# Deformation measurement of circular steel plates using projected fringes 

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#### Abstract

Fringe projection is a versatile method for mapping the surface topography. In this paper, it is used to measure the deformation of steel plates under static penetration. Here, the surface shape changes continuously. Therefore, it is important to minimize the registration time. To achieve this, we apply a method of fringe location with subpixel accuracy that requires only a single exposure for each registration. This is in contrast to phase shifting techniques that require at least three separate exposures.


Keywords Fringe projection • Optical measurements • Steel plates

## 1 Introduction

Projected fringes is a full field method for mapping the topography of surfaces [1-5]. It consists of imaging the fringes projected onto the surface with subsequent processing of the fringe data. In recent years, it has been combined with phase shift techniques [6-12] or Fourier transform methods [13-18]. With some exceptions [19, 20], reports from work with these methods are done inside the optical laboratory. Recently, the

[^0]authors in [21] have developed an optical measurement system to measure surface profile and three-dimensional deformation of small objects simultaneously, by a combination of fringe projection and a two-dimensional digital image correlation technique. Moreover, a Monte Carlo-based uncertainty approach is proposed to estimate overall error in fringe projection in [22].

In this paper, we describe an intensity method based on location of the fringe positions with subpixel accuracy by means of a zero-crossing algorithm. This method is applied to the measurement of deformations of steel plates under static penetration. Since the deformation increases continuously, it is important to have the exposure time as short as possible. The phase shift methods require at least three separate exposures of each deformation state and therefore become inadequate for recording fast dynamic events. The contributions of this article are mainly twofold: (1) a method for full field measurement of the dynamic deformation of steel plates is presented; (2) the experimental results are successfully carried out in mapping the development of the plate deformation.

The remainder of this paper is organized as follows: in Section 2, the measuring system using fringe projection methodology is described. Section 3 presents the test description and a detailed discussion on experimental results is given and we conclude the paper with Section 4.

## 2 System description

The measuring principle is illustrated in Fig. 1. A grating is projected onto the surface at an angle $\theta_{0}$ to the $z$ axis. A charged coupled device (CCD) camera points along the $z$ axis and images the surface with the superposed grating lines. When the object surface deviate from the reference surface, e.g., the $x y$ plane, a projected grating line will be displaced laterally at distance $\Delta u$ as seen by the camera.


Fig. 1 The measuring principle


Fig. 2 Fringe projection geometry

The relation between $\Delta u$ and the corresponding surface deviation $\Delta z$ is given by
$\Delta \mathrm{z}=\frac{x_{0}-x_{R}}{\tan \theta_{0}}=\frac{\Delta \mathrm{u}}{\tan \theta_{0}}$
where $x_{R}$ and $x_{0}$ denote the fringe positions on the reference surface and on the object surface, respectively.

When imaging a grating of period $d_{g}$, the fringe period on the surface is given by
$d_{x 0}=m_{p} \frac{d_{g}}{\cos \theta_{0}}$
where $m_{p}$ is the magnification of the projection unit. When putting $\Delta u=d_{x 0}$, the surface deviation per fringe is given by
$\Delta \mathrm{z}($ per fringe $)=\frac{m_{p} d_{g}}{\sin \theta_{0}}$

As the measuring point moves along the object surface, Fig. 2, the relation between the surface deviation $\Delta z$ and the fringe displacement becomes more complicated [1, p. 234]:
$\Delta \mathrm{z}(x)=\cos \theta_{0}\left[\sin \theta_{0}+\frac{\left(l_{k}-l_{p} \cos \theta_{0}\right) x}{l_{k} l_{p}}\right]^{-1} \cdot \Delta \mathbf{u}(x)$
where $l_{k}$ is the distance between the camera lens and the coordinate center and $l_{p}$ denotes the distance between the projector lens and the coordinate center.

The relation given in Eq. (4) can be written as
$\Delta \mathrm{z}(x)=S\left(\theta_{0}, l_{k}, l_{p}, x\right) \cdot \Delta \mathrm{u}(x)$

Fig. 3 Result from measurement of a rotation of a plane. Thick curve the measurement; thin curve the ideal straight line

where
$S\left(\theta_{0}, l_{k}, l_{p}, x\right)=\cos \theta_{0}\left[\sin \theta_{0}+\frac{\left(l_{k}-l_{p} \cos \theta_{0}\right) x}{l_{k} l_{p}}\right]^{-1}$
is a system function determined by the geometry of the experimental setup. Notice when the camera and the projector are located at the same height above the reference surface $\left(l_{k}-l_{p} \cos \theta_{0}=0\right), S$ becomes equal to $1 / \tan \theta_{0}$, i.e., independent of $x$.

Another reason for not using phase shifting techniques is pointed out by [4] and [10]. Divergent illumination introduces a nonlinear phase into the fringe function. While the system function $S$ in Eq. (6) can be made linear by making $l_{k}-l_{p} \cos \theta_{0}=0$, the phase $\psi(x)=u(x) / d_{x}$ where $d_{x}$ is the fringe pitch which is given by

$$
\begin{equation*}
d_{x}=\frac{1}{f_{0}}\left[1+\frac{x \sin \theta_{0}}{l_{p}}\right]^{2} \tag{7}
\end{equation*}
$$

Fig. 4 The test rig


Here, $f_{0}$ is the fringe frequency in the $x y$ plane for $x=0$, see Eqs. (7.35) and (7.43) of [1]. This results in a phase function with a quadratic dependence on $x$ and cannot be eliminated unless $l_{p} \rightarrow \infty$, i.e., plane wave illumination.

The measurement accuracy is dependent on the accuracy with which the fringe positions are detected. This is done by a zero-crossing algorithm described in [1, p. 274]. By this algorithm, the fringe positions are located with subpixel accuracy [23].

When the fringe positions in both the object and reference image are located, the measuring system determines $\Delta u$ and calculates $\Delta z$ according to Eq. (4). To do this, the mutual fringe order on the reference and the object must be found. In the general case, the zero-order fringe must be pointed out manually. In the present application, the calculations start at the clamped edge of the steel plate where $\Delta u=0$. The measurement algorithm therefore starts automatically without manual assistance. The calculated $\Delta z(x)$ is stored in computer memory. The results can be displayed either as a gray-scale level picture or as a contour map.

A simple and reliable test of this system is to image a plane surface before and after a known tilt. Figure 3 shows the result of such a test. A plane surface of about $1 \mathrm{~m}^{2}$ is imaged before and after a rotation angle of about 8 millirad about the vertical axis. From the smoothness of the measured curve, we can conclude that the zero-crossing algorithm works properly. Also, the ideal straight line in the diagram is drawn. The deviation from this line is seen to be less than about 0.1 mm . This deviation is mainly due to small errors in the setup parameters resulting in a minor error in the system function. It should be noted that the projected fringes in this test had the highest possible quality. One cannot expect such a small measuring error on, e.g., steel plates, but the results of our project indicate a measuring error of about 0.2 mm .

## 3 Test description

The present work was part of a project of testing the behavior of Weldox steel plates under static penetration. Weldox is a trade name of a general structural steel with a high-yield strength. The experimental setup is shown in Fig. 4. The test rig was an Amsler 1,000 tons jack with a length span of $\pm 100 \mathrm{~mm}$. The circular steel plates were mounted by clamping rings at the circumference and the load was applied at the center by a punch rod. The load was applied slowly but continuously and measured by a force sensor. The movement of the punch road was monitored by both a laser distance meter and an inductive probe. The loading stopped automatically when the steel plate was penetrated. Five different model parameters were used including plate thickness, geometry of the replaceable nose, rod diameter, mounting conditions,


Fig. 5 The experimental setup


Fig. 6 a The projected fringes on a plate just before applying the load, b the fringes on the same plate just before penetration


Fig. 7 a The deformation of the imaged part of the plate displayed as a contour map, $\mathbf{b}$ the deformation along a radial line for the same plate
and material quality. By varying these parameters, this test could give valuable information about the behavior of steel plates under static penetration.

Here, we will not go into details of the different findings of the project, but rather present the measuring method used to map the surface profile of the steel plates as the force increased.

### 3.1 Experimental set up for the fringe projection system

Figure 5 shows the experimental setup. The circular steel plate (radius, 250 mm ) is supported along its rim by a circular frame. It is loaded at its center by a pin (cross section,
$1 \mathrm{~cm}^{2}$ ) driven by a hydraulic system. The pin moves slowly downward resulting in an increasing deformation, and finally, the pin penetrates the steel plate. The time lapse from initiating the load to the penetration will among other things depend on the thickness and quality of the steel plate. In our experiments, it lasted typically between 5 and 15 min .

The purpose of the present experiment was to map the plate deformation as a function of the applied load. The projected fringes imaged by the camera were therefore grabbed and stored in the computer each 20th second. At the same instant, the load read off by a force sensor was registered and filed.

The optical system consists of a projector ( 550 W Xenon lamp) with a grating ( 4 lines $/ \mathrm{mm}$ ) and a CCD camera

Fig. 8 The deformation along a radial line plotted at different stages of the plate deformation

Deformation of plate $460-8$

connected to a frame grabber card inside a PC. To get high contrast of the projected fringes, the steel plate was painted white. The system has the following geometric configuration:

Distance, projector-steel plate: $l_{p}=910 \mathrm{~mm}$
Distance, camera-steel plate: $l_{c}=735 \mathrm{~mm}$
Projection angle: $\theta_{0}=53^{\circ}$
Focal length of camera lens: $f=10 \mathrm{~mm}$
Since the deformation is centrally symmetric, we need not measure the whole plate area. The length of the imaged area therefore covered about $3 / 4$ of the diameter.

The first stored image of a measuring sequence was grabbed just before the load was applied. This initial image served as the reference for the subsequent images in the same sequence. After finishing a sequence (i.e., after penetration of the plate), we therefore could map the deformation of the plate as it develops in steps of 20 s .

### 3.2 Experimental results

Figure 6a shows the projected fringes on a plate just before applying the load, while Fig. 6b shows the fringes on the same plate just before penetration. Figure 7a shows the deformation of the imaged part of the plate displayed as a contour map. Figure 7b shows the deformation along a radial line for the same plate.

In Fig. 8, the deformation along a radial line is plotted at different stages of the plate deformation. From this plots, one can see how the deformation develops as a function of time (i.e., load). In this project, 17 different steel plates of different steel plate quality and thickness were measured.

## 4 Conclusion

A method for full field measurement of the dynamic deformation of steel plates is presented. The experimental results were successful in mapping the development of the plate deformation. The only problem was that the white paint teared off just before penetration for some of the plates.

Although the dynamic development in this case was comparatively slow, the experiments have shown that this system is capable of measuring dynamic events at the same speed as the frame rate of the camera. With phase-shift methods, it is common to use four exposures for each surface state, making it at least four times as slow as our method. Future work will investigate on analysis of the measurements based on uncertainty on fringe projection technique and reduction of the noise in the measurements.

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