Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2013, Article ID 169713, 10 pages
http://dx.doi.org/10.1155/2013/169713

Research Article

Stability and l_1-Gain Analysis for Positive 2D Systems with State Delays in the Roesser Model

Zhaoxia Duan,[1] Hamid Reza Karimi,[2] and Zhengrong Xiang[1]

[1] *School of Automation, Nanjing University of Science and Technology, Nanjing 210094, China*
[2] *Department of Engineering, Faculty of Engineering and Science, University of Agder, 4898 Grimstad, Norway*

Correspondence should be addressed to Zhengrong Xiang; xiangzr@mail.njust.edu.cn

Received 8 December 2012; Revised 5 January 2013; Accepted 5 January 2013

Academic Editor: Ligang Wu

This paper considers the problem of delay-dependent stability and l_1-gain analysis for positive 2D systems with state delays described by the Roesser model. Firstly, the copositive-type Lyapunov function method is used to establish the sufficient conditions for the addressed positive 2D system to be asymptotically stable. Then, l_1-gain performance for the system is also analyzed. All the obtained results are formulated in the form of linear matrix inequalities (LMIs) which are computationally tractable. Finally, an illustrative example is given to verify the effectiveness of the proposed results.

1. Introduction

2D systems exist in many practical applications, such as circuits analysis, digital image processing, signal filtering, and thermal power engineering [1–4]. Thus the analysis and synthesis of 2D systems are interesting and challenging problems, and they have received considerable attention; for example, 2D state-space realization theory was researched in [5], the stability and 2D optimal control theory was studied in [6, 7], and H_∞ control and filtering problem for 2D systems were addressed in [8–11]. In addition, linear repetitive processes, a distinct class of 2D systems, have also been investigated. For example, the quasi-sliding mode control problem for linear repetitive processes with unknown input disturbance was solved in [12].

The most popular models of two-dimensional (2D) linear systems were introduced by Roesser [13], Fornasini and Marchesini [5, 14], and Kurek [15]. These models have been extended to positive systems in [16–19]. A positive system means that its state and output are nonnegative whenever the initial condition and input are nonnegative [19–21]. Positive 2D systems are needed in many cases such as the wave equation in fluid dynamics and the heat equation which describes the temperature (using thermodynamic temperature scale) in a given region over time and the Poisson's equation. These facts stimulate the research on 2D positive discrete systems. Reference [22] investigated the choice of the forms of Lyapunov functions for positive 2D Roesser model. The problem of stability analysis for 2D positive systems has been investigated in [17, 23–25]. It should be noted that although positive 2D systems have been discussed in control engineering and mathematics literature recently, there are still many questions which deserve further investigation.

On the other hand, the reaction of real-world systems to exogenous signals is never instantaneous and, always infected by certain time delays. For general systems, even nominal stable systems when were affected by delays may inherit very complex behaviors such as oscillations, instability, and bad performance [26], and delayed systems have attracted many researchers' attention [27–32]. The reachability, minimum energy control, and realization problem for positive 2D discrete-time systems with delays has been analyzed in [18, 33]. And the stability analysis for 2D positive delayed systems has been investigated in [34–36]. In addition, perturbations and uncertainties widely exist in the practical systems. In some cases, the perturbations and unmodeled errors can be merged into disturbances, which can be supposed to be bounded in the appropriate norms. It is important and necessary to establish a criterion evaluating the disturbance attenuation performance for the positive 2D discrete-time

systems. However, to the best of our knowledge, there has been no literature considering the disturbance attenuation performance for positive 2D systems, which motivates the present study.

In this paper, we will study the problem of delay-dependent stability and l_1-gain analysis for positive 2D linear systems with delays. The main theoretical contributions of this paper are as follows (1) We use l_1-gain to evaluate the disturbance attenuation performance of positive 2D linear systems. This important performance is firstly considered for positive 2D systems, and a delay-dependent stability criterion of these systems with state delays is developed. (2) Copositive-type Lyapunov function method is firstly used to analyze delay-dependent stability and l_1-gain performance for positive 2D linear systems. (3) It is significant to characterize conditions under which the positive 2D delayed system is asymptotically stable. All the developed results are expressed in terms of feasibility testing of LMIs which is computationally tractable.

The paper is organized as follows. In Section 2, problem statement and some definitions concerning the positive 2D linear systems with delays are given. In Section 3, some theorems concerning the delay-dependent stability and l_1-gain analysis of positive 2D linear systems are presented. In Section 4, a numerical example is given to illustrate the effectiveness of the proposed results. Finally, concluding remarks are provided in Section 5.

Notations. In this paper, the superscript "T" denotes the transpose. The notation $X > Y$ ($X \geq Y$) means that matrix X-Y is positive definite (positive semidefinite, resp.). $A \geq 0$ (≤ 0) means that all entries of matrix A are nonnegative (nonpositive). $A > 0$ (< 0) means that all entries of matrix A are positive (negative). $R^{n\times m}$ denotes the set of $n \times m$ real matrices. The set of real $n \times m$ matrices with nonnegative entries will be denoted by $R_+^{n\times m}$, R_+^n denotes the set of vectors with nonnegative entries, and the set of nonnegative integers will be denoted by Z_+. The $n \times n$ identity matrix will be denoted by I_n. The l_1 norm of a 2D signal $w(i, j) = [w_1(i, j), w_2(i, j), \ldots, w_m(i, j)]^T$ is given by

$$\left\|w\left(i,j\right)\right\|_1 = \sum_{k=1}^{m} w_k\left(i,j\right). \tag{1}$$

And we say $w(i, j) \in l_1$, if $\|w(i, j)\|_1 < \infty$.

2. Problem Formulation and Preliminaries

Consider the positive 2D Roesser model with state delays [25]:

$$\begin{bmatrix} x^h\left(i+1,j\right) \\ x^v\left(i,j+1\right) \end{bmatrix} = A \begin{bmatrix} x^h\left(i,j\right) \\ x^v\left(i,j\right) \end{bmatrix} + A_d \begin{bmatrix} x^h\left(i-d_h\left(i\right),j\right) \\ x^v\left(i,j-d_v\left(j\right)\right) \end{bmatrix} + Bw\left(i,j\right), \tag{1a}$$

$$z\left(i,j\right) = Hx\left(i,j\right) + Lw\left(i,j\right), \tag{1b}$$

where i and j are integers in Z_+, $x^h(i, j)$ is the horizontal state in $R_+^{n_1}$, $x^v(i, j)$ is the vertical state in $R_+^{n_2}$, $x(i, j)$ is the whole state in R_+^n, $w(i, j) \in R_+^{m_1}$ is the l_1 norm bounded disturbance input, $z(i, j) \in R_+^{m_2}$ is the controlled output, and A, A_d, B, H, $L \geq 0$ are system matrices with compatible dimensions. The matrices are

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \qquad A_d = \begin{bmatrix} A_{d11} & A_{d12} \\ A_{d21} & A_{d22} \end{bmatrix}, \qquad B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}. \tag{2}$$

$d_h(i)$ and $d_v(j)$ are delays along horizontal and vertical directions, respectively. We assume that $d_h(i)$ and $d_v(j)$ satisfy

$$d_{hL} \leq d_h\left(i\right) \leq d_{hH}, \qquad d_{vL} \leq d_v\left(j\right) \leq d_{vH}, \tag{3}$$

where d_{hL}, d_{hH} and d_{vL}, d_{vH} denote the lower and upper delay bounds along horizontal and vertical directions, respectively. The boundary conditions are defined by

$$\begin{aligned} x^h\left(i,j\right) &= h_{ij}, \quad \forall 0 \leq j \leq z_1, \ -d_{hH} \leq i \leq 0, \\ x^h\left(i,j\right) &= 0, \quad \forall j > z_1, \ -d_{hH} \leq i \leq 0, \\ x^v\left(i,j\right) &= v_{ij}, \quad \forall 0 \leq i \leq z_2, \ -d_{vH} \leq j \leq 0, \\ x^v\left(i,j\right) &= 0, \quad \forall i > z_2, \ -d_{vH} \leq j \leq 0, \\ h_{00} &= v_{00}, \end{aligned} \tag{4}$$

where $z_1 < \infty$ and $z_2 < \infty$ are positive integers, $h_{ij} \in R_+^{n_1}$ and $v_{ij} \in R_+^{n_2}$ are given vectors.

Definition 1. The 2D positive system (1a) and (1b) with $w(i, j) = 0$ is said to be asymptotically stable if $\lim_{l\to\infty} X_l = 0$ for all bounded boundary conditions (4), where

$$X_l = \sup\left\{\left\|x\left(i,j\right)\right\| : i + j = l, i, j \geq 1\right\}. \tag{5}$$

Definition 2. For $\gamma > 0$, the system (1a) and (1b) is said to be asymptotically stable with the l_1-gain index γ, if the following conditions hold.

(1) The system (1a) and (1b) with $w(i, j) = 0$ is asymptotically stable.

(2) Under zero boundary conditions, that is, $h_{ij} = 0, v_{ij} = 0$ in (4), it holds that

$$\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left\|z(i,j)\right\|_1 < \gamma\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left\|w\left(i,j\right)\right\|_1, \quad \forall 0 \neq w\left(i,j\right) \in l_1. \tag{6}$$

Remark 3. From (6), we see that γ can characterize the disturbance attenuation performance of the system (1a) and (1b). The smaller the γ is, the better the disturbance attenuation performance is.

3. Main Results

3.1. Stability Analysis. In this subsection, we focus on the problem of delay-dependent asymptotically stability analysis for the positive 2D discrete linear systems with state delays.

Theorem 4. *For given positive constants d_{hL}, d_{hH}, d_{vL}, d_{vH}, the positive 2D system (1a) and (1b) with $w(i,j) = 0$ is asymptotically stable if there exist vectors $p, q, \varsigma_1, \varsigma_2, \zeta \in R_+^n$, such that*

$$\Phi = \operatorname{diag}\left\{\Phi_1, \Phi_2, \ldots, \Phi_n, \Phi_1', \Phi_2', \ldots, \Phi_n', \Phi_1'', \Phi_2'', \ldots, \Phi_n'', \Phi_1''', \Phi_2''', \ldots, \Phi_n'''\right\} < 0, \tag{7}$$

where

$$\Phi_k = \begin{cases} \left(a_k^T - E_k\right)p + \left(a_k^T + \left(d_{hH} - d_{hL}\right)E_k\right)q \\ \quad + E_k\zeta + \left(d_{hH}^2\left(a_k^T - E_k\right) - d_{hH}E_k\right)\varsigma_2 \\ \quad + d_{hH}^2 E_k\varsigma_1, \end{cases} \quad 1 \le k \le n_1,$$

$$\Phi_k = \begin{cases} \left(a_k^T - E_k\right)p + \left(a_k^T + \left(d_{vH} - d_{vL}\right)E_k\right)q \\ \quad + E_k\zeta + \left(d_{vH}^2\left(a_k^T - E_k\right) - d_{vH}E_k\right)\varsigma_2 \\ \quad + d_{vH}^2 E_k\varsigma_1, \end{cases} \quad n_1 + 1 \le k \le n,$$

$$\Phi_k' = \begin{cases} a_{dk}^T p + \left(a_{dk}^T - E_k\right)q + d_{hH}^2 a_{dk}^T\varsigma_2, & 1 \le k \le n_1, \\ a_{dk}^T p + \left(a_{dk}^T - E_k\right)q + d_{vH}^2 a_{dk}^T\varsigma_2, & n_1 + 1 \le k \le n, \end{cases}$$

$$\Phi_k'' = \begin{cases} -E_k\zeta + d_{hH}E_k\left(\varsigma_2 - \varsigma_1\right), & 1 \le k \le n_1, \\ -E_k\zeta + d_{vH}E_k\left(\varsigma_2 - \varsigma_1\right), & n_1 + 1 \le k \le n, \end{cases}$$

$$\Phi_k''' = \begin{cases} -d_{hH}E_k\varsigma_1, & 1 \le k \le n_1, \\ -d_{vH}E_k\varsigma_1, & n_1 + 1 \le k \le n, \end{cases} \tag{8}$$

with $k \in \underline{n} = \{1, 2, \ldots, n\}$, $E_k = [\overbrace{0, \ldots, 0}^{k-1}, 1, \overbrace{0, \ldots, 0}^{n-k}]$, and $a_k(a_{dk})$ represents the kth column vector of matrix $A(A_d)$.

Proof. Choose the following copositive Lyapunov-Krasovskii functional candidate:

$$V(i,j) = V^h(i,j) + V^v(i,j), \tag{9}$$

where

$$\begin{aligned} V^h(i,j) &= \sum_{k=1}^{5} V_k^h(i,j), \\ V_1^h(i,j) &= x^{hT}(i,j)\,p^h, \\ V_2^h(i,j) &= \sum_{r=i-d_h(i)}^{i} x^{hT}(r,j)\,q^h, \\ V_3^h(i,j) &= \sum_{r=i-d_{hH}}^{i-1} x^{hT}(r,j)\,\zeta^h, \\ V_4^h(i,j) &= \sum_{s=-d_{hH}+1}^{-d_{hL}} \sum_{r=i+s}^{i-1} x^{hT}(r,j)\,q^h, \\ V_5^h(i,j) &= d_{hH} \sum_{s=-d_{hH}}^{-1} \sum_{r=i+s}^{i-1} \eta^{hT}(r,j)\,\varsigma^h, \\ V^v(i,j) &= \sum_{k=1}^{5} V_k^v(i,j), \\ V_1^v(i,j) &= x^{vT}(i,j)\,p^v, \\ V_2^v(i,j) &= \sum_{s=j-d_v(j)}^{j} x^{vT}(i,s)\,q^v, \\ V_3^v(i,j) &= \sum_{t=j-d_{vH}}^{j-1} x^{vT}(i,t)\,\zeta^v, \\ V_4^v(i,j) &= \sum_{s=-d_{vH}+1}^{-d_{vL}} \sum_{t=j+s}^{j-1} x^{vT}(i,t)\,q^v, \\ V_5^v(i,j) &= d_{vH} \sum_{s=-dv_H}^{-1} \sum_{t=j+s}^{j-1} \eta^{vT}(i,t)\,\varsigma^v, \\ \eta^h(r,j) &= \left[x^{hT}(r,j) \;\; \delta^{hT}(r,j)\right]^T, \\ \eta^v(i,t) &= \left[x^{vT}(i,t) \;\; \delta^{vT}(i,t)\right]^T, \\ \delta^h(r,j) &= x^h(r+1,j) - x^h(r,j), \\ \delta^v(i,t) &= x^v(i,t+1) - x^v(i,t), \end{aligned} \tag{10}$$

with p^h, q^h, ζ^h, ς_1^h, and $\varsigma_2^h \in R_+^{n_1}$, p^v, q^v, ζ^v, ς_1^v, and $\varsigma_2^v \in R_+^{n_2}$, $\varsigma^h = [\varsigma_1^h \;\; \varsigma_2^h]^T \in R_+^{2n_1}$, and $\varsigma^v = [\varsigma_1^v \;\; \varsigma_2^v]^T \in R_+^{2n_2}$.

Along the trajectory of the system (1a) and (1b), we have

$$\begin{aligned} \Delta V(i,j) &= V^h(i+1,j) - V^h(i,j) + V^v(i,j+1) - V^v(i,j) \\ &= \sum_{k=1}^{5} \Delta V_k^h(i,j) + \sum_{k=1}^{5} \Delta V_k^v(i,j), \end{aligned} \tag{11}$$

where

$$\Delta V_1^h(i,j) = x^{hT}(i+1,j)\,p^h - x^{hT}(i,j)\,p^h,$$

$$\begin{aligned}\Delta V_2^h(i,j) &= \sum_{r=i+1-d_h(i+1)}^{i+1} x^{hT}(r,j)\,q^h - \sum_{r=i-d_h(i)}^{i} x^{hT}(r,j)\,q^h \\ &= x^{hT}(i+1,j)\,q^h - x^{hT}(i-d_h(i),j)\,q^h \\ &\quad + \sum_{r=i+1-d_h(i+1)}^{i} x^{hT}(r,j)\,q^h - \sum_{r=i+1-d_h(i)}^{i} x^{hT}(r,j)\,q^h \\ &\le x^{hT}(i+1,j)\,q^h - x^{hT}(i-d_h(i),j)\,q^h \\ &\quad + \sum_{r=i+1-d_{hH}}^{i} x^{hT}(r,j)\,q^h - \sum_{r=i+1-d_{hL}}^{i} x^{hT}(r,j)\,q^h \\ &= x^{hT}(i+1,j)\,q^h - x^{hT}(i-d_h(i),j)\,q^h \\ &\quad + \sum_{r=i+1-d_{hH}}^{r=i-d_{hL}} x^{hT}(r,j)\,q^h,\end{aligned}$$

$$\Delta V_3^h(i,j) = \sum_{r=i+1-d_{hH}}^{i} x^{hT}(r,j)\,\zeta^h - \sum_{r=i-d_{hH}}^{i-1} x^{hT}(r,j)\,\zeta^h = x^{hT}(i,j)\,\zeta^h - x^{hT}(i-d_{hH},j)\,\zeta^h,$$

$$\begin{aligned}\Delta V_4^h(i,j) &= \sum_{s=-d_{hH}+1}^{-d_{hL}} \sum_{r=i+1+s}^{i} x^{hT}(r,j)\,q^h - \sum_{s=-d_{hH}+1}^{-d_{hL}} \sum_{r=i+s}^{i-1} x^{hT}(r,j)\,q^h \\ &= \sum_{s=-d_{hH}+1}^{-d_{hL}} \left[x^{hT}(i,j)\,q^h - x^{hT}(i+s,j)\,q^h\right] \\ &= (d_{hH}-d_{hL})\,x^{hT}(i,j)\,q^h - \sum_{r=i-d_{hH}+1}^{i-d_{hL}} x^{hT}(r,j)\,q^h,\end{aligned}$$

$$\begin{aligned}&\Delta V_5^h(i,j) \\ &= d_{hH} \sum_{s=-d_{hH}}^{-1} \sum_{r=i+1+s}^{i} \eta^{hT}(r,j)\,\varsigma^h - d_{hH} \sum_{s=-d_{hH}}^{-1} \sum_{r=i+s}^{i-1} \eta^{hT}(r,j)\,\varsigma^h \\ &= d_{hH} \sum_{s=-d_{hH}}^{-1} \left(\eta^{hT}(i,j)\,\varsigma^h - \eta^{hT}(i+s,j)\,\varsigma^h\right) \\ &= d_{hH}^2 \eta^{hT}(i,j)\,\varsigma^h - d_{hH} \sum_{r=i-d_{hH}}^{i-1} \eta^{hT}(r,j)\,\varsigma^h \\ &= d_{hH}^2 \left[x^{hT}(i,j) \;\; x^{hT}(i+1,j) - x^{hT}(i,j)\right] \begin{bmatrix}\varsigma_1^h \\ \varsigma_2^h\end{bmatrix} \\ &\quad - d_{hH} \left[\sum_{r=i-d_{hH}}^{i-1} x^{hT}(i,j) \;\; x^{hT}(i,j) - x^{hT}(i-d_{hH},j)\right] \times \begin{bmatrix}\varsigma_1^h \\ \varsigma_2^h\end{bmatrix},\end{aligned}$$

$$\Delta V_1^v(i,j) = x^{vT}(i,j+1)\,p^v - x^{vT}(i,j)\,p^v,$$

$$\begin{aligned}\Delta V_2^v(i,j) &= \sum_{s=j+1-d_v(j+1)}^{j+1} x^{vT}(i,s)\,q^v - \sum_{s=j-d_v(j)}^{j} x^{vT}(i,s)\,q^v \\ &= x^{vT}(i,j+1)\,q^v - x^{vT}(i,j-d_v(j))\,q^v \\ &\quad + \sum_{s=j+1-d_v(j+1)}^{j} x^{vT}(i,s)\,q^v - \sum_{s=j+1-d_v(j)}^{j} x^{vT}(i,s)\,q^v \\ &\le x^{vT}(i,j+1)\,q^v - x^{vT}(i,j-d_v(j))\,q^v \\ &\quad + \sum_{s=j+1-d_{vH}}^{j} x^{vT}(i,s)\,q^v - \sum_{s=j+1-d_{vL}}^{j} x^{vT}(i,s)\,q^v \\ &= x^{vT}(i,j+1)\,q^v - x^{vT}(i,j-d_v(j))\,q^v \\ &\quad + \sum_{t=j+1-d_{vH}}^{j-d_{vL}} x^{vT}(i,t)\,q^v,\end{aligned}$$

$$\begin{aligned}
\Delta V_3^{\upsilon}(i,j) &= \sum_{s=j+1-d_{\upsilon H}}^{j} x^{hT}(i,s)\,\zeta^{\upsilon} \\
&\quad - \sum_{s=j-d_{\upsilon H}}^{j-1} x^{hT}(i,a)\,\zeta^{\upsilon} \\
&= x^{\upsilon T}(i,j)\,\zeta^{\upsilon} - x^{\upsilon T}(i,j-d_{\upsilon H})\,\zeta^{\upsilon}, \\
\Delta V_4^{\upsilon}(i,j) &= \sum_{s=-d_{\upsilon H}+1}^{-d_{\upsilon L}} \sum_{t=j+1+s}^{j} x^{\upsilon T}(i,t)\,q^{\upsilon} \\
&\quad - \sum_{s=-d_{\upsilon H}+1}^{-d_{\upsilon L}} \sum_{t=j+s}^{j-1} x^{\upsilon T}(i,t)\,q^{\upsilon} \\
&= \sum_{s=-d_{\upsilon H}+1}^{-d_{\upsilon L}} \left[x^{\upsilon T}(i,j)\,q^{\upsilon} - x^{\upsilon T}(i,j+s)\,q^{\upsilon}\right] \\
&= (d_{\upsilon H} - d_{\upsilon L})\,x^{hT}(i,j)\,q^{\upsilon} \\
&\quad - \sum_{t=j-d_{\upsilon H}+1}^{j-d_{\upsilon L}} x^{\upsilon T}(i,t)\,q^{\upsilon}, \\
\Delta V_5^{\upsilon}(i,j) & \\
&= d_{\upsilon H} \sum_{s=-dv_H}^{-1} \sum_{t=j+1+s}^{j} \eta^{\upsilon T}(i,t)\,\varsigma^{\upsilon} \\
&\quad - d_{\upsilon H} \sum_{s=-dv_H}^{-1} \sum_{t=j+s}^{j-1} \eta^{\upsilon T}(i,t)\,\varsigma^{\upsilon} \\
&= d_{\upsilon H} \sum_{s=-d_{\upsilon H}}^{-1} \left(\eta^{\upsilon T}(i,j)\,\varsigma^{\upsilon} - \eta^{\upsilon T}(i,j+s)\,\varsigma^{\upsilon}\right) \\
&= d_{\upsilon H}^2 \eta^{\upsilon T}(i,j)\,\varsigma^{\upsilon} - d_{\upsilon H} \sum_{t=j-d_{\upsilon H}}^{j-1} \eta^{\upsilon T}(i,t)\,\varsigma^{\upsilon} \\
&= d_{\upsilon H}^2 \left[x^{\upsilon T}(i,j) \;\; x^{\upsilon T}(i,j+1) - x^{\upsilon T}(i,j)\right] \\
&\quad \times \begin{bmatrix} \varsigma_1^{\upsilon} \\ \varsigma_2^{\upsilon} \end{bmatrix} \\
&\quad - d_{\upsilon H} \left[\sum_{t=j-d_{\upsilon H}}^{j-1} x^{\upsilon T}(i,j) \;\; x^{\upsilon T}(i,j) - x^{\upsilon T}(i,j-d_{\upsilon H})\right] \\
&\quad \times \begin{bmatrix} \varsigma_1^{\upsilon} \\ \varsigma_2^{\upsilon} \end{bmatrix}.
\end{aligned} \tag{12}$$

Substitute the previously mentioned formulations into (11), and take

$$\begin{aligned}
p &= \begin{bmatrix} p^h \\ p^{\upsilon} \end{bmatrix}, \qquad q = \begin{bmatrix} q^h \\ q^{\upsilon} \end{bmatrix}, \qquad \zeta = \begin{bmatrix} \zeta^h \\ \zeta^{\upsilon} \end{bmatrix}, \\
\varsigma_1 &= \begin{bmatrix} \varsigma_1^h \\ \varsigma_1^h \end{bmatrix}, \qquad \varsigma_2 = \begin{bmatrix} \varsigma_2^h \\ \varsigma_2^h \end{bmatrix}, \\
D_H &= \begin{bmatrix} d_{hH} I_{n_1} & 0 \\ 0 & d_{\upsilon H} I_{n_2} \end{bmatrix}, \qquad D_L = \begin{bmatrix} d_{hL} I_{n_1} & 0 \\ 0 & d_{\upsilon L} I_{n_2} \end{bmatrix}, \\
x(i,j) &= \left[x^{hT}(i,j) \;\; x^{\upsilon T}(i,j)\right]^T, \\
x_d(i,j) &= \left[x^{hT}(i-d_h(i),j) \;\; x^{\upsilon T}(i,j-d_{\upsilon}(j))\right]^T, \\
x_H(i,j) &= \left[x^{hT}(i-d_{hH},j) \;\; x^{\upsilon T}(i,j-d_{\upsilon H})\right]^T, \\
x_s(i,j) &= \left[\sum_{r=i-d_{hH}+1}^{i-1} x^{hT}(r,j) \;\; \sum_{t=j-d_{\upsilon H+1}}^{j-1} x^{\upsilon T}(i,t)\right]^T.
\end{aligned} \tag{13}$$

Then we have

$$\begin{aligned}
\Delta V(i,j) &= x^T(i,j)\left\{\left(A^T - I_n\right)p + \left(A^T + D_H - D_L\right)q \right. \\
&\qquad \left. + \zeta + D_H^2\left(\left(A^T - I_n\right)\varsigma_2 + \varsigma_1\right) - D_H\varsigma_2\right\} \\
&\quad + x_d^T(i,j)\left\{A_d^T p + \left(A_d^T - I_n\right)q + D_H^2 A_d^T \varsigma_2\right\} \\
&\quad + x_H^T(i,j)\left\{-\zeta + D_H(\varsigma_2 - \varsigma_1)\right\} \\
&\quad + x_s^T(i,j)\left\{-D_H\varsigma_1\right\}.
\end{aligned} \tag{14}$$

If condition (7) holds, one obtains

$$\begin{aligned}
&\left(A^T - I_n\right)p + \left(A^T + D_H - D_L\right)q + \zeta \\
&\qquad + D_H^2\left(\left(A^T - I_n\right)\varsigma_2 + \varsigma_1\right) - D_H\varsigma_2 \prec 0, \\
&A_d^T p + \left(A_d^T - I_n\right)q + D_H^2 A_d^T \varsigma_2 \prec 0, \\
&-\zeta + D_H(\varsigma_2 - \varsigma_1) \prec 0, \\
&-D_H\varsigma_1 \prec 0.
\end{aligned} \tag{15}$$

It follows that $\Delta V(i,j) < 0$, which means that

$$V^h(i+1,j) + V^{\upsilon}(i,j+1) < V^h(i,j) + V^{\upsilon}(i,j). \tag{16}$$

Summing up both sides of (16) from D to 0 with respect to i and from 0 to D with respect to j, for any nonnegative integer

$D \geq \max(z_1, z_2)$, one gets

$$\begin{aligned} & V^h(1,D) + V^v(0,D+1) + V^h(2,D-1) \\ & \quad + V^v(1,D) + \cdots + V^h(D+1,0) + V^v(D,1) \\ & = \sum_{i+j=D+1} V^h(i,j) + \sum_{i+j=D+1} V^v(i,j) \\ & = \sum_{i+j=D+1} V(i,j) \\ & \leq V^h(0,D) + V^v(0,D) + V^h(1,D-1) \\ & \quad + V^v(1,D-1) + \cdots + V^h(D,0) + V^v(D,0) \\ & = \sum_{i+j=D} V(i,j). \end{aligned} \tag{17}$$

Then from (9), we can conclude that

$$\lim_{i+j\to\infty} x(i,j) = 0, \tag{18}$$

which implies that the system (1a) and (1b) with $w(i,j) = 0$ is asymptotically stable.

This completes the proof. □

When $d_{hH} = d_{hL} = d_h$, and $d_{vH} = d_{vL} = d_v$, the system (1a) and (1b) with $w(i,j) = 0$ is reduced to the following system:

$$\begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix} = A \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} + A_d \begin{bmatrix} x^h(i-d_h,j) \\ x^v(i,j-d_v) \end{bmatrix}, \tag{19}$$

where d_h and d_v are constant delays along horizontal and vertical directions, respectively, and the boundary conditions are defined in (4). Then we can get the following result.

Corollary 5. *For given positive constants d_h and d_v, the positive 2D system* (19) *is asymptotically stable if there exist vectors p and $q \in R_+^n$, such that*

$$\widetilde{\Phi} = \operatorname{diag}\left\{\Phi_1, \Phi_2, \ldots, \Phi_n, \Phi_1', \Phi_2', \ldots, \Phi_n'\right\} < 0, \tag{20}$$

where

$$\begin{aligned} \widetilde{\Phi}_k &= \left(a_k^T - E_k\right) p + a_k^T q, \\ \widetilde{\Phi}_k' &= a_{dk}^T p + \left(a_{dk}^T - E_k\right) q, \end{aligned} \tag{21}$$

with $k \in \underline{K} = \{1, 2, \ldots, n\}$, $E_k = [\overbrace{0, \ldots, 0}^{k-1}, 1, \overbrace{0, \ldots, 0}^{n-k}]$, p, $q \in R_+^n$, and $a_k(a_{dk})$ represents the kth column vector of matrix $A(A_d)$.

Proof. Choose the following copositive Lyapunov-Krasovskii functional candidate for the system (19):

$$V(i,j) = V^h(i,j) + V^v(i,j), \tag{22}$$

where

$$\begin{aligned} V^h(i,j) &= \sum_{k=1}^{2} V_k^h(i,j), \qquad V_1^h(i,j) = x^{hT}(i,j)\, p^h, \\ V_2^h(i,j) &= \sum_{r=i-d_h}^{i} x^{hT}(r,j)\, q^h, \\ V^v(i,j) &= \sum_{k=1}^{2} V_k^v(i,j), \qquad V_1^v(i,j) = x^{vT}(i,j)\, p^v, \\ V_2^v(i,j) &= \sum_{s=j-d_v}^{j} x^{vT}(i,s)\, q^v, \end{aligned} \tag{23}$$

with $p^h, q^h \in R_+^{n_1}$, $p^v, q^v \in R_+^{n_2}$. Then following the proof line of Theorem 4, the corollary can be obtained. □

3.2. l_1-Gain Analysis. The following theorem establishes sufficient condition of the asymptotical stability with l_1-gain performance for the system (1a) and (1b).

Theorem 6. *For given positive constants d_{hL}, d_{hH}, d_{vL}, d_{vH}, and γ, the positive 2D system* (1a) *and* (1b) *is asymptotically stable with the l_1-gain index γ if there exist vectors $p \in R_+^n$, $q \in R_+^n$, $\varsigma_1 \in R_+^n$, $\varsigma_2 \in R_+^n$, and $\zeta \in R_+^n$, such that*

$$\begin{aligned} \overline{\Phi} = \operatorname{diag}\Big\{ & \overline{\Phi}_1, \overline{\Phi}_2, \ldots, \overline{\Phi}_n, \Phi_1', \Phi_2', \ldots, \Phi_n', \Phi_1'', \Phi_2'', \ldots, \Phi_n'', \\ & \Phi_1''', \Phi_2''', \ldots, \Phi_n''', \mathrm{T}_1, \mathrm{T}_2, \ldots, \mathrm{T}_m \Big\} < 0, \end{aligned} \tag{24}$$

where Φ_k', Φ_k'', and Φ_k''' are denoted as in Theorem 4, and

$$\begin{aligned} & \overline{\Phi}_k \\ & = \begin{cases} \left(a_k^T - E_k\right) p + \left(a_k^T + (d_{hH} - d_{hL}) E_k\right) q + E_k \zeta \\ \quad + \left(d_{hH}^2 \left(a_k^T - E_k\right) - d_{hH} E_k\right) \varsigma_2 \qquad 1 \leq k \leq n_1, \\ \quad + d_{hH}^2 E_k \varsigma_1 + \|h_k\|_1, \\ \left(a_k^T - E_k\right) p + \left(a_k^T + (d_{vH} - d_{vL}) E_k\right) q + E_k \zeta \\ \quad + \left(d_{vH}^2 \left(a_k^T - E_k\right) - d_{vH} E_k\right) \varsigma_2 \qquad n_1 + 1 \leq k \leq n, \\ \quad + d_{vH}^2 E_k \varsigma_1 + \|h_k\|_1, \end{cases} \\ & T_\varepsilon = b_\varepsilon^T p + b_\varepsilon^T q + b_\varepsilon^T D_{hH}^2 \varsigma_2 + \|l_\varepsilon\|_1 - \gamma, \qquad 1 \leq \varepsilon \leq m, \\ & D_H = \operatorname{diag}\left\{d_{hH} I_{n_1}, d_{vH} I_{n_2}\right\}, \end{aligned} \tag{25}$$

with $k \in \underline{n} = \{1, 2, \ldots, n\}$, $\varepsilon \in \underline{m} = \{1, 2, \ldots, m\}$, $E_k = [\overbrace{0, \ldots, 0}^{k-1}, 1, \overbrace{0, \ldots, 0}^{n-k}]$, a_k, a_{dk}, b_k, and h_k represent the kth column vector of matrices A, A_d, B, and H, respectively, and l_ε represents the εth column vector of matrix L.

Proof. It is an obvious fact that (24) implies the following inequality:

$$\Psi = \operatorname{diag}\left\{\Psi_1, \Psi_2, \ldots, \Psi_n, \Psi_1', \Psi_2', \ldots, \Psi_n', \Psi_1'', \Psi_2'', \ldots, \Psi_n'', \Psi_1''', \Psi_2''', \ldots, \Psi_n'''\right\} < 0, \tag{26}$$

where

$$\begin{aligned}
\Psi_k &= \begin{cases} \left(a_k^T - E_k\right)p + \left(a_k^T + \left(d_{hH} - d_{hL}\right)E_k\right)q \\ \quad +\zeta_k + \left(d_{hH}^2\left(a_k^T - E_k\right) - d_{hH}E_k\right)\varsigma_2 \\ \quad +d_{hH}^2 E_k\varsigma_1, \qquad 1 \le k \le n_1, \\ \left(a_k^T - E_k\right)p + \left(a_k^T + \left(d_{vH} - d_{vL}\right)E_k\right)q \\ \quad +\zeta_k + \left(d_{vH}^2\left(a_k^T - E_k\right) - d_{vH}E_k\right)\varsigma_2 \\ \quad +d_{vH}^2 E_k\varsigma_1, \qquad n_1 + 1 \le k \le n, \end{cases} \\
\Psi_k' &= \begin{cases} a_{dk}^T p + \left(a_{dk}^T - E_k\right)q + d_{hH}^2 a_{dk}^T\varsigma_2, & 1 \le k \le n_1, \\ a_{dk}^T p + \left(a_{dk}^T - E_k\right)q + d_{vH}^2 a_{dk}^T\varsigma_2, & n_1 + 1 \le k \le n, \end{cases} \\
\Psi_k'' &= \begin{cases} -\zeta_k + d_{hH}E_k\left(\varsigma_2 - \varsigma_1\right), & 1 \le k \le n_1, \\ -\zeta_k + d_{vH}E_k\left(\varsigma_2 - \varsigma_1\right), & n_1 + 1 \le k \le n, \end{cases} \\
\Psi_k''' &= \begin{cases} -d_{hH}E_k\varsigma_1, & 1 \le k \le n_1, \\ -d_{vH}E_k\varsigma_1, & n_1 + 1 \le k \le n, \end{cases}
\end{aligned} \tag{27}$$

with $k \in \underline{n} = \{1, 2, \ldots, n\}$, $E_k = [\overbrace{0, \ldots, 0}^{k-1}, 1, \overbrace{0, \ldots, 0}^{n-k}]$, and $a_k(a_{dk})$ represents the kth column vector of matrix $A(A_d)$.

By Theorem 4, we can obtain that the system (1a) and (1b) with $w(i, j) = 0$ is asymptotically stable. Now we are in a position to prove that the system (1a) and (1b) has a prescribed l_1-gain index γ for any nonzero $w(i, j) \in l_1$. To establish the l_1-gain performance, we choose the same copositive Lyapunov-Krasovskii functional candidate as in (9) for the system (1a) and (1b). Following the proof line of Theorem 4, we can get that

$$\begin{aligned}
&\Delta V(i, j) + \left\|z(i, j)\right\|_1 - \gamma\left\|w(i, j)\right\|_1 \\
&\quad = x^T(i, j)\left\{\left(A^T - I_n\right)p + \left(A^T + D_H - D_L\right)q \right. \\
&\qquad \left. +\zeta + D_H^2\left(\left(A^T - I_n\right)\varsigma_2 + \varsigma_1\right) - D_H\varsigma_2\right\} \\
&\qquad + x_d^T(i, j)\left\{A_d^T p + \left(A_d^T - I_n\right)q + D_H^2 A_d^T\varsigma_2\right\} \\
&\qquad + x_H^T(i, j)\left\{-\zeta + D_H\left(\varsigma_2 - \varsigma_1\right)\right\} \\
&\qquad + x_s^T(i, j)\left\{-D_H\varsigma_1\right\} \\
&\qquad + w^T(i, j)\left\{B^T\left(p + q + D_H^2\varsigma_2\right)\right\} \\
&\qquad + \left\|Hx(i, j)\right\|_1 + \left\|Lw(i, j)\right\|_1 - \gamma\left\|w(i, j)\right\|_1.
\end{aligned} \tag{28}$$

According to the definition of l_1 norm, one obtains

$$\begin{aligned}
Hx(i, j) &= \begin{bmatrix} h_{1,1} & h_{1,2} & \cdots & h_{1,n_1} & h_{1,n_1+1} & h_{1,n_1+2} & \cdots & h_{1,n} \\ h_{2,1} & h_{2,2} & \cdots & h_{2,n_1} & h_{2,n_1+1} & h_{2,n_1+2} & \cdots & h_{2,n} \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ h_{p,1} & h_{p,2} & \cdots & h_{p,n_1} & h_{p,n_1+1} & h_{p,n_1+2} & \cdots & h_{p,n} \end{bmatrix} \begin{bmatrix} x_1^h(i, j) \\ \vdots \\ x_{n_1}^h(i, j) \\ x_1^v(i, j) \\ \vdots \\ x_{n_2}^v(i, j) \end{bmatrix} \\
&= \begin{bmatrix} h_{1,1}x_1^h(i, j) + \cdots + h_{1,n_1}x_{n_1}^h(i, j) + h_{1,n_1+1}x_1^v(i, j) + \cdots + h_{1,n}x_{n_2}^v(i, j) \\ h_{2,1}x_1^h(i, j) + \cdots + h_{2,n_1}x_{n_1}^h(i, j) + h_{2,n_1+1}x_1^v(i, j) + \cdots + h_{2,n}x_{n_2}^v(i, j) \\ \vdots \\ h_{p,1}x_1^h(i, j) + \cdots + h_{p,n_1}x_{n_1}^h(i, j) + h_{p,n_1+1}x_1^v(i, j) + \cdots + h_{p,n}x_{n_2}^v(i, j) \end{bmatrix}, \\
\left\|Hx(i, j)\right\|_1 &= \sum_{k=1}^{p} h_{k,1}x_1^h(i, j) + \cdots + h_{k,n_1}x_{n_1}^h(i, j) + h_{k,n_1+1}x_1^v(i, j) + \cdots + h_{k,n}x_{n_2}^v(i, j) \\
&= \left(\sum_{k=1}^{p} h_{k,1}\right)x_1^h(i, j) + \cdots + \left(\sum_{k=1}^{p} h_{k,n_1}\right)x_{n_1}^h(i, j) + \left(\sum_{k=1}^{p} h_{k,n_1+1}\right)x_1^v(i, j) \\
&\quad + \cdots + \left(\sum_{k=1}^{p} h_{k,n}\right)x_{n_2}^v(i, j) \\
&= x^T(i, j)\left[\left\|h_1\right\|_1 \ \left\|h_2\right\|_1 \ \cdots \ \left\|h_n\right\|_1\right]^T,
\end{aligned} \tag{29}$$

where h_k represents the kth column vector and $h_{i,j}$ represents the entry located at (i, j) of matrix H. Then, similarly

$$\begin{aligned} \|Lw(i,j)\|_1 &= w^T(i,j)\left[\|l_1\|_1 \;\; \|l_2\|_1 \;\; \cdots \;\; \|l_m\|_1\right]^T, \\ \gamma\|w(i,j)\|_1 &= w^T(i,j)\left[\gamma \;\; \gamma \;\; \cdots \;\; \gamma\right]^T, \end{aligned} \tag{30}$$

where l_k represents the kth column vector of matrix L.

Substituting (29)-(30) into (28) leads to

$$\begin{aligned} &\Delta V(i,j) + \|z(i,j)\|_1 - \gamma\|w(i,j)\|_1 \\ &\quad = x^T(i,j)\left\{\left(A^T - I_n\right)p + \left(A^T + D_H - D_L\right)q + \zeta \right. \\ &\qquad\qquad + D_H^2\left(\left(A^T - I_n\right)\varsigma_2 + \varsigma_1\right) - D_H\varsigma_2 \\ &\qquad\qquad \left. +\left[\|h_1\|_1 \;\; \|h_2\|_1 \;\; \cdots \;\; \|h_n\|_1\right]^T\right\} \\ &\qquad + x_d^T(i,j)\left\{A_d^T p + \left(A_d^T - I_n\right)q + D_H^2 A_d^T \varsigma_2\right\} \\ &\qquad + x_H^T(i,j)\left\{-\zeta + D_H(\varsigma_2 - \varsigma_1)\right\} \\ &\qquad + x_s^T(i,j)\left\{-D_H\varsigma_1\right\} \\ &\qquad + w^T(i,j)\left\{B^T\left(p + q + D_H^2\varsigma_2\right)\right. \\ &\qquad\qquad +\left[\|l_1\|_1 \;\; \|l_2\|_1 \;\; \cdots \;\; \|l_m\|_1\right]^T \\ &\qquad\qquad \left. -\left[\gamma \;\; \gamma \;\; \cdots \;\; \gamma\right]^T\right\}. \end{aligned} \tag{31}$$

If condition (24) holds, we have

$$\begin{aligned} &V^h(i+1,j) - V^h(i,j) + V^v(i,j+1) - V^v(i,j) \\ &\quad + \|z(i,j)\|_1 - \gamma\|w(i,j)\|_1 < 0. \end{aligned} \tag{32}$$

We know that

$$\Delta V(i,j) = V^h(i+1,j) - V^h(i,j) + V^v(i,j+1) - V^v(i,j). \tag{33}$$

For any positive scalars k_h, and $k_v \in Z_+$, it can be verified that

$$\begin{aligned} \sum_{i=0}^{k_h}\sum_{j=0}^{k_v}\Delta V(i,j) &= \sum_{i=0}^{k_h}\sum_{j=0}^{k_v}\left(V^h(i+1,j) - V^h(i,j)\right) \\ &\quad + \sum_{i=0}^{k_h}\sum_{j=0}^{k_v}\left(V^v(i,j+1) - V^v(i,j)\right) \\ &= \sum_{j=0}^{k_v}\left(V^{hT}(k_h+1,j) - V^{hT}(0,j)\right) \\ &\quad + \sum_{i=0}^{k_h}\left(V^{vT}(i,k_v+1) - V^{vT}(i,0)\right). \end{aligned} \tag{34}$$

When k_h and $k_v = \infty$, we have

$$\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left(\|z(i,j)\|_1 - \gamma\|w(i,j)\|_1\right) < \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\Delta V(i,j). \tag{35}$$

The existence of solution for LMI (24) implies that the positive 2D system (1a) and (1b) is asymptotically stable. Together with the zero boundary conditions, one can get

$$\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\Delta V(i,j) = 0. \tag{36}$$

Applying (36) to (35), one has

$$\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\|z(i,j)\|_1 < \gamma\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\|w(i,j)\|_1. \tag{37}$$

By Definition 2, the positive 2D system (1a) and (1b) is asymptotically stable and has the l_1-gain index γ.

This completes the proof. □

Remark 7. In Theorem 6, the disturbance attenuation performance of positive 2D linear systems is analyzed, and sufficient conditions for the existence of l_1-gain performance for positive 2D system to (1a) and (1b) are proposed in terms of LMIs which are computationally tractable. This is also the major contribution of our paper.

4. Numerical Example

Consider the positive 2D system with delays in the Roesser model (1a) and (1b), where

$$A = \begin{bmatrix} 0.10 & 0.20 & \vdots & 0.2 \\ 0.00 & 0.30 & \vdots & 0.10 \\ \cdots & \cdots & \cdots & \cdots \\ 0.00 & 0.10 & \vdots & 0.40 \end{bmatrix},$$

$$A_d = \begin{bmatrix} 0.10 & 0.01 & \vdots & 0.05 \\ 0.10 & 0.02 & \vdots & 0.05 \\ \cdots & \cdots & \cdots & \cdots \\ 0.03 & 0.12 & \vdots & 0.03 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.2 \\ 0.1 \\ 0.1 \end{bmatrix}, \qquad H = \begin{bmatrix} 0.1 & 0 & 0.2 \end{bmatrix}, \qquad L = 0.1,$$

$$d_h(i) = 4 + 2\sin\left(\frac{\pi i}{2}\right), \qquad d_v(j) = 5 + 2\sin\left(\frac{\pi j}{2}\right),$$

$$w(i,j) = e^{-(i+0.5j)}, \tag{38}$$

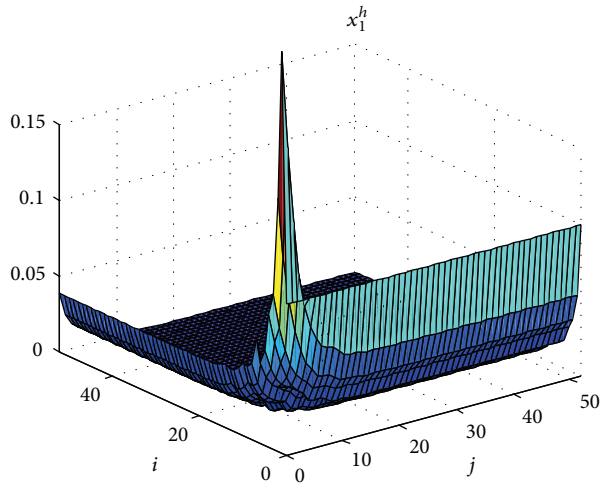

Figure 1: State response of $x_1^h(i, j)$.

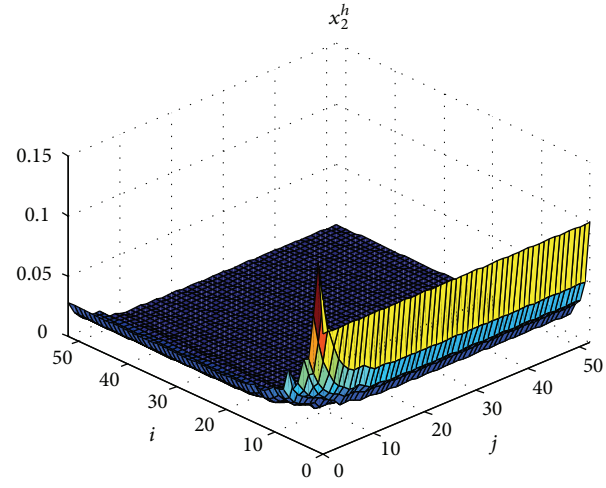

Figure 2: State response of $x_2^h(i, j)$.

where state dimensions are $n_h = 2$ and $n_v = 1$. The boundary conditions are given by

$$x^h(i, j) = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \quad \forall 0 \le j \le 52, \ -d_{hH} \le i \le 0,$$
$$x^v(i, j) = 0.1, \quad \forall 0 \le i \le 52, \ -d_{vH} \le j \le 0. \tag{39}$$

In this example, we can get $d_{hL} = 2$, $d_{hH} = 6$, $d_{vL} = 3$, and $d_{vH} = 7$. Given $\gamma = 4.5$, then by using the LMI Control Toolbox [37] to solve the inequalities in Theorem 6, we can get the following solutions:

$$\begin{aligned} p &= [1.7310 \quad 1.6842 \quad 1.7364]^T, \\ q &= [3.2077 \quad 2.8768 \quad 3.3026]^T, \\ \zeta &= [1.0334 \quad 1.1075 \quad 0.9957]^T, \\ \varsigma_1 &= [0.3675 \quad 0.4042 \quad 0.3552]^T, \\ \varsigma_2 &= [0.0385 \quad 0.0489 \quad 0.0330]^T. \end{aligned} \tag{40}$$

Figures 1, 2, and 3 show the state responses of the system; it can be seen that the corresponding positive 2D system is asymptotically stable. Furthermore, by computing, under zero boundary conditions, we have $\sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\|z(i, j)\|_1 = 4.0206 \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\|w(i, j)\|_1 = 1.0977$. It is obvious that the prescribed l_1-gain performance level $\gamma = 4.5$ is satisfied.

5. Conclusions

This paper has addressed the delay-dependent stability analysis with l_1-gain performance for positive 2D systems with state delays in the Roesser model. A sufficient condition for the existence of the delay-dependent asymptotic stability of positive 2D linear systems with time delays has been established. Copositive-type Lyapunov function method has been used to get a computationally tractable LMI-based sufficient criterion which ensures that the system is asymptotically stable and has a prescribed l_1-gain performance. A numerical example has been given to illustrate the efficiency of the results. Furthermore, our future work will be devoted to the l_1-gain control problem for positive 2D systems with delays.

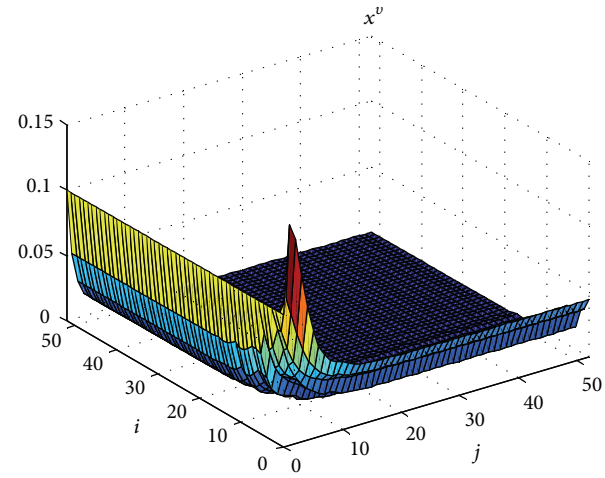

Figure 3: State response of $x^v(i, j)$.

Acknowledgment

This work was supported by the National Natural Science Foundation of China under Grant nos. 60974027 and 61273120.

References

[1] W. S. Lu and A. Antoniou, *Two-Dimensional Digital Filters*, Marcel Dekker, New York, NY, USA, 1992.

[2] R. N. Bracewell, *Two-Dimensional Imaging*, Prentice-Hall Signal Processing Series, Prentice-Hall, Englewood Cliffs, NJ, USA, 1995.

[3] T. Kaczorek, *Two-Dimensional Linear Systems*, vol. 68 of *Lecture Notes in Control and Information Sciences*, Springer, Berlin, Germany, 1985.

[4] L. Xu, M. Yamada, Z. P. Lin, O. Saito, and Y. Anazawa, "Further improvements on Bose's 2D stability test," *International Journal of Control, Automation, and Systems*, vol. 2, no. 3, pp. 319–332, 2004.

[5] E. Fornasini and G. Marchesini, "State-space realization theory of two-dimensional filters," *IEEE Transactions on Automatic Control*, vol. 21, no. 4, pp. 484–492, 1976.

[6] T. Hinamoto, "Stability of 2-D discrete systems described by the Fornasini-Marchesini second model," *IEEE Transactions on Circuits and Systems. I*, vol. 44, no. 3, pp. 254–257, 1997.

[7] M. Bisiacco, "New results in 2D optimal control theory," *Multidimensional Systems and Signal Processing*, vol. 6, no. 3, pp. 189–222, 1995.

[8] J. Wu, X. Chen, and H. Gao, "filtering with stochastic sampling," *Signal Processing*, vol. 90, no. 4, pp. 1131–1145, 2010.

[9] C. L. Du and L. H. Xie, *Control and Filtering of Two-Dimensional Systems*, vol. 278 of *Lecture Notes in Control and Information Sciences*, Springer, Berlin, Germany, 2002.

[10] L. Wu, P. Shi, H. Gao, and C. Wang, "H_∞ filtering for 2D Markovian jump systems," *Automatica*, vol. 44, no. 7, pp. 1849–1858, 2008.

[11] L. Wu, Z. Wang, H. Gao, and C. Wang, "Filtering for uncertain 2-D discrete systems with state delays," *Signal Processing*, vol. 87, no. 9, pp. 2213–2230, 2007.

[12] L. Wu, H. Gao, and C. Wang, "Quasi sliding mode control of differential linear repetitive processes with unknown input disturbance," *IEEE Transactions on Industrial Electronics*, vol. 58, no. 7, pp. 3059–3068, 2011.

[13] R. P. Roesser, "A discrete state-space model for linear image processing," *IEEE Transactions on Automatic Control*, vol. 20, no. 1, pp. 1–10, 1975.

[14] E. Fornasini and G. Marchesini, "Doubly-indexed dynamical systems: state-space models and structural properties," *Mathematical Systems Theory*, vol. 12, no. 1, pp. 59–72, 1978/79.

[15] J. E. Kurek, "The general state-space model for a two-dimensional linear digital system," *IEEE Transactions on Automatic Control*, vol. 30, no. 6, pp. 600–602, 1985.

[16] T. Kaczorek, "Reachability and controllability of non-negative 2D Roesser type models," *Bulletin of the Polish of Sciences: Technical Sciences*, vol. 44, no. 4, pp. 405–410, 1996.

[17] M. E. Valcher, "On the internal stability and asymptotic behavior of 2-D positive systems," *IEEE Transactions on Circuits and Systems. I*, vol. 44, no. 7, pp. 602–613, 1997.

[18] T. Kaczorek, "Reachability and minimum energy control of positive 2D systems with delays," *Control and Cybernetics*, vol. 34, no. 2, pp. 411–423, 2005.

[19] T. Kaczorek, *Positive 1D and 2D Systems*, Springer, London, UK, 2001.

[20] X. Liu, "Constrained control of positive systems with delays," *IEEE Transactions on Automatic Control*, vol. 54, no. 7, pp. 1596–1600, 2009.

[21] L. Farina and S. Rinaldi, *Positive Linear Systems: Theory and Applications*, Pure and Applied Mathematics, John Wiley & Sons, New York, NY, USA, 2000.

[22] T. Kaczorek, "The choice of the forms of Lyapunov functions for a positive 2D Roesser model," *International Journal of Applied Mathematics and Computer Science*, vol. 17, no. 4, pp. 471–475, 2007.

[23] A. Hmamed, M. A. Rami, and M. Alfidi, "Controller synthesis for positive 2D systems described by the Roesser model," in *Proceedings of the 47th IEEE Conference on Decision and Control*, pp. 387–391, Cancun, Mexico, 2008.

[24] T. Kaczorek, "Asymptotic stability of positive 2D linear systems," in *Proceedings in the13th Conference on Computer Applications in Electrical Engineering*, CD-ROM, 2008.

[25] M. Twardy, "An LMI approach to checking stability of 2D positive systems," *Bulletin of the Polish Academy of Sciences Technical Sciences*, vol. 55, no. 4, pp. 385–395, 2007.

[26] J. K. Hale and S. M. V. Lunel, *Introduction to Functional-Differential Equations*, vol. 99 of *Applied Mathematical Sciences*, Springer, New York, NY, USA, 1993.

[27] L. Wu and W. X. Zheng, "Weighted H_∞ model reduction for linear switched systems with time-varying delay," *Automatica*, vol. 45, no. 1, pp. 186–193, 2009.

[28] L. Wu and W. X. Zheng, "Passivity-based sliding mode control of uncertain singular time-delay systems," *Automatica*, vol. 45, no. 9, pp. 2120–2127, 2009.

[29] R. Yang, P. Shi, G. P. Liu, and H. Gao, "Network-based feedback control for systems with mixed delays based on quantization and dropout compensation," *Automatica*, vol. 47, no. 12, pp. 2805–2809, 2011.

[30] X. Su, P. Shi, L. Wu, and Y. D. Song, "A novel approach to filter design for T-S fuzzy discrete-time systems with time-varying delay," *IEEE Transactions on Fuzzy Systems*, vol. 20, no. 6, pp. 1114–1129, 2012.

[31] R. Yang, Z. Zhang, and P. Shi, "Exponential stability on stochastic neural networks with discrete interval and distributed delays," *IEEE Transactions on Neural Networks*, vol. 21, no. 1, pp. 169–175, 2010.

[32] L. Wu, X. Su, P. Shi, and J. Qiu, "Model approximation for discrete-time state-delay systems in the T-S fuzzy framework," *IEEE Transactions on Fuzzy Systems*, vol. 19, no. 2, pp. 366–378, 2011.

[33] T. Kaczorek, "Realization problem for positive 2D systems with delays," *Machine Intelligence and Robotic Control*, vol. 6, no. 2, pp. 61–68, 2004.

[34] T. Kaczorek, "Asymptotic stability of positive 2D linear systems with delays," *Bulletin of the Polish Academy of Sciences: Technical Sciences*, vol. 57, no. 2, pp. 133–138, 2009.

[35] T. Kaczorek, "Independence of asymptotic stability of positive 2D linear systems with delays of their delays," *International Journal of Applied Mathematics and Computer Science*, vol. 19, no. 2, pp. 255–261, 2009.

[36] L. Benvenuti, A. Santis, and L. Farina, *Positive Systems*, Lecture Notes in Control and Information Sciences, Springer, Berlin, Germany, 2003.

[37] S. Boyd, L. E. Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*, vol. 15 of *SIAM Studies in Applied Mathematics*, SIAM, Philadelphia, Pa, USA, 1994.