ON THE ANALYSIS OF A NEW MARKOV CHAIN WHICH HAS APPLICATIONS IN AI AND MACHINE LEARNING

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ABSTRACT

In this paper, we consider the analysis of a fascinating Random Walk (RW) that contains interleaving random steps and random "jumps". The characterizing aspect of such a chain is that every step is paired with its counterpart random jump. RWs of this sort have applications in testing of entities, where the entity is never allowed to make more than a pre-specified number of *consecutive* failures. This paper contains the analysis of the chain, some fascinating limiting properties, and some initial simulation results. The reader will find more detailed results in [12].

Keywords : *Random Walks with Jumps, Random Processes, Ergodic Random Processes*

1. INTRODUCTION

RWs have been studied for more than a century and utilized in a myriad of applications stemming from areas as diverse as biology, computer science, economics and physics. For instance, concrete examples of these applications in biology are epidemics models [2], the Wright-Fisher model, the Moran Model [9] etc... RWs arise in the modeling and analysis of queuing systems [8], ruin problems [11], risk theory [10], sequential analysis and learning theory [4]. In addition to the aforementioned *classical* application of RWs, recent applications include mobility models in mobile networks [5], collaborative recommendation [7], web search algorithms [1], and reliability theory for both software and hardware components [3].

Although Random Walks (RWs) with single-step transitions, such as the ruin problem, have been extensively analyzed [6], problems involving the analysis of RWs containing *interleaving* random steps and random jumps are intrinsically hard. In this paper, we consider the analysis of one such fascinating RW, where every step is paired with its counterpart random jump. Apart from this RW being conceptually interesting, it also has applications in the testing of entities, where the entity is never allowed to make more than a pre-specified number of *consecutive* failures.

To motivate the problem, consider the scenario when we are given the task of testing an error-prone component. At every time step, the component is subject to failure, where the event of failure occurs with a certain probability, q. The corresponding probability of the component not failing[1] is p, where $p = 1 - q$. Further, like all real-life entities, the component can operate under two modes, either in the *Well-Functioning* mode, or in the *Mal-Functioning* mode. At a given time step, we aim to determine if the component is behaving well, i.e, in the *Well-Functioning* mode, or if it is in the *Mal-Functioning* mode, which are the two states of nature. It is not unreasonable to assume that both these hypotheses are mutually exclusive, implying that only one of these describes the state of the component at a given time step, thus excluding the alternative. Let us now consider a possible strategy for determining the appropriate hypothesis for the state of nature.

Suppose that the current maintained hypothesis conjectures that the component is in a *Mal-Functioning* mode. This hypothesis is undermined and systematically replaced by the hypothesis that the component is in its *Well-Functioning* mode if it succeeds to realize a certain number N_1 of successive recoveries (or successes). In the same vein, suppose that the current hypothesis conjectures that the component is in its *Well-Functioning* mode. This hypothesis, on the other hand, is invalidated and systematically replaced by the hypothesis that the component is in its *Mal-Functioning* mode if the component makes a certain number $N_2 + 1$ of successive failures. We shall show that such a hypothesis testing paradigm is most appropriately modeled by a RW in which the random steps and jumps are interleaving. To the best of our knowledge, such a modeling paradigm is novel! Further, the analysis of such a chain is unreported in the literature too.

By way of nomenclature, throughout this paper, we shall

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[1]The latter quantity can also be perceived to be the probability of the component *recovering* from a failure, i.e., if it, indeed, had failed at the previous time instant.

refer to p as the "Reward Probability", and to q as the "Penalty Probability", where $p + q = 1$.

2. PROBLEM FORMULATION

2.1. Specification of the State Space and Transitions

We consider the following RW with Jumps (RWJ) as depicted in Figure 1. Let $X(t)$ denote the index of the state of the walker at a discrete time instant 't'. The details of the RW can be catalogued as below:

1. First of all, observe that the state space of the RW contains $N_1 + N_2 + 1$ states.

2. The states whose indices are in the set $\{1, \ldots, N_1\}$ are paired with their counterpart random jump to state 1.

3. The states whose indices fall in the range between the integers $\{N_1 + 1, \ldots, N_1 + N_2\}$ are paired with *their* counterpart random jump to state $N_1 + 1$.

4. Finally, the state whose index is $N_1 + N_2 + 1$ is linked to both states 1 and $N_1 + 1$.

5. Essentially, whenever the walker is in a state $X(t) = i$ which belongs to the the set $\{1, \ldots, N_1\}$, he has a chance, p, of advancing to the neighboring state $i + 1$, and a chance $q = 1 - p$ of performing a random jump to state 1. Similarly, whenever he is in a state $X(t) = i$ in the set $\{N_1 + 1, \ldots, N_1 + N_2\}$, the walker has a chance, q, of advancing to the neighbor state $i + 1$, and a chance p of operating a random jump to state N_1. However, whenever the walker is in state $N_1 + N_2 + 1$, he has a probability p of jumping to state $N_1 + 1$ and a probability q of jumping to state 1.

6. These rules describe the RWJ compeletely.

The reader will observe a marginal asymmetry in the assignment of our states. Indeed, one could query: Why should do we operate with N_1 and $N_2 + 1$ states in the corresponding modes, instead of N_1 and N_2 respectively? Would it not have been "cleaner" to drop the extra state in the latter case, i.e., to use N_2 states instead of $N_2 + 1$? The reason why we have allowed this asymmetry is because we have consciously intended to put emphasis on the so-called "acceptance state", that counts as unity. Our position is that this is also a more philosophically-correct position – because the acceptance state is really one which has already obtained a success.

We shall now present a detailed analysis of the above RWJ.

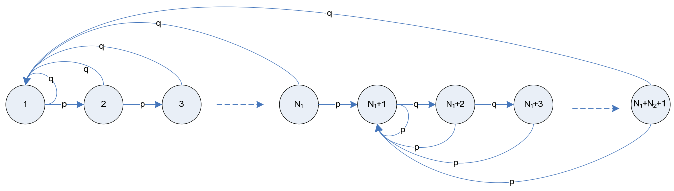


Fig. 1. The state transitions of the Random Walk with Jumps.

3. THEORETICAL RESULTS

The analysis of the above-described RWJ is particularly difficult because the stationary (equilibrium) probabilities of being in any state is related to the stationary probabilities of *non-neighboring* states. In other words, it is not easy to derive a simple difference equation that relates the stationary probabilities of the neighboring states. To render the analysis more complex, we observe that the RW does not possess any time-reversibility properties either!

However, by studying the peculiar properties of the chain, we have succeeded in solving for the stationary probabilities, which, in our opinion, is far from trivial. The proof of the result follows.

Theorem 1 *For the RWJ described by the Markov Chain given in Figure 1, P_1, the probability of the walker being in the* Mal-Functioning *mode, is given by the following expression:*

$$P_1 = \frac{(1 - p^{N_1}) q^{N_2}}{(1 - p^{N_1}) q^{N_2} + p^{N_1 - 1}(1 - q^{N_2+1})}. \tag{1}$$

Similarly, P_2, the probability of being in the Well-Functioning *mode (or exiting from the* Mal-Functioning *mode) is:*

$$P_2 = \frac{(1 - q^{N_2+1}) p^{N_1 - 1}}{(1 - p^{N_1}) q^{N_2} + p^{N_1 - 1}(1 - q^{N_2+1})}. \tag{2}$$

Proof: To prove these results, we shall analyze the properties of the underlying MC that describes the behavior of the walker. By investigating the various transition considerations, we see that matrix of transition probabilities, M, is given by:

$$M = \begin{pmatrix} q & p & 0 & \ldots & 0 & 0 & 0 & \ldots & 0 \\ q & 0 & p & \ldots & 0 & 0 & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ q & 0 & \ldots & 0 & p & 0 & 0 & \ldots & 0 \\ 0 & 0 & \ldots & 0 & p & q & 0 & \ldots & 0 \\ 0 & 0 & \ldots & 0 & p & 0 & q & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \ldots & 0 & p & 0 & \ldots & 0 & q \\ q & 0 & \ldots & 0 & p & 0 & 0 & \ldots & 0 \end{pmatrix}$$

The reader should observe the transitions into the non-adjacent states, i.e., those which represent the jumps.

We shall now compute π_i the stationary (or equilibrium) probability of the chain being in state i. Clearly M represents a single closed communicating class whose periodicity is unity. The chain is thus ergodic, and the limiting probability vector is given by the eigenvector of M^T corresponding to the eigenvalue unity. The vector of steady state (equilibrium) probabilities $\Pi = [\pi_1, \ldots, \pi_{N_1+N_2+1}]^T$ can be thus computed by solving $M^T\Pi = \Pi$.

Consider first the stationary probability of being in state 1, π_1. By expanding the first row we see that this is expressed by the following equation:

$$\begin{aligned}\pi_1 &= q\pi_1 + q\pi_2 + \ldots + q\pi_{N_1} + q\pi_{N_1+N_2+1} \\ &= q\sum_{k=1}^{N_1}\pi_k + q\pi_{N_1+N_2+1}. \end{aligned} \tag{3}$$

For $2 \leq k \leq N_1$, the stationary probability π_k is given by a straightforward first-order difference equation, Eq. (4):

$$\pi_k = p\pi_{k-1}. \tag{4}$$

By applying recurrence, the Eq. (4) can be rewritten as:

$$\pi_k = p^{k-1}\pi_1. \tag{5}$$

By expanding the $(N+1)^{st}$ row, we can see that the probability of being in the "acceptance" state π_{N_1+1} is given by Eq. (6):

$$\pi_{N_1+1} = p\pi_{N_1} + p\pi_{N_1+1} + p\sum_{k=1}^{N_2}\pi_{N_1+1+k}. \tag{6}$$

Again, for $N_1 + 1 \leq k \leq N_1 + N_2 + 1$, the steady probabilities are given by:

$$\pi_k = q\pi_{k-1}. \tag{7}$$

By applying recurrence, Eq. (7) can be written for $1 \leq k \leq N_2 + 1$ as:

$$\pi_{N_1+1+k} = q^k\pi_{N_1+1}. \tag{8}$$

Using Eq. (5) and Eq. (8), and replacing them in Eq. (3) we obtain:

$$\pi_1 = q\sum_{k=1}^{N_1}p^{k-1}\pi_1 + q^{N_2+1}\pi_{N_1+1}. \tag{9}$$

Therefore, we obtain:

$$\pi_1 = (1 - p^{N_1})\pi_1 + q^{N_2+1}\pi_{N_1+1}. \tag{10}$$

From the above, we can deduce the equation that relates π_1 and π_{N_1+1}:

$$\pi_{N_1+1} = \frac{p^{N_1}}{q^{N_2+1}}\pi_1. \tag{11}$$

Consequently, P_1 is given by Eq. (12):

$$\begin{aligned}P_1 &= \sum_{k=1}^{N_1}\pi_k \\ &= \sum_{k=1}^{N_1}p^{k-1}\pi_1 \\ &= \frac{1-p^{N_1}}{1-p}\pi_1. \end{aligned} \tag{12}$$

Similarly, P_2 can be expressed by:

$$\begin{aligned}P_2 &= \sum_{k=N_1+1}^{N_1+N_2+1}\pi_k \\ &= \sum_{k=0}^{N_2}q^k\pi_{N_1+1} \\ &= \frac{1-q^{N_2+1}}{p}\pi_{N_1+1}. \end{aligned} \tag{13}$$

Using the values of P_1 and P_2 and the fact that $P_1 + P_2 = 1$, and after carrying out some simple algebraic manipulations we obtain:

$$\pi_1 = \frac{q^{N_2+1}}{(1-p^{N_1})q^{N_2} + p^{N_1-1}(1-q^{N_2+1})}, \tag{14}$$

and

$$\pi_{N_1+1} = \frac{p^{N_1}}{(1-p^{N_1})q^{N_2} + p^{N_1-1}(1-q^{N_2+1})}, \tag{15}$$

whence:

$$P_1 = \frac{(1-p^{N_1})q^{N_2}}{(1-p^{N_1})q^{N_2} + p^{N_1-1}(1-q^{N_2+1})}, \tag{16}$$

and

$$P_2 = \frac{(1-q^{N_2+1})p^{N_1-1}}{(1-p^{N_1})q^{N_2} + p^{N_1-1}(1-q^{N_2+1})}, \tag{17}$$

which concludes the proof. □

3.1. "Balanced Memory" Strategies

Although the results obtained above are, in one sense, pioneering, the question of understanding *how* the memory of the scheme should be assigned is interesting in its own right. To briefly address this, in this section, we consider the particular case where N_1 and $N_2 + 1$ are both equal to the same value, N. In this case, if $p = q = 1/2$, one can trivially confirm that $P_1 = P_2 = 1/2$, implying that the scheme is not biased towards any of the two modes, the *Mal-Functioning* or the *Well-Functioning* mode. In practice, employing a "Balanced Memory" strategy seems to be a reasonable choice since having equal memory depth (or number of states) for the *Mal-Functioning* and *Well-Functioning* modes eliminates any bias towards any of the conjectured hypotheses.

Theorem 2 *For a "Balanced Memory" strategy in which $N_1 = N_2+1 = N$, the probability, P_1, of being in the* Mal-Functioning *mode, approaches* 0 *as the memory depth N tends to infinity whenever $p > 0.5$. Formally,* $\lim_{N\to\infty} P_1 = 0$.

Proof: Consider the quotient $\frac{P_1}{P_2}$. To prove this result, we first compute its limit as N tends to infinity for $p > 0.5$.

$$\frac{P_1}{P_2} = \frac{(1-p^N)q^{N-1}}{(1-q^N)p^{N-1}}. \tag{18}$$

Dividing the numerator and denominator by p^{2N} we obtain:

$$\frac{P_1}{P_2} = \frac{(1/p^N - 1)(q/p)^{N-1}}{1/p^N - (q/p)^N}. \tag{19}$$

Since $p > 0.5$, we have the condition that $q/p < 1$. Therefore

$$\lim_{N\to\infty} (q/p)^{N-1} = 0.$$

On the other hand, $\lim_{N\to\infty} \frac{(1/p^N-1)}{1/p^N-(q/p)^N} = 1$.

Therefore $\lim_{N\to\infty} \frac{P_1}{P_2} = 0$. Thus, we conclude that $\lim_{N\to\infty} P_1 = 0$, and the result is proved. □

The analogous result for the case when $p < 0.5$ follows.

Theorem 3 *For a "Balanced Memory" strategy in which $N_1 = N_2+1 = N$, the probability, P_1, of being in the* Mal-Functioning *mode, approaches unity as the memory depth N tends to infinity whenever $p < 0.5$. Formally,* $\lim_{N\to\infty} P_1 = 1$.

Proof: The proof is similar to the proof of Theorem 2, except that we rather consider the quotient $\frac{P_2}{P_1}$. By dividing the numerator and denominator by q^{2N}, we get the following expression:

$$\frac{P_2}{P_1} = \frac{(1/q^N - 1)(p/q)^{N-1}}{1/q^N - (p/q)^N}. \tag{20}$$

We remark that $p/q < 1$ for $p < 0.5$, and thus,

$$\lim_{N\to\infty} (p/q)^{N-1} = 0.$$

Moreover, we see that $\lim_{N\to\infty} \frac{(1/q^N-1)}{1/q^N-(p/q)^N} = 1$. Therefore, $\lim_{N\to\infty} \frac{P_2}{P_1} = 0$, and consequently $\lim_{N\to\infty} P_2 = 0$. Hence the result! □

3.2. Symmetry Properties

The MC describing the RW with Jumps is described by its state occupation probabilities and the overall mode probabilities, P_1 and P_2. It is trivial to obtain P_1 from P_2 and vice versa for the symmetric "Balanced Memory" case – one merely has to replace p by q and do some simple transformations. However, the RW also possesses a fascinating property when it concerns the underlying state occupation probabilities – the $\{\pi\}$'s themselves. Indeed, we shall derive one such interesting property of the scheme in Theorem 4, which specifies a rather straightforward (but non-obvious) method to deduce the equilibrium distribution of a "Balanced Memory" scheme possessing a reward probability p from the equilibrium distribution of the counterpart "Balanced Memory" scheme possessing a reward probability of $1-p$.

Theorem 4 *Let $\Pi = [\pi_1, \ldots, \pi_{2N}]^T$ be the vector of steady state probabilities of a balanced scheme characterized by a reward probability p and penalty probability q. Let $\Pi' = [\pi'_1, \ldots, \pi'_{2N}]^T$ be the vector of steady state probabilities of a balanced scheme possessing a reward probability $p' = 1-p$ and penalty probability $q' = 1-q$. Then Π' can be deduced from Π using the following transformation:*

$$\pi'_k = \pi_{\sigma(k)} \textit{ for } k \textit{ with } 1 \le k \le 2N,$$

where σ is a circular permutation of the set S=$\{1, 2, \ldots, 2N\}$ defined by

$$\sigma(k) = \begin{cases} 2N, & \textit{if } k = N \\ (k+N)(\textit{mod } 2N), & \textit{Otherwise} \end{cases}$$

Proof: We shall first prove the theorem for states π'_1 and π'_{N+1}. Using Eq. (14) and (15) and replacing (N_1, N_2+1) by (N, N), we deduce that: $\pi'_1 = \pi_{N+1} = \pi_{\sigma(1)}$ and $\pi'_{N+1} = \pi_1 = \pi_{\sigma(N+1)}$.

Now, consider the hypothesis for k such that $1 < k \le N$. By a simple substitution we see that for all k $(1 < k < N)$, $\sigma(k) = (k+N)(\text{mod } 2N) = k+N$, and for $k = N$, $\sigma(N) = 2N$.

We use Eq. (5) to write

$$\begin{aligned} \pi'_k &= {p'}^{k-1}\pi'_1 \\ &= q^{k-1}\pi_{N+1}. \end{aligned} \tag{21}$$

However, as per Eq. (8), for k such that $2 \le k \le N$:
$q^{k-1}\pi_{N+1} = \pi_{N+k}$.
Therefore, for k such that $2 \le k \le N$:
$\pi'_k = \pi_{N+k} = \pi_{\sigma(k)}$

Now, we treat the case where k is bounded as per $N < k \le 2N$ separately. For this case, first of all, we remark that σ can be expressed differently. Indeed, if k satisfies $N < k \le 2N$, it can be seen that $\sigma(k) = (k+N)(\text{mod } 2N) = k-N$. Considering this, we now apply Eq. (8) to yield:

$$\begin{aligned} \pi'_{N+k} &= {q'}^{k-1}\pi'_{N+1} \\ &= p^{k-1}\pi_1 \\ &= \pi_k. \end{aligned} \tag{22}$$

The result is proven by a straightforward change of variables, since we can easily deduce that for $N < k \le 2N$: $\pi'_k = \pi_{k-N} = \pi_{\sigma(k)}$. □

4. APPLICATION TO COMPONENT TESTING

As briefly alluded to earlier, from a philosophical point of view, the testing of a component can be modeled by the RWJ presented in Section 2.1. At each time step, the entity is either subject to a success or a failure, and is either supposed to be in the *Well-Functioning* or *Mal-Functioning* mode. From a high level perspective, a success "enforces" the hypothesis that the entity is *Well-Functioning* while simultaneously "weakening" the hypothesis that it is *Mal-Functioning*. On the other hand, a failure "enforces" the hypothesis that the entity is deteriorating, i.e *Mal-Functioning*, while "weakening" the hypothesis that it is *Well-Functioning*. It is worth noting that states whose indices are in the set $\{1, \ldots, N_1\}$ serve to memorize the number of consecutive successes that have occurred so far. In other words, if the walker is in state i ($i \in \{1, \ldots, N_1\}$), it implies that we can deduce that the walker has passed the test i consecutive times. Similarly, states whose indices are in the set $\{N_1 + 1, \ldots, N_1 + N_2 + 1\}$ present an

indication of the number of consecutive failures that have occurred. In this case, if the walker is in state $N_1 + i$ where $0 < i \leq N_2 + 1$, we can infer that the walker has made i consecutive failures so far.

We present the following mapping of the states of the RW $\{1, \ldots, N_1 + N_2 + 1\}$ to the set of hypotheses

$$\{\textit{Well-Functioning}, \textit{Mal-Functioning}\}$$

as follows. The mapping is divided into two parts:

Mal-Functioning States: We refer to the states $\{1, \ldots, N_1\}$ as being the so-called *Mal-Functioning* states, because whenever the index $X(t)$ of the current state of the walker is in that set, we conjecture that "the hypothesis that the component is in its *Mal-Functioning* mode" is true. In this phase, the state transitions illustrated in the figure are such that any deviance from the hypothesis is modelled by a successful transition to the neighboring state, while a failure causes a jump back to state 1. Conversely, only a pure uninterrupted sequence of N_1 successes will allow the walker to pass into the set of *Well-Functioning* states.

Well-Functioning states We refer to the states $\{N_1+1, \ldots, N_1+N_2 + 1\}$ as the *Well-Functioning* states, because when in this set of states, we conjecture the hypothesis that the component is in its *Well-Functioning* mode. More specifically, we refer to state $N_1 + 1$ as being an "acceptance" state because, informally speaking, whenever the walker is in that state, the conjectured hypothesis, that the component is *Well-Functioning*, has been confirmed with highest probability. In particular, within theses state, the goal is to detect when the entity deteriorates, causing it to degrade into one of the *Mal-Functioning* states. These states can be perceived to be the "opposite" of the *Mal-Functioning* states in the sense that an uninterrupted sequence of failures is required to "throw" the walker back into the *Mal-Functioning* mode, while a single success reconfirms the conjectured hypothesis that the component is functioning well, forcing the walker to return to the *Well-Functioning* state space.

We hope that this brief summary of the application domain suffices!

5. BRIEF SIMULATION RESULTS

Apart from the above theoretical results, we have also rigorously tested the RWJ which we have studied in various experimental settings. In this section, we present some experimental results for cases where the RWJ has been simulated. The goal of the exercise was to understand the sensitivity of the MC to changes in the memory size, the properties of P_1 as a function of the reward probability and the limiting (asymptotic) behavior of the walk. Although the chain has been simulated for a variety of settings, in the interest of brevity, we present here only a few typical sets of results – essentially, to catalogue the overall conclusions of the investigation.

5.1. P_1 as a Function of p, the Reward Probability

In the first set of experiments, we analyzed the value of P_1 as a function of the reward probability, p, for different memory configurations of a balanced memory set-up. We report here the cases when N was equal to 3, 5 and 10. By observing the plot of P_1 (see Figure 2), we see that this is a monotonically decreasing function of p, which possesses an inflection point at $p = 1/2$, which confirms the conclusions of Theorems 2 and 3. Further, from Figure 2 we see that for values of p such that $p > 0.5$, P_1 decreases significantly and tends towards 0 as we increase N from 3 to 10. Conversely, for values of p such that $p < 0.5$, we observe that P_1 increases and tends towards unity as we increase N from 3 to 10. This too confirm our earlier theoretical results.

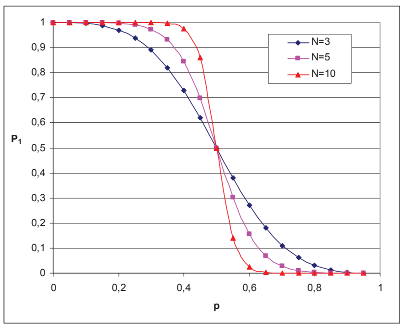
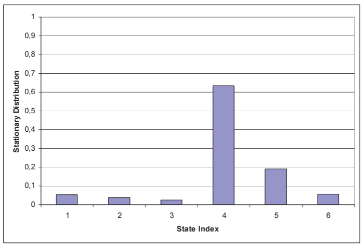


Fig. 2. A plot of P_1 as a function of p, the reward probability.

5.2. Limiting Behavior of the RWJ

In our studies, we were also interested in understanding the limiting behavior of the RWJ. To investigate this, we simulated various balanced memory schemes. Indeed, we see that the series π_i monotonically decreases with the state index for $p < 0.5$. In other words, $\pi_1 > \pi_2 > \ldots \pi_{2N}$. Figure 3 depicts the steady state (equilibrium) distribution associated with two balanced memory chains, each possessing 6 states ($N = 3$). In the first case, the reward probability was $p = 0.3$, and in the second, the reward probability was $p' = 1 - p = 0.7$. The steady probability of each state was estimated by averaging over $1,000$ experiments, each consisting of $100,000$ iterations. The reader will appreciate the confirmation of Theorem 4 as illustrated by Figure 3. In fact, the steady distribution for $p = 0.3$ can be easily deduced from the steady distribution of $p' = 1 - p = 0.7$.



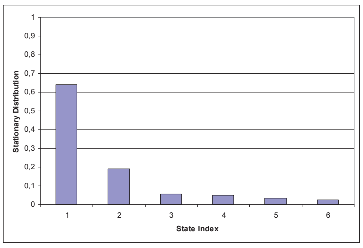


Fig. 3. This figure depicts (a) the stationary distribution for a reward probability $p = 0.7$, and (b) the corresponding stationary distribution for a reward probability $p = 0.3$.

6. CONCLUSIONS

Although Random Walks (RWs) with single-step transitions have been extensively studied [6], problems involving the analysis of RWs that contain interleaving random steps and random "jumps" are intrinsically hard. In this paper, we have considered the analysis of one such fascinating RW, where every step is paired with its counterpart random jump. The paper alludes to the application of the the RW in the testing of entities which constrain the entity to never be allowed to make more than a pre-specified number of *consecutive* failures. The paper contains the detailed analysis of the chain, some fascinating limiting properties, and a few simulations that justify the analytic results. We believe that the entire field of RWs with interleaving steps and jumps is novel, and that this is a pioneering paper in this field. More detailed results of the chain, the simulations, and its potential applications are found in [12].

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