Application of Adaptive Wavelet Networks for Vibration Control of Base Isolated Structures

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Abstract - This paper presents an application of wavelet networks (WNs) in identification and control design for a class of structures equipped with a type of semiactive actuators, which is called magnetorheological (MR) dampers. The nonlinear model is identified based on a WN framework. Based on the technique of feedback linearization, supervisory control and H_∞ control, an adaptive control strategy is developed to compensate for the nonlinearity in the structure so as to enhance the response of the system to earthquake type inputs. Furthermore, the parameter adaptive laws of the WN are developed. In particular, it is shown that the proposed control strategy offers a reasonably effective approach to semi-active control of structures. The applicability of the proposed method is illustrated on a building structure by computer simulation.

Keywords: Vibration control; wavelet network; system identification; H_∞ control, nonlinear structures

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I. Introduction

The protection of civil engineering structures has always been a major concern especially when these structures are built in places prone to hazardous weather conditions (e.g. hurricanes, tsunamis) or in zones of intense seismic activity or when the structure is subjected to heavy loadings (e.g. heavy traffic on a bridge). If a structure is not well protected against these phenomena, they can suffer severe damage, and as a consequence, produce personal injuries or death as could be seen during the earthquakes in Mexico City (1985), Kobe (1995), northwestern Turkey (1999), those that struck southern Asia in 2004 followed by the tsunamis or more recently in China (2008).

In order to make structures more resistant against these phenomena, passive and active dampers were initially proposed. Passive dampers alleviate the energy dissipation of the main structure by absorbing part of the input energy, without the need of external power sources. However, once installed, they are not adaptable to different loading conditions [1]. Active dampers, on the other hand, can respond to variations of the loading conditions and structural dynamics but require large power sources and additional hardware like sensors and DSP's to operate. Active dampers can also inject energy to the structure and may destabilize it in a bounded-input bounded-output sense [2].

Semiactive devices provide an effective solution to overcome the disadvantages of passive and active dampers [3]. They have shown to perform significantly better than passive devices and as well as active devices without requiring large power sources, thus allowing for battery operation [4]. The main characteristics of semiactive devices are the rapid adaptability of their dynamic properties in real time but without injecting any energy into the system. Among diverse semiactive devices, MR fluid dampers are the most attractive and useful ones. MR dampers can generate high yield strength, have low costs of production, require low power, and have fast response and small size. However, they are characterized by a complex nonlinear dynamics (typically hysteresis) which makes mathematical treatment challenging, especially in the modeling and identification of the hysteretic dynamics and the development of control laws for its implementation through magnetorheological (MR) dampers for vibration mitigation purposes. More recently, in [5] a computational algorithm was presented for the modelling and identification of the MR dampers by using wavelet systems to handle the nonlinear terms. By taking into account the Haar wavelets, the properties of integral operational matrix and of product operational Matrix are introduced and utilized to estimate the MR damper parameters easily by considering only the algebraic equations instead of the differential equations of the dynamical system.

On the other hand, wavelet theory is a relatively new and an emerging area in mathematical research [6]. It has been applied in a wide range of engineering disciplines such as signal processing, control engineering, pattern recognition and computer graphics. In the literature, some of the attempts are made in solving surface integral equations, improving the finite difference time domain method, solving linear differential equations and nonlinear partial differential equations, modelling nonlinear semiconductor devices, signal processing and pattern recognition [7]-[14]. The combination of soft computing and wavelet theory has lead to a number of new techniques: wavelet networks (WNs) and fuzzy wavelet [15]. The combination of wavelet theory and neural networks has lead to the development of WNs. WNs are feed forward neural networks using wavelets as activation function. WNs have been used in classification and identification problems with some success. The strength of WNs lies in their capabilities of catching essential features in “frequency-rich” signals. WNs have become popular after the works [16]-[18]. Recently, application of WNs in identification and control design for a class of nonlinear dynamical systems has been investigated in [19].

In recent years, considerable attention has been paid to systematic applications of semiactive linear control algorithms for vibration control of building structures subject to natural hazards, e.g., earthquake and strong wind. A number of control techniques have been developed for vibration control of structures equipped with MR dampers. In [20], the clipped optimal control was one of the first controllers developed for this class of systems. An optimal controller is designed to estimate the force that mitigates the vibrations in the structure and the control signal takes only two values according to an algorithm, in which the MR damper dynamics are ignored. Control techniques based on Lyapunov’s stability theory have been proposed and successfully tested in structures such as buildings, bridges and car suspension systems [21]-[27]. The general control objective is achieved through the choice of control inputs that make the Lyapunov function derivative as negative as possible and consequently obtain the maximum energy dissipation. Other control methods have also been proposed such as bang-bang control ([21], [28]-[29]); sliding mode control ([25], [30]-[31]); backstepping control ([32]-[33]); and intelligent control such as fuzzy logic control ([34]) and neuro fuzzy control ([35]-[36]).

One of the most important tasks in semiactive control system design for structures is the development of an accurate mathematical model of the system equipped with a semiactive control device, e.g., MR damper. However, it is a challenging problem because a building structure with MR dampers is a nonlinear time-varying system, not a linear time-invariant one.

The first idea about combining H_∞ control and adaptive WN is related to the works [13]-[14]. Similar to these works, we use WNs in identification and control design for the building-MR damper structures. The nonlinear model is identified based on a WN framework. Based on the technique of feedback linearization, supervisory control and H_∞ control, an adaptive control strategy is developed to compensate for the nonlinearity in the structure so as to enhance the response of the system to earthquake type inputs. Furthermore, the parameter adaptive laws of the WN are developed. In particular, it is shown that the proposed control strategy offers a reasonably effective approach to semi-active control of structures. The applicability of the proposed method is illustrated on a building structure by computer simulation.

II. Wavelet Networks

The original objective of the theory of wavelets is to construct orthogonal bases of $L_2(\Re)$. These bases are constituted by translation and dilation of the same function $\psi(.)$, namely wavelet function. It is preferable to take $\psi(.)$ localized and regular.

The wavelet subspaces W_j are defined as

$$W_j = \{\psi(2^j x - k) \quad k \in \mathrm{Z}\} \tag{1}$$

which satisfy $W_j \cap W_i = 0, j \neq i$. For each $j \in \mathrm{Z}$, let us consider the closed subspaces $V_j = \cdots \oplus W_{j-2} \oplus W_{j-1}$ of $L_2(\Re)$, where $\oplus$ denotes the direct sum.

The wavelet series representation of the one-dimensional function $f(x)$ is given by

$$f(x)=\sum_{k\in Z} a_{j_0k}\phi_{j_0k} + \sum_{j\geq j_0}\sum_{k\in Z} b_{jk}\psi_{jk}(x) \tag{2}$$

where $\phi_{j_0k}(x)=2^{j_0/2}\phi(2^{j_0}x-k)$, $\psi_{jk}(x)=2^{j/2}\psi(2^j x-k)$ and the wavelet coefficients a_{j_0k} and b_{jk} are

$$a_{j_0k} =< f(x),\phi_{j_0k}(x) >, \tag{3a}$$

$$b_{jk} =< f(x),\psi_{jk}(x) >. \tag{3b}$$

While the function $f(x)$ is unknown, then the wavelet coefficients a_{j_0k} and b_{jk} can not be calculated simply by (3). Since, it is not realistic to use an infinite number of wavelets to represent the function $f(x)$, we consider the following wavelet representation form of the function $f(x)$ [13], [37]-[38]:

$$\hat{f}(x)=\sum_{j=-M_1}^{M_2}\sum_{k=-N_1}^{N_2} b_{jk}\psi_{jk}(x)=\underline{\theta}^T\,\underline{\psi}(x) \tag{4}$$

for some integers M_1, N_1, M_2, N_2, $\underline{\theta}=(b_{M_1N_1},\ldots,b_{M_1N_2},\ldots,b_{M_2N_1},\ldots,b_{M_2N_2})^T$ and $\underline{\psi}(x)=(\psi_{M_1N_1}(x),\ldots,\psi_{M_1N_2}(x),\ldots,\psi_{M_2N_1}(x),\ldots,\psi_{M_2N_2}(x))^T$.

If $\Xi_f(M_1,M_2,N_1,N_2)=f(x)-\hat{f}(x)$ is the Network Error (or approximation error), then it is easy to show that for arbitrary constant $\eta\geq 0$, there exist some constants M_1, N_1, M_2, N_2 such that $\left\|\Xi_f(M_1,M_2,N_1,N_2)\right\|_2\leq\eta$ for all $x\in\Re$ [39]. This means that $f(x)$ can be approximated to any desired accuracy as $\hat{f}(x)$ by a WN with large enough M_1, N_1, M_2, N_2.

The wavelet series representation can be easily generalized to any dimension *n*. For the *n*-dimension case $\underline{x}=[x_1,x_2,\cdots,x_n]^T$, we introduce the wavelet function ([13], [37])

$$\psi(\underline{x})=\psi(x_1,x_2,\ldots,x_n)=\psi(x_1)\psi(x_2)\cdots\psi(x_n) \tag{5}$$

Now, we make a modification to replace the wavelet bases in (4). Then the modified WN becomes

$$\hat{f}(\underline{x})=\sum_{j=-M_1}^{M_2}\sum_{k=-N_1}^{N_2} b_{jk}\psi_{jk}(x)=\sum_{j=-M_1}^{M_2}\sum_{k=-N_1}^{N_2} b_{jk}\prod_{l=1}^{n}\psi_{jk}(x_l)=\underline{\theta}^T\,\underline{\psi}(\underline{x}) \tag{6}$$

where

$$\underline{\theta}=(b_{M_1N_1},\ldots,b_{M_1N_2},\ldots,b_{M_2N_1},\ldots,b_{M_2N_2})^T,$$

$$\underline{\psi}(\underline{x}) = (\psi_{M_1N_1}(\underline{x}),\ldots,\psi_{M_1N_2}(\underline{x}),\ldots,\psi_{M_2N_1}(\underline{x}),\ldots,\psi_{M_2N_2}(\underline{x}))^T .$$

III. System Description

Consider an uncertain n-story building whose base is isolated by means of a passive frictional actuator and an MR damper, as shown in Figure 1. Consider also that the system is perturbed by an incoming earthquake. The system dynamics can be divided into two subsystems, namely, the main structure (S_r) and the base (S_c) [40].

$$S_r : M\,\ddot{X}(t) + C\,\dot{X}(t) + K\,X(t) = [c_1, \underbrace{0,\cdots,0}_{n-1}]^T\,\dot{y}(t) + [k_1, \underbrace{0,\cdots,0}_{n-1}]^T\,y(t) \tag{7a}$$

$$S_c : m\,\ddot{y}(t) + c\,\dot{y}(t) + k\,y(t) + f_{bf}(t) = f_{bg}(t) + f_c(t) \tag{7b}$$

with

$$f_{bf}(t) = c_1(\dot{y}(t) - \dot{x}_1(t)) + k_1(y(t) - x_1(t)) , \tag{7c}$$

$$f_{bg}(t) = -c\,\dot{d}(t) - k\,d(t) + \Phi(\dot{y}(t),\dot{d}(t)) , \tag{7d}$$

$$\Phi(\dot{y}(t),\dot{d}(t)) = -\operatorname{sgn}(\dot{y}(t) - \dot{d}(t))\,[\mu_{\max} - \Delta\mu\, e^{-\nu\left|\dot{y}(t)-\dot{d}(t)\right|}]Q , \tag{7e}$$

where $X(t) = [x_1(t), x_2(t), \cdots, x_n(t)]^T \in \Re^n$ is the structure horizontal absolute displacement vector (measured with respect to an inertial frame), $y \in \Re$ is the horizontal base absolute displacement, $d(t)$ and $\dot{d}(t)$ are the seismic excitation displacement and velocity, respectively, and $f_c(t)$ is the control force. Equation (7c) accounts for the dynamic coupling between the base and the main structure. Equation (7d) describes the forces introduced by the seismic excitation and the base isolation. Equation (7e) describes the dynamics of a frictional base isolator, where μ_{max} is the friction coefficient for high sliding velocity, $\Delta\mu$ is the difference between μ_{max} and the friction coefficient for low sliding velocity, ν is a constant and Q is the force normal to the friction surface. In (7a), M, C and K are square and real matrices representing the mass, damping coefficients and stiffness of the structure, respectively. The structure of those matrices is shown as follows:

$$M = diag\{m_1, m_2, \cdots, m_n\} ,$$

$$C=\begin{bmatrix} c_1+c_2 & -c_2 & \cdots & 0 & 0 \\ -c_2 & c_2+c_3 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & -c_n & c_n \end{bmatrix},$$

$$K=\begin{bmatrix} k_1+k_2 & -k_2 & \cdots & 0 & 0 \\ -k_2 & k_2+k_3 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & -k_n & k_n \end{bmatrix}.$$

Equation (7b) consists of a linear part, described by the mass *m*, damping coefficient *c* and stiffness *k* of the base. $f_{bf}(t)$ represents the linear force caused by the coupling of the base and the main structure. This force is represented by the damping coefficient $c_{bf}=c_1$, the stiffness $k_{bf}=k_1$ and the relative velocity $\dot{y}(t)-\dot{x}_1(t)$ between the base and the first floor of the structure.

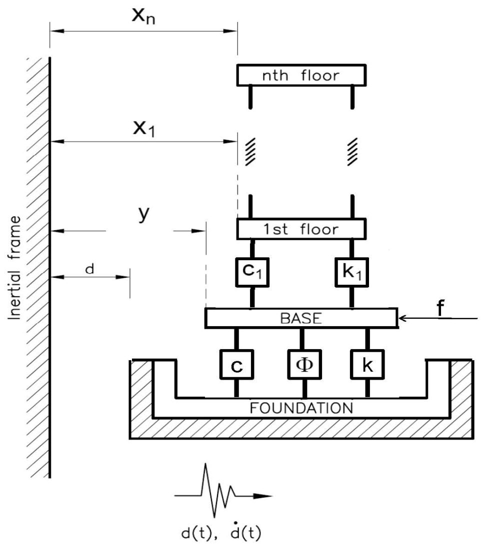

Fig. 1. Base-isolated *n*-story building.

The term $f_c(t)$ in (7b) accounts for the dynamics of the semiactive actuator (MR damper). Such dynamics are given by the Bouc-Wen model, shown in Figure 2, as follows:

$$f_c(t)=-\delta(v(t))\,\dot{y}(t)-\alpha(v(t))\,z(t) \tag{8a}$$

$$\dot{z}(t)=-\beta_1\left|\,\dot{y}(t)\right|z(t)\left|z(t)\right|^{n-1}-\beta_2\;\dot{y}(t)\left|z(t)\right|^{n}+A\,\dot{y}(t) \tag{8b}$$

where $\delta(v(t)) = \delta_a + \delta_b v(t)$, $\alpha(v(t)) = \alpha_a + \alpha_b v(t)$ and $z(t)$ is an unmeasurable evolutionary variable, the parameters β_1, β_2, n and A are constant values that can be used to adjust the shape of the hysteresis loop. The voltage $v(t)$ is the control signal to be generated: it is the input to a PWM system that generates the current, which in turn creates the magnetic filed, used to control the MR damper. The force-velocity relationship of MR dampers exhibits a hysteretic behaviour which is not mathematically easy to model. Fig. 3 shows the typical response of an MR damper under sinusoidal excitation at different levels of magnetic field. Hysteresis can cause serious problems in controlled systems such as instability and loss of robustness.

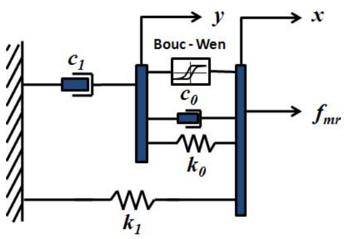

Fig. 2. Bouc-Wen mechanical model.

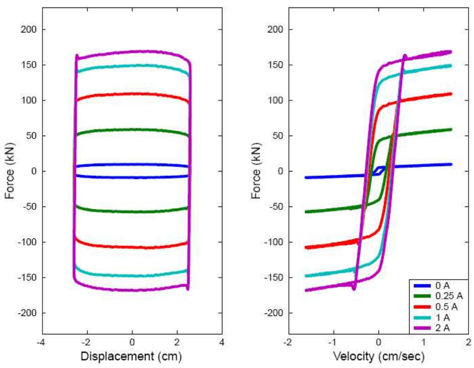

Fig. 3. Typical force-displacement and force-velocity curves of an MR damper.

Due to the base isolation, the movement of the main structure (S_r) is very close to the one of a rigid body. Then it is reasonable to assume that the inter-story motion of the main structure will be much smaller than the absolute motion of the base. Consequently, the following simplified equation of motion of the first floor is obtained:

$$m_1 \ddot{x}_1(t) + c_1 \dot{x}_1(t) + k x_1(t) = c_1 \dot{y}(t) + k_1 y(t) \tag{9}$$

In this work, it is assumed that only state variables of the base and the first floor system are measurable and the unknown seismic excitation $d(t)$ and $\dot{d}(t)$ are bounded and thus the unknown force $f_{bg}(t)$ in (7d) is bounded.

The following propositions about the intrinsic stability of the structure will be used in formulating the control law [40].

Proposition 1. The unforced main structure subsystem, i.e. (7a) with the null coupling term:

$$[c_1,0,...,0]^T\,\dot{y}(t)+[k_1,0,...,0]^T\,y(t)\equiv 0,\ t\geq 0$$

is globally exponentially stable for any bounded initial conditions.

Proposition 2. If the coordinates $(y(t),\dot{y}(t))$ of the base and the coupling term $[c_1,0,...,0]^T\,\dot{y}(t)+[k_1,0,...,0]^T\,y(t)$ are uniformly bounded, then the main structure subsystem is stable and the coordinates $(x(t),\dot{x}(t))$ of the main structure are uniformly bounded for all $t\geq 0$ and any bounded initial conditions.

3.1 RTHT System

The experimental testing of the control performance in civil engineering structures is an important issue in structural control. It is well known that testing vibration reduction systems at large scale structures such as buildings or bridges is rather prohibitive because of the dimensions, the power required to do so and the costs that such tests imply. This is why experiments are usually run at small or mid scale laboratory specimens. Experiments can be performed in one of three ways: shaking table tests, quasi-static tests and pseudo-dynamic or hybrid tests [41].

One significant advantage of hybrid simulation is that it removes a large source of epistemic uncertainty compared to pure numerical simulations by replacing structural element models that are not well understood with physical specimens on the laboratory test floor [42]. There are two main drawbacks with the hybrid test method. Firstly, the method relies on the assumption that the mass of the structure is concentrated at discrete points. Secondly, the loading is applied over a greatly expanded time scale so that time-dependent non-linear behaviour is not correctly reproduced in the physical component.

In hybrid testing, the displacements are imposed on an extended time-scale which typically ranges from 100 to 1000 times the actual earthquake duration to allow for the use of larger actuators without high hydraulic flow requirements, careful observation of the response of the structure during the test, and the ability to pause and resume the experiment. In particular, the method cannot be applied to the testing of highly rate-sensitive components such as visco-elastic dampers and certain active or semi-active structural control devices [43].

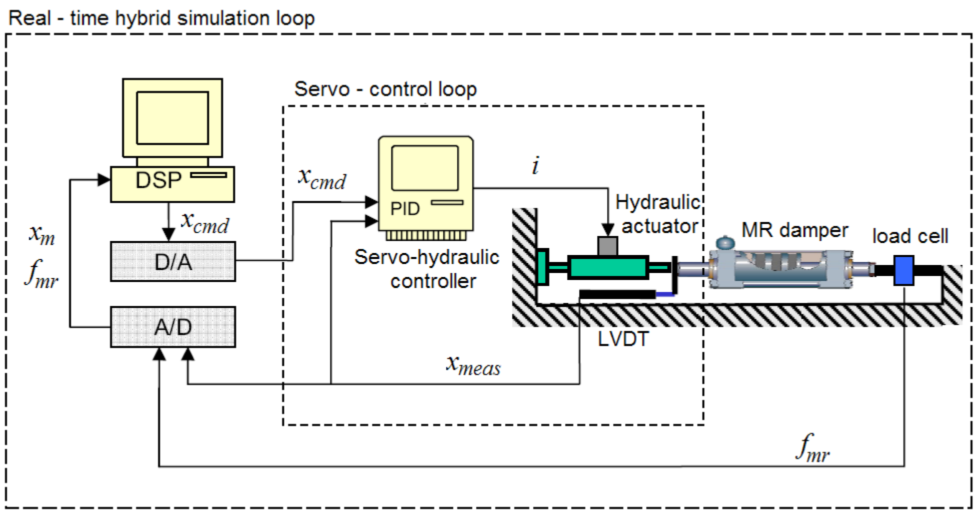

Fig. 4. RTHT system schematic.

Figure 4 shows the experimental environment where the system (1) can be tested. Experiments are executed in a real-time hybrid testing (RTHT) configuration available at the Smart Structures Laboratory, University of Illinois Urbana - Champaign (USA). It consists of a computer that simulates the structure to be controlled and generates the commanding signals (displacements and control signals); a small-scale MR damper that is driven by a hydraulic actuator which in turn is controlled by a servo-hydraulic controller; and DSP, A/D and D/A hardware for signal processing. Sensors available include a linear variable displacement transformer (LVDT) for displacement measurements and a load cell to measure the MR damper force. In Figure 4, x_{cmd} is the commanded displacement, f_{mr} is the MR damper force measured by the load cell, x_{meas} is the displacement measured by the LVDT and i is the control current sent to the hydraulic actuator.

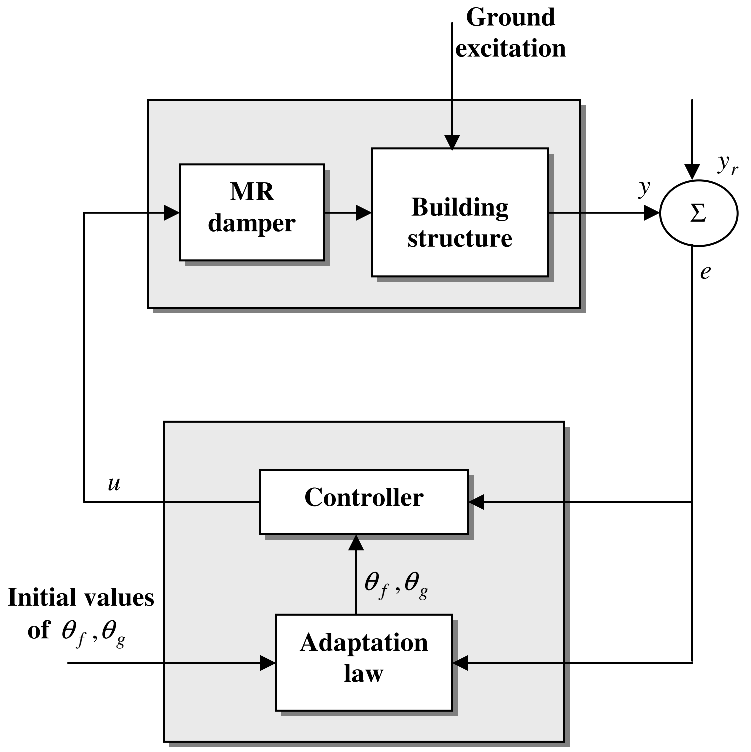

Fig. 5. Adaptive H_∞ control strategy.

IV. Control Design

In this section, it is assumed that the building-MR damper system shown in Figure 5 can be modeled as a single-input-single-output (SISO) nonlinear equation

$$y^{(r)}(t) = F(\underline{y}(t)) + G(\underline{y}(t))u(t) + w(t) \tag{10}$$

where $m < r$ and $(.)^{(i)}$ denotes the ith derivative of $(.)$ and $u(t)$, $y(t)$ and $w(t)$ are the control input, the measured output, and disturbance, respectively. $F(.)$ and $G(.)$ are smooth functions of $\underline{y}(t) = (y(t), y^{(1)}(t), \ldots, y^{(r-1)}(t))^T$.

According to Appendix, taking $x_1(t) = y(t)$, $x_2(t) = y^{(1)}(t)$ up to $x_r(t) = y^{(r-1)}(t)$ yields the extended system model

$$\dot{x}_i(t) = x_{i+1}(t), \quad 1 \le i \le r-1 \tag{11a}$$

$$\dot{x}_r(t) = F(\underline{x}(t)) + G(\underline{x}(t))u(t) + w(t) \tag{11b}$$

$$y(t) = x_1(t) \tag{11c}$$

where $\underline{x}(t) = (x_1(t), \ldots, x_r(t))^T$. We need to know the bounds of $F(.)$ and $G(.)$. That is, we have to make the following assumption.

Assumption 1. There exist functions $F^u(\underline{x}(t))$, $G^u(\underline{x}(t))$ and $G_l(\underline{x}(t))$ such that $|F(\underline{x}(t))| \le F^u(\underline{x}(t))$ and $G_l(\underline{x}(t)) \le G(\underline{x}(t)) \le G^u(\underline{x}(t))$, where $F^u(\underline{x}(t)) < \infty$, $G^u(\underline{x}(t)) < \infty$ and $G_l(\underline{x}(t)) > 0$ for all $\underline{x} \in U_{\underline{x}}$.

All the elements of the state vector $\underline{x}(t)$ are assumed to be available. The objective is to combine the characteristics of wavelets, adaptive control scheme which guarantee that the output $y(t)$ and its derivatives up to order $r-1$ track a given reference signal $y_r(t)$ and its corresponding derivatives up to the order $r-1$, which are assumed all derivatives of the signal $y_r(t)$ to be bounded.

To begin with, the reference signal vector $y_r(t)$ and the tracking error vector $\underline{e}(t)$ will be defined, respectively, as

$$\underline{y}_r = [y_r, y_r^{(1)}, \ldots, y_r^{(r-1)}]^T, \tag{12a}$$

$$\underline{e} = \underline{x} - \underline{y}_r = [e, e^{(1)}, \ldots, e^{(r-1)}]^T. \tag{12b}$$

If the functions $F(.)$ and $G(.)$ are known, then by employing the technique of feedback linearization we can choose the controller $u^*(t)$ to cancel the nonlinearity and achieve the tracking control goal. Specially, let $\underline{k} = [k_1, k_2, \ldots, k_n]$ to be chosen such that all roots of the polynomial $p(s) = s^n + k_n s^{n-1} + \ldots + k_1$ are in the open left-half side of the complex plane and the final control law, shown in Figure 5, is obtained as

$$u(t) = u^e(t) + u^s(t) \tag{13}$$

where

$$u^e(t) = \frac{1}{G(\underline{x}(t))}[-F(\underline{x}(t)) + u^a(t) + y_r^{(n)}(t) - \underline{k}\,\underline{e}(t)] \tag{14}$$

and $u^a(t)$, $u^s(t)$ are two auxiliary controls yet to be specified and the main objective of $u^a(t)$ is to attenuate the effect of disturbance on the tracking error vector and the additional control term $u^s(t)$ is called a supervisory control for the reasons given at the end of this section.

Substituting (13)-(14) into (11) and using (12), we obtain the closed-loop system governed by

$$e^{(n)} + k_n e^{(n-1)} + \ldots + k_1 e = u^a(t) \tag{15}$$

Note that the control signal (13) is useful only if $F(.)$ and $G(.)$ are known exactly. If $F(.)$ and $G(.)$ are unknown, then adaptive strategies must be employed. We employ two adaptive WNs

$$\hat{F}(\underline{x}(t),\underline{\theta}_f(t)) = \underline{\theta}_f^T(t)\,\underline{\psi}_f(\underline{x}(t)) \tag{16}$$

$$\hat{G}(\underline{x}(t),\underline{\theta}_g(t)) = \underline{\theta}_g^T(t)\,\underline{\psi}_g(\underline{x}(t)) \tag{17}$$

to approximate the nonlinear functions $F(.)$ and $G(.)$ of the system, respectively. The optimal weight vectors $\underline{\theta}_f^*$ and $\underline{\theta}_g^*$ are chosen as

$$\underline{\theta}_f^* = \arg\min_{\underline{\theta}_f}\{\max_{\underline{x}}\left|F(\underline{x}(t)) - \underline{\theta}_f^T(t)\psi_f(\underline{x}(t))\right|\} \tag{18}$$

$$\underline{\theta}_g^* = \arg\min_{\underline{\theta}_g}\{\max_{\underline{x}}\left|G(\underline{x}(t)) - \underline{\theta}_g^T(t)\underline{\psi}_g(\underline{x}(t))\right|\}. \tag{19}$$

and the functions $F(.)$ and $G(.)$ which are valid for all $\underline{x}\in U_{\underline{x}}$, which $U_{\underline{x}} \subset \Re^n$ is the compact set of $\underline{x}(t)$, have the following representation

$$F(\underline{x}(t)) = \hat{F}(\underline{x}(t),\underline{\theta}_f^*) + \Xi_f(\underline{x}(t)) = \underline{\theta}_f^{*T}\,\underline{\psi}_f(\underline{x}(t)) + \Xi_f(\underline{x}(t)) \tag{20}$$

$$G(\underline{x}(t)) = \hat{G}(\underline{x}(t),\underline{\theta}_g^*) + \Xi_g(\underline{x}(t)) = \underline{\theta}_g^{*T}\,\underline{\psi}_g(\underline{x}(t)) + \Xi_g(\underline{x}(t)). \tag{21}$$

By using definitions (18)-(19) and the equations (20)-(21), one finds

$$\begin{aligned}\dot{\underline{e}}(t) = A_m\underline{e}(t) + \underline{b}\{&\underline{\theta}_f^{*T}\,\underline{\psi}_f(\underline{x}(t)) + \underline{\theta}_g^{*T}\,\underline{\psi}_g(\underline{x}(t))u(t) + \underline{k}\,\underline{e}(t) - y_r^{(n)}(t) + \Xi_f(\underline{x}(t)) \\ &+ \Xi_g(\underline{x}(t))u(t) + w(t)\}\end{aligned} \tag{22}$$

where

$$A = \begin{bmatrix} 0_{(r-1)\times 1} & I_{r-1} \\ 0 & 0_{1\times(r-1)} \end{bmatrix},\; \underline{b} = [0_{1\times(r-1)} \quad 1]^T.$$

Also, $(A,\underline{b})$ and $(A_1,\underline{b}_1)$ are controllable canonical pairs and it is clear from polynomial $p(s)$ that $A_m = A - \underline{b}\underline{k}$ is Hurwitz. In order to derive the control law, we need the following assumption hold for all $\underline{x}\in U_{\underline{x}}$, $\underline{\theta}_f \in \Omega_b$ and $\underline{\theta}_g \in \Omega_b$, where the constraint set Ω_b for $\underline{\theta} = [\underline{\theta}_f^T, \underline{\theta}_g^T]^T$ is defined as $\Omega_b = \{\left\|\underline{\theta}_f\right\|_2 \le \mathrm{M}_f \;and\; \left\|\underline{\theta}_g\right\|_2 \le \mathrm{M}_g\}$.

Assumption 2. There exists a finite constant $B_w > 0$, such that $\int_0^T w(t)^2 dt \le B_w$.

Remark 2. The effect of $w(t)$, denoting the external disturbance, will be attenuated by the control signal $u^a(t)$, such that the H_∞ control design efficiently deals with the attenuation of $w(t)$ in the error dynamic system (22). Then, the problem under consideration becomes that of finding an adaptive scheme for $u^a(t)$, $\underline{\theta}_f(t)$ and $\underline{\theta}_g(t)$ to achieve the following H_∞ tracking performance:

$$\int_0^T \underline{e}^T Q\ \underline{e}\, dt \le \underline{e}^T(0)P\underline{e}(0) + tr(\underline{\tilde{\theta}}_f(0)\underline{\tilde{\theta}}_f^T(0)) + tr(\underline{\tilde{\theta}}_g(0)\underline{\tilde{\theta}}_g^T(0)) + \gamma^2 \int_0^T w^T w\, dt \qquad \forall\, 0 \le T < \infty \tag{23}$$

where γ is a prescribed attenuation level, and P, Q are arbitrary positive definite weighting matrices.

Theorem 1. *Consider the nonlinear system in (9) with unknown or uncertain* $F(.)$ *and* $G(.)$*, according to Assumption 1. The* H_∞ *tracking performance in (23) is achieved for a prescribed attenuation level* γ *if the following adaptive WN control law is adopted:*

$$\begin{aligned} u(t) &= u^e(t) + u^s(t) \\ &= \frac{1}{\hat{G}(\underline{x}(t), \underline{\theta}_g(t))}\left[u^a(t) - \hat{F}(\underline{x}(t), \underline{\theta}_f(t)) + y_r^{(n)}(t) - \underline{k}\,\underline{e}(t)\right] + u^s(t) \end{aligned} \tag{24}$$

with

$$u^a(t) = \frac{-1}{\beta^2}\underline{b}^T P\underline{e}(t) \tag{25}$$

and

$$u^s(t) = -\frac{\mu_s\,\mathrm{sgn}(\underline{e}^T(t)P\underline{b})}{G_L(\underline{x}(t))}\left(F^U(\underline{x}(t)) + \left|\hat{F}(\underline{x}(t), \underline{\theta}_f(t))\right| + \left|G^U(\underline{x}(t))u^e(t)\right| + \left|\hat{G}(\underline{x}(t), \underline{\theta}_f(t))u^e(t)\right|\right) \tag{26}$$

where

$$\mu_s = \begin{cases} 1 & if \quad \|\underline{e}(t)\| \ge E \\ 0 & if \quad \|\underline{e}(t)\| < E \end{cases} \tag{27}$$

and

$$\dot{\underline{\theta}}(t) = \Pr oj\,(\underline{\theta}(t), \underline{\Pi}(t)) = \underline{\Pi}(t), \tag{28}$$

where β *is an arbitrary parameter and* E *is a constant specified by the designer and* $\underline{\theta}(t)=\left[\underline{\theta}_f^T(t),\underline{\theta}_g^T(t)\right]^T$, $\underline{\Pi}(t)=\left[\underline{\Pi}_f^T(t),\underline{\Pi}_g^T(t)\right]^T$, $\underline{\Pi}_f(t)=2\underline{e}^T(t)P\underline{b}\underline{\psi}_f(\underline{e}(t)+\underline{y}_r(t))$, $\underline{\Pi}_g(t)=2\underline{e}^T(t)P\underline{b}\underline{\psi}_g(\underline{e}(t)+\underline{y}_r(t))u(t)$ *and the positive definite matrix* P *is the solution of the following equation*

$$P A_m + A_m^T P + P\underline{b}\left(\frac{1}{\gamma^2}-\frac{2}{\beta^2}\right)\underline{b}^T P + Q = 0. \tag{29}$$

Remark 3. If the norm of error vector tends to increase, i.e.,$\left\|\underline{e}(t)\right\|\geq E$, then the supervisory control begins to operate to force $\left\|\underline{e}(t)\right\|<E$. In this way, the control is like a supervisor. In this strategy, because of the signal $u^s(t)$ is prepositional to the upper bounds $F^u(\underline{x}(t))$ and $G^u(\underline{x}(t))$ which are usually very large, we choose the signal $u^s(t)$ to operate in the preceding supervisory fashion.

Remark 4. According to Theorem 1 and the compact set $U_{\underline{x}}$, the designer can specify the positive constant E as $E=\left(V(0)/\lambda_{\min}(Q)\right)^{0.5}$.

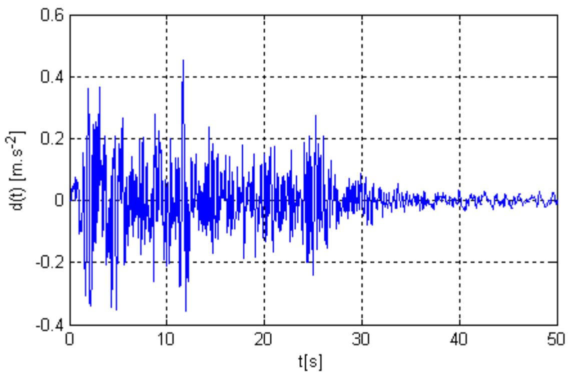

Fig. 6. Time history of the acceleration of the EI Centro earthquake.

V. Numerical Results

The system equations of a seismically excited 10-story building and controllers are implemented in MATLAB/Simulink to evaluate the performance. The horizontal seismic motion is a replica of that of Taft's earthquake shown in Figure 6. The mass and stiffness of the base are $m=6\times10^5$ kg, $k=1.184\times10^7$ N/m, and the base damping coefficient is 0.1, respectively; the main structure stiffness varies linearly from the first floor $k_1 = 9\times10^8$ N/m to the top floor $k_{10} = 4.5\times10^8$ N/m, the damping coefficient is 0.05 and the passive actuator has the following values: $Q = \sum_{i=1}^{10} m_i$, μ_{max} = 0.185, $\Delta\mu$ = 0.09, and ν = 2.0. The parameters of the MR damper are: $\beta_1 = \beta_2 = 3\times10^2\ cm^{-1}$, $A = 120$, $n = 1$, $\delta_a = 3\times10^2\ {}^{kNs}\!/\!_m$, $\delta_b = 1.8\times10^2\ {}^{kNs}\!/\!_m$, $\alpha_a = 4.5\times10^4\ {}^{kNs}\!/\!_m$ and $\alpha_b = 3.6\times10^2\ {}^{kNs}\!/\!_m$.

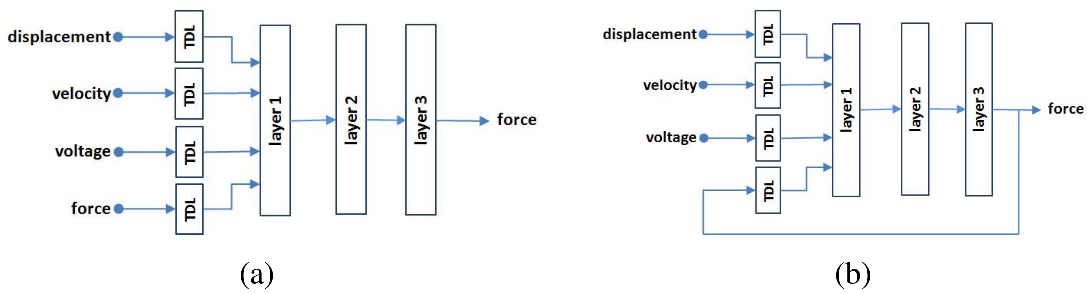

Fig. 7. (a) Schematic of the NN for training, (b) Schematic of the NN model.

5.1 Neural Network Models

The identification of the MR damper system can be replicated using the following four-step procedure: (1) collection of input/output date; (2) selection of a network structure; (3) training of the network; and (4) validation of the trained network. The quality of the trained network is directly related to the quality of the training date. In order to evaluate the feasibility of modeling the MR damper with a neural network, it was trained using data representing different frequencies and voltages. The network takes four inputs: displacement, velocity, voltage and force. The fourth input (force) is fed back from the output. Additionally, the network is dynamic, and the inputs are stored in memory (tapped delay lines, TDL) for a period of time and are updated after each output computation. The structure of the network during the training session is shown in Figure 7(a). This structure allows for fast and more reliable training. Basically, given four inputs, the network must reproduce only the fourth one. The final model is shown in Figure 7(b). Thus, training the network is a two-step procedure in which the first one is to train the network of Figure 7(a) and the second one is to test the network of Figure 7(b) with the same data until a desired performance is achieved. After a trial and error process, it was found that a network with 3 layers (10 neurons in the first layer, 4 neurons in the second one and 1 neuron in the last one) and 4 units-of-time length TDL is good enough to model the MR damper. Sigmoid tangent transfer functions are used in the first two layers while a purely linear transfer function is used in output neuron. Following the training procedure, the estimation of the neural model for a 4 Hz sinusoidal displacement at 3 V is shown in Figure 8 where good performance can be observed.

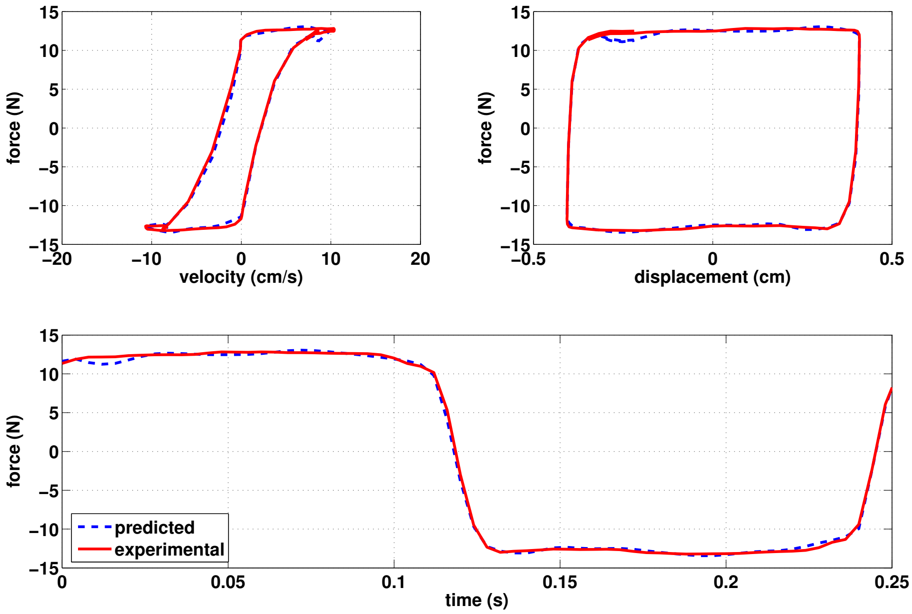

Fig. 8. Predicted and experimental force using a neural network.

5.2 Wavelet Network Models

It is assumed that the coupled building-MR damper structure will be of order 3 as $y^{(3)} = F(\underline{y}) + G(\underline{y})u + w$ with $y_r(t) = 0$. We use the adaptive WN to construct the functions $F(.)$ and $G(.)$ such that the Gaussian wavelet of order 8 is chosen to be the basis of the WN and the constant WN parameters are chosen as $M_1 = M_2 = 7$ and $N_1 = N_2 = 6$. Figure 9 shows the time behaviours of the horizontal base absolute displacement of the first story. It is clear that in the presence of modelling errors the stability of the overall identification scheme is guaranteed and a reduction in absolute displacement is achieved with better results when the active control device is integrated.

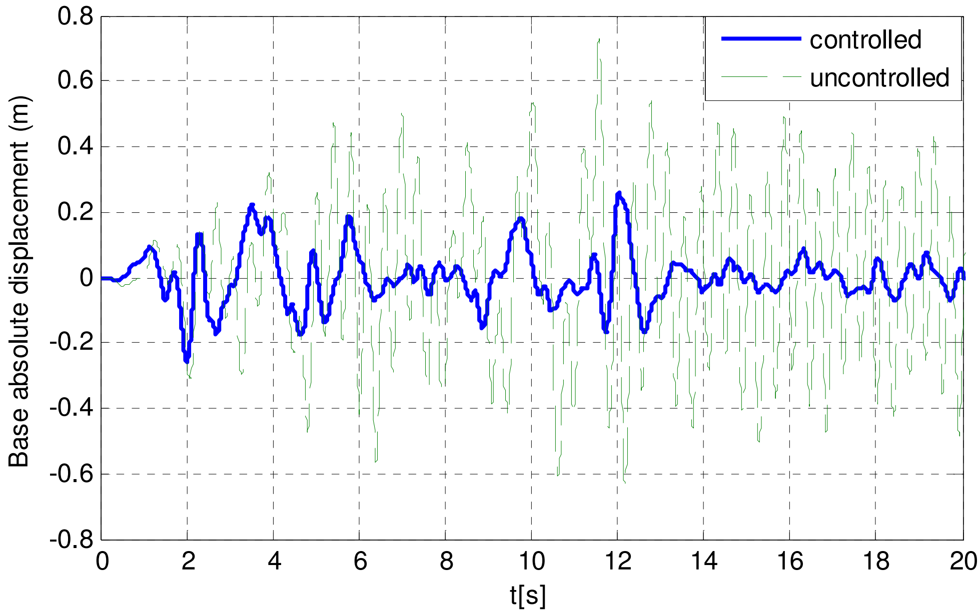

Fig. 9. Horizontal base absolute displacement.

VI. Conclusions

This paper proposed an application of wavelet networks (WNs) in identification and control design for a class of building structures with MR dampers such that stability and a performance on transient response are satisfied simultaneously. The nonlinear model is identified based on a WN framework. By combining the technique of feedback linearization, supervisory control and H_∞ control, an adaptive control strategy with some parameter adaptive laws were developed to compensate for the nonlinearity in the structure so as to enhance the response of the system to earthquake type inputs. It was demonstrated from numerical simulation that the suggested WN methodology was effective to identify and control behaviour of a building-MR damper system.

Appendix

A1. Input-output feedback linearization [44]

Consider the nonlinear single-input-single-output (SISO) plant in the form of

$$\begin{aligned} \dot{x}(t)) &= f(x(t)) + g(x(t))u(t) \\ y(t) &= h(x(t)) \end{aligned} \tag{A1}$$

where $x \in \Re^n$, $u \in \Re$, and $y \in \Re$ denote the state vector, the control input scalar, and the output scalar, respectively. The term $f(x)$ represents the nonlinearities of the plant, $g(x)$ and $h(x)$ the nonlinear output distribution scalar. It is assumed that $f(x)$, $g(x)$ and $h(x)$ are sufficiently continuous functions of x.

In the input-output feedback linearization procedure, the output is differentiated with respect to time several times until the control input u appears. Assume that r is the smallest integer such that the input appears in $y^{(r)}$, then

$$y^{(r)} = L_f^r(h(x)) + L_g(L_f^{r-1}(h(x)))u \tag{A2}$$

where $L_f(.)$ and $L_g(.)$ stand for the Lie derivative of (.) with respect to $f(x)$ and $g(x)$, respectively,

$$\begin{aligned} L_f^0(h(x)) &= h(x) \\ L_f^k(h(x)) &= [\frac{\partial}{\partial x} L_f^{k-1}(h(x))] f(x) \\ L_g(L_f^k(h(x))) &= [\frac{\partial}{\partial x} L_f^k(h(x))] g(x) \end{aligned} \tag{A3}$$

When the relative degree $r < n$, the nonlinear plant (A1) can be transformed, using $z = [y, \dot{y}, \cdots, y^{(r-1)}]^T$ as a part of the new state components, into a normal form as

$$\begin{aligned} \dot{z} &= E_r z + B[a(x) + b(x)u] \\ y &= C z \\ \dot{\eta} &= \omega(x) \end{aligned} \tag{A4a-c}$$

with

$$z = col\{y, \dot{y}, \cdots, y^{(r-1)}\},\ E_r = \begin{bmatrix} 0_{(r-1)\times 1} \vdots I_{(r-1)} \\ 0_{1\times r} \end{bmatrix},$$

$$B = \begin{bmatrix} 0_{(r-1)\times 1} \\ 1 \end{bmatrix},\ C = [1 \vdots 0_{1\times(r-1)}],$$

where $z \in \Re^r$ and $\eta \in \Re^{n-r}$ represent an external part and internal part of the plant dynamics (A1), respectively. Note that the subsystem in a phase variable form (A4a) is simply another expression of (A2), while the subsystem (A4c) does not contain the plant input.

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References

[1] Yoshioka, H., Ramallo, J. & Spencer, B., 'Smart based isolation strategies employing magnetorheological dampers', *J. Engineering Mechanics*, vol. 128, pp. 540–551, 2002.

[2] Spencer, B. and Sain, M., 'Controlling buildings: a new frontier in feedback', *IEEE Control Systems Magazine*, vol. 17, pp. 19–35, 1997.

[3] Dyke, S., Jr., B. S., Sain, M. and Carlson, J., 'An experimental study of MR dampers for seismic protection', *Smart materials and structures*, vol. 7, pp. 693–703, 1998.

[4] Spencer, B. and Song, T., 'New applications and development of active, semi-active and hybrid control techniques for seismic and non-seismic vibration in the USA,' *in Proceedings of International Post-SMIRT Conference Seminar on Seismic Isolation, Passive Energy Dissipation and Active Control of Vibration of Structures*, Cheju, Korea, 1999.

[5] Karimi H.R., Zapateiro M., and Luo N., 'Wavelet-based parameter identification of a nonlinear magnetorheological damper' *Int. J. Wavelets, Multiresoloution and Image Processing,* vol. 7, no. 2, pp. 183-198, 2009.

[6] Burrus C.S., Gopinath R.A. and Guo H., Gopinath R.A. and Guo H., 'Introduction to wavelets and wavelet transforms,' *Prentice Hall*,(New Jersey, 1998).

[7] Kumar D., and Pah N.D., 'Thresholding wavelet networks for signal classification' *Int. J. Wavelets, Multiresoloution and Image Processing,* vol. 2, no. 1, pp. 10-25, 2003.

[8] Maharaj E.A., 'Using wavelets to compare time series patterns,' *Int. J. Wavelets, Multiresoloution and Image Processing*, vol. 3, no. 4, pp. 511-521, 2005.

[9] Sirca G. and Adeli H., 'A neural network-wavelet model for generating artificial accelerograms,' *Int. J. Wavelets, Multiresoloution and Image Processing*, vol. 2, no. 3, pp. 217-235, 2004.

[10] Whitche B. and Craigmile P.F., 'Multivariate spectral analysis using Hilbert wavelet pairs,' *Int. J. Wavelets, Multiresoloution and Image Processing*, vol. 2, no. 4, pp. 567-587, 2004.

[11] Karami A., Karimi H.R., Moshiri B., and Maralani P.J., 'Wavelet-based adaptive collocation method for the resolution of nonlinear PDEs' *Int. J. Wavelets, Multiresoloution and Image Processing*, vol. 5, no. 6, pp. 957-973, 2007.

[12] Karami A., Karimi H.R., Maralani P.J., and Moshiri B., 'Intelligent optimal control of robotic manipulators using wavelets' *Int. J. Wavelets, Multiresoloution and Image Processing*, vol. 6, no. 4, pp. 575 – 592, 2008.

[13] Chen B.S. and Cheng Y.M., 'Adaptive wavelet network control design for nonlinear systems', *Proc. CDC.*, pp. 3224-3329, Kobe, Japan, 1996.

[14] Cheng Y.M., Chen B.S. and Shiau F.Y., 'Adaptive wavelet network control design for nonlinear systems' *Proc. Natl. Counc.*, vol. 22, no. 6, pp. 783-799, 1998.

[15] Thuillard M., 'A review of wavelet networks, wavenets, fuzzy wavenets and their applications', *ESIT*, pp. 5-16, Aachen, Germany, 2000.

[16] Pati, Y.C., Krishnaprasad, P.S., 'Analysis and synthesis of feedforward neural networks using discrete affine wavelet transformations' *IEEE Trans. Neural Networks*, vol. 4, pp. 73-85, 1992.

[17] Postalcioglu S. and Becerikli Y., 2003. 'Nonlinear system modeling by using wavelet networks', *Proc. Int. Conf. Signal Processing*, vol. 1, no. 2, pp. 72-75.

[18] Zhang Q. and Benveniste A., 'Wavelet networks' *IEEE Trans. Neural Networks*, vol. 3, pp. 889-898, 1992.

[19] Karimi H.R., Lohmann B., Moshiri B. and Maralani P.J., 'Wavelet-based identification and control design for a class of non-linear systems' *Int. J. Wavelets, Multiresoloution and Image Processing*, vol. 4, no. 1, pp. 213-226, 2006.

[20] Dyke, S., Spencer, B., Sain, M. and Carlson, J., 'Modeling and control of magnetorheological dampers for seismic response reduction', *Smart Materials and Structures*, vol. 5, pp. 565–575, 1996.

[21] Jansen, L. and Dyke, S., ‘Semiactive control strategies for MR dampers: comparative study’, *J. Engineering Mechanics*, vol. 126, pp. 795–803, 2000.

[22] Yang, G., Large-scale magnetorheological fluid damper for vibration mitigation: modeling, testing and control, PhD thesis, University of Notre Dame, 2001.

[23] Wang, X. and Gordaninejad, F., ‘Lyapunov-based control of a bridge using magnetorheological fluid dampers’, *J. Intelligent Material Systems and Structures*, vol. 13, pp. 415–419, 2002.

[24] Park, C. and Jeon, D., ‘Semiactive vibration control of a smart seat with an MR fluid damper considering its time delay’, *J. Intelligent Material Systems and Structures*, vol. 13, pp. 521–524, 2002.

[25] Luo, N., Villamizar, R., Vehi, J., Rodellar, J. and Ma˜nosa, V., ‘Sliding mode control of structures with uncertain coupled subsystems and actuator dynamics,’ *in Proceedings of the European Control Conference*, Cambridge, U.K., 2003.

[26] Nagarajaiah, S., Narasimhan, S., Agrawal, A. and Tang, P., ‘Semiactive Lyapunov controller for phase II seismic isolated bridge benchmark,’ *in ‘Structures Congress 2006’*, St. Louis, Missouri, U.S.A.

[27] Kim Y., and Langari R., ‘Nonlinear Identification and Control of a Building Structure with a Magnetorheological Damper’ *Proceedings of the American Control Conference*, pp. 3353-3358, 2007.

[28] McClamroch, N., Gavin, H., Ortiz, D. and Hanson, R., ‘Electrorheological dampers and semi-active structural control,’ *in Proceedings of the 33rd Conference on Decision and Control*, 1994.

[29] McClamroch, N. and Gavin, H., ‘Closed loop structural control using electrorheological dampers, *in Proceedings of the American Control Conference*, Seattle, Washington, U.S.A, 1995.

[30] Villamizar, R., Luo, N., Vehi, J. and Rodellar, J., Semiactive sliding mode control of uncertain base isolated structures with actuator dynamics, in ‘Proc. SPIE 10th Annual International Symposium on Smart Structures and Materials’, Warsaw, Poland, 2003.

[31] Moon, S., Bergman, L. and Voulgaris, P., 'Sliding mode control of cable-stayed bridge subjected to seismic excitation', *J. Engineering Mechanics*, vol. 129, pp. 71–78, 2003.

[32] Luo, N., Villamizar, R. and Vehi, J., 'Backstepping control of nonlinear building structures with hysteretic and frictional dynamics,' *in European Control Conference*, Kos, Greece, 2007.

[33] Zapateiro M., and Luo N., Karimi H.R., and Josep Vehí, 'Vibration control of a class of semiactive suspension system using neural network and backstepping techniques' *Mechanical Systems and Signal Processing* (2008), doi: 10.1016/j. ymssp. 2008.10.003.

[34] Kim, H. and Roschke, P., 'Fuzzy control of base-isolation system using multiobjective genetic algorithm', *Computer-Aided Civil and Infrastructure Engineering*, vol. 21, pp. 436–446, 2006.

[35] Schurter, K. and Roschke, P., 'Neuro-fuzzy control of structures using acceleration feedback', *Smart Materials and Structures*, vol. 10, pp. 770–779, 2001.

[36] Li Z., Chang C.C., and Spencer Jr., B.F., 'Intelligent technology-based control of motion and vibration using MR dampers' *Earthquake Engineering and Engineering Vibration*, vol. 1, no. 1, pp. 100-110, 2002.

[37] Xu J.X. and Tan Y., 'Nonlinear adaptive wavelet control using constructive wavelet networks', *Proc. ACC.*, pp. 624-629, Arlington, 2001.

[38] Sureshbabu N. and Farrell J.A., 'Wavelet-based system identification for nonlinear control', *IEEE Trans. Automatic Control*, vol. 44, no. 2, pp. 412-417, 1999.

[39] Delyon B., Juditsky A. and Benveniste A., 'Accuracy analysis for wavelet approximations' *IEEE Trans. Neural Networks*, vol. 6, no. 2, pp. 332-348, 1995.

[40] Luo N., Rodellar J., De la Sen M., Vehi J., 'Output feedback sliding mode control of base isolated structures' *J. The Franklin Institute*, vol. 337, 555–577, 2000.

[41] Shield, C. K., French, C. W. and Timm, J., 'Development and implementation of the effective force testing method for seismic simulation of large-scale structures', *Philosophical Transactions of the Royal Society: Theme Issue on Dynamic Testing of Structures*, vol. 359, pp. 1911–1929, 2001.

[42] Mosqueda, G., Stojadinovic, B. and Mahin, S., '*Implementation and accuracy of continuous hybrid simulation with geographically distributed substructures*,' Technical report, Earthquake Engineering Research Center, University of California, Berkeley, 2005.

[43] Jung R.Y. and Shing P.B., 'Performance evaluation of a real-time pseudodynamic test system', *Eartquake Engineering and StructuralDynamics*, vol. 35, pp. 789-810, 2006.

[44] Khalil H., *Nonlinear Systems*, Third Edition, Prentice Hall, 2002.