

A Geometrical Channel Model for MIMO Mobile-to-Mobile Fading Channels in Cooperative Networks

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Abstract—This paper deals with the modeling and analysis of narrowband multiple-input multiple-output (MIMO) mobile-to-mobile (M2M) fading channels in relay-based cooperative networks. Non-line-of-sight (NLOS) propagation conditions are assumed in the transmission links from the source mobile station to the destination mobile station via the mobile relay. A stochastic narrowband MIMO M2M reference channel model is derived from the geometrical three-ring scattering model, where it is assumed that an infinite number of local scatterers surround the source mobile station, the mobile relay, and the destination mobile station. The complex channel gains associated with the new reference channel model are derived and their temporal as well as spatial correlation properties are explored. General analytical solutions are presented for the four-dimensional (4-D) space-time cross-correlation function (CCF), the three-dimensional spatial CCF, the two-dimensional (2-D) transmit (relay, receive) correlation function (CF), and the temporal autocorrelation function (ACF). Closed-formed expressions for different CCFs under isotropic scattering conditions are also provided in this paper. The proposed reference channel model can be used as a starting point to develop stochastic and deterministic channel simulators. Such channel simulators are not only important for analyzing the dynamic behavior of the MIMO M2M channel capacity but also for the development of future MIMO M2M cooperative communication systems.

Keywords—Amplify-and-forward relay links, mobile-to-mobile fading channels, MIMO channels, space-time correlation function.

I. INTRODUCTION

Recent attempts at combating multipath fading effects along with providing increased mobility support have resulted in the emergence of M2M communication systems in cooperative networks. The use of cooperative diversity protocols [1]–[4] improves the transmission link quality and the end-to-end system's throughput, whereas M2M communication on the other hand, extends the network range/coverage area. The fundamental idea of operation in cooperative networks is to allow mobile stations in the network to relay signals to the final destination. The development and performance investigation of such seemingly simple cooperative networks require a thorough understanding of the M2M fading channel

characteristics. For this reason, there is a need of simple yet efficient M2M fading channel models, providing us with a detailed knowledge about the statistical characterization of the channels.

The idea of introducing M2M communication in non-cooperative networks can be traced back to the work of Akki and Haber [5], [6], which deals with the study of the statistical properties of narrowband single-input single-output (SISO) M2M fading channels under NLOS propagation conditions. Recently, several papers dealing with M2M communication in cooperative networks can also be found in the literature. In various studies pertaining to M2M fading channels in relay-based cooperative networks under NLOS propagation conditions, it has been shown that a double Rayleigh process is the simplest and a well-suited statistical channel model for such channels [7], [8]. The analysis of experimental measurement data for outdoor-to-indoor M2M fading channels presented in [9] verifies the existence of double Rayleigh processes in real-world environments. The authors of [10] have extended the double Rayleigh channel model to the double Rice channel model for LOS propagation environments. Furthermore, several other realistic M2M fading channel models based on the multiple scattering concept [11] are available in the literature for both NLOS and LOS propagation environments [12]–[14]. The M2M fading channel models for relay-based cooperative networks developed so far are for narrowband SISO fading channels. Meaning thereby, the source mobile station, the mobile relay, and the destination mobile station are equipped with only one antenna. However, it is a well-established fact that the gains in terms of channel capacity are larger for MIMO channels as compared to SISO channels [15], [16]. Thus, there is a need to extend the SISO M2M fading channel models to MIMO M2M fading channel models in order to investigate channel capacity and the system performance of cooperative networks.

Another area that requires further attention is the development of simulation models for MIMO M2M fading channels in cooperative networks. Some few techniques for simulating narrowband SISO M2M fading channels in non-

cooperative networks have been proposed in [17]. Various studies have revealed that geometrical channel models are a good starting point for deriving simulation models for MIMO channels. Based on the geometrical two-ring channel models for MIMO fixed-to-mobile (F2M) and/or fixed-to-fixed (F2F) channels [18], [19], the authors of [20] proposed reference and simulation models for narrowband MIMO M2M fading channels in non-cooperative networks. Extensions of MIMO M2M reference and simulation models under LOS propagation conditions have been reported in [21]. In addition to 2-D channel models, 3-D MIMO M2M channel models based on geometrical cylinders can be found in [22]. However, to the best of the authors' knowledge, geometrical channel models for MIMO M2M communication systems in cooperative networks are an unexplored area. This in turn results in a lack of investigation of proper reference and simulation models derived from such geometrical channel models for MIMO M2M fading channels.

Motivated by the discussion presented above, we are addressing in this paper, modeling and simulation approaches for MIMO M2M fading channels in amplify-and-forward relay type cooperative networks. We propose a new reference model for MIMO M2M fading channels in cooperative networks and explore its temporal as well as spatial correlation properties. The scattering environment around the source mobile station, the mobile relay, and the destination mobile station is modeled by a geometrical three-ring scattering model. The proposed geometrical three-ring scattering model is an extension of the geometrical two-ring scattering model presented in [20], where the source mobile station and the destination mobile station are surrounded by rings of scatterers. However, in the proposed extension of the two-ring model, we have a separate ring of scatterers around the mobile relay along with two rings around the source mobile station and the destination mobile station. The reference model is proposed for MIMO M2M fading channels under NLOS propagation conditions. Furthermore, it is assumed that the direct transmission link between the source mobile station and the destination mobile station is blocked by obstacles. We present closed-form expressions for the correlation functions describing the proposed reference model under isotropic scattering conditions.

This article has the following structure: Section II introduces briefly the geometrical three-ring scattering model describing the transmission link from the source mobile station to the destination mobile station via the mobile relay. Based on the geometrical three-ring scattering model, we develop the reference model for MIMO M2M fading channels and study its correlation properties in Section III. Section IV deals with the derivation of closed-form expressions for the correlation functions describing the reference model under isotropic scattering conditions. Section V illustrates the important theoretical results by evaluating the closed-form expressions of the correlation functions presented in Section IV. Finally, concluding remarks are given in Section VI.

II. THE GEOMETRICAL THREE-RING SCATTERING MODEL

In this section, we extend the geometrical two-ring scattering model proposed in [20] to a geometrical three-ring scattering model for narrowband MIMO M2M fading channels in amplify-and-forward relay type cooperative networks. For ease of analysis, we have considered a $2 \times 2 \times 2$ antenna configuration, meaning thereby, the source mobile station, the mobile relay, and the destination mobile station are equipped with two antennas each. For simplicity, we have assumed non-line-of-sight (NLOS) propagation conditions in all the transmission links. It is also assumed that there is no direct transmission link from the source mobile station to the destination mobile station.

Due to high path loss, the contribution of signal power from remote scatterers to the total received power is usually negligible. Therefore, in the proposed three-ring scattering model, we have only assumed local scattering. A total number of M local scatterers around the source mobile station, denoted by $S_S^{(m)}$ ($m = 1, 2, 3, \dots, M$), are positioned on a ring of radius R_S , whereas N local scatterers $S_D^{(n)}$ ($n = 1, 2, 3, \dots, N$) lie around the destination mobile station on a separate ring of radius R_D . Furthermore, the local scatterers $S_R^{(k)}$ ($k = 1, 2, 3, \dots, K$) and $S_R^{(l)}$ ($l = 1, 2, 3, \dots, L$) are located on a third ring of radius R_R around the mobile relay. The number of local scatterers around the mobile relay is $K = L$. It should be pointed out here that $S_R^{(k)} = S_R^{(l)}$ for $k = l$. Throughout this paper, the subscripts S, R, and D represent the source mobile station, the mobile relay, and the destination mobile station. As can be seen from Fig. 1, the symbol $\phi_S^{(m)}$ denotes the angle of departure (AOD) of the m th transmitted wave by the source mobile station, whereas $\phi_D^{(n)}$ represents the angle of arrival (AOA) of the n th received wave at the destination mobile station. Furthermore, the symbols $\phi_{S-R}^{(k)}$ and $\phi_{R-D}^{(l)}$ correspond to the AOA of the k th received wave and the AOD of the l th transmitted wave at the mobile relay, respectively. The mobile relay is positioned at a distance D_{S-R} and an angle γ_S with respect to the source mobile station. While the location of the mobile relay seen from the destination mobile station can be specified by the distance D_{R-D} and the angle γ_D . Furthermore, the source mobile station and the destination mobile station are a distance D_{S-D} apart from each other. It is assumed that the inequalities $\max\{R_S, R_R\} \ll D_{S-R}$, $\max\{R_R, R_D\} \ll D_{R-D}$, and $\max\{R_S, R_D\} \ll D_{S-D}$ hold. The inter-element spacings at the source mobile station, the mobile relay, and the destination mobile station antenna arrays are labeled as δ_S , δ_R , and δ_D , respectively, where it is assumed that these quantities are smaller than the radii R_S , R_R , and R_D , i.e., $\max\{\delta_S, \delta_R, \delta_D\} \ll \min\{R_S, R_R, R_D\}$. With respect to the x -axis, the symbols β_S , β_R , and β_D describe the tilt angle of the antenna arrays at the source mobile station, the mobile relay, and the destination mobile station, respectively. It is further assumed that the source mobile station (mobile relay, destination mobile station) moves with speed v_S (v_R , v_D) in the direction determined by the angle of motion α_S (α_R , α_D).

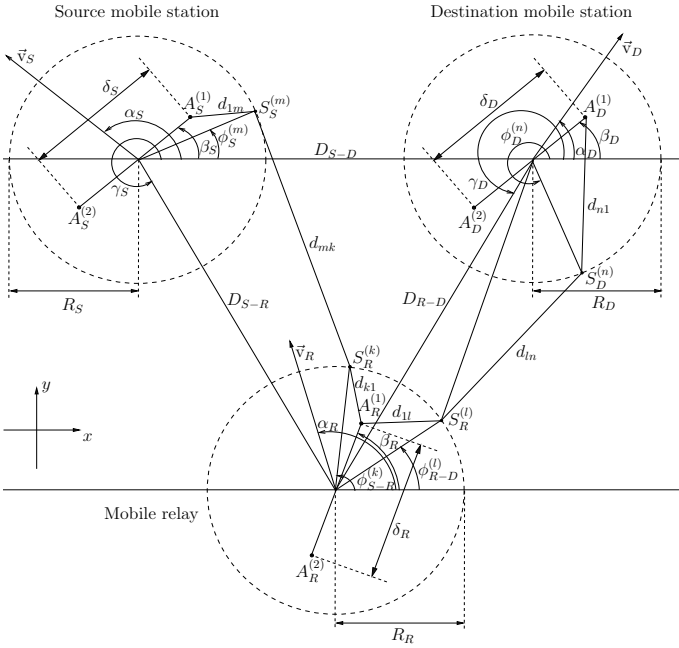


Fig. 1. The geometrical three-ring scattering model for two $2 \times 2 \times 2$ MIMO channels with local scatterers around the source mobile station, the mobile relay, and the destination mobile station.

III. THE REFERENCE MODEL

A. Derivation of the Reference Model

In this section, we develop a reference model for MIMO M2M fading channels in cooperative networks using the geometrical three-ring scattering model shown in Fig. 1. Ignoring the geometrical details, Fig. 1 can be simplified to Fig. 2, in order to understand the overall MIMO channel from the source mobile station to the destination mobile station via the mobile relay. Figure 2 shows that the complete system

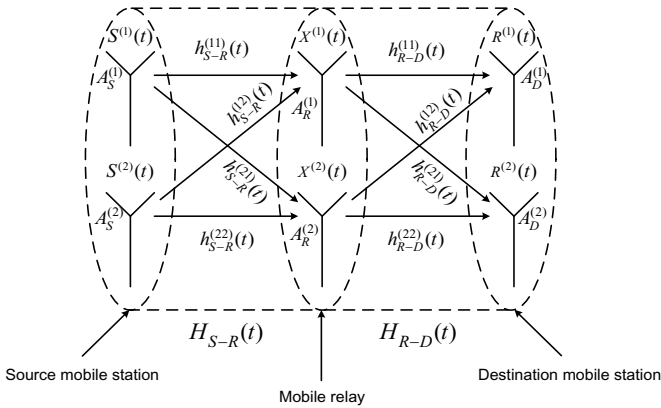


Fig. 2. A simplified diagram describing the overall MIMO channel from the source mobile station to the destination mobile station via the mobile relay.

can be separated into two 2×2 MIMO subsystems. One of the MIMO subsystems (comprising the source mobile station and mobile relay) is denoted by the S-R MIMO subsystem. While the other MIMO subsystem (consisting of the mobile

relay and the destination mobile station) is termed as the R-D MIMO subsystem. The input-output relation for the S-R MIMO subsystem can be expressed as

$$\mathbf{X}(t) = \mathbf{H}_{S-R}(t)\mathbf{S}(t) + \mathbf{N}_R(t) \quad (1)$$

where $\mathbf{X}(t) = [X^{(1)}(t) X^{(2)}(t)]^T$ is a 2×1 received signal vector at the mobile relay, $\mathbf{S}(t) = [S^{(1)}(t) S^{(2)}(t)]^T$ is a 2×1 signal vector transmitted by the source mobile station, and $\mathbf{N}_R(t) = [N_R^{(1)}(t) N_R^{(2)}(t)]^T$ is a 2×1 additive white Gaussian noise (AWGN) vector. In (1), $\mathbf{H}_{S-R}(t)$ is a 2×2 channel matrix, which models the M2M fading channel between the source mobile station and the mobile relay. $\mathbf{H}_{S-R}(t)$ can be expressed as

$$\mathbf{H}_{S-R}(t) = \begin{pmatrix} h_{S-R}^{(11)}(t) & h_{S-R}^{(12)}(t) \\ h_{S-R}^{(21)}(t) & h_{S-R}^{(22)}(t) \end{pmatrix}. \quad (2)$$

Here each $h_{S-R}^{(iq)}(t)$ ($i, q = 1, 2$) represents the diffuse component of the channel describing the transmission link from the source mobile station antenna element $A_S^{(q)}$ to the mobile relay antenna element $A_R^{(i)}$. Considering the geometrical three-ring scattering model shown in Fig. 1, it can be observed that the m th homogeneous plane wave emitted from $A_S^{(q)}$, first encounters the local scatterers $S_S^{(m)}$ around the source mobile station. Furthermore, before impinging on $A_R^{(i)}$, the plane wave is captured by the local scatterers $S_R^{(k)}$ around the mobile relay. It is worth mentioning here that the reference model is based on the assumption that the number of local scatterers, M and K , around the source mobile station and the mobile relay is infinite. The diffuse component $h_{S-R}^{(11)}(t)$ of the transmission link from $A_S^{(1)}$ to $A_R^{(1)}$ can be approximated as [20]

$$h_{S-R}^{(11)}(t) = \lim_{\substack{M \rightarrow \infty \\ K \rightarrow \infty}} \frac{1}{\sqrt{MK}} \sum_{m=1}^M \sum_{k=1}^K g_{mk} e^{j2\pi(f_S^{(m)} + f_{S-R}^{(k)})t} e^{j(\theta_{mk} + \theta_{S-R})} \quad (3)$$

with joint gains $1/\sqrt{MK}$ and joint phases θ_{mk} caused by the interaction of the local scatterers $S_S^{(m)}$ and $S_R^{(k)}$. The joint phases θ_{mk} are assumed to be independent and identically distributed (i.i.d.) random variables, each having a uniform distribution over the interval $[0, 2\pi]$. In (3)

$$g_{mk} = a_m b_k c_{mk} \quad (4a)$$

$$a_m = e^{j\frac{\pi}{\lambda} \delta_S \cos(\phi_S^{(m)} - \beta_S)} \quad (4b)$$

$$b_k = e^{j\frac{\pi}{\lambda} \delta_R \cos(\phi_{S-R}^{(k)} - \beta_R)} \quad (4c)$$

$$c_{mk} = e^{j\frac{2\pi}{\lambda} \{R_S \cos(\phi_S^{(m)} - \gamma_S) - R_R \cos(\phi_{S-R}^{(k)} - \gamma_S)\}} \quad (4d)$$

$$\theta_{S-R} = -\frac{2\pi}{\lambda} (R_S + D_{S-R} + R_R) \quad (4e)$$

$$f_S^{(m)} = f_{S\max} \cos(\phi_S^{(m)} - \alpha_S) \quad (4f)$$

$$f_{S-R}^{(k)} = f_{R\max} \cos(\phi_{S-R}^{(k)} - \alpha_R) \quad (4g)$$

where $f_{S\max} = v_S/\lambda$ ($f_{R\max} = v_R/\lambda$) is the maximum Doppler frequency caused by the motion of the source mobile station

(mobile relay). Furthermore, the symbol λ denotes the carrier's wavelength. The knowledge of the position of the mobile relay with respect to the source mobile station is incorporated in (4d). It should be pointed out here that in the reference model, the AOD $\phi_s^{(m)}$ and the AOA $\phi_{s-R}^{(k)}$ are independent random variables determined by the distribution of the local scatterers around the source mobile station and the mobile relay, respectively.

Replacing a_m and b_k by their complex conjugates a_m^* and b_k^* in (4b) and (4c), respectively, we can obtain the diffuse component $h_{s-R}^{(22)}(t)$ of the $A_s^{(2)} - A_R^{(2)}$ transmission link [20]. The diffuse components $h_{s-R}^{(12)}(t)$ and $h_{s-R}^{(21)}(t)$ can be obtained likewise by substituting $a_m \rightarrow a_m^*$ and $b_k \rightarrow b_k^*$, respectively, in (3) [20].

Similarly, as can be seen from Fig. 2, the input-output relationship of the R-D MIMO subsystem can be written as

$$\mathbf{R}(t) = \mathbf{H}_{R-D}(t)\mathbf{X}(t) + \mathbf{N}_D(t) \quad (5)$$

where $\mathbf{R}(t) = [R^{(1)}(t) R^{(2)}(t)]^T$ is a 2×1 received signal vector at the destination mobile station, $\mathbf{X}(t) = [X^{(1)}(t) X^{(2)}(t)]^T$ is a 2×1 signal vector transmitted by the mobile relay, $\mathbf{H}_{R-D}(t)$ is a 2×2 R-D fading channel matrix, and $\mathbf{N}_D(t) = [N_D^{(1)}(t) N_D^{(2)}(t)]^T$ is a 2×1 AWGN vector. By referring to the previous discussion on the elements of the matrix $\mathbf{H}_{s-R}(t)$, one can easily show that the diffuse component $h_{R-D}^{(11)}(t)$ of the $A_R^{(1)} - A_D^{(1)}$ transmission link can be expressed as

$$h_{R-D}^{(11)}(t) = \lim_{\substack{L \rightarrow \infty \\ N \rightarrow \infty}} \frac{1}{\sqrt{LN}} \sum_{l=1}^L \sum_{n=1}^N g_{ln} e^{j2\pi(f_{R-D}^{(l)} + f_D^{(n)})t} e^{j(\theta_{ln} + \theta_{R-D})} \quad (6)$$

where the term $1/\sqrt{LN}$ and the symbol θ_{ln} correspond to the joint gains and joint phases, respectively, introduced by the local scatterers $S_R^{(l)}$ and $S_D^{(n)}$. Like the joint phases, θ_{mk} , θ_{ln} are also assumed to be i.i.d. random variables with a uniform distribution over the interval $[0, 2\pi]$. Furthermore, in (6)

$$g_{ln} = \nu_l \nu_n \omega_{ln} \quad (7a)$$

$$\nu_l = e^{j\frac{\pi}{\lambda} \delta_R \cos(\phi_{R-D}^{(l)} - \beta_R)} \quad (7b)$$

$$\nu_n = e^{j\frac{\pi}{\lambda} \delta_D \cos(\phi_D^{(n)} - \beta_D)} \quad (7c)$$

$$\omega_{ln} = e^{j\frac{2\pi}{\lambda} \{R_D \cos(\phi_D^{(n)} - \gamma_D) - R_R \cos(\phi_{R-D}^{(l)} - \gamma_D)\}} \quad (7d)$$

$$\theta_{R-D} = -\frac{2\pi}{\lambda} (R_R + D_{R-D} + R_D) \quad (7e)$$

$$f_{R-D}^{(l)} = f_{R_{\max}} \cos(\phi_{R-D}^{(l)} - \alpha_R) \quad (7f)$$

$$f_D^{(n)} = f_{D_{\max}} \cos(\phi_D^{(n)} - \alpha_D) \quad (7g)$$

where $f_{D_{\max}} = v_D/\lambda$ is the maximum Doppler frequency caused by the movement of the destination mobile station. The symbol γ_D in (7d) describes the position of the mobile relay with respect to the destination mobile station. Furthermore, the AOD $\phi_{R-D}^{(l)}$ and the AOA $\phi_D^{(n)}$ are independent random variables determined by the distribution of the local scatterers around the mobile relay and the destination mobile station, respectively.

One can show that the diffuse components $h_{R-D}^{(iq)}(t)$ ($i, q = 1, 2$) of the remaining transmission links from the mobile relay antenna element $A_R^{(q)}$ to the destination mobile station antenna element $A_D^{(i)}$ can easily be obtained from (6) [20].

Finally, substituting (1) in (5) allows us to identify the overall fading channel between the source mobile station and the destination mobile station, i.e.,

$$\begin{aligned} \mathbf{R}(t) &= \mathbf{H}_{R-D}(t)\mathbf{H}_{s-R}(t)\mathbf{S}(t) + \mathbf{H}_{R-D}(t)\mathbf{N}_R(t) + \mathbf{N}_D(t) \\ &= \mathbf{H}_{s-R-D}(t)\mathbf{S}(t) + \mathbf{N}_{R-D}(t) \end{aligned} \quad (8)$$

where $\mathbf{N}_{R-D}(t) = \mathbf{H}_{R-D}(t)\mathbf{N}_R(t) + \mathbf{N}_D(t)$ is the total noise of the system. Whereas $\mathbf{H}_{s-R-D}(t)$ describes completely the reference model of the proposed geometrical three-ring MIMO M2M fading channel and is defined as follows

$$\begin{aligned} \mathbf{H}_{s-R-D}(t) &= \mathbf{H}_{R-D}(t)\mathbf{H}_{s-R}(t) \\ &= \begin{pmatrix} h_{R-D}^{(11)}(t) & h_{R-D}^{(12)}(t) \\ h_{R-D}^{(21)}(t) & h_{R-D}^{(22)}(t) \end{pmatrix} \begin{pmatrix} h_{s-R}^{(11)}(t) & h_{s-R}^{(12)}(t) \\ h_{s-R}^{(21)}(t) & h_{s-R}^{(22)}(t) \end{pmatrix} \\ &= \begin{pmatrix} h_{s-R-D}^{(11)}(t) & h_{s-R-D}^{(12)}(t) \\ h_{s-R-D}^{(21)}(t) & h_{s-R-D}^{(22)}(t) \end{pmatrix}. \end{aligned} \quad (9)$$

Here $h_{s-R-D}^{(iq)}(t)$ ($i, q = 1, 2$) defines the diffuse component of the overall MIMO M2M fading channel, describing the transmission link from the source mobile station antenna element $A_s^{(q)}$ to the destination mobile station antenna element $A_D^{(i)}$ via the mobile relay antenna elements. Expanding (9) allows us to explicitly write the diffuse component $h_{s-R-D}^{(11)}(t)$ of the transmission link from the first antenna element at the source mobile station, $A_s^{(1)}$, to the first antenna element at the destination mobile station, $A_D^{(1)}$, as follows

$$\begin{aligned} h_{s-R-D}^{(11)}(t) &= h_{R-D}^{(11)}(t)h_{s-R}^{(11)}(t) + h_{R-D}^{(12)}(t)h_{s-R}^{(21)}(t) \\ &= \lim_{\substack{K \rightarrow \infty \\ L \rightarrow \infty \\ M \rightarrow \infty \\ N \rightarrow \infty}} \frac{1}{\sqrt{KLMN}} \sum_{k=1}^K \sum_{l=1}^L \sum_{m=1}^M \sum_{n=1}^N a_m \nu_n c_{mk} \omega_{ln} \{b_k \nu_l + b_k^* \nu_l^*\} \\ &\quad e^{j(\theta_{mk} + \theta_{ln} + \theta_{s-R} + \theta_{R-D})} e^{j2\pi(f_s^{(m)} + f_{s-R}^{(k)} + f_{R-D}^{(l)} + f_D^{(n)})t}. \end{aligned} \quad (10)$$

Note that the phases θ_{s-R} and θ_{R-D} in (10) can be set to zero without loss of generality, since the statistical properties of the reference model are not influenced by a constant phase shift. Similarly, the diffuse component $h_{s-R-D}^{(22)}(t)$ of the $A_s^{(2)} - A_D^{(2)}$ transmission link can be expressed as

$$\begin{aligned} h_{s-R-D}^{(22)}(t) &= h_{R-D}^{(21)}(t)h_{s-R}^{(12)}(t) + h_{R-D}^{(22)}(t)h_{s-R}^{(22)}(t) \\ &= \lim_{\substack{K \rightarrow \infty \\ L \rightarrow \infty \\ M \rightarrow \infty \\ N \rightarrow \infty}} \frac{1}{\sqrt{KLMN}} \sum_{k=1}^K \sum_{l=1}^L \sum_{m=1}^M \sum_{n=1}^N a_m^* \nu_n^* c_{mk} \omega_{ln} \{b_k \nu_l + b_k^* \nu_l^*\} \\ &\quad e^{j(\theta_{mk} + \theta_{ln} + \theta_{s-R} + \theta_{R-D})} e^{-j2\pi(f_s^{(m)} - f_{s-R}^{(k)} + f_{R-D}^{(l)} - f_D^{(n)})t}. \end{aligned} \quad (11)$$

Equations (10) and (11) will be used in the next subsection to calculate the 4-D space-time CCF.

B. Correlation Properties of the Reference Model

By definition, the 4-D space-time CCF between the transmission links $A_s^{(1)} - A_D^{(1)}$ and $A_s^{(2)} - A_D^{(2)}$ is equivalent to the correlation between the diffuse components $h_{s-r-d}^{(11)}(t)$ and $h_{s-r-d}^{(22)}(t)$, i.e., [23]

$$\rho_{11,22}(\delta_s, \delta_r, \delta_d, \tau) = E \left\{ h_{s-r-d}^{(11)}(t) h_{s-r-d}^{(22)*}(t + \tau) \right\} \quad (12)$$

where $E \{ \cdot \}$ is the expectation operator. It should be noticed here that the expectation operator is applied to all random variables, i.e., the constant phase shifts $\{\theta_{mk}, \theta_{ln}\}$, the AOD $\{\phi_s^{(m)}, \phi_{r-d}^{(l)}\}$, and the AOA $\{\phi_{s-r}^{(k)}, \phi_d^{(n)}\}$. Substituting (10) and (11) in (12), the 4-D space-time CCF can be expressed as

$$\rho_{11,22}(\delta_s, \delta_r, \delta_d, \tau) = \lim_{\substack{K \rightarrow \infty \\ L \rightarrow \infty \\ M \rightarrow \infty \\ N \rightarrow \infty}} \frac{1}{KLMN} \sum_{k=1}^K \sum_{l=1}^L \sum_{m=1}^M \sum_{n=1}^N E \left\{ a_m^2 v_n^2 (b_k v_l + b_k^* v_l^*)^2 e^{-j2\pi(f_s^{(m)} + f_{s-r}^{(k)} + f_{r-d}^{(l)} + f_d^{(n)})\tau} \right\}. \quad (13)$$

It is important to highlight here, the functions of random variables to which the expectation is applied. We can see that $\{a_m, f_s^{(m)}\}$ and $\{v_l, f_{r-d}^{(l)}\}$ are functions of AOD $\phi_s^{(m)}$ and $\phi_{r-d}^{(l)}$, respectively. While $\{v_k, f_{s-r}^{(k)}\}$ and $\{b_n, f_d^{(n)}\}$ are functions of AOA $\phi_{s-r}^{(k)}$ and $\phi_d^{(n)}$, respectively [20]. If the number of local scatterers approaches infinity, i.e., $(K, L, M, N \rightarrow \infty)$, then the discrete random variables $\phi_s^{(m)}$, $\phi_{s-r}^{(k)}$, $\phi_{r-d}^{(l)}$, and $\phi_d^{(n)}$ become continuous random variables ϕ_s , ϕ_{s-r} , ϕ_{r-d} , and ϕ_d , each of which is characterized by a certain distribution, denoted by $p_{\phi_s}(\phi_s)$, $p_{\phi_{s-r}}(\phi_{s-r})$, $p_{\phi_{r-d}}(\phi_{r-d})$, and $p_{\phi_d}(\phi_d)$, respectively, [20]. The infinitesimal power of the diffuse components corresponding to the differential angles $d\phi_s$, $d\phi_{s-r}$, $d\phi_{r-d}$, and $d\phi_d$ is proportional to $p_{\phi_s}(\phi_s)p_{\phi_{s-r}}(\phi_{s-r})p_{\phi_{r-d}}(\phi_{r-d})p_{\phi_d}(\phi_d)d\phi_s d\phi_{s-r}d\phi_{r-d}d\phi_d$. This implies that when the number of local scatterers approaches infinity, i.e., $(K, L, M, N \rightarrow \infty)$, the infinitesimal power of all the diffuse components becomes equal to $1/(KLMN)$, i.e.,

$$\frac{1}{KLMN} = p_{\phi_s}(\phi_s)p_{\phi_{s-r}}(\phi_{s-r})p_{\phi_{r-d}}(\phi_{r-d})p_{\phi_d}(\phi_d)d\phi_s d\phi_{s-r}d\phi_{r-d}d\phi_d. \quad (14)$$

Thus, we can write the 4-D space-time CCF $\rho_{11,22}(\delta_s, \delta_r, \delta_d, \tau)$ of the reference model given in (13) as

$$\rho_{11,22}(\delta_s, \delta_r, \delta_d, \tau) = \rho_s(\delta_s, \tau) \cdot \rho_r(\delta_r, \tau) \cdot \rho_d(\delta_d, \tau) \quad (15)$$

where

$$\rho_s(\delta_s, \tau) = \int_{-\pi}^{\pi} a^2(\delta_s, \phi_s) e^{-j2\pi f_s(\phi_s)\tau} p_{\phi_s}(\phi_s) d\phi_s, \quad (16)$$

$$\rho_r(\delta_r, \tau) = \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} d\phi_{s-r} d\phi_{r-d} p_{\phi_{s-r}}(\phi_{s-r}) p_{\phi_{r-d}}(\phi_{r-d}) e^{-j2\pi f_{s-r}(\phi_{s-r})\tau} e^{-j2\pi f_{r-d}(\phi_{r-d})\tau} \{b v^*(\delta_r, \phi_{s-r}, \phi_{r-d}) + b^* v(\delta_r, \phi_{s-r}, \phi_{r-d})\}^2, \quad (17)$$

and

$$\rho_d(\delta_d, \tau) = \int_{-\pi}^{\pi} v^2(\delta_d, \phi_d) e^{-j2\pi f_d(\phi_d)\tau} p_{\phi_d}(\phi_d) d\phi_d \quad (18)$$

are the CFs at the source mobile station, mobile relay, and the destination mobile station, and

$$a(\delta_s, \phi_s) = e^{j\frac{\pi}{\lambda} \delta_s \cos(\phi_s - \beta_s)} \quad (19a)$$

$$b v(\delta_r, \phi_{s-r}, \phi_{r-d}) = e^{j\frac{\pi}{\lambda} \delta_r \cos(\phi_{s-r} - \beta_r)} e^{j\frac{\pi}{\lambda} \delta_r \cos(\phi_{r-d} - \beta_r)} \quad (19b)$$

$$v(\delta_d, \phi_d) = e^{j\frac{\pi}{\lambda} \delta_d \cos(\phi_d - \beta_d)} \quad (19c)$$

$$f_s(\phi_s) = f_{s_{\max}} \cos(\phi_s - \alpha_s) \quad (19d)$$

$$f_{s-r}(\phi_{s-r}) = f_{r_{\max}} \cos(\phi_{s-r} - \alpha_r) \quad (19e)$$

$$f_{r-d}(\phi_{r-d}) = f_{r_{\max}} \cos(\phi_{r-d} - \alpha_r) \quad (19f)$$

$$f_d(\phi_d) = f_{d_{\max}} \cos(\phi_d - \alpha_d). \quad (19g)$$

In this article, we refer to the CF at the source mobile station $\rho_s(\delta_s, \tau)$ as the source CF. Similarly, the CF at the mobile relay (destination mobile station) $\rho_r(\delta_r, \tau)$ ($\rho_s(\delta_d, \tau)$) is termed as the relay CF (destination CF). Equation (15) illustrates that the 4-D space-time CCF $\rho_{11,22}(\delta_s, \delta_r, \delta_d, \tau)$ of the reference model can be expressed as the product of the source CF $\rho_s(\delta_s, \tau)$, the relay CF $\rho_r(\delta_r, \tau)$, and the destination CF $\rho_d(\delta_d, \tau)$. Besides, from (16) and (18) it turns out that the source mobile station and the destination mobile station are interchangeable.

The 3-D spatial CCF $\rho(\delta_s, \delta_r, \delta_d)$, defined as $\rho(\delta_s, \delta_r, \delta_d) = E \left\{ h_{s-r-d}^{(11)}(t) h_{s-r-d}^{(22)*}(t) \right\}$ is equal to the 4-D space-time CCF $\rho_{11,22}(\delta_s, \delta_r, \delta_d, \tau)$ of the reference model at $\tau = 0$, i.e.,

$$\begin{aligned} \rho(\delta_s, \delta_r, \delta_d) &= \rho_{11,22}(\delta_s, \delta_r, \delta_d, 0) \\ &= \rho_s(\delta_s, 0) \cdot \rho_r(\delta_r, 0) \cdot \rho_d(\delta_d, 0). \end{aligned} \quad (20)$$

Furthermore, the temporal autocorrelation function (ACF) $r_{h^{(iq)}}(\tau)$ of the diffuse component $h_{s-r-d}^{(iq)}(t)$ of the transmission link from the source mobile station antenna element $A_s^{(q)}$ to the destination mobile station antenna element $A_D^{(i)} \forall i, q \in (1, 2)$ can be given as

$$r_{h^{(iq)}}(\tau) := E \left\{ h_{s-r-d}^{(iq)}(t) h_{s-r-d}^{(iq)*}(t + \tau) \right\}. \quad (21)$$

It is not difficult to show that the temporal ACF $r_{h^{(iq)}}(\tau) \forall i, q \in (1, 2)$ is equal to the 4-D space-time CCF $\rho_{11,22}(\delta_s, \delta_r, \delta_d, \tau)$ of the reference model at $\delta_s = \delta_r = \delta_d = 0$, i.e.,

$$\begin{aligned} r_{h^{(iq)}}(\tau) &= \rho_{11,22}(0, 0, 0, \tau) \\ &= \rho_s(0, \tau) \cdot \rho_r(0, \tau) \cdot \rho_d(0, \tau). \end{aligned} \quad (22)$$

Substituting $\delta_s = 0$, $\delta_r = 0$, and $\delta_d = 0$ in (16), (17), and (18), respectively, gives $\rho_s(0, \tau)$, $\rho_r(0, \tau)$, and $\rho_d(0, \tau)$ as

$$\rho_s(0, \tau) = \int_{-\pi}^{\pi} e^{-j2\pi f_s(\phi_s)\tau} p_{\phi_s}(\phi_s) d\phi_s, \quad (23)$$

$$\begin{aligned} \rho_r(0, \tau) &= \int_{-\pi}^{\pi} \int_{-\pi}^{\pi} e^{-j2\pi f_{s-r}(\phi_{s-r})\tau} e^{-j2\pi f_{r-d}(\phi_{r-d})\tau} \\ &\quad p_{\phi_{s-r}}(\phi_{s-r}) p_{\phi_{r-d}}(\phi_{r-d}) d\phi_{s-r} d\phi_{r-d}, \end{aligned} \quad (24)$$

and

$$\rho_D(0, \tau) = \int_{-\pi}^{\pi} e^{-j2\pi f_D(\phi_D)\tau} p_{\phi_D}(\phi_D) d\phi_D, \quad (25)$$

respectively.

IV. ISOTROPIC SCATTERING SCENARIOS

In this section, the correlation properties of the reference model are studied under isotropic scattering conditions. Isotropic scattering around the source mobile station can be determined by uniformly distributed AOD ϕ_S over the interval $[0, 2\pi)$, i.e.,

$$p_{\phi_S}(\phi_S) = \frac{1}{2\pi}, \quad \phi_S \in [0, 2\pi). \quad (26)$$

Similarly, isotropic scattering around the mobile relay (destination mobile station) can be characterized by a uniform distribution of the AOA ϕ_{S-R} (ϕ_D) along with a uniform distribution of the AOD ϕ_{R-D} over $[0, 2\pi)$. Hence, the distributions $p_{\phi_{S-R}}(\phi_{S-R})$, $p_{\phi_{R-D}}(\phi_{R-D})$, and $p_{\phi_D}(\phi_D)$ can be obtained by replacing the index S by S-R, R-D, and D in (26), respectively. Substituting (26) in (16) and solving the integrals using [24, Eq. (3.338-4)] result in the following closed-form expression of the source CF $\rho_S(\delta_S, \tau)$, i.e.,

$$\rho_S(\delta_S, \tau) = J_0 \left(2\pi \sqrt{\left(\frac{\delta_S}{\lambda}\right)^2 + (f_{S_{\max}} \tau)^2} - 2\frac{\delta_S}{\lambda} f_{S_{\max}} \tau \cos(\alpha_S - \beta_S) \right) \quad (27)$$

where $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind [24]. Furthermore, the destination CF $\rho_D(\delta_D, \tau)$ can be obtained by replacing the index S by D in (27). Substituting $p_{\phi_{S-R}}(\phi_{S-R})$ and $p_{\phi_{R-D}}(\phi_{R-D})$ in (17) allows us to solve the integrals using [24, Eq. (3.338-4)] and write the relay CF $\rho_R(\delta_R, \tau)$ in a closed form as

$$\begin{aligned} \rho_R(\delta_R, \tau) &= 2 \{J_0(2\pi f_{R_{\max}} \tau)\}^2 \\ &+ 2 \left\{ J_0 \left(2\pi \sqrt{\left(\frac{\delta_R}{\lambda}\right)^2 + (f_{R_{\max}} \tau)^2} - 2\frac{\delta_R}{\lambda} f_{R_{\max}} \tau \cos(\alpha_R - \beta_R) \right) \right\}^2 \\ &+ 2 \left\{ J_0 \left(2\pi \sqrt{\left(\frac{\delta_R}{\lambda}\right)^2 + (f_{R_{\max}} \tau)^2} + 2\frac{\delta_R}{\lambda} f_{R_{\max}} \tau \cos(\alpha_R - \beta_R) \right) \right\}^2. \end{aligned} \quad (28)$$

Substituting (27), (28), and the closed-form expression of $\rho_D(\delta_D, \tau)$ in (15) results in the 4-D space-time CCF $\rho_{11,22}(\delta_S, \delta_R, \delta_D, \tau)$ of the reference model in a closed form given in (29). From (29), it follows that the 3-D spatial CCF $\rho(\delta_S, \delta_R, \delta_D)$ of the reference model is a product of four Bessel functions, i.e., $\rho(\delta_S, \delta_R, \delta_D) = \rho_{11,22}(\delta_S, \delta_R, \delta_D, 0) = J_0(2\pi\delta_S/\lambda) \{2J_0(2\pi\delta_R/\lambda)\}^2 J_0(2\pi\delta_D/\lambda)$. In the same way, the temporal ACF $r_{h^{(iq)}}(\tau)$ of the reference model can be written as $r_{h^{(iq)}}(\tau) = \rho_{11,22}(0, 0, 0, \tau) = J_0(2\pi f_{S_{\max}} \tau) \{2J_0(2\pi f_{R_{\max}} \tau)\}^2 J_0(2\pi f_{D_{\max}} \tau)$. A product of

two Bessel functions describes the 2-D spatial CCF and the temporal ACF of the reference model derived from a geometrical two-ring scattering model [20]. For the 3-D spatial CCF and the temporal ACF of the reference model derived from a geometrical three-ring scattering model, a product of four Bessel functions is justified. Since, the geometrical three-ring scattering model is a concatenation of two separate geometrical two-ring scattering models.

V. NUMERICAL RESULTS

The purpose of this section is to illustrate the important theoretical results of the reference and the stochastic simulation model CCFs by evaluating the expressions in (16) and (18). Here, we have discussed numerical results for source CFs and relay CFs. Whereas the results for destination CFs can easily be obtained from source CFs just by replacing the index S by D. The tilt angle β_S (β_R) of the source mobile station (mobile relay) antenna array, the angle of motion α_S (α_R) of the source mobile station (mobile relay), the maximum Doppler frequency $f_{S_{\max}}$ ($f_{R_{\max}}$) caused by the motion of the source mobile station (mobile relay) were selected to be $\beta_S = \beta_R = \pi/2$, $\alpha_S = \pi/4$, $\alpha_R = 0$, and $f_{S_{\max}} = f_{R_{\max}} = 91$ Hz, respectively. The wavelength λ was set to $\lambda = 0.15$ m.

Figures 3–4 present the numerical results in case of isotropic scattering. Figure 3 shows the shape of the source CF $\rho_S(\delta_S, \tau)$ determined by (27). Similarly, the shape of the relay CF $\rho_R(\delta_R, \tau)$ given by (28) is presented in Fig. 4.

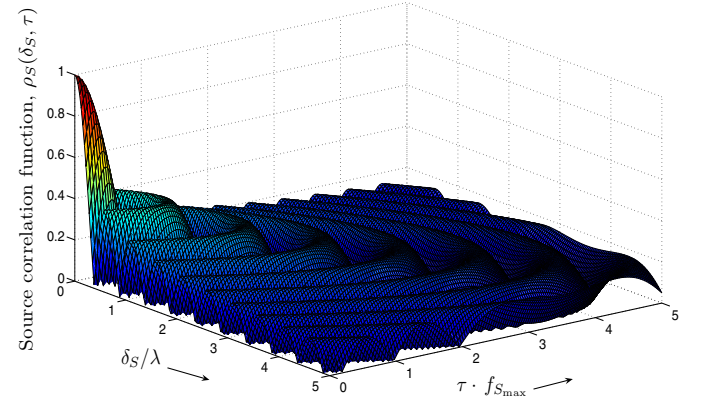


Fig. 3. The source CF $\rho_S(\delta_S, \tau)$ of the 2×2 MIMO M2M reference channel model under isotropic scattering conditions.

VI. CONCLUSION

In this paper, we have derived a reference model for narrow-band MIMO M2M fading channels in relay-based cooperative networks. The starting point for deriving the reference channel model was the geometrical three-ring scattering model, where it is assumed that the local scatterers are located on rings around the source mobile station, the mobile relay, and the destination mobile station. Moreover, the suggested three-ring scattering model came out to be a concatenation of two separate two-ring scattering models. General analytical formulas

$$\begin{aligned}
\rho_{11,22}(\delta_S, \delta_R, \delta_D, \tau) &= 2 \left[\{J_0(2\pi f_{R_{\max}} \tau)\}^2 \right. \\
&+ \left. \left\{ J_0 \left(2\pi \sqrt{\left(\frac{\delta_R}{\lambda}\right)^2 + (f_{R_{\max}} \tau)^2 - 2\frac{\delta_R}{\lambda} f_{R_{\max}} \tau \cos(\alpha_R - \beta_R)} \right) \right\}^2 + \left\{ J_0 \left(2\pi \sqrt{\left(\frac{\delta_R}{\lambda}\right)^2 + (f_{R_{\max}} \tau)^2 + 2\frac{\delta_R}{\lambda} f_{R_{\max}} \tau \cos(\alpha_R - \beta_R)} \right) \right\}^2 \right] \\
&\times J_0 \left(2\pi \sqrt{\left(\frac{\delta_S}{\lambda}\right)^2 + (f_{S_{\max}} \tau)^2 - 2\frac{\delta_S}{\lambda} f_{S_{\max}} \tau \cos(\alpha_S - \beta_S)} \right) J_0 \left(2\pi \sqrt{\left(\frac{\delta_D}{\lambda}\right)^2 + (f_{D_{\max}} \tau)^2 - 2\frac{\delta_D}{\lambda} f_{D_{\max}} \tau \cos(\alpha_D - \beta_D)} \right). \quad (29)
\end{aligned}$$

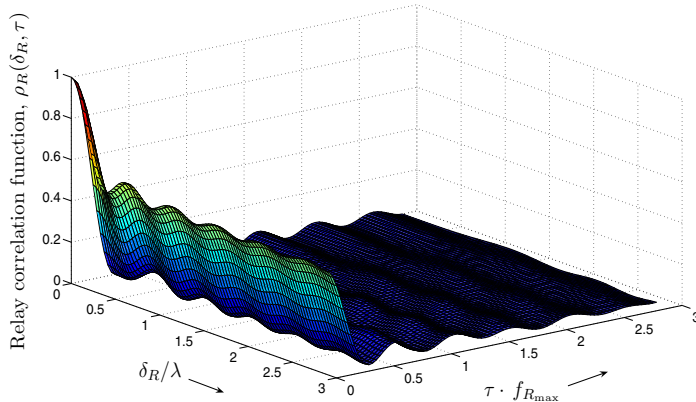


Fig. 4. The relay CF $\rho_R(\delta_R, \tau)$ of the $2 \times 2 \times 2$ MIMO M2M reference channel model under isotropic scattering conditions.

as well as closed-form expressions for the transmit (receive) CF and the relay CF specific to isotropic scattering have been presented. From the developed reference channel model, deterministic channel simulators can be derived. Such channel simulators are useful for analyzing the dynamic behavior of the MIMO channel capacity in relay-based M2M communication systems. In addition, the proposed geometrical three-ring scattering model for narrowband MIMO M2M fading channels can be considered as a starting point for the development and analysis of new channels models for wideband MIMO M2M fading channels in cooperative networks.

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