Level-Crossing Rate and Average Duration of Fades of the Envelope of Mobile-to-Mobile Fading Channels in K-Parallel Dual-Hop Relay Networks

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Abstract— **This paper studies the fading behavior of narrowband mobile-to-mobile (M2M) fading channels in dual-hop distributed cooperative multi-relay systems under non-line-of-sight (NLOS) propagation conditions. M2M fading channels considered here are associated with amplify-and-forward relay networks, where K mobile relays are connected in parallel between the source mobile station and the destination mobile station. Such M2M fading channels are referred to as K-parallel dual-hop relay M2M fading channels. We study the fading behavior of these channels by analyzing the level-crossing rate (LCR) and the average duration of fades (ADF) of the received signal envelope. We derive analytical integral expressions of the aforementioned quantities along with the cumulative distribution function (CDF) of the envelope. These statistical quantities are derived assuming that the underlying stochastic processes are independent but not necessarily identically distributed. In addition, the statistical analysis pertaining to the special case of independent and identically distributed (i.i.d.) processes is also presented in this paper. The validity of the presented expressions is confirmed by simulations. Our results are very beneficial for future performance analysis of overall dual-hop distributed cooperative multi-relay systems.**

I. INTRODUCTION

For decades, diversity techniques have been employed in mobile wireless communication systems in order to mitigate the adverse fading effects. Among several other diversity techniques, the cooperative diversity scheme [1]–[3] has also gained attention for its potential to provide spatial diversity gain. A spatial diversity gain in cooperative diversity systems is achieved if several single-antenna mobile stations in the network share their antennas to form a so-called virtual antenna array [4]. The signal from the source mobile station is thus relayed to the destination mobile station via other mobile stations in the network.

Mobile radio fading channels are usually described by statistical quantities, such as the mean value, the variance, as well as the probability density function (PDF) of the envelope and the phase of the received signal. Unfortunately, these statistical quantities do not give any information about how fast (or slow) the fading channel is changing with time. However, to cope with the problems faced in the development of cooperative diversity systems, a solid knowledge of the underlying fading behavior of the channel is inevitable. Analyzing the LCR and the ADF provides vital information regarding the fading behavior of M2M channels.

Studies pertaining to the statistical analysis of narrowband M2M fading channels under NLOS propagation conditions in a dual-hop relay-based cooperative system with a single relay in the network have shown that a double Rayleigh process is the most suitable statistical channel model for such scenarios [5], [6]. The work presented in [7] is a straightforward extension of the double Rayleigh channel model to the double Rice channel model assuming line-of-sight (LOS) propagation conditions. Several M2M fading channel models have been developed based on the multiple scattering concept proposed in [8] for dual-hop single-relay cooperative systems for both NLOS and LOS propagation environments [9]–[11]. In addition to dual-hop cooperative systems, several papers dealing with the statistical modeling and analysis of multi-hop cooperative systems having K relays connected in series between the source and the destination terminals are available in the literature [12], [13].

This paper studies the fading behavior of M2M fading links in dual-hop amplify-and-forward relay networks, where K mobile relays are connected in parallel between the source mobile station and the destination mobile station. Such M2M fading channels are referred to as K-parallel dual-hop relay M2M fading channels. Each individual transmission link from the source mobile station to the destination mobile station via the kth $(k = 1, 2, \ldots, K)$ relay is modeled as a complex double Gaussian process. Consequently, the overall M2M fading channel taking into account K parallel relay links can be modeled as a sum of K complex double Gaussian processes. The authors of [14] have derived analytical expressions for the characteristic function (CF), the PDF of the envelope as well as the phase, and the mean value along with the variance of K-parallel dual-hop relay M2M fading channels.

In this paper, we fill the information gap regarding the LCR and the ADF of K-parallel dual-hop relay M2M fading channels. Here, we derive analytical integral expressions of these quantities along with the CDF for the envelope of K-parallel dual-hop relay M2M fading channels. The expressions for the LCR, the ADF, and the CDF are derived assuming that the underlying stochastic processes are independent but not necessarily identically distributed. Furthermore, it is shown that the obtained formulae can be reduced to simple and closed-form expressions when i.i.d. processes are taken into account.

The organization of the remaining part of the paper is as follows: In Section II, the reference model for K-parallel dual-hop relay fading channels is developed. Section III deals with the derivation of the LCR and the ADF. Section IV validates the correctness of the obtained analytical expressions by simulations. Finally, some concluding remarks are given in Section V.

II. THE K-PARALLEL DUAL-HOP RELAY FADING CHANNEL MODEL

In this paper, we study the fading behavior of the overall M2M fading channel in a dual-hop cooperative network, where there are K mobile relays connected in parallel between the source mobile station and the destination mobile station. We refer to the resulting overall M2M fading channel as the K-parallel dual-hop relay fading channel. We aim to investigate frequency non-selective K-parallel dual-hop relay fading channels under NLOS propagation conditions in isotropic scattering conditions. Furthermore, we have considered time-division multiple-access (TDMA) based amplify-and-forward relay protocols. In addition, all the mobile stations in the network, i.e., the source mobile station, the destination mobile station, and the K mobile relays do not transmit and receive a signal at the same time in the same frequency band. The propagation scenario considered is presented in Fig. 1.
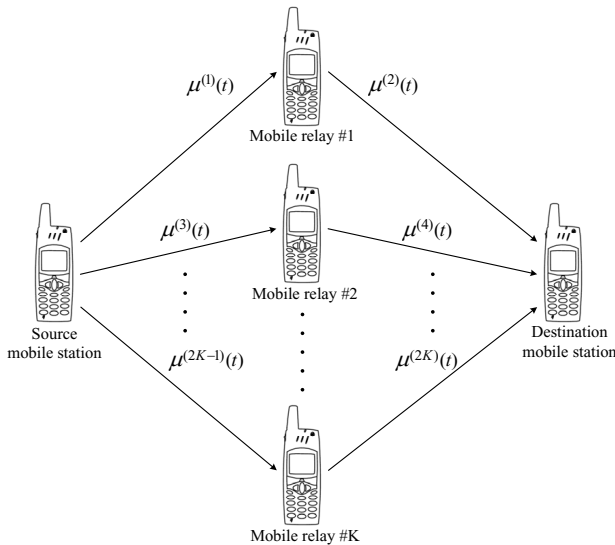

Fig. 1. The propagation scenario describing K-parallel dual-hop relay M2M fading channels.

The overall complex time-varying channel gain associated with K-parallel dual-hop relay fading channels can be written as

$$\chi(t) = \chi_1(t) + j\chi_2(t) = \sum_{k=1}^{K} \varsigma^{(k)}(t) \qquad (1)$$

where $\varsigma^{(k)}(t)$ $(k = 1, 2, \ldots, K)$ describes the fading process in the kth subchannel from the source mobile station to the destination mobile station via the kth mobile relay. Here, the fading process $\varsigma^{(k)}(t)$ is modeled as a weighted zero-mean complex double Gaussian process, i.e.,

$$\varsigma^{(k)}(t) = \varsigma_1^{(k)}(t) + j\varsigma_2^{(k)}(t) = A_{\mathrm{R}^{(k)}} \mu^{(2k-1)}(t) \mu^{(2k)}(t) \quad (2)$$

for $k = 1, 2, \ldots, K$. In (2), $\mu^{(i)}(t)$ $(i = 1, 2, \ldots, 2K)$ is a zero-mean complex Gaussian process with variance $2\sigma_{\mu^{(i)}}^2/\sqrt{K}$. These Gaussian processes $\mu^{(i)}(t)$ are mutually independent, where each one is described by the classical Jakes Doppler power spectral density (PSD). Note that the Gaussian process $\mu^{(i)}(t)$ represents the corresponding scattered component of the subchannel between the source mobile station and the kth mobile relay for $i = 2k-1 = 1, 3, \ldots, (2K-1)$. Analogously, the Gaussian process $\mu^{(i)}(t)$ defines the scattered component of the subchannel between the kth mobile relay and the destination mobile station for $i = 2k = 2, 4, \ldots, 2K$. In (2), $A_{\mathrm{R}^{(k)}}$ is referred to as the relay gain of the kth relay. It is important to state here that the relay gain $A_{\mathrm{R}^{(k)}}$ is only a scaling factor for the variance of the complex Gaussian process $\mu^{(i)}(t)$, i.e., $\mathrm{Var}\{A_{\mathrm{R}^{(k)}}\mu^{(i)}(t)\} = 2(A_{\mathrm{R}^{(k)}}\sigma_{\mu^{(i)}})^2/\sqrt{K}$, where $i = 2, 4, \ldots, 2K$.

The absolute value of the overall complex time-varying channel gain $\chi(t)$ in (1) defines the envelope of K-parallel dual-hop relay fading channels, i.e., $\Xi(t) = |\chi(t)| = \left| \sum_{k=1}^{K} \varsigma^{(k)}(t) \right|$.

III. LCR AND ADF OF K-PARALLEL DUAL-HOP RELAY FADING CHANNELS

This section deals with the derivation of analytical expressions for the LCR and the ADF of the envelope $\Xi(t)$ of K-parallel dual-hop relay fading channels introduced in Section II.

A. Derivation of the LCR

The LCR $N_{\Xi}(r)$ of the envelope $\Xi(t)$ describes the average number of times the process $\Xi(t)$ crosses a certain threshold level from up to down (or from down to up) per second. Mathematically, the LCR $N_{\Xi}(r)$ can be obtained using [15]

$$N_{\Xi}(r) = \int_0^{\infty} \dot{x}\, p_{\Xi\dot{\Xi}}(r, \dot{x})\, d\dot{x} \qquad (3)$$

where $p_{\Xi\dot{\Xi}}(r, \dot{x})$ is the joint PDF of the envelope $\Xi(t)$ and its time derivative $\dot{\Xi}(t)$ at the same time. Throughout this article, the overdot denotes the time derivative. Here, the main problem is to compute the joint PDF $p_{\Xi\dot{\Xi}}(r, \dot{x})$ in order to obtain the LCR $N_{\Xi}(r)$ of the envelope $\Xi(t)$.

For the sake of convenience in deriving the joint PDF $p_{\Xi\dot{\Xi}}(r, \dot{x})$, we first compute the joint CF $\Phi_{\chi_1 \chi_2 \dot{\chi}_1 \dot{\chi}_2}(\omega_1, \omega_2, \dot{\omega}_1, \dot{\omega}_2)$ of the inphase and quadrature components of $\chi(t)$ and $\dot{\chi}(t)$. From (1), it is clear that we can express this joint CF as a product of the joint CFs $\Phi_{\varsigma_1^{(k)} \varsigma_2^{(k)} \dot{\varsigma}_1^{(k)} \dot{\varsigma}_2^{(k)}}(\omega_1, \omega_2, \dot{\omega}_1, \dot{\omega}_2)$ of the inphase and quadrature components of $\varsigma^{(k)}(t)$ and $\dot{\varsigma}^{(k)}(t)$ [16] as

$$\Phi_{\chi_1 \chi_2 \dot{\chi}_1 \dot{\chi}_2}(\omega_1, \omega_2, \dot{\omega}_1, \dot{\omega}_2) = \prod_{k=1}^{K} \Phi_{\varsigma_1^{(k)} \varsigma_2^{(k)} \dot{\varsigma}_1^{(k)} \dot{\varsigma}_2^{(k)}}(\omega_1, \omega_2, \dot{\omega}_1, \dot{\omega}_2).$$

$$(4)$$

By definition, the CF and the PDF form a Fourier transform pair [16]. Thus, given the joint PDF $p_{\varsigma_1^{(k)} \varsigma_2^{(k)} \dot{\varsigma}_1^{(k)} \dot{\varsigma}_2^{(k)}}(y_1, y_2, \dot{y}_1, \dot{y}_2)$ of the inphase and quadrature

components of $\varsigma^{(k)}(t)$ and $\dot\varsigma^{(k)}(t)$, the joint CF $\Phi_{\varsigma_1^{(k)}\varsigma_2^{(k)}\dot\varsigma_1^{(k)}\dot\varsigma_2^{(k)}}(\omega_1,\omega_2,\dot\omega_1,\dot\omega_2)$ can be expressed as

$$\Phi_{\varsigma_1^{(k)}\varsigma_2^{(k)}\dot\varsigma_1^{(k)}\dot\varsigma_2^{(k)}}(\omega_1,\omega_2,\dot\omega_1,\dot\omega_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} dy_1\,dy_2\,d\dot y_1\,d\dot y_2$$
$$\times\, p_{\varsigma_1^{(k)}\varsigma_2^{(k)}\dot\varsigma_1^{(k)}\dot\varsigma_2^{(k)}}(y_1,y_2,\dot y_1,\dot y_2)\, e^{-j(\omega_1 y_1+\omega_2 y_2+\dot\omega_1\dot y_1+\dot\omega_2\dot y_2)} \quad (5)$$

where the joint PDF $p_{\varsigma_1^{(k)}\varsigma_2^{(k)}\dot\varsigma_1^{(k)}\dot\varsigma_2^{(k)}}(y_1,y_2,\dot y_1,\dot y_2)$ is given as follows [10]

$$p_{\varsigma_1^{(k)}\varsigma_2^{(k)}\dot\varsigma_1^{(k)}\dot\varsigma_2^{(k)}}(y_1,y_2,\dot y_1,\dot y_2) = \frac{K}{(2\pi)^2 A_{\mathrm{R}(k)}^2 \sigma_{\mu^{(2k-1)}}^2 \sigma_{\mu^{(2k)}}^2}\int_0^{\infty} d\nu$$
$$\times\, \frac{\nu\, e^{-\frac{\nu^2\sqrt{K}}{2A_{\mathrm{R}(k)}^2\sigma_{\mu^{(2k)}}^2}} e^{-\frac{(y_1^2+y_2^2)\sqrt{K}}{2\sigma_{\mu^{(2k-1)}}^2\nu^2}} e^{-\frac{\nu^2(\dot y_1^2+\dot y_2^2)}{2\left\{\beta_{\mu^{(2k)}}(y_1^2+y_2^2)+\beta_{\mu^{(2k-1)}}\nu^4\right\}}}}{\beta_{\mu^{(2k)}}(y_1^2+y_2^2)+\beta_{\mu^{(2k-1)}}\nu^4}.$$
$$(6)$$

In (6), the quantity $\beta_{\mu^{(i)}}$ $(i=1,2,\ldots,2K)$ is the negative curvature of the autocorrelation function of the inphase and quadrature components of $\mu^{(i)}(t)$ $(i=1,2,\ldots,2K)$. Under isotropic scattering conditions, the quantities $\beta_{\mu^{(i)}}$ can be expressed for M2M fading channels as [17], [18]

$$\beta_{\mu^{(2k-1)}} = 2\left(\sigma_{\mu^{(2k-1)}}\pi\right)^2\left(f_{\mathrm{S_{max}}}^2+f_{\mathrm{R_{max}^{(k)}}}^2\right)/\sqrt{K} \quad (7a)$$
$$\beta_{\mu^{(2k)}} = 2\left(A_{\mathrm{R}(k)}\sigma_{\mu^{(2k)}}\pi\right)^2\left(f_{\mathrm{R_{max}^{(k)}}}^2+f_{\mathrm{D_{max}}}^2\right)/\sqrt{K} \quad (7b)$$

where $f_{\mathrm{S_{max}}}$, $f_{\mathrm{D_{max}}}$, and $f_{\mathrm{R_{max}^{(k)}}}$ are the maximum Doppler frequencies caused by the motion of the source mobile station, the destination mobile station, and the kth mobile relay, respectively.

After substituting (6) in (5), doing some tedious algebraic manipulations, and solving the integrals using [19, Eqs. (3.323-II), (3.338-IV), (3.478-IV), and (6.521-II)], we get the final expression for the joint CF $\Phi_{\varsigma_1^{(k)}\varsigma_2^{(k)}\dot\varsigma_1^{(k)}\dot\varsigma_2^{(k)}}(\omega_1,\omega_2,\dot\omega_1,\dot\omega_2)$ as presented in (8).

Assuming $\varsigma^{(k)}(t)$ $(k=1,2,\ldots,K)$ are i.i.d. complex double Gaussian random processes, which implies that $\Phi_{\varsigma_1\varsigma_2\dot\varsigma_1\dot\varsigma_2}(\omega_1,\omega_2,\dot\omega_1,\dot\omega_2) = \Phi_{\varsigma_1^{(k)}\varsigma_2^{(k)}\dot\varsigma_1^{(k)}\dot\varsigma_2^{(k)}}(\omega_1,\omega_2,\dot\omega_1,\dot\omega_2)$ with $\sigma_{\mu^{(2k-1)}}^2 = \sigma_{\mu^{(1)}}^2$, $\sigma_{\mu^{(2k)}}^2 = \sigma_{\mu^{(2)}}^2$, and $A_{\mathrm{R}} = A_{\mathrm{R}(k)}$ $\forall\, k=1,2,\ldots,K$. Thus, for i.i.d. complex double Gaussian random processes, (4) can simply be expressed as

$$\Phi_{\chi_1\chi_2\dot\chi_1\dot\chi_2}(\omega_1,\omega_2,\dot\omega_1,\dot\omega_2) = \left[\Phi_{\varsigma_1\varsigma_2\dot\varsigma_1\dot\varsigma_2}(\omega_1,\omega_2,\dot\omega_1,\dot\omega_2)\right]^K. \quad (9)$$

Given the joint CF $\Phi_{\chi_1\chi_2\dot\chi_1\dot\chi_2}(\omega_1,\omega_2,\dot\omega_1,\dot\omega_2)$ of the inphase and quadrature components of $\chi(t)$ and $\dot\chi(t)$, we can compute the corresponding joint PDF $p_{\chi_1\chi_2\dot\chi_1\dot\chi_2}(z_1,z_2,\dot z_1,\dot z_2)$ by taking the (complex conjugate of the) inverse Fourier transform of the joint CF $\Phi_{\chi_1\chi_2\dot\chi_1\dot\chi_2}(\omega_1,\omega_2,\dot\omega_1,\dot\omega_2)$ [16], i.e.,

$$p_{\chi_1\chi_2\dot\chi_1\dot\chi_2}(z_1,z_2,\dot z_1,\dot z_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d\omega_1\,d\omega_2\,d\dot\omega_1\,d\dot\omega_2$$
$$\times\, \Phi_{\chi_1\chi_2\dot\chi_1\dot\chi_2}(\omega_1,\omega_2,\dot\omega_1,\dot\omega_2)\frac{e^{-j(\omega_1 z_1+\omega_2 z_2+\dot\omega_1\dot z_1+\dot\omega_2\dot z_2)}}{(2\pi)^4}. \quad (10)$$

Substituting (4) in (10), doing some mathematical manipulations, and solving the resulting integrals using [19, Eq. (3.338-IV)] gives the joint PDF $p_{\chi_1\chi_2\dot\chi_1\dot\chi_2}(z_1,z_2,\dot z_1,\dot z_2)$ in the form

$$p_{\chi_1\chi_2\dot\chi_1\dot\chi_2}(z_1,z_2,\dot z_1,\dot z_2) = \frac{1}{(2\pi)^3}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^{\infty} dy\,d\dot\omega_1\,d\dot\omega_2$$
$$\times\, y\, J_0\left(y\sqrt{z_1^2+z_2^2}\right) g_1(y,\dot\omega_1,\dot\omega_2)\, e^{-j(\dot\omega_1\dot z_1+\dot\omega_2\dot z_2)} \quad (11)$$

where $g_1(\cdot,\cdot,\cdot)$ is defined in (12) and $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind [19].

The joint PDF $p_{\Xi\dot\Xi\Theta\dot\Theta}\left(x,\dot x,\theta,\dot\theta\right)$ of the envelope and the phase of K-parallel dual-hop relay fading channels can now easily be computed by transforming the random variables in (11) from rectangular coordinates $(z_1,z_2,\dot z_1,\dot z_2)$ to polar coordinates $\left(x,\dot x,\theta,\dot\theta\right)$, which results in

$$p_{\Xi\dot\Xi\Theta\dot\Theta}\left(x,\dot x,\theta,\dot\theta\right) = \frac{x^2}{(2\pi)^3}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^{\infty} dy\,d\dot\omega_1\,d\dot\omega_2\, y\, J_0(y x)$$
$$\times\, g_1(y,\dot\omega_1,\dot\omega_2)\, e^{-j\left(\dot\omega_1(\dot x\cos\theta - x\dot\theta\sin\theta)+\dot\omega_2(\dot x\sin\theta + x\dot\theta\cos\theta)\right)} \quad (13)$$

for $x\geq 0$, $|\dot x|<\infty$, $|\theta|\leq\pi$, and $|\dot\theta|<\infty$, where $g_1(\cdot,\cdot,\cdot)$ is given in (12). Finally, integrating (13) over the undesirable variables θ and $\dot\theta$ using [19, Eqs. (3.338-IV), (6.596-I), and (8.464-II)], allows us to express the joint PDF $p_{\Xi\dot\Xi}(x,\dot x)$ of the envelope $\Xi(t)$ and its time derivative $\dot\Xi(t)$ as

$$p_{\Xi\dot\Xi}(x,\dot x) = \frac{1}{\pi}\int_0^{\infty}\int_0^{\infty} dz\,dy\, y\, x J_0(y x)\cos(z\dot x)$$
$$\times\, g_1(y,z\cos\psi,z\sin\psi), \quad x\geq 0,\ |\dot x|<\infty. \quad (14)$$

$$\phi_{\varsigma_1^{(k)}\varsigma_2^{(k)}\dot\varsigma_1^{(k)}\dot\varsigma_2^{(k)}}(\omega_1,\omega_2,\dot\omega_1,\dot\omega_2) = \frac{K}{A_{\mathrm{R}(k)}^2\sigma_{\mu^{(2k-1)}}^2\sigma_{\mu^{(2k)}}^2(\omega_1^2+\omega_2^2)+\left\{\sqrt{K}+\sigma_{\mu^{(2k-1)}}^2\beta_{\mu^{(2k)}}(\dot\omega_1^2+\dot\omega_2^2)\right\}\left\{\sqrt{K}+A_{\mathrm{R}(k)}^2\sigma_{\mu^{(2k)}}^2\beta_{\mu^{(2k-1)}}(\dot\omega_1^2+\dot\omega_2^2)\right\}}.$$
$$(8)$$

$$g_1(y,\dot\omega_1,\dot\omega_2) = \prod_{k=1}^{K}\frac{K}{A_{\mathrm{R}(k)}^2\sigma_{\mu^{(2k-1)}}^2\sigma_{\mu^{(2k)}}^2 y^2+\left\{\sqrt{K}+\sigma_{\mu^{(2k-1)}}^2\beta_{\mu^{(2k)}}(\dot\omega_1^2+\dot\omega_2^2)\right\}\left\{\sqrt{K}+A_{\mathrm{R}(k)}^2\sigma_{\mu^{(2k)}}^2\beta_{\mu^{(2k-1)}}(\dot\omega_1^2+\dot\omega_2^2)\right\}}. \quad (12)$$

Substituting (14) in (3) results in the following final expression for the LCR $N_\Xi(r)$ of the envelope $\Xi(t)$

$$
N_\Xi(r) = \frac{1}{\pi} \int\limits_0^\infty \int\limits_0^\infty \int\limits_0^\infty y\, r\, \dot{x}\, J_0(y\, r)\cos(z\dot{x})
$$
$$
\times\, g_1(y, z\cos\psi, z\sin\psi)\, d\dot{x}\, dz\, dy. \tag{15}
$$

For i.i.d. complex double Gaussian random processes $\varsigma^{(k)}(t)$ $(k = 1, 2, \ldots, K)$, the integral over the variable y in (15) can be solved using [19, Eq. (6.565-IV)]. In this case, the expression of the LCR $N_\Xi(r)$ in (15) reduces to

$$
N_\Xi(r) = \frac{1}{\pi}\cdot\frac{K^K}{2^{K-1}(K-1)!}\cdot\frac{r^K}{\left(A_R\sigma_{\mu^{(1)}}\sigma_{\mu^{(2)}}\right)^{(K+1)}}
$$
$$
\times \int\limits_0^\infty \int\limits_0^\infty \frac{\dot{x}\cos(z\dot{x})}{(g_2(z))^{(K-1)}}\, K_{K-1}\left(\frac{r\, g_2(z)}{A_R\sigma_{\mu^{(1)}}\sigma_{\mu^{(2)}}}\right) d\dot{x}\, dz \tag{16}
$$

where $(\cdot)!$ denotes the factorial, $K_{K-1}(\cdot)$ is the $(K-1)$th-order modified Bessel function of the second kind [19], and

$$
g_2(z) = \sqrt{\left\{\sqrt{K} + \sigma_{\mu^{(1)}}^2\beta_{\mu^{(2)}}z^2\right\}\left\{\sqrt{K} + A_R^2\sigma_{\mu^{(2)}}^2\beta_{\mu^{(1)}}z^2\right\}}. \tag{17}
$$

B. Derivation of the ADF

The ADF $T_{\Xi_-}(r)$ of the envelope $\Xi(t)$ is the expected value of the time intervals over which the process $\Xi(t)$ remains below a certain threshold level r. The ADF $T_{\Xi_-}(r)$ can be calculated by means of [20]

$$
T_{\Xi_-}(r) = \frac{F_{\Xi_-}(r)}{N_\Xi(r)} \tag{18}
$$

where $F_{\Xi_-}(r)$ is the CDF and $N_\Xi(r)$ is the LCR of $\Xi(t)$.

The CDF $F_{\Xi_-}(r)$ of the envelope $\Xi(t)$ is the probability that $\Xi(t)$ remains below the threshold level r (see, e.g., [16]). The PDF $p_\Xi(x)$ of the envelope $\Xi(t)$ required to compute the CDF $F_{\Xi_-}(r)$ can be obtained by integrating (14) over $\dot{x}$ in the interval $(-\infty, \infty)$. With the help of the resulting PDF $p_\Xi(x)$, the final expression of the CDF $F_{\Xi_-}(r)$ of the envelope $\Xi(t)$ can be given as

$$
F_{\Xi_-}(r) = 1 - \frac{2}{\pi}\int\limits_0^\infty \int\limits_0^\infty \int\limits_0^\infty \int\limits_r^\infty y\, x\, J_0(y\, x)\cos(z\dot{x})
$$
$$
\times\, g_1(y, z\cos\psi, z\sin\psi)\, dx\, d\dot{x}\, dz\, dy. \tag{19}
$$

Under the assumption that $\varsigma^{(k)}(t)$ $(k = 1, 2, \ldots, K)$ are i.i.d. complex double Gaussian processes, the product on the right-hand-side in (12) becomes $(\cdot)^K$. Then solving the integral over the variables x and y in (19) allows us to write the CDF $F_{\Xi_-}(r)$ of the envelope $\Xi(t)$ as

$$
F_{\Xi_-}(r) = 1 - \frac{2}{\pi}\cdot\frac{K^K}{2^{K-1}(K-1)!}\cdot\frac{r^K}{\left(A_R\sigma_{\mu^{(1)}}\sigma_{\mu^{(2)}}\right)^K}
$$
$$
\times \int\limits_0^\infty \int\limits_0^\infty \frac{\cos(z\dot{x})}{(g_2(z))^K}\, K_K\left(\frac{r\, g_2(z)}{A_R\sigma_{\mu^{(1)}}\sigma_{\mu^{(2)}}}\right) d\dot{x}\, dz \tag{20}
$$

where $g_2(\cdot)$ is given in (17). It can be shown that (20) reduces to the following closed-form expression

$$
F_{\Xi_-}(r) = 1 - \frac{K^{\frac{K}{2}}\, r^K\, K_K\left(r\sqrt{K}/(A_R\sigma_{\mu^{(1)}}\sigma_{\mu^{(2)}})\right)}{2^{K-1}(K-1)!\left(A_R\sigma_{\mu^{(1)}}\sigma_{\mu^{(2)}}\right)^K}. \tag{21}
$$

The proof of (21) is omitted here for reasons of brevity. However, in the next section, we present numerical results to illustrate that (20) and (21) are equivalent.

IV. NUMERICAL RESULTS

The purpose of this section is to illustrate the theoretical expressions presented in Section III and to validate their correctness with the help of simulations. In this paper, the underlying uncorrelated Gaussian noise processes making up the overall K-parallel dual-hop relay channel are simulated using the sum-of-sinusoids (SOS) concept [18]. The model parameters of the channel simulator were computed by using the generalized method of exact Doppler spread (GMEDS$_1$) [21]. Each Gaussian process $\mu^{(i)}(t)$ $(i = 1, 2, \ldots, 2K)$ was simulated using $N_1^{(i)} = 14 + k$ and $N_2^{(i)} = 14 + k$ for $i = 1, 2, \ldots, 2K$ and $k = 1, 2, \ldots, K$, where $N_1^{(i)}$ and $N_2^{(i)}$ are the number of sinusoids required to simulate the inphase and quadrature components of $\mu^{(i)}(t)$, respectively. The maximum Doppler frequencies caused by the motion of the source mobile station and the destination mobile station, denoted by $f_{S_{max}}$ and $f_{D_{max}}$, respectively, were both set to 91 Hz. Whereas, the maximum Doppler frequencies $f_{R_{max}^{(k)}} = f_{R_{max}}$ caused by the motion of K mobile relays were selected from the set $\{91\text{ Hz}, 125\text{ Hz}, 210\text{ Hz}\}$. The variances of the underlying Gaussian processes are assumed to be equal, i.e., $2\sigma_{\mu^{(i)}}^2 = 2\sigma_\mu^2$ $\forall\; i = 1, 2, \ldots, 2K$. The relay gains associated with the subchannels are also equal, i.e., $A_{R^{(k)}} = A_R$ $\forall\; k = 1, 2, \ldots, K$ unless stated otherwise.

The results presented in Figs. 2–4 show a good fitting of the analytical and simulation results. Figure 2 illustrates the LCR $N_\Xi(r)$ of the envelope $\Xi(t)$ given in (16). It can be seen from Fig. 2 that keeping the relay gains $A_{R^{(k)}}$ and the maximum Doppler frequencies $f_{R_{max}^{(k)}}$ associated with K mobile relays constant, the LCR $N_\Xi(r)$ decreases with the increase in the number of mobile relays in the network. The effect of the maximum Doppler frequencies $f_{R_{max}^{(k)}}$ associated with K mobile relays is noticeable in Fig. 2.
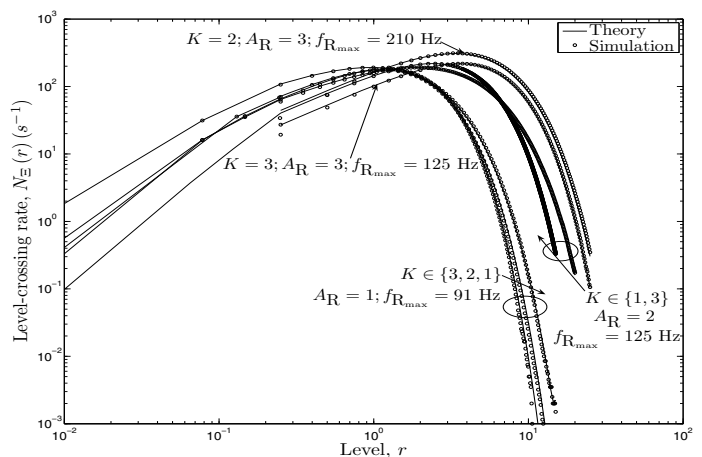

Fig. 2. The LCR $N_\Xi(r)$ of the envelope $\Xi(t)$ of K-parallel dual-hop relay M2M fading channels.

In Fig. 3, the theoretical results of the CDF $F_{\Xi_-}(r)$ of the envelope $\Xi(t)$ described by (20) and (21) are presented. The presented results validate our claim that (20) and (21) are

equivalent. The ADF $T_{\Xi_-}(r)$ of the envelope $\Xi(t)$ described by (18) is evaluated along with the simulation results in Fig. 4. Studying the results presented in Fig. 4 reveals that the ADF $T_{\Xi_-}(r)$ increases at high values of r with the increase in the number of mobile relays keeping the corresponding relay gains $A_{\mathrm{R}^{(k)}}$ and the maximum Doppler frequencies $f_{\mathrm{R}_{\max}^{(k)}}$ constant. At low values of r, however, the ADF $T_{\Xi_-}(r)$ decreases with an increase in the number of mobile relays. We can further deduce from Fig. 4 that the relay gains $A_{\mathrm{R}^{(k)}}$ and the maximum Doppler frequencies $f_{\mathrm{R}_{\max}^{(k)}}$ influence the ADF $T_{\Xi_-}(r)$ of the envelope $\Xi(t)$ significantly.
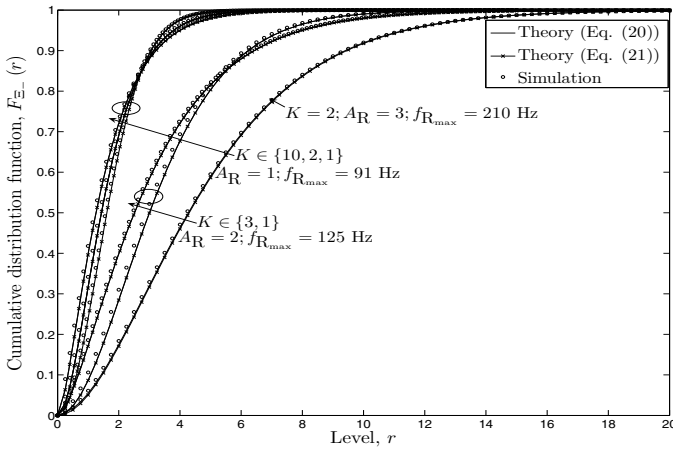

Fig. 3. The CDF $F_{\Xi_-}(r)$ of the envelope $\Xi(t)$ of K-parallel dual-hop relay M2M fading channels.
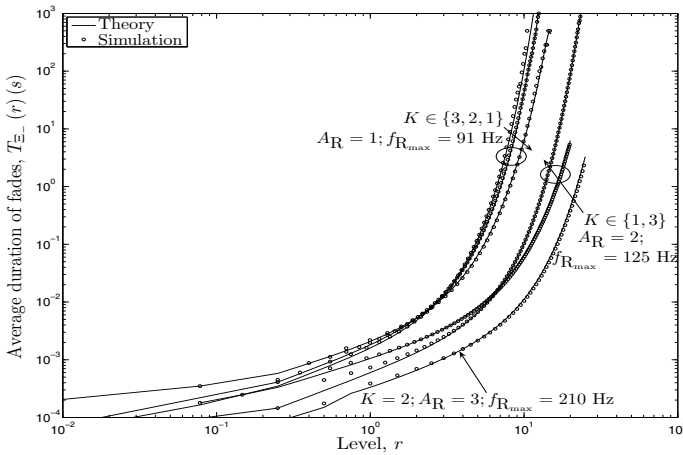

Fig. 4. The ADF $T_{\Xi_-}(r)$ of the envelope $\Xi(t)$ of K-parallel dual-hop relay M2M fading channels.

V. CONCLUSION

In this paper, we have studied the fading behavior of K-parallel dual-hop relay frequency non-selective M2M fading channels under NLOS propagation conditions in isotropic scattering environments. The fading behavior of such M2M fading channels is analyzed by evaluating the LCR and the ADF of the received signal envelope. Here, we presented analytical integral expressions of these quantities along with the CDF associated with the envelope. These expressions are derived assuming that the underlying processes making up such channels are independent but not necessarily identical as well as if they are i.i.d. processes. The theoretical results are validated by simulations. The results presented show that the number of mobile relays K in the network, the relay gains $A_{\mathrm{R}^{(k)}}$, and the maximum Doppler frequencies $f_{\mathrm{R}_{\max}^{(k)}}$ associated with the K mobile relays significantly influence the LCR, the ADF, and the CDF of the received signal envelope of K-parallel dual-hop relay M2M fading channels. These results are of importance to the designers of dual-hop/ multi-hop cooperative multi-relay systems.

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