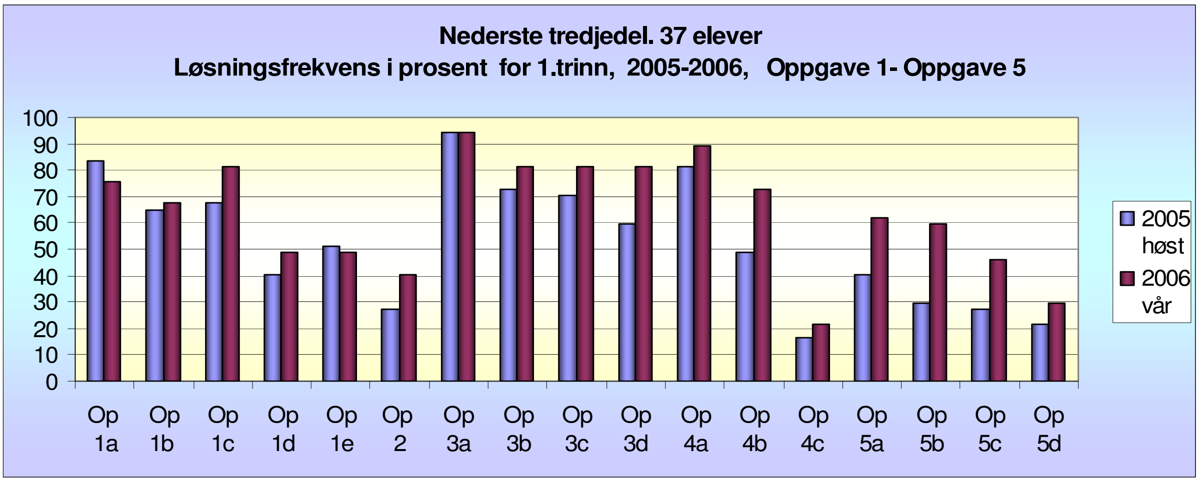


Figure 4.3.1.3.6: 11th grade, solution frequencies for test items 1a – 5d, Nedreste group of students

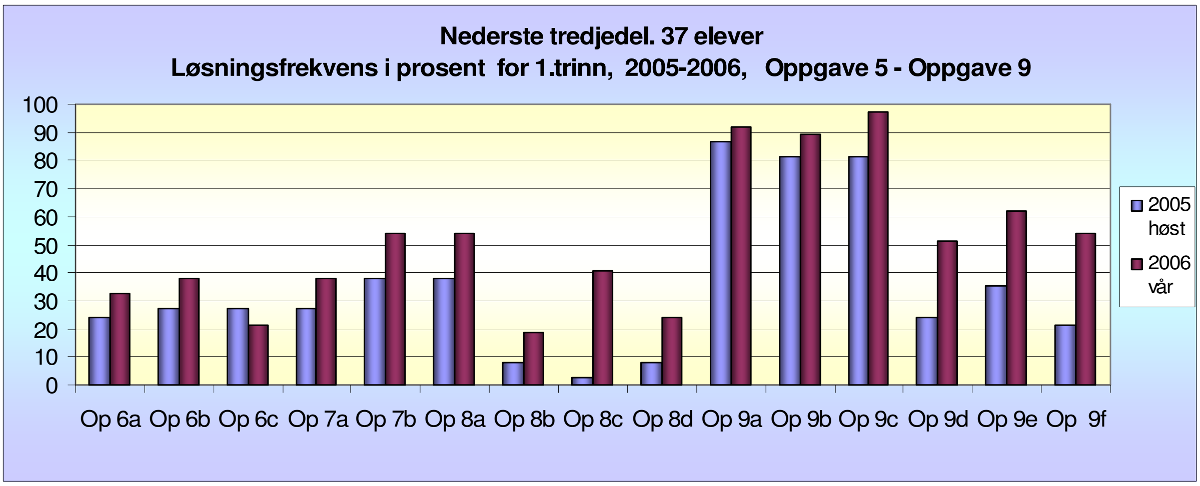


Figure 4.3.1.3.7: 11th grade, solution frequencies for test items 6a – 9f, Nedreste group of students

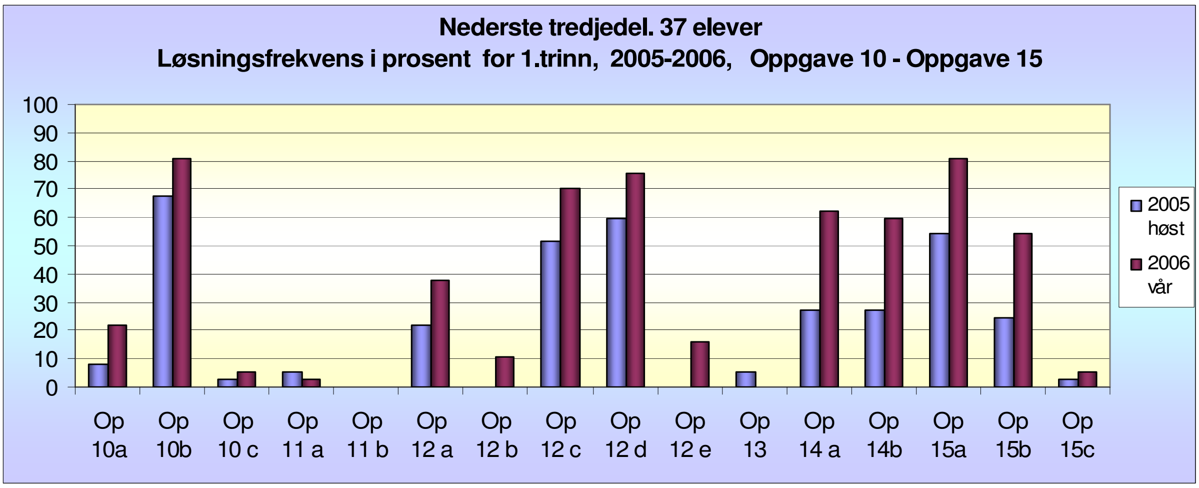


Figure 4.3.1.3.8: 11th grade, solution frequencies for test items 10a – 15c, Nedreste group of students

Looking at the figures above it is easy to notice that some of the test items were quite hard for this group of students. For example less than 5% of the students were able to solve 5 of the items Op 13, Op 11 b, Op 11 a, Op 10 c, Op 15c. In addition 8 of the items were solved by between 10% and 30% of the students in the Nedreste group.

Group of test items with big improvement of the results on the second test

Increase higher than 20 solution frequency points

The students in the Nederste group showed big improvement of the results on the second test for 14 of the test items. The solution frequencies for those items are presented in the table below.

Some of the tasks in this group are algebraic tasks and are related to simplification of algebraic expressions, or to find the value of algebraic expression. It is interesting that in the group of items with big increase were also the tasks Op 14a, and Op14b, related to interpretation of the role of the parameters in the given function. The increase of the solution frequency for those algebraic items was 35 and 32 solution frequency points.

Oppgave		Frekvens i prosent 2005 høst	Frekvens i prosent 2006 vår
Op 3d	Fyll ut helle tabellen	59	81
Op 4b	Sett ring rundt det tallet som ligger nærmest 2,08 i størrelse 209 2,9 2,05 2,1	49	73
Op 5a	Skriv som desimaltall $\frac{3}{10}$	41	62
Op 5b	Skriv som desimaltall $\frac{46}{100}$	30	59
Op 5c	Skriv som desimaltall 45 tusendeler	27	46
Op 8c	Skriv riktig tall i ruta 15 : 10 = $\square$ · 15	3	41
Op 9d	Skriv enklere dersom det er mulig: $2y \cdot y^2$	24	51
Op 9e	Skriv enklere dersom det er mulig: $3a + a^2 + a^2$	35	62
Op 9f	Skriv enklere dersom det er mulig: $5a-2(7-a)+6$	22	54
Op 12c	Hva må x være dersom 3x = 7 og 5y = 11 Hva er da 3x + 5y?	51	70

Op 14a	y = 4 + 2,5x Sett kryss foran det svaret nedenfor som du mener er best: Hva betyr tallet 4 her?	27	62
Op 14b	y = 4 + 2,5x Sett kryss foran det svaret nedenfor som du mener er best Hva betyr tallet 2,5 her?	27	59
Op 15a	Even tenker på to tall. Summen av dem er 19. Differensen mellom dem er 5. Finn tallene.	54	81
Op 15b	Hvordan kan du gå fram for å finne tallene?	24	54

Table 4.3.1.3.4: Group of tasks

Group of test items with decrease in the results on the second test

The students in the Nederste group had lower results for 6 of the test items. The solution frequencies for those items are presented in the table below. In the table below are shown the items with decline bigger than 5 solution frequency points. The biggest decline is for problem Op1a ($\frac{1}{2}+\frac{1}{4}=$) as it is shown in the table 84% of the students (test 1) and 76% of the students (test 2) solved this problem. It is noticeable also that the problem "På en skole er det 10 elever for hver lærer. Hvilke av disse uttrykkene forteller dette?" was among the most difficult for this group of students and was solved by only 5% of the students in the fall and none of the students in the spring.

Oppgave		Frekvens i prosent	
		2005 høst	2006 vår
Op 1a	$\frac{1}{2}+\frac{1}{4}=$	84	76
Op 1e	$60 \cdot 450 =$	51	49
Op 6c	*Anne kjøper bananer i en butikk til 13,50 kr per kg. Hvor mye kan Anne kjøpe for 10,50 kr?*	27	22
Op 11a	For å finne volumet V av en sylinder, har vi denne formelen: $V = \pi r^2 h$. Her er r radius i grunnflata , og høyden er h. Hva skjer med volumet dersom vi dobler både radius r og høyde h?	5	3
Op 13	På en skole er det 10 elever for hver lærer. Hvilke av disse uttrykkene forteller dette? L = antall lærere, E = antall elever. Sett ring rundt det eller de riktige uttrykk.	5	0

Table 4.3.1.3.5: Group of tasks

Group of test items solved by more than half of the students

A group of 26 of the test items Op 1a, Op 1b, Op 1c, Op 3a, Op 3b, Op 3c, Op 3d, Op 4a, Op 4b, Op 5a, Op 5b, Op 7b, Op 8a, Op 9a, Op 9b, Op 9c, Op 9d, Op 9e, Op 9f, Op 10b, Op 12 c, Op 12 d ,Op 14 a, Op 14b, Op 15a, Op 15b. Since this is a big group of tasks, we present here part of them – the tasks with very high results.

This group included 10 test items solved by at least 80% of the students on one of the tests. The solution frequencies for those tests items are presented in the next table. Most of those tasks are algebraic tasks. It is interesting to notice that the algebraic tasks Op 9a, Op9b, related to simplification and task Op 10a, related to evaluation of the algebraic expression $a + b\cdot 2 = 2b + a$, was solved by a very big group of the students. The students did not have items also with the tasks Op 3a-d, which required finding the values of several algebraic expressions.

Oppgave		Frekvens i prosent 2005 høst	2006 vår
Op 1c	900 : 30 =	68	81
Op 3a	Fyll ut hele tabellen ($x=5$)	95	95
Op 3b	Fyll ut hele tabellen ($\frac{x}{2}=12$)	73	81
Op 3c	Fyll ut hele tabellen ($x^2=16$)	70	81
Op 3d	Fyll ut hele tabellen ($4x=2$)	59	81
Op 4a	Sett ring rundt det tallet som ligger nærmest 0,16 i størrelse 0,1 0,2 15 0,21 10	81	89
Op 9a	$2x + 5x$	86	92
Op 9b	$x + x + 2x$	81	89
Op 10b	$a + b{\cdot}2 = 2b + a$ Dette ☐ er alltid sant ☐ er aldri sant ☐ *kan* være sant, nemlig når	68	81
Op 15a	Even tenker på to tall. Summen av dem er 19. Differensen mellom dem er 5. Finn tallene.	54	81

Table 4.3.1.3.6: Group of tasks

Midterste group of students

Midterste is a group of 38 students. The group has average points scores for the two tests approximately the same as the average points scores for all of the 113 students.

Elevgruppe	Antall	Test år	Minimum	Maksimum	Gjennomsnitt	Standardavvik
Midterste tredjedel	38	2005 høst	23	30	26,4	1,7
		2006 vår	23	40	30,4	4,5

Table 4.3.1.3.7: 11th grade, Statistical information for Midterste group

The students in the Midterste group have very close scores on the first test – the minimal score is 23 points and the maximum is 30 points. In 2006 only 6 students in the group has an increase of at least 10 points, so as a total the students' results in 2006 have minimal changes - the average points sum for the Midterste group increased only with 4 points on the second test, so there is not a very big change in the average points score for all of the 38 students in this group.

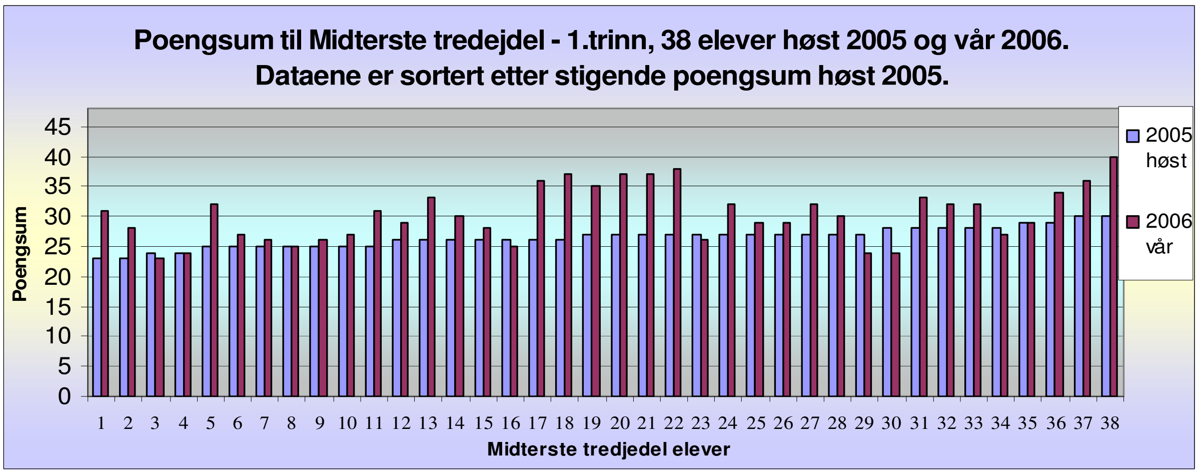


Figure 4.3.1.3.9: 11th grade, Midterste group - points sum for every student in the group (both tests).

A big majority of the students in the group - 29 students increased their score between 1% and 49%. The remaining 9 students (24% - high) decreased their score or didn't change it

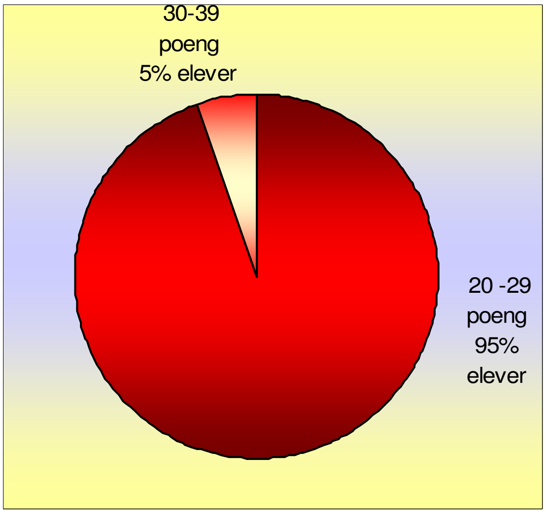


Figure 4.3.1.3.10: 11th grade, Midterste group – test 1, 2005, Distribution of the students' points sum

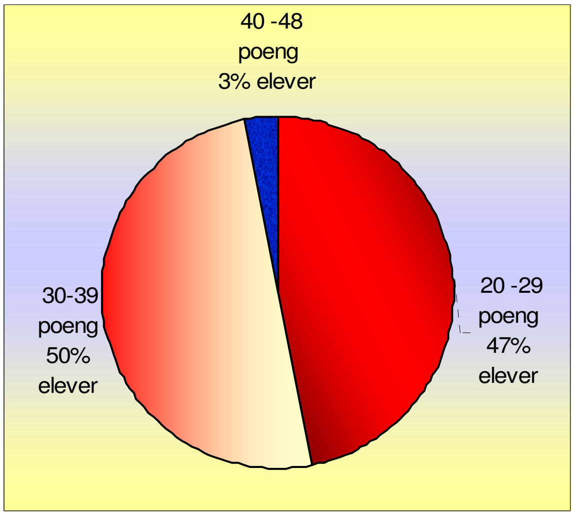


Figure 4.3.1.3.11: 11th grade, Midterste group – test 2, 2006, Distribution of the students' points sum

In the beginning of the school year there were very minimal differences in the group almost all students f the Midterste group of students received points scores between 20 and 29 points, and a small group of 5% of the students got between 30 and 39 points. There were major changes in the group on the second test. At the end of the school year about half of the students had points scores between 20 and 29 points, and half of the students had scores between 30-39 points.

Summary of the students' results on all test items

The following diagrams show the solution's frequency for all of the test items on the tests – Test 1(2005) and Test 2(2006).

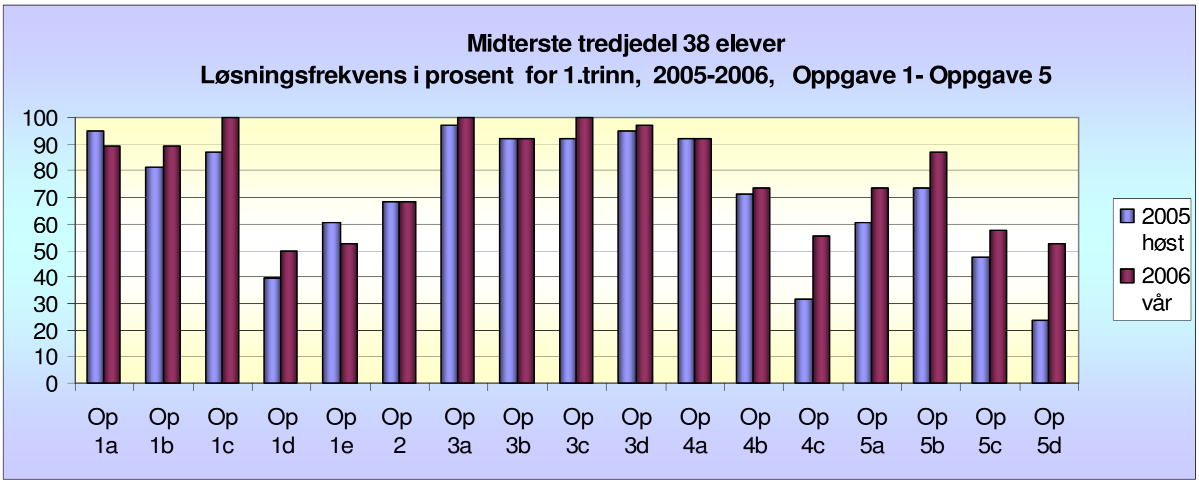


Figure 4.3.1.3.12: 11th grade, solution frequencies for test items 1a – 5d, Midterste group of students

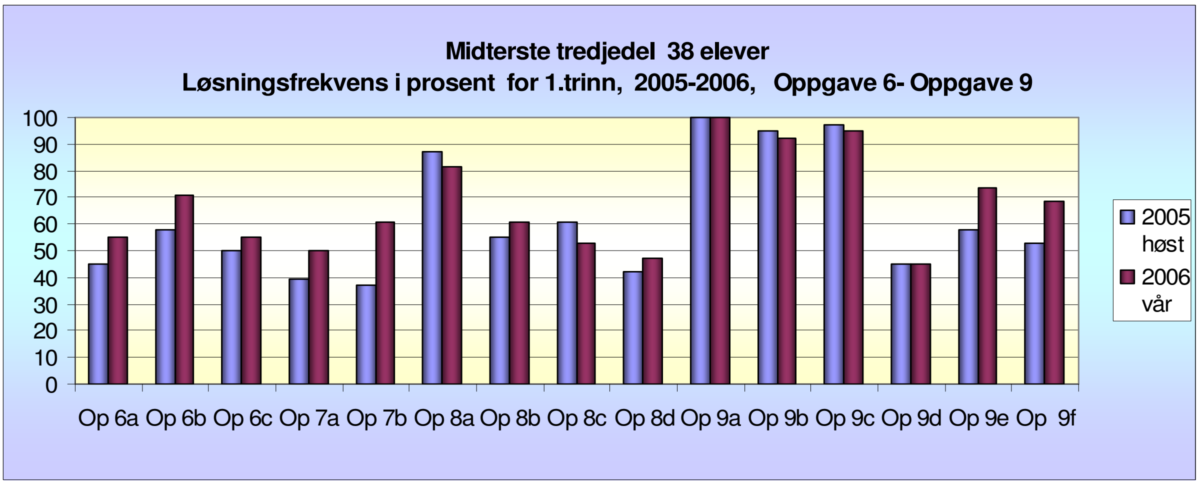


Figure 4.3.1.3.13: 11th grade, solution frequencies for test items 6a – 9f, Midterste group of students

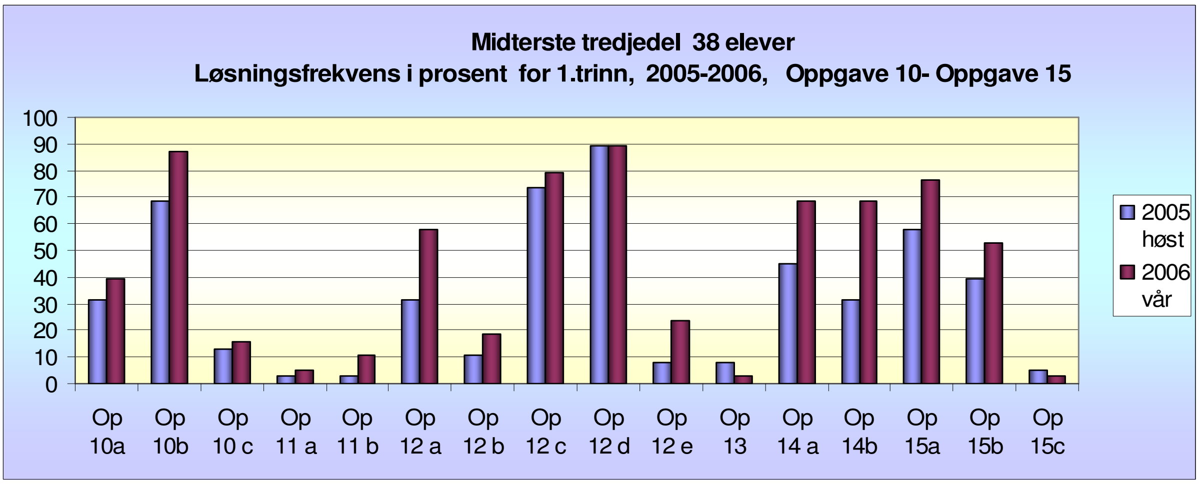


Figure 4.3.1.3.14: 11th grade, solution frequencies for test items 10a – 15c, Midterste group of students

Most of the items were solved by at least half of the students on one of the tests. For a group of eight test items there was big improvement of the results on the second test, and for another group of eight test items there was a decrease in the results on the second test. The different groups of tasks in the tests will be presented next.

Group of test items with big improvement of the results on the second test

Increase of 20 solution frequency points or more

Eight of the test items were solved by at least 7 more students when the students were tested again. The algebraic tasks 23a, 23b, and 23c were the tasks with the biggest increase of the number of students who solved them in the spring.

The biggest increase in the results was on the task Op 23a ($2x + 5x$ …) solved by 16% of the students (test 1) and 84% of the students (test 2). The students had much better skills in solving linear equations. The linear equation $123 + 2x = 195 - x$ was solved by 58% of the students, test 2.

Oppgave		Frekvens i prosent	
		2005 høst	2006 vår
Op 4c	Skriv riktig tall i rutene 5,074 = 5 · 1 + 7 · ☐ + 4 · ☐	32	55
Op 5d	Skriv som desimaltall 28 tideler	24	53
Op 7b	Et terrengløp går i ei løype som er 5,6 km. Hvor mange engelske mil er det? (Ei engelsk mil er 1,609 km).	37	61
Op 10b	a + b·2 = 2b + a Dette ☐ er alltid sant ☐ er aldri sant ☐ kan være sant, nemlig når	68	87
Op 12 a	Hva må x være dersom 123 + 2x = 195 – x	32	58
Op 14 a	y = 4 + 2,5x Sett kryss foran det svaret nedenfor som du mener er best: Hva betyr tallet 4 her?	45	68
Op 14b	y = 4 + 2,5x Sett kryss foran det svaret nedenfor som du mener er best Hva betyr tallet 2,5 her?	32	68
Op 15a	Even tenker på to tall. Summen av dem er 19. Differensen mellom dem er 5. Finn tallene.	58	76

Table 4.3.1.3.8: Group of tasks

Group of test items with decrease in the results on the second test

For seven test items the Midterste group of students showed lower results on the second test.For some of them the decline was minimal. The biggest decline is for problem Op 1e (60 · 450 =), 61% of the students (test 1) and 53% of the students (test 2) solved this problem and for the problem Op 8c (15 : 10 = ☐ · 15), solved by 61% of the students (test 1) and 53% of the students (test 2). It is noticeable also that the problem Op 15c "Even tenker på to tall. Summen av dem er 19. Differensen mellom dem er 5. Hvorfor er det alltid mulig å finne to tall når vi kjenner summen og differensen? " , was difficult also for the Midterste group of students and was solved by only 3% of the students in the spring.

Oppgave		Frekvens i prosent	
		2005 høst	2006 vår
Op 1a	$\frac{1}{2}+\frac{1}{4}=$	95	89
Op 1e	60 · 450 =	61	53
Op 8a	14 : 2 = ☐ · 14	87	82
Op 8c	15 : 10 = ☐ · 15	61	53
Op 9b	$x + x + 2x$	95	92
Op 9c	$t \cdot t \cdot t$	97	95
Op 13	På en skole er det 10 elever for hver lærer. Hvilke av disse uttrykkene forteller dette? L = antall lærere, E = antall elever. Sett ring rundt det eller de riktige uttrykk.	8	3
Op 15c	Even tenker på to tall. Summen av dem er 19. Differensen mellom dem er 5. Hvorfor er det alltid mulig å finne to tall når vi kjenner summen og differensen?	5	3

Group of test items solved by more than half of the students

A group of 38 of the test items were solved by at least half of the students on one of the tests. This group of test items contained 15items which were solved by at least 80% of the students on one of the tests. Those items were mostly algebraic tasks. It is noticeable that the students improved a lot their results on the task of evaluation of $a + b{\cdot}2 = 2b + a$.

Oppgave		Frekvens i prosent	
		2005 høst	2006 vår
Op 1a	$\frac{1}{2}+\frac{1}{4}=$	95	89
Op 1b	$\frac{1}{2}-\frac{1}{3}=$	82	89
Op 1c	900 : 30 =	87	100
Op 3a	Fyll ut hele tabellen (*x=5)*	97	100
Op 3b	Fyll ut hele tabellen ($\frac{x}{2}$ =12)	92	92
Op 3c	Fyll ut hele tabellen (x^2 =16)	92	100
Op 3d	Fyll ut hele tabellen (4*x=2)*	95	97
Op 4a	Sett ring rundt det tallet som ligger nærmest 0,16 i størrelse 0,1 0,2 15 0,21 10	92	92
Op 5b	Skriv som desimaltall $\frac{46}{100}$	74	87
Op 8a	Skriv riktig tall i ruta 14 : 2 = ☐ · 14	87	82
Op 9a	$2x + 5x$	100	100
Op 9b	$x + x + 2x$	95	92
Op 9c	$t \cdot t \cdot t$	97	95
Op 10b	$a + b{\cdot}2 = 2b + a$ Dette ☐ er alltid sant ☐ er aldri sant ☐ *kan* være sant, nemlig når	68	87
Op 12 d	Finn tallet x om x = a + b - c og a = 1, b = 2 og c = 3	89	89

Table 4.3.1.3.9: Group of tasks

Øverste group of students

This is a group that had the highest results to both tests, although the average results of the students increased only with 2,5 points in the spring. Øverste group of students didn't have big differences in the students abilities in the fall of 2005 –the minimum in the group was 30 points, the maksimum 42 points. But the results on the second test show that 8 students decreased their score and the differences in the students' scores were higher – the minimum score in the group was26 points and the maximum score increased to 45 points. The average score of the students in the group for Test 1 was 8 points higher than the average for all 113 students.

Elevgruppe	Antall	Test år	Minimum	Maksimum	Gjennomsnitt	Standardavvik
Øverste tredjedel	38	2005 høst	30	42	34,1	3,1
		2006 vår	26	45	36,6	4,8

Table 4.3.1.3.10: 11th grade, Statistical information for Øverste group

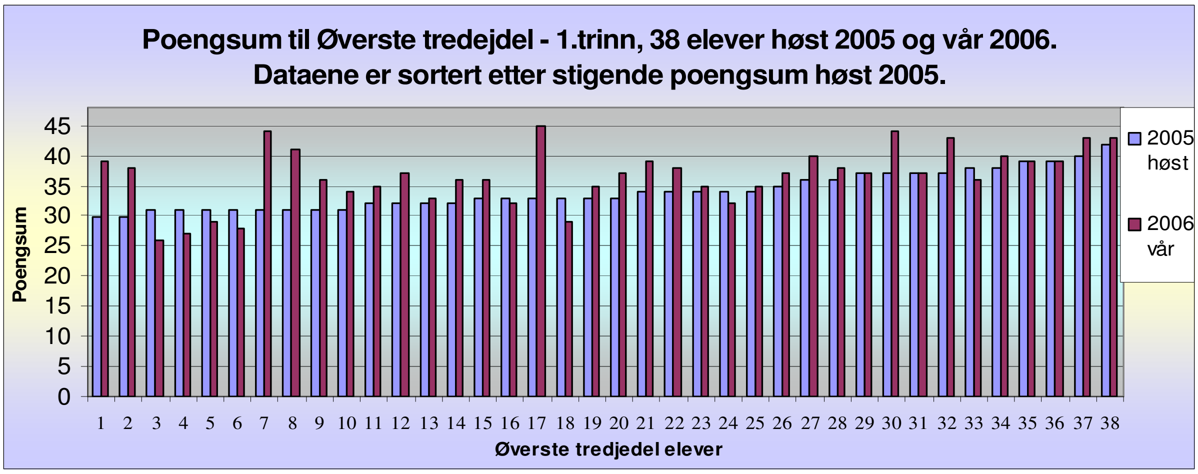


Figure 4.3.1.3.15: 11th grade, Øverste group - points sum for every student in the group (both tests).

A big majority of the students in the group - 26 students increased their score between 1 and 13 points and between 2% and 42%, on average increase 4,6 points, 14% from 33,9 points to 38,5 points. The remaining 12 students (33%) decreased their score or didnot change it – this group is one third of all students. 8 students decreased their score between 1 and 5 points on average decrease 16% from 34,5 to 32,6 points. 4 students didn't change their score.

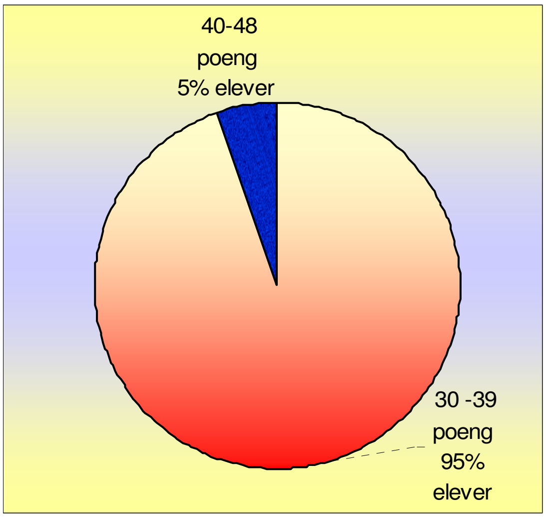


Figure 4.3.1.3.16: 11th grade, Øverste group – test 1, 2005, Distribution of the students' points sum

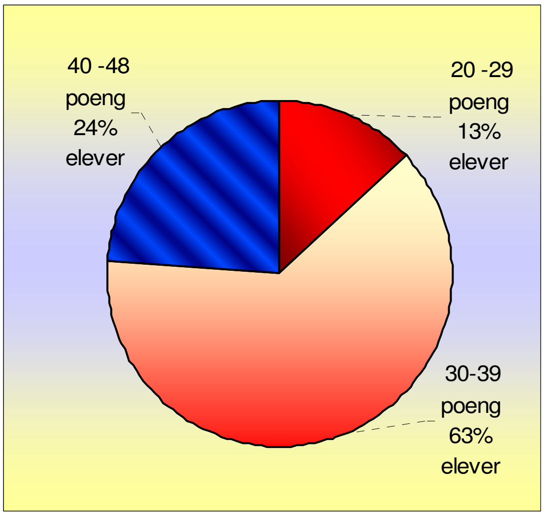


Figure 4.3.1.3.17: 11th grade, Øverste group – test 2, 2006, Distribution of the students' points sum

In the beginning of the school year the largest group was the students with a score between 30 and 39 points – the students with such score were 95% of the students. There were also few students with score between 40 and 48 points.

At the end of the school year 13% of the students had a points score between 20-29 points, the majority of the students had a score between 30-39 points and the rest of 24% of the students showed the highest results between 40 and 48 points.

Summary of the students' results on all test items

The following diagrams show the solution's frequency for all of the test items on the tests – Test 1(2005) and Test 2(2006).

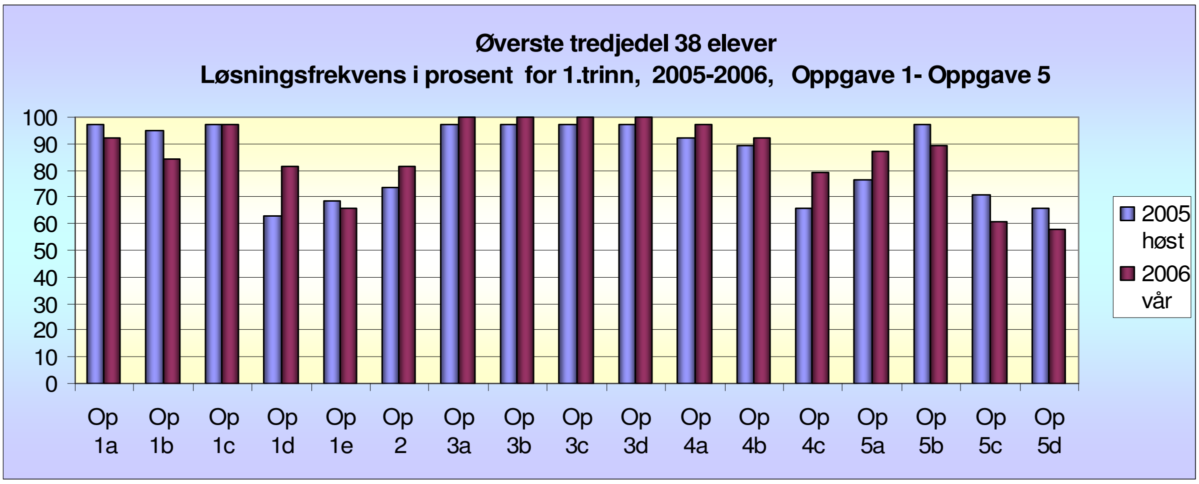


Figure 4.3.1.3.18: 11th grade, solution frequencies for test items 1a – 5d, Øverste group of students

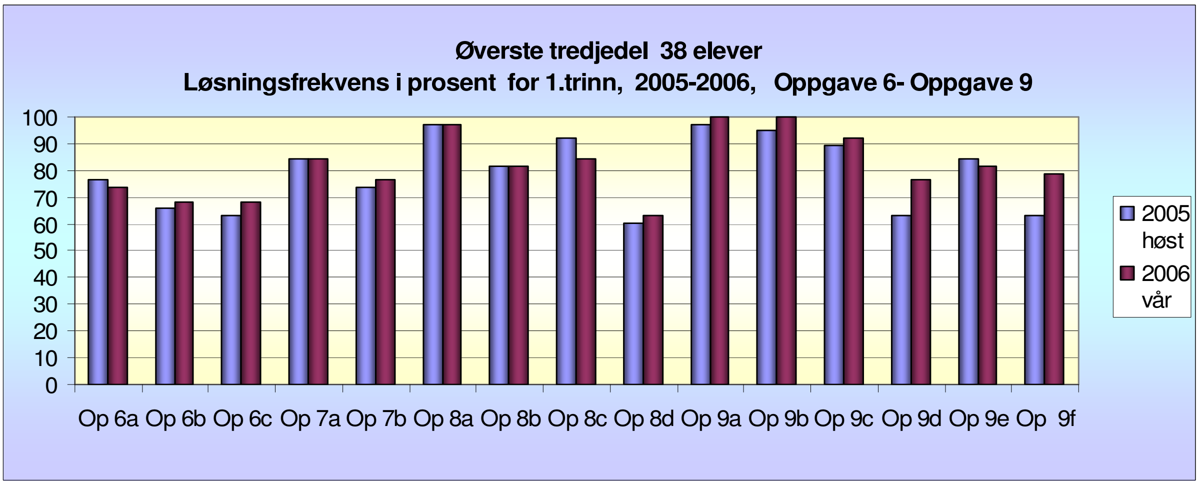


Figure 4.3.1.3.19: 11th grade, solution frequencies for test items 6a – 9f, Øverste group of students

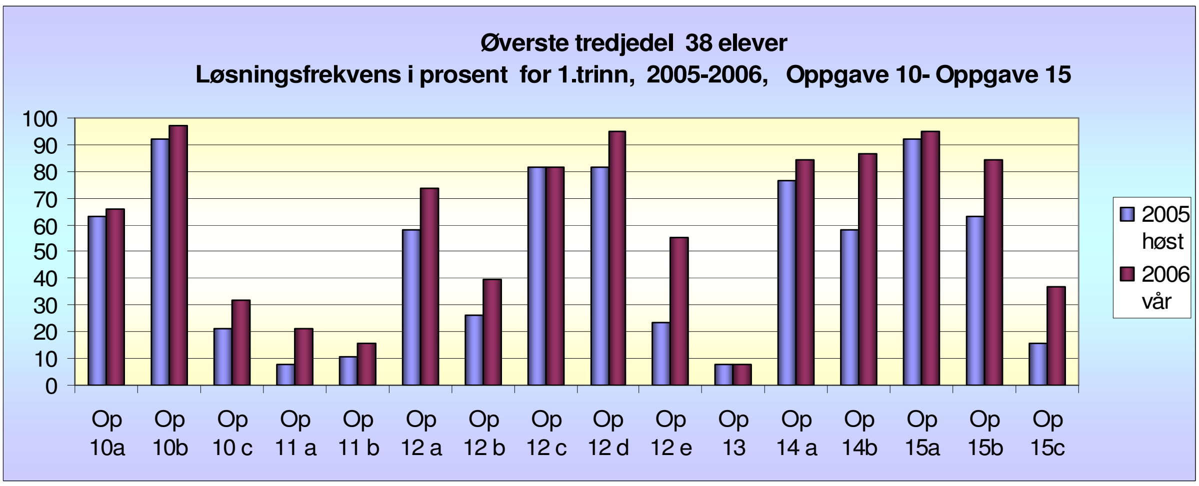


Figure 4.3.1.3.20: 11th grade, solution frequencies for test items 10a – 15c, Øverste group of students

Most of the tasks were solved by at least half of the students. There was a big improvement in the spring for the most difficult tasks Op 10c, Op 11a, Op 11b, Op 13, and Op 15c. The results for those tasks in the Nederste and Midterste groups of students groups are much lower. The majority of the students had very high results for the algebraic tasks.

Group of test items with big improvement of the results on the second test

Increase of 20 solution frequency points or more

For a group of seven test items there was a very significant increase of the solution frequencies, about 20 solution frequency points or more. The finding the value of an algebraic expression when the values of the variables are given for the problem Op 12e (Hva er da verdien av uttrykket $3b^2 - abc$ når a = 3, b = -1 og c = 5.) was a difficult task for the students in the fall, and it was solved only by 24% of the students, but there was a big improvement of the results 55% of the students solved the task in the spring. The students demonstrated also very good improvement for the solution of an open ended task requiring application of problem solving skills. The students demonstrated much better abilities to explain their solutions - on the task Op 15b "Hvordan kan du gå fram for å finne tallene?", and to generalise their solutions - on the task Op 15c "Hvorfor er det alltid mulig å finne to tall når vi kjenner summen og differensen?".

Oppgave		Frekvens i prosent	
		2005 høst	2006 vår
Op 1d	$70 \cdot 0{,}3 =$	63	82
Op 9f	Skriv enklere dersom det er mulig: $5a-2(7-a)+6$	63	79
Op 12 a	Hva må x være dersom $123 + 2x = 195 - x$	58	74
Op 12 e	Hva er da verdien av uttrykket $3b^2 - abc$ når a = 3, b = -1 og c = 5.	24	55
Op 14b	$y = 4 + 2{,}5x$ Sett kryss foran det svaret nedenfor som du mener er best: Hva betyr tallet 2,5 her?	58	87
Op 15b	Hvordan kan du gå fram for å finne tallene?	63	84
Op 15c	Hvorfor er det alltid mulig å finne to tall når vi kjenner summen og differensen?	16	37

Table 4.3.1.3.11: Group of tasks

Group of test items with decrease in the results on the second test

For nine of the test items the students from the Øverste group showed lower results on the second test. For three of those items Op 1e, op6a, Op 9e the results were only 3 solution frequency points lower. In the table below are presented all test items with decline in the results. For Op 1b ($\frac{1}{2} - \frac{1}{3} =$) and Op 5c (Skriv som desimaltall 45 tusendeler.......) there was the biggest decrease in the results on the second test. For some of the items the decrease was very minimal and was between 2 and 5 solution frequency points.

Oppgave		Frekvens i prosent	
		2005 høst	2006 vår
Op 1a	$\frac{1}{2}+\frac{1}{4}=$	97	92
Op 1b	$\frac{1}{2}-\frac{1}{3}=$	95	84
Op 1e	$60 \cdot 450 =$	68	66
Op 5b	Skriv som desimaltall $\frac{46}{100}$	97	89
Op 5c	Skriv som desimaltall 45 tusendeler	71	61
Op 5d	Skriv som desimaltall 28 tideler	66	58
Op 6a	*1 liter hvetemel veier 0,8 kg. Hvor mye veier 0,7 liter hvetemel?*	76	74
Op 8c	$15 : 10 = \square \cdot 15$	92	84
Op 9e	$3a + a^2 + a^2$	84	82

Table 4.3.1.3.12: Group of tasks

Group of test items solved by more than half of the students

The majority of the test items (a group of 42 items) were solved by at least half of the students on one of the tests. The group of students showed very high results on tasks requiring skills for working with simple algebraic expressions as simplification of algebraic expressions and finding the value of an algebraic expression when the values of the variables are given. Most students showed very good skills in solving tasks involving addition and subtraction of fractions, multiplication and division of integers, and multiplication and division of decimals. The solution frequencies for 28 tasks solved by at least 80% of the students on one of the tests are presented in the table below.

Oppgave		Frekvens i prosent	
		2005 høst	2006 vår
Op 1a	$\frac{1}{2}+\frac{1}{4}=$	97	92
Op 1b	$\frac{1}{2}-\frac{1}{3}=$	95	84
Op 1c	900 : 30 =	97	97
Op 1d	$70 \cdot 0{,}3 =$	63	82
Op 2	Finn et tall med to desimaler som ligger mellom 4,755 og 4,762	74	82
Op 3a	Fyll ut hele tabellen (*x=5)*	97	100
Op 3b	Fyll ut hele tabellen ($\frac{x}{2}$ =12)	97	100
Op 3c	Fyll ut hele tabellen (x^2 =16)	97	100
Op 3d	Fyll ut hele tabellen (4*x=2)*	97	100
Op 4a	Sett ring rundt det tallet som ligger nærmest 0,16 i størrelse 0,1 0,2 15 0,21 10	92	97
Op 4b	Sett ring rundt det tallet som ligger nærmest 2,08 i størrelse 209 2,9 2,05 2,1 20,9	89	92
Op 5a	Skriv som desimaltall $\frac{3}{10}$	76	87
Op 5b	Skriv som desimaltall $\frac{46}{100}$	97	89
Op 7a	*1 kg svinekoteletter koster 65,50 kr. Hva koster 0,76 kg?*	84	84
Op 8a	$14 : 2 = \square \cdot 14$	97	97
Op 8b	$14 : \square = 0{,}25 \cdot 14$	82	82
Op 8c	$15 : 10 = \square \cdot 15$	92	84

Op 9a	$2x + 5x$	97	100
Op 9b	$x + x + 2x$	95	100
Op 9c	$t \cdot t \cdot t$	89	92
Op 9e	$3a + a^2 + a^2$	84	82
Op 10b	$a + b{\cdot}2 = 2b + a$ Dette ☐ er alltid sant ☐ er aldri sant ☐ *kan* være sant, nemlig når	92	97
Op 12 c	$3x = 7$ og $5y = 11$ Hva er da $3x + 5y$? $3x + 5y$ =.............................	82	82
Op 12 d	Finn tallet x om x = a + b - c og a = 1, b = 2 og c = 3	82	95
Op 14 a	y = 4 + 2,5x Sett kryss foran det svaret nedenfor som du mener er best Hva betyr tallet 4 her?	76	84
Op 14b	y = 4 + 2,5x Sett kryss foran det svaret nedenfor som du mener er best. Hva betyr tallet 2,5 her?	58	87
Op 15a	Even tenker på to tall. Summen av dem er 19. Differensen mellom dem er 5. Finn tallene.	92	95
Op 15b	Even tenker på to tall. Summen av dem er 19. Differensen mellom dem er 5. Hvordan kan du gjøre for å finne talla?	63	84

Table 4.3.1.3.13: Group of tasks

Comparison of the results for the different groups

Additional information can be presented as a comparison of the results for different groups, who performed the tests. This information shows a comparison of data presented in different parts of the analyses and data concerning the previous school year, which was analysed by Espeland (2006). The following table was made using as source data the results for the group of 206 students (2004-2005), analysed by Espeland (2006), the results for the group of 227 students (test 1, 2005 fall), the results for the group of 126 students (test 2, 2006 spring), and the results for the group of 92 students (both tests, 2005 – 2006). It shows the minimum, maximum, average and the standard deviation for the results of the groups for the first and the second test.

Elevgruppe		Antall	Test år	Minimum	Maksimum	Gjennomsnitt	St.avvik
Elever - Test 1 & Test 2		206	2004 høst	2	41	21,6	8,3
			2005 vår	4	43	24,4	8,3
Alle Elever -Test 1		227	2005 høst	2	42	22,7	8,2
Alle Elever -Test 2		126	2006 vår	9	45	30,2	7,6
Elever - Test 1 & Test 2		113	2005 høst	4	42	26,2	7,3
			2006 vår	10	45	30,5	7,3
Elever Test 1 & Test 2	Nederste tredjedel	37	2005 høst	4	23	17,8	4,1
			2006 vår	10	38	24,3	6,5
	Midterste tredjedel	38	2005 høst	23	30	26,4	1,7
			2006 vår	23	40	30,4	4,5
	Øverste tredjedel	38	2005 høst	30	42	34,1	3,1
			2006 vår	26	45	36,6	4,8

Table 4.3.1.3.14: 11th grade, Statistical information for the group of 206 students (2004-2005) and for 227 students, 126 students, 113 students, Nederste, Midterste and Øverste groups of students.

4.3.2 A comparison with the previous year

4.3.2.1 Results of the group of 227 students

In this part of the analyses we will present the results for a group of 227 students who did the first test. The results of the group of 227 students could not be compared with the results for the group of 126 students who did the test in the spring, because they were two different sample groups.

In the following table are presented the results for the group of all 227 students who did the first test in the beginning of the school year – the average points score, the minimum and the maximum points score for all students. It's important to point out that those results are on all test items included in the test, with maximum possible score on the test 48 points.

Elevgruppe	Antall	Test år	Minimum	Maksimum	Gjennomsnitt	St.avvik
Alle Elever - Test 1	**227**	2005 høst	2	42	22,7	8,2

Table 4.3.2.1.1: 11^{th} grade, Results in points for the group of all students, test 1, fall 2005.
Note: The maximum score a student could receive for this test was 48 points.

4.3.2.2 Solution of different tasks

Summary of the students' results on all test items

The results for a group of 11 tests items show that they were solved by at least 70% of the students. Those were the tasks 1a-1c, 3a-3d, 4a, 9a-9c, 10b. Eight of those tasks had solution frequencies at least 80 solution frequency points. Those tasks were related to computations of arithmetical expressions and operations with simple algebraic expressions. Tasks 10c, 11a, 11b, 12b, 12e, 13, 15c had very low solution frequencies - under 20 solution frequency points.

The following diagrams show the solution frequencies for all test items included in the test in the fall 2005. The students' answers were coded with code 1 for the correct answers or code 2 and 3 for partly correct answers.

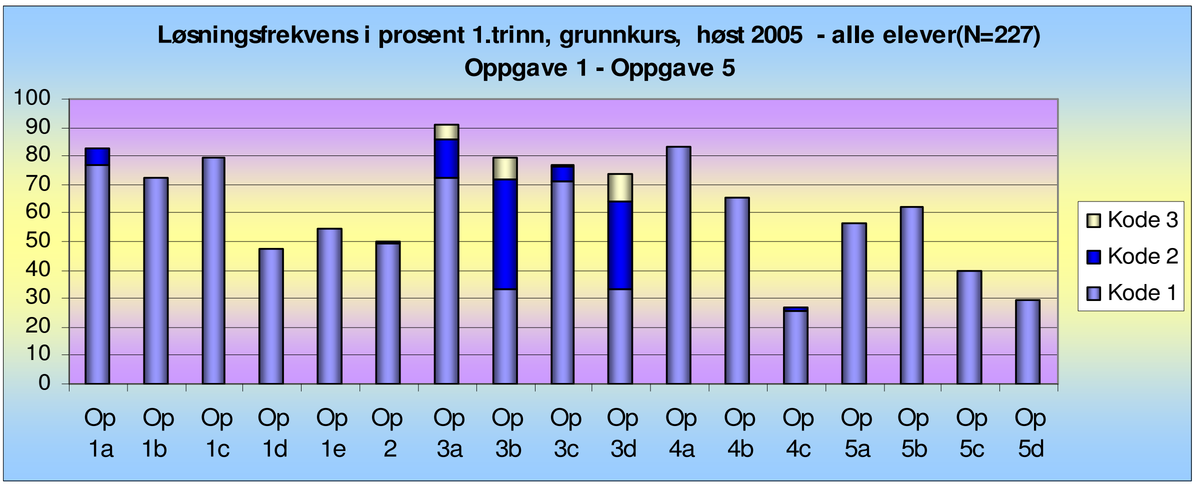


Figure 4.3.2.2.1: 11^{th} grade (N=227) - Solution frequencies in percents on tasks 1a-5d, test 1(2005 fall)

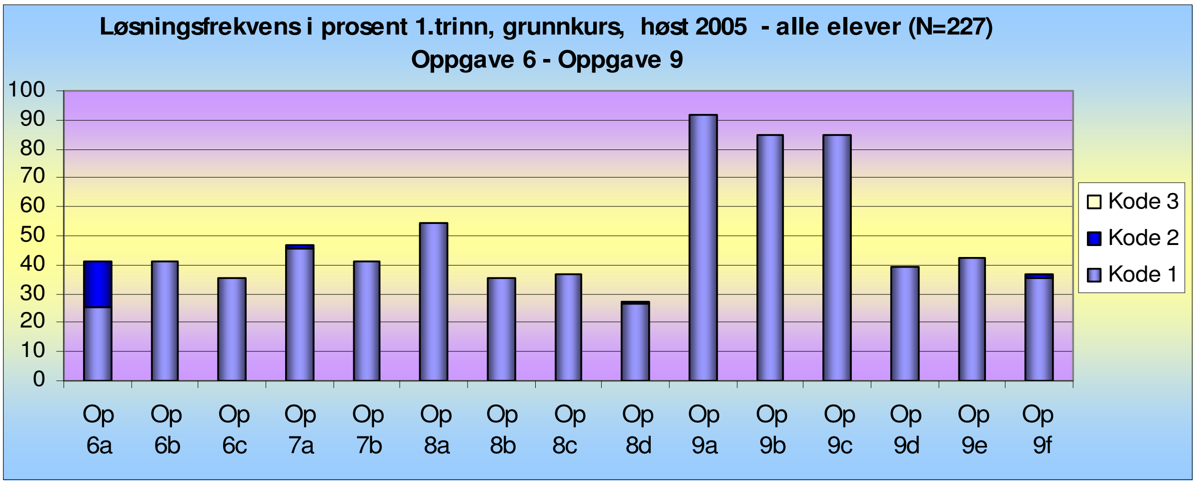


Figure 4.3.2.2.2: 11th grade (N=227) - Solution frequencies in percents on tasks 6a-9f, test 1(2005 fall)

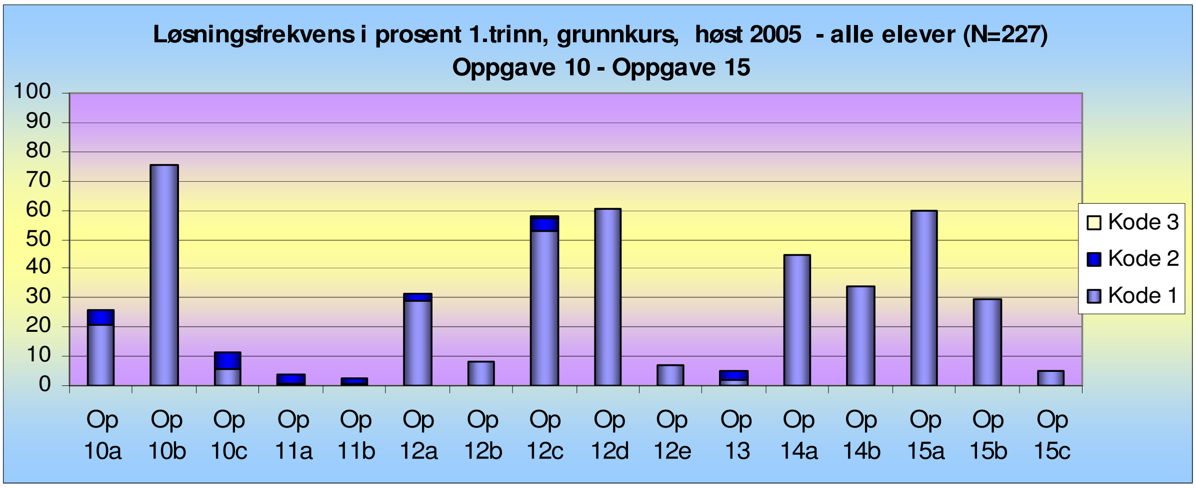


Figure 4.3.2.2.3: 11th grade (N=227) - Solution frequencies in percents on tasks 10a-15c, test 1(2005 fall)

Next we would have a closer look to the students' solutions of a concrete task. The task 12e is related to finding the value of an algebraic expression. After that we would present different groups of tasks.

Finding the value of an algebraic expression

The confusions caused by negatives numbers are studied by variety of scholars. It is almost impossible to connect negative numbers with intuition and common sense, argued Smith (2002). Smith described the problems that students experience when working with negative numbers and summarized that negative numbers probably had caused more headaches to mathematical beginners and to mathematical professionals than any other mathematical form. In addition there is not real life analogy for a negative quantity of anything that doesn't involve mathematical understanding. Smith pointed that negative numbers should be considered important and students should be trained to understand their full complexity, because for example it is confusing for students to learn that in mathematical terminology the word "minus" and the minus sign are used as verbs for the operation of subtraction, but also as adjectives to designate negative quantity.

Oppgave 12e Hva er da verdien av uttrykket $3b^2 - abc$ når $a = 3$, $b = -1$ og $c = 5$.

$3b^2 - abc =$.......................................

Table 4.3.2.2.1: 11th grade (N=227), Task 12e - solution frequency for Test 1 (2005 fall)

Oppgave 12e	frekvens i prosent
	2005 høst
Ikke besvart	41
18	**7**
-12	5
Innsatt tall, men ikke regnet ut	7
andre svar bokstavuttrykk	4
andre svar tallverdier	35

A small group of 7% of the students tested in the fall could do correctly the calculations required by the task. The results show that the majority of the students (52% of the students) received a different result. The variety of the given error answers is very big.

In order to find the value of the algebraic expression $3b^2$- *abc*, the students need to know in what order they should do the calculations. Students need to know in what order they have to do the calculations – that first they have to calculate the value $3b^2$ and the value of the product *abc*. Those operations required the use of parenthesis around the negative numbers, but probably many students omitted the parenthesis. That resulted in large number of students who found incorrect answers, because they didn't know how to use parenthesis.

When solving this problem the students should know that the square of a negative number is a positive number. When a negative number should be subtracted from another number, we have to apply the familiar rule that two negative signs make a positive sign. In this task the values for the variables *a, b* and *c* (the numbers 3, -1 and 5) should be multiplied. The result is a negative number (-15) and because the students have to subtract it, the students have to know how to subtract negative numbers. In algebra it is not necessary to write multiplication sign between unknowns and that is a major difference with arithmetic.

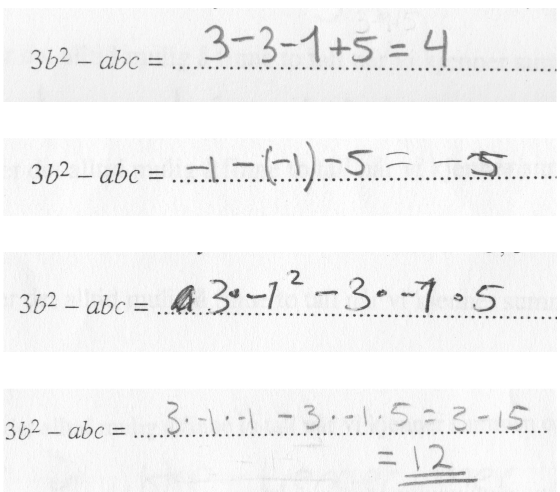


Figure 4.3.2.2.4: Answers given by four students, Task 12e

Students' difficulties

The results show that many students had limitations when finding the value of the given algebraic expression. The possible reasons are that students had problems using the mathematical language in a proper way. The task requires a lot of precision and attention. Students experience problems with doing correctly the substitution of the unknowns with their values. A common reason for mistakes was when the students forgot to use parenthesis around negative numbers when it was necessary. It is possible that those students had problems understanding the arithmetic of negative numbers.

Linchevski & Herscovics (1996) supported the necessity for arithmetic knowledge base, particularly with respect to the use of brackets and the order of operations.

Now we would like to go closer to some groups of tasks from the tests. It is necessary to find which test items were solved by the majority of the students, which tasks were the most difficult, the tasks solved by between 25% and 49% of the students, and the test items with very high results.

Test items with solution frequency between 25 and 49 solution frequency points

The test contained 19 tasks which were solved by between 25% and 49% of the students. In this group are many of the worditems, there are also some algebraic tasks, requiring simplification of expressions and solution of algebraic equations with one unknown, and tasks related to a linear function. This group of problems includes the task Op 15b "Even tenker på to tall. Summen av dem er 19. Differensen mellom dem er 5. Hvordan kan du gå fram for å finne tallene?" solved by 30% of the students.

Oppgave		Frekvens i prosent
		2005
Op 1d	$70 \cdot 0{,}3 =$	47
Op 4c	Skriv riktig tall i rutene $5{,}074 = 5 \cdot 1 + 7 \cdot \square + 4 \cdot \square$	27
Op 5c	Skriv som desimaltall 45 tusendeler	40
Op 5d	Skriv som desimaltall 28 tideler	30
Op 6a	*1 liter hvetemel veier 0,8 kg. Hvor mye veier 0,7 liter hvetemel?*	41
Op 6b	*Kaker skal fylles i bokser, med 0,75 kg i hver. Hvor mange bokser trenger man til 6 kg kaker?*	41
Op 6c	*Anne kjøper bananer i en butikk til 13,50 kr per kg. Hvor mye kan Anne kjøpe for 10,50 kr?*	35
Op 7a	Sett opp et regneuttrykk som passer for å løse oppgaven: (Du skal ikke regne det ut). *1 kg svinekoteletter koster 65,50 kr. Hva koster 0,76 kg?*	47
Op 7b	Sett opp et regneuttrykk som passer for å løse oppgaven: (Du skal ikke regne det ut). *Et terrengløp går i ei løype som er 5,6 km. Hvor mange engelske mil er det? (Ei engelsk mil er 1,609 km).*	41
Op 8b	$14 : \square = 0{,}25 \cdot 14$	35
Op 8c	$15 : 10 = \square \cdot 15$	37
Op 8d	$8 : \frac{1}{2} = 8 \cdot \square$	27
Op 9d	$2y \cdot y^2$	39
Op 9f	$5a - 2(7 - a) + 6$	37
Op 10a	**a** $x + y + z = x + p + z$ Dette $\square$ er alltid sant $\square$ er aldri sant $\square$ *kan* være sant, nemlig når	26
Op 12 a	$123 + 2x = 195 - x$	32
Op 14 a	Om y står for vekta i kg og x står for barnets alder i antall år, så er: $y = 4 + 2{,}5x$ Sett kryss foran det svaret nedenfor som du mener er best: Hva betyr tallet 4 her?	44
Op 14b	Om y står for vekta i kg og x står for barnets alder i antall år, så er: $y = 4 + 2{,}5x$ Sett kryss foran det svaret nedenfor som du mener er best: Hva betyr tallet 2,5 her?	34
Op 15b	Even tenker på to tall. Summen av dem er 19. Differensen mellom dem er 5. Hvordan kan du gå fram for å finne tallene?	30

Table 4.3.2.2.2 : Group of tasks

Group of test items solved by between 50% and 79% of the students

In this group of items are included 12 of the test items with solution frequencies between 50 and 79 solution frequency points. This group included the task 15a "Even tenker på to tall. Summen av dem er 19. Differensen mellom dem er 5. Finn tallene. ", solved by 59% of the students. About half of the students could do correctly the operation "60 · 450 =".

Oppgave		Frekvens i prosent
		2005
Op 2	Finn et tall med to desimaler som ligger mellom 4,755 og 4,762	50
Op 1 b	$\frac{1}{2} - \frac{1}{3} =$	73
Op 10b	$a + b \cdot 2 = 2b + a$ Dette ☐ er alltid sant ☐ er aldri sant ☐ *kan* være sant, nemlig når	76
Op 12 c	$3x = 7$ og $5y = 11$ Hva er da $3x + 5y$?	58
Op 1e	60 · 450 =	55
Op 3 c	Fyll ut hele tabellen: ($x^2 = 16$)	77
Op 3 d	Fyll ut hele tabellen: ($4x = 2$)	74
Op 4b	Sett ring rundt det tallet som ligger nærmest 2,08 i størrelse 209 2,9 2,05 2,1	66
Op 5a	Skriv som desimaltall $\frac{3}{10}$	56
Op 5b	Skriv som desimaltall $\frac{46}{100}$	62
Op 8a	$14 : 2 = \square \cdot 14$	55
Op 15a	Even tenker på to tall. Summen av dem er 19. Differensen mellom dem er 5. Finn tallene.	59

Table 4.3.2.2.3 : Group of tasks

Test items with high results (solution frequency above 80 solution frequency points)

The test contained eight tasks which were solved by at least 80% of the students. Those tasks required skills related to simplification of algebraic expression, skills in working with simple algebraic expressions, comparison of decimal numbers, addition of fractions and calculations with integers. The highest results were the results on the task Op 9a and Op 3a.

Oppgave		Frekvens i prosent
		2005 høst
Op 1 a	$\frac{1}{2} + \frac{1}{4} =$	82
Op 1c	900 : 30 =	80
Op 3 a	Fyll ut hele tabellen: ($x = 5$)	91
Op 3 b	Fyll ut hele tabellen: ($\frac{x}{2} = 12$)	80
Op 4a	Sett ring rundt det tallet som ligger nærmest 0,16 i størrelse 0,1 0,2 15 0,21	83
Op 9a	$2x + 5x$	92
Op 9b	$x + x + 2x$	85
Op 9c	$t \cdot t \cdot t$	85

Table 4.3.2.2.4 : Group of tasks

Test items with minimal results (solution frequency between 0 and 11 solution frequency points)

Here are presented a group of tasks that were the most difficult to solve. Those tasks were solved by not more than 11% of the students. To solve those tasks it is essential to apply abilities for conceptual understanding, abilities to reason mathematically, representation abilities, abilities to manipulate symbolic statements, ability to express mathematical ideas in a written form.

Oppgave		Frekvens i prosent 2005 høst
Op 10 c	$\frac{2x+1}{2x+1+5} = \frac{1}{6}$ Dette ☐ er alltid sant ☐ er aldri sant ☐ *kan* være sant, nemlig når	11
Op 11 a	For å finne volumet *V* av en sylinder, har vi denne formelen: $V = \text{Pi}.r^2h$. Her er *r* radius i grunnflata , og høyden er *h*. Hva skjer med volumet dersom vi dobler både radius *r* og høyde *h*?	4
Op 11 b	Hvordan må høyden endres dersom vi dobler radius *r*, men vil at sylinderen skal ha samme volum *V*?	3
Op 12 b	$\frac{x+1}{x+4} = \frac{4}{5}$	8
Op 13	På en skole er det 10 elever for hver lærer. Hvilke av disse uttrykkene forteller dette? L = antall lærere, E = antall elever. Sett ring rundt det eller de riktige uttrykk.	5
Op 15c	Even tenker på to tall. Summen av dem er 19. Differensen mellom dem er 5. Hvorfor er det alltid mulig å finne to tall når vi kjenner summen og differensen?	5

Table 4.3.2.2.5 : Group of tasks

4.3.2.3 Comparison of the results with the previous year results

In this report the results for all 227 students tested in the fall of 2005 are compared with the previous year results for the group of all 236 students (test 1, fall 2004) reported by Andreassen (2005). Both groups of students took the test early in the fall. This was the time close after the school year started and many students probably were affected by the time of the test. It was a time for many changes for the students being in a new class and school, and having new classmates. Here are presented diagrams that show the results of the comparison between both groups students - all students tested in the fall of 2005 – 227 students, compared with all students tested the previous fall 2004 – 236 students. The data for the group of students tested in 2004 was taken from an SPSS file prepared by Andreassen. This database was not necessary to change – all test items which the students received on the test were included in the data for analyses.

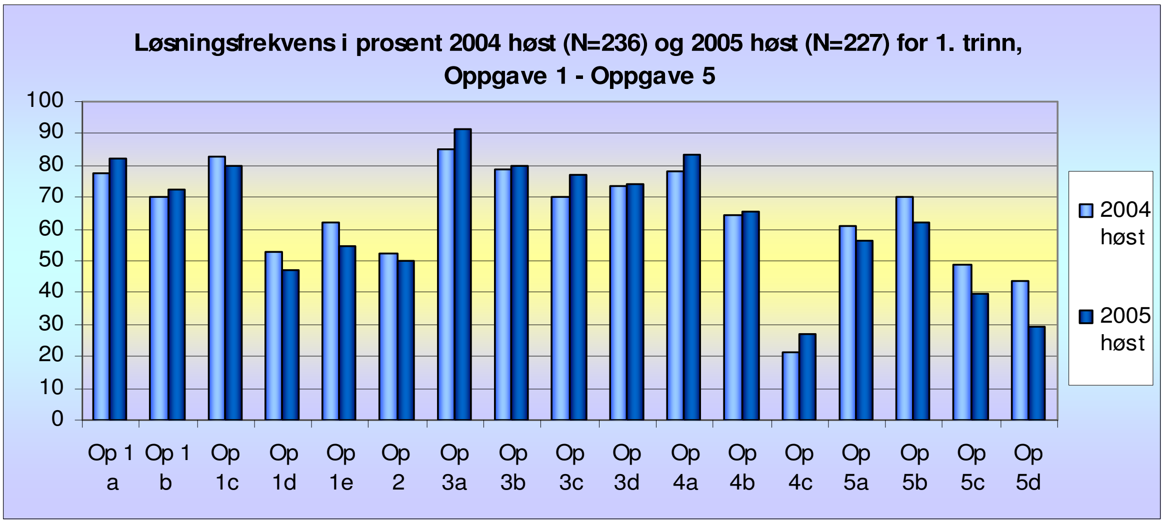


Figure 4.3.2.3.1: 11th grade, solution frequencies for test items 1a – 5d for the group of 236 students (first test - fall 2004) and the solution frequencies for the compared group of 227 students (first test – fall 2005).

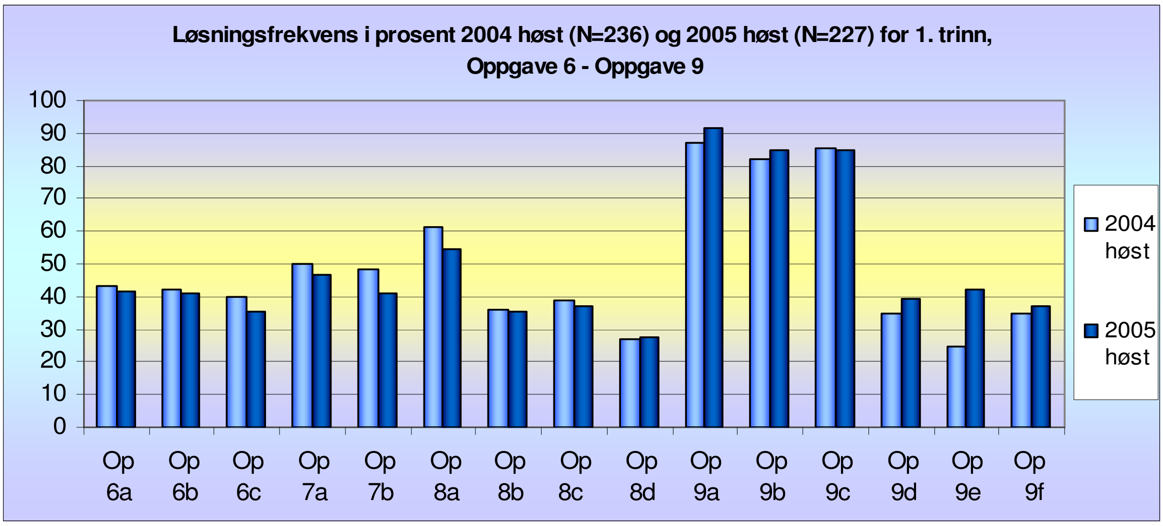


Figure 4.3.2.3.2: 11th grade, solution frequencies for test items 6a – 9f for the group of 236 students (first test - fall 2004) and the solution frequencies for the compared group of 227 students (first test – fall 2005).

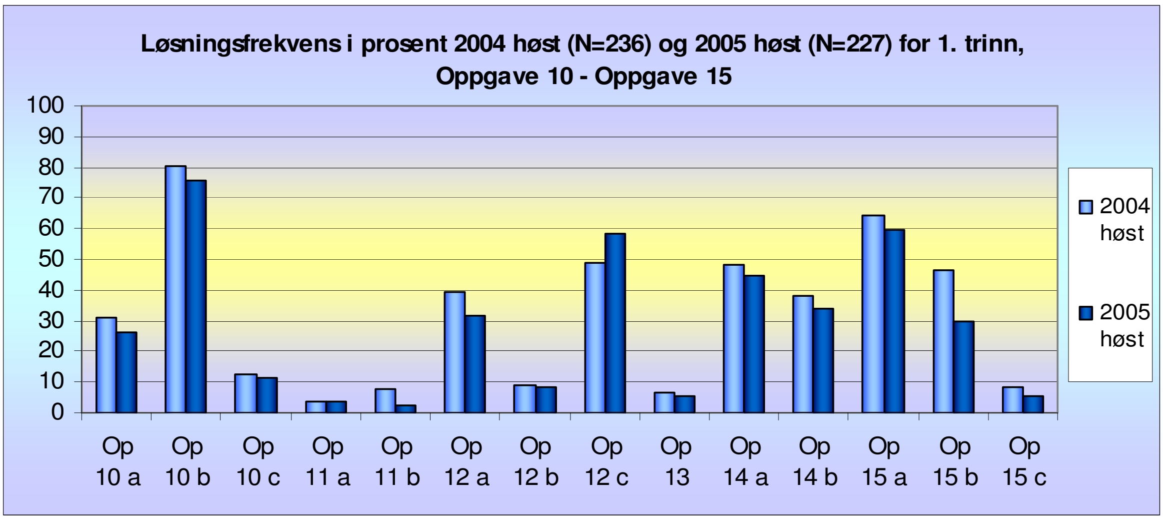


Figure 4.3.2.3.3: 11th grade, solution frequencies for test items 10a – 15c for the group of 236 students (first test - fall 2004) and the solution frequencies for the compared group of 227 students (first test – fall 2005).

In order to be possible to compare the tests results for the both groups was necessary to analyse the students' presentation at equal bases. Tasks 12d and 12e were not included in the comparison - the students did them in the fall of 2005, but the first groups of students tested in 2004 (fall) didn't receive those tasks. The text of problem 11a and 11b has been reformulated in the test done by the students in 2005, the tasks were included in the comparison, because the changes were minimal and in order to make the mathematical problem more clear to the students. The diagrams above include all tasks which were done by both groups of students and the maximum possible score a student could get for those test items was 46 points.

There were tasks which were very difficult for the majority of students. The most difficult were the tasks Op 10c, Op 11a, Op 11b, Op 12b, Op 13 and Op 15c. The tasks 11a, 11b and 15c were the most complex tasks in the test, requiring application of variety of skills – reasoning, generalising, justification of the solution, accuracy when working with algebraic expressions. All those tasks with very low solution rate are tasks with very high number students who try to solve the tasks, but were struggling when trying to answer.

The table presented here gives information what was the minimal, maximum and average solution frequency for all test items included in the comparison. The solution frequency of the items is data with big variations – many of the tasks were very easy for the students, but there were particular problems with very low number of students who could find the solution.

The group of all 227 students who did the test in the fall of 2005 had higher solution frequency for 16 of the tests items, compared with the results for the 236 students from the previous year 2004. Those were allitems from the beginning part of the test, 1a, 1b, 3a - 3d, 4a - 4c, 8d, 9a, 9b, 9d - 9f, 12c. For those 16 test's items the average increase of the solution frequency was 4,7 solution frequency points, where the maximum increase was 17,3 solution frequency points for the problem 9e. Among those tests items were 10items with very high solutions frequency (above 70 solution frequency points) for both groups of students that were compared. The other 6 tests items had solution frequency below 50 solution frequency points on the tests.

The group of 227 students who performed the test in the fall of 2005 had lower results for 30 of the test's items, compared with the first group of 236 students tested in 2004. Those were testsitems 1c, 1d, 1e, 2, 5a - 5d, 6a - 6c, 7a, 7b, 8a - 8c, 9c, 10 a - 10c, 11a, 11b, 12a, 12b, 13, 14a, 14b, 15a - 15c. Only 11 of those items had solution frequency above 50 solution frequency points for the first group of students tested in 2004. The group of 227 students had results for those 30 test items with average decrease of 4,7 solution frequency points compared with the first group of 236 students tested in 2004. The maximum decrease was 16,7 solution frequency points for problem 15b.

The following diagrams show the dividing of the students in subgroups according to their results on the test. The first diagram shows the results for the group of 236 students who did the test in 2004 and the second diagram shows the results for the group of 227 students who did the test in the fall of 2005. The maximum points sum in this comparison is 46 points, because it's necessary to include tests items done by both groups of students. The diagrams and the table give information about each group of students who did the tests in 2004 and in 2005 and how it can be divided in subgroups. The subgroups are – students who received between 1 and 9 points, between 10 and 19 points, between 20 – 29 points, between 30 – 39 points and between 40 and 46 points.

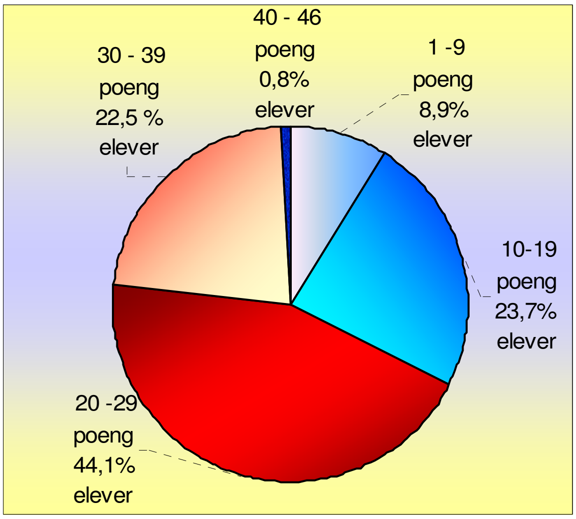


Figure 4.3.2.3.4: 11th grade (N=236) – test 1, 2004, Distribution of the students' points sum in subgroups

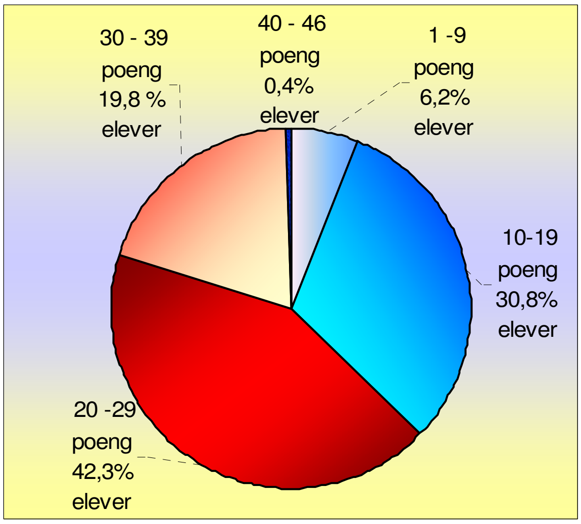


Figure 4.3.2.3.5: 11th grade (N=227) – test 1, 2005, Distribution of the students' points sum in subgroups

Elevgruppe	År	Elever	1 - 9 poeng	10 - 19 poeng	20 - 29 poeng	30 - 39 poeng	40 - 46 poeng
Alle (N=236)	2004	antall	21	56	104	53	2
		frekvens	8,9	23,7	44,1	22,5	0,8
Alle (N=227)	2005	antall	14	70	96	45	1
		frekvens	6,2	30,8	42,3	19,8	0,4

Table 4.3.2.3.1: 11th grade, 236 students (2004) and 227 students (2005): Distribution of the students points sum in percents, results on the first test. Note: the maximum points sum in this comparison is 46 points.

To make those diagrams was necessary to make minimal changes - the items 12d and 12e which were done by the students in 2005 were excluded from the data that was analysed and the students' points score was counted for the test reduced with those two tasks. The database for the 236 students who did the test in the fall of 2004 was not changed - all tasks done by the students were included in this comparison. After that the points score for every student was counted. Than the students were sorted in groups according the points score of each student.

Looking at this data we can't notice very big differences between the group of 236 students who did the test in the fall of 2004 and the group of 227 students who did the test next fall. The group of students with score between 20 and 29 points were the biggest groups both years. If we combine points groups we can find that the majority – 68% of the students (tested the fall 2004) and 73% of the students (tested the fall 2005) had a score between 10 and 29 points, or close to the average 22,7 and 22,1 points for both groups.

The average score for the group of 236 students tested in the fall of 2004 was 22,7 points, the minimum score in this group was 3 points and the maximum score was 42 points. The other big group of 227 students (tested in the fall of 2005) had very similar average score 22,1 points, the minimum score for the group was 2 points and the maximum score 40 points on the test reduced with items 12d and 12e.

5. Discussion

The main purpose of this chapter will be to describe the most important moments of the development during the school year for the groups of students analysed by this study. In addition are discussed the main findings of the longitudinal comparisons of the new results with the previous results. It is important to discuss the results for selected tasks used in the previous studies KIM, Kassel Exeter, and Evaluering or Reform 97.

In the literature review of this study we presented different issues related to the students learning of mathematics. We support the need of integrated and balanced development of all strands of mathematical proficiency - that teachers give enough attention to the learning of facts, skills, concepts and procedures, emphasise the need of reasoning and mathematical writing and pay enough attention to communications. Kilpatrick et al. (2001) recommended that all strands of proficiency grow in balance and supported the idea that mathematics is taught in order to achieve integrated development of all strands of proficiency. Kilpatrick et al. (2001) considered that students can not become proficient in mathematics, if the instruction focuses on one or two of the strands of successful mathematical learning. For example students who can not learn mathematical procedures with conceptual understanding also do not go much further than applying procedures, because they lack flexibility and efficiency in using procedural knowledge when solving problems. We support the view that learning is more effective when students can apply different rules and procedures accurately. Otherwise students experience major difficulties in solving with problems - students might have problems to see certain relationships and waste time by finding simple relations. We consider important the role of the students' attitudes and beliefs, towards the subject of mathematics although our study did not focus on this strand of mathematical proficiency. Kilpatrick et al. (2001) noted that when students achieve better results in the subject of mathematics they develop also better self confidence and more positive attitudes towards mathematics.

The use of diagnostic tests can be considered to be a tool for finding the progression of students' knowledge, finding the common misconceptions that students acquire and how these misconceptions develop. A number of common misconceptions are already described in literature. Some students hold misconceptions as that division is commutative like addition, or that the smaller number can not be divided by the larger number (Booth, 1988; Smith, 2006). Algebra has its roots in arithmetic and depends on a strong arithmetical foundation. If students do not have proper number sense they may experience significant problems in the learning of algebra. Fluency in computing with common fractions can later help students become fluent in computing with rational expressions in algebra. Working with variables is an important part of the learning of algebra. Students need to develop an initial understanding of several different meanings and uses of variables and students need to work frequently with manipulation of symbolic expressions. Fluency in working with symbolic expressions is connected with proper understanding of the order of operations. It is necessary that students are familiar with the distributive, associative, and commutative properties. Solution of equations is an important topic and students have to develop abilities in transforming equations in order to solve them. To learn to use algebra students need mathematical skills in dealing with numbers, beyond basic calculations with whole numbers (MacGregor, 2003). Many items can be solved using algebraic methods, but can be solved successfully also using arithmetical methods. It is important to determine what makes a mathematical problem or activity algebraic, arithmetical or pre-algebraic. It is not the nature of the task, but the nature of the solution methods that matters, argued Van Amerom (2002).

The analyses of this study presented students' results organised in five different parts. The study focused mainly on the results of the groups of 9th and 11th grade students who did both tests and on the comparison of the results with the previous year results for the groups of 9th grade students who did both tests. In addition we compared the results for all participating students in 9th and 11th grade for the test performed at the beginning of the school year with the results of all participating students in 9th and 11th grade on the first test, the previous school year.

In the analyses part we discussed different questions related to the students' difficulties in the transition from arithmetic to algebra. There are presented evidences from the students' results that one of the major obstacles for students was the proper use of algebraic and arithmetical syntax, although the study did not have a specific aim on investigating closely such issues. According to Van Amerom (2002), the symbolizing and the reasoning competencies do not develop at the same pace. This study also provides data that some students even in 11th grade still use very informal way of symbolizing their results and have problems to use properly the algebraic syntax. Those issues were paid more attention in the investigations of the students' answers on the 'Volume tasks' 11a and 11b where we noticed that a major difficulty for the students was to find the correct algebraic formula for volume of a cylinder. Most students tried to construct a formula for the volume of the new cylinders, but experienced problems in analysing the geometrical situations and to use algebraic symbols. The students experience many difficulties in the transition from arithmetic to algebra and that was discussed by different studies (Van Amerom, 2002; Booth, 1988). Booth (1988) considered students' difficulties to be related to the focus of algebraic activity, the mathematical structures in algebraic expressions, the use of notation and convention in algebra and the students' use of informal methods. We consider that the task of exploration of a pattern is very suitable to investigate the level of concept of the letter symbol and the ability to generalise. The analyses of the results of the task 26c (9th grade) showed that only three students (test 2) were able to generalise the problem, although most students were able to find the answer of a similar question in the case of more simple context (task 26a and 26b). Those findings suggest that most students need help in order to develop abilities in generalising. Students need to learn how to construct a general expression using properly algebraic symbolization.

Next we would discuss the students results related to the areas of algebra and numbers. We would focus on the results of the students' groups who performed both tests.

5.1. 9th grade

The test for 9th grade contains test items, designed to assess the mathematical knowledge and abilities of the students in some important topics in algebra and numbers. There are different areas in which the students' mathematical knowledge and abilities were tested. The study has produced a number of results.

5.1.1 Development 9th grade

This section discusses the results of the group of 92 students, in relation to the research questions:

- What are the students' achievements shown in the results on the tests? What is the development in the students' results for the school year?

If we consider the difference of the results on a test item between the second and the first test, we can observe that in general there was a small decline of the results on six of the test items, very minimal increase of the results on 12 test items, there was small or moderate increase of the results on 25 test items and significant increase of the results on 14 test items.

The study found also that the girls had lower average results than the boys. The groups of students with very low point scores (between 1 and 9 points) were substantial and remained similar size groups on both tests. Such very low results had 16% of the students (test1) and 15% of the students (test 2). Those students are students who are achieving minimal results on the tests and potentially many of those students need more special help, so that they do not fall further behind. Although the test contained some very simple items, those students experienced major problems and the results indicated that they lacked basic knowledge and skills in mathematics.

Numbers

The study founds that many students in 9th grade experience serious problems to do simple computational tasks. Especially low are the results of the students from the Nederste group. The results for this group were already reported in details and they suggest that those students have very limited abilities to solve arithmetical tasks.

Integers
Students experience problems in interpreting the symbol and to divide two integers - only about one third of the students were able to answer correctly the expression (3 : 6 =….). The results on the task Op 9 ($60 \cdot 450 = ?$) related to multiplication of two integers are rather low – only 37% of the students (test 1) and 42% of the students (test 2) could do such simple calculation. Big improvement is observed on a some simple tasks, related to positional system of integers - the results on task 16a ($574 = 5\cdot 100 + \square \cdot 10 + 4 \cdot 1$), showed that it was solved by 46% of the students (test 1) and 68% of the students (test 2).

Fractions
The students showed rather low results on tasks involving simple operations with fractions. For example the task requiring subtraction of fractions ($\frac{1}{2} - \frac{1}{3} =$) was solved by 25% of the students (test 1)and 38% of the students (test 2). The results for a task involving addition of fraction ($\frac{1}{2} + \frac{1}{4} =$) were better, 41% of the students (test 1) and 56% of the students (test 2) solved the task, but a big group of students were not able to perform the calculation correctly. The students experienced difficulties also to convert a given fraction into a decimal number. For example the fraction $\frac{3}{10}$ was converted into decimal number by only 36% of the students (test 1) and 38% of the students (test 2) – the most common wrong answer was 3,10 (looking at the number 3 and 10 in the fraction as being a pair of numbers). The task 11 ($\frac{2}{5}$ av en masse er 20 gram. Hva er massen?) is a problem expressed in everyday language and the results show that about half of the students could make sense of the problem and perform the necessary steps to find the answer.

Decimal numbers
There were different tasks involving operations with decimal number and the students solutions varied for similar type of tasks according to the context of the tasks. The tasks involving comparison of a group of decimal numbers and finding the minimum and the

maximum were solved by the majority of the students. The solution frequencies for this type of tasks were high on the tests and varied between 56% and 86%. The students experienced problems to find the number among a group of numbers closest to a given number. About half of the students for example gave erroneously the number 2,05 as the closest to the number 2,08, but the answer 2,1 was found by only 28% of the students (test 1) and 37% of the students (test 2). The students experienced difficulties also in tasks involving comparison of decimal number or calculations with decimal numbers. For example only 22% of the students (test 1) and 29% of the students (test 2) could find a decimal number between the numbers 15,755 and 15,762.

More than half of the students experienced problems to do simple calculations with decimal numbers. The tasks 18 a-d for example are a group of simple tasks and they were solved by a lot less than half of the students. Very problematic for the students was to convert a given decimal fraction, described in words, to a decimal number. For both tasks "Skriv som desimaltall (a) 45 tusendeler, (b) 28 tideler" less than 20% of the students solved those tasks in the fall and 13% of the students solved them in the spring.

Construction of a word problem

The study found high number of 9th grade students who could not construct a word problem with mathematical context describing a given arithmetical expression. The students' difficulties can be related to the complexity of the tasks – that the students needed to use decimal numbers and that could potentially cause difficulties. Most students provided interpretations of the expressions, describing different mathematical context. Few students could construct relevant word problems.

Figure pattern

One of the common topics in arithmetic is to continue a number pattern, or to draw the next term of a figure pattern. Most students could find the solution to a problem related to a figure pattern in the specific cases. We found that most students were able to continue the pattern with more elements and found the number of elements in those cases. The study did not give details about the students' strategies of solution on those tasks, so we do not know how many students used help drawings and counting as a strategy or how many students used a more advanced approach focusing on the structure of the pattern.

Algebra

The students show a significant progress on tasks related to simplification of algebraic expressions. On the first test many of them treated letters as standing for single values, interpreted $2x$ as 20 or gave answers as for example that $2x + 5x$ is compacted to 7 or to $2x5x$. The groups of students who could properly do simplification of algebraic expressions were significantly higher on the second test. On the other hand the results on task 24a (Vurdering av ekvivalens $x + y + z = x + p + z$) showed that many students (26% (test1) and 45% (test 2)) answered that the given relationship can never be true, and such answers suggest that they believed that y and p, being different letters, must represent different numbers. The results on a task related to generalisation of pattern (task 26c) demonstrate that most students experienced problems to generalise. They provided a single symbol as an answer or they treated the unknown n as a specific number, because the students expected a numerical answer.

Simplification of algebraic expressions
In this area we observe a very high increase of the correct students' responses. The highest results were on the task 23a ($2x + 5x$.........), solved by 71% of the students (test 2) and on the task 23b ($x + x + 2x$ ......) solved by 60% of the students (test 2). On the other hand the test item 23d ($2y{\cdot}y^2$........) was a very difficult one. The results show that none of the students (test 1) could simplify this expression and that the task was solved by only 9% of the students (test2).

Operations with simple algebraic expressions
Most students developed much better abilities in operations with simple algebraic expressions, although the results of the group are still not high enough. Less than half of the students were flexible to solve such type algebraic problems requiring simple manipulations. Finding value of an algebraic expression is related to application of computational skills and conceptual knowledge. The tasks 27a, 27b, and 27c are tasks with different structure, and the responses for them show differences. The task 27a ($x = a + b - c$, Dersom $a = 1$, $b = 2$ og $c = 3$ blir $x =$ ....) for example was solved by 11% of the students (test 1) and 37% of the students (test 2). Similar results were the results for tasks 27c ($3x = 7$ og $5y = 11$ Da blir $3x + 5y =$ …), solved by 11% of the students (test 1) and 33% (test 2). A lot more difficult for the students was to find the value of $y = b^3$ when $b = 4$, this was solved by 1% of the students (test 1) and 14% of the students (test 2). Many students answered 12 - such result suggests that the students simply multiplied the numbers 4 and 3. It is useful to ask the question why so many students were not able to solve such simple tasks. It is possible that some students had weak abilities to compute correctly. On the other hand some of the students seemed to have limited conceptual understanding of exponential notation and experienced significant problems to find the values of an expression where a variable is raised to higher order.

Generalisation of a figure pattern
Although most students could find the solution to a problem related to a figure pattern with specific numbers, they could not find the solution to the generalised problem or did not give answers. About one third of the students tried to solve the task in the general case, but most of them provided false generalisations or numerical answers. The students experienced problems to use the variable as describing a general number. Many students provided numerical answers when they were asked to use a variable to describe the pattern in the general case - there was a tendency to use the variable as defining quantities, and not to interpret the variable as a generalised number.

5.1.2 A comparison with the previous year

This section discusses the results of the group of 74 students (2004-2005) and the group of 92 students (2005-2006), in relation to the research question:

- How can the students' results be compared with the results of the other studies in the LCM project?

The comparison called Trends in the students' progress (the section of the analyses 4.2.3) shows that as a whole, the profiles of the two groups are not very different, but the new group of 9th grade students had lower results on most of the tasks. Both groups of students showed similar average results on both tests. In both groups from the 9th grade there has been a positive development. The development of the results for each test item showed that the differences between the two groups of students on the first test were bigger. The group of 74 students had higher results in comparison with the groups of 92 students on 44 test items on the first test (however the differences between the results of the two groups were minimal on 17 of those 44 test items and were less than 5 solution frequency points). The group of 74 students had higher results in comparison with the groups of 92 students on 38 test items on the second test (but on 15 of those 38 tasks the differences of the results between the groups were minimal and were less than 5 solution frequency points). The students' results on the second test show that there were some differences in the results of the two groups of students, but they were not so many as on the first test. The new group of LCM students showed high improvement on a selected group of algebraic tasks related to simplification of algebraic expressions on the second test.

The biggest differences in the students' results on the first test were for test items related to application of conceptual and procedural knowledge on arithmetical tasks as for example operations with fractions, comparison of decimal numbers, operations with decimal numbers, estimation of expression with decimal numbers. Converting fractions into decimals caused more problems to the students in the new group. The results on such tasks were much lower than the results of the compared group of 74 students. The group of 92 students showed much better results on the first test than the compared group of students on four test items related to conceptual understanding. Those tasks were some word problems and one task on evaluation of algebraic relationship.

The results on the second test showed better results for the group of 74 students on a group of eight tasks. Those tasks were related to understanding of numbers and required operations with fractions, operations with decimal numbers, or solving of word problems. For almost all of those tasks the group of 74 students showed higher results also on the first test. It is important to mention that some of the bigger differences between the two groups of students were on tasks requiring application of conceptual knowledge – generalisation of a pattern and construction of a word problem related to a given arithmetical expression. The group of 74 students showed better results on such tasks, but in general the results of both groups on those tasks were low. On the other hand, the results of the groups on the second test show that the group of 92 students showed much higher results on a selected group of tasks related to simplifications of algebraic expressions and on finding the number of elements in the 7th term of a figure pattern.

Additionally the results of those groups of students would be further discussed when presenting comparisons of selected tasks.

5.2. 11th grade

The tasks included in the test of 11th grade were specially selected to assess the mathematical knowledge and abilities of the students in some central topics in algebra and numbers. The study presented analysis of the students' results, focusing on the results of the group of 113 students, i.e. the students participating on both tests during the school year.

Development 11th grade

This section discusses the results of the group of 113 students, in relation to the research questions:

- What are the students' achievements shown in the results on the tests? What is the development in the students' results for the school year?

Numbers

Integers

The students' results on a task 1c of division of integers (900 : 30 =) were very high, but the task is a simple one. Many students could not multiply correctly integers, for example the results on task 1e (60 · 450 =) show that it was solved by 60% of the students (test 1) and 56% of the students (test 2). Many students experienced problems to find the square of the number 24 in task 3b. The tasks 8a - 8d involve application of conceptual abilities and proper understanding of the role of the equation sign. The results on those tasks were not high.

Fractions

The students' results on tasks requiring addition and subtraction of fractions were very high on both tests. It can be noticed that the students improved their abilities to convert fraction into decimal numbers.

Decimal Numbers

There were different tasks involving operations with decimal number and the students solutions varied for similar type of tasks according to the context of the tasks. The tasks involving comparison of a group of decimal numbers and finding a number closest to a given number were solved by most of the students. Many students had difficulties on a task related to conceptual understanding of the positional system of the decimal numbers. The results on task 4c (Skriv riktig tall i rutene $5{,}074 = 5 \cdot 1 + 7 \cdot \square + 4 \cdot \square$) show that groups of 38% of the students (test 1) and 50% of the students (test 2) found the results, but many students provided only one of the answers correctly. Very problematic for the students was to find the square of a given number, as for example in task 3d they had to calculate $(0{,}5)^2$, and it caused major difficulties to many of them to find the exact position of the decimal point. The students' results are low on test items related to application of conceptual abilities - converting given fraction (described in words) to a decimal number and finding a decimal number between two given decimal numbers.

Algebra

Algebraic expressions

Almost all students were able to simplify the expressions in the three items 9a ($2x + 5x...$), 9b ($x + x + 2x...$), and 9c ($t \cdot t \cdot t...$). The other test items 9d, 9e, 9f involve more complex algebraic expressions and many students experienced problems to find a proper way to simplify them. The most difficulties the students experienced to simplify the expression $2y \cdot y^2$ and the expression $5a - 2(7 - a) + 6$. The task 12d (Finn tallet x om $x = a+b - c$ *og* $a=1$, $b=2$ *og* $c=3$) related to finding the value of a simple algebraic expressions was solved by most students. But the students experienced major problems to solve the task 12e (Hva er da verdien av uttrykket $3b^2 - abc$ når $a = 3$, $b = -1$ og $c = 5$.), because of difficulties to calculate with negative number.

Equations and functions

In the case of solution of a linear equation the total results show big improvement on some of the tasks. The students who were able to provide the solution of an equation involving a rational expression were not a big groups of students. The students show significant progress on tasks related to find the solution of a linear equation. The equation (Hva må x være dersom $123 + 2x = 195 - x$) was solved by 35% of the students (test 1) and by 52% of the students (test 2). The progress that is observed was very good, but still many students had problems to solve equations. The test item 12b (Hva må x være dersom $\frac{x+1}{x+4} = \frac{4}{5}$) was a very difficult one and was solved by 12% of the students (test 1) and by 23% of the students (test 2). Very low are the students' results on the problem 10c (Vurdering av $\frac{2x+1}{2x+1+5} = \frac{1}{6}$). The students experienced many difficulties to link an equation to the meaning of the given problem, in the case of task 13 (På en skole er det 10 elever for hver lærer. Hvilke av disse uttrykkene forteller dette?) and the solution of this task required abilities to translate from one symbol system - natural language to another - algebra and generate appropriate algebraic equations.

This study found that the students showed a big increase of the results on items related to interpretation of the role of the parameters in a given linear function in the case of tasks 14a and 14b where the context related to the function ($y = 4 + 2{,}5x$) was described in words.

High increase of the students' results is observed on test items 15a and 15b related to abilities to find a solution and explain solution for open ended task requiring application of problem solving skills. The students' results on the task 15c related to generalisation of the solution, show also an increase of the results, but as a whole the big majority of the students experienced difficulties to give a relevant algebraic answer.

The results on the tasks 11a and 11b ('Volume tasks') showed that very few students could solve such complex items that involve application of a variety of abilities. Most students attempted to solve those tasks, but showed limited abilities to represent and analyse geometrical situations and structures using algebraic language. Many students tried to apply formulas for volume for a new cylinder, but failed to do it correctly.

5.3 Comparison of the results for different tasks

This section discusses the results of different LCM groups of students, in relation to the research question:

- What comparison can be made of the results for selected tasks and the results for similar tasks from studies performed before the curriculum L97 took place?

One of the main aims of this study is to provide comparison of the results for selected tasks and the results for similar tasks from studies performed before the curriculum L97 took place. The selected studies are KIM (1995, 1996), Kassel Exeter (1994), Evaluering av R97 (2002) and CSMS (70- ties). Here would be discussed the results of 9th and 11th grade students, who did both tests for a group of tasks in comparison with the results for similar problems from those projects.

Many test items used in this study were specially selected items and they were already used in previous studies. Such comparison of selected tasks was made also by Espeland (2006). It is known that the other studies used different methods compared with the LCM project. So such comparisons should be made with caution. Although the different studies used different methods, we present such additional comparisons of results for selected tasks. The main point is to present a general picture of the level of the students that are tested now in comparison with the results from previous projects and we do not intend to make detailed analyses.

Further we would present some important notes concerning the methods used by the other projects. The main differences of the methods used by the LCM project and the methods used by the other studies were already discussed by Espeland (2006) and we would also provide some details. We should not forget that all groups of students who participated in the LCM project were tested using the same methods. They were tested at the beginning and at the end of the school year, and they worked on the tests for 45 minutes, with only their teacher present at the time of testing. The students were given the same test, except for some small changes in 2005 spring.

The Kassel Exeter project was an international project that lasted several years and groups of Norwegian students participated in it. Group of 8th grade (corresponding to 9th grade) students were tested in September1994 and the results were analysed by Kristin Hinna (Hinna, 1996). The authors of the report Evaluering av R97 selected a group of tasks from the project Kassel Exeter in order to have a base for comparison of the results from different groups of students, with eight years difference between the testing of them. Both groups of students were in their second year in the upper secondary school. Alseth et al. (2002) noted that the selected area of testing the students' skills and understanding in Numbers was chosen, because it could be viewed as 'fundamental and independent' of the curriculum. The testing of the group of 8th grade students participating in the Evaluering av R97 project was done in September, 2002.

The KIM project was a big study and details about this study were presented in the chapter Literature review. The difficulty in the comparison of the students' results concerning tasks related to numbers is that the group of KIM students, 9th grade was tested in January and February. Our study collected data in the beginning and at the end of the school year. That is why we present the results of the LCM groups of students on both tests. Brekke et al. (2000) analysed the tests results of 10th grade students related to testing the students' knowledge in the area of Algebra. The group of students performed the tests in November and December, 1996, so those students were about half a year older than the 9th grade students who did the test in the spring.

The study CSMS (Concepts in Secondary Mathematics and Science) was a large longitudinal study in Great Britain during 1974 – 1979, with participating big groups of students from 11 to 16 years. Two of the tasks solved by students in this study are presented also in the comparison of the algebraic tasks. It is difficult to compare those students with the groups of Norwegian students, because of cultural differences, differences in the curriculum, and differences in the school systems.

Calculations with numbers

Here we would look at the students' results for selected tasks related to calculations with numbers. This comparison aims to present the main differences in the results of the students participating in the projects Kassel Exeter (1994), Evaluering av R97 (2002) and the LCM study. The table below presents in a comparison of the solution frequencies of the students on selected group of tasks. The tasks were grouped as involving procedural and conceptual knowledge by the authors of the report Evaluering av R97 (2002).

Begrepskunnskap*	Prosedyre Kunnskap*	Kassel Exeter 1994 høst	Evaluering av R97 2002 høst	LCM 9.Trinn 2004 høst 74 elever	LCM 1.vgs 2004 høst 206 elever	LCM 9.Trinn 2005 høst 167 elever	LCM 1.vgs 2005 høst 227 elever
Temperaturen forandrer seg fra -5 C til +8 C. Hva er stigningen i temperaturen?		79	78	88		76	
Uttrykk 20% som en brøk.		63	58	65		56	
En bestemt type penner koster 15 kr for hver. A Hvor mange kan du kjøpe for 200 kr?		68	58	61		60	
B Hvor mye vekslepenger får du tilbake?		70	65	59		61	
	60 . 450=	71	43	34	62	41	55
	25% av 40 km =?	66	66	72		65	
	1/2 + 1/4=?	67	32	50	80	42	82
	1/2 - 1/3=?	60	17	32	72	33	73
	70 . 0,3=	68	37		54		47

Table 5.3.1: Comparison of the results for different tasks - Kassel Exeter 1994, Evaluering av R97 2002, LCM project (Groups: 9th grade 2004 høst, group of 74 students; 9th grade 2005 høst, group of 167 students; 11th grade 2004 høst, group of 206 students; 11th grade 2005 høst, group of 227 students)
*Note: The differentiation of the tasks in the categories conceptual knowledge and procedural knowledge was done by the authors of the report Evaluering av R97 (2003)

The results of the groups of LCM students were lower on almost all tasks on both tests in comparison with the results of the students participating in the Kassel Exeter study. The biggest differences were on tasks involving application of procedural knowledge and skills. The comparison of the students' results provides bases for concerns about the level of fluency of the LCM students in doing simple computational tasks with fractions, decimals and integers. The LCM students, 11th grade did not show good skills in simple computational tasks involving decimals and integers.

The LCM students showed much lower results on tasks involving simple operations with fractions. For example the task requiring subtraction of fractions ($\frac{1}{2}-\frac{1}{3}=$) was solved by less than a third of the LCM students in each group and the same task was solved by 60% of the students tested in 1994. The results on tasks involving addition of fraction ($\frac{1}{2}+\frac{1}{4}=$) and multiplication of integers (60.450 =) show similar tendency – the students participating in the Kassel Exeter project had much better results than the LCM groups of students. Only on one of the procedural tasks (25% av 40 km =?) the results of all groups of students are similar. On two of the tasks related to operations with fractions, the group participating in the Evaluering av R97 study showed lower results than the groups of LCM students. The LCM students, 11th grade tested in the beginning of the school year had lower results than the compared group of students participating in the Kassel Exeter project on two of the tasks requiring simple calculations with numbers(the task 60 . 450= and the task 70 . 0,3 =). The 11th grade students showed very good results when doing simple calculations with fractions.

The results on the four tasks related to conceptual understanding show higher results of the students tested in 1994 compared with the LCM students, tested in 2005 and the students participating in the Evaluering av R97 study, but the differences among the results on those tasks were minimal. The exceptions were the better results of the LCM students (2004) on two of the tasks in this group. So for the tasks related to conceptual understanding there were some differences of the students' performance, but in general the results of all groups of students were rather similar.

Next we would present the students' results on another group of 18 test items, organised in a comparison. The table 5.3.2 contains a variety of arithmetical tasks related to multiplication and division of decimals and integers, positional system (decimal numbers), choice of operation for a word problem, converting a given fraction to a decimal number, estimation of the value of an expression, and comparison of decimal numbers.

Begrepskunnskap	Prosedyre Kunnskap	KIM	LCM	LCM	LCM	LCM	LCM	LCM	LCM	LCM	LCM	LCM
		9.trinn	9.trinn	9.trinn	1.vgs	1.vgs	9.trinn	9.trinn	9.trinn	1.vgs	1.vgs	1.vgs
		1995	2004 høst 74 elever	2005 vår 74 elever	2004 høst 206 elever	2005 vår 206 elever	2005 høst 167 elever	2005 høst 92 elever	2006 vår 92 elever	2005 høst 227 elever	2005 høst 113 elever	2006 vår 113 elever
	$6 \cdot 0{,}5 =$	68	47	50			38	33	46			
	$3 : 6 =$	68	41	45			31	33	36			
	$3 : 0{,}5 =$	51	18	24			13	11	22			
Hvor mye veier et halsbånd?		28	24	24			26	25	13			
Hvor mye koster pølsene?		83	65	61			63	65	65			
Hvor mange kakebokser behøves?		41	22	30	41	54	13	15	22	41	50	59
Regnefortelling 4 : 0.5		27	15	14			5	5	8			
Tall nærmest 13 : 4,32		78	51	64			52	50	61			
Tall nærmest $0{,}73 \cdot 46{,}2$		64	54	64			40	40	55			
Sette inn 2,03 på tallinja		71	84	86			70	67	74			
Sette inn 2,27 på tallinja		67	60	73			51	53	61			
Sette inn 6,4 på tallinja		44	41	41			34	35	36			
Finne minste tall av 5 tall*		79	57	71			55	57	65			
Finne største tall av tre tall **		88	68	68			65	65	74			
Finne største tall av tre tall **		94	89	85			79	71	78			
Skriv som desimaltall: fem tideler (KIM), 3/10 i LCM		94	50	50	64	65	39	36	38	56	59	74
Skriv som desimaltall: elleve tideler (KIM), 28 tideler i LCM		41	30	22	46	43	16	20	13	30	37	47
Skriv som desimaltall: 11 tusendeler (KIM), 45 tusendeler i LCM		47	23	26	50	49	19	17	13	40	49	55

Table 5.3.2: Comparison of the results for different tasks - KIM project and the LCM project (Groups: 9^{th} grade 2004-2005, group of 74 students; 9^{th} grade, group of 167 students 2005 fall, group of 92 students 2005-2006; 11^{th} grade 2004-2005, group of 206 students; 11^{th} grade, group of 227 students 2005 fall, group of 113 students 2005-2006)

* Note: KIM test – in this task the students were only asked to find the minimum, LCM test - in this task the students were asked to find both the minimum and the maximum. The solution frequency in the table is given only for the minimum.

** Note: KIM test – in this task the students were only asked to find the maximum, LCM test – in this task the students were asked to find both the minimum and the maximum. The solution frequency in the table is given only for the maximum.

LCM results - 9^{th} grade

The results of this LCM study and the results of the previous LCM studies for the group of selected tasks show that the LCM students' results are in general lower than the results of the KIM students for the majority of the task in the comparison. Exceptions were the measurement tasks requiring application of conceptual knowledge where all groups of the LCM students showed similar results or in some cases higher results than the KIM students. One of those tasks was 'Sette inn 2,03 på tallinja' the LCM group of 74 students, showed higher results on both tests and the LCM group of 92 students showed results similar to the results of the KIM students.

For some of the tasks we can observe very significant differences. The most major differences among the students' results are found on simple computational tasks, requiring application of procedural knowledge ((a)Task: 3 : 0,5 = …; (b)Task: 3 : 6 =…; (c)Task: 6 · 0,5 =…). Converting fractions into decimals is an area with big differences between the results of the KIM students and the groups of LCM students. On the task related to construction of a word problem related to the arithmetical expression (4 : 0,5) the results of the KIM students were much higher – 27% of the KIM students were able to construct a story and much smaller groups of LCM students showed such abilities. On two of the tasks related to the choice of operation in solution of a word problem, the group of KIM students also showed much higher results.

The differences of the results between the two LCM groups give bases for concerns what longitudinal tendencies of the 9^{th} grade results would develop in the future. This selection of tasks show that the LCM group of 92 students had lower results on most of the tasks in comparison with the LCM group of 74 students. The differences between the results of the two groups were higher than 10 solution frequency points on five of the tasks in this comparison for both tests. On the other hand, the improvement of the results on the group of all tasks was better for the 92 students. For example only on 10 of the tasks in the comparison there was some improvement of the results of the participating group of 74 students, and this improvement was small on 6 of the tasks and moderate on 4 of the tasks. The group of 92 students showed improvement on 14 of those tasks - the improvement of the students' results was small on 9 of those tasks and mostly moderate on the other tasks.

LCM results - 11^{th}grade

In this comparison are included the results of three LCM groups of students, 11^{th} grade on a group of four tasks. Those tasks are simple tasks, requiring application of conceptual knowledge – three of them are related to converting of fractions into decimal numbers, and there is one item related to choice of operation in solving a word problem. The results of those groups of students were included as additional source of information. It was expected that the results of the 11^{h} grade students would be much better than the results of the KIM students, 9^{th} grade. But the results of the 11^{th} grade students on the first test were rather very similar or lower than the results of the KIM students. We need to consider the question why 11^{th} grade students (tested at the beginning of the school year) did not present much better abilities than the groups of 9^{th} grade students tested 9-10 years ago? The results of the second test were also included and nevertheless that there was observed some improvement, those results were not always a lot better than the results of the group of KIM students.

Algebra

In the table presented below we show the students' results from the CSMS study, KIM study (10^{th} grade) and LCM study 9^{th} grade (groups of 74 students, 2005 spring and 92 students, 2006 spring) and 11^{th} grade (groups of 206 students - 2004 fall, 227 students- 2005 fall and113 students - 2006 spring) on a group of algebraic tasks. The table 5.3.3 contains a number of algebraic tasks in the areas equations, simplifications, understanding of variables, and substituting in expressions.

Oppgaver	CSMS 10.trinn 70-tallet	KIM 10.trinn 1996	LCM 9.trinn 2005 vår 74 elever	LCM 1.vgs 2004 høst 206 elever	LCM 9.trinn 2006 vår 92 elever	LCM 1.vgs 2005 høst 227 elever	LCM 1.vgs 2005 høst 113 elever
CSMS:$2a+5a$; KIM: $6n+3n$; LCM; $2x+5x$	86	84	53	86	71	92	95
KIM: $x.x.x$=; LCM: $t.t.t$=		75	38	88	50	85	89
$2y \cdot y^2$ =		51	9	37	9	39	44
Vurdering av ekvivalens $x+y+z=x+p+z$	27	33	11	23	11	26	35
Sette inn verdier $3x=7$ og $5y=11$ i $3x+5y$		70	28	51	33	58	66
Sette inn verdier for $a+b-c$		81	31		37	60	77
KIM: $\frac{x+1}{x+3}=\frac{1}{4}$ LCM: $\frac{x+1}{x+4}=\frac{4}{5}$		9		9		8	12

Table 5.3.3: Comparison of the results for algebraic tasks from different projects: CSMS project; KIM project; LCM project (Groups: 9^{th} grade 2004-2005, group of 74 students; 9^{th} grade 2005-2006, group of 92 students; 11^{th} grade 2004-2005, group of 206 students; 11^{th} grade group of 227 students 2005 fall, group of 113 students 2005-2006)

When we look at the results it can be noticed that the new groups of LCM students in 9^{th} and 11^{th} grade had better results on the selected algebraic tasks than the corresponding groups LCM students, tested the previous school year. However when we consider the results of the LCM students, 9^{th} grade as a whole those results are not that good as the results of the KIM students - in most of the cases the results are much lower. In addition the 11^{th} grade results of the LCM students were higher than the results of the KIM students 10^{th} grade only for about half of the tasks in the comparison.

LCM results - 9^{th} grade

As a whole the results on those tasks show that the LCM students, 9^{th} grade, tested at the end of the school year had lower results than the KIM students, 10^{th} grade, tested in November and December. The observed differences between the LCM and the KIM students are very major for almost all tasks. The LCM students much lower performance on five out of six tasks. The new group of 92 students (spring 2005) showed better results than the compared LCM group of 74 students (spring 2004) on four of the tasks, requiring finding value of expressions and simplifications. Those groups of students are different age and classes, so we expect that there would be differences, but it is not good to notice that the differences are very significant for some of the tasks ((a) Task: $2y \cdot y^2$ =; (b) Task: Vurdering av ekvivalens

$x+y+z=x+p+z$; (c) Task: Sette inn verdier $3x=7$ og $5y=11$ i $3x+5y$); (d) Task: Sette inn verdier for $a+b-c$) .

LCM results - 11th grade

The results for the groups of LCM students, 11th grade in comparison with the results for the groups of KIM students 10th grade show that the LCM students received lower results for about half of their results displayed in the table. The LCM groups of students, 11th grade showed higher results than the compared group of KIM students only on two of the tasks in the comparison related to simplification of algebraic expressions. On most of the tasks the results of LCM students, 11th grade, tested 2005 fall show that the groups of 113 students and 227 students had a little bit better performance than the LCM group of 206 students, 2004 fall. In addition the LCM group of 206 students, 11th grade (2004 fall) had significantly lower results than the compared group of KIM students, 10th grade on three of the tasks ((a) Task: $2y \cdot y^2 =$; (b) Task: Vurdering av ekvivalens $x+y+z=x+p+z$; (c) Task: Sette inn verdier $3x=7$ og $5y=11$ i $3x+5y$). However the two groups of LCM students,11th grade, tested 2005 fall showed better results on those tasks and the differences with the KIM students were not so big. It can be noted also that on the task requiring the students to find the value of the expression $a+b-c$, the LCM group of 227 students (2005 fall) had significantly lower results than the compared group of KIM students.

5.4 Limitations of the study

We defined some important issues and focused on them in the analysis. The given time to do all the work was limited, although we tried to use the time in an effective way. It can be considered that the study has some limitations related to gender issues, lower number of participants for the second test, the size of the groups of students who did both tests, and lack of data concerning the students' attitudes and beliefs towards the subject of mathematics.

This study did not focus on important gender issues. Although there are gender differences we did not provide a detailed comparison of the 9th and 11th grade results for the groups of the boys and the girls. The study provided tables with the average scores of the girls and the boys in both grades, but this is just one of the aspects. More depth of the analyses could be achieved if there was performed an additional analyses of the results for the different gender groups.

Some of the students did not perform the second test, and for the study of progress we had to analyse the subgroups of students that took both tests. The group of 11th grade students who did both tests had only 113 students, although much more students did the first test in the fall of 2005. We provided a comparison of the results for the groups of 227 students (test 1, 2005 fall) and 236 students (test 1, 2004 fall), but we could not compare the development of the students' results for the two groups of 11th grade students who did both tests - the group of 113 students was a much smaller group compared to the group of 202 students, 11th grade, analysed by Espeland (2006).

Another possible limitation of this study is that the analysed groups of students in 9th grade and 11th grade who did both tests are not very big groups of students. We describe different comparisons of the results, but we have to keep in mind that those findings are related only to the results for relatively small groups of students. Although those results give us a lot of information it is important also to study bigger groups of students, coming from more classes.

It is necessary to point also that some bigger studies use as a method double coding of the results by two researchers. In this study we analyse the students' results using as a method coding of the results by one person. The same method of coding was used also by the previous studies in the project. It is always important to consider the coding of the data by a second person in order to achieve better validity and reliability. There is always some chance of mistakes related to the coding of the data that can remain undiscovered, although the results were coded carefully, using long period of time, and there were performed several types procedures in order to check and find the possible mistakes.

Some of the limitations of the study can be associated with the test instrument and the way the students performed the tests. Since we do not point to the students which items in the tests are the most difficult ones, it can be considered that some of the students would not be able to judge the level of the difficulty of the items. As a result those students might use very short time to solve problems, which could be in fact very complex and could require additional attention. It can be considered also that it is possible that some students did not have enough time to try to solve all of the tasks. There is always the possibility that some students needed more than 45 minutes to work. Another issue is related to the time of testing. It was already discussed that some students might not be able to show their best performance short after the school year has started and short before the school year finishes. In addition if the time of performing the test in one class was selected to be the last hour of the day, some students might be tired and be less concentrated.

6. Conclusion

The importance that students develop different mathematical competencies was emphasized by this study. Students need a variety of skills to operate effectively when solving mathematical tasks. The study has recognised the importance of sufficient skills in operating with arithmetical and algebraic problems in a variety of contexts.

The data used in this study provides a very rich source of information. The tests were very carefully designed and big efforts were used to ensure quality of the coding. Very significant time was used for the constructing of the databases, a variety of procedures for analysis of the results and for organising the findings in diagrams and tables.

The study recognises the main achievements and difficulties of the participating groups of 9^{th} and 11^{th} grade students in the areas of algebra and numbers. This study helps to identify certain type of misconceptions, present in the students' answers for some of the tasks. Comparisons of the students' results on the tests describe main issues of the development of the students' results during the school year.

The study founds that many students in 9^{th} grade experience serious problems to do simple computational tasks. High number of 9^{th} grade students experienced problems to do calculations with fractions and decimal numbers. The solution of some of the open ended tasks, as for example the generalisation of a figure pattern and the construction of word problems related to arithmetical expressions were very difficult for the majority of the students in 9^{th} grade. The 9^{th} grade students showed in general lower results than the results of the students from the KIM and the Kassel Exeter projects.

The students' results show that even in 11^{th} grade there are students who experience problems to calculate correctly with decimals and to solve arithmetical tasks in a variety of contexts. The students' results on the arithmetical test items varied, and as a whole they were not high enough. In 11^{th} grade students are expected to be very fluent in doing basic computations - they need a solid foundation and fluency with number operations provides bases to operate fluently on algebraic tasks. Many students in 11^{th} grade experienced problems to compute with decimal numbers, especially when they had to find square of a decimal number. Many students did not demonstrate enough conceptual understanding when solving task related to the positional system of decimal numbers. The results on the tasks involving interpreting of the roles of the constants in a function, show satisfying improvement. The results on the tests show an improvement of the results on tasks as simplification of some algebraic expressions and explanation of a strategy of solution. The students explained their strategy of solution for finding the two numbers on the problem 'Diophantus numbers'. Most of the students, who provided a method of solution, provided an algebraic method and less students gave rhetorical explanations. The 11^{th} grade students showed in general lower results than the results of the students from the KIM and the Kassel Exeter projects.

In addition the study compares the results of the participating groups of 9^{th} and 11^{th} grade students with the results of the students participating the previous school year. The main differences and similarities between the compared groups were described and analysed.

It is very worthwhile to continue the longitudinal project by carrying out testing of new groups students in 9^{th} and 11^{th} grade and do further analyses. It is interesting to observe whether the tendencies found in this study would continue. It is important that the students' results for the three years of the project are compared, analysed and discussed. It is necessary to observe the differences and the similarities of the compared groups of students. It is necessary to describe and analyse the main achievements and main difficulties of the participating groups of students. It is important to look for the main tendencies of the results for the groups of 9^{th} and 11^{th} grade students in the following studies of the longitudinal project. It would be interesting to observe how the collected data from the tests is related to the previous years of the project, find whether there are some significant variations and describe the main conclusions for the results of the longitudinal project.

The study provides a valuable source of information and can be considered to be helpful to teachers. However it is important that diagnostic tests are well designed to pass the needs of the students. Diagnostic tests can be used in different ways. They can be used as a written test or they can be used at the end of the lesson after a teacher organises a discussion with the students around certain mathematical concept.

The aims of the KUL-LCM project are to design and study mathematics teaching development in order to improve mathematical learning experiences of students in class. The results of the tests in the project schools are analysed in order to study the students' achievements and to give a feedback to teachers about the results of their students. The information teachers get is useful as a source providing different comparisons of the new results with the previous year results. The aim is teachers to get opportunities to use the results for the improvement of the mathematics education. The diagnostic tasks included in the tests are useful way to identify the main difficulties of the students and the progress of the students during the school year. Teachers can use the results in order to get information what misconceptions the students hold, investigate further the tables with incorrect answers and utilize this information actively as a helpful tool for identifying what help the students need.

7. Pedagogical Implications

We have discussed already a lot of results of this study. Based on the analyses of the results and on the discussion, several important points can be made.

We consider that the main challenge for teachers is to engage students in learning activities that support the integration of all strands of mathematical competences. Projects (Niss et al., 2002; Kilpatrick et al., 2001) emphasized that conceptual understanding and computational procedures need to be appropriately linked. There is evidence that the disconnections that many students show among their conceptual understanding, procedural fluency, strategic competence, and adaptive reasoning create serious barriers to their progress in learning and using mathematics (Kilpatrick et al., 2001) and such evidence is shown also by our study.

The teachers should help students recognize that there exist obstacles for the learning of algebra. The results show that many students hold misconceptions and demonstrated limited conceptual understanding when solving specific tasks. It is very important that teachers are aware of the problems of the students. We suggest that students are deliberately confronted with certain well-known obstacles of learning algebra, for example the different meanings of letters, the discontinuities between arithmetic and algebra, the interference of the natural language when solving algebraic word problems. If students are given diagnostic tasks in class, and get feedback from the teachers or have the chance to discuss the tasks in class, that would help students develop better conceptual understanding.

The study recognises that teachers need to work further with students to deepen their understanding of fractions and decimals also in grades 9 and 11. The teachers need to help students become more flexible in operating with fractions. This study shows that the 9th grade students experience problems to compute with fractions. It is remarkable also that not enough students in 9th and 11th grades are flexible to calculate with decimal numbers. Similar problems were observed by the previous studies of the project. The deficiency of a solid conceptual foundation can greatly hinder students – for example if students have certain misconception related to numbers they would experience major difficulties when dealing with numbers in variety of problem-solving settings. Kilpatrick et al. (2001) distinguished that to help students learn efficient algorithms for computing with fractions and decimals is one of the most challenging tasks for teachers and pointed that instruction focused only on symbolic manipulation without understanding is ineffective for most students. Kilpatrick et al. (2001) recommended that teachers pay more attention to conceptual understanding and all strands of proficiency.

We recommend the design of more mathematical activities that would help students develop better understanding of models that are useful in problem solving. Problem-solving approach has clear benefits for students – it is important for students to have a structured way of approaching a problem. It is important that students have variety of problem solving strategies at hand and know how to apply them in practice. Especially important is students to learn to be critical to their results and recognize the importance to evaluate their results. The potential advantages of asking students to construct problems and to generalise related problems should not be ignored by the teachers. Problem posing and generalisation helps students to be more creative mathematically and to bring their own ideas into play (Mason et al., 2005). This type of activity requires abilities in relation to the different kinds of mathematical competences and can be a very useful diagnostic exercise (Brekke, 2002). The information gained about the extent to which students can engage in such activity could provide valuable information about their strengths and weaknesses in problem-solving. The data from the tests shows that many students in 9th grade experienced serious problems in constructing a word problem and in generalising figure pattern. 11th grade students demonstrated limited abilities to solve items related to representation and analysis of geometrical situations and structures using algebraic symbols. It is important that the students learn to apply a specific model for constructing a word problem or for problem solving that can help them to work in a more systematic way. It is necessary that students learn to be more critical to their results and be able to evaluate their results. Students need to develop more systematic notations.

It is important to better link the latest research in mathematics education with teachers practise. There is an ongoing debate focused on the forms and approaches to teaching. Ideas how teaching of algebra can be more effective are suggested by a lot of studies. This study recognised the students' difficulties related to the domain of algebra - as problems to solve equations, operations with algebraic expressions and formulas when the unknowns are in second order, to generalise or to use unknowns appropriately. Students experience specific problems when they attempt to solve word problems and problems related to patterns. Teachers play very important role to promote better learning. Research proposed that teachers need to get much better training in mathematics. Grevholm (2004) discussed that it is important to achieve better quality of the teachers' education in mathematics and that there is a great concern regarding the lack of mathematical competence in newly educated teachers. Alseth et al. (2003) discussed the importance that teachers get enough professional training, the importance teachers to base their practise on the use of variety of help materials and the need to supply teachers with more ideas for activities in class.

Further research

Different issues related to students' learning of algebra and numbers has interested researchers for a long time. Questions of critical importance are how to prepare better students for the transition from arithmetic to algebra, and how to improve the students' learning of algebra and numbers. Those questions were addressed by a variety of studies and it is important to have more research. As research findings become better applied in practice, the implications of research can be used to improve significantly the classroom instruction and for the benefits of all students.

There is a special need for research organised locally that focuses on the problems of specific groups of students. It is important for teachers to get additional help and have as guidance results of the students' performance on tests. It is important that teachers and educators work together and cooperate for such research, that the tests are developed to better inform the teachers and that the research finding and implications are better communicated to teachers.

It is valuable to ask students to reflect on their strategies of solution when solving problems. That kind of information helps identifying what were the common methods of solution for selected tasks. The analyses of the students' responses can show whether students are able to explain their strategies of solutions and what problems experienced those students who found incorrect answers.

Teachers and researchers need to build better understanding how students use symbolising and representations, how symbolising and representations enable and constrain learning. It would be good to design test items where symbolizing the solution procedure is at least as important as finding the solution. Perhaps that can be useful for some simple and some more difficult tasks.

It is useful to analyse trends of gender differences and to analyse the students' attitudes and beliefs towards the subject of mathematics. There are many issues related to gender differences and students' attitudes and beliefs that have increasing importance, as for example what roles motivation and self-confidence play in the learning of algebra. It is necessary to identify what factors make some lower-ability students succeed.

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9. Appendix

Masteroppgave i matematikkdidaktikk
Fakultet for realfag
HØGSKOLEN I AGDER – våren 2007

Appendix

Achievement of 9th and 11th Grade Students in Algebra and Numbers

Research based on data from the KUL - LCM project

Emilia Lambeva Log

Appendix

Achievement of 9th and 11th Grade Students in Algebra and Numbers

Research based on data from the KUL - LCM project

Emilia Lambeva Log

Master thesis in mathematical didactics

HØGSKOLEN I AGDER
Fakultet for realfag
Institutt for matematiske fag
MASTER I MATEMATIKKDIDAKTIKK
KRISTIANSAND
2007

Table of content

Appendix 1: Algebra and Numbers 9th grade
Appendix 1.1: Test 2005 høst – 2006 vår

TALL OG ALGEBRA

En undersøkelse, 9. klasse

Navn: .... Dato: ....

Elev nummer .... (fylles ut av skolen)

Elev nummer: .... (fylles ut av skolen)

TALL OG ALGEBRA

En undersøkelse, 9. klasse

Dato: ....   ☐ Gutt   ☐ Jente

Alder: .... år og .... måneder

Lommeregner skal ikke brukes på denne testen.

2 Hva er en kvart (en firedel) av 60 gram?

3 Finn lengden på gardinstoffet Mari må kjøpe, når det skal være fem gardiner, og hvert skal være 1,05 meter.

4 Hva er 25% av 40 km?

5 $\frac{1}{2} + \frac{1}{4} = ?$

6 Temperaturen forandrer seg fra –5 ˚C til +8 ˚C.
Hva er stigningen i temperatur?

7 Uttrykk 20% som en brøk.

8 En bestemt type penner koster 15 kr for hver.

a Hvor mange kan du kjøpe for 200 kr?

b Hvor mye vekslepenger får du da tilbake?

9 $60 \cdot 450 = ?$

10 $\frac{1}{2} - \frac{1}{3} = ?$

11 $\frac{2}{5}$ av en masse er 20 gram. Hva er massen?

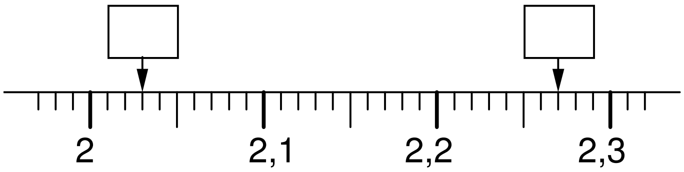
12 Sett *ring rundt* det største tallet og *kryss over* det minste tallet:

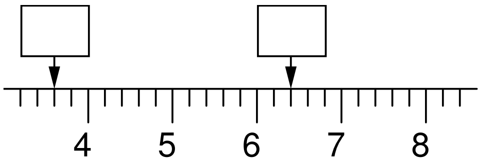
a 0,625   0,25   0,3753   0,125   0,5

b	3,521	3,6	3,75
c	4,09	4,7	4,008

13 Finn et tall med to desimaler som ligger mellom
15,755 og 15,762

14 Les av på følgende skalaer og skriv riktig desimaltall i ruta.





15 a Sett ring rundt det tallet som ligger nærmest i størrelse til 0,16

0,1	0,2	15	0,21	10

b Sett ring rundt det tallet som ligger nærmest i størrelse til 2,08

209	2,9	2,05	2,1	20,9

16 Skriv riktig tall i rutene

a $574 = 5 \cdot 100 + \square \cdot 10 + 4 \cdot 1$

b $5{,}74 = 5 \cdot 1 + 7 \cdot \square + 4 \cdot \square$

17 Skriv som desimaltall

a $\frac{3}{10}$ .... **b** $\frac{46}{100}$ ....

c 45 tusendeler .... **d** 28 tideler ....

18 Skriv svaret som desimaltall.

a $6 \cdot 0{,}5 =$ ………………………………

b $3 : 6 =$ ..………………………………

c $3 : 0{,}5 =$ ………………………………

d $0{,}6 : 0{,}2 =$ …………………………….

19 Til venstre i rammen står et regnestykke. Sett ring rundt det tallet du mener er *nærmest svaret*. Du trenger ikke regne ut svaret.

a

13 : 4,32	0,03	0,3	3	30	300

b

7,5 : 0,24	0,03	0,3	3	30	300

c

0,73 · 46,2	0,03	0,3	3	30	300

20 Sett ring rundt *alle* regneuttrykkene som passer til regneoppgaven:

a *24 halsbånd pakkes i eske. Om 24 halsbånd veier 3 kg, hvor mye veier da ett halsbånd?*

$24 \cdot 3$ $24 : 3$ $3 : 24$ $3 \cdot 24$ $24 - 3$ $3 + 24$

b *1 kg pølser koster 49,50 kr. Per kjøper 1,7 kg. Hvor mye koster det?*

49,50 · 1,7 49,50 : 1,7 1,7 : 49,50 1,7 · 49,50 49,50 – 1,7

c *Kaker skal fylles i bokser, med 0,75 kg i hver. Hvor mange bokser trenger man til 6 kg kaker?*

6 · 0,75 6 : 0,75 0,75 : 6 0,75 · 6 6 – 0,75 6 + 0,75

d *Anne kjøper bananer i en butikk til 13,50 kr per kg. Hvor mye kan Anne kjøpe for 10,50 kr?*

13,50 · 10,50 10,50 : 13,50 13,50 : 10,50 13,50 – 10,50 13,50 + 10,50

21 Lag din egen fortelling som passer til disse regnestykkene:

a $\boxed{4 : 0{,}5}$

Din fortelling:

....

....

....

b $\boxed{5{,}25 \cdot 3{,}28 = 17{,}22}$

Din fortelling:

....

....

....

22 Skriv riktig tall i ruta

a $14 : 2 = \boxed{} \cdot 14$ **b** $14 : \boxed{} = 0{,}25 \cdot 14$

23 Skriv enklest mulig:

a $2x + 5x$ ....

b $x + x + 2x$ ....

c $t \cdot t \cdot t$ ....

d $2y \cdot y^2$ ....

24 **a** $x + y + z = x + p + z$ Dette

☐ er alltid sant ☐ er aldri sant ☐ *kan* være sant, nemlig når ....

b $2a + 3 = 2a - 3$ Dette

☐ er alltid sant ☐ er aldri sant ☐ *kan* være sant, nemlig når ....

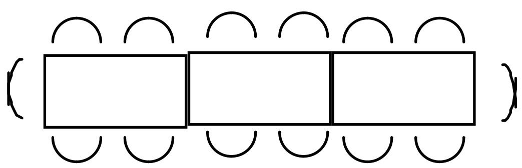
25 Adder tallet x til

a $x + 3y$ ....

b $4x$ ....

c 7 ....

26 Et konfirmasjonsbord er satt sammen av småbord, og rundt er det satt stoler.



a Hvor mange stoler blir det plass til om vi har 4 småbord?..............................

b Enn om vi har 7 småbord? ....

c Dersom vi har n småbord, hvor mange stoler blir det da?...............................

27 **a** $x = a + b - c$ Dersom $a = 1$, $b = 2$ og $c = 3$ blir $x =$.......................

b $y = b^3$ Dersom $b = 4$ blir $y =$ ....

c $3x = 7$ og $5y = 11$ Da blir $3x + 5y =$ ....

30 Marit har tre skåler med nøtter A, B og C. Det er 2 flere i B enn i A. I skål C er det 4 ganger så mange nøtter som i skål A. I alt er det 14 nøtter.
Hvor mange nøtter er det i hver av skålene?

....

Vis eller forklar hvordan du tenkte:

31 5 liter bær veier 10 kg. Hvor mye veier 6 liter bær? ....

Vis hvordan du tenker.

Appendix 1.2: Information about the coding system

Kodeforklaringer - Tall og algebra, 9.trinn

Oppgave	**Kodeforklaringer**
2	Kode 1: 15 eller 15 gram
3	Kode 1: 5,25 m / meter eller 5,25 Kode 2: 1,05 m . 5
4	Kode 1: 10 km eller 10 Kode 2: 25% · 40
5	Kode 1: 3/4 Kode 2: 6/8 eller 12/16
6	Kode 1: 13 ^{o}C eller 13
7	Kode 1: 1/5 Kode 2: 2/10 eller 20/100
8a	Kode 1: 13 penner eller 13
8b	Kode 1: 5 kr eller 5
10	Kode 1: 1/6 Kode 2: 2/12 eller 3/6 - 2/6
11	Kode 1: 50 gram eller 50
12a	Kode 1: Både største og minste tall rett Kode 2: Største tall rett, minste tall feil eller ubesvart Kode 3: Minste tall rett, største tall feil eller ubesvart
12b	Kode 1: Både største og minste tall rett Kode 2: Største tall rett, minste tall feil eller ubesvart Kode 3: Minste tall rett, største tall feil eller ubesvart
12c	Kode 1: Både største og minste tall rett Kode 2: Største tall rett, minste tall feil eller ubesvart Kode 3: Minste tall rett, største tall feil eller ubesvart
14a	Kode 1: Rett tall i begge rutene Kode 2: Rett tall i rute 1, feil tall eller ubesvart i rute 2 Kode 3: Rett tall i rute 2, feil tall eller ubesvart i rute 1
14b	Kode 1: Rett tall i begge rutene Kode 2: Rett tall i rute 1, feil tall eller ubesvart i rute 2 Kode 3: Rett tall i rute 2, feil tall eller ubesvart i rute 2

Oppgave	Kodeforklaringer
16b	Kode 1: Rett tall i begge rutene Kode 2: Rett tall i rute 1, feil tall eller ubesvart i rute 2 Kode 3: Rett tall i rute 2, feil tall eller ubesvart i rute 3
20b	Kode 1: Ring rundt både $49{,}50 \cdot 1{,}7$ og $1{,}7 \cdot 49{,}50$ Kode 2: Ring rundt $49{,}50 \cdot 1{,}7$ eller $1{,}7 \cdot 49{,}50$
21a	Kode 1: Skrev et matematisk problem, som passer til operasjonen 4 : 0,5 i en realistisk sammenheng. Kode 2: Matematisk problem, beskrevet i en realistisk (relevant) sammenheng, men beskrivelsen er ikke presis nok - den har to forskjellige meninger; den beskriver to forskjellige matematiske betydninger, en av dem beskriver ikke operasjonen 4 : 0,5. Kode 11: Feil beskrivelse eller beskriver ikke et matematisk problem i en realistisk sammenheng.
21b	Kode 1: Skrev et matematisk problem, som beskriver operasjon $5{,}25 \cdot 3{,}28 = 17{,}22$ i en realistisk (relevant) sammenheng. Kode 11: Feil beskrivelse eller beskriver ikke et matematisk problem i en realistisk (relevant) sammenheng.
23a	Kode 1: $7x$ Kode 11: Kunne ikke løse oppgaven riktig eller gir tall som resultat
23b	Kode 1: $4x$ Kode 2: $2x+2x$ Kode 11: Kunne ikke løse oppgaven riktig eller gir et tall som resultat
23c	Kode 1: t^3 Kode 11: Kunne ikke løse oppgaven riktig eller gir et tall som resultat
23d	Kode 1: $3a + 2a^2$ Kode 11: Kunne ikke løse oppgaven riktig eller gir et tall som resultat
23e	Kode 1: $2y^3$ Kode 11: Kunne ikke løse oppgaven riktig eller gir et tall som resultat
24a	Kode 1: Riktig kryss og riktig begrunnelse Kode 2: Riktig kryss, men manglende eller utilfredstillende begrunnelse
25a	Kode 1: $2x + 3y$ Kode 2: $x+x+3y$ Kode 11: Kunne ikke løse oppgaven riktig eller gir et tall som resultat

Oppgave	Kodeforklaringer
25b	Kode 1: $5x$ Kode 2: $x+4x$ Kode 11: Kunne ikke løse oppgaven riktig eller gir et tall som resultat
25c	Kode 1: $x+7$ eller $7+x$ Kode 11: Kunne ikke løse oppgaven riktig eller gir et tall som resultat
26c	Kode 1: $n \cdot 4+2$ eller $4 \cdot n+2$ Kode 11: Kunne ikke løse oppgaven riktig eller gir et tall som resultat
27b	Kode 1: 64 Kode 2: $4 \cdot 4 \cdot 4$
30	Kode 1: Riktig svar med riktig forklaring Kode 2: Riktig svar uten forklaring
31	Kode 1: Riktig svar med riktig forklaring Kode 2: Riktig svar uten forklaring

Appendix 1.3: Tables with the solution frequencies for each task on the test – results for a group of 92 students, both tests

Here are presented the tables concerning the results of a group of 92 students, who did the test in the beginning and at the end of the school year 2005-2006. The results were made with the help of the statistical program SPSS.

Oppgave 2 Hva er en kvart (en firedel) av 60 gram?

Oppgave 2	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	16,3	16,3
15	**73,9**	**72,8**
20	5,4	4,3
1,50 eller 1,5	1,1	1,1
15/60	1,1	1,1
25%	1,1	
25		1,1
Andre svar ("1(og) 1 /2"; 45; 1/2)	1,1	3,3

Oppgave 3 Finn lengden på gardinstoffet Mari må kjøpe, når det skal være fem gardiner, og hvert skal være 1,05 meter.

Oppgave 3	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	16,3	14,1
5,25m	**70,7**	**71,7**
1,05m . 5		**1,1**
5,30	2,2	
7,50 eller "7 og en hall"	2,2	2,2
5,10	1,1	1,1
5m og 55cm	1,1	
8,05	1,1	
8,25	1,1	
4,25	1,1	
5,20		3,3
Andre svar(eks. 1,05; 4; 5,05; 5,15; 5,35; 5,50; 7,05)	3,3	6,5

Oppgave 4 Hva er 25% av 40 km?

Oppgave 4	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	22,8	18,5
10 km	**63,0**	**73,9**
25% . 40		**1,1**
25		2,2
7,5	1,1	
15	1,1	1,1
16	1,1	
20	1,1	1,1
30	1,1	
25% . 40 / 100=		1,1
Andre svar(eks."kvart"; 1/2; 8; 12; 7%; 10%; 41cm; 35)	8,7	1,1

Oppgave 5 $\frac{1}{2} + \frac{1}{4} = ?$

Oppgave 5	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	14,1	15,2
3/4	**37,0**	**41,3**
6/8	**4,4**	**13,1**
12/16		**1,1**
1/6	13,0	6,5
2/4	8,7	8,7
2/6	7,6	6,5
1/3 eller 4/12	3,3	1,1
4/8 eller 1/2 eller 3/6	3,3	1,1
2/8	2,2	
2/5	1,1	
1/7	1,1	
2/7	1,1	
1/8	1,1	
5/8	1,1	
1/5		1,1
Andre svar (3/8; 75%; 6/16; "2/4 eller 2/2"; 6)	1,1	4,4

Oppgave 6 Temperaturen forandrer seg fra –5 ˚C til +8 ˚C.
Hva er stigningen i temperatur?

Oppgave 6	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	8,7	7,6
13 grader	**73,9**	**84,8**
14 grader	7,6	2,2
12 grader	3,3	2,2
3 grader	3,3	1,1
Andre svar (7; 9; 15; 18)	3,3	2,2

Oppgave 7 Uttrykk 20% som en brøk.

Oppgave 7	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	33,7	27,2
1/5	**42,4**	**46,7**
2/10 eller 20/100	**12**	**13**
1/4 eller 2/8	5,4	4,4
1/10	1,1	
1/3	1,1	1,1
4/4	1,1	
1/6	1,1	
2/0	1,1	
20%/100%		2,2
100/20		1,1
Andre svar	1,1	4,4

Oppgave 8a En bestemt type penner koster 15 kr for hver.
Hvor mange kan du kjøpe for 200 kr?

Oppgave 8a	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	12,0	13,0
13	**63,0**	**64,1**
12	9,8	5,4
6	3,3	1,1
14	2,2	2,2
11	1,1	2,2
18	1,1	3,3
20	1,1	1,1
195 kr	1,1	
195 penner		1,1
121 penner		1,1
Andre svar (eks. 9; 10; 15; 17; 19; "200:15=")	5,4	5,4

Oppgave 8b En bestemt type penner koster 15 kr for hver
Hvor mye vekslepenger får du da tilbake?

Oppgave 8b	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	18,5	16,3
5 kr	**63,0**	**64,1**
"0 kr"; "ingenting"; "ingen"	8,7	4,4
10 kr	5,4	7,6
20 kr	4,4	2,2
15		2,2
Andre svar (3; 24; 187)		3,3

Oppgave 9 $60 \cdot 450 = ?$

Oppgave 9	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	40,2	35,9
27000	**37,0**	**42,4**
2700	5,4	2,2
27500	2,2	2,2
243000	2,2	
2300	1,1	
2400	1,1	1,1
3024	1,1	
4500	1,1	
5100		2,2
13000	1,1	
16300	1,1	
24300	1,1	1,1
25200	1,1	
32400	1,1	
36000	1,1	
37000	1,1	
54000	1,1	
Andre svar (eks. 1200; 2405; 2450; 2750; 4700; 27300...)		12,9

Oppgave 10 $\frac{1}{2} - \frac{1}{3} = ?$

Oppgave 10	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	46,7	32,6
1/6	**25,0**	**38,0**
1/1 eller 1	8,7	4,4
2/3 eller 4/6	4,3	1,1
5/6	3,3	7,6
1/5	2,2	1,1
0/1	1,1	2,2
0/3	1,1	1,1
0/6	1,1	1,1
0		2,2
1/3 eller 2/6	1,1	2,2
1/4	1,1	1,1
2/6		1,1
Andre svar (eks. 5/12; 2/5; 2/1; 1/4; 9/17; 0/9; 0/-1;…)	4,4	4,4

Oppgave 11 $\frac{2}{5}$ av en masse er 20 gram. Hva er massen?

Oppgave 11	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	33,7	31,5
50 gram	**45,7**	**52,2**
100	7,6	7,6
45	3,3	1,1
10	2,2	1,1
30		1,1
40	1,1	1,1
60		1,1
70	1,1	
80	1,1	
Andre svar (1/5; 2/5; 3; 5/5; 4,5; 25%; 40%)	4,4	3,3

Oppgave 12a Sett *ring rundt* det største tallet og *kryss over* det minste tallet:

0,625 0,25 0,3753 0,125 0,5

Oppgave 12a (minste tall)	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	10,9	5,5
0,125	**56,5**	**65,3**
0,625	1,1	3,3
0,25	8,8	4,4
0,3753	15,2	13
0,5	6,5	7,6
Andre svar (kombinasjoner)	1,1	1,1

Oppgave 12a (største tall)	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	12,0	9,8
0,625	**58,7**	**66,3**
0,25	3,3	2,2
0,3753	6,5	7,6
0,125	1,1	3,3
0,5	13	10,9
Andre svar	4,4	

Oppgave 12b Sett *ring rundt* det største tallet og *kryss over* det minste tallet:

3,521 3,6 3,75

Oppgave 12b (minste tall)	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	19,5	17,4
3,521	**69,6**	**76,1**
3,6	8,7	5,4
3,75		1,1
Andre svar (kombinasjon)	2,2	

Oppgave 12b (største tall)	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	14,1	13,0
3,75	**65,3**	**74**
3,521	8,7	6,5
3,6	10,9	6,5
Andre svar (kombinasjon)	1,1	

Oppgave 12c Sett *ring rundt* det største tallet og *kryss over* det minste tallet:

4,09 4,7 4,008

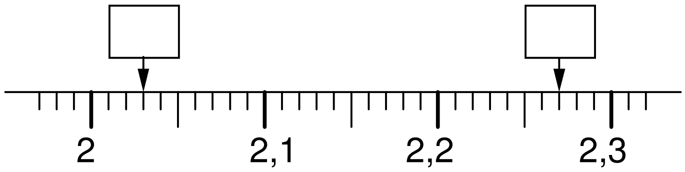
Oppgave 12c (minste tall)	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	18,5	16,3
4,008	**70,7**	**78,3**
4,7	6,5	1,1
4,09	2,2	3,3
Andre svar (kombinasjon)	2,2	1,1

Oppgave 12c (største tall)	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	7,6	6,5
4,7	**77,2**	**85,9**
4,09	8,7	4,4
4,008	5,4	3,3
Andre svar (kombinasjon)	1,1	

Oppgave 13 Finn et tall med to desimaler som ligger mellom 15,755 og 15,762

Oppgave 13	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	35,9	32,6
15,76	**21,7**	**29,3**
15,756 - 15,761	25	25
15,75	2,2	3,3
15,70 og 15,7	3,3	2,2
15 og 15,00	2,2	
15,6 og 15,60	1,1	2,2
15,75 og 15,76	1,1	
15,759 og 15,760		1,1
15,756 og 15,700	1,1	
15,7585 og 15,76	1,1	
Andre svar (eks. 15,80; 15,765; 15,755;…)	5,4	5,4

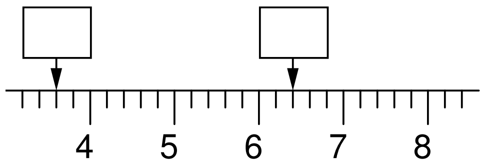
Oppgave 14a Les av på følgende skalaer og skriv riktig desimaltall i ruta.



Oppgave 14a (rute 1)	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	12,0	12,0
2,03	**67,4**	**73,9**
2,3	8,7	4,4
2,4		2,2
3	5,4	3,3
2	3,3	
2,01	1,1	1,1
Andre svar	2,2	3,3

Oppgave 14a (rute 2)	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	15,3	14,2
2,27	**52,2**	**60,9**
2,07	6,5	6,5
2,7	2,2	1,1
2,207	9,8	9,8
7	5,4	2,2
2,0	3,3	
2,26	1,1	1,1
Andre svar	4,4	4,4

Oppgave 14b Les av på følgende skalaer og skriv riktig desimaltall i ruta.



Oppgave 14b (rute 1)	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	15,2	12
3,6	**32,6**	**37**
3,3	15,2	16,3
3,8	25,1	26,1
3	3,3	2,2
3,08	1,1	1,1
8		2,2
3,4	1,1	1,1
Andre svar	6,7	2,2

Oppgave 14b (rute 2)	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	16,3	12
6,4	**34,8**	**35,9**
6,2	41,3	43,5
6,5	2,2	
6,3		1,1
6,8	1,1	1,1
6,02	1,1	1,1
2		3,3
8	2,2	
Andre svar	1,1	2,2

Oppgave 15a Sett ring rundt det tallet som ligger nærmest i størrelse til 0,16

0,1 0,2 15 0,21 10

Oppgave 15a	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	3,3	4,3
0,2	**46,7**	**47,8**
0,1	7,6	10,9
15	3,3	2,2
0,21	39,1	33,7
10		1,1

Oppgave 15b Sett ring rundt det tallet som ligger nærmest i størrelse til 2,08

209 2,9 2,05 2,1 20,9

Oppgave 15b	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	2,2	3,3
2,1	**28,3**	**37,0**
2,9	10,9	5,4
2,05	54,3	50,0
20,9	3,3	3,3
Andre svar (kombinasjon)	1,1	1,1

Oppgave 16a Skriv riktig tall i rutene

$$574 = 5 \cdot 100 + \boxed{} \cdot 10 + 4 \cdot 1$$

Oppgave 16a	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	34,8	25,0
7	**45,7**	**68,5**
60	3,3	3,3
74	3,3	2,2
500	2,2	
Andre svar	9,9	1,1

Oppgave 16b Skriv riktig tall i rutene

$$5{,}74 = 5 \cdot 1 + 7 \cdot \boxed{} + 4 \cdot \boxed{}$$

Oppgave 16b	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	57,6	46,7
0,1 og 0,01	**7,6**	**15,2**
"0,1 og 0,04"; "0,1 og 0,1"; 0,10 og 0,1"	**2,2**	**2,2**
10 og 1	5,4	10,9
"1 og 1"; "1,0 og 1,0"	8,7	12
10 og 100	1,1	1,1
0,01 og 0,001		1,1
-6,26 og 0		1,1
0 og 0		1,1
0 og 1		1,1
0 og 4		1,1
0 og tomt		1,1
Andre svar	15,5	5,4

Oppgave 17 a Skriv som desimaltall

$$\frac{3}{10} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Oppgave 17a	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	31,5	40,2
0,3	**35,9**	**38**
3,10	14,1	6,5
0,33; 0,333; 0,33333	1,1	3,3
3,33 eller 3,333		2,2
33,33		1,1
3,0	3,3	
30; 30,0; 30,00	4,4	2,2
1,3	2,2	
tre tiendeler; 3 tiendeler	2,2	
Andre svar	5,4	6,5

Oppgave 17b Skriv som desimaltall

$$\frac{46}{100} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

Oppgave 17b	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	34,8	45,7
0,46	**37,0**	**28,3**
0,046	2,2	8,7
0,0046	1,1	
46,100	9,8	5,4
46,1	1,1	
46 hundredeler	2,2	
100,46		1,1
10,46		1,1
1,46	1,1	1,1
46 eller 46,0 eller 46,00	4,4	3,3
4600	1,1	
Andre svar	5,5	5,5

Oppgave 17c Skriv som desimaltall

45 tusendeler ....

Oppgave 17c	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	39,1	45,7
0,045	**17,4**	**13,0**
0,0045	14,1	19,6
0,00045		2,2
0,45 eller 0,450 eller 0,4500	3,3	2,2
00,45 eller 000,45	2,2	2,2
4,5 og 4,500	7,6	3,3
45/1000	4,3	1,1
45000,0	2,2	1,1
45,1000	1,1	1,1
450,000 og 450,0	1,1	1,1
45,0	1,1	
Andre svar	6,5	7,6

Oppgave 17d Skriv som desimaltall

28 tideler ....

Oppgave 17d	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	40,2	45,7
2,8	**19,6**	**13,0**
0,28	21,7	29,3
0,028	4,3	1,1
28,0 og 28	4,4	3,3
28/10	4,3	1,1
280,0	2,2	1,1
28,10	2,2	3,3
Andre svar	1,1	2,2

Oppgave 18a Skriv svaret som desimaltall. 6 · 0,5 = ……………………………

Oppgave 18a	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	38,0	35,9
3	**33,7**	**45,7**
0,3 eller 0,30 eller 0,3000	15,2	7,6
30,0	2,2	1,1
6,5	2,2	1,1
3,5	2,2	
9,0	1,1	1,1
0,5	1,1	1,1
Andre svar	4,4	6,5

Oppgave 18b Skriv svaret som desimaltall. 3 : 6 = ..……………………………

Oppgave 18b	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	52,2	38,0
0,5	**32,6**	**35,9**
2 eller 2,0 eller 2,00	2,2	15,2
0,2	2,2	3,3
3 eller 3,0	1,1	3,3
3,6		2,2
0,3	1,1	
0,35	1,1	
0,36	1,1	
Andre svar	6,5	2,2

Oppgave 18c Skriv svaret som desimaltall. 3 : 0,5 = ……………………………

Oppgave 18c	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	65,2	52,2
6	**10,9**	**21,7**
0,6 og 0,60	7,6	1,1
1,5	4,3	10,9
0,5	1,1	1,1
0,05	1,1	1,1
3,5		3,3
0,25		2,2
0,1	1,1	
0,2	1,1	
0,4	7,5	6,6

Oppgave 18d Skriv svaret som desimaltall. 0,6 : 0,2 = …………………………….

Oppgave 18d	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	63,0	53,3
3	**5,4**	**10,9**
0,3	25,0	22,8
0,03	1,1	3,3
0,03333	1,1	
0,4	1,1	1,1
0,2 eller 0,20	2,2	1,1
1,2		2,2
Andre svar (eks. 0; 2,0; 0,15; 12; 6,2; 00,1)	1,1	5,4

Oppgave 19a Til venstre i rammen står et regnestykke. Sett ring rundt det tallet du mener er *nærmest svaret*. Du trenger ikke regne ut svaret.

13 : 4,32	0,03 0,3 3 30 300

Oppgave 19a	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	12,0	13,0
3	**50,0**	**60,9**
0,03	10,9	4,3
0,3	20,7	10,9
30	6,5	7,6
300		3,3

Oppgave 19b Til venstre i rammen står et regnestykke. Sett ring rundt det tallet du mener er *nærmest svaret*. Du trenger ikke regne ut svaret.

7,5 : 0,24	0,03 0,3 3 30 300

Oppgave 19b	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	15,2	16,3
30	**15,2**	**23,9**
0,03	17,4	8,7
0,3	19,6	16,3
3	31,5	34,8
300	1,1	

Oppgave 19c Til venstre i rammen står et regnestykke. Sett ring rundt det tallet du mener er *nærmest svaret*. Du trenger ikke regne ut svaret.

0,73 · 46,2	0,03 0,3 3 30 300

Oppgave 19c	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	15,2	15,2
30	**40,2**	**55,4**
0,03	17,4	8,7
0,3	8,7	6,5
3	5,4	4,3
300	13,0	8,7
"3 og 30"		1,1

Oppgave 20a Sett ring rundt *alle* regneuttrykkene som passer til regneoppgaven:

24 halsbånd pakkes i eske. Om 24 halsbånd veier 3 kg, hvor mye veier da ett halsbånd?

24 · 3   24 : 3   3 : 24   3 · 24   24 – 3   3 + 24

Oppgave 20a	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	10,9	12,0
3 : 24	**25,0**	**13,0**
24 : 3	25,0	42,4
3 : 24 og 24 : 3	18,5	21,7
24 · 3 og 3 · 24	6,5	2,2
24 · 3	3,3	1,1
3 · 24	2,2	1,1
24 : 3 og 3 · 24	2,2	
24 : 3 og 24 – 3	2,2	1,1
24 : 3 og 3 : 24 og 24 - 3	1,1	1,1
Andre svar	3,3	4,4

Oppgave 20b Sett ring rundt *alle* regneuttrykkene som passer til regneoppgaven:

1 kg pølser koster 49,50 kr. Per kjøper 1,7 kg. Hvor mye koster det?

49,50 · 1,7 49,50 : 1,7 1,7 : 49,50 1,7 · 49,50 49,50 – 1,7

Oppgave 20b	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	12,0	14,1
49,50 · 1,7 og 1,7 · 49,50	**28,3**	**39,1**
49,50 · 1,7	**23,9**	**21,8**
1,7 · 49,50	**13**	**4,3**
49,50 : 1,7	12	10,9
1,7 : 49,50	5,4	1,1
49,50 · 1,7 og 1,7 : 49,50	2,2	
49,50 : 1,7 og 1,7 : 49,50	1,1	3,3
49,50 – 1,7	1,1	3,3
49,50 · 1,7 og 49,50 : 1,7		1,1
Andre svar	1,1	1,1

Oppgave 20c Sett ring rundt *alle* regneuttrykkene som passer til regneoppgaven:

Kaker skal fylles i bokser, med 0,75 kg i hver. Hvor mange bokser trenger man til 6 kg kaker?

6 · 0,75 6 : 0,75 0,75 : 6 0,75 · 6 6 – 0,75 6 + 0,75

Oppgave 20c	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	13,0	16,3
6 : 0,75	**15,2**	**21,7**
6 · 0,75 og 0,75 · 6	19,6	22,8
6 · 0,75	18,5	9,8
0,75 · 6	14,1	6,5
6 : 0,75 og 0,75 : 6	8,7	8,7
0,75 : 6	7,6	7,6
6 · 0,75 og 6 : 0,75	1,1	2,2
6 : 0,75 og 0,75 : 6 og 6 - 0,75	1,1	
0,75 : 6 og 6 + 0,75	1,1	
6 – 0,75		2,2
6 + 0,75		1,1
6 · 0,75 og 0,75 : 6		1,1

Oppgave 20d Sett ring rundt *alle* regneuttrykkene som passer til regneoppgaven:

Anne kjøper bananer i en butikk til 13,50 kr per kg. Hvor mye kan Anne kjøpe for 10,50 kr?

$13{,}50 \cdot 10{,}50$ $10{,}50 : 13{,}50$ $13{,}50 : 10{,}50$ $13{,}50 - 10{,}50$ $13{,}50 + 10{,}50$

Oppgave 20d	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	21,7	18,5
10,50 : 13,50	**22,8**	**18,5**
13,50 : 10,50	20,7	23,9
10,50 : 13,50 og 13,50 : 10,50	14,1	14,1
13,50 – 10,50	9,8	15,2
13,50 · 10,50	5,4	4,3
Andre svar	5,5	5,5

Oppgave 21a Lag din egen fortelling som passer til disse regnestykkene:

$\boxed{4 : 0{,}5}$

Oppgave 21a	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	48	43
Riktig målingsdivisjon og realistisk kontekst	**3**	**7**
Riktig målingsdivisjon (kode 2)	**2**	**1**
Urealistisk kontekst	9	12
Subtraksjonsfortelling	2	2
Regnefortelling til det inverse regneuttrykket	11	18
Multiplikasjonsfortelling	3	2
Andre svar	18	11
fortelling (eks. "Hva er 4:0,5.")	4	4

Oppgave 21b Lag din egen fortelling som passer til disse regnestykkene:

$\boxed{5{,}25 \cdot 3{,}28 = 17{,}22}$

Oppgave 21b	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	58	61
Riktig	**13**	**14**
Andre svar	29	25

Oppgave 22a Skriv riktig tall i ruta $14 : 2 = \square \cdot 14$

Oppgave 22a	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	34,8	29,3
0,5	**15,2**	**29,3**
7	35,9	27,2
2	5,4	3,3
0,25	4,3	2,2
0,2	1,1	4,4
28	1,1	1,1
1	1,1	
Andre svar (0; 3,2; 3,5; 4)	1,1	3,3

Oppgave 22b Skriv riktig tall i ruta $14 : \square = 0{,}25 \cdot 14$

Oppgave 22b	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	52,2	53,3
4	**7,6**	**10,9**
0,25	5,4	4,3
2	4,3	5,4
3,5	3,3	4,3
7	3,3	1,1
1	2,2	
0,5	2,2	1,1
15	2,2	
56	2,2	1,1
3	1,1	2,2
25		4,3
5	1,1	3,3
6	1,1	1,1
8		2,2
1/4	1,1	
0,4	1,1	
1,50	1,1	
Andre svar (0,350; 12; 13; 14; 14,25; 16; 2,5; 35; 389; 20; 250; 33; 9)	8,7	5,4

Oppgave 23a Skriv enklest mulig: 2x + 5x ....

Oppgave 23a	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	55,4	18,5
7x	**21,7**	**70,7**
2 + 5	4,4	
6x		2,2
2x + 5x	1,1	2,2
”2x+5x=2x5x”eller ”2x5x”	2,2	
"x.x.x.x.x.x.x=7x	1,1	
"x+x+x+x+x+x+x	1,1	
"xx xxxxx	1,1	
x.x+x.x.x.x.x		1,1
$10x^2$		1,1
"7x, 7^2		1,1
2+5x		1,1
x^7		1,1
$7x^2$		1,1
Andre svar (7; 10; 12; 14; "12.30"; "2.5=10"; "25+52"; 57; "2.2 + 5.5"; "20+50"; "210 + 510=720")	12	

Oppgave 23b Skriv enklest mulig: $x + x + 2x$ ....

Oppgave 23b	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	60,9	21,7
4x	**16,3**	**59,8**
$2x^3$	1,1	4,3
$x^2 + 2x$		3,3
2x+x.x		1,1
3x eller x3	2,2	1,1
x+x+x+x	1,1	
x+x+2.x		1,1
x+x+x+2		1,1
”x” eller ”6x”	2,2	
xx2x	1,1	
2+3x		1,1
2x2 xxx	1,1	
3^x eller 4^x		1,1
$4x^2$		1,1
x^4	1,1	1,1
Andre svar ("1+1+20"; "10+10+210=240"; "2+2+2.2"; "6.6.18"; 2; 6; 12; 20; "3+3+2"; "5+5.25"; 94; 2^3; " + + 2")	13,0	1,1

Oppgave 23c Skriv enklest mulig: $t \cdot t \cdot t$ ....

Oppgave 23c	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	64,1	18,5
t^3	**3,3**	**50,0**
3t eller t3	15,2	26,1
t*t*t eller ttt	2,2	
t eller 1.t	3,3	1,1
3^t	1,1	3,3
x		1,1
tid x tid x tid	1,1	
"1.1.1" eller "2.2.2"	3,3	
Andre svar (3; 6; 9; 14; 41; 20000)	6,5	

Oppgave 23d Skriv enklest mulig: $2y \cdot y^2$ ....

Oppgave 23d	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	70,7	37,0
$2y^3$	**0,0**	**8,7**
4y	1,1	12,0
$2y.y^2$	1,1	10,9
$2y^2$		7,6
$3y^2$	1,1	2,2
2y.y.y	1,1	
y^4		4,3
2.y.y		3,3
$y.y^2$		1,1
yyxyy eller y.y.y.y	2,2	
$2+y^3$		
$2y+y^2$		1,1
y+y.y+y=16y	1,1	3,3
y	1,1	
2y		
3y		2,2
5y	1,1	1,1
6y		1,1
8y		1,1
y^2 eller yy	1,1	1,1
y^3	1,1	
y^8	1,1	1,1
Andre svar	6,5	1,1

Oppgave 24a $x + y + z = x + p + z$ Dette

☐ er alltid sant ☐ er aldri sant ☐ *kan* være sant, nemlig når ....

Oppgave 24a	Frekvens i prosent 2005 høst	2006 vår
Ikke besvart	40,2	20,7
***kan* være sant, nemlig når "p=y"**	**2,2**	**5,4**
***kan* være sant, men mangler kommentar**	**7,6**	**5,4**
alltid sant	21,7	21,7
aldri sant	26,1	44,6
kan være sant, men kommentaren er feil	1,1	2,2
"alltid sant og aldri sant og kan være sant"	1,1	

Oppgave 24b $2a + 3 = 2a - 3$ Dette

☐ er alltid sant ☐ er aldri sant ☐ *kan* være sant, nemlig når ....

Oppgave 24b	Frekvens i prosent 2005 høst	2006 vår
Ikke besvart	41,3	23,9
aldri sant	**34,8**	**48,9**
alltid sant	14,1	20,6
kan være sant	8,7	6,5
"aldri sant og kan være sant"	1,1	

Oppgave 25a Adder tallet *x* til

$x + 3y$ ....

Oppgave 25a – del 1	Frekvens i prosent 2005 høst	2006 vår
Ikke besvart	82,6	52,2
2x + 3y	**3,3**	**19,6**
"x + x + 3y" eller "x + 3y + x"		**2,2**
"x+3y" eller "1.x+3y" eller "3y+x"	4,4	9,8
x+y+y+y		2,2
"x+3y=3yx" eller "x+3y=3xy"	1,1	1,1
x+3yx		1,1
$x+x^2$		1,1
x+y.x.x		1,1
$x+y^3$		1,1
3xy eller 3yx eller $3yx^1$ eller x3y	2,2	2,2
"4xy" eller "4yx"	6,6	6,6

Oppgave 25b Adder tallet *x* til

4*x* ....

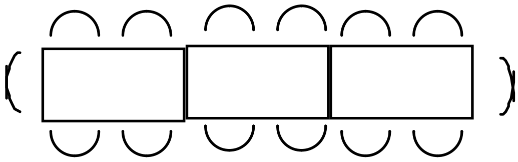
Oppgave 25b	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	79,4	53,3
5x	**3,3**	**28,3**
x+x+x+x	2,2	4,3
4x	4,4	3,3
x+x+x+x=4x	1,1	
4+x	1,1	1,1
x-4x=3x	1,1	
x.10=4x	1,1	
$4+x^2$		1,1
$4x^1$		1,1
$4x^2$	1,1	1,1
4y	1,1	
x.x.x.x	1,1	1,1
x.x.x.x+x		1,1
x^4		2,2
3x		1,1
8x		1,1
Andre svar (5; 20)	2,2	

Oppgave 25c Adder tallet *x* til

7 ....

Oppgave 25c	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	80,4	53,3
7 + x	**5,4**	**21,7**
7x	2,2	9,8
7	4,4	2,2
"4+3" eller "3+4"	1,1	2,2
x+x+x+x+x+x+x=7x	1,1	
x+x+x+x+x+x+x		1,1
x+x+x+x+x+x+x=7		1,1
xxxxxxx	1,1	
$x.x.x.x.x.x.x=x^7$		1,1
x^7		1,1
x-7=6	1,1	
Andre svar	3,3	6,4

Oppgave 26 Et konfirmasjonsbord er satt sammen av småbord, og rundt er det satt stoler.



a Hvor mange stoler blir det plass til om vi har 4 småbord?..............................

Oppgave 26a	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	10	8
18	**61**	**72**
16	8	4
17, 19, 20	4	10
Andre svar	17	6

Oppgave 26 Et konfirmasjonsbord er satt sammen av småbord, og rundt er det satt stoler.

b Enn om vi har 7 småbord? ....

Oppgave 26b	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	17	14
30	**48**	**62**
26	5	3
28	4	
29	4	1
31	3	1
32	2	3
42	5	3
Andre svar	10	12

Oppgave 26 Et konfirmasjonsbord er satt sammen av småbord, og rundt er det satt stoler.

c Dersom vi har *n* småbord, hvor mange stoler blir det da?...............................

Oppgave 26c	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	71	65
***n*.4+2**		**2**
"*n*=*x* bord.4 +2"		**1**
tallverdi (eks. 4, 40; 120, 11; 15;)	19	10
Bokstav (*N; m; x; å; b*)	9	17
"*n*.4"; "4.*n*"		3
bokstavuttrykk ("bord=stol.4(+2)"; ”*n*.6”)		2
"ingen"	1	

Oppgave 27 a $x = a + b - c$ Dersom $a = 1$, $b = 2$ og $c = 3$ blir x=......................

Oppgave 27a	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	71,7	45,7
0	**9,8**	**37,0**
1+2-3	**1,1**	
6	3,3	9,8
24	3,3	1,1
-3		1,1
-1		2,2
Andre svar	11	3,3

Oppgave 27b y = b3 Dersom b = 4 blir **y = ...**

Oppgave 27b	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	72,8	47,8
64	**0,0**	**13,0**
4^3	**1,1**	**1,1**
12 eller "3.4=12"	5,4	18,5
32 eller "4.4.4=y=32" eller "4x4x4x4=32"		5,4
Andre svar	21	14

Oppgave 27c 3x = 7 og 5y = 11 Da blir 3x + 5y = ...

Oppgave 27c	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	78,3	50,0
18	**10,9**	**32,6**
76 eller "21+55"		6,5
3x+5y		2,2
8x	1,1	1,1
8xy	1,1	1,1
Andre svar	8,8	6,6

Oppgave 30 Marit har tre skåler med nøtter *A, B* og *C*. Det er 2 flere i *B* enn i *A*. I skål *C* er det 4 ganger så mange nøtter som i skål *A*. I alt er det 14 nøtter.
Hvor mange nøtter er det i hver av skålene?
....
Vis eller forklar hvordan du tenkte:

Oppgave 30	Frekvens i prosent 2005 høst	2006 vår
Ikke besvart	56,5	60,9
Riktig svar (2 nøtter i skål A, 4 nøtter i skål B, 8 nøtter i skål C) - med forklaring (eks. "Jeg prøvde meg fram med ulike verdier. 2+4+8=14"; "2.4 +2 +2.2=14 nøtter"; "Jeg tok utgangspunkt i et tall og prøvde meg frem.";	**22,8**	**19,6**
Riktig svar (2 nøtter i skål A, 4 nøtter i skål B, 8 nøtter i skål C) - uten forklaring	**7,6**	**7,6**
Riktig svar (2 nøtter i skål A, 4 nøtter i skål B, 8 nøtter i skål C) - med minimal forklaring (eks. "B=2+2=4 A=2 C=8")	**4,4**	**1,1**
Feilsvar (eks. "A=1 B=2 C=4"; "A=2 B=4 C=12"; "A= 2 B=5 C=7"; "A=1 B=8 C=5")	8,7	10,8

Oppgave 31 5 liter bær veier 10 kg. Hvor mye veier 6 liter bær?
....

Vis hvordan du tenker.

Oppgave 31	Frekvens i prosent 2005 høst	2006 vår
Ikke besvart	32,6	30,4
Riktig svar 12kg - med forklaring (eks. "5 l bær veier 10 kg, 1 liter bær veier 2 kg, og da blir det bare 2 kg ekstra."; "10:5=2 6.2=12"; ”10:5=2 1 liter veier 2 kg”; "10:5=2 10+2=12kg")	**21,7**	**32,6**
Riktig svar 12kg - uten forklaring	**17,4**	**13,0**
Riktig svar 12kg - med minimal forklaring (eks. "det veier dobbelt så mye"; "+2 til."; "6.2"; "(11111 + 1) (1111111111 11)"; "Jeg tenkte 10:5=2")	**4,4**	**6,5**
Riktig svar 12kg - med feil forklaring (eks."12 kg veier 6 kg."; "5:10=2kg 2kg . 6=12kg"; "10:5= 2 . 6=12"; "10 : 5 = 2 = 10 . 2"; "det er 2 kg per bær 2.6=12")	**10,9**	**5,4**
Feilsvar (eks. "6.10=60 liter bær"; ”10,5kg 10:5=5"; "11 kg jeg tok 10 kg +1=11 5l=10 6l=11")	13,0	12,0

Appendix 2: Algebra and Numbers 11th grade
Appendix 2.1: Test 2005 høst – 2006 vår

TALL OG ALGEBRA

En undersøkelse, 1. klasse videregående skole

Navn: .... Dato: ....

Elev nummer .... (fylles ut av skolen)

Elev nummer: .... (fylles ut av skolen)

TALL OG ALGEBRA

En undersøkelse, 1. klasse videregående skole

Dato: .... ☐ Gutt ☐ Jente

Alder: .... år og .... måneder

Lommeregner skal ikke brukes på denne testen.

1 **a** $\frac{1}{2}+\frac{1}{4}=$ **b** $\frac{1}{2}-\frac{1}{3}=$

c 900 : 30 = **d** $70 \cdot 0{,}3 =$ **e** $60 \cdot 450 =$

2 Finn et tall med to desimaler som ligger mellom
4,755 og 4,762

3 Fyll ut hele tabellen:

x	$4x$	$\frac{x}{2}$	x^2
2	8	1	4
5			
		12	
			16
	2		

4 a Sett ring rundt det tallet som ligger nærmest 0,16 i størrelse

0,1 0,2 15 0,21 10

b Sett ring rundt det tallet som ligger nærmest 2,08 i størrelse

209 2,9 2,05 2,1 20,9

c Skriv riktig tall i rutene

$$5{,}074 = 5 \cdot 1 + 7 \cdot \boxed{} + 4 \boxed{}$$

5 Skriv som desimaltall

a $\frac{3}{10}$ .... **b** $\frac{46}{100}$ ....

c 45 tusendeler .... **d** 28 tideler ....

6 Sett ring rundt *alle* regneuttrykkene som passer til regneoppgaven:

a *1 liter hvetemel veier 0,8 kg. Hvor mye veier 0,7 liter hvetemel?*

$0{,}8 \cdot 0{,}7$ $0{,}8 : 0{,}7$ $0{,}7 : 0{,}8$ $0{,}8 - 0{,}7$ $0{,}8 + 0{,}7$ $0{,}7 \cdot 0{,}8$

b *Kaker skal fylles i bokser, med 0,75 kg i hver. Hvor mange bokser trenger man til 6 kg kaker?*

$6 \cdot 0{,}75$ $6 : 0{,}75$ $0{,}75 : 6$ $0{,}75 \cdot 6$ $6 - 0{,}75$ $6 + 0{,}75$

c *Anne kjøper bananer i en butikk til 13,50 kr per kg. Hvor mye kan Anne kjøpe for 10,50 kr?*

$13{,}50 \cdot 10{,}50$ $10{,}50 : 13{,}50$ $13{,}50 : 10{,}50$ $13{,}50 - 10{,}50$ $13{,}50 + 10{,}50$

7 Sett opp et regneuttrykk som passer for å løse oppgaven: (Du skal ikke regne det ut).

a *1 kg svinekoteletter koster 65,50 kr. Hva koster 0,76 kg?*

....

b *Et terrengløp går i ei løype som er 5,6 km. Hvor mange engelske mil er det? (Ei engelsk mil er 1,609 km).*

....

8 Skriv riktig tall i ruta

a $14 : 2 = \square \cdot 14$ **b** $14 : \square = 0{,}25 \cdot 14$

c $15 : 10 = \square \cdot 15$ **d** $8 : \frac{1}{2} = 8 \cdot \square$

9 Skriv enklere dersom det er mulig:

a $2x + 5x$ ....

b $x + x + 2x$ ....

c $t \cdot t \cdot t$ ....

d $2y \cdot y^2$ ....

e $3a + a^2 + a^2$ ....

f $5a - 2(7 - a) + 6$ ....

10 **a** $x + y + z = x + p + z$ Dette

☐ er alltid sant ☐ er aldri sant ☐ *kan* være sant, nemlig når

....

b $a + b \cdot 2 = 2b + a$ Dette

☐ er alltid sant ☐ er aldri sant ☐ *kan* være sant, nemlig når

....

c $\frac{2x+1}{2x+1+5} = \frac{1}{6}$ Dette

☐ er alltid sant ☐ er aldri sant ☐ *kan* være sant, nemlig når ....

11 For å finne volumet V av en sylinder, har vi denne formelen: $V = \pi r^2 h$.
Her er r radius i grunnflata , og høyden er h.

a Hva skjer med volumet dersom vi dobler både radius r og høyde h?

....

b Hvordan må høyden endres dersom vi dobler radius r, men vil at sylinderen skal ha samme volum V?

....

12 Hva må x være dersom

a $123 + 2x = 195 - x$ ....

b $\frac{x+1}{x+4} = \frac{4}{5}$ ....

c) $3x = 7$ og $5y = 11$ Hva er da $3x + 5y$?

$3x + 5y =$..

13 På en skole er det 10 elever for hver lærer. Hvilke av disse uttrykkene forteller dette?
L = antall lærere, E = antall elever. Sett ring rundt det eller de riktige uttrykk.

$10L = E$ $\quad$ $10E = L$ $\quad$ $L = 10E$ $\quad$ $E = 10L$ $\quad$ $10L + E$ $\quad$ $11LE$

Oppgave 14 a Vekta av et barn avhenger av alderen. For et barn fra 2 til 10 år kan vi bruke følgende sammenheng som en slags normal:
Om y står for vekt i kg og x står for barnets alder i antall år, så er:

$$y = 4 + 2{,}5x$$

Sett kryss foran det svaret nedenfor som du mener er best:

Hva betyr tallet 4 her?

☐ Barnet er 4 år
☐ Barnet er 4 kg ved fødselen
☐ Barnets vekt øker med 4 kg hvert år
☐ Ingen av delene, det betyr……………………..

Hva betyr tallet 2,5 her?

☐ Barnet er 2,5 år
☐ Barnet er 4 + 2,5 år, altså 6,5 år
☐ Barnet veier 2,5 kg når det blir født
☐ Barnets vekt øker med 2,5 kg hvert år
☐ Ingen av delene, det betyr……………………..

15 ET EKSEMPEL: *Summen* av tallene 6 og 8 er 14. *Differensen* mellom dem er 2.

OPPGAVE: Even tenker på to tall. Summen av dem er 19. Differensen mellom dem er 5.

a Finn tallene.

....

b Hvordan kan du gå fram for å finne tallene?

………………………………………………………………………..

....

…………………………………………………………………………

…………………………………………………………………………

c Hvorfor er det alltid mulig å finne to tall når vi kjenner summen og differensen?

....

…………………………………………………………………………

…………………………………………………………………………

Appendix 2.2: Information about the coding system

Kodeforklaringer Tall og algebra, 1.trinn

Oppgave	Kodeforklaringer
1a	Kode 1: 3/4 Kode 2: 6/8
2	Kode 1: 4,76 Kode 2: 4,760
3a	Kode 1: Riktig tall i alle rutene Kode 2: Feil i en av rutene Kode 3: Feil i to av rutene
3b	Kode 1: Riktig tall i alle rutene Kode 2: Feil i en av rutene Kode 3: Feil i to av rutene
3c	Kode 1: Riktig tall i alle rutene Kode 2: Feil i en av rutene Kode 3: Feil i to av rutene
3d	Kode 1: Riktig tall i alle rutene Kode 2: Feil i en av rutene Kode 3: Feil i to av rutene
4c	Kode 1: Riktig tall i begge rutene Kode 2: Galt tall i en av rutene
7a	Kode 1: 65,50 (kr) · 0,76 (kg) eller 0,76(kg) · 65,50 (kr) Kode 2: 65,50 : 1000 . 760 eller 65,50:100.76
6a	Kode 1: Ring rundt både 0,8 · 0,7 og 0,7 · 0,8 Kode 2: Ring rundt enten 0,8 · 0,7 eller 0,7 · 0,8
9a	Kode 1: $7x$ Kode 11: Kunne ikke løse oppgaven riktig eller gir et tall som resultat
9b	Kode 1: $4x$ Kode 2: $2x+2x$ Kode 11: Kunne ikke løse oppgaven riktig eller gir et tall som resultat
9c	Kode 1: t^3 Kode 11: Kunne ikke løse oppgaven riktig eller gir et tall som resultat

Oppgave	Kodeforklaringer
9d	Kode 1: $2y^3$ Kode 11: Kunne ikke løse oppgaven riktig eller gir et tall som resultat
9e	Kode 1: $3a+2a^2$ Kode 11: Kunne ikke løse oppgaven riktig eller gir et tall som resultat
9f	Kode 1: $7a$-8 Kode 2: $7a$-14+6 Kode 11: Kunne ikke løse oppgaven riktig eller gir et tall som resultat
10a	Kode 1: Riktig kryss og riktig begrunnelse Kode 2: Riktig kryss, men manglende eller utilfredstillende begrunnelse
10c	Kode 1: Riktig kryss og riktig begrunnelse Kode 2: Riktig kryss, men manglende eller utilfredstillende begrunnelse
11a	Kode 1: $8y$ eller verbal svar (eksampel "8 ganger så stort som volumet av sylinderen S") Kode 2: manglende eller utilfredstilende svar (eksampel V = PI · $(2r)^2 \cdot 2h$) eller T = Pi · $4r^2 \cdot 2h$)
11b	Kode 1: 0,25·h eller 1/4·h eller verbal svar (eksampel 1/4 av høyden i S eller "fire ganger så kort / liten" eller "4 ganger mindre enn høyden i S") Kode 2: manglende eller utilfredstillende svar (eksempel h =h/4 eller U= Pi · $(2r)^2 \cdot h/4$ eller U = h ·1/4 eller " "1/4 av S")
12a	Kode 1: 24 Kode 2: 72/3 Kode 3: (195-123)/3
12c	Kode1: 18 Kode 2: 7+11 Kode 3: 3 · 2,33 + 5.2,2 eller 17,99
12d	Kode1: 0 Kode 2: x=1+2-3
13	Kode 1: Ring rundt både $10L=E$ og $E=10L$ Kode 2: Ring rundt enten $10L=E$ eller $E=10L$

Oppgave	Kodeforklaringer
15b	Kode 1: Eleven forklarer metoden for å finne svaret med algebraisk eller verbal begrunnelse uten feil (eksempel "$x+y=19$ $x-y=5$ Innsetningsmetoden" eller "$x+x+5=19$ $2x=19-5$, 14/2=7 7+5=12" eller "Man kan bruke prøve og feile metoden." eller "sette opp flere tall som gir summen 19 og så se hvilke som har differanse 5") Kode 11: Feil eller manglende begrunnelse. (eksempel: "Plusse eller dele.", "Telle seg frem.", "Tenke.", "Regne i hodet.", "Finne to ukjente.", "Gjetting.", "Man kan finne eller man kan ta 5-19.")
15c	Kode 1: Kan generalisere med algebraisk utrykk som løsning, (eksempel: "*Sum - differanse =x;* *x/2 = svar 1*; *svar 1+differanse =svar 2*",eller "Fordi man kan sette opp et likningssett *x+y=summen* ; *x-y=differansen*", eller reflekterer det er mulig å bruke likningsmetoden som forklart i 15b , eksempel - "Fordi da kan man finne det ut ved å gjøre som forklart i b." (Eleven brukte likningsmetode i 15b.), "Du kan sette opp to likninger og regne ut de ukjente."(eleven brukte likningsmetode i 15b.)) Kode 11: Kan ikke generalisere med algebraiske uttrykk som løsning eller feil, manglende, utilfredstillende begrunnelse. (eksempel: "Du kan alltid regne deg til om det er rett eller galt.", "Prøve og feile", "Fordi det er bare å plusse , det er alltid et tall i mellom.")

Appendix 2.3: Tables with the solution frequencies for each task on the test – results for a group of 113 students, both tests

Here are presented the tables concerning the results of a group of 113 students, who did the test in the beginning and at the end of the school year 2005-2006. The results were made with the help of the statistical program SPSS. The solution frequencies of test items 3 and 15 are not included.

Oppgave 1a $\frac{1}{2}+\frac{1}{4}=$

Oppgave 1a	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	0,9	0,9
3/4	**83,2**	**81,4**
6/8 eller 9/12	**8,8**	**4,4**
"2/4 og 1/2	3,5	1,8
"2/6	1,8	6,2
"3/6	0,9	
"5/4	0,9	
" 1/8		0,9
"1/		0,9
"1/6		1,8
"2/3		0,9
"3/8		0,9

Oppgave 1b $\frac{1}{2}-\frac{1}{3}=$

Oppgave 1b	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	1,8	3,5
1/6	**80,5**	**79,6**
1000/6000		**0,9**
5/6	10,6	4,4
1, 1/1 og 6/6	1,8	1,8
1/3 og 2/6	0,9	2,7
0 og 0/6	0,9	3,5
0,83	0,9	
sin 53. hyp/2	0,9	
-1 og -1/1		1,8
andre svar 4/3, 6, -1/6, 2/3	1,8	1,8

Oppgave 1c 900 : 30 =

Oppgave 1c	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	5,3	3,5
30	**84,1**	**92,9**
300	3,5	1,8
3	1,8	0,9
Andre svar	5,4	0,9

Oppgave 1d $70 \cdot 0{,}3 =$

Oppgave 1d	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	18,6	16,8
21	**47,8**	**60,2**
210	1,8	4,4
2,10 og 2,1	4,4	
0,21 og 0,210 og ,210	7,1	1,8
30	2,7	
23 og 23,3 og 23,33 og 23,333	4,4	5,3
233,4 og 234	1,8	
andre svar 17000, 2,7, 0,7, 35, 3,5, 23, 2,333, 70,3, 63,33 osv	11,4	11,4

Oppgave 1e $60 \cdot 450 =$

Oppgave 1e	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	9,7	8
27000	**60,2**	**55,8**
2700	10,6	13,3
270		0,9
27		0,9
24300	2,7	1,8
27500	1,8	
270000	0,9	
2700000	0,9	
3sifra andre svar		1,8
4sifra andre svar	4,4	7,1
5sifra andre svar	8,0	7,1
6sifra andre svar	0,9	1,8
7sifra andre svar		1,8

Oppgave 2 Finn et tall med to desimaler som ligger mellom 4,755 og 4,762

Oppgave 2	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	25,7	9,7
4,76	**55,8**	**63,7**
4,760	**0,9**	
rett med tre desimaler	6,2	11,5
2 desimaler men feil	6,2	4,5
en desimal	1,8	1,8
ikke noe desimaltall	1,8	5,3
ser ingen løsning med to desimaler	0,9	
rett med 5 desimaler	0,9	
to eller tre forskjellige varianter		3,6

Oppgave 4 a Sett ring rundt det tallet som ligger nærmest 0,16 i størrelse

0,1 0,2 15 0,21 10

Oppgave 4a	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	0,9	
0,2	**88,5**	**92,9**
0,1	2,7	0,9
15		
0,21	7,1	4,4
10		
Andre svar	0,9	1,8

Oppgave 4 b Sett ring rundt det tallet som ligger nærmest 2,08 i størrelse

209 2,9 2,05 2,1 20,9

Oppgave 4b	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	0,9	
2,1	**69,9**	**79,7**
209	0,9	
2,9		0,9
2,05	26,5	17,7
20,9	1,8	
Andre svar		1,8

Oppgave 4 c rute 1 Skriv riktig tall i rutene

$$5{,}074 = 5 \cdot 1 + 7 \cdot \boxed{} + 4 \cdot \boxed{}$$

Oppgave 4c rute 1	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	27,4	23
0,01	**36,3**	**50,4**
1/100	**1,8**	
0,1	10,6	9,7
0	4,5	2,7
1	6,2	1,8
10	6,2	7,1
100	1,8	
Andre svar	5,2	5,4

Oppgave 4 c rute 2 Skriv riktig tall i rutene

$$5{,}074 = 5 \cdot 1 + 7 \cdot \boxed{} + 4 \cdot \boxed{}$$

Oppgave 4c rute 2	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	36,3	27,4
0,001	**35,4**	**50,4**
1/1000	**1,8**	
1	8,0	8
0,100 og 00,1 og 0,1	0,9	4,5
10	2,7	
0,01	5,3	4,4
100	0,9	1,8
1000	0,9	
Andre svar	7,9	3,6

Oppgave 5 a Skriv som desimaltall

$\frac{3}{10}$ ....

Oppgave 5a	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	5,3	5,3
0,3	**59,3**	**74,3**
0,333 med ulikt antall desimaler	15,9	13,3
3,33 og 3,333 og 3,3333 osv.	14,1	4,5
0,03	0,9	1,8
Andre svar	4,5	0,9

Oppgave 5 b Skriv som desimaltall

$\frac{46}{100}$ ....

Oppgave 5b	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	10,6	7,1
0,46	**67,3**	**78,8**
0,046	9,7	10,6
4,6	1,8	
Andre svar	10,6	3,6

Oppgave 5 c Skriv som desimaltall 45 tusendeler ....

Oppgave 5c	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	11,5	3,5
0,045	**48,7**	**54,9**
0,0045	23,9	32,7
45/1000	4,4	0,9
45000	1,8	1,8
Andre svar	9,8	6,3

Oppgave 5 d Skriv som desimaltall 28 tideler ....

Oppgave 5d	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	11,5	5,3
2,8	**37,2**	**46,9**
0,28	37,2	43,4
28/10	5,4	
0,028	3,5	2,7
28	1,8	
andre svar	3,6	

Oppgave 6 a Sett ring rundt *alle* regneuttrykkene som passer til regneoppgaven:

1 liter hvetemel veier 0,8 kg. Hvor mye veier 0,7 liter hvetemel?

0,8 · 0,7 0,8 : 0,7 0,7 : 0,8 0,8 – 0,7 0,8 + 0,7 0,7 · 0,8

Oppgave 6a	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	6,2	4,4
0,8 · 0,7 og 0,7 • 0,8	**31,9**	**46,0**
0,8 · 0,7	**11,4**	**7,1**
0,7 · 0,8	**5,4**	**0,9**
0,8 : 0,7	23	13,3
0,7 : 0,8	12,4	9,7
0,8 – 0,7	2,7	11,5
0,8 + 0,7		
0,8 · 0,7 og 0,7 - 0,8		1,8
0,8 · 0,7 og 0,7 : 0,8	2,7	
0,8 : 0,7 og 0,7 · 0,8		0,9
0,8 · 0,7 , 0,8 : 0,7	0,9	
0,8 : 0,7 og 0,8 - 0,7		2,7
0,8 · 0,7 , 0,8 : 0,7 og 0,7 · 0,8		1,8
andre svar	3,6	

Oppgave 6 b Sett ring rundt *alle* regneuttrykkene som passer til regneoppgaven:

Kaker skal fylles i bokser, med 0,75 kg i hver. Hvor mange bokser trenger man til 6 kg kaker?

6 · 0,75 6 : 0,75 0,75 : 6 0,75 · 6 6 – 0,75 6 + 0,75

Oppgave 6b	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	0,9	0,9
6 : 0,75	**50,4**	**59,3**
6 · 0,75	13,3	3,5
6 · 0,75 og 0,75 · 6	12,4	15
0,75 · 6	9,7	5,3
0,75 : 6	4,4	5,3
6 : 0,75 og 0,75 : 6	2,7	3,5
6 : 0,75 og 0,75 · 6	1,8	3,5
0,75 : 6 og 6 - 0,75	0,9	
Andre svar	3,6	3,6

Oppgave 6c Sett ring rundt *alle* regneuttrykkene som passer til regneoppgaven:

Anne kjøper bananer i en butikk til 13,50 kr per kg. Hvor mye kan Anne kjøpe for 10,50 kr?

13,50 · 10,50 10,50 : 13,50 13,50 : 10,50 13,50 – 10,50 13,50 + 10,50

Oppgave 6c	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	8,0	5,3
10,50 : 13,50	**46,9**	**48,7**
13,50 : 10,50	32,7	31,9
10,50 : 13,50,13,50 : 10,50	6,2	3,5
13,50 · 10,50	3,5	4,4
13,50 - 10,50	1,8	3,5
Andre svar	0,9	2,7

Oppgave 7a Sett opp et regneuttrykk som passer for å løse oppgaven: (Du skal ikke regne det ut).

1 kg svinekoteletter koster 65,50 kr. Hva koster 0,76 kg?

Oppgave 7a	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	5,3	4,4
Multiplikasjon	**50,4**	**54,9**
1kg / 65,50kr. = 0,76kg / *x* kr		**0,9**
65,50:100.76		0,9
Divisjon	36,3	25,7
Andre svar: (eks. "0,24.65,50=x+65,50"; "1kg-0,76kg=svar . 65,5"; "1000g : 65,5=S S.760g="; "1:0,76""65,50:10=? ?.0,76=";	7,9	13,2

Oppgave 7 b Sett opp et regneuttrykk som passer for å løse oppgaven: (Du skal ikke regne det ut).

Et terrengløp går i ei løype som er 5,6 km. Hvor mange engelske mil er det?
(Ei engelsk mil er 1,609 km).

Oppgave 7b	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	3,5	3,5
Divisjon	**49,6**	**63,7**
Multiplikasjon	34,5	29,2
1,609: 5,6	7,1	0,9
Andre svar: (eks. " "3 engelske mil"; " 3,5 engelske mil"; "3,8"; "5"; "5-6 engelske mil")	5,4	2,7

Oppgave 8 a Skriv riktig tall i ruta

$$14 : 2 = \boxed{} \cdot 14$$

Oppgave 8a	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	8,8	9,7
0,5	**74,3**	**77,9**
7	6,2	4,4
0,7	3,5	
2	2,7	3,5
0,2	1,8	0,9
Andre svar (-1 ; 14; 8; 1; 0,02; 0,25; 1,75))	2,7	3,6

Oppgave 8 b Skriv riktig tall i ruta

$$14 : \boxed{} = 0,25 \cdot 14$$

Oppgave 8b	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	16,8	19,5
4	**48,7**	**53,1**
1/0,25		**0,9**
0,25	8	6,2
3,5	3,5	4,4
25	2,7	0,9
49	2,7	0,9
1/4	1,8	2,7
2	1,8	0,9
2,5	1,8	0,9
7	1,8	0,9
Andre svar (0,2; 0,4; 1,75; 14; 3; 4,5; 35; 42; 50; 56; 6; 60; 8 …)	10,6	8,7

Oppgave 8 c Skriv riktig tall i ruta

$$15 : 10 = \square \cdot 15$$

Oppgave 8c	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	16,8	22,1
0,1	**51,3**	**59,3**
1/10	**0,9**	
1,5	6,2	2,7
10	4,4	3,5
0,33	2,7	0,9
0,15	1,8	
0,25	1,8	1,8
0,3	1,8	
Andre svar (eks. 0,05; 0,11; 0,17; 0,75; 2; 3; 6; 7,5; 13; 1001; 0; 2/3)	12,4	9,7

Oppgave 8d Skriv riktig tall i ruta

$$8 : \frac{1}{2} = 8 \cdot \square$$

Oppgave 8d	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	23,9	18,6
2	**36,3**	**45,1**
2/1	**0,9**	
0,5	23,9	20,4
1/2	2,7	7,1
1,5	1,8	2,7
3	1,8	
16	1,8	
Andre svar (0,05; 0,1; 0,16; 4; 12; 20;…)	7,1	6,2

Oppgave 9 a Skriv enklere dersom det er mulig:

$2x + 5x$ ....

Oppgave 9a	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	1,8	0,9
7x	**94,7**	**97,4**
10x	0,9	
2x+5x	0,9	
8x	0,9	0,9
2	0,9	
10x^2		0,9

Oppgave 9 b Skriv enklere dersom det er mulig:

$x + x + 2x$ ....

Oppgave 9b	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	2,7	
4x	**89,4**	**93,8**
2x+2x	**0,9**	
2x^3	2,7	0,9
3x	1,8	
x^2+2x	1,8	1,8
Andre svar (3x; 5x; 2x^2; 46)	0,9	3,6

Oppgave 9 c Skriv enklere dersom det er mulig:

$t \cdot t \cdot t$ ....

Oppgave 9c	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	1,8	
t^3	**89,4**	**94,7**
3t	7,1	3,5
Andre svar (3t^2; t^2; x)	1,8	1,8

Oppgave 9 d Skriv enklere dersom det er mulig:

$2y \cdot y^2$ ....

Oppgave 9d	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	3,5	0,9
2y^3	**44,3**	**57,5**
2y.y^2	23,0	15,9
2y^2	16,8	13,3
3y^2	5,3	6,2
2y+y^2	3,5	1,8
Andre svar	3,6	4,4

Oppgave 9 e Skriv enklere dersom det er mulig:

$3a + a^2 + a^2$ ....

Frekvens i prosent for de ulike svartypene		
Oppgave 9e	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	3,5	0,9
$3a+2a^2$	**59,3**	**72,6**
$3a+a^4$	13,3	11,5
$5a^2$	4,4	2,7
$3a^5$	4,4	4,4
$3a+a^2$	2,7	
$3a^4$	2,7	3,5
Andre svar	9,7	4,4

Oppgave 9 f Skriv enklere dersom det er mulig:

$5a - 2(7 - a) + 6$ ....

Oppgave 9f	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	11,5	7,1
7a-8	**42,5**	**66,4**
7a-14+6	**1,8**	
5a -14+2a+6	**1,8**	
5a+2a-8		**0,9**
3a-8	7,1	7,1
6a-8	4,4	4,4
3a+20	3,5	
7a+20	3,5	0,9
7a-1	2,7	
andregradsuttrykk	1,8	2,7
andre svar	19,5	10,6

Oppgave 10 a $x + y + z = x + p + z$ Dette

☐ er alltid sant ☐ er aldri sant ☐ *kan* være sant, nemlig når ....

Oppgave 10a	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	3,5	1,8
***kan* være sant, nemlig når "p=y"**	**29,2**	**39,8**
kan være sant, nemlig når "y og p=samme tall"		**0,9**
***kan* være sant, men mangler kommentar**	**5,3**	**1,8**
alltid sant	13,3	10,6
aldri sant	44,2	42,5
kan være sant, men kommentaren er feil	3,6	1,8
Andre svar (eks. "alltid sant og aldri sant og kan være sant")	0,9	0,9

Oppgave 10 b $a + b \cdot 2 = 2b + a$ Dette

☐ er alltid sant ☐ er aldri sant ☐ *kan* være sant, nemlig når ....

Oppgave 10b	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	2,7	0,9
alltid sant	**76,1**	**88,5**
aldri sant	17,7	8
kan være sant	2,7	2,7
Andre svar ("alltid sant og kan være sant")	0,9	

Oppgave 10c $\frac{2x+1}{2x+1+5} = \frac{1}{6}$ Dette

☐ er alltid sant ☐ er aldri sant ☐ *kan* være sant, nemlig når ....

Oppgave 10c	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	5,3	2,7
***kan* være sant når x=0**	**9,7**	**15,0**
***kan* være sant, men mangler kommentar**	**2,7**	**2,7**
alltid sant	31,9	38,9
aldri sant	48,7	37,2
kan være sant, men med feil kommentar	1,8	3,6

Oppgave 11a For å finne volumet V av en sylinder, har vi denne formelen: $V = \pi r^2 h$.
Her er r radius i grunnflata , og h er høyden av sylinderen.

Sett at vi har en sylinder vi kaller S, med et volum y. Sett at vi lager en ny sylinder vi kaller T, med dobbelt så stor radius og dobbelt så stor høyde som sylinderen S. Hva blir volumet av denne sylinderen T?

Oppgave 11a		Frekvens i prosent	
		2005 høst	2006 vår
Ikke besvart		24	18
Kategori 1	**"V=8y"; "T=8y"**	**1**	**2**
Kategori 2	**"8 ganger så stort som volumet av sylinderen S"; "Volumet blir 8 ganger så stort"**		**2**
Kategori 3	**"V=PI(2r) ^{2}.2h" ; " T=Pi.4r^2.2h" ; "T=2PI.2r^2.2h"**	**3**	**6**
Kategori 4	**"Volumet av sylinderen T = volumet av sylinderen S.2^3"; "Volumet til S.2^3"**	**2**	
Kategori 5	eks. "Pi.2r^2.2h"; "PI.r^2.2.h.2"; "4y"; "4 ganger så stor"	17	22
Kategori 6	eks. "V= PI.r^2.h.2" ;"2y"; "Volumet blir det dobbelte";	19	19
Kategori 7	To forskjellige varianter i et svar: (eks. "T=2y T: V=PI. 2r^2.2h")	7	6
Kategori 8	eks. "y^2" ; "2y^2"; "S/T=y/y^2"; "V=PI.r^4.h^2"; V(T)= (PI.r^2.h) 2	7	15
Kategori 9	Andre svar	20	10

Oppgave 11b For å finne volumet V av en sylinder, har vi denne formelen: $V = \pi r^2 h$.
Her er r radius i grunnflata , og h er høyden av sylinderen.

Sett at S har radius r og høyde h. Vi vil nå lage en ny sylinder U. Den skal ha dobbelt så stor radius som S, men likevel ha samme volumet som S. Hvor stor må høyden i U da være?

Oppgave 11b		Frekvens i prosent 2005 høst	2006 vår
Ikke besvart		44	28
Kategori 1	**"1/4h" ; "0,25h" ; " fire ganger så liten" ; "4 ganger mindre enn høyden i S" ; "1/4 av høyden i S" osv.**	**1**	**8**
Kategori 2	**"H=H/4" ; "1/4 av S" ; "U=h.1/4"**	**3**	**1**
Kategori 3	**"u= Pi. (2r) ^{2}.h/4"**	**1**	
Kategori 4	"U=2r^2.(1/2)h.PI" ; " V=PI.2r^2. 0,5h" ;	4	11
Kategori 5	"1/2h" ; "0,5h" ; "halvparten av h i S" ; "høyden i U=1/2høyden i S"	28	33
Kategori 6	" Dobbelt så høy som S"; "hU=hS.2" ; "2.h" ; "Pi.r^2.2.h"	3	3
Kategori 7	Andre svar	17	17

Oppgave 12a Hva må x være dersom $123 + 2x = 195 - x$

Oppgave 12a	Frekvens i prosent 2005 høst	2006 vår
Ikke besvart	29,2	20,4
24	**34,5**	**52,2**
72/3	**2,7**	**1,8**
(195-123)/3		**0,9**
72	7,1	1,8
36	3,5	1,8
106	1,8	2,7
14	1,8	0,9
2	0,9	1,8
Andre svar: (eks. -24; -28; 12; 14; 15; 18; 23; 34; 57)	18,5	15,8

Oppgave 12b Hva må x være dersom $\frac{x+1}{x+4} = \frac{4}{5}$

Oppgave 12b	Frekvens i prosent 2005 høst	2006 vår
Ikke besvart	65,5	54,9
11	**12,4**	**23,0**
3	5,3	4,4
"x=3 x=1" ; "x=3 og 1"	1,8	
"3/1"	0,9	2,7
1	0,9	1,8
"3 og 1"	0,9	4,4
andre svar tallverdier	6,2	8,8
andre svar bokstavuttrykk	6,2	

Oppgave 12c Hva må x være dersom

$3x = 7$ og $5y = 11$ Hva er da $3x + 5y$?

Oppgave 12c	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	6,2	6,2
18	**64,6**	**75,2**
7+11	**3,6**	**1,8**
17,99	**0,9**	
"76" ; "21 + 55" ;	12,3	11,5
andre svar tallverdier	7,2	4,5
andre svar bokstavuttrykk (eks. "18xy"; "8xy"; "3x5y")	5,4	0,9

Oppgave 12d Finn tallet x om $x = a+b - c$ *og* a=1, b=2 *og* c=3
x=…………………

Oppgave 12d	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	14,2	7,1
0	**77,0**	**86,7**
6	2,7	2,7
andre svar (eks. 1, 11; 2; 18; 0,2; 0,5, y; "1 eller -3")	6,2	3,6

Oppgave 12e Hva er da verdien av uttrykket $3b^2 – abc$ når $a = 3$, $b = -1$ og $c = 5$.
$3b^2 – abc$ =………………………………….

Oppgave 12e	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	27,4	15,0
18	**10,6**	**31,9**
-12	8,8	7,1
2	5,3	
Innsatt tall, men ikke regnet ut		4,5
andre svar bokstavuttrykk	4,5	
andre svar tallverdier	43,4	41,6

Oppgave 13 På en skole er det 10 elever for hver lærer. Hvilke av disse uttrykkene forteller dette?
L = antall lærere, E = antall elever. Sett ring rundt det eller de riktige uttrykk.

$10L = E$ $\quad$ $10E = L$ $\quad$ $L = 10E$ $\quad$ $E = 10L$ $\quad$ $10L + E$ $\quad$ $11LE$

Oppgave 13	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	7,1	4,4
10*L* = *E* og *E* = 10*L*	**1,8**	**1,8**
Enten 10*L* = *E* eller *E* = 10*L*	**5,3**	**1,8**
10*E* = *L* og *L* = 10*E*	44,2	65,5
10*E* = *L*	21,2	8,8
L = 10*E*	15,9	9,7
10*E* = *L og L* = 10*E* og 11*LE*	1,8	6,2
Andre svar	2,7	1,8

Oppgave 14 a Vekta av et barn avhenger av alderen. For et barn fra 2 til 10 år kan vi bruke følgende sammenheng som en slags normal:
Om y står for vekt i kg og x står for barnets alder i antall år, så er:

$$y = 4 + 2,5x$$

Sett kryss foran det svaret nedenfor som du mener er best:

Hva betyr tallet 4 her?

☐ Barnet er 4 år
☐ Barnet er 4 kg ved fødselen
☐ Barnets vekt øker med 4 kg hvert år
☐ Ingen av delene, det betyr……………………..

Oppgave 14a	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	6,2	3,5
Barnet er 4 kg ved fødselen	**49,6**	**71,7**
Barnets vekt øker med 4 kg hvert år	26,5	17,7
Barnet er 4 år	8,8	1,8
"Ingen av delene, det betyr..." med kommentarer	7,1	4,4
"Ingen av delene, det betyr..." uten kommentarer	0,9	0,9
"Barnet er 4 år" og "Barnets vekt øker med 4 kg hvert år"	0,9	

Oppgave 14b Vekta av et barn avhenger av alderen. For barn i skolealder, kan vi bruke følgende sammenheng som en slags normal:
Om y står for vekt i kg og x står for barnets alder i antall år, så er:

$$y = 4 + 2{,}5x$$

Sett kryss foran det svaret nedenfor som du mener er best:
Hva betyr tallet 2,5 her?

- ☐ Barnet er 2,5 år
- ☐ Barnet er 4 + 2,5 år, altså 6,5 år
- ☐ Barnet veier 2,5 kg når det blir født
- ☐ Barnets vekt øker med 2,5 kg hvert år
- ☐ Ingen av delene, det betyr……………………..

Oppgave 14b	Frekvens i prosent	
	2005 høst	2006 vår
Ikke besvart	8,8	2,7
Barnets vekt øker med 2,5 kg hvert år	**38,9**	**71,7**
Barnet er 2,5 år	27,4	16,8
Barnet var 2,5 kg når det blir født	12,4	7,1
Barnet er 4 + 2,5 år, altså 6,5 år	3,5	0,9
"Ingen av delene, det betyr..." med kommentar	6,2	0,9
"Ingen av delene, det betyr..." uten kommentarer	1,8	
Svar med kommentar	0,9	

Appendix 3: Comparison of results for selected tasks – tasks included in both tests for 9th grade and 11th grade

Oppgave 23a Skriv enklest mulig: 2x + 5x ....

Oppgave 9a Grunnkurs / Oppgave 23a 9.trinn	Frekvens i prosent		Frekvens i prosent	
	LCM 9.trinn 2005 høst	LCM 9. trinn 2006 vår	LCM 1.vgs 2005 høst	LCM 1.vgs 2006 vår
Ikke besvart	55	18	2	1
7x	**22**	**71**	**95**	**98**
2x+5x og 2+5	6	2	1	
Svar med x^2 (2x5x; $7x^2$; $10x^2$)	2	2		1
Andre svar (6x; 8x;10x; xx+xxxxx; 2x5x …)	3	7	2	1
Setter inn tall for den variable	12		1	

Oppgave 23b Skriv enklest mulig: $x + x + 2x$ ....

Oppgave 9b Grunnkurs / Oppgave 23b 9.trinn	Frekvens i prosent		Frekvens i prosent	
	LCM 9.trinn 2005 høst	LCM 9. trinn 2006 vår	LCM 1.vgs 2005 høst	LCM 1.vgs 2006 vår
Ikke besvart	61	22	3	
4x	**16**	**60**	**89**	**94**
$2x^3$	1	4	3	1
x^2+2x		3	2	2
Setter inn verdi for den variable	13	1	1	
Andre svar (3x; 2x+2x)	2	1	3	2
Andre svar	7	9		2

Oppgave 23c Skriv enklest mulig: $t \cdot t \cdot t$ ....

Oppgave 9c Grunnkurs / Oppgave 23c 9.trinn	Frekvens i prosent		Frekvens i prosent	
	LCM 9.trinn 2005 høst	LCM 9. trinn 2006 vår	LCM 1.vgs 2005 høst	LCM 1.vgs 2006 vår
Ikke besvart	64	18	2	
t^3	**3**	**50**	**89**	**95**
3t eller t3	15	26	7	4
Andre svar (eks. $3t^2$; t; x)	8	6	2	2
Andre svar tallverdi	10			

Oppgave 23d Skriv enklest mulig: $2y \cdot y^2$ ....

Oppgave 9d Grunnkurs / Oppgave 23d 9.trinn	Frekvens i prosent		Frekvens i prosent	
	LCM 9.trinn 2005 høst	LCM 9. trinn 2006 vår	LCM 1.vgs 2005 høst	LCM 1.vgs 2006 vår
Ikke besvart	71	37	4	1
$2y^3$	**0**	**9**	**44**	**58**
4y	1	12		
$2y.y^2$	1	11	23	16
$2y^2$		8	17	13
$3y^2$	1	2	5	6
$2y+y^2$		3	4	2
Andre svar	21	17	4	4
Andre svar tallverdi	5	1		

Oppgave 24a $x + y + z = x + p + z$ Dette

☐ er alltid sant ☐ er aldri sant ☐ *kan* være sant, nemlig når ....

Oppgave 10a Grunnkurs / Oppgave 24a 9.trinn	Frekvens i prosent		Frekvens i prosent	
	LCM 9.trinn 2005 høst	LCM 9. trinn 2006 vår	LCM 1.vgs 2005 høst	LCM 1.vgs 2006 vår
Ikke besvart	40	21	4	2
***kan* være sant, nemlig når "p=y"**	**2**	**5**	**29**	**41**
***kan* være sant, men mangler kommentar eller kommentaren er feil**	**9**	**8**	**6**	**3**
alltid sant	22	22	13	11
aldri sant	26	45	44	43
Andre svar (eks."aldri sant og kan være sant")	1		1	1

Oppgave 27 a $x = a + b - c$ Dersom $a = 1$, $b = 2$ og $c = 3$ blir x=.......................

Oppgave 12d Grunnkurs / Oppgave 27a 9.trinn	Frekvens i prosent		Frekvens i prosent	
	LCM 9.trinn 2005 høst	LCM 9. trinn 2006 vår	LCM 1.vgs 2005 høst	LCM 1.vgs 2006 vår
Ikke besvart	72	46	14	7
0	**10**	**37**	**77**	**87**
"1+2-3"	**1**			
6	3	10	3	3
andre svar	14	8	6	4

Oppgave 27c 3x = 7 og 5y = 11 Da blir 3x + 5y = ...

Oppgave 12c Grunnkurs Oppgave 27c 9.trinn	Frekvens i prosent		Frekvens i prosent	
	LCM 9.trinn	LCM 9. trinn	LCM 1.vgs	LCM 1.vgs
	2005 høst	2006 vår	2005 høst	2006 vår
Ikke besvart	78	50	6	6
18	**11**	**33**	**65**	**75**
17,99			**1**	
"76" ; "21 + 55" ;		7	12	12
andre svar tallverdier	8	5	11	6
andre svar bokstavuttrykk (eks. "18xy"; "8xy"; "3x5y")	3	6	5	1

Oppgave 20c Sett ring rundt *alle* regneuttrykkene som passer til regneoppgaven:

Kaker skal fylles i bokser, med 0,75 kg i hver. Hvor mange bokser trenger man til 6 kg kaker?

6 · 0,75 6 : 0,75 0,75 : 6 0,75 · 6 6 – 0,75 6 + 0,75

Oppgave 6b Grunnkurs / Oppgave 20c 9.trinn	Frekvens i prosent		Frekvens i prosent	
	LCM 9.trinn	LCM 9. trinn	LCM 1.vgs	LCM 1.vgs
	2005 høst	2006 vår	2005 høst	2006 vår
Ikke besvart	13	16	1	1
6 : 0,75	**15**	**22**	**50**	**59**
6 · 0,75	19	10	13	4
6 · 0,75 og 0,75 · 6	20	23	12	15
0,75 · 6	14	7	10	5
0,75 : 6	8	8	4	5
6 : 0,75 og 0,75 : 6	9	9	3	4
Andre svar	3	7	6	7

Oppgave 20d Sett ring rundt *alle* regneuttrykkene som passer til regneoppgaven:

Anne kjøper bananer i en butikk til 13,50 kr per kg. Hvor mye kan Anne kjøpe for 10,50 kr?

13,50 · 10,50 10,50 : 13,50 13,50 : 10,50 13,50 – 10,50 13,50 + 10,50

Oppgave 6c Grunnkurs/ Oppgave 20d 9.trinn	Frekvens i prosent		Frekvens i prosent	
	LCM 9.trinn	LCM 9. trinn	LCM 1.vgs	LCM 1.vgs
	2005 høst	2006 vår	2005 høst	2006 vår
Ikke besvart	22	19	8	5
10,50 : 13,50	**23**	**19**	**47**	**49**
13,50 : 10,50	21	24	33	32

10,50 : 13,50 og 13,50 : 10,50	14	14	6	4
13,50 . 10,50	5	4	4	4
13,50 - 10,50	10	15	2	4
Andre svar	5	5	1	3

Oppgave 5 $\frac{1}{2} + \frac{1}{4} = ?$

Oppgave 1a Grunnkurs / Oppgave 5 9.trinn	Frekvens i prosent		Frekvens i prosent	
	LCM 9.trinn	LCM 9. trinn	LCM 1.vgs	LCM 1.vgs
	2005 høst	2006 vår	2005 høst	2006 vår
Ikke besvart	14	15	1	1
3/4	**37**	**41**	**83**	**81**
6/8 eller 9/12 eller 12/16	**4**	**14**	**9**	**4**
1/2; 2/4; 4/8; 3/6	12	10	4	2
Andre svar	33	20	3	12

Oppgave 10 $\frac{1}{2} - \frac{1}{3} = ?$

Oppgave 1b Grunnkurs / Oppgave 10 9.trinn	Frekvens i prosent		Frekvens i prosent	
	LCM 9.trinn	LCM 9. trinn	LCM 1.vgs	LCM 1.vgs
	2005 høst	2006 vår	2005 høst	2006 vår
Ikke besvart	47	33	2	4
1/6	**25**	**38**	**81**	**80**
1000/6000				**1**
5/6	3	8	11	4
1; 1/1 eller 6/6	9	4	2	2
2/3 eller 4/6	4	1	1	
Andre svar	12	16	5	10

Oppgave 9 $60 \cdot 450 = ?$

Oppgave 1e Grunnkurs / Oppgave 9 9.trinn	Frekvens i prosent		Frekvens i prosent	
	LCM 9.trinn	LCM 9. trinn	LCM 1.vgs	LCM 1.vgs
	2005 høst	2006 vår	2005 høst	2006 vår
Ikke besvart	40	36	10	8
27000	**37**	**42**	**60**	**56**
2700	5	2	11	13
4sifra andre svar	4	3	4	7
5sifra andre svar	9	1	12	9
Andre svar	4	15	3	7

Appendix 4: Letter to a school with 11th grade results

Til lærerne knyttet til KUL-LCM-prosjektet – skole 36.

Her kommer resultatene fra testene som ble gjennomført i dette prosjektet skoleåret 2005/2006.
Test 1 ble gjennomført rett etter skolestart høsten 2005, og Test 2 rett før skoleslutt våren 2006. For å kunne vurdere utviklingen har jeg sammenlignet resultatet fra begge testene. Testen ble ikke forandret i løpet av året. Test 1 er dermed identisk med Test 2. Dette er også samme test som ble gjennomført våren 2005.

Elevantallet på 9. trinn høsten 2005 er 227, og elevantallet våren 2006 er 126. Antall elever som tok begge testene (Test 1 og Test 2) er 113. Grunnen til det store avviket i antall her er at ved en skole ble ikke testen gjennomført for alle elevene våre 2006. Dette skyldtes en feil.

Maksimal oppnåelig poengsum er 48 poeng. Jeg har brukt samme poengbergning og samme kodesystem som høsten 2004 og våren 2005. Elevene har fått ett poeng for både korrekte og delvis eller nesten korrekte svar.

Elevene som ikke møtte på en av testene står med tomme celler i tabellen.

Vi takker for deres medvirkning ved undersøkelsen, og håper at denne tilbakemeldingen er til nytte for dere i arbeidet.

Kristiansand, 28. november 2006

Emilia

Resultater for hele 1. trinnet i undersøkelsen.

Gjennomsnitt, høyeste og laveste poengsum på Test 1 (2005 høst), Test 2 (2006 vår) og testene fra 2004 høst og 2005 vår.

Elevgruppe	Test år	Elevantall	Gjennomsnitt	Maksimum	Min
Elever - Test1 & Test 2	2004 høst	206	21,6	41	2
Elever - Test1 & Test 2	2005 vår	206	24,4	43	4
Alle elever - Test 1	2005 høst	227	22,74	42	2
Alle elever - Test 2	2006 vår	126	30,21	45	9
Elever - Test1 & Test 2	2005 høst	113	26,2	42	4
Elever - Test1 & Test 2	2006 vår	113	30,5	45	10

Tabell 1: Poengsum (Gjennomsnitt, Maksimum, Minimum) testene fra 2004 høst og 2005 vår, Test 1 (2005 høst- alle elever N=227), Test 2 (2006 vår - alle elever N=126) , Test 1 (2005 høst elever, som tok begge tester N=113), Test 2 (2006 vår elever , som tok begge tester N=113)

Gjennomsnitt, høyeste og laveste poengsum på de to testene (Test 1 (N=227) og Test 2 (N=126)) sett i forhold til kjønn.

År	Kjønn	Antall	Gjennomsnitt	Maksimum	Minimum
2005 høst	Alle Gutter Test 1	97	23,48	40	2
2005 høst	Alle Jenter Test 1	130	22,18	42	5
2006 vår	Alle Gutter Test 2	54	32,35	44	17
2006 vår	Alle Jenter Test 2	71	28,54	45	9

Tabell 2: : Poengsum Test 1 (2005 høst, antall elever 227), Test 2 (2006 vår, antall elever 126) delt etter kjønn (Merknad: Test 2 - En elev (ukjent kjønn) er ikke inkludert.)

Gjennomsnitt, høyeste og laveste poengsum på de to testene sett i forhold til kjønn for elever som har gjort både Test 1 og Test 2 (N=113).

År	Kjønn	Antall	Gjennomsnitt	Maksimum	Minimum
2005 høst	Gutter -Test1 & Test 2	46	27,57	40	4
2005 høst	Jenter - Test1 & Test 2	67	25,27	42	7
2006 vår	Gutter - Test1 & Test 2	46	32,48	44	17
2006 vår	Jenter - Test1 & Test 2	67	29,13	45	10

Tabell 3: Poengsum Test 1 (2005 høst, antall elever 113), Test 2 (2006 vår, antall elever 113) delt etter kjønn

Sammenligning oppgave for oppgave

Diagrammene under viser sammenligning av resultatene for de to testene for N=113 elever som hadde gjort Test 1 (2005 høst) og Test 2 (2006 vår). Kode 1, kode 2 og kode 3 er slått sammen.

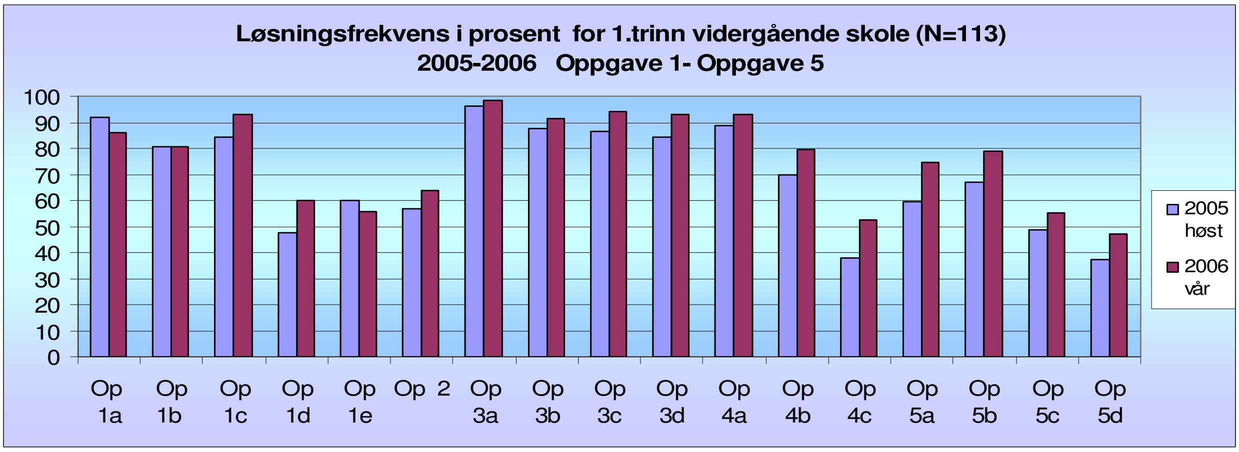


Diagram 1: Løsningsfrekvens i prosent for N=113 elever 2005-2006, Oppgave 1 – Oppgave 5

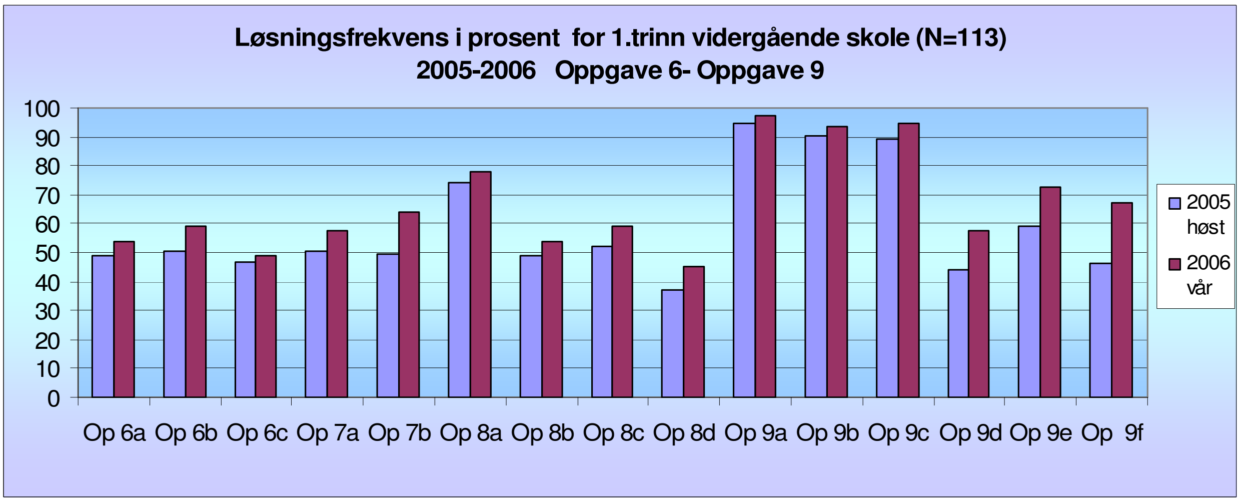


Diagram 2: Løsningsfrekvens i prosent for N=113 elever 2005-2006, Oppgave 6 – Oppgave 9

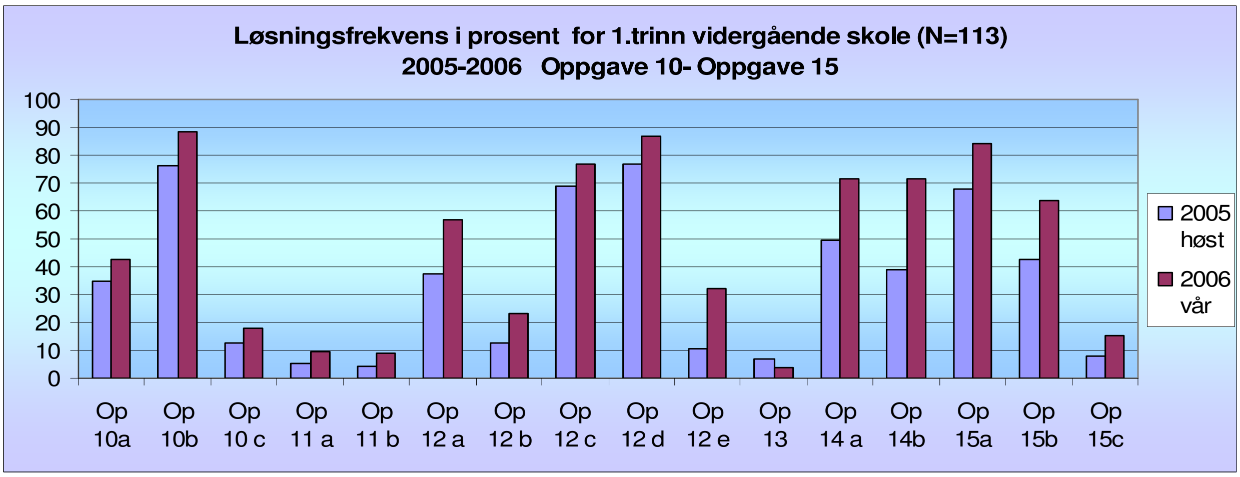


Diagram 3: Løsningsfrekvens i prosent for N=113 elever 2005-2006, Oppgave 6 – Oppgave 9

Løsningsfrekvens i prosent på hver oppgave 2005, for alle elever som har gjort Test 1 (2005 høst) N=227.

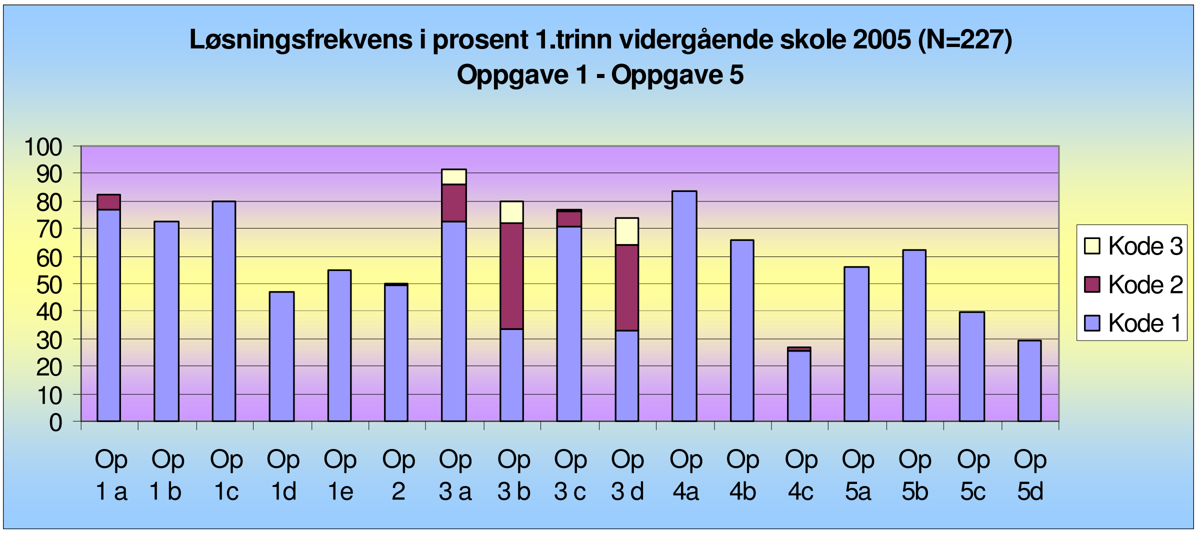


Diagram 4: Løsningsfrekvens i prosent for N=227 elever, Test 1 (2005 høst), Oppgave 1– Oppgave 5

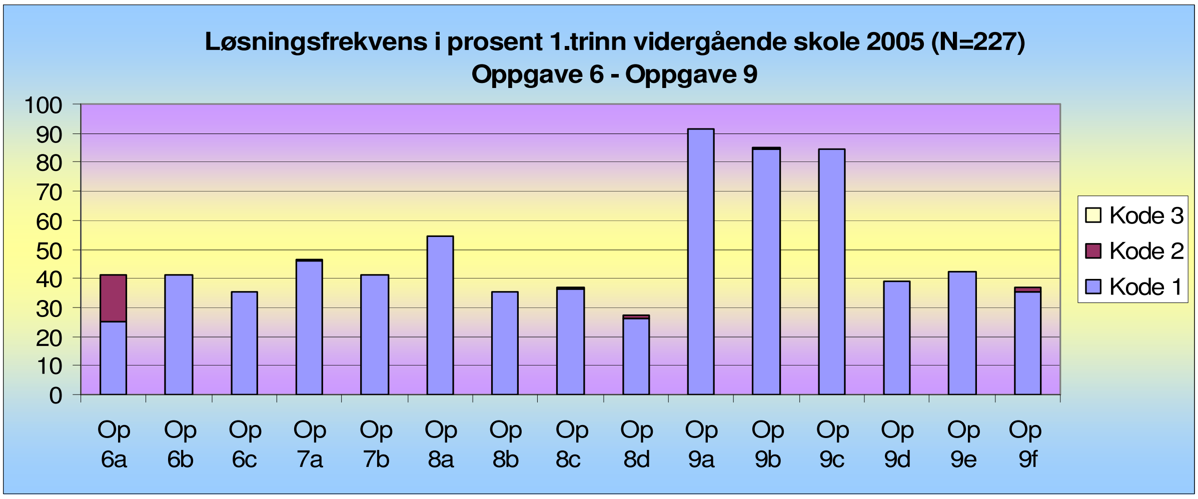


Diagram 5: Løsningsfrekvens i prosent for N=227 elever, Test 1 (2005 høst), Oppgave 6– Oppgave 9

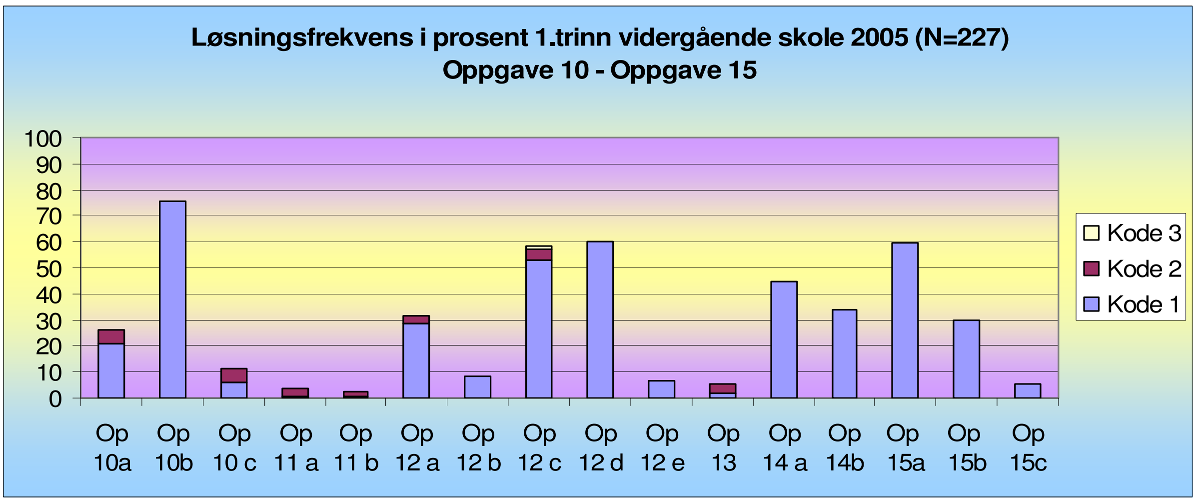


Diagram 6: Løsningsfrekvens i prosent for N=227 elever, Test 1 (2005 høst), Oppgave 10– Oppgave 15

1. trinn, Vidergående skole 36, Gruppe mxa

Elevnummer	Poengsum Kode: 1+2+3 2005	Poengsum Kode: 1+2+3 2006	Feilsvar Kode 11 2005	Feilsvar Kode 11 2006	Ikke besvart Kode 0 2005	Ikke besvart Kode 0 2006
1	39	39	5	9	4	0
2	17	30	30	17	1	1
3	27	35	12	11	9	2
4	30	36	11	10	7	2
5	13	24	30	16	5	8
6	17	20	19	22	12	6
7	40	43	5	5	3	0
8	17	30	29	15	2	3
9	28	24	20	20	0	4
10	32	35	10	11	6	2
11	28		10		10	
12	32	37	15	10	1	1
13	35	37	12	10	1	1
14	25		22		1	
15	25	32	18	8	5	8
16	29	29	14	19	5	0
17	33	36	12	11	3	1
18	25	27	15	16	8	5
19	24	23	19	23	5	2
20	38	36	10	12	0	0
21	23	28	21	17	4	3
22	27	37	12	9	9	2
23	28		18		2	
24	7	10	21	21	20	17
25	27	37	20	10	1	1
26		20		16		12
27		25		15		8
28		33		14		1
29		36		12		0
30		18		29		1
31		34		11		3
Antall Elever	**25**	**28**	**25**	**28**	**25**	**28**
Gjennomsnitt	**26,64**	**30,39**	**16,4**	**14,25**	**4,96**	**3,36**
Minimum	**7**	**10**	**5**	**5**	**0**	**0**
Maksimum	**40**	**43**	**30**	**29**	**20**	**17**

Tabell 4: Skole 36, 1. trinn, Gruppe mxa, 2005-2006