Further Results on Mathematical Models of Vehicle Localized Impact

W. Pawlus, J.E. Nielsen, H.R. Karimi and K.G. Robbersmyr

Abstract— **In this paper, we propose a method of modeling for vehicle crash systems based on viscous and elastic properties of the materials. This paper covers an influence of different arrangement of spring and damper on the models' response. Differences in simulating vehicle – to – rigid barrier collision and vehicle – to – pole collision are explained. Comparison of the models obtained from wideband (unfiltered) acceleration and filtered acceleration is done. At the end we propose a model which is suitable for localized collisions simulation.**

I. INTRODUCTION

This paper deals with establishing an appropriate mathematical model representing vehicle soft impacts such as localized pole collisions. In simulation of the vehicle collision, elements which exhibit viscous and elastic properties are used. Models utilized by us consist of energy absorbing elements (EA) and masses connected to their both ends. We focus on finding a model with such an arrangement of springs, dampers and masses, which simulated, will give a response similar to the car's behavior during the real crash.

Due to the fact that real crash tests are complex and complicated events, their modeling is justified and advisable. Every car which is going to appear on the roads has to conform to the worldwide safety standards. However, crash tests consume a lot of effort, time and money. The appropriate equipment and qualified staff is needed as well. Therefore our goal is to make possible simulation of a vehicle crash on a personal computer.

Approach presented here – mathematical modeling of a crash event with the equations of motion which can be solved explicitly with closed form solutions – is different that the methods which have been shown in [1] – [4]. In order to simulate the collision of a car the software based on FEM (Finite Element Method) was utilized. After the creation of 3D – CAD and FE models the crash simulations were performed. Results obtained showed good correlation between the test and model responses. When it comes to determining crush stiffness coefficients, in [5] it is presented a method which employs CRASH3 computer program. Vehicle structure was modeled as a homogenous body and then the comparative analysis of the crash response of

vehicles tested in both full – overlap and partial – overlap collisions was done.

A lumped parameter modeling (LPM) is an another way of approximation of the vehicle crash. It is an analytical method of formulating a model which can be further used for simulation of a real event. It allows us to establish dynamic equations of the system – differential equations – which give the complete description of the model's behavior, see the references [6] and [7].

To be able to analyze a given collision, it is often enough (and more efficient) not to examine the complicated crash pulse but just to study its approximation. Those approximated functions were compared to experimental pulses in [8]. Subsequently they were tested to obtain different models' responses which were compared to the original pulse. Results confirmed that the crash pulse approximation is a reasonable method to simplify the collision analysis. Recently, the Haar wavelet-based performance analysis of the safety barrier for use in a full-scale test was proposed in [9]. In [10], a basic mathematical model is proposed to represent a collision together with its analysis. The main part of this research is devoted to methods of establishing parameters of the vehicle crash model and to real crash data investigation, i.e. – creation of a Kelvin model (spring and damper connected in parallel with mass) for a real experiment, its analysis and validation. After model's parameters extraction a quick assessment of an occupant crash severity is done. Finally, the dynamic response of such a system was similar to the car's real behaviour in the time interval which corresponds to the collision's duration. Parameters of this assembly (spring stiffness and damping coefficient) were obtained analytically with closed – form solutions according to [11].

In this paper, we present a process of improving the accuracy of the vehicle crash model. We start with simulation of the vehicle to pole impact by using the Kelvin model (spring and damper in parallel connected to mass). Afterwards, by filtering the crash pulse data, more accurate response of the system is obtained. Model establishment is done one more time. This allows us to compare what are the crash models for both raw and filtered data and to decide which of them is more suitable to represent vehicle to pole collision.

W. Pawlus, J.E. Nielsen, H.R. Karimi and K.G. Robbersmyr are with the Department of Engineering, Faculty of Engineering and Science, University of Agder, N-4898 Grimstad, Norway (e-mail: hamid.r.karimi@uia.no).

II. EXPERIMENTAL SETUP

In the experiment conducted by UiA [12] the test vehicle, a standard Ford Fiesta 1.1 L 1987 model was subjected to a central impact with a vertical, rigid cylinder at the initial impact velocity v_0 = 35 km/h. Mass of the vehicle (together with the measuring equipment and dummy) was 873 kg. Experiment's scheme is shown in Fig. 1.
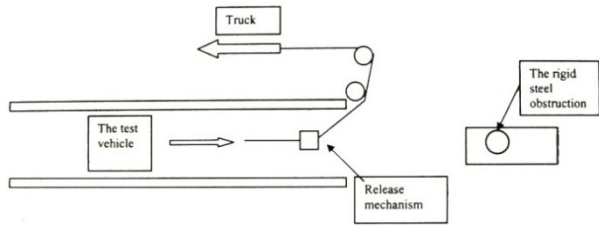

Fig. 1.   Scheme of the test collision [12]

Vehicle accelerations in three directions (longitudinal, lateral and vertical) together with the yaw rate at the center of gravity were measured. Using normal – speed and high – speed video cameras, the behavior of the obstruction and the test vehicle during the collision was recorded. Fig. 2 shows one of the stages of the crash.

Fig. 2.   Car is undergoing a deformation

III. RAW DATA ANALYSIS - KELVIN MODEL

According to [10] the Kelvin model shown in Fig. 3 has been proposed to represent the vehicle to pole collision. Symbols used: k – spring stiffness, c – damping coefficient, m – mass, V_0 – initial impact velocity.
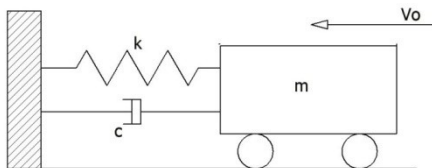

Fig. 3.   Kelvin model.

Known parameters of the model are:
m = 873 kg – mass
V_0 = 10.8 m/s – initial impact velocity.
Parameters which we obtain from the crash pulse analysis (acceleration of the car in the x – direction – longitudinal) shown in Fig. 4 are listed in Table I.
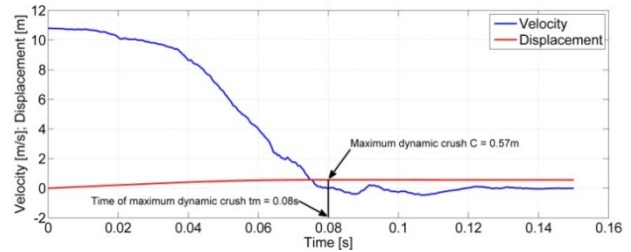

Fig. 4.   Raw data analysis

By following [11] (method of calculating damping factor ζ and natural frequency f is covered in [10]) spring stiffness k and damping coefficient c of the Kelvin model are defined to be:

$$k = 4\pi^2 f^2 m = 4\pi^2 (2.9375 Hz)^2 \cdot 873 kg = 297392 N / m$$
$$c = 4\pi f \zeta m = 4\pi \cdot 2.9375 Hz \cdot 0.1 \cdot 873 kg = 3223 N - s / m$$

Validation of the model has been done in Matlab Simulink software – the response of the Kelvin model with above estimated parameters is shown in Fig. 5.
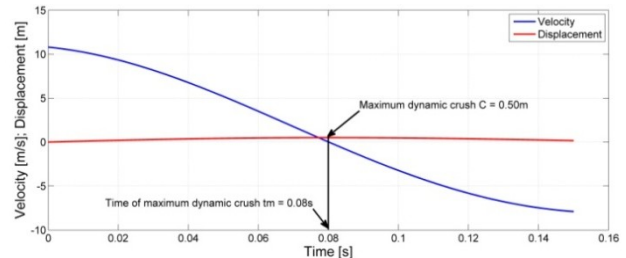

Fig. 5.   Kelvin model's response – raw data

Comparison of dynamic crush and time of dynamic crush from the crash pulse analysis and Kelvin model response is done in Table I.

TABLE 1
COMPARISON BETWEEN CAR'S AND KELVIN
MODEL'S RESPONSE – RAW DATA

Parameter	Crash pulse analysis	Kelvin model
Dynamic crush C [m]	0.57 m	0.50 m
Time of dynamic crush t_m [s]	0.08 s	0.08 s

Remark 1. Since the raw data has been used above, the discrepancy between the real initial impact velocity (which is $V_0 = 9.86$ m/s) and initial impact velocity obtained from the raw data analysis (which is $V_0 = 10.80$ m/s) is visible. Therefore to eliminate inaccuracies in modeling caused by this velocity difference we need to filter the acceleration measurements.

IV. ACCELERATION MEASUREMENTS FILTERING

Digital filtering method has been used here – according to [13]. Frequency response corridors for an appropriate channel class are specified in this standard. Since our goal is to analyze the crash pulse (i.e. integration for velocity and displacement) we select the channel class CFC 180. Filter utilized by us was Butterworth 3rd order lowpass digital filter with cut – off frequency $f_N = 300$ Hz. Comparison between the wideband data and data filtered with this method is shown in Fig. 6.
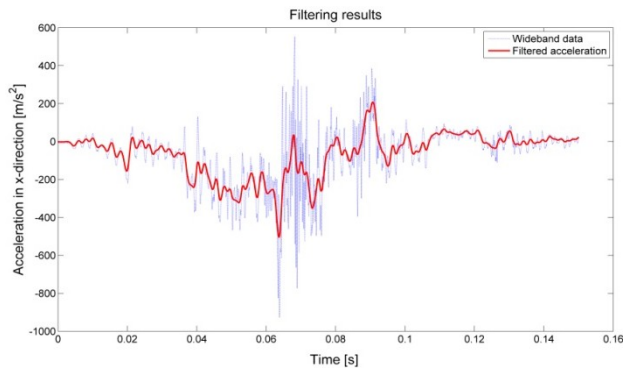

Fig. 6. Butterworth 3rd order filtering results.

In Fig. 7 the comparison in the frequency domain between the raw and filtered acceleration is presented.
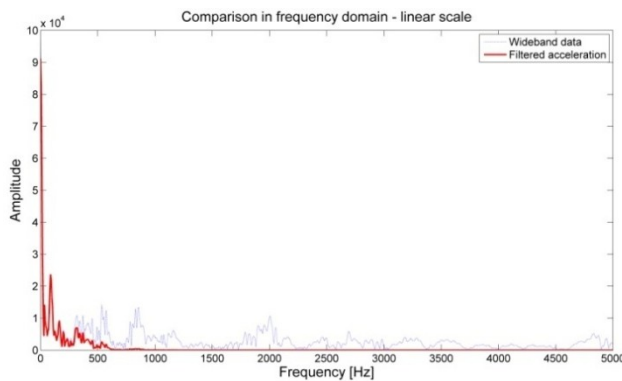

Fig. 7. Frequency analysis of crash pulses.

Since the scale is linear, we clearly see that the filtering helped us to get rid of the high frequency components of the crash pulse. This makes its analysis more efficient and gives us results which better correspond to the reality than the ones obtained from wideband data (velocity and displacement). This has crucial influence on our further

considerations because in order to develop a good model, we need to have at our disposal real parameters of the crash test (e.g. initial velocity).

V. FILTERED DATA ANALYSIS

A. Kelvin model

Let us determine what is the maximum dynamic crush and the time at which it occurs for the filtered data.
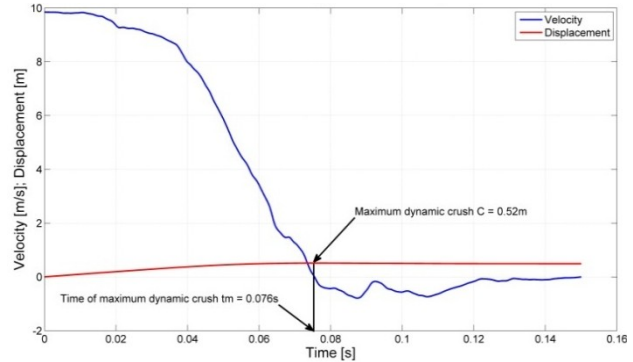

Fig. 8. Filtered data analysis.

Parameters which we obtain from the crash pulse analysis (acceleration of the car in the x – direction – longitudinal) shown in Fig. 8 are listed in Table II.

Proceeding in the same manner as in Section 3, we obtain the following parameters of the Kelvin model:

$$k = 4\pi^2 f^2 m = 344150 N / m$$
$$c = 4\pi f \zeta m = 2427 N - s / m$$

Kelvin model response for those parametrs is shown in Fig. 9.

Comparison between the model and reality for the filtered data is done in Table II.

TABLE II
COMPARISON BETWEEN CAR'S AND KELVIN MODEL'S
RESPONSE – FILTERED DATA

Parameter	Crash pulse analysis	Kelvin model
Dynamic crush C [m]	0.520 m	0.430 m
Time of dynamic crush t_m [s]	0.076 s	0.076 s

Filtering the data has improved our calculations – we have obtained the real value of the initial velocity $V_0 = 9.86$ m/s. However, we observe a larger discrepancy between the dynamic crush from the acceleration's integration and model's prediction than for the raw data. This allows us to

claim that since the method utilized in both of those cases remains the same and accuracy of our calculations has increased because of the data filtering, the Kelvin model is not suitable for modeling the impact examined by us. For that reason we investigate a simpler model which consists of spring and mass only.
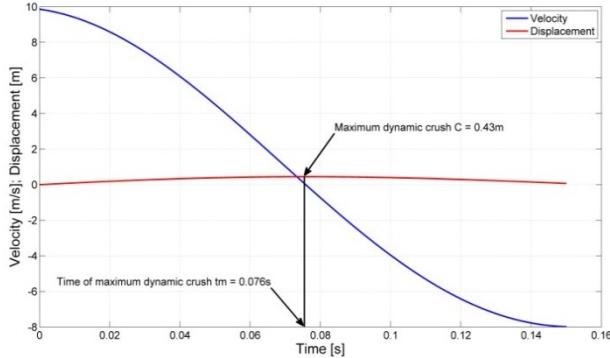

Fig. 9. Kelvin model's response – filtered data.

B. Spring – mass model

The motion of this system is a non – decayed oscillatory one (sinusoidal) because there is no damping in it [11]. This arrangement is shown in Fig. 10. Symbols: k – spring stiffness, m – mass, a – absolute displacement of mass m.
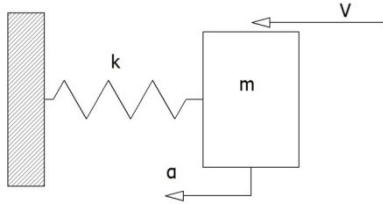

Fig.10. Spring – mass model.

Let us introduce the following denotation:
V – initial barrier impact velocity [m/s]
f – structural natural frequency [Hz].
Response of this system is characterized by the following equations:

$$\ddot{\alpha}(t) = -V\omega_e \sin(\omega_e t) \qquad (1)$$

$$\dot{\alpha}(t) = V \cos(\omega_e t) \qquad (2)$$

$$\alpha(t) = \frac{V}{\omega_e} \sin(\omega_e t) \qquad (3)$$

which represent deceleration, velocity and displacement, respectively. Furthermore we define:

$$C = \frac{V}{\omega_e} \qquad (4)$$

$$t_m = \frac{\pi}{2\omega_e} \qquad (5)$$

$$\omega_e = \sqrt{\frac{k}{m}} \qquad (6)$$

as maximum dynamic crush, time of maximum dynamic crush and system's circular natural frequency, respectively. To investigate what are parameters C and t_m of such a model, first we need to find the spring stiffness k. By substituting (6) to (4) and rearranging one gets:

$$k = \frac{V^2}{C^2} m \qquad (7)$$

From Fig. 8 it is obtained C = 0.52 m and V = 9.86 m/s for filtered data. Therefore,

$$k = \frac{(9.86m/s)^2}{(0.52)^2} 873kg = 313878N/m$$

$$t_m = \frac{\pi}{2\omega_e} = \frac{\pi}{2\sqrt{\frac{k}{m}}} = \frac{\pi}{2\sqrt{\frac{313878N/m}{873kg}}} = 0.083s$$

Spring – mass model's response for above spring stiffness k (initial velocity and mass of the car remain the same) is shown in Fig. 11.
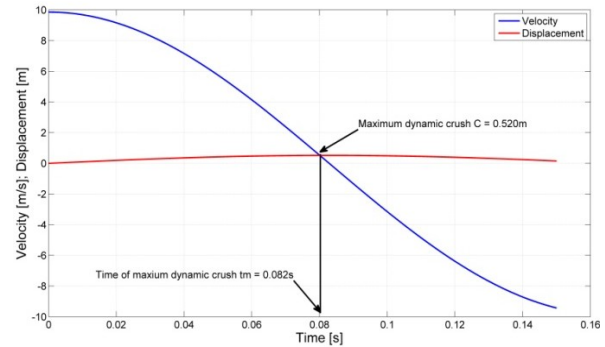

Fig. 11. Spring – mass model's response.

Let us again compare what is the dynamic crush and the time at which it occurs for the data analysis and model.

TABLE III
COMPARISON BETWEEN CAR'S AND SPRING
– MASS MODEL'S RESPONSE – FILTERED DATA

Parameter	Crash pulse analysis	Spring - mass model
Dynamic crush C [m]	0.520 m	0.520 m
Time of dynamic crush t_m [s]	0.076 s	0.082 s

Results obtained in this step are the best. The dynamic crush estimated by the spring – mass model is exactly the same as the reference dynamic crush of a real car. When it comes to the time when it occurs, the difference between the model and reality is less than 1%. This model gives us good approximation of the car's behavior during the crash. It is a

particular case of a Kelvin model in which damping has been set to zero as well as of a Maxwell model in which damping is going to infinity. Since the method for finding the parameters of the Kelvin model does not provide satisfactory results, it is advisable to use a different model.

VI. MAXWELL MODEL

The arrangement in which spring and damper are connected in series to mass is called Maxwell model – Fig. 12.
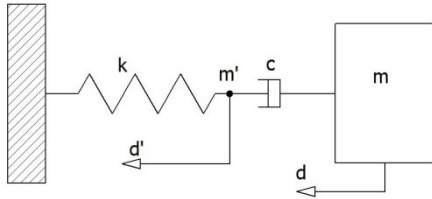
Fig. 12. Maxwell model – m' designates Zero Mass.

To derive its equation of motion it is proposed to place small mass *m'* between spring and damper. By doing this, the inertia effect which occurs for the spring and damper is neglected and the system becomes third order differential equation which can be solved explicitly [11]. According to Fig. 12 we define *d* and *d'* as absolute displacement of mass *m* and absolute displacement of mass *m'*, respectively. We establish the following equations of motion (EOM):

$$m\ddot{d} = -c(\dot{d} - \dot{d}') \tag{8}$$

$$m'\ddot{d}' = c(\dot{d} - \dot{d}') - kd' \tag{9}$$

By differentiating (8) and (9) w.r.t. time and setting m' = 0 we obtain:

$$m\dddot{d} = -c(\ddot{d} - \ddot{d}') \tag{10}$$

$$0 = c(\ddot{d} - \ddot{d}') - k\dot{d}' \tag{11}$$

We sum up both sides of (10) and (11) and rearrange:

$$\dot{d}' = \frac{-m}{k}\dddot{d} \tag{12}$$

We substitute (12) into (8) and finally obtain the undermentioned EOM:

$$\dddot{d} + \frac{k}{c}\ddot{d} + \frac{k}{m}\dot{d} = 0 \tag{13}$$

Therefore, characteristic equation of the Maxwell model is:

$$s\left[s^2 + \frac{k}{c}s + \frac{k}{m}\right] = 0 \tag{14}$$

In this system, the rebound of the mass depends on the sign of the discriminant Δ of the quadratic equation in brackets. For positive Δ there is no rebound, i.e. $\left(\frac{k}{c}\right)^2 > 4\frac{k}{m}$. In this case, roots of the characteristic equation (14) are, respectively, $s_0 = 0$ and two negative real roots $s_1 = a + b$ and $s_2 = a - b$, where $a = \frac{-k}{2c}$ and $b = \sqrt{\left(\left(\frac{k}{2c}\right)^2 - \frac{k}{m}\right)}$.

On the other hand for negative Δ the rebound occurs when $\left(\frac{k}{c}\right)^2 < 4\frac{k}{m}$. In this case, roots of the characteristic equation (14) will be $s_0 = 0$ and two complex roots $s_1 = a + ib$ and $s_2 = a - ib$, where $a = \frac{-k}{2c}$ and $b = \sqrt{\left(\frac{k}{m} - \left(\frac{k}{2c}\right)^2\right)}$.

In a Maxwell model, the mass may not rebound from the obstacle. It means that its displacement increases with time to an asymptotic value. The parameter which determines whether the rebound will occur or not is damping coefficient. When it is less than a limiting one (named transition damping coefficient c^*), the mass will be constantly approaching an obstacle, whereas when it is higher, there will exist a dynamic crush at a finite time. Another boundary situation is for damping coefficient c = ∞. Then the Maxwell model degenerates into spring – mass system. To determine the value of transition damping coefficient we assume that Δ=0, or equivalently $\frac{k}{c} = 2\sqrt{\frac{k}{m}}$, and

$$c^* = \frac{\sqrt{km}}{2} \tag{15}$$

Indeed, for $c < c^*$ we have $\Delta > 0$ – it means no dynamic crush at a finite time.
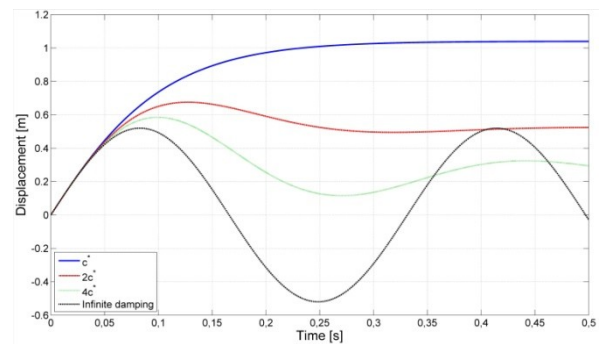
Fig. 13. Maxwell model responses for different values of damping.

We are able to assess what should be the minimal damping, which we add to the simple spring – mass model mentioned above, which will produce the dynamic crush not extended in infinite period of time. According to (15), for model and crash test being analyzed in Section 5B, we calculate the transition damping coefficient:

$$c^* = \frac{\sqrt{313878\,N/m \cdot 873\,kg}}{2} = 8277\,N-s/m$$

For every damping greater than this value, the Maxwell model formed from the spring – mass model from Section 5B, will give us the response more and more similar to the spring – mass model characteristics presented in Fig. 11, as it is shown in Fig. 13.

It is noting that the final displacement (or asymptotic value – for transition damping coefficient) achieved by the mass in this model is characterized by the equation (V_0 – initial impact velocity, m – mass, c – damping coefficient):

$$crush = \frac{V_0 m}{c} \qquad (16)$$

This system is appropriate for simulating soft impacts or offset impacts because the time of dynamic crush is longer than for Kelvin model. If we assume the same parameters for both models, e.g.:

k = 100 N/m, c = 15 N-s/m, m = 5 kg, v_0 = 10 m/s

In Fig. 14, it is seen that for the Maxwell model the dynamic crush occurs later than for the Kelvin model.
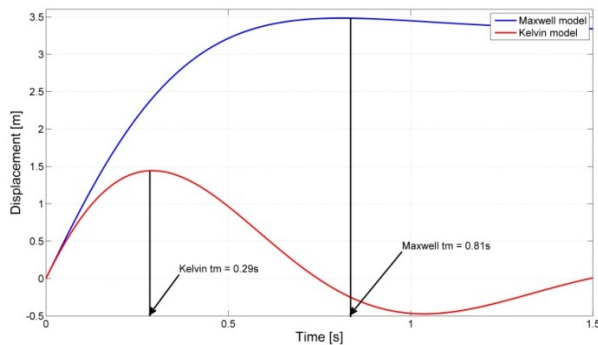

Fig. 14. Maxwell and Kelvin models' responses comparison.

This is an analog situation to the real crash: in a vehicle – to – rigid barrier collision (Kelvin model) the whole impact energy is being consumed faster, therefore the crash is more dynamic than the vehicle – to – pole collision (Maxwell model) – under the assumption that we compare the same cars with the same initial impact velocities – as in the example above. It is noting that we do not investigate here the magnitude of the displacement of both models – as we can see for the same parameters it is higher for the Maxwell model. Above example just illustrates the dynamic response of those two systems and in order to apply those two models to the real crash one needs to assess what is spring stiffness and damping coefficient of both of them separately.

VII. CONCLUSION

In this paper, we studied a process of improving the accuracy of the vehicle crash model. First, we simulated the vehicle under a pole impact by using the Kelvin model (spring and damper in parallel connected to mass). Afterwards, by filtering the crash pulse data, more accurate response of the system was obtained. Model establishment was done one more time. Finally, we compared the crash models for both raw and filtered data and it was concluded which of them is more suitable to represent vehicle to pole collision.

The obtained results indicate that the Kelvin model is not appropriate for simulation of the collision which we deal with. Based on the Section 6 for the data prepared in the proper way, we establish a proper model. An extension of our analysis to the Maxwell model is still under consideration as a part of our further work plan. Due to the fact that the comparative analysis of a spring – mass system and car's behavior in the collision turned out to be appropriate, further improvement of the model is advisable.

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