The Influence of Severity of Fading on the Statistical Properties of the Capacity of Nakagami-m Channels with MRC and EGC

Gulzaib Rafiq
Faculty of Engineering and Science,
University of Agder, Service Box 509,
N-4898 Grimstad, Norway.
Email: gulzaib.rafiq@uia.no

Valeri Kontorovich
Centro de Investigación y
de Estudios Avanzados, CINVESTAV
Mexico City, Mexico.
Email: valeri@cinvestav.mx

Matthias Pätzold
Faculty of Engineering and Science,
University of Agder, Service Box 509,
N-4898 Grimstad, Norway.
Email: matthias.paetzold@uia.no

***Abstract*—In this article, we have studied the statistical properties of the capacity of Nakagami-m channels when spatial diversity combining, such as maximal ratio combining (MRC) and equal gain combining (EGC), is employed at the receiver. The presented results provide insight into the statistical properties of the channel capacity under a wide range of fading conditions in wireless links using L-branch diversity combining techniques. We have derived closed-form analytical expressions for the probability density function (PDF), cumulative distribution function (CDF), level-crossing rate (LCR), and average duration of fades (ADF) of the channel capacity. The statistical properties of the capacity are studied for different values of the number of diversity branches and for different severity levels of fading. The analytical results are verified with the help of simulations. It is observed that increasing the number of diversity branches increases the mean channel capacity, while the variance and ADF of the channel capacity decreases. Moreover, systems in which the fading in diversity branches is less severe (as compared to Rayleigh fading) have a higher mean channel capacity. The presented results are very helpful to optimize the design of the receiver of wireless communication systems that employ spatial diversity combining.**

I. Introduction

The received signal impairments, caused by multipath fading in wireless communication systems, can be reduced by diversity combining methods, such as MRC and EGC [1], [2]. In diversity combining schemes, the received signals in different diversity branches are combined in a way that results in an increased signal-to-noise ratio (SNR) [1], [2]. Hence, such methods increase the system throughput and therefore enhance the overall system performance. Due to these advantages, numerous papers have been published dealing with the system performance and the capacity analysis of Rayleigh and Rice channels (see, e.g., [3]–[6] and the references therein). On the other hand, the Nakagami-m process is considered to be a more general channel model as compared to Rayleigh and Rice models because it can be used to study the scenarios where the fading is more (or less) severe as compared to Rayleigh fading. The generality of this model also derives from the fact that it incorporates Rayleigh and Rice models as special cases. Results pertaining to the statistical analysis of the signal envelope and the system performance analysis for MRC and EGC in Nakagami-m channels can be found in [7], [8]. However, to the best of the authors' knowledge, there is still a gap of information regarding the statistical analysis of the capacity of Nakagami-m channels with MRC and EGC. The aim of this paper is to fill in this gap.

This paper deals with the derivation and analysis of the PDF, CDF, LCR, and ADF of the channel capacity of Nakagami-m channels for both MRC and EGC. The PDF can be helpful to analyze the mean channel capacity and the variance of the channel capacity, while the LCR and ADF of the channel capacity give an insight into the temporal behavior of the channel capacity. We have studied the statistical properties of the channel capacity for different values of the number of diversity branches L and for different values of m controlling the severity of fading in Nakagami-m channels. We have also included the results for Rayleigh channels (which arise for the case when $m = 1$) for comparison purposes. It is observed that for both MRC and EGC, an increase in the number of diversity branches L increases the mean channel capacity, while the variance and the ADF of the channel capacity decrease. Moreover, an increase in the severity of fading results in a decrease in the mean channel capacity, however the variance and ADF of the channel capacity increase. It is also observed that at lower signal levels, the LCR is higher for channels with smaller values of the number of diversity branches L or higher severity levels of fading than for channels with higher values of L or lower severity levels of fading.

The rest of the paper is organized as follows. The MRC and EGC schemes in Nakagami-m channels are briefly reviewed in Section II and III, respectively. In Section IV, we present the derivation of the statistical properties of the capacity of Nakagami-m channels with MRC. The statistical properties of the capacity of Nakagami-m channels with EGC are discussed in Section V. The theoretical and simulation results are analyzed and illustrated in Section VI. Finally, the conclusions are presented in Section VII.

II. Nakagami-m Channels with MRC

The signal envelope in the lth branch of an L-branch diversity system can be characterized by a Nakagami-m process

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$\zeta_l(t)$. The PDF $p_{\zeta_l}(z)$ of the Nakagami-m process $\zeta_l(t)$ is given by [9]

$$p_{\zeta_l}(z) = \frac{2m_l^{m_l} z^{2m_l-1}}{\Gamma(m)\Omega_l^{m_l}} e^{-\frac{m_l z^2}{\Omega_l}}, \quad z \geq 0 \tag{1}$$

for $l = 1, 2, \ldots, L$, where $\Omega_l = E\left\{z^2\right\}$, $m_l = \Omega_l^2 \left/ \text{Var}\left\{z^2\right\}\right.$, and $\Gamma(\cdot)$ represents the gamma function [10]. In order to generate Nakagami-m processes $\zeta_l(t)$, we have used the following relation [11]

$$\zeta_l(t) = \sqrt{\sum_{i=1}^{2\times m_l} \mu_{i,l}^2(t)} \tag{2}$$

where $\mu_{i,l}(t)$ $(i = 1, 2, \ldots, 2 \times m_l)$ are the underlying independent and identically distributed (i.i.d.) Gaussian processes, and m_l is the parameter of the Nakagami-m distribution associated with the lth branch. The parameter m_l controls the severity of the fading. Increasing the value of m_l decreases the severity of fading associated with the lth branch and vice versa. In this article, we have assumed that $\Omega_l = 2m_l\sigma_0^2$ for the sake of simplicity. Here, σ_0^2 denotes the variance of the underlying Gaussian processes $\mu_{i,l}(t)$ in $\zeta_l(t)$. In an MRC diversity system, the instantaneous SNR $\gamma(t)$ at the combiner output can be expressed as [1]

$$\gamma(t) = \frac{P_s}{N_0} \sum_{l=1}^{L} \zeta_l^2(t) = \gamma_s \Lambda(t) \tag{3}$$

where $\gamma_s = P_s / N_0$ can be termed as the average SNR of each branch and $\Lambda(t) = \sum_{l=1}^{L} \zeta_l^2(t)$. Here, P_s represents the total transmitted power per symbol, and N_0 denotes the variance of the additive white Gaussian noise (AWGN). The PDF $p_\Lambda(z)$ of the process $\Lambda(t)$ can be expressed using [12, Eq. (2)] as

$$p_\Lambda(z) = \frac{z^{\sum_{l=1}^{L} m_l - 1} e^{-\frac{z}{2\sigma_0^2}}}{(2\sigma_0^2)^{\sum_{l=1}^{L} m_l}\, \Gamma\left(\sum_{l=1}^{L} m_l\right)}, \quad z \geq 0. \tag{4}$$

Under the assumption of isotropic scattering, the joint PDF $p_{\Lambda\dot{\Lambda}}(z, \dot{z})$ of $\Lambda(t)$ and its time derivative $\dot{\Lambda}(t)$ at the same time t can be written as [8]

$$p_{\Lambda\dot{\Lambda}}(z, \dot{z}) = p_\Lambda(z) \frac{1}{\sqrt{8\pi z \sigma_{\dot{\zeta}_l}^2}} e^{-\frac{\dot{z}^2}{8z\sigma_{\dot{\zeta}_l}^2}}, \quad z \geq 0, |\dot{z}| < \infty. \tag{5}$$

Here, $\sigma_{\dot{\zeta}_l}^2 = (\pi f_{\max})^2\, \Omega_l \,/\, m_l$ denotes the variance of the process $\dot{\zeta}_l(t)$ [8], where $\dot{\zeta}_l(t)$ represents the time derivative of the process $\zeta_l(t)$, and $f_{\max}$ is the maximum Doppler frequency. In Section IV, we will use the results presented in (4) and (5) to analyze the statistical properties of the capacity of Nakagami-m channels with MRC.

III. Nakagami-m Channels with EGC

The instantaneous SNR $\gamma(t)$ at the combiner output in an L-branch EGC diversity system is given by [1]

$$\gamma(t) = \frac{P_s}{LN_0} \left(\sum_{l=1}^{L} \zeta_l(t)\right)^2 = \frac{\gamma_s}{L} \Lambda(t) \tag{6}$$

where $\Lambda(t) = \left(\sum_{l=1}^{L} \zeta_l(t)\right)^2$, and $\zeta_l(t)$ represents the received signal envelope in the lth Nakagami-m branch. We again proceed by first finding the PDF $p_\Lambda(z)$ of the process $\Lambda(t)$ and the joint PDF $p_{\Lambda\dot{\Lambda}}(z, \dot{z})$ of the process $\Lambda(t)$ and its time derivative $\dot{\Lambda}(t)$. However, finding the PDF of a sum of Nakagami-m processes $\sum_{l=1}^{L} \zeta_l(t)$ is still an open problem, and hence the PDF $p_\Lambda(z)$ of $\Lambda(t)$ is thus unknown. One of the remedies for this problem is to use an appropriate approximation to the sum $\sum_{l=1}^{L} \zeta_l(t)$ to find the PDF $p_\Lambda(z)$ (see, e.g., [9], [13] and the references therein). In this article, we have approximated the sum of Nakagami-m processes $\sum_{l=1}^{L} \zeta_l(t)$ by another Nakagami-m process $S(t)$ with parameters m_S and Ω_S, as suggested in [13]. Hence, the PDF $p_S(z)$ of $S(t)$ can be obtained by replacing m_l and Ω_l in (1) by m_S and Ω_S, respectively, where $\Omega_S = E\left\{S^2(t)\right\}$ and $m_S = \Omega_S^2 \left/ \left(E\left\{S^4(t)\right\} - \Omega_S^2\right)\right.$. The quantity $E\left\{S^n(t)\right\}$ can be calculated using [13]

$$E[S^n(t)] = \sum_{n_1=0}^{n} \sum_{n_2=0}^{n_1} \cdots \sum_{n_{L-1}=0}^{n_{L-2}} \binom{n}{n_1} \binom{n_1}{n_2} \cdots \binom{n_{L-2}}{n_{L-1}} \times E[\zeta_1^{n-n_1}(t)] E[\zeta_2^{n_1-n_2}(t)] \cdots E[\zeta_L^{n_{L-1}}(t)]$$

where

$$E[\zeta_l^n(t)] = \frac{\Gamma(m_l + n/2)}{\Gamma(m_l)} \left(\frac{\Omega_l}{m_l}\right)^{n/2}, \quad l = 1, 2, \ldots, L. \tag{7}$$

By using this approximation for the PDF of a sum $\sum_{l=1}^{L} \zeta_l(t)$ of Nakagami-m processes and applying the concept of transformation of random variables [14, Eq. (7–8)], the PDF $p_\Lambda(z)$ of the squared sum of Nakagami-m processes $\Lambda(t)$ can be expressed using $p_\Lambda(z) = 1 \left/ (2\sqrt{z})\right.\, p_S\left(\sqrt{z}\right)$ as

$$p_\Lambda(z) \approx \frac{m_S^{m_S} z^{m_S - 1}}{\Gamma(m_S)\Omega_S^{m_S}} e^{-\frac{m_S z}{\Omega_S}}, \quad z \geq 0. \tag{8}$$

The joint PDF $p_{\Lambda\dot{\Lambda}}(z, \dot{z})$ can now be expressed with the help of [11, Eq. (13)], (8), and by using the concept of transformation or random variables [14, Eq. (7–8)] as

$$p_{\Lambda\dot{\Lambda}}(z, \dot{z}) \approx \frac{e^{-\frac{\dot{z}^2}{8Lz\sigma_{\dot{\zeta}_l}^2}}}{\sqrt{8\pi z L \sigma_{\dot{\zeta}_l}^2}}\, p_\Lambda(z), \quad z \geq 0, |\dot{z}| < \infty. \tag{9}$$

Using (8) and (9), the statistical properties of the capacity of Nakagami-m channels with EGC will be analyzed in Section VI.

IV. Statistical Properties of the Capacity of Nakagami-m Channels with MRC

The instantaneous channel capacity $C(t)$ for the case when diversity combining is employed at the receiver can be expressed as [15]

$$C(t) = \log_2\left(1 + \gamma(t)\right) \quad \text{(bits/s/Hz)} \tag{10}$$

where $\gamma(t)$ represents the instantaneous SNR given by (3) and (6) for MRC and EGC, respectively. The expression in (10) can be considered as a mapping of the random process

$\gamma(t)$ to another random process $C(t)$. Hence, the statistical properties of the instantaneous SNR $\gamma(t)$ can be used to find the statistical properties of the channel capacity. The PDF $p_\gamma(z)$ of the instantaneous SNR $\gamma(t)$ can be obtained using the relation $p_\gamma(z) = (1/\gamma_s)\, p_\Lambda(z/\gamma_s)$. Thereafter, applying the concept of transformation of random variables, the PDF $p_C(r)$ of the channel capacity $C(t)$ is obtained using $p_C(r) = 2^r \ln(2)\, p_\gamma(2^r - 1)$ as follows

$$p_C(r) = \frac{2^r \ln(2)(2^r-1)^{\sum_{l=1}^{L} m_l - 1}}{\Gamma\left(\sum_{l=1}^{L} m_l\right)(2\sigma_0^2\gamma_s)^{\sum_{l=1}^{L} m_l}} e^{-\frac{(2^r-1)}{2\sigma_0^2\gamma_s}}, \quad r \geq 0. \tag{11}$$

The CDF $F_C(r)$ of the channel capacity $C(t)$ can be found using the relationship $F_C(r) = \int_0^r p_C(x)dx$ [14]. After solving the integral, the CDF $F_C(r)$ of $C(t)$ can be expressed as

$$F_C(r) = 1 - \frac{1}{\Gamma\left(\sum_{l=1}^{L} m_l\right)} \Gamma\left(\sum_{l=1}^{L} m_l, \frac{(2^r-1)}{2\sigma_0^2\gamma_s}\right), \quad r \geq 0 \tag{12}$$

where $\Gamma(\cdot,\cdot)$ represents the incomplete gamma function [10, Eq. (8.350-2)].

The LCR of the channel capacity defines the average rate of up-crossings (or down-crossings) of the channel capacity through a certain threshold level [16]. In order to find the LCR $N_C(r)$ of the channel capacity $C(t)$, we first need to find the joint PDF $p_{C\dot{C}}(z,\dot{z})$ of the channel capacity $C(t)$ and its time derivative $\dot{C}(t)$. The joint PDF $p_{C\dot{C}}(z,\dot{z})$ can be obtained using $p_{C\dot{C}}(z,\dot{z}) = (2^z \ln(2))^2 p_{\gamma\dot{\gamma}}(2^z-1, 2^z \dot{z} \ln(2))$, where $p_{\gamma\dot{\gamma}}(z,\dot{z}) = \left(1/\gamma_s^2\right) p_{\Lambda\dot{\Lambda}}(z/\gamma_s, \dot{z}/\gamma_s)$. The expression for the joint PDF $p_{C\dot{C}}(z,\dot{z})$ can be written as

$$p_{C\dot{C}}(z,\dot{z}) = \frac{2^z \ln(2)}{\sqrt{(2^z-1)\, 8\pi\sigma_{\dot{\zeta}_l}^2 \gamma_s}} e^{-\frac{(2^z \ln(2)\dot{z})^2}{8\gamma_s \sigma_{\dot{\zeta}_l}^2 (2^z-1)}} p_C(z) \tag{13}$$

for $z \geq 0$ and $|\dot{z}| < \infty$. The LCR $N_C(r)$ can now be obtained by solving the integral in $N_C(r) = \int_0^\infty \dot{z}\, p_{C\dot{C}}(r,\dot{z})d\dot{z}$. After some algebraic manipulations, the LCR $N_C(r)$ can finally be expressed in closed form as

$$N_C(r) = \sqrt{\frac{2\sigma_{\dot{\zeta}_l}^2 \gamma_s (2^r-1)}{\pi 2^{2r}(\ln(2))^2}}\, p_C(r), \quad r \geq 0. \tag{14}$$

The ADF of the channel capacity denotes the average duration of time over which the channel capacity is below a certain threshold level [16]. The ADF $T_C(r)$ of the channel capacity $C(t)$ can be obtained using $T_C(r) = F_C(r)/N_C(r)$ [1], where $F_C(r)$ and $N_C(r)$ are given by (12) and (14), respectively.

V. Statistical Properties of the Capacity of Nakagami-m Channels with EGC

For the case of EGC, the PDF $p_\gamma(z)$ of the instantaneous SNR $\gamma(t)$ can be obtained by substituting (8) in $p_\gamma(z) = (1/\acute{\gamma}_s)\, p_\Lambda(z/\acute{\gamma}_s)$, where $\acute{\gamma}_s = \gamma_s/L$. Thereafter, the PDF $p_C(r)$ is obtained by applying the concept of transformation of random variables on (10) as

$$\begin{aligned} p_C(r) &= 2^r \ln(2)\, p_\gamma(2^r-1) \\ &\approx \frac{2^r \ln(2)(2^r-1)^{m_S-1}}{\Gamma(m_S)\,(\acute{\gamma}_s\,\Omega_S/m_S)^{m_S}} e^{-\frac{m_S(2^r-1)}{\acute{\gamma}_s\Omega_S}}, \quad r \geq 0. \end{aligned} \tag{15}$$

By integrating the PDF $p_C(r)$, the CDF $F_C(r)$ of the channel capacity $C(t)$ can be obtained using $F_C(r) = \int_0^r p_C(x)dx$ as

$$F_C(r) \approx 1 - \frac{1}{\Gamma(m_S)} \Gamma\left(m_S, \frac{m_S(2^r-1)}{\acute{\gamma}_s\Omega_S}\right), \quad r \geq 0. \tag{16}$$

The joint PDF $p_{C\dot{C}}(z,\dot{z})$ for the case of EGC can be obtained using $p_{C\dot{C}}(z,\dot{z}) = (2^z \ln(2))^2 p_{\gamma\dot{\gamma}}(2^z-1, 2^z\dot{z}\ln(2))$ and $p_{\gamma\dot{\gamma}}(z,\dot{z}) = \left(1/\acute{\gamma}_s^{\,2}\right) p_{\Lambda\dot{\Lambda}}(z/\acute{\gamma}_s, \dot{z}/\acute{\gamma}_s)$ as

$$p_{C\dot{C}}(z,\dot{z}) \approx \frac{2^z \ln(2)}{\sqrt{(2^z-1)\, 8\pi L\sigma_{\dot{\zeta}_l}^2 \acute{\gamma}_s}} e^{-\frac{(2^z \ln(2)\dot{z})^2}{8L\acute{\gamma}_s \sigma_{\dot{\zeta}_l}^2 (2^z-1)}} p_C(z) \tag{17}$$

for $z \geq 0$ and $|\dot{z}| < \infty$. Now by employing the formula $N_C(r) = \int_0^\infty \dot{z}\, p_{C\dot{C}}(r,\dot{z})d\dot{z}$, the LCR $N_C(r)$ of the channel capacity $C(t)$ can be approximated in closed form as

$$N_C(r) \approx \sqrt{\frac{2\sigma_{\dot{\zeta}_l}^2 \acute{\gamma}_s (2^r-1)}{\pi}} \frac{m^{m_S}(2^r-1)^{m_S}}{\Gamma(m_S)\,(\acute{\gamma}_s\Omega_S)^{m_S}} e^{-\frac{m_S(2^r-1)}{\acute{\gamma}_s\Omega_S}} \tag{18}$$

for $z \geq 0$. By using $T_C(r) = F_C(r)/N_C(r)$, the ADF $T_C(r)$ of the channel capacity $C(t)$ can be obtained, while $F_C(r)$ and $N_C(r)$ are given by (16) and (18), respectively.

VI. Numerical Results

This section aims to analyze and to illustrate the analytical findings of the previous sections. The correctness of the analytical results will be confirmed with the help of simulations. For comparison purposes, we have also shown the results for Rayleigh channels (obtained for $m_l = 1$, $\forall l = 1, 2, \ldots, L$). Moreover, we have also presented the results for the classical Rayleigh channels which arise when $L = 1$ and $m_l = 1$. The underlying Gaussian processes $\mu_{i,l}(t)$ $(i = 1, 2, \ldots, 2 \times m_l)$ are generated using the sum-of-sinusoids method [17]. The model parameters were calculated using the generalized method of exact Doppler spread (GMEDS$_1$) [18]. The number of sinusoids for the generation of the Gaussian processes $\mu_{i,l}(t)$ was chosen to be $N_i = 29$. The SNR γ_s was set to 15 dB, the maximum Doppler frequency $f_{\max}$ was 91 Hz, and the parameter σ_0 was equal to unity. Finally, using (3), (6), and (10), the simulation results for the statistical properties of the channel capacity $C(t)$ of Nakagami-m channels with MRC and EGC were obtained.

Figures 1 and 2 present the PDF $p_C(r)$ of the capacity of Nakagami-m channels with MRC and EGC, respectively, for different values of the number of diversity branches L and severity parameters m_l. It is observed that in both cases an increase in the number of diversity branches L increases the mean channel capacity. However, the variance of the channel capacity decreases. This fact is specifically highlighted in

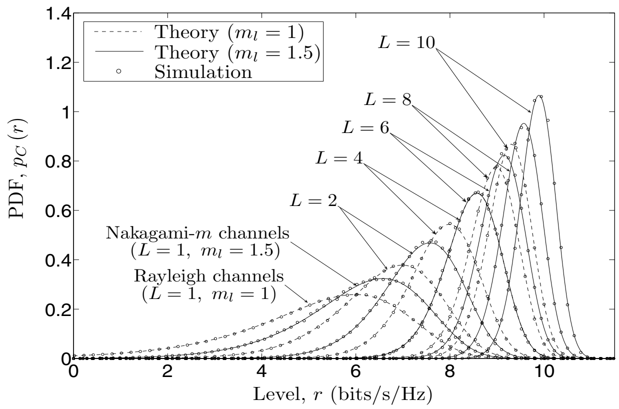

Fig. 1. The PDF $p_C(r)$ of the capacity of Nakagami-m channels with MRC.

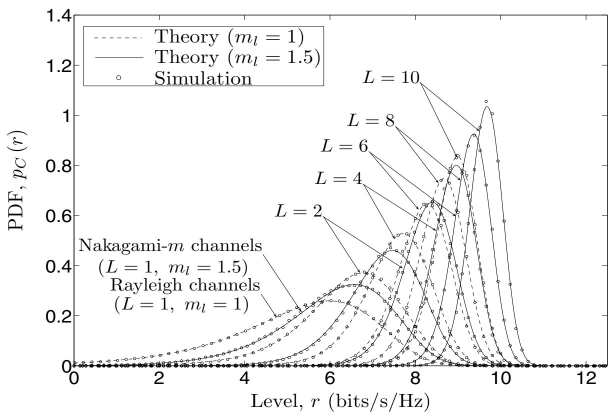

Fig. 2. The PDF $p_C(r)$ of the capacity of Nakagami-m channels with EGC.

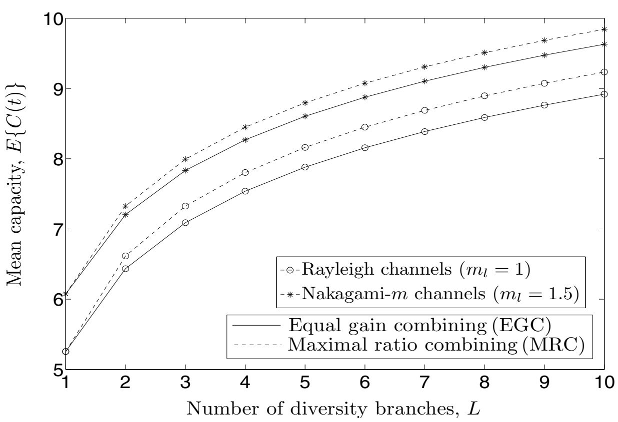

Fig. 3. Comparison of the mean channel capacity of Nakagami-m channels with MRC and EGC.

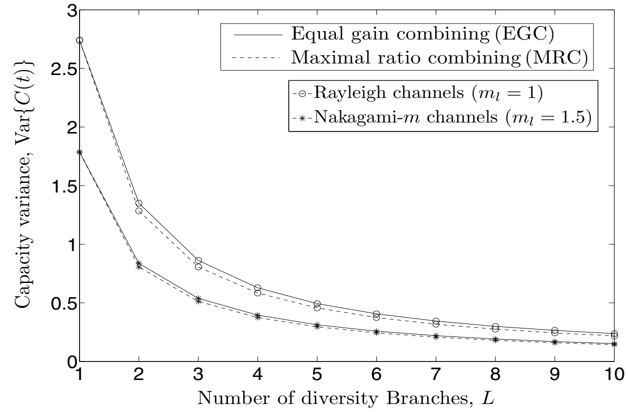

Fig. 4. Comparison of the variance of the channel capacity of Nakagami-m channels with MRC and EGC.

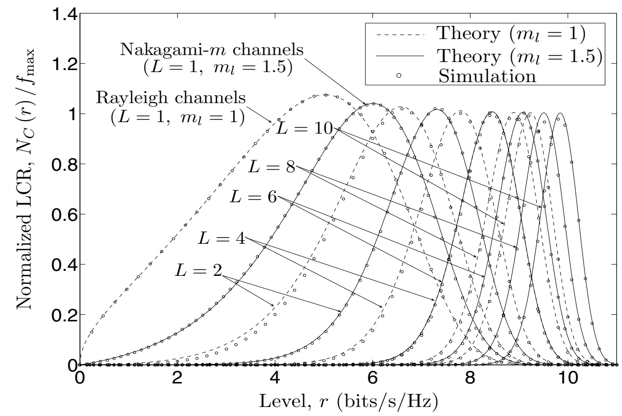

Fig. 5. The normalized LCR $N_C(r)/f_{\max}$ of the capacity of Nakagami-m channels with MRC.

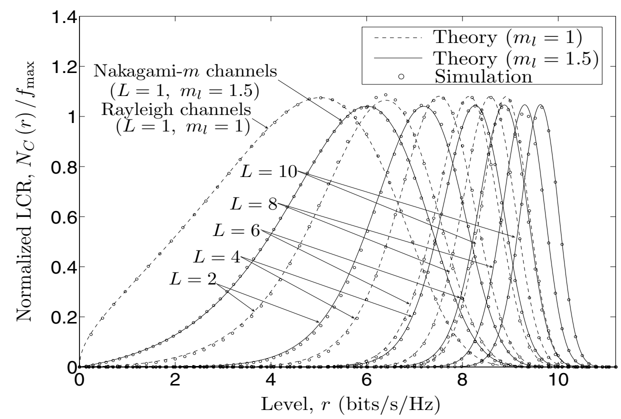

Fig. 6. The normalized LCR $N_C(r)/f_{\max}$ of the capacity of Nakagami-m channels with EGC.

Figs. 3 and 4, where the mean channel capacity and the variance of the capacity of Nakagami-m channels, respectively, is studied for different values of the number of diversity branches L and severity parameters m_l. It can be observed that the mean channel capacity and the variance of the capacity of Nakagami-m channels are quite different from those of Rayleigh channels. Specifically for both MRC and EGC, if the branches are less severely faded ($m_l = 1.5,\ \forall l = 1, 2, \ldots, L$) as compared to Rayleigh fading ($m_l = 1,\ \forall l = 1, 2, \ldots, L$), then the mean channel capacity increases, while the variance of the channel capacity decreases. The LCR $N_C(r)$ of the capacity of Nakagami-m channels with MRC and EGC is shown in Figs. 5 and 6 for different values of the number of diversity branches L and severity parameters m_l. It can be seen in these two figures that at low signal levels r, the LCR $N_C(r)$ of the channels with lower values of the number of diversity branches L is higher as compared to that of the channels with higher values of L. However, the converse statement is true

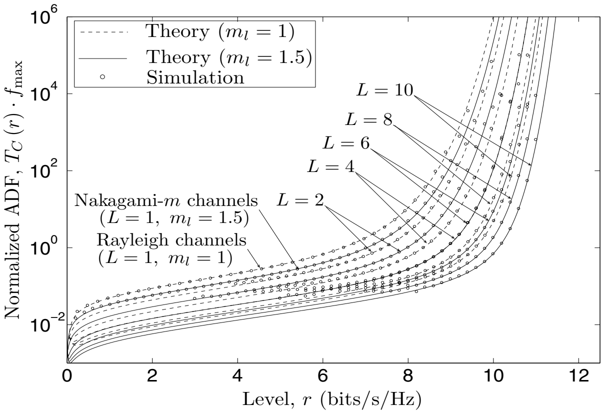

Fig. 7. The normalized ADF $T_C(r) \cdot f_{\max}$ of the capacity of Nakagami-m channels with MRC.

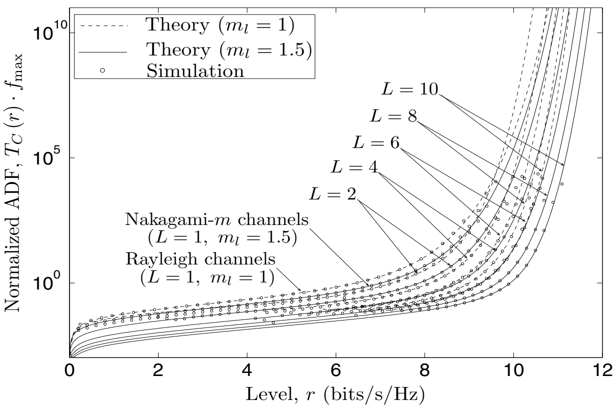

Fig. 8. The normalized ADF $T_C(r) \cdot f_{\max}$ of the capacity of Nakagami-m channels with EGC.

for high signal levels r.

The ADF $T_C(r)$ of the capacity of Nakagami-m channels with MRC and EGC is shown in Figs. 7 and 8, respectively. The results show that an increase in the number of diversity branches decreases the ADF of the channel capacity. Moreover, an increase of the severity parameters m_l results in an increase in the ADF of the channel capacity. The analytical expressions are verified using simulations, whereby a very good fitting is found.

VII. Conclusion

This article presents a statistical analysis of the capacity of Nakagami-m channels for MRC and EGC diversity schemes. We have derived closed-form analytical expressions for the PDF, CDF, LCR, and ADF of the channel capacity of Nakagami-m channels with MRC and EGC. The presented results show that the number of diversity branches L and the severity of fading have a significant influence on the channel capacity. Specifically, increasing the number of diversity branches increases the mean channel capacity, while the variance and the ADF of the channel capacity decreases. Moreover, an increase in the severity of fading in diversity branches results in a decrease in the mean channel capacity. However, the ADF and the variance of the channel capacity increases. It is also observed that at lower signal levels, the LCR is higher for channels with smaller values of the number of diversity branches L or higher severity levels of fading than for channels with higher values of L or lower severity levels of fading. The analytical findings are verified using simulations, where a very good agreement between the theoretical and simulation results was observed.

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