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A fuzzy logic approach to modeling a vehicle crash test

Research article

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Abstract: This paper presents an application of fuzzy approach to vehicle crash modeling. A typical vehicle to pole collision is described and kinematics of a car involved in this type of crash event is thoroughly characterized. The basics of fuzzy set theory and modeling principles based on fuzzy logic approach are presented. In particular, exceptional attention is paid to explain the methodology of creation of a fuzzy model of a vehicle collision. Furthermore, the simulation results are presented and compared to the original vehicle's kinematics. It is concluded which factors have influence on the accuracy of the fuzzy model's output and how they can be adjusted to improve the model's fidelity.

Keywords: Vehicle crash • Fuzzy logic • Modeling

1. Introduction

Vehicle collision is a phenomenon which is extremely complex from the dynamic point of view. There are a lot of vehicle elements and joints which interact with each other during a crash. Furthermore, they all undergo deformation caused by the impact energy transformation, therefore they cannot be assumed to be perfectly rigid. This complicates the mathematical description, analysis, and simulation of this type of event. According to [1] two approaches to mathematical modeling of real world systems can be distinguished:

1. Mathematical approach – the fundamental laws of physics (*e.g.* Newton's Laws or conservation principle) are used to derive dynamics of a phenomenon or system.

2. System identification – experimental approach. System is examined by performing on it experiments and subsequently model parameters are estimated. They are selected to minimize an error between a real system's output and the one predicted by a model.

The second methodology is more appropriate for modeling complex systems because it does not investigate their detailed mathematical specification but, on the other hand, it allows one to create their "black box" models. This approach will be followed in this paper.
Vehicle users safety is one of the great concerns of everyone who is involved in the automotive industry. However, crash tests are complex and complicated experiments. Therefore it is advisable to establish a vehicle crash model and use its results instead of a full-scale experiment measurements

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to predict car's behavior during a collision. This will help to increase safety of all road users: car drivers and their passengers, as well as vulnerable road users (VRUs) such as motorcyclists and pedestrians. This task involves a number of correlated issues with many different approaches and methodologies. There are three main ideas proposed in [2]: safer behavior, safer infrastructure and safer vehicles. The ideas applicable to the last topic are discussed in this study.

Nowadays we can distinguish two main approaches in the area of vehicle crash modeling. The first one utilizes FEM (Finite Element Method) software, whereas the second way is called LPM (Lumped Parameter Modeling). The major advantage of a FEM model is its capability to represent geometrical and material details of the structure. The major disadvantage of FE method is its cost and the fact that it is time-consuming. To obtain good correlation of a FEM simulation with test measurements, extensive representation of the major mechanisms in the crash event is required. This increases costs and the time required for modeling and analysis. On the other hand, in a typical lumped parameter model, used for a frontal crash, the car can be represented as a combination of masses, springs and dampers. The dynamic relationships among the lumped parameters are established using Newton's laws of motion and then the set of differential equations is solved using numerical integration techniques. The major advantage of this technique is the simplicity of modeling and the low demand on computer resources. The problem with this method is obtaining the values for the lumped parameters, *e.g.* mass, stiffness, and damping. There is a number of methods which can be applied to assess parameters of such models (stiffness, damping) basing on the real crash data. One of them is fitting the models' responses to the real car's displacement - see [3–5]. The advantage of such a methodology is the fact that models can be easily created, without a lot of computational effort. However, a serious drawback with using this method is that the established models are valid only for a collision scenario for which they have been formulated. This makes them impossible to use to represent different crash tests. Therefore, particular attention is being paid to the estimation of nonlinear parameters of viscoelastic models as well and to ahead prediction of vehicle kinematics – refer to [6–8]. It is done in order to provide for a wider range of crash events which can be simulated by using one model only. Moreover, applying the nonlinear models of vehicle crashes increases their accuracy and improves the simulation results.

Because of the fact that crash pulse is a complex signal, it is justified to simplify it. One solution for this is covered in [9]. References [10–12] talk over commonly used ways of describing a collision – *e.g.* investigation of tire marks or the crash energy approach. In the most recent scope of research concerning crashworthiness it is to define a dynamic vehicle crash model which parameters will be changing according to the changeable input (*e.g.* initial impact velocity). One of such trials is presented in [13]. In addition to this work, in [14] one can find a complete derivation of vehicle collision mathematical models composed of springs, dampers and masses with piecewise nonlinear characteristics of springs and dampers.

References [15–19] discuss usefulness of neural networks and fuzzy logic in the field of modeling of crash events. Fuzzy logic together with neural networks and image processing have been employed in [20] to estimate the total deformation energy released during a collision. However, the number of publications regarding fuzzy logic application to vehicle crash modeling is limited. On the other hand, fuzzy controllers are thoroughly described as vehicle path planners ([21] and [22]) or as a technology utilized in damping reduction strategies in vehicle active suspension systems ([23] and [24]). Fuzzy logic application is not limited to land mobile robots - some functions of railway and underwater vehicles can be successfully assisted by it as well ([25] and [26]).

The work presented in this study offers considerable improvement of simulation outcomes as compared to the standard lumped parameter modeling of viscoelastic systems discussed above. The major improvement is observed in the accuracy of the results – kinematics of a reference vehicle is reproduced with higher degree of fidelity than in a typical lumped parameter model. Simultaneously, the current study can be considered as a continuation and further enhancement of mathematical models of vehicle crashes based on system identification and "black box" modeling. The advantage offered here is that the fuzzy logic approach allows to simulate any type of vehicle crash (frontal impact, oblique collision, *etc.*), since it is purely based on signals only. It does not involve much of computational complexity and offers quicker performance as a typical neural-network based methodology. Finally, the significant enhancement of vehicle crash modeling shown in this work, as compared to the previously mentioned approaches, is that a successfull model is obtained without complex and complicated mathematical analysis and formulation of differential equations. The method shown here uses only inputs and outputs of the system and represents a full-scale vehicle collision by the set of fuzzy rules which relates those inputs and outputs without the need of extremely thorough mathematical derivation. For this reason this field of research is worth researching since it offers satisfactory results at a reasonable level of modeling complexity. Hence, fuzzy logic models of vehicle collisions may be used in an early design stage of vehicles to assess overall behavior of a given vehicle involved in a collision and estimate impact severity for its occupants.

The most important contribution of this paper is the application of artificial intelligence methods including fuzzy logic to create a "black-box" model of a vehicle collision and validation of the obtained simulation results with the full-scale experimental data analysis. Novelty of this research is related to the application of a regular fuzzy logic modeling method to a real-world problem which has not been widely explored by this approach so far.

2. Fuzzy logic modeling methodology

2.1. The fuzzy sets basics

Fuzzy logic was first proposed in [27]. This notion was explained by fuzzy sets which are means to represent uncertainty [28]. In probability theory, the uncertainty is assumed to be a random process. In opposite to that, the fuzzy set theory considers not all uncertainties random – *e.g.* imprecision, vagueness, and lack of information can be successfully modeled by fuzzy logic. According to [29] fuzzy models are used wherever it is difficult to create a mathematical model, but the actions can be described in a qualitative way, by using fuzzy rules. They are applied for processes that have strong cross-coupling, nonlinear relationships between quantities, large distortions and time delays. In order to create a fuzzy model of a given system, the following steps should be taken:

1. Defining fuzzy rules.
2. Defining membership functions for inputs and outputs.
3. Fuzzification of inputs to develop conclusions.
4. Applying rules to develop conclusions.
5. Combining conclusions to obtain final output distribution.
6. Output defuzzification to obtain a crisp value.

The above procedure can be visually represented as shown in Figure 1.

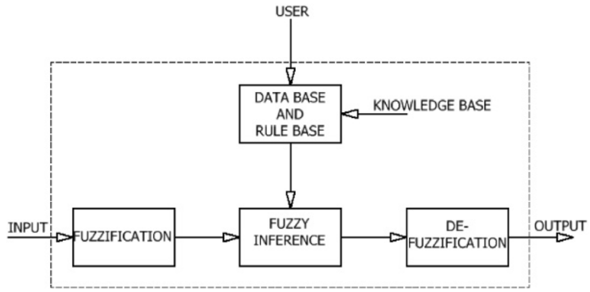

Figure 1. Structure of the fuzzy system.

2.2. Methodology of creating a vehicle crash fuzzy model

The aim of the model established by using fuzzy sets theory is to reproduce kinematics of a car involved in a crash event. The fuzzy system from Figure 1 is depicted in Figure 2 as "Fuzzy Model". The collision measurements (acceleration, velocity, and displacement) are inputs to this system. The predicted output is subsequently feed back to be compared with the reference, original vehicle behavior. Thanks to the feedback loop, the fuzzy system in fact controls the error between the actual and desired system's response. The main idea of this reasoning is shown in Figure 2. The aim of the fuzzy model is to increase the change of the output δ_u when the difference between the reference and actual response is negative and vice versa. In other words – it minimizes the error e and the rate of change of error δ_e. Thanks to this operation it is possible to predict kinematics of a car involved in a crash event.

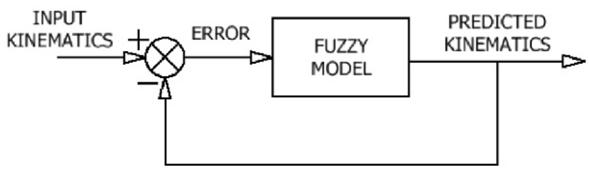

Figure 2. Scheme of the vehicle crash fuzzy model.

2.3. Fuzzy rules

To describe a system and perform inference, rules such as "If A then Z" (implication $A \rightarrow Z$) are used. A is referred to as an antecedent and Z is known as a consequent, where both A and Z are fuzzy sets. Such linguistic rules are called Mamdani-type ones. Mamdani model is a set of rules in which every rule defines one fuzzy point in the domain. They were named after E.H. Mamdani ([30] and [31]) who first used this kind of statement in a fuzzy rule-base to control a plant. The other commonly used model is Takagi-Sugeno one, which has a function in the conclusion (consequence) instead of a fuzzy set. The quantities which are provided as inputs to the fuzzy model (denoted as "ERROR" in Figure 2) are: the difference between the desired input and actual output as well as the rate of change of this error. The tabular structure of the linguistic fuzzy rulebase is presented in Table 1. Letters stand for

Table 1. Linguistic rulebase for the vehicle crash fuzzy model.

Change of error δ_e	Error e						
	$B-$	$M-$	$S-$	0	$S+$	$M+$	$B+$
$B-$	$B-$	$B-$	$B-$	$B-$	$M-$	$S-$	0
$M-$	$B-$	$B-$	$B-$	$M-$	$S-$	0	$S+$
$S-$	$B-$	$B-$	$M-$	$S-$	0	$S+$	$M+$
0	$B-$	$M-$	$S-$	0	$S+$	$M+$	$B+$
$S+$	$M-$	$S-$	0	$S+$	$M+$	$B+$	$B+$
$M+$	$S-$	0	$S+$	$M+$	$B+$	$B+$	$B+$
$B+$	0	$S+$	$M+$	$B+$	$B+$	$B+$	$B+$

big (B), medium (M), and small (S), respectively, whereas the signs denote whether a given quantity is positive or negative.

The rules should be interpreted as *e.g.* "IF the error e is medium negative AND the rate of change of error δ_e is small positive THEN the change of output δ_u is small negative". The surface obtained from the above table is shown in Figure 3. Please note that δ_u denotes the change of the output, δ_e – rate of change of the error, and e – error itself. It is noting that the axes of this graph are unitless. That is because of its application to different data sets (acceleration, velocity, and displacement). In each of those cases, the output and error are expressed in g, km/h, and cm, respectively.

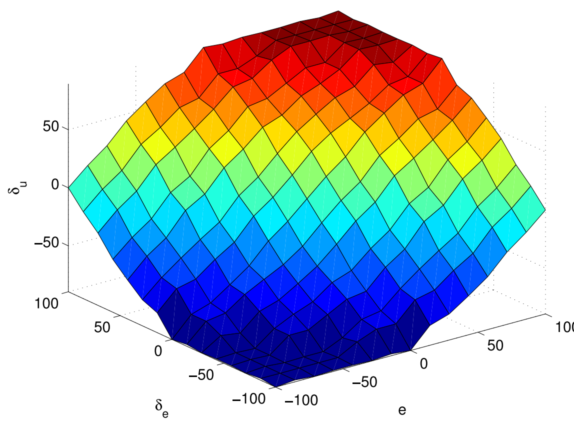

Figure 3. Correlation of the fuzzy model's inputs and output.

2.4. Membership functions

In conventional set theory it is possible to classify elements only as members or not members of a given set. In the fuzzy set theory, however, the membership of a given element to a given set is characterized by the value of the so called

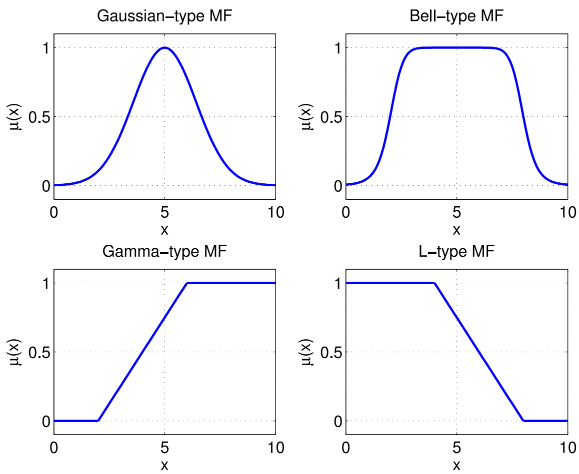

Figure 4. Exemplary membership functions.

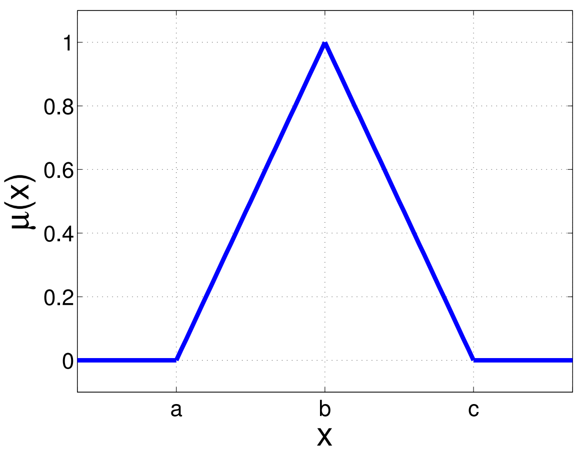

Figure 5. T-type (triangle) membership function.

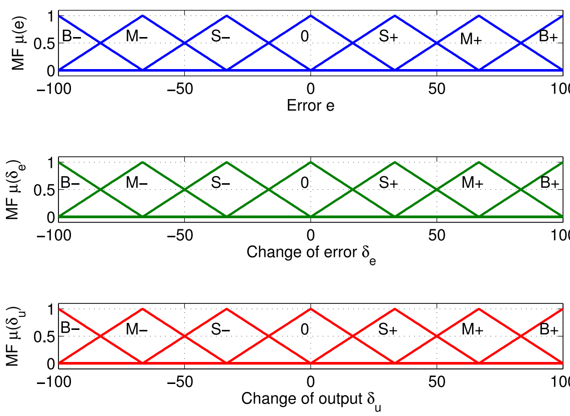

Figure 6. Membership functions of the fuzzy model.

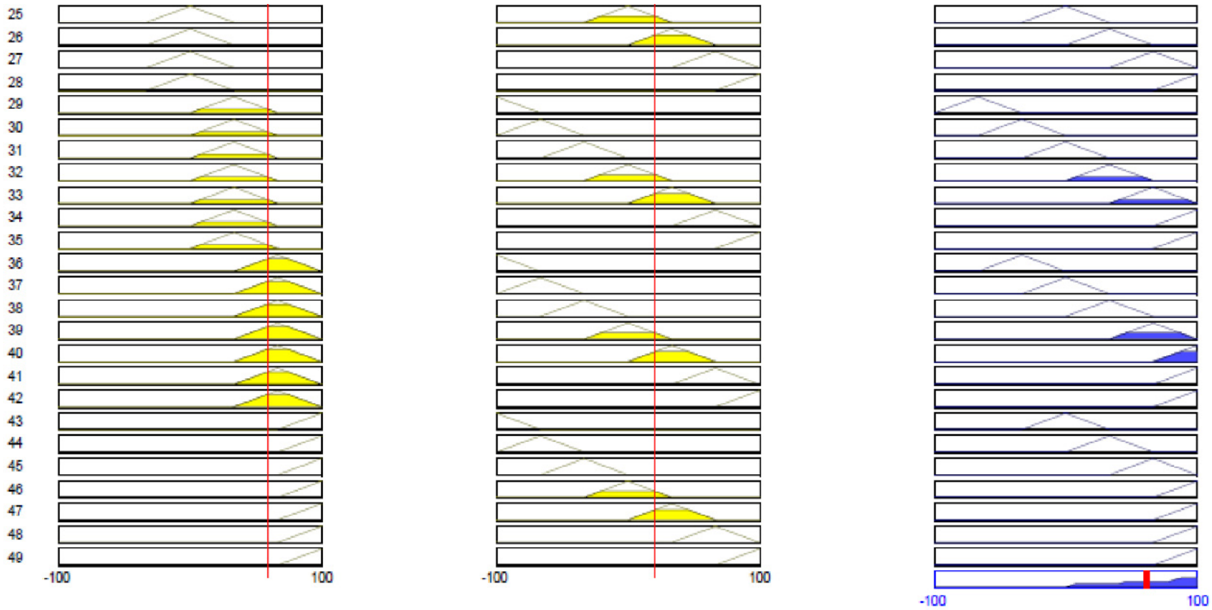

Figure 7. Inference results for arbitrary values of inputs.

membership function μ (abbreviated as MF), which ranges from 0 to 1. Some typical membership functions are shown in Figure 4. In this work, the so called T-type MF (or triangle MF) is used. This type of MF is often used in various applications and simultaneously offers a simple computational apparatus [32]. It is illustrated in Figure 5 and expressed by the following formula:

$$t(x,a,b,c) = \begin{cases} 0 & \text{for } x \le a \\ \frac{x-a}{b-a} & \text{for } a < x \le b \\ \frac{c-x}{c-b} & \text{for } b < x \le c \\ 0 & \text{for } x > c. \end{cases} \quad (1)$$

Taking advantage of the shape of the crash pulse plotted in Figure 11 and minimal and maximal values achieved by those plots, it was decided that the values of the inputs: error e and change of error δ_e, as well as the values of the change of output δ_u lie within the limits of $< -100; 100 >$. The obtained membership functions are presented in Figure 6.

2.5. Inference

To assess what a degree of a truth level is for each individual rule, the inference should be performed. It is a process of mapping membership values from the input windows, through the rulebase, to the output window [28]. As shown in Section 2.3 in this study there are presented rules which contain an internal logical "AND" expression, however, between particular rules there is logical "OR". Mathematically, the first operation can be explained as intersection of two fuzzy sets A and B:

$$\mu_{A \cap B}(u) = min\{\mu_A(u), \mu_B(u)\} \text{ for all } u \in U. \quad (2)$$

On the other hand, the second operation can be characterized as union of the two fuzzy sets:

$$\mu_{A \cup B}(u) = \mu_{A+B}(u) = max\{\mu_A(u), \mu_B(u)\} \text{ for all } u \in U. \quad (3)$$

Therefore, finally, for the rules stated as:

$$\text{OR IF } e \text{ is A AND } \delta_e \text{ is B THEN } \delta_u = \text{C} \quad (4)$$

the whole so called max-min inference process is given by the following equation:

$$\mu_C(\delta_u) = max\{min\{\mu_A(e), \mu_B(\delta_e)\}\}. \quad (5)$$

The results of inference for some exemplary values ($e = 58.7$ and $\delta_e = 20.9$) are shown in Figure 7. Please note that not all the rules are presented for the sake of simplicity – because of the logical "AND" between two inputs e (1st column) and δ_e (2nd column), no output δ_u (3rd column) has been produced for rules less than 32. This graph illustrates relations described in Equation 2-5.
The value of $\delta_u = 61.8$ was found by using the min-max inference technique together with the center of area method in the defuzzification process (which will be explained later). Simulations were performed in MATLAB™ software.

2.6. Defuzzification

Defuzzification is the procedure of acquiring the crisp value representing the fuzzy output set obtained in the inference process. The most well known defuzzification technique is called center of area method. It can be explained as ([28]):

$$\text{Crisp output value} = \frac{\text{Sum of first moments of areas}}{\text{Sum of areas}}. \tag{6}$$

Equations for a continuous and discrete system are, respectively:

$$u(t) = \frac{\int u\mu(u)du}{\int \mu(u)du} \tag{7}$$

$$u(kT) = \frac{\sum_{i=1}^{n} u_i\mu(u_i)}{\sum_{i=1}^{n} \mu(u_i)}. \tag{8}$$

According to [29] the advantage of this method is that all active rules are part of the defuzzification process. It provides greater sensitivity of the fuzzy model to the changes in input data. However, the drawback of this approach is its computational complexity.

3. Experimental setup description

The data used by us come from the typical vehicle to pole collision. The initial velocity of the car was 35 km/h, and the mass of the vehicle (together with the measuring equipment and dummy) was 873 kg. During the test, the acceleration at the center of gravity in three dimensions (x – longitudinal, y – lateral and z – vertical) was recorded. The yaw rate was also measured with a gyro meter. Using normal speed and high-speed video cameras, the behavior of the safety barrier and the test vehicle during the collision was recorded – see Figure 8 to Figure 10.

3.1. Crash pulse analysis

Having at our disposal the acceleration measurements from the collision, we are able to describe in details motion of the car. Since it is a central impact, we analyze only the pulse recorded in the longitudinal direction (x-axis). By integrating car's deceleration we obtain plots of velocity and displacement, respectively – see Figure 11. At the time when the relative approach velocity is zero (t_m), the maximum dynamic crush (d_c) occurs. The relative velocity in the rebound phase then increases negatively up to the final separation (or rebound) velocity, at which time a vehicle rebounds from an obstacle. When the relative

Figure 8. Car before a collision.

Figure 9. The moment of impact.

Figure 10. Car's deformation.

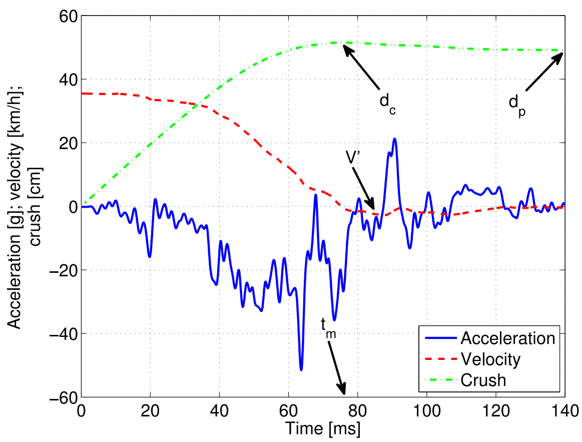

Figure 11. Real car's kinematics.

Table 2. Relevant parameters characterizing the real collision

Parameter	Value
Initial impact velocity V $[km/h]$	35
Rebound velocity V' $[km/h]$	3
Maximum dynamic crush d_c $[cm]$	52
Time when it occurs t_m $[ms]$	76
Permanent deformation d_p $[cm]$	50

acceleration becomes zero and relative separation velocity reaches its maximum recoverable value we have the separation of the two masses.

4. Simulation results

The created fuzzy model which was used to simulate a vehicle to pole collision is illustrated in Figure 12. It was applied to predict the reference vehicle's kinematics – results of this operation are presented in Figure 13, Figure 14, and Figure 15, respectively. The output of the fuzzy model closely follows the reference signals. It was shown that the complexity of the examined characteristics does not affect the accuracy of the prediction. High degree of fidelity is achieved for a relatively simple plot (displacement) as well as for a rapidly changing course (acceleration).

5. Further validation

In order to verify if the proposed fuzzy logic model is capable to represent a different type of collision than the

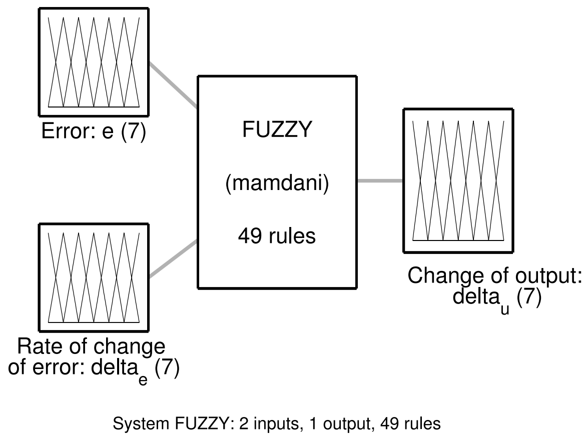

Figure 12. Fuzzy model of a vehicle crash.

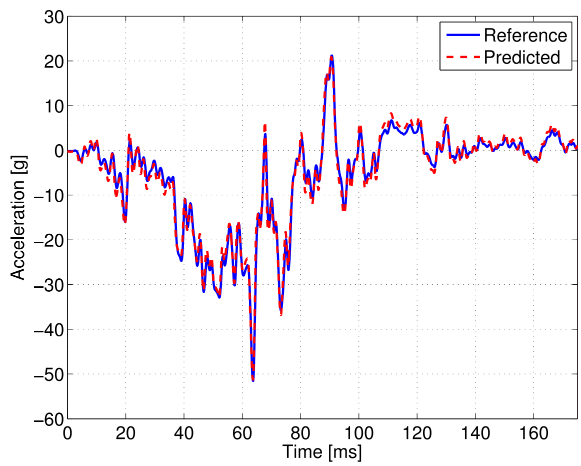

Figure 13. Simulation results for acceleration reproduction.

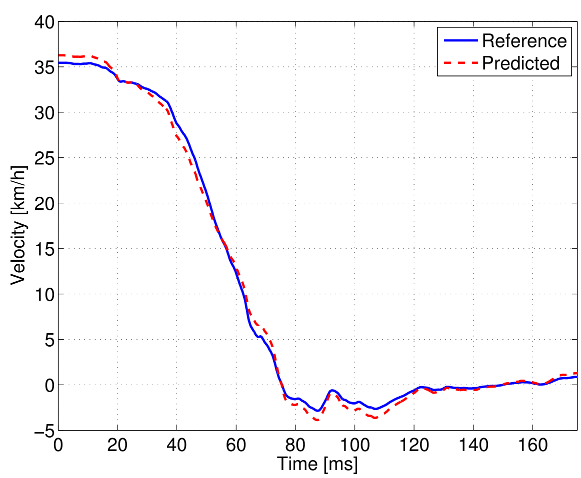

Figure 14. Simulation results for velocity reproduction.

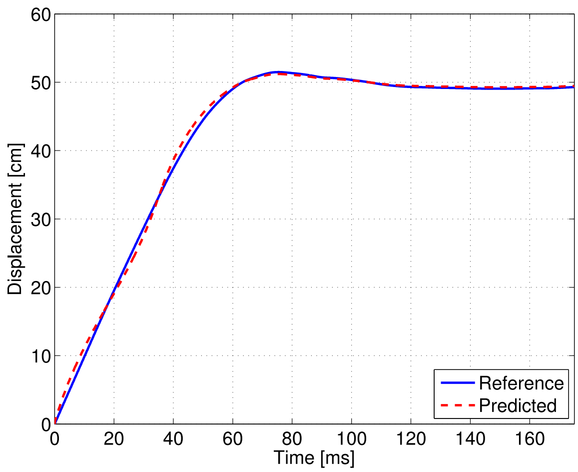

Figure 15. Simulation results for displacement reproduction.

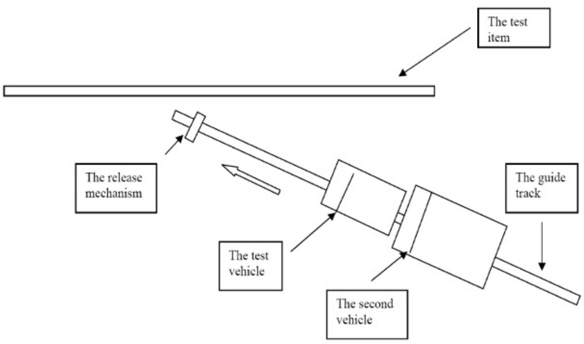

Figure 16. Scheme of the experiment.

one already presented in Section 3 it is suggested to verify its performance to reproduce kinematics of a vehicle involved in a different crash scenario.

5.1. Vehicle oblique collision

A typical vehicle to safety barrier collision is selected to provide us with additional data sets. A new, additional experimental setup description is covered in details in [33]. It is a typical high-speed vehicle to safety barrier oblique collision – scheme showing the layout of the test setup is illustrated in Figure 16.

The vehicle has an initial velocity of 104 km/h while impacting the barrier at the angle of $\Psi = 20°$. Its total mass including the measuring equipment and dummy was determined to be 893 kg. During the test, the acceleration at the center of gravity (COG) in three dimensions (x-longitudinal, y-lateral and z-vertical) was recorded. The yaw rate was also measured with a gyro meter. The safety barrier and car themselves are shown in Figure 17 and

Figure 17. Safety barrier – location of impact.

Figure 18, respectively. Using normal-speed and high-speed video cameras (recording rate was 250 frames per second), the behavior of the test vehicle during the collision was recorded – see Figure 19.

5.2. Analysis of vehicle kinematics

Having at our disposal the acceleration measurements from the collision, we are able to describe in details motion of the car. Since it is an oblique impact, we analyze only the pulses recorded in the longitudinal (x-axis) and lateral (y-axis) directions as well as the yaw rate. By integrating car's deceleration we obtain plots of velocity and displacement, respectively – see Figure 20. At the time

Figure 18. Car used in experiment.

when the lateral velocity component is zero, the vehicle starts to move completely alongside the safety barrier. Results shown in Figure 20 are already plotted for the convenience in the global reference frame. The particular components (X-longitudinal and Y-lateral, respectively) of the initial velocity are determined by applying a simple trigonometric relationships (initial impact velocity is $v_0 = 104$ $[km/h]$ and the angle of impact is $\Psi = 20°$):

$$v_X = v_0 \cdot \cos\Psi = 98\ [km/h] \tag{9}$$

$$v_Y = v_0 \cdot \sin\Psi = 36\ [km/h] \tag{10}$$

It is noted that the negative value of the Y-direction velocity component showed in Figure 20 is related to the assumed global reference frame – see Figure 21. Its center is located directly in the first point of contact between the vehicle and the barrier.

5.3. Results of validation

Here are presented the results of applying the created fuzzy model in the same way as already shown in Section 4. The estimated signals of acceleration, velocity, and displacement are compared to the reference ones and are shown in Figure 22, Figure 23, Figure 24, respectively. It is shown that the overall behavior of the estimated acceleration curves follow the reference ones. Consequently, the similarities between estimated and reference velocities as well as displacements are observed. The discrepancies are observed in the velocity and displacements plots, however they stay within the reasonable limits. Please note that to visualize the effectiveness of the method presented in this work, the results are compared with the ones presented in [34]. In [34] vehicle crash was modeled as a viscoelastic

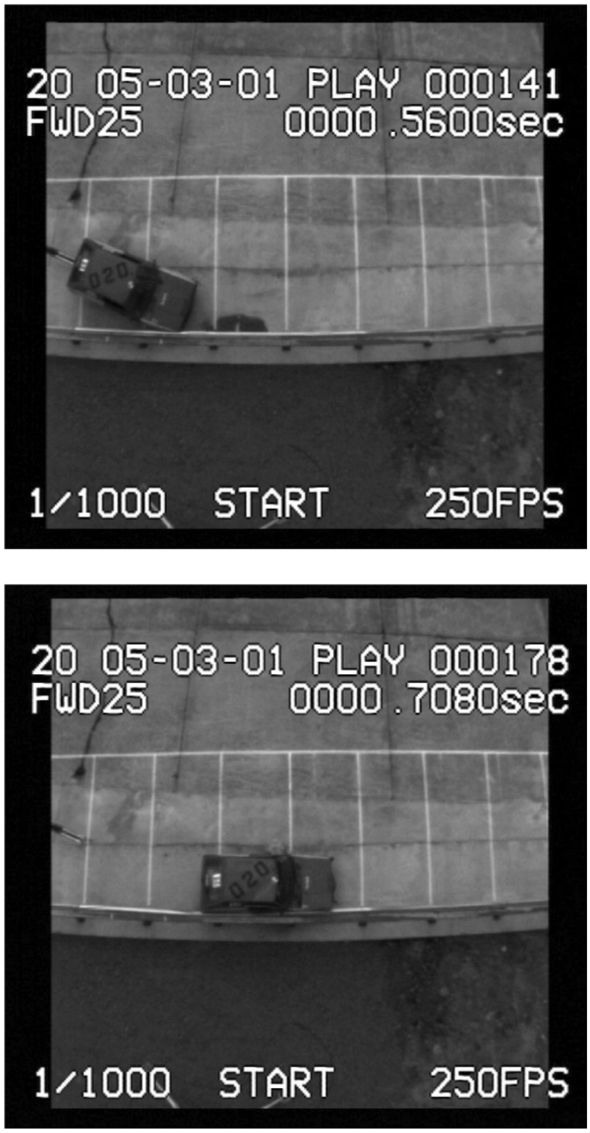

Figure 19. Subsequent steps of the crash test.

system consisting of a mass, spring, and damper in two different arrangements (parallel connection: so called Kelvin model, and in series connection: so called Maxwell model). Parameters of those models were estimated by fitting their dynamic equations of motion to the reference displacement of the vehicle. It is noting that those parameters were constant throughout the simulation. Responses of the two different models are shown in Figure 25 and Figure 26. Calculating the root-mean-square errors for each of the methods (y_i – reference value, $\hat{y}_i$ – estimated value):

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{n}} \tag{11}$$

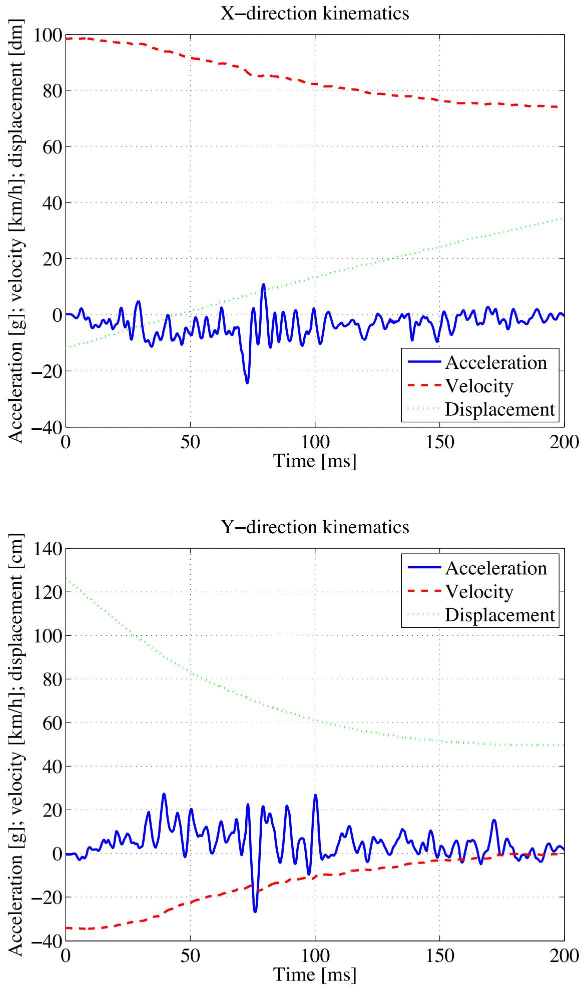

Figure 20. Complete kinematics of the experimental vehicle.

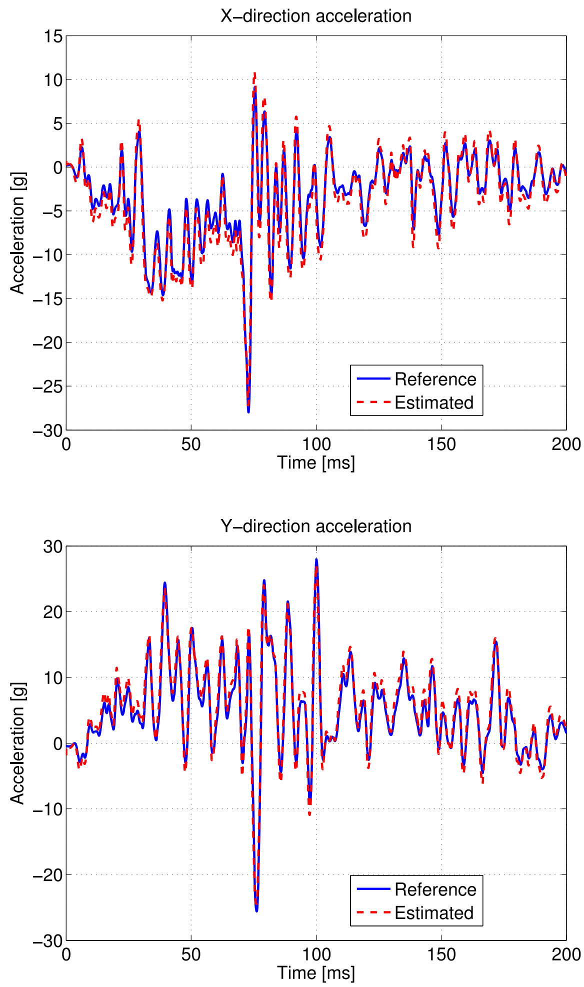

Figure 22. Comparative analysis of acceleration pulses of a vehicle involved in oblique collision.

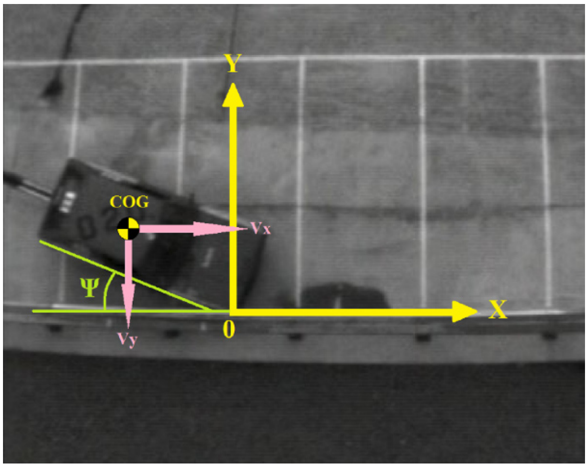

Figure 21. Vehicle moving in the global reference frame.

yields the results shown in Table 3. The value of the root-mean-square error determines the average difference between the reference and estimated value.

Table 3 clearly points out the significant improvement in the results of modeling vehicle crash by using fuzzy logic-based method described in this work with respect to the results yielded by typical lumped-parameter models. It is observed that for both frontal and oblique collisions the factor which plays the most important role during a collision (*i.e.* acceleration) follows closely the reference one yielding low values of RMSE. RMSE for velocities and displacements are also at low level, as compared to the typical lumped parameter viscoelastic models. Above considerations explicitly show the benefit of the current method and enhancement of vehicle crash modeling outcomes.

Table 3. Root-mean-square errors (RMSE).

Quantity	Acceleration $[g]$	Velocity $[km/h]$	Displacement $[cm]$
Kelvin model	10.49	16.81	24.60
Maxwell model	8.60	4.02	2.21
Fuzzy logic approach: Frontal impact	1.39	0.45	1.12
Fuzzy logic approach: Planar impact - X-direction	1.21	2.73	6.38
Fuzzy logic approach: Planar impact - Y-direction	1.44	2.66	5.67

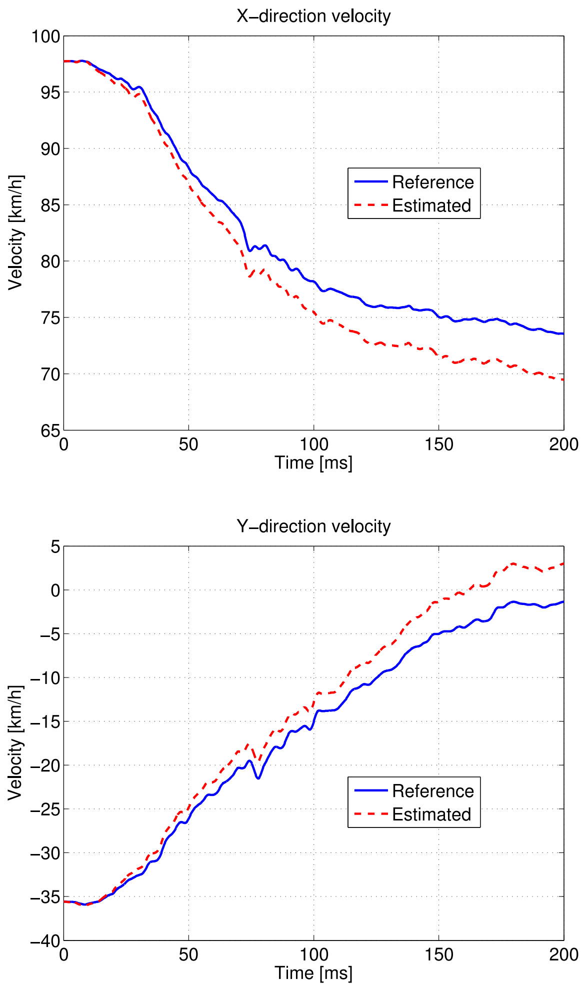

Figure 23. Comparative analysis of velocities of a vehicle involved in oblique collision.

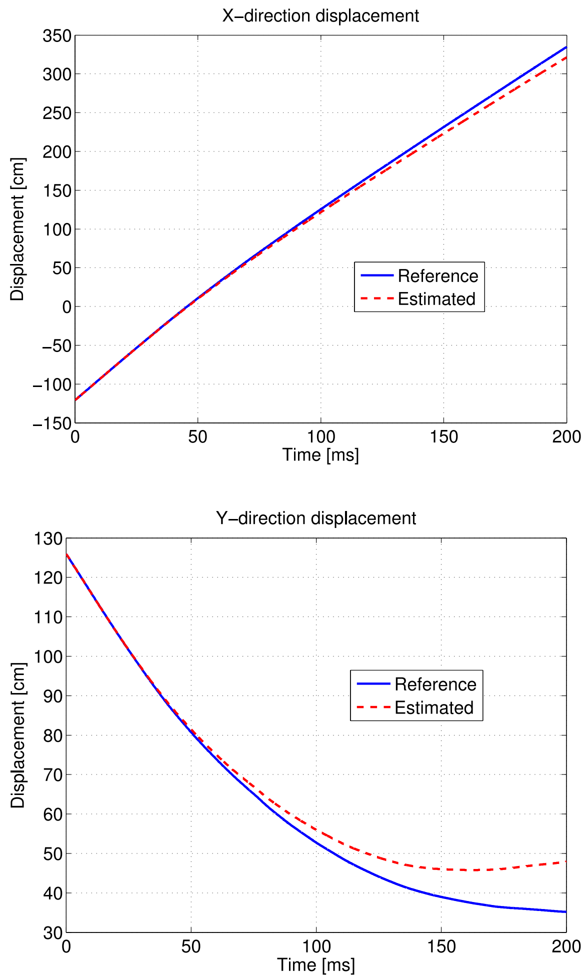

Figure 24. Comparative analysis of displacements of a vehicle involved in oblique collision.

6. Conclusions and future works

The methodology presented in this study proves usefulness of the fuzzy logic application to vehicle crash modeling. The results obtained in this work were compared to the results of By using the fuzzy approach to vehicle crash modeling a lot of different crash scenarios may be successfully simulated, regardless of their type, initial impact velocity or impact angle. This makes the current study a valuable contribution to modeling and simulation of vehicle behavior throughout a collision. Care should be taken while selecting the membership functions' ranges as well as their density. Higher MF density offers higher sensitivity of the modeling therefore it should be used for the applications in which it is crucial to capture the rapidly changing system's output. To obtain an even more precise fuzzy model's

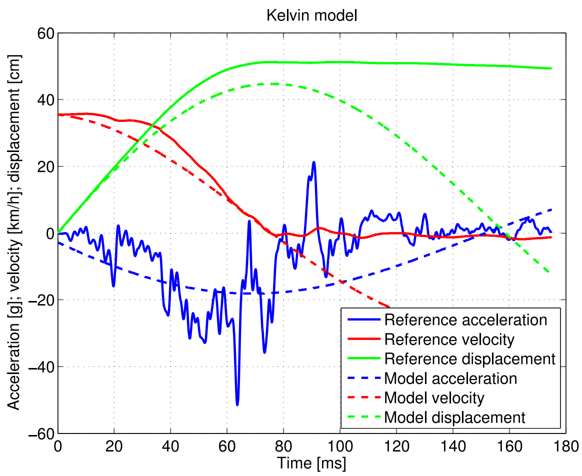

Figure 25. Kelvin model performance comparison [34].

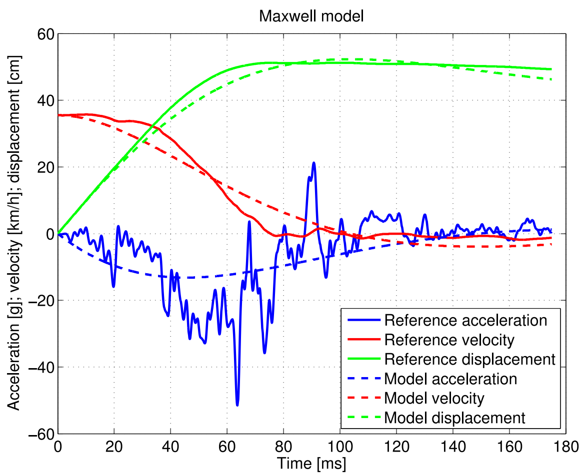

Figure 26. Maxwell model performance comparison [34].

output, the number of rules can be increased, as well as the number of labels for each variable. The factor which plays a crucial role in fuzzy modeling is also a shape of MF - it should be adjusted individually to the nature of an event being modeled. Thus, to achieve a better response of a fuzzy model it is advisable to increase its complexity: the number of MFs, the number of rules as well as to verify which shape of MF is the most suitable for the vehicle crash modeling. In the wider perspective the methodology discussed in this paper may be used as a tool for safety assessment of a vehicle crash depending on number of inputs like the type of collision, vehicle initial impact velocity or impact angle. Those various factors may be linked by the fuzzy logic approach with determination of occupants severity, creating a reliable and effective knowledge base. Finally, it is advisable to investigate performance of neuro-fuzzy inference systems in the area of vehicle crash modeling. They offer a strong potential in the ahead prediction of signals which makes them appropriate tools for prediction of crash pulses. Such enhanced modeling methodology would ultimately increase safety of vehicle occupants, since more extensive work would be possible to be carried out in the early design stage due to those mentioned efficient and effective modeling methodologies.

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