Wind Turbine Modeling Using The Bond Graph

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Abstract— **This paper addresses the problem of bond graph methodology as a graphical approach for modeling wind turbine systems. In this case, we consider the modeling of a wind turbine system with individual pitch control scheme and the interaction with tower motions. Two different bond graph models are presented, one complex and one simplified. Furthermore, the purpose of this paper is not to validate a specific wind turbine model, but rather show the difference between modeling with a classical mechanical method and by using the bond graph approach. Simulation results illustrate the simplified system response obtained using implementation of the governing equations in MATLAB/Simulink and is compared with a bond graph implementation in the simulation program 20-sim.**

I. INTRODUCTION

The demand for energy world wide is increasing every day. In these green times renewable energy is a hot topic all over the world. Wind energy is currently the most popular energy sector. The growth in wind power industry has been tremendous over the last decade, according to [1] it has (on average) doubled every third year. Up to early 2010 the world wide capacity was 159,213 MW.

Whenever we are talking about wind turbine (WT) control systems, the turbine model becomes a critical part of the discussion. Over the years it has been some discussions about how to model the WT accurately. In [2]-[4] they perform dynamic analysis on a one-mass-model, in [5]-[8] they examine a two-mass-model. In [9] they use actual measured data from a WT and compares it with both a one-mass and a two-mass-model. They validate the model using a recorded case obtained in a fixed speed, stall regulated WT. In [10] a six, three and a two-mass model are compared. They argue that a six mass model is needed for the precise transient analysis of wind turbine generator systems (WTGS), and they develop a way to transform a six mass model into a two mass model. The goal here is not to use the model in the control scheme, but in the use of transient stability analysis of gridconnected WTGS.

The aforementioned references only consider WTs with collective pitch control (CPC) or to check the transient stability, but they give a good starting point on how to model a WT with individual pitch control (IPC). In [11], the modeling problem is approached in a different way. Here they consider the turbine as a complex flexible mechanism, and uses the finite-element-method (FEM) multibody approach. An aero-servo-elastic model is introduced, which consists of the aerodynamic forces from the wind, the servo dynamics from the different actuators and the elasticity in the different joints and the structure.

As seen above there are many types of WT models, ranging from single mass, one state model to multiple mass models. In a simulation point of view it is desirable that the model is as simple as possible and can capture as much of the dynamics that appear in reality. This is an absolute demand, another important issue is to keep the central processing unit (CPU) labor to a minimum. E.g. when dealing with hardware in the loop (HIL) simulation, it is necessary to download the model to for example a programmable logic controller (PLC). This argues for the importance of having a fast C-code. This brings us to the use of the bond graph (BG) methodology. The BG provides with a systematic way to model dynamic systems. Things that potentially can have a negative effect on the execution of the C-code can for example be algebraic loops and differential causality on the different elements in the system. It is a quite intuitive way in setting up the bonds and connecting the elements, this will be discussed in a later section. The outcome from the BG is a set of first order differential equations, which afterwards can be used for controller design.

The WTGS can be divided into several subsections, see Fig. 1. The subsystems emphasized in this paper are the mechanical system and the tower motion.

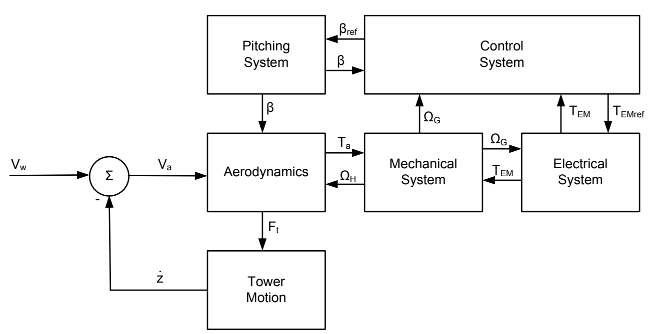

Fig. 1. Setup for WTGS

The system setup is adopted from [12], where V_w is wind speed, V_p is wind speed for power production, $\dot{z}$ is tower speed, F_t is force acting on the tower (thrust force), β_{ref} is pitch angle reference, β is actual pitch angle, T_a is aerodynamic torque, Ω_H is hub speed, Ω_G is generator speed, T_{EMref} is generator torque reference and T_{EM} is actual generator torque.

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The expression for power produced by the wind is given by [13]:

$$P_a = \frac{1}{2}\rho\pi R^2 v^3 C_p(\lambda,\beta) \quad (1)$$

And the tip-speed ratio (TSR) λ is defined as:

$$\lambda = \frac{v_b}{v} \quad (2)$$

From Eq. 1 we can find the aerodynamic torque and the force acting on the tower as follows:

$$T_a = \frac{1}{2}\rho\pi R^3 v^2 C_p(\lambda,\beta) \quad (3)$$

$$F_t = \frac{1}{2}\rho\pi R^2 v^2 C_T(\lambda,\beta) \quad (4)$$

where P_a is the aerodynamic power, ρ is the air density, R is the blade radius, v is the wind speed, C_p and C_T are both functions of the tip-speed ratio and the pitch angle and v_b is the tip speed of the blade.

This paper starts with an introduction to WT modeling. Section II gives a short overview on how the BG methodology works and the different elements. Section III gives the tower motion and WT model. The procedure here is to first make a simplified WT model and write down the differential equations using Newton's second law. Based on this simplified model a BG is made. The argument is; if we can make a BG of the simplified model, then we are ready to make a BG for the more complex WT. At last in this section a BG for the full WT is presented. Section IV states the simulation results and section V gives the conclusion and states some suggestions regarding future work.

II. INTRODUCTION TO BOND GRAPH

BG is a graphical way of modeling physical systems. All these physical systems have in common the conservation laws for mass and energy. BGs, originated by Paynter [14] in 1961, deals with the conservation of energy. This gives a unified approach to modeling physical systems. Further follows a short introduction to this modeling tool, more information can be found in [15].

Within physical systems, energy is transported from one item to another. This energy is either stored or converted to other forms. But the important thing is that it can not dissipate. If the energy is changing in one place, it also changes in an opposite way at another location. The definition of power is the change in energy (E) relative to time:

$$P = \frac{d}{dt}(E) \quad (5)$$

This power is transferred between the different parts in the physical system with the use of power bonds, see Fig. 2. In BG notation the definition of power is effort multiplied with flow. In for example electric systems this means voltage multiplied with current, and in mechanical systems force multiplied with velocity.

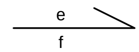
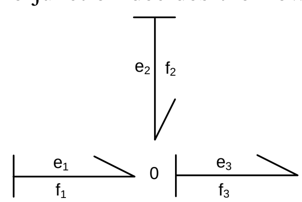


Fig. 2. Power bond

A. System Elements

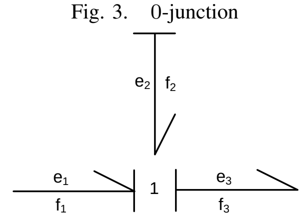
In BG modeling there are a total amount of nine different elements. We will also here introduce the causality assignments, but first we have to explore the cause and effect for each of the basic BG elements. Only elements with its preferred causality will be discussed.

1) Junctions: There are two different types of junctions that connects the different parts in a BG, namely the 0-junction and the 1-junction. The 0-junction is a effort equalizing connection, Fig. 3 represent Eq. 6. Since the efforts are the same, only one bond can decide what it is. This is seen by the causality stoke, the bond which has its causality stroke closest to the junction decides the effort. The 1-junction is a flow equalizing connection, Fig. 4 represent Eq. 7. Since the flows are the same, only one bond can decide what it is. The bond which has its causality stroke furthest away from the junction decides the flow.



$$e_1 = e_2 = e_3 \quad (6)$$
$$f_3 = f_1 + f_2$$

Fig. 3. 0-junction



$$f_1 = f_2 = f_3 \quad (7)$$
$$e_3 = e_1 + e_2$$

Fig. 4. 1-junction

2) Source Element: We can divide the source elements into two different kinds, effort- and flow-source. The effort source gives an effort into the system, then it is up to the system to decide the flow. This is what is meant by cause and effect, and its vice versa for the flow source. Fig. 5 shows how the causality is indicated on the graphical elements. If the vertical line is closest to the junction, then this element decides the effort, furthest away from the junction decides the flow. For the source elements these causality assignments are fixed.

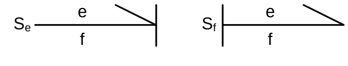
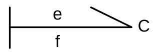


Fig. 5. Effort and flow source with their causality assignments

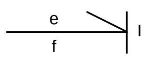
3) Compliance Element: The causality assignment for the C-element has two possibilities, but one is preferred in contrast to the other. This is discussed at the end of this section. The preferred case is seen in Fig. 6 and its corresponding equation in Eq. 8. We see from both the equation and the figure that flow is given to the element/equation and it gives the effort in return.



$$e = \frac{1}{C}\int f dt = \frac{q}{C} \quad (8)$$

Fig. 6. Example of compliance element with integral causality

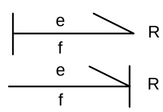
4) Inertia Element: There are two choices for the causality assignment for the I-element, as for the C-element, also here one is preferred over the other. The preferred case is seen in Fig. 7 and its corresponding equation in Eq. 9.



$$f = \frac{1}{I}\int e dt = \frac{p}{I} \quad (9)$$

Fig. 7. Example of inertia element with integral causality

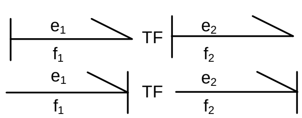
5) Resistive Element: It is a bit more freedom when it comes to the causality assignment for the R-element. Its equation does not include any integration or derivation, only an algebraic expression. The two causality choices is shown in Fig. 8 and its corresponding equation in Eq. 10.



$$e = Rf \quad (10)$$
$$f = \frac{1}{R}e$$

Fig. 8. Example of resistive element

6) Transformer: The transformer element can work in two ways; either it transforms a flow to another flow or it transforms an effort to another effort. Fig. 9 represents Eq. 11-12, m is the transformation ratio.



$$e_1 = me_2 \quad (11)$$
$$f_2 = mf_2$$

$$e_2 = \frac{1}{m}e_1 \quad (12)$$
$$f_1 = \frac{1}{m}f_2$$

Fig. 9. Example of the two transformers

7) Gyrator: The gyrator can work in two ways; either it transform a flow into an effort or it transform an effort into a flow. Fig. 10 represents Eq. 13-14, r is the gyrator ratio.

The importance of integral causality is nicely explained in [16]. First imagine a step in effort is imposed on a C-element, this means the causality stroke in Fig. 6 needs to be changed and we need to rewrite the corresponding equation. Now the flow output is proportional to the derivative of the input effort. From calculus we know that the derivative of a step function at the beginning is infinite, i.e. this do not give any physical meaning. We can imagine a simple electric circuit containing a voltage source coupled with a capacitor, if a step input were to be imposed on the voltage source the

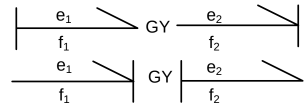


Fig. 10. Example of the two gyrators

$$e_1 = rf_2 \quad (13)$$
$$e_2 = rf_1$$

$$f_1 = \frac{1}{r}e_2 \quad (14)$$
$$f_2 = \frac{1}{r}e_1$$

capacitor would experience a very high current and it would blow up. Another major advantage is the ability to easily spot algebraic loops in large dynamic systems. Algebraic loops can for example occur in a mechanical system if there are more than two dampers and they are not independent of each other. These loops can be hard to spot by inspecting the governing equations or the block diagram. If we are inspecting the BG model and we see that the causality stokes on the R-elements are different, this implies that they are not independent of each other, i.e there exist algebraic loops between them. When we know exactly where these algebraic loops are, we can fix them by for example adding an extra I- or a C-element. And then we can give this extra element a value such that it does not influence the rest of the system. Algebraic loops do not necessarily mean that the simulation will crash, but it might, especially if there are nonlinearities in the dampers. And a smooth simulation is always preferred. The procedure on how to extract the algebraic and dynamic equations from a BG is not included in this short overview. It can be done in a very systematic way and it will partly be shown in the next section.

III. MODEL DESCRIPTION

Fig. 11 shows a sketch of a WT [10]. It consists of six inertias which are; the three blades, hub, gearbox and generator. The inputs are wind speed and electro magnetic torque. To derive the dynamic equations for this model using Newton's second law can be quite hard, and one can easily make some mistakes. This is why the differential equations are derived for the simplified case. The different parameters are explained in Tab. I. Fig. 12 shows a three mass sketch of

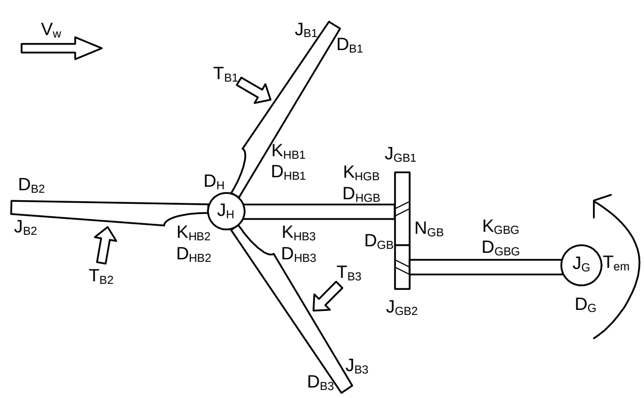


Fig. 11. Wind turbine model

a WT. The sketch consists of a hub, gearbox and generator. Inputs are aerodynamic torque and electro magnetic torque.

If the MATLAB/Simulink simulation result corresponds to the 20-sim result, then we are ready to make our BG model based on Fig. 11. More information about the simulation program 20-sim can be found in [17].

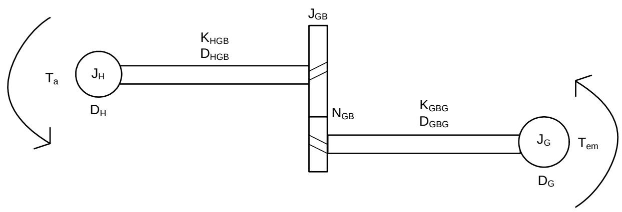


Fig. 12. Simplified wind turbine model

TABLE I
ABBREVIATIONS IN CONNECTION WITH FIG. 11

T_a	Aerodynamic torque
T_{B1-3}	Blade torque 1-3 from wind
D_{B1-3}	Blade 1-3 damping
D_{HB1-3}	Damping between hub and blade 1-3
K_{HB1-3}	Stiffness between hub and blade 1-3
J_H	Hub inertia inertia
D_H	Hub damping
D_{HGB}	Damping between hub and gearbox
K_{HGB}	Stiffness between hub and gearbox
J_{GB1}	Gearwheel 1 inertia
J_{GB2}	Gearwheel 2 inertia
N_g	Gearbox ratio
D_{GB}	Gearbox damping
D_{GBG}	Damping between gearbox and generator
K_{GBG}	Stiffness between gearbox and generator
J_G	Generator inertia
D_G	Generator damping
T_{em}	Electro magnetic torque

A. Tower Motion

It is assumed that the tower movement does not influence the mechanical system, it only affects its input, i.e. the wind speed. The BG model of the tower motion can be seen in Fig. 14. The starting point in making the BG model is first to identify which elements experience the same flow (1-junction) and which experience the same effort (0-junction). The hub is connected to ground through a spring and a damper. The graph in Fig. 13 is a simplification of Fig. 13. We assume zero input force from the ground and whenever there are "through going" bonds on a junction we can eliminate them.
The dynamic equation from the BG model shown in Fig. 14 is:

$$\dot{p}_2 = S_e - R\frac{p_2}{I} - \frac{q_3}{C} \quad (15)$$

$$\dot{q}_3 = \frac{p_2}{I} \quad (16)$$

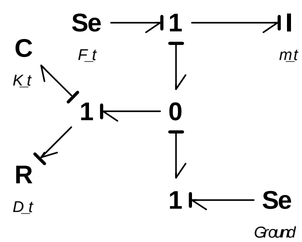


Fig. 13. Bond graph of tower motion

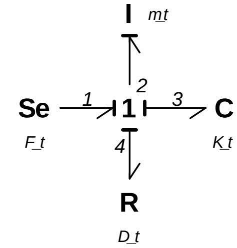


Fig. 14. Simplified bond graph of tower motion

We can rewrite Eq. 15-16 in a non-BG notation:

$$m_t\ddot{z} = F_t - D_t\dot{z} - K_t z \quad (17)$$

where m_t is the tower mass, F_t is the force acting on the tower from the wind, D_t is the tower damping and K_t is the tower stiffness.

B. Dynamic Equations of Simplified WT Using Newton's second law

These governing equations are made by using Newton's second law on rotational form. They are derived by considering each individual inertia, starting with the hub and working our way though the mechanical model. Underneath follow the equations.
Hub:

$$\begin{aligned} J_H\ddot{\omega}_H &= T_a - D_H\dot{\omega}_H \\ &- D_{HGB}\left(\dot{\omega}_H - \dot{\omega}_{GB}\right) \\ &- K_{HGB}\theta_1 \end{aligned} \quad (18)$$

Speed difference between hub and gearbox;

$$\dot{\theta}_1 = \dot{\omega}_H - \dot{\omega}_{GB} \quad (19)$$

Gearbox:

$$\begin{aligned} J_{GB}\ddot{\omega}_{GB1} &= D_{HGB}\left(\dot{\omega}_H - \dot{\omega}_{GB}\right) \\ &+ K_{HGB}\theta_1 \\ &- D_{GBG}N_g\left(N_g\dot{\omega}_H - \dot{\omega}_G\right) \\ &- H_{GBG}N_g\theta_2 \end{aligned} \quad (20)$$

Speed difference between gearbox and generator:

$$\dot{\theta}_2 = N_g\dot{\omega}_{GB} - \dot{\omega}_G \quad (21)$$

Generator:

$$\begin{aligned} J_G\ddot{\omega}_G &= -T_{em} \\ &+ D_{GBG}\left(N_g\dot{\omega}_{GB} - \dot{\omega}_G\right) \\ &+ K_{GBG}\theta_2 - D_G\dot{\omega}_G \end{aligned} \quad (22)$$

C. BG Model of Simplified WT

The starting point now is exactly the same as for the tower motion. The 1-junctions indicate the different velocities and the 0-junctions the different forces. For example the 0-junction between bond nr. 4 and 8 indicate the first flexible shaft, this junction is connected to three 1-junctions. This means three different velocities, hub speed, gearbox speed and their difference.

The number of state equations are equal the number of dynamic elements in the system. We have three I-elements and two C-elements, which give the total amount of five state equations with the state vector x.

$$x \;=\; [p_2 \; q_7 \; p_9 \; q_{14} \; p_{16}]^T \tag{23}$$

In a non-BG notation this is:

$$x \;=\; [J_H\Omega_H \; \theta_1 \; J_{GB}\Omega_{GB} \; \theta_2 \; J_G\Omega_G]^T \tag{24}$$

where Ω_H, Ω_{GB} and Ω_G are rotational speeds for the hub, gearbox and generator, respectively. For small systems these state equations are found quite fast directly from the bond graph. But we can also choose to get them directly from the simulation program 20-sim.

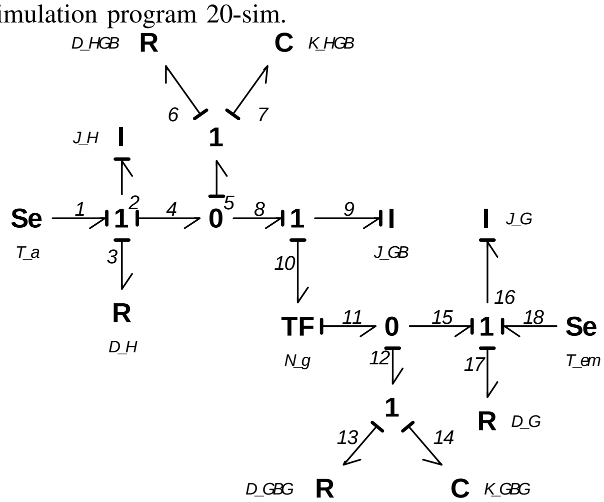


Fig. 15. BG representation of the simplified WT model.

D. BG Model of WT

The BG model representing the mechanical system presented in Fig. 11 can be seen in Fig. 16. Here the inputs are wind speed minus tower movement on each blade and generator torque. The wind speed is fed through a modulated gyrator (MGY) which transforms flow into effort according to a formula embedded inside the gyrator (Eq. 4). This transformation is dependent on the blades pitch angle (not a constant), hence the modulated gyrator and not an ordinary gyrator.

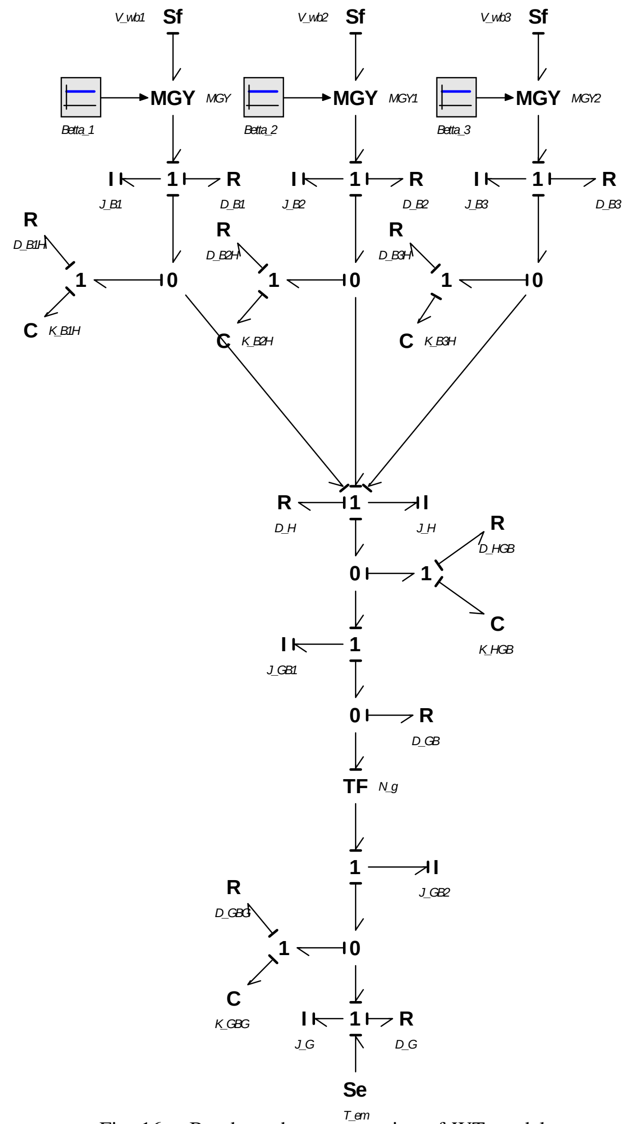


Fig. 16. Bond graph representation of WT model

We can also note that all the R-,I and C-elements have the same causality, respectively. This is what we desire, that each element have their preferred causality. Since the R-element is not a dynamic element, the causality is not that important. But it is preferred that they have the same causality, whether it is integral or derivative is not of major importance.

IV. SIMULATION

To validate that the MATLAB/Simulink model and the BG model are the same interpretation of the mechanical system (Fig. 12), we set the inputs to zero and give an initial value for the hub rotational speed. All the other values are non realistic WT parameters, they are assigned arbitrary values.

If the dynamic behavior of the different rotational speeds are the same, then the two models are considered equal. The plots for Ω_H, Ω_{GB} and Ω_G are shown in Fig. 17-18.

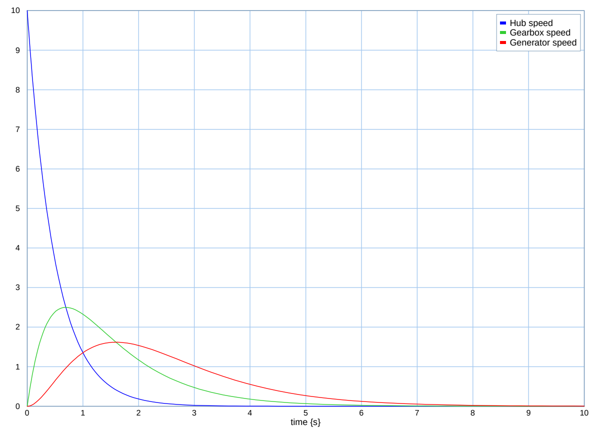


Fig. 17. Simulation with 20-sim

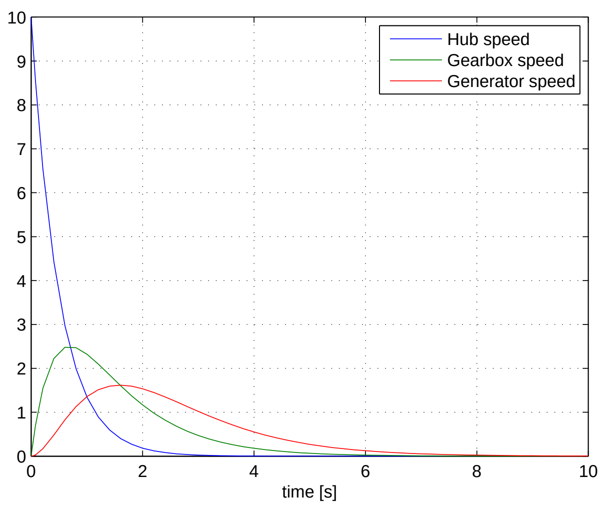


Fig. 18. Simulation with MATLAB/Simulink

V. CONCLUSIONS AND FUTURE WORK

A. Conclusions

The purpose of this paper was to make a bond graph (BG) model for a wind turbine (WT) system. This is done by first making a BG model for a simplified WT, the result from this simulation is then compared with the model made with a classical approach. The simulation results from the two approaches are the same and this confirms one of the benefits of BG approach as a generally usable approach to modeling physical systems of arbitrary types.

B. Future Works

A natural next step will be to expand the model to also include the pitching system, electric system and the aerodynamics. When the entire model of the WTGS is made we are ready to explore how to mix BG with control theory.

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