A study of Wavelets for edge detection of anatomical structure on medical images

by

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Abstract

Osteoporosis is a bone disease that makes your bone crumble away. To find out how thick your bone structure is, medical personnel uses x-ray images and measure the thickness of your bones. Today this is done manually by the use of a ruler. In this thesis we have examined the use of Wavelets for edge detection in order to find a better and smarter way of measuring the bone thickness digitally.

Wavelets are a relatively new mathematical method that allows you to split up and examine a signal. Three different Mother Wavelets; Haar, Mexican Hat and Morlet were examined to find the one that was best suited for finding edges. Tests have been performed involving the different Mother Wavelets. They all have their own qualities, and are to some extent able to detect edges, but Mexican Hat proved to be the best.

We came up with our own theory for how to use Mexican Hat to detect the edges of objects in images. The theory was named "Champagne glass" theory, and it tells you something about which scaling you need to use in order to best find the edges.

The task was to find the edges of bone structure on x-ray images. With our methods we were able to find the outer edges of the bone structure and to some extent the inner edges. The problem with the inner edges is that they are very diffuse, and it is hard to state where the edge actually is.

Preface

This is a master thesis in Information and Communication Technology at Agder University College. It is the final assignment and it lasts for one semester.

Our employer is Sørlandet Hospital, Kristiansand, and Dr. Glenn Haugeberg. He has been helpful when writing title and definition for this thesis. For medical support his knowledge has been helpful. His information was also helpful for us to understand where and how our results might be used.
Our teaching supervisor, Professor Per Henrik Hogstad, has been a great resource to us. He was the one who introduced us to the assignment. His inspiration and motivation had a positive effect on us, and was important for our decision to undertake this assignment. Introduction to the subject was also given by Prof Hogstad during a series of lectures. Results obtained during the study, was always discussed with Prof Hogstad. New tasks to solve and methods to use were discussed before further action was taken.

A CD containing our Java application and MathCAD program is attached to this report. This report and some images are also on the CD.

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1 Introduction

X-ray images are often taken to determine whether a person has a broken bone or not, and from the image it is easy to answer yes or no. But x-ray images are not only taken to evaluate whether a person has broken a bone or not. Many people suffer from osteoporosis, and medical personnel can use x-ray images to determine the bone mineral density or how much bone structure is left [4] [8]. The human bone is hollow and could be considered a tube. For people with osteoporosis the bone structure is decaying from inside the bone making it hollower. When medical personnel are looking at x-ray images they can determine how much bone structure that is left and how much is decaying [6]. Today a normal way to measure the thickness of the bone is to look at the image and use a ruler.

Measuring the bone thickness with a ruler is an old fashioned way in these days of technology, but there has not been a better alternative. One way to measure the bone thickness with the help of computer, would be to measure the pixel values. The pixel value would be a lot higher where the bone is compared to the rest of the image. A problem with this technique is that there might be a lot of noise on the image. As mentioned before the bone can be looked at as a tube. The outside is sharp and smooth and the edges are easy to find, and therefore easy to start measuring. The inside however is very rugged, and hard to find a smooth line to measure. This is what makes is really hard to determine the thickness of the bone.

With the help of a mathematical function called Wavelets we will examine whether it is possible to determine the edges on the bone, and thereby be able to find the thickness of the bone.

Wavelets is a rather new science compared to more traditional sciences, but it is gaining more and more followers as the opportunities for possible use reveal itself. To learn more about Wavelets you can look at Amaras Introduction to Wavelets [10]. Wavelets also have a forum [15]. For more information about the use for Wavelets for edge detection in medical images, you can look at former papers written at Agder University College [14]. There is also some information at Professor Hogstads web pages [12].

In this report you will first be given a short introductory to signal analyzes, what Wavelets is and what it can do. Different Mother Wavelets will be introduced together with theories for how edge detection might be possible. Tests will be conducted on these different theories. Discussions will be made in order to enlighten certain areas, and to provide positive and negative sides. In the end there will be a conclusion to whether or not it is possible to detect the edges and if so what method that worked best.

1.1 Premises

Both of us finished bachelor of computer science at Agder University College in Grimstad. This is our finale thesis to complete the master education at the same University College.

The thesis requires groundings in mathematics and Java programming. Neither of us has any experience using Wavelets. This has never been a part of our education. Because of this lack of knowledge, we will get lectures by University Science teacher Prof Hogstad. Refreshing our knowledge about Fourier-transformation would also be of interest when learning about Wavelets. This will probably ease our learning, and make our learning-curve steeper. Simultaneously he will demonstrate an application for us to implement our solution. This requires some knowledge about Java. We have both some experience using Java during the education the first three years. In addition, one of us has some experience from later projects and personal use.

Medically, we have no previous knowledge. Thus, for medically inspiration and sustentation, Glenn Haugeberg is our teaching supervisor, and will be available for guidance.

1.2 Problem definition

In images a lot of information is hidden to the naked eye, but the information is still there. X-ray images (perhaps other images) show the bone structure in the body. The thickness of a bone is related to bone mass and is hard to determine quantitatively for a human. An approach to solve this problem is the use of Wavelets. This assignment will concentrate on using different types of Wavelets that can be used to determine edges on x-ray (perhaps other) of the bone. This information should then be used for measurements for the thickness of the cortical bone, the amount of porosity and eventually also to determine bone loss related to development of erosions around the joint in patients with rheumatoid arthritis.

If time permits, a closer look at finding edges in blood veins in the liver would be of interest, since this will have a similar approach.

2 Signal analysis

2.1 Overview

According to Oxford dictionary, a signal is a sequence of electronic impulses or radio waves transmitted or received. So a signal can be anything that can be looked at in the form of waves and, or impulses. This can be TV signals, radio signals or communication signals. Since we already have said that TV can be regarded as signals, we can draw the conclusion that images can be regarded as signals.

In many occasions it can be of interests to split the signal into its components. This can be either to find certain components that we want to examine closer, or to see if there are components that can be removed without loss to the quality of the signal. There can also be components that are desired to remove to enhance an effect in the image. Some components are just causing noise in the picture and are also desirable to remove. To split the signal in its components there are two different mathematical ways, one is called Fourier analyze and the other Wavelet.

2.2 Fourier

Founded, by the French mathematician; Jean Baptiste Joseph Fourier (1768 - 1830) [3].

Equation 1: The Fourier Transform for a one dimensional function [13]

$$f(x) \equiv \int_{-\infty}^{\infty} f^{\cdot}(u)\, e^{j2\pi ux}\, du$$

Equation 2: The inverse Fourier transformation [13]

$$f^{\cdot}(x) \equiv \int_{-\infty}^{\infty} f(x)\, e^{-j2\pi ux} dx$$

Fourier proved in 1822 that all periodical functions can be expressed as a sum of different sinus and cosine with different frequencies and amplitude. Such Fourier sums are genius when it comes to finding different frequencies or amplitudes in a signal. Fourier can be and is used for removing certain part of a signal, like compressing a sound file. What Fourier cannot do is tell us anything about where the frequencies occurred or where the amplitude occurred. So if you for example have a signal, Fourier can determine that the signal consists of two different frequencies, but there is no telling where in the signal the frequencies occur. Fourier can however put together the different parts into the original signal [1]. Fourier theory showed that almost all functions can be replaced by a set of sinus and cosine functions [13].

2.3 Wavelet

Wavelet is composed by the words wave and let, which can be translated to small waves or waves that only go a certain distance. And that is what Wavelets are all about. In Fourier sinus and cosine are used as basis functions, while Wavelets uses these small Wavelets as basis functions. The most distinct difference between Fourier and Wavelets is that Wavelets can be used to find out where in the signal a specific part is [1]. To give an example you can think of a bike. If you take the bike apart you end up with all the bike parts in front of you. You have two wheels, a frame, a handle and a bike seat and so on. Wavelet can not only say which parts the bike consists of, but also contains information about where on the bike the parts are located. There are two main methods of Wavelets, separated into continuous Wavelets transform (CWT) and discrete Wavelets transform (DWT).

Equation 3: The continuous time Wavelet transform (CWT) of **f(x)** with respect to a Wavelet **Ψ(x)** [12]

$$W(a,b) \equiv \int_{-\infty}^{\infty} \psi(a,b,x)\, f(x)\, dx$$

Equation 4: The Wavelet when it is scaled, **a**, and translated, **b** [12]

$$\psi(a,b,x) \equiv \left(|a|\right)^{\frac{-1}{2}} \Psi\left[\frac{(x-b)}{a}\right]$$

2.3.1 History

Wavelet is a relatively new mathematical method compared to more traditional mathematical theories. The theory has derived from Fourier's work in the 17^{th} century. The first one to work on something that resembles Wavelets like we know them today was Haar in 1909, but since then it lay dead until people like Daubechies, Mallat and Meyer revived the interest for Wavelets in 1980's -1990's. Since then the use of Wavelets has escalated and is now in use in almost any scientific research. Areas of use are; mathematics, communication, electro, economy, quantum physics, signaling and information technology [11].

2.3.2 DWT or CWT

There are two main methods of Wavelets transformations, the discrete transform and the continuous transform. The CWT is a continuous transform which is a continuous function of time and their transforms are a continuous function of frequency. DWT is not continuous like CWT, but uses none overlapping blocks, and the frequencies are typically spaced at unit powers of two. Continuous transform means a lot more heavy calculations than the discrete transform. CWT therefore requires a lot more computer power and will take more time. Since this project is not dependant on a time efficient algorithm we will focus the rest of the paper on CWT since this also offers the best accuracy.

2.3.3 Wavelets vs. Fourier

Both Fourier and Wavelets are used for signal analysis. If we think in terms of a picture, both methods can find out that the picture consists of a boy and his dog. They can both take the boy, or the dog out of the picture, or both. Both methods also have the capabilities to reassemble the picture to its original state after taking it apart. What Fourier cannot do is determine where in the picture the boy is and where the dog is. This feature is possible with the Wavelets transform. When determining the edges on bone structures in x-ray images, it is vital to know where in the image the bone is. Therefore Wavelets is the only real alternative for us in this research paper, and we will focus on Wavelets from here on out.

2.4 Mother Wavelets

The function Ψ is called Mother Wavelets as it forms the basis for all the functions that can be created by changing a, b, or both, in the function **Ψa,b(x)**. The index **a** is called dilation scale, and **b** is called translation. When **a=1** and **b=0** you get **Ψ(x)** which is the Mother Wavelet at basis position.

Examples of different mother Wavelet: Haar, Morlet, Mexican Hat and Daubechies

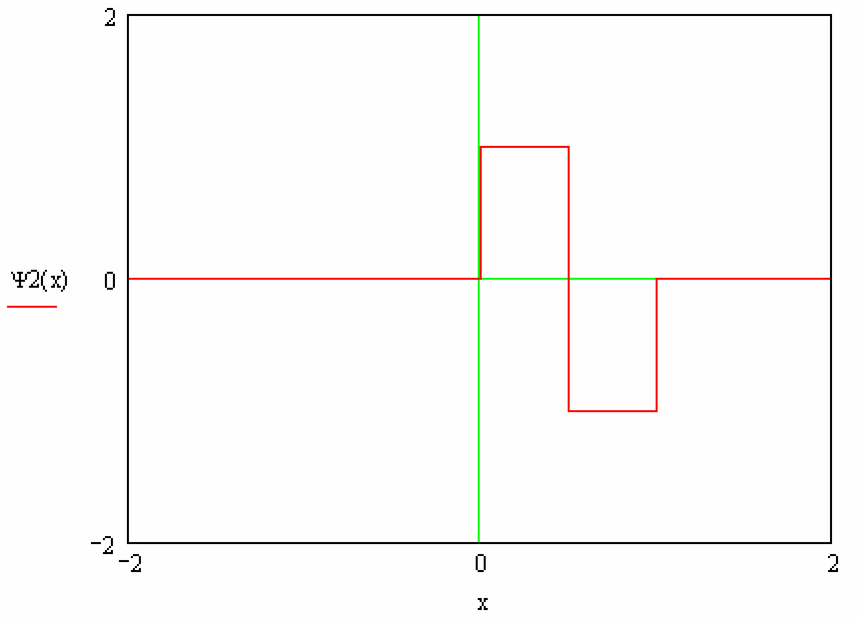
When we have e.g. Mexican Hat, the **Ψ(x)** is replaced by a formula for that specific Wavelet. This one can, in turn, be stretched and translated. How much this mother Wavelet is stretched and translated is determined by **a**, and **b**.

2.4.1 Haar

The Haar Wavelet is one of the simplest Wavelets, but still it is very much used.

Equation 5: Haar Mother Wavelet [12]

$$\Psi 2(t) \equiv \left| \begin{array}{l} 1 \quad \text{if} \quad 0 < t \le \frac{1}{2} \\ (-1) \quad \text{if} \quad \frac{1}{2} < t \le 1 \\ 0 \quad \text{otherwise} \end{array} \right.$$

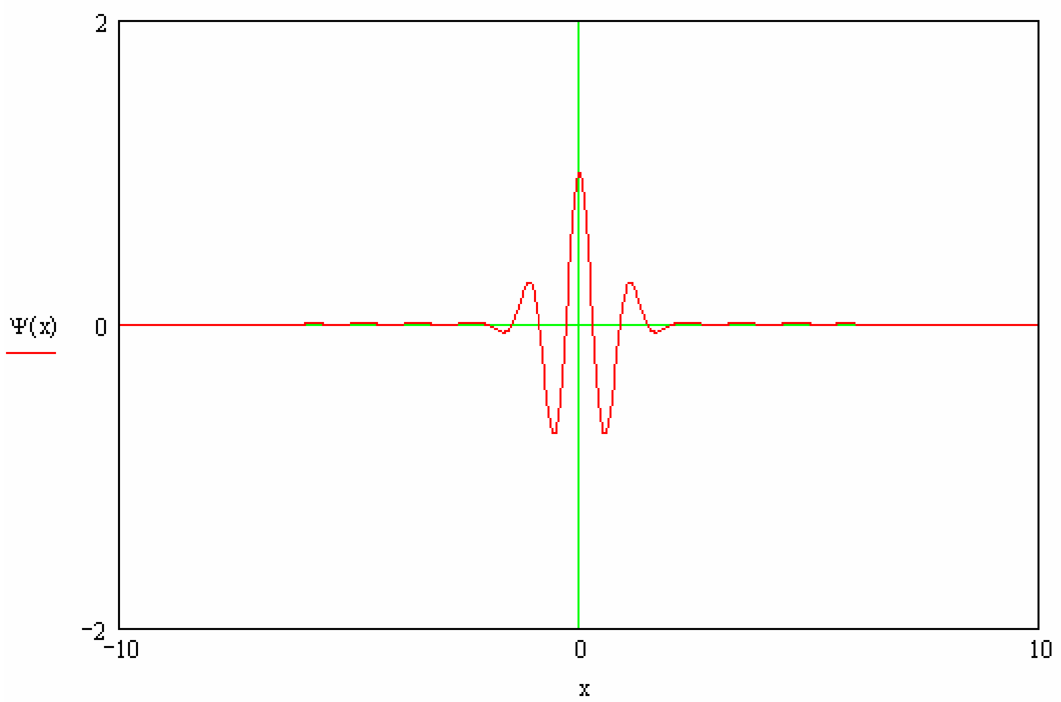


Graph 1: Graph visualizing the Haar Mother Wavelet

2.4.2 Morlet

Equation 6: Morlet Mother Wavelet [12]

$$\Psi(x) \equiv e^{-x^2}\cos\left(\pi\sqrt{\frac{2}{\ln(2)}}\,x\right)$$

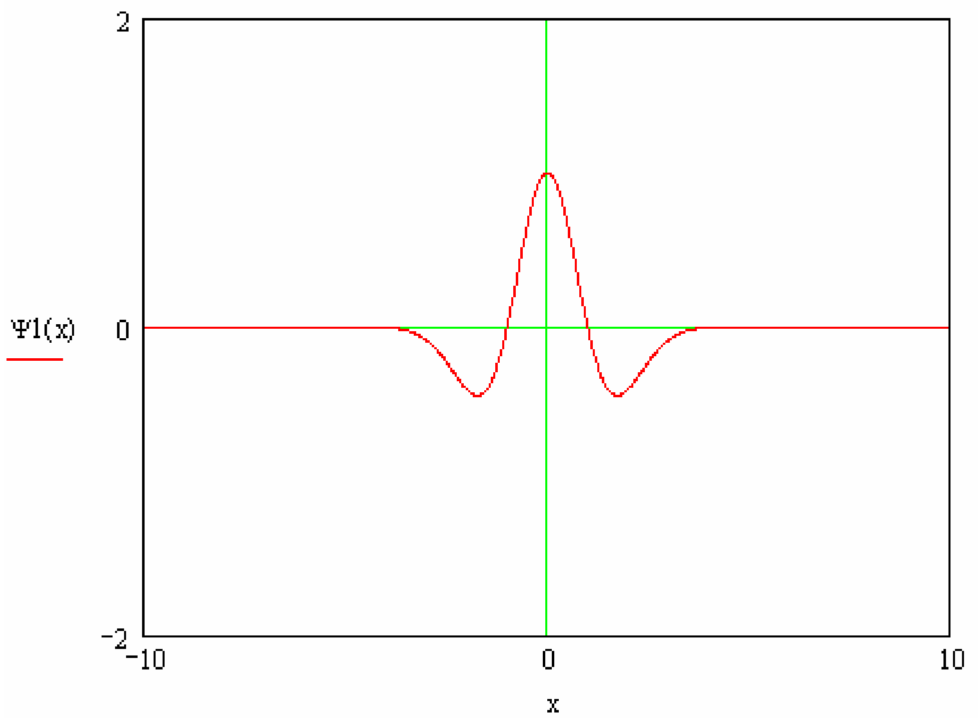


Graph 2: Graph visualizing the Morlet Mother Wavelet

2.4.3 Mexican hat

Equation 7: Mexican Hat Mother Wavelet [12]

$$\Psi 1(x) \equiv \left(1 - x^2\right) e^{\frac{-x^2}{2}}$$



Graph 3: Graph visualizing the Mexican Hat Mother Wavelet

3 Methods

3.1 Overview

There are several methods for finding the edges in images, but many of them have flaws which make them unfit, or inaccurate. Therefore this paper is about trying to find edges in images using Wavelets. Several different Wavelets will be tried out to see which one provides the best result.

3.2 Theory on finding edges

There are different methods to finding edges in images. In this assignment the focus will be to find the edges with the help of Wavelets. However Wavelet is not a term for finding edges but a tool which might be helpful. There are many ways of finding the edges with the help of Wavelets all depending what kind of edges you need to find and what Wavelet you use. When you transform an image with Wavelets, you shoot a beam across the image (taking out one single line). Then you let the Wavelet translate that line, transforming the signal.

A narrow Wavelet is excellent for finding sharp edges, since the edge will have a great impact on the transformed wave. But a narrow Wavelet is also very easily affected by noise in the image, and all edges are not always sharp and clear. The hope is therefore to find a method of using wider Wavelets to find the edges. Since there are many types of Wavelets (Mother Wavelets) and each Wavelet has its own qualities there might be many ways that will lead to a solution. A technique that works on one type of edges might not work on another type of edges. There is also the matter of which scaling of the Wavelet to use, and here too, it depends on the type and width of the edges.

To find the Wavelet that will be most useful, and to find the correct scaling to use in each case we have a couple of theories that might help us. One is to see if the energy in the transformed signal can give us a clue to which scaling to use and maybe even give a solution to finding the edges. Then we have a theory about weighting the different Maximums created by the transformed wave and a homemade theory called Champagne glass theory

3.3 The energy of a transformed signal

Disturbances in pictures may cause deviation and errors in transformed signals. Short or none-scaled Wavelets detect small changes in images, like disturbance, very easy. In conversation with our teaching supervisor, we hope to find a wider Wavelet to detect edges in images. What scale to use, is based on an analysis of the energy of a transformed signal. A wider Wavelet will ignore disturbances because it spans a wide area.

3.3.1 Energy distribution in a signal

Below the equation for the distributed energy at a certain scale and translation is shown.

Equation 8: The energy distribution of a signal [12]

$$\frac{W(a,b)^2}{a^2}$$

The Wavelet transformed of a signal returns a certain value. When this is multiplied with it self and divided by the scale number squared, we get a new value; the energy for this scale at a certain translation. In other words, it gives information about how much of **Ψa,b(x)** that exist in the original function **f**. The equation gives no contribution to the energy until the Wavelet is transformed over an edge or other changes in an image.

This value represents the contribution of a certain translation at a certain scale. The distribution is often shown as a grey-scale-image (image shown below). Negative values result in darker areas, and positive result in lighter areas. When zero is returned, grey scale 128, the middle grey scale, is set.

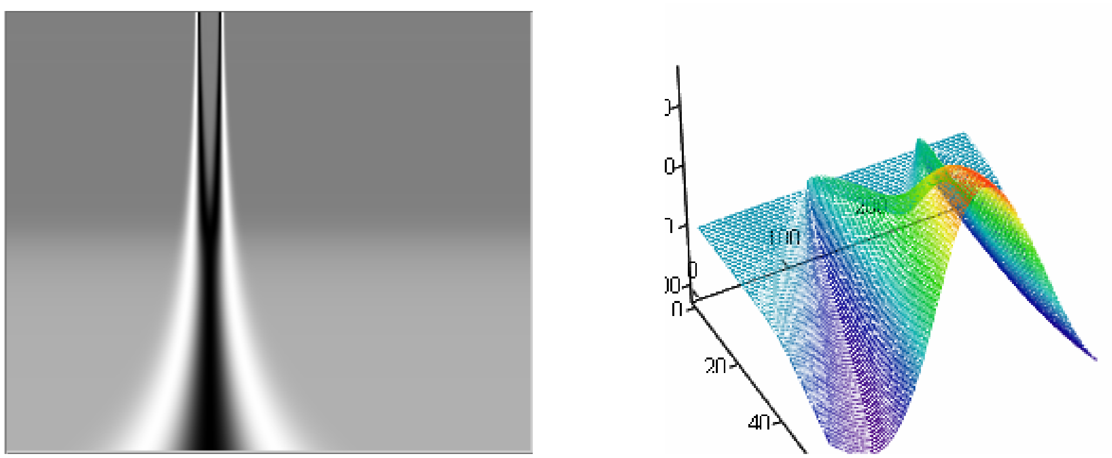


Figure 1: The energy distribution of a transformed signal in the Java application (left) and MathCAD (right)

When two peaks (like the top of the java image) occur, the scaled Wavelet covers only one edge in an image. Further down, the separated black areas on the top goes together as one. This indicates that the Wavelet starts covering the whole line.

3.3.2 Total Energy of a signal

For each scaling, the sum of all translations (the width of the image) decides the energy for this scale.

Equation 9: Total energy

$$\int_{imgStart}^{imgEnd} \frac{W(a,b)^2}{a^2}\, db$$

The difference between "imgEnd" and "imgStart" represent the width of the image.

Using low scales causes the transformed signal to be affected by disturbances in the original signal. The total energy might be used to find a scale value ignoring disturbances in the original signal.

3.4 Weighting

We try to get to the exact value for the edge start/end, using the scale of the Wavelet with total energy at maximum. Weighting are used to approach this value.

Weighting maximums

The wave created by the transformation has three maximums. Maximum **A**, and **C** have the same height which is not equal to maximum **B**. By weighting these different we hope to find a formula that will enable us to find the edges of the transformed object.

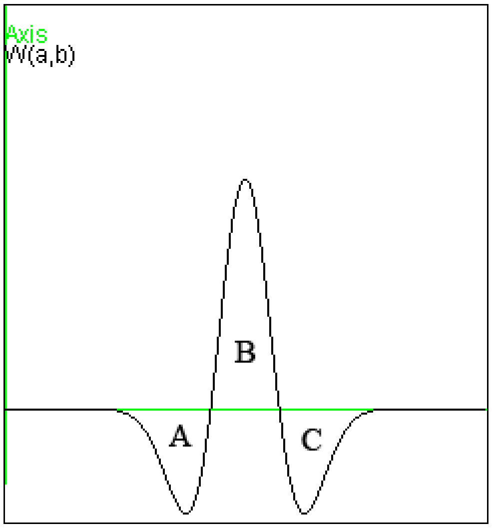


Figure 2: A transformed signal using Wavelet scale where the energy is on top (from java application)

Weighting areas

Using the same scale value as before, the areas are now weighted against each other. The first area, A, are weighted against the next area, B. A point between the maximums, of each area, is decided using the relation between the areas. Theory indicates that the point is moved towards the maximum point of area B.

3.5 Champagne glass theory

When you transform an object with a width of 1 pixel or more, using the Mexican hat Wavelet, you end up with an image (the CWT energy distribution) that somehow resembles a champagne glass. The champagne glass theory was based on the images created when transforming with Mexican Hat. The theory can also be used on Morlet, but Morlet does not create perfect champagne glass images.

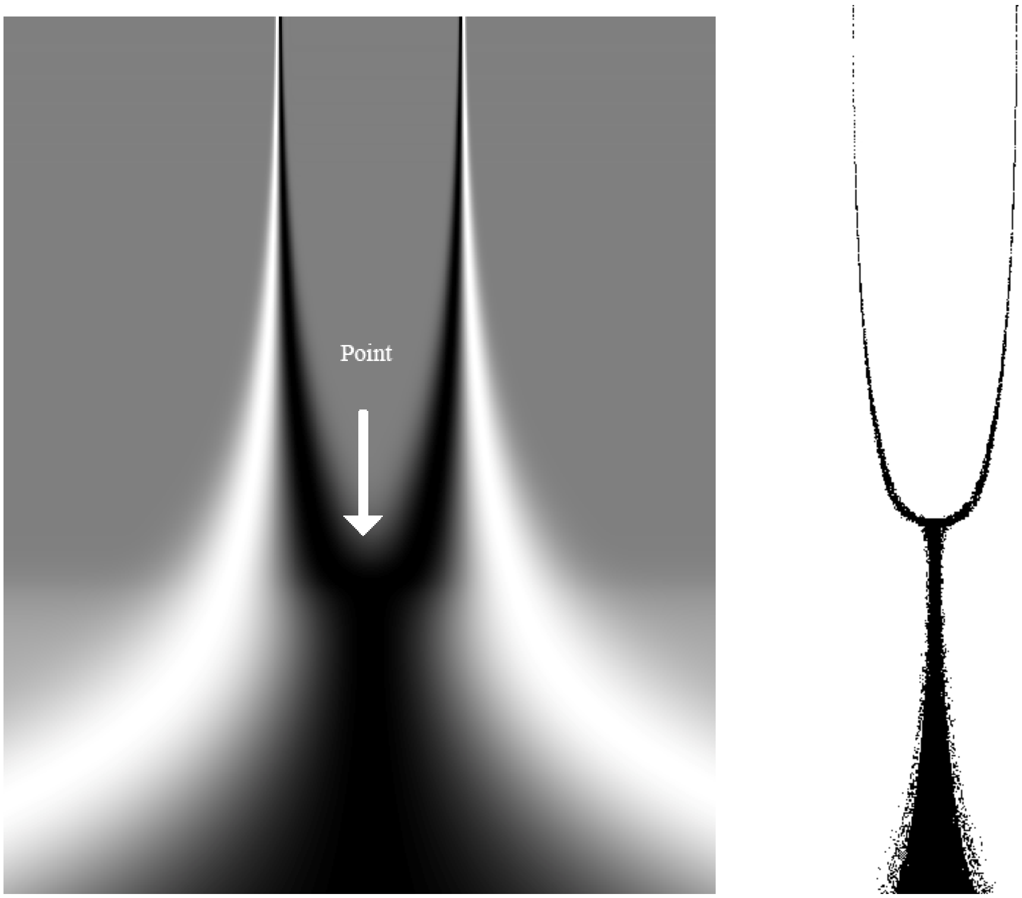


Figure 3: These images was the origin for the "champagne glass" theory

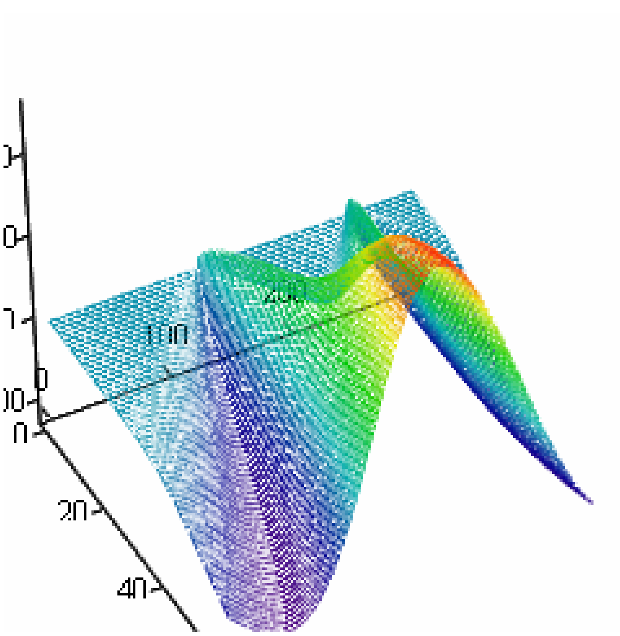


Figure 4: The same image as above. The energy distribution visualized using MathCAD

The white area (figure 3), is where the first part of the Wavelet hits the object that we are transforming, and the black area is where the top of the Wavelet hits. On the other side it is the top of the Wavelet that goes out of the object first. Then the smaller part, or bottom of the Wavelet, leaves the object. The area between the black and white lines is grey, because the contribution from the positive and negative peaks neutralizes each other. At the top of the picture the Wavelet is clearly smaller than the line that is being transformed, thus creating two sharp distinct lines. As we go further down the transformed picture the lines become more and more diffuse and in the end they melt together. As they melt together the width of the Wavelet is now broader than the object that we are translating the Wavelet over. The theory of the Champagne glass is that if you take the scaling in the bottom of the champagne glass right before the two black areas melt into one, you get the best scaling for finding the edges of the line. The area is marked with a "Point" in figure 3. Since each line of pixels in the transformed image represents a scaling, you can count the number of lines in the image to find the scaling in the bottom of the Champagne glass.

3.6 Convolution

Disturbances in an image often result in oscillation in the original signal. These oscillations can be reduced by using a mathematical method called convolution. The definition of convolution h(t) is shown below [2]:

Equation 10: Convolution

$$h(t) \equiv \int_0^t f(\tau)\, g(t-\tau)\, d\tau$$

Here f(t) is the original signal, and g(t) is a constructed signal. The convolution of these two functions, tell us something about how much of g(t) it is in f(t) as g(t) is shifted over the function f(t) [9]. We will create a function g(t) that help us get a smoother curve of the original signal. This way, some disturbance is ignored.

4 The application

4.1 Overview

We used two different programs to gain knowledge and control over the Wavelets, the first was MathCAD, and the other was a self developed JAVA application. With these two programs we had a good platform to explore all the different options and theories surrounding the Wavelet image transform.

4.2 MathCAD

MathCAD is a mathematical program that is used to calculate different mathematical equations. The version used was MathCAD 2001 i Professional. We used this program to explore how the mathematical equations of Wavelets looked when graphed. It also gave us a chance to test how the parameters in the formulas affected the graphs. This gave us a better understanding of the Wavelets, how they were built up and how they looked when graphed.

MathCAD is also a good tool for writing advanced mathematical formulas, something which is difficult to do in Word. Therefore we used MathCAD to write the formulas and the copied and pasted them into our document.

Our mentor and teaching supervisor Associated Professor Hogstad created a program in MathCAD where we could load an image into the program and test some of our theories, and easily manipulate the different variables involved. This came in addition to our JAVA application, and worked as a safety to ensure that we were on the right track, and that the formulas where written correctly.

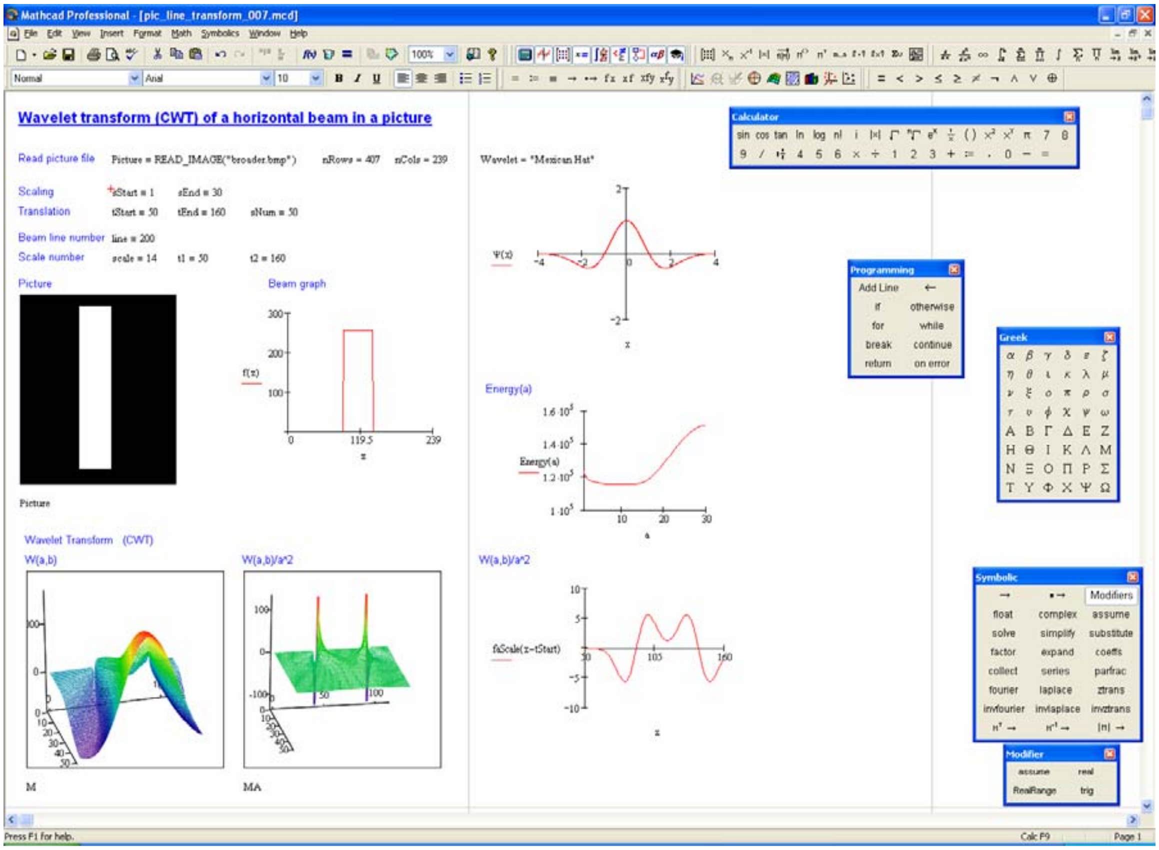


Figure 5: The MathCAD program used for this assignment

4.3 JAVA Application

To have an environment where images could be loaded and Wavelets could be tested a JAVA program was written. The JAVA application was first written by our mentor Professor Hogstad. We were given an introduction in what worked, and how it worked. After that, it was up to us to further enhance the application to suit our needs.

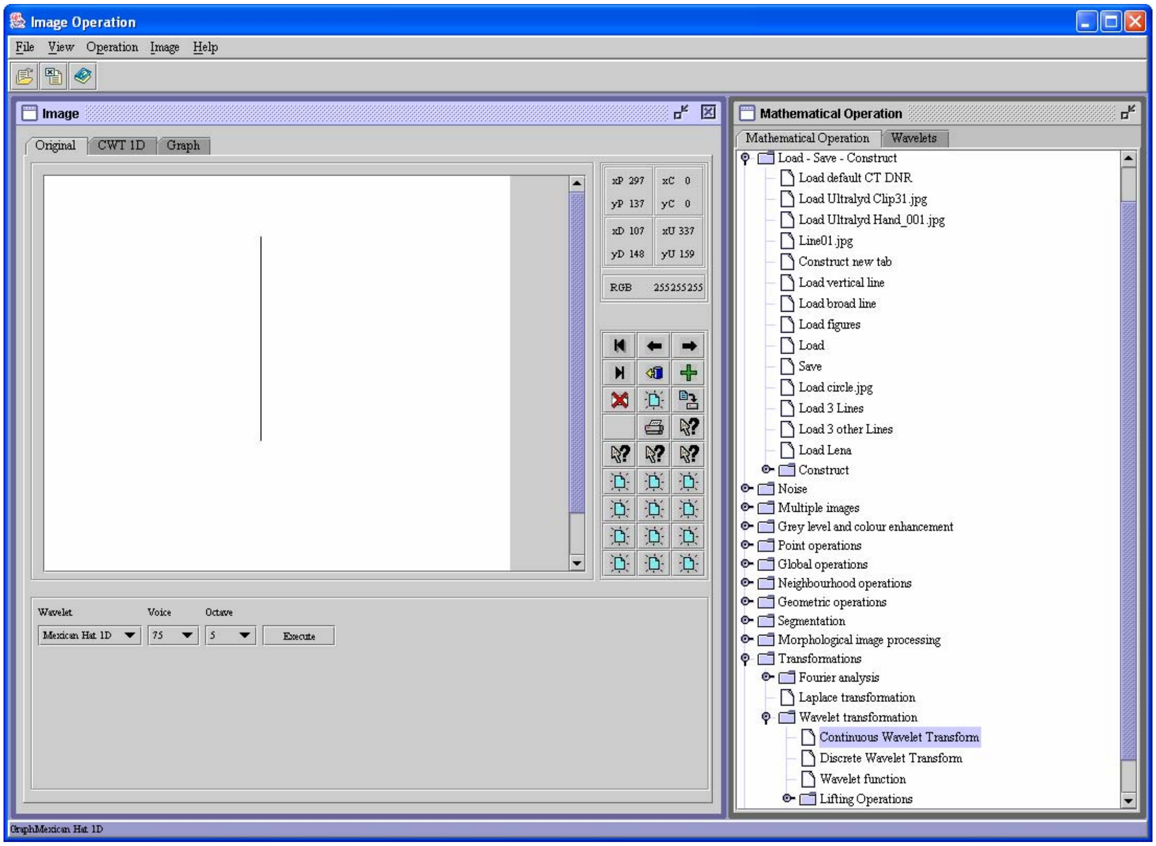


Figure 6: The Java application developed for this assignment

The framework, and user interface was already done. Some adjustments and extensions were necessary to gain information for us to analyse. After enhancing the application we ended up with a program with the possibilities to load an image and choose from three different Mother Wavelets to translate over it. Those Mother Wavelets were; Morlet, Mexican Hat and Haar. We could manipulate the number of Voice and Octave that we wanted to use. The transformed image came up as a separate image within the application. The energy distribution for each scale and translation was shown as a greyscale in that image; this feature was already implemented in the application. A table gave us the opportunity to see each pixel value in the image, and the RGB-value of each of these pixels.

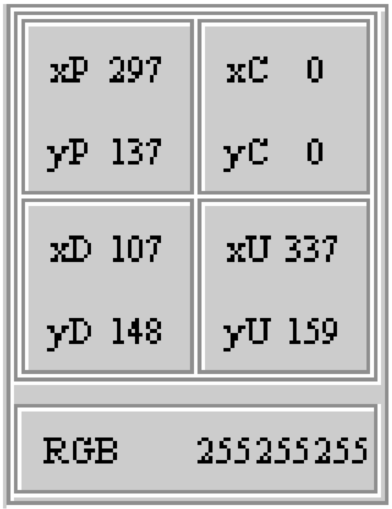


Figure 7: The panel presenting pixel values, and position for the mouse pointer

Different graphs were necessary to get information of behaviour of Wavelets. This included the Wavelet-transformed, $W_{(a,b)}$, $W_{(a,b)}^2$, $W_{(a,b)}^2/a^2$, and the sum of all translations of $W_{(a,b)}^2/a^2$. The last function is also known as the total energy of a certain scale.
An image was created to visualise the behaviour of the above functions. Some of these functions were rescaled to fit to the screen. The graphs were for visualisation purposes and therefore not necessary to keep in real size. Together with the information written to the system, we are able to find the information we needed for our research.

Some variables are not possible to change during runtime. These variables must be set before compiling and running the program. These variables are, what scale to use, the points set from which the areas will be calculated, and which graphs to show.

5 Results

5.1 Overview

We have tested three types of Mother Wavelet in order to find the one best suited to find edges. One of the Wavelets called Mexican Hat has been tested for several different theories on finding the edges. Due to the complexity of x-ray images with all the noise and vague edges we have started testing our theories on images with an object with a sharp edge. When our theories worked on a simple black line on a white background we increased the width of the line to see if it still worked. Once we had a theory that we were sure would work perfect, plan was to try it on x-ray images.

5.2 Energy of a transformed signal

The total energy curve are studied for three different Wavelets; Haar, Morlet, and Mexican hat. Tests are performed on simple images of lines with different widths (Attachment A).

5.2.1 Mexican hat

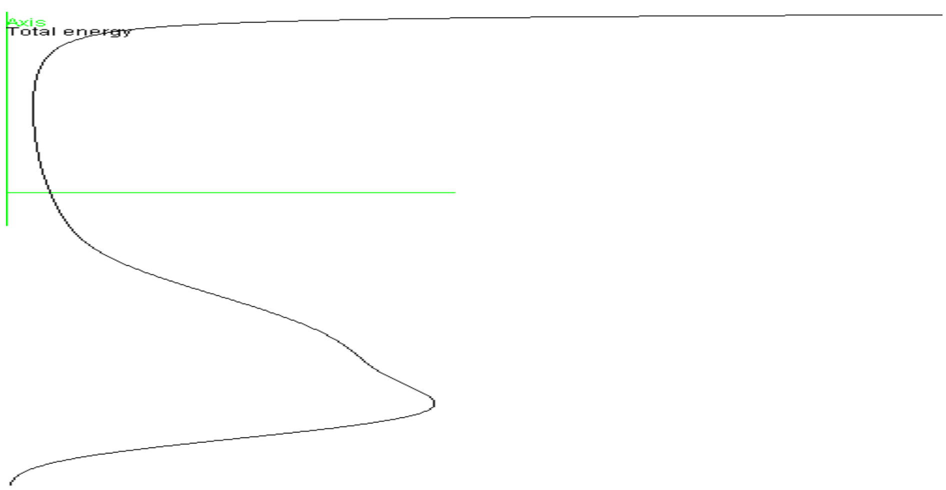
The total energy has a characteristic curve. It starts out with a high value for none scaling Wavelet. The energy then decrease for each scale until the point where the scaled

Wavelet covers the whole line. Further, the curve increases to a certain point and then decrease again. The curves are shown below (graph 4 – 10).

Simple line

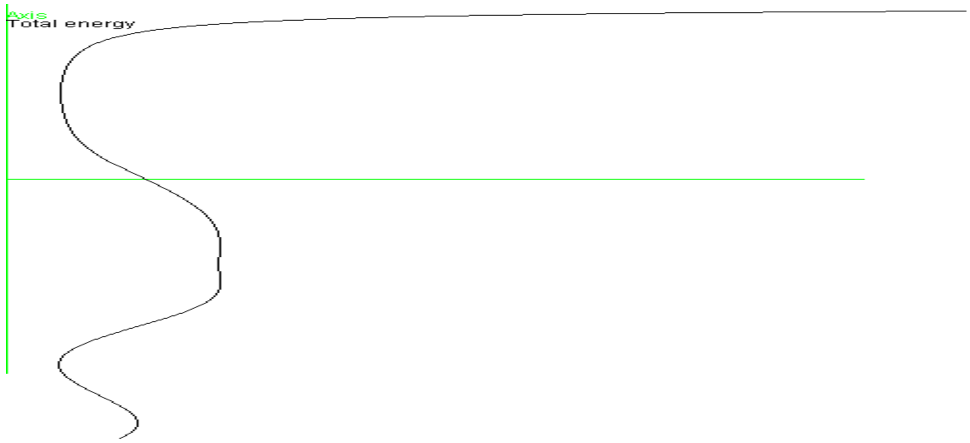
Graph 4: The energy curve when transforming a simple line, one pixel wide

Broad line



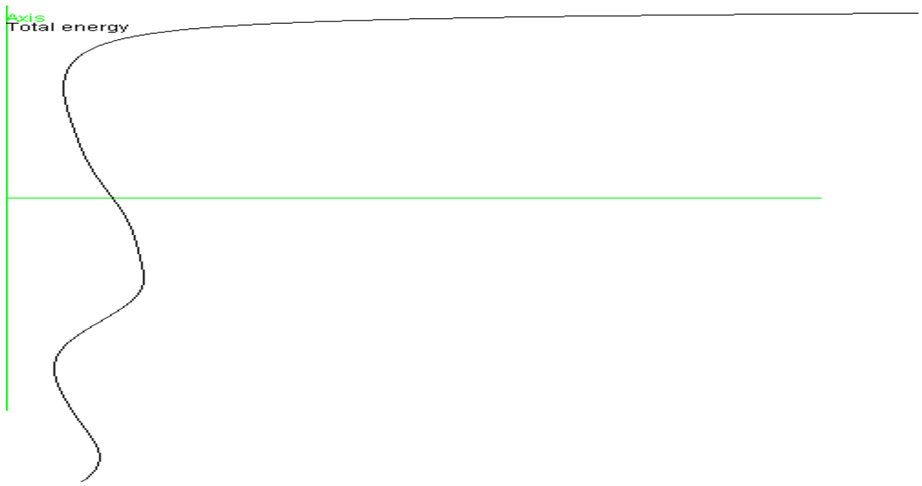
Graph 5: The energy curve when transforming a broad line

Double broad line



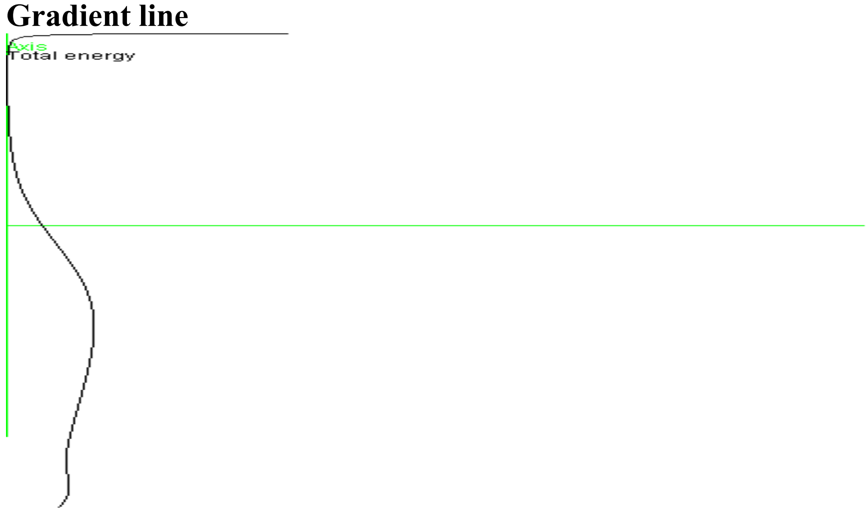
Graph 6: The energy curve when transforming to broad lines

Broad and thin line



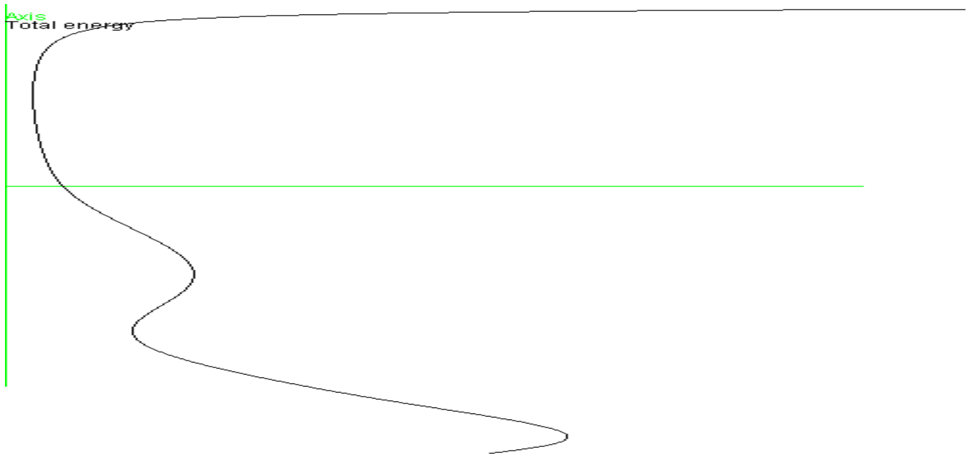
Graph 7: The energy curve when transforming one thin and one broad line

Gradient line



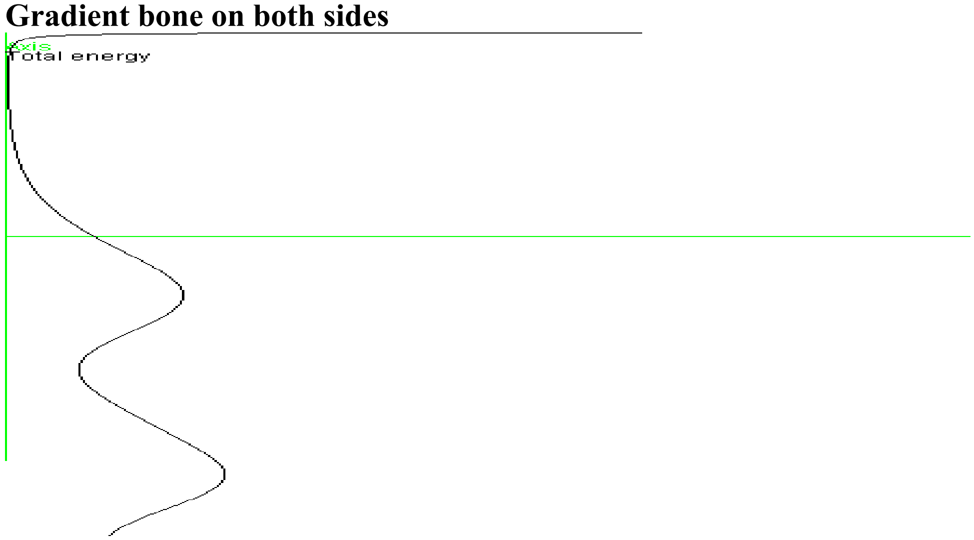
Graph 8: The energy curve when transforming a gradient line

Gradient outer line of bone



Graph 9: The energy curve when transforming two lines with outer edge gradient

Gradient bone on both sides

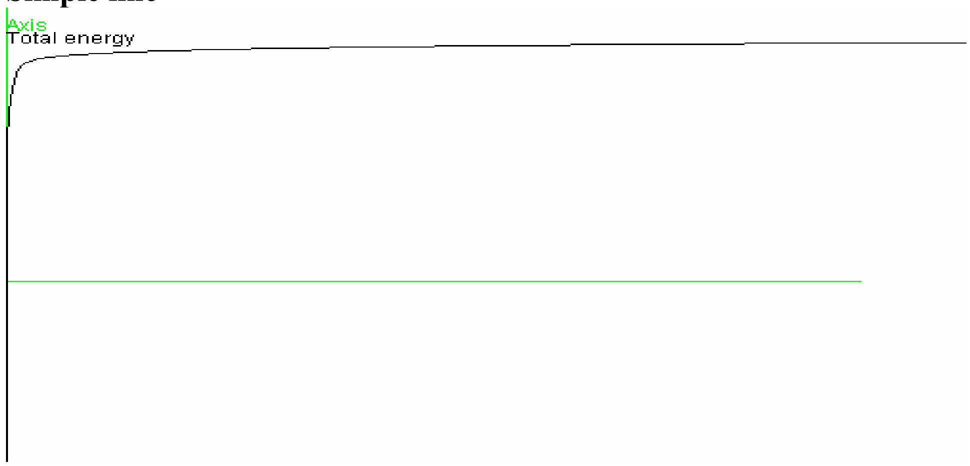


Graph 10: The energy curve when transforming two lines gradient on both sides

5.2.2 Morlet

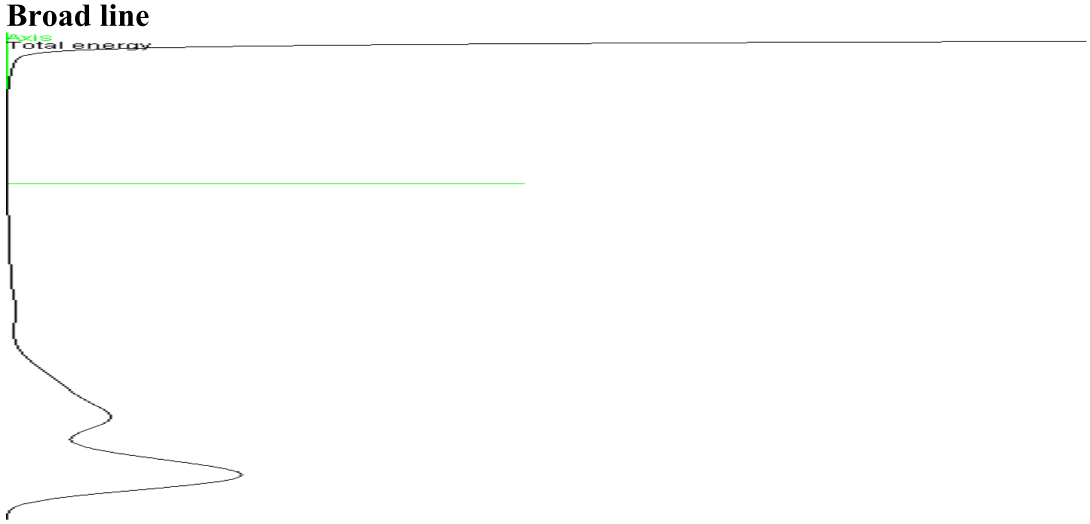
This curve start with a high value for none scaled Morlet. Then the curve decrease to a low value and keeps this value a while before it starts a small increase. The curve then start decrease after a short while, before it starts increasing and decreasing again at the end (graph 11-17).

Simple line



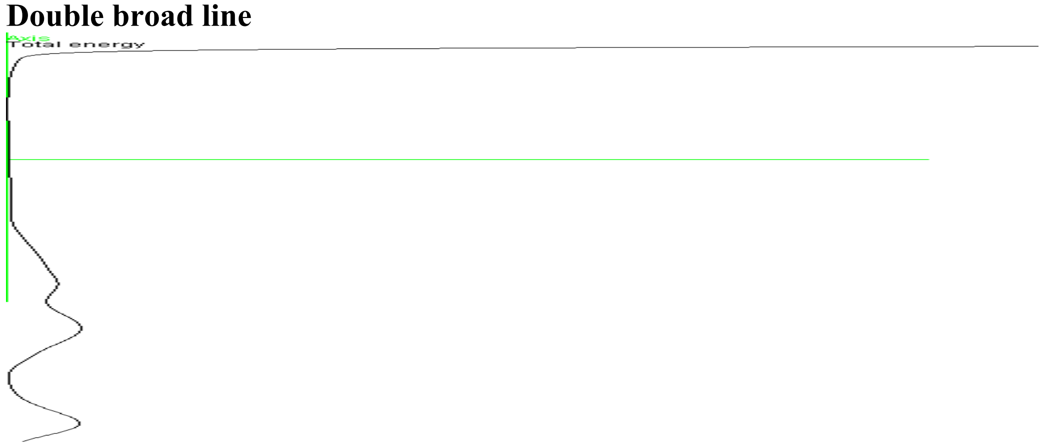
Graph 11: The energy curve when transforming a simple line

Broad line



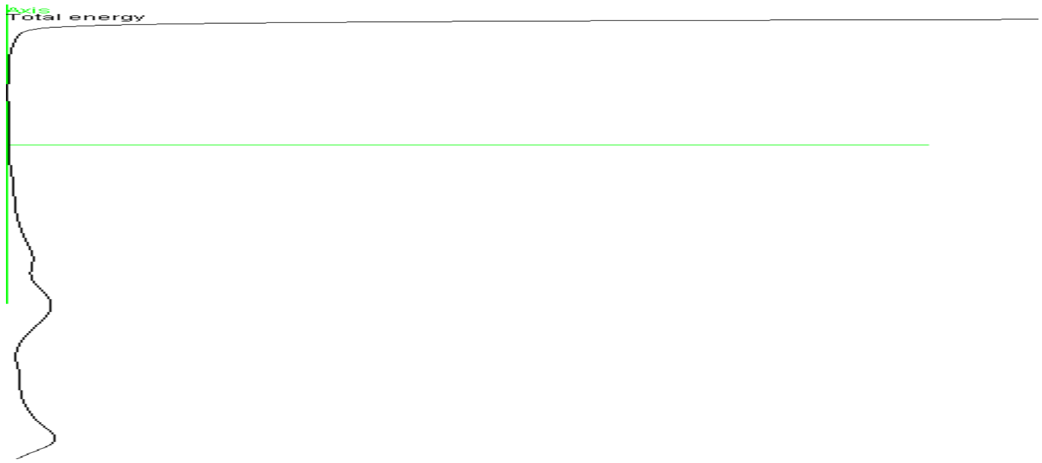
Graph 12: The energy curve when transforming a broad line

Double broad line



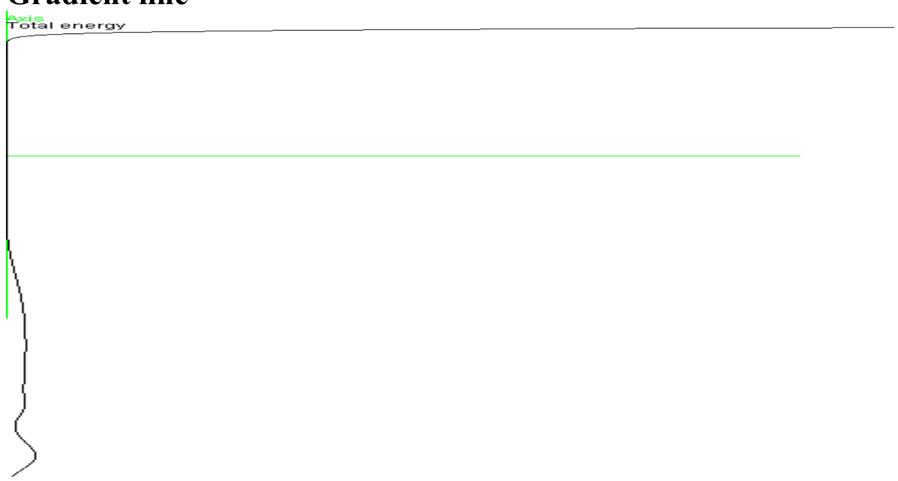
Graph 13: The energy curve when transforming two broad lines

Broad and thin line



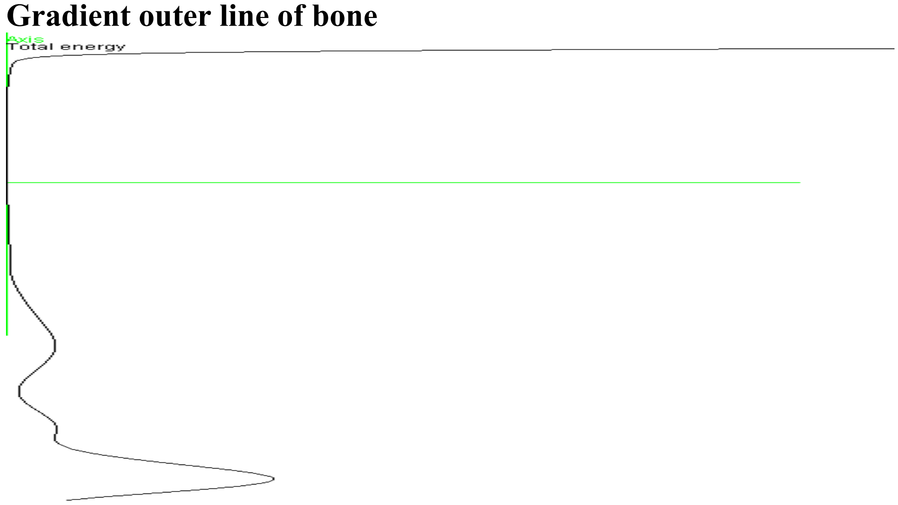
Graph 14: The energy curve when transforming one broad and one thin line

Gradient line



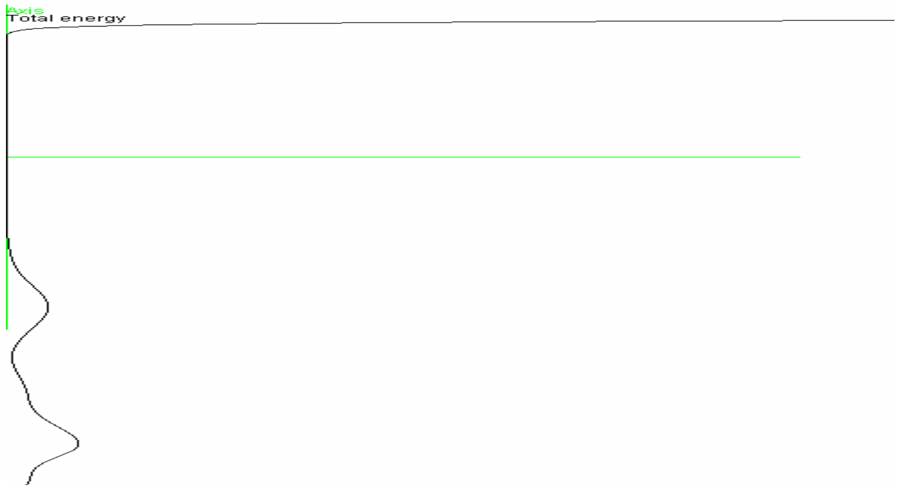
Graph 15: The energy curve when transforming a gradient line

Gradient outer line of bone



Graph 16: The energy curve when transforming two lines with outer edge gradient

Gradient bone on both sides

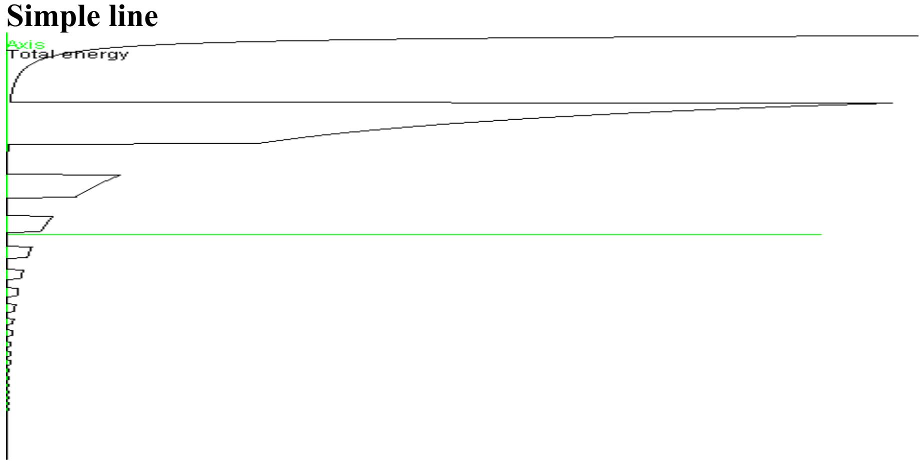


Graph 17: The energy curve when transforming two lines gradient on both sides

5.2.3 Haar

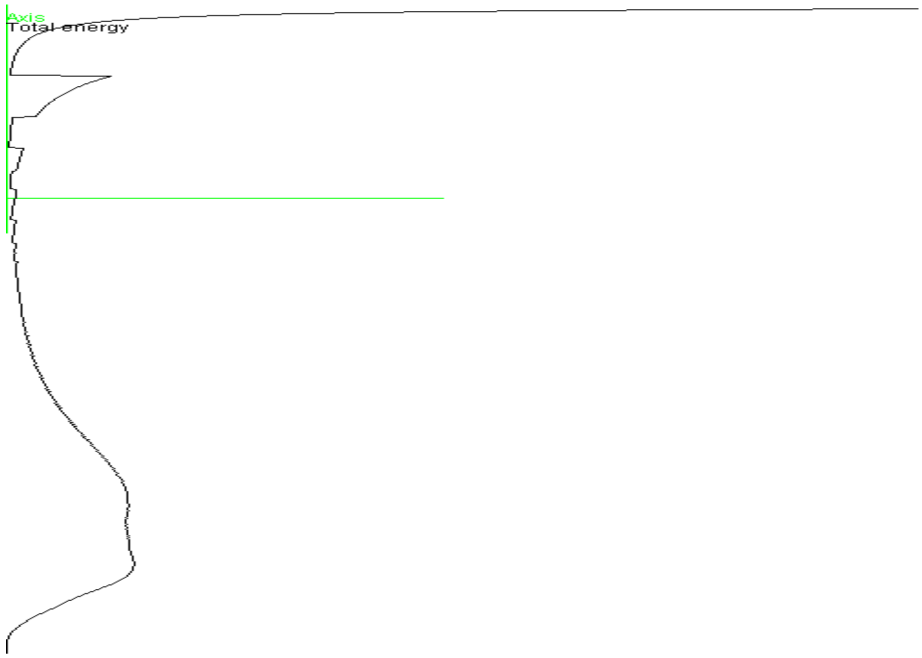
The total energy curve decrease for the first scales. Further it flattens out, before increasing at the end. The curve has rectangular impulses over the line. These impulses are huge at short scales, but get smaller as the scales get wider (graph 18 - 24).

Simple line



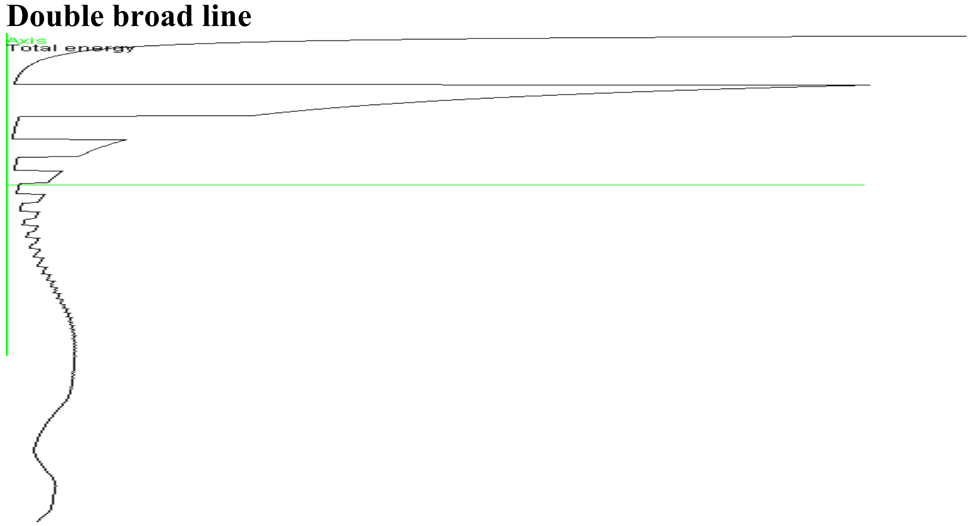
Graph 18: The energy curve when transforming a simple line

Broad line



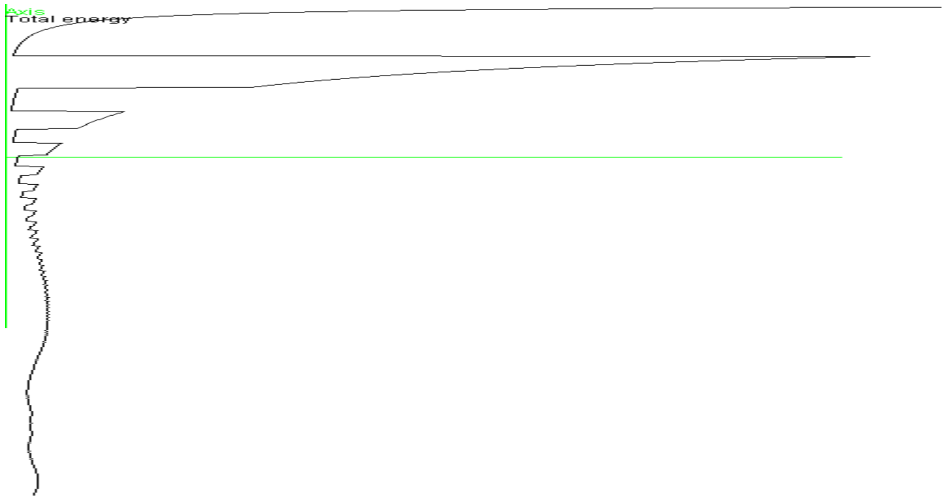
Graph 19: The energy curve when transforming a broad line

Double broad line



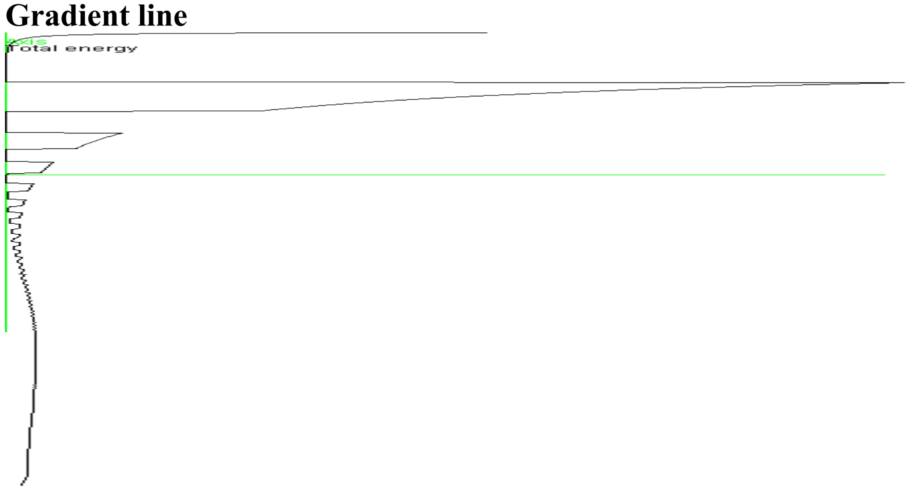
Graph 20: The energy graph when transforming two broad lines

Broad and thin line



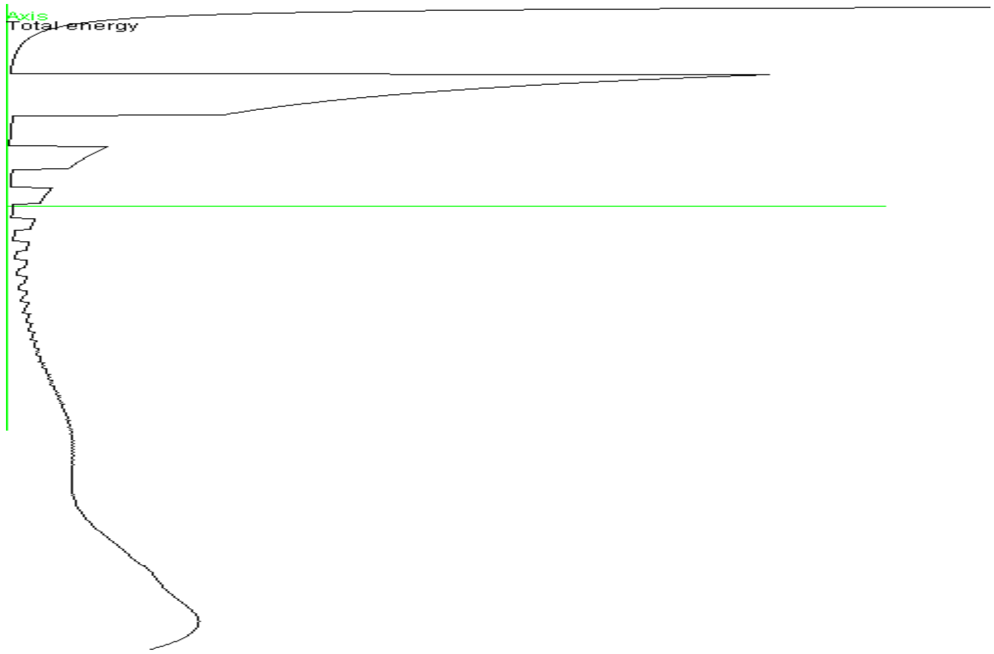
Graph 21: The energy graph when transforming one broad and one thin line

Gradient line



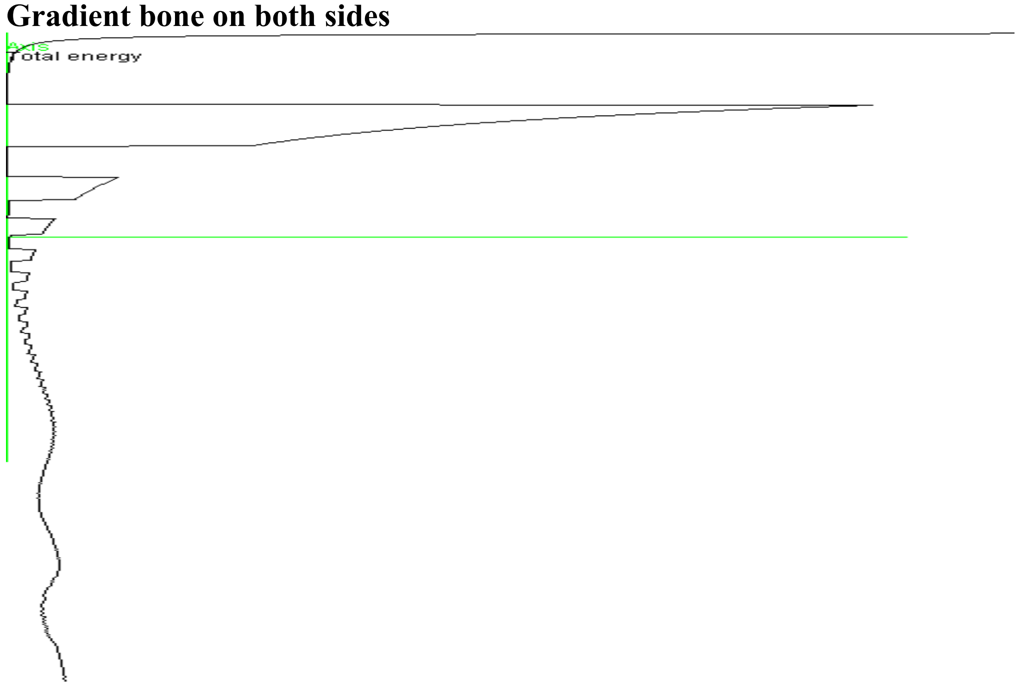
Graph 22: The energy graph when transforming a gradient line

Gradient outer line of bone



Graph 23: The energy graph when transforming two lines with outer edge gradient

Gradient bone on both sides



Graph 24: The energy graph when transforming two lines gradient on both sides

5.2.4 Discussion of the energy

The Mexican hat has more distinct marked maximums in the energy graph compared to the other Wavelets. When presenting this to our teaching supervisor, he recommended us to pay further attention to the Mexican Hat Wavelet.

When there is more than one line in an image, several maximums appear at the total energy curve. This led us to check new images. The new images contain only one line. Each image has different widths on the line. Using these images, we will make further study on the total energy curve using the Mexican Hat.

The testing images are very wide. This is to avoid a Wavelet to cover the outer line of the images when the line, in the image, gets very wide.

Table 1: The width of the line and its max energy

Width	Max energy
126	521
94	485
33	360
19	290

5.3 Using max total energy

5.3.1 Overview

We wanted to see if there was a relationship between the scaling which gave the max total energy and the width of the object transformed. Since we needed a broad Wavelet in order to fix the problem with noise that narrow Wavelets would run into, max total energy came up as an option. Now it turned out that the Wavelet which gives that highest energy is the smallest, most narrow Wavelet. The graph of the total energy shows that the most narrow Wavelets gives the highest total energy, but the graph then quickly decreases. After having gone far down the graph starts to increase again, when and how much is depended on the width of the object being transformed. When we are talking about the max energy, we are referring to the second peak in the total energy graph and not the first peak at scaling 1.

5.3.2 Possible uses of max total energy

We did not know if there was any connection between the total max energy and the width of the object being transformed, but we had a few ideas as to where there might be a connection. One was that there should be a direct connection between the width of the Wavelet at the scaling with max energy and the object that was transformed. This theory was rejected when we saw the total energy graph, since the max energy was at scaling 1, and the second peak in the graph was at a scaling broader than the object. Other theories were that it would be a link between the heights of the Wavelet as it ran over the object and the points where it crossed the x-line. The two above mentioned theories goes under the theory of Weighting. Another similar theory was that there might be a link between the areas created by the Wavelet and the points where it crosses the x-line.

5.3.3 Max energy scale

Analysing the total energy at max, results in a certain scale. This scale is analysed on different images. These are images of straight lines on a black or white background. The widths of the lines are different on every image. This time each image contains only one line. The crossing point of the x-axis is now in our interest.

Table 2: Start and end point for edges, and the crossing point of x-axis in MathCAD and the Java application

Original			Transformed in MathCAD			Transformed in Java		
Image	**Start**	**End**	**Max Energy**	**Start**	**End**	**Max Energy**	**Start**	**End**
blackWhite.bmp	125	-	31	121	-	518	125	-
whiteBlack.bmp	125	-	31	128	-	518	125	-
thin.bmp	117	117	0	116	118	1	116	118
broad.bmp	108	131	11	106,5	133	296	103	137
broader.bmp	91	139	24	88,5	142	434	61,5	171

5.3.4 Weighting

The weighting theory was based on the scaling which gave the max total energy. We wanted to see if there was any connection between the scaling which gave the highest total energy and the width of the objects transformed.

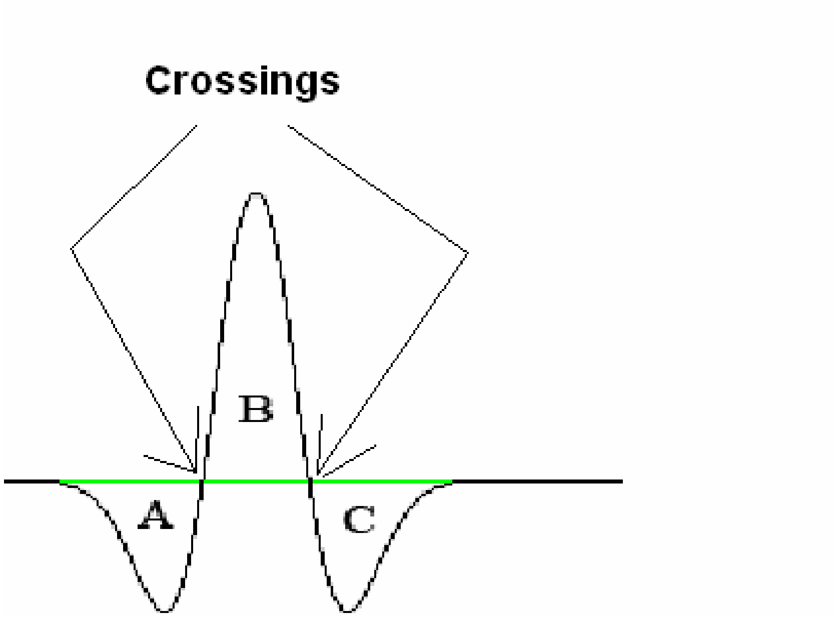


Figure 8: The areas and crossing points used for further calculations

The area A = area C, and the areas A and C is almost exactly half of area B, which means area A + C = B.

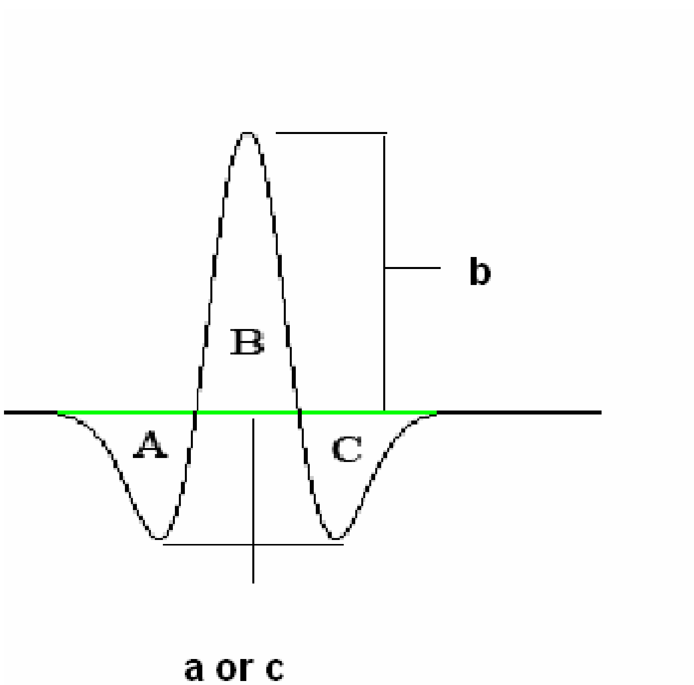
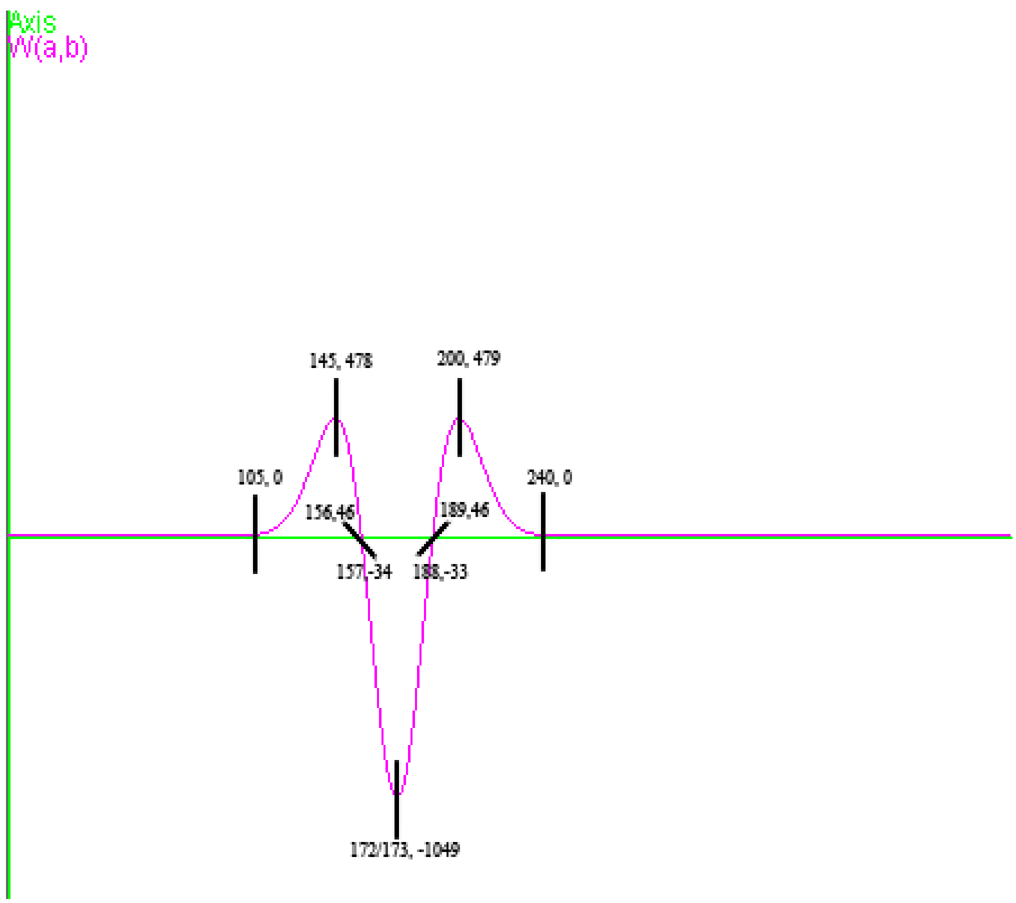


Figure 9: Maximum points used for further calculations

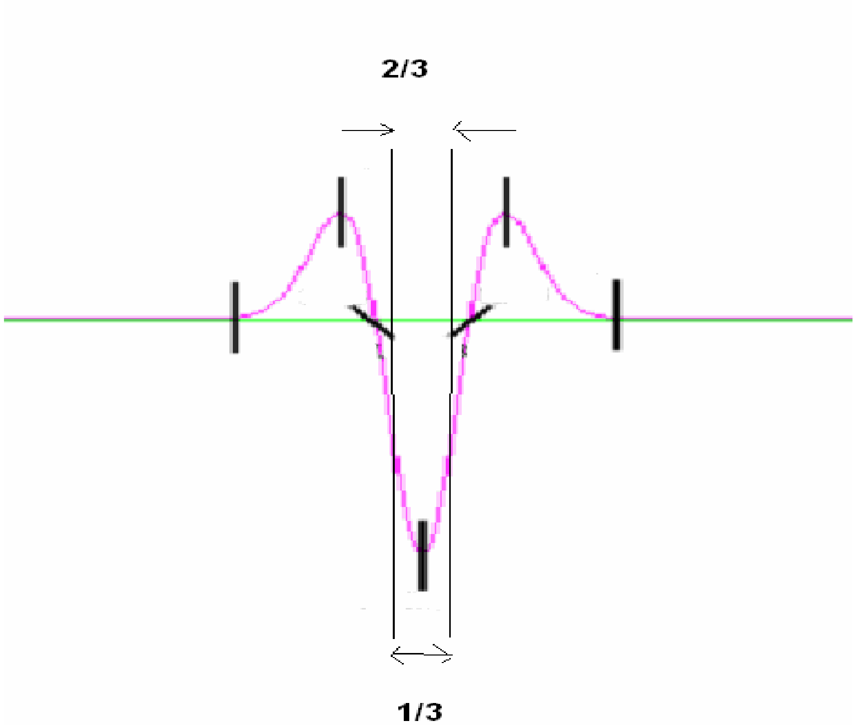
The height of area B, **b,** was also approximately twice the height of **a,** and **c**. On the first objects that we transformed the crossings of the Wavelet was too broad to fit with the edges of our objects. With this in mind we wanted to weight the area B twice as much as areas A and C in order to find the edges of our object.

We knew the top points of the areas A, B and C, which you can read of the data from the graph. Since the area of B and the height of **b** was twice that of A, **a** and C, **c** we wanted to weight the importance of the top B point twice as much, and this would result in two new points inside the crossings.



Graph 25: The transformed signal of an image of a broad line with values used for further calculations

The first maximum value was between 144 and 147, with a y-value of 478, the medium value 145 was used in further calculations. The second maximum value was between 172 and 173, with the according y value of -1049. The third and last maximum was found between 198 and 201, with a y value of 479, the medium value of 200 was used for further calculations.



Graph 26: The points for the edges was moved towards the maximum

Table 3: Different position for maximums at x-axis

2. Maximum	172
Minus 1. Maximum	145
Difference	27

The distance between 1st and 2nd Maximum was calculated to 27 pixels. Since the 2nd Maximum is supposed to be weighted twice as much as the 1st Maximum, we need to move 2/3's way from the 1st Maximum point toward the 2nd Maximum point. This gives us an edge at point 163. When the same procedure is repeated at the other side it gives us the second edge at 182. At the original image the edge starts at 163 and ends at 182 which give us a very accurate answer.

This procedure was also tried on other objects of various widths to see if the theory and method was valid or if it was a lucky shot.
Width of original object tested.

Table 4: Calculated edge for a broad line

	x-value	y-value	Calculated edge	Actual edge
1.Maximum	144-147	478		
			163	163
2.Maximum	172-173	-1049		
			182	182
3.Maximum	198-201	479		

Object, that was wider than original object.

Table 5: Calculated edge for a very broad line

	x-value	y-value	Calculated edge	Actual edge
1.Maximum	746-750	-1140		
			896	908
2.Maximum	968-970	2521		
			1042	1033
3.Maximum	1188-1192	-1140		

As we can see from the tables the theory did not work out when the width of the objects were increased. The error was not off by a couple of pixels, but missed the edges by as much as 12 pixels which is way of.

5.3.5 Discussion

The theory of finding the edges by using the scaling with the maximum energy and then weighting seemed to work fine after testing only one object width. When the same theory was executed on objects with different widths it did not work anymore. As we can see from the x-value in the table the maximum y value spreads over several x values. This makes it hard to decide which x values to use in the calculations. In order to see if we could find the edges by using different x-values, all possibilities were tried out. However

none seemed to get close enough to give a satisfying result. When the theory failed on one width we quickly conducted tests on a lot of widths in order to find a trend. Result was that we must have been lucky to find it on one of the widths since no other widths matched.
The broader the object became the more inaccurate the x-value peaks became and the more we missed the actual edges. There might be the possibility that other more advanced mathematical formulas might still give hope to finding the edges by weighting. However we found no such formula, and after having missed the edges by as much as 12 pixels, this theory was concluded as non functional.

5.4 Champagne glass

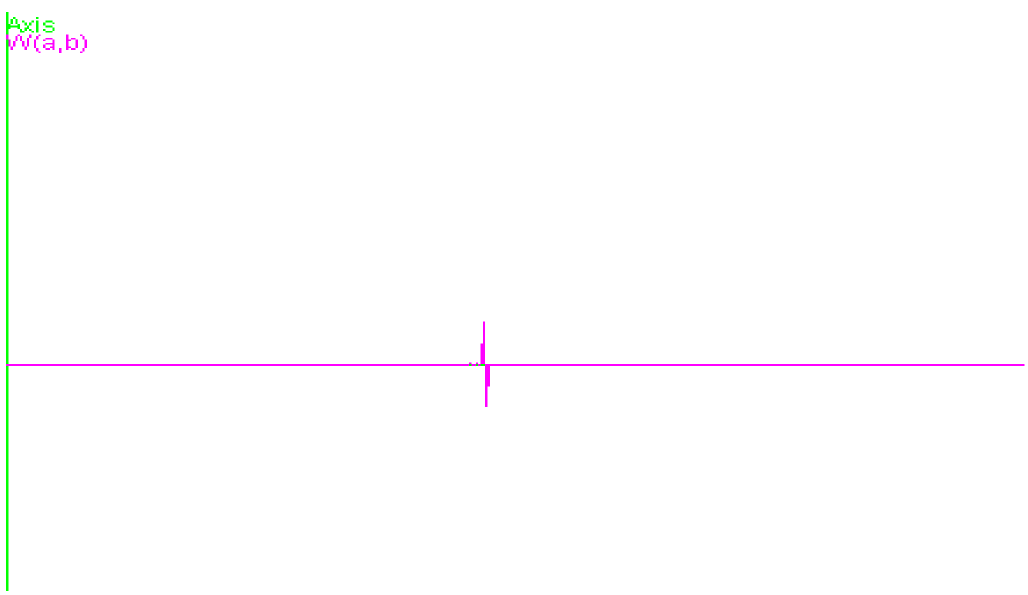
The results in the next chapters come from testing on the Champagne glass theory. First we ran the Wavelets across a line in the image to create the transformed Wavelet image. Then by looking at the transformed image we could decide which scaling that was in the bottom of the "Champagne glass". We then ran the Wavelet across the same line in the picture but this time with a scaling matching the Champagne glass theory. The pictures of the graphs represent the Wavelet as it moves over the line in the original image. The total energy is also graphed in the same picture.

5.4.1 Haar

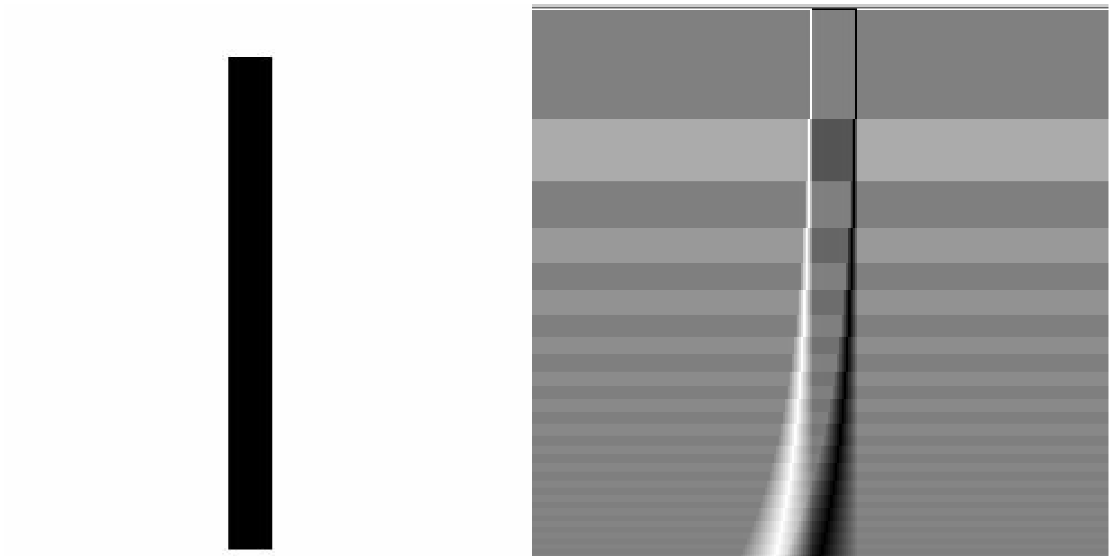
Figure 10: Haar transformation of a simple line

Table 6: Data for Haar transformation of simple line

	Wavelet	Image	Scaling
	Haar	simpleline	10
First edge	207		
second edge	207		
First edge with Wavelet	205		
second edge with Wavelet	205		



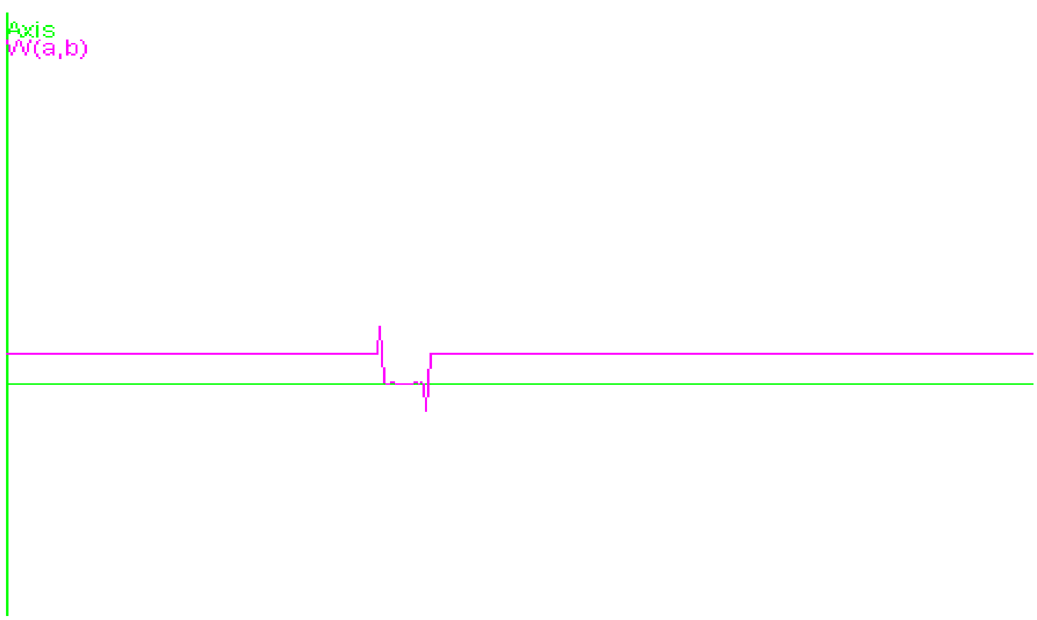
Graph 27: The transformed signal of simple line using Haar



Graph 28: Haar transformation of a broad line

Table 7: Data for Haar transformation of broad line

	Wavelet	Image	Scaling
	Haar	broad	100
First edge	163		
second edge	182		
First edge with Wavelet	163		
second edge with Wavelet	182		



Graph 29: The transformed signal of a broad line using Haar

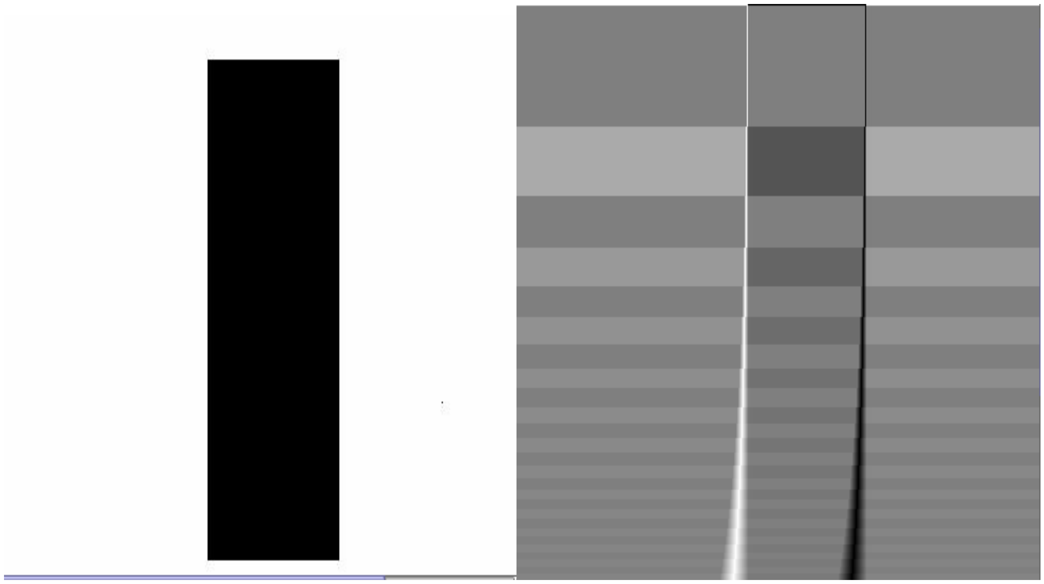
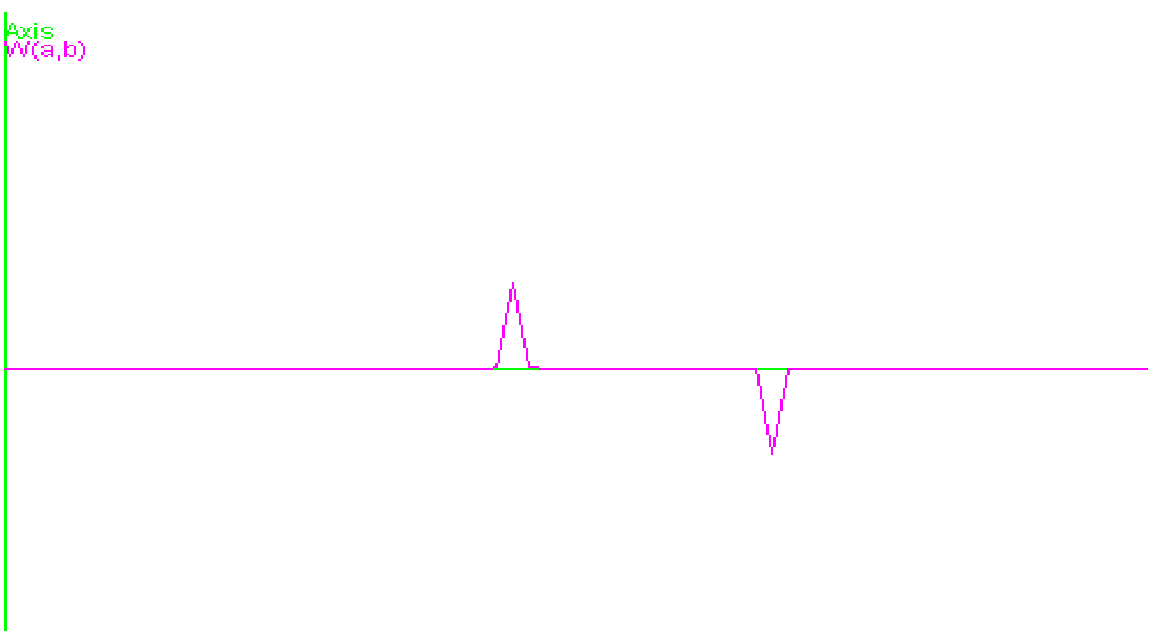


Figure 11: Haar transformation of a broader line

Table 8: Data for Haar transformation of broader line

	Wavelet	Image	Scaling
	Haar	broader	285
First edge	226		
second edge	337		
First edge with Wavelet	226		
second edge with Wavelet	338		



Graph 30: The transformed signal of a broader line using Haar

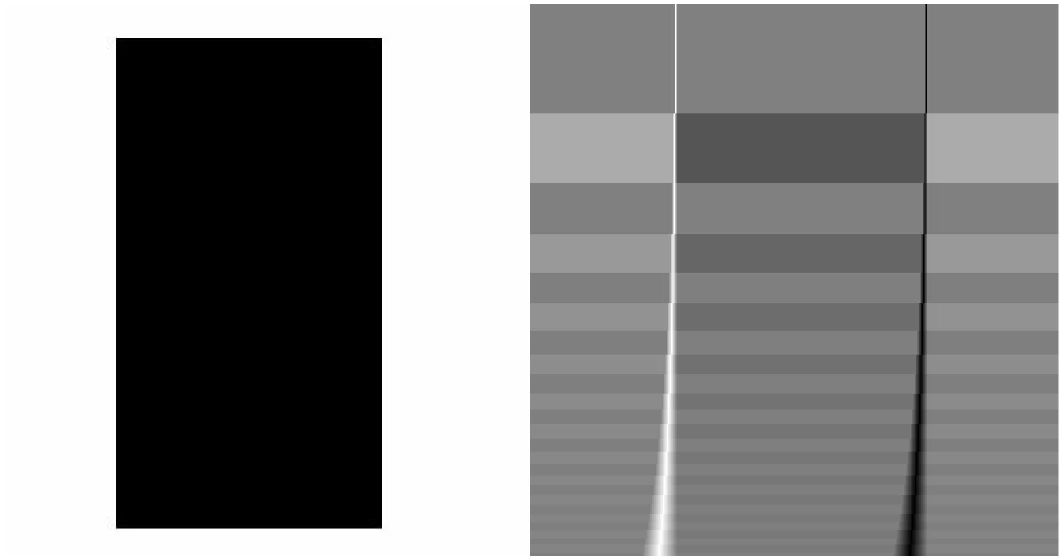
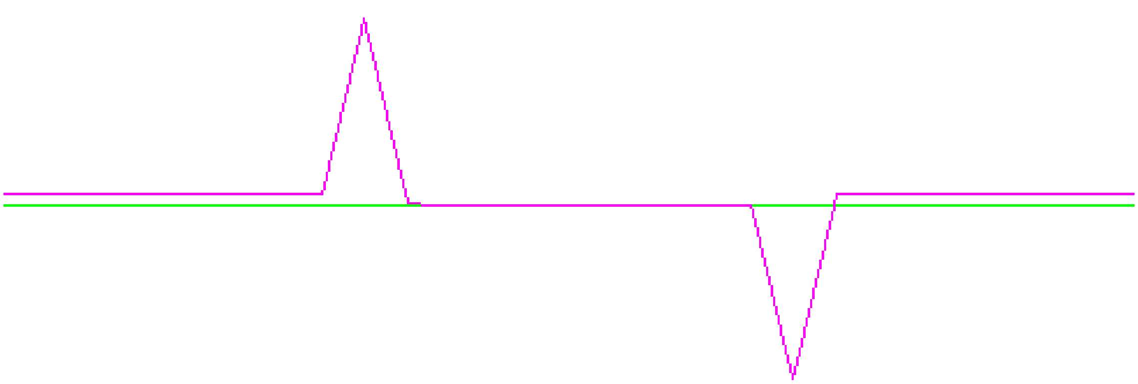


Figure 12: Haar transformation of pretty broad line

Table 9: Data for Haar transformation of simple line

	Wavelet	Image	Scaling
	Haar	prettybroad2	390
First edge	399		
second edge	583		
First edge with Wavelet	399		
second edge with Wavelet	583		



Graph 31: The transformed signal of pretty broad line using Haar

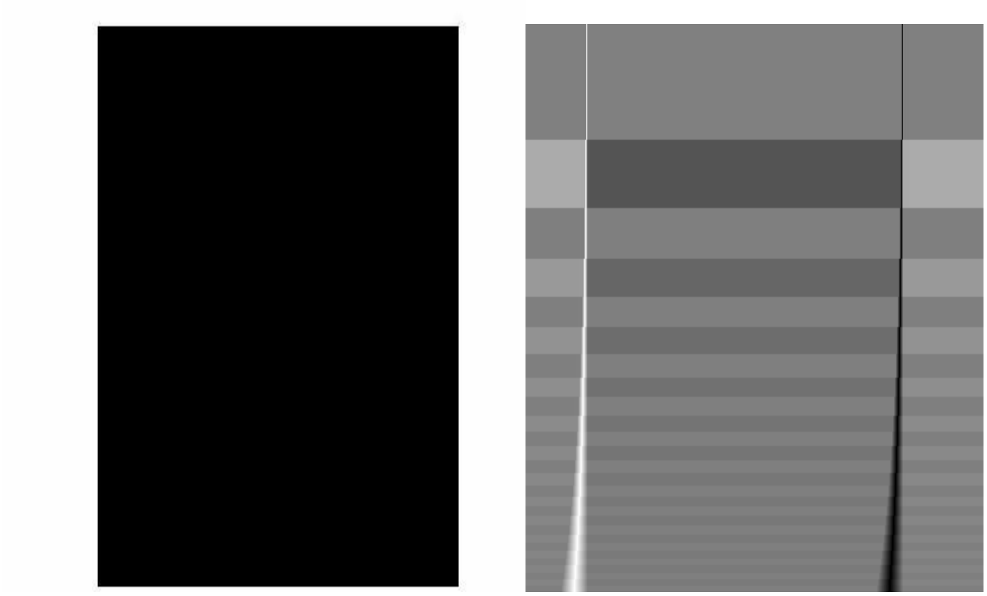
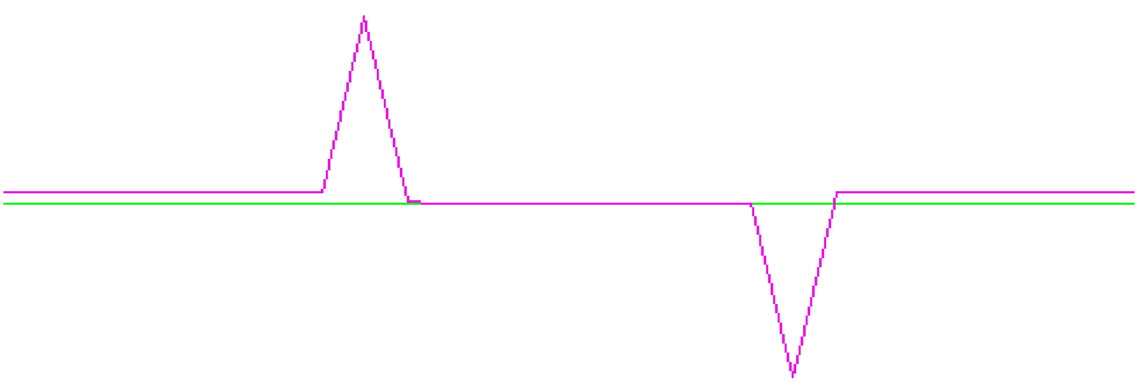


Figure 13: Haar transformation of very broad line

Table 10: Data for Haar transformation of simple line

	Wavelet	Image	Scaling
	Haar	verybroad2	450
First edge	333		
second edge	664		
First edge with Wavelet	333		
second edge with Wavelet	665		



Graph 32: The transformed signal of very broad line using Haar

5.4.2 Mexican Hat

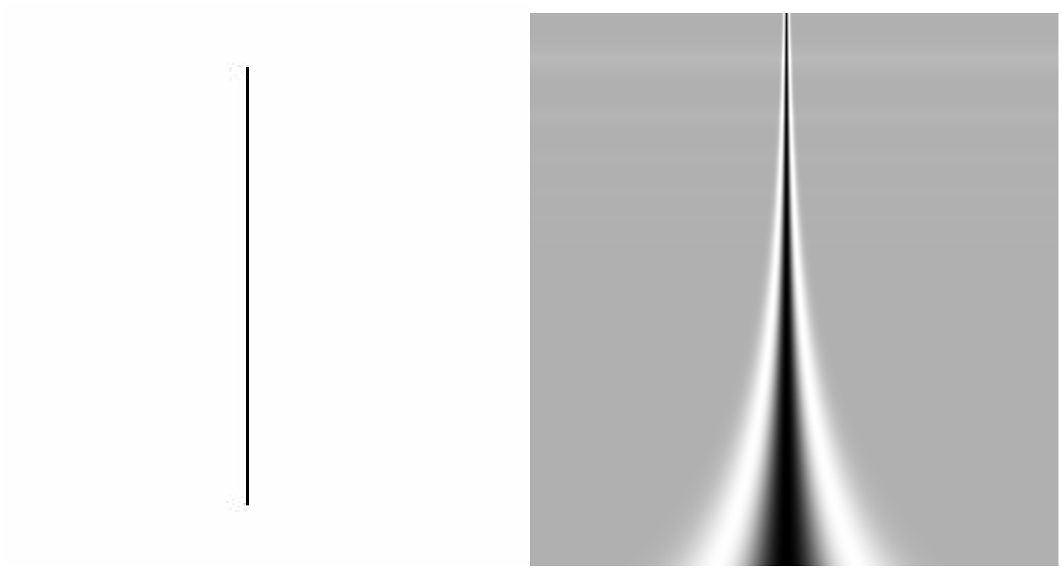
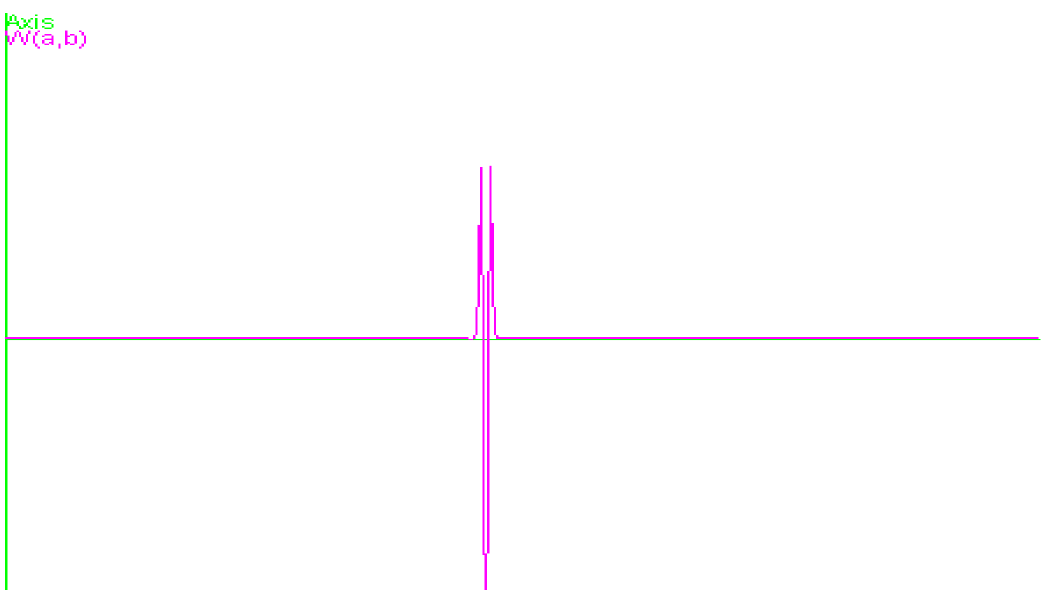


Figure 14: Mexican Hat transformation of a simple line

Table 11: Data for Mexican Hat transformation of simple line

	Wavelet	Image	Scaling
	Mexican	simpleline	10
First edge	207		
second edge	207		
First edge with Wavelet	207		
second edge with Wavelet	207		



Graph 33: The transformed signal of simple line using Mexican Hat

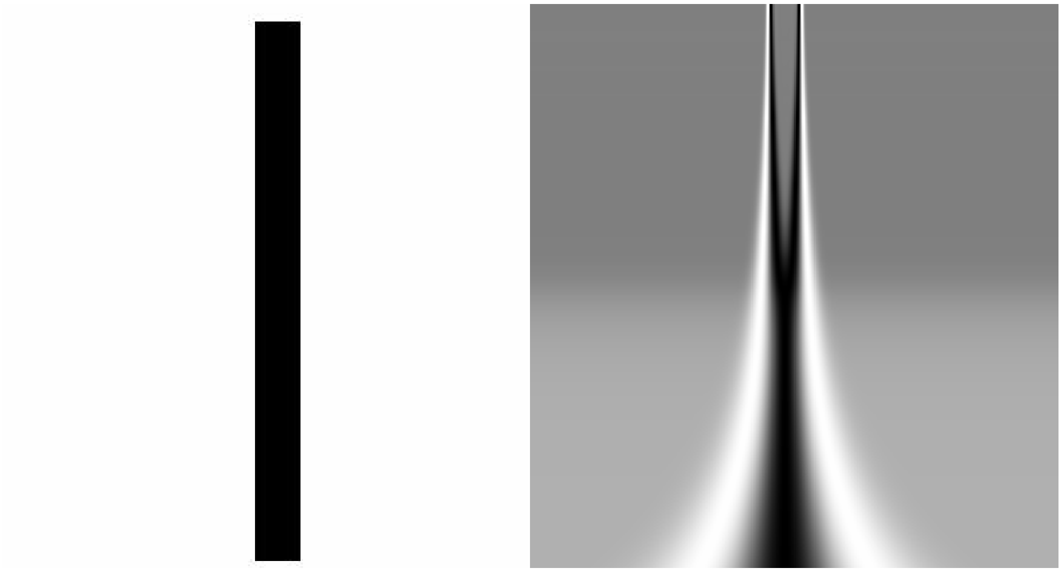
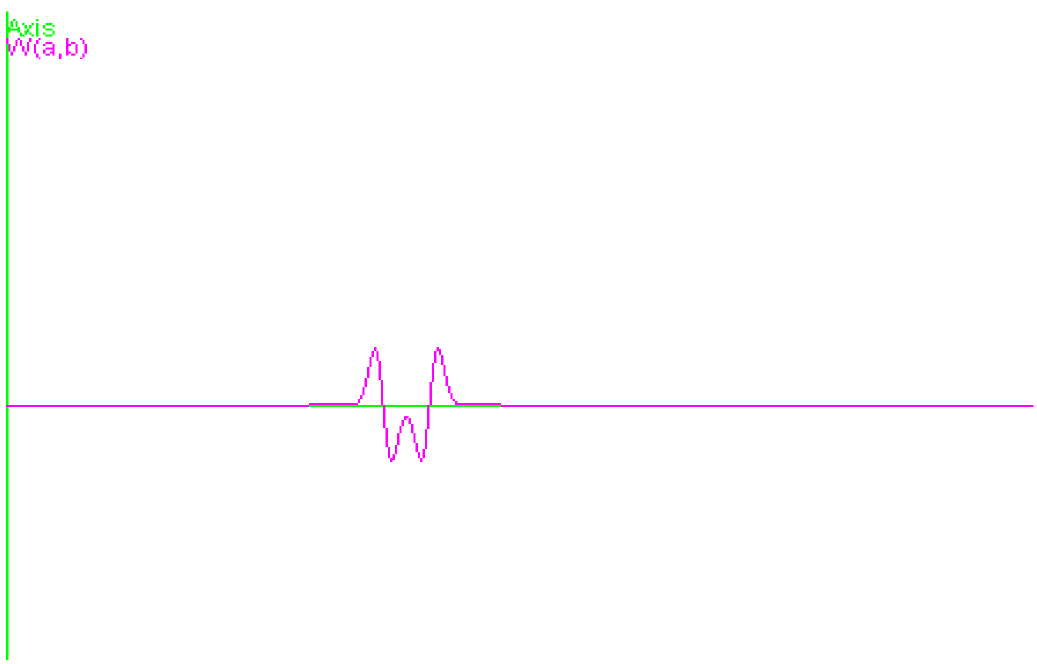


Figure 15: Mexican Hat transformation of a broad line

Table 12: Data for Mexican Hat transformation of broad line

	Wavelet	Image	Scaling
	Mexican	broad	140
First edge	163		
second edge	182		
First edge with Wavelet	163		
second edge with Wavelet	182		



Graph 34: The transformed signal of broad line using Mexican Hat

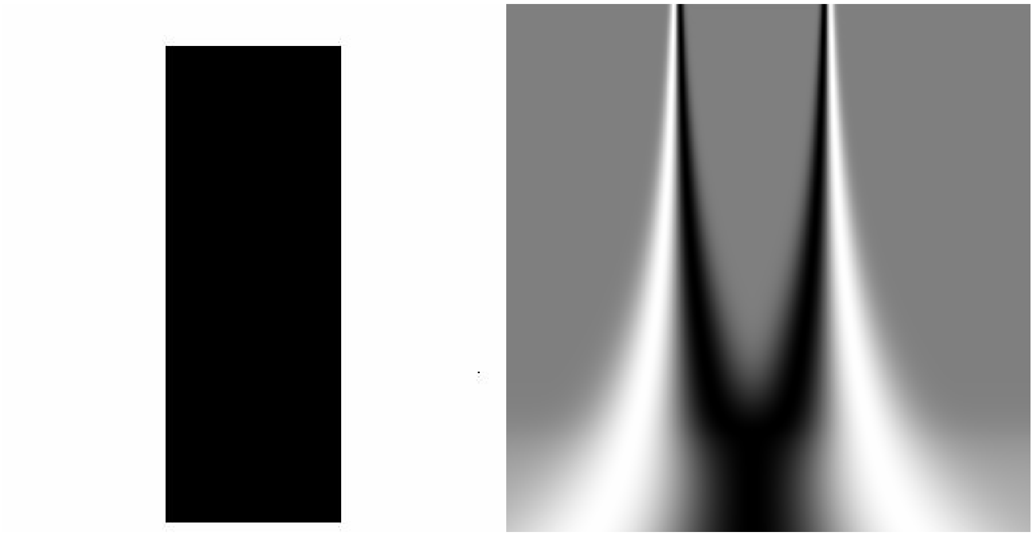
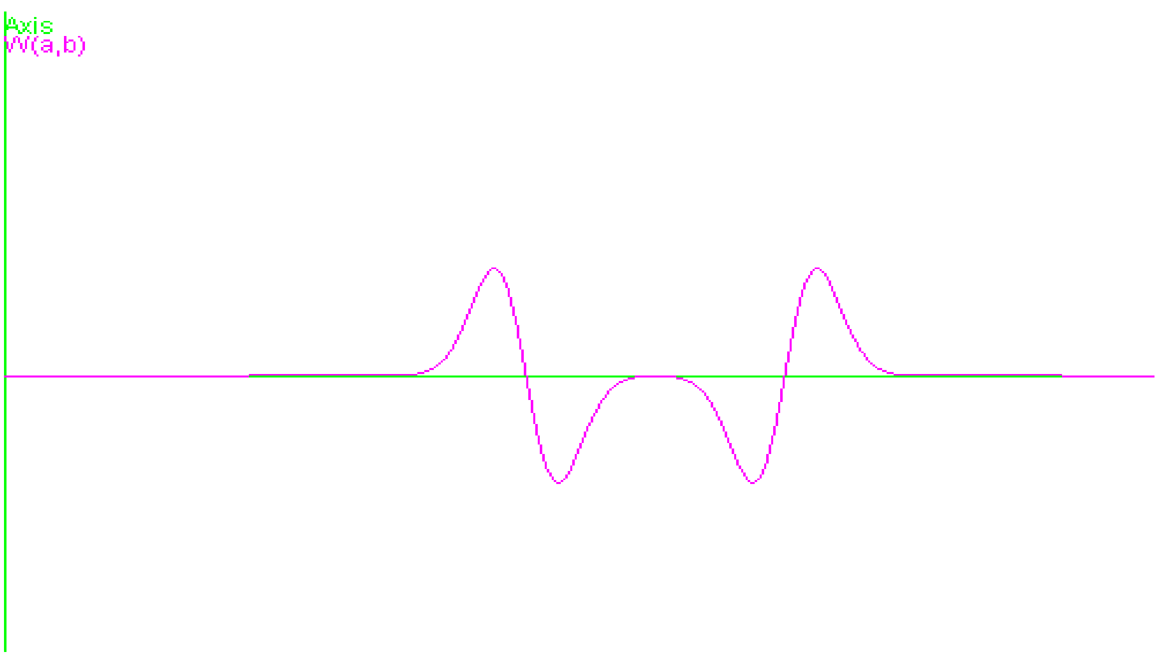


Figure 16: Mexican Hat transformation of broader line

Table 13: Data for Mexican Hat transformation of simple line

	Wavelet	Image	Scaling
	Mexican	broader	285
First edge	226		
second edge	337		
First edge with Wavelet	226		
second edge with Wavelet	337		



Graph 35: The transformed signal of broader line using Mexican Hat

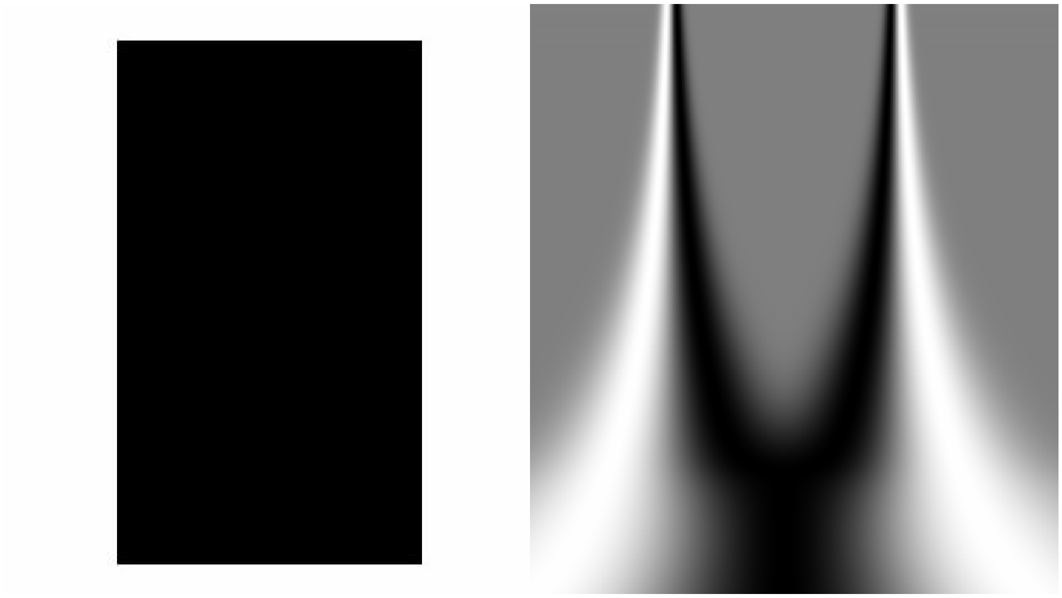
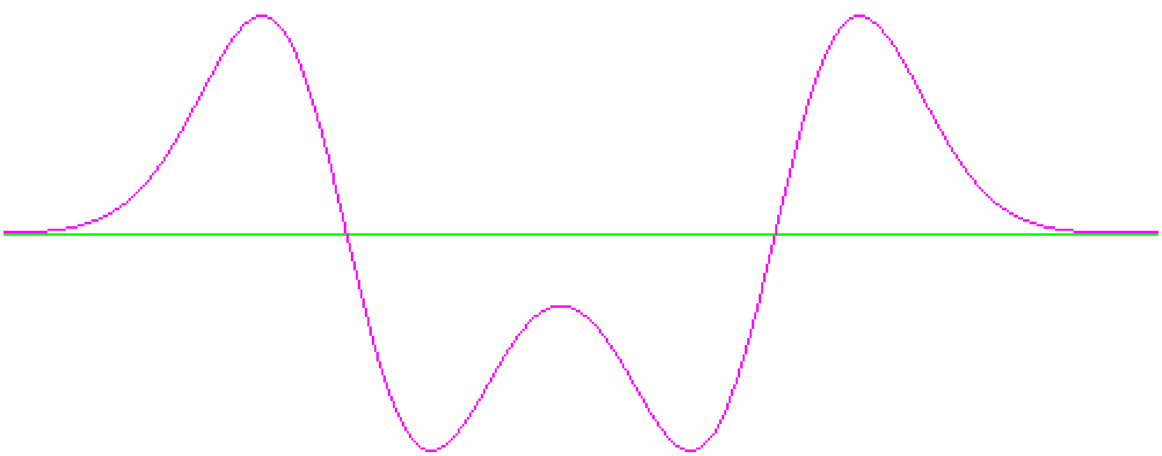


Figure 17: Mexican Hat transformation of pretty broad line

Table 14: Data for Mexican Hat transformation of pretty broad line

	Wavelet	Image	Scaling
	Mexican	prettybroad2	390
First edge	399		
second edge	583		
First edge with Wavelet	399		
second edge with Wavelet	583		



Graph 36: The transformed signal of pretty broad line using Mexican Hat

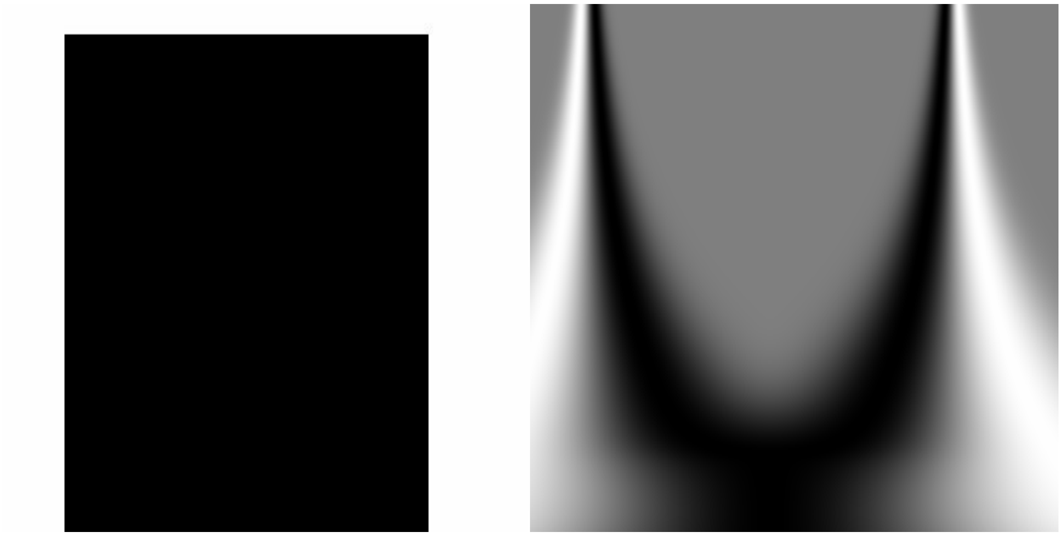
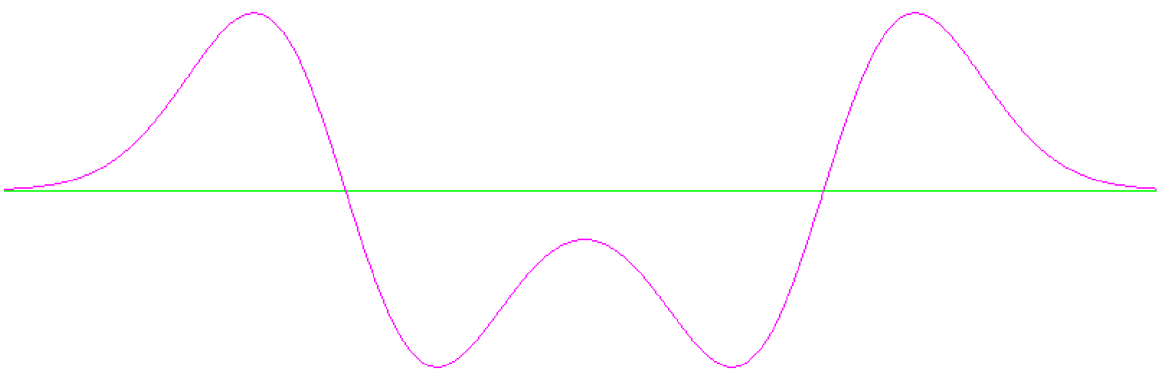


Figure 18: Mexican Hat transformation of very broad line

Table 15: Data for Mexican Hat transformation of very broad line

	Wavelet	Image	Scaling
	Mexican	verybroad2	450
First edge	333		
second edge	664		
First edge with Wavelet	333		
second edge with Wavelet	664		



Graph 37: The transformed signal of very broad line using Mexican Hat

5.4.3 Morlet

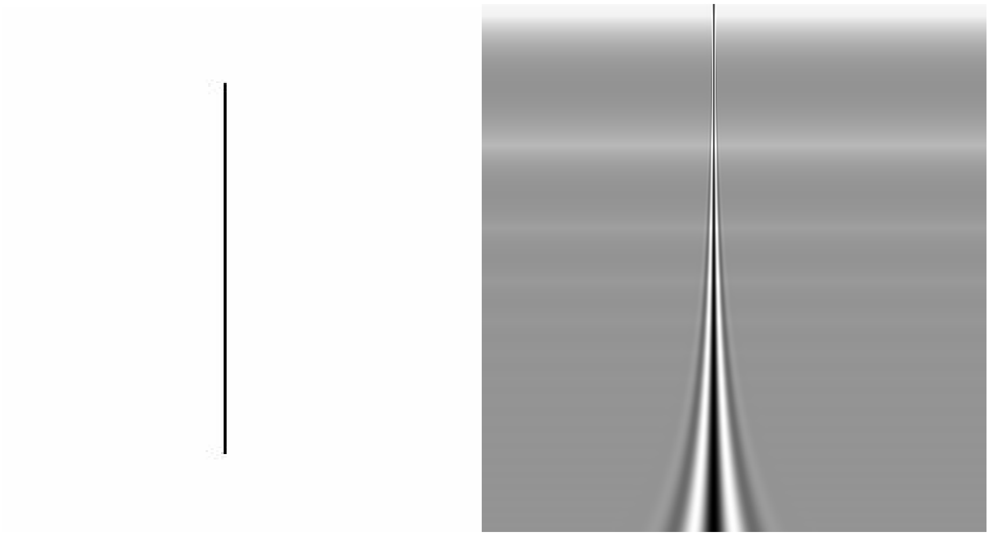
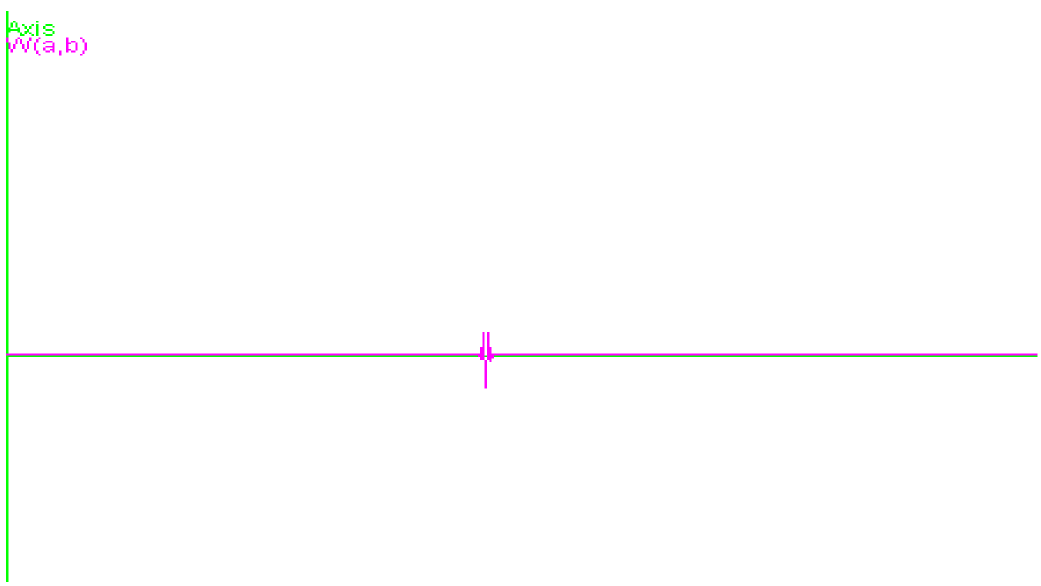


Figure 19: Morlet transformation of a simple line

Table 16: Data for Morlet transformation of simple line

	Wavelet	Image	Scaling
	Morlet	simpleline	50
First edge	207		
second edge	207		
First edge with Wavelet	207		
second edge with Wavelet	207		



Graph 38: The transformed signal of simple line using Morlet

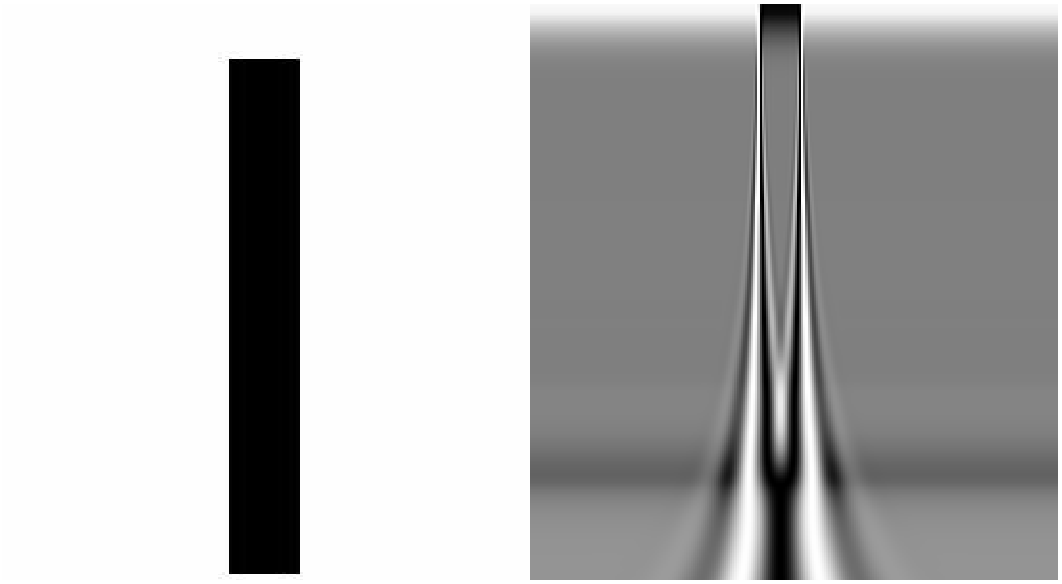
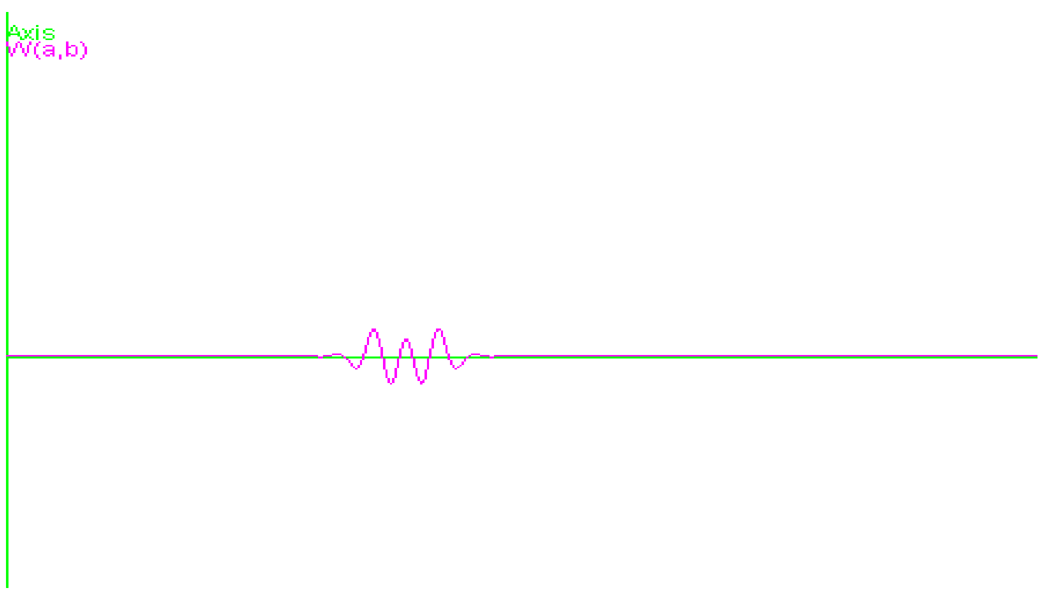


Figure 20: Morlet transformation of a broad line

Table 17: Data for Morlet transformation of broad line

	Wavelet	Image	Scaling
	Morlet	broad	280
First edge	163		
second edge	182		
First edge with Wavelet	163		
second edge with Wavelet	182		



Graph 39: The transformed signal of broad line using Morlet

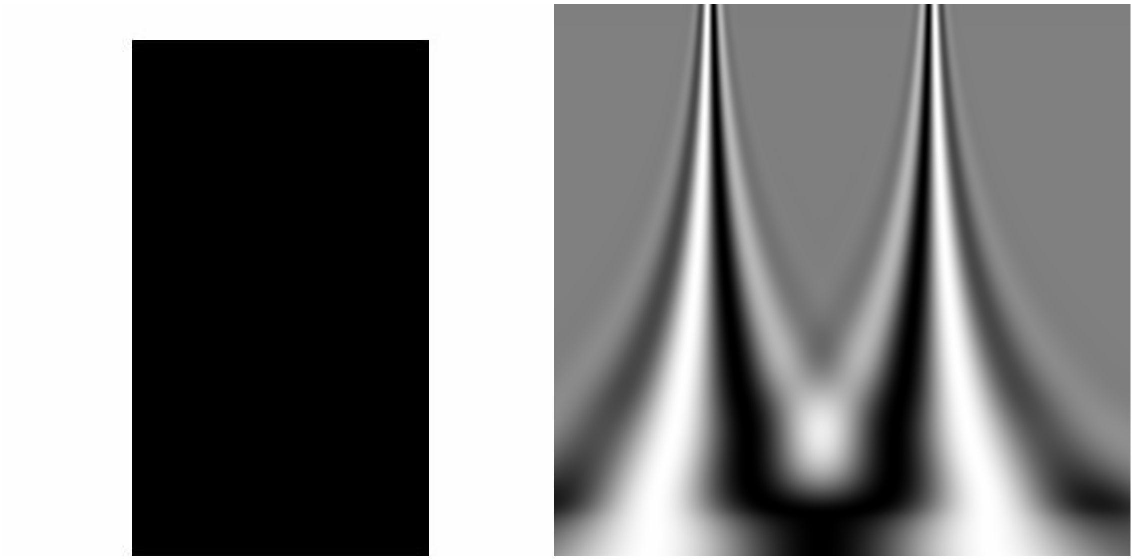
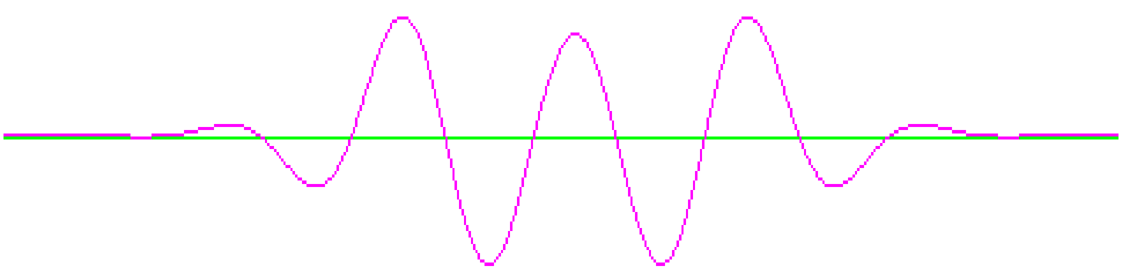


Figure 21: Morlet transformation of broader line

Table 18: Data for Morlet transformation of broader line

	Wavelet	Image	Scaling
	Morlet	broader	450
First edge	226		
second edge	337		
First edge with Wavelet	226		
second edge with Wavelet	337		



Graph 40: The transformed signal of broader line using Morlet

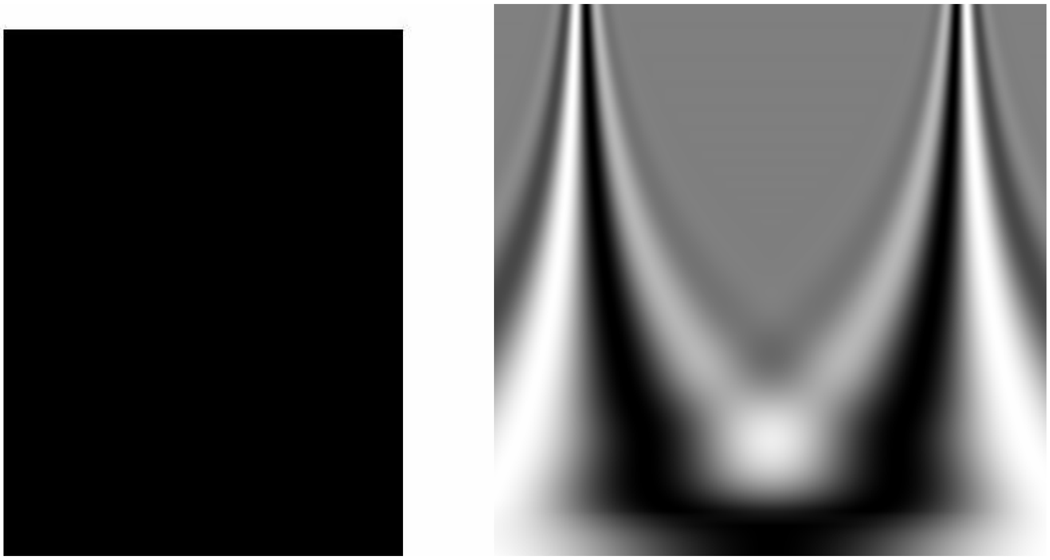
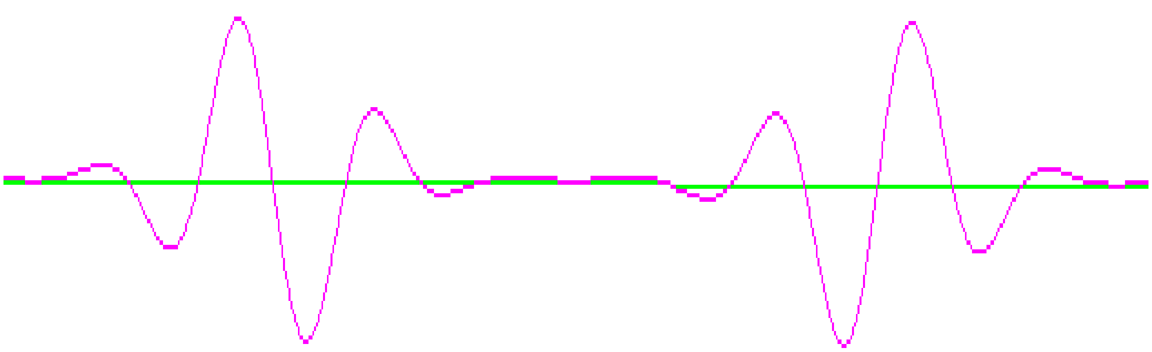


Figure 22: Morlet transformation of very broad line

Table 19: Data for Morlet transformation of very broad line

	Wavelet	Image	Scaling
	Morlet	verybroad2	450
First edge	333		
second edge	664		
First edge with Wavelet	333		
second edge with Wavelet	664		



Graph 41: The transformed signal of very broad line using Morlet

5.4.4 Finding edges

When you look at the graph of the Wavelet in the picture you can see that it moves along a line. Where it crosses this x-axis you will find the edges of the object that you are transforming. You can either read this of the graph or look at the values of the graph to get an even more detailed and accurate answer.

5.4.4.1 Haar

With Haar the graph goes down to the x-axis and lays close to the x-axis for the width of the object and then goes into another peak. Here the edges can be found at the end of the first peak and beginning of the second peak. It is not always easy to determine where the edges are with this Wavelet, and as the results show you can sometimes miss with a pixel or two.

With Haar you do not get the form of a Champagne glass when transforming a line from the original image. It does not matter how broad or narrow the object you are trying to find is. The transformed images from Haar always look like a distorted image of your original image. The values used for the scaling was mainly based on the scales used when transforming with Mexican Hat. This may account for some of the inaccuracy in finding the edges, but more testing with different scales did not show an improvement in the accuracy. Without the reference in scaling from Mexican Hat there would be no way of determining what the scaling could be when using the Haar Wavelet. On the other hand you could probably use one scaling for most any object of any width as long as it was narrower the object, because Haar sharp and distinct edges.

5.4.4.2 Mexican Hat

It is very easy to find the edges when the Mexican Hat Wavelet is used, since it only crosses the x-line twice, and this is where the edges are. If you look at the values of the graph you can easily determine where the edges are, down to an accuracy of a pixel. If x = 90 has an y = 24 and x = 91 has an y = -30, then you know the edge must be somewhere between x = 90 and x = 91. This also makes it easier to calculate the correct edge values.

5.4.4.3 Morlet

With Morlet you will find the edge the same way, and just as accurate as with the Mexican Hat Wavelet. But if you look at the Morlet Wavelet you will see that it not only crosses the x-line twice, but ten times. This makes it a lot harder to determine the edges since you do not know which crossing is the correct one. An advantage with Morlet however is that if you are trying to find the edges of a really narrow object you can use a higher scaling with Morlet than with Mexican Hat. With a higher scaling it is easier to remove noise in the image.

5.4.4.4 New scale

When the graph crosses the x-line the values might be for example x = 100 y = 23 and for x = 101 y = -30. This means that the edge will be somewhere between x = 100 and x = 101. In an attempt to get y values closer to zero we tried to look for a more accurate scale. All values for each scale from a transformed signal are examined. Then we looked for the scaling which gave y values closest to zero. Finding the scaling that gave us y values did not have a great impact on the accuracy on the edge detection. It did not matter if there was a small variance in the scaling, as long as it was smaller than the objects width.

We wanted to see if there was a correlation between the new and more accurate scales that we had found and the scaling which gave the maximum energy. This was to see if there might be a possibility to find the scaling by knowing the maximum energy instead of the width of the object.

Table 20: The value at the edges of different lines using the new scale

width of line	max energy scale	new scale	left edge value	right edge value
19	290	219	0,0718	0,3224
33	360	268	1,8789	1,3416
94	485	366	0,001	1,2826
126	521	395	0,5333	0,2017

The correlation between the new scale and max energy scale is 0,999703. This number is calculated using Microsoft Excel. Here we can see that there is an almost perfect correlation between the most accurate scaling and the scaling which gave the max energy.

5.4.5 Discussion

After testing these three Mother Wavelets on the Champagne theory, Mexican Hat sticks out as the supreme Wavelet to use. Using Haar there is no way of determining the best scale to use and the accuracy is less than with the two other Wavelets. So with the Haar Wavelet ruled out we were left with Mexican Hat and Morlet. They both find the edges with accuracy down to one pixel, and you can find the scaling to use on both of them. Difference is that Mexican Hat has fewer waves and is thereby simpler. With Morlet it is not as obvious which one of the crossings is the one that states the edges. With Mexican Hat this is very easy since it only crosses the x-line twice and there you have the edges.

If you need to find the edges on a really narrow object, the Morlet offers a chance to use a higher scaling. This might be an advantage if there is a lot of noise in the original image, since a higher scaling is more likely to rule out single pixels of noise. This advantage is on the other side only with really narrow objects with only a few pixels width. When it comes to finding the thickness in a bone on x-ray images it is hard to say how broad the bones will be. Therefore this advantage has a minimal effect and does not make Morlet a more usable Wavelet than Mexican hat.

A correlation was found between the scaling which produces the maximum energy and the scaling which gives the most accurate result. This might be used to produce a table, matrix or formula to find the best scaling based on the maximum energy scaling. This way you do not need to look at the transformed image first, or know the width of the object beforehand. This link between the maximum energy and the best scaling was found when transforming single object with two distinct edges. With more diffuse objects with multiple edges there might not be a correlation anymore.

5.5 Correlation

To see if there were any connection between the width of the object we were transforming and the scaling used, we ran a correlation test. We already knew the width of the objects that we were testing on, and we used the scaling parameters we found from the champagne glass theory.

Table 21: Scaling for different widths using champagne glass theory

Object Width	Scaling
1	10
19	140
111	285
184	390
331	450

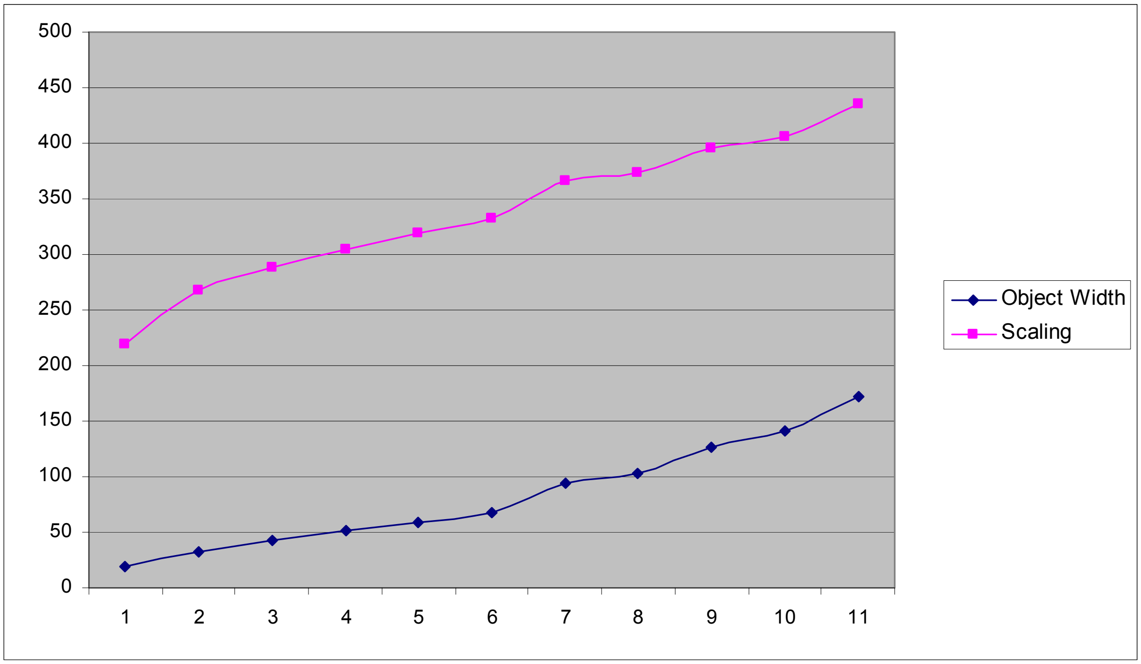
We used results gathered from the Mexican Hat Wavelet, and the correlation factor was 0.927681. When the number 1 means that there is a perfect correlation, 0.927681means that there is definitely a connection but it is not perfect. There are many reasons why it is not a perfect correlation. One is the obvious that there are very few test widths, and that the spacing between the different widths is a bit arbitrary. Another reason why there might not be a perfect correlation is that we have included the object with width 1. There is no champagne glass created when transforming this object. So the scaling for this object is chosen for no other reason than the fact that we can find the edge very accurate with it. Since the scales are picked out by eyesight in the bottom of the Champagne glass there might be the possibility of human error. The scales are not based by any mathematical formula but by human eyesight and intuition. The fact that there are more than one scale that are suited to find the edges, makes it irrelevant to find that one perfect scaling.

In an attempt to improve our correlation number we created many more objects to transform in order to get more data to our correlation function.

Table 22: Scaling using champagne glass theory on different widths

Object Width	Scaling
19	219
33	268
43	288
51	305
59	319
68	333
94	366
103	374
126	396
141	406
172	436

We dropped the object width of 1 and added more objects with more level spacing between them. Now the correlation factor was now 0.97146064 which is a clear improvement from earlier, but still not perfect. If you look at the graph you can easily see that they follow each other.



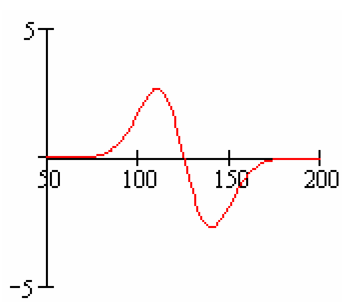
Graph 42: Correlation between object width and scaling

5.5.1 Forecast

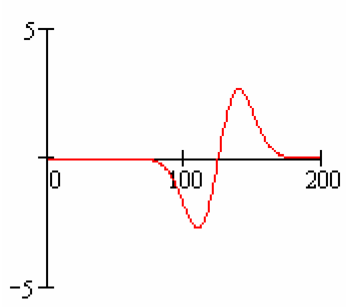
Based on the data used to find the correlation we used the Excel function Forecast to see if we could somehow calculate a scaling for a random width. This formula worked pretty well for objects with widths over 30, and not so well for Widths under 10. Reason for this was that our data starts with a width of 19 and a scaling for 219. This means that when the mean linear graph, from which the forecasted number is picked, is created, it does not take into consideration the fact that a width of 1 equals a scaling 10. This means that since we excluded the width of 1, the correlation of our data improved but the plunge in the scaling graph that should have been there was removed, thus making the forecast formula useless for really narrow objects. On the other side the forecast formula worked really well on objects with a width of 30 or more. Based on the width of the object we were now able to calculate a scaling that would find the edges. In order to use the forecast you would have to know the width of the object you want to find the edges on. In this assignment we wanted to find the edges in order to be able to find the width of the object. This means that forecast can not be used for finding the correct scaling since we do not know the width of the object beforehand.

5.6 Using scale decided from max total energy

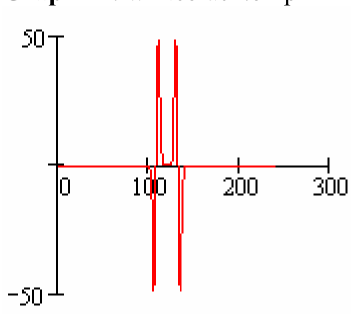
We now knew that when the Wavelet was just as big as, or a little bit smaller, than the object, it would result in finding the edges. The problem was to find a way of finding the correct scaling. This is when the scale that gave the max energy came back into use. With this point as a reference one could now use it to find the best suited scaling. MathCAD here provided us with a method to quickly find the scaling. When the scale using max total energy is found, one lower scale is tested until the whole Wavelet is inside the object. That is, when the Wavelet transformed is approximately zero between the two edges. The following graphs are results obtained using MathCAD.



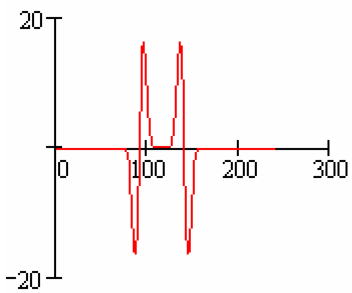
Graph 43: blackwhite.bmp



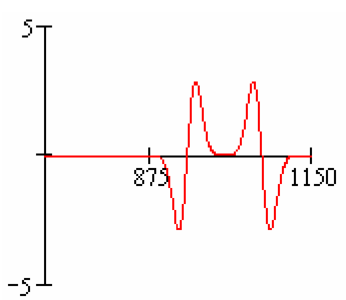
Graph 44: whiteblack.bmp



Graph 45: broad.bmp



Graph 46: broader.bmp



Graph 47: prettybroadline4.jpg

Table 23: The edges found using the new scale in MathCAD

Original			Transformed in MathCAD		
Image	**Start**	**End**	**New Scale**	**Start**	**End**
blackWhite.bmp	125	-	12	125	-
whiteBlack.bmp	125	-	12	125	-
broad.bmp	108	131	1	108	131
broader.bmp	91	139	5	91	139
broadest.bmp	938	1062	8	938	1062

5.6.1 Discussion

Knowing what scaling to use was discovered by examining the Champagne glass theory, but that theory offered no solution on how to find the scaling except looking at the transformed images. Now with the help of MathCAD and the scaling which gave the max energy we have a method for calculating the best suited scaling. This solution has only been tested in MathCAD and has yet to be implemented and tested in Java. It has only been tested with the Mexican Hat Wavelet and we do not know what results Haar and Morlet would give. Using this method helped us find the edges exactly where they are. When the whole Wavelet is inside a line, the behaviour of the transformed signal is the same as it is for images with only one edge.

5.7 Convolution

Convolution is a common method for smoothing out noise in signals. With the help of convolution we might be able to make the signal from the images much easier to find the edges on. This is not very important when you have a sharp clear object on a one colored background. Once you get images like the x-ray images there will be a lot of noise in the signal and this might affect us when trying to decide the edges.

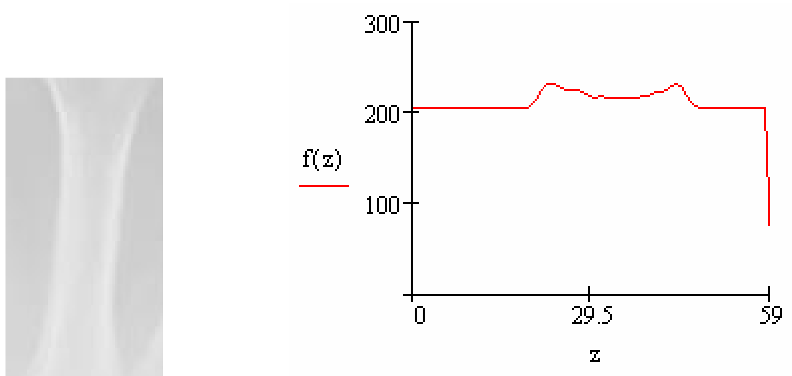
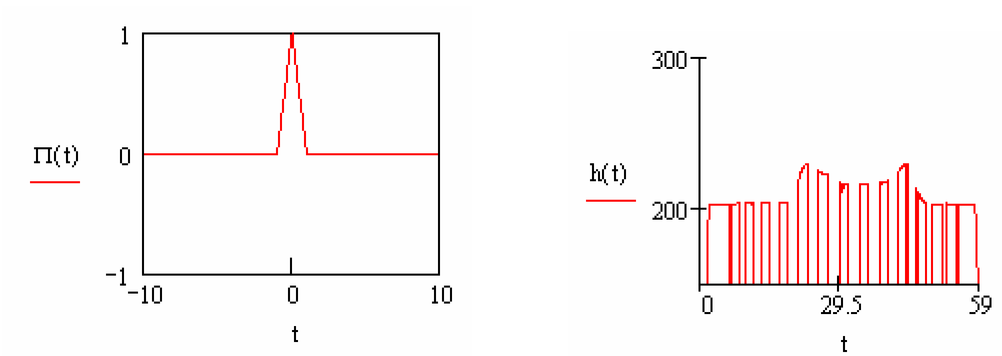
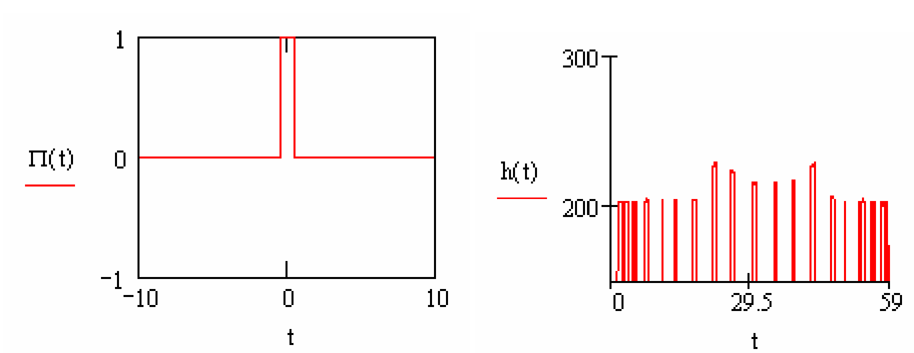


Figure 23: Original image of a bone and the function obtained from the image



Graph 48: Convolution between the original signal and a triangular impulse.



Graph 49: Convolution between the original signal and a rectangular impulse

If we ignore the empty interval (see h(t)) we can see that by using convolution on the original signal gives a new, smoother signal. With the help of interpolation the empty spaces between the pulses can be erased.

5.8 X-ray images

5.8.1 Overview

X-ray images are the kind of images used for observing the bone structure in the human body. These images need to be digitalized in order for us to work on them with our computer. The images should then have a high resolution and a minimum loss of information in them. This can for example be .bmp images or other high quality formats. We got an image from the Sørlandet Hospital in Kristiansand showing the bone structure of a hand. It is with this image that we are supposed to find the edges of the bone for further being able to determine the width of the bone.

5.8.2 The image

Figure 24: X-ray image used for further testing

The image above is the one we got from the Hospital. With so much information in the image we found it best to cut out the piece that was essential for us to simplify our task. The only part we needed was one of the fingers from the middle of the hand. This is the first finger bone after the wrist.

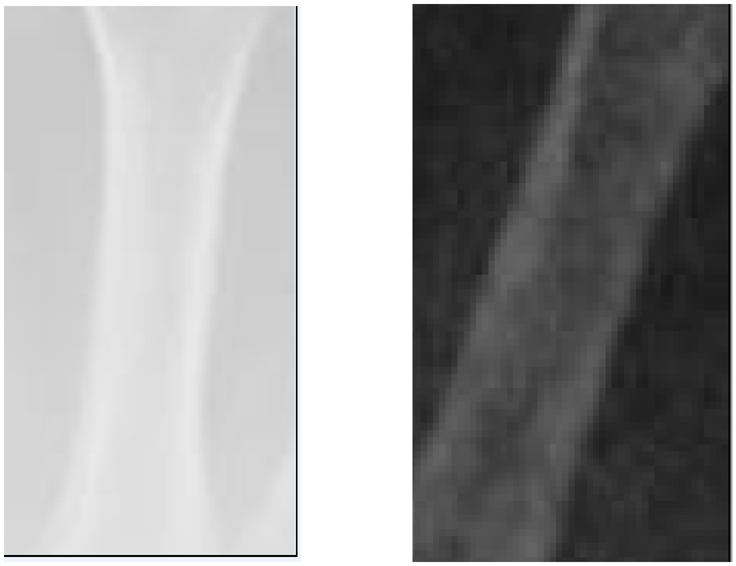


Figure 25: The difference when using dark folio on top of image

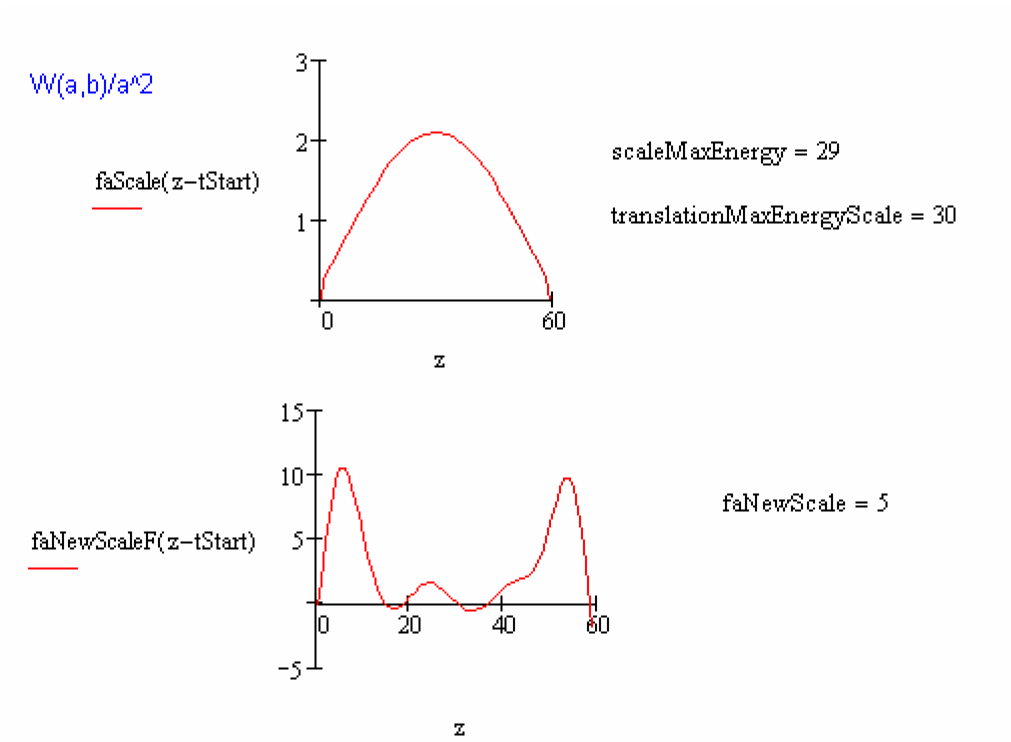
When we had these images of the bone of only one finger we had eliminated a lot of disturbing elements. On one of the images some sort of dark folio is placed on top of the x-ray image to undertone the light grey area.

5.8.3 Defining the edges

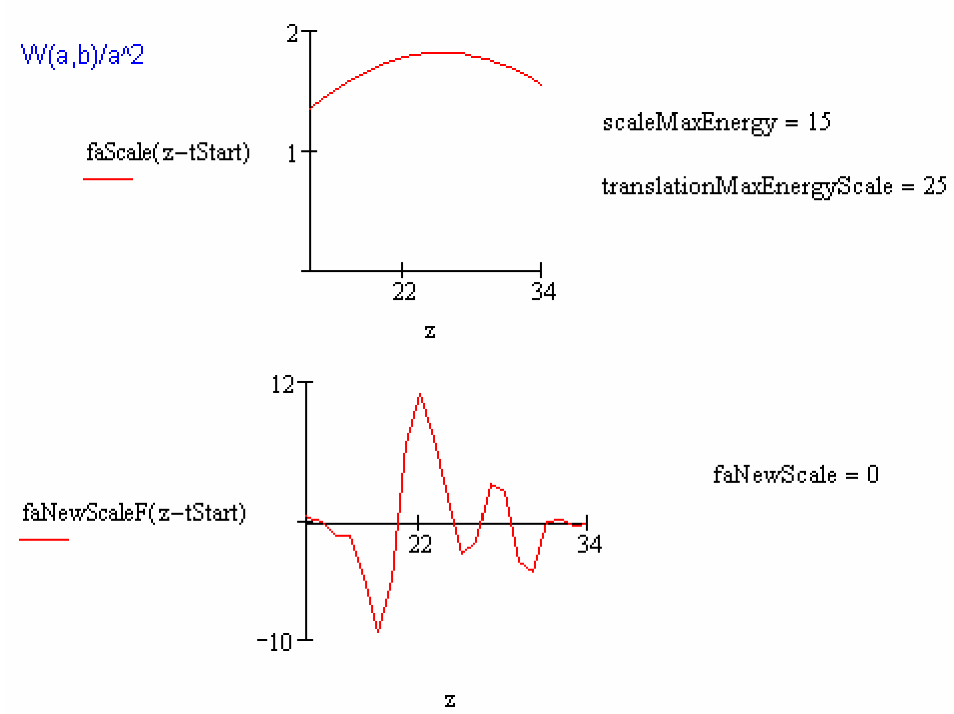
When you look at the image of the finger it is pretty clear where the outer edges of the bone structure goes. The bone is hollow like a tube and on the inside it is filled with a fluid. This is where the real problem arises. The inside is diffuse and hard to determine the crossing between the bone and the fluidly mass on the inside. To be on the safe side a discussion with medical personnel to define the correct edges is needed.

5.8.4 Finding the edges with MathCAD

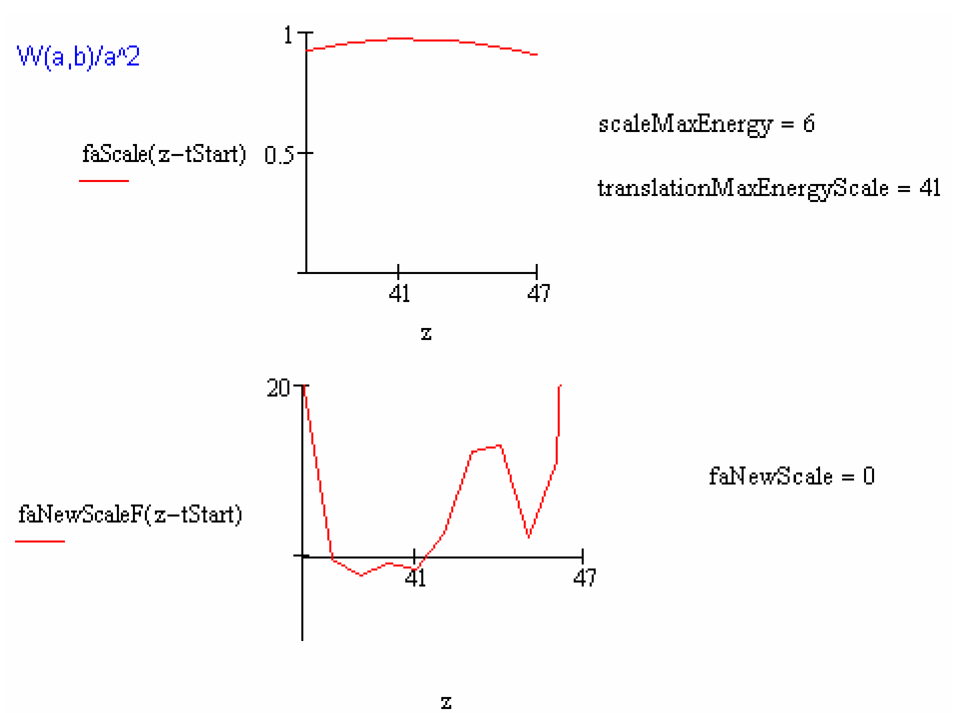
We were given a program from our mentor Professor Hogstad, in MathCAD in which we could try to find the edges of objects. We tried the program on the x-ray image of the finger with the light grey background



Graph 50: The entire x-ray image transformed in MathCAD



Graph 51: Left side of the bone transformed in MathCAD



Graph 52: Right side of the bone transformed in MathCAD

MathCAD offer the possibility to transform the whole image or just the left or right part of it. Above are the results after transforming with the different options. The scaling used is decided by MathCAD based on the scaling which provides the maximum energy. The program finds the scaling with the maximum energy and then test one and one scaling lower till it finds a scaling which allows the Wavelet to get inside the object.

MathCAD provided different results when you transformed the whole image at the same time and when you concentrated on one side. When you only transform one side of the image you get the most accurate results. It is still not easy to determine the inner edges, but the outer edges of the bone can be plotted quite accurate.

Convolution could maybe help clean up the signal some, and this might provide more accurate answers. This was however not done in these tests.

5.8.5 Finding the edges with Java

We still did not have a good method for finding the best suited scaling except looking at the transformed image and use the Champagne glass theory. First we tried the bone image without the folio on it.

5.8.5.1 Bone image without folio

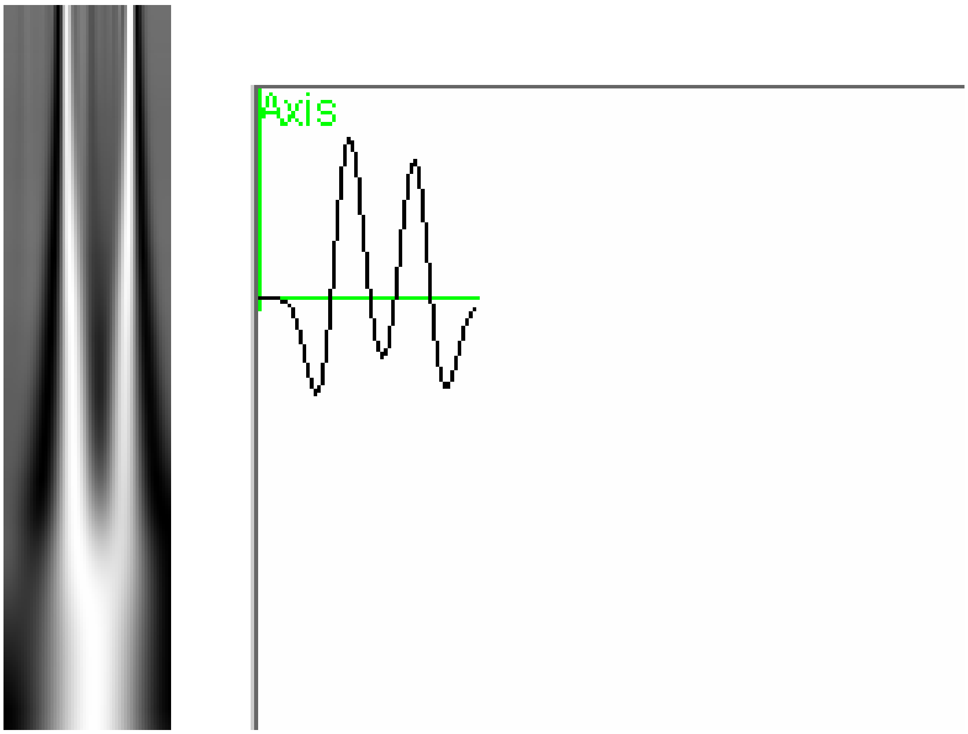
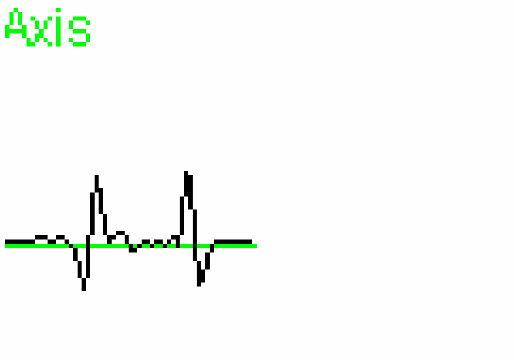


Figure 26: The transformed image and graph based on champagne theory

That gave us a scaling of 165 which resulted in the graph next to the transformed image. Looking at this graph and looking at the data behind the graph gave us suggested edges at pixels 19 and 46. This cohered with the edges of the bone structure, for the outer edges on each side of the bone. There was however no way of determining the inner edges of the bone. According to the Champagne glass theory to find the edges the scaling had to be at a scaling which was narrower then the object. To find the inner edges the object would in this case than be much narrower then the whole bone, and so the scaling that could find the outer edges might not work sufficient enough to find the inner edges.

The scaling was set to 20 in hope that this might find the inner edges. This number was set from experience that a object of about 5 pixels should have a scaling at about 20. The attempt did not give satisfying results, and it seemed that only more noise had been admitted into our graph. It was now not only hard to find the inner edges, even the outer edges now diverged from the actual edges.



Graph 53: The transformed graph using scale 20

5.8.5.2 Bone image with folio

An idea was to try an image with the grey area under toned in order to get a more distinct transition between the bone and the grey area surrounding it. From the image it looked as the fluidly mass inside the bone was toned down as well, which could be to our advantage.

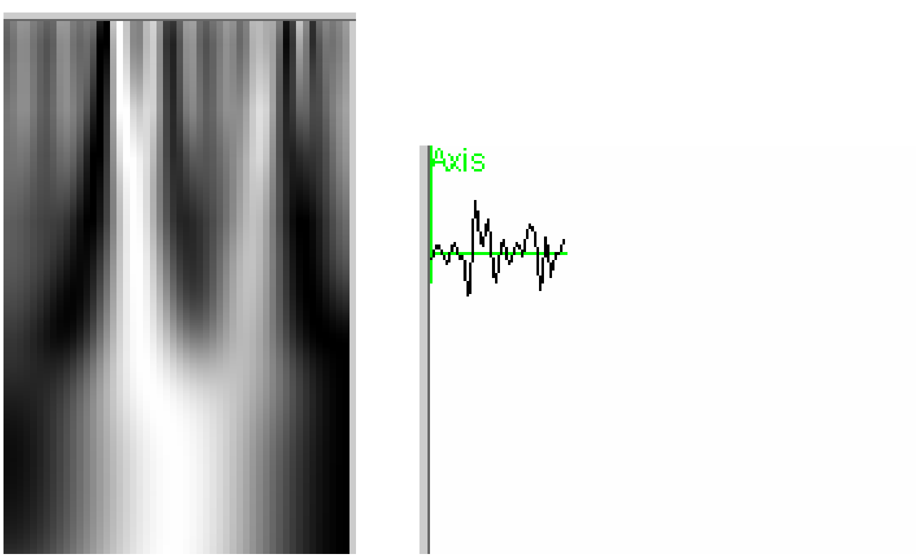


Figure 27: The transformed image and graph based on champagne theory

The graph shows that again a lot of noise has affected the graph, but this time the edges cohered with the actual edges. The pixels given by the graph stated that the outer edges of the bone on this image were 16 and 40. There was also a indication that the inner edges of the bone was 23-24 on the left side and 36 on the right side. This to seem to cohered with the actual edges.

5.8.6 Discussion

There are many problems when determining the edges on actual x-ray images. When you have a clear sharp line it is fairly easy to see the edge, but at the x-ray images the edges are very diffuse. There is also the fact that we are looking for many different edges in the same image. You have the outer edges of the bone and the inner edges of the bone. The outer edges turned out to be easy to find with the help of Champagne glass theory. On the inner edges one needed a narrower scaling in order to find the edges. It was also necessary to use the image with the folio on since the other image was to light. It seemed that it was necessary to transform the image twice. First to find the outer edges with a broad Wavelet that would eliminate noise. Then, once you knew the outer edges you could use a narrow Wavelet to find the inner edges. This would create a lot of noise in the graph, but since you already knew the outer edges you knew what to look for.

An option would be to make the images larger before transforming them in order to make the width of the object larger. This would enable you to use a wider Wavelet to find both inner and outer edges. When we tried to enlarge the images the quality of the image decreased strongly, making it impossible to use.

The Wavelet used to find the edges on the x-ray was Mexican Hat. This one was used since it had provided us with the best results for the Champagne glass tests.

When you have a black object on a white background the Champagne glass image is black with white coloring around it. Because the bone is white on a darker background the coloring on the transformed images will also be the opposite.

6 Discussion

We have tested several theories on how to find edges on images. In the beginning there was a hope that the maximum total energy would help us find the edges. This theory was eventually concluded as non functional. Not even with the help of weighting by area or by the heights of maximums did we get the wanted results from using max energy scaling. At one point there seemed to be hope, but it turned out as more object widths was explored that it only worked on objects of a certain width. The theory was only tested with the Wavelet known as Mexican Hat, and other Wavelets might have proved to give a better result. We did however not find anything that indicated that another Wavelet would work better. And when the results diverged as much as they did from the actual edges a decision was made to abandon these theories surrounding max energy.

A method that proved much more fruitful was the theory we decided to call the Champagne glass theory. The reason for this name was that the transformed image on an object resembled a champagne glass. If we used the scaling at the bottom of the champagne glass on the transformed image, we got a scaling that was almost as wide as the object that we wanted to find the edges on. This proved to give quite accurate results on finding the edges. The scaling was now directly related to the width of the object for which we wanted to find the edges. At small objects width there will still be the problem that the Wavelet will be narrow, which again may cause noise problems. The scaling used was picked out on eyesight from the transformed image and not calculated. This was a problem since for every new image we had to first look at the transformed image, pick out a scaling and go back and implement this in our code. We found a correlation between the image width and scaling, and with the help of the forecast function in Excel, we where able to determine an appropriate scaling. This again provided that we knew the width of the image that we wanted to find the edges on. Since the point with finding the edges, was to help finding the width of the object, we would not be able to know the width beforehand. In the end an idea to find the correlation between the max energy scale and the champagne scale was introduced. Fortunately we did not have the time to implement and try out this. In MathCAD another possible solution presented itself. Here we could find the max scaling and MathCAD could calculate down one scaling at the time until the Wavelet was narrower then the object.

When we had found the right Wavelet to use and a method for finding the edges on sharp objects, time was come to test it on the x-ray images. X-ray images are much more complicated than black objects on a white background. There are no clear sharp edges, but instead the edges are foggy and diffuse. There are also a lot of edges, and we quickly realized that we had to cut out the part of the image that we wanted to transform over. Another solution to this problem would of course be to make an algorithm that would only transform over a certain part of the image. The fact that the bones on the images were so diffuse caused many other problems. The outer edges of the bone were maybe not clear and sharp but still you could see where the edge went. On the inside of the bone it was hard to tell the bone from the fluidly mass inside the bone. Here medical personnel would have to assist us by telling us what is defined as bone and what is defined as fluidly mass.

Finding both the outer edges and the inner edges of the bone we had to transform the picture twice. One time with a broad Wavelet that would eliminate all the noise, and easily find the outer more distinct edges. Once we were sure about the outer edges we transformed the image again, this time with a much smaller Wavelet. Since we already knew the outer edges, we knew what to look for in order to find the inner edges. The first image we tried, the one without the folio turned out to give to much noise to our graph and also it was inaccurate. Thought was that because the background was light grey and the bone was light there would not be a distinct line between the bone and the background. The mass inside the bone also had a grey tone to it. When the folio covered the image the actual bone was enhanced compared to the background. With the folio on there was still a lot of noise in our graph, but it was much more accurate and possible to find the edges.

Convolution might solve some of the problems with noise in the images, and thoughts of running a convolution algorithm over the image before transforming it. This would smooth out a lot of the noise and maybe make it easier to determine the edges. A problem with convolution would be that the inner edge of the bone is so faint that it might be smoothed out by the convolution algorithm.

7 Conclusion

The task of this assignment was to find a method for finding the edges in medical images with the help of Wavelets. Beforehand we got information from our mentor that based on earlier projects Mexican Hat, Morlet and Haar would probably be the most suited Wavelets. Through the project we have tried out those three different Wavelets, and tests indicated that Mexican Hat was easy to use, and would produce wanted results.

X-ray images are complicated images with weak and diffuse edges and lots of noise in them. Therefore in order to find a good method for finding edges, tests where first conducted on simple sharp objects. Several methods for finding the edges were tested, but the one that proved must fruitful we named "Champagne glass theory". The basis in this theory was that the best scaling to use for finding the edges is the scaling that produce the Wavelet that is just big enough to be covered by the object on which the edges shall be detected.

The "Champagne glass theory" offers no solution on how to find the best suited scaling other than to view the transformed image and pick out the scaling that is in the bottom of the glass. A possible solution is to find the proven correlation between the scaling which gives the maximum energy and the scaling in the bottom of the champagne glass.

Our theories on finding edges with the help of "Champagne glass theory" worked perfect on sharp edges, and tests conducted on the x-ray image proved that we were able to find the outer edges of the bone. The inner edges are more diffuse, and although they were detected, we can not yet conclude whether or not the method is good enough.

Further work to be done would be to find a method for calculating the scaling to use, and conduct more tests. In cooperation with medical personnel the inner edges needs to be defined. A method that lets you transform only a selected area and not the whole line across the image would also be needed.

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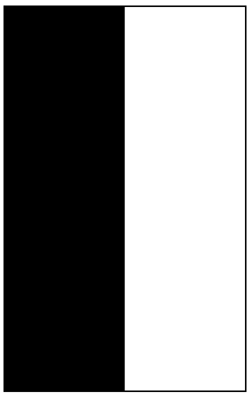
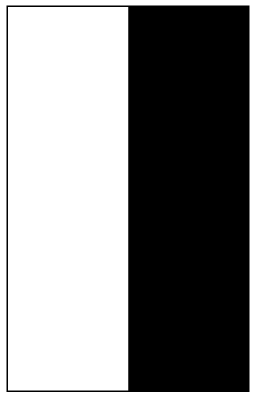
Attachments

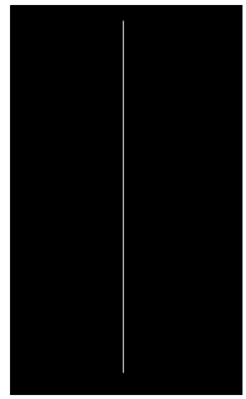
Images used for testing:



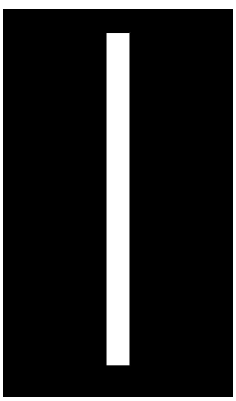
blackWhite.bmp



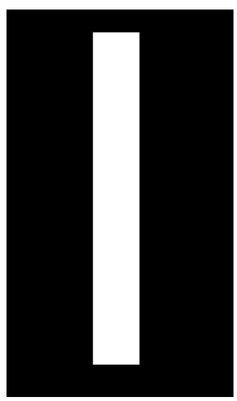
whiteBlack.bmp



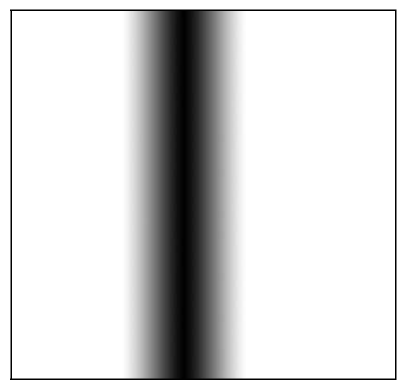
thin.bmp



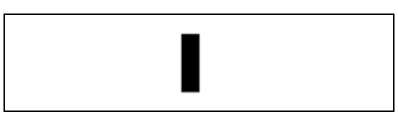
broad.bmp



broader.bmp

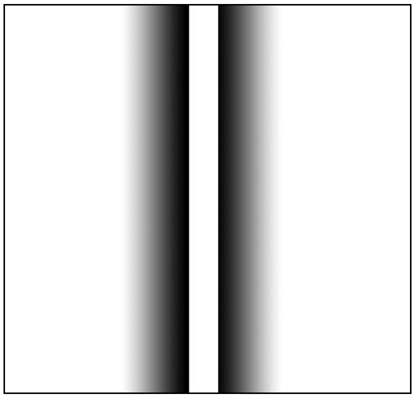


gradient wide line

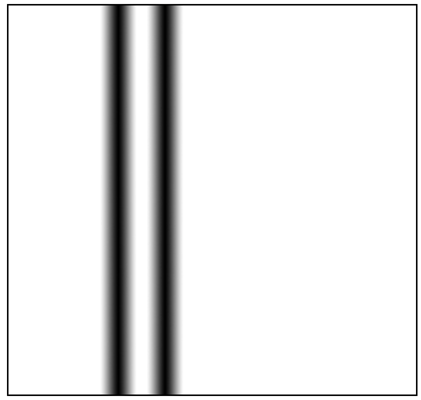


Prettybroadline4_thinner.jpg

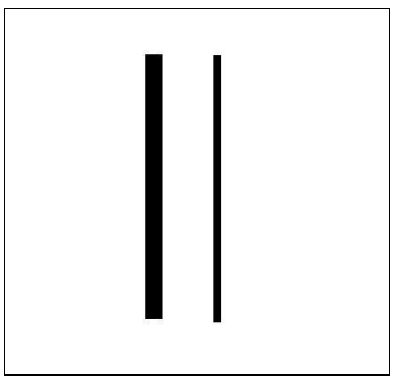
prettybroadline4 inverted



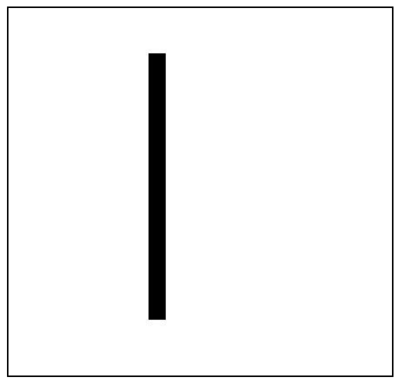
Gradient split



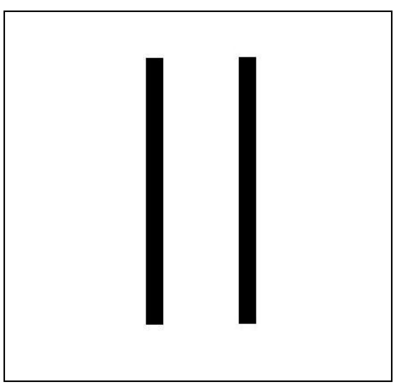
two gradient lines



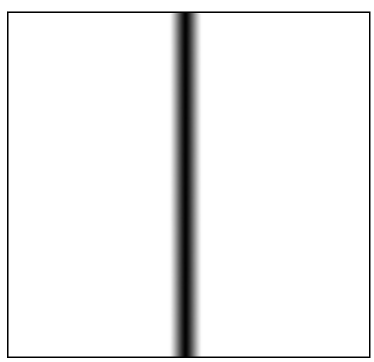
Broad and thin line



one single broad line



Two broad lines



Line gradient on both sides